American Economic Journal: Macroeconomics 2020, 12(1): 104–152
https://doi.org/10.1257/mac.20180383

Temporary Price Changes, Inflation Regimes, and the Propagation of Monetary Shocks†

By Fernando Alvarez and Francesco Lippi*

We present a sticky price model that features the coexistence of many price changes, most of which are temporary, with a modest flexibility of the aggregate price level. Stickiness is introduced in the form of a price plan, namely a set of two prices: either price can be charged at any moment but changing the plan entails a menu cost. We analytically solve for the optimal plan and for the aggregate output response to a monetary shock. We present evidence consistent with the model implications using scanner data, as well as Consumer Price Index data across a wide range of inflation rates. (*JEL* D22, E31, E52, L11, O11, O23)

Empirical analyses of price setting behavior have long documented that the time series of several prices display temporary deviations from a recurrent price level termed the *reference price*, which is often measured as the modal price in a given time period. This observation gives rise to *transitory price changes*, i.e., temporary deviations above or below the reference price. Kehoe and Midrigan (2015) concludes that over 70 percent of price changes in the Bureau of Labor Statistics (BLS) data are temporary. Such transitory price changes do not fit neatly in simple sticky price models, though they might be important since they make a large difference in the measured frequency of price changes. This difference is apparent in the way different authors have approached the treatment of sales in the data: some authors, such as Bils and Klenow (2004), count sales as price changes since they view temporary price changes as a source of price flexibility. Others, such as Golosov and Lucas (2007) or Nakamura and Steinsson (2008), exclude sales from the counting of price changes believing they are not a useful instrument to respond to aggregate shocks.[1]

* Alvarez: Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637 (email: f-alvarez1@uchicago.edu); Lippi: Luiss University and Einaudi Institute for Economics and Finance (EIEF), Via Sallustiana 62, 00184 Rome, Italy (email: flippi@luiss.it). Virgiliu Midrigan was coeditor for this article. First draft May 2015, previously titled "Price plans and the real effects of monetary policy." We are grateful to seminar participants at University of Minnesota, University of Chicago, Northwestern University, EIEF, Booth, Toulouse, UPenn, Nova, Banque de France, European Central Bank, and the 2015 Hydra conference, and to our discussants John Leahy, Oleksiy Kryvtsov, and Nir Jaimovich for many useful comments. Part of the research for this paper was sponsored by the ERC advanced grant 324008. Jean Flemming, David Argente, and Francisca Sara provided excellent research assistance.

† Go to https://doi.org/10.1257/mac.20180383 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] Klenow and Kryvtsov (2008, 871) puts it very clearly: "Focusing on regular prices begs the question of whether one should exclude sale prices for macro purposes. This is not obvious. First, sales may have macro content. Items may sell at bigger discounts when excess inventory builds up or when inflation has been low."

This paper analyzes a stylized version of the model in Eichenbaum, Jaimovich, and Rebelo (2011), a model that produces transitory, as well as persistent, price changes. The objective is to provide a theoretical benchmark to assess whether the temporary price changes matter for the propagation of monetary shocks. The model extends Golosov and Lucas (2007) by assuming that upon paying the menu cost the firm can choose two prices, instead of one, say $\mathcal{P} \equiv \{p_H, p_L\}$. We call the set $\mathcal{P}$ a price plan, which is a singleton in the standard model, and the firm is free to change prices as many times as it wishes within the plan. Instead, changes of the plan require paying a fixed cost. We analytically solve for the optimal policy of a representative firm, for several cross-sectional statistics (e.g., frequency and size of temporary and reference price changes) and for the cumulative impulse response function of output to a once and for all monetary shock.

To highlight the importance of the model with temporary price changes, and whether it is consequential to abstract from them in the modeling of the monetary transmission mechanism, we compare the cumulated effect of a once and for all monetary shock on output in two models: a model with temporary prices and a standard menu-cost model. Since the menu-cost model cannot accommodate both a large number of price changes *and* a small number of reference price changes, the comparison hinges on what is being kept constant across models. We establish analytical results for two interesting cases. If the models feature the same number of *total price changes*, then the real effect of the monetary shock is larger in the model with temporary prices than in the standard menu-cost model. This happens since temporary price changes are a nonnegligible fraction of total price changes and they turn out to be an imperfect tool to respond to permanent policy shocks. The model with temporary price changes thus rationalizes the coexistence of a high frequency of price changes with a slow propagation of monetary shocks. Instead, if the models feature the same number of *reference price changes*, then the effect of a monetary shock is smaller in the model with temporary prices than in the standard menu-cost model. We find this last result useful since the frequency of reference price changes is easily estimated in actual data, and the practice of filtering out the high frequency price changes is often used in calibrations. Thus, using a canonical menu-cost model without temporary prices, and calibrating it to *all price changes* or to just *reference price changes*, leads to either underestimation or overestimation of the real effect of a monetary shock. Such differential effects, together with the inability of the simpler menu-cost model to account for both the large number of price changes and small number of reference price changes, imply that one should not abstract from temporary price changes.

A few previous contributions studied the relevance of temporary price changes using microfounded sticky price models. Guimaraes and Sheedy (2011) develops a model of sales where the firm's profit maximizing behavior implies a randomization between a regular price and a low price. They assume that the timing for the adjustment of the regular price follows an exogenous rule à la Calvo. The real effects of monetary shocks in their model are essentially identical to the real effects produced by a Calvo model. This is because their model features a strong strategic substitutability in the firm's price setting decisions that, in the quantitative parametrization chosen by the authors, completely mutes the individual firm's incentive to use less sales in order to respond to, e.g., a positive monetary shock. Such effect is not present in our framework since the firm's pricing decision is not subject to any form of strategic interaction with other firms. Our paper is also related to Kehoe and Midrigan (2015), who set up a model where the firm faces a regular menu cost for permanent price changes and a smaller menu cost for temporary price changes (which are reversed after one month). Their model implies that firms will not use the temporary price changes to respond to monetary policy shocks, from which they conclude that temporary price changes are not a relevant measure of the firm's price flexibility. Our result is different because the implementation of a temporary price change is not automatically reversed.[2] One novelty compared to those papers is that in our model, firms use the temporary price changes to respond to shocks so that temporary prices cannot be ignored in order to understand the transmission of monetary policy shocks. Analyzing empirically whether firms use temporary price changes to respond to the aggregate macroeconomic conditions, as in the recent papers by Kryvtsov and Vincent (2014) and Anderson et al. (2015), is a useful way to select between these alternative models.

Next we illustrate the organization of the paper and give a preview of the main results. The paper has three main parts: the first one analyzes the firm's problem and its aggregation, deriving several price-setting statistics for the steady state. The second one compares such statistics with data taken from the United States, Chile, and Argentina. The third part of the paper characterizes the propagation of a monetary shock and discusses the implications of temporary price changes for macroeconomics.

The first part of the paper sets up the price-setting problem and derives the firm's optimal policy (Section I). The decision rules determine when to change plans and how to change prices within plans, which can be expressed in terms of four thresholds. In Proposition 1 and Proposition 2, we solve for the optimal decision rules analytically for the case of near-zero inflation. In Proposition 3, we solve for the case of very high inflation showing that the problem becomes the one studied by Sheshinski and Weiss (1977), i.e., one where inflation becomes the sole motive for price adjustment and idiosyncratic shocks play no role in the optimal decisions. For the intermediate cases of moderate inflation rates, this section provides a system of four equations whose solution yields the optimal decision rules. Section II characterizes the model behavior in the steady state. In Proposition 4, we discuss the invariant distribution of the relevant state, and a key statistic: the frequency of plan changes. Proposition 5 and Proposition 6 analyze the frequency of price and plan changes at zero inflation. We consider the continuous-time limit as well as its discrete-time counterpart, which is important to clarify and quantify the sense in which the model with plans generates many price changes that are temporary in nature. The results of these propositions are illustrated in Section IIIA. Proposition 7 and Proposition 8 analyze the time spent at reference prices and how the frequency of reference price changes is related to the frequency of price plans in the menu-cost model and in the

[2] See Section V and online Appendix H for a version of our model that, like Kehoe and Midrigan's paper, assumes that even the price changes within the plan are costly.

plan model, in all cases for inflation close to zero. In Proposition 9, we characterize the size of price changes around zero inflation in terms of the optimal thresholds.

The second part of the paper, in Section III, compares the steady-state predictions of the model with evidence gathered using the micro scanner price data from the United States and Chile, and CPI data from Argentina. For the latter country, we are able to document how several statistics on temporary price changes change as inflation ranges from low to very high values. We show that such patterns are broadly consistent with the predictions of the model. In particular, we compare the predictions of the menu-cost model and of the model with plans for the frequency of all price changes, of reference price changes, as well as the degree to which prices come back to old values, across a wide range of inflation rates. We also evaluate the degree of asymmetry of the model compared with the data for moderate inflation rates, and for robustness compare it with data from the Billion Prices Project.

The third part of the paper consists of Section IV, which analytically derives the cumulative output response of the economy to an unexpected small monetary shock. The results in this section allow several comparisons with the size of the real effects produced by the canonical menu-cost model. We first develop a method to analytically evaluate the area under the cumulative impulse response, obtaining a semiclosed-form solution. In Proposition 10, we specialize the analysis for an economy with a near-zero inflation and obtain a simple expression for the area under the IRF for both the menu-cost model and the plan model. Importantly, this proposition shows that the real effect of monetary shocks depend on the frequency of plan changes, as opposed to the frequency of price changes. Furthermore, in Proposition 11, we show that a large part of this effect occurs on impact. We complement these results for low inflation with a quantitative analysis of moderate inflations. Section IVB compares the real effects in the menu-cost model with those in the model with plans, highlighting the importance of temporary price changes. In particular, Proposition 12 compares these two model economies keeping the same number of price plans in each of them, and Table 2 explores other relevant cases. Section V concludes by discussing the scope and robustness of our results.

I. Economic Environment and the Firm's Problem

Consider a firm whose (log) profit-maximizing price at time t, $p^*(t)$, follows the process

$$dp^*(t) \;=\; \mu dt + \sigma dW(t), \tag{1}$$

where $W(t)$ is a standard Brownian motion with variance σ^2 per unit of time and the drift is given by the inflation rate μ. A firm that charges the (log) price $p(t)$ at time t has a loss, relative to what it will get charging the desired price $p^*(t)$, equal to $B\left(p(t) - p^*(t)\right)^2$ where B is a constant that relates to the curvature of the profit function.[3]

[3] See Appendix B in Alvarez and Lippi (2014) for a derivation of these expressions as a second-order approximation to the general equilibrium problem in which firms face a CES demand for their goods.

The firm maximizes the present value of profits, discounted at rate $r \geq 0$. At any moment of time the firm has a price plan available. A price plan is given by two numbers $\mathcal{P} \equiv \{p^L, p^H\}$ so that the firm can charge either (log) price in this set at t, i.e., $p(t) \in \mathcal{P}$ at t. At any time the firm can pay a cost ψ and change its price plan to any $\mathcal{P} \in \mathbb{R}^2$. We let $\mathcal{P}_i$ be the ith price plan and let τ_i be the stopping time at which this ith price plan was chosen, so this plan will be in effect between τ_i and τ_{i+1}. The stopping times and the price plans can depend on all the information available until the time they are chosen. The problem for the firm is to choose the stopping times τ_i when price plans are changed (for $i = 1, 2, \ldots$), choose the two prices in each price plan $\mathcal{P}_i$, as well as the price $p(t) \in \mathcal{P}_i$ to charge at any $\tau_i < t < \tau_{i+1}$.

The firm maximizes the value of profits discounted at the rate r. The state of the problem is given by the triplet: $\{p^*(t), p^L, p^H\}$, where $p^*(t)$ is the current desired price level, and where $\mathcal{P} = \{p^L, p^H\}$ is the price plan currently available containing a low and a high price (p^L and p^H, respectively). Letting $p^* = p^*(0)$ denote the current desired price level (where we normalize the current time by 0), we can write the firm's problem as

$$V(p^*, p^L, p^H)$$
$$= \min_{\{\tau_i, \mathcal{P}_i\}_{i=1}^{\infty}} E\left[\sum_{i=1}^{\infty} \int_{\tau_{i-1}}^{\tau_i} e^{-rt} \min_{p(t) \in \mathcal{P}_{i-1}} B\big(p(t) - p^*(t)\big)^2 dt + \sum_{i=1}^{\infty} e^{-r\tau_i}\psi \,\middle|\, p^* = p^*(0), \mathcal{P}_0\right]$$

where $\tau_0 = 0$ and $\{\tau_i, \mathcal{P}_i\}_{i=1}^{\infty}$ are the (stopping) times and the corresponding price plans. The key novel element compared to the standard menu-cost problem is the min operator which appears inside the square bracket: at each point in time the firm can freely choose to charge any of the prices specified by the plan; for instance, the plan P_0 lets the firm freely choose either p^L or p^H at any point in time.

Normalization of the Value Function.—Note that $V: \mathbb{R}^3 \to \mathbb{R}$ is symmetric in the following sense: $V(p^*, p^L, p^H) = V(p^* + \Delta, p^L + \Delta, p^H + \Delta)$ for all $\{\Delta, p^*, p^L, p^H\}$, a property that follows directly from the fact that the argument of the period return function is the difference between the price charged $p(t)$ and the desired price $p^*(t)$. Notice that the law of motion of the ideal price is irrelevant for this symmetry property, which amounts to a normalization of the desired (log) price level $p^*(t)$ with respect to the desired price level at the beginning of each plan $p^*(\tau_i)$. Using this symmetry property, we can rewrite the normalized value function for the ith plan, with a current desired price $p^*(t)$ and desired price $p^*(\tau_i)$ when the plan started, as

$$v(g(t); \ell, h) \equiv V\big(p^*(t), p^*(\tau_i) + \ell, p^*(\tau_i) + h\big) \quad \text{with} \quad g(t) \equiv p^*(t) - p^*(\tau_i)$$

where $\ell < h$, $p^L \equiv p^*(\tau_i) + \ell$, $p^H \equiv p^*(\tau_i) + h$. In words, g measures the current desired price relative to the desired price at the time of the last change in plans (we will omit the "current period" argument t from here on). With this new definition, the state g is reset to zero every time a new price plan is chosen. We refer to the state g as the *desired price*, where it is understood that the desired price is normalized by the level $p^*(\tau_i)$ observed at the beginning of each plan.

A. *The Firm's Optimal Policy*

Since at any time price plans can be changed by paying a cost ψ, the value function must satisfy the following equation for all g:

$$rv(g;\ell,h) = \min\left(\min_{g^*\in\{\ell,h\}} B(g-g^*)^2 + \mu v'(g;\ell,h) + \frac{1}{2}\sigma^2 v''(g;\ell,h), \min_{\{\ell' h'\}} v(0,\ell',h') + \psi\right),$$

where the outer min operator describes the firm's optimal choice between sticking to the current plan versus changing the plan, and g follows the law of motion $dg(t) = \mu dt + \sigma dW(t)$.

We look for an optimal policy that is described by four numbers: $\underline{g} < \ell < h < \bar{g}$, where $\underline{g}$ and $\bar{g}$ denote the boundaries of the inaction region. Simple algebra shows that $\hat{g} \equiv (\ell+h)/2$ is the threshold below which it is optimal for the firm to charge the low price within the plan.[4] The firm's value function then is

$$(2) \quad rv(g;\ell,h) = \min\begin{cases} B(g-\ell)^2 + \mu v'(g;\ell,h) + \frac{\sigma^2}{2}v''(g;\ell,h) & \text{optimal for } g \in [\underline{g},\hat{g}] \\ B(g-h)^2 + \mu v'(g;\ell,h) + \frac{\sigma^2}{2}v''(g;\ell,h) & \text{optimal for } g \in [\hat{g},\bar{g}] \\ r\left[\min_{\{\ell' h'\}} v(0,\ell',h') + \psi\right] & \text{optimal for } g \notin [\underline{g},\bar{g}] \end{cases} \cdot$$

The oscillations of g about the threshold $\hat{g}$ will generate a price change within the plan. When g crosses either of the barriers, $\underline{g}$ and $\bar{g}$, the price plan is changed and a price change occurs.

To determine the optimal policy parameters, $\underline{g}, l, h, \bar{g}$, we use the following optimality conditions, where we use that $g = 0$ at the beginning of a plan, i.e., at the time when the optimal prices are chosen, and a prime denotes derivatives with respect to g:

$$(3) \quad v(\underline{g};\ell,h) = \psi + v(0;\ell,h), \quad v(\bar{g};\ell,h) = \psi + v(0;\ell,h), \quad \text{value matching}$$

$$(4) \quad v'(\underline{g};\ell,h) = 0, \quad v'(\bar{g};\ell,h) = 0, \quad \text{smooth pasting}$$

$$(5) \quad \frac{\partial v(0;\ell,h)}{\partial \ell} = 0, \quad \frac{\partial v(0;\ell,h)}{\partial h} = 0. \quad \text{optimal prices}$$

Fixing the value of ℓ, h, the set of equation (3) and (4) are the familiar value-matching and smooth-pasting conditions for a fixed-cost problem (see Dixit 1991). Lastly, the prices ℓ, h within the plan should be optimally decided, which

[4] Notice $\min_{g^*\in\{\ell,h\}}(g-g^*)^2 = (g-\ell)^2$ if $g \le \hat{g}$ where $\hat{g} \equiv (\ell+h)/2$ and $(g-h)^2$ otherwise. See Appendix A for the closed-form solution of the value function.

requires equation (5) to hold. These prices are the main novelty compared to a traditional menu-cost problem in which only one price is allowed. In this modified problem, the firm can spread profit losses using two prices, instead of one as in the canonical menu-cost problem. Analysis of the first-order condition gives that the optimal prices satisfy

$$(6)\quad \ell = \frac{E\left[\int_0^\tau e^{-rt}\iota(t)g(t)\,dt \,\middle|\, g(0)=0\right]}{E\left[\int_0^\tau e^{-rt}\iota(t)\,dt \,\middle|\, g(0)=0\right]},\ h = \frac{E\left[\int_0^\tau e^{-rt}(1-\iota(t))g(t)\,dt \,\middle|\, g(0)=0\right]}{E\left[\int_0^\tau e^{-rt}(1-\iota(t))\,dt \,\middle|\, g(0)=0\right]},$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g} < g(t) < \hat{g}$ and zero otherwise, and all expectations are conditional on $g(0) = 0$, i.e., the value at the start of the plan (see Appendix B for analytic equations to solve for the optimal prices).

