Monetary Non-Neutrality in the Cross-Section

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Abstract

I model the heterogeneous effect of monetary policy on employment and consumption across households who participate in segmented labor markets, earning and spending their income in different industries. Monetary policy – which is an aggregate demand shifter – has a larger effect on the employment of households with flatter labor supply curves, that is, of households who have stickier wages, work in sticky-price or labor-intensive industries, or have large expenditure shares on these industries. In the aggregate, the ability of producers and consumers to shift expenditure towards primary factors with flatter supply curves increases the real effects of monetary policy. Calibrating the model to the US economy reveals significant heterogeneity in the cumulatice response of employment to monetary policy across occupations, varying from 2.5% (farming) to 4% (entertainment) for a 1% nominal interest rate cut. Ignoring the presence of semi-fixed assets, heterogeneous price adjustment frequencies, and input-output linkages would significantly reduce this cross-sectional range.

1 Introduction

Monetary policy has the mandate to stabilize aggregate inflation and aggregate employment. But, as a side effect of its action on aggregates, does monetary policy also have a differential impact on the labor income and cost of living of different households? At present, we have no established theory and little empirical evidence of how monetary policy affects different occupations, geographic regions, or demographic groups, depending on which industries they earn income and purchase goods from. In this paper I study the aggregate and cross-sectional effects of monetary policy in an economy with multiple heterogeneous workers and industries, where industries hire workers, capital, and intermediate inputs in different proportions, while households participate in segmented labor markets and purchase different bundles of goods. I show that a monetary expansion reallocates demand towards price-rigid and

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supply-elastic goods, because their price increases by less. This in turn bids up the labor demand of households who make these goods, and the real income of those who buy them.

Compared with the HANK literature (Kaplan et al. (2018), Auclert (2019)) – which focuses on the households' consumption-saving problem while modeling cross-sectional differences in labor income as exogenous to monetary policy – I highlight that monetary policy endogenously affects labor income and consumption prices across households regardless of their consumption-saving patterns. My framework is essentially a currency union model, where heterogeneous households are subject to the same central bank. Compared to the currency union literature, where households face heterogeneous shocks but respond in the same way to monetary policy (Gali and Monacelli (2008)), I emphasize that the employment and income of different households respond differently to monetary policy when the households' labor supply curves have different slopes (Benigno (2004) makes a similar point in a simpler setting).

In Section 2 I stary by illustrating the key economic mechanism through a simple two-household model. Specifically, I highlight how the usual Phillips curve intuition from aggregate models carries over to a cross-section of households. In aggregate models, the effect of monetary policy on prices and output is determined by the slope of the Phillips curve (that is an aggregate supply curve). Whenever the Phillips curve is flatter, monetary policy has a larger effect on output and a smaller effect on prices. The same economic mechanism is at work in the cross-section. Monetary policy is an aggregate demand shifter, therefore it has a larger effect on employment, and a smaller effect on wages, for households with flatter labor supply curves. Like the aggregate Phillips curve, the slope of household-specific supply curves depends on nominal rigidities and factor supply elasticities in a way that I make precise.

I then introduce the full framework in Section 3, which features many heterogeneous industries and households. Households have different types, meaning that they participate in segmented labor markets and differ in their consumption preferences, ownership of industries and capital assets, wage rigidity, and labor supply elasticity. Industries hire different bundles of labor types, capital assets, and intermediate inputs, and they face different nominal rigidities and demand elasticities. Workers are allowed to move freely across industries, but they cannot change the type of labor they are endowed with. Labor types thus represent segmented labor markets, which could correspond to different occupations, or industry-occupation pairs, demographic groups, residents of different regions, etc. Capital assets in the model represent any fixed or semi-fixed factors of production (such as equipment, land, structures, but also entrepreneurial know-how, etc.).

I summarize the log-linearized equilibrium of the economy into four equations: factor supply, relative factor demand, aggregate demand, and changes in the households' net asset positions. Factor supply is described by a set of sector-by-factor Phillips curves, which relate inflation in the various industries with the employment gaps of all primary factors. In turn, the relative demand for primary factors depends on the aggregate output gap, on relative prices, and on changes in asset positions across households who consume different bundles of goods. Finally, the aggregate output gap and the households' net asset positions are then pinned down by the households' consumption-saving decision given interest rates (set by the central bank).

In Section 4 I combine the Phillips curves with the relative demand equation, to derive the equilibrium response of employment and consumption to monetary policy across households. On impact a monetary expansion increases the demand for all goods and factors proportionally, because final users increase spending proportionally on all goods.[1] In general equilibrium, however, this uniform impact effect propagates in asymmetric ways. Changes in aggregate demand increase the relative price of goods and factors with steeper Phillips curves, which in turn affects the relative demand for primary factors and the relative income of households due to expenditure switching and income reallocation. Changes in relative demand then feed back into relative prices, and so on. The equilibrium fixed point is given by a cross-sectional multiplier, which depends on the product of Phillips curve slopes and relative demand elasticities.

Like in the representative agent benchmark (Gali (2015); Woodford (2003)), the Phillips curve slopes depend on sectoral nominal rigidities and factor supply elasticities. A primary factor has flatter Phillips curves when it is supplied more elastically, or it is complementary with elastically supplied factors, etc., or when it has a stickier price, it is employed by sectors with stickier prices, or whose customers have stickier prices, etc. Heterogeneous nominal rigidities or supply elasticities (in a network-adjusted sense) are necessary for monetary policy to affect relative prices. As the relative price of flat-Phillips-curve goods falls after a monetary expansion, producers and consumers shift expenditure towards them – as governed by the relevant demand elasticities – thereby boosting the employment of the primary factors used more intensively in the production of these goods.

By contrast, monetary policy has no redistributive effects if all primary factors face the same (network-adjusted) supply elasticities and nominal rigidities. By themselves, home bias in consumption, heterogeneous industry size and centrality, or heterogeneous demand elasticities do not imply any redistributive effects of monetary policy.

While the Phillips curve slopes are sufficient to determine the direction of cross-sectional employment responses, the direction of income responses is ambiguous, as it depends on the response of both prices and quantities – and we just established that the two are negatively correlated in the cross-section. The employment effect dominates if and only if primary factors are aggregate substitutes.

These cross-sectional effects of monetary policy also have aggregate implications. Proposition 4 shows that, like in models with a single primary factor, the aggregate monetary non-neutrality depends on the slope of the Phillips curve for the GDP deflator. With multiple heterogeneous primary factors, the GDP deflator is less sensitive to the employment gaps of primary factors with flatter Phillips curves. Moreover, a monetary expansion reallocates employment precisely towards these factors, as we just discussed. Therefore, through a composition effect, the aggregate Phillips curve is flatter compared to an economy with a single factor of production. In turn, a flatter aggregate Phillips curve means more aggregate monetary non-neutrality.

Nonetheless, the difference between aggregate non-neutrality in the representative agent approximation vs the

[1] To focus on propagation through the production side, I assume homothetic preferences and production, uniform MPCs, and no portfolio heterogeneity.

heterogeneous agents economy is small, because it is a quadratic function of the cross-sectional dispersion in price rigidities and factor supply elasticities relative to the average. By contrast, the cross-sectional effects are much larger as they increase linearly with the underlying parameter heterogeneity.

Can monetary policy be paired with appropriate fiscal policies in order to eliminate its cross-sectional effects? I show that this is possible when there is no heterogeneity in the supply elasticity of primary factors, using an appropriate combination of production and consumption taxes. First, zero-sum production subsidies must be set to replicate the change in relative marginal costs across industries that would result from a proportional increase in the wages of all primary factors. Second, zero-sum lump-sum taxes must be set so that the agents' incomes move proportionately. Under this tax and transfer scheme monetary policy has no cross-sectional effects, and the evolution of all aggregate variables is the same as in the representative-agent equilibrium. However this is a complex policy scheme, which requires knowledge of the full input-output structure and of the agents' ownership share in the various industries.

A key implication of the theory is that there is a negative cross-sectional correlation between the response of prices and employment to monetary policy. This is in line with the empirical evidence that I present in Section 5.2. I start with some stylized facts about household heterogeneity, where households are classified according to the occupation of the primary earner. Merging employment data by industry and occupation from the American Community Survey (ACS) with industry-level measures of price adjustment frequencies from Pasten et al. (2019), I find that different occupations are employed by industries with widely different degrees of price stickiness and capital intensity. By contrast, using data from the Consumer Expenditure Survey (CEX), I find little heterogeneity in the price stickiness or capital intensity of consumption baskets. I then revisit the traditional aggregate business cycle facts (Cooley and Prescott (1995)) for a cross-section of occupations. Using time series of employment and wages from the Current Population Survey (CPS), and industry-level prices from the Bureau of Labor Statistics (BLS), I find that occupations with more cyclical employment have less cyclical wages, and the prices of the goods that they produce are also less cyclical.

Finally Section 6 calibrates the model to the US economy, finding large redistributive effects of monetary policy in the cross-section. I measure household-level non-neutrality based on the impulse responses of employment to a 1% monetary shock. In the baseline model the cumulative employment response varies from 2.5% to 4% across occupations, and the impact responses exhibit a similar dispersion. Different nominal rigidities in the sectors that hire different occupations are an important driver of cross-sectional effects, which are also amplified by input-output linkages. Heterogeneous complementarity with semi-fixed assets is another important driver of cross-sectional heterogeneity.[2]

[2]Like in representative agent models, the presence of input output linkages also flattens the Phillips curve, thereby increasing aggregate non-neutrality.

1.1 Related Literature

Adopting a similar input-output framework, but with flexible prices, Baqaee and Farhi (2018) study the propagation and aggregation of exogenous productivity and markup shocks in economies with multiple primary factors and a general input-output network. Different from Baqaee and Farhi (2018), in my framework markups change endogenously due to sticky prices, and I provide an explicit solution for changes in relative employment and income across households (while Baqaee and Farhi (2018) characterize them only implicitly).

In a related paper, La'O and Morrison (2023) study optimal monetary policy when earnings inequality is (exogenously) correlated with aggregate demand. In this paper I show that monetary policy endogenously affects earnings inequality, but I do not solve for the optimal policy.

The HANK literature (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2021)) also considers New Keynesian frameworks with heterogeneous households, focusing on the effects of monetary policy across households through their consumption-saving decisions. In this paper instead I focus on the endogenous redistributive effects of monetary policy on the households' labor demand and consumption prices.

A large literature studies the implications of input-output networks for aggregate monetary non-neutrality in models with only one factor of production (Basu (1995), Carvalho (2006), Nakamura and Steinsson (2010), Pasten et al. (2019), LaO and Tahbaz-Salehi (2019), Rubbo (2023)), while multi-country New Keynesian models (Aoki (2001); Benigno (2004), Devereux and Engel (2003), Huang and Liu (2007), Gali and Monacelli (2008)) allow for many primary factors – the workers in each country – but adopt simpler Armington-style production structures, often with symmetric nominal rigidities and labor supply elasticities. This paper jointly accounts for multiple primary factors of production and a quantitatively realistic input-output network, proposing a novel characterization of cross-sectional monetary non-neutrality which uses industry-by-primary factor Phillips curve slopes as sufficient statistics.

Based on detailed micro-level data from Denmark and Sweden, Huber et al. (2020) and Coglianese et al. (2021) empirically document heterogeneous effects of monetary policy when households derive income and consumption from different industries. Minton and Wheaton (2022) also document heterogeneous effects of monetary policy across states with different downward wage rigidity, as proxied by more or less binding minimum wages. These empirical studies confirm the theoretical predictions of this paper.

2 A simple model of cross-sectional non-neutrality

In this section I illustrate how heterogeneous Phillips curves determine the response of wages and employment to monetary policy across households in a simple model. The same economic forces determine heterogeneous effects of monetary policy in the general framework, introduced in Section 3. Examples **??**, **??** and **??** below further illustrate how other dimesions of heterogeneity, such as heterogeneous exposure of households to industries through consumption and ownership, shape aggregate demand elasticities and cross-sectional consumption responses.

I start by defining worker-specific Phillips curves, and relating them with the cross-sectional response of wages and employment to monetary policy. I then show that heterogeneous Phillips curves also make aggregate employment more responsive to monetary policy. Finally, I consider which primitive parameters determine the slope of worker-specific Phillips curves.

Environment Consider an economy with two households (indexed by h, k) and one final good. The two households supply differentiated types of labor, and labor markets are fully segmented. For simplicity, in this example I assume that households and firms fully discount the future and there is no trade in financial assets (I relax both assumptions in the general model). The economy starts out in a steady-state where the households have equal income shares and zero asset positions. An unanticipated monetary shock changes the aggregate demand for labor.

For now I do not fully specify preferences and wage setting, but simply assume that the required wages w_h of the two households depend on their own labor supply ℓ_h and on aggregate wages, with household-specific elasticities κ_h^ℓ and κ_h^p:

$$w_h = \kappa_h^\ell \ell_h + \kappa_h^p \bar{w} \tag{1}$$

where I denote by $\bar{x} \equiv \frac{1}{2}(x_1 + x_2)$ for any variable x and lower case letters denote log-changes relative to the initial steady-state.

The final good is produced using the labor of both households. For now I do not fully specify the production function but simply assume that the relative demand for the two workers depends on their relative wages, with elasticity θ:

$$\ell_h - \ell_k = -\theta (w_h - w_k) \tag{2}$$

Phillips curves and cross-sectional non-neutrality Equation (1) is a wage Phillips curve, tracing out a supply relation, while equation (2) traces out a demand curve. Combining the two allows us to solve for the households' equilibrium employment relative to the aggregate:

$$\ell_h - \bar{\ell} = -\frac{1}{2} \frac{\theta \bar{\kappa}^\ell}{1 + \theta \bar{\kappa}^\ell \left(1 + \frac{1}{4} \frac{\kappa_h^\ell - \kappa_k^\ell}{\bar{\kappa}^\ell} \frac{\kappa_h^p - \kappa_k^p}{1 - \bar{\kappa}^p}\right)} \left(\frac{\kappa_h^\ell - \kappa_k^\ell}{\bar{\kappa}^\ell} + \frac{\kappa_h^p - \kappa_k^p}{1 - \bar{\kappa}^p}\right) \bar{\ell} \tag{3}$$

Equation (3) shows that, when aggregate employment increases, relative employment declines for the household with a steeper Phillips curve, whose wages respond more to its own employment ($\kappa_h^\ell > \kappa_k^\ell$) or to aggregate wages ($\kappa_h^p > \kappa_k^p$). This is intuitive: the two households are subject to the same aggregate demand shock ($\bar{\ell}$), but one of them has a steeper labor supply curve. Hence for this household the same shift in demand moves wages by more, and employment by less.

In a similar way we can solve for cross-sectional consumption. The households' budget constraints imply

$$c_h - \bar{c} = w_h - \bar{w} + \ell_h - \bar{\ell} = -\frac{1}{2}\frac{(\theta-1)\,\bar{\kappa}^{\ell}}{1+\theta\bar{\kappa}^{\ell}\left(1+\frac{1}{4}\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}\frac{\kappa_h^{p}-\kappa_k^{p}}{1-\bar{\kappa}^{p}}\right)}\left(\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}+\frac{\kappa_h^{p}-\kappa_k^{p}}{1-\bar{\kappa}^{p}}\right)\bar{\ell} \tag{4}$$

Equation (4) shows that cross-sectional consumption and employment have the same sign if and only if workers are substitutes ($\theta - 1 > 0$). This is intuitive: as we explained before, employment and wages move in opposite ways due to expenditure switching. As usual, the employment effect dominates if and only if workers are substitutes.

Cross-sectional heterogeneity and aggregate non-neutrality I now study the response of aggregate employment to a 1% change in nominal GDP, given by

$$d\log nGDP = \bar{w} + \bar{\ell}$$

We can use equation (1) to solve for aggregate wages in terms of aggregate and cross-sectional employment:

$$\bar{w} = \frac{\bar{\kappa}^{\ell}}{1-\bar{\kappa}^{p}}\left[\bar{\ell}+\frac{1}{4}\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}\left(\ell_h-\ell_k\right)\right] = \frac{\bar{\kappa}^{\ell}}{1-\bar{\kappa}^{p}}\left[\bar{\ell}+Cov\left(\frac{\kappa^{\ell}}{\bar{\kappa}^{\ell}},\ell\right)\right]$$

Combining with the solution for cross-sectional employment in (2) yields

$$\bar{\ell} = \frac{d\log nGDP}{1+\frac{\bar{\kappa}^{\ell}}{1-\bar{\kappa}^{p}}\left[1-\frac{1}{4}\frac{\theta\bar{\kappa}^{\ell}}{1+\theta\bar{\kappa}^{\ell}\left(1+\frac{1}{4}\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}\right)}\left(\left(\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}\right)^2+\frac{\kappa_h^{p}-\kappa_k^{p}}{1-\bar{\kappa}^{p}}\frac{\kappa_h^{\ell}-\kappa_k^{\ell}}{\bar{\kappa}^{\ell}}\right)\right]} \tag{5}$$

Equation (5) shows that aggregate employment is more sensitive to changes in nominal demand when the Phillips curve slopes are heterogeneous, and this effect is stronger when the two labor types are better substitutes (i.e. when θ is large). Intuitively, if $\theta > 0$ final good producers substitute towards the cheaper labor type, thereby reallocating expenditure towards the factor of production with a flatter supply curve. Expenditure switching flattens the aggregate supply curve through a composition effect, so that the economy responds more to changes in aggregate demand induced by monetary policy.

Nominal rigidities, labor supply elasticities, and the slope of the Phillips curve To wrap up the analysis of this simple model, I relate the Phillips curve parameters κ_h^{ℓ} and κ_h^{p} with the primitives of a standard New Keynesian model.

