EIEF WORKING PAPER SERIES

Einaudi Institute for Economics and Finance

EIEF Working Paper 17/01
January 2017

Ambiguous Policy Announcements

by
Claudio Michelacci
(EIEF and CEPR)
Luigi Paciello
(EIEF and CEPR)

Ambiguous Policy Announcements

Claudio Michelacci
EIEF and CEPR

Luigi Paciello*
EIEF and CEPR

December 8, 2017

Abstract

We study the effects of an announcement of a future shift in monetary policy when agents face Knightian uncertainty about the commitment capacity of the monetary authority. Households are ambiguity-averse and are differentially exposed to inflation due to differences in wealth. In response to the announcement of a future loosening in monetary policy, only wealthy households (creditors) will act as if the announcement will be fully implemented, due to the potential loss of wealth from the prospective policy easing. And when creditors believe the announcement more than debtors, their expected wealth losses are larger than the wealth gains that debtors expect. Hence the economy responds as if aggregate net wealth falls, which attenuates the effects of the announcement. We study the quantitative properties of the model in a liquidity trap after allowing for a realistic characterization of households' wealth portfolio.

*We thank Manuel Amador, Gene Ambrocio, Philippe Andrade, Francesco Caselli, Martin Ellison, Gaetano Gaballo, Nicola Gennaioli, Luigi Guiso, Cosmin Ilut, Javier Suarez, Giovanni Violante, and Mirko Wiederholt for useful comments, as well as seminar participants at all places where versions of this paper have been presented. We acknowledge the financial support of the European Research Council under ERC Advanced Grant 293692 and ERC Starting Grant 676846. E-mail: c.michelacci1968@gmail.com, gigipaciello@gmail.com. Postal address: EIEF, Via Sallustiana, 62, 00187 Roma, Italy.

1 Introduction

Policy makers often use announcements of future reform of economic institutions or changes in fiscal or monetary policy to stimulate the economy in the short run. The credibility of these announcements is sometimes uncertain and the policies typically carry important redistributive implications. For example during the Great Recession, with nominal short-term interest rates at the zero lower bound, central banks have relied extensively on announcements of future monetary policy changes to raise current inflation and stimulate the economy, a practice generally known as forward guidance. The credibility of these announcements is uncertain because they may appear to be in contrast with the legally stated primary objective of the central bank (price stability) and it is well known that inflation tends to redistribute wealth from creditors to debtors (Fisher 1933, Doepke and Schneider 2006, and Adam and Zhu 2016). In this paper we show that when agents are ambiguity-averse, these policy announcements can have little and sometimes even unintended effects in the period before the new policy is actually implemented.[1]

We consider the impact of monetary policy announcements in a new Keynesian model, where changes in nominal interest rates affect real interest rates due to sticky prices. We analyze the effects of announcements of future changes in nominal interest rates in an economy where agents have well defined expectations about the future dynamics of the economy in the absence of the announcement, while they face Knightian uncertainty about the credibility of the announcement. In this sense the announcements are "ambiguous". Households differ in their net financial asset position and are ambiguity-averse, using a worst-case criterion to assess the credibility of the announcement. In response to the announcement of a future monetary easing, just a fraction of households in the economy will act as if the announcement will be implemented and among them there are more households with great net financial wealth (in brief creditors) than households with little or negative net financial wealth (debtors), since their worst-case scenario is that real rates, hence financial income, will actually fall. And if creditors ascribe greater credibility to the announcement than debtors do, the wealth losses they expect to incur are larger than the gains that debtors expect, so the economy behaves as if expected aggregate net wealth falls.[2] We refer to this fall in expected aggregate net wealth as the *misguidance* effect, which is

[1] Ambiguity aversion with some associated Knightian uncertainty is a natural paradigm for characterizing the behavior of agents who are unable to assess the exact probability of some future contingencies, which is likely when agents have to deal with unfamiliar news, such as announcements about future unconventional policies in an unusual economic environment.

[2] Hereafter, for expositional simplicity, we refer to the worst-case beliefs of agents that drive their choices as their *beliefs* or *expectations*. In adopting this terminology, we follow the empirical literature documenting that agents act on the basis of their self-reported beliefs and expectations, see for example Hurd (2009), Kézdi and Willis (2011), Armantier, de Bruin Wändi, Topa, van der Klaauw, and Zafar (2015), and Gennaioli, Ma, and Shleifer (2016).

generally due to an adverse (endogenous) correlation between agents' wealth and the change in their (worst-case) beliefs. When financial wealth is concentrated enough, the misguidance effect can be so strong to dominate the intertemporal substitution effect on consumption typically emphasized by the literature, and lead to a contraction in activity due to lack of aggregate demand. Generally, when a policy easing is announced, the real rate expected by creditors is lower than that expected by debtors. This produces a rebalancing in the financial asset positions of households and can even cause credit crunches characterized by a collapse of financial markets, which happens because households get rid of their financial assets in order to get fully insured against future monetary policy uncertainty.

In the case of an announcement of a future monetary policy tightening (a rise in future real rates), debtors are the most likely to take the announcement as credible and for them the increase in future rates reduces consumption through both substitution and income effects. So aggregate consumption and output unambiguously fall.

We study the importance of the mechanism for an economy in a liquidity trap. We show that the misguidance effect is mitigated but still present when households can trade in real assets as well as in long-term nominal bonds. When announcing a future monetary policy easing, forward guidance and fiscal policies are complementary in stimulating the economy since the misguidance effect can be mitigated by policies which redistribute wealth from creditors to debtors at the time of the announcement so as to anticipate to today the redistribution induced by the future monetary easing.

We use the start of forward guidance by the ECB on 4 July 2013 to study the quantitative importance of the mechanism.[3] For the effect to be present (i) the central bank should announce a commitment to a future policy and (ii) ambiguity-averse households should doubt about the credibility of the announcement, which are both likely to apply in the specific episode: the announcement was generally perceived as a commitment by the ECB on keeping future interest rates low and there was (and still there is) substantial debate on whether a future monetary policy easing would imply a violation of the ECB mandate for price stability.[4] We study the effects of the ECB forward guidance announcement in

[3]On that date the ECB Governing Council announced that "it expected the key ECB interest rates to remain at present or lower levels for an extended period of time."

[4]The international press generally reported the statement by the Governing Council by saying that "the ECB will commit to keeping interest rates low" (see for example "https://www.ft.com/content/827ca972-e4d5-11e2-875b-00144feabdc0")—even if verbs like "pledge", "vow" and "commit" were not used in the original official statement by the ECB. Indeed, after the announcement, long-term government bond yields and EONIA swap rates fell by 5-10 basis points at maturities between 2 and 4 years (see Coeuré (2013), ECB (2014), and Picault (2017)), while inflation expectations were revised upward (Andrade and Ferroni 2016), which is consistent with the "Odyssean" interpretation that the announcement implied a commitment on a future monetary policy easing. During the crisis the ECB has been often accused of violating her mandate for ensuring price stability, even by the President of the Deutsche Bundesbank, Jens Weidmann, generating doubts about the future of the ECB as evidenced by the pronounced increased dispersion in how much European households trust the ECB, see Guiso, Sapienza, and Zingales (2016).

our economy, which we assume is initially in a liquidity trap. We allow agents to trade in short and long term nominal bonds as well in equity. We use data from the Household Finance and Consumption Survey (HFCS) and match the entire distribution of European households' financial assets. To calibrate the amount of uncertainty resulting from the announcement, we use a Difference-in-Differences strategy based on quarterly micro data. We construct a measure of the inflation expectations of households and find that in response to the ECB announcement creditor households experienced a relative increase in their inflation expectations, which indicates the presence of a misguidance effect. We then calibrate the ECB announcement to match the increased correlation between the inflation expectations of households and their financial asset position as implied by our Dif-in-Dif estimates. We compare the response of our economy where agents are ambiguity averse and face uncertainty about the credibility of the announcement with the full credibility benchmark where all agents accord full credit to the announcement. We find that in our model the effect of the ECB announcement on output is considerably dampened by comparison with the benchmark, despite a sizable increase in the price of long-term bonds and equity as well as in the turn-over in financial markets. Under our preferred parametrization, output increases by around a percentage point before implementation, against a gain of around 2.5% under the full credibility benchmark.

The literature. There is a vast literature on the effects of policy choices under lack of commitment, see Kydland and Prescott (1977) for a seminal contribution, Golosov and Tsyvinski (2008) for a review of the literature and Cooley and Quadrini (2004) and Golosov and Iovino (2017) for some recent study of optimal monetary policy and social insurance, respectively. This paper is first in studying the effects of monetary policy under lack of commitment when agents face Knightian uncertainty about the commitment capacity of the policy maker.[5]

There is a growing literature on optimal monetary policy in a liquidity trap (Eggertsson and Woodford 2003) as well as on the effects of forward guidance (Del Negro, Giannoni, and Patterson 2015). For conventional new Keynesian sticky-price models it is a puzzle why forward guidance has been little effective in stimulating the economy and several papers have proposed explanations for the puzzle. Wiederholt (2014) emphasizes that with dispersed information about the state of the economy (at least some) agents could interpret the announcement of future low interest rates as a bad signal of the underlying state, while Andrade, Gaballo, Mengus, and Mojon (2015) study the empirical implications of this mechanism once allowing for heterogenous beliefs on the duration of the liquidity trap. Angeletos and Chen (2017) further notice that dispersed information dampens the response

[5]In modelling, we take this uncertainty as given; see Barthelemy and Mengus (2017) for an analysis of how the monetary authority can increase the credibility of forward guidance by signalling her type before the economy enters a liquidity trap.

to forward guidance also through strategic complementarity in the action of agents, while Gaballo (2016) links the dispersion of information after forward guidance to the communication capacity of the monetary authority. In our model there is complete information on the state of the economy, but agents are ambiguity averse and doubt the credibility of the announcement, which interacts with the redistribution of wealth induced by monetary policy. Kaplan, Moll, and Violante (2016b) and McKay, Nakamura, and Steinsson (2015) stress heterogeneity in the marginal propensity to consume out of current income and emphasize that forward guidance could be little effective in stimulating the economy because agents at their financial or liquidity constraint cannot increase their consumption. Our mechanism is similar in that, in response to an inflationary announcement, it endogenously generates a small consumption response of poor households, but it does not rely on financial constraints, which as emphasized by Werning (2015) could actually be relaxed by forward guidance. García-Schmidt and Woodford (2015), Gabaix (2016), and Farhi and Werning (2017) replace rational expectations with some form of bounded rationality and obtain a muted response to forward guidance.[6] Ambiguity aversion and Knigthian uncertainty are also a departure from rational expectations, but they lead to the novel finding that forward guidance causes an adverse correlation between agents' wealth and their (worst-case) beliefs.

At least since Fisher (1933) it has been known that expansionary monetary policy redistributes wealth from creditors to debtors. It has also been observed that such redistribution could expand aggregate demand because agents may differ in marginal propensity to consume out of wealth (as first posited by Tobin, 1982), or in portfolio liquidity or term structure, as in Kaplan, Moll, and Violante (2016a) and Auclert (2015) respectively. Here we focus on the redistribution of expected wealth induced by news about future policies, which, under ambiguity aversion, is a negative-sum game because the net losers from the redistribution tend to believe the news more strongly than the net winners.

Other papers have shown the relevance of ambiguity aversion to business cycle analysis. Ilut and Schneider (2014) show that shocks to the degree of ambiguity can drive the business cycle, Backus, Ferriere, and Zin (2015) examine asset pricing, Ilut and Saijo (2016) focus on firm dynamics while Ilut, Krivenko, and Schneider (2016) devise methods suitable for stochastic economies where ambiguity-averse agents differ in their perception of exogenous shocks, and study the implications for precautionary savings and asset premiums. Here instead we focus on the effects of policy announcements, and more generally news about the future, and how they interact with wealth inequality and redistribution.

Section 2 characterizes the economy. Section 3 studies monetary policy announcements in a simple case. Section 4 extends the model, which is calibrated in Section 5 to quantify

[6] While the literature on unconventional monetary policy with bounded rationality has typically focused on forward guidance, Iovino and Sergeyev (2017) analyze quantitative easing.

the effects of forward guidance by the ECB in Section 6. Section 7 concludes. The Appendix contains details on theoretical derivations, data and model computation.

2 The model

We consider an analytically tractable New Keynesian model in discrete time. Several assumptions of the model are relaxed in Section 3.4, while Section 4 extends the model for a quantitative analysis. The economy is populated by a unit mass of households, indexed by $x \in [0,1]$, who are ambiguity-averse and differ only in net financial wealth, $a_{xt} \in [\underline{a}_t, \bar{a}_t]$, which is invested in one-period bonds paying a real interest rate r_t in period t. There is a unit mass of firms that demand labor to produce intermediate goods sold under monopolistic competition; prices are sticky. The nominal interest rate is adjusted to achieve the inflation target set by a monetary authority which has an unambiguous mandate to maintain price stability. The monetary authority has always complied with this mandate over the years. We focus on the short run response of the economy, when the monetary authority suddenly and unexpectedly announces that in the future it will deviate from its historical mandate and households doubt whether the authority will act as announced. Hereafter the convention is that, unless otherwise specified, variables are real—measured in units of the final consumption good.

Households Household $x \in [0,1]$ is infinitely-lived, with a time-t one-period-ahead subjective discount factor β_t. Her per period preferences over consumption c_{xt} and labor l_{xt} are given by

$$U(c_{xt}, l_{xt}) = \frac{\left(c_{xt} - \psi_0 \, \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma}, \tag{1}$$

with $\psi_0, \psi > 0$ and $\sigma > 1$. When all households share the same beliefs, these preferences (Greenwood, Hercowitz, and Huffman 1988) guarantee that the economy is characterized by a representative household, which is a canonical benchmark in the new Keynesian literature. Financial markets are incomplete, in that households can only invest in a one-period bond, which, at time t, pays (gross) return r_t per unit invested. Households can borrow freely by going short on the asset. The labor market is perfectly competitive, so households take the wage w_t as given. At each point in time t, household x chooses the triple $\{c_{xt}, l_{xt}, a_{xt+1}\}$ subject to the budget constraint

$$c_{xt} + a_{xt+1} \leq w_t \, l_{xt} + r_t \, a_{xt} + \lambda_t, \tag{2}$$

where a_{xt+1} measures the units invested in bonds at time t, while λ_t denotes (lump sum) government transfers (specified below).

Monetary policy rule The (gross) interest rate paid in period t is given by $r_t = R_{t-1}/\Pi_t$, where $\Pi_t = p_t/p_{t-1}$ is gross inflation realized in period t and R_{t-1} is the (gross) nominal interest rate set by the monetary authority at period $t-1$ according to the rule

$$R_t = \min\left\{1, \frac{1}{\beta_t}\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi}\right\}, \tag{3}$$

where $\phi > 1$, $1/\beta_t$ represents the natural rate of interest, and Π_t^* is the time-t *inflation target*, which we assume is equal to one in steady state, $\bar{\Pi}_t^* = 1$.

Firms The final consumption good is produced by a (representative) competitive firm, which uses a continuum of varieties $i \in [0,1]$ as inputs according to

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \tag{4}$$

where y_{it} is the amount of variety i used in production. The variety i is produced only by a firm i, which uses a linear-in-labor production function, so that $y_{it} = \ell_{it}$, where ℓ_{it} denotes firm i's demand for labor whose unit cost is w_t. Firm $i \in [0,1]$ sets the nominal price for its variety p_{it} to maximize expected profits at the beginning of the period, $d_{it} \equiv y_{it}\,(p_{it}/p_t - w_t)$, taking as given the demand schedule by the competitive firm, the aggregate nominal price, p_t, and the wage rate, w_t. We assume firm i chooses its nominal price at time t, p_{it}, after the monetary authority has set the inflation target Π_t^*, but before any time-t policy announcement. Finally, we posit initially that the government owns all the firms in the economy and rebates profits back to households in lump-sum fashion, so that $\lambda_t = \int_0^1 d_{it}\,di$.

Market clearing In equilibrium, output Y_t is equal to aggregate consumption $C_t \equiv \int_0^1 c_{xt}\,dx$, so that $Y_t = C_t$, and labor demand is equal to labor supply, $\int_0^1 \ell_{it} di = \int_0^1 l_{xt} dx$. Since bonds are in zero net supply, clearing the financial market requires that $\int_0^1 a_{x,t} = 0$.

Steady state At $t = 0$, the economy is initially in a steady state with $\beta_t = \beta < 1$, where a monetary authority with an unambiguous mandate for price stability has always set $\Pi_t^* = 1$, and households expect Π_t^* to remain equal to one also in any future t, implying $\bar{r} = \bar{R} = 1/\beta$ and $\bar{\Pi} = 1$, where the upper bar denotes the steady state value of the corresponding quantity.

Policy announcement At $t = 0$ (after firms have set their nominal price), the monetary authority announces that in period $T > 0$, and only at T, the inflation target will deviate from full price stability, implying that $\Pi_T^* = \varepsilon$, and $\Pi_t^* = 1$ for all $t \neq T$. If $\varepsilon > 1$, the announcement is *inflationary*; if $\varepsilon < 1$, it is *deflationary*. On the basis of the announcement, household $x \in [0,1]$ makes her decisions on consumption, labor supply and saving, while firm $i \in [0,1]$ supplies any amount demanded at its set price.

Ambiguity aversion and uncertainty Households are ambiguity-averse as in the multiple priors utility model of Gilboa and Schmeidler (1989), whose axiomatic foundations are provided by Epstein and Schneider (2003). Households fully understand the wording of the policy announcement, but they are uncertain about whether the monetary authority will actually deviate from her historical mandate for price stability at time T. In particular, households assume that the monetary authority will set the inflation target at time T, Π_T^*, to minimize

$$\boldsymbol{L} = (1 - \Pi_T^*)^2 + \gamma\,(\varepsilon - \Pi_T^*)^2\,, \tag{5}$$

where $\gamma \in R^+$ measures the *credibility* of the monetary authority and ε is the monetary announcement, with the convention that $\varepsilon = 1$ denotes no announcement. The first term in (5) is the cost to the authority of deviating from price stability, the second is the credibility cost of reneging the announcement. The credibility parameter γ is fully known to the monetary authority, so households infer that

$$\Pi_T^* = \frac{1 + \gamma\varepsilon}{1 + \gamma}. \tag{6}$$

Households have multiple priors about the probability distribution of γ and we start assuming that γ could be any random variable on the positive real line. Given (6), then households conclude that Π_T^* could be any random variable with support $\Omega \subseteq \mathcal{S}_{T-1}$ where

$$\mathcal{S}_{T-1} = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}]. \tag{7}$$

When the announcement is inflationary, $\varepsilon > 1$, Π_T^* could have any value in $\mathcal{S}_{T-1} = [1, \varepsilon]$; when it is deflationary, $\varepsilon < 1$, Π_T^* could have any value in $\mathcal{S}_{T-1} = [\varepsilon, 1]$.

The utility of household x is given by the sum of the felicity from time-t consumption and labor plus the expected continuation utility, which is evaluated at the household's worst-case scenario on the realizations of the inflation target. Formally, we assume that preferences at time t order future streams of consumption, $\mathbf{C_t} = \{c_s(h^s)\}_{s=t}^{\infty}$, and labor supply, $\mathbf{L_t} = \{l_s(h^s)\}_{s=t}^{\infty}$, so that utility is defined recursively as

$$V_t(\mathbf{C_t}, \mathbf{L_t}) = U(c_t(h^t), l_t(h^t)) + \beta_t \min_{\Omega \subseteq \mathcal{S}_t,\ G \in \mathscr{P}(\Omega)} \int_\Omega V_{t+1}(\mathbf{C_{t+1}}, \mathbf{L_{t+1}})\, G(d\Pi_{t+1}^*), \tag{8}$$

where $h^t = \{\Pi_{-\infty}^*, ..., \Pi_{t-1}^*, \Pi_t^*\}$ denotes history up to time t, and Ω is the support of the probability distribution G that household x ascribes to the realizations of the inflation target one period ahead, Π_{t+1}^*. Household x chooses consumption plans, $c_t(h^t)$, labor supply $l_t(h^t)$ and savings $a_{t+1}(h^t)$ to maximize (8). We assume that, $\forall t$, household $x \in [0, 1]$ can condition her choices to the entire history up to time t, h^t, which is fully characterized by the observed realizations of Π_t^* up to t. Expected utility arises when the household is forced to take Ω and the associated probability distribution G as given. Under ambiguity aversion, the household chooses a support Ω and an associated probability distribution G, so as to minimize the continuation utility V_{t+1} (worst case criterion). In particular, the support Ω is chosen among the possible realizations of the inflation target at $t+1$, denoted by $\mathscr{S}_t$, and G from the set of all probability distributions with support Ω, denoted by $\mathscr{P}(\Omega)$. There is no uncertainty about the inflation target at $t < T-1$ or at $t \geq T$. So we have $\mathscr{S}_t = 1\ \forall t \neq T-1$ and $\Pi_t^* = 1$ with probability one for all $t \neq T$. There is instead lack of confidence in the probability assessment of the inflation target at T, Π_T^*, so that the set $\mathscr{S}_{T-1}$ is given by (7). Notice that if the realizations of the inflation target affect the consumption and labor streams of different households differently, these preferences will give rise to actions that are taken *as if* households hold heterogeneous beliefs. Finally, notice that since the set $\mathscr{S}_t$ is common to all households $x \in [0,1]$, they all face the same uncertainty.[7] We can now define an equilibrium as follows:

Equilibrium An equilibrium is a set of beliefs, quantities, and prices such that, $\forall t$,

1. Each *household* $x \in [0,1]$ chooses c_{xt}, l_{xt}, and a_{xt+1} to maximize (8), which also determines her beliefs about the support for the next-period realizations of the inflation target, $\Omega_{xt} \subseteq \mathscr{S}_t$, and the associated probability distribution $G_{xt} \in \mathscr{P}(\Omega_{xt})$;

2. The *monetary authority* sets the nominal interest rate R_t as in (3);

3. Each *firm* $i \in [0,1]$ sets the price $p_{it} = p_t$ optimally, after the inflation target for the period has been determined (but before any policy announcement);

4. The labor market, the goods market, and the financial market *all clear* at wage w_t, inflation Π_t, and interest rate r_t.

