The Optimal Fiscal Rule: Evidence from Italian Municipalities

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Abstract

Fiscal rules are implemented in more than 106 countries with the goal of containing governments' tendency to overaccumulate debt. However, the majority of them is based on a rule of thumb. This research has a double objective. Firstly, we test the effectiveness of existing fiscal rules by using a quasi-experimental design in Italy. In 1999, the central government imposed municipal fiscal rules which were relaxed in 2001 for municipalities not above 5,000 inhabitants. By performing a difference-in discontinuity, we show that this relaxation worsen the deficit thorough an increase in current expenditures and that the effect is homogeneous in debt. Secondly, since existing fiscal rules are effective, we study the optimal one when governments are subject to hyperbolic discounting and persistent spending shocks. In fact, since shocks are private information, there is a trade-off between flexibility and discretion. We derive the optimal rule and its implementation through a limit in the spending growth of the government based on its previous growth.

1 Introduction

In the last forty years, the level of debt of sovereign countries has been consistently increasing because of a secular expansion of government spending not supported by an equal increase in tax revenues.

The main reason is embedded in political economy and, in particular, in political polarization and turnover which cause an overspending bias. In fact, incumbent governments do not fully value the spending choice of different governments and tend to overspend at the expenses of future policy-makers. This behaviour can be approximated through a hyperbolic discounting factor (Persson and Svensson 1989 [32]; Alesina and Tabellini 1990 [32]). While each government is subject to an overspending bias when in power, it prefers that futures governments act only in function of their true spending needs (i.e. hyperbolic discounting factor equal to one).

In order to reach this objective, it is necessary to add some constraints to the ability of the current government to accumulate debt and run deficit. For this reason, many governments introduced fiscal rules. The International Monetary Fund (IMF) defines fiscal rules as "long-lasting constraints on fiscal policy through numerical limits on budgetary aggregates" (Budina et al., 2012 [12]). Although more than 106 countries introduced these type of regulations, they are largely based on a rule of thumb, mostly because, until recently, the theoretical literature on optimal fiscal rules developed models too abstract and simplified for an application in the real life. In particular, since the real spending needs are neither observable nor contractible, the social planner cannot constraint the government to take optimal spending decisions. This impossibility leads to a key trade-off between commitment and discretion. On one side, fiscal rules limit the incentive of governors towards excessive spending. On the other side, they constraint the ability of policymakers to respond to unexpected shocks. Indeed, a high threshold could enable the government to cope with an increase in its necessities but could also incentives a significantly overstatement of its needs. Hence, it is crucial to elaborate an optimal fiscal rule.

However, before implementing the optimal rule, it is necessary to understand if the regulations already in place, independently from being optimal, are effective. In fact, in case of no compliance, other enforcement actions would be required before the implementation of the optimal rule.

Performing this type of assessment is often hard since the decision to adopt a fiscal rule is endogenous (Alesina and Perotti, 1996 [3], Alesina and Passalacqua, 2016 [1]).

Hence, our first contribution consists in solving the endogeneity problem by relying on the Italian municipal fiscal rules (Domestic Stability Pact (DSP)) between 1999 and 2004. The target of the DSP in the period of interest is homogenous over the years. It established a limit in the growth of the fiscal gap (saldo finanziario), defined as revenues minus current expenditures net of some components.[1] Moreover, from 2001 onward, the central government relaxed the DSP for all municipalities with no more than 5,000 inhabitants. Hence, our research investigates the effects caused by the relaxation of the DSP by exploiting this Italian quasi-experimental setting. To implement this analysis we cannot adopt a regression discontinuity design (RDD) because there exists another policy which increases sharply the mayor's salary at the same threshold. We cannot use a difference-in-difference (DiD) design neither, since large municipalities cannot be assumed to be a good counterfactual for small ones. Therefore, following Grembi et al (2016) [25], we perform a difference-in-discontinuities which combines two sources of variation: before/after 2001 and just below/just above 5,00 inhabitants. Since the wage increase remained constant in 1999-2004, the effect linked to the wage policy can be removed by taking the difference between the pre-treatment and post-treatment discontinuities around the 5,000 inhabitants' threshold.

First, we analyze the effect on the two main variables of interest: fiscal gap (target of the DSP) and deficit (overall measure of municipality's budget position). We find that the relaxation of the DSP translates into a worsening of the fiscal gap (about 45 euros per capita) and of the deficit (about 58 euros per capita) over the four years period. In order to understand the channels through which the DSP constraints the local government's budget, we perform the same analysis by using as dependent variable all components of municipalities' financial reports. The results seem to be driven

[1] To be precise, the revenues were considered equal to the first four titles of the revenues (titolo I, II, III, IV) net of the state-aided component of the IRPEF (compartecipazione irpef), state transfers, real-estate and financial disposal, new debt and capital transfers made by the state. Expenditures were defined as current expenditures net of interest payments, expenditures paid by using European funds enlarged for that specific purpose, natural disasters and some components of the expenses for administrative elections.

by an increase in current expenditures of about 32 euros per capita.
In the second part of our empirical analysis we demonstrate that the effect of the relaxation of the DSP is homogenous in municipalities' debt position.
Lastly, different robustness checks validate the idea that the results are not driven by the sample size, the 5,000 inhabitants cutoff, or sample selections. Overall, our analyses confirm the effectiveness of Italian fiscal rules and leave room for the application of the optimal fiscal rule.

Therefore, the second contribution of this research consists in developing an optimal fiscal rule and its implementation. By building upon the works of Amador, Werning, and Angeletos (2006) [6] and Halac and Yared (2014) [27], we develop a discrete-time dynamic model with a spending-biased government. The government has a logarithmic utility and is subject to persistent random spending shocks. These shocks (θ) represent the real social value of spending. They are observable only by the incumbent government and non-contractible. All governments are forward-looking but value less the spending made by the other governments.[2] Consequently, the incumbent government tends to overspend when in power.
First, we derive the optimal fiscal rule. Since we assume that θ is neither observable nor contractible, the planner must induce the governments to truthfully reveal their realized spending needs by solving a principle-agent problem. We assume that Halac and Yared's (2014) [27] result holds and that the sequential optimal rule consists in establishing a maximum spending on the base of a θ contingent threshold $\theta_p(\theta_0)$. We compute the optimal threshold which, because of the logarithmic utility, is independent from the wealth level. Moreover, under no persistency it is constant and equal to the one found by Felli, Piguillem and Shi (2021) [21] in the case with no default.
Second, we derive the implementation of the optimal fiscal rule as a limit in the growth of the government spending proportional to its previous one. It represents our main contribution to the theoretical field since we create a direct mapping between theoretical model and empirical data by solving two of the main problems faced when matching theoretical fiscal rules and real life data.

[2] To be precise, every unit of spending transforms into one unit of consumption when the government is in power but it delivers only $0 < \beta < 1$ units of consumption when the government is out of office.

The first problem, as showed by Felli, Piguillem and Shi (2021) [21], is the dependence of the fiscal rule on the level of total wealth, in particular on debt. This dependency increases the complexity of the implementation since obtaining the data and processing them requires a high amount of time and they are not always available. Fixing the threshold in terms of growth allows to make the implementation independent from the total wealth. Indeed, the second part of the empirical analysis provides a preliminary support to the model by proving that, as expected, the effect of the relaxation of the fiscal rules expressed in terms of growth is not heterogeneous in municipalities' debt position.

The second problem is the dependence of the fiscal rules on previous spending shocks. In order to solve this issue, we integrate over θ_{-2} by using the unconditional distribution. In this way, we remove the role of past histories and find a one-to-one mapping between the observed previous growth rate and θ_{-1}.

Lastly, in order to implement the optimal fiscal rule in real life, we need to know the value of the theoretical parameters in the model. To this end, we calibrate the model by using municipalities'unconstrained spending decisions. We obtain two main findings. First, under persistent spending needs, the optimal threshold is increasing in previous shocks. This result is consistent with the idea that a municipality with high spending needs today is more likely to face high spending needs tomorrow.

Second, the optimal limit in the growth according to municipalities' previous growth factor incentives governments to cut their spending whenever possible since virtuous municipalities are rewarded by being allowed higher freedom. Lastly, it is worth underlining that, while the Italian rule is imposing a growth limit as the optimal rule, it is still sub-optimal. In fact, it sets a unique growth factor limit for all municipalities. By doing so, it penalizes virtuous municipalities which are constrained in their spending possibilities, independently from having or not the possibility of affording higher outlay. As a consequence, the DSP removes a key instrument to promote a cut in government's spending.

The research is organized as follows. Section 1.1 reviews the literature. Section 2 describes the Italian institutional framework. Section 3 presents the Empirical model elaborated to assess the effectiveness of the DSP and section 4 reports the data. Section 5 shows the empirical results and section 6 describes some robustness checks. Section 7 develops the theoretical model and section 8 reports the results of the calibration. Section 9 evaluates the quantitative application. Section 10 concludes.

1.1 Literature Review

This research relates to two strands of literature. First, it connects to the literature which analyzes the optimal trade-off when agents discount the future quasi-hyperbolically. The main assumption in this literature is that agents (or governments) are tempted to overspend. Ideally a social planner would like to constraint agents' spending exactly to their true needs. However this solution would not be applicable since these agents are subject to random spending needs, which are neither observable nor contractible. Consequently, it is desirable to endow them with some discretion. This literature has originated from Amador et al. (2006) [6] and has been extended by Halac and Yared by introducing persistent shocks when preferences are logarithmic (2014) [27] and by considering endogenous interest rates(2018) [26]. Lastly, Felli, Piguillem,and Shi (2021) [21] built a continuous time model with political turnover and possibility of default. Our model build on Halac and Yared 's [27] [26] work but, differently from them, proposes an implementation of the optimal rule in terms of limit in government spending growth on the base of their previous growth. In this way we can create a direct mapping between the theoretical model and the empirical data.

Second, we contribute to the literature that analyses the effectiveness of fiscal rules. Previous results are heterogeneous. While some studies confirm that the implementation of fiscal rules determines lower budget imbalances, others emphasize the reasons for which they may be ineffective (Alesina and Perotti 1999 [2]).

The same ambivalence holds for subanational fiscal rules. Eichengreen and von Hagen (1996) [20] and Rodden (2004) [36] underline the importance of these rules in case of severe fiscal imbalances and high decentralization. Moreover, local governments have incentives to free ride on fiscal discipline. The reasons may be different: relying on a common pool of national resources (Weingast, 2006) [39], being "too big to fail" (Wildasin, 1997) [40], private creditors' beliefs that central government will bail them out (Dafflon, 2002) [17]. Instead, Milesi-Ferretti (2004) [29] argues that subanational fiscal rules might lead to unwanted outcomes as creative accounting and window dressing.

The empirical evidences mainly focus on two types of assessments: a cross-country comparison in specific regions, such as the European Union (Debrun et al. 2008 [18]) or Latin America (Gavin and Perotti 1997 [23]), and an analysis of local governments, mostly in the United States (Poterba 1994 [34]; 1996 [33]).

However, there are many papers that analysed the Italian DSP, as well. Patrizii et al. (2006) [31] addressed the ability of regions and local governments to meet the DSP requirements and Brugnano and Rapallini (2010)[35] assessed the effects of the DSP on local public borrowing requirements from 1999 to 2005. Our analysis is closer to the one by Grembi et al(2016) [25]. In fact, we implement their methodology over the same framework. However, with the aim of being as faithful as possible to the Italian rules, we use a different sample[3] and a more specific measure of fiscal gap. These changes and other smaller modifications lead to very different results.

2 Institutional Framework

2.1 Italian Municipalities

The Italian institutional framework consists of three sub-national levels: Regions, Provinces and Municipalities. The current study focuses on the lowest level of the government represented by the municipalities.

[3] We remove Sicily from the sample since from 2002 onward all regions with special autonomy (Regioni a Statuto Speciale) were allowed to set their own fiscal rules. Moreover, in order to follow the spirit of the DSP, we perform most of our analysis by using measure in cash basis.

In the late 1900 and early 2000s Italy experienced, thorough the change of the title V of the Constitution, a phenomenon of decentralization which enabled municipalities to have partial control over both, local revenues and expenditures.

In the period of interest, on the revenues side, mayors could vary ICI within a bracket from 0.4 to 0.7 percent of the legal home value, and IRPEF within a bracket from 0 to 0.5 percent of taxable income. Moreover, local governments could also set freely other local taxes (e.g. building rights or the occupation of public areas) and fees and tariffs for the services they provided (e.g waste management). Finally, still nowadays, Italian towns are characterized by a sizable level of tax evasion, which the mayor can decide to fight.

On the expenditure side, municipalities also had room for adjustment since about one-third of expenditures were classified as not rigid. For instance, as underlined by Chiades and Mengotti (2013) [14], municipalities could reduce the expenditures by outsourcing part of their services (e.g. child-care provided by private firms). Furthermore, Bandiera, Prat, and Valletti (2009) [8] showed how comparable Italian municipalities can pay very differently for similar goods, and they interpreted it as evidence of passive waste. This implies that mayors could reduce passive waste in order to adjust the financial position of the municipality.

Moreover, municipalities were authorized to have a higher control over their debt by getting access to financial instruments previously precluded. For example, they were allowed to issue bonds directly on the market and to carry out debt restructuring operations.

It is worth noticing that the population size of the municipalities is still determining a vast array of policies as the wage of the mayor and of the members of the executive committee. In particular, the administrators of municipalities above 5,000 inhabitants enjoy a significant increase in their wage. In 2005, a new law maintained the same population thresholds but reduced the wages, for each threshold, by 10 percent. To have a better idea, Table A1 summarizes the main differences determined by the population thresholds and their changes over time.

2.2 The Domestic Stability Pact

In 1997 the European Union passed the Stability and Growth Pact (SGP). It consists of a set of fiscal rules designed to prevent European countries from spending beyond their means. The SGP only holds central governments. However, since the budget of local governments is a crucial component of the total budget of the state, the control of the public finances requires the cooperation of a wide range of entities and not just the commitment of the central government. For this reason, in the late 1900s, many countries as Spain, Germany, The Netherlands, Belgium, Austria, Finland and France adopted subnational rules called Domestic Stability Pacts (DSPs) in order to reduce the incentives for local governments to run deficits. In 1999, through the article 28 of the annual budget law 448/1998, Italy implemented the DSP (Patto di Stabilità Interno) designed as a set of constraints on the level of deficits and/or expenditures of local authorities. The DSP covered any level of the sub-national governments (Regions, Provinces and Municipalities) but the typologies of the constraints and targets were heterogenous across different subnational levels.

The DSP changed almost every year with the enactment of the annual budget law. As this work focuses on Italian municipalities, Table A2 summarizes the main differences in the evolution of the DSP at the municipality level from 1999 until 2015. After 2015, there has been a significant change in the formulation of governmental accounting which led to a verry different implementation of the limits imposed on the municipalities.

