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International Portfolio Allocation under Model Uncertainty†

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This paper revisits an old argument, hedging real exchange rate risk, as an explanation of the international home bias in equity. In a dynamic model, the relevant risk to be hedged is the long-run risk as opposed to the short-run risk. Domestic equity is indeed a good hedge with respect to long-run real-exchange-rate risk. Two new frameworks are able to explain a large share of the observed US home bias: a model with Hansen-Sargent preferences in which agents fear model misspecification and a model with Epstein-Zin preferences. These two models are also immune to the risk-free rate puzzle. (*JEL* C58, F31, G11, G15)

The lack of international diversification in equity portfolios is one of the most persistent observations in international finance. Investors hold a large share of their wealth in domestic securities, more than what would be dictated by the share of these securities in the world market. This is known as the "the home-bias puzzle" (French and Poterba 1991, Tesar and Werner 1995).[1]

We address this puzzle by revisiting an old and popular argument, dating back to the work of Adler and Dumas (1983), based on hedging real exchange rate (RER) risk. Recently, this explanation has been dismissed for two reasons which both point against the ability of the model to match the data. On the one side, it has been argued that the risk to be hedged is quantitatively too small to explain a significant

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[1] An incomplete list of papers that have tried to address the home-bias puzzle are: Benigno and Kücük-Tuger (2008); Bottazzi et al. (1996); Coeurdacier (2009); Coeurdacier and Gourinchas (2009); Coeurdacier, Kollmann, and Martin (2008); Cole and Obstfeld (1991); Engel and Matsumoto (2009); Epstein and Miao (2003); Heathcote and Perri (2009); Kollmann (2006); Obstfeld and Rogoff (2001); Pavlova and Rigobon (2007); Uppal (1993); Van Nieuwerburgh and Veldkamp (2009).

departure from full international diversification. Indeed, the relevant covariance between real exchange rate and the cross-country excess return in the equity market is negligible once taken conditional on the returns on other assets traded (see van Wincoop and Warnock 2010). On the other side, the explanatory power of the model can increase by raising the coefficient of risk aversion. However, with standard isoelastic preferences, high values of the risk aversion coefficient are linked to low values of the intertemporal elasticity of substitution, and therefore the model might produce counterfactual implications for the mean of the risk-free rate—the so-called risk-free rate puzzle (Weil 1989).

In this work we revive hedging RER as a relevant channel by providing two new arguments. First, we stress that what is really important is the medium- to long-run real exchange rate risk as opposed to the short-run risk emphasized by the existing literature. At longer horizons, we show that hedging RER risk is empirically relevant, even conditional on other excess returns, and equities are a good hedge for this risk. This is key for our model to succeed in explaining the home-bias puzzle. Second, we show that there are three models which share the same implications for the steady-state international portfolio allocation conditionally to the same processes for the risks to be hedged and for the excess returns on the asset traded.[2] The three models differ in the specification of preferences: (i) isoelastic expected-utility, (ii) Hansen-Sargent, and (iii) Epstein-Zin preferences. As discussed above, model (i) is in general subject to the risk-free rate puzzle, while the two alternative frameworks present a disconnection between the parameter measuring risk aversion and the intertemporal elasticity of substitution. In this way, they can be immune to the puzzle.

In particular, we show that under models (ii) and (iii), RER risk is still relevant even when the intertemporal elasticity of substitution is unitary in contrast to model (i). Specifically, in model (ii), building on Hansen and Sargent (2005, 2007), agents have doubts about the true probability distribution and are averse to these doubts. These agents have unitary elasticity of intertemporal substitution and unitary risk aversion, and the relevance of RER risk is related to the degree of ambiguity aversion. With Epstein and Zin (1989) preferences, the elasticity of intertemporal substitution is decoupled from the degree of risk aversion, which depends here on the temporal resolution of uncertainty and governs the relevance of the RER risk in this latter model.

The three models entail different reasons for why real exchange rate risk is relevant. Agents enter financial markets with different appetites for state-contingent wealth, and asset trading provides a way to reduce such differences.

With standard isoelastic preferences, the appetite for wealth is driven by the marginal utility of consumption and indeed asset trading helps to reduce the idiosyncratic movements in the marginal utilities across countries. In this case and under a non-unitary intertemporal elasticity of substitution, real exchange rate

[2] Our approach differs from most of the existing literature which finds portfolio shares as a function of primitive parameters, like the risk-aversion coefficient, the share of traded goods, or the trade cost. This is clearly a desirable feature of general equilibrium models, but it has the drawback of hiding the hedging relationships based on observable variables that are at the root of the portfolio decisions. van Wincoop and Warnock (2010) show that the covariances between the asset returns and the sources of risk implied by these models are often counterfactual; once data restrictions on asset prices are considered, these models fail to solve the portfolio home-bias puzzle.

movements produce idiosyncratic variations in the marginal utilities of real wealth that need to be hedged.

In the model with doubts and ambiguity aversion, differences in the appetite for wealth can also be driven by differences in the beliefs. In particular, with Hansen-Sargent preferences, beliefs depend on the worst-case scenario from the point of view of utility, which, in our model, depends only on consumption. It follows that ambiguity-averse agents are sensitive to the news regarding their consumption profiles. Therefore, cross-country variations in beliefs translate into news about the cross-country differences in consumption growth. The real exchange rate is a source of fluctuations for relative consumption growth and, through this channel, plays a role in the model.

Finally, under Epstein-Zin preferences, the stochastic discount factor includes an additional driver of the appetite of wealth which depends on the temporal resolution of uncertainty. Cross-country differences in this component also map into real exchange rate risk. In these three models (although for different reasons, as explained above), an appreciation of the home-country real exchange rate, i.e., an increase in the domestic price level relative to the foreign one expressed in units of domestic currency, raises the appetite for state-contingent wealth for the domestic agent relative to the foreign one. If at the same time the excess return of domestic over foreign equity increases, then domestic equity represents a good hedge with respect to the movements in the real exchange rate and domestic agents would like to hold more of this asset because it pays well when needed. Another finding of our work is the empirical result that nondiversifiable labor-income risk is not sufficient to explain asset home bias, confirming the previous work of Baxter and Jermann (1997) and in contrast to recent findings of Coeurdacier and Gourinchas (2009). The latter assumes that cross-country excess returns on equity are a good proxy of the cross country returns on domestic human wealth. We do not make this assumption and therefore we find that labor-income risk is not much correlated with excess returns on the equity market.

The structure of this paper is the following. The next section gives the main intuition of the results and discusses more extensively the contribution of the paper with respect to related literature. Section II presents and discusses the empirical evidence supporting our main results. Section III presents the theoretical discussion, presenting the three competing models and contrasting the relative implications for optimal portfolio allocation.

I. Main Results, Intuition, and Related Literature

In this section, we describe the main results of the paper, provide their intuition, and discuss the comparisons with related literature. We present three alternative models implying the same optimal international portfolio allocation: (i) a standard expected-utility model with isoelastic consumption utility and constant relative risk aversion; (ii) a model with log-utility in consumption, where agents face and fear model uncertainty as in the framework of Hansen and Sargent (2005, 2007); and (iii) a model with unitary intertemporal elasticity of substitution and nonexpected utility as in Epstein and Zin (1989) and Kreps and Porteus (1978). This paper argues that the three models can explain a good portion of the international portfolio diversification puzzle. However, model (i) is less desirable because it relies on implausibly low values for the intertemporal elasticity of substitution, thereby implying a too high real risk-free rate.

Under the three models considered, departures from full international portfolio diversification arise for two classical reasons which have been extensively investigated in the literature: hedging labor-income risk, on the one side; and hedging real-exchange-rate risk, on the other side. This paper shows that the three models share the same equation determining the steady-state portfolio allocation as it follows:[3]

$$\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{\lambda}}^{full} - s_{\xi}\frac{\beta}{1-\beta}\Sigma_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{l,t+1}) - s_c\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\Sigma_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{q,t+1}), \tag{1}$$

where $\bar{\boldsymbol{\lambda}}$ is a vector capturing the cross-country differences in portfolio holdings and $\bar{\boldsymbol{\lambda}}^{full}$ represents the steady-state vector of portfolio holdings under full diversification.[4] Details on the underlying models and derivations are presented in Section IV. Deviations from a fully diversified portfolio arise because of the co-movements between the vector of excess returns, $\mathbf{exr}_{t+1}$, and labor-income risk, $\varepsilon_{l,t+1}$, and between the same vector and real-exchange-rate risk, $\varepsilon_{q,t+1}$. In equation (1), s_{ξ} is the steady-state ratio between labor income and financial wealth and s_c is the steady-state ratio between consumption and financial wealth; β is the time discount factor in consumer's preferences; $\boldsymbol{\Sigma}_t$ is the time $-t$ conditional variance-covariance matrix of the vector of excess returns $\mathbf{exr}_{t+1}$; and $E_t(\cdot)$ is the standard expectations operator. Parameter γ is the key parameter of our analysis, and is model specific. It represents the risk-aversion coefficient and at the same time the inverse of the intertemporal elasticity of substitution in the model with standard isoelastic consumption utility (model i), the degree of ambiguity aversion under Hansen-Sargent preferences (model ii), the risk-aversion coefficient under Epstein-Zin preferences (model iii), which is, in this case, decoupled from the inverse of the intertemporal elasticity of substitution (unitary in our case).[5]

Under model (i), equation (1) has been investigated in the literature in one form or another with unsatisfactory results. On the one hand, there is controversy on the sign and magnitude of the comovements between excess returns and labor-income risk. Studies like Baxter and Jermann (1997) have shown that this covariance is

[3] The result of equivalance in the portfolio allocation across the three models holds only for the steady-state portfolio allocations and conditionally on the same processes for excess returns, real exchange rate risk and labor income risk. While models (ii) and (iii) imply the same equilibrium along all the other dimensions of the equilibrium allocation, model (i) has in general different implications.

[4] We consider trade in two equities and two bonds (one for each country) and therefore three excess returns. The vector $\bar{\boldsymbol{\lambda}}^{full}$ has three dimensions and implies zero bond holdings and an equal split of equity between countries. See Section IV for details. In particular $\bar{\boldsymbol{\lambda}}^{full} = \mathbf{0}$.

[5] In model (iii), given unitary intertemporal elasticity of substitution, γ also captures the risk attitude toward the temporal resolution of uncertainty, with $\gamma > 1$ (<1) implying a preference for an early (late) resolution of uncertainty. Notice also that model (ii), Hansen-Sargent preferences, is defined only when $\gamma \geq 1$, whereas the other two models just require $\gamma \geq 0$. However, $\gamma \geq 1$ is the empirically relevant range to consider as will be shown later.

small or even of the wrong sign pointing toward a worsening of the international portfolio diversification puzzle. Others, as Coeurdacier and Gourinchas (2009), have instead argued that it can provide the right comovements and the significant magnitude to explain home bias in portfolio holdings. We contribute to this debate by showing that our model-consistent measure of labor-income risk delivers weak comovements with the relevant excess returns, and therefore it is not sufficient to explain significant departures from a fully diversified portfolio.

What is left in (1) is hedging real exchange rate risk. However, along this dimension, two strong arguments against its relevance have been proposed in the literature. First, when the menu of assets traded internationally is rich enough to include risk-free nominal bonds and equities, home bias in equities arises when the excess return of domestic versus foreign equity covaries in a significant way with the component of the RER risk, which is orthogonal to what is already explained by the excess returns in the international bond market. This conditional covariance turns out to be much smaller than the unconditional one, as shown by van Wincoop and Warnock (2010). The RER risk considered in this case is related to one-period-ahead changes in the real exchange rate.[6] Second, even when the covariance is small, the RER risk component in (1) can be made as large as needed by increasing parameter γ. However, with the standard isoleastic preferences of model (i), the values of γ needed to produce some home-bias in the equity portfolio are not only implausibly high as values measuring risk aversion, but they also produce very low values for the intertemporal elasticity of substitution, and are therefore subject to the risk-free rate puzzle.

In this work, we overturn these two arguments with three new results. First, we show that in a dynamic model $\varepsilon_{q,t+1}$ is not just the short-run real exchange risk on which the literature has focused so far, but instead measures the present discounted value of surprises in the real exchange rate, and therefore captures also the medium-to long-run risk. Second, we show that at medium to long horizons domestic equities still represent a good hedge with respect to RER risk, even conditioning on other excess returns. At the same time we confirm the result of the literature that domestic equities are not a good hedge at short horizons, but this is not what really matters in (1). Finally under models (ii) and (iii), the parameter γ is decoupled from the intertemporal elasticity of substitution and can rise without falling in the risk-free rate puzzle, as is discussed in Barillas, Hansen, and Sargent (2009), among others. Hansen-Sargent preferences are particularly appealing because high values for γ can be justified as plausible or not depending on the model itself rather than relying on ad hoc calibrations, since γ can be related to the probability that agents can detect the difference between the subjective and the reference model. According to this criterion, we find that reasonable values of γ are able to explain a substantial degree of home bias in US equity holdings.

Although the three models share the same implications for the steady-state portfolio allocation, the mechanisms at work behind each model are of a completely different nature to deserve further discussion. A fundamental principle of finance,

[6] Indeed, the few contributions that take the same methodological perspective as ours and focus on the hedging relationships that underlie portfolio choices (such as Coeurdacier and Gourinchas 2009, and van Wincoop and Warnock 2010) typically use static models, which, by construction, neglect any possible source of RER risk related to the medium and long run.

based on the arbitrage theory, is that investors want to hold assets that pay well when needed. A measure of the appetite for wealth is given by the stochastic discount factor (SDF). The higher the stochastic discount factor in a particular state of nature, the higher the appetite for wealth of the agent in that state. Therefore, when agents have different SDF, they might exploit trade in assets in order to hedge such differences. Indeed, when perfect risk-sharing is achieved, the wedge between the SDF across different agents is completely eliminated. In particular, in model (i), perfect risk-sharing requires equalization of the stochastic discount factors evaluated in units of the same consumption index:

$$m_{t+1} = m^*_{t+1} \frac{q_t}{q_{t+1}}, \tag{2}$$

where m_{t+1} is the real SDF for evaluating wealth in states of nature at time $t+1$ for the household of a generic country H; m^*_{t+1} is the respective factor for the household in country F; and q_t is the real exchange rate. In model (i), the stochastic discount factors depend on the marginal utilities of consumption in the respective countries. Given this preference specification, as we show in Section IV, deviations from full risk-sharing can arise from three sources of risks:

$$\ln m_{t+1} + \Delta \ln q_{t+1} - \ln m^*_{t+1} \simeq m_e(\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{\lambda}}^{full})\mathbf{exr}_{t+1} + m_q \varepsilon_{q,t+1} + m_l \varepsilon_{l,t+1}, \tag{3}$$

where the coefficients m_e, m_q, and m_l depend on structural model parameters.[7] Departures from full diversification are optimal when excess returns, $\mathbf{exr}_{t+1}$, display some covariance with the other two sources of risk. In particular, the real exchange rate risk, $\varepsilon_{q,t+1}$, is relevant only as long as the intertemporal elasticity of substitution (and therefore the risk-aversion coefficient) is different from 1, otherwise $m_q = 0$. In this model, the intution for why real exchange rate risk matters relies on the fact that, with nonunitary intertemporal elasticity of substitution, fluctuations in the real exchange rate produce idiosyncratic variations in the way households belonging to different countries evaluate future wealth. These idiosyncratic movements can be hedged through asset markets if excess returns and RER risk covary. Under log-utility, instead, substitution and income effects cancel each other out and relative-inflation risk does not imply differences in the marginal utility of consumption across countries, once evaluated in the same units of consumption goods.