3 Solution of the model

We start by assuming that the policy announcement at $t = 0$ is about the next-period inflation target Π_1^*, so that $T = 1$. We further assume that there are only two types of households differing only in initial financial wealth.[8] A fraction (half) of households are

[7] There is empirical evidence suggesting that more educated individuals and those with greater financial literacy are characterized by smaller ambiguity when investing in financial markets and dealing with financial institutions, see Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016). Here we do not allow for exogenous differences in ambiguity to better isolate the effects of wealth inequality on households' choices.

[8] Both assumptions are relaxed in the quantitative model of Section 4. To keep the notation consistent throughout the paper, we have described the economy for general T and for an arbitrary distribution of households' assets a_{xt}. In this simple model the assumption $T = 1$ entails only a minor loss of generality, because firms adjust prices in every period so output can respond just at $t = 0$. The time horizon of the announcement will matter in the quantitative model because in that case prices are adjusted slowly.

creditors, $j = c$, with wealth equal to $a_{x0} = a_{c0} = B > 0 \ \forall x \in [0, 1/2]$, and the remaining fraction are *debtors*, $j = d$, with financial wealth $a_{x0} = a_{d0} = -B < 0$, $\forall x \in [1/2, 1]$. Here B denotes the amount of initial *financial imbalances* in the economy. First we prove some preliminary results that clarify the functioning of the model. Then we characterize household optimal choices. Finally, we solve for equilibrium aggregate output Y_0 and end-of-period financial imbalances B' at $t = 0$.

3.1 Preliminary results and the full credibility benchmark

Figure 1 shows the timeline of the experiment. At the announcement, $t = 0$, prices are predetermined at a value normalized to one, $p_0 = 1$. The analysis focuses on characterizing output at time zero, Y_0 which is determined, given sticky prices, by the saving decisions of creditors, a_{c1}, and debtors, a_{d1}. Clearing the financial market implies that $a_{c1} = -a_{d1} = B'$, where B' denotes the amount of financial imbalances at the end of period zero. In the

Figure 1: Timing

following periods, $t \geq 1$, firm $i \in [0, 1]$ sets its price p_{it} to maximize (expected) profits, $d_{it} \equiv y_{it} \left(p_{it}/p_t - w_t \right)$, taking as given the demand for the variety of the competitive firm, which has the conventional form:

$$y_{it} = Y_t \left(\frac{p_{it}}{p_t} \right)^{-\theta}.$$

The resulting optimal nominal price is a markup over firm i's expected nominal wage:

$$p_{it} = \frac{\theta}{\theta - 1} E_{it}[w_t \, p_t] \quad \forall i \in [0, 1], \tag{9}$$

which immediately implies $p_{it} = p_t \ \forall i$. Also, since firms set their price after observing Π_t^*, pricing decisions are taken under perfect information $\forall t \geq 1$, so (9) implies that

$$w_t = \frac{\theta - 1}{\theta}, \quad \forall t \geq 1. \tag{10}$$

The utility in (1), together with the preferences in (8), further implies that the labor supply of a household of type $j = c, d$ solves a simple static maximization problem, yielding the familiar condition

$$\psi_0 \, l_{jt}^{\psi} = w_t. \tag{11}$$

This implies that all households (independently of wealth and beliefs) supply the same labor, which, given that aggregate labor supply equals output, yields $l_{jt} = Y_t$, $\forall j$. This, together with (10) and (11), immediately proves that:

Lemma 1 *Output Y_t converges back to steady state at $t = 1$, so that $Y_t = \bar{Y}$ $\forall t \geq 1$.*

In the Appendix we use Lemma 1 together with the monetary rule in (3) to prove the following lemma which implies that the interest rate r_1 moves one-for-one with the inflation target Π_1^*:

Lemma 2 *At any point in time $t \geq 0$, inflation is equal to the inflation target, $\Pi_t = \Pi_t^*$, and the nominal interest rate remains unchanged at its steady state value, $R_t = \bar{R}$.*

Before solving the model, we characterize the properties of the economy in the canonical benchmark in which all households fully believe the announcement. Let

$$N(Y) \equiv Y - \psi_0 \, \frac{Y^{1+\psi}}{1+\psi},$$

denote output net of the effort cost of working, which in equilibrium is just a monotonically increasing transformation of output Y.[9] We also denote by $\bar{N} \equiv N(\bar{Y})$ the steady state value of $N(Y)$ and by $N_0 \equiv N(Y_0)$ the value of $N(Y)$ at time zero. In the appendix we write the Euler equation of consumption for creditors $j = c$ and debtors $j = d$ when households accord full credibility to the announcement. After imposing that at time $t = 0$ the good and the financial market clear, we prove that

Proposition 1 (The full credibility benchmark) *If all households fully believe the announcement, then $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$. Thus output Y_0 is a strictly increasing function of ε and is independent of initial imbalances B. The new steady-state financial imbalances after implementation, B'/ε, are strictly positive and decrease (relative to B) if the announcement is inflationary, $\varepsilon > 1$, while they increase if the announcement is deflationary, $\varepsilon < 1$.*

3.2 Household problem

Let $\varepsilon^{\tau} = \Pi_1^*$ denote the next period inflation target, where τ measures (in percentage) how much of the announcement ε will be implemented in period one. Given (7), households

[9] Notice that $N'(Y) > 0$ when $w < 1$, which is implied by (10).

think τ could have any value on the unit interval. The optimal choice for labor in (11) and the lump-sum transfer λ_t imply that the income of households differs just because of differences in capital income. Moreover, at $t = 0$ the household knows that the economy is back to steady state at $t = 1$ (Lemma 1), so that the only uncertainty faced by the household is about the next period interest rate r_1, which given Lemma 2 is determined by the next period inflation target ε^{τ}. Given these considerations we can think that at $t = 0$ household $j = c, d$, with initial wealth a_{j0}, chooses her optimal end-of-period-zero savings a' to solve

$$V\left(a_{j0}\right) = \max_{a'} \left\{ \frac{\left(N_0 + \bar{R}\, a_{j0} - a'\right)^{1-\sigma}}{1-\sigma} + \beta \min_{\Upsilon \subseteq [0,1],\ G \in \mathscr{P}(\Upsilon)} \int_{\Upsilon} \bar{V}\left(\frac{a'}{\varepsilon^{\tau}}\right) G(d\tau) \right\}, \tag{12}$$

where the continuation utility is

$$\bar{V}\left(a\right) = \frac{\left[\bar{N} + (\bar{R} - 1)\, a\right]^{1-\sigma}}{(1-\sigma)(1-\beta)}, \tag{13}$$

which measures the next period present discounted value of utility of a household who enters the period with wealth a. The household faces Knightian uncertainty about τ and chooses its support $\Upsilon \subseteq [0, 1]$ as well its associated probability distribution $\mathscr{P}(\Upsilon)$. For any end-of-period-zero savings a', inflation affects the real wealth of the household at the beginning of period one, which explains the argument of $\bar{V}$ in (12). The minimization problem in (12) under the household's optimal savings a' determines the worst case beliefs of household j, in brief her *beliefs*. Clearly $\bar{V}(\cdot)$ is an increasing function, $\bar{V}' > 0$, so higher inflation (higher ε^{τ}, given Lemma 2) lowers continuation utility when $a' > 0$, and increases it when $a' < 0$. If $a' = 0$, households' utility is unaffected by inflation. The general idea is that higher inflation is good if the household is a debtor at the end of period zero $a' < 0$, while it is bad if she is a creditor $a' > 0$. Then the minimization problem in (12) immediately implies that:

Proposition 2 (Household beliefs) *A household-j's beliefs about next period inflation Π_1^* depend on the announcement, ε, and her end-of period savings, a'. When $a' = 0$, beliefs are indeterminate. If $a' \neq 0$, they are degenerate and equal to $\varepsilon^{\tau(a',\varepsilon)}$ where*

$$\tau(a', \varepsilon) = \mathbb{I}(\varepsilon > 1) \times \mathbb{I}(a' > 0) + \mathbb{I}(\varepsilon < 1) \times \mathbb{I}(a' < 0), \tag{14}$$

in which $\mathbb{I}$ denotes the indicator function.

Proposition 3 implies that there are three possible equilibrium types of households: there could be *trusting* households who act as if the announcement will be fully implemented for sure ($\tau = 1$); *skeptical* households who act as if the announcement will never be implemented ($\tau = 0$); and finally there could be *Zen* households who have reached "complete peace of mind" about future monetary policy choices since they have no savings at the end of the period, $a' = 0$, so their (worst-case) belief about τ is indeterminate. If the announcement is inflationary, $\varepsilon > 1$, trusting households are those with $a' > 0$, while households with $a' < 0$ are skeptical. If the announcement is deflationary, $\varepsilon < 1$, households with $a' > 0$ are skeptical, while those with $a' < 0$ fully trust the announcement.

The right hand side of (12) is continuous in the end of period household's savings a' with a kink at $a' = 0$ if $\varepsilon \neq 1$. The kink arises because after an announcement, $\varepsilon \neq 1$, the expected return on assets discontinuously falls when a' switches from negative to positive—due to the shift in beliefs implied by (14). It is easy to check that the derivative of the right hand side of (12) with respect to a' is globally strictly decreasing in a', with a (possible) discontinuity point at $a' = 0$, which guarantees a unique solution for a' in (12). A household chooses $a' = 0$ when this derivative changes sign at $a' = 0$, which, if $\varepsilon \neq 1$, can happen for a non degenerate set of values of time zero net output N_0. In particular, after substituting (14) into (12) we conclude that the household optimally chooses $a' = 0$ for any N_0 in the interval

$$\mathscr{I}(a_0, \varepsilon) = [\underline{I}(a_0, \varepsilon), \overline{I}(a_0, \varepsilon)] \tag{15}$$

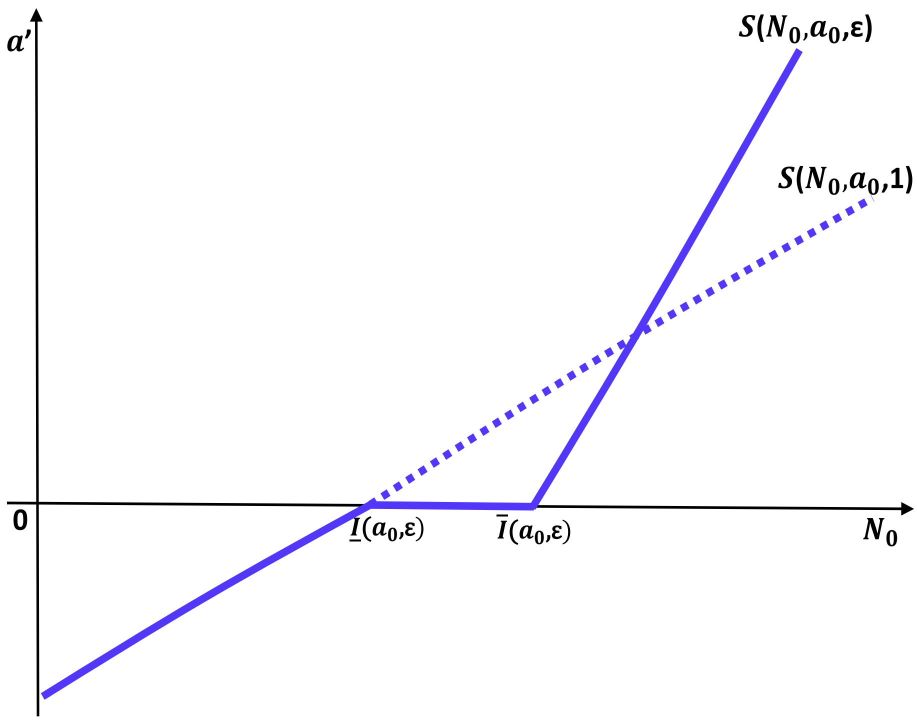
where $\underline{I}(a_0, \varepsilon) = \min\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}a_0$ while $\overline{I}(a_0, \varepsilon) = \max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}a_0$. Notice that the set of values of N_0 in (15) is decreasing in a_0, which follows from household's desire to smooth consumption when income falls. When N_0 is out of the interval $\mathscr{I}(a_0, \varepsilon)$, the optimal choice of savings is determined by a conventional first order condition, obtained by setting to zero the derivative of the right hand side of (12) with respect to a'. After noticing that the beliefs τ in (14) are constant when a' is away from zero and after some simple algebra we can conclude that:

Proposition 3 (Optimal savings) *The optimal end-of-period savings of a household are given by the following function $S(N_0, a_0, \varepsilon)$ of current net output N_0, her initial savings a_0, and the announcement ε:*

$$a' = S(N_0, a_0, \varepsilon) \equiv \begin{cases} 0 & \text{if } N_0 \in \mathscr{I}(a_0, \varepsilon), \\ \frac{N_0 + \bar{R}a_0 - \varepsilon^{\frac{\tau(a', \varepsilon)}{\sigma}}\bar{N}}{\left(\bar{R}-1\right)\varepsilon^{-\left(1-\frac{1}{\sigma}\right)\tau(a', \varepsilon)} + 1} & \text{if } N_0 \notin \mathscr{I}(a_0, \varepsilon). \end{cases} \tag{16}$$

The function $S(N_0, a_0, \varepsilon)$ is plotted in Figure 2. The dotted line corresponds to the policy before the announcement, $\varepsilon = 1$; the solid line to the policy after an inflationary announcement, $\varepsilon > 1$. Household's savings a' are (weakly) increasing in N_0, because the household saves more when output increases temporarily, to smooth consumption. In the absence of announcements, $\varepsilon = 1$, the marginal propensity to consume (say $1 - S_{N_0}$) is independent of household's savings a', which is due to the preferences in (1). After an

Figure 2: Optimal savings, before and after an inflationary announcement



Notes: The dotted line is the household savings policy in (16) before the announcement, $\varepsilon = 1$. The solid line is the policy function after an inflationary announcement $\varepsilon > 1$.

announcement, $\varepsilon \neq 1$, the expected interest rate falls when a' switches from negative to positive and thereby the saving function gets steeper when a' turns positive (provided that $\sigma > 1$). The set of values of $N_0 \in \mathscr{I}(a_0, \varepsilon)$ that lead to zero savings, $a' = 0$, corresponds to the horizontal line in the figure. Before the announcement, this set is degenerate. After the announcement, it is non degenerate because a household with $a' > 0$ believes that the return on savings is strictly lower than a household with $a' < 0$, which leads to a kink in the continuation utility in (13). For given N_0, the effects of an inflationary announcement, $\varepsilon > 1$, on savings are generally ambiguous since the savings function S remains unchanged for $a' < 0$ while it shifts to the right and gets steeper for $a' > 0$. This follows from the fact that only households with $a' > 0$ act as if the announcement will be fully implemented. But with $a' > 0$, the income and substitution effects of a fall in interest rates work in opposite directions: if a' is small, the substitution effect prevails and S shifts down (locally), implying a reduction in savings and an increase in consumption for given N_0; if a' is large, the fall in interest rates causes a large fall in future capital income, so the income effect prevails, and S shifts up (locally), implying a fall in consumption for given N_0. Graphically this means that the solid line lies below the dotted line for a positive but small enough a' while it lies above when a' is large enough.[10]

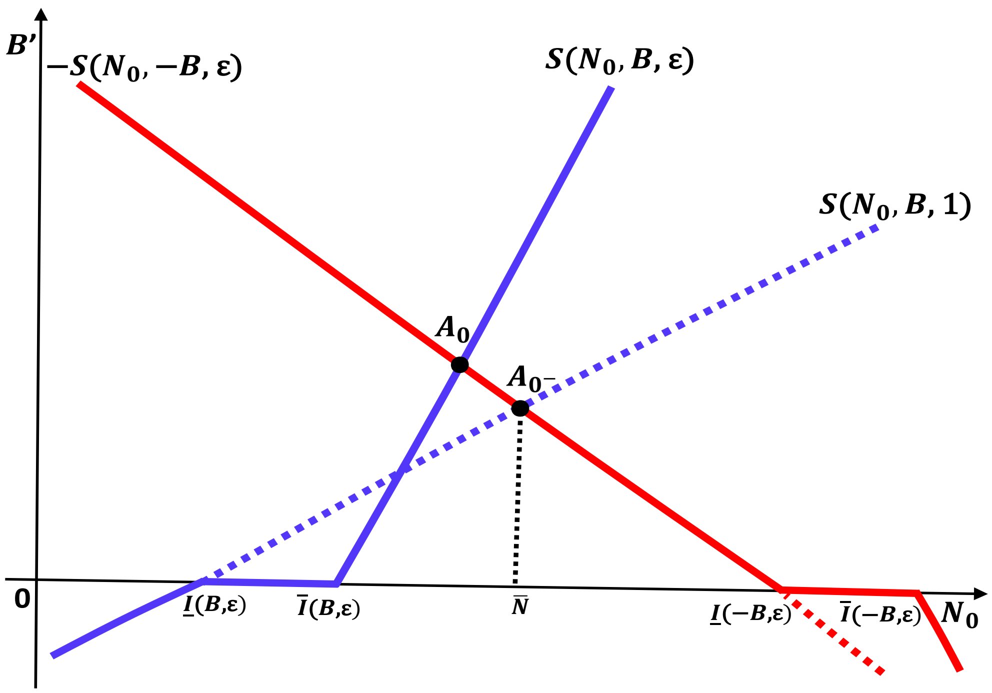
3.3 Equilibrium

In equilibrium both household types $j = c, d$ maximize (12) and financial markets clear. Given (16), this implies that B' and N_0 should satisfy the following system of equations:

$$B' = S(N_0, B_0, \varepsilon), \tag{17}$$
$$B' = -S(N_0, -B_0, \varepsilon). \tag{18}$$

Equation (17) can be interpreted as a demand for assets of creditor households $j = c$: the demand for assets is increasing in time-zero net output N_0, because creditors want to save more when output increases. It corresponds to the blue line in Figure 3: the dotted line for the situation before the announcement $\varepsilon = 1$, the solid line for the situation after an inflationary announcement $\varepsilon > 1$. The horizontal blue segment identifies the set of values of $N_0 \in \mathscr{I}(B, \varepsilon)$, such that creditors $j = c$ choose $B' = 0$ after the announcement. By the

Figure 3: Clearing of financial markets after an inflationary announcement



Notes: The equilibrium of financial markets before the announcement, $\varepsilon = 1$, is A_{0^-} at net output $\bar{N}$. The equilibrium after the inflationary announcement $\varepsilon > 1$ is A_0.