Equation (6) shows that the optimal prices are a weighted average of the desired prices $g(t)$ over the periods in which they will apply (as measured by the indicator function ι). Simple closed-form solutions can be computed for special cases, for instance, in the case of a small inflation (technically $\mu/\sigma^2 \to 0$) discussed in the next subsection, or in the deterministic problem obtained when inflation diverges ($\mu/\sigma^2 \to \infty$), discussed in Section IC. Yet another closed-form solution for equation (6) is discussed in online Appendix G, where we study a "Calvo" version of our problem in which plans change at an exogenous rate λ.

B. *The Small Inflation Case*

This section discusses the optimal decision rules for the case of zero inflation. We first show that around zero inflation several features of the optimal decision rules are not sensitive to inflation. This result justifies the use of the optimal rules derived for the zero inflation case in a range of low inflation rates, a case that is suitable for most developed economies. Next we derive closed-form solutions for the optimal decision rules. We have the following result (see Appendix D for the proof).

PROPOSITION 1: *Let $\ell(\mu)$, $h(\mu)$, $\bar{g}(\mu)$, $\underline{g}(\mu)$ denote the optimal thresholds that solve equation* (4) *and equation* (5) *when the inflation rate is μ. We have*

(*i*) *At $\mu = 0$, the width of the inaction region, $\bar{g} - \underline{g}$, and the width between the high and the low price, $h - \ell$, have a zero sensitivity with respect to inflation, i.e.,*

$$(7)\qquad h'(0) - \ell'(0) = \bar{g}'(0) - \underline{g}'(0) = 0.$$

(*ii*) *The optimal prices display "front loading," i.e., a positive elasticity with respect to inflation at $\mu = 0$, i.e.,*

$$(8)\qquad \hat{g}'(0) \equiv \frac{h'(0)+\ell'(0)}{2} = \ell'(0) = h'(0) > 0.$$

Panel A. Optimal thresholds: $\underline{g} < \ell < \hat{g} < h < \bar{g}$

Panel B. Width of ranges: $\bar{g} - \underline{g}$, $h - \ell$ versus $\hat{g}$

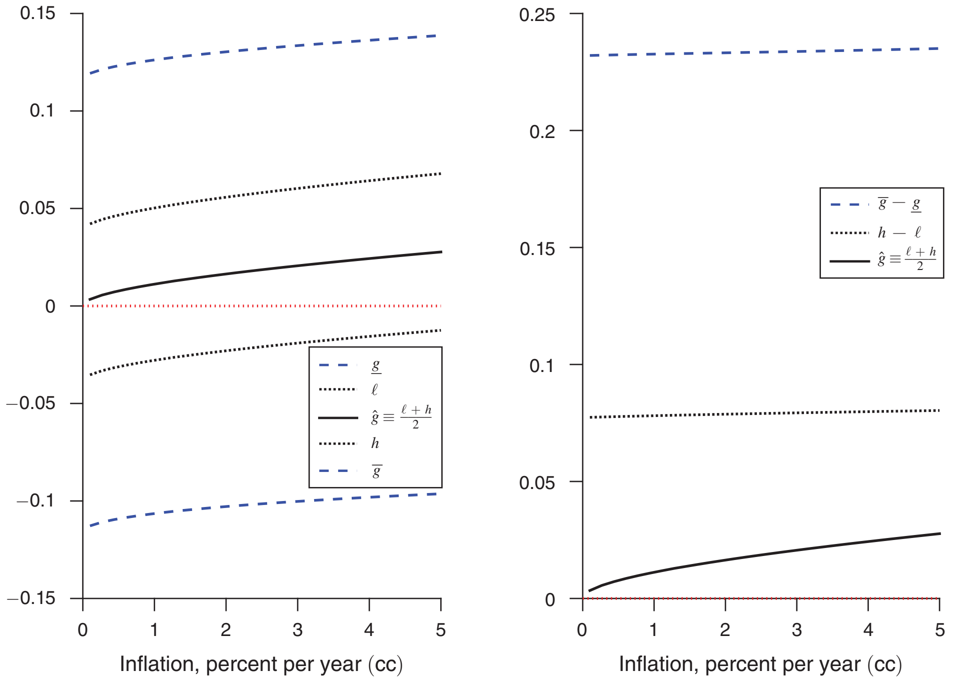

Figure 1. Optimal Decisions at Low Inflation

(*iii*) *The number of price changes per period*, $N(\mu)$, *has a zero sensitivity with respect to inflation, i.e.*, $N'(0) = 0$.

The proposition implies that, as long as $\mu > 0$, every optimal plan must start with the low price ℓ since $g = 0$ by definition at the beginning of a plan and $\hat{g} > 0$. It also shows that a small inflation, namely a small inflation rate above $\mu = 0$, will have a modest effect on the width of the inaction region as well as on the average frequency of price changes, N. These findings, common to several models where idiosyncratic shocks are present, justify using the limiting case of a zero inflation as a benchmark for a range of low inflation rates. Figure 1 illustrates the result of the proposition using numerical results for a calibrated version of the model. It is apparent that close to zero inflation the optimal prices are symmetrically distributed around zero, i.e., that $\ell(\mu) \approx -h(\mu)$, and so are the optimal inaction thresholds $\underline{g}(\mu) \approx -\bar{g}(\mu)$. Moreover, all policy variables have a positive sensitivity with respect to inflation (front loading, point (ii) of the proposition). Panel B shows that the width of the inaction intervals, $\bar{g}(\mu) - \underline{g}(\mu)$, and the width between the high and the low price $h(\mu) - \ell(\mu)$, i.e., the size of a price change within the plan, are insensitive to inflation (point (i) of the proposition). The third part of the proposition states that the frequency of price changes $N(\mu)$, as measured by the number of price changes per year, is also insensitive to inflation at $\mu = 0$. This feature is in stark contrast with inflation behavior at high inflation, discussed in Section IC, where

$N(\mu)$ has an elasticity of $2/3$ with respect to inflation. Both the low elasticity at low inflation and the $2/3$ elasticity at high inflation find strong support in the data, as will be shown in Section III.

We now present an analytic characterization of the optimal thresholds at zero inflation. It is straightforward to notice that zero inflation implies the following symmetry features of the optimal policy $\underline{g}(\mu) = -\bar{g}(\mu)$, and $\ell(\mu) = -h(\mu)$. We first establish the following intermediate result.

LEMMA 1: *Assume $\mu = 0$ and let $h = -\ell$ be the optimal decision rule for the high price within a plan given a barrier $\bar{g}$ for change of plans. The optimal price h is given by a function ρ of the variable $\phi \equiv r\bar{g}^2/\sigma^2$, satisfying:*

$$h = \bar{g}\rho(\phi),$$

$$\textit{where} \quad \rho(\phi) = \frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)} \quad \textit{with } \rho(0) = \frac{1}{3} \textit{ and } \rho'(\phi) < 0.$$

This lemma shows that the ratio $h/\bar{g}$ is equal to $1/3$ for small $\phi \equiv r\bar{g}^2/\sigma^2$, and an even smaller fraction for larger values.[5]

We now turn to the characterization of the optimal decision concerning the width of the inaction range, $\bar{g}$ given h. This involves solving the value function explicitly, which is shown to be differentiable at $g = 0$ in spite of the fact that the objective function is not. The following proposition, together with Proposition 1, shows that indeed the optimal policy is given by the (symmetric) thresholds $h, \bar{g}$ and provides a complete characterization.

PROPOSITION 2: *Assume $\mu = 0$. The optimal policy rule is given by the symmetric thresholds $\bar{g} = -\underline{g}$ and $h = -\ell$. The value of $\bar{g}$ is the unique solution to the equation*

$$\eta^2 r\frac{\psi}{B} = \kappa(\eta\bar{g}),$$

$$\textit{with} \quad \eta \equiv \sqrt{2r/\sigma^2} \quad \textit{and} \quad \kappa(x) \equiv \left[1 - 2\rho\left(\frac{x^2}{2}\right)\right]\left[x^2 - 2x\frac{(e^x + e^{-x} - 2)}{(e^x - e^{-x})}\right],$$

where the function $\rho(\cdot)$ is given in Lemma 1. The function κ is strictly increasing, with $\kappa(0) = 0$, $\lim_{x\to\infty}\kappa(x) = \infty$. For small values, we have $\kappa(x) = x^4/36 + o(x^4)$, and for large values $\kappa(x)/x^2 \to 1$ as $x \to \infty$. As shown in Lemma 1, the value of h is given by $h = \bar{g}\rho(\eta^2\bar{g}^2/2)$.

Propostion 2 provides simple approximate solutions for $\bar{g}$ and h, which are accurate for small values of $(r^2/\sigma^4)\psi/B$, thus for small values of the fixed cost ψ

[5] Indeed, as ϕ becomes arbitrarily large, as implied by very large values of $\bar{g}$ (hence of the fixed cost ψ), then the $h/\bar{g}$ converges to zero and h converges to $\sigma/\sqrt{2r}$.

and/or a small value of r. In this case, we can disregard the terms of order higher than x^4 and write

$$h = \frac{1}{3}\bar{g} \quad \text{and} \quad \bar{g} = \left(18\frac{\psi}{B}\sigma^2\right)^{1/4} \tag{9}$$

Note also that this approximation for $\bar{g}$ does not invoke r.[6] It is interesting to compare the expression for $\bar{g}$ in equation (9) with the one obtained in the standard menu-cost model, which we refer to as the Golosov-Lucas model, or GL model for short.[7] In the GL model, $h = 0$ since each price plan has only one price. The expression for $\bar{g}$ in such a model is identical except that instead of the factor 18, it has a factor 6, or in other words, it will lead to the same value of $\bar{g}$ if it had a fixed cost three times higher. This is intuitive: if the firm has the same fixed cost and has access to the price plan, then it chooses to have a wider band, by a factor of $3^{1/4}$ or approximately 32 percent wider than in the case without access to price plans. Note that otherwise the formula for the inaction threshold is the same quartic root expression as in Barro (1972) or Dixit (1991). Indeed, the quartic root is not obvious at all in this context, since the period objective function is not quadratic—as in these two papers—it is given by $(h - |g|)^2$, which includes the absolute value.[8]

C. *The High Inflation Case*

This section briefly discusses the limit of the model for "high inflation," which occurs if μ/σ^2 diverges. This case provides a good description for economies in which inflation μ is high relative to the volatility of the marginal costs σ. We have the following result (see Appendix D for the proof).

PROPOSITION 3: *Consider a steady-state model with $r \to 0$ in the limiting case where $\mu/\sigma^2 \to \infty$. We have the following optimal decision rules:*

(*i*) *All plans have identical duration τ given by*

$$\tau = 2\left(\frac{3\psi\mu^{-2}}{B}\right)^{1/3}. \tag{10}$$

(*ii*) *The optimal low and high prices within the plan are*

$$\ell = \frac{\tau}{4}\mu, \quad h = \frac{3\tau}{4}\mu. \tag{11}$$

(*iii*) *All plans begin with $g(0) = 0$ and are terminated when $g(\tau) = \tau\mu$.*

[6] Indeed, by differentiating the equation for the optimal $\bar{g}$ with respect to r, one can show that $\partial\bar{g}/\partial r = 0$ when evaluated at $r = 0$.

[7] The hallmark of the GL model is the joint presence of idiosyncratic shocks and a menu-cost friction.

[8] For instance, Dixit (1991) finds that when the period objective function is purely $|g|$, the approximation for the optimal rule has a cubic root.

The proposition shows that with high inflation, the model becomes deterministic as in the Sheshinski and Weiss (1977) model where $\sigma = 0$. As in that model, the frequency of price changes $N(\mu) = 1/\tau$ has an elasticity of $2/3$ with respect to inflation. The model thus converges to the Sheshinsky Weiss model with the only difference, irrelevant for the interpretation of the facts, that the menu cost is paid every two price changes. Contrast this result with the one obtained for zero inflation where we highlighted the lack of sensitivity of the frequency of price changes to inflation (see Proposition 1). In Section III, we will show how these predictions are validated in the data as inflation moves from low to very high values.

II. Model Behavior in the Steady State

This section computes some statistics for the frequency and size of price changes observed in a steady state, i.e., under the invariant distribution of desired prices. For the limiting case of zero inflation, which provides a good benchmark for low inflation regimes, we present an analytic characterization of size and frequency of the price changes triggered by the change of plans as well as of the price changes that occur *within* the plan. We also discuss the notion of *reference* price change, a benchmark statistic in the empirical literature.

Let μ denote the steady-state inflation rate and $f(g)$ denote the invariant density function of the desired prices $g \in [\underline{g}, \bar{g}]$. Moreover, let $N_p(\mu)$ denote the frequency of *plan* changes per period, i.e., the frequency with which g hits either plan-resetting barriers. We have the following proposition.

PROPOSITION 4: *Given the inflation rate μ and policy parameters $\ell(\mu)$, $h(\mu)$, $\bar{g}(\mu)$, $\underline{g}(\mu)$. Define $\xi \equiv -2\mu/\sigma^2$. We have that*

(*i*) *the density for the desired prices is given by*

$$f(g) = \begin{cases} A\left(e^{-\xi \underline{g}} - e^{-\xi g}\right) & \text{for } g \in [\underline{g}, 0] \\ A\dfrac{e^{-\xi \underline{g}} - 1}{1 - e^{-\xi \bar{g}}}\left(e^{-\xi g} - e^{-\xi \bar{g}}\right) & \text{for } g \in [0, \bar{g}] \end{cases} \tag{12}$$

$$\text{where} \quad A = \left(-\underline{g}\, e^{-\xi \underline{g}} - \bar{g}\frac{e^{-\xi \underline{g}} - 1}{1 - e^{-\xi \bar{g}}} e^{-\xi \bar{g}}\right)^{-1}.$$

(*ii*) *The frequency of plan changes per period, $N_p(\mu)$, is*

$$N_p(\mu) = \frac{\mu\left(e^{-\xi \bar{g}} - e^{-\xi \underline{g}}\right)}{\bar{g}\left(e^{-\xi \underline{g}} - 1\right) - \underline{g}\left(e^{-\xi \bar{g}} - 1\right)}. \tag{13}$$

The density function for the desired prices in equation (12) shows that for finite values of μ/σ^2, the density has zero mass at the boundaries of the support, $\{\underline{g}, \bar{g}\}$. This feature, common to models with idiosyncratic shocks ($\sigma^2 > 0$) is key to understand the small impact response of small shocks (see Alvarez, Lippi, and Passadore

2017). As $\mu/\sigma^2 \to 0$, the distribution has a triangular tent shape. As μ/σ^2 diverges, the shape of the distribution converges towards a rectangle, i.e., more and more mass piles up near the inaction boundaries, so the impact response of the model converges to the one of the Caplin-Spulber model. Equation (13) gives a closed-form solution for the number of plan changes per period as a function of inflation and of the optimal policy rule. Simple analysis reveals that this function has a zero sensitivity with respect to inflation at $\mu = 0$, a feature that echoes the behavior of the total number of price changes established in part (iii) of Proposition 1.

A. *Analytic Results for an Economy with Low Inflation*

This subsection presents several analytic results that are a useful guidance for economies with low inflation. Technically, we consider an economy with $\mu > 0$ but small (i.e., $\mu \downarrow 0$), arguing that this benchmark is accurate in a range of small inflation rates. We first present results on the frequency of various types of price changes that appear in the model, i.e., the frequency of temporary price changes (price changes within a plan) and other notions of low frequency price changes, such as reference price changes. Then, we briefly discuss the model's implications for the size of price changes. For completeness, in the online Appendix F we show that, unlike the menu-cost model, the plan model has a hazard rate of price changes that is decreasing in its duration, and we give a closed-form expression for it. Indeed, for short duration we get that the hazard rate $h(t)$ is approximately $h(t) \approx 1/(2t)$.[9]

On the Prevalence of Temporary Prices.—We now discuss some analytic results on the "prevalence" of reference prices, namely an analysis of how much time the actual prices will spend at the modal price (defined over a period of length T). This statistic is of interest because it has been analyzed empirically both by Eichenbaum, Jaimovich, and Rebelo (2011) and by Kehoe and Midrigan (2015).

We start by setting up a discrete-time/discrete-state representation of the model, for three reasons. First, in the continuous-time version, the expected number of price changes within a price plan diverges to $+\infty$ (see equation (15) below for a proof). Second, the source of the empirical study in Eichenbaum, Jaimovich, and Rebelo (2011) comes from a grocery chain where price changes are decided (and recorded) weekly. Third, a finite number of total price changes per period allows us to compare the effect of introducing price plans into an otherwise stylized version of the Golosov-Lucas model by keeping the *total* number of price changes fixed.

Discrete-Time Version of the Model.—The discrete-state/discrete-time representation has time periods of length Δ and the normalized desired price following

$$g(t+\Delta) - g(t) = \begin{cases} +\sqrt{\Delta}\sigma & \text{with probability } 1/2 \\ -\sqrt{\Delta}\sigma & \text{with probability } 1/2 \end{cases}. \tag{14}$$

[9] This is because when the ideal price is close to $\hat{g}$, and it crosses this barrier, it triggers a price change within the plan. This price has about half a chance to be reverted in a very short time.

We assume that g reaches $\pm\bar{g}$ after an integer number of periods (or steps); we define this value as $\bar{n} = \bar{g}/\left[\sqrt{\Delta}\,\sigma\right]$, an integer greater than or equal to 2 (a requirement that allows us to have price changes within a plan). Let $g(t)$ follow equation (14) for $-\bar{g} < g < \bar{g}$ and let $\tau(\bar{g})$ be the stopping time denoting the first time at which $|g(t)|$ reaches $\bar{g}$.

We define N to be the total number of price changes per unit of time, N_p the number of price plans per unit of time, and N_w the number of price changes per unit of time without a price plan change, so that $N = N_w + N_p$. The next proposition characterizes the expected number of plans per unit of time, N_p.

PROPOSITION 5: *Let $\Delta > 0$ be the length of the discrete-time period. Assume that $\bar{g}/\left(\sigma\sqrt{\Delta}\right)$ is an integer larger than 2. Given the threshold $\bar{g}$, the number of plan changes per unit of time is $N_p = \sigma^2/\bar{g}^2$.*

It is immediate to realize that N_p is independent of Δ, and hence its value coincides with the number of adjustments of the continuous time model, i.e., the limit for $\Delta \to 0$. We also notice that N_w depends only on N_p and Δ, and no other parameters. This is because the computation for N_w requires knowing the number of steps $\bar{n}$ that are necessary to get from $g = 0$ to $|g| = \bar{g}$ (at which time the plan is terminated), which is $\bar{n} = \sqrt{N_p/\Delta}$ as shown in the proposition. Thus, the value of N_w, which in principle depends on $\bar{g}, \sigma, \Delta$, can be fully characterized in terms of two parameters only: N_p and Δ.

We now establish two inequalities bounding N_w as a function of the length of the time period Δ, and of the number of price plans per unit of time N_p. The inequality follows directly from Doob's uncrossing inequality applied to our setup.