Under Hansen-Sargent preferences, model (ii), agents in the two countries may use different subjective probability distributions. The first important consequence is that perfect risk sharing requires the equalization of the subjective stochastic discount factors evaluated under the same probability measure,

$$g_{t+1} m_{t+1} = g^*_{t+1} m^*_{t+1} \frac{q_t}{q_{t+1}},$$

[7] When markets are incomplete, agents try to minimize the projection of the deviations from zero, described by equation (3), on the space spanned by the asset returns.

where g_{t+1} and g^*_{t+1} are the changes of measure from the subjective distributions (which are country-specific) to the reference one (which is instead common across countries). Accordingly, agents face an additional hedging motive with respect to model (i), driven by the possible difference in the subjective probability measures; differences in the appetite for wealth now arise not only from possible differences in the marginal utilities of consumption across countries—first hedging motive, coming from (3)—but also from differences in the probabilities that agents assign to states of nature. More specifically, when agents are ambiguity averse, such subjective probabilities reflect the fear of model misspecification and are related to revisions in the path of consumption growth. Ambiguity-averse agents will assign a high subjective probability to those states of nature in which their consumption growth is lower than in the other country, and will therefore want to invest more in assets that pay well precisely in those states of nature. In a log-linear approximation, this additional hedging motive can be written as a linear function of the same three sources of risk identified above:

$$\text{(4)} \quad \ln g_{t+1} - \ln g^*_{t+1} \simeq (\gamma - 1)\left[g_e\left(\overline{\boldsymbol{\lambda}} - \overline{\boldsymbol{\lambda}}^{full}\right)\mathbf{exr}_{t+1} + g_q \varepsilon_{q,t+1} + g_l \varepsilon_{l,t+1}\right],$$

where, in this model, γ, with $\gamma \geq 1$, denotes the degree of ambiguity aversion, which we assume to be equal across countries ($\gamma = \gamma^*$), and g_e, g_q, and g_l are parameters. The optimal portfolio now seeks to "minimize" the sum of equations (3) and (4). We show that $g_l = m_l$ and $g_e = m_e$, and therefore the two sources of risk behind the two hedging motives are collinear, implying the same portfolio, with respect to labor-income risk. On the contrary, g_q is always different from m_q; hence, as long as there are fluctuations in the real exchange rate, the degree of ambiguity aversion does affect the optimal portfolio allocation. In particular, in the paper, we restrict attention to the case of log utility and therefore, $m_q = 0$. With the standard preferences of model (i), this would imply no role for the RER risk. Instead, with model uncertainty, real exchange rate risk still matters because g_q is different from zero. Why it matters depends only on the desire to hedge differences in the distorted beliefs rather than in the subjective evaluations of future wealth as in the case of model (i). Since models (i) and (ii) coincide along every dimension when $\gamma = 1$ (i.e., when the coefficient of risk aversion of model (i) and the degree of ambiguity aversion of model (ii) are unitary), and since in both cases raising γ does not affect the optimal portfolio that hedges labor-income risk, it is not just a coincidence that the portfolio implications of the two models map into one another also for nonunitary but equal values of γ.

Finally, it is a well-known result of the literature that, under a unitary intertemporal elasticity of substitution, Epstein-Zin and Hansen-Sargent preferences coincide when the risk-aversion coefficient of the former model is interpreted as the degree of ambiguity aversion in the latter, and therefore it is not surprising that models (ii) and (iii) have the same portfolio implications. However, while with Hansen-Sargent preferences the desire to hedge RER risk arises from differences in the probability distributions that agents assign to a given state of nature, with Epstein-Zin preferences what matters is the need to hedge differences in the subjective stochastic discount factors across countries. In particular, agents are no longer indifferent with respect to the temporal resolution of uncertainty (i.e., they prefer an earlier resolution of uncertainty when γ is higher) and fear bad news with respect to their future consumption path which they would like to risk-share. As with Hansen-Sargent preferences, this additional hedging motive translates into hedging real exchange rate fluctuations.

As already discussed, this paper is related to the classical literature which has addressed the international portfolio diversification puzzle under standard preferences, but it is also related to the literature on ambiguity aversion and portfolio choices. In this respect, the most relevant papers are Epstein and Miao (2003) and Uppal and Wang (2003). Epstein and Miao (2003) incorporate Knightian uncertainty in a multi-agent model by allowing for multiple priors. They explain home bias in asset holdings by assuming heterogeneity among agents concerning the degree of ambiguity about returns. They do not consider natural asymmetries coming from the open-economy dimension, and therefore $\varepsilon_{q,t+1} = \varepsilon_{l,t+1} = 0$ in their model. Instead, in our model, the implied under-diversification is not the result of an asymmetric attitude toward ambiguity across agents, since we assume $\gamma = \gamma^*$, but rather the consequence of natural open-economy asymmetries, which we show are indeed relevant in the data. Uppal and Wang (2003), instead, analyze a single-agent partial-equilibrium model with trading in multiple assets, which is indeed useful to understand the investment decisions of a single agent, but less so for the portfolio choices of different agents trading with each other, and therefore to address the international home-bias puzzle. As in Epstein and Miao (2003), they too build key asymmetries in the degrees of ambiguity aversion, though with respect to different assets rather than across different agents.[8] Uppal and Wang (2003) build on the framework proposed by Hansen and Sargent (2005, 2007), as we do, but in order to obtain tractability they adopt a noninnocuous modification of the original problem, as done also in Maenhout (2004, 2006). This makes our model and theirs not comparable.[9] Our approach, instead, adheres completely to the methodology of Hansen and Sargent (2005, 2007). In this respect, a further contribution of this paper is to derive a simple solution of nonlinear robust-control problems through approximation methods.[10] Our paper is also the first to study the implications of Epstein-Zin preferences for portfolio allocations in an international context.[11]

[8] Kirabaeva (2007) adopts a similar perspective, within a partial-equilibrium multi-agent multiple-asset model with "smooth ambiguity preferences" à la Klibanoff, Marinacci, and Mukerji (2005), and shows that an undiversified portfolio can arise as long as the agents are more optimistic and confident about the returns of the domestic asset relative to the foreign one.

[9] Indeed, they transform a constant lagrange multiplier into a time-varying function of the value function to get a closed-form solution. Pathak (2002) describes the transformation employed by Maenhout (2004, 2006) and Uppal and Wang (2003) as poorly motivated and explains in detail the unappealing consequences.

[10] Vardas and Xepapadeas (2004) also investigate the implications of multiplier preferences a la Hansen and Sargent (2005, 2007) for the optimal (not necessarily international) portfolio allocation, within a partial equilibrium multiple-asset model, and show that ambiguity aversion can affect the optimal total holdings of risky assets. Another somewhat related contribution, although using a very different approach, is Ben-Haim and Jeske (2003), which also addresses the home-bias puzzle in a partial equilibrium model incorporating Knightian uncertainty. Rather than using any parametric model of ambiguity aversion, however, Ben-Haim and Jeske (2003) take a nonprobabilistic approach, which makes their analysis not comparable to ours.

[11] Colacito and Croce (2011) use Epstein-Zin preferences in a two-country model to study the cross-country correlation between stochastic discount factors. In their context, however, consumption processes are exogenous; there is no international trade in goods and portfolio allocation is not explicitly taken into account.

II. Empirical Evidence

In this section, we evaluate the portfolio implication of equation (1) using empirical models for the processes of the excess returns, labor-income risk, and real exchange rate risk. In a more extensive form, the three equations nested in (1) can be written as

$$(5)\qquad \overline{\alpha}_H^b + \overline{\alpha}_F^b = \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{l,t+1}, \hat{r}^e_{H,t+1} - \hat{r}^b_{H,t+1} | exr^b_{t+1}, exr^e_{t+1})}{\mathrm{var}_t(\hat{r}^e_{H,t+1} - \hat{r}^b_{H,t+1} | exr^b_{t+1}, exr^e_{t+1})} + \frac{s_c}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{q,t+1}, \hat{r}^e_{H,t+1} - \hat{r}^b_{H,t+1} | exr^b_{t+1}, exr^e_{t+1})}{\mathrm{var}_t(\hat{r}^e_{H,t+1} - \hat{r}^b_{H,t+1} | exr^b_{t+1}, exr^e_{t+1})}$$

$$(6)\qquad \overline{\alpha}_H^e = \frac{1}{2} + \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{l,t+1}, \hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^b_{t+1}, exr^{eb}_{t+1})}{\mathrm{var}_t(\hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^b_{t+1}, exr^{eb}_{t+1})} + \frac{s_c}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{q,t+1}, \hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^b_{t+1}, exr^{eb}_{t+1})}{\mathrm{var}_t(\hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^b_{t+1}, exr^{eb}_{t+1})}$$

$$(7)\qquad \overline{\alpha}_F^b = -\frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{l,t+1}, \hat{r}^{b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^b_{H,t+1} | exr^{eb}_{t+1}, exr^e_{t+1})}{\mathrm{var}_t(\hat{r}^{b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^b_{H,t+1} | exr^{eb}_{t+1}, exr^e_{t+1})} - \frac{s_c}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\frac{\mathrm{cov}_t(\varepsilon_{q,t+1}, \hat{r}^{b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^b_{H,t+1} | exr^{eb}_{t+1}, exr^e_{t+1})}{\mathrm{var}_t(\hat{r}^{b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^b_{H,t+1} | exr^{eb}_{t+1}, exr^e_{t+1})}.$$

In the above equations, $\hat{r}^e_H$, $\hat{r}^{e^*}_F$, $\hat{r}^b_H$, and $\hat{r}^{b^*}_F$ denote real log-returns in the domestic equity, foreign equity, domestic bond, and foreign bond, respectively. Moreover, exr^{eb}, exr^e, and exr^b denote the excess returns on domestic equity (over domestic bonds), foreign equity, and foreign bonds, respectively:

$$(8)\qquad exr^{eb}_{t+1} \equiv \hat{r}^e_{H,t+1} - \hat{r}^b_{H,t+1}$$

$$(9)\qquad exr^e_{t+1} \equiv \hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1}$$

$$(10)\qquad exr^b_{t+1} \equiv \hat{r}^{b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^b_{H,t+1}.$$

The vector $\mathbf{exr}_{t+1}$ in (1) is defined as $\mathbf{exr}_{t+1} \equiv [exr^{eb}_{t+1}, exr^e_{t+1}, exr^b_{t+1}]'$, and $\varepsilon_{l,t+1}$ denotes labor-income risk, which in our model is given by

$$(11)\qquad \varepsilon_{l,t+1} = \sum_{j=0}^{\infty}\beta^k\big[E_{t+1}(\Delta\hat{\xi}^R_{t+1+k} - \Delta\hat{q}_{t+1+k}) - E_t(\Delta\hat{\xi}^R_{t+1+k} - \Delta\hat{q}_{t+1+k})\big],$$

and depends on the surprises in the present-discounted value of the cross-country differences in labor-income growths, in units of the same consumption basket; $\hat{\xi}^R_{t+1}$ denotes the cross-country difference in labor income and $\hat{\xi}^R_{t+1} - \hat{q}_{t+1}$ represents indeed the cross-country difference evaluated in units of the same consumption basket, through the real exchange rate. Finally, $\varepsilon_{q,t+1}$ denotes the real exchange rate risk, which depends on the time- t surprises in real exchange rate changes:

$$\varepsilon_{q,t+1} \equiv \sum_{k=0}^{\infty} \beta^k \big[E_{t+1} \Delta \hat{q}_{t+1+k} - E_t \Delta \hat{q}_{t+1+k} \big]. \tag{12}$$

In particular, we show that, differently from the current literature, which has mainly focused on "static" two-country models, our *dynamic* model implies that the relevant source of real exchange rate risk is not simply related to one-period changes, $\Delta \hat{q}_{t+1}$, but rather to revisions in the full future path of the real exchange rate. In the next section, we will stress that our focus on medium- to long-run risk is critical in order to explain home bias in asset holdings.

Equations (5)–(7), together with the identity

$$\overline{\alpha}_H^e + \overline{\alpha}_F^e + \overline{\alpha}_H^b + \overline{\alpha}_F^b = 1, \tag{13}$$

determine the shares of wealth $\overline{\alpha}_H^e$, $\overline{\alpha}_F^e$, $\overline{\alpha}_H^b$, $\overline{\alpha}_F^b$ that the representative agent in country H holds in domestic equities, foreign equities, domestic bonds, and foreign bonds, respectively.[12] In particular, equation (5) determines the share of financial wealth invested in the bond market. When $s_\xi \neq 0$ and $\gamma = 1$, and when $\varepsilon_{l,t+1}$ co-varies positively with the excess return of domestic equity over domestic bonds, domestic agents will take an overall long position in the bond markets $(\overline{\alpha}_H^b + \overline{\alpha}_F^b > 0)$. In this case, indeed, in the face of a bad shock to labor income, domestic bonds pay relatively better than equities; bonds are a better hedge with respect to labor-income risk. When, instead, real exchange rate risk matters, and in particular when $\gamma > 1$ and $\varepsilon_{q,t+1}$ co-varies positively with the excess return of domestic equity over domestic bonds, domestic agents will further increase their long position in the bond markets $(\overline{\alpha}_H^b + \overline{\alpha}_F^b > 0)$ since, in the face of a bad shock to the real exchange rate, bonds pay relatively better than equities.

Equation (6), instead, determines the optimal portfolio diversification between domestic and foreign equities. Absent RER risk or when $\gamma = 1$, the excess return on foreign equity should co-vary positively with labor-income risk, to obtain home bias in equity. In this case, the return on domestic equity increases, relative to that on foreign equity, when, indeed, domestic agents receive a bad shock to their labor income. This makes domestic equity a better hedge against labor-income risk relative to foreign equity and points toward explaining the home bias in equity holdings.[13] When γ is, instead, different from the unitary value, and in the most relevant case of $\gamma > 1$, the share invested in foreign equities further decreases when foreign

[12] The portfolio allocation of the agent in country F follows similarly.

[13] A popular argument for international diversification being worse is the neoclassical model of Baxter and Jermann (1997) in which labor income and dividends are correlated. In this case, the above covariance would be negative, implying even larger holdings of foreign assets. Heathcote and Perri (2009), instead, show a case in which the correlation can become positive in a model with capital accumulation and home bias in consumption preferences. Furthermore, Coeurdacier and Gourinchas (2009) discuss several theoretical cases that can rationalize a positive covariance and then imply home-bias in equity. See also Coeurdacier, Kollmann, and Martin (2008) and Engel and Matsumoto (2009).

equities are not a good hedge with respect to RER risk. This turns out to be the case when, in the face of a bad shock on the real exchange rate, foreign equities pay little relative to domestic equities.

Analogously, equation (7) describes the position taken in the foreign bond market and as a consequence in the domestic bond market, given the overall position implied by (5). When the covariance between $\varepsilon_{l,t+1}$ and the excess return of the foreign bond with respect to the domestic bond is positive, then foreign bonds do not pay well when needed. In this case, the domestic agent would like to take a short position in the foreign bond market ($\overline{\alpha}_F^b < 0$). Similarly when the covariance between real exchange rate risk, $\varepsilon_{q,t+1}$, and the excess return of the foreign bond with respect to the domestic bond is positive.[14]

In our empirical analysis, we study three alternative cases concerning the asset structure. The first case assumes that the only asset available for international trade is equity (henceforth *Asset Menu I*) as in Baxter and Jermann (1997). When bonds are not traded, i.e., $\overline{\alpha}_H^b = \overline{\alpha}_F^b = 0$, the only relevant equation is (6), which together with (13) determines the split between domestic and foreign equities. In this case, variance and covariances will not be conditional on other excess returns, but just on time– t information. In the second case, we allow for trade in both risky and riskless securities, but restrict the latter to an overall balanced position, i.e., $\overline{\alpha}_H^b = -\overline{\alpha}_F$, (*Asset Menu II*), as in van Wincoop and Warnock (2010) and Coeurdacier and Gourinchas (2009). In this case, therefore, a long position on domestic bonds necessarily implies a short position of equal magnitude on foreign bonds. The relevant conditions will be (6) and (7) involving only two excess returns (exr_{t+1}^e, exr_{t+1}^b): covariances and variances will be conditional on the remaining excess return. Finally, we consider the general case given by equations (5), (6), and (7), in which bond and equity instruments are both available, and leveraged positions between risky and riskless assets are also allowed for (*Asset Menu III*).

A. *Data*

To evaluate the empirical implications of equation (1), we collect and use quarterly data for the G7 countries, over the sample 1980:I–2007:IV. We consider the United States as the home country and the aggregation of the remaining G7 countries as the foreign country.[15]

We define the CPI index for the foreign country, expressed in USD, as

$$S_t P_t^* = \sum_i \omega_{i,t} S_{i,t} P_{i,t},$$

[14] Note that this does not necessarily imply a long position in the domestic bond market. Indeed, the overall position depends on equation (5), as previously discussed.