[10] In response to a deflationary announcement $\varepsilon < 1$, a' unambiguously increases for given N_0. In this case the function S remains unchanged for $a' > 0$ while it shifts to the left and gets flatter for $a' < 0$. This is because the income and substitution effects of higher interest rates work in the same direction when $a' < 0$, in making debtors both more willing to save and poorer.

same logic, equation (18) characterizes the supply of assets by debtor households $j = d$: the supply of assets is decreasing in N_0, as debtors want to borrow less (save more) when time-zero output is higher. It corresponds to the red negatively sloped line in Figure 3, where the horizontal red segment identifies the values of $N_0 \in \mathscr{I}(-B, \varepsilon)$, such that debtors optimally choose $B' = 0$ when $\varepsilon > 1$. The equilibrium is represented by the point where the two schedules cross. This corresponds to point A_{0^-} before the announcement, with net output equal to $\bar{N}$, while it corresponds to A_0 after the announcement. As it can be inferred from Figure 3, the system (17)-(18) has two properties: (i) it has at least one non-negative solution, $B' \geq 0$, which follows from the fact that $\underline{I}(B, \varepsilon) < \overline{I}(-B, \varepsilon)$; and (ii) $B' > 0$ requires that

$$B > \frac{|\varepsilon^{\frac{1}{\sigma}} - 1|\bar{N}}{2\bar{R}}, \tag{19}$$

which corresponds to the condition $\underline{I}(-B, \varepsilon) > \overline{I}(B, \varepsilon)$. In brief we have proved that:

Proposition 4 (Equilibrium) *An equilibrium always exists. In equilibrium, creditors and debtors never switch their net financial asset position: if $B > 0$, then $B' \geq 0$. If* (19) *holds, then the financial market is active, $B' > 0$, otherwise we have a credit crunch equilibrium, $B' = 0$, where net output N_0 can be any value in the interval*

$$[\underline{I}(-B, \varepsilon), \overline{I}(B, \varepsilon)]. \tag{20}$$

When initial financial imbalances B are so small that (19) fails, both households types $j = c, d$ completely undo their financial positions, $B' = 0$, and become Zen households with undeterminate beliefs about future monetary policy. In this *credit crunch* equilibrium, net output N_0 can be any value in the interval (20), which implies that output at $t = 0$ can be neither too low (so that debtors do not find it optimal to borrow) nor too high (so that creditors do not find optimal to lend).[11] In the rest of the analysis we focus on the Pareto efficient equilibrium level of net output that corresponds to $\overline{I}(B, \varepsilon)$—i.e. to the upper value of the interval in (20). If (19) holds, we know that $B' > 0$ and we can use (17) and (18) to solve for N_0 after using the fact that households $j = c$ have beliefs $\tau(B', \varepsilon)$, while households $j = d$ have beliefs $\tau(-B', \varepsilon)$, see Proposition 4. In discussing the resulting equilibrium value of N_0, we define two useful statistics. The first is the *average credibility* of the announcement:

$$\overline{\tau} = \frac{\tau(B', \varepsilon) + \tau(-B', \varepsilon)}{2}. \tag{21}$$

[11]For example, in a credit crunch equilibrium caused by an inflationary announcement $\varepsilon > 1$, output never falls, $\underline{I}(-B, \varepsilon) > \bar{N}$, and it is always lower than the output level of the benchmark New Keynesian model characterized in Proposition 1, as $\overline{I}(B, \varepsilon) < \varepsilon^{\frac{1}{\sigma}}\bar{N}$.

The second is the *correlation* between households' wealth and their perception of the announcement's credibility:

$$\rho = \frac{\tau(B', \varepsilon) - \tau(-B', \varepsilon)}{2\overline{\tau}} \in [-1, 1]. \tag{22}$$

When $\rho > 0$, creditors believe the announcement more than debtors; and conversely when $\rho < 0$; $\rho = 0$ means that all households share the same beliefs. In an equilibrium with $B' > 0$, only one type of household believes the announcement, so that $\overline{\tau} = 1/2$: if the announcement is inflationary, creditors believe it, so that $\rho = 1$; if it is deflationary, debtors believe it, so that $\rho = -1$. In general we have that:

$$\rho = 1 - 2\mathbb{I}(\varepsilon < 1), \tag{23}$$

which will play a key role in the determination of time zero output. After some algebra we can then prove that:

Proposition 5 (Equilibrium output) *If* (19) *fails, the Pareto-efficient equilibrium level of net output is equal to* $N_0 = \overline{I}(B, \varepsilon) = \max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}a_0$. *If* (19) *holds, then net output is given by*

$$N_0 = N_0(\varepsilon, \overline{\tau}, \rho) \equiv \left[\varphi\, \tilde{\varepsilon}^{\,\frac{1+\rho}{\sigma}} + (1 - \varphi)\, \tilde{\varepsilon}^{\,\frac{1-\rho}{\sigma}}\right] \bar{N} - \mu B, \tag{24}$$

where $\tilde{\varepsilon} \equiv \varepsilon^{\overline{\tau}}$ *measures the announcement rescaled by its average credibility* $\overline{\tau} = \frac{1}{2}$, *while*

$$\varphi \equiv \frac{1 + (\bar{R} - 1)\, \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma}-1\right)}}{2 + (\bar{R} - 1)\left[\tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma}-1\right)} + \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma}-1\right)}\right]} \quad \in [0, 1],$$

$$\mu \equiv \frac{\bar{R}\,(\bar{R} - 1)\left[\tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma}-1\right)} - \tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma}-1\right)}\right]}{2 + (\bar{R} - 1)\left[\tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma}-1\right)} + \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma}-1\right)}\right]} > 0.$$

where ρ *is the (endogenous) equilibrium correlation between households' wealth and their perception of the announcement's credibility, as given in (23).*

The Pareto efficient equilibrium level of output is decreasing in the initial level of financial imbalances B. The first term on the right-hand side of (24) characterizes the intertemporal substitution effect on consumption. This term is always positive, it is scaled by the average credibility of the announcement $\tilde{\varepsilon} \equiv \varepsilon^{\overline{\tau}}$, and it is independent of B. It is also greater than $\bar{N}$ if the announcement is inflationary $\varepsilon > 1$, while it is smaller than $\bar{N}$ if the announcement is deflationary $\varepsilon < 1$. The second term, equal to $-\mu B$, characterizes what we call the *misguidance effect*: the effects on consumption of redistributing expected future income from one household type to the other when households are ambiguity averse. This term is generally negative because expected losses in future capital income by a group of households are larger than the expected gains of income by the other group due to heterogeneous beliefs. The term is zero only when $B = 0$, because no income is redistributed. It would also be zero if $\rho = 0$, because in this case the income losses expected by the household type that loses from the redistribution (creditors when $\varepsilon > 1$, debtors when $\varepsilon < 1$) are exactly equal to the gains expected by the other type. And zero-sum transfers of wealth between household types have no effect on aggregate consumption, because under homogenous beliefs all households have the same marginal propensity to consume—due to the utility function in (1) and the absence of financial constraints. So $\rho = 0$ would imply $N_0 = \tilde{\varepsilon}^{\frac{1}{\sigma}} \bar{N}$, as in a standard representative-household New Keynesian model in response to an announcement $\tilde{\varepsilon} = \varepsilon^{\bar{\tau}}$, which is as in the full credibility benchmark (see Proposition 1) once the announcement is rescaled for its average credibility $\bar{\tau}$. But with $B > 0$ and once we take into account that the correlation between changes in beliefs and households initial asset position ρ is given by (23), we see that the term $-\mu B$ is strictly negative and more negative the greater the amount of initial imbalances B. Since this term is linear in B, we have that when initial imbalances B are large enough, an inflationary announcement $\varepsilon > 1$ can even be contractionary, making $N_0(\varepsilon, \bar{\tau}, \rho)$ smaller than $\bar{N}$. The next proposition summarizes this discussion:

Proposition 6 (The effects of wealth inequality) *After an inflationary announcement, $\varepsilon > 1$, output Y_0 increases less than in the full credibility benchmark. This difference is increasing in B, and Y_0 can even decrease relative to steady state output $\bar{Y}$ if B is large enough. In response to a deflationary announcement, $\varepsilon < 1$, Y_0 always decreases. The decrease is larger the larger is B; and if B is large enough, Y_0 decreases more than in the full credibility benchmark.*

Finally, for completeness, we compare the steady state imbalances that result when the announcement is implemented, B'/ε, with the corresponding imbalances in the canonical New Keynesian model, where the announcement is fully credited by all households:

Proposition 7 (Aggregate rebalancing) *After an inflationary announcement, $\varepsilon > 1$, the new steady state financial imbalances after implementation, B'/ε, always decrease ($B'/\varepsilon < B$), and they decrease more than in the full-credibility benchmark. After a deflationary announcement $\varepsilon < 1$, there are two (strictly positive) thresholds $\tilde{B}_1$ and $\tilde{B}_2$, with $\tilde{B}_1 < \tilde{B}_2$, such that for $B < \tilde{B}_1$, B'/ε falls; for $B \in [\tilde{B}_1, \tilde{B}_2]$, B'/ε increases, but less than in the full-credibility benchmark; and for $B > \tilde{B}_2$, B'/ε increases, and more than in the benchmark.*

3.4 Discussion

We now briefly discuss some properties and extensions of the analytical model, with the theoretical details reported in the Appendix. Section 6 studies the quantitative implications of all these extensions.

Long vs short term nominal bonds We assumed that households can save or borrow just in a one period bond that pays a pre-specified nominal interest rate (short term nominal bonds). But as shown in Lemma 2 the nominal interest rate remains unchanged over time. This means that households disagree just on future expected inflation (not on future short term nominal interest rates), so allowing households to trade in nominal bonds at different maturities would have no equilibrium effects.

Real asset In the Appendix we study an extension of the model where households can also trade in a real asset which is in fixed supply (say a Lucas tree) and pays with certainty a per period return equal to $\beta^{-1} - 1$. Households face some convex costs in adjusting their holdings of the real asset. When households disagree on the expected real return of financial assets, trading in the real asset is profitable. After a monetary announcement (either inflationary $\varepsilon > 1$ or deflationary $\varepsilon < 1$), the expected real interest rate on financial assets is generally lower for creditors than for debtors (see Proposition 3), so the real asset tends to be reallocated from debtors to creditors. Compared with the baseline model, after an announcement, output is higher and a credit crunch equilibrium with $B' = 0$—which tends to arise when adjustments costs are small enough—is more likely. But the misguidance effect is still present: output is decreasing in the initial financial imbalances B and, after an inflationary announcement, it is always smaller than in the full credibility benchmark.

Government bonds We assumed that the supply of bonds is entirely determined by households. In practice households also also hold government bonds in their portfolio and as emphasized by Ricardo (1888) and Barro (1974) government bonds are not net wealth. So the financial assets of households should be measured net of government bonds, which makes households poorer and inflationary announcements more likely to be expansionary.

Policy implications When debtors and creditors share the same beliefs, they also have the same marginal propensity to consume and redistributing wealth has no effects on aggregate consumption. But in response to an inflationary announcement, their beliefs are different and taxing today the creditors to transfer the resulting income to the debtors is expansionary. Intuitively redistributive policies are equivalent to reducing the level of initial imbalances B, which makes forward guidance more expansionary.

Liquidity traps For expositional simplicity we assumed that the economy is initially in a steady state equilibrium. The analysis would go through almost unchanged if considering an economy which is initially in a liquidity trap, say because at time zero the household discount factor β_0 is so high that the nominal interest rate in (3) is at the zero lower bound—while the economy is back to steady state in period one with $\beta_t = \beta < 1$, $\forall t \geq 1$.

Modeling of ambiguity aversion Households have Maximin preferences as in Gilboa and Schmeidler (1989) but, after an inflationary announcement, the expected inflation of creditors would respond more than the expected inflation of debtors also under alternative models of ambiguity aversion, including the multiplier preferences proposed by Hansen and Sargent (2001, 2008), whose axiomatic foundations are provided by Strzalecki (2011). Essentially different models of ambiguity aversion generate similar results provided that, in response to the monetary announcement, they generate the same response in average expected inflation and in the correlation between households' wealth and their perception of the announcement's credibility, as parameterized by ρ. In practice, in the quantitative analysis of Section 4, we target them both using micro level evidence on expected inflation, and because of this we believe that the qualitative as well as the quantitative results of the analysis are little sensitive to the specific modeling of ambiguity aversion.[12]

Different source of uncertainty We assumed that households face no uncertainty before the announcement and just doubt the credibility of the monetary authority γ. An alternative would be that households are initially uncertain about future inflation, for example because they doubt about the inflation target that the monetary authority will set in period T, so that in the absence of any announcements the support of the feasible values of Π_T^* is $\mathcal{S}_{T-1} = [\hat{\Pi}^l, \hat{\Pi}^h]$. In this environment an announcement can reduce the initial disagreement in the (worst-case) inflation expectations of debtors and creditors and an inflationary announcement can be highly expansionary on aggregate demand. To see this, assume that the credibility parameter, $\gamma > 0$, is fully known and that households think that, after an announcement ε, the central bank sets Π_T^* to minimize

$$\boldsymbol{L}_1 = \left(\hat{\Pi} - \Pi_T^*\right)^2 + \gamma\left(\varepsilon - \Pi_T^*\right)^2, \tag{25}$$

where $\hat{\Pi} \in [\hat{\Pi}^l, \hat{\Pi}^h]$ is the inflation target about which households face Knightian uncertainty. Given (25) households infer that

$$\Pi_T^* = \frac{\hat{\Pi} + \gamma\varepsilon}{1 + \gamma}$$

[12] Literally, a credit crunch equilibrium can no longer arise under a smooth model of ambiguity aversion. Yet, the same forces that lead to a credit crunch equilibrium under Maximin preferences would cause the end-of-period financial imbalances B' to get concentrated close to zero.

which implies that the support of the feasible values of Π_T^* becomes equal to

$$\mathcal{S}_{T-1} = \left[\frac{\hat{\Pi}^l + \gamma\varepsilon}{1+\gamma}, \frac{\hat{\Pi}^h + \gamma\varepsilon}{1+\gamma} \right]. \tag{26}$$

This means that the announcement reduces the disagreement in inflation expectations between creditors and debtors which falls from $\hat{\Pi}^h - \hat{\Pi}^l$ to $(\hat{\Pi}^h - \hat{\Pi}^l)/(1+\gamma)$. Moreover, in response to an inflationary announcement, the inflation expectations of debtors increase by $\gamma(\varepsilon - \hat{\Pi}^l)/(1+\gamma)$ while those of creditors increase just by $\gamma(\varepsilon - \hat{\Pi}^h)/(1+\gamma)$, implying a fall in expected (real) interest rates larger for debtors than for creditors, which is highly expansionary on aggregate demand—as it would be a negative ρ with $\varepsilon > 1$ in (24).

In practice households face uncertainty about both the future inflation target $\hat{\Pi}$ and the credibility of the announcement γ. The relative response of inflation expectations of creditors and debtors identifies the empirically relevant source of uncertainty. If uncertainty is mostly about the credibility of the monetary authority γ, as in (7), after an inflationary announcement, the inflation expectations of creditors increase more than those of debtors. When instead uncertainty is mostly about the future inflation target, as in (26), the disagreement in inflation expectations between creditors and debtors falls after the announcement. The evidence below indicates that, at the start of forward guidance by the ECB, there was substantial uncertainty about the credibility of the announcement, which is coherent with the large disagreement among European households in how much the ECB could be trusted, see for example Guiso, Sapienza, and Zingales (2016).

Bounds on credibility We assumed that households regard as possible any γ between zero (no credibility) and infinity (full credibility). When credibility is bounded to be in the interval $[\gamma^l, \gamma^h]$, the set of inflation targets Π_T^* that households regard as feasible after an inflationary announcement $\varepsilon > 1$ is given by

$$\mathcal{S}_{T-1} = \left[\frac{1+\gamma^l\varepsilon}{1+\gamma^l}, \frac{1+\gamma^h\varepsilon}{1+\gamma^h} \right]. \tag{27}$$

In practice the lower bound of (27) determines the increase in expected inflation common to all households, while the size of the interval measures the uncertainty following the announcement. After forward guidance, the interval in (27) can be identified by using evidence on changes in average expected inflation and in the correlation between expected inflation and the financial asset position of agents. Knowing the interval in (27) is enough to characterize the effects of the announcement in the model, but notice that, without any additional assumptions, the bounds on credibility, γ^l and γ^h, and the announcement ε are not separately identified. To achieve identification in the quantitative analysis below, we will make the additional assumption that full credibility is possible, $\gamma^h = \infty$, which is redundant for the purpose of estimating the effect of the announcement in our economy, but has the advantage of including the full credibility benchmark as part of the possible paths of the economy.

4 The quantitative model

We now extend the model to evaluate quantitatively how ambiguity aversion can alter the effects of an announcement about the future path of short-term nominal interest rates. For this purpose we extend the model by allowing for (i) a general distribution of households' financial assets which consist of short-term and long-term nominal bonds as well of firm equity; (ii) sticky prices à la Rotemberg (1982); (iii) a liquidity trap; and (iv) a fiscal authority which finances interest rate payments on bonds through lump-sum taxes. Extension (i) is needed to match the observed distribution of households' assets and their exposure to monetary policy changes; (ii) to obtain a conventional New-Keynesian Phillips curve; (iii) to characterize the state of the (European) economy at the time of the start of forward guidance by the ECB; and (iv) to allow for a positive net supply of bonds. We next describe the economy, then characterize the equilibrium in the liquidity trap before the announcement, and finally turn to the response of the economy after the announcement.

4.1 Assumptions

Assets The wealth of household x at the beginning of period t is characterized by the triple $\boldsymbol{\omega}_{xt} = (a_{xt}, b_{xt}, e_{xt})$ where: (i) a_{xt} are the consumption units invested by the household at $t-1$ in a one-period nominal bond paying a nominal return R_{t-1} at t, which we will refer to as *short-term (nominal) bonds*; (ii) b_{xt} are the number of annuities held by the household each paying a nominal amount ν in every period, which we will refer to as *long-term (nominal) bonds*; and (iii) e_{xt} are the *equity* shares owned by the household, each paying dividends d_t at t. The price of a long-term bond is denoted by q_t^b, that of equity by q_t^e. Notice that the instantaneous return of short-term bonds is affected by both nominal interest rates and inflation, while long-term bonds have an instantaneous return equal to

$$\nu_t = \frac{\nu}{p_t},$$

which is affected just by the price level p_t. Finally equity is a real asset whose instantaneous return depends on the aggregate conditions of the economy. We allow a_{xt}, b_{xt} and e_{xt} to be positive or negative and just to be constrained by the household's natural borrowing limit. Household x pays the following convex costs (measured in consumption units) for adjusting her holdings of long-term bonds b_{xt} and equity e_{xt}:

$$\chi\left(b_{xt+1}, b_{xt}, e_{xt+1}, e_{xt}\right) = \frac{\overline{\chi}_b}{2}\left(\frac{b_{xt+1}-b_{xt}}{b_{xt}}\right)^2 |b_{xt}| + \frac{\overline{\chi}_e}{2}\left(\frac{e_{xt+1}-e_{xt}}{e_{xt}}\right)^2 |e_{xt}|, \tag{28}$$

with $\overline{\chi}_b, \overline{\chi}_e \geq 0$. The household-$x$ budget constraint is now given by

$$\begin{aligned} & c_{xt} + a_{xt+1} + q_t^b\, b_{xt+1} + q_t^e\, e_{xt+1} + \chi(b_{xt+1}, b_{xt}, e_{xt+1}, e_{xt}) \\ & \leq w_t\, l_{xt} + r_t\, a_{xt} + \left(q_t^b + \nu_t\right)\, b_{xt} + \left(q_t^e + d_t\right) e_{xt} - \varsigma_t, \end{aligned} \tag{29}$$

where ς_t is a lump-sum tax specified below and $r_t = R_{t-1}/\Pi_t$ with $\Pi_t = p_t/p_{t-1}$ denoting gross inflation.

Financial markets The net supply of short-term and long-term bonds is equal to A and B, respectively. Bonds are issued by the government which levies a lump-sum tax on each household $x \in [0,1]$ to finance debt payments so that

$$\varsigma_t = (r_t - 1)\, A + \nu_t\, B, \tag{30}$$

implying that government debt does not increase aggregate net wealth as in Barro (1974). Market clearing implies that $\int_0^1 a_{xt}\, dx = A$, $\int_0^1 b_{xt}\, dx = B$ and $\int_0^1 e_{xt}\, dx = 1$, $\forall t$.

Firms Each firm $i \in [0,1]$ incurs a per period fixed operating cost ζ, which we use to calibrate the aggregate value of equity to the data. As in Rotemberg (1982), firm i can set her time-t nominal price p_{it} subject to the following convex adjustment costs:

$$\kappa\left(\pi_{it}, Y_t\right) = \frac{\kappa_0}{2}\left(\pi_{it}\right)^2 Y_t, \tag{31}$$

where $\pi_{it} = (p_{it} - p_{it-1})/p_{it-1}$, Y_t is aggregate output and $\kappa_0 > 0$.

Liquidity trap The economy is in steady state at $t = 0$ and we follow Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011) in assuming that the nominal interest rate is pushed to the zero lower bound by a temporary (unforeseen) increase in the households' subjective discount factor β_t which evolves as follows:

$$\beta_t = \begin{cases} \hat{\beta} & \text{if} \;\; t = 0, 1, ...t_\beta \\ \beta & \text{otherwise} \end{cases} \tag{32}$$

with $\hat{\beta} > 1 > \beta$ which, given (3), leads to $R_t = 1$ $\forall t = 0, 1, ...t_\beta$ when $\Pi_t^* = 1$ $\forall t$.