The first significant change is related to the municipalities subject to the regulation. The DSP was initially enforced on all municipalities but in 2001-2010 it was limited only to those above 5,000 inhabitants. In 2011, the constraints were extended to all municipalities above 1,000 inhabitants. After 2015, the DSP has been abolished and replaced with a new set of balanced budget rules for all municipalities. In order to assess if they were above or below the threshold, the regulation considered ISTAT intercensal estimations of the resident population of two years before (e.g for 2001 it considered the population of 1999 at December 31^{st}). The central government did not specify the reasons behind the creation and modification of the thresholds. Different reasons may have led to these decisions. It could have been the result of a trade-off between providing relief to small municipality and guaranteeing an overall compliance to the SGP or it may have been due to the difficulty of monitoring small municipalities and to the small impact of their financial decisions on the overall national budget.

The second consistent change in the DSP is about the financial components considered in order to determine the objectives of the law and their achievement. In particular, the financial reports of the municipalities include budgetary indicators on both accrual (competenze and accertamenti) and cash bases (pagamenti in conto competenza and in conto residuo, riscossioni in conto competenza and conto residuo). Over time, the DSP focused on one type of measure, both or on a mix of the two. From 1999 until 2002 the DSP constraint was only cash based while from 2003 until 2007 the regulations were not only in terms of cash but also of accrual. Lastly, from 2008 until 2015, the DSP considered capital expenditures on a cash basis while current expenditures on an accrual basis.

The third important change regards the target set by the DSP. The main objective for 1999-2015 was in terms of deficit while in 2005-2006 it concerns an expenditure cap. However, DSP targets' specification was much more complex. In fact, during the years, different items were net out. For example, until 2004 capital expenditures were excluded from the expenditures analysed while in 2005 they represented a crucial component. Moreover, the objectives were calculated on the base of different past values and the limits themselves changed over time: in some years municipalities had to cap the growth of their target while in others they were asked to cut the target. Independently from the reference years used for the computation, there was always a lag of at least two years due to the time needed to certify the official budget data. For example, in 1999 the DSP rule imposed a zero-growth limit with respect to the deficit of 1997 while in 2009 it required an improvement with respect to the average of the deficit between 2003-2005.

The fourth relevant change in the DSP concerns the penalty system. Initially, the only sanction for non-compliant municipalities was the threat of paying the quota that was directly imputable to the municipality of any fine that European authorities would impose to Italy for non-compliance with the SGP. In 2002 more stringent sanctions were introduced in order to increase the enforcement of the DSP. The entity and type of sanctions changed over time. They included cuts on intergovernmental grants, limitation in staff recruitment and current expenditures. Moreover, most of the years the non-complainants were limited in the possibility of contracting new debt. In 2008 the sanctions have been made harsher by introducing a 30 % cut in the wage paid to mayors and municipal councillors of non-compliant municipalities. The strong use of cut in central transfers as enforcement is the main reason for the recurrent exclusion of intergovernmental transfers from the computation of the target.

It is worth to underline that from 2002 regions with special autonomy (Regioni a Statuto Speciale) were allowed to set their own fiscal rules for municipal governments.

Since after 2004 the DSP has been modified repeatedly in its basic and core characteristics, we focus our analysis on the period 1999-2004. In the period of interest, as summarised by table 1, the rule was expressed as a limit in the growth of the fiscal gap with respect to two years before the actual budget year.

Moreover, in order to assess the effect of the DSP in this interval of time, we pay particular attention in verifying exactly which items were included in the computation of the fiscal gap (saldo finanziario). In order to do so, we complement the study with the technical reports on municipalities' financial statements. Those reports are made by experts with the aim of verifying the financial position of the municipalities and their compliance with the DSP. In these documents not only is reported the final amount of the fiscal gap, but also the single elements of the financial reports that have been considered in order to make the assessment.[4] In particular, in the period of interest, the target of the law was the fiscal gap (saldo finanziario) defined as revenues minus current expenditures

[4] In order to have a deeper understanding of the structure of a municipality's technical report, see https://www.comune.alessandria.it/rendiconto-2004

Table 1: DSP for Italian municipalities evolution from 1999 until 2004

Year	Municipalities covered	Target	Limit and Reference Year
1999	All	Fiscal gap	zero growth wrt fiscal gap 1997
2000	All	Fiscal gap	zero growth wrt fiscal gap 1998
2001	>5000 [1]	Fiscal gap	max. 3% growth wrt fiscal gap 1999
2002	>5000	Fiscal gap Current expenditures cap	max 2.5% growth wrt fiscal gap 2000
2003	>5000	Fiscal gap	zero growth wrt fiscal gap 2001
2004	>5000	Fiscal gap	zero growth wrt fiscal gap 2002

NOTES: (1) In order to assess if the municipalities were above or below the threshold, the regulation considered ISTAT intercensal estimations of the resident population of two years before (e.g for 2001 it considered the population at the end of 1999).

net of some components. On one side, the revenues considered were equal to the first four titles of the revenues (titolo I, II, III, IV) net of the state-aided component of the IRPEF (compartecipazione irpef), state transfers, real-estate and financial disposal, new debt and capital transfers made by the state. On the other side, the expenditures examined were equal to the current expenditures net of the interest payments, expenditures paid by using European funds enlarged for that specific purpose, natural disasters and some components of the expenses for administrative elections.

2.3 Debt Rules Over Time

The DSP did not impose any direct limit on the level of debt of the municipalities. However, a ministerial decree of 1995 was already regulating their debt position. Even after the introduction of the DSP, the typology of the rule remained unchanged while the level of the target was modified over the years. Table A3 reports the main changes.

The law established a numerical ceiling to the level of debt set as the ratio of interest expenditure to current revenue. More in depth, the amount of interest paid by the municipalities net of the interest contributions received from central and regional governments could not be higher than a certain percentage of the first three components of the revenues (tax revenues, government transfers, non-tax revenues) of two years before. Until 2003, the percentage of the target was set to 25%. In 2004, the threshold was reduced to 12% only to be increased again in 2006 to 15%. In 2011, a new law was issued in order to facilitate the objective of reducing municipalities' debt. It established a decrease in the target to 8% in 2012, to 6% in 2013 and to 4% from 2014. However, these very tight limits were never implemented for the following years since in 2013 a new law maintained the threshold to 8% with the aim of favouring capital investments at the municipal level. In 2015, law 190/2014 raised the threshold to 10%.

Especially during the first years of the legislation, the limit was not binding since municipalities were enjoying more fiscal autonomy and relatively low interest rates. In practice, only the municipalities close to default were constrained.

Moreover, in 2001, the constitutional amendment, authorized the issue of new debt only to finance capital investment (golden rule).

Lastly, from 2003, all municipalities that were not complying with the DSP were not allowed to issue new debt the following year.

3 Empirical Model

3.1 Analytical Framework

In this section, we formalize the evaluation framework that allows us identifying the effectiveness of Italian fiscal rules in limiting municipalities' public spending. Performing this type of assessment is often very hard since the decision to adopt a fiscal rule is endogenous (Alesina and Perotti, 1996; Alesina and Passalacqua, 2015). In order to solve this problem, we exploit a quasi-experimental setting linked to a structural change in Italian fiscal rules. In particular, we focus on the effects caused by the decision of the central government of relaxing, from 2001 onward, the DSP for all municipalities with no more than 5,000 inhabitants.

This analysis cannot be performed with a difference-in-difference strategy (before and after

2001) since it cannot be assumed that small cities are a good counterfactual for large cities, i.e it is hard to believe that the parallel trend assumption holds.
In the absence of other policies changing across the 5,000 inhabitant's threshold, this institutional setup would be appropriate for a sharp RDD approach applied in 2001. However, as can be seen in section 2.1, the exact same threshold was also used to establish a wage increment in the salary of the mayor and the executive committee. Different studies, as that one of Gagliarducci and Nannicini (2013) [22], showed that higher wages attract individuals with a higher level of education and that, once elected, perform better. Hence, it can be easily argued that the presence of a pre-existing policy using the same threshold compromises the continuity assumption and, consequently, the possibility of performing an RDD analysis. In fact, it would be impossible to disentangle the effect of the wage policy from those due to the relaxation of the DSP.

In order to solve this problem, following Grembi et al (2016) [25], we perform a difference-in-discontinuities (diff-in-disc). This methodology combines two sources of variation: the before/after 2001 (typical of the DiD) and the just below/just above 5,00 inhabitants (typical of the RDD). The wage increase has been introduced in 1960 and remained constant in real terms over the period of interest. Hence, the effect linked to the wage policy can be removed by taking the difference between the pre-treatment and post-treatment discontinuities around the 5,000 inhabitants' threshold. It is important to underline that the diff-in-disc design, as the RDD, is a local analysis. Hence, it can only help identifying the effect of relaxing fiscal rules for small municipalities.

For the present study, we fix the population level at the year 1999. This is the first year used for the current analysis and also the one established by the DSP in order to compute the inhabitant's threshold in 2001. Fixing the population level solves multiple potential issues. First of all, it removes any sorting problem since the DSP with the new threshold regulation was only enacted in December 2000 and there was not previous notice of the change in the threshold. Moreover, it dismisses any worry regarding the possibility that the number of inhabitants was influenced by the DSP and the consequent policies enacted by the municipalities. A possible concern about fixing the population, might be linked to the potential loss of track of those municipalities which, by changing their level of inhabitants, moved above and below the 5,000 threshold during the period of interest 2001-2004. In fact, the position with respect to the threshold was computed by looking at the resident population of two years before. For example, Cavallermaggiore's population in 2000 was equal to 4979 and in 2001 equal to 5041; hence Cavallermaggiore was free from the need to comply the DSP in 2002 but had to comply in 2003. In order to avoid this problem, we removed from the dataset those 42 municipalities which moves above and below the threshold. In this way, fixing the population does not compromise any dummy based on being above or below the cut-off.

The overall analysis is limited to the period between 1999 and 2004. In fact, as described in section 2.2, after this interval the DSP was repeatedly modified in its basic and core characteristics. These continuous changes increase the difficulties in disentangling the effect of the DSP. Moreover, as explained in section 2.1, after 2005 there has also been a 10% reduction in the wages paid to the majors and, consequently, to the executive committee. This change might lead to some additional compounding effects hard to disentangle.

Furthermore, following Grembi et al(2016) [25], we restrict the sample to Italian municipalities between 3,400 and 7,100 inhabitants. This decision is due to two main reasons:

1. The local nature of the difference-in-discontinuities design
2. The presence of additional compounding factors when exiting the range of 3,000-10,000 inhabitants (see section 2.1).

3.2 Empirical Specification

We use two main specifications to perform the diff-in-disc analysis. The first one estimates the average effect caused by the relaxation of the DSP.

$$Y_{it} = \gamma_i + \lambda_t + \sum_{k \neq 2000} \delta_k I_{t=k} P_{99} + \sum_{k \neq 2000} \xi_k I_{t=k} P_{99} S_i + \beta T_t S_i + \epsilon_{it} \quad (1)$$

γ_i represents the municipality fixed effect and λ_t the time fixed effect. S_i is a dummy taking value one for the cities of at most 5,000 inhabitants and T_t indicates the post-treatment period (after 2000). Lastly, P_{99} represents the population normalized and fixed at 1999 level (i.e $Population_{1999}$ - 5,000). β is the parameter of interest and represents the effect of being after the introduction of the population threshold and not above 5,000 inhabitants. By looking at this coefficient, it is possible to assess the effectiveness of the DSP overall, during the period 2001-2004.

The second specification returns coefficients representing the annual result of the relaxation of the fiscal rules. The following analysis enables to understand the time pattern of the effect. In particular, if the effect of the relaxation of the DSP is heterogeneous across years.

$$Y_{it} = \gamma_i + \lambda_t + \sum_{k \neq 2000} \delta_k I_{t=k} P_{99} + \sum_{k \neq 2000} \xi_k I_{t=k} P_{99} S_i + \sum_{k \neq 2000} \beta_k I_{t=k} S_i + \epsilon_{it} \tag{2}$$

This model differs from the previous one for the introduction of multiple parameters of interest represented by the interaction term $S_i * I_{t=k}$ which indicates for every year the effect of not being above the 5,000 inhabitants' threshold. In order to avoid a problem of multicollinearity, we remove the coefficient of the year 2000. Hence, all the other terms must be interpreted in deviation from this year.

By using the diff-in-disc design, we also test if the effect of the DSP is heterogeneous in municipalities' debt level. In order to assess this hypothesis, we introduce in the previous model an additional dummy representing the weight of indebtedness of the municipality.

$$\begin{aligned} Y_{it} = \gamma_i + \lambda_t + \sum_{k \neq 2000} \delta_k I_{t=k} P_{99} + \sum_{k \neq 2000} \xi_k I_{t=k} P_{99} S_i + \sum_{k \neq 2000} \theta_k I_{t=k} D_{99} \\ + \sum_{k \neq 2000} \alpha_k I_{t=k} D_{99} P_{99} + \sum_{k \neq 2000} \mu_k I_{t=k} D_{99} S_i P_{99} + +\beta T_t S_i + \iota D_{99} T_t S_i + \epsilon_{it} \end{aligned} \tag{3}$$

We create a variable equal to the ratio of the debt level (functional and financial debt) over total revenues. D_{99} is a dummy taking value one when the debt ratio of the municipality is above the median level. The ratio is fixed at 1999 in order to remove any problem of reversed causality (see Section 2.3. The terms of interest of the regression above are two: β which represents the effect of being after the introduction of the population threshold and not above the 5,000 inhabitants and ι which captures the additional effect of not being subject to rule while having a high level of debt. The decision to use debt instead of interest rates or debt repayment is strictly linked to the structure of the theoretical models. In fact, the literature (Felli, Piguillem and Shi (2021) [21]; Amador, Werning, and Angeletos (2006) [6]; Halac and Yared (2014) [27]) as well as the model that we develop (see section 7) derived that government's spending decision is a ratio of total wealth (i.e $\tau + h - b$). However, in municipalities' financial reports often the information about municipalities' debt level (Quadro 8) is missing. This causes a significant reduction in the sample. In order to verify that the findings are not driven by the smaller sample size or a voluntary decision of the municipalities not to report debt (possible problem of self-selection), we reproduce the same analysis by using data about debt service (i.e the amount spent to pay debts' principal and interests). Even though this variable is not perfectly matching the debt level, it constitutes a good proxy and enables us to perform analysis on the whole sample.

4 Data

The present study uses municipalities' financial reports (certficato di conto consultivo) from the Italian Ministry of the Interior (Ministero dell'Interno), which are the only data available regarding municipalities' balance sheet for the period of interest 1999-2004. In particular, the accounting measures include information both on accrual and cash bases. Hence, we create two datasets which include the same municipalities. One reports the variables on cash basis and the other on accrual basis. We complement the database with the annual population level from the intercensal study of the Italian Statistical Office (Istat).