For simplicity I assume that each family h has GHH preferences over consumption (C) and labor (L), described by the utility function

$$\mathcal{U}_h\left(C_h, L_h\right) \equiv C_h - \frac{L^{1+\varphi_h}}{1+\varphi_h} \tag{6}$$

where households can have different Frisch elasticities of labor supply (given by $\frac{1}{\varphi_h}$). The final good is produced using the labor of both households, with equal expenditure shares and elasticity of substitution θ:

$$Y = \frac{1}{2}\left(L_h^{\frac{\theta-1}{\theta}} + L_k^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{7}$$

I consider two different specifications for nominal rigidities, to isolate the role of heterogeneity in the households' exposure through the employment versus the consumption channel.

To study the employment channel I assume that there is a continuum of final good varieities, of whom only a fraction δ can set prices after observing changes in wages, and households purchase a bundle of these varieties. The final good price changes by

$$p^Y = \delta \bar{w}$$

Moreover each household is comprised of a continuum of workers, of whom only a fraction δ_h can set their nominal wage after observing labor demand and consumption prices. Final good producers purchase a CES aggregator of labor services across household members.[3]

The utility function (6) implies the consumption-leisure tradeoff

$$w_{hi} - p^Y = \varphi_h \ell_{hi}$$

for the workers i in household h who can update their wages, and $w_{hi} = 0$ for the others. On average across the members of household h we have

$$w_h = \delta_h \varphi_h \ell_h + \delta_h p^Y = \delta_h \varphi_h \ell_h + \delta \delta_h \bar{w}$$

so that $\kappa_h^\ell = \delta_h \varphi_h$ and $\kappa_h^p = \delta \delta_h$. Hence households with more elastic labor supply or stickier wages have a flatter Phillips curve.

To study the implications of heterogeneous nominal rigidities in consumption, instead, I assume that households have the same wage adjustment probability δ_W but have access to different final good retailers. Only a fraction δ_h^R of the retailers that household h buys from can update their price after observing nominal GDP. Retailers rebate their profits to the households proportionately to their purchasing shares. In this setting the consumption-leisure tradeoff implies

$$w_h = \delta_W \varphi_h \ell_h + \delta_W \delta_h^R \bar{w}$$

so that $\kappa_h^\ell = \delta_W \varphi_h$ and $\kappa_h^p = \delta_W \delta_h^R$. Hence households with more elastic labor supply or stickier consumption prices have a flatter Phillips curve.

Overall, the elasticities κ_h^ℓ of wages with respect to employment depend on the households' labor supply elas-

[3]See Section 3.2 for a fully-specified model of price and wage setting.

ticities and wage adjustment probabilities, whereas the elasticities κ_h^p with respect to aggregate wages depend on the households' wage adjustment probabilities and on the price flexibility of their consumption bundles.

3 General environment

This section sets up a New Keynesian model with monopolistic competition and sticky prices, featuring multiple heterogeneous households, industries and primary factors. Sections 3.1 through 3.3 lay out the assumptions about preferences, production and policy instruments and derive optimality conditions for consumers and producers; section 3.4 defines the general equilibrium; and sections 3.5 and 3.6 describe the log-linearized model.

3.1 Final users

The final users are households, investment providers, and the government. There are N_w household types and N_f capital asset types, each with a corresponding set of investment providers. Final users are indexed by h or $f \in \{1, ..., N_w + N_f + 1\}$. The set of households is denoted by $\mathcal{N}_w$, and the set of capital types is denoted by $\mathcal{N}_f$. The price of factor h is W_h, and the quantity supplied is L_h.

Remark 1. Different types of labor and capital need not be employed by different industries. Rather, they correspond to segmented factor markets. For example, worker types could represent regions or occupations between which there are no worker flows at business cycle frequencies. Likewise, capital assets could represent equipment and structures which cannot be repurposed in the short run.

3.1.1 Households

Preferences Each household type $h \in \mathcal{N}_w$ has a representative agent, who supplies a distinct labor type and whose per-period preferences are described by the utility function

$$U_h = \sum_t \rho_h^t \left[\frac{C_{ht}^{1-\gamma_h}}{1-\gamma_h} - \frac{L_{ht}^{1+\varphi_h}}{1+\varphi_h} \right] \tag{8}$$

where ρ_h is the discount factor of household h. All households enjoy consumption (C) and dislike labor (L), with heterogeneous income effects in labor supply γ_h and Frisch elasticities $\frac{1}{\varphi_h}$. Consumption aggregators $C_{ht} \equiv \mathcal{C}_{ht}(c_{1ht}, ..., c_{Nht})$ are homothetic over the N goods produced in the economy, and can differ across households. Consumption preferences $\mathcal{C}_{ht}$ are time-varying, to account for changes in expenditure shares across goods at constant prices.

Budget constraints Households maximize the present discounted value of per-period utility flows, with a common discount factor ρ, subject to type-specific budget constraints

$$P_{ht}^{C}C_{ht} \leq W_{ht}L_{ht} + (1+I_t)\,B_{ht-1} - B_{ht} + \sum_{a\in\mathcal{A}}\left[D_{a,t}Q_{ha,t-1} + P_{a,t}\left(Q_{ha,t-1} - Q_{ha,t}\right)\right] + \tilde{T}_{ht} \tag{9}$$

where P_{ht}^{C} is the price index implied by the consumption aggregator $\mathcal{C}_h$, W_{ht} is the nominal wage earned by workers of type h, I_t is the nominal interest rate between $t-1$ and t, and $\tilde{T}_{ht}$ is an exogenous lump-sum transfer. Households can trade a set $\mathcal{A}$ of assets, where each asset $a \in \mathcal{A}$ has dividend D_{at} and price P_{at} at time t, and Q_{at} is the quantity of asset a held by household h. Households can also trade nominal bonds B_t.

The results below hold for any set of available assets and subject to any borrowing constraints. The set of available assets includes firms and non-labor primary factors, which have dividends $d_{it} \equiv \Pi_{it} - \mathcal{T}_{it}$ and $d_{ft} \equiv W_f L_f$ respectively, where Π_{it} are sector i profits net of lump-sum taxes $\mathcal{T}_{it}$ paid by firms to the government. Each household type h owns shares in all the industries $i \in \mathcal{N}$ in the economy, as described by the ownership matrix $\hat{\Xi}$ whose elements $\hat{\Xi}_{ih}$ denote the share of profits from industry i accruing to type-h agents. Likewise, the matrix $\left(\hat{\Xi}_{hf}^{L}\right)$ denotes the shares of income from each asset f that is rebated to each household type h.

I denote by

$$T_{ht} = (1+I_t)\,B_{ht-1} - B_{ht} + \sum_{a\in\mathcal{A}} P_{a,t}\left(Q_{ha,t-1} - Q_{ha,t}\right) + \sum_{a\in\mathcal{A}\setminus\mathcal{N}_f\cup\mathcal{N}} D_{a,t}Q_{ha,t-1} + \tilde{T}_{ht}$$

so that the households' budget constraint becomes

$$P_{ht}^{C}C_{ht} \leq \sum_{f\in\mathcal{N}_w\cup\mathcal{N}_f} \hat{\Xi}_{hf}^{L} W_{ht}L_{ht} + \sum_{i\in\mathcal{N}} \hat{\Xi}_{ih}\left(\Pi_{it} - \mathcal{T}_{it}\right) + T_{ht}$$

Consumption-leisure tradeoff At each time t, the optimal consumption-leisure tradeoff satisfies the first order condition

$$C_{ht}^{\gamma_h} L_{ht}^{\varphi_h} = \frac{W_{ht}}{P_{ht}^{C}} \tag{10}$$

The flexible nominal wage W_{ht} is defined as the value W_{ht} which satisfies the consumption-leisure tradeoff (10) given consumption, labor demand and prices. Importantly, W_{ht} has no data counterpart. The model counterpart of wages in the data is the sticky nominal wage paid by the firms, introduced in Section 3.2 below.

Consumption-saving decision I do not specify the households' Euler equations, steady-state asset holdings, and borrowing constraints explicitly. Rather, following Auclert et al. (2018), I take as sufficient statistics the derivatives of consumption with respect to current and future income and real interest rates that are implied by the households' consumption-saving problem.

Definition 1. The *intertemporal marginal propensities to consume* $\mathcal{J}^{h,Y}_{t\tau}$ are equal to the fraction of income changes at time τ that households of type h spend at time t

$$\mathcal{J}^{h,Y}_{t,\tau} = \frac{d\log C_{ht}}{d\log\left(\frac{W_{h\tau}L_{h\tau}+\sum_{a\in\mathcal{A}}D_{a\tau}+\tilde{T}_{h\tau}}{P^C_\tau}\right)}$$

The derivatives of household consumption with respect to real interest rates $\{r_\tau\}_{\tau=0}^{\infty}$ at constant real income are denoted by

$$\mathcal{J}^{h,r}_{t,\tau} = \frac{\partial\log C_{ht}}{\frac{\partial r_\tau}{1+r_\tau}}$$

The intertemporal mnarginal propensities to consume must satisfy

$$\sum_{t=0}^{\infty}\mathcal{J}^{h,Y}_{t,\tau} = 1$$

for all households h and times τ.

3.1.2 Capital utilization

I adopt a stylized model of investment, with the objective to deliver capital supply curves with constant elasticity φ_f. Importantly, to simplify the analysis I abstract from the intertemporal dimension of the investment problem, which would determine a different sensitivity of investment and consumption demand to monetary policy. I assume that investment fully depreciates within one period, so that changes in interest rates have no direct effect on the relative price of consumption and investment goods.

Each capital asset f is produced by combining a fixed endowment ($\bar{K}_f$) with an investment good I_f. At each period, the supply of asset f is given by (omitting time subscripts for legibility)

$$L_f = \left[(1+\varphi_f)\,I_f\right]^{\frac{1}{1+\varphi_f}}\,\mathcal{E}_f\bar{K}_f$$

where $\mathcal{E}_f$ is a shock to the supply of capital. For convenience I assume that the investment component I_f fully depreciates from one period to the next, while the endowment component $\bar{K}_f$ never depreciates.

In turn investment is produced with constant returns to scale, using as inputs a combination of primary factors (L_{fht}) and intermediate goods (X_{fit}), according to the production function

$$I_{ft} = G_{ft}(\{L_{fht}\},\{X_{fit}\})$$

The production function G_{ft} is time-varying, to allow for changes in expenditure shares at constant prices.

There are N_f investment producing sectors, one for each asset type f, who sell the investment good at marginal cost P^I_f to capital retailers. Retailers purchase capital endowments from the agents, combine them with the investment good, and sell capital services to the firms at a rental rate W_f in a perfectly competitive market. Capital retailers are owned by the agents in proportion to their ownership shares $\mathcal{Z}$ in the capital endowments, and rebate their profits accordingly.

Profit maximization yields the capital supply curves

$$U_f^{\varphi_f} = \frac{W_f}{P_f}\bar{K}_f \tag{11}$$

where

$$U_f \equiv \left[\left(1+\varphi_f\right) I_f\right]^{\frac{1}{1+\varphi_f}}$$

can be interpreted as a measure of capital utilization. As a result, the payments $W_f L_f$ to factor f are divided between investment expenditures, given by

$$\frac{1}{1+\varphi_f} W_f L_f \tag{12}$$

and profits of the investment producers, given by

$$\frac{\varphi_f}{1+\varphi_f} W_f L_f \tag{13}$$

Profits in turn are rebated to households according to their ownership shares of investment firms, denoted by $\mathcal{Z}$. Hence the share of income from factor $f \in \mathcal{N}_f$ rebated to households of type $h \in \mathcal{N}_w$ is given by $\mathcal{W}_{fh} = \mathcal{Z}_{fh}\frac{\varphi_f}{1+\varphi_f}$, while the share of income from factor f that goes to investment into factor $f' \in \mathcal{N}_f$ is $\mathcal{W}_{ff'} = \frac{1}{1+\varphi_f}$ if $f = f'$ and $\mathcal{W}_{ff'} = 0$ otherwise.

Like flexible wages in the consumption-leisure tradeoff (10), flexible rental rates are defined as the value W_{ft} that satisfies the utilization equation (10) given capital demand and prices. These flexible rental rates have no data counterpart. The model counterpart of the rental rates measured in the data is the sticky rental rate paid by the firms, introduced in Section 3.2 below.

I treat investment producing sectors $f \in \mathcal{N}_f$ as final users, with wealth effects $\gamma_f = 0$ and income transfers $T_f \equiv 0$.

3.2 Production

There are N good-producing industries in the economy (indexed by $i, j \in \{1, ..., N\}$). Within each industry there is a continuum of firms, producing differentiated varieties.

All firms z in industry i have the same constant returns to scale production function

$$Y_{izt} = G_i(\{L_{ihzt}\}_{h\in\mathcal{N}_h\cup\mathcal{N}_f}, \{X_{ijzt}\}_{j=1}^{N})$$

where L_{ihzt} is the quantity of primary factor h hired by firm z in industry i at time t, and X_{ijzt} is the quantity of intermediate input j used by the firm.

Customers (consumers and other producers) buy a CES bundle of sectoral varieties, with elasticity of substitution ϵ_i. The industry output is given by

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

and the implied sectoral price index is

$$P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

The government provides time-varying proportional input subsidies, fully financed through lump-sum taxes on profits. Following a standard practice in the literature, I assume that steady-state subsidies are set to eliminate the markup distortions arising from monopolistic competition.

All producers minimize costs given input prices. With constant returns to scale marginal costs are the same for all firms within a sector i, and they all use inputs in the same proportions. The marginal cost of sector i, denoted by MC_i, is the solution of the cost minimization problem (omitting time subscripts for legibility)

$$MC_i = min_{\{X_{ij}\},\{L_{ih}\}} \sum_{h \in \mathcal{N}_h \cup \mathcal{N}_f} W_h L_{ih} + \sum_{j=1}^{N} P_j X_{ij} \quad s.t. \ G_i \left(\{L_{ih}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ij}\}_{j=1}^{N} \right) = 1 \tag{14}$$

Producers are subject to nominal rigidities, modeled à la Calvo. In every sector i and at each period, only a randomly-selected fraction δ_i of the firms can update their price. They set it to maximize the present discounted value of future profits, conditional on being unable to update their price:

$$P_{it}^* = \frac{\epsilon_{it}}{\epsilon_{it} - 1} \frac{\mathbb{E} \sum_s \left[\frac{1}{1+R_{t,s}} (1-\delta_i)^s Y_{izt+s}(P_{it}^*) (1 - \tau_{it+s}) MC_{it+s} \right]}{\mathbb{E} \sum_s \left[\frac{1}{1+R_{t,s}} (1-\delta_i)^s Y_{izt+s}(P_{it}^*) \right]} \tag{15}$$

where the demand functions are given by $Y_{izt+s}(P_{zt}) = Y_{it+s} \left(\frac{P_{zt}}{P_{it+s}} \right)^{-\epsilon_{it}}$. The firms z who cannot adjust their price instead keep it unchanged ($P_{izt} = P_{izt-1}$), and must absorb any cost changes in their markup μ_{izt}.

Sticky factor prices To model sticky factor prices I assume that primary factors (workers and capital assets) are first purchased by marketplaces, who then sell their services to producers in all the different sectors. Each marketplace deals with only one primary factor.

Marketplaces are treated like any other industry. In particular there is a continuum of marketplaces for each type, with fixed unit mass, facing Calvo-style price rigidities. The marketplaces rebate profits to final users according to their ownership shares of primary factors $\hat{\Xi}^L$.

Primary factor types are fixed, hence wages differ across primary factors. Nonetheless workers and investment providers can freely move across marketplaces, therefore all units of a given primary factor earn the same flexible wage. By contrast, different marketplaces who hire the same factor type might charge different prices, due to the Calvo friction. We assume that ownership of each marketplace is equally shared among the relevant final users, so that in practice all units of each primary factor h earn the same (sticky) wage, equal to the average price charged by h-specific marketplaces.

Retailers To streamline the notation below, it is convenient to augment the set of industries with one retailer for each final user. Retailers assemble consumption, investment and government spending bundles and sell them to the relevant final user, to whom they also rebate their profits. Importantly, these fictitious retailer sectors are distinct from actual retailers in the data. Their price is equal to the price index for the relevant final user. Without loss of generality, we model relative preference shocks for consumers, investment producers, or the government, as relative demand shocks in the corresponding retailers' production function.

Remark 2. Given our modeling of retailers and factor marketplaces, the overall number of sectors in the economy is $\bar{N} = N + 2N_w + 2N_f + 1$ (the actual production sectors, the factor marketplaces for labor and capital, and the retailers for consumption, investment and government spending).

3.2.1 Aggregation

Our goal is to relate changes in aggregate inflation, as measured by the GDP deflator, with changes in aggregate output, as measured by real GDP. These aggregate quantities are defined below. Changes in real GDP and the GDP deflator are measured relative to the initial steady-state, denoted by starred variables.

Definition 2. Nominal GDP is given by total final expenditures:

$$GDP = \sum_{h \in \mathcal{N}_w} P_h^C C_h + \sum_{f \in \mathcal{N}_f} P_f^I I_f + G_t$$

Definition 3. Infinitesimal changes in real GDP around the initial steady-state are denoted by $d \log Y_t$, and are equal to the share-weighted sum of changes in aggregate consumption, aggregate investment, and government spending:

$$d \log Y_t = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log C_{ht} + \sum_{f \in \mathcal{N}_f} \frac{P_f^* I_f^*}{GDP^*} d \log I_{ft} + \frac{dG_t}{GDP^*}$$

Definition 4. Infinitesimal changes in the GDP deflator around the initial steady-state are denoted by $d \log P_t^Y$, and are equal to the share-weighted change in the price indices of the final users:

$$d \log P_t^Y = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log P_{ht}^C + \sum_{f \in \mathcal{N}_f} \frac{P_f^{*C} I_f^*}{GDP^*} d \log P_{ft}^C + \frac{G}{GDP^*} d \log P_G$$

Remark 3. Around an efficient equilibrium, changes in real GDP equal the income-weighted sum of changes in the quantities of primary factors, plus the change in aggregate productivity:

$$d\log Y_t = \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} \frac{W_h^* L_h^*}{\sum_{f\in\mathcal{N}_w\cup\mathcal{N}_f} W_f^* L_f^*} d\log L_{ht} + \sum_{i=1}^{N} \frac{P_i^* Y_i^*}{GDP^*} d\log A_{it}$$

3.3 Monetary policy

At each period t the central bank sets the nominal interest rate I_{t+1} on risk-free nominal bonds, where I denote $i_{t+1} \equiv \log(1 + I_{t+1})$. All results below hold regardless of the policy rule. In the calibration section I assume a standard Taylor rule.