4.2 The economy before the announcement

We start characterizing the equilibrium of the economy at $t \geq 0$, after the shock to β_t has been realized, under the assumption that monetary policy follows the interest rate rule in (3) with $\Pi_t^* = 1$, implying that $R_t = \hat{R}_t$, $\forall t$, where

$$\hat{R}_t \equiv \min\left(1, \frac{1}{\beta_t}\,{\Pi_t}^{\phi}\right). \tag{33}$$

There is perfect foresight and household-x chooses consumption, labor supply, and financial assets holdings to maximize $\forall t$

$$V_t(\boldsymbol{\omega}_{xt}; \hat{R}_t) = \max_{\{c_{xs}, l_{xs}, \boldsymbol{\omega}_{xs+1}\}_{s \geq t}} E_{xt}\left[\sum_{s=t}^{\infty} \beta_{ts}\, U(c_{xs}, l_{xs})\right], \tag{34}$$

subject to the flow budget constraint in (29). Here $\beta_{tt} = 1$ and $\beta_{ts} = \prod_{u=t}^{s-1} \beta_u$ for $s > t$ denote the subjective discount factor from $s \geq t$ to t, while $E_{xt}[\cdot]$ is the expectation operator at time t conditional on household-x's beliefs.[13] The value function is indexed to the expected future profile of nominal interest rates, as specified by (33).

Given the current and past nominal prices set by the firm, firm-i's profits are equal to

$$d_t(p_{it}, p_{it-1}) = \left(\frac{p_{it}}{p_t} - w_t\right)\left(\frac{p_{it}}{p_t}\right)^{-\theta} Y_t - \kappa(\pi_{it}, Y_t) - \zeta. \tag{35}$$

At t, firm-i sets nominal prices to maximize the present discounted value of profits

$$W_t(p_{it-1}; \hat{R}_t) = \max_{\{p_{is}\}_{s \geq t}} E_{ft}\left[\sum_{s=t}^{\infty} m_{ts}\, d_s(p_{is}, p_{is-1})\right], \tag{36}$$

where $m_{tt} = 1$ and $m_{ts} = \left(\prod_{u=t+1}^{s} r_u\right)^{-1}$ for $s > t$ denote the value at t of one unit of income received at $s \geq t$ and $E_{ft}[\cdot]$ is the time-t expectation operator conditional on firms' beliefs.[14] The solution to the firm's problem implies symmetric pricing, $p_{it} = p_t$ $\forall i$, which, after maximizing (36), can be used to derive the following standard new-Keynesian Phillips curve:

$$1 - \kappa_0\,(\Pi_t - 1)\,\Pi_t + \kappa_0\, E_{ft}\left[m_{t,t+1}\,(\Pi_{t+1} - 1)\,\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = \theta\,(1 - w_t). \tag{37}$$

[13] This is redundant notation under perfect foresight, but it will be useful once allowing for uncertainty.

[14] Again, this is redundant notation under perfect foresight, but it will be useful when we will allow for uncertainty.

Using symmetry, individual firm profits are equal to aggregate profits, $d_{it} = D_t$, where

$$D_t = \left[1 - w_t - \frac{\kappa_0}{2}\left(\Pi_t - 1\right)^2 - \zeta\right] Y_t. \tag{38}$$

Clearing of the goods market implies that

$$Y_t = C_t + \int_0^1 \chi\left(b_{xt+1}, b_{xt}, e_{xt+1}, e_{xt}\right) dx + \kappa(\pi_t, Y_t) + \zeta, \tag{39}$$

where $C_t \equiv \int_0^1 c_{xt} dx$ denotes aggregate consumption. The equilibrium of the economy in a liquidity trap is then characterized by the following Lemma fully proved in the Appendix:

Lemma 3 (Equilibrium under homogeneous beliefs) *When* β_t *evolves as in* (32)*, the economy is fully characterized by the tuple* $\left[D_t, Y_t, C_t, \Pi_t, R_t, w_t, r_t, q_t^b, q_t^e\right]$, *where (i)* D_t *and* Y_t *are given by* (38) *and* (39)*; (ii) inflation* Π_t *satisfies the Phillips curve in* (37) *under perfect foresight; (iii) the nominal interest rate is given by* (33)*; (iv) the interest rate satisfies the identity* $r_t = R_{t-1}/\Pi_t$*; (v) aggregate labor supply and consumption solve a representative household problem that yields* (11) *and*

$$C_t = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \left(\sum_{s=t}^{\infty} \beta_{ts}^{\frac{1}{\sigma}} m_{ts}^{1-\frac{1}{\sigma}}\right)^{-1} \sum_{s=t}^{\infty} m_{ts} \left(\frac{\psi_0 \psi}{1+\psi} Y_s^{1+\psi} + D_s\right); \tag{40}$$

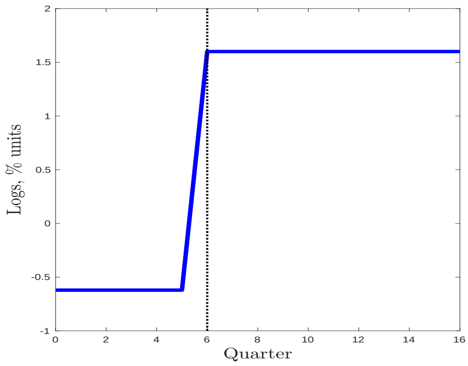
and (vi) there is no trade in long-term bonds and equity at the equilibrium prices that satisfy

$$q_t^b = \frac{\nu_{t+1} + q_{t+1}^b}{r_{t+1}}, \tag{41}$$

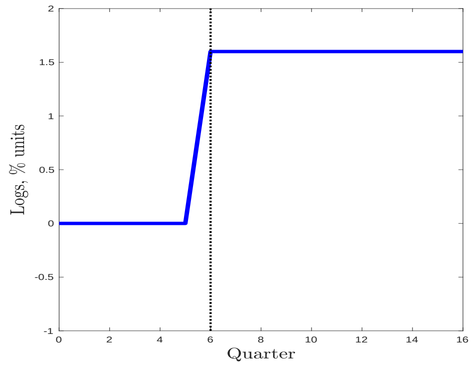
$$q_t^e = \frac{d_{t+1} + q_{t+1}^e}{r_{t+1}}. \tag{42}$$

Points (i)-(iv) have been proved above. Point (v) and (vi) follow from the absence of disagreement among households about the future evolution of nominal interest rates, inflation and aggregate output. In absence of disagreement, all households share the same marginal propensity to consume, so the aggregate economy is characterized by a representative household (point v), and households can achieve their desired consumption profile by using just short term nominal bonds, so the adjustments costs in (28) discourage households from trading in long-term bonds and equity (point vi). Figure 4 characterizes key properties of the baseline economy before the announcement. The shock to the subjective discount factor β_t causes a recession and a deflation until time t_β. At $t = t_\beta + 1$, β_t returns to its steady state value β, and the economy is back to steady state. This follows from the fact that the monetary policy in (33) is forward looking and the distribution of financial assets

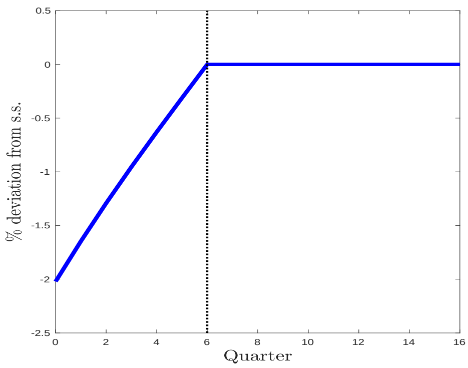
Figure 4: The baseline economy before the announcement



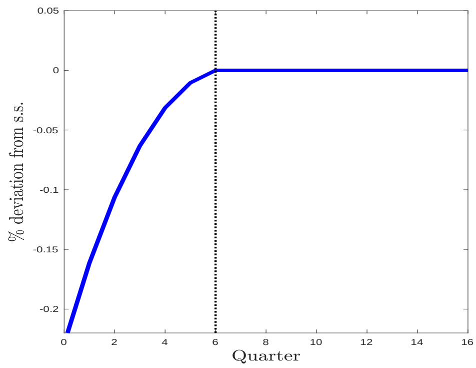
(a) Natural interest rate, $1/\beta_t$



(b) Nominal interest rate, R_t



(c) Output, Y_t



(d) Inflation, Π_t

Notes: The natural interest rate and the nominal interest rate are in logs in percentage, equal to $-100 \times \ln(\beta_t)$ and $100 \times \ln(R_t)$ respectively. Output Y_t and inflation Π_t are in percentage deviation from their steady state value. The vertical dashed lines denote the time $t_\beta + 1$ when the economy exits the liquidity trap. The economy is calibrated as in Table 1.

has no effect on aggregate output, consumption, and inflation, so there are no endogenous state variables relevant for the aggregate economy.[15] We state this result formally in the following Proposition:

Proposition 8 (The baseline economy before the announcement) *When β_t evolves as in* (32) *and agents have perfect foresight about the path of $R_t = \hat{R}_t$, output is unaffected by the household distribution of assets, and the economy is back to steady state at time t_β+1. The equilibrium nominal interest rate, inflation and output are such that:* $\forall t = 0, 1, ...t_\beta$, $R_t = 1$, $\Pi_t < 1$ *and* $Y_t < \bar{Y}$; *while* $\forall t > t_\beta$, $R_t = 1/\bar{\beta}$, $\Pi_t = 1$ *and* $Y_t = \bar{Y}$.

[15]Notice that over the transition the distribution of financial assets moves, but this has no feedbacks on aggregate output, consumption and inflation.

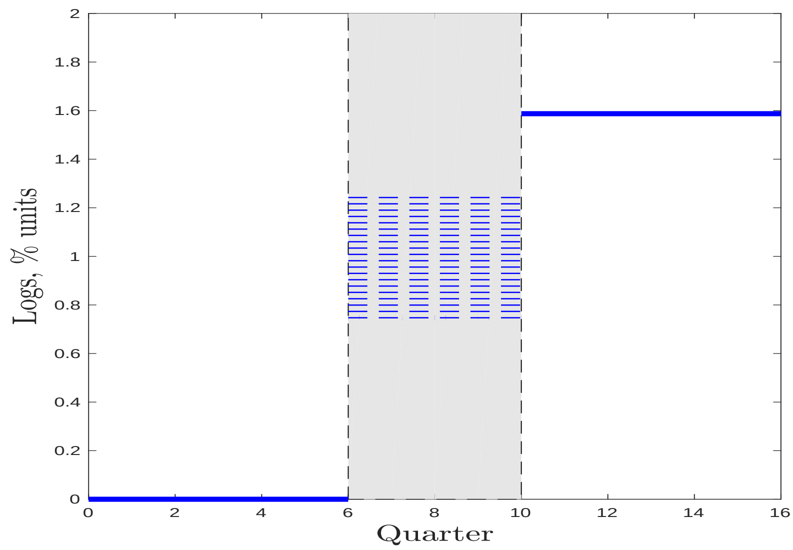
4.3 The economy after the announcement

At $t = 0$ after the shock to the path of the discount factor β_t has been realized, the monetary authority promises that it will keep nominal interest rates "low for longer" by announcing that, when the economy has exited the liquidity trap and until time $t_r > t_\beta$, it will keep nominal interest rates at $R^* < \bar{R}$, which is lower than the level $\bar{R}$ implied by the policy in *normal-times* as given by (33) (see Proposition 8). Due to the announcement, the economy may exit the liquidity trap before $t_\beta + 1$ and we denote by $T \leq t_\beta + 1$ the (endogenous) first date when $\hat{R}_T$ in (33) is greater than one. After using the equilibrium values of $\hat{R}_t$ implied by Proposition 8, we obtain that the resulting announced path for the nominal interest rate R_t^a is a monotonically increasing step function such that

$$R_t^a = \begin{cases} 1 & \text{if } \ t = 0, 1, ...T - 1, \\ R^* & \text{if } \ t = T, T + 1, ...t_r, \\ \bar{R} & \text{if } \ t > t_r. \end{cases} \tag{43}$$

Figure 5 shows different profiles of R_t^a associated with different values of R^*. The shaded area corresponds to the time interval that starts at T and ends at t_r, over which the monetary authority has announced that it will depart from its policy rule in (33).

Figure 5: Multiple priors on R_t



Notes: The nominal interest rate is expressed in logs in percentage, i.e. $100 \times \log(R_t)$. The solid blue line corresponds to R_t for $t < T$ and $t > t_r$. The horizontal dashed blues lines in the time interval from T to t_r, shaded in grey, correspond to different values of R^* over which agents have multiple priors.

Ambiguity Households and firms doubt about the credibility of the announcement and whether the monetary authority will deviate from her normal interest rate rule in (33). As a result, households and firms face uncertainty about the value of the nominal interest rate R^* that the monetary authority will set from time T until t_r and have multiple priors about it. For simplicity we assume that there is uncertainty just about R^* and, as discussed in Section 3.4, we assume that full credibility is possible but also that there is a lower bound on the credibility of the monetary authority. In particular, households think that R^* could be any random variable with support $\Omega \subseteq \{R^l, R^h\}$, with R^l corresponding to the value of R^* announced by the central bank.[16] The larger the difference $R^h - R^l$, the greater the amount of uncertainty.

The household problem At $t \geq T$ all uncertainty about R^* is resolved, and household-x solves her problem under perfect foresight. At $t < T$, household-x faces uncertainty about the nominal interest rate R^* that the central bank will set from T to t_r and takes her decisions under the worst case possible realization of R^*, so households act as if they disagree about the evolution of the economy after T. Notice that household-x faces no uncertainty about the realization of equilibrium quantities until $T-1$, the only uncertainty is about her continuation utility at T, which is function both of the realization of R^* as well as of her wealth at T as characterized by the vector $\boldsymbol{\omega}_{xT}$. We denote this continuation utility by $V_T(\boldsymbol{\omega}_{xT}; R^*)$, which is constructed analogously to $V_t(\boldsymbol{\omega}_{xt}; \hat{R}_t)$ in (34). After remembering that $T \leq t_\beta + 1$, so that $\beta_t = \hat{\beta}, \forall t \leq T-1$ (see (32)), we obtain that the value of the problem of household-x at $t = 0$ satisfies

$$V_0(\boldsymbol{\omega}_{x0}) = \max_{\{c_{xt}, l_{xt}, \boldsymbol{\omega}_{xt+1}\}_{t=0}^{T-1}} \left\{ \sum_{t=0}^{T-1} \hat{\beta}^t U(c_{xt}, l_{xt}) + \hat{\beta}^T \min_{\Omega \subseteq \{R^l, R^h\}, G \in \mathscr{P}(\Omega)} \int_\Omega V_T(\boldsymbol{\omega}_{xT}; R^*) G(dR^*) \right\}, \tag{44}$$

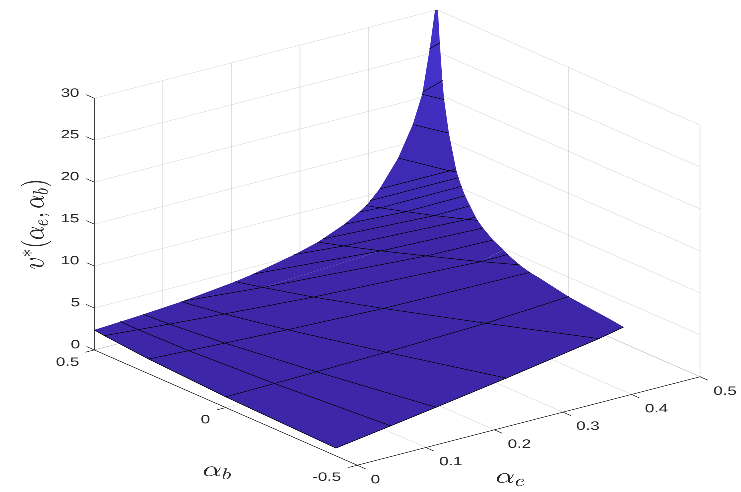
where the maximization is subject to the flow budget constraint in (29). The minimization in (44) just says that household-x internalizes that her consumption, labour and portfolio choices will affect her worst-case beliefs about the realization of R^* at $t = T$. Given aggregate prices and the functions $V(\boldsymbol{\omega}_{xT}; R^*)$, household x could be (i) skeptical, (ii) trusting, or (iii) Zen. Household x is is skeptical and behaves as if $R^* = R^h$ if $V(\boldsymbol{\omega}_{xT}; R^h) < V(\boldsymbol{\omega}_{xT}; R^l)$; she is trusting and behaves as if $R^* = R^l$ if $V(\boldsymbol{\omega}_{xT}; R^h) > V(\boldsymbol{\omega}_{xT}; R^l)$; she is Zen if her wealth at T is such that

$$\boldsymbol{\omega}_{xT} \in \left\{ \boldsymbol{\omega}^* \in \mathbb{R}^3 : V(\boldsymbol{\omega}^*; R^l) = V(\boldsymbol{\omega}^*; R^h) \right\}. \tag{45}$$

[16] Given the relatively rich characterization of households' portfolios that contain three assets, here we assume that the set of the possible realizations of R^* is discrete. This simplifies the analysis, given that differently from the analytical model of Section 3, the continuation value functions after T of households and firms are not generally concave in the future choices of the monetary authority.

To characterize the set of portfolio allocations in (45), that make household-x Zen we define household-x's wealth at the end of period $T-1$ as equal to $\upsilon_{xT-1} \equiv a_{xT} + q^b_{T-1} b_{xT} + q^e_{T-1} e_{xT}$.[17] We also define the share of household-x's wealth invested at the end of period $T-1$ in bonds and equity as equal to $\alpha^b_{xT-1} \equiv (q^b_{T-1} b_{xT})/\upsilon_{xT-1}$ and to $\alpha^e_{xT-1} \equiv (q^e_{T-1} e_{xT})/\upsilon_{xT-1}$, respectively. For given portfolio shares α^b and α^e, we find a unique value of wealth $\upsilon^*(\alpha^b, \alpha^e)$ which belongs to the set in (45)—and makes the household indifferent about future monetary policy choices about R^*. υ^* is strictly positive because an expansionary monetary policy increases labor income, which is beneficial to all households. So only sufficiently wealthy households can become indifferent about future choices of the monetary authority. Generally, for given portfolio shares α^b and α^e, households are trusting if their wealth is above $\upsilon^*(\alpha^b, \alpha^e)$; they are skeptical if their wealth is below $\upsilon^*(\alpha^b, \alpha^e)$; they are Zen if their wealth is exactly equal to $\upsilon^*(\alpha^b, \alpha^e)$. The resulting function $\upsilon^*(\alpha^b, \alpha^e)$ is plotted in Figure 6. Wealth is expressed in units of yearly steady state labor income. $\upsilon^*(\alpha^b, \alpha^e)$ is increasing in

Figure 6: Portfolio shares and worst-case beliefs



Notes: For given portfolio shares α^b and α^e the figure plots the threshold level of wealth $\upsilon^*(\alpha^b, \alpha^e)$ that belongs to the set in (45). Wealth is expressed in units of steady state yearly labor income. For given α^b and α^e, households with wealth above υ^* act as if $R^* = R^l$; those with wealth below υ^* act as if $R^* = R^h$; those with wealth equal to υ^* are Zen. All parameter values are as in Table 1.

both α^b and α^e. When all household's wealth is invested in short-term bonds, $\alpha^b = \alpha^e = 0$, υ^* is equal to 1.2 times steady state yearly labor income. When household's wealth is invested just in long-term bonds and equity in equal proportions, $\alpha^b = \alpha^e = 1/2$, then υ^* is equal to 30 times steady state yearly labor income. Generally υ^* is more sensitive to α^b

[17]Notice that $\boldsymbol{\omega} \in \mathbb{R}^3$ is a triple fully characterizing a household's wealth, while υ is a scalar measuring the value of a household's wealth.

than to α^e, because monetary policy affects less the return on equity than the return on long-term bonds, which follows from the fact that dividends respond little to monetary policy changes—due to the negative co-movement between aggregate consumption and firms mark-ups under sticky prices—,while the return on long-term bonds is directly affected by inflation. For example, if all wealth is invested in long-term bonds, $\alpha^b = 1$ and $\alpha^e = 0$, υ^* is equal to 12 times steady state yearly labor income, while if $\alpha^b = 0$ and $\alpha^e = 1$, υ^* becomes equal to 105 times steady state yearly labor income.

The firm problem At $t < T$, firms face the same uncertainty as households about the evolution of the economy at $t \geq T$. At $t = 0$, firm i set prices $\{p_{it}\}_{t=0}^{T-1}$ to maximize the sum of the present value of profits

$$W_0\left(p_{i-1}\right) = \max_{\{p_{it}\}_{t=0}^{T-1}} \left\{ \sum_{t=0}^{T-1} m_{0t}\, d_t(p_{it}, p_{it-1}) + m_{0T} \min_{\Omega \subseteq \{R^l, R^h\}, G \in \mathscr{P}(\Omega)} \int_{\Omega} W_T\left(p_{iT-1}; R^*\right)\, G(dR^*) \right\}, \tag{46}$$

where $p_{i-1} = 1\ \forall i$, $d_t(p_{it}, p_{it-1})$ is given in (35) and $W_T(p; R^*)$ denotes the equity value of the firm at the beginning of period T, which is constructed as in (36). Firms are identical and set the same price $p_{it} = p_t, \forall i$, so that (37) holds $\forall t \leq T$, with firms' expectations determined by their worst-case beliefs. At our parameter values, $W_T\left(p; R^*\right)$ is decreasing in R^*, because a higher interest rate reduces the present value of profits.[18] Therefore, firms will make their pricing decisions under a worst-case probability distribution that assigns full probability to the case $R^* = R^h$.