[15] In particular, we use data on aggregate nominal compensation of employees, from the OECD Quarterly National Accounts (**OCOS02B, where ** is the two-letter country code), the Consumer Price Indexes from the IFS database (**I64 … F), nominal returns on short-term treasury bills from the IFS database (**I60C …), nominal National Price and Gross Return indexes on the domestic stock market, from MSCI Barra (MS****L), in local currency, and bilateral nominal exchange rate vis-à-vis the USD, constructed using the domestic stock-price indexes in USD, from the MSCI Barra (MS****$). Moving from the monthly National Price and Gross Return indexes from MSCI database, we construct series for the quarterly nominal returns on equity ($R_{i,t}^e$) following Campbell (1999).

in which $P_{i,t}$ is the CPI in local currency for country i, $S_{i,t}$ is the bilateral nominal exchange rate between the local currency in country i and the dollar (US dollars for one unit of local currency), and $\omega_{i,t}$ is the GDP-weight of country i relative to the aggregation of the G6 countries at time t:[16]

$$\omega_{i,t} = \frac{GDP_{i,t}}{\sum_i GDP_{i,t}}.$$

Accordingly, the real exchange rate between the US and the G6 countries is simply computed as

$$\hat{q}_t = \log\left(\frac{S_t P_t^*}{P_t}\right) = \log\left(\frac{\sum_i \omega_{i,t} S_{i,t} P_{i,t}}{P_t}\right),$$

where P_t is the CPI index for the United States.

Analogously, we compute nominal labor income in US dollars for the foreign country as

$$S_t W_t^* \overline{l}_t^* = \sum_i \omega_{i,t} S_{i,t} W_{i,t} \overline{l}_{i,t},$$

in which $W_{i,t}$ is the nominal wage of country i and $\overline{l}_{i,t}$ is hours worked in country i. We measure nominal compensation $W_{i,t}\overline{l}_{i,t}$ using data on aggregate nominal compensation of employees in country i. Accordingly, relative labor income in units of US dollars is the log difference between the aggregate nominal compensation in the US and in the other G7 countries

$$\log\left(\frac{W_t \overline{l}_t}{S_t W_t^* \overline{l}_t^*}\right) = \log\left(\frac{W_t \overline{l}_t}{P_t}\frac{P_t}{S_t P_t^*}\frac{P_t^*}{W_t^* \overline{l}_t^*}\right) = \log\left(\frac{\xi_t}{q_t \xi_t^*}\right) = \hat{\xi}_t^R - \hat{q}_t,$$

where we have defined $\xi_t \equiv W_t \overline{l}_t / P_t$, $\xi_t^* \equiv W_t^* \overline{l}_t^* / P_t^*$, $\hat{\xi}_t^R \equiv \ln \xi_t - \ln \xi_t^*$.

Given nominal quarterly returns on the stock market, defined by $R_{i,t}^e$ for each country i and R_t^e for the US, and nominal quarterly returns on bonds, defined by $R_{i,t}^b$ for each country i and R_t^b for the US, we can obtain the real returns as $r_{i,t}^b \equiv R_{i,t}^b P_{i,t-1}/P_{i,t}$ and $r_{i,t}^e \equiv R_{i,t}^e P_{i,t-1}/P_{i,t}$ for each country i and for the US. Using those, we construct the three excess returns

$$exr_t^{eb} \equiv \hat{r}_{H,t}^e - \hat{r}_{H,t}^b = \log\left(\frac{r_t^e}{r_t^b}\right)$$

$$exr_t^e \equiv \hat{r}_{F,t}^{e^*} + \Delta\hat{q}_t - \hat{r}_{H,t}^e = \log\left(\frac{\sum_i \omega_{i,t} r_{i,t}^e \frac{q_{i,t}}{q_{i,t-1}}}{r_t^e}\right)$$

$$exr_t^b \equiv \hat{r}_{F,t}^{b^*} + \Delta\hat{q}_t - \hat{r}_{H,t}^b = \log\left(\frac{\sum_i \omega_{i,t} r_{i,t}^b \frac{q_{i,t}}{q_{i,t-1}}}{r_t^b}\right).$$

[16] To check for robustness, we repeated the analysis using average GDP-weights as an alternative aggregation methodology, as in Coeurdacier and Gourinchas (2009), and using both aggregate and per capita levels for the quantity variables. None of our results are significantly affected.

TABLE 1—SOME DATA STATISTICS (*Annual rates*)

	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot)$	$\rho(\cdot,\Delta\hat{\xi}^R-\Delta\hat{q})$	$\rho(\cdot,\Delta\hat{q})$
$\Delta\hat{\xi}^R - \Delta\hat{q}$	0.77	13.05	0.02	1.00	−0.44
$\Delta\hat{q}$	0.17	11.35	0.18	−0.44	1.00
$\hat{r}_F^{e*} + \Delta\hat{q} - \hat{r}_H^e$	0.70	13.54	0.11	−0.53	0.44
$\hat{r}_F^{b*} + \Delta\hat{q} - \hat{r}_H^b$	0.98	10.72	0.03	−0.92	0.72
$\hat{r}_H^e - \hat{r}_H^b$	6.35	15.85	0.00	−0.03	−0.14

Note: Means and standard deviations are in percentage points.

Table 1 reports some summary statistics. We report the average level $\mu(\cdot)$ and the standard deviation $\sigma(\cdot)$, both annualized and in percentage points, the serial correlation coefficient $\rho(\cdot)$ and the correlation with one-period changes in relative labor income, $\rho(\cdot,\Delta\hat{\xi}^R-\Delta\hat{q})$, and in the real exchange rate, $\rho(\cdot,\Delta\hat{q})$. These unconditional correlations already suggest that domestic equity seems a poor hedge against labor income risk, relative to foreign stocks, while both domestic equity and domestic bonds seem somewhat useful in providing the right co-movement in hedging real exchange rate fluctuations. In the next sections we will refine and articulate these results.

In order to evaluate the optimal portfolio allocation implied by our model, we need to calibrate the steady-state ratio of consumption to financial wealth, s_c. To this end, we use the average financial wealth-to-disposable income ratio for the US computed by Bertaut (2002), and the average consumption-to-disposable income ratio for the US, computed using data on personal consumption of non-durable goods and personal disposable income.[17] The former, on a quarterly frequency, amounts to about 20, while the latter to around 0.3: by using these numbers we get a calibrated consumption-to-wealth ratio $s_c = 0.3/20 = 0.015$. We calibrate the quarterly time discount factor following Tallarini (2000) and Barillas et al. (2009): $\beta = 0.995$. Using the value of s_c obtained above, we derive the model-consistent steady-state value of the labor income-to-financial wealth ratio, by using $s_\xi = s_c - (1-\beta)/\beta = 0.01$.

B. *The Statistical Model*

We define the data vector $\mathbf{y}_t \equiv \left[\Delta\hat{\xi}_t, \Delta\hat{\xi}_t^*, \Delta\hat{q}_t, exr_t^{eb}, exr_t^e, exr_t^b, \hat{r}_{H,t}^b\right]'$, which collects the relevant variables involved in our analysis, and estimate the following VAR(p) model:

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{14}$$

in which $\mathbf{e}_t$ is a $k \times 1$ random vector (with $k = 7$) distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. Since we are interested

[17] Data for disposable income and nondurable consumption for the United States are taken from the Federal Reserve Economic Dataset, FRB of St. Louis, "Real Disposable Personal Income," series ID: DPIC96, and "Real Personal Consumption Expenditures: Nondurable Goods," series ID: PCNDGC96. We checked the sensitivity of our results to the use of different measures of consumption, including services and durables, and found that our qualitative results are unaffected.

in characterizing the medium- and long-horizon properties of the data, and given the quarterly frequency, we set the lag length to $p = 4$, at which standard Durbin-Watson and White tests confirm lack of serial correlation and heteroskedasticity of residuals.[18] Using the above, we can construct the implied sources of risk ($\varepsilon_{l,t+1}$ and $\varepsilon_{q,t+1}$) and evaluate the conditional moments needed to validate the theoretical implications (5)–(7).

In particular, using the companion form for the above VAR(p)

$$\mathbf{Y}_t = \mathbf{M} + \mathbf{B}\mathbf{Y}_{t-1} + \mathbf{C}\mathbf{e}_t,$$

and denoting with $\boldsymbol{\iota}_x$ an appropriate column vector that "picks up" variable x from the data vector $\mathbf{y}_t$ (for example: $\boldsymbol{\iota}_q'\mathbf{y}_t = \Delta\hat{q}_t$), and with $\mathbf{I}_{kp}$ the $k \cdot p \times k \cdot p$ identity matrix, we can construct labor-income risk as

$$\begin{aligned}\varepsilon_{l,t+1} &\equiv \sum_{j=0}^{\infty}\beta^k(E_{t+1} - E_t)\big(\Delta\hat{\xi}^R_{t+1+k} - \Delta\hat{q}_{t+1+k}\big)\\ &= (\boldsymbol{\iota}_\xi - \boldsymbol{\iota}_{\xi^*} - \boldsymbol{\iota}_q)'\mathbf{C}'(\mathbf{I}_{kp} - \beta\mathbf{B})^{-1}\mathbf{C}\mathbf{e}_{t+1}\end{aligned}$$

and real-exchange-rate risk as

$$\varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty}\beta^k(E_{t+1} - E_t)\Delta\hat{q}_{t+1+k} = \boldsymbol{\iota}_q'\mathbf{C}'(\mathbf{I}_{kp} - \beta\mathbf{B})^{-1}\mathbf{C}\mathbf{e}_{t+1}.$$

Figure 1 displays the dynamic paths of the two relevant sources of risk implied by the estimated VAR (14). The top panel confirms the basic insight of Table 1 also in a dynamic setting: labor-income risk tends to co-move in a negative way with respect to the real-exchange-rate risk, with a correlation of about -0.444. While one-period changes in the real exchange rate are less volatile than changes in relative labor income (see Table 1), however, labor-income risk is basically as volatile as RER risk. The bottom panel shows a first assessment of the differences between short-run RER risk (one-period changes in the real exchange rate, $\Delta\hat{q}_{t+1}$) and the model-specific RER risk (revisions in the discounted stream of future RER changes, $\varepsilon_{q,t+1}$). In particular, the bottom panel shows that the two notions of RER risk can have rather different empirical features: the short-run risk is substantially more stable than the long-run risk (standard deviation of about 11.3 as opposed to about 15, in annualized percentage terms), and is only partially correlated with the latter (the correlation is about 0.65). This descriptive evidence points already to potentially different implications for optimal portfolios depending on which notion of RER

[18] Standard statistical criteria for lag selection give different prescriptions in this case (e.g., Akaike's Final Prediction Error criterion tends to point to two/three lags, depending on the informational content of the data vector, while Schwarz's Bayesian Criterion to only one), although the relevant statistics for both criteria show little variability across lag specifications. The choice of our benchmark specification is therefore primarily disciplined by the goal of a comprehensive characterization of the dynamic properties of the data, especially at medium- and long-term horizons. In Benigno and Nisticò (2009), however, we explored different specifications of the statistical model, both in terms of lag length and informational content of the data vector, without finding substantial changes in our qualitative results.

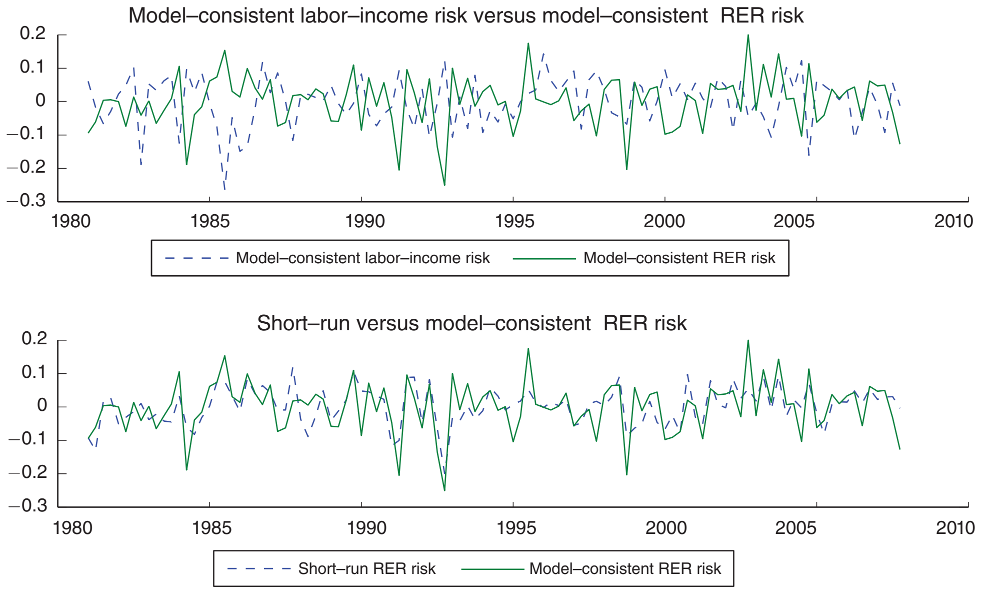

Figure 1. Short-run Real Exchange Rate Risk ($\Delta \hat{q}_{t+1}$), Model-consistent Real Exchange Rate Risk ($\varepsilon_{q,t+1}$), and Labor-Income Risk ($\varepsilon_{l,t+1}$)

risk is the relevant one. Such implications of the relevant sources of risk for optimal portfolios are the subject of the next two subsections.

C. *The Role of Labor-Income Risk*

Equation (1) shows that the optimal portfolio allocation consists of two components: one related to hedging labor-income risk, and the other related to hedging real exchange rate risk. In this section, we evaluate the empirical relevance of the first component. In particular, this component can be useful in explaining the home-bias puzzle in equity holdings as long as the covariance between the present discounted value of domestic versus foreign labor income and the excess return of foreign versus domestic equity is positive.

This hedging motive, which is common to all three models regardless of the interpretation or specific value of γ, has been emphasized by several studies without reaching a clear consensus. Baxter and Jermann (1997) show that when equity is the only asset that can be traded internationally, the presence of nondiversifiable income risk actually implies a foreign-equity bias. On the other hand, Bottazzi, Pesenti, and van Wincoop (1996) and more recently Julliard (2003) and Coeurdacier and Gourinchas (2009) bring evidence supporting the view that hedging against labor-income risk can explain some degree of home bias in equity holdings. Heathcote and Perri (2009) and Coeurdacier and Gourinchas (2009), moreover, discuss some theoretical examples that can produce the required co-movements to explain home bias.

We analyze this interaction in the context of our dynamic model. Using the output of the VAR, we construct the surprises in the path of relative labor income across countries, and compute the time-t conditional moments needed. For comparisons

Table 2—The Empirical Role of Labor-Income Risk

	Asset Menu I	*Asset Menu II*	*Asset Menu III*
Conditional covariance-to-variance ratios of LIR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	−0.782	−0.268	−0.365
$\hat{r}^{b*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{b}_{H,t+1}$	—	−1.287	−1.271
$\hat{r}^{e}_{H,t+1} - \hat{r}^{b}_{H,t+1}$	—	—	−0.206
Optimal portfolio allocation under rational expectations			
$\overline{\alpha}^{e}_{F}$	1.276	0.766	0.862
$\overline{\alpha}^{e}_{H}$	−0.276	0.234	0.342
$\overline{\alpha}^{b}_{F}$	—	1.277	1.262
$\overline{\alpha}^{e}_{H} + \overline{\alpha}^{e}_{F}$	1.000	1.000	1.204
$\overline{\alpha}^{b}_{H} + \overline{\alpha}^{b}_{F}$	—	0.000	−0.204
$\overline{\alpha}^{e}_{F} + \overline{\alpha}^{b}_{F}$	1.276	2.043	2.124

Notes: LIR denotes labor-income risk; *Asset Menu I:* equities only; *Asset Menu II:* equities and balanced bonds; *Asset Menu III:* general model with equities and bonds. $\overline{\alpha}^{e}_{F}$ denotes the share of wealth invested in foreign equity; $\overline{\alpha}^{e}_{H}$ denotes the share of wealth invested in domestic equity; $\overline{\alpha}^{b}_{F}$ denotes the share of wealth invested in foreign bonds; $\overline{\alpha}^{e}_{H} + \overline{\alpha}^{e}_{F}$ measures the overall share of wealth invested in equity assets; $\overline{\alpha}^{b}_{H} + \overline{\alpha}^{b}_{F}$ measures the overall share of wealth invested in debt instruments; and $\overline{\alpha}^{e}_{F} + \overline{\alpha}^{b}_{F}$ measures the overall share of wealth invested in foreign assets.

with existing literature, however, we also compute the unconditional covariance-to-variance ratios, obtained through straightforward OLS projection of the surprises in relative labor income on the excess returns of the assets available for trade.[19] In the simple case of *Asset Menu I* (only equities), we obtain

$$\varepsilon_{l,t+1} = -\underset{(0.087)}{0.539} \cdot \left(\hat{r}^{e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}\right) + u_{l,t+1},$$

where the standard error is reported in parenthesis. The unconditional covariance-to-variance ratio is negative, statistically significant, and economically rather large. The implication is that hedging labor-income risk does not produce home bias in equity, but rather implies a *foreign* equity bias. This result, on the one hand, supports Baxter and Jermann (1997), and, on the other hand, weakens the argument of Heathcote and Perri (2009).