3.4 Equilibrium

The equilibrium concept adapts the definition in Baqaee and Farhi (2020) to account for the endogenous determination of markups given pricing frictions and shocks. Given sectoral markups, all markets must clear; and the evolution of markups must be consistent with Calvo pricing and the realization of monetary policy.

Definition 5. At each period t, for given a path of interest rates $\{I_{t+s}\}_{s=0}^{\infty}$, sectoral probabilities of price adjustment δ_i, the general equilibrium is given by a vector of firm-level markups μ_{ift}, a vector of sectoral prices P_{it}, a vector of factor-specific nominal wages W_{ht}, a vector of factor supplies L_{ht}, a vector of sectoral outputs Y_{it}, a matrix of intermediate input quantities X_{ijt}, and a matrix of final demands C_{iht} such that: (i) a fraction δ_i of firms in each sector i charges the profit-maximizing price given by (15); (ii) the markup charged by adjusting firms is given by the ratio of the profit-maximizing price and marginal costs, while the markups of non-adjusting firms are such that their price remains constant; (iii) agents maximize utility subject to their budget constraints; (iv) producers in each sector i minimize costs and charge the relevant markup; and (v) markets for all goods and factors clear.

Remark 4. This equilibrium concept nests the standard one with flexible prices, which is obtained as a special case when $\delta_i = 1$ for every sector i.

3.5 Log-linearized model

I now derive a log-linear approximation of the model around an initial efficient steady state, where industries make zero profits net of taxes and subsidies ($\Pi_i^* - \mathcal{T}^*{}_i = 0\ \forall i$), transfers are $\{\bar{T}_{ht}\}_{t=0}^{\infty}$ for each household h, and nominal GDP is normalized to 1 ($\sum_h P_h^* C_h^* + \sum_f P_f^* I_f^* = 1$). Starred variables denote the steady-state, lower case letters denote log-deviations from the steady-state, and vectors are boldfaced. I denote level changes in transfers with respect to the intial equilibrium path by $t_{ht} = T_{ht} - \bar{T}_{ht}$.

Variables Table 1 defines the endogenous variables of the model.

Employment	$\boldsymbol{\ell}_t = \begin{pmatrix} \ell_{1t} & ... & \ell_{N_w+N_f,t} \end{pmatrix}^T$
Consumption	$\boldsymbol{c}_t = \begin{pmatrix} c_{1t} & ... & c_{N_w+N_f,t} \end{pmatrix}^T$
Factor prices	$\mathbf{w}_t = \begin{pmatrix} w_{1t} & ... & w_{N_w+N_f,t} \end{pmatrix}^T$
Inflation rates	$\boldsymbol{\pi}_t = \begin{pmatrix} \pi_{1t} & ... & \pi_{Nt} \end{pmatrix}^T, \ \pi_{it} \equiv p_{it} - p_{i,t-1}$
Transfers	$\mathbf{t}_t = \begin{pmatrix} t_{1t} & ... & t_{N_w t} \end{pmatrix}^T$

Table 1: Model variables

Remark 5. Table 1 defines employment for primary factors, rather than for sectors. In this environment, knowing factor-level employment is sufficient to characterize the evolution of prices, employment and output in all sectors.

Parameters I now introduce notation for four sets of parameters, which describe the log-linearized model: expenditure shares (α, β and Ω); income rebates (Ξ^L and Ξ); demand and supply elasticities; and sectoral price adjustment probabilities. The properties of the matrices α, β, Ω, Ξ^L and Ξ are discussed in CITE. Tables 2 and 3, and Figure 1, summarize the notation.

Input shares	$\Omega \in \mathbb{R}^{\bar{N} \times \bar{N}}$
Expenditure on primary factors	$\alpha \in \mathbb{R}^{\bar{N} \times N_w + N_f}$
Final use bundles	$\beta \in \mathbb{R}^{\bar{N} \times N_w + N_f + 1}$
Leontief inverse	$\Psi \equiv (I - \Omega)^{-1}$
Income shares of final users	$\mathbf{s} \in \mathbb{R}^{N_w + N_f + 1}$, $\mathcal{S} \equiv diag\,(\mathbf{s})$
Domar weights	$\bar{\mathbf{\Psi}}^T = \mathbf{s}^T \Psi_C$
Factor income shares	$\bar{\mathbf{\Psi}}_L^T \equiv \mathbf{s}^T \Psi_{CL}$, $\mathcal{L} \equiv diag\left(\bar{\mathbf{\Psi}}_L\right)$
Factor ownership	$\hat{\Xi}^L \in \mathbb{R}^{N_w + N_f + 1 \times N_w + N_f}, \ \Xi^L \equiv \hat{\Xi}^L + \bar{\mathbf{T}}\mathbf{s}^T$
Sector ownership	$\Xi \in \mathbb{R}^{\bar{N}, N_w + N_f + 1}, \Xi \equiv \hat{\Xi} + \bar{\mathbf{T}}\mathbf{s}^T$

Table 2: Input-output definitions and income rebates

Wealth effects in factor supply	$\Gamma \in \mathbb{R}^{(N_w + N_f) \times (N_w + N_f)}, \ \Gamma \equiv diag\,(\gamma_1, ..., \gamma_{N_w}, 0, ..., 0)$
Factor supply elasticities	$\Phi \in \mathbb{R}^{N_w + N_f}, \ \Phi \equiv \Gamma \mathcal{S}^{-1} \mathcal{W} \mathcal{L} + diag\,(\boldsymbol{\varphi})$
Demand elasticities	$\theta^i_{jk} \equiv \frac{d \log \frac{X_{ij}}{X_{ik}}}{d \log \frac{P_j}{P_k}}$
Price adjustment probabilities	$\Delta \equiv diag\left(\hat{\delta}_1, ..., \hat{\delta}_N\right)$

Table 3: Input-output definitions

Remark 6. The matrix Δ in Table 3 contains an increasing and convex transformation of the primitive Calvo parameters

$$\hat{\delta}_i\left(\rho, \delta_i\right) \equiv \frac{\delta_i\left(1-\rho(1-\delta_i)\right)}{1-\rho\delta_i(1-\delta_i)}$$

which corresponds to the slope of sectoral Phillips curves with respect to real marginal costs (see Gali (2015); Woodford (2003)).

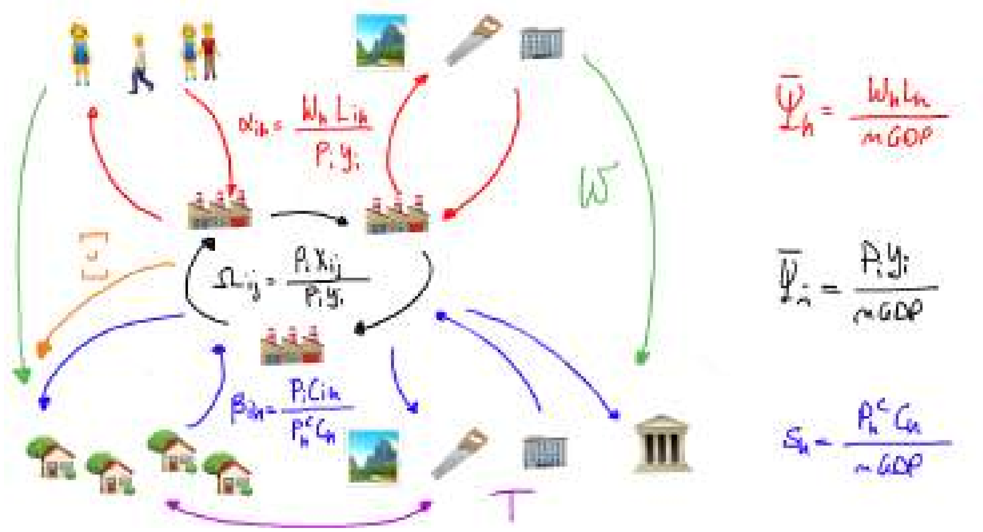

Figure 1: Log-linearized model

Aggregation Following definition 4, log-changes in the GDP deflator π^Y are a given by weighted average of sectoral inflation rates, with weights equal to the aggregate final expenditure shares $\bar{\boldsymbol{\beta}}$:

$$\pi^Y = \sum_{i=1}^{\bar{N}} \bar{\beta}_i \pi_i$$

In turn, aggregate final expenditure shares are a weighted average of the expenditure shares of the various final users, with weights given by final use shares:

$$\bar{\beta}_i \equiv \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} s_h \beta_{ih}$$

Following Definition 3 and Remark 3 , aggregate output is equal to aggregate employment (including all factors, not just labor). It is given by a weighted average of factor-level employment, with weights equal to factor income shares:

$$\bar{y} = \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} \bar{\Psi}_h \ell_h$$

3.6 Log-linearized equilibrium

The equilibrium of the economy is characterized by five equations: price setting, factor supply, relative factor demand, aggregate demand, and changes in the households net asset positions. Section 3.6.1 derives the factor supply and pricing equations and combines them to derive sector-by-factor Phillips curves. Section 3.6.2 discusses the relative demand equation. Section 3.6.3 introduces the aggregate demand equation and characterizes the changes in net asset positions. Section 4 below then combines relative demand with the Phillips curves to characterize the response of factor-level employment to monetary policy in general equilibrium.

3.6.1 Sector-by-factor Phillips curves

The factor supply equation

$$\mathbf{w}_t = \Phi \boldsymbol{\ell}_t + \underline{\beta}^T \mathbf{p}_t + \Gamma \mathcal{S}^{-1} \mathbf{t}_t \tag{16}$$
$$\underline{\beta}^T \equiv (I - \Gamma) \beta^T + \Gamma \mathcal{S}^{-1} \Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right) \Psi^{-1}$$

relates primary factor prices $\mathbf{w}$ with factor employment $\boldsymbol{\ell}$, prices $\underline{\beta}^T \mathbf{p}$ and transfers $\mathbf{t}$. The factor supply equation is obtained by log-linearizing and combining the consumption-leisure equation (10) and the budget constraint (9), and stacking them together with the log-linearized capital utilization equations (11). The price indices $\underline{\beta}^T \mathbf{p}$ are the relevant factor-specific weighted average prices to compute real factor returns, as implied by the consumption price indices faced by different households and utilization producers and by their asset ownership shares (which govern wealth effects in labor supply). Note that changes in net asset positions and exogenous transfers (captured by $\mathbf{t}_t$) cause wealth effects in labor supply, as households with higher disposable income supply less labor at a given real wage.

The pricing equation

$$\mathbf{p}_t = \mathcal{P}_L \mathbf{w}_t + \left(I - \mathcal{P}\Psi^{-1}\right) \left(\mathbf{p}_{t-1} + \mathbb{E}\boldsymbol{\pi}_{t+1}\right) \tag{17}$$
$$\mathcal{P} \equiv \Delta \left(I - \Omega\Delta\right)^{-1}$$

instead is obtained by log-linearizing and combining the evolution of marginal costs (14) and the optimal pricing condition (15).

The pass-through matrix $\mathcal{P}$ is an $\bar{N} \times \bar{N}$ matrix whose (i, j)-th element describes the propagation of a cost shock to sector j into the price of sector i. I also denote by

$$\mathcal{P}_L \equiv \mathcal{P}\alpha$$

the $\bar{N} \times N_H + N_F$ block of this matrix corresponding to primary factors, and by

$$\mathcal{P}_{C,:} \equiv \beta^T \mathcal{P}, \ \underline{\mathcal{P}}_{C,:} \equiv \underline{\beta}^T \mathcal{P}$$

the $N_H + N_F \times \bar{N}$ block of this matrix corresponding to final users.

In the definition of $\mathcal{P}$, the price adjustment probability Δ governs the response of prices to changes in marginal costs. The matrix $(I - \Omega\Delta)^{-1}$ is similar to the Leontief inverse, although it discounts supplier sectors by their price adjustment probability. Its (i, j)-th element corresponds to the propagation of a cost shock in sector j into the marginal cost of sector i, either directly or through intermediate suppliers, suppliers' suppliers, etc.

The matrix $\mathcal{P}_L$ denotes the component of the cost-passthrough matrix $\mathcal{P}$ corresponding to the factor marketplaces, and it describes the pass-through of factor prices into good prices (directly, and through input-output linkages). The response to lagged prices and expected inflation instead is mediated by the matrix $\mathcal{P}\Psi^{-1}$. The matrix Ψ^{-1} maps price wedges into the underlying changes in marginal costs, while the matrix $\mathcal{P}$ describes the pass-through of desired price changes into actual prices, directly and indirectly through the input-output network. If prices were fully flexible we would have $\mathcal{P}\Psi^{-1} = \mathbb{I}$, which means that current prices do not depend on past prices or expected future inflation. If instead prices are fully rigid we have $\mathcal{P} = \mathbb{O}$, so that prices are constant over time ($\mathbf{p}_t = \mathbf{p}_{t-1}$, and $\mathbb{E}\boldsymbol{\pi}_{t+1} = \mathbf{0}$).

Going back to simplified environment from Section 2, $\mathcal{P}_L\Phi$ is a generalized version of the wage elasticity κ^ℓ, while $\underline{\mathcal{P}}_{CL}$ is a generalized version of the price elasticity κ^p. Like we did in Section 2, Proposition **??** combines the supply and pricing equations in (**??**) to derive sector-by-factor Phillips curves – that is, macro-supply equations which relate prices in each sector with the employment of each primary factor. Proposition **??** expresses their slopes as a function of primitives, such as factor supply elasticities, sectoral price adjustment probabilities, and input-output linkages.

Proposition 1. *Sectoral inflation evolves according to*

$$\boldsymbol{\pi}_t - \rho\mathbb{E}\boldsymbol{\pi}_{t+1} = \kappa\left[\boldsymbol{\ell}_t + \Phi^{-1}\Gamma\mathcal{S}^{-1}\mathbf{t}_t\right] - \mathcal{V}\left(\mathbf{p}_{t-1} + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) \tag{18}$$

where

$$\begin{aligned} &\kappa \equiv \mathcal{P}_L\left(I - \underline{\mathcal{P}}_{CL}\right)^{-1}\Phi && \in \mathbb{R}^{\bar{N}\times(H+F)} \\ &\mathcal{V} \equiv \mathcal{P}\Psi^{-1} - \kappa\Phi^{-1}\underline{\beta}^T\left(I - \mathcal{P}\Psi^{-1}\right) && \in \mathbb{R}^{\bar{N}\times\bar{N}} \end{aligned} \tag{19}$$

The matrix $\mathcal{V}$ is such that $\sum_j \mathcal{V}_{ij} = 0$, $(I - \mathcal{V})_{ij} \in [0, 1]$ and, as long as no sector has fully flexible prices ($\delta_i \neq 1$ $\forall i$), the matrix $I - \mathcal{V}$ is invertible.

In an environment with multiple sectors and factors, inflation in each sector depends on the employment gap of all the factors. This is captured by the slope κ in equation (18). Current inflation also responds to lagged prices and expected future inflation, with elasticity $-\mathcal{V}$.

The slope κ is key to the cross-sectional effects of monetary policy, because it determines the effect of changes in employment on relative prices. The elements κ_{ih} capture the response of prices in sector i to the employment of factor h. Equation (19) decomposes the slope κ in three parts:

$$\kappa = (\text{factor price pass-through})\,(\text{factor price Phillips curves})\,(\text{supply elasticity}) \tag{20}$$

I discuss each of the three terms in turn.

The matrix Φ captures heterogeneity in factor supply elasticities. The elements Φ_{hk} tell us by how much the real wage[4] of factor h needs to increase if the employment of factor k increases by 1%. Φ_{hk} depends on Frish elasticities and wealth effects in labor supply. Both can vary across primary factors, depending on the households' preferences and ownership of capital assets, and on the investment technology.

The pass-through $\mathcal{P}_L$ of factor prices into good prices captures heterogeneity in the nominal rigidities of primary factors, and of the sectors which employ them. The price pass-through of factor h into industry i is given by $(\mathcal{P}_L)_{ih} \equiv \left(\Delta\,(I-\Omega\Delta)^{-1}\,\alpha\right)_{ih}$. The matrix of factor input shares α captures the direct effect of factor prices on sectoral marginal costs, while the geometric sum $(I-\Omega\Delta)^{-1} = \Omega\Delta + (\Omega\Delta)^2 + ...$ describes the indirect effect through intermediate inputs. The n-th term of the geometric sum, $(\Omega\Delta)^n_{ij}$, tells us the effect of an increase in the marginal cost of sector j on sector i's marginal cost, through all supply chains of length n starting with j and ending with i. Along the path, each supplier is weighted according to its input shares Ω, and discounted by the probability of price adjustment Δ. Thus, factors h which are used intensively by industries (including labor unions) with flexible prices, or whose customers have flexible prices, etc., have a higher price pass-through and stepper slopes $\kappa_{:,h}$

The multiplier $(I-\underline{\mathcal{P}}_{CL})^{-1}$ captures heterogeneity in the nominal rigidly of the sectors which different final users own and buy from. This multiplier maps changes in real factor prices into changes in nominal factor prices. As factor prices increase, good prices also increase, so that the nominal change in factor prices must be larger than the real change. The effect of factor prices on final prices is given by $\underline{\mathcal{P}}_{CL}$. For a given increase in factor prices, consumption (investment) prices increase more for final users who spend more on flex-price goods (or goods that are produced using flex-price intermediates, etc), so that the primary factors that they supply will have steeper Phillips curves. In addition, through a wealth effect in labor supply, households who own a smaller share of profit income also have steeper wage Phillips curves.[5]

[4]Given our notation (see Section 3.5), nominal wages mean the counterfactual flexible wages that satisfy equation (10) given factor demand and good prices. These are different from the actual (sticky) factor prices, which correspond to the first $H+F$ entries in the vector $\boldsymbol{\pi}$.