PROPOSITION 6: *Let $\Delta > 0$ be the length of the time period, and $\bar{g}$ be the width of the inaction band. The expected number of price changes within a plan N_w (per unit of time) has the following bounds*:

$$\frac{1}{\sqrt{\frac{\Delta}{N_p}} + \frac{\Delta}{2}\left[\frac{1+\sqrt{\Delta N_p}}{1-\sqrt{\Delta N_p}}\right]} \le N_w \le 2\sqrt{\frac{N_p}{\Delta}} - \frac{N_p}{2}. \tag{15}$$

Note that both the lower and upper bounds for N_w are increasing in N_p and decreasing in Δ. As $\Delta \to 0$, then $N_w \to \infty$, and indeed N_w behaves as $\Delta^{-1/2}$ for small values of Δ.[10]

Next we use the lower bound in equation (15) to derive an approximation for N as a function of Δ and N_p, which is accurate for small Δ. To this end, we first define

[10] Note also that both bounds for N_w are increasing in N_p, at least for small N_p. This is because as N_p becomes large, the price gap is reset to values closer to zero more often, which is the time when price changes without a price plan tend to happen. Finally, note that fixing $\Delta > 0$, and letting $N_p \to 0$ then $N_w \to 0$, and hence $N = N_p + N_w \to 0$. Summarizing, letting $N \equiv N_w + N_p = \mathcal{N}(\Delta, N_p)$, we have $\mathcal{N}(0, N_p) = \infty$ for $N_p > 0$ and $\mathcal{N}(\Delta, 0) = 0$ for $\Delta > 0$, with the upper bound and lower bound of $\mathcal{N}(\Delta, N_p)$ being increasing in N_p and decreasing in Δ.

the function $N = \mathcal{N}(\Delta, N_p)$, which gives the total number of price changes as a function of Δ and N_p. This implicitly defines the function $N_p = \mathcal{N}_p(\Delta, N)$. We have the following approximation:

$$(16) \quad N \approx \sqrt{\mathcal{N}_p/\Delta} \quad \text{for small } \Delta \qquad \text{or, formally,} \qquad \lim_{\Delta \to 0} \frac{\mathcal{N}_p(\Delta, N)}{\Delta N^2} = 1.$$

The previous results provide novel insights on the measurement of flexibility for an actual economy featuring both temporary and permanent price changes. It is common practice to measure the flexibility of an economy by the measured frequency of the price changes, at least since Blinder (1994). Indeed, this statistic is central in Klenow and Malin (2010), who also carefully distinguish between permanent and short-lived price changes. A relevant result in our paper, presented below in Proposition 10, is key to interpret the importance of temporary versus permanent price changes. The result will establish that it is the number of permanent changes N_p (i.e., plan changes) that concur to determine the output effect of a monetary shock, not the overall number of price changes.

Fraction of Time Spent at the Reference Price.—Reference prices are defined as the modal price during an interval of time, say during $[0, T]$, a concept introduced by Eichenbaum, Jaimovich, and Rebelo (2011). In this section, we analyze the "prevalence" of reference prices: the idea is to highlight that while there are many price changes during a time interval, prices spend a large fraction of time at the modal value during this interval, i.e., prices are often at the reference price. The comparison between the large frequency of price changes and the prevalence of reference prices captures the idea that prices return to the previous values.

To analyze this effect, we compute a statistic that depends on two parameters: a time interval of length T and a fraction $\alpha \in [1/2, 1]$. The statistic $F(T, \alpha)$ is the fraction of sample periods of length T in which the firm price spends at least αT time at the modal price. The parameter T is the time-window chosen by the statistician who measures references prices in the data, for instance T is a quarter in Eichenbaum, Jaimovich, and Rebelo (2011). The parameter α defines what fraction of time prices spend at the modal value in a sample path of length T. We will show that it is possible to have $F(T, \alpha) \approx 1$ even for α close to one, and at the same time that we have an arbitrarily large number of price changes, N. We have the following result.

PROPOSITION 7: *Fix $\sigma^2 > 0$ and let $\Delta \geq 0$ be the length of the time period, with $\Delta = 0$ denoting the case of a Brownian motion. Consider an interval length $T > 0$, a fraction $1/2 \leq \alpha < 1$, and a number $0 < \epsilon \leq 1$. Then there exists a threshold value $G > 0$ such that for all $\bar{g} \geq \sigma G$, then $F(T, \alpha) \geq 1 - \epsilon$. The threshold G depends on ϵ, α, and T, but it is independent of σ.*

In words, the proposition states that for any fraction $\alpha \in (1/2, 1)$, it is possible to choose a value of $\bar{g}$ large enough so that the price will be at the reference price at least a fraction $1 - \epsilon$ of the times. Notice by equation (16) that a given $\bar{g}$ is consistent with a very large number of price changes (as Δ is small). Thus, our model can simultaneously have prices spending a large fraction of time at the reference price, as well as a very large number of price changes, a feature that is apparent in the microdata.

Frequency of Reference Price Changes: Model with Plans versus Menu-Cost Model.—Next we compare the duration of *Reference Prices* in the model with N_p plans per unit of time, with the duration of *Reference Prices* in a model without plans with N^{GL} price changes per unit of time, imposing that $N^{GL} = N_p$. For short, we refer to the first model as the plans model and to the second model as the *GL* model—for Golosov and Lucas.[11] We fix the same initial condition $p^*(0)$, and the same sample path for the Brownian shocks, so that $\mathbf{p}^* \equiv [p^*(t)]$ for $t \geq 0$ is the same for both models. We specialize the comparison for the case where inflation is positive, but arbitrarily small.[12] We have the following result.

PROPOSITION 8: *Let N^{GL} be the number of price changes in a menu-cost model and N_p the number of plan changes in a plans model. Assume $N^{GL} = N_p$ and fix an arbitrary interval $[T_1, T_2]$ with $0 \leq T_1 < T_2$ and a path $\mathbf{p}^*$. Let the duration of the modal price in the interval $[T_1, T_2]$ be $D(T_1, T_2; \mathbf{p}^*)$ for the plans model and $D_\mu^{GL}(T_1, T_2; \mathbf{p}^*)$ for the GL model. We have:* $\lim_{\mu \to 0} D_\mu(T_1, T_2; \mathbf{p}^*) \leq \lim_{\mu \to 0} D_\mu^{GL}(T_1, T_2; \mathbf{p}^*) \leq 2D(T_1, T_2; \mathbf{p}^*)$.

Note that since the inequalities in Proposition 8 hold for any path $\mathbf{p}^*$, they also hold for the average and median durations. Computing the frequency of price changes as the reciprocal of its duration, the proposition implies that the number of reference price changes in GL (N_r^{GL}) is smaller than in a model with plans (N_r), namely

$$N_r^{GL} < N_r < 2N_r^{GL}. \tag{17}$$

The intuition behind the proof of this result is that if the number of price changes in the GL model equals the number of plan changes in the plan model, then the latter will have more reference price changes than the GL model due to the price changes that occur within each plan, in fact, up to twice as many.

The result in Proposition 8 that when the GL and the plans model have the same number of plan changes, the GL model will have fewer reference price changes than the plan model has important implications to calibrate the model. In particular, this means that if one wants to calibrate both models to have the same number of reference price changes, then the frequency of price changes in the GL model must be higher than the frequency of plans changes in the model with plans. This result will be useful to compare the effect of monetary policy across models, as we will see later.

[11] In particular, we require that $\sigma^2/\bar{g}^2$ be the same for both models, and ensure this by choosing a larger value for the fixed cost in the plans model. Using the expression for the case where $r \downarrow 0$, it can be seen that this requires a fixed cost in the plans model that is a third of the cost in the GL model.

[12] More precisely, we use the limit as $\mu \to 0$ to break some ties that occur only when $\mu = 0$.

On the Size of Price Changes.—Consider the distribution of price changes, Δp, in a model with positive but arbitrarily small inflation. Notice that at $\mu = 0$, the firm is indifferent between ℓ and h when starting a plan, but this indeterminacy is resolved as we take limit $\mu \downarrow 0$, which ensures the firm's optimal price at the start of the plan is ℓ. Recall from Proposition 1 that ℓ and h, and the width of the interval between prices is insensitive to μ at low inflation. We have the following result.

PROPOSITION 9: *Let $E[|\Delta p|]$ measure the size of price changes, as measured by the mean absolute value of price changes Δp. The size of price changes within a plan, $(2/3)\bar{g}$, is equal to the size of price changes between plans. Thus, the mean absolute size of price changes is $E[|\Delta p|] = (2/3)\bar{g}$.*

It is interesting to compare these predictions to the data. In most micro datasets, it is found that the size of price changes excluding sales is smaller than the size of all price changes, as summarized by Klenow and Malin (2010). In scanner datasets, the size of *Reference* price changes is smaller than the size of *all* price changes, suggesting that temporary price changes are larger than reference price changes.[13] In the more encompassing BLS data, however, the size of *Reference* price changes is essentially identical to the size of all price changes, as in Table 4 of Kehoe and Midrigan (2015), where it is equal to 0.11 in both samples.

III. Comparing the Model Predictions versus the Data

This section presents a comparison of some steady-state moments of the model, focusing on statistics related to the temporary price changes, using different datasets. First, we use weekly scanner data by Eichenbaum, Jaimovich, and Rebelo (2011) for the United States and by Elberg (2014) for Chile, two economies with relatively low and stable inflation. Second, we use Argentina's CPI data over the 1988–1997 period, featuring inflation rates from 2 to 5,000 percent per year, to test the model's predictions at different inflation rates.

A. *Model Predictions versus the US and Chilean Weekly Scanner Data*

We begin by presenting a comparison of the model predictions with the US and Chilean data using weekly scanner data. The first two columns of Table 1 report selected price-setting statistics taken from Eichenbaum, Jaimovich, and Rebelo's (2011) seminal paper, based both on their primary source as well as on Dominick's data, their alternate source. The third column presents similar statistics computed by Elberg (2014) for Chile. An advantage of the latter dataset is that it is based on 8 chains containing more than 36 million price quotes; another advantage is that sales are less prominent outside the United States, so that the Chilean data facilitate the comparison with the Argentine data used below.

[13] See Table 2 in Eichenbaum, Jaimovich, and Rebelo (2011), where the standard deviation of reference price changes is 0.14 versus 0.20 log points for all price changes.

The next columns report five alternative parametrizations of our model that depend *only* on two parameters: the length of the decision period, Δ, which we assume to be weekly as in the scanner datasets of Eichenbaum et al. and Elberg (i.e., $\Delta = 1/52$), and the frequency of plan changes, N_p. We consider five values for N_p, reported in the top row of Table 1. Once this is done, the model is fully parametrized.[14] The results are computed for an economy with low inflation (namely $\mu = 0$, virtually identical results obtain for inflation equal to 4 percent).

Table 1 reports several moments of interest, including statistics for the "reference price" defined as the modal price in a quarter. We also compute a measure of the prevalence of reference prices, namely the fraction of time that prices spend at the reference price, as well as the fraction of time spent below it. The second to last row of the table shows that the stickier the plan (i.e., the smaller N_p), the larger the fraction of time that prices spend at their modal value, consistently with the theoretical result in Proposition 7. In the model, the remaining time is equally split in visits to other prices that are "above" as well as "below" the reference price.

Comparison with Eichenbaum, Jaimovich, and Rebelo (2011).—The fourth and fifth columns of Table 1, corresponding respectively to $N_p = 5.8$ and $N_p = 3.3$, roughly match the two datasets studied by Eichenbaum, Jaimovich, and Rebelo (2011). The model does a good job at matching the total number of price changes, the probability of price changes, the time (fraction of weeks per quarter) spent at the reference price, as well as the time spent below the reference price. These two parametrizations, however, produce too many reference price changes than observed in the Eichenbaum, Jaimovich, and Rebelo (2011) data. A closer match of the frequency of reference price changes, especially the one measured on Dominick's data, is obtained by setting $N_p = 0.4$ (second to last column of Table 1). This case preserves the model ability to generate a small number of reference price changes (about 1.2 per year) with a much higher number of total price changes, about 5 times higher. This parametrization also matches the fraction of time spent at the reference price (0.88 versus 0.77). However, the parametrization misses on the absolute value of the number of price changes, a failure that may be less dramatic than it seems if one considers EichenbaumJaimovich, and Rebelo's (2011) warning that the number of price changes in the scanner data is likely to be upward biased due to the presence of measurement error.

Comparison with Elberg (2014).—The sixth column of Table 1, corresponding to $N_p = 1.4$, matches closely the statistics of the Chilean data on the four independent dimensions of the statistics we considered (the last four rows of the table).

Notice that the case with $N_p = 1.4$, which closely matches Chilean data, falls in between the case $N_p = 3.3$ and $N_p = 0.4$, which are the ones that bracket the observations for the United States, as explained above. Because of this, we will use the parametrization with $N_p = 1.4$ to discuss the relevance of temporary price changes for the propagation of monetary shocks in Section IVB. We stress

[14] It is straightforward to use equation (9) and Proposition 5 to map these values to different primitives, e.g., different values of the fixed cost ψ.

Table 1—Summary Statistics on Price-Setting Behavior (weekly model)

	Weekly data			Model with plans[d]				
	EJR[a]	EJR[b]	Elb.[c]					
Number of plan changes per year (N_p)	—	—	—	5.8	3.3	1.4	0.4	0.2
Number of price changes per year ($N = N_p + N_w$)	20	13	9.3	20	15	10	5.1	3.7
Number of ref. price changes per year (N_r)	1.1	1.3	2.1	4.1	3.2	2.4	1.2	0.8
Prob. of price changes (p. week)	0.43	0.24	0.18	0.35	0.27	0.18	0.10	0.07
Prob. of ref. price change (p. quarter)	0.27	0.33	0.52	0.93	0.80	0.59	0.29	0.20
Fraction of weeks at ref. price	0.62	0.77	0.80	0.50	0.62	0.75	0.88	0.91
Fraction of weeks below the ref. price	0.30	0.16	0.13	0.25	0.17	0.12	0.06	0.04

Notes: The reference price is defined as the modal price in a quarter. The EJR data are taken from Section D and Tables 1, 3, and 5 in Eichenbaum, Jaimovich, and Rebelo (2011).

[a] Primary dataset

[b] Dominicks' data

[c] Data for Chile taken from Table 2 in Elberg (2014) scanner dataset from multiple retailers.

[d] Results from a discrete-time weekly model, i.e., $\Delta = 1/52$, with zero inflation.

that, independently of one's preferred calibration, the model successfully reproduces the main feature of the data, i.e., the coexistence of a small frequency of reference price changes with a much higher frequency of price changes, that is about five times higher.

B. *Model Predictions versus Argentine CPI Data at Different Inflation Rates*

We now turn to using nine years of microdata that underlie the CPI in Argentina from December 1988 to September 1997, a period with very large variations of inflation rates. This data gives qualitative support for the implications of the plan model at both high and low inflation rates. In particular, we calibrate the model to low inflation and then freeze its parameters. We then perform a comparative statics exercise changing *only* the (steady-state) inflation rate, keeping all other parameters constant to test the model ability to replicate the empirical patterns observed in the data.

We compare the statistics in the Argentine data, computed for every nonoverlapping four-month period, with the same statistic computed for the invariant distribution of the plan model and for the invariant distribution of the menu-cost model (see online Appendix I or Alvarez et al. (2018) for a description of the data). Since prices are gathered every two weeks, we use a discrete-time version of the models with $\Delta = 1/26$. The first type of statistics measures the frequency of price changes and the frequency of reference price changes, where reference prices are defined as the modal price in a four-month period. The second type of statistics measures the extent to which at the time of price changes prices "come back" to old values recently used.

At small inflation rates, the plan model displays reference prices that are sticky—roughly with duration equal to half of the duration of the price plans—and prices that come back to previous values. At low inflation rates, the menu-cost model displays neither of these features. Instead, as inflation rates increase, the plan model and the menu-cost model both converge to a menu-cost model with no idiosyncratic shocks, and thus prices are not coming back to old values. We find that the level as well as the pattern of these statistics for the plan model are closer to the ones in the Argentine data than those of the menu-cost model across a large range of inflation rates.

In each nonoverlapping four-month period, we compute the annual continuously compounded inflation across all store-product combinations. We use continuously compounded inflation to be consistent with the model, where the log ideal price has a constant drift μ. The choice of four months is a compromise so that we compare the average inflation in a period in the data with the steady state in the model. Additionally, since we compute reference prices—see their definition below—we need a reference interval of time to define them.[15] For these statistics, we use a discrete-time version of both models where the period is two weeks, i.e., $\Delta = 1/26$ of a year. In each of the two models, we need to set a value for σ and ψ/B so that, with a 2 percent inflation ($\mu = 0.02$) the average number of price changes and the standard deviation of price changes are equal to ones in the Argentine data for inflation rate of that magnitude, which is achieved in the last four years of our sample.

It is instructive to consider the variation in the inflation rates in the Argentine data we use. On the one hand, inflation is extremely high in the first three years of the sample, and there is even a period with a hyperinflation. On the other hand, about a year after the stabilization plan of April 1991, there is almost price stability. For instance, the four-month period with the highest inflation rate has an annualized continuously compounded inflation rate of 792 percent during the second four months of 1989. Indeed, the entire year of 1989 has a continuously compounded inflation of 405 percent, which amounts to an annual inflation rate of about 5,600 percent.[16] The average continuously compounded inflation during the three years between 1994 to 1996 is 0.93 percent. Recall that we measure inflation in continuously compounded annualized terms throughout to be consistent with the model.

The first two statistics for which we compare the Argentine data with data generated by the two models are two measures of the frequency of price changes. The first one is the probability of price changes, estimated as the average fraction of price changes in each two-week period across all good $\times$ outlet combinations, excluding substitutions.[17] The second statistic is the probability of a reference price change, estimated as the fraction of good $\times$ outlets in which there is a reference price different from the one in the previous four months. The reference price in a given four-month period is the modal price for that good $\times$ outlet combination. Each of the panels of Figure 2 displays a scatter plot for the probability of price changes and the average inflation for each of the 36 nonoverlapping four-month periods with a red diamond. The figure also plots the corresponding statistic for the plan's model and for the menu-cost model. In both models, as the steady-state inflation increases, the benefit of increasing prices is higher, and thus prices are changed more often. Technically, the ideal price hits the top barriers more often, even if the barriers are

[15] Given the periodicity of our price data (two weeks) and the one used in US studies (weekly for the scanner data in Eichenbaum, Jaimovich, and Rebelo 2011 and monthly CPI data in Kehoe and Midrigan 2015), a reference interval of four months is a reasonable compromise. Eichenbaum, Jaimovich, and Rebelo (2011) uses a reference period of three months and Kehoe and Midrigan (2015) uses one of five months.

