Dynamic Debt Deleveraging and Optimal Monetary Policy*

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Abstract

This paper studies optimal monetary policy under dynamic debt deleveraging once the zero bound is binding. Unlike much of the existing literature, the natural rate of interest is endogenous and depends on macroeconomic policy. We provide micro-foundation for debt deleveraging based both on household over accumulation of debt and leverage constraint on banks; and show that they are isomorphic in our proposed post-crisis New Keynesian model, thus integrating two popular narrative for the crisis. Optimal monetary policy successfully raises the natural rate of interest by creating an environment that speeds up deleveraging, thus endogenously shortening the duration of the crisis and a binding zero bound. Inflation should be front loaded. Fiscal-policy multipliers can be even higher than in existing models, but depend on the way in which public spending is financed.

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1 Introduction

This paper proposes a tractable post-crisis version of the canonical New Keynesian model. By post-crisis we mean the period after 2008 when several central banks had to cut their short-term interest rate to zero. While all models involve shortcomings, the standard New Keynesian model came under criticism for a few key abstractions which turned out to be important omissions to understand some elements of the crisis. First, in its most basic form it features only a single risk-free short-term interest rate. Second, there is no explicit banking in the model and there is only a single representative agent without a meaningful distinction between borrowers and savers. Finally, the shocks are somewhat reduced form which makes it difficult to pinpoint what – exactly – was the trigger of the crisis that started in 2008. These omissions are among the elements we wish to integrate into a standard New Keynesian post-crisis model. We propose to do so with only minor addition in complexity.

Our model builds on a rich literature developed before and after the crisis. The zero lower bound on short-term nominal interest was certainly not an unfamiliar concept to economists prior to the crisis. Moreover, right after it, the literature has expanded very quickly on the basis of what had already been developed. There is, for example, a relatively large literature on monetary and fiscal policy subject to the zero interest rate bound (see for example Eggertsson and Woodford (2003), Adam and Billi (2006), Eggertsson (2008) as pre-crisis examples and Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011) and Werning (2011) for post crisis examples). Generally, however, these papers take the shock that leads the economy to the zero bound – shock to the "natural rate of interest" – as given. Hence the duration of the trap under some basic policy specifications is purely exogenous and the duration of the exogenous forces that perturb the economy – usually given by preference shocks – does not have much meaningful interaction with the policy chosen. Here, instead, we wish to model this origin in a more explicit and tractable way and make the duration of the negative natural rate of interest – and therefore the crisis – endogenous and a function of policy.

Recently a literature has started to emerge that tries to model in greater detail how the economy finds itself up against the zero bound, the very origin of the current global economic crisis. We see the literature as focusing mainly on two narratives. One powerful narrative is that the source is a deleveraging cycle on the household side (for recent theoretical contributions inspired by the crisis see e.g. Eggertsson and Krugman (2012), Hall (2011), Guerrieri and Lorenzoni (2012) and Rognlie, Shleifer and Simsek (2014), while Mian and Sufi (2011) provide extensive empirical evidence for this mechanism).[1] Another powerful narrative traces the origin of the crisis to turbulences in the banking sector (see e.g. Curdia and Woodford, 2010, Gertler and Kiyotaki, 2010).[2]

Consider first the household debt-deleveraging story: We have a period of too much optimism about debt, in which debtors borrow and spend aggressively via a process of leveraging (piling up debt). Since one person's debt is another's asset, creditors have to

[1] See also Geanakoplos (2010), and references therein, although he, and the literature cited, does not emphasize the connection of the leverage cycle to the interest rate channel as we do here and as the literature above does. Thus he does not focus as much on the interaction of the leverage cycle and the zero bound which is the central focus here.

[2] See also Andrès et al. (2013) and De Fiore and Tristani (2012, 2013) for alternative approaches.

be induced to spend less via high real interest rates. Then there is a "Minsky moment" (Eggertsson and Krugman, 2012) in which people realize things have gone too far – that all the newly issued debt may in fact not be sustainable – and we move from a process of leveraging to deleveraging, i.e. the overextended agents need to pay down their debt. But the problem is that this process is not symmetric, because the central bank may not be able to cut the interest rate enough to induce sufficient spending by those that are not too deep into debt because of the zero bound. Hence, one way to explain a drop in the natural rate of interest is to say that debtors – as a group – are trying to deleverage very fast, so that the real interest rate needs to fall to negative levels to get the savers to spend enough to sustain full employment. A negative natural rate of interest rate can make the zero bound binding. This in turns creates problems for macroeconomic policy. In earlier work on deleveraging, such as Eggertsson and Krugman (2012), the deleveraging shock corresponds to a sudden drop in borrowing capacity that the borrower must satisfy. Their main focus, however, is on an example in which this adjustment takes place in only one period, "the short-run" because the borrower is at a corner solution. Here, instead, our main focus is on relaxing this assumption so that the process of deleveraging smoothly takes place over several periods – determined endogenously – as a result of the optimal deleveraging decisions of the households.[3]

Consider now the banking turbulence story: There is a crisis in the interbank market that increases the cost of funding that banks face. An example could include some shock to the banks' capital or a need to reduce leverage ratios. The banks' capital constraint tightens during a period of stress, leading banks to be less willing to lend, and thus triggering a downturn. As it will turn out, however, the mechanism through which this affects the macro economy will be largely analogous to the household debt deleveraging story already outlined. Indeed we will show that in terms of aggregate variables such as output, inflation and interest rate they are isomorphic. From the perspective of the most baseline New Keynesian perspective, therefore, there is no particular reason to insist on choosing one over the other, and we will refer to both as "dynamic deleveraging". Our prior is that both played an important role in the crisis.

Within our framework we generalize the standard New Keynesian (NK) prototype model (such as for example illustrated in Clarida, Galì and Gertler, 1999, Woodford, 2003, and Galì, 2008) as one that involves exactly the same pair of equations, familiar to many readers, namely the IS and the AS equations which are typically summarized as follows (denoting output in log deviation from steady state with, $\hat{Y}_t$, inflation with π_t, the nominal interest rate with i_t and steady state inflation by π)

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(i_t - E_t(\pi_{t+1} - \pi) - r_t^n)$$

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta(\pi_{t+1} - \pi)$$

[3]We indeed expound here one suggested extension discussed in the Web Appendix of Eggertsson and Krugman (2012) which, however, delivers a less compact model due to a different specification of preferences and production. Moreover, they do not provide explicit microfoundations for the banking sector which, in our case, allows us to nest both the household deleveraging story as well as the banking story within a single framework. Finally, they are silent on the welfare implications of alternative policies. Another closely related paper is Curdia and Woodford (2010) which we also build upon. Their focus, however, is mostly on shocks to the aggregate banking system, moreover they do not focus on sub-optimal monetary and fiscal policy at the zero bound.

where $\beta, \sigma, \kappa > 0$ are coefficients. The only difference between our current model and the benchmark is that r_t^n (which has the interpretation of being the natural rate of interest) is now an endogenous variable that depends on the level of private debt. In the paper we will show how this variable is determined in equilibrium by a system of equations that depends on the households' level of indebtedness, among other things, as well as on the spread between the risk-free interest rate and a risky, and accordingly higher, interest rate that borrowers need to pay. In the case of household debt deleveraging shock it corresponds to a "shock" to the "safe level" of debt, giving the household an incentive to pay down their debt to a new steady state. In case of a banking shock, it corresponds to a shock to their required leverage ratio or cost of equity financing which will curtail their lending to a new steady state. In the transition period we show that the natural rate of interest can be temporarily negative.

One relatively minor difference between our model and the standard one is that it is written in terms of inflation in deviation from steady-state inflation, π, which may be positive: a reasonable number, for example, would be 2% in the US. What this implies is that the recession at the zero-lower bound does not need to be associated with actual deflation, only that inflation has to be below the target of the central bank. Some authors have claimed that the lack of deflation during the crisis following 2008 suggested a major failure of the canonical New Keynesian model. Our proposed model fixes this problem. A more important advantage of our framework is that the explicit introduction of borrowing and lending allows a more disciplined calibration of the shock triggering the Great Recession. In much of the earlier literature (e.g. Eggertsson, 2011) the driving force is an unobserved preference shock, calibrated so as to generate the Great Recession. Here, instead, we have two more observables. First, there is an endogenous level of debt held by the households (or alternatively some measure of bank leverage if one take a banking perspective). Second, there is an interest rate for borrowing which is different from the risk free interest rate. Those two variables allow for a relatively straightforward calibration of the shock as we will see. We can then ask if the shocks calibrated to match these new observables can generate the Great Recession. The short answer is yes.

The first main conclusion of the paper is that the duration of a negative natural rate of interest is now endogenous – rather than depending only on exogenous preference shocks or an implicitly specified "short-run" – and is dependent on the stance of policy. Under a monetary policy regime that targets high enough inflation to avoid the zero bound, for example, the economy will experience a shorter duration of a negative natural rate of interest than if the policy regime is insufficiently stimulating. The intuition for this is simple: In a recession there is a drop in overall income, hurting borrowers' ability to pay down their debt, which means that the process of deleveraging will be slower than if the recession is avoided via aggressive enough monetary and fiscal policy. Since it is the deleveraging process that drives the reduction in the natural rate of interest, this affects how long the natural rate of interest stays below its steady state.

The second key result of the paper is to some extent a corollary of the first. Endogenous deleveraging will in general amplify the effect of policy at the zero bound. Why? Policy will now not only dampen the crisis today, as the previous literature has emphasized, but also shorten its duration by directly affecting the natural rate of interest. Consider the nominal interest rate path under a policy that tries to stabilize inflation, and the output gap assuming either dynamic deleveraging or exogenous preference shocks. We find that optimal monetary policy under dynamic deleveraging prescribes a shorter duration at the zero bound than if the crisis was driven by exogenous preference shocks, precisely because it will have a direct effect on the natural rate of interest: Optimal policy is powerful enough to "jump start" the economy and thus lead to a more rapid normalization of the nominal interest rate than would otherwise have occurred.

Our third result is that we are able to explicitly derive a social welfare function inside our heterogenous agent model. While the standard New Keynesian model involves only output and inflation, the social welfare function in our model involves an additional term because we have different agents in the model, i.e., both borrowers and savers, and we assume incomplete insurance between the two. Relative to the standard objective, we find that this additional term gives the government even further reason to engage in aggressive countercyclical policy. A key reason for this is that borrowers tend to suffer more in a recession driven by debt deleveraging and thus have higher marginal utility of income. Meanwhile borrowers have more to gain from inflationary policy that savers as inflation lowers the real value of their debt, lowers the real interest rate paid on that debt moving forward, and increases their labor income when the marginal value of extra income in especially high for borrowers.

A fourth result that emerges is that we find that optimal monetary policy in a liquidity trap under dynamic deleveraging will prescribe excess inflation, and possibly output above potential, well above the inflation target, even during the period in which the zero bound is binding and the natural rate of interest is negative. This is partly explained by the fact that social welfare now takes account the social benefit of redistribution in response to the shock, but also to some extent to the fact that an endogenous natural rate of interest prescribes even more aggressive policy action than in the standard model so as to speed up the recovery.

Finally, we study the effects of fiscal policy under dynamic deleveraging. There we find that it can be even more effective than has been found in the previous literature, since a fiscal expansion speeds up the deleveraging cycle. Crucially, however, the effectiveness of policy depends on how it is financed.

This work is organized as follows. Section 2 first describes dynamic deleveraging in a simple endowment economy to clarify some key assumptions, then it presents two alternative models of banking which are isomorphic in terms of their conclusions. Finally, it discusses the general model. Section 3 illustrates the log-linear version of the general model. Section 4 presents the calibration of the model and studies the positive implications of dynamic debt deleveraging in comparison with the standard NK model. Section 5 characterizes optimal monetary policy under commitment and compares the result with the standard NK model. Section 6 investigates fiscal-policy multipliers and finally Section 7 concludes.

2 Model

2.1 Dynamic deleveraging in an endowment economy

We start out by showing a simple example of dynamic deleveraging in an endowment economy. This is helpful to clarify the role of some key assumptions in the general environment we propose in the next section. Imagine a closed-economy endowment model with two agents, a borrower and a saver. They have the following utility functions

$$E_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \log C_t^j \qquad \text{where } j = b \text{ or } s$$

in which C_t^j is consumption of agent j and $0 < \beta^j < 1$ is the discount factor, with $\beta^s \geq \beta^b$. They make their consumption choices subject to a standard budget constraint

$$\frac{b_t^j}{1+r_t^j} = b_{t-1}^j + C_t^j - \frac{1}{2}Y + T_t^j \tag{1}$$

where b_t^j is one-period risk-free real debt of agent j and r_t^j is the associated interest rate and Y is the endowment that remains constant. We adopt the notation that b_{t-1}^j is the amount of debt contracted in period $t-1$ and repaid in period t (inclusive of interest payments) expressed in units of the consumption good. Hence the real value of debt contracted in period t is $b_t^j/(1+r_t^j)$. We also adopt the notation that a positive number for b_t^j denotes debt, while a negative one an asset. T_t^j is a lump-sum transfer out of the control of the agent.

Let us define the risk-free real interest rate by $1 + r_t$. We now consider the following function for the interest rate faced by each agent j

$$1 + r_t^j = \begin{cases} 1 + r_t & \text{if } b_t^j \leq \bar{b}_t \\ (1+r_t)(1+\phi(b_t^j - \bar{b}_t)) & \text{if } b_t^j > \bar{b}_t \end{cases} \tag{2}$$

This relationship, shown in Figure 1, says that if the borrower's debt level is below $\bar{b}_t$ then he faces the risk-free rate $1 + r_t$. If he borrows more than $\bar{b}_t$, however, he needs to pay a premium above the risk-free rate given by $1 + \phi(b_t^b - \bar{b}_t)$. Hence as the borrowing increases, so does the rate the borrower needs to pay. This can be thought of as a generalization of the strict borrowing-limit in Eggertsson and Krugman (2012) where $b_t \leq \bar{b}_t$. That constraint is obtained in the limit as $\phi \to \infty$ since in that case the borrower will never exceed the borrowing limit; while in our case the borrower may choose to do so but at the expenses of paying some premium over the risk-free rate.

Given the simple structure outlined above an equilibrium is a collection of stochastic processes $\{C_t^b, C_t^s, r_t^b, r_t^s, b_t^b\}$ that satisfies the following five equations

$$\frac{1}{C_t^s} = \beta^s(1+r_t^s)E_t\frac{1}{C_{t+1}^s} \tag{3}$$

$$\frac{1}{C_t^b} = \beta^b(1+r_t^b)E_t\frac{1}{C_{t+1}^b} \tag{4}$$

$$1 + r_t^b = \begin{cases} 1 + r_t^s & \text{if } b_t^b \leq \bar{b}_t \\ (1+r_t^s)(1+\phi(b_t^b - \bar{b}_t)) & \text{if } b_t^b > \bar{b}_t \end{cases} \tag{5}$$

$$C_t^s + C_t^b = Y \tag{6}$$

$$b_t^b = (1 + r_t^b)[b_{t-1}^b + C_t^b - \frac{1}{2}Y] \tag{7}$$

where the first two equations are the consumption Euler equations of the saver and the borrower respectively. The third equation determines the spread between the rate faced by the borrower and the lender; it follows directly from (2). The fourth equation is the resource constraint and finally the last equation is the budget constraint of the borrower. Some details on how we arrive at these equations are in the footnote.[4]

It should be apparent from the equations above that the steady state is relatively straightforward to derive. The first two equations imply that the real interest rate faced by each of the agents is given by their respective discount factors in steady state, i.e., $1 + r^s = (\beta^s)^{-1}$ and $1 + r^b = (\beta^b)^{-1}$. This, then, is enough to pin down the steady-state equilibrium debt given by equation (5) so that

$$b^b = \bar{b} + \phi^{-1}\left(\frac{\beta^s}{\beta^b} - 1\right)$$

which is shown with b_{high}^b in Figure 1. The steady state debt just derived suggests that the borrower will borrow above the threshold $\bar{b}$ to an extent so that the borrowing rate r^b equals the borrower's discount rate. Notice the contrast to Eggertsson and Krugman (2012), which can be obtained as a special case when $\phi \to \infty$ and the debt limit is binding so that $b^b = \bar{b}$ and the borrower is at a corner solution.

The key thought experiment we want to consider is the case when the debt limit $\bar{b}$ goes from some "high" level to a "low" one, i.e. b goes from $\bar{b}^{high}$ to $\bar{b}^{low}$, an experiment sometimes referred to as a Minsky moment. This is shown in Figure 1. In the previous literature, such as Eggertsson and Krugman (2012), then by assumption the household pays down their debt immediately. Here, however, the borrower is no longer at a corner, instead, he is satisfying the consumption Euler equation (4). In the absence of any deleveraging, the borrower is faced with a higher borrowing cost than before, as shown in point B in Figure 1. The higher borrowing cost, however, gives the borrower the incentive to pay down his debt over time, to deleverage. The optimal dynamic path of deleveraging – which is in sharp contrast to the immediate deleveraging in Eggertsson and Krugman (2012) – can be explicitly derived by solving the dynamic equations (3)-(7), the solution of which we turn to next. The dynamic deleveraging is what moves the borrower from point B down to point C in Figure 1 where once again he faces an interest rate that is equal to the inverse of his discount factor, $(\beta^b)^{-1}$.