[5]This is because, with sticky prices, profits are negatively correlated with employment. Therefore, as employment rises, firm owners become poorer and are willing to supply labor at a lower wage.

Examples Examples 1, 2 , and 3 derive sector-by-factor Phillips curves in three simple economies, to illustrate the role of input-output linkages and fixed factors. For simplicity in these example I assume full discounting of the future ($\rho = 0$).

Example 1. Upstream workers have flatter Phillips curves

Consider and economy with two sectors (I and F, standing for intermediate and final goods) and two households, 1 and 2, as in Figure 2. Household 1 works in the intermediate good sector, while household 2 works in the final good sector. Wages are flexible, and both sectors have the same price adjustment probability δ. The intermediate good sector uses only labor in production, while the final good sector uses labor and intermediate goods with expenditure shares α_F and $1 - \alpha_F$, and elasticity of substitution θ. Both households have the same inverse Frish elasticity φ, and there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$).

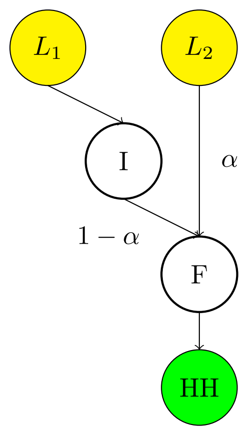

Figure 2: Two-stage vertical chain

Following Proposition 1, the Phillips curve slopes for intermediate and final goods are given by

$$\begin{pmatrix} \kappa_{I1} & \kappa_{I2} \\ \kappa_{F1} & \kappa_{F2} \end{pmatrix} = \frac{\varphi\delta}{1-\bar{\delta}} \begin{pmatrix} 1-\alpha_F\delta & \alpha_F\delta \\ (1-\alpha_F)\,\delta & \alpha_F \end{pmatrix}$$

where

$$\bar{\delta} \equiv \delta\left[(1-\alpha_F)\,\delta + \alpha_F\right]$$

Note that prices in the intermediate good sector depend on employment of the final good workers, because changes in the wages of final workers affect consumption prices and hence the desired wage of intermediate good workers. Also note that the Phillips curve for the final sector – which coincides with the consumer price Phillips curve – has a flatter slope for intermediate workers than for final workers. This is because changes in the wage of intermediate workers are only partially passed through by the intermediate good sector.

Example 2. Capital intensity and Phillips curve slopes

Consider the economy in Example 4. Think of the two primary factors as capital and labor (labeled K and L). Capital is more inelastically supplied, with inverse elasticity $\varphi_K > \varphi_L$. Capital utilization is produced using the consumption bundle, and workers have no wealth effects in labor supply ($\gamma = 0$). Following Proposition 1, the Phillips curve slopes for the two goods are given by

$$\begin{pmatrix} \kappa_{1L} & \kappa_{1K} \\ \kappa_{2L} & \kappa_{2K} \end{pmatrix} = \frac{\delta}{1-\delta}\left[\begin{pmatrix} \alpha_1\varphi_L & (1-\alpha_1)\,\varphi_K \\ \alpha_2\varphi_L & (1-\alpha_2)\,\varphi_K \end{pmatrix} + \delta\,(\alpha_1-\alpha_2)\begin{pmatrix} 1-\beta_1 \\ -\beta_1 \end{pmatrix}\begin{pmatrix} -\varphi_L & \varphi_K \end{pmatrix}\right]$$

If the employment of labor and capital were to increase proportionately, prices would increase by more in the more capital-intensive sector:

$$(\boldsymbol{\kappa}_{2:} - \boldsymbol{\kappa}_{1:})\,\mathbf{1} = \delta\,(\alpha_1-\alpha_2)\,(\varphi_K-\varphi_L) > 0 \iff \alpha_1 > \alpha_2$$

Moreover, averaging out the two sectors according to their consumption shares we find that the slope of the GDP deflator is steeper for capital than for labor:

$$\frac{\kappa_L^Y}{s_L} = \frac{\delta}{1-\delta}\varphi_L < \frac{\kappa_K^Y}{s_K} = \frac{\delta}{1-\delta}\varphi_K$$

Example 3. Complementarity with fixed assets

Consider the economy in Figure 3. The economy has two cities (New York and Boise), and households in both cities are employed in the local construction sector. Housing is built using local labor and land, with substitution elasticity θ. Land is scarce in New York, so that the steady-state expenditure share on land is larger there, and the labor share is smaller ($\alpha_{NY} < \alpha_B$). Households have preferences over housing in both cities, with substitution elasticity σ. For simplicity, assume that all households have the equal expenditure shares on the two cities. Labor is more elastically supplied than land, with inverse elasticity $\varphi_L < \varphi_K$. All wages and land prices are flexible, while house prices are sticky, with the same adjustment probability δ in the two cities.

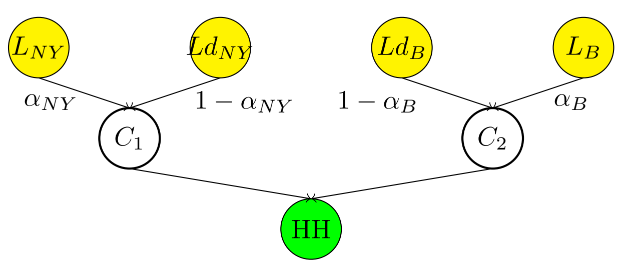

Figure 3: NYC vs Boise

Following Proposition 1, the Phillips curve slopes for the two cities are given by

$$\begin{pmatrix} \boldsymbol{\kappa}_{NY,:} \\ \boldsymbol{\kappa}_{B,:} \end{pmatrix} = \frac{\delta}{1-\delta} \begin{pmatrix} \left(1-\frac{1}{2}\delta\right)\alpha_{NY}\varphi_L & \left(1-\frac{1}{2}\delta\right)(1-\alpha_{NY})\varphi_K & \frac{1}{2}\delta\alpha_B\varphi_L & \frac{1}{2}\delta(1-\alpha_B)\varphi_K \\ \frac{1}{2}\delta\alpha_{NY}\varphi_L & \frac{1}{2}\delta(1-\alpha_{NY})\varphi_K & \left(1-\frac{1}{2}\delta\right)\alpha_B\varphi_L & \left(1-\frac{1}{2}\delta\right)(1-\alpha_B)\varphi_K \end{pmatrix}$$

Like in Example 2, a uniform increase in the employment of all factors would raise prices by more in New York:

$$(\boldsymbol{\kappa}_{NY:} - \boldsymbol{\kappa}_{B:})\,\mathbf{1} = \delta(\alpha_B - \alpha_{NY})(\varphi_K - \varphi_L) > 0$$

Likewise, the slope of the GDP deflator is steeper after a uniform increase in labor and land employment in New York compared to Boise:

$$\kappa_{NY}^Y - \kappa_B^Y = \frac{1}{2}\frac{\delta}{1-\delta}(\alpha_{NY} - \alpha_B)(\varphi_L - \varphi_K)$$

3.6.2 Relative demand

Following Lemma **??** in CITE, the relative demand for primary factors evolves according to

$$\begin{aligned} \boldsymbol{\ell}_t &= \mathbf{1}\bar{y}_t + \mathcal{D}_p\mathbf{p}_t + \mathcal{D}_T\mathcal{S}^{-1}\mathbf{t}_t & (21) \\ \mathcal{D}_p &\equiv -\mathcal{L}^{-1}\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, S^{-1}\Xi_L^T\right)\right)^{-1}\left[Cov_{\mathbf{s}}\left(\Psi_{CL}^T, \beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right) + \Theta\left(\Psi_{:,L}, I\right)\right] \\ \mathcal{D}_T &\equiv \mathcal{L}^{-1}\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, S^{-1}\Xi_L^T\right)\right)^{-1} Cov_s\left(\Psi_{CL}^T, I\right) \end{aligned}$$

The relative demand slope $\mathcal{D}_p$ generalizes the micro-level elasticity of substitution θ from the simplified framework in Section 2. The substitution operator Θ is defined in Rubbo (2024). The operator $\Theta(\Psi_{:,L}, I)$ captures an aggregate elasticity of substitution between primary factors, defined as the response of relative factor demand to changes in relative good prices. The relative demand for primary factors demand also depends on income reallocation across final users who consume different bundles of goods. This is described by the multiplier $\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, S^{-1}\Xi_L^T\right)\right)^{-1}$, which captures reallocation of factor income, and by the covariance $Cov_{\mathbf{s}}\left(\Psi_{CL}^T, \beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right)$, which captures changes in profit rebates relative to consumption prices. Income reallocation becomes irrelevant if all final users have the same expenditure shares ($\beta = \bar{\boldsymbol{\beta}}\mathbf{1}^T$), and we have $\mathcal{D}_p \equiv -\mathcal{L}^{-1}\Theta(\Psi_{:,L}, I)$, $\mathcal{D}_T = \mathbb{O}$.

Examples Examples 1 and **??** illustrate how the matrix $\mathcal{D}_p$ in equation (**??**) maps micro-level elasticities of substitutions and expenditure shares into an aggregate elasticity of substitution between primary factors.

Example 4. Micro-level and aggregate elasticities of substitution

Consider the economy depicted in Figure **??**, with two sectors (1 and 2) and two primary factors (H and F). H and F could represent labor and capital, or households in different occupations, etc.

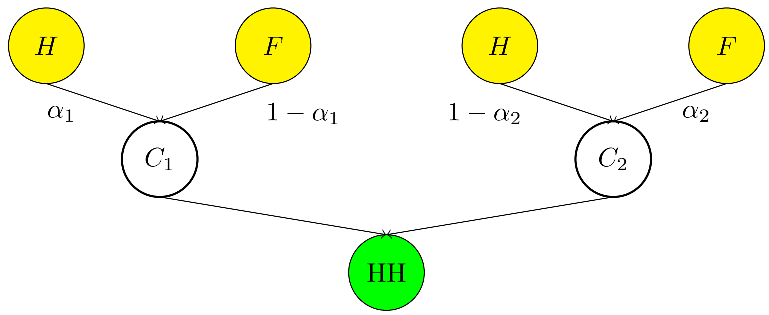

Figure 4: Two-good, two-workers economy

Both sectors use both primary factors, but in different proportions, as captured by different expenditure shares $\alpha_1 \neq \alpha_2$ on factor H). Both sectors have CES production functions with elasticity of substitution θ. All final users have the same expenditure shares β_1 and $1-\beta_1$ on the two final goods, with substitution elasticity σ.

The relative demand equation (**??**) allows us to map micro-level substitution elasticities θ and σ into aggregate elasticities of substitution between H and F. These are captured by the matrix $\mathcal{D}_p$, which in our example is given by

$$\mathcal{D}_p = -\begin{pmatrix} \frac{1}{s_H} \\ -\frac{1}{1-s_H} \end{pmatrix} \begin{pmatrix} A\theta & -A\theta & B\sigma & -B\sigma \end{pmatrix}$$
$$A \equiv \beta_1 \alpha_1 (1-\alpha_1) + (1-\beta_1) \alpha_2 (1-\alpha_2)$$
$$B \equiv \beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)$$

As in equation (**??**), the elements $(\mathcal{D}_p)_{hi}$ tell us by how much the demand of factor h relative to the aggregate changes with the relative price of good i. Note that $\mathcal{D}_p$ only depends on relative factor prices and relative final good prices. Therefore

$$\ell_H - \ell_F = -\frac{\beta_1 \alpha_1 (1-\alpha_1) + (1-\beta_1) \alpha_2 (1-\alpha_2)}{s_H (1-s_H)} \theta (w_H - w_F) - \frac{\beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)}{s_H (1-s_H)} \sigma (p_1 - p_2)$$

To obtain a counterpart of the demand equation (2) from Section 2 we need additional assumptions about the passthrough of wages into prices. Assuming that $\mathbf{p}_{t-1} = 0$, $\rho = 0$ and both consumption goods have the same Calvo parameter δ, the pricing equation becomes

$$p_i = \delta (\alpha_i w_H + (1-\alpha_i) w_F) \;\; i = 1, 2$$

and we obtain the demand equation

$$\ell_H - \ell_F = -\bar{\theta} (w_H - w_F)$$

where

$$\bar{\theta} \equiv \left[1 - \frac{\beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)^2}{s_H (1-s_H)}\right]\theta + \frac{\beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)^2}{s_H (1-s_H)}\sigma\delta$$

The aggregate elasticity $\bar{\theta}$ combines substitution elasticities in production and consumption. It discounts the consumption elasticity by the price adjustment probability in the final good sectors, because this dampens the response of final good prices to changes in the workers' relative wage, and hence the amount of substitution by final consumers. If the two final goods goods use factors in the same proportions ($\alpha_1 = \alpha_2$) the aggregate elasticity $\bar{\theta}$ equals the production elasticity θ, because substitution across final goods has no effect on the relative demand for different workers.

Example 5. Home bias in consumption and relative demand

Consider the economy in Example 4, but now set $\alpha_1 = 1$ and $\alpha_2 = 0$ and allow the two households to have different final expenditure shares β_H and β_F on good 1. Assume for simplicity that $\beta_H = 1 - \beta_F$, so that the households have equal income shares. This could represent a currency union, where countries produce and consume different goods. There is home bias in consumption whenever $\beta_H > \frac{1}{2}$. Both households have the same elasticity of substitution σ between goods.

Like in Example 4, we want to use the matrix $\mathcal{D}_p$ to derive an aggregate elasticity of substitution between H and F that is the counterpart of the parameter θ in Section 2.

Assuming that each household owns the final sectors in proportion to their consumption shares, $\mathcal{D}_p$ is given by

$$\mathcal{D}_p = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left[\left(\beta_H - \frac{1}{2}\right)\begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix} - \sigma\beta_H \begin{pmatrix} 0 & 0 & 1 & -1 \end{pmatrix}\right]$$

so that

$$\ell_H - \ell_F = 2\left[\left(\beta_H - \frac{1}{2}\right)(w_H - w_F) - \sigma\beta_H (p_1 - p_2)\right]$$

Again, in order to relate relative labor demand with relative wages we need to make assumptions about the passthrough if wages into prices. Maintaining the same assumptions as in example 4 we have

$$\ell_H - \ell_F = -\bar{\theta}(w_H - w_F)$$

where

$$\bar{\theta} \equiv 2\beta_H \sigma\delta - (2\beta_H - 1)$$

is the counterpart of the demand elasticity θ from section 2. When households have equal expenditure shares on the two goods we recover the substitution elasticity in consumption, $\bar{\theta} = \sigma$. Otherwise, if the wage w_H increases relative to w_F, this raises the demand for the good on which household H has a larger expenditure share. If there is home bias in consumption ($\beta_H > \frac{1}{2}$) the demand for H's own product is increasing in H's wage, which dampens the expenditure switching effect. Vice versa, income reallocation and expenditure switching reinforce each other when there is reverse home bias ($\beta_H < \frac{1}{2}$).

3.6.3 Aggregate demand and net asset positions

Following Definition 1 and the budget constraints (9), changes in net asset positions $\{\mathbf{t}_t\}_{t=0}^{\infty}$ must satisfy

$$\begin{aligned} \mathcal{S}^{-1} t_{ht} = & \sum_{\tau=0}^{\infty} \mathcal{J}_{t\tau}^{h,Y} \frac{1}{s_h} \left[\Xi_{h:}^{L} \mathcal{L} \boldsymbol{\ell}_{\tau} + \left(\mathcal{S}^{-1} \Xi_{h:} diag\left(\bar{\Psi}\right) - \Psi_{h:} \right) \Psi^{-1} \mathbf{p}_{\tau} + \tilde{t}_{h\tau} \right] \\ & - \frac{1}{s_h} \left[\Xi_{h:}^{L} \mathcal{L} \boldsymbol{\ell}_{t} + \left(\Xi_{h:} diag\left(\bar{\Psi}\right) - \Psi_{h:} \right) \Psi^{-1} \mathbf{p}_{t} + \mathcal{S}^{-1} \tilde{t}_{ht} \right] + \\ & + \sum_{\tau=0}^{\infty} \left[\mathcal{J}_{t\tau}^{h,r} + \sum_{z=0}^{\infty} \mathcal{J}_{tz}^{h,Y} \mathcal{S}^{-1} d_{h,z\tau} \right] dr_{\tau} \end{aligned} \tag{22}$$

where I denoted by

$$\mathbf{d}_{h,z\tau} \equiv \sum_{a \in \mathcal{A} \setminus \mathcal{N}_f \cup \mathcal{N}} \frac{D_{a,z} Q_{ha,z-1}}{\sum_{a' \in \mathcal{A} \setminus \mathcal{N}_f \cup \mathcal{N}} D_{a',z} Q_{ha',z-1}} \frac{d \log D_{a,z}}{d\mathbf{r}_{\tau}}$$

the log-change in dividend income of all assets except firms and capital.

Aggregating the budget constraints of all final users weighted by income shares yields the aggregate demand equation

$$\bar{y}_t = \sum_{\tau=0}^{\infty} \left\{ \left[\mathbb{E}_s \mathcal{J}_{t\tau}^{:,r} + \sum_{z=0}^{\infty} Cov_s \left[\mathcal{J}_{tz}^{:,Y}, \mathcal{S}^{-1} \mathbf{d}_{z\tau} \right] \right] dr_{\tau} + \mathbb{E}_s \left(\mathcal{J}_{t\tau}^{:,Y} \right) \bar{y}_{\tau} + Cov_s \left[\mathcal{J}_{t\tau}^{:,Y}, \mathcal{S}^{-1} \Xi_L \mathcal{L} \boldsymbol{\ell}_{\tau} + \left(\mathcal{S}^{-1} \Xi diag\left(\bar{\Psi}\right) - \Psi_{C:} \right) \Psi^{-1} \mathbf{p}_{\tau} + \right. \right. \tag{23}$$

4 Monetary non-neutrality: cross-section and aggregate

In this section I characterize the response of factor-level employment to monetary policy in general equilibrium. The dynamics of the economy are governed by the Phillips curves (18), the relative demand equation (21), the evolution of net asset positions (22), and the aggregate demand equation (23). Lagged prices ($\mathbf{p}_{t-1}$) are a state variable.