In Table 2, we report the time-t conditional covariance-to-variance ratios related to labor-income risk, and the portfolio allocation implied by the first component in equation (1), i.e., the optimal overall allocation under the assumption $\gamma = 1$. In particular, the first column of Table 2 confirms the finding of Baxter and Jermann (1997) that the portfolio diversification puzzle is even worse than expected.

This result has been recently challenged, within a static framework, by Coeurdacier and Gourinchas (2009). They emphasize that, with a richer set of asset traded, variances and covariances should be computed conditional on the other asset returns. In particular, they claim that results would be overturned when moving from *Asset Menu I* to *Asset Menu II*. They provide empirical support to this

[19] The unconditional covariance-to-variance ratios would be appropriate in our case if the process $\mathbf{y}_t$ were in fact a multivariate white noise. Our data, however, do not support this representation. This is particularly relevant for the real exchange rate, since a pure random walk assumption, as discussed in the recent work of Backus et al. (2010), would be inconsistent with the widely documented violation of the uncovered interest parity.

claim (i.e., a positive covariance between excess returns on foreign versus domestic equity and labor income risk).

The second column of Table 2 shows that the covariance between labor-income risk and the excess return on equities is instead still inconsistent with an explanation of the home-bias based on hedging labor-income risk, regardless of the tradability of riskless bonds. Such substantial difference in the results, however, does not originate from the fact that our model is dynamic, but rather from the different empirical definition of labor income risk.

Coeurdacier and Gourinchas (2009), indeed, use the unexpected component of the (home relative to foreign) return-to-labor, constructed as

$$(15)\qquad \hat{r}^{w}_{t+1} - E_t\hat{r}^{w}_{t+1} = \sum_{k=0}^{\infty}\rho^k(E_{t+1} - E_t)(\Delta\hat{\xi}^{R}_{t+1+k} - \Delta\hat{q}_{t+1+k}) - \sum_{k=1}^{\infty}\rho^k(E_{t+1} - E_t)(\hat{r}^{e}_{H,t+1+k} - \Delta\hat{q}_{t+1+k} - \hat{r}^{e^*}_{F,t+1+k}),$$

where $\rho \equiv 1 - s_c$ is a constant of linearization that depends on the average consumption-wealth ratio. This measure is borrowed from Campbell (1996) and relies on two implicit and critical assumptions. First, it is assumed that there exists a market for domestically tradeable claims on the stream of future labor-income flows, which implies that the return to labor is computed in analogy to the return on any financial asset, using the log-linear approximation of Campbell and Shiller (1988).[20] Second, the term on the second line of equation (15) arises because innovations in the future relative returns on domestic human wealth are assumed to be equal to those in the future excess returns on domestic versus foreign equities.[21]

With this definition, it follows that the return-to-labor is likely to be positively related, by construction, with the excess return on foreign versus domestic equity. This result is questionable, indeed Lustig and Van Nieuwerburgh (2008) perform a simple consumption growth accounting exercise and show that innovations in future returns on human wealth are actually negatively correlated with innovations in future returns on financial assets.[22]

We do not make either of the assumptions above. Instead, in our framework, the relevant measure of nondiversifiable labor-income risk is directly implied by the theoretical model, and corresponds to the revision in the present discounted value of cross-country labor income $\varepsilon_{l,t+1}$, as shown by equation (11).[23] It is worth noticing that our measure of labor-income risk is instead similar to those used by Shiller (1995) and Baxter and Jermann (1997), which coincide with the first

[20] The first unappealing implication of this approach, in a two-country world, is that the claims on human wealth would have to be restricted to domestic trade only (otherwise labor-income risk could also be diversified internationally).

[21] This is a strong assumption, as discussed by Campbell (1996).

[22] Van Nieuwerburgh, Lustig, and Verdelhan (2010), similarly, estimate an affine-yield model on bond yields and stock returns, and document a very weak correlation between expected returns on human wealth and those on equities.

[23] Note that in a first-order approximation (which is all that is needed to evaluate the relevant orthogonality conditions and derive the steady-state portfolio allocation), expected excess returns are always zero, so the last terms in (15) would drop even if we did make the two assumptions discussed above.

Table 3—Loadings of Excess Returns on Real Exchange Rate Depreciations

Loadings of:	*Asset Menu I*	*Asset Menu II*	*Asset Menu III*
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	0.365	0.021	−0.026
	(0.072)	(0.068)	(0.071)
$\hat{r}^{b*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{b}_{H,t+1}$	—	0.747	0.781
	—	(0.086)	(0.086)
$\hat{r}^{e}_{H,t+1} - \hat{r}^{b}_{H,t+1}$	—	—	−0.098
	—	—	(0.048)

Notes: Standard errors in parentheses. Dependent variable is $\Delta\hat{q}_{t+1}$.

summation on the right-hand side of (15).[24] Using this definition, we find that domestic equity is not a good hedge, reinforcing Baxter and Jermann's (1997) result even if we condition on bond returns.

The third column of Table 2 displays the results for *Asset Menu III*, and shows that allowing for leveraged positions between equity and riskless assets does not substantially alter the results.

D. *The Role of Real Exchange Rate Risk*

In the above section, we showed that there is no support to the view that domestic equity is a good hedge against nondiversifiable labor-income risk to explain the home bias in US equity holdings. We now move to analyze the empirical relevance of the second component in equation (1), related to real exchange rate risk.

The role of hedging real exchange rate fluctuations as an explanation for the home-bias puzzle has been recently questioned by van Wincoop and Warnock (2010) and Coeurdacier and Gourinchas (2009). Their main argument is based on the evidence that the covariance between real exchange rate changes and the excess return on foreign versus domestic equity becomes negligible once this covariance is taken conditional on other returns, like the excess return on riskless bonds. This observation comes from the results of a simple OLS regression between one-period ahead changes in the real exchange rate and the vector of excess returns,

$$\Delta\hat{q}_{t+1} = \kappa_q + \psi'_q \mathbf{exr}_{t+1} + u_{q,t+1}, \tag{16}$$

reported in Table 3. While the loading of the excess returns on foreign equity is significant and positive if equity is the only tradeable asset, once the vector of excess returns is augmented to include the excess return on foreign versus domestic bonds, the covariance-to-variance ratio becomes negligible.

In a static, rational-expectations model, such small covariances (provided they are of the right sign) would require an extremely high γ to justify the hedging role of domestic equities, which, however, would open room for other puzzles, like the aforementioned risk-free rate puzzle.

[24] Indeed, the only difference between (11) and the measure in Shiller (1995) and Baxter and Jermann (1997) is the discount parameter. While they use $\rho \equiv 1 - s_c$, we use the time discount factor β. Numerically, however, β and ρ are very close numbers.

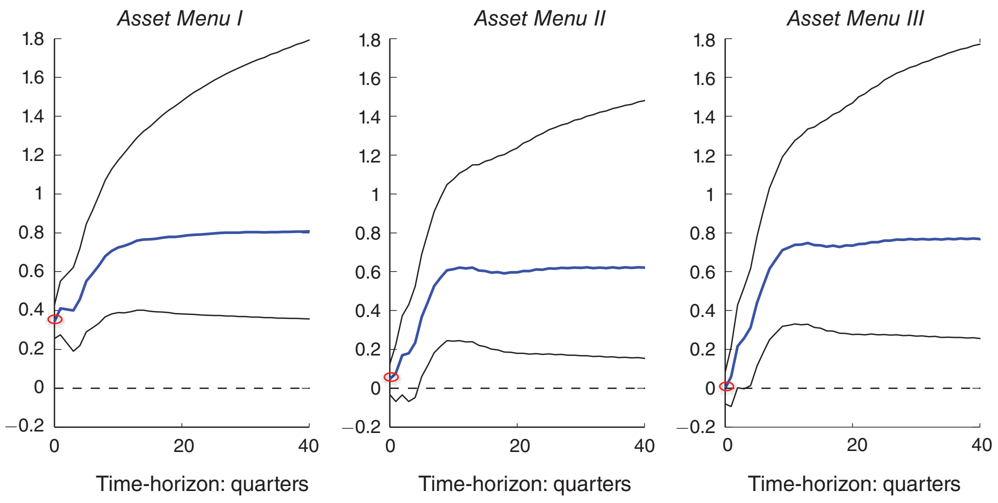

Figure 2. The Covariance-to-Variance Ratio between $\Delta E_{q,t+1}\hat{q}_{t+1+k}$ and exr^{e}_{t+1}, for Increasing k (*horizontal axis*)

Notes: Asset Menu I: equities only. *Asset Menu II*: equities and balanced bonds. *Asset Menu III*: general model with equities and bonds. Thin lines: 68 percent confidence bands.

Instead, our dynamic model gives a new role to real exchange rate risk. What matters is not only the current real exchange rate risk, the *short-run* risk, but also the revisions in the entire future expected path of the real exchange rate, the *long-run* risk.

To study whether shifting from a short-run to a long-run perspective affects the hedging properties of equity with respect to real exchange rate risk, we start writing equation (12) in terms of levels instead of growth rates:

$$\varepsilon_{q,t+1} \equiv \sum_{k=0}^{\infty} \beta^{k} \Delta E_{t+1} \Delta \hat{q}_{t+1+k} = (1 - \beta) \sum_{k=0}^{\infty} \beta^{k} \Delta E_{t+1} \hat{q}_{t+1+k}, \tag{17}$$

in which $\Delta E_{t+1}(\cdot) \equiv E_{t+1}(\cdot) - E_{t}(\cdot)$ denotes the time-$t+1$ revisions in conditional expectations.

By looking at different terms in the summation above, we can investigate the comovement between asset returns and surprises in the real exchange rate path at different time horizons. In particular, we can evaluate whether the hedging properties of equity and bonds change when the risk to be hedged is farther away in the future, as opposed to very soon. To this end, given our estimated model (14), we compute the time—$t+1$ news about the real exchange rate k periods ahead—given by $\Delta E_{t+1}\hat{q}_{t+1+k}$—and for each time horizon we evaluate the covariance-to-variance ratios with respect to all excess returns of interest, conditional on time-t information and on the residual asset space: $\Sigma_{t}^{-1} E_{t}(\Delta E_{t+1}\hat{q}_{t+1+k} \cdot \mathbf{exr}_{t+1})$.

Figure 2 plots the covariance-to-variance ratios (and their one-standard-deviation confidence bands) of RER risk at different horizons k with the current excess return on foreign versus domestic equity. Figure 3 does the same for the excess return on foreign versus domestic bonds, for the two asset menus which include bonds (*II* and *III*).

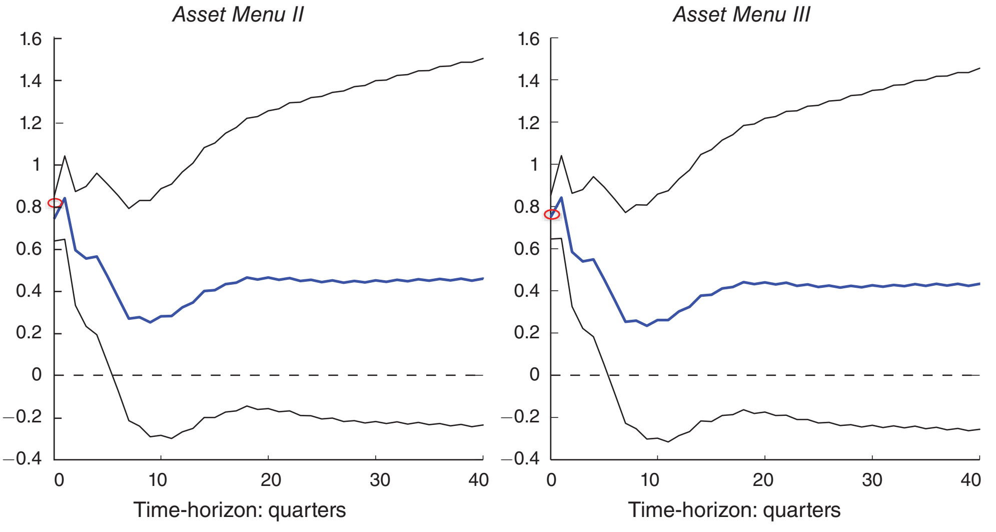

Figure 3. The Covariance-to-Variance Ratio between $\Delta E_{t+1}\hat{q}_{t+1+k}$ and exr^{b}_{t+1}, for Increasing k (*horizontal axis*)

Notes: Asset Menu II: equities and balanced bonds. *Asset Menu III*: general model with equities and bonds. Thin lines: 68 percent confidence bands.

The first point in each plot, marked with a red circle and corresponding to $k = 0$, measures the covariance-to-variance ratio implied by a static model, in which only the short-run risk matters. Moving from the left to the right panel of Figure 2, the first point drops from about 0.4 to virtually zero, implying that the hedging power of equity against real exchange rate risk fades away, when we condition on other excess returns and, in particular, on bonds. This is the core of the results of van Wincoop and Warnock (2010) and Coeurdacier and Gourinchas (2009).

The novel insight of Figure 2, however, is that at longer horizons the hedging properties of equity sharply improve, even when we condition on other excess returns.[25] The confidence bands reveal that (perhaps not surprisingly) the precision of the estimated covariance-to-variance ratios decreases as the time-horizon rises, reflecting the limited sample size and the parsimonious specification of the statistical model. However, most of the increase in the estimates' uncertainty occurs at time horizons higher than about 15 quarters, and mainly with respect to the upper bound of the credible set. In order to scrutinize further the effect of sample uncertainty on this result, we compute the relative contribution of each time horizon k to the point estimate of the overall covariance-to-variance ratio, for each asset menu. The results are displayed in Figure 4, and show that the covariance-to-variance ratio between the excess return on foreign equity and RER risk is mainly driven by the co-movement with the news on the real exchange rate over *medium* time horizons (within about ten quarters). Over such medium horizons, as shown by

[25] A recent literature documents the quantitatively substantial implications of long-run risk for asset valuation, in the context of nonexpected utility frameworks. See, among others, Hansen, Heaton, and Li (2008), who also provide an interpretation related to model uncertainty.

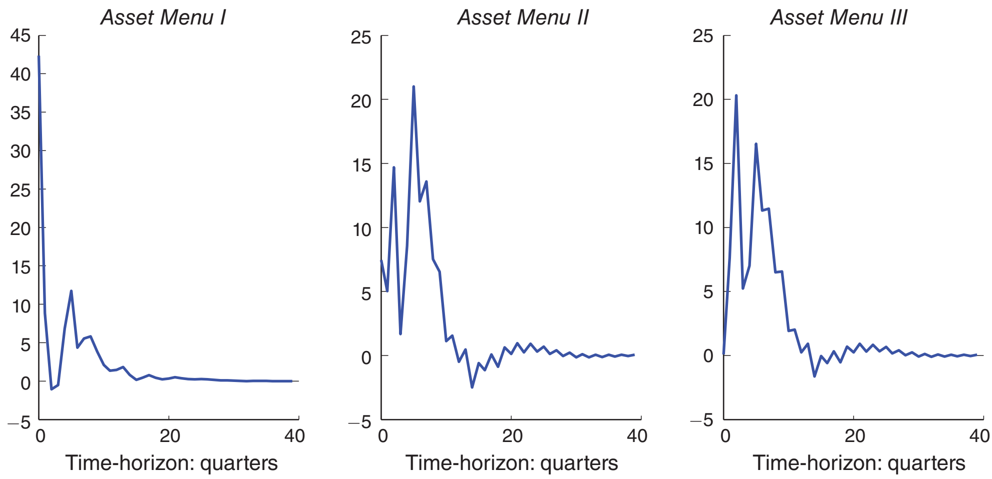

Figure 4. The Relative Contribution of Each Time-Horizon to the Covariance-variance Ratio between $\varepsilon_{q,t+1}$ and exr^{e}_{t+1}

Notes: Asset Menu I: equities only. *Asset Menu II*: equities and balanced bonds. *Asset Menu III*: general model with equities and bonds. Relative contribution in percentage points.