Figure 2 shows the path of each of the endogenous variables for illustrative values of the parameters which will generally take the same form for a finite ϕ.[5] The deleveraging is accomplished over a period of time, which is determined optimally by the borrower as seen in the third panel on the first column, where private debt to output falls from 108%

[4]The first-order conditions are derived via writing up a Lagrangian. Here we make the simplifying assumption that the borrower takes b_t^b in the interest-rate premium function as exogenous (corresponding to aggregate debt in the economy). In the general model we allow the spread function to depend upon both individual and aggregate debt. We also make the assumption that the spread between the two interest rates is rebated lump sum to the saver which is why no lump sum transfer appears in (7). In the general model we put a little more structure on this by creating notation for banks, and assuming that the banks are owned by the savers.

[5]Illustrative parameters assumed: $\phi = 0.055$, $Y = 1$, $\beta^b = 0.9796$, $\beta^s = 0.9852$, $\bar{b}^{high} = .9773$, $\bar{b}^{low} = .78$. The model is log-linearized around the steady state to generate the figures.

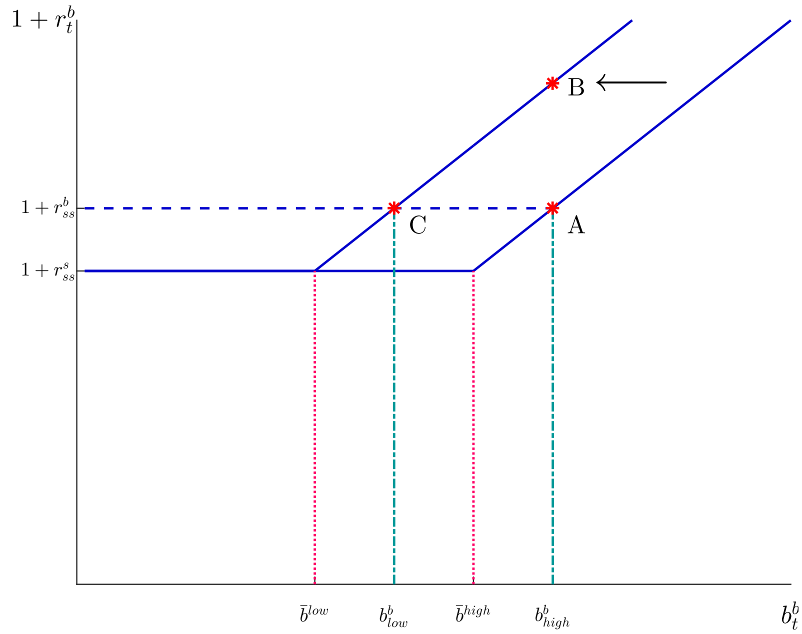

Figure 1: Plot of the function characterizing the cost of borrowing: equation (2) when $\bar{b} = \bar{b}^{high}$ and when $\bar{b} = \bar{b}^{low}$ with $\bar{b}^{high} > \bar{b}^{low}$. The initial steady state is A when $\bar{b} = \bar{b}^{high}$. As $\bar{b}$ moves to $\bar{b}^{low}$, the equilibrium moves to B and then to the final steady state C along the shifted line. Note that b_{high}^b (b_{low}^b) is the steady-state level of debt when the threshold is $\bar{b}^{high}$ ($\bar{b}^{low}$).

to 88%.[6] The borrower deleverages by cutting consumption and gradually paying down his debt. What induces the borrower to deleverage is the rise in the interest rate he faces given by r_t^b as shown in the second panel on the first column of Figure 2. Meanwhile, to make up for this drop in spending the saver needs to correspondingly increase his own spending (since all output is consumed). For this to happen we observe that while the borrower's interest rate rises, the saver's interest rate drops in order to induce the saver to make up for the decline in spending by the borrower. The interest rate faced by the saver may even reach negative levels for a large enough shock to $\bar{b}$. Since the saver's rate is the risk-free short-term interest rate, which will correspond to the nominal interest rate set by the central bank in a more general setting, this will have major implications for monetary policy as we will soon see.

A few comments are now appropriate. First, observe that since the speed of deleveraging – as determined by how long the agent takes to reach their new level of steady-state debt – is optimally determined in this economy, it is perhaps not hard to imagine for the reader at this point that this speed may be affected by macroeconomic policy, an insight we will soon confirm once we introduce endogenous production and endogenous macroeconomic policy. Crucially this implies that the duration of negative real interest rate for a risk-less asset will be endogenous, and this will be critical to many of our results. Second, note that there is nothing in our experiment that depends on the gap between β^b and β^s to be large, which is vividly shown in Figure 1. Even if this gap is small, as long as $\bar{b}^{high}$ falls to $\bar{b}^{low}$, then a spread

[6]It should be noted that the shock $\bar{b}$ expressed as a ratio of output –the variable $\bar{b}^{gdp}$– moves from 97.73% to 78%.

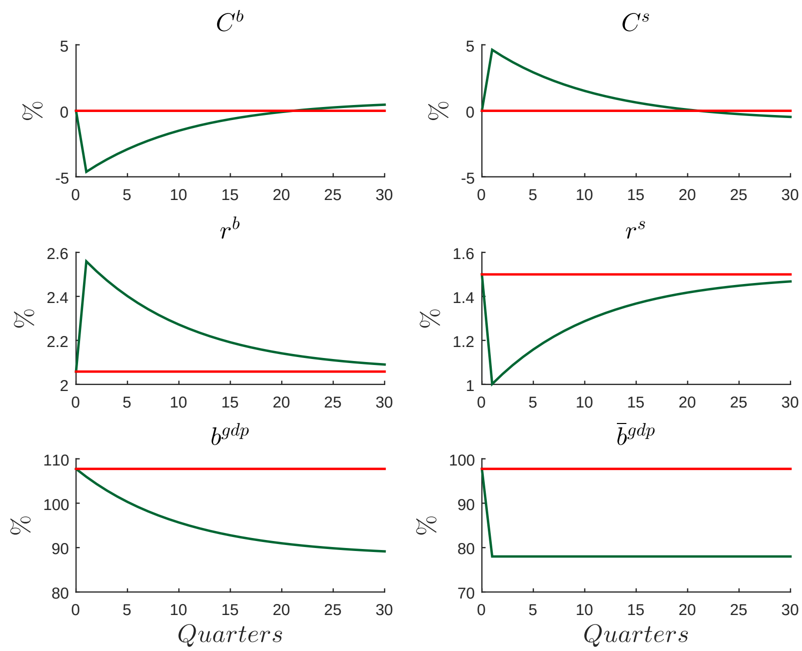

Figure 2: Responses following a deleveraging shock, $\bar{b}$ moves from $\bar{b}^{high}$ to $\bar{b}^{low}$, in the endowment-economy model. Variables are: consumption of borrowers (C^b), consumption of savers (C^s), real interest rate on borrowing (r^b), real interest rate on saving (r^s), debt of borrowers with respect to output (b^{gdp}), the risk-free borrowing threshold with respect to output ($\bar{b}^{gdp}$). C^b, C^s, are in percentage deviation with respect to the initial steady state; r^b, r^s, b^{gdp} and $\bar{b}^{gdp}$ are in percent and at annual rates.

will open to exactly the same extent and the borrower will deleverage. In other words, the dynamics of the deleveraging are independent of the difference between β^b and β^s, only the steady state depends on this difference. In fact, even if $\beta^b \to \beta^s$ exactly the same thought experiment can be done as we have already done. The reason this observation is important is that it is convenient to assume that $\beta^b \to \beta^s$ for some (though not all) aspects of our analysis. In particular, this assumption means that we can derive social welfare in a more tractable way as we will soon see.[7] Note that in the case in which $\beta^b \to \beta^s$ borrowing and lending is no longer motivated by differences in discount factors. What defines borrowers and lenders in this case is the initial asset distribution, whereby some agents are born with debt, and others with assets.[8] Before moving on to a more general setting, we now provide more microfoundations for the spread function between borrowing and lending.

2.2 Banking and deleveraging

The purpose of the last section was to illustrate a basic debt-deleveraging mechanism in an endowment economy, to motivate our general environment coming in the next section. A key input in the analysis was the presence of a borrowing cost, which was reflected in a

[7]Indeed when $\beta^b < \beta^s$ aggregate welfare cannot be written in a recursive way.

[8]In our example we can assume that debtors start from a level of debt $b^b = \bar{b}^{high}$. Observe that while there are initial conditions for debt consistent with lower values of the debt, it can be no higher than this value in steady state. Taking this initial value as given, then, and assuming a debt deleveraging shock, the new steady state will be uniquely defined as $b^b = \bar{b}^{low}$, precisely as in our exercise above.

spread the borrower had to pay relative to the return saver obtained on his savings in the case debt went above a certain "safe" level. Left unstated, however, is where this spread comes from. In the general model of next section, we will specify the spread function in a relatively broad way. Here we put more microfounded structure on borrowing and lending, with explicit banking technology, to help the reader interpreting the general function we present in the next section.

Broadly, we think of the spread function as stemming from two main sources. One reflects that there is only a certain "safe" level of debt over which lending to a particular group of individuals is risky and default might result. The other reflects cost of capital constraints faced by the financial system as a whole. As we will see, we do not need to take a stance on which is the ultimate source. Let us start with the first interpretation.

A bank is a technology that transforms deposit contracts of some people into loan contracts of others. We assume only banks can produce loan contracts (i.e. the household cannot lend directly to one another). Consider the profits of a bank issuing loans to a group of individuals j (it can be a continuum of measure one), which are financed via deposits to individual(s) i (here the identity of the depositors is not important, they could be many or one, for we assume all depositors receive the same risk-free deposit rate which is determined in equilibrium and is independent of the identity of the depositor). Imagine that within the group of loans j, there is a probability $\gamma_t(l_t(j), \bar{b}_t^j, b_t^b)$ that each loan is not repaid so that in aggregate $\gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j)$ are the resources lost by the intermediary in the lending activity. This extra costs can be already predicted at time t. Intermediaries are unable to distinguish ex ante who among the borrowers of type j will default. Assume now that this probability is higher the further away the loan is from what the bank deems to be "safe", i.e. $\bar{b}_t^j$. Similarly the higher is the aggregate level of debt in the economy b_t^b, the higher is the probability of default. The terms of both the loan and the deposit contracts are determined in period t to be paid out in period $t+1$. At the end of period t some people abscond with the money, in a way we will be precise below. In period $t+1$ the remaining loans are collected and deposits paid.

The profit of a bank offering loan contracts of type j and issuing deposits i is

$$d_t(i) - l_t(j) - \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j) + E_t R_{t,t+1}\{(1+r_t^b)l_t(j) - (1+r_t^d)d_t(i)\} \tag{8}$$

where $R_{t,t+1}$ is a stochastic discount factor used to price the real value of next-period income flows.

The problem of the bank can be greatly simplified by the following assumption: Suppose that if there are profits of these loan contracts, then the bank pays the profits to its owner (the representative saver) in period t and only holds enough assets at the end-of-period to pay off the depositor(s) in period $t+1$. This implies that $(1+r_t^b)l_t(j) = (1+r_t^d)d_t(i)$ so that the last term of the profit function drops out. Furthermore, using this to substitute out for $d_t(i)$, we can simplify (8) to

$$\left\{\frac{r_t^b - r_t^d}{1+r_t^d}\right\} l_t(j) - \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)l_t(j)$$

in which case the problem of the bank is simply to choose how much to lend to borrowers j

(which is financed by "hiring" depositors at the rate r_t^d). This yields the first order condition

$$\frac{r_t^b - r_t^d}{1 + r_t^d} = \gamma_t^1 \tag{9}$$

where $\gamma_t^1 \equiv \frac{\partial \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)}{\partial l_t(j)} l_t(j) + \gamma_t(l_t(j), \bar{b}_t^j, b_t^b)$. The above problem looks exactly the same as in the previous section if we make one additional assumption: "Fraud opportunities" and thus default arrives exogenously to the savers when they can "pose" as borrowers. In this case the proceeds of the fraud show up in the exogenous lump sum term in equation (1), while the borrowers budget constraint remains unchanged. A "Minsky moment" can then be defined as a sudden reduction in $\bar{b}_t^j$ which is the perceived borrowing capacity of the group of borrowers of type j, which is also the borrowing capacity of the economy as a whole and it shows up exactly in the same fashion as we have already analyzed.[9]

Consider now an alternative environment, namely that the spread reflects some cost of funding which the banks face, this was our second interpretation of the shock triggering the crisis. One example of such a constraint is capital requirement, that is, the bank needs to hold a certain capital, k_t^s, as a fraction ζ of its outstanding borrowing, b_t^b, i.e.

$$k_t^s \geq \zeta \frac{b_t^b}{1 + r_t^b}$$

in which ζ is the inverse of the leverage ratio of the bank. The bank raises this capital from savers, so we need to adjust the saver's budget constraint to reflect this while the borrower's one remains unchanged.[10] Suppose lending some capital to the bank is completely risk-free from the perspective of the savers, so that $r_t^k = r_t^s$. In writing the bank's problem, let us now imagine, as in Jermann and Quadrini (2012) or Justiniano et al (2014), that there is some cost of equity financing by the bank beyond r_t^k. In particular suppose that there is a function $f(\cdot)$ which is weakly convex and captures the cost of equity financing above a certain threshold $\bar{k}$, with the property that $f(1) = 0$. The profit of the bank can now be written as

$$\Psi_{t+1} = (1 + r_t^b)\tilde{b}_t^b + (1 + r_t^s)\tilde{b}_t^s - (1 + r_t^s)k_t^s \left[1 + \frac{1}{\zeta} f\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right)\right], \tag{10}$$

where we have appropriately re-scaled the function $f(\cdot)$ by ζ and defined $\tilde{b}_t^b \equiv b_t^b/(1+r_t^b)$ and $\tilde{b}_t^s \equiv b_t^s/(1 + r_t^s)$. Considering that the capital-requirement constraint binds in equilibrium and that $\tilde{b}_t^b + \tilde{b}_t^s = k_t^s$, it is easy to show that the first-order condition of the optimization problem implies

$$(1 + r_t^b) = (1 + r_t^s) \left[1 + F\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right)\right],$$

in which

$$F\left(\frac{(1 + r_t)k_t^s}{\bar{k}}\right) \equiv f\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right) + \frac{(1 + r_t^s)k_t^s}{\bar{k}} f'\left(\frac{(1 + r_t^s)k_t^s}{\bar{k}}\right).$$

[9]The first order condition (9) is in fact equivalent to the type of friction we assumed in equation (2). To see this rewrite (9) as $1 + r_t^b = (1 + r_t^d)(1 + \gamma_t^1)$ which reduces to (5) if we assume that $\gamma_t = b_t^b - \bar{b}_t$.

[10]In particular the saver's budget constraint (1) should be written as $b_t^s/(1 + r_t^s) - k_t^s = b_{t-1}^s - (1 + r_{t-1}^k)k_{t-1}^s + C_t^s - (1/2)Y + T_t^s$.

Using again the fact that the capital requirement binds in equilibrium $(1+r^s_t)k^s_t = \zeta b^b_t$, we can further write the above first-order condition as

$$(1+r^b_t) = (1+r^s_t)\left[1+F\left(\frac{\zeta b^b_t}{\bar{k}}\right)\right].$$

This, once again, gives a spread between the lending and the deposit rate which is a function of the aggregate level of debt in the economy as in (2). Here, however, it comes about due to capital constraints of the banks. In particular we can see that this spread may be rising either because of an abrupt change in the required leverage ratio of the banks (an increase in ζ), or because of an increase in the cost of the bank's equity financing (a fall in $\bar{k}$). Any of the interpretations outlined above is valid for the general spread function that we choose in the setup of the next section.

We will now extend the simple example discussed in the past two subsections into a more general setting. The main new elements of this environment relative to the simple example are that we will introduce endogenous production and explicitly model monetary and fiscal policy.

2.3 General Environment

Imagine a closed economy lived in by a continuum of agents on a unitary interval. People are grouped into "savers" of mass $1-\chi$, denoted by the subscript s, and "borrowers" of mass χ, denoted by the subscript b. Utility of a generic agent is given by

$$E_t \sum_{T=t}^{\infty} (\beta^j)^{T-t} \left[1-\exp(-zC^j) - \frac{(L^j_T)^{1+\eta}}{1+\eta}\right] \text{ where } j = s \text{ or } b \tag{11}$$

in which E_t denotes the standard expectation operator; z is a positive parameter, β^j is the intertemporal discount factor in preferences, with $0 < \beta^j < 1$, and C is a consumption bundle

$$C \equiv \left[\int_0^1 C(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where $C(i)$ is the consumption of a generic good i. There is a continuum of goods produced on the interval $[0,1]$; θ is the intratemporal elasticity of substitution between goods with $\theta > 1$; L^j is hours worked. We have chosen an exponential utility in consumption, common in applied finance (see e.g. Calvet, 2001), as it makes aggregation a bit simpler. In a companion appendix we show how the model can be solved for GHH preferences. As we already noted in section (2.1), it helps deriving a social welfare function to assume that $\beta^s \longrightarrow \beta^b$, an abstraction we will maintain for the rest of the paper.

Agents are subject to the following budget constraint

$$\frac{B^j_t}{1+i^j_t} = B^j_{t-1} + P_t C^j_t - W^j_t L^j_t - \Psi^j_t - \Gamma^j_t + T^j_t \tag{12}$$

where B^j, if positive, is nominal debt and conversely asset if negative. P_t is the price index associated with the consumption bundle C, W^j denotes wage specific to labor of quality j;

Ψ^j are profits from operating firms which produce goods while Γ_t^j are profits from financial intermediation (and/or fraud of the kind we discussed in last section); T_t^j are lump-sum taxes.