[16] The figure comes from the following calculation: 5,600 $\approx$ $[\exp(405/100) - 1] \times 100$.

[17] We include all price changes, even those that have a sale flag.

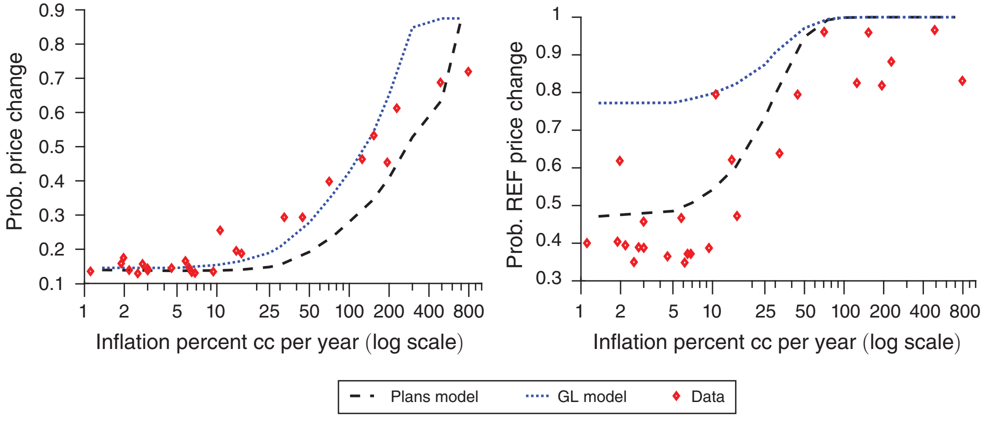

Figure 2. Probability of Price Changes versus Inflation

Notes: Figure on the left shows the probability of a price change per two-week period, averaged during four-month period in the data. Figure on the right shows the probability of a price change per four-month period. Reference prices are computed as modal prices in a four-month period.

wider as inflation increases. Both models show patterns of the probability of price changes for different inflation rates roughly similar to the one in the Argentine data. Instead, for the probability of reference price changes, the plan model displays a pattern much closer to the data than the one for the menu-cost model. In the menu-cost model, the probability of reference price changes varies with inflation much less than in the data.

Figure 3 displays the time spent at reference prices in the Argentine data and in each of the two models. The time spent at the reference price is defined for each good × outlet and four-month nonoverlapping time period. For each one, we compute the fraction of two-week periods for which the price equals the reference price. The statistic displayed is the average across good × outlets for each four-month period. In the models, it is the average under the invariant distribution. Both models display a pattern similar to the one in the Argentine data—as inflation increases, prices tend to go up, and hence the time spent at the reference price decreases with the steady-state inflation. Yet, the values for the plan model are closer to the ones in the data than those for the menu-cost model—recall that the models are calibrated so that at 2 percent inflation, they have the same total number of price changes as that in the data.

Figure 4 displays two statistics that measure whether the values of prices right after a price change have been used as prices in the recent past. The first statistic is the fraction of price changes in a two-week period for which the values of the prices after the change have not been used for the good × outlet in the past year. For each good × outlet and each four-month period, we compute the average of this fraction across the eight, two-week periods. The statistic displayed is the average across all good × outlets in each nonoverlapping four-month period. A related statistic, which is not displayed in the figure but reported in Tables J1 and J2 in the online Appendix under "novelty index," is the fraction of prices that in a given period are new, i.e., that have not been used by that good × outlet for the last year. The second

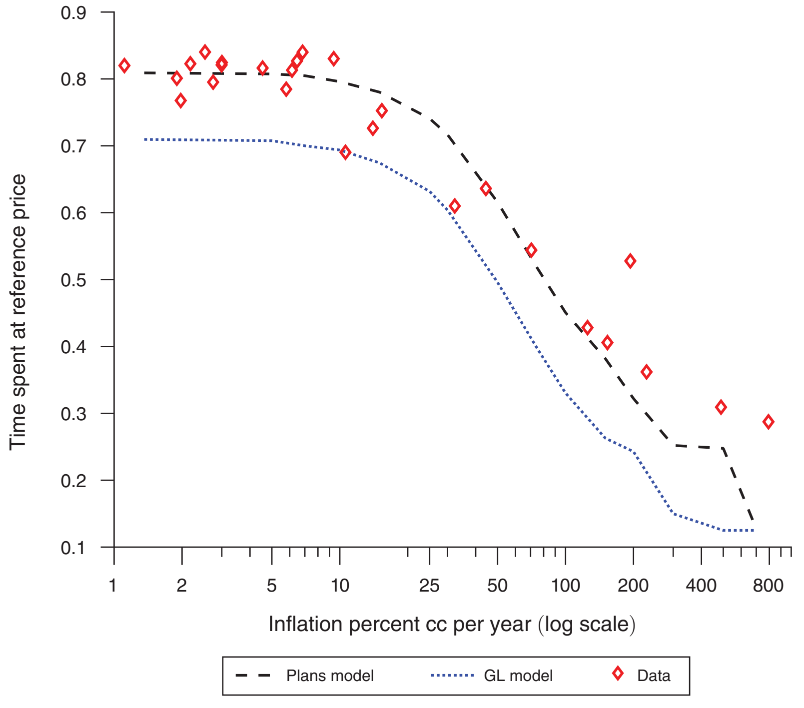

FIGURE 3. PREVALENCE OF REFERENCE PRICES VERSUS INFLATION

Notes: Time spent at reference price is measured as a fraction of the four-month period. Reference prices are computed as modal prices in a four-month period for each product $\times$ store combination.

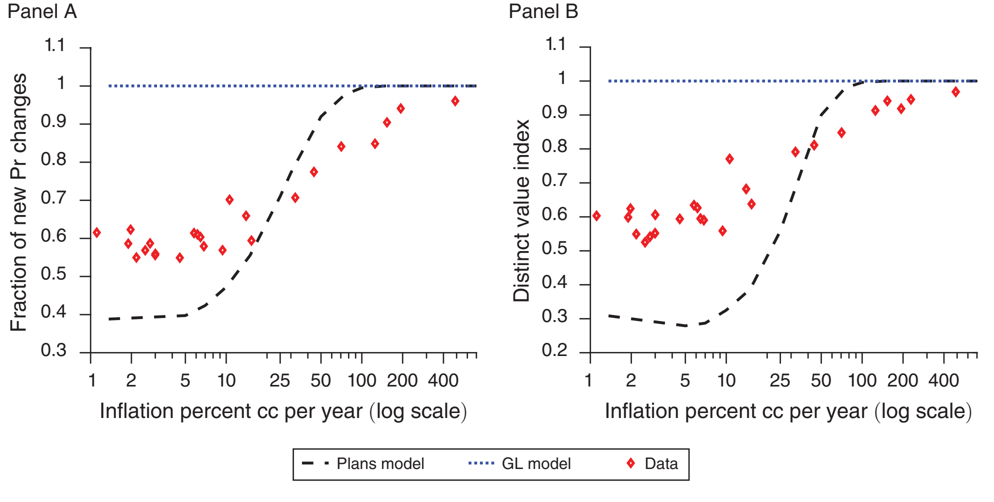

FIGURE 4. INDICATORS OF COMEBACK PRICES VERSUS INFLATION

Notes: Figure on the left shows the fraction of price changes where the new price has not been observed in the last year. Figure on the right shows the distinct value index, defined as (number of distinct prices during the four-month period $-$ 2) divided by (number of price spells during the four-month period $-$ 2). Numerator and denominator computed for the eight, two-week periods in each four-month period.

statistic is the distinct value index, the ratio between the normalized number of distinct prices used in the last four-month period relative to the normalized number of price spells.[18] We compute the statistic only for four-month periods where there are three or more price spells, since with fewer spells prices can't return to an old value. The statistic displayed for each four-month nonoverlapping period is the average across all good × outlet combinations. We normalize the number of distinct prices and the number of price spells by subtracting two from each so that the distinct value index varies between 0 and 1, taking the value of 1 if after a change in prices the values are different, and 0 if there are at most two values for prices that are used in all price spells.

Comparing the two statistics with the data, it is clear that the model with plans has the same qualitative patterns as the Argentine data, i.e., for low inflation, price changes feature prices that have been used in the past, but as inflation increases, there are more distinct or new prices. In the plan model, the logic is clear: as inflation increases, the ideal price has a larger positive drift, and thus it hits the upper barrier much more often than the lower barrier. While the qualitative behavior of both statistics is the same for the plan model as for the data, the model displays a wider range of variation than the data. Nevertheless, the pure menu-cost model is clearly at odds with the data, since essentially all price changes lead to new values for the prices at all inflation rates.

C. *Asymmetry and Sales*

In this section, we discuss the extent to which prices spend an equal amount of time above or below reference prices, whether the degree of asymmetry is related to "sales," and more broadly, whether temporary price changes and sales are exactly the same phenomenon. In the Argentine CPI data, at least for low inflation rates, the frequency of prices that are higher than reference prices is equal to the frequency of prices that are lower than reference prices, and hence it is consistent with the symmetry of the model at low inflation. The same pattern emerges using internet-scraped data for Argentina in a later period, i.e., 2008–2010. Additionally, using internet-scraped data for another four countries, we find that the frequency of sales is correlated with the degree of asymmetry: countries with higher fraction prices with sales flags (such as the United States) also have prices spending more time below reference prices. This correlation also holds for each country across different goods categories. From this discussion, we conclude that temporary price changes and sales are not exactly the same phenomenon.

We measure the degree of symmetry by the fraction of time that the price of a given good × outlet spends below the reference price, relative to the time that the price spends at a price different from the reference price in each nonoverlapping four-month period. At zero inflation rate the plan model is symmetric, so prices are equally likely to be above than below the reference price. It turns out that the Argentine CPI data is also quite symmetric. For this statistic, it is important

[18] A price spell is the interval with the same continuous price. In four months, with prices gathered every two weeks, the number of price spells is between 1 and 8.

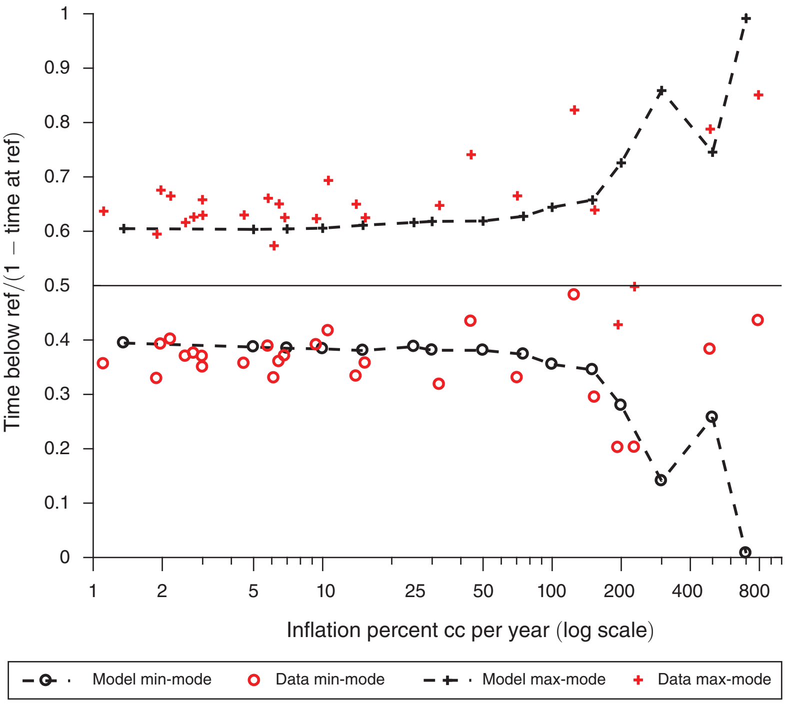

Figure 5. Fraction of Time Spent below Reference Prices

Note: Fraction of time spent below reference prices, out of the time spent outside reference prices.

to specify what is the value of the reference price if there are multiple modes for the prices in a four-month period. In Figure 5, we plot the fraction of time below reference prices, divided by the time not at the reference price, using two versions of the value of reference prices: one that uses the maximum mode as a reference price and the other that uses the minimum mode as the reference price. We compute the time spent below the reference price in the two alternative ways for simulated data obtained from the plan model as well as for the CPI. It can be seen that outside high inflation rates, the average of the two values for this statistic is close to 0.5, both for the Argentine CPI data and for the model. Table J5 and Table J6 in online Appendix J display the data for both definitions of reference prices.

We complement the empirical analysis by using data scraped from internet outlets, taken from the Billion Prices Project (BPP) developed by Cavallo and Rigobon (2016). These data contain daily prices from selected internet outlets for Argentina, Brazil, Chile, Colombia, and the United States for the years 2008, 2009, and 2010.[19] In Figure 6, we display the time spent by prices below reference prices during the years 2008–2010 for each country $\times$ goods category in the vertical axis, and the fraction of times that the prices in country $\times$ goods category

[19] We sample the BPP data every two weeks to make it comparable with the CPI data from Argentina, which we use above. The source of this data, the data itself, and its detailed description can be found at http://www.thebillionpricesproject.com/our-research/.

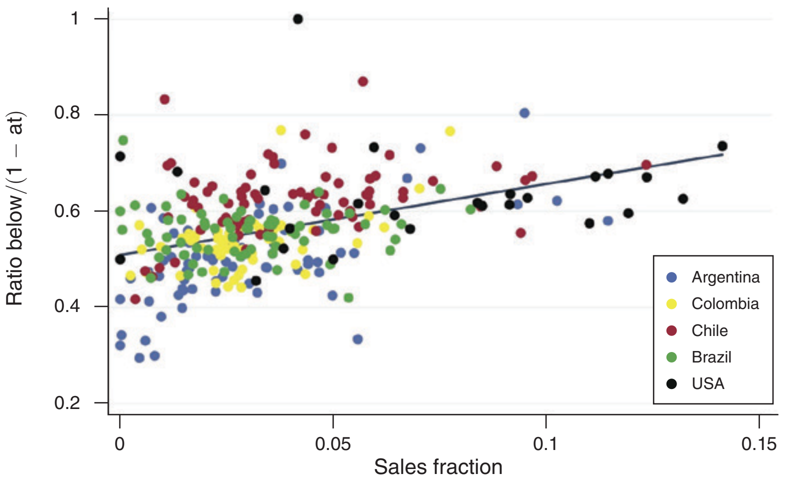

Figure 6. Normalized Fraction of Prices below Reference versus Sales

Notes: Each dot is the average of the statistic for a good category × country during the years 2008, 2009, and 2010. The underlying data are prices from few internet stores in each country, as produced by the Billion Price Project. The statistics are computed by first sampling these data every two weeks. In the vertical axis, we display the fraction of time prices that are below reference prices divided by the time prices spent outside reference prices. Reference prices are computed as the average of the maximum mode and the minimum mode.

display a sales flag. Reference prices are computed as the average of the maximum mode and the minimum mode. There are two clear facts that emerge from Figure 6. First, while the average of the fraction of time spent below reference prices is higher than 50 percent, which is the value that corresponds to the symmetric case, the differences are small. For instance, using an unweighted average across all countries and categories, the fraction of time spent below reference prices is 60 percent. Detailed statistics for these data can also be found in Table J11 and Table J12 in online Appendix J. Second, it is clear that the fraction of time where goods have a sales flag is positively correlated with the degree of asymmetry, in particular with the fraction of time spent below reference prices.

IV. Output Response to a Monetary Shock

This section studies the propagation of a monetary shock in a menu-cost models with plans. In particular, we consider an economy in steady state, i.e., with an invariant cross-sectional distribution of desired prices, and analyze the effect of an unexpected once and for all monetary shock of size $\delta > 0$ on output. We consider the impulse response of output to such a shock, and focus on the area below such impulse response function as a summary measure of the propagation mechanism. As in Alvarez, Le Bihan, and Lippi (2016), such a measure, combining the persistence of the response to the shock with the intensity (size) of the response, is convenient because it is easier to characterize than the full profile of the impulse response. This section provides a general framework, analytic results for the case of zero inflation, an accurate benchmark for low inflation rates, and numerical results for any inflation rate.

Let $P(t,\delta)$ denote the aggregate price level t periods after a monetary shock of size δ:

$$P(t,\delta) = \Theta(\delta) + \int_0^t \theta(s,\delta)\, ds. \tag{18}$$

The notation in equation (18) uses that the aggregate price level, just before the shock, is normalized to zero so that $\Theta(\delta)$ is the instantaneous jump in the price level at the time of the monetary shock and $\theta(\delta,t)$ is the contribution to the price level at time t.

We consider models where the effect of output is proportional to the difference between the monetary shock and the price level, i.e., denoting by $Y(t,\delta)$ the impulse response of aggregate output t units of time after the shock of size δ as: $Y(t,\delta) = (1/\epsilon)(\delta - P(t,\delta))$, where ϵ is an elasticity that maps the increase in real balances (or real wages) into increases in output. We denote the cumulated output response following a small monetary shock of size δ as follows:

$$\mathcal{M}(\delta) = \int_0^\infty Y(t,\delta)\, dt \equiv \int_0^\infty \frac{1}{\epsilon}\big(\delta - P(t,\delta)\big)\, dt. \tag{19}$$

Our approach to characterize equation (19) is to compute the corresponding cumulated output measure for each firm, as indexed by its desired price g, and then aggregate over firms using the steady-state distribution $f(g)$ from Proposition 4.

The Firm's Expected Contribution to Cumulative Output.—Consider the optimal policy parameters $\{\underline{g}, \bar{g}, \ell, h\}$, let $\hat{g} \equiv (\ell + h)/2$ and define the *price gap* as the difference between the charged price and the desired price. Denoting price gaps by $\hat{p}(t)$, we have

$$\hat{p}(t) = h + (\ell - h)\iota(t) - g(t), \quad \text{for } \tau_i \leq t < \tau_{i+1}, \tag{20}$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g} < g(t) < \hat{g}$ and zero otherwise, already introduced above.[20] The price gap measures the firm's deviation from the static profit maximizing price (in log points), so that a firm with a negative gap is charging a low price (i.e., it has a low markup) and thus contributes to above average output.

Define the expected cumulated output for a firm with desired normalized price g:

$$\hat{m}(g) = -E\left[\int_0^\tau \hat{p}(t)\, dt \,|\, g(0) = g\right], \tag{21}$$

where τ is the stopping time indicating the next change of price plans. Appendix C provides an analytic solution for the expected value $\hat{m}(g)$ by solving the associated stochastic differential equation for the benchmark case with no discounting $(r \to 0)$.

[20] This follows from the definition and simple algebra: $\hat{p}(t) \equiv p(t) - p^*(t) = [p^*(\tau_i) + h + (\ell - h)\iota(t)] - [p^*(\tau_i) + g(t)]$, which gives the equation in the text.