Figure 2, the estimated covariance-to-variance ratios are not excessively affected by sample uncertainty.

Figure 3, instead, shows that the hedging properties of riskless bonds are affected in the opposite direction by longer time horizons. Indeed, while the covariance-to-variance ratio for $k = 0$ is significantly positive and large, as the time horizon rises, the point estimate decreases and the effects of sample uncertainty become stronger. The one-standard-deviation confidence interval widens to include zero after about five quarters.

Finally, to give further robustness to our results, we compute the covariance-to-variance ratio of real exchange rate risk with the excess return on foreign versus domestic equity also for the United States, vis-à-vis each of the other G7 countries, and report the results in Figure 5. As the first points in the figure show (marked with a red circle), when $k = 0$, the covariance-to-variance ratio between RER risk and the excess return on the equity of each of the G6 countries is basically zero, consistent with the results of van Wincoop and Warnock (2010); the hedging properties of US equity against *short-run* RER risk are very poor, regardless of the currency in which the alternative asset is denominated. However, the figure also reveals that as the relevant time horizon rises, the covariance-to-variance ratio with respect to all countries (with the notable exception of Japan) becomes clearly positive; the hedging properties of US equity against *long-run* RER risk are substantially stronger.[26]

[26] In Figure 2, the aggregation of the country-specific variables, discussed in Section IIA, implies that the agent is allocating his/her wealth between domestic assets, on the one side, and a given portfolio of foreign assets, on the other side, with weights equal to the GDP weights used in the aggregation. This further implies that the covariance-to-variance ratio displayed in Figure 2 also reflects the co-movements between the bilateral exchange rates and returns across different pairs of countries. These additional components explain why the pattern displayed in Figure 2 does not coincide with the average pattern implied by Figure 5.

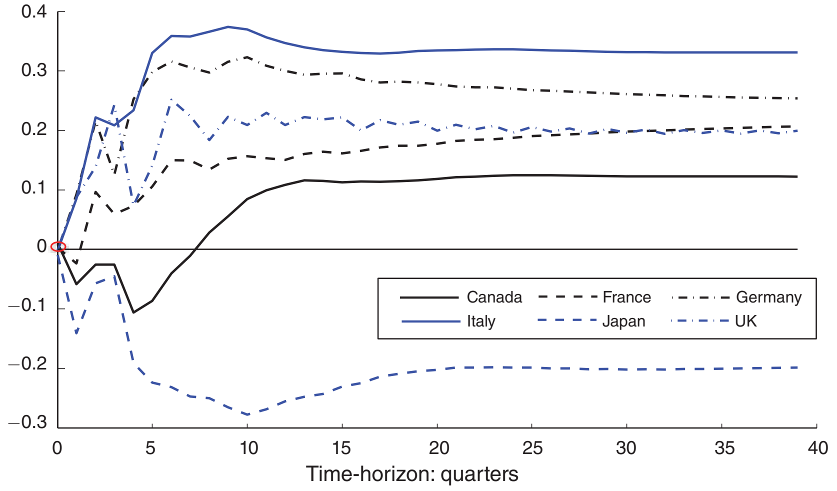

FIGURE 5. THE COVARIANCE-TO-VARIANCE RATIO BETWEEN $\Delta E_{t+1}\hat{q}_{t+1+k}$ AND exr^{e}_{t+1}, FOR INCREASING k (*horizontal axis*)

Notes: The US vis-à-vis each of the other G7 countries. *Asset Menu III*: general model with equities and bonds.

E. *The Role of* γ

We now merge the results of the previous sections and discuss the role of γ in equation (1). In particular, γ measures risk aversion and the inverse of the elasticity of intertemporal substitution in model (i), ambiguity aversion in model (ii), and the attitude toward the temporal resolution of uncertainty in model (iii).

Table 4 reports the covariance-to-variance ratios of the two sources of risk implied by our model (labor-income and real exchange rate), with the relevant excess returns, for the three asset menus that we consider. In particular, the empirical co-movements, with respect to the excess return on foreign equity, reveals that domestic equity—relative to foreign—is qualitatively useful to hedge against real exchange rate risk (positive ratio), while not against labor-income risk (negative ratio), also when riskless bonds are available for trade. The two sources of risk, therefore, drive the optimal portfolio allocation in opposite directions. However, the third column of Table 4 shows that the empirical co-movements of exr^{e} with real exchange rate risk are twice as strong as those with labor-income risk (three times stronger in *Asset Menu II*), implying that hedging RER risk can potentially become the main driver of the optimal portfolio allocation, provided that the relative component in equation (1) is relevant enough. Similarly, co-movements with exr^{b} imply that domestic bonds—relative to foreign—represent a useful hedge against real exchange rate risk (positive ratio), but a bad hedge against labor-income risk (negative ratio). In the general case of *Asset Menu III*, finally, Table 4 also implies that equity assets are a relatively better hedge against labor-income risk (negative co-movements with exr^{eb}), while debt instruments are relatively better to hedge real exchange rate risk (positive ratio).

Figure 6 shows the implications of these empirical co-movements for the optimal portfolio allocation by increasing γ, and under the three asset menus considered.

Table 4—The Empirical Role of Real Exchange Rate Risk

	Asset Menu I	*Asset Menu II*	*Asset Menu III*
Conditional covariance-to-variance ratios of LIR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	−0.782	−0.268	−0.365
$\hat{r}^{b*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{b}_{H,t+1}$	—	−1.287	−1.271
$\hat{r}^{e}_{F,t+1} - \hat{r}^{b}_{H,t+1}$	—	—	−0.206
Conditional covariance-to-variance ratios of RERR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	0.809	0.622	0.769
$\hat{r}^{b*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{b}_{H,t+1}$	—	0.467	0.444
$\hat{r}^{e}_{F,t+1} - \hat{r}^{b}_{H,t+1}$	—	—	0.311

Notes: LIR denotes labor-income risk; RERR denotes real exchange rate risk; *Asset Menu I:* equities only; *Asset Menu II:* equities and balanced bonds; *Asset Menu III:* general model with equities and bonds.

The first point of each line captures the case of $\gamma = 1$ which is driven by only labor-income risk—the first component of the right-hand side of (1). As γ increases, the second component of (1), related to real exchange rate risk, becomes progressively more important.

Figure 6 shows that, as γ rises, the share of wealth allocated to domestic equity ($\overline{\alpha}^{e}_{H}$) sharply increases, regardless of the specific asset structure (up to 87 percent for *Asset Menu I*, 112 percent for *Asset Menu II*, and 99 percent for *Asset Menu III*). In particular, in the general case of *Asset Menu III*, when $\gamma = 5$, the share of wealth allocated to domestic equity reaches about 89 percent, explaining a large proportion of the home bias found in the data, and in contrast with the 35 percent of the benchmark case with $\gamma = 1$.

The three models under analysis can be successful in explaining the home bias in the international portfolio allocation. However, as already discussed, under the standard isoelastic expected utility of model (i), γ corresponds to the inverse of the intertemporal elasticity of substitution. By raising γ, the mean of the risk-free real rate increases and the model falls in the risk-free rate puzzle. Instead, in model (ii) and (iii), there is an additional degree of freedom; the intertemporal elasticity of substitution is assumed to be unitary, while γ measures the degree of ambiguity aversion and the risk-aversion coefficient, respectively.

III. The Models

In this section, we discuss, in detail, the three alternative preference specifications and show that they deliver the same steady-state portfolio allocation given by equation (1). We consider a model with two countries, denoted domestic (H) and foreign (F), each populated by a representative agent. Representative agents supply a fixed amount of labor. In each country, there is a continuum of firms producing a continuum of goods in a market characterized by monopolistic competition. All goods are traded. Households enjoy consumption of both domestic and foreign goods and can trade in a set of financial assets. Specifically, there are four assets traded in the international markets: two risk-free nominal bonds, denominated in each currency, and two equity assets, representing claims on the dividends of domestic and foreign firms, respectively.

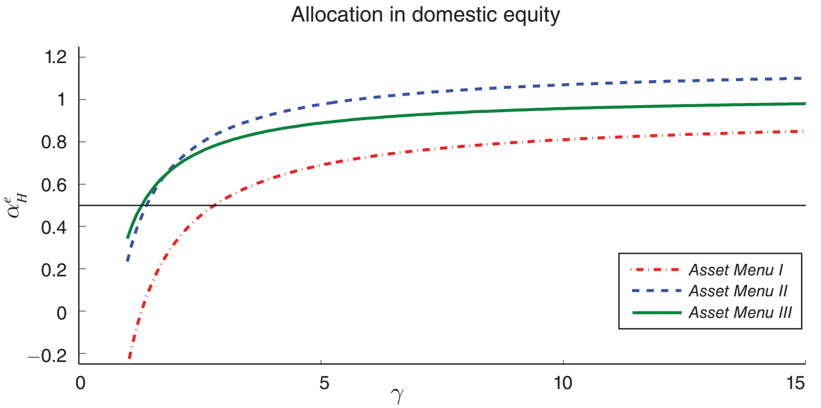

FIGURE 6. OPTIMAL PORTFOLIO ALLOCATION: THE EFFECT OF FEARS OF MODEL MISSPECIFICATION

Notes: Asset Menu I: equities only. *Asset Menu II:* equities and balanced bonds. *Asset Menu III:* general model with equities and bonds. $\alpha_H^e = 0.5$ implies full diversification in equity holdings.

A. *Three Alternative Preference Specifications*

We consider the following alternative specifications of preferences: model (i) a standard model with isoelastic consumption utility and constant relative risk aversion, model (ii) a model with log-utility in consumption in which agents face and fear model uncertainty as in the framework developed by Hansen and Sargent (2005, 2007), and model (iii) a model with unitary intertemporal elasticity of substitution and nonexpected utility as in Epstein and Zin (1989) and Kreps and Porteus (1978).

In model (i), the representative agent maximizes the present discounted values of the utility flow, which is isoelastic with respect to current consumption

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{c_t^{1-\rho}}{1 - \rho}\right\}, \tag{18}$$

where β, with $0 < \beta < 1$, is the intertemporal discount factor; ρ, with $\rho > 0$, is the risk aversion coefficient and the inverse of the intertemporal elasticity of substitution; c_t is consumption at date t; and $E_{t_0}(\cdot)$ is the time $-t_0$ expectation operator. Preferences are similar in the foreign country in reference to foreign consumption.

In model (ii), model uncertainty is assumed. Agents are endowed with some "reference" probability distribution, which is common across countries, but which they do not trust, and might instead act using a nearby distorted "subjective" distribution, possibly specific to each country. In this case, the representative agent in the domestic economy maximizes utility given by

$$\tilde{E}_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t\right\}, \tag{19}$$

where the utility flow is logarithmic in current consumption, and where, now, $\tilde{E}_{t_0}(\cdot)$ is the time $-t_0$ expectation operator taken with respect to the distorted probability measure. We assume that the distorted measure is absolutely continuous with respect to the "reference" measure and such that the expected utility can be written in terms of the "reference" distribution as

$$\tilde{E}_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t\right\} = E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln c_t\right\},$$

where G_t is a nonnegative martingale with $G_{t_0} = 1$ that acts as a change of measure from the "approximating" to the "subjective" distribution. The representative agent in the other country has similar preferences but a possibly different expectation operator $\tilde{E}^*_{t_0}(\cdot)$.

Notice that this way of modelling model uncertainty is observationally equivalent to a model with preference shocks, as in Pavlova and Rigobon (2007), but in which the preference shocks are restricted to be a martingale. Moreover, in our context, the preference shocks are not exogenous but will correspond to the optimizing behaviour of an "evil" agent that manipulates the distorted beliefs to minimize the above utility. Indeed, in this environment, we consider the sophisticated agents of the robust-control theory of Hansen and Sargent (2005, 2007). These agents fear model misspecification, and seek decision rules that are robust to it. Following Hansen and Sargent (2005, 2007), we can regard such a robust decision-making process as a two-player game between the representative household and an "evil" agent. The household surrounds the reference model with a set of alternative distributions, in which he/she believes the true one lies. The "evil" agent will choose the most unfavorable distribution in this set, and the household will act accordingly. To choose the worst-case distribution, the "evil" agent seeks to minimize the utility of the decision maker under an entropy constraint which defines the size of the set of alternative models, and imposes a bound on the allowed discrepancy between the distorted and the approximating measures. Hansen and Sargent (2005) propose an alternative formulation of this problem in which the entropy constraint is added to the utility of the agent to form a modified objective function[27]

$$(20) \qquad E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln c_t\right\} + \theta E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \beta E_t(g_{t+1} \ln g_{t+1})\right\},$$

where $\theta > 0$ is a penalty parameter on discounted relative entropy, and where g_t is the martingale increment with the properties that

$$(21) \qquad G_{t+1} = g_{t+1} G_t$$

$$(22) \qquad E_t g_{t+1} = 1.$$

The problem of the "evil" agent becomes that of choosing the path $\{g_t\}$ to minimize (20) under the constraints (21) and (22). Higher values of θ imply less fear

[27] Hansen and Sargent (2005) label this preference structure "multiplier preferences." See Strzalecki (2011) for a complete axiomatization.

of model misspecification, because the "evil" agent is penalized more by raising entropy when minimizing the utility of the decision maker. When θ goes to infinity, the optimal choice of the "evil" agent is to set $g_{t+1} = 1$ at all times, meaning that the optimal distortion is zero. The rational expectations equilibrium with standard log utility of model (i) is nested under this limiting case. In this context, the decision maker chooses sequences for consumption and portfolio shares to maximize (20), taking into account the minimizing action of the "evil" agent.

In model (iii), we consider a particular form of Epstein-Zin preferences with unitary intertemporal elasticity of substitution. Here, the indirect utility, v_t, can be written in a recursive form

$$v_t = c_t^{1-\beta}\left(\left[E_t(v_{t+1})^{1-\gamma}\right]^{1/(1-\gamma)}\right)^{\beta}, \tag{23}$$

in which γ, with $\gamma > 0$, measures risk aversion and the attitude toward the temporal resolution of uncertainty.

It is established in the literature (see among others Barillas et al. 2009) that the solution of the inner minimization problem of model (ii), in which the "evil" agent chooses distortions to minimize the utility of the decision maker, implies a transformation of the original utility function (20) into a nonexpected recursive utility function of the form (23), where the parameter γ is the following monotonic transformation of θ:[28]

$$\gamma = 1 + \frac{1}{(1 - \beta)\theta}. \tag{24}$$

It follows that under this restriction, and in the range $\gamma \geq 1$, models (ii) and (iii) imply the same equilibrium allocations of quantity and prices.[29] Moreover, model (i) will coincide with (ii) and (iii) under the assumptions: $\rho = 1$, $\theta \rightarrow \infty$, $\gamma = 1$ for each respective model. However, we will show that model (i) implies the same steady-state portfolio allocation of models (ii) and (iii), conditional on the same processes for the excess return, real exchange rate risk, and labor-income risk, when $\rho = \gamma$ and θ is consistent with (24) for the same γ.

B. *The Model Economy*

The consumption index c is a CES aggregator of domestic (c_H) and imported (c_F) goods:

$$c \equiv \left[n^{1/\vartheta}(c_H)^{(\vartheta-1)/\vartheta} + (1 - n)^{1/\vartheta}(c_F)^{(\vartheta-1)/\vartheta}\right]^{\vartheta/(\vartheta-1)},$$

[28] Barillas et al. (2009) discuss the relation between fear of model misspecification and the class of risk-adjusted preferences described in Kreps and Porteus (1978) and Epstein and Zin (1989). See also Strzalecki (2009) for an analysis on how models of ambiguity aversion imply different preferences for the timing resolution of uncertainty.