As explained in section 3.1, we restrict the sample to municipalities between 3,400 and 7,100 inhabitants. Moreover, since from 2002 regions with special autonomy were able to set their own fiscal rules, we drop all municipalities belonging to these regions. In addition, we remove all those municipalities that had incompleteness or inconsistencies in the financial report during the six-year period. In particular, we remove all municipalities which reported for at least one-year taxes, fees and tariff, total revenues, total expenditures or total transfers equal to zero. We also remove those municipalities for which we miss information about either the accrual or the cash basis. e.g. San Giorgio Su Legnano reported in 1999 zero state transfers on an accrual basis but 1,368,239.45 on a cash basis of which 628,967.55 on accrual account and 739,271.90 on residual account. Lastly, as explained in section 3.1, we remove the municipalities that move around the 5,000 inhabitants' threshold over time.

The main variable of interest is the fiscal gap, which is the target of the DSP. The measure that I construct is equal to current expenditures minus total revenues net of some financial components. It aims at being as close as possible to the legal one, defined in section 2.2. However, municipalities' financial reports do not include some additional sub classifications related to municipalities'expenditures. For this reason, we compute the outlays of the municipalities as current expenditures net of interest repayments. This change does not compromise the final result but it might lead to a possible overestimation of the fiscal gap and may reduce the statistical significance of the estimated coefficient since the introduction of additional outlays increases the level of noise in the data. The other variable of interest is the deficit, which is defined as total expenditures minus total revenues. It represents an overall measure of municipality's budget position. This variable, given its aggregate nature, is easily subject to noise. However, it is useful in order to make an overall assessment of municipalities' financial situation. Additionally, we perform the same analysis by using as dependent variable all the components of municipalities' financial reports in order to understand the channels through which the DSP constraints the local government's budget. In the specific, we divide expenditures into current outlays, capital outlays and debt service and revenues into taxes, fees and tariffs, transfers from the government and other revenues.
All data are expressed in per capita and real terms by using 2021 as base year.

The dataset includes 861 municipalities for a total of 5166 observations. Table 2, reports the average values of the main outcome variables in cash basis for municipalities above and below 5,000 inhabitants. All values are in per capita and real terms by using 2021 as base year. The descriptive statistics seem to provide preliminary evidences that the DSP may be effective in improving municipalities' financial position. In fact, municipalities above 5,000 inhabitants have on average a lower deficit and fiscal gap paired with lower outlays and higher revenues. On the contrary, municipalities below 5,000 inhabitants enjoy on average higher transfers from the government. This result may be due to the use of central transfers' cut as e nforcement method against non-compliers. In Table C1, we show that the same pattern is confirmed also for the variable in accrual basis.

In order to assess if the effect of the DSP is heterogeneous in municipalities' debt level, we include in the dataset two additional stock variables: financial and operating debt (i.e accounts payable and arrears on municipal expenditures' payments). However, it is important to underline that municipalities' financial reports (certficato di conto consultivo) are self-reported. Hence, it is not unusual to find missing or wrong values. In particular, as explained in section 3.2, the report related to the municipality's debt position (Financial reports, Quadro 8) contains a large number of missing and unreliable values which consistently reduce the sample to 503 municipalities, of which 256 are treated after 2001 and 247 are in the control group. In total we have 3018 observations. We show in Table C2, that, even with the reduced sample, the summary statistics manifest a similar pattern.

Table 2: Descriptive Statistics

	Above 5000	Below 5000
A: Fiscal discipline		
Fiscal gap	69.48	96.53
Deficit	-40.70	-33.22
B: Expenditures		
Current outlays	750.70	760.08
Capital outlays	337.31	357.41
Debt service	109.60	107.46
C: Revenues		
Taxes	297.14	282.03
Fees and Tariffs	91.26	85.85
Government transfers	255.78	271.47
Other revenues	646.30	662.00
Observations	2,508	3,108

NOTE: The municipalities have a population between 3,400 and 7,100 inhabitants. All values are in cash basis, per capita and real terms by using 2021 as base year.

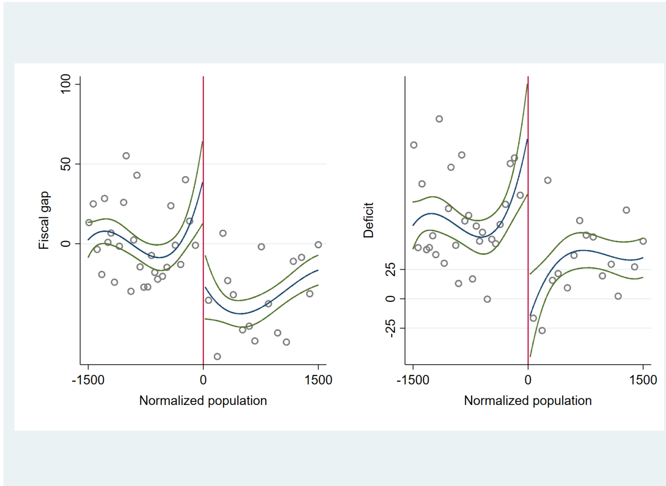
5 Empirical Results

5.1 Baseline Specification

The graphs below show a graphical representation of a difference-in-discontinuity design. They report scatters and third-degree polynomial fits of the differences between each post-2001 outcome value and each pre-2001 value. The wage increase policy has been introduced in 1960 and remained constant in real terms over time. Hence, by taking the difference between the pre-treatment and post-treatment period, we remove the effect linked to the wage policy. In this way, any discontinuity at the 5,000 threshold observed in the graph can be imputable to the relaxation of the DSP. Figure 1 reports the variables of fiscal discipline: fiscal gap and deficit. Both measures exhibit a sharp jump at the cut-off indicating a worsening of the financial position of the municipalities not subject to the rule.

Figure 2 and Figure 3 allow to shed some lights on the composition of the fiscal adjustment.

Figure 1: Difference-in-Discontinuities for deficit and fiscal gap



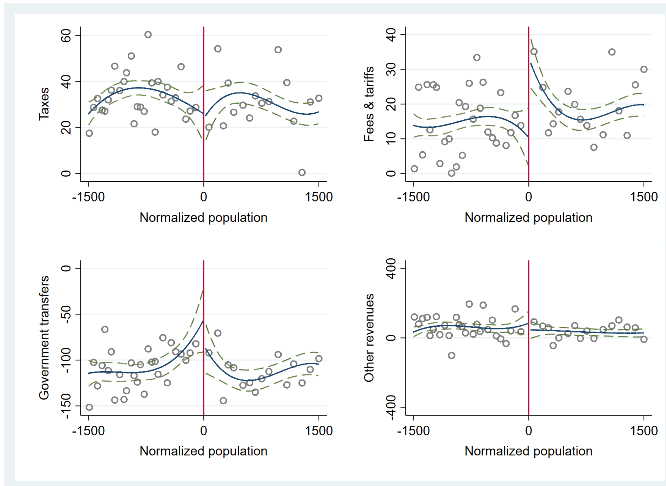
NOTE - The vertical axis reports the difference of each post-rule (i.e 2001, 2002, 2003, 2004) outcome value and each pre-rule (i.e 1999-2000) outcome value. The horizontal axis reports the 1999 population normalized (i.e actual population size-5000). The central line is a third-order polynomial fit; the lateral lines represent the 95 percent confidence interval. The scatter points are grouped in 50 bins.

Figure 2 shows the behaviour of the different revenues (taxes, fees and tariffs, transfers from the government and other revenues) around the threshold. Fees and Tariffs appear to be lower for those municipalities not subject to the rule. However, the result may be driven by some outliers just above the cut-off.

Figure 3 disentangles the impact of the relaxation of the DSP on three different expenditure types (current outlays, capital outlays and debt service). The municipalities not subject to the rules appear to spend significantly more compared to the municipalities just above the threshold.

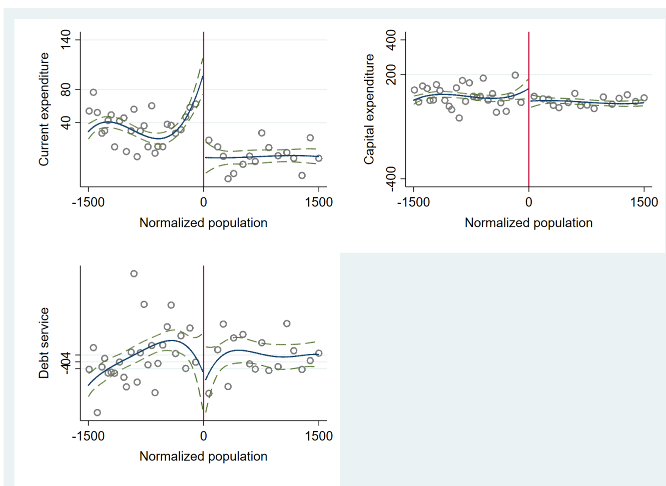
Moreover, for many variables it is possible to see a sharper change closer to the 5,000 cut-off. This phenomenon may be a potential validation of the existence of a spending bias. In fact, those municipalities closer to the cut-off have a higher probability of crossing the inhabitants' threshold. Hence, it is possible that those governments are subject to a higher hyperbolic discounting rate which leads them to overspend more in comparison to smaller municipalities which are less likely to cross the threshold.

Figure 2: Difference-in-Discontinuities for revenues outcomes



NOTE - The vertical axis reports the difference of each post-rule (i.e 2001, 2002, 2003, 2004) outcome value and each pre-rule (i.e 1999-2000) outcome value. The horizontal axis reports the 1999 population normalized (i.e actual population size-5000). The central line is a third-order polynomial fit; the lateral lines represent the 95 percent confidence interval. The scatter points are grouped in 50 bins.

Figure 3: Difference-in-Discontinuities for expenditures outcomes



NOTE - The vertical axis reports the difference of each post-rule (i.e 2001, 2002, 2003, 2004) outcome value and each pre-rule (i.e 1999-2000) outcome value. The horizontal axis reports the 1999 population normalized (i.e actual population size-5000). The central line is a third-order polynomial fit; the lateral lines represent the 95 percent confidence interval. The scatter points are grouped in 50 bins.

Overall, the graphical representation suggests a positive and significant impact of the DSP over municipalities' fiscal discipline through a reduction of their current expenditures.

The statistical analysis confirms the graphical result. Table 3 contains the main diff-in-disc estimations obtained by using the specification in equation 1. The coefficients represents the average effect of the relaxation of the DSP. The change in the DSP leads to an economic and statistically significant worsening of the fiscal gap (44.89 euros per capita) and deficit (58.20 euros per capita) due to an increase in current expenditures by 32.37 euros per capita. The presence of high standard errors is mainly due to the high level of noise of the data which are self-reported.

Table 3: Diff-in-disc estimates of the effect of relaxing fiscal rules

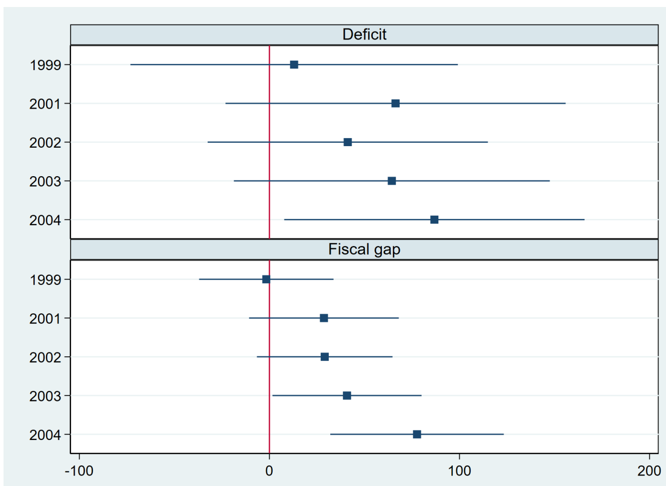
Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	58.20 (21.36)	44.89 (14.13)		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	38.57 (11.61)	23.42 (13.73)	-0.48 (4.86)	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	-3.68 (8.09)	-5.84 (3.97)	20.78 (13.85)	15.60 (29.59)

NOTE: The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All measures are in cash basis.The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5000 inhabitants and after 2001 (i.e S_iT_t). The estimates are retrieved by using the specification in equation (18).

Figure 4, Figure 5 and Figure 6 show point estimators and confidence intervals for the annual effect of fiscal rules' relaxation obtained by using the specification in equation 2. All the coefficients must be interpreted in deviation from the year 2000. In particular, Figure 4 reports these parameters for the two measures of fiscal discipline (fiscal gap and deficit), Figure 5 depicts those of the different revenues' components and Figure 6 shows the coefficients of the expenditures' types. The analysis confirms the same narrative depicted by the graphical representation. Moreover, it suggests that the effects of the DSP's relaxation were stronger during the final years (2003-2004). Additional analysis would be needed in order to understand the exact reasons behind the heterogeneity of the effect over time. In fact, there are different potential explanations. On one hand, it might be the result of a tightening of the enforcement rule. On the other hand, given the complexity of the regulations, it might be the consequence of a learning process linked to a gradual understanding of the rules and of their implementation. Lastly, it could be due to an exponential effect of the worsening of municipalities' financial condition.

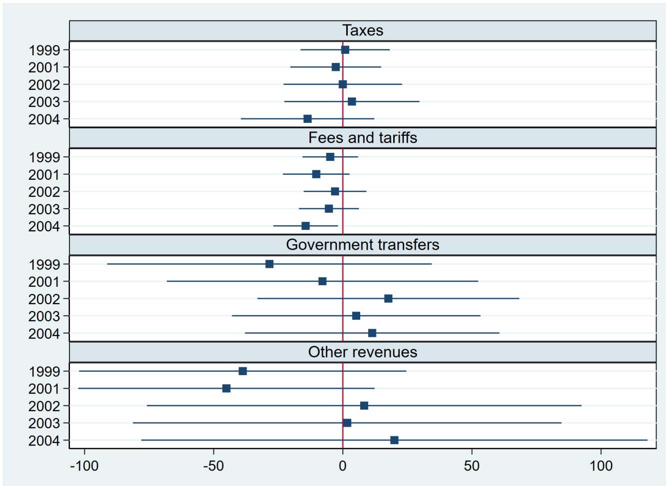
Moreover, the absence of coefficients statistically different from zero for the year 1999 supports the existence of a parallel trend before 2001, once it is net out the wage change at the threshold.

Figure 4: Annual Difference-in-Discontinuities for deficit and fiscal gap



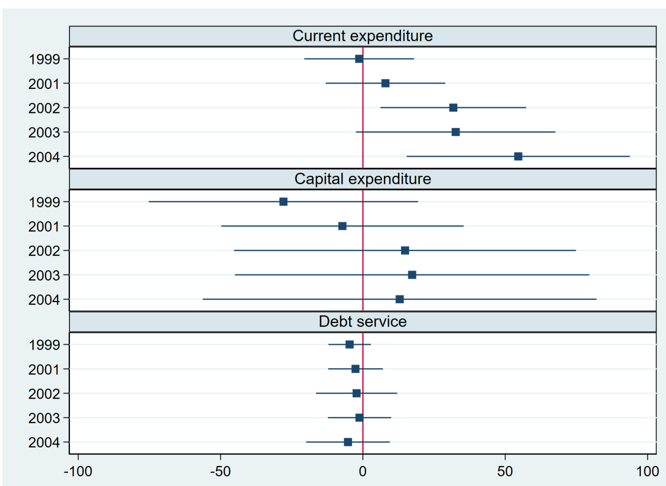
NOTE -The figure reports the point estimates and the relative confidence intervals of the annual impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants. The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004).All measures are in cash basis, per capita and real term (2021 is the base year). The estimates are retrieved by using the specification in equation (19), hence all coefficients must be interpreted as a deviation from the year 2000.