Equilibrium Appendix C details how we solve the model using global non-linear methods. Basically the model is solved by backward induction: we first use Lemma 3 to characterize the equilibrium at $t \geq T$ for each R^* after all uncertainty is resolved and then solve for the equilibrium at $t < T$ by aggregating the choices of households and firms as determined by (44) and (46), respectively. T is the endogenously determined first date when the economy exits the liquidity trap after the announcement. Notice that at $t < T$ agents act as if they "agree to disagree" about the evolution of aggregate quantities at $t \geq T$, while agents fully agree on the value of T and all aggregate quantities before T.

5 Calibration

The model is calibrated at the quarterly frequency. Table 1 reports the parameter values and targets used in the calibration.

[18]Under our calibration, profits fall in response to a policy easing—due to the costs of adjusting prices—, but the present value of profits is affected less by this reduction in profits than by the lower interest rates.

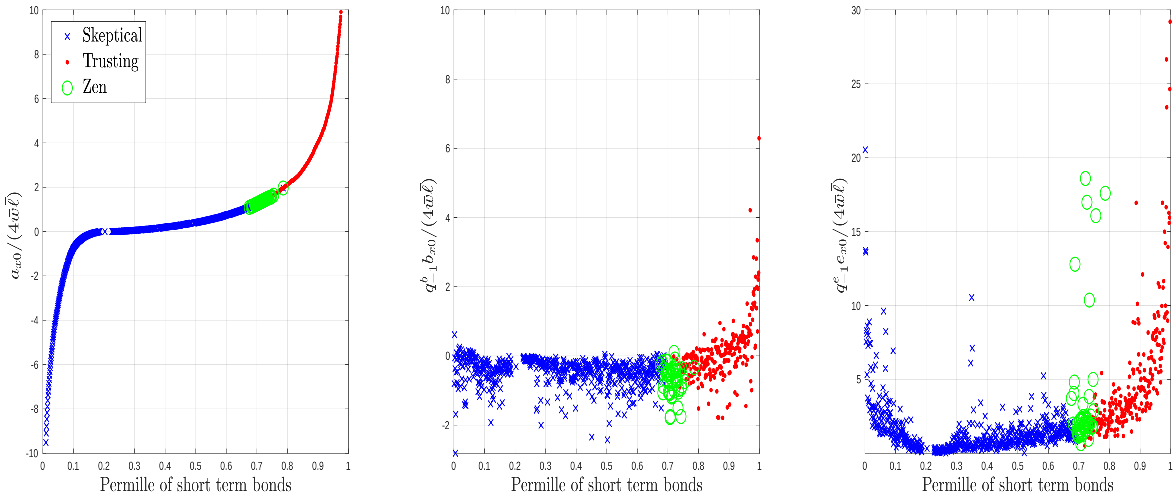
Table 1: Baseline calibration

Model		Data	
Parameter	Value	Moment	Value
σ	2	Elasticity of intertemporal substitution	0.5
ψ	0.5	Frisch elasticity of labor supply	2
ψ_0	0.6667	Labor supply normalization in steady-state	1
θ	3	Micro-estimates	3
κ_0	30	Slope of Phillips curve	0.1
ϕ	1.5	Taylor rule response to inflation	1.5
$\overline{\chi}_b$	0.91	Mean ratio of adjustment costs to value of transactions in LT bonds	0.6%
$\overline{\chi}_e$	1.04	Mean ratio of adjustment costs to value of transactions in equity	0.6%
A	3.07	Net supply of ST bonds divided by yearly average labor income	1.15
B	-0.72	Net supply of LT bonds divided by yearly average labor income	-0.27
ζ	0.22	Value of equity divided by yearly average labor income	2.67
β	0.9843	Ratio of labor income over consumption expenditures	0.85
ν	0.0160	Equalization of nominal returns on LT and ST bonds	0.016
$\hat{\beta}$	1.00625	Percentage fall in Euro Area GDP in 2012	2%
t_β	5	Expected length of Euro Area 2012 recession	6
t_r	9	Maximal response of forward rates to FG	10
R^h	1.0144	Average response of inflation expectations to FG	10 bp
R^l	1.0084	Maximal difference in the response of inflation expectations to FG	20 bp

Preferences and technologies Following Guvenen (2006) and Keane and Rogerson (2012) we set the elasticity of intertemporal substitution (EIS) to 0.5 and the Frisch elasticity of labor supply to 2. We normalize steady-state labor supply to one, which determines the scaling factor of the utility function ψ_0. We set $\theta = 3$, consistent with micro level evidence on the elasticity of substitution across varieties (Nevo 2001; Chevalier, Kashyap, and Rossi 2003; and Broda and Weinstein 2006) and in the range of values typically used in macro models, see for example Midrigan (2011). The parameter governing the cost of price adjustment κ_0 is used to match the elasticity of inflation to current marginal cost in the Phillips curve θ/κ_0, which we set at 0.1 as in Schorfheide (2008). We use the standard value $\phi = 1.5$ in the Taylor rule in (3). The parameters governing the cost of adjusting the holdings of long-term bonds and equity, $\overline{\chi}_b$ and $\overline{\chi}_e$, are set so that, for each asset, adjustment costs during the six quarters after the announcement are equal to 0.6% of the value of the transactions occurred, which is in line with the estimates by Barber and Odean (2000) and Bonaparte and Cooper (2009).

Financial assets and returns We parameterize the initial joint distribution of short-term bonds, long-term bonds and equity over a support of $n = 1000$ discrete points of equal mass (a_i, b_i, e_i), $i = 1, 2 \ldots n$. Data on short-term bonds, long-term bonds and equity are from the Household Finance and Consumption Survey (HFCS), see the Appendix for full details. Each a_i corresponds to a permille of the distribution of short-term bonds held by households in the Euro-Area. b_i and e_i are determined to match the average value of household's wealth invested in long-term bonds $q^b_{-1} b_i$ and equity $q^e_{-1} e_i$ associated with the permille a_i. Short-term bonds are equal to the wealth invested by the household directly or indirectly (through mutual or pension funds) in currency, deposits, and short-term bonds (with remaining maturity less than 3 years and a half) minus the sum of non-mortgage debt plus outstanding balance of adjustable interest rate mortgages. Long-term bonds are equal to the wealth invested by the household (directly or indirectly) in long-term bonds (with remaining maturity greater than 3 years and a half) minus the outstanding balance of fixed interest rate mortgages. Equity corresponds to household's wealth invested (directly or indirectly) in private businesses, publicly traded companies, and real estate properties used for productive purposes. The fixed cost of production ζ is used to match the ratio of the aggregate value of equity with yearly labor income in the HFCS. We set β to match a ratio of labor income over consumption expenditures of 0.85, which corresponds to the value obtained from the Euro Area Accounts (EAA) in 2012. This yields a steady state return on savings of 6.5%, which is the approximate real return from investing in the stock market in the Euro Area. The nominal return on long-term bonds ν is normalized so that in steady states it is equal to the nominal return on short-term bonds. Since nominal prices are initially normalized to one, this also implies that $q^b_{-1} = 1$. Panel (a) of Figure 7 plots the permilles of the distribution of short-term bonds a_i. The associated average value of long-term bonds $q^b_{-1} b_i$ and equity $q^e_{-1} e_i$ are plotted in panels (b) and (c), respectively. All values are scaled by average yearly labor income. As it is well known, total household's wealth $\upsilon = a + q^b_{-1} b + q^e_{-1} e$ is highly dispersed and concentrated in the right tail of the distribution, with standard deviation and skewness (again scaled by average yearly labor income) equal to 23 and 116, respectively.

Figure 7: The distribution of financial assets from HFCS



(a) Short-term bonds, a_i (b) Long-term bonds, $q^b_{-1}b_i$ (c) Equity, $q^e_{-1}e_i$

Notes: Panel (a) plots the permilles of the distribution of short-term bonds held by European households a_i, $i = 1, 2 \ldots n$; the y-axis has been truncated to the left and to the right for illustrative purposes. Panels (b) and (c) plot the average value of long-term bonds and equity associated to each permille of the distribution of short-term bonds, equal to $q^b_{-1}b_i$ and $q^e_{-1}e_i$, respectively. All values are scaled by average annual labor income and are from the HFCS, see the Appendix for details.

Liquidity trap We set the discount factor at $t = 0, \ldots t_\beta$ to $\hat{\beta} = 1.005$ so that, before the announcement, at $t = 0$ output is 2% below its steady state value, roughly in line with the fall of Euro Area GDP in 2012. The liquidity trap is assumed to last six quarters, $t_\beta = 5$, to match the average expected duration of the recession from the Euro Area Survey of Professional Forecasters (SPF) in 2013, before the start of forward guidance by the ECB.

Forward guidance announcement To quantify the (possible) increase in disagreement among European households about expected future inflation after forward guidance, we rely on micro level evidence for Italy.[19] We interpret self reported inflation expectations as measuring the beliefs on the basis of which individuals act—say about the inflation that arises under the nominal interest rate R^* that solves the maxmin household problem in (44).[20] The Appendix details the source and construction of the variables used. The data are quar-

[19] The Appendix also reports country level evidence for the Euro Area. For countries we do not have information on expected inflation but just on the fraction of households who think that inflation will increase in the next year relative to the past year.

[20] Hurd (2009) reviews the evidence supporting the claim that the subjective probabilities of households explain well their behavior. Self reported expectations are biased and heterogeneous across households but they tend to have strong power in predicting household's behavior; for direct evidence about the effects of subjective beliefs see Kézdi and Willis (2011) and Armantier et al. (2015) who focus on households' financial decisions and Gennaioli, Ma, and Shleifer (2016) who focus on corporate investment.

terly and the sample covers the period 2012:I-2014:II. The end of the sample is dictated by the start of the ECB's Quantitative Easing program in 2015:I. For each Italian province we calculate the pre-announcement (in 2012) fraction of households with a positive net financial asset position (creditor households). Expected inflation is measured two quarters ahead. In each province i and quarter t, we construct a measure of the (average) inflation expectation bias of agents in the province by calculating $\hat{\pi}_{it} \equiv E_{it}[\pi_{it+2}] - \pi_{it+2}$ where $E_{it}[\pi_{it+2}]$ and π_{it+2} are expected inflation and realized future inflation, respectively. To evaluate whether, in response to forward guidance, the inflation expectations have increased more for creditor than for debtor households, we run the following Difference-in-Differences regression:

$$\hat{\pi}_{it} = \overline{\varrho} F_i + \varrho F_i \times \mathbb{I}_{t \geq t_0} + \varrho_x X_{it} + \epsilon_{it} \tag{47}$$

where F_i is equal to the (standardized) proportion of creditor households in the province. The controls X_{it} include a full set of time and province dummies. $\mathbb{I}_{t \geq t_0}$ is a dummy equal to one in the quarter of the announcement (t_0=2013:III) and in all subsequent quarters, zero in previous quarters. The coefficient $\overline{\varrho}$ measures the average effect of F_i on inflation expectations. The Difference-in-Differences coefficient ϱ measures the increase in the effect of F_i on inflation expectations in the quarters after the announcement. The results from estimating (47) are reported in Table 2, which indicate that, after forward guidance, the inflation expectations have increased more for creditor than for debtor households: provinces with two-standard-deviations more of creditor households—which represents two thirds of the cross sectional variation—experience an increase of around 20 basis points more in their inflation expectations. We take this as a (conservative) measure of the disagreement about expected inflation between trusting and skeptical households that emerges at $t = 0$ after the announcement. We use this information to identify the difference $R^h - R^l$, which is implicitly set to target a difference of 20 basis points between trusting and skeptical households in the yearly inflation they expect between T and $T + 3$.

To separately identify R^l and R^h we also target the change in average expected inflation after forward guidance, as obtained from Inflation Linked Swaps (ILS).[21] The data indicate that expected inflation has increased by around 10 basis points at a time horizon of two years which is in line with the evidence by Andrade and Ferroni (2016). We require R^l and R^h to be such that at $t = 0$ after the announcement, the yearly expected inflation between T and $T + 3$ has increased on average across households by 10 basis points. Finally we set $t_r = 9$, which coincides with the peak in the response of instantaneous forward rates to forward guidance, see Coeuré (2013), ECB (2014), and Picault (2017).

[21] An ILS is a contract, which involves an exchange of a fixed payment (the so-called "fixed leg" of the swap) for realised inflation over a predetermined horizon.

Table 2: Effects of forward guidance on expected inflation, Micro Evidence

VARIABLES	$\hat{\pi}_{it}$
Announcement-dummy $\times$ F_i (coefficient ϱ)	.10***
	.04
Effect of financial position F_i (coefficient $\overline{\varrho}$)	.02
	.02
R-squared	.35
No. of observations	1082
No. of i units	110

Notes: Results from regression (47). The regression includes year and individual fixed effect. The dependent variable is $\hat{\pi}_{it}$, in (47). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

6 Quantitative results

Panels (a)-(c) of Figure 7 show which households are skeptical (in blue), trusting (in red) or Zen (in green). Skeptical households tend to be the poorest households in the economy at $t = 0$, they roughly represent two thirds of the population and account for 57 percent of steady aggregate consumption. Trusting households are among the wealthiest households in the economy and represent around 26 percent of the population. The remaining (middle wealth) households are Zen. The correlation between (worst-case) beliefs of households and wealth is not perfect because the entire household portfolio also matters.

The blue solid lines in Figure 8 characterize the dynamics of the aggregate economy after the announcement before all uncertainty is resolved at time T which is pinpointed by the black vertical dotted line in each panel. Under our calibration we have $T = t_\beta + 1$, which implies that, in the absence of the announcement, the economy would be back to steady state at T (see Proposition 8). At $t < T$ after the announcement, *all* households act as if the nominal interest rate (panel a) at T will be lower than its steady state level while output (panel b) and inflation (panel c) are expected to be greater, yet there is disagreement on magnitudes. The regions shaded in grey illustrate the Knightian uncertainty about the dynamics of the economy from T onwards causing the disagreement among households before T: under the optimal worst-case beliefs, trusting and skeptical households act as if, at $t \geq T$, the economy will evolve along the upper or the lower contour of the region, with the red lines with o-markers characterizing the expectations of trusting households. Generally, $\forall t$, the red lines correspond to the *full credibility* benchmark where all firms and households accord full credibility to the announcement—fully believing that $R_t = R^l$ for

$\forall t = T, \ldots, t_r$. Quantitatively, trusting households act as if the nominal interest rate at T is 76 basis points below steady state and output 161 basis points above steady state, while for skeptical households the expected reduction in the nominal interest rate and the expected increase in output (both relative to steady state) are of 16 and 5 basis points, respectively.

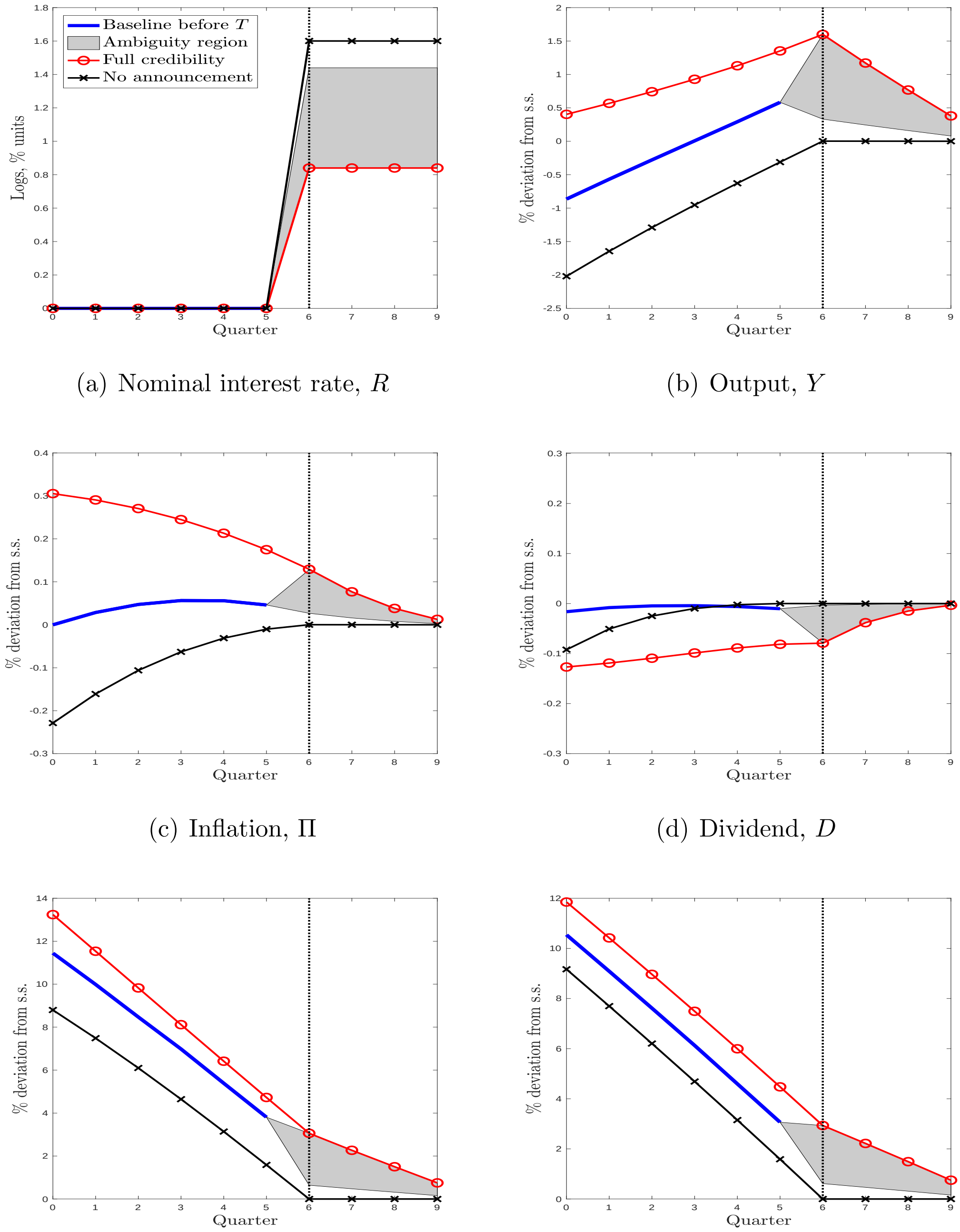
The response of output and inflation to the announcement in the interim before T is smaller than in the full credibility benchmark. The black lines with x-markers characterize the economy without announcement, as in Figure 4. This *no-announcement* economy represents the relevant benchmark to evaluate the effects of the announcement. Relative to the no-announcement benchmark, output in the model increases on impact by just above one percent compared with a 2.5% increase in the full credibility benchmark. When analyzing the cumulated response of output in the six quarters before T, we find that the response in the model is equal to 6.6 percent which represents 43 percent of the response in the full credibility benchmark, while the analogous proportion for the inflation response is 37 percent. Despite the muted response in output and inflation the response of the prices of long-term financial assets is sizable with a bigger increase in equity prices (panel e) than in long-term bond prices (panel f): when compared with the no-announcement benchmark, equity prices increase by about 2.5% at the time of the announcement, while nominal bond prices rise by 1.5%.[22] This happens because equity is a relatively good hedge against future monetary policy uncertainty, which follows from the observation that uncertainty about dividends (panel d) is relatively small as indicated by the range of the associated gray region. It is standard to evaluate the effects of forward guidance by looking at the response of nominal yields at long-term maturities, for example at three or four years. To evaluate the performance of the model along this dimension, we calculate the nominal internal rate of return i_s from investing in long-term bonds at $t = 0$ and then selling them at time s for $s = 12$ (three years) or $s = 16$ (four years).[23] At the three-year maturity, i_s falls by 12 basis point while at the four-year maturity the fall is by 10 basis points, which is reasonably in line with the existing evidence on the response of long-term government bond yields and EONIA swap rates to forward guidance by the ECB (see Coeuré (2013) and ECB (2014)).

The sources of the dampening There are three reasons why output responds less in the model than in the full credibility benchmark. First, the share of skeptical households is larger, implying a smaller average reduction in expected future nominal interest rates. Second, firms act as if the announcement will not be fully implemented and, since

[22]There is evidence that stock markets have responded positively to the start of forward guidance by the ECB. For example the Euro Area Dow Jones Euro Stock Price Index published by Datastream increased by more than 4 percent in the month after the start of forward guidance by the ECB.

[23]In practice i_s solves the equation $q_0^b p_0 = \nu \sum_{n=1}^{s} (1+i_s)^{-n} + (1+i_s)^{-s} q_s^b p_s$, where $\nu = \bar{R}$ implies $q_s^b p_s = 1$ for $s > t_r$, so that for $s = 12$ or $s = 16$ the internal rate of return i_s is immune to the disagreement in expectations among households in the economy.