[29] In the range $0 < \gamma < 1$, instead, model (ii) is not defined.

in which n, with $0 < n < 1$, is the weight given to the consumption of domestic goods; and ϑ, with $\vartheta > 0$, is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes c_H and c_F are Dixit-Stiglitz aggregators of the continuum of differentiated goods produced in country H and F, respectively:

$$c_H \equiv \left[\int_0^1 c(h)^{(\sigma_t-1)/\sigma_t} dh\right]^{\sigma_t/(\sigma_t-1)} \qquad c_F \equiv \left[\int_0^1 c(f)^{(\sigma_t-1)/\sigma_t} df\right]^{\sigma_t/(\sigma_t-1)},$$

where σ_t is the time-varying elasticity of substitution across the continuum of measure one of goods produced in each country, with $\sigma_t > 1$, for all t. The appropriate consumption-based price indices expressed in units of the domestic currency are defined as

$$P \equiv \left[n(P_H)^{1-\vartheta} + (1 - n)(P_F)^{1-\vartheta}\right]^{1/(1-\vartheta)}, \tag{25}$$

with

$$P_H \equiv \left[\int_0^1 p(h)^{1-\sigma_t} dh\right]^{1/(1-\sigma_t)} \qquad P_F \equiv \left[\int_0^1 p(f)^{1-\sigma_t} df\right]^{1/(1-\sigma_t)}.$$

A similar structure of preferences holds for the foreign agent marked with the appropriate asterisks. In particular, the weight n^* in the foreign consumption index might not be equal to n. In the case in which $n > n^*$, home bias in consumption arises. More generally, when $n \neq n^*$, there are deviations from purchasing power parity and fluctuations in the real exchange rate, even if the law-of-one price holds for each traded good. In our model, this is a possible source of risk to be hedged through portfolio choices.[30]

In each country, there is a continuum of firms of measure one producing the goods in a monopolistic-competitive market. A domestic firm of type h has a constant-return-to-scale production technology $y_t(h) = Z_t^{\phi} l_t^{1-\phi}$, where Z_t is a natural resource available in the country and l_t denotes labor which is employed at the wage rate W_t; ϕ is a parameter with $0 < \phi \leq 1$. When $\phi = 1$, the model collapses to an endowment economy. The variables Z_t and Z_t^*, in the foreign economy, are exogenous stochastic processes.

Prices are set without frictions and the law-of-one price holds. Equilibrium implies that prices are equalized across all firms within a country and set as a time-varying markup μ_t, with $\mu_t \equiv \sigma_t/[(\sigma_t - 1)(1 - \phi)] > 1$, over nominal marginal costs

$$P_{H,t} = \mu_t \frac{W_t l_t}{y_{H,t}},$$

[30] Notice that we could have alternatively modeled fluctuations in the real exchange rate through deviations from the law-of-one price without affecting our conclusions.

implying that the wage payments are inversely related to the markup:

$$W_t l_t = \frac{P_{H,t} y_{H,t}}{\mu_t}.$$

Firms make profits and distribute them in the form of dividends. The aggregate dividends in the domestic economy are given by

$$D_{H,t} = P_{H,t} y_{H,t} - W_t l_t = \frac{(\mu_t - 1)}{\mu_t} P_{H,t} y_{H,t},$$

which displays a positive correlation between dividends and the markup. The existence of nondiversifiable labor income is another source of risk to be hedged through portfolio choices.[31] The markups, μ_t and μ_t^*, in the foreign economy are exogenous stochastic processes and allow for a possible negative correlation between labor income and equity returns.[32]

The market for foreign goods works in a similar way with the appropriate modifications.

There are two equity markets—one for each country—with shares that are traded internationally. The market prices for equity shares in local currency are $V_{H,t}$ and $V_{F,t}^*$ for the domestic and foreign country, respectively. Households can also trade in two risk-free nominal bonds, denominated in units of the two currencies. The flow-budget constraint of the domestic agent is

$$(26)\quad B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V_{F,t}^* \leq R_{H,t}^b B_{H,t-1} + S_t R_{F,t}^{b^*} B_{F,t-1} + x_{H,t-1}(V_{H,t} + D_{H,t}) + x_{F,t-1} S_t (V_{F,t}^* + D_{F,t}^*) + W_t l_t - P_t c_t,$$

in which $B_{H,t}$ and $B_{F,t}$ are the amounts of one-period nominal bonds, in units of the two currencies, held at time t; $R_{H,t}^b$ and $R_{F,t}^{b^*}$ are the risk-free returns from period $t-1$ to period t, in the respective currencies; $x_{H,t}$ and $x_{F,t}$ are the shares of the domestic and foreign equity, respectively, held by the domestic agent. Finally S_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. The flow-budget constraint (26) can be written in a more compact form and in real terms (in units of the domestic consumption index) as

$$(27)\quad a_t = r_{p,t} a_{t-1} + \xi_t - c_t,$$

where we have defined

$$a_t \equiv \frac{B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V_{F,t}^*}{P_t}$$

[31] When $\phi = 1$, we are in a pure endowment economy, in which all income is diversifiable. In this case, μ_t goes to infinity.

[32] Markup shocks can fall in the category of redistributive shocks, discussed by Coeurdacier, Kollmann, and Martin (2008) and Coeurdacier and Gourinchas (2009).

and

$$r_{p,t} = \alpha^b_{H,t-1} r^b_{H,t} + \alpha^b_{F,t-1} r^{b^*}_{F,t} \frac{q_t}{q_{t-1}} + \alpha^e_{H,t-1} r^e_{H,t} + \alpha^e_{F,t-1} r^{e^*}_{F,t} \frac{q_t}{q_{t-1}}.$$

In the definition above, $\alpha^b_{H,t}$, $\alpha^b_{F,t}$, $\alpha^e_{H,t}$, $\alpha^e_{F,t}$ represent the shares of wealth that the domestic agent invests in the domestic bond, foreign bond, domestic equity, and foreign equity, respectively, satisfying the following restriction:

$$\alpha^b_{H,t} + \alpha^b_{F,t} + \alpha^e_{H,t} + \alpha^e_{F,t} = 1. \tag{28}$$

Moreover $r^b_{H,t}$, $r^{b^*}_{F,t}$, $r^e_{H,t}$, and $r^{e^*}_{F,t}$ are the respective real returns.[33] The variable ξ_t denotes nondiversifiable real labor income, defined as $\xi_t \equiv W_t l_t / P_t$, and q_t is the real exchange rate defined as $q_t \equiv S_t P^*_t / P_t$.

The domestic agents optimization problem is to choose consumption and the portfolio allocations to maximize their intertemporal objective function—(18) for model (i), (20) for model (ii), and (23) for model (iii)—under the flow-budget constraint (27) and appropriate no-Ponzi game conditions.

C. *Optimality Conditions*

The optimality condition with respect to consumption implies an orthogonality condition, in expectation, between the real stochastic discount factor and the real portfolio return

$$\tilde{E}_t(m_{t+1} r_{p,t+1}) = 1. \tag{29}$$

A similar condition applies to the foreign economy:

$$\tilde{E}^*_t(m^*_{t+1} r^*_{p,t+1}) = 1. \tag{30}$$

The optimality conditions with respect to the portfolio allocation imply a set of four restrictions for each agent, one for each asset, given by:

$$\tilde{E}_t(m_{t+1} r^b_{H,t+1}) = 1, \qquad \tilde{E}^*_t\left(m^*_{t+1} r^b_{H,t+1} \frac{q_t}{q_{t+1}}\right) = 1, \tag{31}$$

$$\tilde{E}_t\left(m_{t+1} r^{b^*}_{F,t+1} \frac{q_{t+1}}{q_t}\right) = 1, \qquad \tilde{E}^*_t(m^*_{t+1} r^{b^*}_{F,t+1}) = 1, \tag{32}$$

$$\tilde{E}_t(m_{t+1} r^e_{H,t+1}) = 1, \qquad \tilde{E}^*_t\left(m^*_{t+1} r^e_{H,t+1} \frac{q_t}{q_{t+1}}\right) = 1, \tag{33}$$

$$\tilde{E}_t\left(m_{t+1} r^{e^*}_{F,t+1} \frac{q_{t+1}}{q_t}\right) = 1, \qquad \tilde{E}^*_t(m^*_{t+1} r^{e^*}_{F,t+1}) = 1. \tag{34}$$

[33] Lower case variables denote the real counterpart of the respective upper case variable. Please refer to the Appendix for details on the derivations and definitions.

The above formulation of the orthogonality conditions nest the three alternative preference specifications. Model (i) implies $\tilde{E}_t(\cdot) = \tilde{E}_t^*(\cdot) = E_t(\cdot)$, and the domestic and foreign stochastic discount factors are defined by

$$(35) \qquad m_{t+1} \equiv \beta\left(\frac{c_t}{c_{t+1}}\right)^{\rho} \qquad m_{t+1}^* \equiv \beta\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\rho}.$$

In model (ii), the subjective expectation operators apply, and the stochastic discount factors are given by

$$(36) \qquad m_{t+1} \equiv \beta\frac{c_t}{c_{t+1}} \qquad m_{t+1}^* \equiv \beta\frac{c_t^*}{c_{t+1}^*}.$$

In model (iii), instead, $\tilde{E}_t(\cdot) = \tilde{E}_t^*(\cdot) = E_t(\cdot)$, and the stochastic discount factors are defined by

$$(37) \qquad m_{t+1} = \beta\frac{c_t}{c_{t+1}}\left(\frac{v_{t+1}^{1-\gamma}}{\left[E_t(v_{t+1})^{1-\gamma}\right]}\right) \qquad m_{t+1} = \beta\frac{c_t^*}{c_{t+1}^*}\left(\frac{v_{t+1}^{*1-\gamma}}{\left[E_t(v_{t+1}^*)^{1-\gamma}\right]}\right).$$

Notice, moreover, that in model (ii) the martingale increments g_{t+1} and g_{t+1}^* provide a mapping between conditional expectations under the (country-specific) "distorted" measure and the (common) "approximating" one:

$$(38) \qquad \tilde{E}_t(X_{t+1}) = E_t(g_{t+1}X_{t+1}) \qquad \tilde{E}_t^*(X_{t+1}) = E_t(g_{t+1}^*X_{t+1}),$$

for a generic random variable X_{t+1}. Such distortions in beliefs are chosen optimally by the "evil" agent, and are such that

$$(39) \qquad g_{t+1} = \left(\frac{v_{t+1}^{1-\gamma}}{\left[E_t(v_{t+1})^{1-\gamma}\right]}\right) \qquad g_{t+1}^* = \left(\frac{v_{t+1}^{*1-\gamma}}{\left[E_t(v_{t+1}^*)^{1-\gamma}\right]}\right),$$

as shown, among others, by Barillas, Hansen, and Sargent (2009). Equation (39), therefore, implies that models (ii) and (iii) share the same set of orthogonality conditions (29)–(34), once they are evaluated under the "reference" probability measure.

Equilibrium in the goods market requires the production of each good to be equal to world consumption

$$y_{H,t} = c_{H,t} + c_{H,t}^* \qquad y_{F,t}^* = c_{F,t} + c_{F,t}^*.$$

The labor markets are in equilibrium at the exogenously supplied quantities of labor

$$l_t = \bar{l}_t \qquad l_t^* = \bar{l}_t^*,$$

where $\overline{l}_t$ and $\overline{l}_t^*$ are exogenous stochastic processes. Bonds are in zero-net supply worldwide:

$$B_{H,t} + B_{H,t}^* = B_{F,t} + B_{F,t}^* = 0.$$

Equity shares sum to one:

$$x_{H,t} + x_{H,t}^* = x_{F,t} + x_{F,t}^* = 1.$$

Given the path of the stochastic disturbances $\{\overline{l}_t, \overline{l}_t^*, Z_t, Z_t^*, \mu_t, \mu_t^*\}$, an equilibrium is an allocation of quantities $\{c_t, c_{H,t}, c_{F,t}, c_t^*, c_{H,t}^*, c_{F,t}^*, \alpha_{H,t}^b, \alpha_{F,t}^b, \alpha_{H,t}^e, \alpha_{F,t}^e, \alpha_{H,t}^{b^*}, \alpha_{F,t}^{b^*}, \alpha_{H,t}^{e^*}, \alpha_{F,t}^{e^*}, a_t, a_t^*\}$ and prices $\{r_{H,t}^b, r_{F,t}^{b^*}, r_{H,t}^e, r_{F,t}^{e^*}, q_t, P_{H,t}/P_{F,t}, W_t/P_t, W_t^*/P_t^*\}$ such that each agent's consumption, portfolio shares, and wealth are optimal given prices, and goods, labor, and asset markets are in equilibrium.

Although we have written a general equilibrium model, in the next section, we show that we do not really need to solve the entire model to understand the determinants of the portfolio allocation. Instead, we can isolate a block of the general equilibrium conditions to determine the portfolio shares $\{\alpha_{H,t}^b, \alpha_{F,t}^b, \alpha_{H,t}^e, \alpha_{F,t}^e, \alpha_{H,t}^{b^*}, \alpha_{F,t}^{b^*}, \alpha_{H,t}^{e^*}, \alpha_{F,t}^{e^*}\}$ by taking as given the path of returns $\{r_{H,t}^b, r_{F,t}^{b^*}, r_{H,t}^e, r_{F,t}^{e^*}\}$, the real exchange rate q_t, and the processes of nondiversifiable labor incomes $\{\xi_t, \xi_t^*\}$. The optimal portfolio allocation, therefore, depends on the co-movements between these sources of risk.[34]

D. *Portfolio Choices with Isoelastic Preferences*

Under model (i), we can characterize the equilibrium portfolio allocation combining equations (31)–(34) to obtain the following set of orthogonality conditions:

$$E_t\left[\left(m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}}\right)\left(r_{H,t+1}^e - r_{H,t+1}^b\right)\right] = 0 \tag{40}$$

$$E_t\left[\left(m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}}\right)\left(r_{F,t+1}^{e^*} \frac{q_{t+1}}{q_t} - r_{H,t+1}^e\right)\right] = 0 \tag{41}$$

$$E_t\left[\left(m_{t+1} - m_{t+1}^* \frac{q_t}{q_{t+1}}\right)\left(r_{F,t+1}^{b^*} \frac{q_{t+1}}{q_t} - r_{H,t+1}^b\right)\right] = 0, \tag{42}$$

where the stochastic discount factors are given in (35). The above conditions require the cross-country difference in the real stochastic discount factors—evaluated in terms of domestic consumption—to be orthogonal to three relevant excess returns: the excess return of domestic equity on domestic bonds, the excess return on foreign

[34] Empirical restrictions of this kind should apply to any general equilibrium model of international portfolio allocation. Indeed, recent papers in the literature characterize the optimal portfolio allocation in terms of primitive parameters or shocks, yet without considering such empirical restrictions. Along this dimension, they would be less successful. See van Wincoop and Warnock (2010) and Coeurdacier and Gourinchas (2009) for a related argument and for models that are, instead, evaluated under data restrictions.

versus domestic equity, and the excess return on foreign versus domestic bonds. Indeed, conditions (40)–(42) and restriction (28) are sufficient to characterize the equilibrium portfolio allocation.

Given the assumption of incomplete markets, we cannot solve for the optimal portfolio allocation in nonlinear closed form.[35] However, we can still derive many insights by using the approximation methods developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010). As a first step, we solve for the paths of consumption and wealth, given returns and the steady-state portfolio shares, using a *first-order* approximation of the Euler equations and the budget constraints. In particular, letting variables with hats denote log deviations from the steady state and variables with upper bars denote the steady-state level, this yields

$$(43)\quad \hat{m}_{t+1} + \Delta\hat{q}_{t+1} - \hat{m}^*_{t+1} = -(\rho\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1}) = -\rho\frac{(1-\beta)}{\beta s_c}\overline{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - \rho\frac{s_\xi}{s_c}\varepsilon_{l,t+1} - (\rho - 1)\varepsilon_{q,t+1},$$

where a superscript R denotes the difference between the domestic and the respective foreign variable; s_ξ is the steady-state ratio between nontraded income and financial wealth, which is common across countries and given by $s_\xi \equiv \overline{\xi}/\overline{a}$; and s_c is the steady-state ratio between consumption and financial wealth and such that $s_c = (1-\beta)/\beta + s_\xi$; the vector of excess returns $\mathbf{exr}_{t+1}$ is defined in (8), (9), and (10); labor-income risk and real exchange rate risk are defined in (11) and (12), respectively. Moreover, $\overline{\boldsymbol{\lambda}} \equiv \overline{\boldsymbol{\alpha}} - \overline{\boldsymbol{\alpha}}^*$ captures the cross-country difference in optimal portfolios and is given by[36]

$$(44)\qquad \overline{\boldsymbol{\lambda}} = \begin{bmatrix} 2(\overline{\alpha}^e_H + \overline{\alpha}^e_F) - 2 \\ 2\overline{\alpha}^e_F - 1 \\ 2\overline{\alpha}^b_F \end{bmatrix}.$$

Equation (43) captures the familiar hedging motive, related to the wedge between the marginal utilities of consumption, and is the equivalent to equation (3), with the coefficients now defined by

$$m_e \equiv -\rho\frac{1-\beta}{\beta s_c} \qquad m_l \equiv -\rho\frac{s_\xi}{s_c} \qquad m_q \equiv 1-\rho,$$

and where clearly $\overline{\boldsymbol{\lambda}}^{full} = \mathbf{0}$.