Given $\hat{m}(g)$ and the steady-state density of desired prices in equation (12), $f(g)$, the cumulated aggregate output following a small monetary shock $\delta > 0$ is $\mathcal{M}(\delta) = \int_{\underline{g}+\delta}^{\bar{g}} f(g-\delta)\hat{m}(g)\,dg$ which, for a small $\delta > 0$, we approximate by

$$\mathcal{M}(\delta) = \delta\mathcal{M}'(0) + o(\delta) = -\delta\int_{\underline{g}}^{\bar{g}} f'(g)\hat{m}(g)\,dg + o(\delta). \tag{22}$$

Equation (22) lends itself to straightforward numerical analysis since we have analytic expressions for each of its components, as shown below. Next we derive a closed-form analytic expression for the case of zero inflation. This result provides a useful benchmark for low inflation economies. We then return to the general case with inflation and present some numerical results on the effect of μ on $\mathcal{M}(\delta)$.

A. *Analytic Results at* $\mu = 0$ *and Sensitivity to Inflation*

This section specializes the model to $\mu = 0$ to provide a simple analytic characterization of the cumulated output effect. We assume that the economy is in a steady state when an unexpected shock occurs. The shock permanently increases money, nominal wages, and aggregate nominal demand by δ log points. We use the function $\hat{m}$ derived above, as well as the invariant distribution of normalized desired prices $f(g)$, to compute the cumulative impulse of aggregate output for a once and for all shock to the money supply of size δ for the special case in which inflation is zero. We summarize the solution of the integration in equation (22) for the $\mu = 0$ case in the proposition, where we use $N_p = \sigma^2/\bar{g}^2$ to denote the expected number of plan changes per period (see Proposition 5).

PROPOSITION 10: *The cumulative output effect after a small monetary shock* δ *is* $\mathcal{M}(\delta) = \delta\mathcal{M}'(0) + o(\delta)$ *where*

$$\mathcal{M}'(0) = \frac{\bar{g}^2}{18\sigma^2} = \frac{1}{18N_p}. \tag{23}$$

This proposition shows that the cumulated output effect is a decreasing function of the number of plan changes per period, N_p. It is interesting that only the number of *plan* changes, not the *total* number of price changes, appears in the formula. Below we will use this proposition to discuss the relevance of temporary price changes in macroeconomics. Next we provide a result that highlights one key difference between a canonical menu-cost model and a model with plans, namely the impact response of aggregate prices to a monetary shock.

Impact Effect.—Consider the impact of a shock on the price level in the model with price plans and zero inflation, so that $\ell = -h$ and $\hat{g} = 0$. On impact there are two types of price changes, those that come with a change of price plan, and those within the existing price plans. The mass of price changes triggered by a change of price plans is second order, as in canonical menu-cost models. In spite of this, we show next that an aggregate shock triggers a nonnegligible response of the aggregate price level on impact, a result triggered by price changes *within* the plan.

Let $\tilde{\Theta}(\delta)$ denote the impact effect on prices due to price changes within the existing price plan, which is given by the mass of firms whose negative desired price $g < 0$ becomes positive following the shock times the size of their price change, $2h$. The next proposition summarizes the main result.

PROPOSITION 11: *Let $\mu = 0$, δ be an aggregate nominal shock, and $\tilde{\Theta}(\delta)$ denote the impact effect on prices due to price changes within the existing price plans. We have*

$$\lim_{\delta \to 0} \lim_{r \to 0} \frac{\tilde{\Theta}(\delta)}{\delta} = \lim_{\delta \to 0} \lim_{\frac{\psi}{B} \to 0} \frac{\tilde{\Theta}(\delta)}{\delta} = \frac{2}{3}.$$

Thus, for either a small discount rate r or a small fixed cost ψ/B, the response of the aggregate price level on impact is $2/3$ of the monetary expansion. This result is in sharp contrast with the zero impact effect that is a pervasive feature of time-dependent and state-dependent models.[21] It is intuitive that a zero density at the boundaries of the inaction region implies that a small shift of the support of the distribution, say of size δ, triggers a very small mass of adjustments in canonical menu-cost models, since this mass is roughly given by the product between the (near zero) density at the boundary and δ. While this basic mechanism continues to hold in the model with plans, concerning the mass of firms that adjust their plan, the key difference is that in the model with plans there is a nonnegligible mass of firms that change their price *within* the plan.

How Inflation Affects the Cumulated Output Response.—Next we briefly return to the general case with nonzero inflation and discuss how inflation affects the cumulated output effect. Figure 7 reports the cumulated output effect at various inflation rates computed by a numerical evaluation of equation (22). Each point of the curves in the figure is obtained by solving the model at a given inflation rate, keeping all other fundamental parameters constant (such as ψ, B, σ, r).

The blue solid line plots the cumulated output effect, $\mathcal{M}$, as a function of the inflation rate μ in a range from 0 to 35 percent, relative to the value of $\mathcal{M}$ computed at zero inflation. It appears that the curve is flat around zero, a result that shows that, as proved analytically for several of the model's endogenous variables, the zero inflation case provides an accurate approximation for a nonnegligible range of small inflation rates. In Alvarez and Lippi (2019), we provide an analytic result on the sensitivity of the cumulated output response to inflation, confirming the robustness of this finding. As inflation gets into high values the cumulative effect drops, mostly as a result of more frequent plan changes, as suggested by the N_p term, which

[21] This result can be seen analytically from equations (19) and (20) in Caballero and Engel (2007). See Proposition 1 in Alvarez, Lippi, and Passadore (2017) for analytic results on a large class of models in which the impact effect is second order.

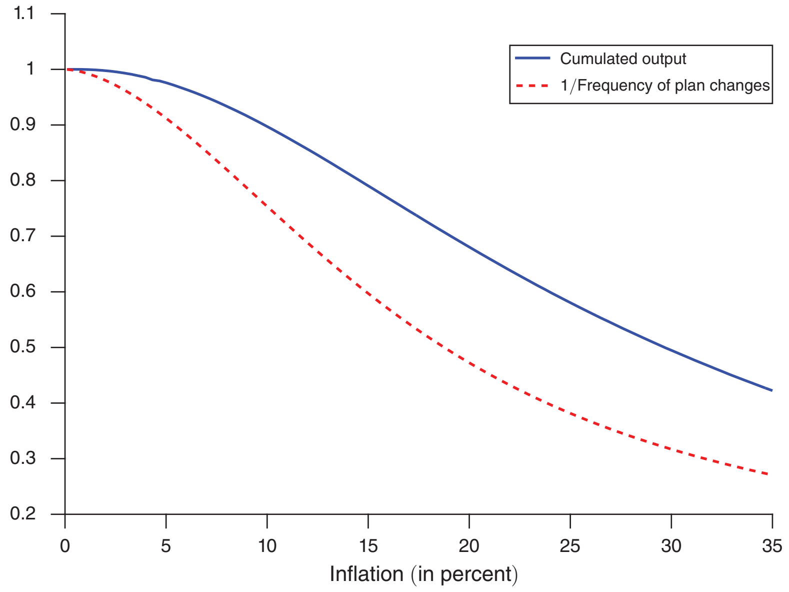

Figure 7. Cumulated Output Response, $\mathcal{M}'(0;\mu)$, at Different Inflation Rates μ

appears in the denominator of equation (23). When inflation equals 30 percent, the cumulated output effect is about 50 percent smaller than the effect at zero inflation. The red dashed line in Figure 7 plots $1/N_p(\mu)$ as a function of inflation (normalized to 1 at $\mu = 0$). This curve can be used to gauge what part of the reduction is due to the higher frequency of plan changes. The gap between the solid line and the dashed line indicates that other forces are at work, such as changes in the shape of the distribution of desired price changes, to mitigate the "output reducing" effect of higher inflation.

B. *Do Temporary Price Changes Matter for Macro?*

This section uses the theoretical results gathered so far to discuss an important substantive macroeconomic question: do temporary price changes matter for the transmission of monetary shocks? To be even more prosaic, suppose we have a simple menu-cost model and some data, containing both temporary and permanent price changes. Should the model be calibrated to match the total number of price changes or only the number of permanent price changes? Our model with plans of course embeds both temporary and permanent price changes and will be used to answer these questions. We will use the model with plans as a laboratory of the "true" monetary response and derive the implications for what approximate simple calibration of the menu-cost model, e.g., including or excluding temporary price changes, best matches the true response of the economy.

We begin the analysis by computing the cumulated output effect in a standard Golosov-Lucas model (GL for short) with a threshold for price adjustment equal to $\bar{g}$. The cumulated output effect is $\mathcal{M}_{GL}(\delta) = \int_{-\bar{g}}^{\bar{g}} m(g+\delta) f(g)\, dg$, which is given by $\mathcal{M}_{GL}(\delta) = \delta \mathcal{M}'_{GL}(0) + o(\delta)$ where

$$\mathcal{M}'_{GL}(0) = \int_{-\bar{g}}^{\bar{g}} m'(g) f(g)\, dg = \frac{\bar{g}^2}{6\sigma^2} = \frac{1}{6N^{GL}}, \tag{24}$$

where we used that the expected number of price changes per period in GL is $N^{GL} = \sigma^2/\bar{g}^2$. The next proposition compares the real cumulative effects in the model with plans to those in the GL model.

PROPOSITION 12: *Assume $\mu = 0$ and let N_p be the mean number of plan changes per period and N^{GL} be the mean number of price changes in the canonical menu-cost model without plans. The ratio of the cumulative output responses in the two models is*

$$\lim_{\delta \downarrow 0} \lim_{\frac{\psi}{B} \downarrow 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \lim_{\delta \downarrow 0} \lim_{r \downarrow 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \frac{3N_p}{N^{GL}}.$$

The proposition is extremely useful because, for small fixed cost ψ/B or small discount factor r, it shows that the only parameters that matter for the comparison between the GL model and the model with plans are the number of plan changes (N_p) and the number of price changes in the GL model (N^{GL}).[22]

In Table 2, we present three alternative comparisons. As a benchmark, and for consistency with the analysis of the previous section, we use a weekly plans model ($\Delta = 1/52$) with a total number of about two reference price changes per year ($N_r = 2.4$, see the second row of the table). Note that given these data, the plans model is fully determined. Instead, the critical choice concerns the frequency of price changes to be used for the GL model. Next we discuss different possible parametrizations of the GL model matching the plans model, respectively, on the total number of price changes, on the number of reference price changes or on the number of plan changes.

The first column of Table 2 assumes that the *total* number of price changes in the GL model is the same as in the model with plans, $N^{GL} = N$, equal to ten price adjustments per year. The second row of the table shows that this corresponds to about 1.4 plan changes per year, so that a straightforward application of Proposition 12 implies that the output effect in the model with plans is 2.5 times bigger than in the GL model (first column, third row). The economics behind this result is that the model with plans is stickier: many of its price changes are temporary, and revert to baseline. In comparison, a GL model with the same number of *total* price changes has no temporary price changes, which implies more flexibility of the aggregate price level and a smaller output effect.

An analytic approximation highlights how the total number of price changes N and the length of time period Δ affect the result of the first column

[22] We refer to these magnitudes as "parameters" since one can always choose the underlying fixed costs to obtain those objects as the optimal firm choices.

Table 2—Plans Model versus Menu-Cost Model Matching Selected Moments

	$N^{GL} \approx N = 10$	$N_r^{GL} \approx N_r = 2.4$	$N^{GL} \approx N_p = 1.4$
Plans model	$N_p = 1.4, N_r = 2.4$	$N_p = 1.4, N_r = 2.4$	$N_p = 1.4, N_r = 2.4$
Menu cost	$N^{GL} = 11, N_r^{GL} = 3$	$N^{GL} = 3.1, N_r^{GL} = 2.2$	$N^{GL} = 1.4, N_r^{GL} = 1.4$
Ratio: $\frac{\mathcal{M}_{GL}}{\mathcal{M}_{\text{Plans}}}$	$\frac{3N_p}{N^{GL}} = \frac{3 \times 1.4}{11} \approx 0.40$	$\frac{3N_p}{N^{GL}} = \frac{3 \times 1.4}{3.1} \approx 1.35$	$\frac{3N_p}{N^{GL}} = 3$

Notes: All results refer to a discrete-time model where the time period is one week, i.e. $\Delta = 1/52$ and inflation is zero. The reference price is defined as the modal price in a quarter.

(where $\Delta = 1/52$). Imposing that $N = N^{GL}$, applying Proposition 12 and rewriting N_p in terms of N and Δ, by the approximation in equation (16), we can write

$$\lim_{\delta \to 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \frac{3N_p}{N^{GL}} = \frac{3\mathcal{N}_p(\Delta, N)}{N} \approx 3N\Delta. \tag{25}$$

Indeed, as $\Delta \to 0$ the ratio of the real effects goes to zero, i.e., the model with plans can be made arbitrarily stickier than the menu-cost model.

Two alternative comparisons are presented in the second and third columns of Table 2. Both comparisons assume that the *total* number of price changes in the menu-cost model is matched to a measure of low-frequency price changes: the number of reference price changes (second column) or the number of plan changes (third column). The motivation for these comparisons is to understand the consequences of the practice, followed by several economists, to calibrate the menu-cost model after discarding from the data some temporary price fluctuations, as in, e.g., Golosov and Lucas (2007) and Nakamura and Steinsson (2008).[23] The second column presents a calibration where the menu-cost model is calibrated to match the number of *reference* price changes observed in the data (formally: $N_r^{GL} = N_r$). The third row of the table shows that in this case the menu-cost model overestimates the effects produced by a model with plans by about 35 percent for the parametrization chosen. The third column finally assumes that the number of price changes in the menu-cost model, N^{GL}, is matched to the number of plan changes, N_p. Naturally the model with plans will feature more price changes per period, due to the presence of the temporary changes. This comparison yields a clear analytic result: setting $N^{GL} = N_p$ in Proposition 12 shows that the high frequency price changes provide a significant amount of flexibility to respond to the shock in the short run compared to the menu-cost model. The cumulative output effect in the plans model under this assumption is $1/3$ of the effect of the menu-cost model. The mechanism behind this result relies on the larger response of prices on impact that occurs in the model with plans (see Proposition 11).

Altogether, the analysis suggests two main points. First, taking into account the nature of price changes, temporary versus reference, is essential for a proper quantification of the monetary transmission mechanism. Failing to do so leads to a substantial underestimation of the effectiveness of monetary shocks, as illustrated in

[23] The heuristic behind this practice is that temporary price changes are not seen as an ordinary form of price adjustment.

the first column of Table 2. This result justifies Kehoe and Midrigan's (2015) view that "prices are sticky after all," since a model with plans can indeed produce many price changes (arbitrarily many as Δ gets smaller) and yet deliver a large output effect. Second, our model shows that approximating the effects of temporary price changes by calibrating a standard menu cost to selected low-frequency moments for price changes can also be misleading. The model with plans shows that the output effect of monetary shocks in a model with N_r reference price changes is smaller than the output effect in a menu-cost model with the same number of reference price changes.

V. Conclusions, Robustness, and Extensions

This paper showed that the introduction of a two-price plan in a standard menu-cost environment generates a persistent "reference" price level and many short-lived deviations from it, as seen in many datasets. We also showed that modeling temporary price changes substantially alters the real effect of monetary shock. In the continuous time model, the real effect of monetary shocks is inversely proportional to the number of plan changes, and independent of the number of price changes. Thus, one can have a very modest amount of aggregate price response to a monetary shock and simultaneously have an arbitrarily large number of price changes. Our preferred exercise is that the net effect, over and above the standard menu-cost model is to add some extra flexibility to the aggregate price level, and hence to have a smaller real effect relative to this benchmark. Yet this conclusion, as illustrated in our characterizations, depends on the exact nature of the comparison, i.e., what is being kept constant when comparing a model with plans to one without them.

In this concluding section, we discuss two substantive extensions that are useful to clarify which results are robust to changes in the modeling environment and which ones are not. The extensions, whose analytical details and formal propositions are given in dedicated appendices, are of intrinsic interest and also allow us to clarify the origin of the differences between our contribution and some important related papers in the literature.

Price Plans à la Calvo.—Online Appendix G analyzes the consequences of introducing price plans $P \equiv \{p_H, p_L\}$ in a setting where the adjustment times for the plan are exogenous and follow the standard Calvo model in which the probability of adjusting the plan has a constant hazard rate. The model retains tractability, and we can derive the optimal prices within the plan and analytically characterize the hazard rate of price changes, which is also decreasing as for the model in the main text.[24] There are two main differences of the "Calvo" version of the model compared to the version in the main body of the paper. The first difference is that the ratio of the area under the impulse response of output of an economy with price plans and one without, but with the same number of price plans, is $1/2$ instead of $1/3$ as in the model in the body of the paper. The second difference is that the Calvo model

[24] In the Calvo version of the model, the hazard rate function is simply $h(t) = 1/(2t) + \lambda$, where λ is the Poisson arrival rate of a price plan change.

shows that some implications of the model in the main body of the paper for the size and the duration of price changes, measured at and just prior to a plan change, are not robust to the specification of the cost of plan changes. Because of this lack of robustness, we did not focus on such statistics. In particular, the behavior of the size and duration of price changes prior (and at) the time of a plan change differ in the two versions of the price plan model. In the benchmark version of the model in the main body of the paper, since a price plan ends when the normalized desired price is large, the duration of a price spell immediately before a plan change cannot be very small. In other words, just before a change of the plan, there cannot be many temporary price changes. Related to this feature, the size of a price change that coincides with a plan change should be large. Both implications come from the fact that in the model in the body of the paper price plans are changed when the (absolute value of the) normalized desired price reaches a threshold $\bar{g}$. In principle, one may try to devise a test of these implications by identifying the times at which price plans have occurred. Yet these implications are not central to models with price plans and temporary price changes; instead, they come from the particular mechanism assumed to change price plans, namely by paying a fixed cost. If instead price plans are changed at exponentially distributed time, i.e., à la Calvo, there are no implications of the typical length of price changes before a price change, and price changes can be small even if they coincide with a plan change.

Costly Price Changes within the Plan.—Online Appendix H discusses a modified model, which assumes that price changes within the plan are not free, although they are cheaper than changes of the plan. This extension is useful to understand the differences with the model of Kehoe and Midrigan (2015), who allow for *costly* temporary price changes that *automatically* revert to the baseline price after one period. The main finding of this extension is a "continuity property" of our main result, namely that the model with price plans continues to feature a lot of additional price flexibility even if the price changes (within the plan) are not free.[25] Of course for the price plan to deliver a significant amount of additional flexibility, the cost of price changes within the plan must be small, an assumption that is consistent with the large number of temporary price changes in the data. This extension highlights the important role of the assumption of the automatic-price reversion in the model of Kehoe and Midrigan: under this assumption, it becomes very costly for the firm to use the temporary price change to track the permanent monetary shock, since this implies paying the temporary cost several times (to offset the automatic reversal).