The nominal interest rate i_t^j is specific to the agent. We now write down a more general function for their interest-rate costs motivated by the banking technology outlined in the last section. Savers, which in equilibrium hold assets, will once again get the risk-free rate which we now specify in nominal term i_t. Instead borrowers, which in equilibrium are going to contract debt, face a borrowing cost

$$1 + i_t^j = (1 + i_t)\tilde{\phi}\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right) \tag{13}$$

which is proportional to the saving rate through a premium captured by the function $\tilde{\phi}(\cdot,\cdot,\cdot)$. The borrowing premium depends on agent j's real debt, defined as $b_t^j \equiv B_t^j/P_t$, in reference to a level $\bar{b}_t$ which represents the maximum amount of real debt that can be considered risk-free at a certain point in time. The premium is also a function of the aggregate debt (per borrowers) given by $b_t = \left(\int_\chi b_t^j dj\right)/\chi$ again in reference with the same level $\bar{b}_t$. Finally, we may also have an exogenous shift in this function captured by ζ_t, which could be modeled as the leverage ratio in our model of banking detailed in the previous section (the parameter ζ) or the cost of equity financing (the parameter $\bar{k}$). When the individual and aggregate debt levels are equal to $\bar{b}_t$, borrowing and saving rates coincide, this is similar to the inflection point in Figure 1. It is required that $\tilde{\phi}(1,1,\zeta) = 1$ in which ζ is the initial steady state of ζ_t. Furthermore, we assume that it is always the case that $\tilde{\phi}(\cdot,\cdot,\cdot) \geq 1$. The borrowing premium is also non-decreasing with the increasing borrowing of agent j, i.e. we assume that the derivative of the function with respect to the first argument is non-negative, $\tilde{\phi}_{b^j}(\cdot,\cdot,\cdot) \geq 0$. Moreover, at the risk-free level $\bar{b}_t$, the marginal cost of increasing the individual borrowing capacity is zero, i.e. $\tilde{\phi}_{b^j}(1,\cdot,\cdot) = 0$, which is a sort of optimality condition at the individual level when borrowing is at the risk–free threshold. Finally, the borrowing premium is also non-decreasing with the increasing aggregate borrowing, meaning that the derivative of the function with respect to the second argument is non-negative $\tilde{\phi}_b(\cdot,\cdot,\cdot) \geq 0$. [11]

The above framework implies first-order conditions associated with the optimal choices of consumption, labor and asset holdings of savers and borrowers which are left to the Appendix.

On the production side, we assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor, $Y(i) = L(i)$. Here we make another key simplifying assumption. We assume that production is a Cobb-Douglas indexes of the two types of labor as $L(i) = (L^s(i))^{1-\chi}(L^b(i))^\chi$.[12] Given this technology, labor compensation for each type of worker is equal to total compensation $W_jL_j = WL$ where the aggregate wage index is appropriately defined by $W =$

[11] We further assume that $\tilde{\phi}_b(1,1,\zeta) > 0$ and $\tilde{\phi}_{b^i,b}(1,1,\zeta) + \tilde{\phi}_{b^i,b^i}(1,1,\zeta) > 0$ where $\tilde{\phi}_{b^i,b}(\cdot,\cdot,\zeta)$ and $\tilde{\phi}_{b^i,b^i}(\cdot,\cdot,\zeta)$ are second derivatives.

[12] This assumption makes our model a bit simpler than, for example, Curdia and Woodford (2011) and Eggertsson and Krugman (2012). In the latter work there is a labor supply effect of deleveraging which this assumption allows us to abstract from.

$(W^s)^{1-\chi}(W^b)^{\chi}$. This structure will greatly facilitate the aggregation of the model together with the assumption of exponential consumption utility.

Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta}Y$ where aggregate output is

$$Y_t = (1-\chi)C_t^s + \chi C_t^b. \tag{14}$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1-\alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price which is going to apply at a generic future period T with a probability α^{T-t}. Furthermore this price is going to be indexed to the inflation target over the period given by Π^{T-t}. A constant subsidy τ on firms' revenues is in place. The implied first-order conditions of the firms are shown in the Appendix.

To complete the characterization of the model we specify fiscal policy and assume that

$$T_t^j = \tau P_t Y_t \tag{15}$$

for each agent j implying the government budget constraint

$$(1-\chi)T_t^s + \chi T_t^b = \tau P_t Y_t. \tag{16}$$

The model is closed with the specification of monetary policy. Details on the model's non-linear equilibrium conditions are given in the Appendix.

3 Characterization: Post-crisis New-Keynesian model

Once we take a log-linear approximation of the equilibrium conditions around the initial steady state in which $\bar{b}_t = \bar{b}^{high}$ and $\zeta_t = \zeta$, our model takes a simple form and can represent a stylized version of New-Keynesian models with heterogenous agents and financial frictions. We denote the steady state by omitting time subscript to the variable in question.

The Euler equations of savers implies

$$E_t\hat{C}_{t+1}^s - \hat{C}_t^s = \sigma[\hat{\imath}_t - E_t(\pi_{t+1} - \pi)] \tag{17}$$

where we have defined $\hat{\imath}_t \equiv \ln(1+i_t)/(1+i)$, $\pi_t \equiv \ln \Pi_t$, $\pi \equiv \ln \Pi$ and $\sigma \equiv 1/(zY)$. In particular, we set $\hat{C}_t^j \equiv (C_t^j - C^j)/Y$ for each $j = s, b$.

A first-order approximation of the Euler equation of the borrowers implies

$$E_t\hat{C}_{t+1}^b - \hat{C}_t^b = \sigma\left[\hat{\imath}_t^b + \upsilon\left(\hat{b}_t - \hat{d}_t\right) - E_t(\pi_{t+1} - \pi)\right] \tag{18}$$

where we have further defined $\hat{\imath}_t^b \equiv \ln(1+i_t^b)/(1+i)$, $\hat{b}_t \equiv (b_t - \bar{b}^{high})/\bar{b}^{high}$, $\hat{d}_t \equiv (\bar{b}_t - \bar{b}^{high})/\bar{b}^{high} + (\zeta_t - \zeta)/\zeta$ while $\upsilon \equiv \varepsilon_b(1,\zeta) > 0$ in which $\varepsilon\left(b_t/\bar{b}_t, \zeta\right)$ is a function discussed in the Appendix.[13]

It is key to note that $\hat{d}_t$ can vary both for shifts in $\bar{b}_t$ and ζ_t in an isomorphic way so that there is no clear distinction between the two interpretations given in Section 2.2

[13]Note that the assumption already made that $\tilde{\phi}_{b^i,b}(1,1,\zeta) + \tilde{\phi}_{b^i,b^i}(1,1,\zeta) > 0$ implies that $\upsilon > 0$. Furthermore, without losing generality, we have assumed the normalization $\varepsilon_b(1,\zeta) = \varepsilon_\zeta(1,\zeta)$.

stemming from either tighter borrowing or lending constraints. In what follows, we focus on a deleveraging shock triggered by a fall in $\bar{b}_t$.

The spread between borrowing and saving rates can be approximated as

$$\hat{\imath}_t^b = \hat{\imath}_t + \varphi(\hat{b}_t - \hat{d}_t) \tag{19}$$

where $\varphi \equiv \phi_b(1, \zeta) > 0$ is the steady-state elasticity of the premium function $\phi(b_t/\bar{b}_t, \zeta_t)$ with respect to real debt and $\phi(b_t/\bar{b}_t, \zeta_t) \equiv \tilde{\phi}\left(b_t/\bar{b}_t, b_t/\bar{b}_t, \zeta_t\right)$.[14]

A first-order approximation of the budget constraint of the borrowers delivers

$$\hat{b}_t = \frac{1}{\beta}(\hat{b}_{t-1} + \beta\hat{\imath}_t^b - (\pi_t - \pi)) + \frac{(1+i)}{\tilde{b}}(\hat{C}_t^b - \hat{Y}_t), \tag{20}$$

where $\hat{Y}_t \equiv (Y_t - Y)/Y$ and $\tilde{b} \equiv \bar{b}^{high}/Y$.

Goods market equilibrium implies

$$\hat{Y}_t = \chi\hat{C}_t^b + (1-\chi)\hat{C}_t^s. \tag{21}$$

Equations (17), (18) together with (19), (20) and (21) constitute the aggregate demand block of the model.

In a log-linear approximation, the supply block can be derived to obtain the standard New-Keynesian Phillips curve

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta E_t(\pi_{t+1} - \pi) \tag{22}$$

where we have defined $\kappa \equiv (1-\alpha)(1-\alpha\beta)(\eta + \sigma^{-1})/\alpha$.[15]

Equations (17), (18) together with (19), (20), (21) and (22) determine the equilibrium allocation for $\left\{\pi_t, \hat{C}_t^b, \hat{C}_t^s,\ \hat{Y}_t, \hat{\imath}_t^b, \hat{\imath}_t, \hat{b}_t\right\}_{t=t_0}^{\infty}$ given the specification of monetary policy and given exogenous process $\hat{d}_t$ and initial condition $\hat{b}_{t_0-1}$.

3.1 A parallel with the textbook New-Keynesian model

Before going further it is useful to now explore the interpretation of the results we have already obtained in the linearized model. In particular we can now show that the model we have just sketched out generalizes the standard New-Keynesian model common in economic textbooks. To see this, let us combine equations (17), (18), (19) and (21) to yield

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t - E_t(\pi_{t+1} - \pi) - r_t^n) \tag{23}$$

where r_t^n is now given by

$$r_t^n \equiv -\chi(\upsilon + \varphi)\left(\hat{b}_t - \hat{d}_t\right). \tag{24}$$

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[14] Note that $\phi_b(1,\zeta) = \tilde{\phi}_{b^j}(1,1,\zeta) + \tilde{\phi}_b(1,1,\zeta)$. Since $\tilde{\phi}_{b^j}(1,1,\zeta) = 0$, the assumption already made that $\tilde{\phi}_b(1,1,\zeta) > 0$ is needed to obtain $\varphi > 0$.

[15] Given the preferences' specification assumed, the way wealth is distributed across the two types of agents does not enter directly into the log-linear version of the AS equation, as instead in Curdia and Woodford (2010).

Meanwhile the AS equation is exactly the same as in the standard model, as we see in (22). However, in the New-Keynesian benchmark model r_t^n is exogenous so that given a policy rule for the nominal interest rate one can characterize an equilibrium in the model. Here, however, r_t^n is endogenously determined. Hence, even if demand is still determined by the real interest rate and expected future income, the level of private indebtedness will now shift this standard demand relationship. In particular if the real value of private debt, $\hat{b}_t$, is above the "target" value $\hat{d}_t$ then private debt is too high, triggering a negative shock to the natural rate of interest, i.e., a negative shock to demand. Equivalently if $\hat{b}_t$ is below $\hat{d}_t$ then there is extra room for the indebted agents to spend, which acts as a positive demand shock.

Observe, however, that although $\hat{d}_t$ is exogenous in the model, $\hat{b}_t$ is endogenously determined. To solve for this variable, however, we need to solve the entire model using the set of equations summarized in the last section.

4 Dynamic debt deleveraging

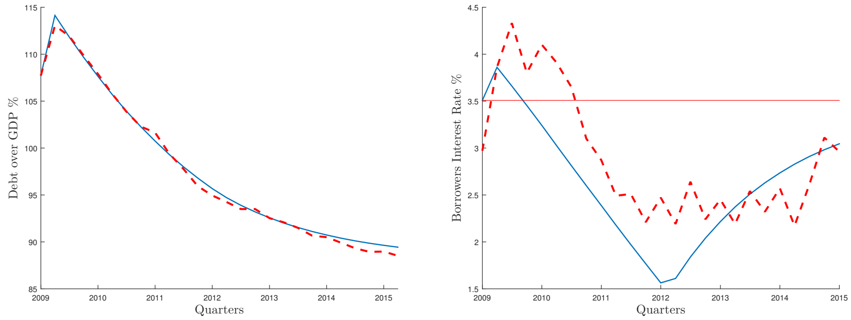
4.1 Calibration

While the main contribution of the paper is not a quantitative evaluation, it is useful to parameterize the model to organize the following discussion and get a rough sense of the order of magnitudes the debt deleveraging mechanisms can deliver. One of the key attractions of the standard New Keynesian model is the short distance between the prototype pre-crisis model and medium-scale models estimated for policy simulations. As we hope is transparent, our extension is tractable enough so that it can be easily scaled up to a estimated medium-size model, e.g. in the tradition of Smets and Wouters (2007).

We choose the parameters of the model from the existing literature when possible. We then pick the size of the shock (i.e. b^{high} to b^{low}), as well as two parameters specific to our model, ϕ and υ, to match the degree of household debt deleveraging observed since 2008. The variables in the model we look to proxy in the data, are household debt, b_t, and the borrowing rate, i_t^b. To match the evolution of these model variables to their data counterparts as closely as possible we use as a criterion mean squared errors. The strategy and data details are described in the Appendix. The data series are shown in figure 3 with the dashed lines showing the data and the solid line the model output once the shock and the parameters ϕ and υ have been calibrated. Table 1 shows all the parameter values of our calibration.

Table 1: Parameters under Benchmark Model

Parameter	Value	Source or Target
Intertemporal elasticity of substitution in consumption	$\sigma = 0.66$	Smets and Wouters (2007)
Inverse of the Frisch elasticity of labor supply	$\eta = 1$	Justiniano et al. (2015)
Slope of the AS equation	$\kappa = 0.02$	Eggertsson and Woodford (2003)
Share of borrowers	$\chi = 0.61$	Justiniano et al. (2015)
Steady-state inflation rate	$\Pi = 1.005$	Corresponding to 2% at annual rates: Justiniano et al. (2015)
Intertemporal discount factor	$\beta = .9963$	Match real interest rate of 1.5%: Domeij and Ellingsen (2015)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Parameter of spread function	$\phi = 0.055$	Match the data
Parameter of the borrowers' Euler equation	$\upsilon = 0.159$	Match the data.
Initial real debt	$\bar{b}^{high} = 4.0869$	Initial debt over GDP at 107.73%
Final real debt	$\bar{b}^{low} = 3.3384$	Final debt over GDP at 88%



(a) **(b)**

Figure 3: Figure (a) shows the dynamic of nominal debt as a percentage of GDP implied by the model (blue continuous line) in comparison with the historical values of US nominal debt as a percentage of GDP (red dotted line). Figure (b) shows the dynamic of the borrowers' interest rate implied the model (blue continuous line) in comparison with the US borrowers' interest rate (red dotted line). Both variables are in %.

A typical calibration approach is to pick parameters and shocks to match some specific features of the data, and then explore the implications for other observables. Figure 4 shows an exercise of this kind, assuming that the central bank attempts to target two percent inflation unless it is precluded from doing so due to the ZLB (in which case $i_t^s = 0$). In this figure we inserted the shocks calibrated as described above and explore transition dynamics for the other key endogenous variables, given the parameters assumed in Table 1. The top row of Figure 4 shows the output of the model while the bottom row shows US data counterparts from 2007 to 2015.[16] There are a few takeaways. First, we see that the debt deleveraging shock, chosen to measure actual debt deleveraging in the US data, generates strong enough downward pressures on the nominal interest rate for the zero bound to be binding. Second, the resulting recession is roughly of the same order as seen in the data. Third, the model generates a drop in inflation, with actual deflation only appearing in a single quarter, but then remaining below target until the ZLB stops being binding. In short, the model generates a benchmark that at least in broad strokes paints a picture of movements of the key model variables that are of similar order as observed in the US economy following the crisis of 2008. It now becomes interesting to explicitly uncover the underlying dynamics that generate this outcome and also explore the possible role for policy.

Before moving on it is worth pointing out a key feature of the data the model misses. The short term nominal interest rate starts to rise at the end of 2012 in the model, hence the recession lasts roughly 3 years. While actually the recession (as measured in deviation of output from trend) did not last much longer than this, the bottom panel shows that the short term nominal interest rates remained at zero until the end of 2015, full three years longer. One reason for this failure of the model, is that the spread between the borrowing and lending rate largely subsided by 2012, which the model interprets as saying that there is no longer need for negative real rates to achieve its inflation target. Adding more internal propagation to the model may resolve this issue, since we are here only considering the most stripped down variation of the New Keynesian model that in general fails to generate

[16] See details in Appendix B.

persistent responses to most shocks. Another way to rationalize the fact that the lift-off in the simulation is earlier than in the data is that our model disregards some long-run secular forces which have the effect of lengthening the duration of zero-lower bound, a theme of the recent literature on secular stagnation (see e.g. Eggertsson and Mehrotra, 2014, Benigno and Fornaro, 2015). Under this interpretation, we can think of the dynamics documented in this paper as those that were specific to the financial crisis that could be layered on top of the slower moving forces that the secular stagnation literature is about.