[35] van Wincoop and Warnock (2010) work with a closed-form solution, but in a partial-equilibrium, two-period model. Coeurdacier and Gourinchas (2009), Coeurdacier, Kollmann, and Martin (2008), Heathcote and Perri (2009), and Kollmann (2006) obtain closed-form solutions by assuming that markets are locally complete.

[36] In a symmetric steady state, where $\overline{A} = \overline{S}\,\overline{A}^*$, it can be shown that $\overline{\alpha}^{e*}_i = 1 - \overline{\alpha}^e_i$ and $\overline{\alpha}^{b*}_i = -\overline{\alpha}^b_i$, for $i = H,F$.

As a second step, we use equation (43) and a *second-order* approximation of the orthogonality conditions (40)–(42) to determine the steady-state portfolio shares as a function of prices, returns, and nondiversifiable labor income.[37] The optimal steady-state portfolio shares, hence, satisfy the following equation:

$$\overline{\boldsymbol{\lambda}} = -s_{\xi}\frac{\beta}{1-\beta}\Sigma_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{l,t+1}) - s_c\frac{(\rho-1)}{\rho}\frac{\beta}{1-\beta}\Sigma_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{q,t+1}). \tag{45}$$

The above equation coincides with equation (1) discussed in Section II, when indeed ρ replaces γ.

E. *Portfolio Choices under Model Uncertainty and Epstein-Zin Preferences*

Since models (ii) and (iii) are equivalent, we are going to focus only on the derivation of the equilibrium portfolio allocations under model (ii). Under model uncertainty conditional expectations under the "reference" and "subjective" distributions are linked by the mapping of (38); therefore, we can write conditions (40)–(42) as

$$E_t\left[\left(g_{t+1}m_{t+1} - g_{t+1}^{*}m_{t+1}^{*}\frac{q_t}{q_{t+1}}\right)\left(r_{H,t+1}^{e} - r_{H,t+1}^{b}\right)\right] = 0$$

$$E_t\left[\left(g_{t+1}m_{t+1} - g_{t+1}^{*}m_{t+1}^{*}\frac{q_t}{q_{t+1}}\right)\left(r_{F,t+1}^{e^*}\frac{q_{t+1}}{q_t} - r_{H,t+1}^{e}\right)\right] = 0$$

$$E_t\left[\left(g_{t+1}m_{t+1} - g_{t+1}^{*}m_{t+1}^{*}\frac{q_t}{q_{t+1}}\right)\left(r_{F,t+1}^{b^*}\frac{q_{t+1}}{q_t} - r_{H,t+1}^{b}\right)\right] = 0.$$

The above set of equations implies the three restrictions needed to determine the portfolio allocation. In a second-order approximation, and in compact form, they read as

$$E_t[(\hat{g}_{t+1}^{R} - (\Delta\hat{c}_{t+1}^{R} - \Delta\hat{q}_{t+1}))\cdot\mathbf{exr}_{t+1}] = \mathbf{0}. \tag{46}$$

With model uncertainty and fears of model misspecification, the cross-country difference in the appetite for wealth is given by the sum of a first hedging motive, described by the wedge between the marginal utilities of consumption, and a second hedging motive given by the wedge between the distortions in the subjective beliefs.

[37] Details of the derivation are shown in the Appendix.

Since we assume logarithmic utility, the first hedging motive implies only two sources of risk:

$$(47)\qquad \hat{m}_{t+1} + \Delta\hat{q}_{t+1} - \hat{m}^*_{t+1} = -(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})$$
$$= -\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - \frac{s_\xi}{s_c}\varepsilon_{l,t+1},$$

and no role for the real-exchange-rate risk.

The second hedging motive depends on the factor $\hat{g}^R_{t+1}$, which measures the cross-country difference between the subjective distortions. Since g_{t+1} and g^*_{t+1} are given by (39) and continuation values scaled by consumption can be written as

$$\frac{v_t}{c_t} = \left[E_t\left(\frac{v_{t+1}}{c_{t+1}}\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\right]^{\beta/(1-\gamma)},$$

it is easy to see that g_{t+1} and g^*_{t+1} can be related to the current and future consumption path. In particular, in a first-order approximation, which suffices to evaluate (46), we can write

$$\hat{g}_{t+1} = -(\gamma - 1)\sum_{k=0}^{\infty}\beta^k\Delta E_{t+1}\Delta\hat{c}_{t+1+k},$$

in which $\hat{g}_{t+1}$ increases when the agent fears bad news with respect to the consumption-growth profile.[38] Higher values of g_{t+1} imply that the agent is assigning a higher probability to those states of nature where there is bad news on the consumption-growth profile. When g_{t+1} increases, the appetite for receiving additional wealth increases as well. In this case, the agent would like to hold assets that pay well when there is indeed bad news about the consumption-growth profile. The above derivations apply also to the foreign agent. In the symmetric case in which $\gamma = \gamma^*$, we can show that the optimal relative distortion depends negatively on the surprises in the consumption-growth differential across countries:

$$(48)\qquad \hat{g}^R_{t+1} = -(\gamma - 1)\sum_{k=0}^{\infty}\beta^k\Delta E_{t+1}\Delta\hat{c}^R_{t+1+k}$$
$$= -(\gamma - 1)\left[\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} + \frac{s_\xi}{s_c}\varepsilon_{l,t+1} + \varepsilon_{q,t+1}\right],$$

where the second equality substitutes for relative consumption growth using a first-order approximation of the Euler equations and the budget constraints. Equation (48) corresponds to equation (4).

[38] Hansen, Heaton, and Li (2008) show how to derive g_{t+1} as a closed-form solution including risk-premia terms, which, however, are not important in our approximation for computing the steady-state portfolio shares.

Under model uncertainty and fears of model misspecification, the cross-country difference in the appetite for wealth is given by the sum of (47)—capturing the first hedging motive described by the wedge between the standard marginal utilities of consumption—and (48)—capturing the additional hedging motive given by the wedge between the subjective distortions.

As shown by equaton (48), this second hedging motive is driven by three sources of risk: fluctuations in the excess returns, the labor-income risk, and fluctuations in the real exchange rate. As argued in Section II, the coefficients related to excess returns and labor-income risk are the same for the two hedging motives: $m_e = g_e = -(1 - \beta)/\beta s_C$ and $m_l = g_l = -s_\xi/s_c$. It follows that, absent real exchange rate risk, the optimal portfolio allocation under model uncertainty is the same as the one under no model uncertainty. The degree of ambiguity aversion γ would be irrelevant in this case. On the contrary, the coefficient on real exchange rate risk is different across the two components: $m_q = 0$ and $g_q = -1$. As a consequence, under log-utility, ambiguity aversion implies the existence of an additional channel driving the optimal steady-state portfolio shares. Such additional channel depends on the covariances between the excess returns and the surprises in the real exchange rate, and on the degree of ambiguity aversion γ.

Notice that the sum of (47) and (48) implies (43) when γ is replaced by ρ. It follows that also model (ii) implies the steady-state portfolio allocation (45), when γ replaces ρ. Therefore, models (i), (ii), and (iii) are equivalent with respect to their steady-state portfolio implications, conditionally on the same processes for the excess returns, labor-income, and RER risks.

F. *Calibrating γ Using Detection Error Probabilities*

An appealing feature of model (ii), compared to model (iii), is that the parameter γ, which measures the concerns for model misspecification, can be calibrated within the model, using its stochastic properties. We follow Anderson, Hansen, and Sargent (2003) and Hansen and Sargent (2007) in using detection error probabilities.

Let us call model A the approximating model and model $B(\gamma)$ the worst-case model associated with a specific γ. Agents start with the belief that the models are equally likely. That is, they assign 50 percent prior probability to each model. After having seen T observations, they can perform a likelihood ratio test for distinguishing the two models. Under the hypothesis that model A is correct, we denote, with $p_A(\gamma)$, the probability that a likelihood ratio test would instead falsely say that model $B(\gamma)$ generated the data. Conversely, we denote, with $p_B(\gamma)$, the probability that a likelihood ratio test would falsely say that model A generated the data, when in fact model $B(\gamma)$ is correct. The detection error probability, then, is the weighted average of $p_A(\gamma)$ and $p_B(\gamma)$ with the weights given by the prior probabilities:

$$p(\gamma) = \frac{1}{2}(p_A(\gamma) + p_B(\gamma)).$$

The detection error probability is a decreasing function of γ, since a larger γ (and therefore a smaller θ) implies a lower penalization upon relaxing the entropy constraint in equation (20). Indeed, a higher γ implies a wider entropy ball, inside which the consumer allows the evil agent to choose the worst-case distortion, and ultimately a consumer more afraid of misspecification. Accordingly, higher values of γ imply a larger divergence between the worst-case model and the approximating one, and is therefore less probable that the likelihood-ratio test will favor the wrong model. When $\gamma = 1$, on the contrary, the two models are equivalent and $p(\gamma)$ is therefore equal to $1/2$.

It is important to notice that the mapping between γ and $p(\gamma)$, is model-specific and varies in different contexts. This is why the plausibility of a given value of γ, as a measure of the concern about model misspecification, should be appropriately determined in terms of the detection error probability that it implies, which can instead be regarded as a context-invariant measure.

In our context the approximating model is given by the VAR in (14):

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t,$$

where $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. We can show that the worst-case models, associated with specific values of γ and γ^*, imply a distortion in the mean of the VAR, and take the form

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{w}(\gamma) + \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t \tag{49}$$

for consumers in country H and

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{w}^*(\gamma^*) + \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t \tag{50}$$

for those in country F, where $\mathbf{w}(\gamma)$ and $\mathbf{w}^*(\gamma^*)$ are the optimal distortions in the mean.[39]

We simulated 100,000 samples, each of size 112 observations (corresponding to the sample 1980:1–2007:IV that we use in the VAR estimation), and computed the detection error probabilities associated with the approximating and the worst-case models, by varying the parameters γ and γ^*. The results are displayed in Figure 7.

Figure 7 then shows the detection error probabilities, $p(\gamma)$ and $p(\gamma^*)$, plotted against γ and γ^*, and the associated discounted relative entropies, defined by:

$$\eta_{t_0} \equiv E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t E_t(g_{t+1} \ln g_{t+1})\right\},$$

and analogously for $\eta^*_{t_0}$.

We follow Anderson, Hansen, and Sargent (2003), Maenhout (2006), and Barillas et al. (2009), and consider alternative models whose detection error probabilities are

[39] Please refer to the Appendix for details on the derivation.

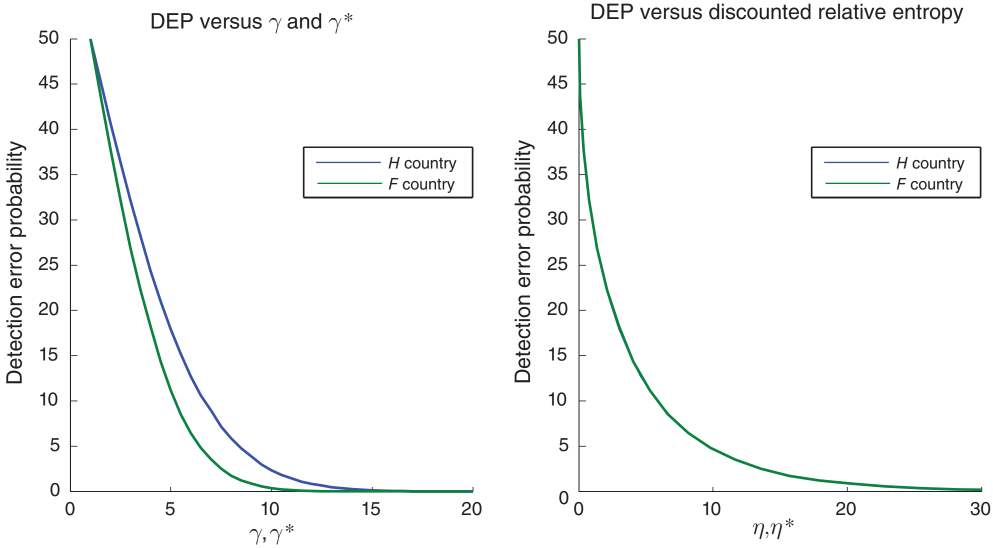

Figure 7. Detection Error Probabilities (DEP) versus Fear of Model Misspecification (γ *and* γ ***, *left panel*) and versus Discounted Conditional Relative Entropy (η *and* η^*, *right panel*)

around 10 percent as "difficult to detect." Figure 6 has shown that values of γ or γ^* around five are sufficient to get the most of the model fit in terms of home-bias in equity holdings. Figure 7 then shows that values of γ and γ^* between 5 and 7 are still associated with detection error probabilities around 0.10.

The degree of ambiguity aversion needed to explain the empirical facts is therefore consistent with conservative values of the detection error probabilities, thus validating the empirical relevance of the model's implications. Given that the left panel shows that for similar detection error probability γ and γ^* are very close, we can also conclude that the assumption $\gamma = \gamma^*$ is generally innocuous.[40]

IV. Conclusions

The observation that international investors hold a disproportionate share of their wealth in domestic rather than foreign assets is one of the most persistent facts in international finance. This is named the international home-bias puzzle, which the literature has been dealing with for a couple of decades.

This paper revives an old argument, hedging real exchange rate risk, within a dynamic general equilibrium model of portfolio and consumption choices, with incomplete markets, under three alternative preference specifications: a standard model with isoelastic utility, a model with Hansen-Sargent preferences for robustness, and a model with Epstein-Zin preferences. We show that these three models share the same implications for international portfolios. We characterize them in terms of covariances between measurable sources of risk to be hedged (nondiversifiable

[40] At the threshold value of 10 percent, the values for γ and γ^* are, respectively, 7.5 and 5.5.

labor-income risk and real exchange rate risk) and a vector of cross-country excess returns, and evaluate their empirical relevance using financial and macro data on the G7 countries.

From an empirical perspective, our results suggest that, contrary to what has been claimed in recent related contributions, hedging nondiversifiable labor-income risk does not seem sufficient to account for the lack of international portfolio diversification. Indeed, in a setting in which equity is the only available asset, correlations in financial data support a large foreign-equity bias, as in Baxter and Jermann (1997). Adding further assets does not help in identifying a clear role for this risk in explaining the home-bias puzzle, once the former is measured in a model-consistent way. On the other hand, hedging real exchange rate risk is able to generate a substantial equilibrium home bias in equity holdings. Most importantly, the models with Hansen-Sargent and Epstein-Zin preferences are not subject to the risk-free rate puzzle. Moreover, we show that the relevant notion of real exchange rate risk in a dynamic model is not simply related to one-period changes (short-run risk), but rather to news about the full path of the RER (long-run risk). This distinction turns out to be empirically relevant; the hedging properties of domestic equities, which were shown to be very weak toward short-run RER risk, substantially improve when evaluated with respect to longer time horizons.

From a theoretical perspective, we show that the relevance of the RER risk depends crucially on a structural parameter, γ, whose economic interpretation marks the difference among the competing models. Indeed, although the three models share the same implications for the optimal portfolio allocation, the theoretical mechanisms behind the relevance of the real exchange rate risk are of completely different nature. In model (i), the stronger relevance of RER risk implied by a higher γ is due to the lower elasticity of intertemporal substitution that it implies (equal to $1/\gamma$), which introduces a relative inflation risk that reduces current domestic consumption (relative to foreign) when the RER appreciates. In model (ii), instead, an increase in γ means a higher degree of ambiguity aversion, which introduces a cross-country wedge in the subjective probability measures, in the form of downward revisions in current domestic consumption (relative to foreign). In model (iii), finally, a higher γ captures a preference for an earlier resolution of uncertainty, which implies that revisions in the future consumption profile widen the cross-country difference in the subjective stochastic discount factors that the agents seek to hedge.