Appendix A. Solving the Value Function in Closed Form

Let the optimal prices within the plan be denoted by ℓ, h, and the optimal plan-resetting thresholds $\underline{g}, \bar{g}$. The value function $v(g; \ell, h)$ is given by two piecewise component functions v_0, for $g \in [\underline{g}, \hat{g}]$ and v_1 for $g \in (\hat{g}, \bar{g}]$. These functions

[25] This result, formally given in Proposition 18, extends the result of Proposition 11. The economics is that even in the presence of a small adjustment cost within the plan there is a nonnegligible mass of firms in the neighborhood of the price-adjustment threshold (within a plan).

must satisfy the following conditions, where primes denote the partial derivatives with respect to the first argument of the function:

$$\frac{\partial v_0(0;\ell,h)}{\partial \ell} = 0, \qquad \frac{\partial v_0(0;\ell,h)}{\partial h} = 0,$$

$$v_0(\underline{g};\ell,h) = \psi + v_0(0;\ell,h), \qquad v_1(\bar{g};\ell,h) = \psi + v_0(0;\ell,h),$$

$$v_0'(\underline{g};\ell,h) = 0, \qquad v_1'(\bar{g};\ell,h) = 0,$$

$$\lim_{g\uparrow\hat{g}} v_0(g;\ell,h) = \lim_{g\downarrow\hat{g}} v_1(g;\ell,h), \qquad \lim_{g\uparrow\hat{g}} v_0'(g;\ell,h) = \lim_{g\downarrow\hat{g}} v_1'(g;\ell,h),$$

which is a system of eight equations in eight unknowns $\ell, h, \underline{g}, \bar{g}$ and the four unknowns from the two second-order differential equations for v_0 and v_1.

The solution of the value function v_0 in inaction is given by the sum of a particular solution and the solution to the homogenous function. The particular solution $v^p(g)$ is

$$v_0^p(g;\ell,h) = \frac{B}{r}\left[\ell^2 + \frac{2\mu}{r}\left(\frac{\mu}{r} - \ell\right) + \frac{\sigma^2}{r} + 2g\left(\frac{\mu}{r} - \ell\right) + g^2\right];$$

the homogeneous solution $v_0^f(g)$, for $g > 0$, is

$$\text{(A1)} \quad v_0^f(g;\ell,h) = \xi_{0,1}e^{\eta_1 g} + \xi_{0,2}e^{\eta_2 g} \text{ where } \eta_i = \frac{-\mu \pm \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}, \; i = 1,2.$$

Equivalent expressions are obtained for v_1. Thus, the solution for v is

$$\text{(A2)} \quad v(g;\ell,h) = \begin{cases} \frac{B}{r}\left[\ell^2 + \frac{2\mu}{r}\left(\frac{\mu}{r} - \ell\right) + \frac{\sigma^2}{r} + 2g\left(\frac{\mu}{r} - \ell\right) + g^2\right] & \\ \quad + \xi_{0,1}e^{\eta_1 g} + \xi_{0,2}e^{\eta_2 g} & \text{for all } g \in \left[\underline{g}, \frac{\ell+h}{2}\right] \\ \frac{B}{r}\left[h^2 + \frac{2\mu}{r}\left(\frac{\mu}{r} - h\right) + \frac{\sigma^2}{r} + 2g\left(\frac{\mu}{r} - h\right) + g^2\right] & \\ \quad + \xi_{1,1}e^{\eta_1 g} + \xi_{1,2}e^{\eta_2 g} & \text{for all } g \in \left(\frac{\ell+h}{2}, \bar{g}\right] \end{cases}.$$

Appendix B. Solving the Optimal Prices within the Plan

We solve for ℓ, h, given a value of the inaction thresholds $\underline{g}, \bar{g}$. Consider a firm that has just reset its price plan and that *takes as given* the value of the plan-resetting thresholds. Let τ be the stopping time associated with a change in the plan (which occurs as either $\underline{g}$ or $\bar{g}$ is hit). The relevant objective function for this problem, until the next time the plan is changed, is

$$\min_{\ell,h} E\left[\int_0^\tau e^{-rt}\min\left(\left(\ell - g(t)\right)^2, \left(h - g(t)\right)^2\right)dt \,\middle|\, g(0) = 0\right].$$

Note that this is a quadratic minimization problem, with a convex objective function. The first-order conditions for this problem for ℓ and h give equation (6).

Consider the expected discounted values that appear in the numerator and denominator of the expression that determines ℓ equation (6), as a function of an arbitrary initial g:

$$n_\ell(g) \equiv E\left[\int_0^\tau e^{-rt}\iota(t)g(t)dt \,\middle|\, g(0) = g\right], \quad d_\ell(g) \equiv E\left[\int_0^\tau e^{-rt}\iota(t)\,dt \,\middle|\, g(0) = g\right].$$

We are interested in evaluating them at $g = 0$ to get: $\ell = n_\ell(0)/d_\ell(0)$.

PROPOSITION 13. *Consider the steady-state problem with* $r \to 0$. *Given two arbitrary thresholds* $\underline{g} < \bar{g}$ *defining the stopping times for a change of plan, the optimal low and high prices within the plan,* ℓ *and* h *respectively, solve the system of equations*

$$\ell = \frac{n_\ell(0)}{d_\ell(0)} \quad \text{and} \quad h = \frac{n_h(0)}{d_h(0)},$$

where $\hat{g} \equiv (\ell + h)/2$, $\xi \equiv -2\mu/\sigma^2$, *and*

$$n_\ell(0) = \frac{\underline{g}}{\mu\xi} + \frac{\underline{g}^2}{2\mu} + C\left(1 - e^{\xi\underline{g}}\right), \tag{B1}$$

$$\text{with} \quad C = \frac{1}{\mu}\left(\frac{\frac{\underline{g} - \hat{g}}{\xi} + \frac{\underline{g}^2 - \hat{g}^2}{2} + \frac{\hat{g}\xi + 1}{\xi^2}\left(1 - e^{\xi(\bar{g} - \hat{g})}\right)}{e^{\xi\underline{g}} - e^{\xi\bar{g}}}\right),$$

$$d_\ell(0) = \frac{\underline{g}}{\mu} + A\left(1 - e^{\xi\underline{g}}\right), \tag{B2}$$

$$\text{with} \quad A = \frac{1}{\mu}\left(\frac{\underline{g} - \hat{g} + \frac{1 - e^{\xi(\bar{g} - \hat{g})}}{\xi}}{e^{\xi\underline{g}} - e^{\xi\bar{g}}}\right),$$

$$n_h(0) = F\left(1 - e^{\xi\underline{g}}\right), \tag{B3}$$

$$\text{with} \quad F = \frac{1}{\mu}\left(\frac{\frac{\bar{g} - \hat{g}}{\xi} + \frac{\bar{g}^2 - \hat{g}^2}{2} + \frac{\hat{g}\xi + 1}{\xi^2}\left(1 - e^{\xi(\bar{g} - \hat{g})}\right)}{e^{\xi\bar{g}} - e^{\xi\underline{g}}}\right),$$

$$d_h(0) = E\left(1 - e^{\xi\underline{g}}\right), \tag{B4}$$

$$\text{with} \quad E = \frac{1}{\mu}\left(\frac{\bar{g} - \hat{g} + \frac{1 - e^{\xi(\bar{g} - \hat{g})}}{\xi}}{e^{\xi\bar{g}} - e^{\xi\underline{g}}}\right).$$

Ordinary Differential Equation (*ODE*) *for* $n(g)$.—Likewise, for $r \to 0$ the function $n_\ell(g)$ solves the ODE

$$0 = \begin{cases} g + n'\mu + \frac{\sigma^2}{2} n'' & \text{for } \underline{g} < g < \hat{g} \\ 0 + n'\mu + \frac{\sigma^2}{2} n'' & \text{for } \hat{g} < g < \bar{g} \end{cases}. \tag{B5}$$

We solve for $n(g)$ using the boundary conditions $n(\underline{g}) = n(\bar{g}) = 0$. This gives

$$n_\ell(g) = \begin{cases} \frac{1}{\mu\xi}(\underline{g} - g) + \frac{1}{2\mu}(\underline{g}^2 - g^2) + C\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ D\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases}. \tag{B6}$$

To pin down C and D, use value matching and smooth pasting at $g = \hat{g}$ to get equation (B1).

ODE for $d(g)$.—The function $d(g)$ solves the ODE (where obvious, we omit the ℓ subscript in what follows). Focus on the steady-state case, i.e., $r \to 0$, we get (since τ is finite so that $d(g) < \infty$):

$$0 = \begin{cases} 1 + d'\mu + \frac{\sigma^2}{2} d'' & \text{for } \underline{g} < g < \hat{g} \\ 0 + d'\mu + \frac{\sigma^2}{2} d'' & \text{for } \hat{g} < g < \bar{g} \end{cases}. \tag{B7}$$

We solve for $d(g)$ using the boundary conditions $d(\underline{g}) = d(\bar{g}) = 0$. This gives

$$d_\ell(g) = \begin{cases} \frac{1}{\mu}(\underline{g} - g) + A\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ B\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases}. \tag{B8}$$

To pin down A and B, use value matching and smooth pasting at $g = \hat{g}$, to get equation (B2).

Appendix C. Solution for the Cumulated Output Effect

Using the price-gap definition given in the text, we have

$$\hat{m}(g) = -E\left[\int_0^\tau \left(h + (\ell - h)\iota(t) - g(t)\right) dt \,|\, g(0) = g\right]$$

so that $\hat{m}(g)$ solves the following ODE: $0 = g - \ell + \hat{m}'\mu + (\sigma^2/2)\hat{m}''$ for $g \in (\underline{g}, \hat{g})$ and $0 = g - h + \hat{m}'\mu + (\sigma^2/2)\hat{m}''$ for $g \in (\hat{g} < \bar{g})$, where the function $\hat{m}(g)$ is continuous and differentiable at $\hat{g}$, with boundary conditions $\hat{m}(\underline{g}) = \hat{m}(\bar{g}) = 0$. This gives

$$\hat{m}(g) = \begin{cases} \frac{g - \underline{g}}{\mu}\left(\ell - \frac{1}{\xi}\right) + \frac{\underline{g}^2 - g^2}{2\mu} + A\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ \frac{g - \bar{g}}{\mu}\left(h - \frac{1}{\xi}\right) + \frac{\bar{g}^2 - g^2}{2\mu} + B\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases}, \tag{C1}$$

where $\xi \equiv -\frac{2\mu}{\sigma^2}$.

Use continuity and differentiability at $\hat{g}$ to solve for A and B. Continuity gives

$$\left(\hat{g}-\underline{g}\right)\left(\ell-\frac{1}{\xi}\right)+\frac{\underline{g}^{2}-\bar{g}^{2}}{2}+\mu A\left(e^{\xi\hat{g}}-e^{\xi\underline{g}}\right) = \left(\hat{g}-\bar{g}\right)\left(h-\frac{1}{\xi}\right)+\mu B\left(e^{\xi\hat{g}}-e^{\xi\bar{g}}\right).$$

Differentiability gives

$$B = A+\frac{\ell-h}{\mu\xi e^{\xi\hat{g}}},$$

which gives a simple linear system of two equations in A, B.

Appendix D. Proofs

PROOF OF PROPOSITION 1:

(i) The symmetry of the quadratic period losses implies $-\ell(-\mu) = h(\mu)$ and $-\underline{g}(-\mu) = \bar{g}(\mu)$. Assuming differentiability we get $\ell'(-\mu) = h'(\mu)$ and $\underline{g}'(-\mu) = \bar{g}'(\mu)$, which gives equation (7) at $\mu = 0$.

(ii) With a slight abuse of notation, let $v(g;\ell,h,\mu)$ denote the value function with inflation explicitly added as a fourth argument. Totally differentiate the value-matching condition in equation (3) with respect to inflation and evaluate at $\mu = 0$:

$$\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial g}\frac{\partial \underline{g}}{\partial\mu}+\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\ell}\frac{\partial\ell}{\partial\mu}+\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial h}\frac{\partial h}{\partial\mu}+\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\mu}$$

$$= \frac{\partial v(0;\ell,h,\mu)}{\partial\ell}\frac{\partial\ell}{\partial\mu}+\frac{\partial v(0;\ell,h,\mu)}{\partial h}\frac{\partial h}{\partial\mu}+\frac{\partial v(0;\ell,h,\mu)}{\partial\mu}.$$

Notice that the three terms in the second line are equal to zero due to the optimality of ℓ and h and the symmetry of v with respect to zero inflation. The first term of the first equation is also zero due to the optimal choice of $\underline{g}$. We thus have

$$\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\ell}\frac{\partial\ell}{\partial\mu}+\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial h}\frac{\partial h}{\partial\mu}+\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\mu} = 0.$$

The key to the proof is to note that $\partial v(\underline{g};\ell,h,\mu)/\partial h = 0$ since at g the price h is not affecting the return, so it has a zero impact on the value function for $g \in (\underline{g},0)$ and it also has a zero effect at $g = 0$ due to optimality. Then we get

$$\frac{\partial\ell}{\partial\mu} = -\frac{\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\mu}}{\frac{\partial v(\underline{g};\ell,h,\mu)}{\partial\ell}} > 0$$

since the derivative $\partial v(\underline{g};\ell,h,\mu)/\partial\mu$ is negative (lower inflation at $\underline{g}$ lowers the expected losses for $g \in (\underline{g},0)$ and has a zero effect at $g = 0$ due to symmetry). By the same logic, $\partial v(\underline{g};\ell,h,\mu)/\partial\ell > 0$ since a higher price at $\underline{g}$ worsens the losses (the firm is about to set a lower, not a higher, price at that point).

(iii) Symmetry of the period losses implies that $N(\mu) = N(-\mu)$. Assuming differentiability then gives $N'(\mu) = -N'(-\mu)$, which at $\mu = 0$ gives the result in the proposition. ∎

PROOF OF LEMMA 1:

Let the functions $a(g)$ and $d(g)$ denote respectively the numerator and denominator of equation (6) for the optimal price h in the case of $\mu = 0$. These functions solve the following ODEs and boundary conditions:

$$ra(g) = |g(t)| + \tfrac{\sigma^2}{2}a''(g)$$

$$\text{for all} \quad g \in [-\bar{g},\bar{g}], \quad g \neq 0, \quad a(-\bar{g}) = a(\bar{g}) = 0, \quad \text{and} \quad a'(0) = 0;$$

$$rd(g) = 1 + \tfrac{\sigma^2}{2}d'(g) \qquad \text{for all} \quad g \in [-\bar{g},\bar{g}], \quad d(-\bar{g}) = d(\bar{g}) = 0.$$

First we develop the expressions for a. The function a must be symmetric around $g = 0$ so that $a(g) = a(-g)$ for all $g \in [0,\bar{g}]$, thus,

$$a(g) = \begin{cases} +g/r + A_1 e^{\sqrt{2r/\sigma^2}\,g} + A_2 e^{-\sqrt{2r/\sigma^2}\,g} & \text{if } g \in [0,\bar{g}] \\ -g/r + A_2 e^{\sqrt{2r/\sigma^2}\,g} + A_1 e^{-\sqrt{2r/\sigma^2}\,g} & \text{if } g \in [-\bar{g},0] \end{cases}.$$

The boundary conditions $a'(0) = 0$ and $a(\bar{g}) = 0$ give

$$1 = r(A_2 - A_1)\sqrt{2r/\sigma^2}, \quad 0 = \bar{g} + rA_1 e^{\sqrt{2r/\sigma^2}\,\bar{g}} + rA_2 e^{-\sqrt{2r/\sigma^2}\,\bar{g}}.$$

Hence,

$$-\bar{g} - \frac{e^{-\sqrt{2r/\sigma^2}\,\bar{g}}}{\sqrt{2r/\sigma^2}} = rA_1\left(e^{\sqrt{2r/\sigma^2}\,\bar{g}} + e^{-\sqrt{2r/\sigma^2}\,\bar{g}}\right),$$

$$r(A_1 + A_2) = \frac{1}{\sqrt{2r/\sigma^2}} + 2rA_1 = \frac{1}{\sqrt{2r/\sigma^2}} - 2\frac{\bar{g} + \dfrac{e^{-\sqrt{2r/\sigma^2}\,\bar{g}}}{\sqrt{2r/\sigma^2}}}{e^{\sqrt{2r/\sigma^2}\,\bar{g}} + e^{-\sqrt{2r/\sigma^2}\,\bar{g}}},$$

since we are interested in

$$ra(0) = r(A_1 + A_2) = \bar{g}\left[\frac{1}{\sqrt{2r/\sigma^2}\,\bar{g}} - 2\frac{1 + \dfrac{e^{-\sqrt{2r/\sigma^2}\,\bar{g}}}{\sqrt{2r/\sigma^2}\,\bar{g}}}{e^{\sqrt{2r/\sigma^2}\,\bar{g}} + e^{-\sqrt{2r/\sigma^2}\,\bar{g}}}\right].$$

For d we have, as shown in Alvarez, Le Bihan, and Lippi (2016) that

$$rd(0) = \frac{e^{\sqrt{2r/\sigma^2}\,\bar{g}} + e^{-\sqrt{2r/\sigma^2}\,\bar{g}} - 2}{e^{\sqrt{2r/\sigma^2}\,\bar{g}} + e^{-\sqrt{2r/\sigma^2}\,\bar{g}}}.$$

We can write

$$ra(0) = \bar{g}\frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}\right)}, \quad rd(0) = \frac{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2}{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}}.$$

Thus,

$$h = \frac{ra(0)}{rd(0)} = \bar{g}\rho(\phi) = \bar{g}\frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)}.$$

The properties of ρ follow directly from this expression. The limit as $\phi \to 0$ follows by letting $x = \sqrt{2\phi}$ into the expression for $h/\bar{g}$ and expanding the exponentials, canceling to obtain

$$\frac{h}{\bar{g}} = \frac{e^x - e^{-x} - 2x}{x\left[e^x + e^{-x} - 2\right]} = \frac{2\left(x + x^3/3! + x^5/5! + \cdots\right) - 2x}{x\left[2\left(1 + x^2/2 + x^4/4! + \cdots\right) - 2\right]}$$

$$= \frac{(2/3!) + x^2(2/5!) + x^4(2/7!) + \cdots}{(2/2!) + x^2(2/4!) + x^4(2/6!) + \cdots}.$$

Taking the limit $x \to 0$, we obtain $h/\bar{g} = (2/3!)/(2/2!) = 1/3$. That ρ is decreasing it follows by inspection of the previous expression since each of the coefficients of x is smaller in the numerator. That the limit as $\phi \to 0$ of $\rho(\phi)\sqrt{2\phi} \to 1$ follows immediately since $\phi > 0$. This also implies that $\rho \to 0$ as $\phi \to \infty$. ∎

PROOF OF PROPOSITION 2:

We first establish an intermediate result in the following lemma.