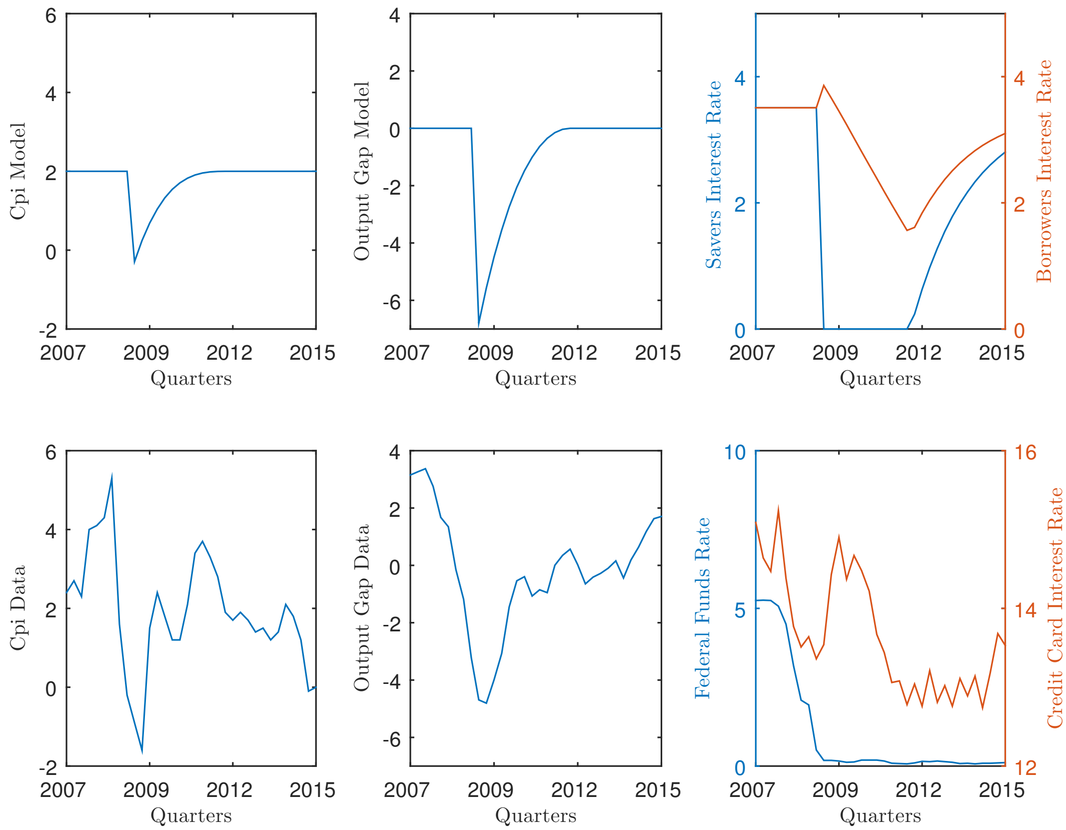


Figure 4: The first row plots the model impulse responses of inflation, output gap, and interest rates (continuos line: savers' rate; dashed line: borrowers' rate) following the deleveraging shock. The second row plots the respective data counterparts.

4.2 Deleveraging when there are no frictions

With an interesting numerical example as a benchmark, that at least captures the quantitative movements in some key variables, we can now study the interaction between *endogenous deleveraging* and policy. Figure 5 starts with an example in which the central bank successfully targets two percent inflation, and the zero bound is not imposed. Equivalently we can interpret this as the equilibrium allocation if all prices were flexible. This experiment is helpful to clarify the main forces in the model in response to a deleveraging shock if other frictions play no role.

In response to a deleveraging shock we see that first there is an increase in the spread between the borrowing and the saving rate, i_t^b and i_t respectively, that is triggered by the

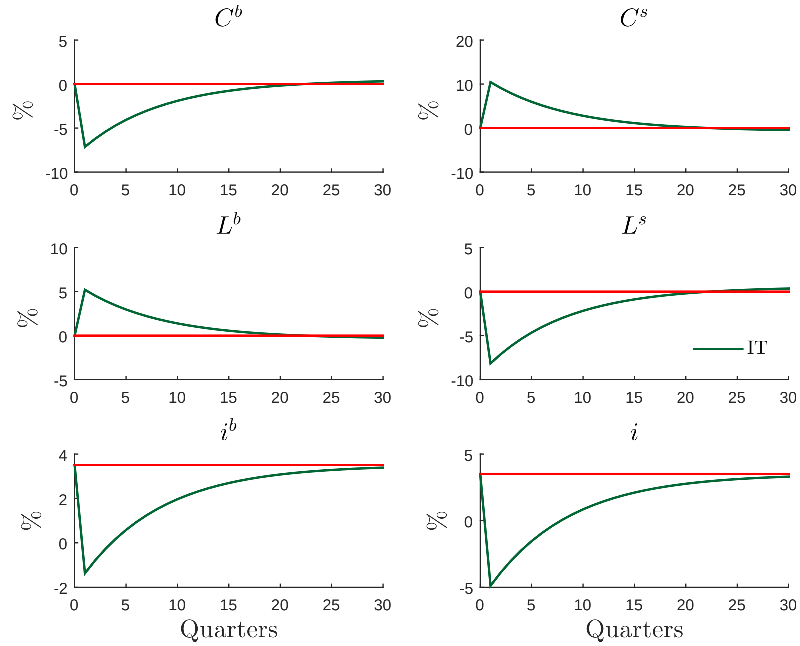


Figure 5: Responses following a deleveraging shock when the central bank can target constant inflation (or prices are flexible) without taking in consideration the zero-lower bound (line "IT"). Variables are: consumption of borrowers (C^b), consumption of savers (C^s), hours worked of borrowers (L^b), hours worked of savers (L^s), nominal interest rate on borrowing (i^b), nominal interest rate on saving (i). C^b, C^s, L^b, L^s are in percentage deviation with respect to the steady state; i^b and i are in percent and at annual rates.

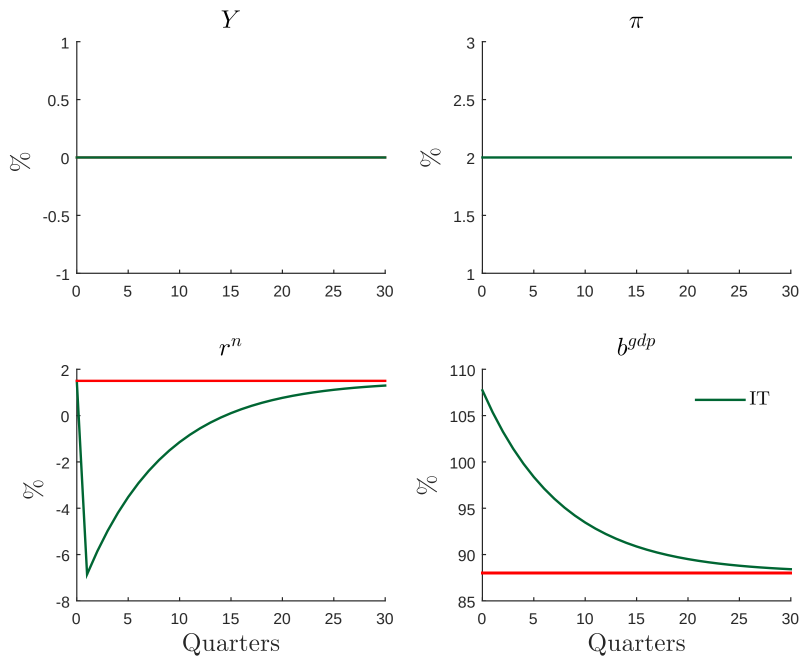


Figure 6: Responses following a deleveraging shock when the central bank can target constant inflation (or prices are flexible) without taking in consideration the zero-lower bound (line "IT"). Variables are: output (Y), inflation rate (π), natural rate of interest defined as in (24) (r^n), aggregate debt to GDP (b^{gdp}). Y is in percentage deviation with respect to the steady state; π, r^n and b^{gdp} are in percent and at annual rates.

exogenous shock $\bar{b}_t$. In response to this the borrowers find it optimal to start paying down their debt – deleverage – so we see a decline in their outstanding debt b_t in Figure 6. How do the borrowers deleverage? In equilibrium they do so in two ways: By cutting down their consumption, C_t^b, and by increasing their hours worked, L_t^b. Note, however, that this is perfectly offset by a drop in hours by the savers and an increase in their consumption. It is clear why the borrowers decide to deleverage – they are facing higher borrowing costs. But why do the savers increase their consumption and cut back their hours? The reason is that the risk-free interest rate, i_t^s, declines, which means that consumption today is now relatively less expensive than it was before. This price change is a key to understand the problem we shall see once we add more frictions to the model, because if there is a bound on how much this interest rate can decline (due to the zero bound) that can create serious problems for macroeconomic management. We can also see that the saver finds it in his interest to cut back hours. The reason is that the higher consumption of the saver reduces his marginal utility of consumption, in turn reducing his incentive to supply work.[17]

Figure 6 shows by how much the real interest rate needs to drop for output to remain unchanged: by about 6 percentage points. The real interest rates that are consistent with this equilibrium, however, are negative. For a central bank that targets inflation at 2 percent (as we assume here) this means that if the natural rate of interest is below -2% then the zero bound becomes binding and the equilibrium adjustment we have just explored is not feasible. This is the case we now turn to.

4.3 Dynamic deleveraging at the zero bound

The key to the adjustment mechanism in response to a deleveraging shock, outlined in the last section, was that, in response to cutbacks in spending by the borrower, the risk-free interest rate declines, which induced the savers to make up for this drop in spending. As revealed in Figure 6, however, this adjustment implies a negative interest rate faced by the saver, which we assume is the rate controlled by the central bank. At two percent inflation target, the real saving rate can be at -2 percent, even if the nominal rate is zero. It can't go any further than that, however, which is needed in our example, as shown in Figure 6. Hence the zero bound is violated.

The first row in Figure 4 already showed us the drop in output and inflation triggered by the fact that the central bank cannot accommodate the shock.[18] Figure 7 clarifies that the fact that the central bank hits the ZLB introduces an important endogenous component to the dynamic deleveraging relative to when the ZLB was not imposed. Because the ZLB is now binding, this reduces output and thus the income of the borrowers. This, in turn, implies a slowdown in the pace of deleveraging. A simple way of seeing this is to compute the statistic (24) that maps into the natural rate of interest of the standard New-Keynesian model. The key point is that this process is now endogenous and, as we can see in Figure 7, the crisis means that it recovers more slowly (green line) than if policy had been able to accommodate it fully. What this means is that endogenous deleveraging increases the

[17]Real wages of savers increase following the shock, and offset in part the wealth effect on their labor supply. Without the increase in real wages, labor supply would fall twice as low.

[18]In Figure 7, and in what follows, the inflation-targeting policy considering the zero lower bound is defined as $\pi_t = \pi$ whenever $i_t > 0$ otherwise $i_t = 0$.

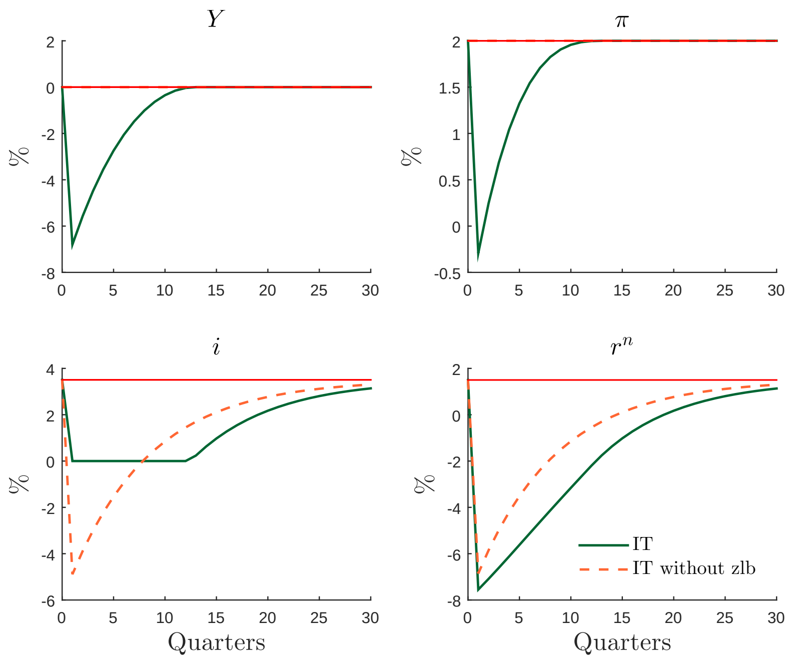


Figure 7: Responses following a deleveraging shock if the central bank can target constant inflation by taking in consideration (line "IT") and without taking in consideration (line "IT without zlb") the zero-lower bound. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, r^n, i and are in percent and at annual rates.

persistence of the crisis by creating a feedback between falling in income and a slowdown in deleveraging. This will have important policy implication, as we soon shall see.

Another way to see the feedback between the ZLB and debt deleveraging is to conduct a slightly different thought experiment by comparing our model to the standard New-Keynesian one.

4.4 Zero inflation target with and without endogenous deleveraging

Consider the following thought experiment: Let us extract the real interest rate – or the natural rate of interest – from the model of Section 3. This is the natural rate of interest in our model in the case that monetary policy is able to target inflation at two percent when we ignore the zero bound. Putting it differently, we can directly back this variable out of equation (24) and this variable is what is shown in Figure 6.

If we treat this sequence of numbers as an exogenous variable into the standard NK model described in Section 3.1 and ignore the zero bound we obtain exactly the same solution as before, namely no output gap and inflation at target. But we can now also impose the zero bound in that model too, and compare with our previous solution of Section 4.3, and ask what happens. In the standard NK model, we are keeping the natural rate of interest as purely exogenous and it follows the path shown in Figure 6. By comparing the two outcomes we are then seeing what making the natural rate of interest endogenous does to our solution.

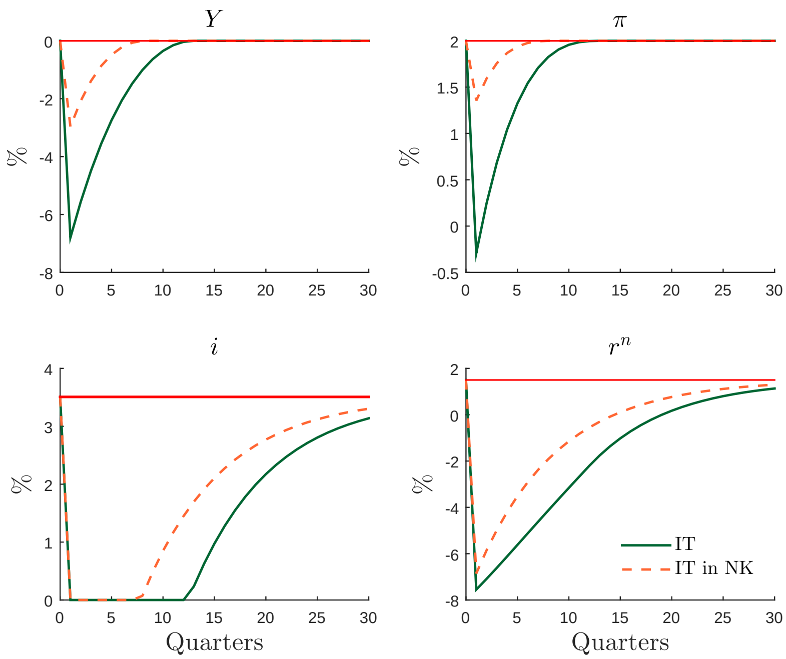


Figure 8: Comparison between the responses to a deleveraging shock in the deleveraging model, under inflation targeting and considering the zero-lower bound (line "IT"), with those of the benchmark New-Keynesian model of Section 3.1, under inflation targeting and considering the zero-lower bound (line "IT in NK"). (The responses of the two models coincide under inflation targeting without considering the zero-lower bound). Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

That is, we can infer to what extent it matters that in our new model the drop in output leads to an endogenous propagation by making it harder for the borrowers to deleverage, thus delaying the recovery of the natural rate of interest to its steady state and prolonging the crisis.

Figure 8 shows the evolution of output, inflation, the natural interest rate and the nominal interest rate in the standard NK model (orange dashed line) and compares it to the dynamic deleveraging model (green line) where we have constructed the shocks as described above. We parameterize the NK model exactly like our current one, using the mapping shown in Section 3.1. We see that, with endogenous deleveraging, both the effects on output and inflation are bigger than in the standard case (and note that in the absence of the zero bound each of the variables would have behaved exactly the same). The reason is that under dynamic deleveraging then the output slack lowers the natural rate of interest further, and makes it more persistent, leading to the zero bound being even more binding. Since aggregate demand depends on the current and expected future nominal interest rate, expected inflation and expected output, this feeds into lower demand today, thus lower output and inflation and so on. We see that quantitatively this effect is quite large, the inflation and output drop is more than double what it is without dynamic debt deleveraging, and that the zero lower bound is binding for several more quarters once the endogenous persistence of the natural rate of interest is taken into account.

Hence we conclude that adding dynamic deleveraging can have large impacts on the actual dynamics at the zero bound, both in terms of the persistence of the crisis (for a given shock as measured by the natural rate of interest) and its impact. But what is the implication of this for policy? That is the issue we now analyze.

5 Optimal policy under dynamic deleveraging

To consider optimal monetary policy, we first need to study how social welfare looks in our model. We consider a benevolent policymaker maximizing the utility of the households in the economy

$$W_t = E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{T-t} \left[(1-\tilde{\chi}) \left(U(C_t^s) - V(L_t^s) \right) + \tilde{\chi} \left(U(C_t^b) - V(L_t^b) \right) \right] \right\} \tag{25}$$

for a generic weight $\tilde{\chi} \in (0,1)$. The heterogeneity in the model give rise to some special consideration relative to the prototype New Keynesian model. Recall that our deleveraging experiment brings the economy from one distribution of wealth to another. We approximate our model around the efficient steady state and make the assumption that the economy will reach the efficient allocation in the long run. This allows us to cleanly focus on the relevant short-run trade-offs triggered by debt deleveraging. We can rationalize the long run allocation as efficient by making an appropriate choice of the weight $\tilde{\chi}$. The efficient steady state is implicitly defined by the first-order conditions of the maximization problem of (25) under the resource constraint

$$Y_t = (L_t^s)^{1-\chi} (L_t^b)^{\chi} = (1-\chi) C_t^s + \chi C_t^b. \tag{26}$$

implying the proportionality of the marginal utility of consumption between borrowers and savers

$$\frac{U_c(C_t^s)}{U_c(C_t^b)} = \frac{\tilde{\chi}}{(1-\tilde{\chi})} \frac{(1-\chi)}{\chi}, \tag{27}$$

through the parameter $\tilde{\chi}$ in relation to χ.