Figure 5: Annual Difference-in-Discontinuities for revenues outcomes



NOTE -The figure reports the point estimates and the relative confidence intervals of the annual impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants. The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004).All measures are in cash basis. The estimates are retrieved by using the specification in equation (19), hence all coefficients must be interpreted as a deviation from the year 2000.

Figure 6: Annual Difference-in-Discontinuities for expenditures outcomes



NOTE -The figure reports the point estimates and the relative confidence intervals of the annual impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants. The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004).All measures are in cash basis. The estimates are retrieved by using the specification in equation (19), hence all coefficients must be interpreted as a deviation from the year 2000.

5.2 Accounting Measures in Accrual Basis

In addition, we verify if the relaxation of the DSP has an impact on municipalities' accounting variables expressed in accrual basis. In fact, until 2003 the DSP limited only the accounting measures in cash basis. However, it could be as well that by pursuing the objective of respecting the regulation in cash basis, municipalities improve their position in accrual basis as well.

Table 4 contains the main diff-in-disc estimations obtained by using the specification in equation 1. The coefficients represent the average effect of the relaxation of the DSP. The change in the fiscal rule appears to produce an effect similar to the one found for the cash basis components. The relaxation of the DSP leads to a worsening of the fiscal gap (30.12 euros per capita) due to an increase in the current expenditure of the unconstrained municipalities (32.19 euros per capita).

Moreover, the level of transfers received by the municipalities below 5,000 inhabitants is significantly higher with respect to those of the constrained municipalities (13.50 euros per capita). This result is consistent with the design of the law which established the use of cut in central transfers as enforcement mechanism. The same feature can also be found in the cash basis analysis where unconstrained municipalities receive, almost at a significant level (p-value of 0.133), around 20 euros more per capita in comparison with municipalities above 5,000 inhabitants.

Lastly, the change in deficit is not statistically significant. This may be due to multiple factors. On one hand, the presence of noise. On the other hand, it may be the result of a strategical behaviour of the municipalities. In fact, municipalities are not allowed to spend more than what is registered in the financial reports in accrual basis. Hence, it may be that they are inflating their spending needs in order to keep a margin for potential additional outlays. However, in practice, municipalities are not always carrying these additional expenditures out because of the DSP's stricter constraint in cash basis. The descriptive statistics seem to support this theory. In fact, by looking at both financial bases, the pattern of the deficit seems to be the same by being lower for those municipalities above 5,000 inhabitants. However, while the average deficit in cash basis is positive (i.e total expenditures higher than total revenues), the one in accrual basis is negative.

Table 4: Diff-in-disc estimates of the effect of relaxing fiscal rules, accrual basis

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	5.35	30.12		
	(6.15)	(15.51)		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	37.48	-20.75	1.06	
	(12.63)	(15.97)	(4.95)	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	-3.38	-2.78	13.50	15.98
	(7.84)	(2.56)	(6.54)	(42.85)

NOTE: The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All measures are in accrual basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5000 inhabitants and after 2001 (i.e $S_i T_t$). The estimates are retrieved by using the specification in equation (18).

5.3 Controlling for Debt

In the following subsection, by using equation 3, we test if the effect of the DSP is heterogeneous in municipalities' debt level. The results presented in Table 5 show that, even by controlling for municipalities debt's ratio, the relaxation of the DSP causes a worsening of municipalities' financial position with a fiscal gap increase equal to 75.98 euros per capita. The other coefficients are not significant, probably because of the higher standard deviation caused by the smaller sample.

Moreover, as explained in section 3.2, we perform the same analysis by using debt service as proxy for debt. Table 6 confirms the results presented above. In particular, even after controlling for municipalities' debt service ratio, the relaxation of the DSP increases municipalities' deficit (about 84 euros per capita) and fiscal gap (about 50.50 euros per capita) with respect to a baseline situation. The main driver of these changes is an increase in municipalities' current expenditures by about 41 euros per capita. Moreover, also under this new specification, the effect of the relaxation

Table 5: Diff-in-disc estimates of the effect of relaxing fiscal rules, controlling for debt

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
S˙i*T˙t	70.09	75.98		
	(45.24)	(23.29)		
S˙i*T˙t*D˙99	-11.5	-29.97		
	(63.14)	(34.06)		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
S˙i*T˙t	23.94	-23.90	-1.80	
	(21.92)	(41.96)	(9.74)	
S˙i*T˙t*D˙99	31.47	67.71	9.55	
	(30.20)	(60.91)	(13.51)	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
S˙i*T˙t	-18.85	-8.79	24.77	-11.28
	(17.91)	(5.99)	(28.30)	(52.67)
S˙i*T˙t*D˙99	5.13	3.89	6.25	49.12
	(17.92)	(9.52)	(38.02)	(73.78)

NOTE - The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and with data about their financial and operational debt. It covers a time period of 6 years (1999-2004). All variables are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants and after 2001 (i.e S_iT_t) and the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5000 inhabitants, after 2001 and with a debt ratio higher than the median(i.e $D_{99}S_iT_t$). All estimates are retrieved with the specification in equation (20).

of the fiscal rules is not heterogeneous in municipalities' debt service position.

Table 6: Diff-in-disc estimates of the effect of relaxing fiscal rules, controlling for debt service

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
S˙i*T˙t	83.61	50.54		
	(33.73)	(22.23)		
S˙i*T˙t*D˙99	-46.58	-10.24		
	(43.21)	(28.51)		
Panel B: Expenditures				
	Current outlays	Capital outlays		
S˙i*T˙t	41.11	0.75		
	(23.24)	(29.51)		
S˙i*T˙t*D˙99	-7.79	37.05		
	(27.78)	(42.74)		
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
S˙i*T˙t	-9.74	-8.95	24.09	-25.83
	(11.35)	(6.19)	(21.70)	(38.31)
S˙i*T˙t*D˙99	11.63	6.60	-9.34	59.28
	(16.15)	(7.96)	(27.65)	(53.97)

NOTE - The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All variables are in cash basis, per capita and real terms (2021 is the base year). The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5000 inhabitants and after 2001 (i.e S_iT_t) and the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5000 inhabitants, after 2001 and with a debt service ratio higher than the median(i.e $D_{99}S_iT_t$). All estimates are retrieved with the specification in equation (20).

6 Robustness Checks

In this section, we present some additional analysis in order to verify that the results obtained are not driven by the sample size or due to random chance.

6.1 Optimal Bandwidth

The choice of the bandwidth plays a key role in RDD type analysis. Consequently, it is crucial to check that the result is not driven by the choice of the sample. The algorithm developed by Cattaneo, Jansson and Ma (2020) [13] can be used to select the optimal bandwidth on which to perform the analysis. Given the high number of fixed effects used in the specifications (e.g. 6 time fixed effect and 861 municipality fixed effect), the significant restriction of the sample may compromise the results. For this reason, following Grembi et al (2016) [25], we perform the same analysis by using an alternative specification which is not relying on fixed effects.

The first specification allows to assess the average effect of the relaxation of the DSP.

$$Y_{it} = \delta_0 + \delta_1 P_{99} + S_i(\gamma_0 + \gamma_1 P_{99}) + T_t[\alpha_0 + \alpha_1 P_{99} + S_i \beta_0 + \beta_1 P_{99})] + \xi_{it} \tag{4}$$

Where, following a consistent notation, P_{99} represents the population normalized, S_i is a dummy taking value one for the municipalities of at most 5,000 inhabitants and T_t indicates the post-treatment period (after 2000). The coefficient of interest is β_0 which represents the effect of being after the introduction of the population threshold and not above the 5,000 inhabitants. The results reported in Table D1 and Table D2 confirm the previous outcomes by matching, almost perfectly, all the estimates, both in cash and accrual basis. Therefore, it can be concluded that the fiscal rules elaborated by the DSP are effective.

The second specification introduces in the model above an additional dummy representing the weight of indebtedness of the municipality. It allows to test the heterogeneity in the effect of the relaxation of the DSP with respect to municipalities' debt position.

$$\begin{aligned} Y_{it} = {} & \delta_0 + \delta_1 P_{99} + S_i(\gamma_0 + \gamma_1 P_{99}) + T_t[\alpha_0 + \alpha_1 P_{99} + S_i(\beta_0 + \beta_1 P_{99})] \\ & + D_t[\eta_0 \delta_0 + \eta_1 P_{99} + S_i(\eta_3 + \eta_4 P_{99} + T_t[\eta_5 + \eta_6 P_{99} + S_i(\eta_7 + \eta_8 P_{99}]] + \xi_{it} \end{aligned} \tag{5}$$

As in the previous case, D_{99} is a dummy variable taking value one when the debt position of the municipality is higher than the median. In order to be consistent with the analysis performed in section 5, we carry out the assessments by using both the debt ratio and the debt service ratio. Table D3 and Table D4 validate the results found with specification 3 and strengthen the empirical validation of the model.

6.2 Placebo Tests

We perform two placebo tests in order to verify that the results obtained are not due to random chance rather than a causal relationship. The first test is performed over a sample including only municipalities above 5,000 inhabitants while the second consists only of municipalities below the cut-off. In order to avoid that the size of the sample could influence the result, we pre-select the optimal bandwidth by using Cattaneo, Jansson and Ma (2020)'s [13] algorithm and, then, we implement equation 4. Both the false thresholds are selectdd in order to use a cut-off which has never been used by the DSP but, as in the case of the true threshold, is used by the wage's policies. The first test is implemented over all municipalities between 6,500 and 15,000 inhabitants for a total of 714 municipalities and the false threshold is set at 10,000 inhabitants. The second test is performed over a sample including all municipalities between 1,000 and 5,000 inhabitants for a total of 2473 municipalities and the false threshold is set at 3,000 inhabitants. As shown in Table D5 and Table D6, around the false cut-offs there is no significant effect for any of the variables of interest.

7 Theoretical Model

In this section, by building on Halac and Yared's (2014, 2018) [27] [26] work, we develop a dynamic model with hyperbolic discounting, logarithmic utility and persistent shocks. First, we briefly describe the model's fundamental and rules-free equilibrium. Then, we discuss the optimal fiscal rule and its implementation.

7.1 Environment

Time is discrete and infinite, t $\in$ [0,∞). At every time period t $\geq$ 0 there is an incumbent government which is taking the spending decisions.

Each government is subject to a real spending need (θ) which is observed only by the incumbent government and which is non-contractible. This assumption is not particularly demanding. In fact, it is plausible to believe that the incumbent has additional or more specific information compared to the other parties. Moreover, even if the shocks were observable or ex-post verifiable, the spending needs may not be contractible. For example, it would be unfeasible to write a policy rule that constraints a particular political party, even by knowing that it tends to overspend.
θ is a random variable which can take values within a bounded set $\theta \in [\underline{\theta}, \overline{\theta}]$. It follows a log-normal distribution subject to persistent shocks.

$$log\theta_{t+1} = (1 - \rho)\mu_\theta + \rho log\theta_t + \epsilon$$

where: $0 \leq \rho < 1$ and $\epsilon \sim$ N(0, σ^2). The stationary distribution of this process implies a mean μ_θ and a variance of $\frac{\sigma^2}{1-\rho^2}$. We assume $\mu_\theta = \frac{-\sigma^2}{2(1-\rho^2)}$ in order to have an $E(\theta_t) = 1$. Hence, the process of θ becomes:

$$log\theta_{t+1} = \frac{-\sigma^2}{2(1+\rho)} + \rho log\theta_t + \epsilon$$

Government's preferences are represented by a logarithmic utility which depends on the level of the government's expenditure and on the value that it attributes to its spending need. The higher is the spending need/taste θ, the higher is the marginal utility obtained by spending one additional unit.

$$u(\theta_t, g_t) = \theta_t log(g_t)$$

All governments, whether incumbent or opposition, are forward-looking and discount the future at a discounting rate δ. However, the incumbent government values the spending by future governments less. In order to depict this behaviour we introduce an additional hyperbolic-discounting factor $0 < \beta < 1$ at which the current government discounts the future. To be precise every unit of spending transforms into one unit of consumption when the government is in power but it delivers only β units of consumption when the government is out of office. Hence, the parameter β captures the political frictions in the economy. It includes both the rate of political turnover and the rate of political polarization. The higher is the level of political polarization, the higher is the difference in the objectives of the two governments and, consequently, the lower is the value attached by the incumbent government to other policy-makers' expenditures. In this context, β will be lower since one unit of future spending generates a lower utility. In the same way, the higher is the rate of political turnover, the higher is the probability that the incumbent government will loose the power and the possibility of taking part to the spending decisions. Consequently, also in this case, the incumbent discounts the future more, i.e β will be lower.

7.2 Rules-Free Equilibrium

Before proceeding to the analysis of the optimal fiscal rule, it is useful to discuss the equilibrium decisions of the incumbent government without the presence of rules.

The incumbent government solves the following maximization problem:

$$\begin{aligned} w(\theta, b) = \quad & \max_{g,b'} (g) + \beta\delta E[v(\theta', b')] \\ \text{s.t.} \quad & g \leq \tau - b + \frac{b'}{1+r} \end{aligned} \tag{6}$$

$$v(\theta, b) = \theta log(g^*(\theta, b)) + \delta E[v(\theta', b'^*(\theta, b))] \tag{7}$$

The government maximizes its utility subject to a standard budget constraint where τ represents government's revenues, b the value of outstanding government bonds and r the risk-free rate. It is relevant to underline two components. First, future government's spending needs are unknown and this is why there is an expectation over the future θ'. Second, the incumbent government discounts the ex ante continuation value by an additional factor β. For each θ, the continuation value is equal to equation (2). The incumbent government values all non-me governments in the same way and it is passively subject to their spending decision. For this reason after the government looses power, it discounts all future allocations only by δ and in equation (2) $g^* = g^*(\theta, b)$.

Let h be equal to the future discounted wealth, i.e $h = \dfrac{\tau}{r}$, the solution of this problem is:

$$g(\theta, b) = \gamma(\theta)[\tau + h - b]] \tag{8}$$

where

$$\gamma(\theta) = \frac{\theta}{\theta + \beta\delta E[B(\theta')|\theta]} \tag{9}$$

$$v(\theta, b) = A(\theta) + B(\theta) log(\tau + h - b) \tag{10}$$

$$w(\theta, b) = A^w(\theta) + B^w(\theta) log(\tau + h - b) \tag{11}$$

$$A(\theta) = \theta log(\gamma(\theta) + \delta E[A(\theta')|\theta] + \delta B(\theta') log((1 + r)(1 + r)(1 - \gamma(\theta')))|\theta] \tag{12}$$

$$B(\theta) = \theta + \delta E[B(\theta')|\theta] \tag{13}$$

Proof: See Appendix E.1

By looking at $\gamma(\theta)$ it is possible to see that government's spending solution is different from the one of the planner who sets β equal to one.