Section 4.1 studies cross-sectional non-neutrality, as captured by the response of factor-level employment to changes in aggregate employment and asset positions. Section **??** instead illustrates aggregate monetary non-neutrality, defined as the change in real GDP relative to nominal GDP. All results in this section build upon the Phillips curves (18) and the relative demand equation (21). Therefore they hold for given changes in net asset positions, lagged prices, and inflation expectations, which of course are endogenous to interest rates. To fully solve the model one needs to also account for the evolution of net asset positions (22), and the aggregate demand equation (23). I do so in the calibration section, where I compute the impulse responses of all variables to monetary shocks.

4.1 Cross-sectional non-neutrality

Proposition 2 combines the Phillips curves (18) with the relative demand equation (**??**), to express cross-sectional employment as a function of the aggregate output gap, net asset positions, lagged prices, and inflation expectations.

Proposition 2. *It holds that*

$$\boldsymbol{\ell}_t = \left(I - \mathcal{D}_p \kappa\right)^{-1} \left[\mathbf{1}\tilde{y}_t + \left(\mathcal{D}_p \kappa \Phi^{-1} \Gamma + \mathcal{D}_T\right) \mathcal{S}^{-1} \mathbf{t} + \mathcal{D}_p \left(I - \mathcal{V}\right) \left(\mathbf{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{t+1}\right)\right] \tag{24}$$

Proof. Equation (24) follows immediately from equations (18) and (**??**). □

In equation (24), the *cross-sectional multiplier* $(I - \mathcal{D}_p\kappa)^{-1}$ captures a feedback loop between changes in relative factor demand and changes in factor prices. The effect of factor demand on relative prices is captured by the slope κ of sectoral Phillips curves. As we will see, heterogeneity in κ across sectors is key to the cross-sectional effect of monetary policy. In turn, the effect of relative price changes on cross-sectional labor demand is captured by the matrix $\mathcal{D}_p$ from equation (**??**).

In commenting Proposition 2 I focus on the aggregate employment term in (24). The remaining terms describe the employment response to changes in asset positions, relative price changes coming from past monetary shocks (as captured by lagged prices $\mathbf{p}_{t-1}$) and expected future inflation.

The loading on aggregate employment has two components: a direct effect, captured by the vector $\mathbf{1}$, and a propagation effect, captured by the cross-sectional multiplier $(I - \mathcal{D}_p\kappa)^{-1}$. To understand the direct effect, note that a monetary expansion increases the demand for all final users proportionately on impact (i.e. parallel to the vector $\mathbf{1}$). This is because preferences and production are homothetic, and agents were optimizing their initial expenditures.

However, when Phillips curve slopes differ across primary factors, due to heterogeneous nominal rigidities or supply elasticities, even a proportional increase in labor supply will affect relative factor prices. Changes in relative prices in turn affect relative real incomes, and trigger expenditure switching by producers and consumers. This then feeds back into relative factor demand, as captured by the cross-sectional multiplier $(I - \mathcal{D}_p\kappa)^{-1}$. Overall, demand falls for factors who have steeper Phillips curves, or who are employed by the same sectors as steep-Phillips curve factors, etc.

Corollary 1 highlights the necessary conditions for changes in aggregate output to affect cross-sectional employment.

Corollary 1. *Changing the aggregate output gap does not affect relative employment across primary factors if either:*

1. sectors use primary factors in equal proportions ($\alpha = \alpha \mathbf{1} \mathbf{s}^T$);

2. a proportional increase in the employment of all primary factors does not affect relative prices ($\kappa \mathbf{1} \propto \mathbf{1}$).

If there is no variation in factor intensity across sectors (condition 1), then changes in relative prices have no effect on factor demand and $\mathcal{D}_p = \mathbb{O}$. Conversely, if a proportional increase in the employment of all primary factors has no effect on relative prices (condition 2), then we also have no feedback effect on relative demand ($\mathcal{D}_p \mathbf{1} = \mathbf{0}$). Note that Corollary 1 holds regardless of demand elasticities. Hence heterogeneous demand elasticities per-se do not imply a cross-sectional effect of monetary policy.

In this section I focused on the effect of monetary policy on cross-sectional employment. Cross-sectional consumption instead does not only depend on employment, but also on income from primary factors, profit rebates, and consumption or investment prices. I discuss cross-sectional consumption in Section **??**.

4.1.1 Examples

The examples below illustrate how various dimensions of heterogeneity shape cross-sectional monetary non-neutrality in a few simple economies. To simplify the exposition I assume $\rho = 0$, $\mathbf{T} \equiv \mathbf{0}$, and $\mathbf{p}_{t-1} = \mathbf{0}$.

Example 6. Input-output linkages

Let's return to the vertical economy in Example 1, with $\alpha_F = \frac{1}{2}$. The relative response of employment to monetary policy in the intermediate and final sectors is given by

$$\ell_I - \ell_F = \frac{\varphi\theta}{1 + \varphi\theta\left(\frac{\bar{\delta}}{\delta} + \frac{1}{4}\frac{\delta(1-\delta)^2}{1-\bar{\delta}}\right)} \frac{1-\delta}{1-\bar{\delta}} \bar{y}$$

To draw a parallel with the simple framework in Section 2, it is as if the intermediate good workers had Phillips curve parameters $\kappa_1^\ell = \delta\varphi$ and $\kappa_1^p = \delta^2$, while the final good workers had parameters $\kappa_2^\ell = \varphi$ and $\kappa_2^p = \delta$. These parameters govern the response of the prices that final producers face, and expenditure switching between the two workers happens only in final good production. The relative price change of intermediate and final goods in response to a uniform change in the employment gap of the two agents is proportional to $1 - \delta$, and hence the employment response is also proportional to $1 - \delta$. Like in Section 2 the employment response is scaled by the elasticity of substitution θ, which governs expenditure switching by final producers.

Example 7. Micro-level vs aggregate demand elasticities

Consider the economy in Example 4, with two sectors using two primary factors in different proportions.

Assume that the two primary factors have supply elasticities φ_H and φ_F and wage adjustment probabilities δ_H and δ_F, while final good prices have the same adjustment probability δ.

Like in the simple model from Section 2 the cross-sectional responses of employment and consumption to monetary policy depend on the parameters κ_h^ℓ and κ_h^p of the wage Phillips curves, and on the aggregate substitution elasticity between the two primary factors. The Phillips curve parameters are the same as in Section 2, while in

Example ?? we showed that the aggregate substitution elasticity is given by

$$\bar{\theta} \equiv \theta \left(1 - \frac{\beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)^2}{s_1 (1-s_1)}\right) + \sigma\delta \frac{\beta_1 (1-\beta_1)(\alpha_1 - \alpha_2)^2}{s_1 (1-s_1)}$$

The cross-sectional responses of employment and consumption are then given by equations (3) and (4) in Section 2, replacing θ with $\bar{\theta}$.

Example 8. NYC vs Boise

Consider the economy in Example 3, and assume that the supply of land is fully inelastic ($\varphi_K \to \infty$). We want to study whether employment is more sensitive to monetary policy in NYC or in Boise.

In this economy, cross sectional employment is given by

$$\ell_B - \ell_{NY} \propto \theta (\sigma\delta - \theta)(\alpha_B - \alpha_{NY}) \bar{y}$$

Therefore employment in NYC is more sensitive to changes in aggregate demand $\bar{y}$ if and only if $\sigma\delta < \theta$, that is, if housing is less substitutable across the two cities than labor and land are substitutable in the production of housing services. Intuitively, this means that agents prefer to improve the housing stock in their current location, by using labor more intensively on the same amount of land, rather than moving to a more land-abundant location. The housing stock is always more sensitive to monetary policy in Boise than in NYC – because Boise faces a more elastic housing supply – however if $\sigma\delta < \theta$ construction employment is more sensitive in NYC, because a monetary expansion increases the labor share by more.

4.2 Aggregate non-neutrality

4.2.1 Static case

Section 4.1 discussed how monetary policy impacts the relative employment and consumption of heterogeneous households. I now consider the implications of households heterogeneity for aggregate monetary non-neutrality. To build intuition I start by considering a static economy, with full discounting of the future ($\rho = 0$), no asset trade ($\mathbf{t} \equiv \mathbf{0}$), and $\mathbf{p}_{t-1} = \mathbf{0}$. Section 4.2.3 extends the analysis to the dynamic economy.

In the static economy, I define aggregate monetary non-neutrality as the response of real GDP ($\bar{y}$) relative to nominal GDP. Applying Definitions 2 and 3 yields

$$d \log nGDP = p^Y + \bar{y} \tag{25}$$

Denoting the slope of the Phillips curve for the GDP deflator by

$$\boldsymbol{\kappa}^Y \equiv \mathbf{s}^T \beta^T \kappa \in \mathbb{R}^{1, N_w + N_f}$$

under our assumptions we have

$$d \log nGDP_t = \boldsymbol{\kappa}^Y \boldsymbol{\ell}_t + \bar{y}_t \tag{26}$$

Let's start from a benchmark model with a single household and a single primary factor ($N_w = 1$, $N_f = 0$). In this case we have $\ell_t = \bar{y}_t$ and equation (26) immediately implies

$$\bar{y}_t = \frac{d \log nGDP_t}{1 + \kappa^Y} \tag{27}$$

Equation (27) highlights the key role of the Phillips curve slope κ^Y. Prices respond more when the Phillips curve is steeper, and therefore output responds less, for a given increase in nominal GDP.

The heterogeneous agents counterpart of equation (27) is given by

$$\bar{y}_t = \frac{d \log nGDP}{1 + \bar{\kappa}^Y + Cov_s\left(\frac{\boldsymbol{\kappa}^Y}{\mathbf{s}}, \frac{\boldsymbol{\ell}}{\bar{y}}\right)} \tag{28}$$

where $\bar{\kappa}^Y = \sum_h \kappa_h^Y$ denotes the response of the GDP deflator to a proportional 1% increase in the employment gap of all factors. In equation (28), aggregate monetary non-neutrality depends both on the aggregate slope $\bar{\kappa}^Y$, and on the cross-sectional covariance between the Phillips curve slopes $\frac{\boldsymbol{\kappa}^Y}{\mathbf{s}}$ and factor employment $\frac{\boldsymbol{\ell}}{\bar{y}}$. The aggregate Phillips curve is flatter – and monetary policy has larger real effects – when employment increases more for factors h with a flatter slope κ_h^Y.

While it is hard to determine the sign of this covariance in general, in Section 4.1 we saw that the demand for primary factors with steeper Phillips curves tends to decline after a monetary expansion. Thus we can expect the covariance to be negative. The examples in Section 4.1.1 illustrate that this is the case in some relevant special cases. Hence the presence of heterogeneous nominal rigidities and labor supply elasticities tends to increase the aggregate monetary non-neutrality.

4.2.2 Examples

The examples below specialize equation (28) to the simple economies in Sections **??** and 4.1.1, showing how different dimensions of heterogeneity across agents affect aggregate non-neutrality. Like in Section 4.1.1, we assume $\rho = 0$ and $\mathbf{p}_{t-1} = \mathbf{0}$.

Example 9. Nominal rigidity

Consider the economy in Examples **??** and **??**, with two agents and one final good. In the aggregate, the ability to shift demand towards the worker with stickier wages increases monetary non-neutrality, compared to an economy where both workers have the same wage rigidity, equal to the average $\bar{\delta}$. The response of aggregate employment to a 1% increase in the money supply is given by

$$\bar{y} = \frac{1}{1 + \varphi \frac{\bar{\delta}}{1-\bar{\delta}} \left[1 - \frac{\varphi \theta \bar{\delta}}{1+\varphi \theta \bar{\delta}} \left(\frac{\delta_{flex} - \delta_{sticky}}{\bar{\delta}} \right)^2 \right]} > \frac{1}{1 + \varphi \frac{\bar{\delta}}{1-\bar{\delta}}}$$

Aggregate monetary non-neutrality is increasing in the elasticity of substitution θ, which governs the ability of the economy to reallocate demand towards the sticky worker, and it is the same as with a representative agent only when production is Leontief.

Example 10. Labor supply elasticity

Consider now the economy in Example 2. The response of aggregate employment to a 1% increase in the money supply is given by:

$$\bar{y} = \frac{1}{1 + \bar{\varphi} \frac{\delta}{1-\delta} \left[1 - \frac{\bar{\varphi} \theta \delta}{1+\bar{\varphi} \theta \delta} \left(\frac{\varphi_I - \varphi_E}{\bar{\varphi}} \right)^2 \right]} > \frac{1}{1 + \bar{\varphi} \frac{\delta}{1-\delta}}$$

The intuition is very similar to Example 9: the ability of the economy to reallocate demand towards the more elastic worker increases aggregate non-neutrality, compared to an economy where both workers have the average labor supply elasticity $\bar{\varphi}$. Again, heterogeneity increases aggregate non-neutrality by more when workers are more substitutable, and it has no effect when production is Leontief.

4.2.3 Dynamic case

Similar to the static setting in Section 4.2.1, I define aggregate monetary non-neutrality as the relative impact response of real and nominal GDP.

In the one-sector, representative agent model, it is well-known that inflation and employment decay over time at the same rate as the monetary shock. With multiple industries and primary factors, instead, inflation and employment in general do not decay at constant rates, even when the monetary shocks do. This makes it harder to compare the impulse responses of inflation and employment analytically.

To address the issue, Proposition 3 shows that there exists a set of inflation indices which do decay at constant rates. This result is useful to characterize the dynamic evolution of all other inflation indices as well, as they can be written as linear combinations of these special indices.

Proposition 3. *Let*

$$\mathcal{M} \equiv \left(I - \alpha \underline{\Psi}_{C,:} \right) \Psi^{-1}$$

Denote by $\{\boldsymbol{\zeta}_i\}_{i=1}^N$ the left eigenvectors of the matrix $\mathcal{M}$, and its eigenvalues by $\{\nu_i\}_{i=1}^N$. Consider the price indices $\pi_t^{\zeta_i} \equiv \boldsymbol{\zeta}_i^T \boldsymbol{\pi}_t$, defined by the eigenvectors $\boldsymbol{\zeta}_i$. The impact response $\pi_0^{\zeta_i}$ to future output gaps ℓ_t decays at constant *rate ξ_i with time t, and the law of motion for $\pi_t^{\zeta_i}$ is given by:*

$$\pi_t^{\zeta_i} = \xi_i \pi_{t-1}^{\zeta_i} + \sum_{s=0}^{\infty} (\rho\xi_i)^s \boldsymbol{\kappa}^{\zeta_i} \mathbb{E}\boldsymbol{\ell}_{t+s}$$

where κ^{ζ_i} is the slope of the Phillips curve defined by the index $\pi_t^{\zeta_i}$. The slope κ^{ζ_i} is equal to

$$\boldsymbol{\kappa}^{\zeta_i} \equiv \frac{\xi_i}{\nu_i} \boldsymbol{\zeta}_i^T \Delta (I - \Delta)^{-1} \Xi^L \alpha \Phi$$

Moreover it holds that $\nu_i \geq 0 \ \forall i$, and the constant ξ_i satisfies

$$\xi_i = \frac{1 + \rho + \nu_i - \sqrt{(1 + \rho + \nu_i)^2 - 4\rho}}{2\rho} \in (0, 1)$$

Building on Proposition 3, one can characterize the impact response of any price index $\pi_t^{\phi} \equiv \boldsymbol{\phi}^T \pi_t$, where $\boldsymbol{\phi} \in \mathbb{R}^{\bar{N}}$ is a vector of sectoral weights. For example, for $\boldsymbol{\phi} = \mathbf{s}^T \beta^T$ the index π_t^{ϕ} corresponds to the GDP deflator π_t^Y, while for $\boldsymbol{\phi} = \mathbf{e}_i$ the index π_t^{ϕ} corresponds to inflation in industry i. Note that any index π^{ϕ} can be written as a linear combination of the eigen-indices $\left\{\pi_t^{\zeta_i}\right\}_{i=1}^{N}$, with some coefficients $\{x_i\}_{i=1}^{N}$. Hence, assuming $\pi_{-1} = \mathbf{0}$, we have

$$\pi_0^{\phi} = \sum_t \rho^t \left[\sum_i \xi_i^t x_i \kappa^{\zeta_i}\right] \mathbb{E}\boldsymbol{\ell}_t \tag{29}$$

In equation (**??**), the response of current inflation π_0^{ϕ} to employment gaps t-periods ahead is given by

$$\kappa_t^{\phi} \equiv \sum_i \xi_i^t x_i \kappa^{\zeta_i} \tag{30}$$

Definition 6. I denote by $\{\kappa_t\}_{t=0}^{\infty}$ the t-periods ahead slope, whose elements $(\kappa_t)_{ih}$ are the impulse response of inflation in sector i at time 0 to the employment of factor h at time $t \geq 0$. Given any sectoral weights ϕ, the elasticity of current inflation π_0^{ϕ} to a proportional increase in the employment gap of all factors t periods ahead is denoted by

$$\bar{\kappa}_t^{\phi} = \sum_h \kappa_{ht}^{\phi}$$

Remark 7. Two special cases are worth mentioning. First, in the static economy with $\rho = 0$ and $\mathbf{p}_{-1} = \mathbf{0}$, the slope κ derived in Proposition 1 coincides with the coefficient κ_0 derived from the eigendecomposition (29). Second, in a representative agent economy with only one good ($N_w = \bar{N} = 1$, $N_f = 0$) it holds that $\kappa_t = \kappa_0 \ \forall t$.