Once we plug into (27) the steady-state levels of consumption of borrowers and savers (given respectively by (A.22) and (A.23) in the Appendix) reached at the end of the deleveraging period, it is easy to see that there is only one value of $\tilde{\chi}$ which makes (27) consistent with this final steady state. This is the value we pick for our welfare weights in (25). It also implies that the optimal inflation rate coincides with the inflation target Π.[19]

We take a second-order approximation of (25) around the efficient steady state. In the Appendix, we show that it is equivalent to the following quadratic loss function

$$L_{t_0} = \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\hat{Y}_t^2 + \chi(1-\chi)\lambda_c (\tilde{C}_t^b - \tilde{C}_t^s)^2 + \lambda_\pi (\pi_t - \pi)^2 \right] \right\}. \tag{28}$$

[19] If we choose an alternative weight $\tilde{\chi}$ the final steady state will be inefficient creating an incentive for policy to deviate from the inflation target Π in order to correct for the inefficient – final – distribution of wealth. The presence of this "long-run" incentive is not convenient since it will blur the understanding of the optimal adjustment following a deleveraging shock. Moreover, to deal with a distorted steady state, we have to take a more complex approximation procedure through a second-order approximations of the equilibrium conditions implied by the optimization problem of private agents and by the resource constraints. This procedure comes at the cost of a less neat analysis without the benefits of getting much additional insights.

The benevolent policymaker is concerned about the deviations of output and inflation from their respective steady states as is standard in the literature. However, there is an additional and new term in the loss function capturing the deviations of consumption of the borrowers and savers from their respective efficient steady state. We have defined $\tilde{C}_t^j \equiv (C_t^j - \bar{C}^j)/Y$ for each j where $\bar{C}^j$ is indeed the efficient steady-state level.

We can also write these latter terms with respect to the initial steady state obtaining equivalently

$$L_{t_0} = \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^b - \hat{C}_t^s - c^R)^2 + \lambda_\pi(\pi_t - \pi)^2 \right] \right\} \tag{29}$$

where c^R captures the relative difference between the initial and final steady-state consumption of borrowers and savers defined as $c^R \equiv [(C^b - \bar{C}^b) - (C^s - \bar{C}^s)]/Y$.

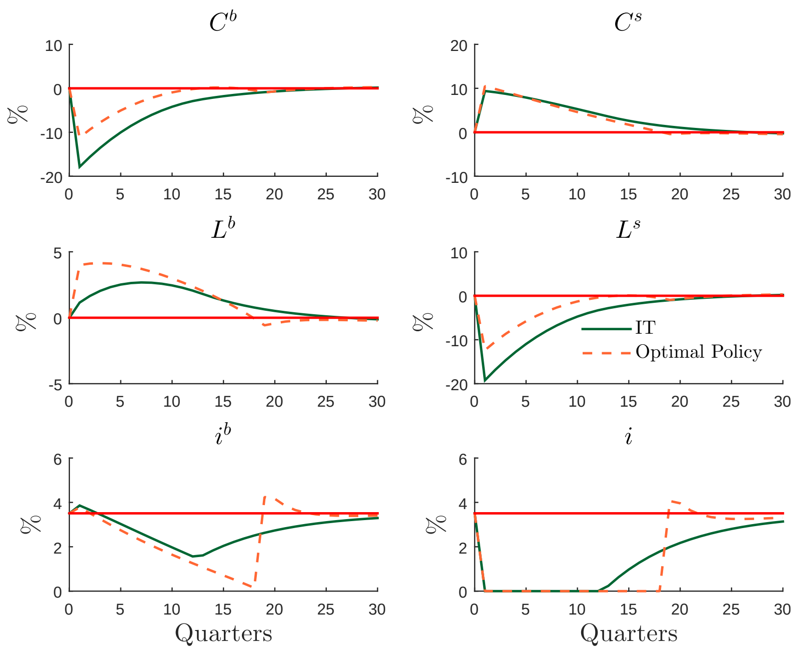


Figure 9: Responses following a deleveraging shock under optimal monetary policy with commitment (line "Optimal Policy") in comparison to inflation-targeting policy (line "IT") taking in consideration the zero-lower bound. Variables are: consumption of borrowers (C^b), consumption of savers (C^s), hours worked of borrowers (L^b), hours worked of savers (L^s), nominal interest rate on borrowing (i^b), nominal interest rate on saving (i). C^b, C^s, L^b, L^s are in percentage deviation with respect to the steady state; i^b and i are in percent and at annual rates.

Optimal monetary policy minimizes (29). The policymaker would like to keep inflation and output on target and at the same time achieve the efficient levels of consumption for the two agents. However, the three objectives can only be simultaneously reached in the long run. As shown in Section 4 and in particular in Figure 5, a deleveraging shock under an inflation-targeting policy produces short-run divergences between the consumption of borrowers and savers which are in contrast with the objective (28), even without taking into account the zero-lower bound. Adding the zero-lower bound makes things worse since output drops as shown in Section 4.3 and Figure 7.

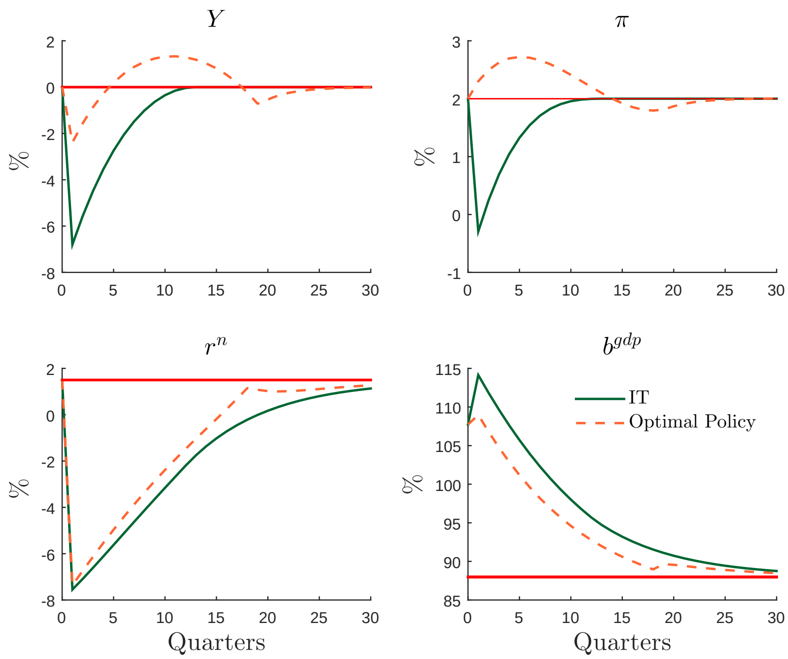


Figure 10: Responses following a deleveraging shock under optimal monetary policy with commitment (line "Optimal Policy") in comparison to inflation-targeting policy (line "IT") taking in consideration the zero-lower bound. Variables are: output (Y), inflation rate (π), natural rate of interest defined as in (24) (r^n), aggregate debt to GDP (b^{gdp}). Y is in percentage deviation with respect to the steady state; π, r^n and b^{gdp} are in percent and at annual rates.

Optimal policy under commitment minimizes the loss function (29) by choosing the sequences $\left\{\pi_t, \hat{C}_t^b, \hat{C}_t^s,\ \hat{Y}_t, \hat{\imath}_t^b, \hat{\imath}_t, \hat{b}_t\right\}_{t=t_0}^{\infty}$ under the constraints (17), (18),(19), (20), (21) and (22) given exogenous process $\hat{d}_t$ and initial condition $\hat{b}_{t_0-1}$, taking into account the zero-lower bound constraint on the nominal interest rate. Details on the first-order conditions of the optimal policy problem are left to the Appendix.

Figures 9 and 10 show the responses of some variables of interest to a permanent shock on $\bar{b}_t$, and therefore on $\hat{d}_t$, under both optimal policy and inflation targeting considering the zero bound on nominal interest rates. Optimal policy has a larger effect in the model than the case in which it targets constant inflation. The way optimal policy works is to a large extent similar to that in the standard NK model (see e.g. Eggertsson and Woodford, 2003). In particular, as shown in Figure 9, optimal policy involves committing to keep the nominal interest rate low for a substantial period of time longer than if the central bank is an inflation targeter.[20] The result of this commitment is an output boom and inflation during and after the trap. A key difference is that this commitment is even stronger than in the standard model because it implies an accommodation that is forceful enough that inflation overshoots the two-percent target throughout the duration of the zero bound. In our numerical example we see that inflation never undershoots it target at the ZLB, instead, it reaches almost 3 percent. This feature of optimal policy is new and different relatively to the standard model, which we will now elaborate on.

[20] In Section 5.3 we will provide an interesting counterexample.

5.1 Comparison to optimal policy in the standard NK model

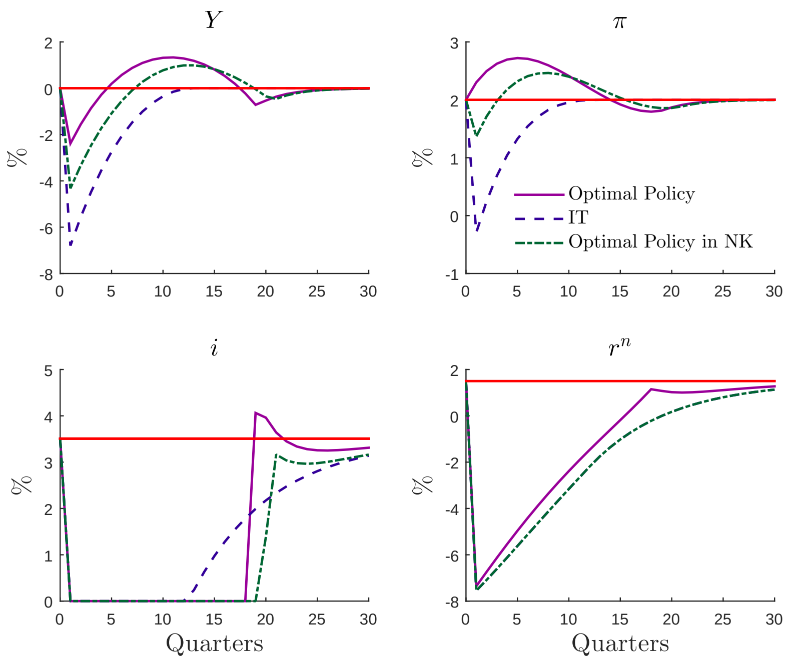


Figure 11: Comparison between the responses to a deleveraging shock. "Optimal Policy": optimal policy under commitment in the deleveraging model. "IT": inflation-targeting policy considering the zero-lower bound in the deleveraging model coinciding with the same policy in the standard NK model. "Optimal Policy in NK": optimal policy in the standard NK model. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

To compare the optimal policy in the standard NK model to ours, we compute the natural rate of interest from equation (24) assuming that the central bank follows a strict inflation target and hits the zero bound. This is the basic exercise we carried out in Figure 7. If we feed this process into the standard NK model and assume that it is exogenous, it is easy to see that the solution of the NK model, under inflation targeting and considering the zero-lower bound, is exactly the same as we already have in our model as shown in Figure 7. To see this, notice that both models satisfy exactly the same equations (22) and (23) and that the process r_t^n across the two simulations is by construction exactly the same. This baseline case – which is the same across the two models – is reported in Figure 11 via a blue dashed line. It is interesting now to ask: How does the optimal monetary policy look across the two models? And should we expect it to be different?

First, one important difference is that the loss function of the benchmark NK model corresponds to (29) but with $\lambda_c = 0$. Second, in our setting, policy will take into account that its actions will endogenously affect the deleveraging process (and the natural rate of interest) a consideration not present in the standard model. Figure 11 shows that the implications of the two optimal policies are quite different in terms of output and inflation. In our dynamic deleveraging model, optimal policy (line "Optimal Policy") is aggressive enough to bring about an immediate increase in inflation, thus overshooting the implicit inflation target of the central bank by a significant amount. In the benchmark New-Keynesian model

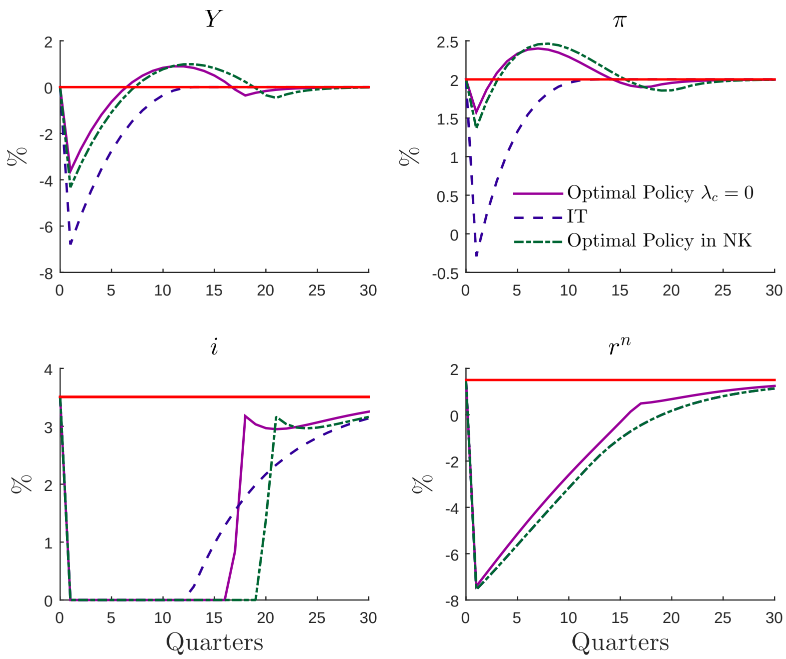


Figure 12: Comparison between the responses to a deleveraging shock. "Optimal Policy $\lambda_c = 0$": optimal policy under commitment in the deleveraging model when $\lambda_c = 0$ in (29). "IT": inflation-targeting policy considering the zero-lower bound in the deleveraging model coinciding with the same policy in the standard NK model. "Optimal Policy in NK": optimal policy in the standard NK model. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

(line "Optimal Policy in NK"), inflation overshoots the target less aggressively and with some delay while recovery peaks later. The most interesting feature in the comparison is the behavior of the nominal interest rate. Optimal policy in our model implies an earlier lift-off than in the standard model, even if it is consistent with a smaller drop in output and inflation. How is it possible then that there is more expansion in output and inflation under our model than in the benchmark NK case? The endogeneity of the natural rate of interest explains this. The zero bound policy in the deleveraging model speeds up the deleveraging and mitigates the fall in the natural rate of interest (as shown in the right-bottom panel) resulting in a more expansionary policy once evaluated in terms of output and inflation. There are no such feedback effects between policy and the natural rate in the New-Keynesian model.

The difference in optimal policy is partially explained by the endogenous feedback between policy and the natural rate of interest under debt deleveraging, and partially by the different objective functions the government has in the two models. Figure 12 assumes that policy is set using the same objective: as already noted our more general model has an objective that coincides with the standard one when $\lambda_c = 0$ in (29) (in which case the policymaker does not care about the distribution of income across the two agents). As we can see, optimal policy (but with the same objective as the standard NK model, the line "Optimal Policy $\lambda_c = 0$") yields more similar dynamics to the standard NK model (the line "Optimal Policy NK") with three important differences remaining. First, both inflation and output overshoot their long term value earlier than in the standard case (and before the zero bound stops being binding). Second, we see that the optimal policy has the effect of increasing the natural rate of interest above the level we feed exogenously into the NK model (in the latter it corresponds to the green-dotted line in the panel on the natural rate). Third, and this is related to the second point, we see that the optimal policy now prescribes a substantially shorter duration of the zero interest rate than in the NK model while achieving a similar pattern of inflation and output. The reason for this last point is not that the policy is less aggressive. Instead – it is again because it is successful in endogenously raising the natural rate of interest and generating an output boom and inflation that the liftoff of rates is now earlier than it was in the absence of the policy easing, a point we return to in Section 5.3.

An important conclusion that emerges from the analysis, then, is that incorporating heterogeneity between borrowers and savers has important implication for optimal policy due to its welfare evaluation. In particular inflation policy becomes more attractive than in the standard model. This stems from the fact that borrowers are suffering more than savers in a debt deleveraging cycle, and hence their marginal utility is higher, and from the fact that the government cares about how the cost of the recession is shared across agents. The utility of borrowers, in turn, can be improved with higher inflation relative to the standard model. Even if it comes at the expense of the savers, a government maximizing welfare of the form (28) will pursue stronger inflation policy. In our model the labor market is perfectly flexible so that one way in which the borrower can react to the shock is by increasing labor supply. With more realistic frictions it is likely that the borrowers ability to deleverage by increasing labor supply significantly decreases, thus making the case for inflation even stronger.

One interesting implication of our results is that our model provides theoretical rationale for inflation policy based upon special consideration for borrowers, beyond the traditional case made in the modern ZLB literature. The improved welfare of borrowers as a consequence of inflation was in fact a key rationale for the inflationary policy pursued by the US government during the Great Depression in 1933 (see Eggertsson, 2008).