Figure 8: Impulse responses to the forward guidance announcement



(a) Nominal interest rate, R

(b) Output, Y

(c) Inflation, Π

(d) Dividend, D

(e) Equity price, q^e

(f) Nominal bond price, pq^b

Notes: The blue solid lines correspond to the economy after the announcement, the black solid line to the benchmark with no announcement, the red dashed lines to the full credibility benchmark, where all households and firms act as if $R_t = R^l$, $\forall t = T, \dots t_r$. The black vertical dotted lines pinpoint T. Variables are in deviation from their steady state value with the exception of the logged nominal interest rate which is in levels (multiplied by 100). The parameter values are as in Table 1.

firms are forward-looking, the lower expected future inflation leads to lower inflation today which implies a lower aggregate demand because of the higher interest rates r_t at $t < T$.

Finally there is the misguidance effect, that arises because skeptical households are not randomly selected in the population—which corresponds to the effects of ρ in the analytical model of Section 2. To decompose the difference between the response of output in the full credibility benchmark and the response in the model, we construct two intermediate benchmarks. In the first, we assume that firms fully trust the announcement and that skeptical households are a random proportion of the population equal to 57 percent, with the remaining households being trusting. We call this the *average credibility* benchmark. In the second benchmark we assume instead that firms believe that the nominal interest rate will be set at $R_t = R^h$, $\forall t = T, \ldots t_r$, as in the equilibrium of the model. We call this the average credibility benchmark with *skeptical firms*. By comparing the response under full-credibility with the response under average-credibility, we identify the effect of differences in the average credibility of the announcement. By comparing the response in the two average credibility benchmarks we identify the effect of firms' beliefs. Finally by taking the difference between the response in the average credibility benchmark with skeptical firms and the response in our model we measure the misguidance effect. Table 3 shows how the three channels account for the dampening in the output response on impact (first row) and in the cumulated response in the six quarters before T (second row). Differences in average credibility account for slightly more than a half of the overall dampening in the output response, while the misguidance effect accounts for 24-29 percent. Differences in firm beliefs explain relatively less and matter more for the impact effect than for the cumulated response.

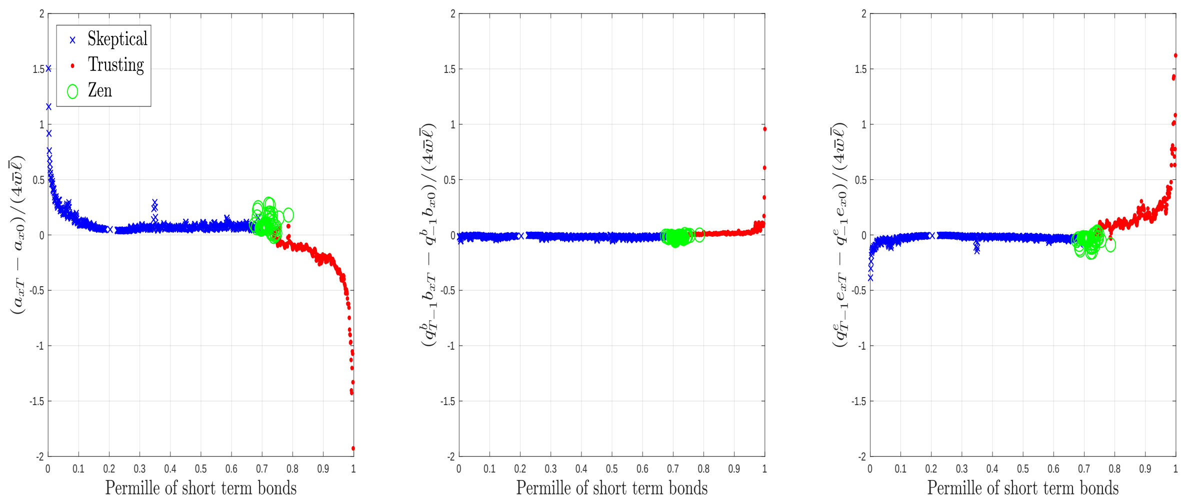
Table 3: Decomposing the dampening of the output response

	Average credibility $(y^f - y^a)/(y^f - y^m)$	Skeptical firms $(y^a - y^s)/(y^f - y^m)$	Misguidance $(y^s - y^m)/(y^f - y^m)$	Total
Impact	0.53	0.23	0.24	1.00
Cumulated	0.56	0.15	0.29	1.00

Notes: The first row decomposes the dampening of the output response on impact, the second focuses on the cumulated response in the interim before T. Responses (either on impact or cumulated) are calculated as a log difference with respect to the no-announcement benchmark and are denoted by y^i, where $i = f, a, s, m$ stands for the response under full-credibility, average-credibility, average credibility with skeptical firms, and the model, respectively. The dampening is measured by the difference $y^f - y^m$.

Rebalancing The turnover in financial markets increases after the announcement because trading in financial assets is profitable when households act based on different beliefs. As a result there is a rebalancing in the net as well as in the gross financial asset position of households. Figure 9 characterizes the changes in the value of household's wealth invested in short term bonds (panel a), long term bonds (panel b) and equity (panel c) as a function of the permille of the initial distribution of short term bonds to which the household belongs to. Skeptical households are in blue, Zen households in green, and trusting households in red. Skeptical and Zen households tend to buy short-term bonds and sell equity as well as long term bonds, while trusting households tend to reduce their holdings of short term bonds to buy long term bonds and especially equity. The turnover in the equity market increases more markedly than the turnover in the market for long term bonds and there is a substantial amount of deleveraging among the most indebted households in the economy. The effects of forward guidance on the turnover in financial markets has been little studied, but some existing (preliminary) evidence is consistent with the predictions of the model. According to ECB (2015), total turnover has increased in all money market segments in the months after the start of forward guidance by the ECB, on average by around 6%.[24] There is similar evidence for the US (Kreicher and McCauley 2016), which might indeed indicate that increased turn-over in money markets is a general feature of forward guidance. The

Figure 9: Changes in households' wealth between time -1 and $T-1$



(a) Short-term bonds (b) Long-term bonds (c) Equity

Notes: The x-axis identifies the permille of the initial distribution of short-term bonds to which the household belongs to, while the y-axis in panels a-c shows the change (between time -1 and $T-1$) in the amount of wealth invested by the household in short term bonds $(a_{xT-1} - a_{x0})/(4\bar{w}\bar{\ell})$, long-term bonds $(q^b_T b_{xT} - q^b_{-1} b_{x0})/(4\bar{w}\bar{\ell})$, and equity $(q^T_e e_{xT-1} - q^e_{-1} e_{x0})/(4\bar{w}\bar{\ell})$, respectively.

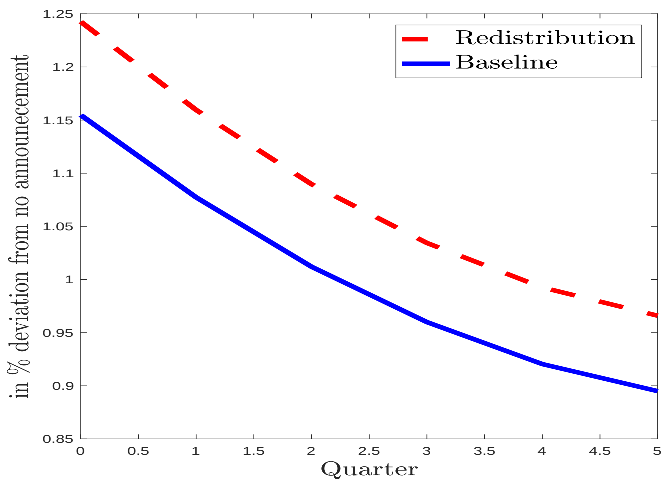
Euro Area stock market turnover ratio has also increased by around 5 percentage points

[24]The secured segment of the money market recorded the largest year-on year increase in the 12 months after the announcement up by 24%. The turnover in the foreign exchange swap market, which can be taken as an instrument to hedge against future inflation risk, rose by around 10% on average in the twelve months after the announcement with larger increases at long maturities.

in the quarters following forward guidance, while the Euro Area Accounts (EAA) indicate an acceleration in the process of household deleveraging after July 2013: the Euro-area household debt over GDP ratio was stable in the two quarters before the announcement and it has then fallen by more than three percentage points in the four quarters after the announcement.

Redistributive policies In our model, when debtors and creditors share the same beliefs, they also have the same marginal propensity to consume and redistributing wealth has no effects on aggregate consumption. But after the announcement, their beliefs are different and taxing today the wealthy to transfer the resulting tax income to the poor is expansionary. This implies that fiscal redistributive policies are complementary with forward guidance in stimulating the economy. The general idea is that forward guidance is more expansionary when financial imbalances are smaller (see Proposition 6). To highlight this point, at the time of forward guidance we tax by 20% the short-term bonds held by the wealthiest 20% of households in the economy. The resulting tax revenue is then redistributed as a lump sum transfer to all other households in the economy in the form of short-term bonds, so that the net supply of short term bonds remains unchanged. Figure 10 compares the output response after redistribution (red dashed line) with the response before redistribution (blue solid line). Responses are calculated as a difference with respect to the no-announcement benchmark. Fiscal redistribution increases the response by around 10% with a cumulated effect on output in the quarters before T which goes up by more than a half of a percentage point.

Figure 10: Output response to forward guidance after fiscal redistribution



Notes: Responses are calculated as a difference with respect to the no-announcement benchmark. The solid blue line corresponds to the baseline model as in Figure 8, the red dashed line to the response after taxing the short term bonds held by the wealthiest 20% of households in the economy. The parameter values are as in Table 1.

7 Conclusions

We have characterized the equilibrium of a new Keynesian model in which ambiguity-averse households with heterogeneous wealth use a worst-case criterion to judge the credibility of monetary policy announcements. An announcement of monetary loosening is less expansionary in our framework than under full credibility for two reasons. First, just a fraction of households in the economy will act as if the announcement will be fully implemented. Secondly, this fraction is not randomly selected in the population because wealthy creditor households are more prone to act as if the announcement will be implemented, due to the potential loss of wealth from the prospective policy easing. And when creditors believe the announcement more than debtors, their expected wealth losses are larger than the wealth gains that debtors expect. Hence the economy responds as if there is a fall in perceived aggregate wealth, which if large enough can even cause a contraction in aggregate demand. To gauge the importance of these effects, we have calibrated the model to the Euro Area in a liquidity trap after allowing for a relatively rich characterization of the wealth portfolio of European households. We find that forward guidance is substantially less expansionary than in the full credibility benchmark. Despite the muted effects on output and inflation, forward guidance has significant effects on both the price of all long term financial assets and the turnover in financial markets.

In the paper, we have focused the analysis on the effects of monetary policy announcements, but the same logic would apply to announcements about any future policy that, if implemented, would generate winners and losers, such as pension reforms, or revisions to competition, innovation or fiscal policy, or changes to labor market institutions like unemployment insurance and job protection. Generally, announcements of future reforms that will redistribute wealth if implemented, tend to have little and sometimes even unintended perverse effects when agents are ambiguity-averse, because the net losers from the redistribution tend to give more credit to the announcement than the net winners.

We have emphasized an interesting complementarity between fiscal policy and monetary policy announcements. In our model, fiscal transfers and also their timing affect the formation of households' beliefs, so governments can use them strategically to enhance the credibility and effectiveness of announcements. We think this is an important mechanism that requires further investigation. In practice households differ in the level and composition of their wealth, as well as in their marginal propensity to consume, degree of ambiguity aversion, financial literacy, labor income, and human capital. Under ambiguity aversion, this heterogeneity has a first-order effect on the formation of households' (worst-case) beliefs and thereby on the effects of policy announcements. As a result the menu of policies available to make announcements more effective is potentially large.

References

Adam, K. and J. Zhu (2016). Price level changes and the redistribution of nominal wealth across the Euro Area. *Journal of the European Economic Association 14*(4), 871–906.

Andrade, P. and F. Ferroni (2016). Delphic and Odyssean monetary policy shocks: Evidence from the Euro-Area. Discussion Papers in Economics, University of Surrey 12/16.

Andrade, P., G. Gaballo, E. Mengus, and B. Mojon (2015). Forward Guidance and heterogeneous beliefs. Banque de France Working Paper 573.

Angeletos, G.-M. and L. Chen (2017). Forward guidance without common knowledge. NBER Working Paper 22785.

Armantier, O., B. de Bruin Wändi, G. Topa, W. van der Klaauw, and B. Zafar (2015). Inflation expectations and behavior: Do survey respondents act on their beliefs? *International Economic Review 56*(2), 505–536.

Auclert, A. (2015). Monetary policy and the redistribution channel. Mimeo Princeton.

Backus, D., A. Ferriere, and S. Zin (2015). Risk and ambiguity in models of business cycles. *Journal of Monetary Economics 69*, 42–63.

Barber, B. M. and T. Odean (2000). Trading is hazardous to your wealth: The common stock investment performance of individual investors. *Journal of Finance 55*(2), 773–806.

Barro, R. J. (1974). Are government bonds net wealth? *Journal of Political Economy 82*, 1095–1117.

Barthelemy, J. and E. Mengus (2017). The signaling effect of raising inflation. HEC Paris Research Paper ECO/SCD-2016-1162.

Bonaparte, Y. and R. Cooper (2009). Costly portfolio adjustment. NBER Working Paper 15227.

Broda, C. and D. E. Weinstein (2006). Globalization and the gains from variety. *Quarterly Journal of Economics 121*, 541–585.

Chevalier, J. A., A. K. Kashyap, and P. E. Rossi (2003). Why don't prices rise during periods of peak demand? Evidence from scanner data. *American Economic Review 93*, 15–37.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011). When is the government spending multiplier large? *Journal of Political Economy 119*(1), 78–121.

Coeuré, B. (2013). The usefulness of Forward Guidance. Technical report, Speech before the Money Marketeers Club of New York, 26 September 2013.

Cooley, T. F. and V. Quadrini (2004). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Economic Theory 118*(2), 174–208.

Dimmock, S. G., R. Kouwenberg, O. S. Mitchell, and K. Peijnenburg (2016). Ambiguity aversion and household portfolio choice puzzles: Empirical evidence. *Journal of Financial Economics 119*(3), 559–577.

Doepke, M. and M. Schneider (2006). Inflation and the redistribution of nominal wealth. *Journal of Political Economy 114*(6), 1069–1097.

Eggertsson, G. B. and M. Woodford (2003). Zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity 2003*(1), 139–233.

Epstein, L. G. and M. Schneider (2003). Recursive multiple-priors. *Journal of Economic Theory 113*(1), 1–31.

Farhi, E. and I. Werning (2017). Monetary policy, bounded rationality, and incomplete markets. NBER Working Paper 23281.

Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica 1*(4), 337–357.

Gabaix, X. (2016). A behavioral New Keynesian model. NBER Working Paper 22954.

Gaballo, G. (2016). Rational inattention to news: The perils of forward guidance. *American Economic Journal: Maxroeconomics 8*(1), 42–97.

García-Schmidt, M. and M. Woodford (2015). Are low interest rates deflationary? A paradox of perfect-foresight analysis. NBER Working Paper 21614.

Gennaioli, N., Y. Ma, and A. Shleifer (2016). Expectations and investment. In M. Eichenbaum and J. Parker (Eds.), *NBER Macroeconomics Annual 2015*, pp. 379–431. University of Chicago Press.

Gilboa, I. and D. Schmeidler (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics 18*(2), 141–153.

Golosov, M. and L. Iovino (2017). Social insurance, information revelation, and lack of commitment. Mimeo, Bocconi University.

Golosov, M. and A. Tsyvinski (2008). Optimal fiscal and monetary policy without commitment. In *New Palgrave Dictionary of Economics, Monetary Economics*, pp. 283–286.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review 78*(3), 402–17.

Guiso, L., P. Sapienza, and L. Zingales (2016). Monnet's error? *Economic Policy 31*(86), 247–297.

Guvenen, F. (2006). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics 53*(7), 1451–1472.

Hansen, L. P. and T. J. Sargent (2001). Robust control and model uncertainty. *American Economic Review 91*, 60–66.

Hansen, L. P. and T. J. Sargent (2008). *Robustness*. Princeton university press.

Honkkila, J. and I. K. Kavonius (2013). Micro and macro analysis on household income, wealth and saving in the Euro Area. Working Paper Series European Central Bank 1619.

Hurd, M. D. (2009). Subjective probabilities in household surveys. *Annual Review of Economics 1*, 543–562.

Ilut, C., P. Krivenko, and M. Schneider (2016). Uncertainty aversion and heterogeneous beliefs in linear models. *Mimeo Duke University*.

Ilut, C. and H. Saijo (2016). Learning, confidence, and business cycles. *Mimeo Duke University*.

Ilut, C. and M. Schneider (2014). Ambiguous business cycles. *American Economic Review 104*(2), 2368–2399.

Iovino, L. and D. Sergeyev (2017). Quantitative easing without rational expectations. Manuscript, Bocconi University.

Kaplan, G., B. Moll, and G. Violante (2016a). Monetary policy according to HANK. NBER Working Paper 21897.

Kaplan, G., B. Moll, and G. Violante (2016b). A note on unconventional monetary policy in HANK. Mimeo, New York University and Princeton University.

Keane, M. and R. Rogerson (2012). Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature 50*(2), 464–476.

Kézdi, G. and R. J. Willis (2011). Household stock market beliefs and learning. NBER Working Paper 17614.

Kreicher, L. and R. McCauley (2016). Asset managers, eurodollars and unconventional monetary policy. Working Paper 578, Bank For International Settlements.

Kydland, F. E. and E. C. Prescott (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy 85*(3), 473–492.

McKay, A., E. Nakamura, and J. Steinsson (2015). The power of Forward Guidance revisited. Mimeo, Columbia University.

Midrigan, V. (2011). Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica 79*(4), 1139–1180.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *American Economic Review 69*, 307–342.

Picault, M. (2017). Pricing the ECB's Forward Guidance with the EONIA swap curve. *International Journal of Finance and Economics 22*(2), 129–138.

Ricardo, D. (1888). Essay on the funding system. In J. McCulloch (Ed.), *The Works of David Ricardo*. London: John Murray.

Rotemberg, J. (1982). Sticky prices in the United States. *Journal of Political Economy 90*, 1187–1211.

Schorfheide, F. (2008). DSGE model-based estimation of the New Keynesian Phillips curve. *FRB Richmond Economic Quarterly 94*(4), 397–433.

Strzalecki, T. (2011). Axiomatic foundations of multiplier preferences. *Econometrica 79*, 47–73.

Del Negro, M., M. P. Giannoni, and C. Patterson (2015). The Forward Guidance puzzle. Mimeo, New York Fed.

ECB (2014). The ECB's Forward Guidance. *Monthly Bulletin April*, 65–73.

ECB (2015). *Euro Money Market Study 2014*. European Central Banck, available at www.ecb.europa.eu.

Tobin, J. (1982). *Asset Accumulation and Economic Activity: Reflections on Contemporary Macroeconomic Theory*. University of Chicago Press.

Werning, I. (2015). Incomplete markets and aggregate demand. NBER Working Paper 21448.

Wiederholt, M. (2014). Empirical properties of inflation expectations and the Zero Lower Bound. *Goethe University Frankfurt*.

APPENDIX

Section A contains some proofs and theoretical derivation, Section B describes the data, Section C discusses computational details, while Section D contains some algebra.

A Theoretical derivations

This section contains the proofs of some results stated in the main text, including a full characterization of the model in Section 3 when (i) agents can trade in real assets as well as in short-term nominal bonds and (ii) ambiguity aversion is modeled as in Hansen and Sargent (2001, 2008).