Proposition 4 derives a dynamic analog of equation (28). Following definition 6, the impact response of the GDP deflator is given by

$$\pi_0^Y = \sum_{t=0}^{\infty} \rho^t \kappa_t^Y \left[\boldsymbol{\ell}_t + \Phi^{-1} \Gamma \mathcal{S}^{-1} \mathbf{t}_t\right] \tag{31}$$

Using equations (31) and (25), and assuming $\mathbf{p}_{-1} = \mathbf{0}$, we can express the impact response of aggregate output as a function of the coefficients $\left\{\kappa_t^Y\right\}_{t=0}^{\infty}$ and of the rate of decay of the employment impulse-response, $\frac{\boldsymbol{\ell}_t}{\bar{y}_0}$:

$$\bar{y}_0 = \frac{nGDP_0 - \sum_{t=0}^{\infty} \rho^t Cov_s\left(\frac{\boldsymbol{\kappa}_t^Y}{\mathbf{s}}\Phi^{-1}\Gamma, \mathcal{S}^{-1}\mathbf{t}_t\right)}{1 + \sum_{t=0}^{\infty} \rho^t \boldsymbol{\kappa}_t^Y \frac{\boldsymbol{\ell}_t}{\bar{y}_0}} \tag{32}$$

Proposition **??** moves from equation (32) to derive a dynamic counterpart of equation (28).

Proposition 4. *The impact response of employment to an increase in the money supply m_0 is given by*

$$\bar{y}_0 = \frac{nGDP_0 - \sum_{t=0}^{\infty} \rho^t Cov_s\left(\frac{\boldsymbol{\kappa}_t^Y}{\mathbf{s}}\Phi^{-1}\Gamma, \mathcal{S}^{-1}\mathbf{t}_t\right)}{1 + \sum_t \rho^t \left(\bar{\kappa}_t^Y + Cov_s\left(\frac{\kappa_{ht}^Y}{s_h}, \frac{\ell_{ht}}{\bar{y}_t}\right)\right)\frac{\bar{y}_t}{\bar{y}_0}} \tag{33}$$

As a special case, in the one-sector, representative-agent economy ($N_w = \bar{N} = 1$, $N_f = 0$, see Gali (2015); Woodford (2003)) the slopes κ_t^Y are constant over time ($\kappa_t^Y \equiv \kappa^Y \ \forall t$). Moreover, employment decays at the same rate as the monetary shock. Therefore if the shock decays at a constant rate η we also have $\frac{y_t}{y_0} = \eta^t$, and equation (32) simplifies to

$$y_0 = \frac{nGDP_0}{1 + \frac{\kappa^Y}{1-\rho\eta}}$$

In an economy with multiple primary factors, instead, aggregate non-neutrality is higher when employment increases relatively more for factors with flatter Phillips curves, at each given period ($Cov_s\left(\frac{\kappa_{ht}^Y}{s_h}, \frac{\ell_{ht}}{\bar{y}_t}\right) < 0$ in equation (33)). Moreover, aggregate non-neutrality is also higher when households with stronger wealth effects see a decline in the present value of their net asset positions ($\sum_{t=0}^{\infty} \rho^t Cov_s\left(\frac{\boldsymbol{\kappa}_t^Y}{\mathbf{s}}\Phi^{-1}\Gamma, \mathcal{S}^{-1}\mathbf{t}_t\right) < 0$).

4.3 Eliminating the cross-sectional effects of monetary policy

Proposition 5 characterizes a set of taxes and subsidies which eliminates the cross-sectional effects of monetary policy on employment and income, when all primary factors have the same wealth effects and Frish elasticities in labor supply. As it is clear from equation (34) this scheme requires industry-specific subsidies, along with agent-specific transfers, which depend on the details of the input-output structure.

Proposition 5. *Suppose that all primary factors have the same wealth effects and Frish elasticities in labor supply ($\gamma_h = \gamma$, $\varphi_h = \varphi \ \forall h$). Then there exists a set of input subsidies $\{\tau_i\}_{i=1}^N$ and lump-sum taxes $\{T_h\}_{h=1}^H$, such that monetary policy has no effect on cross sectional employment and incomes when complemented with these taxes and subsidies. The relative input subsidies paid to industries i and j are proportional to the differential exposure of prices in the two industries to total labor costs:*

$$\tau_i - \tau_j \propto \left(\Delta\left(I - \Omega\Delta\right)^{-1}\bar{\alpha}\right)_i - \left(\Delta\left(I - \Omega\Delta\right)^{-1}\bar{\alpha}\right)_j \tag{34}$$

where $\bar{\alpha}_i = \sum_h \alpha_{ih}$. Lump-sum taxes are then set so that all the agents' incomes, including labor and profit rebates, change proportionately. The level of the subsidies is pinned down by imposing that lump-sum taxes sum to 0 and the government budget is balanced. Under these taxes and subsidies, the impulse responses of sectoral inflation and aggregate real GDP to monetary policy are the same as with a representative agent, with sectoral labor shares given by $\bar{\alpha}$.

5 Empirical evidence

5.1 Data

I follow the BEA 3-digit industry classification and the IPUMS 2010 2-digit classification of occupations, excluding the non-employed and active military. This leaves us with 71 industries and 22 occupations.

Occupation level expenditure shares on different consumption goods are computed from the Consumer Expenditure Survey (CEX) for the year 2007, following the methodology in Borusyak and Jaravel (2021). The probability that workers in each occupation are hired by each industry is computed based on the American Community Survey (ACS) for the year 2012. Estimates of industry-level price adjustment probabilities are computed by Pasten et al. (2017), based on micro data underlying sectoral price series published by the BLS. Estimates of occupation-level wage rigidity are based on Grigsby et al. (2019). As their dataset only reports wages adjustment probabilities for five broad industries – not by occupation – I proxy for occupation level wage adjustment frequencies by averaging industry level frequencies, weighted by the probability that each occupation is hired by the given industry (computed based on ACS data).

Inflation series at the industry level are based on the Producer Price Index (PPI) database published by the BLS. Time series of occupation-level employment, hours worked, hourly wages and weekly earnings are provided by the Current Population Survey (CPS).

5.2 Stylized facts

Based on data for the US, this Section documents heterogeneity in the degree of nominal rigidity faced by workers in different occupations, both on the consumption side, through the price stickiness of the goods they buy, and on the employment side, through the price stickiness of the goods they make. It then moves on to connect these micro-level facts with the cyclical volatility of prices and employment. It first documents large heterogeneity in the cyclical volatility of industry-level prices, which – as expected – is strongly correlated with estimates of industry-level price flexibility based on micro data. It then documents significant heterogeneity also in the cyclicality of employment across different occupations, and shows that it is negatively correlated with the average price cyclicality of the goods that they produce.

5.2.1 Nominal rigidities across occupations

Figure 5 shows that there is substantial heterogeneity in the price adjustment frequency of the industries which hire different occupations. This is computed by averaging industry level frequencies, weighted by the probability that a given occupation is hired by each industry.

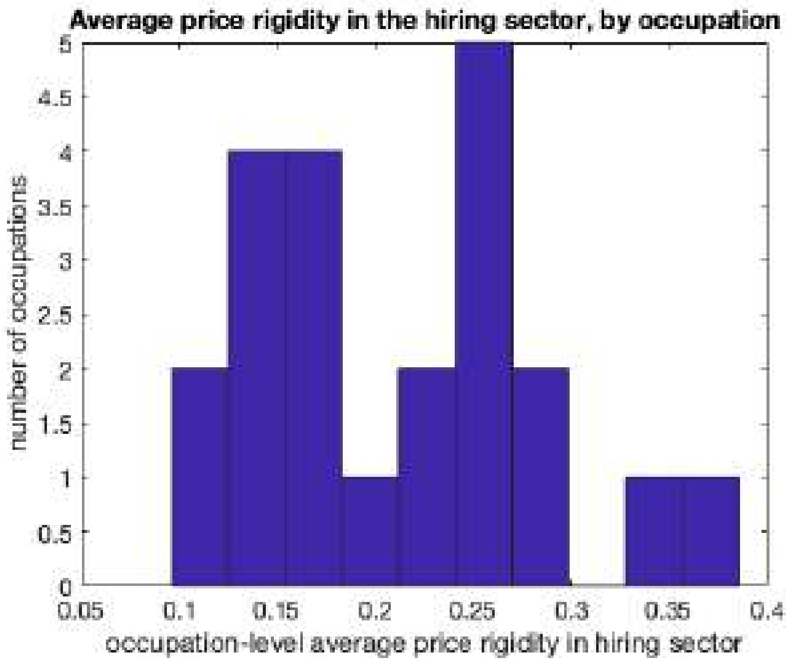

Figure 5: Average price adjustment probability in the hiring sector, by occupation

Based on the available data, other elements of heterogeneity seem less relevant. Figure 6 plots the average price adjustment probability of goods purchased by individuals in different occupations, weighted by consumption shares.

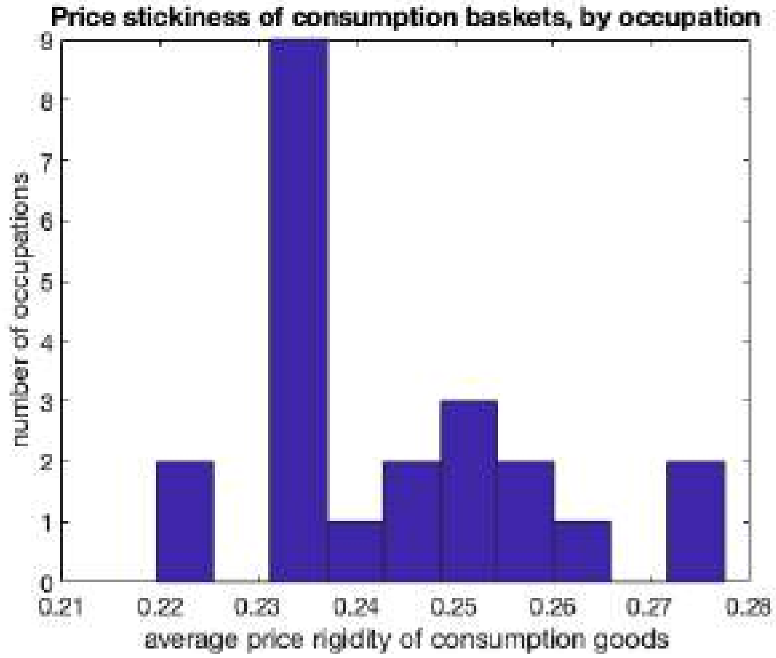

Figure 6: Average price stickiness of goods consumed, by occupation

The figure displays little heterogeneity, suggesting that this is not an important driver of cross-sectional non-neutrality from a quantitative point of view. Finally, Figure 7 plots the distribution of occupation-level wage rigidity computed based on Grigsby et al. (2019). Again, there is relatively little heterogeneity across occupations, likely due to lack of detailed occupation level data.

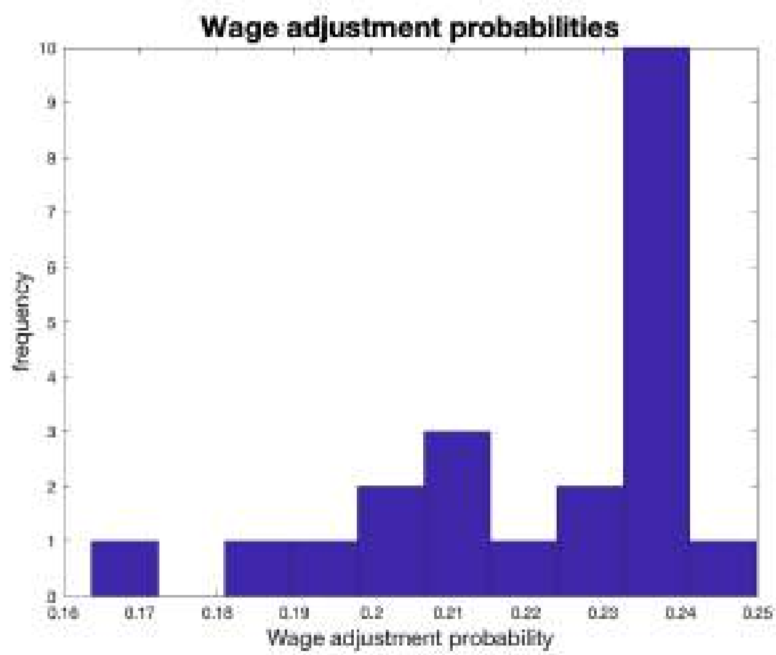

Figure 7: Distribution of wage adjustment probabilities across occupations

5.2.2 Cross-sectional cyclicality of prices and employment

This section presents a cross-sectional revisit of business cycle facts, in the spirit of Cooley and Prescott (1995). The first column of Tables 4 and 5 report standard deviations of hp-filtered employment and wages at the occupation level, while the remaining columns report their correlations with leads and lags of aggregate employment.

Figures 8 through 10 highlight some notable features of the tables.

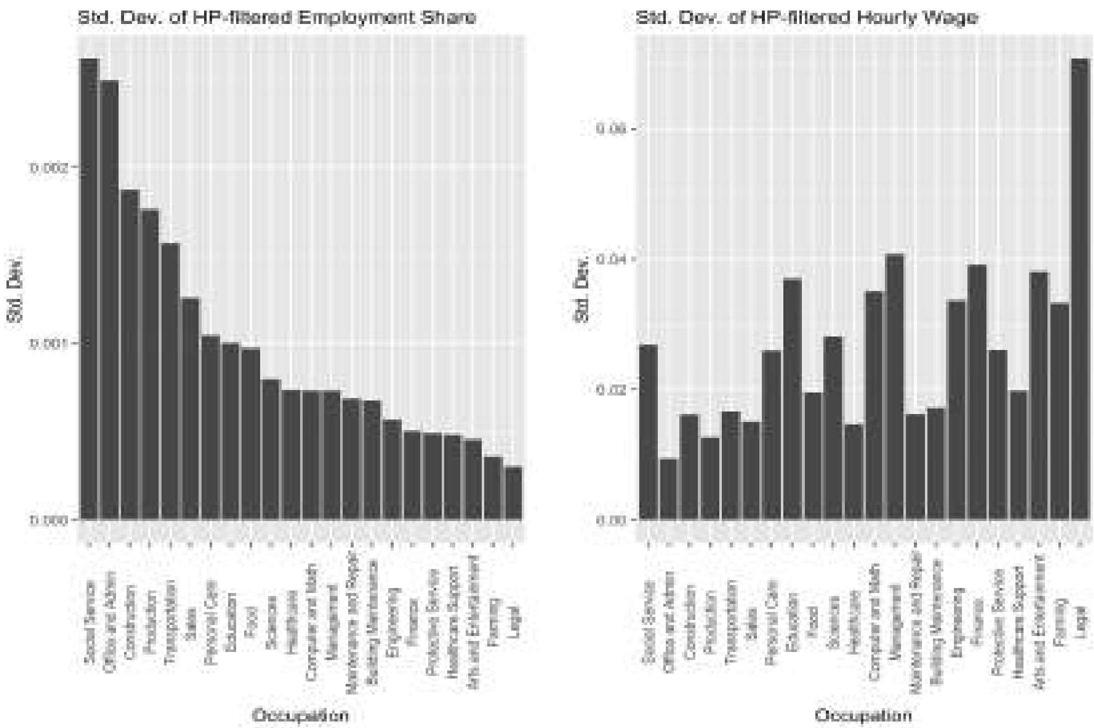

Figure 8:

Series	Occupation	Std Dev	Lag 4	Lag 3	Lag 2	Lag 1	Contemp	Lead 1	Lead 2	Lead 3	Lead 4
Employment Share	Construction	0.0019	0.30	0.47	0.59	0.72	0.82	0.81	0.73	0.62	0.50
	Production	0.0018	0.09	0.29	0.50	0.68	0.79	0.76	0.67	0.54	0.38
	Sales	0.0013	0.26	0.37	0.43	0.51	0.62	0.57	0.52	0.46	0.33
	Transportation	0.0016	0.37	0.41	0.43	0.45	0.43	0.38	0.24	0.08	-0.03
	Engineering	0.0006	0.36	0.44	0.49	0.44	0.41	0.33	0.18	0.07	0.02
	Maintenance and Repair	0.0007	0.14	0.25	0.32	0.37	0.40	0.39	0.38	0.36	0.29
	Sciences	0.0008	0.35	0.39	0.40	0.41	0.38	0.30	0.20	0.14	0.11
	Management	0.0007	0.14	0.23	0.30	0.35	0.38	0.31	0.26	0.17	0.02
	Social Service	0.0026	0.59	0.57	0.53	0.45	0.36	0.21	0.05	-0.10	-0.21
	Office and Admin	0.0025	-0.19	-0.06	0.07	0.17	0.24	0.30	0.33	0.35	0.32
	Building Maintenance	0.0007	0.02	0.09	0.11	0.15	0.21	0.22	0.24	0.27	0.26
	Arts and Entertainment	0.0005	0.02	0.02	0.05	0.13	0.17	0.15	0.10	0.05	-0.02
	Legal	0.0003	-0.18	-0.13	-0.05	0.00	0.05	0.10	0.18	0.23	0.26
	Education	0.0010	0.08	0.04	0.01	-0.01	-0.04	-0.06	-0.10	-0.12	-0.14
	Personal Care	0.0010	-0.13	-0.09	-0.06	-0.05	-0.04	-0.02	0.01	0.06	0.12
	Farming	0.0004	-0.15	-0.14	-0.12	-0.10	-0.07	-0.01	-0.01	0.05	0.07
	Food	0.0010	-0.19	-0.17	-0.10	-0.08	-0.08	-0.02	0.02	0.05	0.09
	Finance	0.0005	-0.08	-0.13	-0.16	-0.15	-0.12	-0.12	-0.08	-0.04	-0.07
	Computer and Math	0.0007	0.04	-0.03	-0.08	-0.10	-0.14	-0.13	-0.11	-0.11	-0.08
	Healthcare Support	0.0005	-0.06	-0.08	-0.13	-0.17	-0.19	-0.20	-0.14	-0.06	0.01
	Healthcare	0.0007	-0.08	-0.19	-0.26	-0.28	-0.27	-0.20	-0.17	-0.12	-0.10
	Protective Service	0.0005	-0.32	-0.35	-0.35	-0.35	-0.29	-0.22	-0.11	0.02	0.08

Table 4: Correlation values of employment share with FRED employment share.