5.2 Real versus nominal debt

A common rationale for inflation policy during the Great Depression, was the fact that debt was contracted in nominal terms, and thus its real value had increased as a result of the deflation in 1929-33. Inflation, in contrast, would depreciate the real value of this debt back to its original level if aggressive enough. While this provides one rationale for inflation, there are two other forces in our model that work in the same direction. Inflation also reduces the real interest rate of debt being rolled forward, moreover, it stimulates output thus increasing the income of the borrowers. Both effects will improve the ability of the borrowers to repay the debt. Which effect is stronger? To disentangle the importance of these channels we can abstract from the fact that the debt is contracted in nominal terms and instead index it to inflation. Figure 13 shows that the differences between the two cases is quite marginal with inflation and output a bit lower when debt is real. This suggests that in our example inflation is desirable mainly because it supports income and reduces real interest rate on debt rolled forward. Hence from a quantitative point of view, the inflation policy just shown is not being delivered so as to depreciate the real value of debt of the borrowers via unexpected inflation.

It is worth noting, however, that a key abstraction of our model is that all debt is one

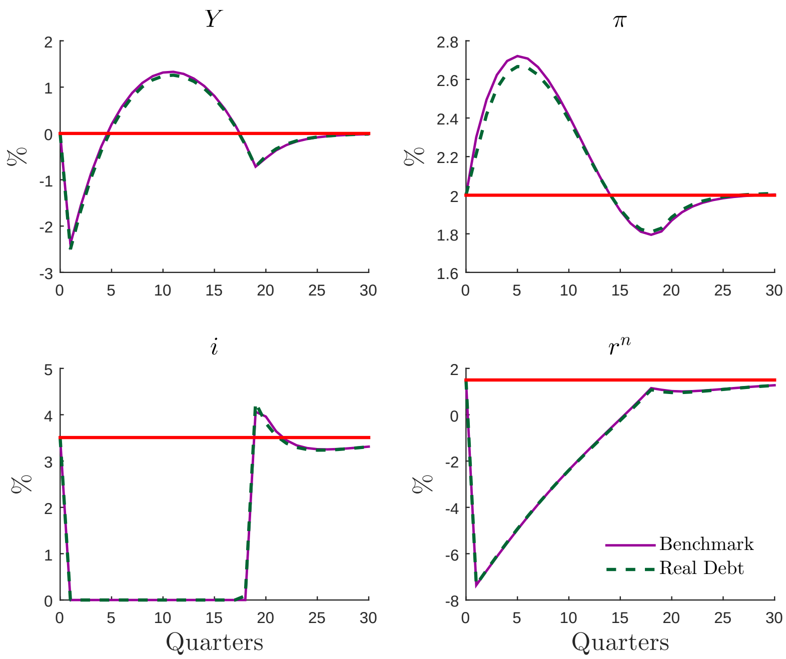


Figure 13: Responses following a deleveraging shock under optimal monetary policy with commitment when debt is nominal (line "Benchmark") in comparison to the case in which debt is real (line "Real Debt") taking in consideration the zero-lower bound. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

period debt. An interesting extension is to consider how the results are affected by instead introducing long term debt, which is better consistent with the data. While this may provide stronger incentives for the government to inflate –to generate redistribution– it will also lessen the government incentive to keep the short-term real interest rate low on freshly issued debt (as much of the interest rates are predetermined). Which effect is quantitatively stronger, in a more general quantitative model, remains to be seen.

5.3 A special case of earlier liftoff under optimal policy than inflation targeting

An important result we documented is that optimal policy implies a shorter duration at the ZLB when the natural rate of interest is endogenous, than in the standard model (see Section 5.1). This is because optimal policy leads to a faster normalization in the natural rate of interest than otherwise would have occurred. One of the key lessons from the standard literature is that the optimal commitment typically implies a longer duration at the ZLB than if policy was set to hit the inflation target of the central bank as soon as the ZLB is no longer binding. This result provided one rationale for several central banks announcing that they would keep short rates low beyond where people had expected them prior to those announcements. This has been often referred to as "forward guidance". In some cases, for example, central banks explicitly tied themselves not to raise rates until some calendar date far into the future (e.g. the Bank of Canada). The objective was to increase demand.

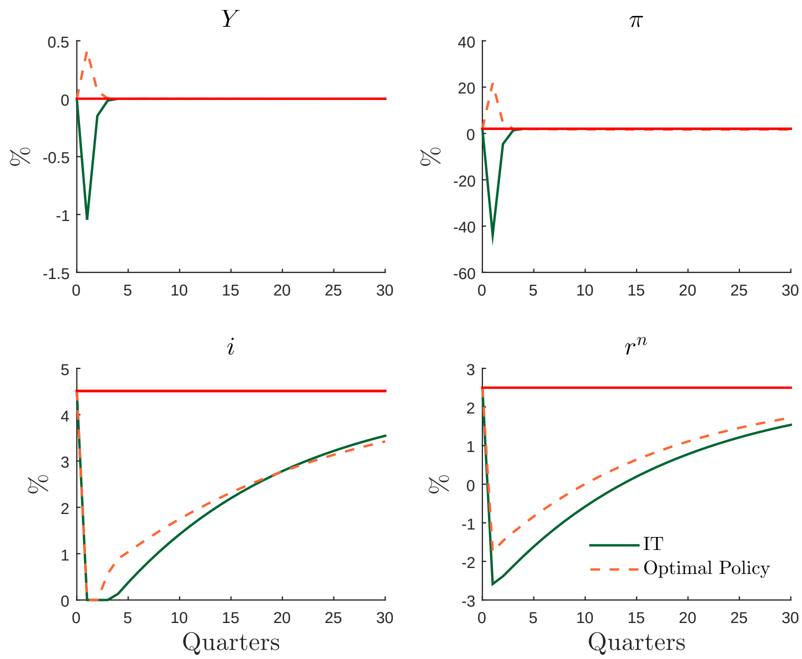


Figure 14: Responses following a deleveraging shock under optimal monetary policy with commitment (line "Optimal Policy") in comparison to inflation-targeting policy (line "IT") taking in consideration the zero-lower bound for the special parametrization of footnote 5.3. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

One interesting aspect of our theory is that optimal policy may in principle no longer prescribe a longer duration at the ZLB. This, in turn, might in principle suggest that "forward guidance" in terms of longer duration of short-term nominal rates at zero is counterproductive.

While we have been unable to provide to settle this question, numerical experiments suggest that we need relatively extreme parameter configuration for the optimal commitment to imply shorter duration at the ZLB than an simple inflation target. In particular one needs to assume very high degree of price flexibility. Figure 14 shows one such example.[21] The Figure shows that deflation under the inflation-targeting policy is quite deep and that the substantial increase in inflation under optimal policy is key to reduce the costs of deleveraging and contain significantly the fall in the natural rate of interest. This makes possible an earlier liftoff for optimal policy than under inflation targeting. It remains to be seen if such examples can be constructed under less extreme parameter configurations, but our theory of endogenous natural rate of interest at least provides for this as a theoretical possibility, unlike the standard model.

[21]The parametrization used in Figure 14 is: $\kappa = 10$, $\beta = 0.9938$, $\varphi = 0.0055$, $\upsilon = 0.0011$, $\bar{b}^{high} = 0.2601$ corresponding to a debt-to-GDP ratio of 10%, $\bar{b}^{low} = 0.0260$ corresponding to a debt-to-GDP ratio of 1%. All the other parameters are as in Table 1.

6 Government Spending

Monetary policy works through a commitment to actions in the future that may be dynamically inconsistent. Accordingly, many are skeptical about the extent to which it has an impact which, in any event, is highly dependent on the credibility of the central bank. A policy that is not subject to this problem to the same extent is fiscal policy, because it involves taking direct actions today. Moreover, it has been shown to be highly effective in the standard model when monetary policy is not capable of generating the appropriate commitment, with very high multipliers of government spending, see e.g. Eggertsson (2011) and Christiano et al (2012). It is of interest to see how endogenous debt deleveraging affect these conclusions. Here we consider a simple experiment in which monetary policy is constrained by targeting the inflation target of two percent, while we model fiscal policy as being able to react directly to the shock. We define G the public expenditure which now enters aggregate demand

$$Y_t = (1-\chi)C_t^s + \chi C_t^b + G_t.$$

We assume that public expenditure is financed with lump-sum taxes. In particular we set the following distribution of taxes between borrowers and savers

$$T_t^b = \frac{\omega}{\chi} G_t + \tau P_t Y_t$$

$$T_t^s = \frac{1-\omega}{1-\chi} G_t + \tau P_t Y_t$$

implying the government budget constraint

$$(1-\chi)T_t^s + \chi T_t^b = G_t + \tau P_t Y_t.$$

The parameter ω, with $0 < \omega < 1$ determines who is paying for public spending. When $\omega = 0$, the savers pay. When $\omega = 1$, the borrowers pay while when $\omega = \chi$ all pay in equal shares. There are a few changes to account for in our model, given the above specification, which are detailed in the Appendix.

We repeat the experiment of Section 4.4 where our model and the standard NK model are aligned in implying the same responses under inflation targeting assuming also the zero-lower bound. In this environment, we study the effects of an increase of government expenditure in both models. In particular we set $G_t = \psi Y_t$ and calibrate ψ, eventually with different values in the two models, in a way that the response of public spending in the first period is 3% in both models. Given this endogenous and same impulse of fiscal policy, Figure 15 shows the responses of output, inflation, nominal interest rate and the natural rate with and without public spending in the two models. In Figure 15, for the deleveraging model, we assume that public spending is financed equally across savers and borrowers, i.e. $\omega = \chi$. In Figure 16, we repeat the experiment for only the deleveraging model when $\omega = 0, \chi$ or 1.[22]

Looking at the first-period response of output, we see that in both models, by construction, output drops by 6.79%. With a 3% increase in public spending in the first period, output drops by 3.15% in our model, when assuming that financing of public spending is

[22] It should be noted that the value of ψ depends on the different assumptions on ω.

equally shared across borrowers and savers ($\omega = \chi$). This implies a first-period multiplier equal to 1.21. In the standard New-Keynesian model, the drop in output with a 3% increase in public spending is 3.62% implying a multiplier of 1.05. A model in which the dynamic of the natural rate of interest is endogenous and agents pay for spending in equal shares thus implies a larger multiplier than in the standard model. However, if considering a different redistribution of taxes in our model, we obtain a drop in output of only 1.76% if savers finance it all, with a larger multiplier of 1.67. Instead, if taxes are levied on borrowers, output drops by more, 4.58%, implying multiplier lower than one, 0.736.

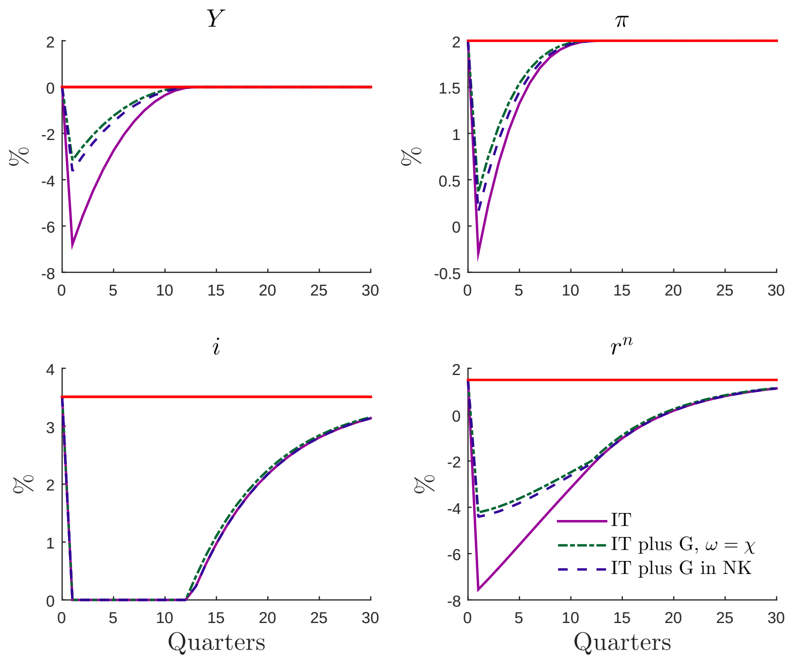


Figure 15: Comparison between the responses to a deleveraging shock with and without public spending. Line "IT": inflation-targeting policy in the model of this paper without public spending which coincides with the same policy under the benchmark NK model. Line "IT plus G, $\omega = \chi$": inflation-targeting policy in the model of this paper with public spending and equal financing across agents ($\omega = \chi$). Line "IT plus G in NK": inflation-targeting policy in the NK model with public spending. Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

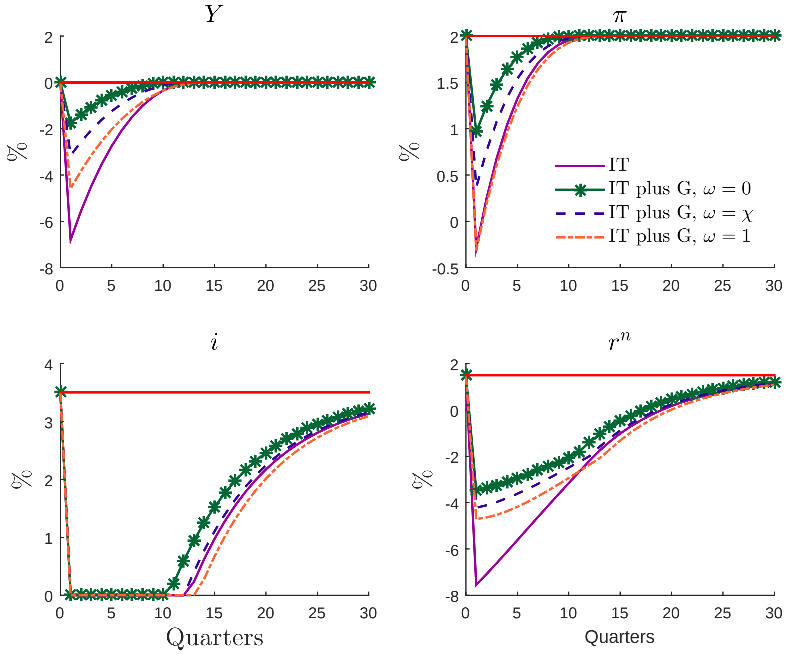


Figure 16: Comparison between the responses to a deleveraging shock with and without public spending. Line "IT": inflation-targeting policy in the model of this paper without public spending. Line "IT plus G, ($\omega = 0$)": inflation-targeting policy in the model of this paper with public spending and financing all on savers ($\omega = 0$). Line "IT plus G, ($\omega = \chi$)": inflation-targeting policy in the model of this paper with public spending and equal financing across savers and borrowers savers ($\omega = \chi$). Line "IT plus G, ($\omega = 1$)": inflation-targeting policy in the model of this paper with public spending and financing all on borrowers ($\omega = 1$). Variables are: output (Y), inflation rate (π), nominal interest rate on savings (i), natural rate of interest defined as in (24) (r^n). Y is in percentage deviation with respect to the steady state; π, i and r^n are in percent and at annual rates.

7 Conclusions

In this paper we have extended the standard New Keynesian model to take into account dynamic deleveraging. In doing so we provide a relatively general framework which we hope will be useful for further applications. We kept the analysis as simple as possible to provide a workhorse post-crisis model.

We have largely limited our focus to deleveraging shocks at the household side but we have also remarked on the isomorphism with credit shocks capturing changes in the degree of leverage of intermediaries or other financial constraints. The analysis can be extended to study other sources of disturbances, like the more standard productivity and cost-push shocks.

One main extension, as we see it, is to take the framework we develop and enlarge it into a medium scale DSGE model that can be estimated. We have chosen not to do so here, in order to obtain a tractable model that allows sharp analytic predictions about optimal policy and clearly generalizes the existing literature on the zero bound. We hope future research takes this analysis one step further into a fully estimated model, e.g., along the lines recently pursued by Justiniano et al. (2014).

Finally, there could be many applications of the approach developed here to open economies or currency areas, to study the endogeneity of a country's deleveraging embedded in an international transmission mechanism. Benigno and Romei (2014), Bhattarai et al. (2015) and Fornaro (2014) are examples in this direction.

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A Details of the general model of section 2.3

Households choose consumption and working hours to maximize utility (11) under the flow budget constraint (12) and an appropriate borrowing-limit condition. As outlined in Section 2.1 we now make the simplifying assumption that $\beta^s \to \beta^b = \beta$, in which case borrowing and lending are still well defined but determined by initial conditions.

The Euler's equation of savers implies

$$U_c(C_t^s) = \beta(1+i_t)E_t\left\{U_c(C_{t+1}^s)\frac{P_t}{P_{t+1}}\right\}. \tag{A.1}$$

Borrowers are not price takers with respect to the borrowing cost since they understand that it will be affected by their individual debt decision. For each individual j belonging to the class of borrowers the following Euler equation can be derived:

$$U_c(C_t^j) = \beta\frac{(1+i_t^j)}{1-\tilde{\epsilon}\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)}E_t\left\{U_c(C_{t+1}^j)\frac{P_t}{P_{t+1}}\right\}, \tag{A.2}$$

where the function $\tilde{\epsilon}\left(\cdot;\cdot,\cdot\right)$ captures the elasticity of the premium with respect to individual real debt and is defined by

$$\tilde{\epsilon}\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right) \equiv \frac{b_t^j}{\bar{b}_t}\frac{\tilde{\phi}_{b^j}\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)}{\tilde{\phi}\left(\frac{b_t^j}{\bar{b}_t}, \frac{b_t}{\bar{b}_t}, \zeta_t\right)}.$$

In equilibrium, borrowers are identical and choose the same level of debt $b_t^j = b_t$. The Euler equation (A.2) can be simplified to

$$U_c(C_t^b) = \beta\frac{(1+i_t^b)}{1-\epsilon\left(\frac{b_t}{\bar{b}_t}, \zeta_t\right)}E_t\left\{U_c(C_{t+1}^b)\frac{P_t}{P_{t+1}}\right\}, \tag{A.3}$$

where $\epsilon\left(b_t/\bar{b}_t, \zeta_t\right) \equiv \tilde{\epsilon}\left(b_t/\bar{b}_t, b_t/\bar{b}_t, \zeta_t\right)$. In the same way, the relationship between borrowing and saving rates can be written as

$$(1+i_t^b) = (1+i_t)\cdot\phi\left(\frac{b_t}{\bar{b}_t}, \zeta_t\right)$$

where $(1+i_t^j) = (1+i_t^b)$ for each j belonging to the mass of borrowers and where we have further defined $\phi(b_t/\bar{b}_t, \zeta_t) \equiv \tilde{\phi}(b_t/\bar{b}_t, b_t/\bar{b}_t, \zeta_t)$.