A.1 Proofs of results

Proof of Lemma 2. $R_{-1} = \bar{R}$ because the economy is initially in a steady state. Given the timing of the monetary announcement, prices do not respond at $t = 0$ so $\Pi_0 = \Pi_0^* = 1$, which given (3) yields $R_0 = \bar{R}$. Lemma 1 implies that the economy is back to steady state starting from $t = 1$ so it must be that the (real) interest rate is back to steady state starting from $t = 2$, $r_t = \bar{R}\ \forall t \geq 2$. By assumption we also have $\Pi_t = \Pi_t^* = 1$, $\forall t \geq 2$ so we have $R_{t-1} = r_t = \bar{R}\ \forall t \geq 2$, which immediately gives $R_t = \bar{R}\ \forall t$. And this together with (3) also implies that $\Pi_t = \Pi_t^*\ \forall t \geq 0$. ■

Proof of Proposition 1. Under full credibility, household $j = c, d$ solves the problem

$$\max_{\{c_{js}, l_{js}, a_{js+1}\}_{s\geq 0}} \sum_{s=0}^{\infty} \beta^s\, U(c_{js}, l_{js}),$$

subject to the budget constraint in (2). The first order condition for the consumption choices of household j at $t = 0$ yields the Euler condition

$$\left(c_{j0} - \psi_0 \frac{l_{j0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta\, r_1 \left(c_{j1} - \psi_0 \frac{l_{j1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{48}$$

where $r_1 = \bar{R}\,\varepsilon^{-1}$, which uses full credibility and Lemma 2. Output Y_0 can be obtained using the market clearing condition for final consumption

$$Y_0 = \frac{c_{c0} + c_{d0}}{2},$$

where, given Lemma 1, c_{j0} and c_{j1} should satisfy

$$c_{j0} = Y_0 + \bar{R}\, a_{j0} - a_{j1} \quad \text{and} \quad c_{j1} = \bar{Y} + (\bar{R} - 1)\varepsilon^{-1}\, a_{j1} \quad \forall j = c, d. \tag{49}$$

We can substitute (49) into (48), and use the conditions for financial market clearing at $t = -1$, $a_{c0} = -a_{d0} = B$, and at $t = 0$, $a_{c1} = -a_{d1} = B'$. Since $l_{jt} = Y_t$, we obtain that

(48) evaluated for $j = c$ and for $j = d$ yields

$$\frac{\bar{N} + (\bar{R} - 1)\varepsilon^{-1} B'}{N_0 + \bar{R}B - B'} = \varepsilon^{-\frac{1}{\sigma}} \tag{50}$$

and

$$\frac{\bar{N} - (\bar{R} - 1)\varepsilon^{-1} B'}{N_0 - \bar{R}B + B'} = \varepsilon^{-\frac{1}{\sigma}}, \tag{51}$$

respectively. After solving for N_0, we obtain

$$N_0 = \bar{N}\varepsilon^{\frac{1}{\sigma}}, \tag{52}$$

which can be substituted into (50) to solve for B' to obtain

$$B' = \frac{\bar{R}\,B}{(\bar{R} - 1)\,\varepsilon^{\frac{1}{\sigma} - 1} + 1} > 0. \tag{53}$$

This means that $B'/\varepsilon - B < 0$ if $\varepsilon > 1$, and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$, which completes the proof. ■

Proof of Proposition 7. The proof proceeds in three steps. We characterize (i) the full-credibility (FC) benchmark, (ii) an inflationary announcement $\varepsilon > 1$, and (iii) a deflationary announcement $\varepsilon < 1$.

FC benchmark The properties of the FC benchmark are given in Proposition 1, which implies that $N_0 = \varepsilon^{\frac{1}{\sigma}}\bar{N}$; $B' > 0$; $B'/\varepsilon - B < 0$ if $\varepsilon > 1$; and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$. Finally (53) implies that

$$B'/\varepsilon - B = \frac{\varepsilon^{-\frac{1}{\sigma}}\,N_0 - \bar{N} + B\,\left[\varepsilon^{-\frac{1}{\sigma}}\left(\bar{R} - \varepsilon\right) - (\bar{R} - 1)\right]}{\bar{R} - 1 + \varepsilon^{1 - \frac{1}{\sigma}}} \tag{54}$$

where $N_0 = \varepsilon^{\frac{1}{\sigma}}\bar{N}$.

Case $\varepsilon > 1$ If (19) fails we have a credit crunch equilibrium, $B'/\varepsilon = 0$, which immediately implies a larger fall in B'/ε than in the FC benchmark. If (19) holds, then $B' > 0$ and we can use (17) and (18) under Proposition 4 to show that B'/ε still satisfies (54). After using (24) under $\bar{\tau} = 1/2$ and $\rho = 1$ we also obtain

$$N_0 = \bar{N}\,(\varphi\,\varepsilon^{\frac{1}{\sigma}} + 1 - \varphi) - \mu B < \varepsilon^{\frac{1}{\sigma}}\bar{N}$$

where the last inequality follows from the fact that $\varphi \in [0, 1]$ and $\mu > 0$. This together with (54), proves in general that, $\forall B$, B'/ε falls more than in the FC benchmark.

Case $\varepsilon < 1$ Proposition 6 implies that if (19) fails, we have a credit crunch equilibrium, $B'/\varepsilon = 0$. If (19) holds, $B'/\varepsilon > 0$, which from (23) implies $\bar{\tau} = 1/2$ and $\rho = -1$, that can be substituted into (18) and (24) to obtain

$$B'/\varepsilon - B = \frac{N_0 - \bar{N}}{\varepsilon \bar{R}} + \left(\varepsilon^{-1} - 1\right) B, \tag{55}$$

and

$$N_0 = \bar{N}\left[\varphi + (1-\varphi)\,\varepsilon^{\frac{1}{\sigma}}\right] - \mu B, \tag{56}$$

with

$$\varphi = \frac{1 + (\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}-1}}{2 + (\bar{R}-1)\,\left(1+\varepsilon^{\frac{1}{\sigma}-1}\right)} \quad \in [0,1],$$

$$\mu = \frac{\bar{R}(\bar{R}-1)\left(\varepsilon^{\frac{1}{\sigma}-1} - 1\right)}{2 + (\bar{R}-1)\,\left(1+\varepsilon^{\frac{1}{\sigma}-1}\right)} > 0.$$

By combining (55) with (56), we conclude that $B'/\varepsilon - B < 0$ if

$$B < \tilde{B}_1 \equiv \frac{\bar{N}\,(1-\varepsilon^{\frac{1}{\sigma}})}{2\,\bar{R} - (\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}} - \varepsilon\,(1+\bar{R})}, \tag{57}$$

where $B = \tilde{B}_1$ satisfies the condition (19) for a No-Credit-Crunch equilibrium, which generally implies that, $\forall B < \tilde{B}_1$, B'/ε falls. For $B \geq \tilde{B}_1$ (19) is satisfied and we can use (17) to write

$$B'/\varepsilon - B = \frac{\bar{N} - \varepsilon^{-\frac{1}{\sigma}}\,N_0 + B\,\left[\varepsilon^{-\frac{1}{\sigma}}\left(\bar{R}-\varepsilon\right) - (\bar{R}-1)\right]}{\bar{R} - 1 + \varepsilon^{1-\frac{1}{\sigma}}}. \tag{58}$$

where N_0 is given by (56). Comparing (54) with (58), we immediately conclude that B'/ε increases less than in the FC benchmark if and only if $N_0 > \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$. Since N_0 in (56) is decreasing in B, we can then conclude that B'/ε increases less (more) than in the FC benchmark if and only if $B < \tilde{B}_2$ ($B > \tilde{B}_2$) where

$$\tilde{B}_2 \equiv \bar{N}\,\frac{[1 + (\bar{R}-1)\varepsilon^{\frac{1}{\sigma}-1}]\,(1-\varepsilon^{\frac{1}{\sigma}})}{\bar{R}\,(\bar{R}-1)\,(1-\varepsilon^{\frac{1}{\sigma}-1})}$$

is the value of B at which $N_0 = \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$. Remember that at $B = \tilde{B}_1$ we have $B'/\varepsilon - B = 0$ and that N_0 in (56) is strictly decreasing in B. So from (58) we conclude that at $B = \tilde{B}_1$, $N_0 > \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$, which immediately implies that $\tilde{B}_2 > \tilde{B}_1$. ■

Proof of Lemma 3. Points i-iv are proved in the main text. Maximizing (34) with respect to l_{xs} yields (11), while the first order condition for a_{xt+1} yields $\forall t \geq 0$

$$\left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta\, E_{xt}\left[r_{t+1}\left(c_{xt+1} - \psi_0 \frac{l_{xt+1}^{1+\psi}}{1+\psi}\right)^{-\sigma}\right], \tag{59}$$

After using (59) and the perfect foresight assumption, maximizing with respect to z_{xt+1}, $\forall z = b, e$, yields

$$\chi_{z_{xt}}\left(b_{xt}, b_{xt-1}, e_{xt}, e_{xt-1}\right) = E_{xt-1}\left[\left(\frac{q_t^z + r_t^z}{r_t} - q_{t-1}^z\right) + \frac{\chi_{z_{xt}}\left(b_{xt+1}, b_{xt}, e_{xt+1}, e_{xt}\right)}{r_t}\right], \tag{60}$$

which should hold $\forall t > 0$, where r_t^z is the instantaneous return of asset z at time t—equal to ν_t if $z = b$, to d_t if $z = e$,—while the subscript to the function χ denotes the variable with respect to which the partial derivative of χ is calculated. The condition in (60) determines the optimal rate of growth of z_{xt} by equating the marginal cost of adjusting the holdings z_{xt} at $t-1$ to the sum of the expected future return of z_{xt} plus the marginal effect of today investment in z_{xt} on tomorrow adjustment costs of the household. Under perfect foresight (60) says that all assets should pay the same return at the margin, which after using the conditions for market clearing of financial markets implies that there is no trade in either long-term bonds or equity, $b_{xt} = b_{xt+1}$ and $e_{xt} = e_{xt+1}$, $\forall x$ and $\forall t$ at the equilibrium prices q_t^b and q_t^e that satisfy (41) and (42). We now use (11), (59), and the intertemporal budget constraint of household-x to solve for c_{xt} as equal to

$$c_{xt} = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \frac{r_t a_{xt} + \sum_{s=t}^{\infty} m_{ts}\left[\frac{\psi_0 \psi}{1+\psi}(Y_s)^{1+\psi} + d_s e_{xs} + \nu_s b_{xs} - \varsigma_s\right]}{\sum_{s=t}^{\infty} \beta_{ts}^{\frac{1}{\sigma}} \left(m_{ts}\right)^{1-\frac{1}{\sigma}}}, \tag{61}$$

which uses the fact that there is no trade in either long-term bonds or equity so that adjustment costs are zero. We then aggregate the resulting consumption choices of all households x to calculate aggregate consumption, $C_t \equiv \int_0^1 c_{xt} dx$. After using the clearing conditions of financial markets and the fact that (30) implies that $\forall t$

$$r_t A + \sum_{s=t}^{\infty} m_{ts}\left(\nu_s B - \varsigma_s\right) = 0,$$

we finally obtain that C_t is given by (40), which concludes the proof. ■

A.2 The model of Section 3 with real assets

We first characterize the economy, second we state the main results of the analysis, which we then formally prove.

Assumptions There is a real asset (say a Lucas tree) in fixed supply H with

$$\frac{h_{c0} + h_{d0}}{2} = H. \tag{62}$$

where h_{j0} denotes the amount of real assets initially owned by households of type $j = c, d$. A unit of the real asset yields a per-period return $\bar{R} - 1$ for sure with $\beta\bar{R} = 1$. Adjusting the holding of the real asset from h to $h^{'}$ involves convex adjustment costs to the household given by

$$X\left(h' - h, h\right) = \frac{x_0}{2}\left(\frac{h' - h}{h}\right)^2 h.$$

Adjustment costs are in consumption units (and enter the utility function). We define net output of an economy with H available units of real assets as equal to

$$N^H(Y, H, h_c', h_c, h_d', h_d) \equiv Y - \psi_0 \frac{Y^{1+\psi}}{1+\psi} + \left(\bar{R} - 1\right) H - \frac{1}{2} X\left(h_c' - h_c, h_c\right) - \frac{1}{2} X\left(h_d' - h_d, h_d\right)$$

where the last two terms represent current period adjustment costs of households of type c and d, respectively. We set h_{d0}, h_{c0}, and ψ_0 (while leaving all other quantities unchanged) to guarantee that in the initial steady state, the consumption level of creditors $j = c$ and debtors $j = d$ is unchanged relative to the baseline economy. This requires having $h_{d0} = h_{c0} = H$ and setting ψ_0 so that

$$\bar{N} \equiv N(\bar{Y}) = N^H(\bar{Y}, H, h_c, h_c, h_d, h_d) \tag{63}$$

where $\bar{N} \equiv N(\bar{Y})$ denotes steady state net output in the baseline model. Without loss of generality we normalize the supply of the real asset to one, $H = 1$. We denote by q_t the price of the real asset at time t, which is equal to one in steady state, $q = 1$. Notice that in period one we are back to steady state so that $q_t = 1$, $\forall t \geq 1$. All the other assumptions are as in the baseline model. We denote by Δ the units of the real asset which are reallocated from one household type to the other in period zero immediately after the announcement:

$$\Delta = |h_{c1} - h_{c0}| = |h_{d1} - h_{d0}| \leq h_{d0}\mathbb{I}(h_{c1} - h_{c0} > 0) + h_{c0}\mathbb{I}(h_{d1} - h_{d0} > 0), \tag{64}$$

where $\mathbb{I}$ denotes the indicator function. We use the notation $X(\Delta) = X\left(\Delta, 1\right)$ and $X'(\Delta) = X_1\left(\Delta, 1\right)$. Net output a time zero is equal to $N_0 \equiv N^H(Y_0, H, h_{c1}, h_{c0}, H - h_{c1}, H - h_{c0})$.

Main result The availability of the real asset allows agent to trade to exploit tradable opportunities arising from their disagreement about the expected real return of the financial asset. We prove that:

Proposition 9 (Equilibrium with real assets) *The equilibrium of the model with real assets has the following features:*

1. *The real asset is not traded, if agents act as if there is no disagreement about the expected future return on the financial asset (as for example in the full credibility benchmark).*

2. *When adjustment costs are sufficiently small (x_0 small enough), the equilibrium features a credit crunch where $B' = 0$. A Credit crunch equilibrium is more likely than in the baseline model.*

3. *After any monetary announcement (either inflationary $\varepsilon > 1$ or deflationary $\varepsilon < 1$), the real asset is reallocated from debtors, $j = d$, to creditors $j = c$.*

4. *The Pareto efficient equilibrium level of net output, N_0, is always higher than in the baseline model. The difference is higher, the lower the adjustment costs (lower x_0). If adjustment costs are infinity, $x_0 = \infty$, we have the equilibrium of the baseline model.*

5. *The equilibrium amount of net output, N_0, is decreasing in the level of initial imbalances B and net output contracts when B is large enough.*

Proof of Proposition 9. We separately prove each of the statements of the proposition.

Proof of point 1 At $t = 0$, the Euler equation for the holdings of real assets for creditors , h_{c1}, and for debtors, , h_{d1}, read as follows:

$$q_0 + X_1 \left(h_{c1} - h_{c0}, h_{c0}\right) = \frac{\left[c_{c0} - \psi_0 \frac{l_{c0}^{1+\psi}}{1+\psi} - X\left(h_{c1} - h_{c0}, h_{c0}\right)\right]^{\sigma}}{\left(c_{c1} - \psi_0 \frac{l_{c1}^{1+\psi}}{1+\psi}\right)^{\sigma}} \tag{65}$$

$$q_0 + X_1 \left(h_{d1} - h_{d0}, h_{d0}\right) = \frac{\left[c_{d0} - \psi_0 \frac{l_{d0}^{1+\psi}}{1+\psi} - X\left(h_{d1} - h_{d0}, h_{d0}\right)\right]^{\sigma}}{\left(c_{d1} - \psi_0 \frac{l_{d1}^{1+\psi}}{1+\psi}\right)^{\sigma}} \tag{66}$$

where we used $\beta \bar{R} = 1$ and the fact that the economy is back to steady state in period one, which implies $q_1 = 1$ and no adjustment in the holdings of the real asset at $t \geq 1$. The conditions (65) and (66) together with (62) immediately imply that $h_{j1} - h_{j0}$, $j = c, d$ can be different from zero if only if the (percentage) changes in the marginal utility of consumption are different for the two household types, which can happen either in a credit-crunch equilibrium ($B' = 0$) or if $B' > 0$ and households behave as if they have different beliefs about the return of the financial asset at $t = 1$, $\rho \neq 0$. To formally see the latter statement write the Euler equations for the choice of financial assets for creditors a_{c1} and for debtors a_{d1} at $t=0$, which read as follows:

$$\frac{\left[c_{c0}-\psi_0\frac{l_{c0}^{1+\psi}}{1+\psi}-X\left(h_{c1}-h_{c0},h_{c0}\right)\right]^{\sigma}}{\left(c_{c1}-\psi_0\frac{l_{c1}^{1+\psi}}{1+\psi}\right)^{\sigma}}=\varepsilon^{\bar{\tau}(1+\rho)}, \tag{67}$$

$$\frac{\left[c_{d0}-\psi_0\frac{l_{d0}^{1+\psi}}{1+\psi}-X\left(h_{d1}-h_{d0},h_{d0}\right)\right]^{\sigma}}{\left(c_{d1}-\psi_0\frac{l_{d1}^{1+\psi}}{1+\psi}\right)^{\sigma}}=\varepsilon^{\bar{\tau}(1-\rho)}. \tag{68}$$

where again we used $\beta\bar{R}=1$ and the the fact that the economy is back to steady state in period one. Conditions (67) and (68) immediately imply that the (percentage) changes of the marginal utility of consumption are different for the two household types if and only if $\rho\neq 0$.

Proof of point 2 To prove the point we show that if

$$B>\frac{\left|\varepsilon^{\frac{1}{\sigma}}-1\right|\bar{N}}{2\bar{R}}+\frac{\left|\varepsilon^2-1\right|+\left(1+\varepsilon^{\frac{1}{\sigma}}\right)\left(\bar{R}-1\right)\left|\varepsilon-1\right|}{4x_0\bar{R}} \tag{69}$$

holds, we have that financial markets are active $B'>0$, otherwise we have a credit crunch equilibrium, $B'=0$. By comparing (19) with (69), we then immediately see that a credit crunch equilibrium is more likely when households can trade in real assets and that this is the only equilibrium when x_0 is small enough.

At time zero, total output (equal to the sum of output produced with labor and output of the Lucas'trees) satisfies the aggregate resource constraint so that

$$\left(\bar{R}-1\right)H+Y_0=\frac{c_{c0}+c_{d0}}{2}.$$

Notice that adjustment costs do not enter the aggregate resource constraint because they enter each household utility function. Clearing of the labor market implies that $\forall t\ Y_t=l_{jt}$, $\forall j=c,d$. Let's start assuming that the equilibrium features $B'>0$. Then creditors behave as if their consumption at $t=0$ and $t=1$ is equal to

$$c_{c0} = \left(\bar{R}-1\right)H+Y_0+\bar{R}\,a_{c0}-a_{c1}-q_0\cdot\left(h_{c1}-h_{c0}\right), \tag{70}$$

$$c_{c1} = \left(\bar{R}-1\right)h_{c1}+\bar{Y}+\frac{\bar{R}-1}{\max\left\{1,\varepsilon\right\}}a_{c1}, \tag{71}$$

where ε denotes the monetary announcement and we used the fact that the economy is back to steady state at $t=1$, that $\bar{\tau}=1/2$ and that ρ is still given by (23). By an identical logic, debtors behave as if their consumption at $t = 0$ and $t = 1$ is equal to

$$c_{d0} = \left(\bar{R} - 1\right) h_{d0} + Y_0 + \bar{R}\, a_{d0} - a_{d1} - q_0 \cdot (h_{d1} - h_{d0}), \tag{72}$$
$$c_{d1} = \left(\bar{R} - 1\right) h_{d1} + \bar{Y} + \left(\bar{R} - 1\right) \frac{a_{d1}}{\min\{1, \varepsilon\}}. \tag{73}$$

By combining (67) with (65) and after using (62), $\bar{\tau} = 1/2$ and ρ in (23), we finally obtain

$$q_0 + X' (h_{c1} - h_{c0}) = \max\{1, \varepsilon\}. \tag{74}$$

Analogously by combining (68) with (66), and after using (62) and (23) we obtain

$$q_0 - X'(h_{c1} - h_{c0}) = \min\{1, \varepsilon\}. \tag{75}$$

The system given by (74) and (75) immediately implies that

$$\Delta = h_{c1} - h_{c0}. \tag{76}$$

with

$$\Delta = \frac{|\varepsilon - 1|}{2x_0} \tag{77}$$
$$q_0 = \frac{1 + \varepsilon}{2} \tag{78}$$

Now we can solve for the equilibrium in the market for the financial asset. We use (76), the conditions for financial market clearing at $t = -1$, $a_{c0} = -a_{d0} = B$, and at $t = 0$, $a_{c1} = -a_{d1} = B'$ together with (67) and (68) to obtain

$$\frac{\bar{N} + \frac{\overline{R}-1}{\max\{1,\varepsilon\}} B' + (\overline{R} - 1)\Delta}{N_0 + \bar{R}B - B' - q_0\Delta} = \frac{1}{\max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}} \tag{79}$$

$$\frac{\bar{N} - \frac{\overline{R}-1}{\min\{1,\varepsilon\}} B' - (\overline{R} - 1)\Delta}{N_0 - \bar{R}B + B' + q_0\Delta} = \frac{1}{\min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}}, \tag{80}$$

where $\bar{N}$ is steady state net output and N_0 denotes net output at $t = 0$, while Δ and q_0 are given by (77) and (78), respectively. (79) and (80) implicitly define two linear schedule that can be plotted having B' on the x-axis and N_0 on the y-axis. The schedule implied by (79) is positively sloped, that defined by (80) is negatively sloped, which guarantees a unique intersection. The equilibrium features $B' > 0$ if the intercept on the y-axis of the schedule defined by (80) is above the intercept on the y-axis of the schedule defined by

(79). The intercept on the y-axis of the schedule in (80) is given by

$$N_0^{AH} = \bar{R}B + \min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}\left[\bar{N} - (\overline{R} - 1)\Delta\right] - q_0\Delta$$

The intercept on the y-axis of (79) is given by

$$N_0^{BH} = -\bar{R}B + q_0\Delta + \max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}\left[\bar{N} + (\overline{R} - 1)\Delta\right] \tag{81}$$

The condition $N_0^{AH} \geq N_0^{BH}$ is equivalent to

$$B > \frac{\left|\varepsilon^{\frac{1}{\sigma}} - 1\right|\bar{N}}{2\bar{R}} + \frac{q_0\Delta}{\bar{R}} + \frac{\left(1 + \varepsilon^{\frac{1}{\sigma}}\right)}{2\bar{R}}\left(\bar{R} - 1\right)\Delta \tag{82}$$

After using (77) and (78), we finally obtain the condition in (69): if (69) fails we have a credit crunch equilibrium with $B' = 0$ where any net output in the interval $[N_0^{AH}, N_0^{BH}]$ can be sustained as an equilibrium. This concludes the proof for this point.