LEMMA 2: *Let $h \geq 0$ be an arbitrary width for the reset prices. We derive an equation solving for the optimal inaction threshold $\bar{g}$, where we write the width of* *the price threshold as $h = \gamma \bar{g}$ for a constant $0 \leq \gamma$. The optimal inaction threshold $\bar{g}$ must solve:*

$$\eta^2 r \frac{\psi}{B} = \varphi(x;\gamma) \tag{D1}$$

$$\text{with} \quad x \equiv \eta\bar{g} \quad \text{and} \quad \varphi(x;\gamma) \equiv (1-2\gamma)\left(x^2 - 2x\frac{[e^x + e^{-x} - 2]}{[e^x - e^{-x}]}\right).$$

The function $\varphi(x;\gamma)$ is: (i) strictly increasing in $x \geq 0$ for each $0 \leq \gamma < 1/2$, (ii) strictly decreasing in γ for each $x > 0$, and (iii) for $0 \leq \gamma < 1/2$, then $\lim_{x\to\infty} \varphi(x;\gamma)/x^2 = 1$, *and* $\lim_{x\to 0} \varphi(x;\gamma)/\left(x^4/12\right) = 1$.

Using Lemma 2, we obtain the function κ by simply replacing ρ into φ and using that $x^2/2 \equiv (\eta\bar{g})^2/2 = r\bar{g}^2/\sigma^2 \equiv \phi$, where ϕ is defined in Proposition 1. Since $\varphi(x,\gamma)$ is increasing in x and decreasing in γ, and since ρ is decreasing in γ, then κ is strictly increasing in x. Since $\rho'(0)$ is finite, then we can just substitute the limit value of $\rho(0) = 1/3$ and obtain $\kappa(x) = (1-2/3)x^4/12 + o(x^4) = x^4/36 + o(x^4)$. For large x, we established that $\rho(x) \to 0$, and hence $\lim_{x\to\infty} \kappa(x)/x^2 = \lim_{x\to\infty} \varphi(x)/x^2 = 1$. ∎

PROOF OF LEMMA 2:

The solution of the value function in inaction is given by the sum of a particular solution and the solution to the homogenous function. The particular solution $v^p(g)$ is

$$v^p(g) = \frac{B}{r}\left[g^2 + h^2 - 2|g|h + \frac{\sigma^2}{r}\right]$$

and the homogeneous solution $v^f(g)$, for $g > 0$, is

$$v^f(g) = A_1 e^{-\eta g} + A_2 e^{\eta g} \qquad \text{where} \quad \eta = \sqrt{2r/\sigma}. \tag{D2}$$

Thus, the solution of v is

$$v(g;h) = \begin{cases} (B/r)g^2 - (B2h/r)g + (B/r)\left[h^2 + \frac{\sigma^2}{r}\right] & \\ \quad + A_1 e^{\sqrt{2r/\sigma^2}\,g} + A_2 e^{-\sqrt{2r/\sigma^2}\,g} & \text{for all } g \in (0,\bar{g}] \\ (B/r)g^2 + (B2h/r)g + (B/r)\left[h^2 + \frac{\sigma^2}{r}\right] & \\ \quad + A_2 e^{\sqrt{2r/\sigma^2}\,g} + A_1 e^{-\sqrt{2r/\sigma^2}\,g} & \text{for all } g \in [-\bar{g},0) \end{cases}. \tag{D3}$$

Value matching and smooth pasting are

$$A_1 + A_2 + \psi = (B/r)\bar{g}^2 - (B2h/r)\bar{g} + A_1 e^{\sqrt{2r/\sigma^2}\,\bar{g}} + A_2 e^{-\sqrt{2r/\sigma^2}\,\bar{g}}, \tag{D4}$$

$$0 = 2(B/r)\bar{g} - (B2h/r) + \sqrt{2r/\sigma^2}\left[A_1 e^{\sqrt{2r/\sigma^2}\,\bar{g}} - A_2 e^{-\sqrt{2r/\sigma^2}\,\bar{g}}\right]. \tag{D5}$$

If v is differentiable at $g = 0$, then equation (5) implies

$$\sqrt{2\left(r/\sigma^2\right)}\,[A_1 - A_2] = B2\left(h/r\right). \tag{D6}$$

For $r > 0$, we can rewrite this system as

$$a_1 + a_2 + r\frac{\psi}{B} = \left(\bar{g} - 2h\right)\bar{g} + a_1 e^{\eta\bar{g}} + a_2 e^{-\eta\bar{g}}, \tag{D7}$$

$$0 = 2\left(\bar{g} - h\right) + \eta\left[a_1 e^{\eta\bar{g}} - a_2 e^{-\eta\bar{g}}\right], \tag{D8}$$

$$a_1 - a_2 = 2h/\eta. \tag{D9}$$

Solving for a_1 and replacing it, we get

$$r\frac{\psi}{B} = \left(\bar{g} - 2h\right)\bar{g} + a_2\left(e^{\eta\bar{g}} + e^{-\eta\bar{g}} - 2\right) + 2h\left(\frac{e^{\eta\bar{g}} - 1}{\eta}\right),$$

$$a_2 = -2\frac{\bar{g} + h\left(e^{\eta\bar{g}} - 1\right)}{\eta\left[e^{\eta\bar{g}} - e^{-\eta\bar{g}}\right]}.$$

Solving for $\bar{g}$, we get

$$r\frac{\psi}{B} = \left(\bar{g} - 2h\right)\bar{g} + 2h\left(\frac{e^{\eta\bar{g}} - 1}{\eta}\right) - \frac{e^{\eta\bar{g}} + e^{-\eta\bar{g}} - 2}{e^{\eta\bar{g}} - e^{-\eta\bar{g}}}\,2\left[\frac{\bar{g}}{\eta} + h\left(\frac{e^{\eta\bar{g}} - 1}{\eta}\right)\right]. \tag{D10}$$

Thus, we can define

$$\eta^2 r\frac{\psi}{B} = \varphi\left(x;\gamma\right) \qquad \text{with} \quad x \equiv \eta\bar{g}, \quad h \equiv \gamma\bar{g},$$

and $\varphi(\cdot)$ defined as

$$\varphi\left(x;\gamma\right) \equiv x\left(x - 2\gamma x\right) + 2\gamma x\left(e^x - 1\right) - 2\frac{e^x + e^{-x} - 2}{e^x - e^{-x}}\left(x + \gamma x\left(e^x - 1\right)\right). \tag{D11}$$

Rewrite the first term as: $x\left(x - 2\gamma x\right) = x^2\left(1 - 2\gamma\right)$ and collecting the remaining terms we have (after simple algebra):

$$2\gamma x\left(e^x - 1\right)\left[\frac{1 - e^x + e^{-x} - 2}{e^x - e^{-x}}\right] - 2\left[\frac{e^x + e^{-x} - 2}{e^x - e^{-x}}\right]x = \left(2\gamma - 1\right)\left[\frac{e^x + e^{-x} - 2}{e^x - e^{-x}}\right]2x.$$

Thus, we can write

$$\varphi\left(x;\gamma\right) \equiv \left(1 - 2\gamma\right)\left(x^2 - 2\frac{\left[e^x + e^{-x} - 2\right]}{\left[e^x - e^{-x}\right]}x\right).$$

To see that φ is increasing in x, note that

$$\frac{\partial\varphi\left(x;\gamma\right)}{\partial x} \equiv \left(1 - 2\gamma\right)\left(2\frac{\left[e^x + e^{-x} - 2\right]}{\left[e^x - e^{-x}\right]}\right)\left(x\frac{\left[e^x + e^{-x}\right]}{\left[e^x - e^{-x}\right]} - 1\right) \geq 0$$

since

$$2\frac{[e^x + e^{-x} - 2]}{[e^x - e^{-x}]} = 2\frac{(e^x - 1)(1 - e^{-x})}{[e^x - e^{-x}]} \geq 0$$

and

$$x\frac{[e^x + e^{-x}]}{[e^x - e^{-x}]} = \frac{x + x^3/2! + x^5/4! + x^7/6! + \cdots}{x + x^3/3! + x^5/5! + x^7/7! + \cdots} \geq 1.$$

To see that $\lim_{x\to\infty}\varphi(x;\gamma) = \infty$, note that

$$\lim_{x\to\infty}\frac{\varphi(x;\gamma)}{x} = \lim_{x\to\infty} x - 2\lim_{x\to\infty}\frac{[e^x + e^{-x} - 2]}{[e^x - e^{-x}]} = \infty - 2 = \infty$$

and thus

$$\lim_{x\to\infty}\frac{\varphi(x;\gamma)}{x^2} = 1.$$

Finally to obtain the expansion for small x, we write

$$\begin{aligned}
\frac{\varphi(x;\gamma)}{(1-2\gamma)} &= x^2 - 2\frac{e^x + e^{-x} - 2}{e^x - e^{-x}}x \\
&= x^2\left(1 - 2\frac{1}{x}\left[\frac{x^2/2 + x^4/4! + \cdots}{x + x^3/3! + x^5/5! + \cdots}\right]\right) \\
&= x^2\left(\frac{x^4/3! + x^6/5! + \cdots - x^4(2/4!) - x^6(2/6!) + \cdots}{x^2 + x^4/3! + x^6/5! + \cdots}\right) \\
&= x^4\frac{1}{12} + o(x^4). \blacksquare
\end{aligned}$$

PROOF OF PROPOSITION 3:

Fix a sequence of stopping times in the firm's sequence problem. The firm's expected losses within the plan are

$$\min_x E\left(B\int_0^\tau e^{-rt}(x - g(t))^2\,dt\,\middle|\,g(0) = 0\right) \quad \text{with } x = \{\ell, h\},$$

where τ is the length of the plan and $g(t) \equiv p^*(t) - p^*(0)$. Recall that for $\mu > 0$ all plans begin with $g = 0$ and price ℓ, and switch to the high price h when $g \in (\hat{g}, \bar{g})$, where $\hat{g} \equiv (\ell + h)/2$. Let us consider the steady-state case where $r \to 0$ rewrite

$$\begin{aligned}
B\min_{\ell,h}\int_0^\tau E\Big(\iota(t)\big(\ell^2 + g^2(t) - 2g(t)\ell\big) \\
+ \big(1 - \iota(t)\big)\big(h^2 + g^2(t) - 2g(t)h\big)\,\big|\,g(0) = 0\Big)\,dt,
\end{aligned}$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g} < g(t) < \hat{g}$ and zero otherwise. Computing the expectations using the law of motion for $g(t) = g(0) + \mu t + \sigma \int_0^t dW(s)$, we get

$$B \min_{\ell,h} \int_0^\tau \left(\iota(t)\left(\ell^2 + \mu^2 t^2 - 2\mu t\ell + \sigma^2 t\right) + \left(1 - \iota(t)\right)\left(h^2 + \mu^2 t^2 - 2\mu t h + \sigma^2 t\right)\right) dt.$$

We conjecture (and later verify) that the optimal rule is linear in inflation, so that $\ell = \mu\tilde{\ell}$, $h = \mu\tilde{h}$, which implies $\hat{g}/\mu = \left(\tilde{\ell} + \tilde{h}\right)/2 \equiv h$. We can rewrite the problem as

$$\mu^2 B \min_{\tilde{\ell},\tilde{h}} \int_0^\tau \left(\iota(t)\left(\tilde{\ell}^2 + t^2 - 2t\tilde{\ell} + \frac{\sigma^2}{\mu^2}t\right) + \left(1 - \iota(t)\right)\left(\tilde{h}^2 + t^2 - 2t\tilde{h} + \frac{\sigma^2}{\mu^2}t\right)\right) dt.$$

For $\mu \to \infty$ the σ/μ terms becomes infinitesimal, thus $\tilde{\ell}$, $\tilde{h}$ solve the following deterministic problem (we omit the $\mu^2 B$ scaling term):

$$\text{(D12)} \qquad \min_{\tilde{\ell},\tilde{h}} \int_0^{\tau_1} \left(\tilde{\ell}^2 + t^2 - 2\tilde{\ell}t\right) dt + \int_{\tau_1}^{\tau} \left(\tilde{h}^2 + t^2 - 2\tilde{h}t\right) dt,$$

where τ_1 is the time elapsed until $g(t)$ hits $\hat{g}$ starting from $g(0) = 0$, or $\tau_1 = (\ell + h)/2\mu = \left(\tilde{\ell} + \tilde{h}\right)/2$. Using $\tau_1 = h$ in the first-order conditions for the optimal ℓ and h in equation (6) immediately gives equation (11), which verifies the conjecture that ℓ and h are linear in μ.[26]

Let us now turn to the optimal choice of τ, the duration of a plan. In the deterministic problem obtained when $\mu^2/\sigma^2 \to \infty$, the firm optimal choice of τ solves (again we assume $r \to 0$)

$$\min_\tau \frac{1}{\tau}\left(\psi/B + \int_0^{\tau/2}\left(\frac{\tau\mu}{4} - \mu t\right)^2 dt + \int_{\tau/2}^{\tau}\left(\frac{\tau\mu 3}{4} - \mu t\right)^2 dt\right);$$

symmetry of the period losses and optimal prices immediately gives

$$\min_\tau \frac{1}{\tau}\left(\psi/B + 2\int_0^{\tau/2}\left(\frac{\tau\mu}{4} - \mu t\right)^2 dt\right).$$

The first-order condition gives the optimal duration of each price within the plan $\tau/2 = \left(3\psi\mu^{-2}/B\right)^{1/3}$, which shows the elasticity of the frequency of price changes, $1/\tau$ with respect to inflation is $2/3$, as in the Sheshinski and Weiss (1977) model. ∎

PROOF OF PROPOSITION 4:

First we derive the invariant distribution of desired prices, whose density we denote by $f(g)$. Recall $dg(t) = \mu dt + \sigma dW$. The Kolmogorov forward equation gives $0 = \mu f' + \left(\sigma^2/2\right)f''$. Given the repricing rule with boundaries $\left(\underline{g}, \bar{g}\right)$ and return point $g^* \in \left(\underline{g}, \bar{g}\right)$. Note $g^* = 0$ in our model as the desired normalized price is zero at the beginning of a new plan. For $g \in \left(\underline{g}, g^*\right)$, the

[26] The expressions for the optimal ℓ and h can equivalently be obtained taking first-order conditions of equation (D12) (note that the derivatives of the extremes of integration will cancel out at $\tau_1 = h$).

density $f(g)$ solving ODE and boundary $f(\underline{g}) = 0$ is $f(g) = A\left(e^{-\xi \underline{g}} - e^{-\xi g}\right)$ with $\xi \equiv -2\mu/\sigma^2$. For $g \in (g^*, \bar{g})$, the density $f(g)$ solving ODE, boundary $f(\bar{g}) = 0$, and continuity at g^* is $f(g) = C\left(e^{-\xi g} - e^{-\xi \bar{g}}\right)$ with $C \equiv A\, \frac{e^{-\xi \underline{g}} - e^{-\xi g^*}}{e^{-\xi g^*} - e^{-\xi \bar{g}}}$. Use that the density integrates to 1 to find A: $1 = \int_{\underline{g}}^{g^*} f(g)\, dg + \int_{g^*}^{\bar{g}} f(g)\, dg$, after simple algebra, we get $A = \left((g^* - \underline{g})e^{-\xi \underline{g}} - (\bar{g} - g^*)\, \frac{e^{-\xi \underline{g}} - e^{-\xi g^*}}{e^{-\xi g^*} - e^{-\xi \bar{g}}}\, e^{-\xi \bar{g}}\right)^{-1}$. Next we compute the number of plan changes per unit of time. Let the function $\mathcal{T}(\tilde{p})$ the expected time until g first reaches $\bar{g}$ or $\underline{g}$. The average number of plan adjustments, denoted by $N_p = 1/\mathcal{T}(0)$. The function $\mathcal{T}(g)$ satisfies the ODE: $0 = 1 - \mu\mathcal{T}'(g) + (\sigma^2/2)\mathcal{T}''(g)$ with boundary conditions $\mathcal{T}(\underline{g}) = \mathcal{T}(\bar{g}) = 0$, where $\xi \equiv -2\mu/\sigma^2$. This gives $\mathcal{T}(g) = A_0 + \frac{g}{\mu} + A_1 e^{-\xi g}$ where $A_1 = -\frac{\bar{g} - \underline{g}}{\mu\left(e^{-\xi \bar{g}} - e^{-\xi \underline{g}}\right)}$, $A_0 = -A_1 e^{-\xi \bar{g}} - \frac{\bar{g}}{\mu}$, which gives equation (13). ∎

PROOF OF PROPOSITION 5:

Let $T(g)$ be the expected time until the next change of price plan, i.e., until $|g_n|$ reaches $\bar{g}$. We can index the state by $i = 0, \pm 1, \ldots, \pm \bar{n}$. We have the discrete-time version of the Kolmogorov backward equation (KBE):

$$T_i = \Delta + \tfrac{1}{2}\left[T_{i+1} + T_{i-1}\right] \quad \text{for all } i = 0, \pm 1, \pm 2, \ldots, \pm(\bar{n} - 1),$$

and at the boundaries, we have $T_{\bar{n}} = T_{-\bar{n}} = 0$. We use a guess and verify strategy, guessing a solution of the form:

$$T_i = a_0 + a_2 i^2 \quad \text{for all } i = 0, \pm 1, \pm 2, \ldots, \pm \bar{n},$$

for some constants a_0, a_2. Inserting this into the KBE, we obtain

$$a_0 + a_2 i^2 = \Delta + \frac{1}{2}\left[a_0 + a_2(i+1)^2 + a_0 + a_2(i-1)^2\right]$$
$$\text{for all } i = 0, \pm 1, \pm 2, \ldots, \pm(\bar{n} - 1)$$

so that $a_2 = -\Delta$. Using this value, into the equation for the boundary condition, we get

$$a_0 - \Delta(\bar{n})^2 = 0 \quad \Rightarrow \quad a_0 = \Delta(\bar{n})^2$$

and since $\bar{n}\sqrt{\Delta}\sigma = \bar{g}$ and $T_0 = a_0$, we have

$$T_0 = a_0 = \Delta(\bar{n})^2 = \Delta\left(\frac{\bar{g}}{\sqrt{\Delta}\sigma}\right)^2 = \left(\frac{\bar{g}}{\sigma}\right)^2 = \frac{1}{N_p}. \ ∎$$

PROOF OF PROPOSITION 6:

The proof for the inequalities in equation (15) considers the case $\bar{n} \geq 2$ (see the discussion following equation (14)), which is equivalent to $N_p \Delta \leq 1/4$. The proof revolves on two lemmas, stated below, whose full-length proof is given in the online Appendix. The first lemma provides an upper bound to the number of price changes within a plan.