Series	Occupation	Std Dev	Lag 4	Lag 3	Lag 2	Lag 1	Contemp	Lead 1	Lead 2	Lead 3	Lead 4
Hourly Wage	Construction	0.0161	0.15	0.07	-0.01	-0.07	-0.18	-0.29	-0.37	-0.43	-0.47
	Production	0.0127	0.06	0.01	0.00	-0.07	-0.10	-0.14	-0.19	-0.19	-0.22
	Sales	0.0150	-0.15	-0.11	-0.08	-0.05	-0.01	0.05	0.04	0.08	0.06
	Transportation	0.0166	-0.15	-0.14	-0.15	-0.19	-0.18	-0.16	-0.18	-0.17	-0.16
	Engineering	0.0336	-0.01	0.00	0.00	-0.01	-0.01	-0.02	0.01	0.02	0.02
	Maintenance and Repair	0.0162	-0.00	0.07	0.15	0.15	0.18	0.12	0.02	-0.06	-0.15
	Sciences	0.0281	-0.12	-0.07	0.01	-0.01	0.01	0.01	-0.01	-0.03	-0.05
	Management	0.0407	-0.03	0.03	0.04	0.07	0.11	0.11	0.09	0.08	0.12
	Social Service	0.0268	-0.14	-0.13	-0.10	-0.08	-0.05	-0.05	-0.02	-0.05	-0.04
	Office and Admin	0.0094	-0.37	-0.32	-0.23	-0.18	-0.16	-0.16	-0.17	-0.13	-0.14
	Building Maintenance	0.0171	-0.04	-0.02	0.05	0.04	-0.02	-0.06	-0.10	-0.11	-0.18
	Arts and Entertainment	0.0381	-0.03	-0.05	-0.04	-0.01	-0.03	0.01	-0.02	-0.02	-0.04
	Legal	0.0708	-0.03	-0.03	-0.02	-0.01	-0.03	-0.06	-0.08	-0.11	-0.09
	Education	0.0370	-0.07	0.01	0.05	0.13	0.15	0.16	0.15	0.12	0.08
	Personal Care	0.0259	-0.06	-0.03	0.01	0.04	0.05	0.05	0.07	0.06	0.00
	Farming	0.0333	-0.17	-0.12	-0.06	-0.08	-0.08	-0.12	-0.17	-0.17	-0.19
	Food	0.0195	-0.06	-0.01	0.00	0.03	-0.01	-0.04	-0.08	-0.04	-0.05
	Finance	0.0391	-0.04	0.05	0.09	0.06	0.05	0.05	0.09	0.08	0.11
	Computer and Math	0.0350	-0.05	-0.04	0.04	0.05	0.05	0.05	0.06	0.03	0.02
	Healthcare Support	0.0198	-0.03	-0.01	-0.05	-0.06	-0.08	-0.15	-0.15	-0.12	-0.11
	Healthcare	0.0147	-0.17	-0.16	-0.09	-0.09	-0.04	-0.08	-0.10	-0.10	-0.15
	Protective Service	0.0261	0.15	0.18	0.22	0.19	0.16	0.15	0.04	0.00	-0.08

Table 5: Correlation values of employment share with FRED employment share.

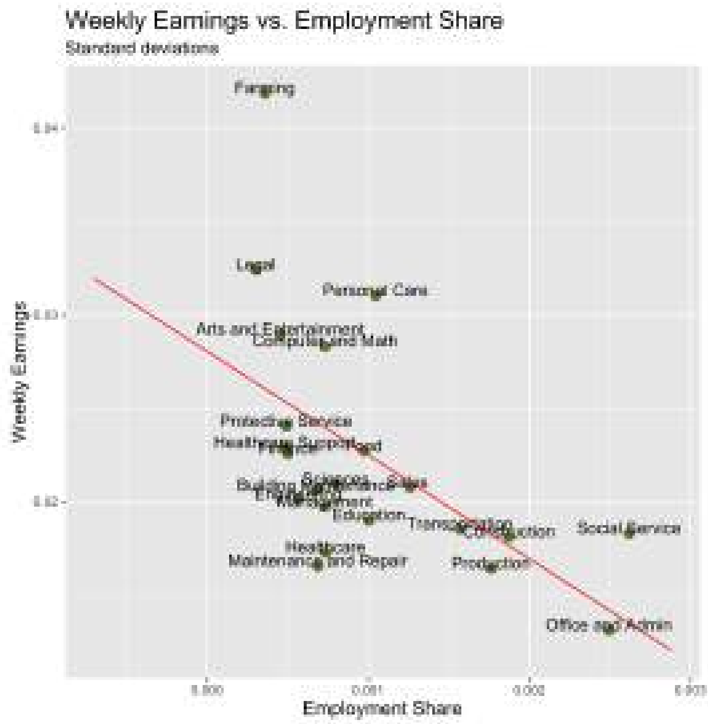

Figure 9:

Figure 8 shows that there is significant heterogeneity in the cyclical volatility of employment and wages at the occupation level, and the two are negatively correlated (figure (9)). A similar patterns emerges when measuring the correlation of the cyclical component of occupation-level employment and wages with aggregate employment. Occupations with more positive employment correlation tend to have more negative wage correlation (figure 10).

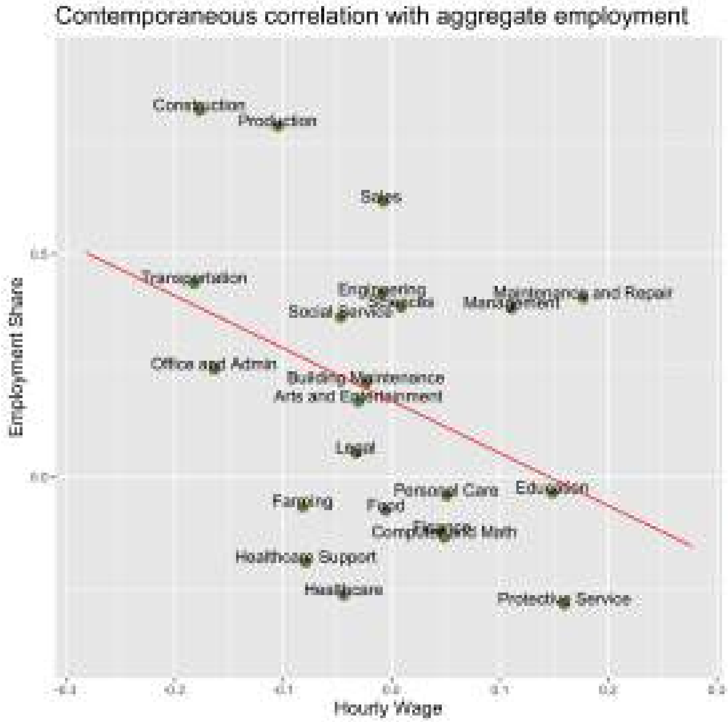

Figure 10:

The negative relation between the volatility of prices (wages) and quantities (employment) in Figure 8 and 10 presents an analogy with aggregate business cycle comovement, and it is consistent with the implications of the theoretical model presented below.

5.3 Empirical impulse-responses

TO BE COMPLETED

6 Calibration

This section calibrates the model to the US economy. To quantify the cross-sectional effects of monetary policy, Section **??** computes occupation-level impulse-responses of employment to a 1% shock to the nominal interest rate. The model predicts substantial heterogeneity. In order to isolate its origins, Sections **??** and **??** compare the baseline model with alternative calibrations, which shut down heterogeneity through input-output linkages and complementarity with fixed assets.

6.1 Data

I calibrate the preference parameters in Table 3 to conventional values. The discount factor is $\rho = 0.9975$, and the wealth effect in labor supply is set to $\gamma = 1$ for all agents. Occupation-specific inverse Frish elasticities are calibrated based on Webber (2018). Final consumption shares β for each occupation are computed by combining data from the BEA input-output accounts and the Consumer Expenditure Survey (CEX) for the year 2007, following the methodology in Borusyak and Jaravel (2021). The production parameters are calibrated as described in Section 5.1. The elasticities of substitution in production and consumption are set to consensus values in the literature. The substitution elasticity between consumption goods is $\sigma = 0.9$,[6] the elasticity of substitution between labor and intermediate inputs is $\theta_L = 0.5$, and [7] the elasticity of substitution across intermediate inputs is $\theta = 0.1$.

I calibrate ownership shares in firms and capital assets by combining data on asset ownership across income quintiles from the Survey of Consumer finances with income data at the occupation level from the ACS.

6.2 Model-implied impulse responses

6.2.1 Risk free asset, no borrowing constraints

I start by considering a model where households can freely trade a risk-free asset, with no borrowing constraints. Therefore all households must be on their Euler equations:

$$\Gamma\left(\mathbf{c}_{t+1} - \mathbf{c}_t\right) = \mathbf{1} i_{t+1} - \beta_H^T \boldsymbol{\pi}_{t+1}$$

where the H subscript denotes the subset of a vector or matrix corresponding to households. Combining the Euler equations with the budget constraints and the relative demand equation implies the following dynamic equilibrium system:

$$\begin{cases} \boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1} = \hat{\kappa}\left[\mathbf{1}\bar{y} + \left[\mathcal{D}_T + \Phi^{-1}\Gamma\right]\mathcal{S}^{-1}\mathbf{t}_t\right] - \hat{\mathcal{V}}\left(\mathbf{p}_{t-1} + \rho \mathbb{E}\boldsymbol{\pi}_{t+1}\right) & \text{Phillips curves} \\ \bar{y}_{t+1} = \bar{y}_t + \frac{\mathbf{s}_H^T\left(I + \mathcal{S}_H^{-1}\Xi_{HL}\mathcal{L}\mathcal{D}_T^{:,H}\right)^{-1}}{\mathbf{s}_H^T\left(I + \mathcal{S}_H^{-1}\Xi_{HL}\mathcal{L}\mathcal{D}_T^{:,H}\right)^{-1}\mathbf{1}}\left[\Gamma_H^{-1}\mathbf{1} i_{t+1} - \hat{\beta}^T \boldsymbol{\pi}_{t+1}\right] & \text{Aggregate Euler equation} \\ \mathcal{S}_H^{-1}\left(\mathbf{t}_{t+1} - \mathbf{t}_t\right) = \left(I - \frac{\left(I + \mathcal{S}_H^{-1}\Xi_{HL}\mathcal{L}\mathcal{D}_T^{:,H}\right)\mathbf{1}\mathbf{s}_H^T}{\mathbf{s}_H^T\left(I + \mathcal{S}_H^{-1}\Xi_{HL}\mathcal{L}\mathcal{D}_T^{:,H}\right)^{-1}\mathbf{1}}\right)\left(I + \mathcal{S}_H^{-1}\Xi_{HL}\mathcal{L}\mathcal{D}_T^{:,H}\right)^{-1}\left[\Gamma_H^{-1}\mathbf{1} i_{t+1} - \hat{\beta}^T \boldsymbol{\pi}_{t+1}\right] & \text{Asset trades} \\ i_{t+1} = \phi_\pi \pi_t^Y + \phi_y \bar{y}_t + \epsilon_{yt} & \text{Policy rule} \end{cases}$$

where

$$\hat{\kappa} \equiv \mathcal{P}_L\left[I - \Phi\left(\mathcal{D}_p + \Phi^{-1}\underline{\beta}^T\right)\mathcal{P}_L\right]^{-1}\Phi$$

$$\hat{\mathcal{V}} \equiv \mathcal{P}\Psi^{-1} - \hat{\kappa}\left(\mathcal{D}_p + \Phi^{-1}\underline{\beta}^T\right)\left(I - \mathcal{P}\Psi^{-1}\right)$$

$$\hat{\beta}^T \equiv \Gamma_H^{-1}\beta_H^T + \left[\mathcal{S}_H^{-1}\left(\Xi_{HL}\mathcal{L}\mathcal{D}_p\Psi + \Xi_H\bar{\boldsymbol{\Psi}}\otimes\right) - \Psi_{H:}\right]\Psi^{-1}$$

Figure 12 plots occupation-level impulse responses of employment to a 1% shock to the Taylor rule. The blue

[6] Atalay (2017), Herrendorf et al. (2013), and Oberfield and Raval (2014) estimate it to be slightly less than one.
[7] This is consistent with Atalay (2017), who estimates this parameter to be between 0.4 and 0.8.

lines correspond to employment in each given occupation, while the red lines plot the impulse-response of aggregate employment. The calibrated employment responses have very different magnitudes across occupations. The next paragraphs study how different dimensions of heterogeneity (wage rigidity, labor supply elasticity, position in the input-output network) affect cross-sectional non-neutrality.

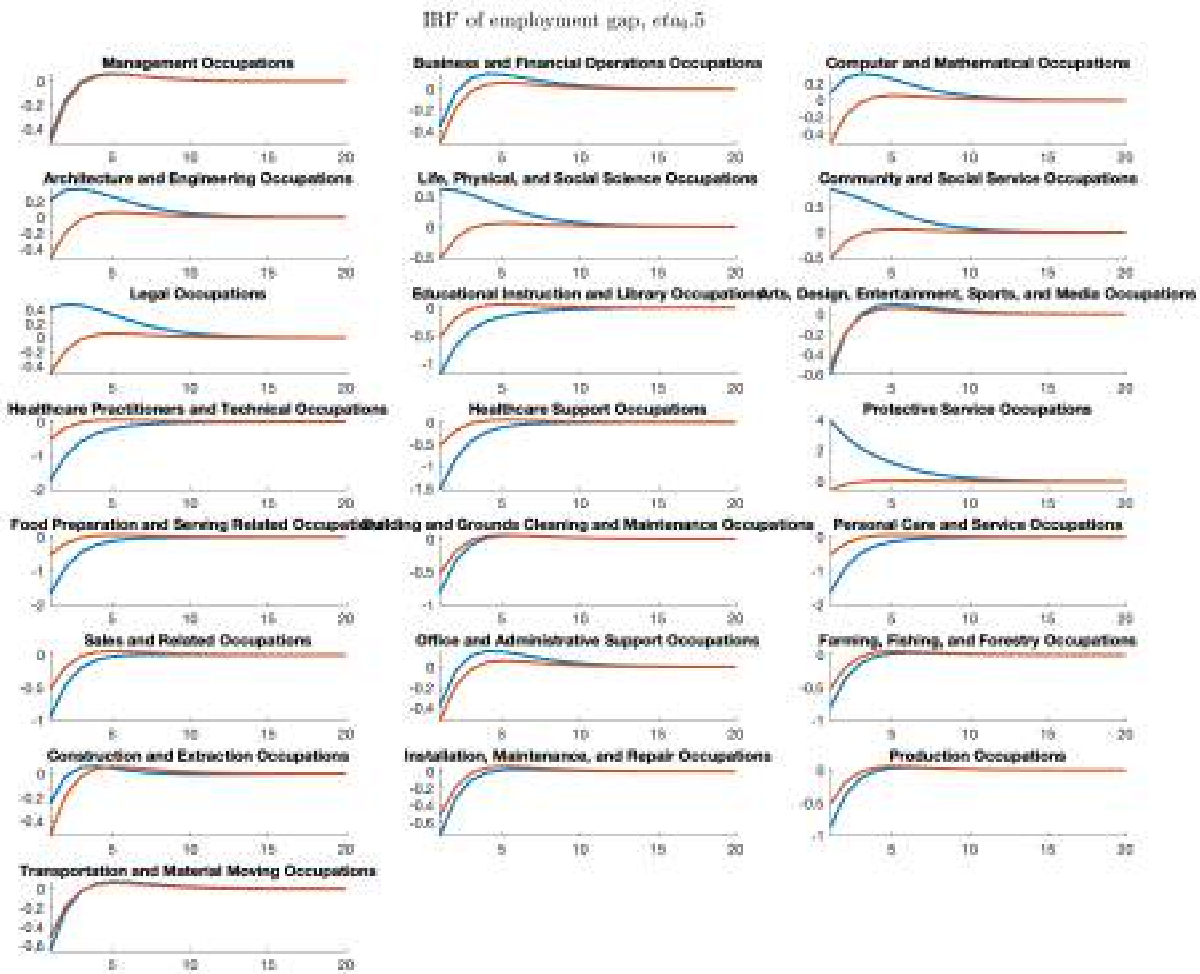

Figure 11: Impact responses of employment to a 1% increase in the money supply.

Nominal rigidities and semi-fixed assets Figures **??** and **??** compare the baseline model with alternative models without heterogeneity in nominal rigidities or without semi-fixed assets.