7.3 The Optimal fiscal rule

If the planner could observe θ, only equation 7 would be necessary. However, since we assume that θ is neither observable nor contractible, the planner must induce the governments to truthfully reveal their realized θ. In order to solve this problem, we must solve a principle-agent problem. We assume that Halac and Yared's(2014) [27] result holds. Hence, the sequential optimal rule consists in establishing a maximum spending on the base of a θ contingent threshold $(\theta_p(\theta))$. That being the case, we impose that all agents with $\theta_t \leq \theta_p(\theta_{t-1})$ are free to choose their spending and all those with $\theta_t \geq \theta_p(\theta_{t-1})$ must spend the same as the $\theta_p(\theta_{t-1})$ agent. Consequently, given a previously observed value θ_0, the spending function becomes:

$$g(\theta,b) = \begin{cases} \gamma(\theta)[\tau+h-b] \quad if\ \theta \leq \theta_p(\theta_0) \\ \\ \gamma(\theta_p(\theta_0))[\tau+h-b] \quad if\ \theta \geq \theta_p(\theta_0) \end{cases} \tag{14}$$

Also $B(\theta)$ is piecewise and must satisfy:

$$g(\theta,b) = \begin{cases} \theta + E[B(\theta')|\theta] \quad if\ \theta \leq \theta_p(\theta_0) \\ \\ \theta + E[B(\theta')|\theta] \quad if\ \theta \geq \theta_p(\theta_0) \end{cases} \tag{15}$$

However, notice that the coefficient is the same in both cases. This result is due to two reasons. The first one is the choice of a logarithmic utility which separates the resource components from the consumption ratio component. The second one is linked to the fact that, independently of being constrained or not, the incumbent government values consumption at rate θ. As a result, the solution for γ found in section 7.2 is the same.

The only difference is embedded in the constant $A(\theta)$ since now the government takes into account that tomorrow it may fall into the constrained zone. Once that we have the conditional expectation of A, we can compute the ex-ante expectation and use it to choose the optimal rule. The optimal threshold takes value:

$$\theta_p(\theta_0) = \beta E[\theta|\theta \geq \theta_p(\theta_0)|\theta_0] \frac{E[B(\theta')|\theta_p(\theta_0)]}{E[B(\theta')|\theta' \geq \theta_p(\theta_0)|\theta_0]} \tag{16}$$

Proof: See Appendix E.2

Note that, because of the logarithmic utility, the optimal threshold is independent from the total wealth level. Moreover, under no persistence, i.e $\rho = 0$, $E[B(\theta)]$ is constant and we get back to the rule elaborated by Felli, Piguillem and Shi (2021) in the case with no default.

7.4 Implementation of the Optimal Fiscal Rule

To implement $\theta_p(\theta_0)$ there are several alternatives often found in the observed fiscal rules. The easiest application consists in the creation of a spending limit.

$$g(\theta, b) = \begin{cases} g(\theta, b) \quad if\ \theta \leq \theta_p(\theta_{-1}) \\ g(\theta_p(\theta_{-1}), b) \quad if\ \theta \geq \theta_p(\theta_{-1}) \end{cases} \tag{17}$$

This type of rule often finds a practical implementation in real life, especially in the form of a deficit limit. However, its application is linked to two main challenges. First, it depends on the level of total wealth. This increases the complexity of the implementation since obtaining the data to compute this measure and processing them requires a high amount of time. Second, because of a persistent process of θ, the rule depends on the previous period spending shock which is still private knowledge.

In order to solve these two problems, we propose a different implementation. Following the approach used by the Italian DSP, we implement the optimal fiscal rule in terms of limit in governments spending growth on the basis of their previous growth. This alternative approach allows to solve both the problems presented above. First, the use of a threshold in terms of growth allows to make the implementation independent of debt.

$$\nu(\theta, \theta_{-1}) \leq \gamma(\theta_p(\theta_{-1}))\frac{\beta E[B(\theta)|\theta_{-1}]}{\theta_{-1}} = \bar{\nu}(\theta_{-1}) \tag{18}$$

Where $\bar{\nu}(\theta_{-1})$ represents the growth limit on the basis of the previous shock history.

It is important to underline that, although the high variance does not enable us to draw a final conclusion, the empirical result in section 5.3 seem to support the theoretical model elaborated in section 7. In fact, the empirical result shows that the effect of the relaxation of the Italian fiscal rule, expressed as growth limit, is homogenous in municipalities' debt position. This is exactly what we would expect according to the theory since governments' spending decision is independent from municipalities' total wealth when expressed in terms of growth.

Second, by integrating over θ_{-2} using the unconditional distribution we remove the role of past histories and we find a one-to-one mapping between the observed previous growth rate and level of θ. In fact, from the data we can observe the growth rate of a government spending but no the histories of θ that led it there. $\tilde{\nu}$ represents the previous period spending growth independent from the previous histories of θ.

$$\int \nu(\theta_{-1}, \theta_{-2}) dF^u(\theta_{-2}) = \gamma(\theta_{-1}) \int \frac{\beta E[B(\theta_{-1})|\theta_{-2}]}{\theta_{-2}} dF^u(\theta_{-2}) = \hat{\nu}(\theta_{-1}) \tag{19}$$

Consequently, the implementation of the optimal rule can take the form of a limit in the growth of the current spending proportional to the previous one.

$$\bar{\nu}(\tilde{\nu}_{-1}) = k(\tilde{\nu}_{-1})\tilde{\nu}_{-1} \tag{20}$$

Note that stating that k depends on the previous period growth factor $\tilde{\nu}$, is equivalent to say that it depends on the previous realization of θ since there exists a one to one mapping between the two variables. In particular, the limit in the growth is equal to:

$$k(\tilde{\nu}_{-1}(\theta_{-1})) = \frac{\gamma(\theta_p(\theta_{-1}))\frac{E[B(\theta)|\theta_{-1}]}{\theta_{-1}}}{\gamma(\theta_{-1})\int_{\underline{\theta}}^{\bar{\theta}} \frac{E[B(\theta)|\theta_{-1}]}{\theta_{-1}} dF^u(\theta_{-1})} \tag{21}$$

Proof: See Appendix E.3

We can think about the Italian fiscal rule as a unique growth factor limit in municipalities' growth.

$$\bar{\nu}(\tilde{\nu}_{-1}) = k\tilde{\nu}_{-1} \tag{22}$$

Since the implementation that we have just derived sets k dependent on the previous growth factor, it is easy to see that the Italian fiscal rule is sub-optimal.

8 Calibration

Before implementing the optimal fiscal rules elaborated in section 7, we need to know the parameters of the model. For this reason, in this section we report the results of the calibration of $\sigma, \rho, \beta, \delta$.

We assume that the distribution of government's spending needs (θ) is a truncated log-normal in the domain $\theta \in [\underline{\theta}, \overline{\theta}]$ subject to persistent shocks. It generates a stationary distribution $f^u(\theta) = LN\left(\frac{-\sigma^2}{2(1-\rho^2)}, \frac{\sigma^2}{1-\rho^2}\right)$ and a conditional distribution $f^c(\theta) = LN((1-\rho)\mu + (\theta), \sigma^2)$. In order to estimate the true distribution of the spending needs, we need to observe municipalities' unconstrained spending decisions. For this reason, the data sample includes data from 2001 until 2004 about all municipalities below 5,000 inhabitants.

We calibrate δ directly from the data. In fact, since the planner is the national government, its discount rate must be equal to the interest rate that the Italian central government pays on its debt. In order to compute the other three parameters, we match three moments in the data. In particular, we take advantage of the spending patterns in the rules-free equilibrium by using the first and second moment of the spending ratio and the variance of the spending growth factor. Appendix F describes the specification and variables used for the calibration. Table 7 reports the calibrated parameters and the corresponding data moments. The discount rate is in line with the literature and ρ and β seem to confirm the existence of a government spending bias and of persistent spending needs.

Table 7: Parameters and moments

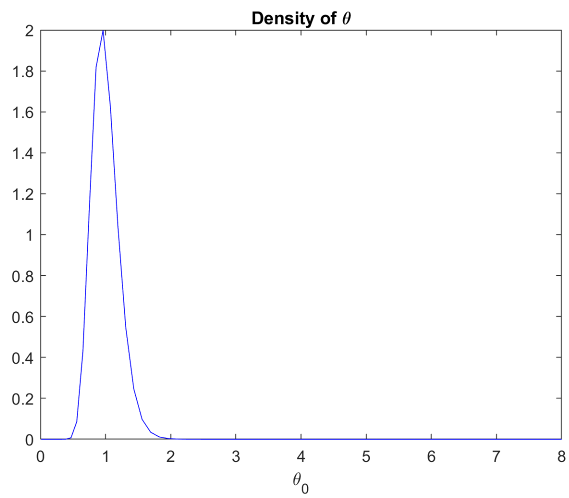
Parameter	Symbol	Value	Moment
Discount rate	δ	0.96	Interest rate
Present bias	β	0.88	Average spending ratio
Persistency	ρ	0.44	Dispersion in the spending growth factor
Distribution std dev	σ	0.19	Second moment of the spending ratio

9 Policy Analysis

In this section we present the optimal fiscal rule and its implementation computed by using the parameters retrieved through the calibration.

Figure 7 shows the distribution of government's spending needs. The distribution is narrowed around 1, taking values mainly between 0.5 and 2.

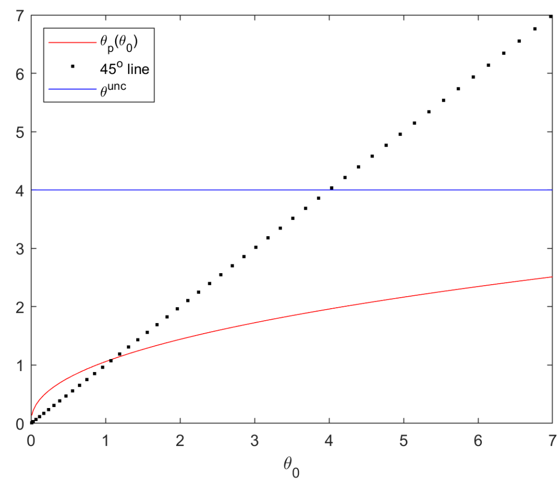
Figure 7: Density of government's spending needs (θ)



First we compute the values of the optimal threshold according to the different θ_0. In figure 8 it is possible to see that, under persistency, $\theta_p(\theta_0)$ is increasing in θ (red line). This result is consistent with the idea that a municipality with high spending needs today is more likely to face high spending needs tomorrow. Hence, the social planner sets a high $\theta_p(\theta_0)$ to allow the incumbent government to meet municipalities' true spending needs. On the contrary, without persistency, the optimal threshold is constant (blue line) since knowing the previous θ does not allow to make any inferences about the future.

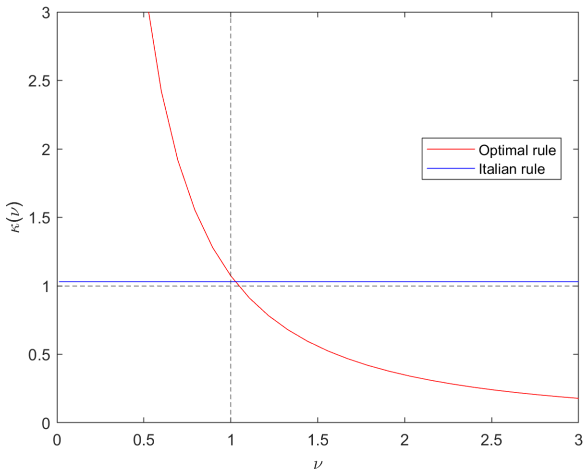
Then, we assess the optimal limit in the growth factor k according to municipalities' previous growth factor. The red line in figure 9 shows the optimal rule elaborated in section 7. It is easy to see that this specification of k incentives municipalities to cut their spending whenever possible.

Figure 8: Optimal threshold



In order to understand the reasoning behind the rule, it is useful to make some examples. A government which increased its expenditures by 50 % with respect to the previous period will be constrained to halve its spending in the next one. Vice-versa, a municipality which decreased its pay out by 50 % will be able to grow freely in the next period. In this way virtuous municipalities are rewarded by being allowed higher freedom. The blue line depicts the Italian fiscal rule. It is constant since the DSP sets the same limit k for all municipalities, independently from their previous growth. It stands out that no government is incentivized to cut its spending. Indeed, virtuous governments are penalised the most since they are constrained in their spending without considering their possibility of affording higher outlays. Indeed, this problem has been underlined by many Italian municipalities over the years. This visual result confirms the conclusion drawn in section 7 about the sub-optimality of the DSP. By implementing the rule developed in section 7.4 we would have all the advantages linked to the application of a fiscal rule in growth terms while promoting a virtuous behavior.

Figure 9: The optimal growth limit (k) in the growth space (ν)



10 Conclusion

Fiscal rules are usually considered as a possible solution to the consistent increase in the level of public debt over GDP. We tackle this question from a double perspective.

In the first part of the study, we address it from an empirical point of view. We use Italian quasi-experimental evidence and we show that fiscal rules enforced by a national government can be effective in causing a reduction in the accumulation of debt by local governments. Moreover, we find that the fiscal adjustment occurs through a decrease in municipalities' current expenditures. This result confirms that there is room to implement an optimal fiscal rule.
Using data about one country allows to increase the internal validity of the work by overcoming the limits of cross-countries analysis but at the cost of its external validity. However, we believe that Italy, by being a country known for its poor legal enforcement, especially between 1999 and 2004, could represent a benchmark. Hence, fiscal rules may be useful also in regulatory environment with serious commitment issues and, even more, in countries with a strong legal enforcement. Lastly, our results suggest that municipal policy makers prefers to cut expenditures rather than rise taxes in order to comply with the national rules. Although the standard errors don't allow to draw a strong

conclusion, future researches might focus on understanding the political incentives that drive the choice around this trade-off.

In the second part of the study, we tackle the question from a theoretical point of view. Since fiscal rules appear to be effective, we develop a new optimal fiscal rule in terms of limit in government spending growth on the basis of its previous one. This new formulation enables to remove the dependence of the fiscal rule on government's total wealth and to create a direct mapping between its previous growth factor and its spending need. Moreover, it incentives governments to cut their spending since virtuous municipalities are rewarded with higher freedom. Lastly, we show that, while the Italian rule imposed a limit in the growth rate as the optimal rule, it was still sub-optimal since it set a unique growth factor limit for all municipalities. At this stage our model constitutes a first step towards developing a theory that can help providing precise quantitative prescriptions for real-life case studies. However, in order to reach this objective, some additional features should be implemented as endogenous taxation and a partition of the spending bias between political turnover and political friction.