The methodological contribution of the paper goes beyond the analysis of the home-bias puzzle. The class of preferences that we suggest, in fact, produces a perturbation of the equilibrium stochastic discount factor, which decouples the attitudes toward intertemporal substitution with those toward ambiguity, and can prove useful in addressing other failures of standard preference specifications along the asset-price dimension.[41] Indeed, it has been shown in closed-economy settings that disentangling the elasticity of intertemporal substitution from the degree of risk aversion helps in accounting for the equity premium puzzle. Once

[41] See for a discussion Backus, Routledge, and Zin (2005).

we open the economy to international trade in assets, there are additional puzzling features of financial data, among which the foreign equity- and bond-premia puzzles and the Backus-Smith anomaly are notable examples.[42] All these stylized facts imply restrictions on the stochastic discount factor that standard preferences cannot meet at the same time, and that might be all reconnected to some common misspecification.[43] The modification of the stochastic discount factor that our preference specification implies is a promising tool to correct this misspecification and build macro models whose predictions are closer to the empirical implications of financial data.

Appendix

A. *Some Useful Definitions*

To get equation (27), we have defined

$$A_t \equiv B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V^*_{F,t},$$

and

$$R_{p,t} \equiv \alpha^b_{H,t-1} R^b_{H,t} + \alpha^b_{F,t-1} R^{b^*}_{F,t} \frac{S_t}{S_{t-1}} + \alpha^e_{H,t-1} R^e_{H,t} + \alpha^e_{F,t-1} R^{e^*}_{F,t} \frac{S_t}{S_{t-1}},$$

with

$$R^e_{H,t} \equiv \frac{V_{H,t} + D_{H,t}}{V_{H,t-1}},$$

$$R^{e^*}_{F,t} \equiv \frac{V^*_{F,t} + D^*_{F,t}}{V^*_{F,t-1}},$$

and

$$B_{H,t} \equiv \alpha^b_{H,t} A_t,$$

$$S_t B_{F,t} \equiv \alpha^b_{F,t} A_t,$$

$$x_{H,t} V_{H,t} \equiv \alpha^e_{H,t} A_t,$$

$$x_{F,t} S_t V^*_{F,t} = \alpha^e_{F,t} A_t.$$

[42] Barillas et al. (2009) discusses the implications of model uncertainty for the equity premium puzzle; Piazzesi and Schneider (2007) studies the slope of the yield curve with Epstein-Zin preferences. Ilut (2008) studies how ambiguity aversion can help explain the uncovered-interest-rate puzzle.

[43] All excess-return puzzles, for example, imply "high" lower bounds on the volatility of the equilibrium stochastic discount factor, as discussed for the equity premium by Hansen and Jagannathan (1991).

Analogously for the foreign country:

$$B^*_{H,t} \equiv \alpha^{b^*}_{H,t} A^*_t S_t,$$

$$B^*_{F,t} \equiv \alpha^{b^*}_{F,t} A^*_t,$$

$$x^*_{H,t} V_{H,t} \equiv \alpha^{e^*}_{H,t} A^*_t S_t,$$

$$x^*_{F,t} V^*_{F,t} = \alpha^{e^*}_{F,t} A^*_t.$$

B. *The Equilibrium Portfolio with Isoelastic Preferences*

Here, we derive the optimal portfolio allocation for the case with isoelastic preferences, equation (45).

In what follows, a variable with an "upper-bar" denotes the symmetric steady state, and a "hat" denotes the log-deviation with respect to such steady state. A first-order approximation of the Euler conditions (29) and (30) implies

$$\rho E_t \Delta \hat{c}_{t+1} = E_t \hat{r}_{p,t+1}, \tag{A1}$$

$$\rho E_t \Delta \hat{c}^*_{t+1} = E_t \hat{r}^*_{p,t+1}. \tag{A2}$$

In particular, the portfolio returns can be approximated to first order as

$$\hat{r}_{p,t+1} = \hat{r}^{\,b}_{H,t+1} + \overline{\boldsymbol{\alpha}}' \mathbf{exr}_{t+1},$$

$$\hat{r}^*_{p,t+1} = \hat{r}^{\,b}_{H,t+1} + \overline{\boldsymbol{\alpha}}^{*\prime} \mathbf{exr}_{t+1} - \Delta \hat{q}_{t+1},$$

where we have defined

$$\overline{\boldsymbol{\alpha}} \equiv \begin{bmatrix} \overline{\alpha}^b_F \\ \overline{\alpha}^e_H + \overline{\alpha}^e_F \\ \overline{\alpha}^e_F \end{bmatrix} \qquad \overline{\boldsymbol{\alpha}}^* \equiv \begin{bmatrix} \overline{\alpha}^{b^*}_F \\ \overline{\alpha}^{e^*}_H + \overline{\alpha}^{e^*}_F \\ \overline{\alpha}^{e^*}_F \end{bmatrix}, \tag{A3}$$

and the vector of excess returns as

$$\mathbf{exr}_t \equiv \begin{bmatrix} \hat{r}^{\,b^*}_{F,t} + \Delta \hat{q}_t - \hat{r}^{\,b}_{H,t} \\ \hat{r}^{\,e}_{H,t} - \hat{r}^{\,b}_{H,t} \\ \hat{r}^{\,e^*}_{F,t} + \Delta \hat{q}_t - \hat{r}^{\,e}_{H,t} \end{bmatrix}.$$

In a first-order approximation, the no-arbitrage conditions imply that excess returns have zero conditional means, $E_t \mathbf{exr}_{t+1} = \mathbf{0}$. It follows, using equations (A1) and

(A2), that the cross-country differential in the expected consumption growth depends on the expected depreciation in the real exchange rate

$$\text{(A4)} \qquad \rho E_t \Delta \hat{c}^R_{t+1} = E_t \Delta \hat{q}_{t+1},$$

where an upper-script R denotes the difference between the domestic and foreign variables.

A first-order approximation of the flow budget constraint (27), together with the budget constraint of the foreign agent, implies

$$\text{(A5)} \qquad \beta \hat{a}^R_t = \hat{a}^R_{t-1} + \overline{\boldsymbol{\lambda}}' \mathbf{exr}_t + \Delta \hat{q}_t + \beta s_\xi \hat{\xi}^R_t - \beta s_C \hat{c}^R_t,$$

where s_ξ is the steady-state ratio between nontraded income and financial wealth, given by $s_\xi \equiv \overline{\xi}/\overline{a}$, which is equal in the two countries; s_c is the steady-state ratio between consumption and financial wealth and such that $s_c = (1 - \beta)/\beta + s_\xi$. Moreover, the vector $\overline{\boldsymbol{\lambda}}$ is defined as

$$\text{(A6)} \qquad \overline{\boldsymbol{\lambda}} \equiv \begin{bmatrix} 2\overline{\alpha}^b_F \\ 2(\overline{\alpha}^e_H + \overline{\alpha}^e_F) - 2 \\ 2\overline{\alpha}^e_F - 1 \end{bmatrix}.$$

The set of difference equations (A4) and (A5) can be solved forward to obtain relative consumption and relative wealth $(\hat{c}^R_t, \hat{a}^R_t)$ as a function of the states $(\hat{a}^R_{t-1}, \hat{q}_{t-1})$ and the processes of excess returns, relative non-diversifiable income and the real exchange rate $\{\mathbf{exr}_t, \hat{\xi}^R_t, \hat{q}_t\}$. In particular, we obtain

$$\text{(A7)} \quad (\rho \hat{c}^R_t - \hat{q}_t) = \rho \frac{(1-\beta)}{\beta s_c} (\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \rho \frac{(1-\beta)\xi}{\beta s_c} \overline{\boldsymbol{\lambda}}' \mathbf{exr}_t$$
$$+ \rho \frac{(1-\beta)}{s_c} E_t \sum_{T=t}^{\infty} \beta^{T-t} (\hat{\xi}^R_T - \hat{q}_T)$$
$$+ (1-\beta)(\rho - 1) E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{q}_T$$

and

$$\text{(A8)} \quad (\hat{a}^R_t - \hat{q}_t) = (\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \overline{\boldsymbol{\lambda}}' \mathbf{exr}_t + s_\xi (\hat{\xi}^R_t - \hat{q}_t) + s_c \frac{\rho - 1}{\rho} \hat{q}_t$$
$$- (1-\beta) s_\xi E_t \sum_{T=t}^{\infty} \beta^{T-t} (\hat{\xi}^R_T - \hat{q}_T)$$
$$- (1-\beta) s_c \frac{\rho - 1}{\rho} E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{q}_T.$$

Using the above, we can derive the hedging motive (43):

$$
\text{(A9)} \qquad -(\rho\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1}) = -\rho\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\,\mathbf{exr}_{t+1} - \rho\frac{s_\xi}{s_c}\varepsilon_{l,t+1} - (\rho-1)\varepsilon_{q,t+1},
$$

where $\varepsilon_{l,t+1}$ is defined as in (11) and $\varepsilon_{q,t+1}$ as in (12).

To determine the portfolio shares we use a second-order approximation of the moment conditions (40)–(42). In particular, we just need three restrictions to determine the vector $\bar{\boldsymbol{\lambda}}$:

$$
E_t[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^{\,e}_{H,t+1} - \hat{r}^{\,b}_{H,t+1})] = 0,
$$

$$
E_t[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^{\,e^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{\,e}_{H,t+1})] = 0.
$$

$$
E_t[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^{\,b^*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{\,b}_{H,t+1})] = 0,
$$

We can now use equation (A9) in the conditions above and solve for the steady-state vector of portfolio shares and obtain equation (45).

C. *The Equilibrium Portfolio with Ambiguity Aversion*

Under model uncertainty and ambiguity aversion, when risk aversion and intertemporal elasticity of substitution are unitary, it is still true that (A4) and (A5) hold, although now $\rho = 1$. This implies that the solution for relative consumption and relative wealth now reads

$$
\text{(A10)} \qquad (\hat{c}^R_t - \hat{q}_t) = \frac{(1-\beta)}{\beta s_c}(\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_t + \frac{(1-\beta)s_\xi}{s_c}E_t\sum_{T=t}^{\infty}\beta^{T-t}(\hat{\xi}^R_T - \hat{q}_T),
$$

$$
\text{(A11)} \qquad (\hat{a}^R_t - \hat{q}_t) = (\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \bar{\boldsymbol{\lambda}}'\mathbf{exr}_t + s_\xi(\hat{\xi}^R_t - \hat{q}_t) - (1-\beta)s_\xi E_t\sum_{T=t}^{\infty}\beta^{T-t}(\hat{\xi}^R_T - \hat{q}_T),
$$

and implies that the wedge between the SDF's is

$$
\text{(A12)} \qquad -(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1}) = -\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - \frac{s_\xi}{s_c}\varepsilon_{l,t+1},
$$

where there is no role for the RER $\varepsilon_{q,t+1}$.

However, we can use (A10) and (A11) to write (48) as

$$\hat{g}_{t+1}^{R} = -(\gamma - 1)\frac{(1 - \beta)}{\beta s_c}\overline{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - (\gamma - 1)\frac{s_\xi}{s_c}\varepsilon_{l,t+1} - (\gamma - 1)\varepsilon_{q,t+1}.$$

Therefore, the left-hand side of the orthogonality condition (46) can be written as

$$\hat{g}_{t+1}^{R} - (\Delta\hat{c}_{t+1}^{R} - \Delta\hat{q}_{t+1}) = -\gamma\frac{(1 - \beta)}{\beta s_c}\overline{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1}$$

$$- \gamma\frac{s_\xi}{s_c}\varepsilon_{l,t+1} - (\gamma - 1)\varepsilon_{q,t+1},$$

from which it follows that (46) implies equation (45), in which, indeed, γ replaces ρ.

D. *Derivation of Equations* (49)–(50)

Consider the companion form of the approximating model

$$\mathbf{Y}_t = \mathbf{M} + \mathbf{B}\mathbf{Y}_{t-1} + \mathbf{C}\mathbf{e}_t,$$

the vector of shocks $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. Accordingly, the probability density of $\mathbf{e}_t$, denoted by $f(\mathbf{e}_t)$, is proportional to

$$\exp\left(-\frac{1}{2}\mathbf{e}_t'\boldsymbol{\Omega}^{-1}\mathbf{e}_t\right).$$

The approximating and the worst-case models are linked through the martingale increments g and g^* for the agents of country H and F, respectively. We showed, in Section V, that in a first-order approximation g and g^* are related to the revisions in the expected future path of the respective consumption growth:

$$\hat{g}_t = -(\gamma - 1)\sum_{k=0}^{\infty}\beta^k\left(E_t\Delta\hat{c}_{t+k} - E_{t-1}\Delta\hat{c}_{t+k}\right)$$

$$\hat{g}_t^* = -(\gamma^* - 1)\sum_{k=0}^{\infty}\beta^k\left(E_t\Delta\hat{c}_{t+k}^* - E_{t-1}\Delta\hat{c}_{t+k}^*\right),$$

in which we are allowing for different γ and γ^*.

Using equations (A1)–(A2) and a first-order approximation of the flow-budget constraint (27), we can solve for the growth rate of domestic consumption, as a function of steady-state portfolio shares, asset returns and labor income. It follows that we can write g and g^* as linear combinations of the VAR innovations:

$$\hat{g}_t = -(\gamma - 1)\mathbf{z}(\gamma)'\mathbf{e}_t$$

$$\hat{g}_t^* = -(\gamma^* - 1)\mathbf{z}^*(\gamma^*)'\mathbf{e}_t,$$

in which vectors $\mathbf{z}$ and $\mathbf{z}^*$ depend on γ and γ^* through the steady-state portfolio shares. Indeed, simple algebra shows that

$$\mathbf{z}(\gamma) \equiv \frac{1 - \beta}{\beta s_c} \boldsymbol{\iota}_{exr} \overline{\boldsymbol{\alpha}}(\gamma) + \mathbf{H}'\iota_r + \frac{s_\xi}{s_c} \mathbf{H}'(\boldsymbol{\iota}_\xi - \boldsymbol{\iota}_r)$$

for country H, and

$$\mathbf{z}^*(\gamma^*) \equiv \frac{1 - \beta}{\beta s_c} \boldsymbol{\iota}_{exr} \overline{\boldsymbol{\alpha}}^*(\gamma^*) + \mathbf{H}'(\boldsymbol{\iota}_r - \boldsymbol{\iota}_q) + \frac{s_\xi}{s_c} \mathbf{H}'(\boldsymbol{\iota}_\xi + \boldsymbol{\iota}_q - \boldsymbol{\iota}_r),$$

for country F, in which $\overline{\boldsymbol{\alpha}}$ and $\overline{\boldsymbol{\alpha}}^*$ are defined in (A3), and $\mathbf{H} \equiv \mathbf{C}'(\mathbf{I} - \beta\mathbf{B})^{-1}\mathbf{C}$.

It follows that the probability distribution of the distorted model for the agent in country H, denoted by $\tilde{f}(e_t)$, is given by

$$\tilde{f}(\mathbf{e}_t) \equiv f(\mathbf{e}_t) \cdot g_t \propto \exp\left(-\frac{1}{2} \mathbf{e}_t' \boldsymbol{\Omega}^{-1} \mathbf{e}_t\right) \exp(-(\gamma - 1)\mathbf{z}(\gamma)'\mathbf{e}_t).$$

Completing the square finally allows us to write $\tilde{f}(\mathbf{e}_t)$ as

$$\tilde{f}(\mathbf{e}_t) \propto \exp\left(-\frac{1}{2}(\mathbf{e}_t - \mathbf{w}(\gamma))'\boldsymbol{\Omega}^{-1}(\mathbf{e}_t - \mathbf{w}(\gamma))\right),$$

in which $\mathbf{w}(\gamma \equiv -(\gamma - 1)\boldsymbol{\Omega}\mathbf{z}(\gamma)$ is the mean distortion implied by the preference for robustness. Similarly, the distorted probability distribution function for the agent in country F, $\tilde{f}^*(\mathbf{e}_t)$, is given by

$$\tilde{f}^*(\mathbf{e}_t) \equiv f(\mathbf{e}_t) \cdot g_t^* \propto \exp\left(-\frac{1}{2}(\mathbf{e}_t - \mathbf{w}^*(\gamma^*))'\boldsymbol{\Omega}^{-1}(\mathbf{e}_t - \mathbf{w}^*(\gamma^*))\right),$$

in which $\mathbf{w}^*(\gamma^*) \equiv -(\gamma^* - 1)\boldsymbol{\Omega}\mathbf{z}^*(\gamma^*)$.

Equations (49)–(50) directly follow.

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