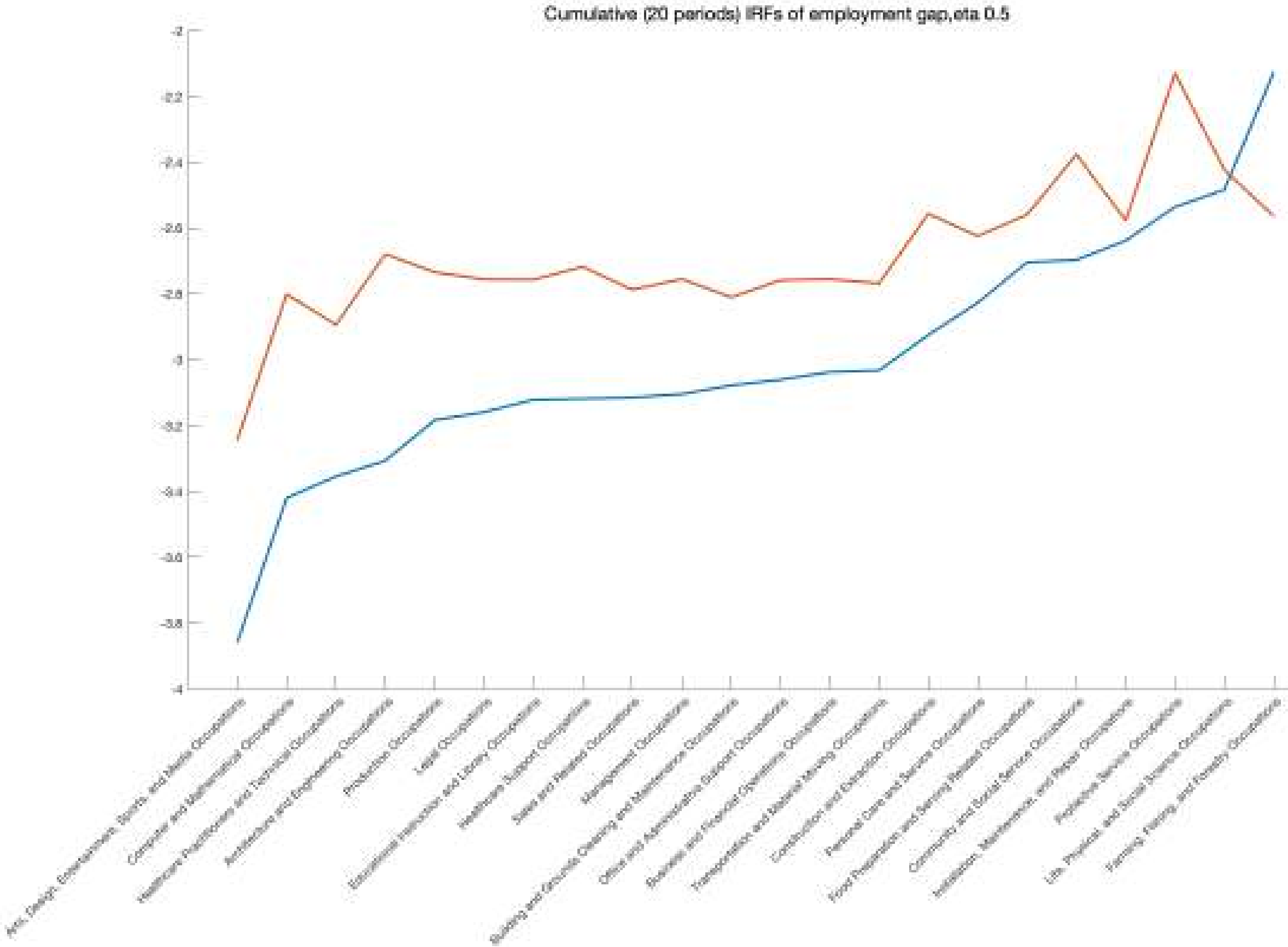

Figure 12: Impulse-responses of occupation-level employment to a 1% increase in the money supply. The red lines correspond to aggregate employment.

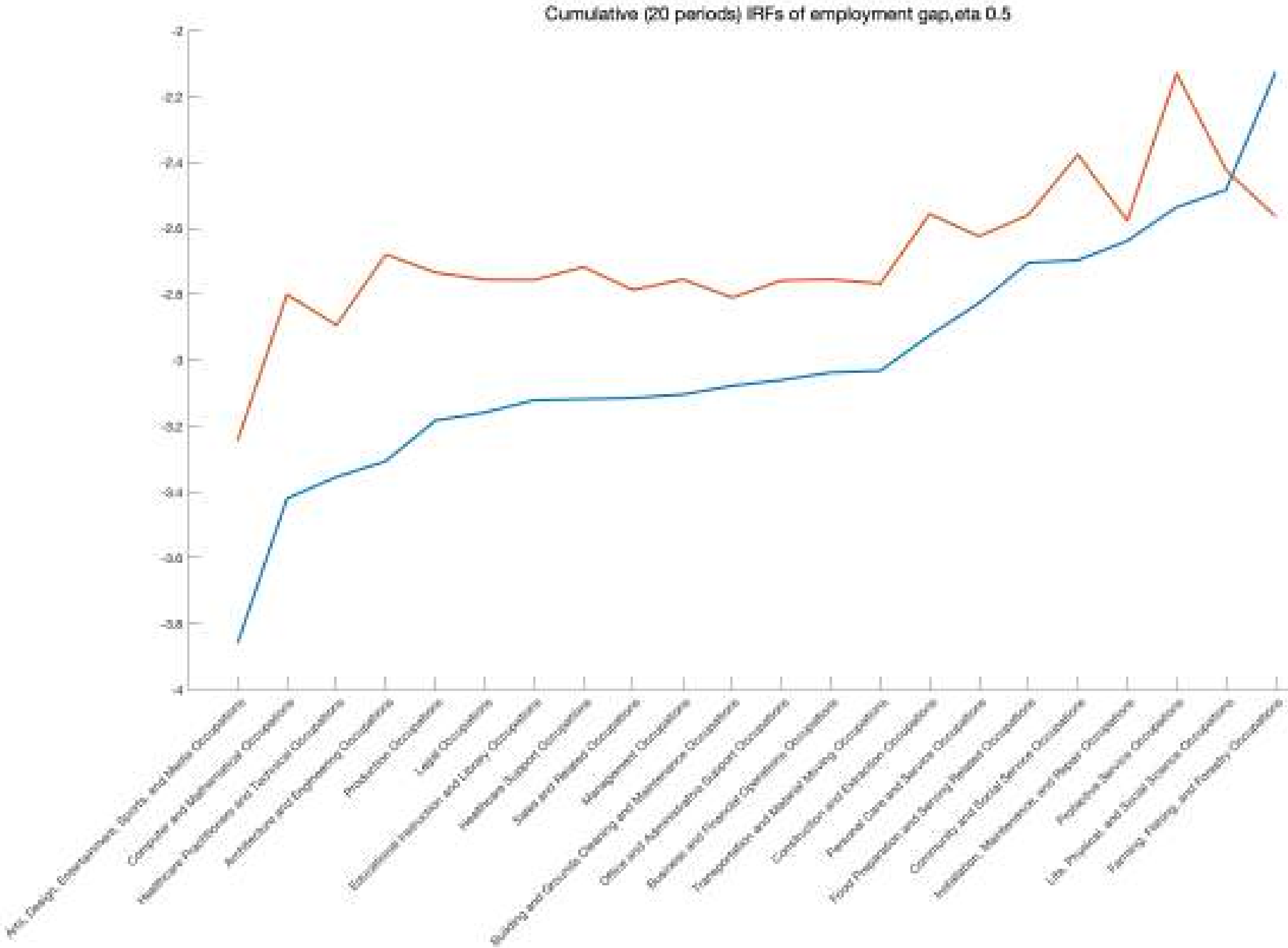

Figure 13: Impact responses of employment to a 1% increase in the money supply.

Input-output linkages Figure 14 compare the baseline model with an alternativ models without input-output linkages.

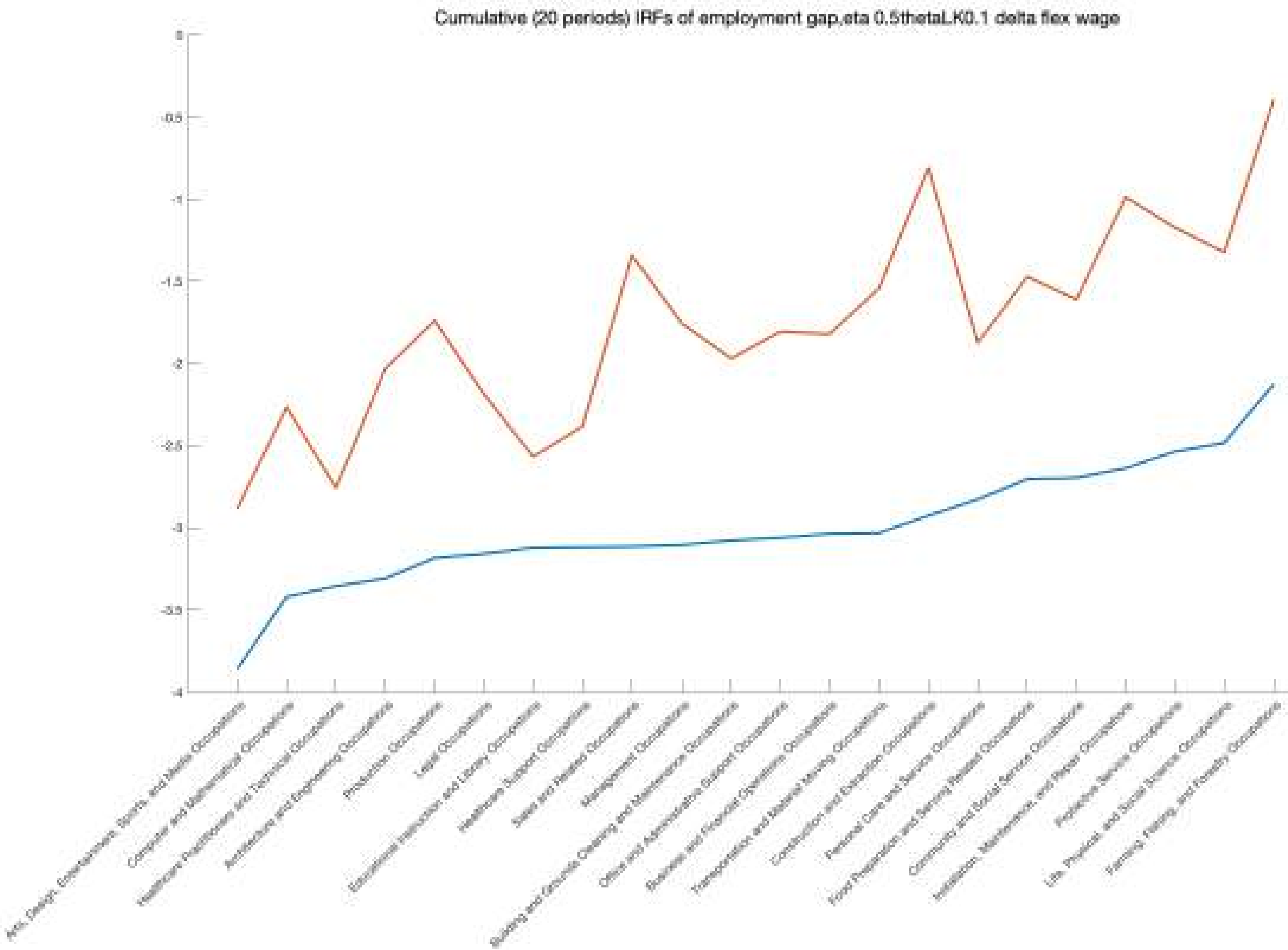

Figure 14: Impact-responses of employment to a 1% increase in the money supply

6.2.2 General HANK model

In this section I compute impulse responses of employment and consumption to a 1% shock to the Taylor rule in a version of the general consumption-savings model in equations (22) and (23). I assume that MPCs only depend on income levels, and combine data on marginal propensities to consume across the income distribution from Patterson (2023) with income data from the ACS to construct average MPCs at the occupation level. I then use equation (**??**) to calibrate the intertemporal response of consumption to interest rates ($\mathcal{J}^R$).

7 Conclusion

Based on a New Keynesian model with multiple heterogeneous households and industries, I show that monetary policy has heterogeneous effects on real income across households.

A monetary expansion increases labor demand relatively more for households that face more nominal rigidity in employment and/or consumption, and for households that have a more elastic labor supply, or whose labor is complementary with elastically supplied production factors. These households end up having flatter Phillips curves, and – like in the textbook representative agent model – a flat Phillips curve is associated with larger real effects of monetary policy. By contrast, demand-side heterogeneity – through heterogeneous substitution elasticities or home bias in consumption – by itself does not imply any cross-sectional effects of monetary policy.

Calibrating the model to US national account data reveals sizable heterogeneity in the employment response to monetary shocks across occupations.

References

Aoki, K. (2001): "Optimal monetary policy responses to relative-price changes," *Journal of Monetary Economics*, 48, 55–80.

Atalay, E. (2017): "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics*, 9, 254–280.

Auclert, A. (2019): "Monetary Policy and the Redistribution Channel," *American Economic Review*, 109, 2333–2367.

Auclert, A., B. Bardoczy, M. Rognlie, and L. Straub (2021): "Using the SequenceâSpace Jacobian to Solve and Estimate HeterogeneousâAgent Models," *Econometrica*, 89, 2375–2408.

Auclert, A., M. Rognlie, and L. Straub (2018): "The Intertemporal Keynesian Cross," Tech. Rep. w25020, National Bureau of Economic Research, Cambridge, MA.

Baqaee, D. R. and E. Farhi (2018): "Macroeconomics with Heterogeneous Agents and Input-Output Networks," Working Paper 24684, National Bureau of Economic Research.

——— (2020): "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 135, 105–163.

Basu, S. (1995): "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *The American Economic Review*, 85, 512–531.

Benigno, P. (2004): "Optimal monetary policy in a currency area," *Journal of International Economics*, 63, 293–320.

Borusyak, K. and X. Jaravel (2021): "The Distributional Effects of Trade: Theory and Evidence from the United States," Tech. Rep. w28957, National Bureau of Economic Research, Cambridge, MA.

Carvalho, C. (2006): "Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks," *The B.E. Journal of Macroeconomics*, 6, 1–58.

Coglianese, J., M. Olsson, and C. Patterson (2021): "Monetary policy and the labor market: a quasi-experiment in sweden," *Working paper*.

Cooley, T. F. and E. C. Prescott (1995): "1. Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton University Press, 1–38.

Devereux, M. B. and C. Engel (2003): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *The Review of Economic Studies*, 70, 765–783.

Gali, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton ; Oxford: Princeton University Press, second edition ed.

Gali, J. and T. Monacelli (2008): "Optimal monetary and fiscal policy in a currency union," *Journal of International Economics*, 76, 116–132.

Grigsby, J., E. Hurst, and A. Yildirmaz (2019): "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data," Tech. Rep. w25628, National Bureau of Economic Research, Cambridge, MA.

Herrendorf, B., R. Rogerson, and Ã. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–2789.

Huang, K. X. D. and Z. Liu (2007): "Business cycles with staggered prices and international trade in intermediate inputs," *Journal of Monetary Economics*, 54, 1271–1289.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary Policy According to HANK," *American Economic Review*, 108, 697–743.

La'O, J. and W. Morrison (2023): "Optimal Monetary Policy with Redistribution," *Working Paper*.

LaO, J. and A. Tahbaz-Salehi (2019): "Optimal Monetary Policy in Production Networks," *SSRN Electronic Journal*.

Minton, R. and B. Wheaton (2022): "Minimum Wages and the Rigid-Wage Channel of Monetary Policy," *Working Paper*.

Nakamura, E. and J. Steinsson (2010): "Monetary Non-Neutrality in a Multisector Menu Cost Model," *Quarterly Journal of Economics*, 125, 961–1013.

Oberfield, E. and D. Raval (2014): "Micro Data and Macro Technology," NBER Working Paper 20452, National Bureau of Economic Research, Inc.

Pasten, E., R. Schoenle, and M. Weber (2017): "Price Rigidity and the Origins of Aggregate Fluctuations," Working Paper 23750, National Bureau of Economic Research.

——— (2019): "The propagation of monetary policy shocks in a heterogeneous production economy," *Journal of Monetary Economics*.

Patterson, C. (2023): "The Matching Multiplier and the Amplification of Recessions," *American Economic Review*, 113, 982–1012.

Rubbo, E. (2023): "Networks, Phillips Curves, and Monetary Policy," *Econometrica*, 91, 1417–1455.

Webber, D. A. (2018): "Employment Adjustment over the Business Cycle: The Impact of Competition in the Labor Market," Tech. Rep. 11887, Institute of Labor Economics (IZA).

Woodford, M. (2003): *Interest and prices: foundations of a theory of monetary policy*, Princeton, N.J. ; Woodstock, Oxfordshire [England]: Princeton University Press.

Appendix

A. Income and real rate Jacobians

In this section I derive equations (22) and (23), and provide a proof of equation (**??**).

Each household maximizes the present value of utility (8) subject to the budget constraint (9) and to any portfolio adjustment costs and/or borrowing constraints $\sum_a P_a Q_a + B \geq \underline{A}$. We can always break down the household problem in two steps. First, the household chooses the optimal consumption path and asset portfolios for given labor supply (suppressing household subscripts for legibility):

$$\mathcal{W}(L) = \max_{\{C\},\{Q_a\},\{B\}} \sum_{t=0}^{\infty} e^{-\rho t} \left(\frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \right) \tag{35}$$

subject to borrowing constraints and portfolio adjustment costs. Then, the household solves

$$\max_{\{L\}} \mathcal{W}(L)$$

It follows immediately from the envelope theorem that this second problem implies the consumption-leisure tradeoff (10) as a first order condition.

Let's then focus on the maximization problem (35). For given labor supply, we can define intertemporal marginal propensities to consume and consumption responses to interest rates for each household type as in (1). If we could measure both from the data, we could use them directly in the calibration.[8] In practice, it is hard to estimate consumption responses to interest rates at constant income. I follow Auclert et al. (2018) and exploit the relation between marginal propensities to consume and interest rate responses in equation (**??**). This relation holds under the assumption that $\underline{A} = 0$ in the borrowing constraint.

To derive the relation, start from the budget constraints in equation (9), dropping the time and household subscripts for convenience:

$$C = \frac{W}{P^C} L + \sum_{a \in \mathcal{A}} \left[\frac{D_a + P_a}{P_{a-1}} - (1+I) \right] \frac{P_{a-1}}{P^C} Q_{a,-1} + (1+I) \frac{A_{-1}}{P^C} - \frac{A}{P^C} + \tilde{T}$$

where total assets A are defined as

$$A \equiv B + \sum_{a \in \mathcal{A}} P_a Q_a$$

[8]Technically, MPCs could depend on labor supply L. However, the error from using MPCs computed when L is at its steady-state value is second order.