The optimal supply of labor implies that the marginal rate of substitution between labor and consumption is equated to the real wage

$$\frac{V_l(L_t^j)}{U_c(C_t^j)} = \frac{W_t^j}{P_t}. \tag{A.4}$$

for each agent j.

On the production side, we assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor, $Y(i) = L(i)$. Here we make another key simplifying assumption.[23] We assume that production is a Cobb-Douglas indexes of the two types of labor as $L(i) = (L^s(i))^{1-\chi}(L^b(i))^{\chi}$. Given this technology, this implies that labor compensation for each type of worker is equal to total compensation $W_j L_j = WL$ where the aggregate wage index is appropriately defined by $W = (W^s)^{1-\chi}(W^b)^{\chi}$. This structure will greatly facilitate the aggregation of the model.

Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta} Y$ where aggregate output is

$$Y_t = (1-\chi)C_t^s + \chi C_t^b.$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1-\alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price which is going to apply at a generic future period T with a probability α^{T-t}. Furthermore this price is going to be indexed to the inflation target over the period given by Π^{T-t}. Adjusting firms choose prices to maximize the presented discounted value of the profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[(1+\tau)\Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i) \right]$$

where λ_t is a linear combination of the marginal utilities of real income of the two agents, $\lambda_t = [(1-\chi)U_c(C_t^s) + \chi U_c(C_t^b)]$, which is used to evaluate profits, since these are risk-shared across agents. Moreover τ is a constant subsidy on firms' revenues. The first-order condition of the optimal pricing problem implies

$$\frac{P_t^*}{P_t} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta-1} Y_T \right\}} \tag{A.5}$$

where $\mu \equiv \theta/[(\theta-1)(1+\tau)]$ and in equilibrium $P_t(i) = P_t^*$ since all firms adjusting their prices fix it at the same price. The remaining fraction α of firms, not chosen to adjust their prices, indexes their previously adjusted prices to the inflation target $\bar{\Pi}$. Calvo's model further implies the following law of motion for the general price index

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \Pi^{1-\theta}. \tag{A.6}$$

We assume that utility from consumption is exponential $u(C^j) = 1 - \exp(-zC^j)$ for some positive parameter z while disutility of working is isoelastic $v(L^j) = (L^j)^{1+\eta}/(1+\eta)$. These are convenient assumptions for aggregation and tractability purposes. We can see this by taking a weighted average of (A.4), for $j = s, b$, with weights $1-\chi$ and χ, to obtain

$$\frac{L_t^{\eta}}{z \exp(-zY_t)} = \frac{W_t}{P_t}, \tag{A.7}$$

[23] This assumption makes our model a bit simpler than, for example, Curdia and Woodford (2011) and Eggertsson and Krugman (2012). In the latter work there is a labor supply effect of deleveraging which this assumption allows us to abstract from.

where aggregate output and labor are related through $Y_t \Delta_t = L_t$ and Δ_t is an index of price dispersion defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di,$$

which follows the law of motion

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta} \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}. \tag{A.8}$$

To complete the characterization of the model we specify fiscal policy and assume that

$$T_t^j = \tau P_t Y_t$$

for each agent j implying the government budget constraint

$$(1-\chi)T_t^s + \chi T_t^b = \tau P_t Y_t.$$

The model is closed with the specification of monetary policy.

A.1 Equilibrium conditions: A summary

Here, we describe the equilibrium conditions of our model in a more synthetic way. On the demand side, Euler equations of savers and borrowers are

$$U_c(C_t^s) = \beta(1+i_t)E_t\left\{U_c(C_{t+1}^s)\frac{1}{\Pi_{t+1}}\right\}, \tag{A.9}$$

$$U_c(C_t^b) = \beta\frac{(1+i_t^b)}{1-\epsilon\left(\frac{b_t}{\bar{b}_t},\zeta_t\right)}E_t\left\{U_c(C_{t+1}^b)\frac{1}{\Pi_{t+1}}\right\}, \tag{A.10}$$

where $\Pi_t \equiv P_t/P_{t-1}$.

Borrowing and saving rates are related through

$$(1+i_t^b) = (1+i_t)\cdot\phi\left(\frac{b_t}{\bar{b}_t},\zeta_t\right). \tag{A.11}$$

The dynamic of borrowing is described by the flow budget constraint of the borrowers

$$\frac{b_t}{1+i_t^b} = \frac{b_{t-1}}{\Pi_t} + C_t^b - Y_t \tag{A.12}$$

which follows from (12) where we have substituted in (15) and firms' profits, given by $\Psi_t^j = (1+\tau)P_tY_t - W_tL_t$ noting that $W_tL_t = W_t^jL_t^j$. Moreover, we have set $\Gamma_t^j = 0$ since intermediaries are held only by savers.[24]

[24]It should be noted that if profits of intermediation were also rebated to the borrowers, the relevant interest rate in (A.12) would be an appropriately weighted average of borrowing and saving rates.

Goods market equilibrium connects borrowers' and savers' consumption to real output

$$Y_t = (1-\chi)C_t^s + \chi C_t^b. \tag{A.13}$$

The supply side of the model is characterized by the standard New-Keynesian aggregate-supply equation, written in a recursive form, obtained by combining equations (A.5), (A.6), (A.7) together with $Y_t = \Delta_t L_t$

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t}, \tag{A.14}$$

where F_t and K_t satisfy:

$$F_t = \lambda_t Y_t + \alpha\beta E_t\left\{F_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1}\right\}, \tag{A.15}$$

$$K_t = \mu\frac{\lambda_t \Delta_t^\eta Y_t^{1+\eta}}{z\exp(-zY_t)} + \alpha\beta E_t\left\{K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta}\right\}, \tag{A.16}$$

in which

$$\lambda_t = z[(1-\chi)\exp(-zC_t^s) + \chi\exp(-zC_t^b)], \tag{A.17}$$

and

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^\theta \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}. \tag{A.18}$$

The above set of 10 equations (A.9) to (A.18) determines the equilibrium allocation for the following stochastic processes of 11 endogenous variables $\{C_t^b, C_t^s, i_t, i_t^b, b_t,\ Y_t, \Pi_t, F_t, K_t, \lambda_t, \Delta_t\}_{t=t_0}^\infty$ given initial condition on b_{t_0-1} and Δ_{t_0-1} together with a policy rule and for given exogenous sequence $\{\bar{b}_t, \zeta_t\}_{t=t_0}^\infty$ considering the zero lower bound on the nominal interest rate $i_t \geq 0$.

A.2 Steady State

Of particular importance is the steady state implied by the above equilibrium conditions, since we are approximating our model through log-linear approximations. We consider an initial steady state in which $\bar{b}_t = \bar{b}^{high}$, $\zeta_t = \zeta$ and monetary policy sets inflation rate to the target $\Pi_t = \Pi$. It clearly follows from (A.18) that $\Delta_t = 1$. In this steady state, the Euler equations of the savers, (A.9), and borrowers, (A.10), imply, respectively, that

$$(1+i) = \beta^{-1}\Pi, \tag{A.19}$$

and

$$(1+i^b) = \beta^{-1}\Pi\left(1-\epsilon\left(\frac{b}{\bar{b}^{high}}, \zeta\right)\right) \tag{A.20}$$

while the borrowing premium is given by

$$\frac{(1+i^b)}{(1+i)} = \phi\left(\frac{b}{\bar{b}^{high}}, \zeta\right), \tag{A.21}$$

following equation (A.11).

Combining (A.19) to (A.21) we get

$$\frac{\phi\left(\frac{b}{\bar{b}^{high}},\zeta\right)}{1-\epsilon\left(\frac{b}{\bar{b}^{high}},\zeta\right)}=1$$

which implicitly defines the level of debt b, for each borrower, with respect to the risk-free threshold $\bar{b}^{high}$. In particular, under minor restrictions on the functions $\phi(\cdot)$ and $\epsilon(\cdot)$, b can be set equal to $\bar{b}^{high}$ implying that $\phi(\cdot)=1$ so that borrowing and saving rates are equal in the steady state, $i^b=i$, while $\epsilon(\cdot)=0$. In terms of the original function describing the borrowing premium, as shown in (13), these results are consistent with the assumptions already made that $\tilde{\phi}(1,1,\zeta)=1$ and $\tilde{\phi}_{b^j}(1,1,\zeta)=0$, where the latter captures the fact that for each single borrower a change in their debt position with respect to the risk-free threshold has zero marginal effect on the premium.

Having determined the steady-state level of debt, we obtain the consumption of each borrower from (A.12)

$$C^b=Y-\frac{(1-\beta)}{\Pi}\bar{b}^{high},$$

while from the aggregate resource constraint (A.13), we obtain consumption of savers

$$C^s=Y+\frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\bar{b}^{high}.$$

Given the policy rule $\Pi_t=\Pi$, the aggregate-supply block of the model, characterized by equations (A.14)–(A.16), implies that steady-state output is determined by

$$\frac{Y^{\eta}}{z\exp(-zY)}=1,$$

where we have also assumed a subsidy on firms' revenues equal to $\tau=1/(\theta-1)$ such that $\mu=1$.

An important implication of our preference specification is that the steady-state level of output is independent of the distribution of wealth, and therefore of the debt deleveraging process. In particular, we are interested in studying the effects of a permanent reduction in $\bar{b}$ from $\bar{b}^{high}$ to $\bar{b}^{low}$. Following this shock consumption of savers and borrowers converge to new levels defined by

$$\bar{C}^b=Y-\frac{(1-\beta)}{\Pi}\bar{b}^{low}, \tag{A.22}$$

$$\bar{C}^s=Y+\frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\bar{b}^{low}. \tag{A.23}$$

B Calibration

To find new observables to calibrate the shock, we rely on our interpretation of the model as being driven by debt deleveraging on the household side, but an alternative would be to look at measures of disturbances in the banking system. As an empirical proxy for the household debt, we build a series of household debt over GDP, using the nominal debt series of Households and Nonprofit Organizations in the Fred website.[25] It is shown in Figure 17. For this figure we illustrate three basic trends, in order to assess what debt level can be considered "reasonable" post crisis. As an empirical proxy for borrowers interest rate we use the Commercial Bank Credit Card Interest Rate. We show the borrowing rate in Figure 3, panel (b). [26]

Figure 17: Plot of US Private Debt over GDP in percent. The Figure also shows trends for different subsamples.

[25]Following Eggertsson, Ferrero and Raffo (2014), we approximate the GDP with the sum of Consumption and Gross Investment from the NIPA tables.

[26]We took the series for the Account Interest Assessed. We chose the Commercial Bank Credit Card Interest Rate series as it features an opposite pattern of cyclicality with respect to Federal Funds Rate after the 2008. This measure of spread was fluctuating around 10%. Since in our model the steady state spread is zero, we demeaned the data spread using the historical mean from 1995 to the first quarter of 2009. To compute the borrowers interest rate we add this demeaned spread to the Federal Funds rate. It is important to underline that having a positive spread in the steady state will not influence our results.

Table 2: Parameters under Benchmark Model

Parameter	Value	Source or Target
Intertemporal elasticity of substitution in consumption	$\sigma = 0.66$	Smets and Wouters (2007)
Inverse of the Frisch elasticity of labor supply	$\eta = 1$	Justiniano et al. (2015)
Slope of the AS equation	$\kappa = 0.02$	Eggertsson and Woodford (2003)
Share of borrowers	$\chi = 0.61$	Justiniano et al. (2015)
Steady-state inflation rate	$\Pi = 1.005$	Corresponding to 2% at annual rates: Justiniano et al. (2015)
Intertemporal discount factor	$\beta = .9963$	Match real interest rate of 1.5%: Domeij and Ellingsen (2015)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Parameter of spread function	$\phi = 0.055$	Match the data
Parameter of the borrowers' Euler equation	$\upsilon = 0.159$	Match the data.
Initial real debt	$\bar{b}^{high} = 4.0869$	Initial debt over GDP at 107.73%
Final real debt	$\bar{b}^{low} = 3.3384$	Final debt over GDP at 88%

The model and data are at a quarterly frequencies. The calibrated parameters, shown in Table 1, are largely standard and taken directly from the literature as cited in the table.[27] Particular to our model are the parameters ϕ and υ which govern the spread function. The main new element of the calibration is the choice of shock which will be a one time reduction in $\bar{b}$ from $\bar{b}^{high}$ to $\bar{b}^{low}$ and the implication this has for the new observables we have introduced. We use the data on debt to discipline the choice of $\bar{b}^{high}$ to $\bar{b}^{low}$. We set the initial debt, $\bar{b}^{high} = 4.0869$ to match the value of the debt over GDP in the second quarter of 2008.[28] We set the final debt, $\bar{b}^{low} = 3.3384$ to match debt over GDP equal to 88%, observed in last quarter of 2015 (when the Federal Reserve increased its benchmark rate). This is also an interesting benchmark for a reason shown in Figure 17. This figure draws a trend line for the increase in the private debt to output ratio estimated for the period 1987-2000. The date 2001 marks the break point at which we see a very rapid increase in debt, a period many have associated with "bubble". The value 88% corresponds, as seen in the figures, to the value of this trend estimated on 1987-2000 data in 2015 which happens to coincide exactly with the observed value of real debt over GDP at that time.[29] While this is only one illustrative we experiment with other values as further discussed below. The two key parameters that are left to be determined are ϕ and υ. These parameters capture the characteristics of the function (13) that determines the discrepancy between borrowing and lending rates and the extent to which households internalize this in their optimizing decisions (which in turn determine the speed of debt deleveraging). Our strategy is to pick these two parameters to match as closely as we can the data in Figure 3 using as a criterion Minimum Mean Square Error of the data relative to the model. This procedure results in $\phi = .055$ and $\upsilon = .159$.[30]

By construction the model matches the data in Figure 3 relatively well since we have chosen ϕ and υ to minimize the distance of the model from the data . Let us now look at what happens to variables we have not tried explicitly to match, feeding the shock into the model, i.e. b^{high} falls to b^{low}. Figure 4 in the text shows the model and data. As empirical measure we use annual percentage change in CPI for inflation and detrended GDP, through HP filter, for the deviation of output from potential. The short term nominal interest rate – i.e. the risk-free rate paid by the saver – is the Federal Funds rate.The duration of the output contraction is about three years, which is similar to our measure of the output gap

[27]Except for, perhaps, the fraction of borrowing and lending where we rely on Justiniano, Primiceri and Tambalotti (2015).

[28]Debt over GDP was equal to 107.73%

[29]We consider private debt over GDP at quarterly frequency from the first quarter of 1952 to the third quarter of 2015. We divide this series into four subsamples: i) from the first quarter of 1952 to the second quarter of 1964, ii) from the third quarter of 1964 to the second quarter of 1984, iii) from the third quarter of 1984 to the fourth quarter of 1986 and iv) from the first quarter of 1987 to the fourth quarter of 2000. We choose these subsamples since they have different linear trends. Excluding the third subsample, we compute the linear trend of all these subsamples. We take the stance to consider the third subsample as a discontinuity jump due to the short time span. Finally, we expand the linear trend of the fouth subsample up to the third quarter of 2015.

[30]We create a grid of ϕ and υ. For every couple we simulate our model, assuming that the central bank targets the inflation and that the nominal interest rate cannot go below zero. We compute the square difference of the deleveraging in our model and in data as well as the square difference of the borrowers' interest rate in the data and in our model. We pick the couple that minimize the sum of these square differences, that is $\phi = .055$ and $\upsilon = .159$.

according to the HP filter, which shows output back at trend around 2012.[31] As noted in the text the key discrepancy between the model and the data is the Federal Funds rate. We have also experimented with increasing the duration of the recession by having $\bar{b}^{low}$ lower. Another natural benchmark, relative to the one we choose, is the value of household debt over GDP in 2001 which was 76.5%. Accordingly, we have re-estimated the values of υ and ϕ. This adjustment does indeed lengthen the duration of the zero-lower bound by about three quarters. While output and inflation moves are of similar order, this parametrization does a little bit worse in terms of matching the spreads and the debt deleveraging which is why we focus on the numerical example considered in the text.

C Derivation of the loss function (28)

In this section we show the derivations of the second-order approximation of the welfare (25). The approximation is taken with respect to an efficient steady state. This efficient steady state maximizes (25) under the resource constraint (26).

At the efficient steady state the following conditions hold

$$
\begin{aligned}
(1-\tilde{\chi})\bar{U}_c^s &= (1-\chi)\bar{\lambda}; \\
\tilde{\chi}\bar{U}_c^b &= \chi\bar{\lambda}; \\
(1-\tilde{\chi})\bar{V}_l^s &= (1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}; \\
\tilde{\chi}\bar{V}_l^b &= \chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}
\end{aligned}
$$

where all upper bars denote steady-state values and $\bar{\lambda}$ is the steady-state value of the lagrange multiplier associated with the constraint (26). Note that the above conditions imply $\bar{U}_c^s/\bar{U}_c^b = (1-\chi)\tilde{\chi}/[\chi(1-\tilde{\chi})]$ so that an appropriately chosen $\tilde{\chi}$ determines the efficient distribution of wealth.