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Appendix

A Appendix

Institutional Framework

Table A1: Legislative population thresholds for Italian Municipalities

Population	Mayor's wage until 2005	Mayor's wage after 2005 [1]	Executive committee's wage (%)	Executive committee's size	Council size	Electoral rule
Below 1,000	1,291	1,162	15	4	12	Single
1,001-3,000	1,446	1,301	20	4	12	Single
3,001-5,000	2,169	1,952	20	4	16	Single
5,001-10,000	2,789	2,509	50	4	16	Single
10,001-15,000	3,099	2,788	55	6	20	Single
15,001-30,000	3,099	2,788	55	6	20	Runoff
30,001-50,000	3,460	3,114	55	6	30	Runoff
50,001-100,000	4,132	3,114	55	6	30	Runoff
100,001-250,000	5,010	4,508	55	10	40	Runoff
250,001-500,000	5,784	5,205	75	12	46	Runoff
Above 500,000	7,7798	7,018	75	14-16	50-60	Runoff

NOTES- *Population* is the number of resident inhabitants as measured by the last available census. *Wage of mayor* refers to the monthly gross wage of the mayor in euro at 2000 prices. *Wage of executive committee*is the monthly gross wage of the members of the executive committee as a percentage of the mayor's one. *Executive Committee's Size* is the maximum allowed number of executives appointed by the mayor. *Council Size* is the number of seats in the City Council. Since 1993, *Electoral Rule* can be either single round (with 60% premium) or runoff (with 66% premium) plurality voting.(1) With art. 1 law 266/2005, mayor's wage was reduced by 10%.

Table A2: DSP for Italian municipalities evolution from 1999 until 2015

Year	Municipalities covered	Target	Limit and Reference Year	Expenditures Considered	Revenues excluded	Expenditures excluded	Financial components considered	Sanctions
1999	All	Fiscal gap	zero growth wrt fiscal gap 1997	Current expenditures	Central transfers	Capital expenditures, interest and debt repayment	Cash basis	
2000	All	Fiscal gap	zero growth wrt fiscal gap 1998	Current expenditures	Central transfers	Capital expenditures, interest and debt repayment	Cash basis	Paymet of the European sanctions
2001	>5000^1	Fiscal gap	max. 3% growth wrt fiscal gap 1999	Current expenditures	Central transfers	Capital expenditures, interest and debt repayment	Cash basis	
2002	>5000	Fiscal gap Current expenditures cap	max 2.5% growth wrt fiscal gap 2000	Current expenditures	Central transfers	Capital expenditures, interest and debt repayment	Cash basis	Cut of central transfers
2003	>5000	Fiscal gap	zero growth wrt fiscal gap 2001	Current expenditures	Central transfers	Capital expenditures, interest and debt repayment	Cash and accrual basis	Cut of central transfers, limitation on debt
2004	>5000	Fiscal gap	zero growth wrt fiscal gap 2002	Current expenditures	Central transfers, financial receivables, real estate disposal	Capital expenditures, interest and debt repayment	cash and accrual basis	Limitation on current expenditures, on staff recruitment and on debt
2005[2]	>5000	Total expenditures cap	Lower than avarage of expenditures 2001-2003 increased by 10%	Current and capital expenditures	fiancial receivables	Debt repayment, personnel costs	Cash and accrual basis	Limitation on current expenditures, on staff recruitment and on debt

NOTES: (1) In order to assess if the municipalities were above or below the threshold, the regulation considered ISTAT intercensal estimations of the resident population of two years before (e.g for 2001 it considered the population at the end of 1999).(2) The annual budget law reduced the threshold to 3000 inhabitants but the new limit was never implemented in practice.

Table A2: DSP for Italian municipalities evolution from 1999 until 2015

Year	Municipalities covered	Target	Limit and Reference Year	Expenditures Considered	Revenues excluded	Expenditures excluded	Financial components considered	Sanctions
2006	>5000	Total expenditures cap	Current expenditures decreased by 6.5%, capital expenditures decreased by 8.1% wrt 2004	Current and capital expenditures	financial receivables	Debt repayment, personnel costs	Cash and accrual basis	Limitation on current expenditures, on staff recruitment and on debt
2007ˆ[1]	All	Fiscal gap	improvement wrt average fiscal gap 2003-2005[2]	Current and capital expenditures	Financial receivables	Debt repayment	On cash and accrual basis	Cuts of central transfers,limitation on current expenditures and on staff recruitment.
2008	All	Fiscal gap	improvement wrt average fiscal gap 2003-2005[3]	Current and capital expenditures	Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures and staff recruitment, payment of a sanction by the administrators
2009	>5000	Fiscal gap	improvement wrt average fiscal gap 2003-2005. Different rules for municipalities in better conditions [4]	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures and staff recruitment, impossibility of contracting new debt, payment of a sanction by the administrators
2010	>5000	Fiscal gap	improvement wrt average fiscal gap 2003-2005. Different rules for municipalities in better conditions [5]	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[7]

NOTES: (1) In order to assess if the municipalities were above or below the threshold, the regulation considered ISTAT intercensal estimations of the resident population of two years before (e.g for 2007 it considered the population at the end of 2005). (2) The entity of the improvement depends on the sign of the fiscal gap between 2003-2005.(3) The municipalities who had a positive fiscal gap (i.e revenues ¿ expenditures) were not required to make any improvement. (4) There were different thresholds according to two criteria: the compliance to the rules of 2007 and the sign of the fiscal gap. The municipalities that respected the DSP the previous year could exclude from the capital expenditures up to 4% of the outstanding payments (residui passivi) of the previous two years. The municipalities that did not respect the pact but had a positive fiscal gap did not have to make any improvement. The municipalities that respected the pact and had a positive fiscal gap were allowed to worsen their fiscal gap. (5) There were different thresholds according to two criteria: the compliance to the rules of 2007 and the sign of the fiscal gap. The municipalities that respected the DSP the previous year could exclude up to 0.75% in outstanding payments (residui passivi) of the capital expenditures recorded two years before. The municipalities that did not respect the pact but had a positive fiscal gap did not have to make any improvement. The municipalities that respected the pact and had a positive fiscal gap were allowed to worsen their fiscal gap.

Table A2: DSP for Italian municipalities evolution from 1999 until 2015

Year	Municipalities covered	Target	Limit and Reference Year	Expenditures Considered	Revenues excluded	Expenditures excluded	Financial components considered	Sanctions
2011	>5000	Fiscal gap	fiscal gap not lower than a fixed percentage of the average current expenditures 2006-2008	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[2]
2012	>5000	Fiscal gap	fiscal gap not lower than a fixed percentage of the average current expenditures 2006-2008 [1]	Current and capital expenditures	Central transfers, financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[2]
2013	>1000	Fiscal gap	fiscal gap not lower than a fixed percentage of the average current expenditures 2007-2009 minus the reduction of central government transfers	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[2]
2014	>1000	Fiscal gap	fiscal gap not lower than a fixed percentage of the average current expenditures 2009-2011 minus the reduction of central government	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[2]
2015	>1000	Fiscal gap	fiscal gap not lower than a fixed percentage of the average current expenditures 2010-2012 minus the reduction of central	Current and capital expenditures	Central transfers, Financial receivables	Debt repayment	Capital expenditures on a cash basis, current expenditures on an accrual basis	Cuts of central transfers, limitation on current expenditures, staff recruitment and debt, payment of a sanction by the administrators[2]

NOTES: (1) The municipalities that are "virtuous" had to pursue a fiscal gap=0. (2) The mayor and the councilors receive a salary cut of 30%.

Table A3: Evolution of debt limits' regulations

Legal reference	Limit [1]	Variable limited	Reference revenues
D. Lgs 1 77/1995 art. 46 D. Lgs 267/2000 art. 2000	25%	Interest expenditures net of interest contributions from the center and regional governments	First three components of the revenues
Law 31/2004 art. 1/44	12%		
Law 296/2006 art.1/698	15%		
D. Lgs 183/2011 art. 1, D. Lgs 147/2013 art.1	8%		
Law 190/2014 art.1	10%		

NOTES: (1) The debt limit must be respected the year in which new debt is issued.

B Appendix

Data Sources

Table B1: Variables' description and sources

Variable	Definition and measure	Source
Deficit	Total expenditures - Total revenues	IMI, Financial reports: author's calculations
Fiscal gap	Current expenditures - Total revenues net of central transfers, capital revenues, issue of new debt and interest repayment [1]	IMI Financial reports: author's calculations
Total outlays	Total outlays	IMI Financial reports: Quadro 3
Current outlays	Current expenditures	IMI Financial reports: Quadro 3, 4
Capital outlays	Capital expenditures	IMI Financial reports: Quadro 3, 5
Debt services	Interest and installment payment	IMI Financial reports: Quadro 3, 4
Total revenues	Total revenues	IMI Financial reports: Quadro 2
Taxes	Tax revenues	IMI Financial reports: Quadro 2
Fees and Tariffs	Revenues from fees and tariffs	IMI Financial reports: Quadro 2
Revenues, Title 1	Revenues from taxes, fees and tariffs	IMI Financial reports: Quadro 2
Revenues, Title 2	Total transfers received	IMI Financial reports: Quadro 2
Revenues, Title 3	Total non-tax revenues	IMI Financial reports: Quadro 2

NOTES- *IMI* stands for Italian Ministry of the Interior and FR for fianancial reports. Successively, all values have been expressed in per capital and in real terms by using 2021 as base year. (1) More specifically, the fiscal gap is equal to the first four titles of the revenues (titolo I, II, III, IV) net of compartecipazione irpef, transfers, debt collection, real-estate and financial disposal, capital transfers made by the state minus current expenditures net of interest payments.

Table C1: Variables' description and sources (continuation)

Variable	Definition and measure	Source
Revenues, Title 4	Capital revenues	IMI Financial reports: Quadro 2
Revenues from new loans and services for third parties	Revenues from new loans and services for third parties	IMI Financial reports: Quadro 2
Other revenues	Other revenues	IMI Financial reports: author's calculation
Financial debt	Financial debt at the end of the year	IMI Financial reports: Quadro 8
Operating debt	Operating debt at the end of the year	IMI Financial reports: Quadro 8
Total debt	Total debt including financial operating debt and cash advances	IMI Financial reports: Quadro 8
Population	Intercensal population of the municipalities	ISTAT

NOTES- *IMI* stands for Italian Ministry of the Interior. Successively, all values have been expressed in per capital and in real terms by using 2021 as base year.

C Appendix

Summary Statistics

Table C1: Descriptive Statistics, accrual basis

	Above 5000	Below 5000
A: Fiscal discipline		
Fiscal gap	75.36	101.58
Deficit	16.91	21.19
B: Expenditures		
Current outlays	779.66	791.36
Capital outlays	429.54	444.51
Debt service	110.13	108.23
C: Revenues		
Taxes	299.87	286.03
Fees and Tariffs	96.85	93.16
Government transfers	232.65	248.97
Other revenues	753.79	763.62
Observations	2,058	3,108

NOTE: The municipalities have a population between 3,400 and 7,100 inhabitants. All values are in accrual basis, per capita and real terms by using 2021 as base year.

Table C2: Descriptive Statistics, debt reduced sample on cash basis

	Above 5000	Below 5000
A: Fiscal discipline		
Fiscal gap	53.7	96.65
Deficit	-45.86	-30.04
B: Expenditures		
Current outlays	767.02	773.49
Capital outlays	352.64	364.63
Debt service	116.59	114.81
C: Revenues		
Taxes	310.27	289.85
Fees and Tariffs	91.98	87.71
Transfers	250.12	270.80
Other revenues	679.59	688.46
Observations	1,482	1,536

NOTE: The municipalities have a population between 3,400 and 7,100 inhabitants. All values are in cash basis, per capita terms and real terms by using 2021 as base year.

D Appendix

Robustness Checks

Table D1: Diff-in-disc estimates of the effect of relaxing fiscal rules

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	58.20	44.89		
	(21.32)	(14.11)		
h	402.47	317.23		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	38.57	23.42	-0.48	
	(11.59)	(13.71)	(4.85)	
h	242.15	247.30	282.52	
Panel B: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	-3.68	-5.84	20.78	15.60
	(8.08)	(3.97)	(13.83)	(29.54)
h	201.19	374.62	380.39	329.87

NOTE - The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All measures are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants and after 2001 (i.e S_iT_t). The estimates are retrieved by using the specification in equation (21) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

Table D2: Diff-in-disc estimates of the effect of relaxing fiscal rules, accrual basis

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	5.35	30.12		
	(6.14)	(15.48)		
h	431.67	403.44		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	37.47	-20.75	1.06	
	(15.94)	(37.35)	(4.95)	
h	368.10	428.08	418.51	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	-3.38	-2.77	13.50	15.98
	(7.84)	(2.55)	(6.53)	(42.78)
h	299.17	476.37	382.37	489.88

NOTE - The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All measures are in accrual basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants and after 2001 (i.e S_iT_t). The estimates are retrieved by using the specification in equation (21) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

Table D3: Diff-in-disc estimates of the effect of relaxing fiscal rules, controlling for debt

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
S˙i*T˙t	70.09	75.98		
	(45.03	(23.19)		
S˙i*T˙t*D˙99	-11.49	-29.95		
	(62.85)	(33.89)		
h	384.21	420.21		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
S˙i*T˙t	23.94	-23.90	-1.80	
	(21.82)	(41.77)	(9.69)	
S˙i*T˙t*D˙99	31.47	67.71	9.55	
	(30.06)	(60.63)	(13.44)	
h	294.55	376.68	268.13	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
S˙i*T˙t	-18.85	-8.78	24.77	-11.28
	(17.83)	(5.96)	(28.17)	(52.43)
S˙i*T˙t*D˙99	5.13	3.88	6.25	49.12
	(22.70)	(9.47)	(37.85)	(73.44)
h	256.74	279.51	416.84	385.82

NOTE: The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and with information about their financial and operational debt levels. It covers a time period of 6 years (1999-2004). All variables are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants and after 2001 (i.e S_iT_t) and the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants, after 2001 and with a debt ratio higher than the median. The estimates are retrieved by using the specification in equation (22) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

Table D4: Diff-in-disc estimates of the effect of relaxing fiscal rules, controlling for debt service

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
S˙i*T˙t	83.61	50.54		
	(33.64)	(22.18)		
S˙i*T˙t*D˙99	-46.58	-10.24		
	(43.10)	(28.43)		
h	402.47	317.22		
Panel B: Expenditures				
	Current outlays	Capital outlays		
S˙i*T˙t	41.11	0.75		
	(23.18)	(29.43)		
S˙i*T˙t*D˙99	-7.79	37.05		
	(27.71)	(42.62)		
h	247.30	328.80		
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
S˙i*T˙t	-9.74	-8.95	24.09	-25.83
	(11.32)	(6.18)	(21.64)	(38.20)
S˙i*T˙t*D˙99	11.63	6.60	-9.34	59.28
	(16.01)	(7.93)	(27.57)	(53.82)
h	201.19	374.62	380.39	323.27