LEMMA 3: *Let $\Delta > 0$ be the length of the time period, and $\bar{g}$ be the width of the inaction band. Let n_w be the expected number of price changes during a price plan. We have*

$$n_w \leq \frac{2}{\sqrt{\Delta}} \frac{1}{\sqrt{N_p}} - \frac{1}{2}. \tag{D13}$$

Hence, the total number of price changes per unit of time within price plans, denoted by N_w, and equal to $n_w N_p$, satisfies:

$$N_w \leq 2\sqrt{\frac{N_p}{\Delta}} - \frac{N_p}{2},$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/\left(\sigma\sqrt{\Delta}\right) = 1/\sqrt{N_p \Delta}$ is an integer larger than 2.

The second lemma yields a lower bound on the number of price changes within a plan.

LEMMA 4: *The expected number of price changes per unit of time within a plan N_w has the following lower bound:*

$$N_w \geq \frac{1}{\sqrt{\dfrac{\Delta}{N_p} + \dfrac{\Delta}{2}\left[\dfrac{1+\sqrt{\Delta N_p}}{1-\sqrt{\Delta N_p}}\right]}},$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/\left(\sigma\sqrt{\Delta}\right) = 1/\sqrt{N_p \Delta}$ is an integer larger than 2. ∎

PROOF OF PROPOSITION 7:

We fix an interval $[0, T]$ and index each price-path in the interval by ω, so the prices for this path are denoted by $p(\omega, t)$ for each $t \in [0, T]$. We let Ω_T be the measure of these sample paths. We will fix a path ω and define three concepts. First, we define the set of prices observed in a interval of length T for a given price path ω: $\mathcal{P}(\omega) \equiv \{y : y = p(\omega, t) \text{ for some } t \in [0, T]\}$. Second, we define the modal price in an interval of length T for a given price path ω, or the reference price

$$p^{ref}(\omega) \equiv \text{mode of } \mathcal{P}(\omega).$$

Third, we define the duration of the reference price as the time spent at the modal price in $[0, T]$ for a given sample path ω:

$$d^{ref}(\omega) \equiv \int_0^T \mathbf{1}_{\{p(\omega,t) = p^{ref}(\omega)\}}\, dt.$$

Finally, the statistic $F(T, \alpha)$ measures the mass of price paths of length T for which the duration of the reference price is higher than αT:

$$F(T,\alpha) = \Omega_T\left(\omega : d^{ref}(\omega) \geq \alpha T\right).$$

The proof proceeds by first defining a subset of the path at which the price spent at least αT of the time at the reference price. This will give a lower bound for F. The advantage is that this lower bound is easier to compute. Then we will show the proposition for the lower bound. We first consider the most delicate case, i.e., the continuous time case with $\Delta = 0$. We note that, without loss of generality given the symmetry in the model, we will consider that at the beginning $[0, T]$ the normalized desired price $g(0)$ is positive. Using the invariant distribution for the normalized desired prices, and conditioning that $g = g(0) > 0$, we have that it has density $f(g) = 2/\bar{g} - 2g/\bar{g}^2$. Fixing $g = g(0) > 0$, we can consider the path of price that will follow during $[0, T]$. If $0 < g(\omega, t) < \bar{g}$ for all $0 < t < \alpha T$, then the price will remain at $p(\omega, t) = p_-^*(\omega) + h$ where $p_-^*(\omega)$ is the ideal price at the start of the current price plan corresponding to this price path. Thus, if $\alpha > 1/2$ the reference price in this path is $p_-^*(\omega) + h$. If otherwise for $g(\omega, t) = 0$ or $g(\omega, t) = \bar{g}$ at some $0 < t < \alpha T$, then the reference price may be a different number. If g reaches the upper bound, there will be a new price plan. If g reaches zero, then there will be a price change within the plan. Notice that in either of these two events it is possible that $p_-^*(\omega)$ will still be the reference price (depending of what happens subsequently); we will ignore this possibility so that we obtain a lower bound on F. We denote our lower bound as $\tilde{F}(T, \alpha)$, which is given by

$$\begin{aligned} F(T,\alpha) &\geq \tilde{F}(T,\alpha) \\ &\equiv \int_0^{\bar{g}} \Pr\left\{0 < B(t) < \frac{\bar{g}}{\sigma} \text{ for all } t \in [0, \alpha T] \mid B(0) = \frac{g}{\sigma}\right\} f(g)\, dg, \end{aligned}$$

where B is a standard Brownian motion (BM). We compute the lower bound for the probability of a BM not hitting a barrier as follows. First, we denote this probability as

$$Q\left(\alpha T, \frac{\bar{g}}{\sigma} \middle| \frac{g}{\sigma}\right) \equiv \Pr\left\{0 < B(t) < \frac{\bar{g}}{\sigma} \text{ for all } t \in [0, \alpha T] \;\middle|\; B(0) = \frac{g}{\sigma}\right\}$$

so we can write

$$\tilde{F}(T,\alpha) = \int_0^{\bar{g}} Q\left(\alpha T, \frac{\bar{g}}{\sigma} \;\middle|\; \frac{g}{\sigma}\right) f(g)\, dg.$$

We break the interval $[0, \bar{g}]$ in three parts. Let $n \geq 2$ be an integer and let $a = (1/n)(\bar{g}/\sigma)$ so that

$$Q\left(\alpha T, \frac{\bar{g}}{\sigma} \;\middle|\; \frac{g}{\sigma}\right) \geq \begin{cases} 0 & \text{if } \frac{g}{\sigma} \in [0, a) \\ Q(aT, 2a \mid a) & \text{if } \frac{g}{\sigma} \in [a, a(n-1)] \\ 0 & \text{if } \frac{g}{\sigma} \in (a(n-1), an] \end{cases},$$

where the inequality for the middle range follows immediately by the assumption that $\bar{g}/\sigma = na$ for $n \geq 2$. The density for the first hitting time of either of two barriers for a BM, from which we can obtain Q, is as follows:

$$Q(\alpha T, na|a)$$

$$= \frac{2\pi}{a^2n^2}\sum_{j=0}^{\infty}(2j+1)(-1)^j \cos\left[\pi(2j+1)\frac{(n-2)}{2n}\right]\int_{\alpha T}^{\infty} \exp\left(-\frac{(2j+1)^2\pi^2 t}{2n^2a^2}\right) dt$$

$$= \sum_{j=0}^{\infty}(-1)^j \cos\left[\pi(2j+1)\frac{(n-2)}{2n}\right]\frac{4}{(2j+1)\pi} \exp\left(-\frac{(2j+1)^2\pi^2\alpha T}{2n^2a^2}\right),$$

and for $n = 2$ we get

$$Q(\alpha T, 2a|a) = \frac{2\pi}{a^2 4}\sum_{j=0}^{\infty}(2j+1)(-1)^j\int_{\alpha T}^{\infty} \exp\left(-\frac{(2j+1)^2\pi^2 t}{8a^2}\right) dt$$

$$= \sum_{j=0}^{\infty}(-1)^j \frac{4}{(2j+1)\pi} \exp\left(-\left(\frac{(2j+1)\pi}{2}\right)^2 \frac{\alpha T}{2a^2}\right).$$

Clearly $Q(\alpha T, 2a|a)$ is increasing in a, since for larger a the BM starts further away from the two barriers. Using Gregory-Leibniz's formula for π, for any $\alpha T > 0$, we have

$$\lim_{a\to\infty} Q(\alpha T, 2a|a) = \sum_{j=0}^{\infty}(-1)^j\frac{4}{(2j+1)\pi} = 1.$$

Thus, for any $\delta > 0$, we can find an A_δ such that $Q(\alpha T, 2a|a) \geq 1-\delta$ for $a > A_\delta$.

We also have

$$\int_0^a f(g)\,dg + \int_{(n-1)a}^{na} f(g)\,dg = \frac{2}{n}.$$

Thus,

$$\tilde{F}(T,\alpha) \geq \left[1-\frac{2}{n}\right]Q(\alpha T, 2a|a) \geq \left[1-\frac{2}{n}\right](1-\delta) \geq 1-\epsilon.$$

Hence, setting n and a large enough, we show the desired result. In particular, set n and δ to satisfy

$$\frac{n}{n-2} < \frac{1}{1-\epsilon} \quad \text{and} \quad 0 < \delta < 1 - \frac{n}{n-2}(1-\epsilon)$$

and let G satisfy $G \geq nA_\delta$. ∎

PROOF OF PROPOSITION 8:

We let $\{\tau_i\}_{i=0}^{\infty}$ denote the times at which the plans in the plans model (PL model) change, which are also the times at which the prices change in the GL model for the path $\mathbf{p}^*$. For each $i \geq 0$, define the interval $[t_i, t_{i+1}] \equiv [\tau_i, \tau_{i+1}] \cap [T_1, T_2] = \emptyset$. In the GL model, there is only one price in the interval $[t_i, t_{i+1}]$. In the PL model, there are, at most, two difference prices in the interval $[t_i, t_{i+1}]$. Thus, the duration of mode in the interval $[t_i, t_{i+1}]$ is at most the same for the PL model than the GL model, and at least half for the PL than the GL model, where the minimum duration is achieved if each of the two prices in the PL model have exactly the same duration. Thus, defining the $D^{PL}[a, b; \mathbf{p}^*]$ and $D^{GL}[a, b; \mathbf{p}^*]$ as the duration of the mode on the interval $[a, b]$ for path $\mathbf{p}^*$. We thus have $D^{PL}[t_i, t_{i+1}; \mathbf{p}^*] \leq D^{GL}[t_i, t_{i+1}; \mathbf{p}^*] \leq 2D^{PL}[t_i, t_{i+1}; \mathbf{p}^*]$ for all i.

Since $\mu > 0$, but small, then the prices that correspond to different intervals $[t_i, t_{i+1}]$ with nonempty intersection with $[T_1, T_2]$ are different, both in the PL model and in the GL model. Thus, the duration of the mode in $[T_1, T_2]$ can be computed as the highest duration across all intervals $[t_i, t_{i+1}]$. Thus, for reference prices, we define: $D^{PL}[T_1, T_2; \mathbf{p}^*] \equiv \max_i D^{PL}[t_i, t_{i+1}; \mathbf{p}^*]$ and $D^{GL}[T_1, T_2; \mathbf{p}^*] \equiv \max_i D^{GL}[t_i, t_{i+1}; \mathbf{p}^*]$. Taking the maximum in the previous inequality, we have: $\max_i D^{PL}[t_i, t_{i+1}; \mathbf{p}^*] \leq \max_i D^{GL}[t_i, t_{i+1}; \mathbf{p}^*] \leq 2\max_i D^{PL}[t_i, t_{i+1}; \mathbf{p}^*]$, which gives the desired result. ∎

PROOF OF PROPOSITION 9:

Within a plan, the size of price increases is $\Delta p = h - \ell = \bar{g}/3 - (-\bar{g}/3) = (2/3)\bar{g}$, while the size of price decreases is $\Delta p = -(2/3)\bar{g}$. Thus, the mean absolute value of price changes within the plan is $E[|\Delta p|] = (2/3)\bar{g}$. Next the size of price increases between plans is $\Delta p = (1/3)\bar{g}$ (plan ends hitting upper barrier), while the size of price decreases is $\Delta p = -\bar{g}$ (plan ends hitting lower barrier). Thus, the mean absolute value of price changes between plans is $E[|\Delta p|] = (2/3)\bar{g}$. ∎

PROOF OF PROPOSITION 10:

Note that the invariant density function when $\mu = 0$ is triangular,[27] namely $f(g) = (\bar{g} - |g|)/\bar{g}^2$ for $g \in (-\bar{g}, \bar{g})$. Recall that at $\mu = 0$, then $\underline{g} = -\bar{g}$, $\ell = -h$, so that $\hat{g} = 0$, and $h = \bar{g}/3$. The ODE solved by $\hat{m}$ becomes $0 = g - \ell + (\sigma^2/2)\hat{m}''$ for $g \in (\underline{g}, 0)$ and $g - h + (\sigma^2/2)\hat{m}''$ for $g \in (0, \bar{g})$, where the function $\hat{m}(g)$ is continuous and differentiable at $\hat{g}$, with boundary conditions $\hat{m}(-\bar{g}) = \hat{m}(\bar{g}) = 0$. This gives

(D14)
$$\hat{m}(g) = \frac{-g^2}{\sigma^2}\left(\frac{\bar{g} + g}{3}\right) \quad \text{for } g \in (-\bar{g}, 0),$$
$$\hat{m}(g) = \frac{g^2}{\sigma^2}\left(\frac{\bar{g} - g}{3}\right) \quad \text{for } g \in (0, \bar{g}).$$

[27] This follows since the invariant density solves the Kolmogorov forward equation: $f''(g) = 0$, which immediately implies the linearity, with the boundary conditions $f(\bar{g}) = 0$.

Next we use equation (23) and integrate $\mathcal{M}'(0) = \int_{\underline{g}}^{\bar{g}} f'(g)\hat{m}(g)\,dg$. Simple analysis gives $\mathcal{M}'(0) = \bar{g}^2/(18\sigma^2)$ or, using that $N_p = \sigma^2/\bar{g}^2$, we write $\mathcal{M}'(0) = 1/(18N_p)$. ∎

PROOF OF PROPOSITION 11:

Using the expression for $f(g)$ given above, we obtain

$$\tilde{\Theta}(\delta) = h\left[1 - \frac{\delta^2}{2\bar{g}^2} - \frac{(\bar{g}-\delta)^2}{\bar{g}^2}\right]. \tag{D15}$$

Note that one can understand this simple expression by computing the fraction of firms with normalized desired price g that the shock shifts from a negative to a positive desired price. For a small δ, this fraction is $f(0)\delta$. The effect on price of this is $2hf(0)\delta$. Thus, we have:

$$\tilde{\Theta}(\delta) = \tilde{\Theta}(0) + \tilde{\Theta}'(0)\delta + o(\delta) = 2hf(0)\,\delta + o(\delta) = 2\frac{h}{\bar{g}}\delta + o(\delta). \tag{D16}$$

We notice that the first equality in equation (D16) will hold for other cases, i.e., even if f is different (see for instance the extension that assumes costly price changes within the plan, developed in online Appendix H). The proof of the proposition follows immediately from equation (D15) after replacing the optimal value of h for $r \to 0$ or $h = \bar{g}/3$. ∎

PROOF OF PROPOSITION 12:

The proof is immediate using the result of Proposition 10 and equation (24). ∎

REFERENCES

Alvarez, Fernando, Martin Beraja, Martín Gonzalez-Rozada, and Pablo Andrés Neumeyer. 2018. "From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models." *Quarterly Journal of Economics* 134 (1): 451–505.

Alvarez, Fernando, Hervé Le Bihan, and Francesco Lippi. 2016. "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach." *American Economic Review* 106 (10): 2817–51.

Alvarez, Fernando, and Francesco Lippi. 2014. "Price Setting with Menu Costs for Multiproduct Firms." *Econometrica* 82 (1): 89–135.

Alvarez, Fernando, and Francesco Lippi. 2019. "The Analytic Theory of a Monetary Shock." EIEF Working Papers Series 1910.

Alvarez, Fernando, and Francesco Lippi. 2020. "Temporary Price Changes, Inflation Regimes, and the Propagation of Monetary Shocks: Dataset." *American Economic Journal: Macroeconomics*. https://doi.org/10.1257/mac.20180383.

Alvarez, Fernando, Francesco Lippi, and Juan Passadore. 2017. "Are State- and Time-Dependent Models Really Different?" *NBER Macroeconomics Annual 2016*, Vol. 31, edited by Martin Eichenbaum and Jonathan A. Parker, 379–457. Chicago: University of Chicago Press.

Anderson, Eric, Benjamin A. Malin, Emi Nakamura, Duncan Simester, and Jón Steinsson. 2015. *Informational Rigidities and the Stickiness of Temporary Sales—Staff Report 513*. Minneapolis, MN: Federal Reserve Bank.

Barro, Robert J. 1972. "A Theory of Monopolistic Price Adjustment." *Review of Economic Studies* 39 (1): 17–26.

Bils, Mark, and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112 (5): 947–85.

Blinder, Alan S. 1994. "On Sticky Prices: Academic Theories Meet the Real World." In *Monetary Policy*, edited by N. Gregory Mankiw, 117–54. Chicago: University of Chicago Press.

Caballero, Ricardo J., and Eduardo M.R.A. Engel. 2007. "Price Stickiness in *Ss* models: New Interpretations of Old Results." *Journal of Monetary Economics* 54 (S): S100–121.

Cavallo, Alberto, and Roberto Rigobon. 2016. "The Billion Prices Project: Using Online Prices for Measurement and Research." *Journal of Economic Perspectives* 30 (2): 151–78.

Dixit, Avinash. 1991. "Analytical Approximations in Models of Hysteresis." *Review of Economic Studies* 58 (1): 141–51.

Eichenbaum, Martin, Nir Jaimovich, and Sergio Rebelo. 2011. "Reference Prices, Costs, and Nominal Rigidities." *American Economic Review* 101 (1): 234–62.

Elberg, Andrés. 2014. "Heterogeneous Price Dynamics, Synchronization, and Retail Chains: Evidence from Scanner Data." *Emerging Markets Finance and Trade* 50 (6): 137–53.

Golosov, Mikhail, and Robert E. Lucas, Jr. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy* 115 (2): 171–99.

Guimaraes, Bernardo, and Kevin D. Sheedy. 2011. "Sales and Monetary Policy." *American Economic Review* 101 (2): 844–76.

Kehoe, Patrick, and Virgiliu Midrigan. 2015. "Prices Are Sticky after All." *Journal of Monetary Economics* 75: 35–53.

Klenow, Peter J., and Oleksiy Kryvtsov. 2008. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123 (3): 863–904.

Klenow, Peter J., and Benjamin A. Malin. 2010. "Microeconomic Evidence on Price-Setting." In *Handbook of Monetary Economics*, Vol. 3A, edited by Benjamin Friedman and Michael Woodford, 231–84. Amsterdam: North Holland.

Kryvtsov, Oleksiy, and Nicolas Vincent. 2014. "On the Importance of Sales for Aggregate Price Flexibility." Bank of Canada Working Paper 2014-45.

Nakamura, Emi, and Jón Steinsson. 2008. "Five Facts about Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics* 123 (4): 1415–64.

Sheshinski, Eytan, and Yoram Weiss. 1977. "Inflation and Costs of Price Adjustment." *Review of Economic Studies* 44 (2): 287–303.