By taking a second-order expansion of the utility flow around the efficient steady-state, we obtain

$$
\begin{aligned}
U_t = {} & \bar{U} + (1-\tilde{\chi})\left[\bar{U}_c^s(C_t^s - \bar{C}^s) + \frac{1}{2}\bar{U}_{cc}^s(C_t^s-\bar{C}^s)^2\right] + \\
& + \tilde{\chi}\left[\bar{U}_c^b(C_t^b - \bar{C}^b) + \frac{1}{2}\bar{U}_{cc}^b(C_t^b-\bar{C}^b)^2\right] + \\
& - (1-\tilde{\chi})\left[\bar{V}_l^s(L_t^s - \bar{L}^s) + \frac{1}{2}\bar{V}_{ll}^s(L_t^s-\bar{L}^s)^2\right] - \\
& - \tilde{\chi}\left[\bar{V}_l^b(L_t^b - \bar{L}^b) + \frac{1}{2}\bar{V}_{ll}^b(L_t^b-\bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3)
\end{aligned}
$$

[31] To be clear, we do not think this is the most reasonable estimate of the output gap, but we use it here since it is very transparent and widely used, and thus helpful for illustrative purposes. We took the series of GDP as previously defined from 1990 till the last data available. We divided it by the CPI and we de-trend it by using the HP filter. Since this series is at quarterly frequency we set the multiplier λ_{HP} equal to 1600.

where an upper-bar variable denotes the efficient steady state while $\mathcal{O}(||\xi||^3)$ collects terms in the expansion which are of order higher than the second. We can use the steady-state conditions to write the above equation as

$$
\begin{aligned}
U_t &= \bar{U} + (1-\chi)\bar{\lambda}\left[(C_t^s - \bar{C}^s) + \frac{1}{2}\frac{\bar{U}_{cc}^s}{\bar{U}_c^s}(C_t^s - \bar{C}^s)^2\right] + \\
&\quad + \chi\bar{\lambda}\left[(C_t^b - \bar{C}^b) + \frac{1}{2}\frac{\bar{U}_{cc}^b}{\bar{U}_c^b}(C_t^b - \bar{C}^b)^2\right] + \\
&\quad -(1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}\left[(L_t^s - \bar{L}^s) + \frac{1}{2}\frac{\bar{V}_{ll}^s}{\bar{V}_l^s}(L_t^s - \bar{L}^s)^2\right] - \\
&\quad -\chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}\left[(L_t^b - \bar{L}^b) + \frac{1}{2}\frac{\bar{V}_{ll}^b}{\bar{V}_l^b}(L_t^b - \bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3).
\end{aligned}
$$

Note that for a generic variable X, we have

$$
X_t = \bar{X}\left(1 + \tilde{X}_t + \frac{1}{2}\tilde{X}_t^2\right) + \mathcal{O}(||\xi||^3)
$$

where $\tilde{X}_t \equiv \ln X_t/\bar{X}$ and moreover recall that

$$
Y_t = \chi C_t^s + (1-\chi)C_t^b.
$$

We can write the above approximation as

$$
\begin{aligned}
U_t &= \bar{U} + \bar{\lambda}\bar{Y}\left[\tilde{Y}_t + \frac{1}{2}\tilde{Y}_t^2\right] - \frac{1}{2}\bar{\lambda}z\left[(1-\chi)(C_t^s - \bar{C}^s)^2 + \chi(C_t^b - \bar{C}^b)^2\right] + \\
&\quad -\chi\bar{\lambda}\bar{Y}\left[\tilde{L}_t^s + \frac{1}{2}(1+\eta)(\tilde{L}_t^s)^2\right] \\
&\quad -(1-\chi)\bar{\lambda}\bar{Y}\left[\tilde{L}_t^b + \frac{1}{2}(1+\eta)(\tilde{L}_t^b)^2\right] + \mathcal{O}(||\xi||^3),
\end{aligned}
\tag{C.24}
$$

where we have also used the fact that with the preference specification used $\bar{U}_{cc}^s/\bar{U}_c^s = \bar{U}_{cc}^b/\bar{U}_c^b = -z$ and $\bar{V}_{ll}^s\bar{L}^s/\bar{V}_l^s = \bar{V}_{ll}^b\bar{L}^b/\bar{V}_l^b = \eta$. Note that the efficient steady state of output is equal also to the intial steady state of output. Therefore, in what follows we can use the fact that $\bar{Y} = Y$ and also clearly $\tilde{Y}_t = \hat{Y}_t$.

Notice that conditions (A.4) and (A.7) imply

$$
\frac{(L_t^s)^{1+\eta}}{z\exp(-zC_t^s)} = \frac{(L_t^b)^{1+\eta}}{z\exp(-zC_t^b)} = \frac{(\Delta_t Y_t)^{1+\eta}}{z\exp(-zY_t)}
$$

where we have used $W_tL_t = W_t^sL_t^s = W_t^bL_t^b$ and $L_t = \Delta_t Y_t$. The above equations imply exactly that

$$
\tilde{L}_t^s = \tilde{\Delta}_t + \hat{Y}_t - \frac{z}{1+\eta}[(C_t^s - \bar{C}^s) - (Y_t - Y)],
$$

$$
\tilde{L}_t^b = \tilde{\Delta}_t + \hat{Y}_t - \frac{z}{1+\eta}[(C_t^b - \bar{C}^b) - (Y_t - Y)].
$$

and therefore that

$$\tilde{L}_t^s = \tilde{\Delta}_t + \hat{Y}_t - \frac{\sigma^{-1}}{1+\eta}(\tilde{C}_t^s - \hat{Y}_t),$$

$$\tilde{L}_t^b = \tilde{\Delta}_t + \hat{Y}_t - \frac{\sigma^{-1}}{1+\eta}(\tilde{C}_t^b - \hat{Y}_t).$$

where $\tilde{C}_t^b \equiv (C_t^b - \bar{C}^b)/Y$ and $\tilde{C}_t^s \equiv (C_t^s - \bar{C}^s)/Y$. Moreover,

$$\tilde{C}_t^s - \hat{Y}_t = -\chi(\tilde{C}_t^b - \tilde{C}_t^s)$$

$$\tilde{C}_t^b - \hat{Y}_t = (1-\chi)(\tilde{C}_t^b - \tilde{C}_t^s)$$

which can be substituted into (C.24) to obtain

$$\begin{aligned} U_t &= \bar{U} - \frac{1}{2}\bar{\lambda}Y\left\{(\eta+\sigma^{-1})\cdot\hat{Y}_t^2 + \chi(1-\chi)\sigma^{-1}(\tilde{C}_t^b - \tilde{C}_t^s)^2 + \right. \\ &\quad \left. +\chi(1-\chi)\frac{\sigma^{-2}}{(1+\eta)}(\tilde{C}_t^b - \tilde{C}_t^s)^2\right\} - \bar{\lambda}Y\cdot\hat{\Delta}_t + \mathcal{O}(||\xi||^3). \end{aligned}$$

Note that

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^\theta \Delta_{t-1} + (1-\alpha)*\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}$$

By taking a second-order approximation of $\hat{\Delta}_t$, as it is standard in the literature and integrating appropriately across time, we obtain that

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{(\pi_t-\pi)^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

We can therefore write

$$W_{t_0} = -\bar{\lambda}(\eta+\sigma^{-1})Y\cdot\frac{1}{2}E_t\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

where

$$L_t = \hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\tilde{C}_t^b - \tilde{C}_t^s)^2 + \lambda_\pi(\pi_t-\pi)^2$$

where we have defined

$$\lambda_c \equiv \frac{\sigma^{-1}(1+\eta)+\sigma^{-2}}{(1+\eta)(\eta+\sigma^{-1})}$$

$$\lambda_\pi \equiv \frac{\theta}{\kappa}.$$

D First-order conditions of optimal policy under commitment

In this section, we characterize the optimal policy problem in details.

Optimal monetary policy under commitment minimizes the loss function

$$L_{t_0} = \frac{1}{2} E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^b - \hat{C}_t^s - c^R)^2 + \lambda_\pi(\pi_t - \pi)^2 \right] \right\} \tag{D.25}$$

where c^R captures the relative difference between the initial and final steady-state consumptions of borrowers and savers defined as $c^R \equiv [(C^b - \bar{C}^b) - (C^s - \bar{C}^s)]/Y$. The minimization constrained by the following set of structural equations of the model:

$$\hat{Y}_t = \chi \hat{C}_t^b + (1-\chi)\hat{C}_t^s \quad (\lambda_1) \tag{D.26}$$

$$E_t \hat{C}_{t+1}^b - \hat{C}_t^b = \sigma[\hat{\imath}_t^b - E_t(\pi_{t+1} - \pi) + \upsilon(\hat{b}_t - \hat{d}_t)] \quad (\lambda_2) \tag{D.27}$$

$$E_t \hat{C}_{t+1}^s - \hat{C}_t^s = \sigma[\hat{\imath}_t^s - E_t(\pi_{t+1} - \pi)] \quad (\lambda_3) \tag{D.28}$$

$$\hat{C}_t^b = \frac{\bar{b}}{(1+i)}(\hat{b}_t - (\hat{\imath}_t^b)) - \frac{\bar{b}}{\beta(1+i)}(\hat{b}_{t-1} - (\pi_t - \pi)) + \hat{Y}_t \quad (\lambda_4) \tag{D.29}$$

$$\hat{\imath}_t^b = \hat{\imath}_t^s + \varphi\left(\hat{b}_t - \hat{d}_t\right) \quad (\lambda_5) \tag{D.30}$$

$$\pi_t - \pi = \kappa \hat{Y}_t + \beta E_t(\pi_{t+1} - \pi) \quad (\lambda_6) \tag{D.31}$$

$$-\hat{\imath}_t^s + \hat{\imath}_{ss,t} \leq 0. \quad (\lambda_7) \tag{D.32}$$

Note that in each of the above equations we have written on the right the respective Lagrange multiplier.

First-order conditions of the optimal policy problem are:

$$\hat{Y}_t: \qquad \hat{Y}_t + \lambda_{1,t} - \lambda_{4,t} - k\lambda_{6,t} = 0 \tag{D.33}$$

$$\hat{C}_t^s: \qquad -(\chi(1-\chi)\lambda_c)\left(\hat{C}_t^b - \hat{C}_t^s - \hat{C}_t^R\right) - (1-\chi)\lambda_{1,t} - \lambda_{3,t} + \frac{\lambda_{3,t-1}}{\beta} \tag{D.34}$$

$$\hat{C}_t^b: \qquad (\chi(1-\chi)\lambda_c)\left(\hat{C}_t^b - \hat{C}_t^s - \hat{C}_t^R\right) - (\chi)\lambda_{1,t} - \lambda_{2,t} + \frac{\lambda_{2,t-1}}{\beta} + \lambda_{4,t} = 0 \tag{D.35}$$

$$\hat{\pi}_t: \qquad \lambda_\pi(\pi_t - \pi) + \sigma\frac{\lambda_{2,t-1}}{\beta} + \sigma\frac{\lambda_{3,t-1}}{\beta} - \frac{\bar{b}}{(1+i)\beta}\lambda_{4,t} + \lambda_{6,t} - \lambda_{6,t-1} = 0 \tag{D.36}$$

$$\hat{\imath}_t^s: \qquad -\lambda_{3,t}\sigma - \lambda_{5,t} - \lambda_{7,t} = 0 \tag{D.37}$$

$$\hat{\imath}_t^b: \qquad -\lambda_{2,t}\sigma + \frac{\bar{b}}{(1+i)}\lambda_{4,t} + \lambda_{5,t} = 0 \tag{D.38}$$

$$\hat{b}_t: \qquad -\frac{\bar{b}}{(1+i)}\lambda_{4,t} + \frac{\bar{b}}{(1+i)}E_t\lambda_{4,t+1} - \phi\lambda_{5,t} - \sigma\upsilon\lambda_{2,t} = 0. \tag{D.39}$$

$$\lambda_{7,t}(-\hat{\imath}^s_t + \hat{\imath}_{ss,t}) = 0. \tag{D.40}$$

The set of first-order conditions together with the equilibrium constraints is solved using a solution method which takes into account the zero lower bound (see also Eggertsson and Woodford, 2003).

E Model with public expenditure

In this section, we discuss in details the extension of Section 6, in which we add public expenditure.

The steady state of consumption for borrowers and savers and output is now defined by the following equations

$$C^s = Y + \frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\bar{b}^{high} - T^s$$

$$C^b = Y - \frac{(1-\beta)}{\Pi}\bar{b}^{high} - T^b$$

$$\frac{Y^\eta}{z\exp(-z(Y-G))} = 1,$$

which can be written as

$$C^s = Y + \frac{(1-\beta)}{\Pi}\frac{\chi}{1-\chi}\tilde{b}Y - \frac{1-\omega}{1-\chi}s_gY$$

$$C^b = Y - \frac{(1-\beta)}{\Pi}\tilde{b}Y - \frac{\omega}{\chi}s_gY.$$

$$\frac{Y^\eta}{z\exp(-z(Y-s_gY))} = 1.$$

where $s_g = G/Y$, $\tilde{b} = \bar{b}^{high}/Y$.

We calibrate the share of public spending over GDP at $s_g = 0.3$. Given the other parameters of Table 1, we can compute the steady-state of the model.

In a log-linear approximation around the steady state, the model can be written through the following set of equations

$$E_t\hat{C}^b_{t+1} = \hat{C}^b_t + \sigma\left[\hat{\imath}^b_t - E_t(\pi_t - \pi) + \lambda\left(\hat{b}_t - \hat{d}_t\right)\right] \tag{E.41}$$

$$E_t\hat{C}^s_{t+1} = \hat{C}^s_t + \sigma(\hat{\imath}_t - E_t\hat{\pi}_{t+1}) \tag{E.42}$$

$$\hat{\imath}^b_t = \hat{\imath}_t + \upsilon(\hat{b}_t - \hat{d}_t) \tag{E.43}$$

$$\hat{Y}_t = \chi\hat{C}^b_t + (1-\chi)\hat{C}^s_t + g_t \tag{E.44}$$

$$\hat{C}^b_t = \frac{\bar{b}}{1+i}(\hat{b}_t - \hat{\imath}^b_t) - \frac{\bar{b}}{\beta(1+i)}(\hat{b}_{t-1} - (\pi_t - \pi)) + \hat{Y}_t - \hat{t}^b_t \tag{E.45}$$

$$\pi_t - \pi = \kappa \left(\hat{Y}_t - \frac{\sigma^{-1}}{\sigma^{-1} + \eta} g_t \right) + \beta E_t(\pi_{t+1} - \pi) \tag{E.46}$$

where

$$g_t = \hat{t}_t \tag{E.47}$$

$$\hat{t}_t^b = \frac{\omega}{\chi} \hat{t}_t \tag{E.48}$$

$$\hat{t}_t^s = \frac{1 - \omega}{1 - \chi} \hat{t}_t \tag{E.49}$$

and we have further defined the following variables

$$g_t = \frac{(G_t - G)}{Y} \quad \hat{t}_t = \frac{T_t - T}{Y}.$$

Note that the parameter ω is between 0 and 1. When $\omega = 0$, the burden of public-spending financing is on the savers. When $\omega = 1$ all the financing is on borrowers while when $\omega = \chi$ is uniform across agents.

In Figures (12) and (14), first we compute the natural rate of interest in the deleveraging model under inflation targeting and assuming that $g_t = 0$. This is given by

$$r_t^n = -\chi(\varphi + \upsilon) \left(\hat{b}_t - \hat{d}_t \right). \tag{E.50}$$

We then input the same r_t^n in the following NK model with public spending

$$(\pi_{t+1} - \pi) = \kappa \left(\hat{Y}_t - \frac{\sigma^{-1}}{\sigma^{-1} + \eta} g_t \right) + \beta E_t(\pi_{t+1} - \pi) \tag{E.51}$$

$$\hat{Y}_{t+1} - g_{t+1} = \hat{Y}_t - g_t + \sigma \left[\hat{\imath}_t - E_t \pi_{t+1} - r_t^n \right] \tag{E.52}$$

Given $g_t = 0$, the inflation-targeting policy of our model under zero-lower bound is equivalent to the inflation-targeting in the NK model under zero-lower bound, for the path of inflation, output and nominal interest rate. This is shown in Figure (12) with the lines IT and IT in NK.

We then assume the following process for g_t

$$g_t = -\psi \hat{Y}_t.$$

and consider a deleveraging shock of the same magnitude as before, again in the case in which the central bank targets inflation and hits the zero bound. Considering the same deleveraging shock and same r_t^n computed above, we set the parameter ψ in both models in such a way that the initial response of g_t is 3% in the first period in both models. This means eventually that ψ is chosen in a different way in both models. In particular we repeat this experiment for the three different cases, i.e. when $\omega = 0, \chi, 1$, in the deleveraging model, implying therefore different choices of ψ.

Finally, it should be noted that when $g_t \neq 0$ the natural rate of interest is computed as

$$\tilde{r}_t^n = -\chi(\varphi + \upsilon) \left(\hat{b}_t - \hat{d}_t \right) - \sigma^{-1}(E_t g_{t+1} - g_t)$$

in the deleveraging model while is given by

$$r_t^{\prime n} = r_t^n - \sigma^{-1}(E_t g_{t+1} - g_t)$$

in the NK model.