NOTE - The sample is composed by municipalities with a population between 3,400-7,100 inhabitants and covers a time period of 6 years (1999-2004). All variables are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 5,000 inhabitants and after 2001 (i.e S_iT_t) and the impact of relaxing fiscal rul,es on policy outcomes for municipalities with no more than 5,000 inhabitants, after 2001 and with a debt service ratio higher than the median (i.e $S_iT_tD_{99}$). The estimates are retrieved by using the specification in equation (22) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

Table D5: Diff-in-disc estimates of the effect of relaxing fiscal rules, 10,000 cut-off

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	-1.00	2.24		
	(18.03)	(13.88)		
h	1143.98	269.45		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	7.21	3.53	-6.44	
	(15.04)	(16.98)	(6.99)	
h	465.79	589.49	869.78	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	-5.42	2.63	2.23	-0.75
	(7.84)	(5.45)	(12.55)	(28.32)
h	445.12	453.83	612.45	771.69

NOTE - The sample is composed by municipalities with a population between 6,500-15,000 inhabitants and covers a time period of 6 years (1999-2004). All measures are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 10,000 inhabitants and after 2001 (i.e S_iT_t). The estimates are retrieved by using the specification in equation (21) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

Table D6: Diff-in-disc estimates of the effect of relaxing fiscal rules, 3,000 cut-off

Panel A: Fiscal Discipline				
	Deficit	Fiscal gap		
Estimate	-11.26	-5.24		
	(24.39)	(11.06)		
h	334.65	329.94		
Panel B: Expenditures				
	Current outlays	Capital outlays	Debt service	
Estimate	-13.39	-14.28	-0.15	
	(8.62)	(15.87)	(3.09)	
h	431.29	337.03	505.8	
Panel C: Expenditures				
	Taxes	Fees and tariffs	Government transfers	Other revenues
Estimate	3.38	-0.09	-1.83	-27.09
	(4.62)	(2.46)	(8.89)	(20.20)
h	317.89	353.59	393.90	358.02

NOTE - The sample is composed by municipalities with a population between 1,000-5,000 inhabitants and covers a time period of 6 years (1999-2004). All measures are in cash basis. The table reports estimates of the impact of relaxing fiscal rules on policy outcomes for municipalities with no more than 3,000 inhabitants and after 2001 (i.e S_iT_t). The estimates are retrieved by using the specification in equation (21) and the optimal bandwidth is computed by using Cattaneo, Jansson and Ma (2020)'s algorithm. h represents the optimal bandwidth.

E Appendix

Theoretical Model

E.1 Rules-Free Equilibrium

We guess the following solution:

$g(\theta, b) = \gamma(\theta)[\tau + h - b]$

$v(\theta, b) = A(\theta) + B(\theta)log(\tau + h - b)$

$w(\theta, b) = A^w(\theta) + B^w(\theta)log(\tau + h - b)$

By taking v as given the incumbent government solves the following problem:

$$\max_{b'}\{\theta log\left(\tau - b + \frac{b'}{1+r}\right) + \beta\delta E[v(\theta', b')]\}$$

By taking the FOC and using the envelope condition, we obtain the following Euler equation:

$$\frac{\theta}{g} = \beta\delta(1+r)\frac{E[B(\theta')|\theta]}{\tau + h - b}$$

The budget constraint implies that b'=(1+r)[g-τ+b]. Hence, by using the fact that $h = \frac{\tau}{r}$, we can write:

$$\tau + h - b' = \tau + h - (1+r)[g - \tau + b] = \tau + h - (1+r)[\gamma(\theta)(\tau + h - b) - \tau + b] =$$

$$= -(1+r)\gamma(\theta)(\tau+h-b)+\tau+h-(1+r)(b-\tau)] = -(1+r)\gamma(\theta)(\tau+h-b)+(1+r)(\tau+h)-(1+r)b = (1+r)[1-\gamma(\theta)](\tau+h-b)$$

By replacing the last equation in the Euler Equation guessed, we obtain:

$$\frac{\theta}{\gamma(\theta)(\tau + h - b)} = \beta\delta(1+r)\frac{E[B(\theta')|\theta]}{(1+r)[1-\gamma(\theta)](\tau + h - b)}$$

Solving the last equation for $\gamma(\theta)$ generates equation 9 :

$$\gamma(\theta) = \frac{\theta}{\theta + \beta\delta E[B(\theta')|\theta]}$$

Now we have to verify the guesses for the value functions which are:

$A(\theta) = \theta \log(\gamma(\theta)) + \delta E[A(\theta')|\theta] + \delta E[B(\theta') \log((1+r)(1-\gamma(\theta')))|\theta]$

$B(\theta) = \theta + \delta E[B(\theta')|\theta]$

Using the last equation we can compute:

$$E[B(\theta')|\theta] = E[\theta'|\theta] + \delta E[E[B(\theta'')|\theta']|\theta]$$

In order to prove them. we can use the process for θ. In fact, note that the n periods ahead outcome path is given by:

$$\log \theta_{t+n} = (1-\rho^n)\mu_\theta + \rho^n \log \theta_t + \sum_{i=0}^{n-1} \rho^i \epsilon_{t+n-i}$$

Then,

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta} \theta^{\rho^n} E[\exp(\sum_{i=0}^{n-1} \rho^i \epsilon_{t+n-i})|\theta_t]$$

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta} \theta^{\rho^n} E[\prod_{i=0}^{n-1} \exp(\rho^i \epsilon_{t+n-i})|\theta_t]$$

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta} \theta^{\rho^n} E[\prod_{i=0}^{n-1} (\exp(\epsilon_{t+n-i}))^{\rho^i}|\theta_t]; \quad n \geq 1$$

We can use the properties of a lognormal distribution.

If $\epsilon_{t+n-i} \sim N(0,\sigma^2)$, then $\exp(\epsilon_{t+n-i}) \sim LN(0,\sigma^2)$.

If $\exp(\epsilon_{t+n-i}) \sim LN(0,\sigma^2)$, then $\exp(\epsilon_{t+n-i})^{\rho^i} \sim LN(0,(\rho^i)^2\sigma^2)$.

Finally, if each $\exp(\epsilon_{t+n-i})^{\rho^i} \sim LN(0,(\rho^i)^2\sigma^2)$ then $\prod_{i=0}^{n-1}(\exp(\epsilon_{t+n-i}))^{\rho^i} \sim LN(0, \sum_{i=0}^{n-1}(\rho^i)^2\sigma^2)$.

It follows then that $E[\prod_{i=0}^{n-1}(\exp(\epsilon_{t+n-i}))^{\rho^i}|\theta_t] = E[\prod_{i=0}^{n-1}(\exp(\epsilon_{t+n-i}))^{\rho^i}] = \exp(\sum_{i=0}^{n-1}(\rho^i)^2\sigma^2/2)$

Replacing this in the last equation, we obtain:

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta}\theta_t^{\rho^n}\exp(\sum_{i=0}^{n-1}(\rho^i)^2\sigma^2/2)$$

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta}\theta_t^{\rho^n}\exp(\sum_{i=0}^{n-1}\rho^{2i}\sigma^2/2)$$

$$E[\theta_{t+n}|\theta_t] = e^{(1-\rho^n)\mu_\theta}\theta_t^{\rho^n}\exp\left(\frac{1-\rho^{2n}}{1-\rho^2}\frac{\sigma^2}{2}\right)$$

To solve for B we need to compute:

$$B(\theta) = \theta + \sum_{n=1}^{\infty}\delta^n E[\theta_n|\theta] = \sum_{n=0}^{\infty}\delta^n\theta^{\rho^n}\exp\left(\frac{1-\rho^{2n}}{1-\rho^2}\frac{\sigma^2}{2} + (1-\rho^n)\mu_\theta\right)$$

Replacing μ_θ we obtain:

$$B(\theta) = \theta + \sum_{n=1}^{\infty}\delta^n\theta^{\rho^n}\exp\left(\frac{\sigma^2}{2(1-\rho^2)}[1-\rho^{2n}-(1-\rho^n)]\right)$$

$$B(\theta) = \theta + \sum_{n=1}^{\infty}\delta^n\theta^{\rho^n}\exp\left(\frac{\sigma^2}{2(1-\rho^2)}[\rho^n-\rho^{2n}]\right)$$

$$\begin{aligned} B(\theta) &= \theta + \sum_{n=1}^{\infty}\delta^n\theta^{\rho^n}\exp\left(\frac{\sigma^2\rho^n}{2}\frac{(1-\rho^n)}{(1-\rho^2)}\right) \\ &= \theta + \sum_{n=1}^{\infty}\delta^n\left(\theta\exp\left(\frac{\sigma^2}{2}\frac{(1-\rho^n)}{(1-\rho^2)}\right)\right)^{\rho^n} \\ &= \sum_{n=0}^{\infty}\left(\delta\theta^\rho\exp\left(\frac{\rho\sigma^2}{2}\right)\right)^n \quad \text{this step is wrong} \\ &= \frac{1}{1-\delta\theta^\rho\exp\left(\frac{\rho\sigma^2}{2}\right)} \end{aligned}$$

The last is true only if $\delta\theta^\rho\exp\left(\frac{\rho\sigma^2}{2}\right) < 1$

It is straightforward to verify that the last satisfies the guess for $B(\theta)$.

Moreover, with this shaper characterization we can compute spending and savings ratios. Replacing $B(\theta)$ in equation 9 we obtain:

$$\gamma(\theta) = \frac{\theta(1-\delta\rho)}{\theta[1-\rho\delta(1-\beta)] + \beta\delta\frac{(1-\rho)\mu}{1-\delta}}$$

$$\gamma(\theta) = \frac{\theta(1-\delta)(1-\delta\rho)}{\theta[1-\rho\delta(1-\beta)](1-\delta) + \beta\delta(1-\rho)\mu}$$

The above is the consumption ratio out of total wealth. In addition, we can define the savings ratio $s(\theta) = 1 - \gamma(\theta)$, that would b:

$$s(\theta) = \frac{\beta\delta[(1-\rho)\mu + \rho\theta]}{\theta[1-\rho\delta(1-\beta)](1-\delta) + \beta\delta(1-\rho)\mu}$$

E.2 The Optimal Fiscal Rule

The introduction of a θ contingent threshold $\theta_p(\theta_0)$ limiting the maximum spending causes a change only in the constant $A(\theta)$.

The equation for $A(\theta)$ is the following:

$$A(\theta) = \theta\log(\gamma(\theta)) + \delta E[A(\theta')|\theta] + \delta E[B(\theta')\log((1+r)(1-\gamma(\theta')))|\theta]$$

Without the threshold, this constant would solve:

$$E[A(\theta)|\theta_0] = E[\theta\log(\gamma(\theta))|\theta_0] + \delta E[E[A(\theta')|\theta]|\theta_0] +$$

$$\delta E[E[B(\theta')\log(1-\gamma(\theta'))|\theta]|\theta_0] + \delta E[E[B(\theta')|\theta]|\theta_0]\log(1+r)$$

By using the law of iterated expectations to simplify the double expectations, we get:

$$E[A(\theta)|\theta_0](1-\delta) = E[\theta\log(\gamma(\theta))|\theta_0] + \delta E[B(\theta')\log(1-\gamma(\theta'))|\theta_0] + \delta\frac{E[\theta'|\theta_0]}{1-\delta}\log(1+r)$$

When we introduce a threshold $\theta_p(\theta_0)$, we have to take into account that tomorrow the agent may fall in the constrained zone. Hence, we differentiate two cases:

if $\theta \leq \theta_p$:

$$A^-(\theta) = \theta \log(\gamma(\theta)) + \delta E[A(\theta')|\theta] + E[B(\theta')\log(1+r)|\theta] +$$

$$\delta \int_{\underline{\theta}}^{\theta_p(\theta)} B(\theta') \log(1-\gamma(\theta'))dF(\theta'|\theta) + \delta \int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta') \log(1-\gamma(\theta_p(\theta)))dF(\theta'|\theta)$$

if $\theta \geq \theta_p$:

$$A^+(\theta) = \theta \log(\gamma(\theta_p(\theta_0))) + \delta E[A(\theta')|\theta] + E[B(\theta')\log(1+r)|\theta] +$$

$$\delta \int_{\underline{\theta}}^{\theta_p(\theta)} B(\theta') \log(1-\gamma(\theta'))dF(\theta'|\theta) + \delta \int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta') \log(1-\gamma(\theta_p(\theta)))dF(\theta'|\theta)$$

All other terms are not effected by the presence of a threshold. For example:

$$\int_{\underline{\theta}}^{\theta_p} A(\theta')dF(\theta'|\theta) + \int_{\theta_p}^{\bar{\theta}} A(\theta')dF(\theta'|\theta) = \int_{\underline{\theta}}^{\bar{\theta}} A(\theta')dF(\theta'|\theta) = E[A(\theta')|\theta]$$

Consequently, the function $A(\theta)$ solves the following equation, where we simplify the notation by calling the equations above as $A^-(\theta)$ and $A^+(\theta)$.

$$E[A(\theta)|\theta_0] = \int_{\underline{\theta}}^{\theta_p(\theta_0)} A^-(\theta)dF(\theta|\theta_0) + \int_{\theta_p(\theta_0)}^{\bar{\theta}} A^+(\theta)dF(\theta|\theta_0)$$

Up to know we have computed the conditional expectation of $A(\theta)$ when there is a threshold θ_p. By using the previous equations we can compute the ex ante expectation of $A(\theta)$. It can be easily seen that integrating and adding the different areas generates a similar pattern for the term involving $(1+r)$. As a result, the coefficient $A(\theta)$ solves:

$$E[A(\theta)|\theta_0](1-\delta) = \int_{\underline{\theta}}^{\theta_p} \theta \log(\gamma(\theta))dF(\theta|\theta_0) + \int_{\theta_p}^{\bar{\theta}} \theta \log(\gamma(\theta_p))dF(\theta|\theta_0)$$

$$\delta \int_{\underline{\theta}}^{\theta_p} \left[\int_{\underline{\theta}}^{\theta_p(\theta)} B(\theta') \log(1-\gamma(\theta'))dF(\theta'|\theta) + \int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta') \log(1-\gamma(\theta_p))dF(\theta'|\theta) \right] dF(\theta|\theta_0)$$

$$+\delta\int_{\theta_p}^{\bar{\theta}}\left[\int_{\underline{\theta}}^{\theta_p(\theta)} B(\theta')\log(1-\gamma(\theta'))dF(\theta'|\theta)+\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')\log(1-\gamma(\theta_p))dF(\theta'|\theta)\right]dF(\theta|\theta_0)$$

$$+\delta\frac{E[\theta'|\theta_0]}{1-\delta}\log(1+r)$$

This expression is particularly complex since an agent can be bound today, tomorrow, both or never. First, we regroup the terms:

$$E[A(\theta)|\theta_0](1-\delta)=\int_{\underline{\theta}}^{\theta_p(\theta_0)}\theta\log(\gamma(\theta))dF(\theta|\theta_0)+\int_{\theta_p(\theta_0)}^{\bar{\theta}}\theta\log(\gamma(\theta_p))dF(\theta|\theta_0)+\delta\frac{E[\theta'|\theta_0]}{1-\delta}\log(1+r)$$

$$\delta\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\theta}}^{\theta_p(\theta)} B(\theta')\log(1-\gamma(\theta'))dF(\theta'|\theta)dF(\theta|\theta_0)+\delta\int_{\underline{\theta}}^{\bar{\theta}}\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')\log(1-\gamma(\theta_p(\theta)))dF(\theta'|\theta)dF(\theta|\theta_0)$$

In order to optimize this functional equation, we use Gateaux derivatives. This technique consists in perturbing the function $\theta_p(\theta)$ in all directions $\psi(\theta)$ by a factor ϵ and evaluate it t $\epsilon = 0$.
By calculating the derivative and evaluating it at $\epsilon = 0$ we obtain:

$$\frac{\psi(\theta_0)\int_{\theta_p(\theta_0)}^{\bar{\theta}}\theta dF(\theta|\theta_0)}{\gamma(\theta_p)}-\delta\int_{\underline{\theta}}^{\bar{\theta}}\frac{\psi(\theta)\int_{\theta_p(\theta)}^{\bar{\theta}}B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))}dF(\theta|\theta_0)=0;\quad\forall\theta_0$$

Note that we have disregarded the derivatives with respect to the limits of integration because they cancel out.
The above equation is useful, but it still presents the problem of an arbitrary directional function ψ. In order to solve this problem, we use the fact that the foc must hold for all θ_0. Thus, we can integrate the last equation over all θ_0 by using its invariant distribution (which we assume exists.)
By calling the invariant $\tilde{f}(\theta)$ and integrating the above, we obtain:

$$\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\psi(\theta_0)\int_{\theta_p(\theta_0)}^{\bar{\theta}}\theta dF(\theta|\theta_0)}{\gamma(\theta_p)}-\delta\int_{\underline{\theta}}^{\bar{\theta}}\frac{\psi(\theta)\int_{\theta_p(\theta)}^{\bar{\theta}}B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))}dF(\theta|\theta_0)\right]d\tilde{F}(\theta_0)=0 \tag{23}$$

We, then, insert the integral operator and obtain:

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\psi(\theta)\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))} dF(\theta|\theta_0)d\tilde{F}(\theta_0) = E\left[E\left[\frac{\psi(\theta)\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))}|\theta_0\right]\right]$$

By the law of iterated expectations we know that $E[E[X|Y]] = E[X]$. Consequently, it can be written:

$$E\left[E\left[\psi(\theta)\frac{\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))}|\theta_0\right]\right] = E_\theta\left[\psi(\theta)\frac{\int_{\theta_p(\theta)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta)}{1-\gamma(\theta_p(\theta))}\right] = E\left[\psi(\theta_0)\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta_0)}{1-\gamma(\theta_p(\theta_0))}\right]$$

Using this last equation in equation (21) we can write:

$$\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\psi(\theta_0)\int_{\theta_p(\theta_0)}^{\bar{\theta}} \theta dF(\theta|\theta_0)}{\gamma(\theta_p)} - \frac{\delta\psi(\theta_0)\int_{\theta_p(\theta_0)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta_0)}{1-\gamma(\theta_p(\theta_0))}\right] d\tilde{F}(\theta_0) = 0$$

$$\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} \theta dF(\theta|\theta_0)}{\gamma(\theta_p)} - \delta\int_{\theta_p(\theta_0)}^{\bar{\theta}} \frac{B(\theta')}{1-\gamma(\theta_p(\theta_0))} dF(\theta'|\theta_0)\right] \psi(\theta_0)d\tilde{F}(\theta_0) = 0$$

Since $\tilde{f}(\theta_0) \geq 0$ and the above equation must be true for all directional variations with $\psi \geq 0$, by the Fundamental Lemma of calculus of variations, it must be true that the term inside the integral is identically zero for all θ_0, and thus the foc becomes:

$$\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} \theta dF(\theta|\theta_0)}{\gamma(\theta_p(\theta_0))} - \delta\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} B(\theta')dF(\theta'|\theta_0)}{1-\gamma(\theta_p(\theta_0))} = 0; \qquad \forall\theta_0$$

In order to characterize a sharp solution, We replaced γ and B above with the expressions found before.

$$\int_{\theta_p(\theta_0)}^{\bar{\theta}} \theta f(\theta|\theta_0)d\theta\frac{1-\gamma(\theta_p(\theta_0))}{\gamma(\theta_p(\theta_0))} = \delta\int_{\theta_p(\theta_0)}^{\bar{\theta}} B(\theta')f(\theta'|\theta_0)d\theta'$$

Note that:

$$\frac{1-\gamma(\theta_p(\theta_0))}{\gamma(\theta_p(\theta_0))} = \frac{\beta\delta E[B(\theta')|\theta_p(\theta_0)]}{\theta_p(\theta_0)}$$

Also, dividing both sides of the first equation by $1-F(\theta|\theta_0)$, we can write:

$$\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} \theta f(\theta|\theta_0)d\theta}{1-F(\theta|\theta_0)} = E\left[\theta|\theta \geq \theta_p(\theta_0)|\theta_0\right]$$

$$\frac{\int_{\theta_p(\theta_0)}^{\bar{\theta}} B(\theta')f(\theta'|\theta_0)d\theta'}{1-F(\theta|\theta_0)} = E\left[B(\theta')|\theta' \geq \theta_p(\theta_0)|\theta_0\right]$$

Putting all three in the first one we have:

$$E\left[\theta|\theta \geq \theta_p(\theta_0)|\theta_0\right] \frac{\beta\delta E[B(\theta')|\theta_p(\theta_0)]}{\theta_p(\theta_0)} = \delta E\left[B(\theta')|\theta' \geq \theta_p(\theta_0)|\theta_0\right].$$

It can be reorganized to deliver:

$$\theta_p(\theta_0) = \beta E\left[\theta|\theta \geq \theta_p(\theta_0)|\theta_0\right] \frac{E[B(\theta')|\theta_p(\theta_0)]}{E\left[B(\theta')|\theta' \geq \theta_p(\theta_0)|\theta_0\right]}$$

Note that in general $E[B(\theta')|\theta_p(\theta_0)] \neq E\left[B(\theta')|\theta' \geq \theta_p(\theta_0)|\theta_0\right]$. Thus, this term does not cancels out.

E.3 Implementation of the Optimal Fiscal Rule

First of all notice that, as stated in section 7.4, the use of a threshold in terms of growth allows to make the implementation independent from debt.

In fact, the spending is:

$$g(\theta, b) = \gamma(\theta)\left[\tau + h - b\right]$$

And recall that, when $(1+r)\delta = 1$, the spending growth can be written as:

$$\nu(\theta, \theta_{-1}, b, b_{-1}) = \frac{g(\theta, b)}{g(\theta_{-1}, b_{-1})} = \frac{\gamma(\theta)[\tau + h - b])}{\gamma(\theta_{-1})[\tau + h - b_{-1}]} = \gamma(\theta)\frac{\beta E[B(\theta)|\theta_{-1}]}{\theta_{-1}}$$

We implement the optimal rule in terms of limit in the spending growth. Hence, government's spending is constraint as follow 18:

$$\nu(\theta,\theta_{-1}) \leq \gamma(\theta_p(\theta_{-1}))\frac{\beta E[B(\theta)|\theta_{-1}]}{\theta_{-1}} = \bar{\nu}(\theta_{-1})$$

Although this solution solves the problem of fiscal policies' dependence on total wealth, $\bar{\nu}(\theta_{-1})$ is still contingent on θ_{-1}. Hence, we focus on finding an implementation able to capture this dependency in the empirical data.

Note that the previous growth is:

$$\nu(\theta_{-1},\theta_{-2}) = \gamma(\theta_{-1})\frac{\beta E[B(\theta_{-1})|\theta_{-2}]}{\theta_{-2}}$$

Integrating over θ_{-2} by using the unconditional distribution, we obtain 19:

$$\int \nu(\theta_{-1},\theta_{-2})dF^u(\theta_{-2}) = \gamma(\theta_{-1})\int \frac{\beta E[B(\theta_{-1})|\theta_{-2}]}{\theta_{-2}}dF^u(\theta_{-2}) = \hat{\nu}(\theta_{-1})$$

Where $\tilde{\nu}(\theta_{-1})$ is the unconditional distribution of the previous growth factors, which can be directly observed in the data. In fact, from the data we can observe the growth rate of a government spending but no the histories of θ that led it there. Notice that in this way we obtain a one-to-one mapping between government's previous growth rate $\tilde{\nu}$ and its spending need θ_{-1}.

Consequently we can implement an optimal fiscal rule where the growth of the current spending is proportional to the previous one.

$$\bar{\nu}(\hat{\nu}_{-1}) = \kappa(\hat{\nu}_{-1})\hat{\nu}_{-1}$$

Thus, the problem reduces to find the proportion $\kappa(\bar{\nu}_{-1})$. From equation (19) it is clear that there is a one to one mapping from $\bar{\nu}$ to θ_{-1}. Hence, replacing (18) and (19) in the last, $\kappa(\bar{\nu}_{-1})$ must satisfy:

$$\gamma(\theta_p(\theta_{-1}))\frac{\beta E[B(\theta)|\theta_{-1}]}{\theta_{-1}} = \kappa(\hat{\nu}_{-1})\gamma(\theta_{-1})\int \frac{\beta E[B(\theta_{-1})|\theta_{-2}]}{\theta_{-2}}dF^u(\theta_{-2}) \quad \Rightarrow$$

$$\kappa(\hat{\nu}_{-1}(\theta_{-1})) = \frac{\gamma(\theta_p(\theta_{-1}))\frac{E[B(\theta)|\theta_{-1}]}{\theta_{-1}}}{\gamma(\theta_{-1})\int \frac{E[B(\theta_{-1})|\theta_{-2}]}{\theta_{-2}}dF^u(\theta_{-2})}$$

Note that, since in the last equation the integration is over the unconditional distribution, it is the same as writing:

$$\kappa(\hat{\nu}_{-1}(\theta_{-1})) = \frac{\gamma(\theta_p(\theta_{-1}))\frac{E[B(\theta)|\theta_{-1}]}{\theta_{-1}}}{\gamma(\theta_{-1})\int \frac{E[B(\theta)|\theta_{-1}]}{\theta_{-1}} dF^u(\theta_{-1})} \tag{24}$$

F Appendix

Calibration

In this section we report the formulas and data used to estimate the four parameters of interest: $\delta, \beta, \sigma, \rho$.

We calibrate δ directly from the data. Since the planner is the Italian government, we set the discount rate equal to the interest rate that the Italian central government pays on its debt.

$$\delta = \frac{1}{1+r} \tag{25}$$

In order to obtain the other three parameters, we use the model-generated moments of the rules-free equilibrium. In particular, We rely on the moments of the spending ratios to compute the value of β and σ and on the variance of the spending growth to estimate ρ.
The effectiveness of this approach is due to multiple reasons. On one hand, since we are talking about a stationary distribution, we can assume that all municipalities follow the same distribution of θ. Hence, we can omit the indexes t and i from the the spending ratios in the data. On the other hand, $E(\theta) = 1$ and δ is directly calibrated from the data. Hence, by studying the theoretical expression of γ (9), it can be seen that the average of the spending ratio is determined by β and its variance by σ.
Consequently, the hyperbolic discounting β is obtained by matching the first moment of the spending ratios (equation 26) and the variance of $\theta\sigma$ is determined by using the second moment of the spending ratios (equation 27).
Lastly, since there is some degree of persistence, we need to estimate ρ. In order to do so, we use the variance of the spending growth factor (equation 28). In fact, the presence of persistency generates a difference between the distibution of the spending ratios and spending growth. Indeed, in the absence of persistency, i.e $\rho = 0$, the spending growth would be the result of a random shock. In order to do so, we use the variance of the spending growth factor (equation 28).In this case, however, the spending growth reflects the conditional distribution. For this reason, first, we integrate over

the conditional distribution and then over the unconditional distribution in order to match the data.

$$E[\gamma(\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} \gamma(\theta) f^u(\theta) d\theta = \frac{1}{N} \sum \hat{\gamma}(\theta) \tag{26}$$

$$E[\gamma(\theta)^2] = \int_{\underline{\theta}}^{\bar{\theta}} \gamma(\theta)^2 f^u(\theta) d\theta = \frac{1}{N} \sum \hat{\gamma}(\theta)^2 \tag{27}$$

$$\begin{aligned} E[\nu(\theta)^2] - E[\nu(\theta)]^2 & \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\bar{\theta}} \gamma(\theta)^2 f^c(\theta|\theta_{-1}) d\theta \right] \left(\frac{\beta E[B(\theta)|\theta_{-1}]}{\theta_{-1}}\right)^2 f^u(\theta_{-1}) d\theta_{-1} \\ &= \frac{1}{N} \sum \hat{\nu^2} - \frac{1}{N} \sum (\hat{\nu})^2 \end{aligned} \tag{28}$$

Table F1 includes all the details on data sources and measurement.

Table F1: Description and source of the variables used for the calibration

Variable	Definition and measure	Source
Interest rate (r)	10-year Italian bond's interest rate averaged over 2001-2004	FED
Total wealth $(h + \tau - b)$	(1+r) $\frac{Total\ Revenues}{r}$	IMI
Spending growth factor $(\hat{\nu})$	$\frac{Total\ Spending_t}{Total\ Spending_{t-1}}$	IMI

NOTES- *IMI* stands for Italian Ministry of the Interior.

Lastly, we verify that theoretical and empirical distributions match not only in their first and second moments but also in their whole distribution. The figures below show the unconditional distribution of governments' spending ratios (γ) and spending growth (ν) derived from the model (figure F1) and from the data (figure F2). By looking at the graphs it results clear that the matching is verified.

Figure F1: Unconditional theoretical distribution of γ and ν

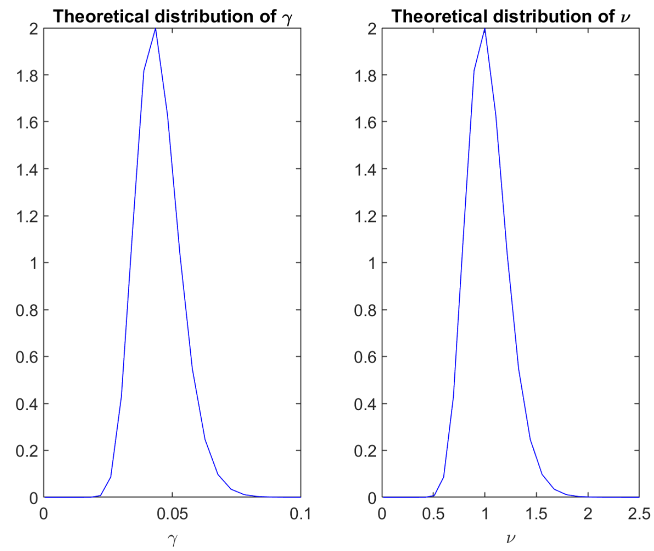

Figure F2: Unconditional empirical distribution of γ and ν