Proof of point 3 The proof follows directly from (76) and the definition of Δ in (64).

Proof of point 4 If $B' > 0$, start by noticing that in the baseline model with just one financial asset we have that net output is equal to

$$N_\varepsilon = \varphi_\varepsilon\bar{N} - \mu_\varepsilon B, \tag{83}$$

where

$$\varphi_\varepsilon = \frac{1 + (\bar{R} - 1)\,\varepsilon^{\frac{1}{\sigma} - 1} + \bar{R}\varepsilon^{\frac{1}{\sigma}}}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma} - 1}}$$

$$\mu_\varepsilon \equiv \frac{\bar{R}\,(\bar{R} - 1)\left|1 - \varepsilon^{\frac{1}{\sigma} - 1}\right|}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma} - 1}}.$$

Let's now consider the case of a deflationary announcement $\varepsilon < 1$. From solving (80) and (79) and after some algebra we obtain

$$N_0 = N_\varepsilon + \frac{\left(\bar{R} - 1\right)\left[1 + \left(2\bar{R} - 1\right)\varepsilon^{\frac{1}{\sigma} - 1}\right]}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma} - 1}}(1 - q_0)\Delta \tag{84}$$

where N_ε is net output in the baseline model as given in (83). After using (77) and (78), (84) immediately proves point 4 for the case of a deflationary announcement with $B' > 0$.

Now let's consider the case $\varepsilon > 1$. After solving (80) and (79) we obtain

$$N_0 = N_\varepsilon + \frac{\left(\bar{R}-1\right)\left[1 + \left(2\bar{R} - 1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]}{1 + \bar{R} + \left(\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}}\left(q_0 - 1\right)\Delta \tag{85}$$

where N_ε is again given in (83). After using (77) and (78), (85) proves point 4 for the case of an inflationary announcement with $B' > 0$. For the case of a credit crunch equilibrium ($B' = 0$), the result follows from comparing N_0^{BH} in (81) with $\bar{I}(B, \varepsilon)$ in (20).

Proof of point 5 When $B' > 0$, the proof follows from using (84) when $\varepsilon < 1$ and (85) when $\varepsilon > 1$, after remembering the definition of N_ε in (83). When the equilibrium features a credit crunch equilibrium, the proof follows directly from the definition of N_0^{BH} in (81). ■

A.3 The model of Section 3 with an alternative modeling of ambiguity aversion

Here we briefly discuss the properties of the analytical model of Section 3, when we assume that households have multiplier preferences with respect to ambiguity as in Hansen and Sargent (2001, 2008). We assume households have a reference probability distribution $\hat{G}(\Pi_1^*)$ on the support of the next period realization of the inflation target Π_1^* given by

$$\mathcal{S}_0 = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}]. \tag{86}$$

For example, it is reasonable to think that $\hat{G}(\Pi_1^*)$ is uniform over $\mathcal{S}_0$—which would correspond to a diffuse prior on the credibility parameter γ of the central bank. Agents consider the possibility that $\hat{G}$ may not be the appropriate distribution to characterize the realizations of the inflation target in period one and consider alternative models as parameterized by different distributions G's with support $\mathcal{S}_0$. Rather than by (12), the problem of household $j = c, d$, with initial wealth a_{j0}, now reads as follows

$$V\left(a_{j0}\right) = \max_{a'}\left\{\frac{\left(N_0 + \bar{R}\,a_{j0} - a'\right)^{1-\sigma}}{1-\sigma} + \beta\min_{G}\left[\int_{\mathcal{S}_0}\bar{V}\left(\frac{a'}{\Pi_1^*}\right)dG(\Pi_1^*) + \hat{\lambda}\,R(G|\hat{G})\right]\right\}, \tag{87}$$

where the continuation utility is still given by (13) so that

$$\bar{V}\left(a\right) = \frac{\left[\bar{N} + (\bar{R} - 1)\,a\right]^{1-\sigma}}{(1-\sigma)(1-\beta)},$$

while the divergence between the probability distributions G and $\hat{G}$ is measured by using relative entropy

$$R(G|\hat{G}) = \int_{\mathcal{S}_0}\log\frac{dG(\Pi_1^*)}{d\hat{G}(\Pi_1^*)}dG(\Pi_1^*)$$

and $\hat{\lambda}$ is a parameter measuring the penalty to the discrepancy R. Let $g(\Pi_1^*) = G'(\Pi_1^*)$ and $\hat{g}(\Pi_1^*) = \hat{G}'(\Pi_1^*)$ denote the density functions of the distribution functions G and $\hat{G}$, respectively. Then the first order condition with respect to $g(\Pi_1^*)$ reads as follows

$$\bar{V}\left(\frac{a'}{\Pi_1^*}\right) + \hat{\lambda}\log\left(\frac{g(\Pi_1^*)}{\hat{g}(\Pi_1^*)}\right) + \hat{\lambda} = 0,$$

which implies that agents act according to the following probability density function for the realization of the next period inflation target:

$$g(\Pi_1^*) = \frac{\hat{g}(\Pi_1^*)}{\displaystyle\int_{\mathcal{S}_0} \exp\left(\frac{\bar{V}\left(\frac{a'}{\Pi_1^*}\right) - \bar{V}\left(\frac{a'}{s}\right)}{\hat{\lambda}}\right)\hat{g}(s)ds}. \tag{88}$$

This uses the fact that, by definition, we have $\int_{\mathcal{S}_0} g(\Pi^*)d\Pi^* = \int_{\mathcal{S}_0} \hat{g}(\Pi_1^*)d\Pi^* = 1$. After using (88) to replace $g(\Pi_1^*)$ in (87), we obtain that the problem of the household becomes equal to

$$V\left(a_{j0}\right) = \max_{a'}\left\{\frac{\left(N_0 + \bar{R}\,a_{j0} - a'\right)^{1-\sigma}}{1-\sigma} + \beta\int_{\mathcal{S}_0}\bar{V}\left(\frac{a'}{\Pi_1^*}\right)g(\Pi_1^*)dG(\Pi_1^*)\right\},$$

with $g(\Pi_1^*)$ given by (88). This analysis yields two general conclusions. The first is that, after an inflationary announcement $\varepsilon > 1$, creditors act as if their expected inflation responds more than the expected inflation of debtors, as in the baseline model. This follows directly from (88) which implies that agents with $a' > 0$ give more probability weight to large realizations of Π_1^*, that cause them lower continuation utility $\bar{V}\left(a'/\Pi_1^*\right)$. The two different specification of preferences could cause different responses to the monetary announcement of the correlation between households' wealth and their perception of the announcement's credibility—say they can generate a different value of ρ. In practice, in the quantitative analysis of Section 4, we target this correlation using micro level evidence on expected inflation and because of this we believe that the two formulations yield very similar qualitative and quantitative results.

B Data appendix

We describe the sources of our data for realized and expected inflation in the Italian provinces, and in the Euro Area, as well as the net financial assets of European households.

B.1 Italian data

Our Italian data come from ISTAT's Survey of Inflation Expectations conducted by the Bank of Italy and Sole24Ore (Italy's main daily business paper), and from the Bank of

Italy's Survey of Household Income and Wealth.

Realized inflation at province level is taken directly from ISTAT's "I.Stat" online archive. We use the general price index, pgen in the ISTAT database. Realized inflation in the province corresponds to the yearly log-difference of pgen in the province. We take yearly log-differences because the ECB monitors price stability on the basis of the annual rate of change in HICP and because of the working of the inflation expectations question (see below).

Expected inflation measures 2 quarters ahead expected inflation, averaging the reported estimates of all observations in the province in the Survey of Inflation Expectations. The disaggregated province level data are confidential data kindly made available to us by the Bank of the Italy. The Survey has been conducted quarterly since 1999, in March, June, September and December. The sample comprises about 800 companies, operating in all industries including construction. Individuals are asked to predict the price inflation 6 months ahead, answering the following question: "[If the survey is conducted in June 2013] What do you think consumer price inflation in Italy, measured by the 12-month change in the Harmonized Index of Consumer Prices (HICP), will be in December 2013?". Note that the individuals in the survey are all asked to predict the evolution of the same index (HICP at the national level) . In practice, therefore we are assuming that the replies of respondents in the survey in that province reflect the average beliefs of agents in the province.

Net Financial Assets (NFA) Our data on the Italian households' NFA come from the Survey of Household Income and Wealth (SHIW), administered by the Bank of Italy on a representative sample of Italian households. The survey, which is biannual, collects detailed data on households' finances. Each wave surveys about 8,000 households, which, applying weights provided by SHIW (mnemonic Pesofit in SHIW), are fully representative of the Italian resident population. To increase sample size, we use both the 2010 and the 2012 waves. NFA is calculated as the difference between the sum of households' holdings of postal deposits, saving certificates and CDs (mnemonic shiwaf1 in SHIW), government securities (mnemonic shiwaf2) and other securities (mnemonic shiwaf3) minus the sum of their financial liabilities to banks and other financial companies (mnemonic shiwpf1), trade debt (mnemonic shiwpf2) and liabilities to other households (mnemonic shiwpf3).

Creditor households are those with positive NFA (see the construction of the variable NFA for details).

Fraction of creditor households For each province we calculate the pre-announcement fraction of creditor households, based on the 2010 and 2012 waves of SHIW, weighting each household according to the weights provided by SHIW (mnemonic Pesofit).

Inflation expectation bias In each province i and quarter t, we calculate the difference between expected inflation and future realized inflation.

Table A1: Descriptive statistics

VARIABLES	(1) mean	(2) sd	(3) N	(4) min	(5) max
A) Italian Micro data					
Pre-announcement fraction of creditor households	0.66	0.13	1078	0.32	0.94
Pre-announcement fraction of creditor households, divided by SD	-0.13	1.00	1078	-2.76	2.00
Inflation rate in province π_{it}	1.77	1.24	1078	-0.47	4.76
Two quarters ahead expected inflation, $E_{it}[\pi_{it+2}]$	2.02	1.23	1078	-10	8.72
Two quarters ahead realized inflation, π_{it+2}	1.15	1.16	1078	-9.62	4.53
Inflation expectation bias, $\hat{\pi}_{it}$	0.86	0.74	1078	-3.61	6.79
Year	2012.80	0.75	1082	2012	2014
B) Euro Area data					
Net per capita financial assets	1.91	1.57	100	-0.42	4.67
Net per capita financial assets, divided by SD	1.22	1	100	-0.27	2.98
Fraction of households who think inflation will increase in next 12 months	15.99	6.08	100	6	38.10
Inflation rate in country	1.65	0.77	100	-0.05	3.09
Change in Country Inflation rate	-0.34	0.60	100	-2.39	1.18
Year	2012.80	0.75	100	2012	2014

Notes: Quarterly data over the sample period 2012:I-2014:II. Realized inflation comes from ISTAT. Data on expected inflation are based on confidential data from the Bank of Italy-Sole 24Ore survey on expectations. The Net Financial Asset position of households is calculated using the 2010 and 2012 waves of the Survey of Household Income of Wealth (SHIW). Euro Area data come from ECB, Joint Harmonized Programme of Business and Consumer Surveys by European Commission and Eurosystem Household Finance and Consumption Survey (HFCS).

B.2 Euro Area Data: realized and expected inflation

The data are for the Euro 11 countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

Core Inflation is the yearly log differences in the Harmonised Index of Consumer Prices (HICP), net of energy and unprocessed food, multiplied by 100, taken from the Eurostat data warehouse available at "http://ec.europa.eu/eurostat/".

Fraction of households who think inflation will increase in next 12 months come from the European Commission's Business and Consumer Surveys. The key advantage of the Consumer Survey is that it directly asks households for their expectations about future inflation, which distinguishes it sharply from the commonly used Survey of Professional Forecasters. Sample size varies with country. Price expectations are derived from the question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall. The fraction of households who think inflation will increase in next 12 months is tteh fraction of households who selecting option i). The series are seasonally adjusted by the Commission.

B.3 European households' Financial Assets

We use information from the Euro Area Accounts (EAA) and the Eurosystem Household Finance and Consumption Survey (HFCS) to calculate the financial asset positions of European households. The HFCS collects fully harmonized data on households' portfolio asset allocation of households and consumption expenditures in the Euro-11 countries (except Ireland). Wealthy individuals are over-sampled for better characterization of the right tail of the income and wealth distribution of the ten countries we consider. The structure of the HFCS resembles that of the US Survey of Consumer Finances. To account for measurement error and missing observations, HFCS reports five separate imputation replicates (implicates) for each record. All statistics are calculated by the procedure recommended by HFCS: for each implicate we calculate the desired statistic using HFCS weights (mnemonic hw0010) and then average across the five implicates (mnemonic im0100). The survey was carried out in 2010 except in Finland and the Netherlands, where it was done in 2009, and in Spain (2008). As discussed in Honkkila and Kavonius (2013) and Adam and Zhu (2016), HFCS contains no information on the amount of currency held by the household. So we use EAA to impute household's holdings of currency in HFCS.

Short Term Financial Assets is calculated as the difference between Short-term financial assets and Short-term financial liabilities. We impute the wealth invested by investment funds (mutual funds and pensions funds) in equity as opposed to bonds as well as the share of wealth invested by households in short-term bonds as opposed to long-term bonds. According to data from the ECB data warehouse at https://www.ecb.europa.eu/press/pr/stats/, the share of wealth of invested funds invested in equity was equal to 0.4076 in 2013:II. We set the share of short-term bonds over total bonds as equal to the share of government bonds with remaining maturity less than 3 years and a half, which in 2014 (the first year available) was equal to 0.407 (see http://ec.europa.eu/eurostat/web/government-finance-statistics/data/database). Short-term financial assets are calculated as the sum of (i) Deposits (mnemonic da2101); (ii) $0.407 \times (1 - 0.4076) \times$ Mutual funds (mnemonic da2102); (iii) $0.407\times$ Bonds (mnemonic da2103); (iv) Managed accounts (mnemonic da2106); (v) Money owed to households (mnemonic da2107); (vi) Other assets (mnemonic da2108); and (vii) $0.407 \times (1 - 0.4076)\times$ Voluntary pensions plus whole life insurance (mnemonic da2109). The sum is then divided by per household average labor income in the Euro Area (mnemonic di1100). To impute household's holdings of currency, Deposits divided by labor income are rescaled to have an average value of 1.498 which is the analogous value in 2013:II from the EAA—obtained by dividing the variable Currency and deposits by Compensation of employees. Short-term financial liabilities are calculated as equal to the sum of the outstanding balance of (i) Adjustable interest rate mortgages on household's main residence (mnemonic dl1110); (ii) Adjustable interest rate mortgages on other properties (mnemonic dl1120); and (iii) Other non mortgage debt (mnemonic dl1200). The resulting sum is then divided by per household average labor income in the Euro Area (mnemomic di1100). The amount of adjustable-rate mortgages is the sum of the outstanding balance of all mortgages (mnemonics hb1701-hb1703 for household main residence; mnemonics hb3701-hb3703 for other properties) with an adjustable interest rate (mnemonics hb1801-hb1803 for household main residence; mnemonics hb3801-hb3803 for other properties).

Long Term Financial Assets is calculated as the difference between Long-term financial assets and Long-term financial liabilities. We impute the wealth invested by investment funds (mutual funds and pensions funds) in equity as opposed to bonds as well as the share of wealth invested by households in short-term bonds as opposed to long-term bonds. According to data from the ECB data warehouse at https://www.ecb.europa.eu/press/pr/stats, the share of wealth of invested funds invested in equity was equal to 0.4076 in 2013:II. We set the share of short-term bonds over total bonds as equal to the share of government bonds with remaining maturity less than 3 years and a half, which in 2014 (the first year available) was equal to 0.407 (see http://ec.europa.eu/eurostat/web/government-finance-statistics/data/database). Long-term financial assets are the sum of (i) $(1 - 0.407) \times (1 - 0.4076)\times$ Mutual funds (mnemonic da2102); (ii) $(1 - 0.407)\times$ Bonds (mnemonic da2103); and (iii) $(1-0.407)\times(1-0.4076)\times$ Voluntary pensions plus whole life insurance (mnemonic da2109). This sum is divided by average labour income (mnemonic di1100). Long-term financial liabilities are the sum of (i) Fixed interest rate mortgages on household's main residence (mnemonic dl1110); and (ii) Fixed interest rate mortgages on other properties (mnemonic dl1120). The resulting sum is again divided by per household average labor income in the Euro Area. The amount of fixed-rate mortgages is the sum of the outstanding balance of all mortgages (mnemonics hb1701-hb1703 for household main residence; mnemonics hb3701-hb3703 for other properties) with a fixed rate (mnemonics hb1801-hb1803 for household main residence; mnemonics hb3801-hb3803 for other properties).

Equity is calculated as the sum of (i) Non self-employment private business (mnemonic da2104); (ii) Shares of publicly traded companies (mnemonic ds2105); (iii) $0.4076\times$ Mutual funds (mnemonic da2102); (iv) $0.4076\times$ Voluntary pensions plus whole life insurance (mnemonic da2109); (v) Value of self-employment businesses (mnemonic da1140); (vi) $0.4\times$ Real estate properties excluding household's main residence (mnemonic da1120). The resulting sum is again divided by per household average labor income in the Euro Area. We follow Kaplan, Moll, and Violante (2016a) in assuming that a 40 percent of real estate (after excluding main residence) is productive capital, which reflects the fact that part of the housing stock owned by households is commercial space rented out to businesses or a direct input into production.

Total Wealth is the sum of Short-Term Financial Assets plus Long-Term Financial Assets plus Equity.

Consumption expenditures is the sum of the expenditures during the last 12 months on food and beverages at home (mnemonic hi0100) and on food and beverages outside the home (mnemonic hi0200).

Average labor income in the Euro Area is the average of the employee income of all household members (mnemonic di1100) for all households whose head is aged 20-65 (mnemonic ra0300). The resulting average labor income is EUR 21,631.

B.4 Robustness with Euro Area data

To check robustness about the increase in disagreement among European households about expected future inflation after forward guidance we also analyzed country level evidence for the Euro 11 Area. For countries we do not have information on expected inflation but just on the fraction of households who think that inflation will increase in the next year relative to the past year, which we denote by $P\left(E_{it}[\pi_{it+4}]-\pi_{it}\right)$. Then we calculate

$$\hat{\pi}_{it} \equiv P\left(E_{it}[\pi_{it+4}]-\pi_{it}\right)-\vartheta\left(\pi_{it+4}-\pi_{it}\right) \tag{89}$$

where ϑ will be estimated. To evaluate whether, in response to forward guidance, the inflation expectation bias has increased more for creditor households than for debtor households, we run the same Difference-in-Differences regression as in (47), but where F_i is now equal to the (standardized) average per capita Net Financial Asset of households in the country. The controls X_{it} includes a full set of time and country dummies and also the realized future inflation which allows to estimate ϑ in 89. Table A2 reports the results from estimating (47) with the Euro 11 data. Column 1 reports the results discussed in the main text, column 2 reports the result with the sample of countries. The evidence indicates that the inflation expectations have become more correlated with the NFA of households and this conclusion is confirmed when focusing on the sample of Euro 11 countries.

Table A2: FG Effects on expected inflation bias

VARIABLES	**Micro Evidence** $\hat{\pi}^c_{it}$	**EURO 11** $\hat{\pi}^p_{it}$
Announcement-dummy $\times$ F_i (coefficient ϱ)	.10***	2.1***
	.04	.71
Effect of financial position F_i (coefficient $\overline{\varrho}$)	.02	10.39***
	.02	.95
Future changes in inflation $\pi_{it+4}-\pi_{it}$, γ		2.33***
		.75
R-squared	.35	.97
No. of observations	1,078	100
No. of i units	110	10

Notes: Results from regression (47). All regressions include year and individual fixed effect. The dependent variable is $\hat{\pi}^j_{it}$, in (47) and (89). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA in the province in column (1) or the (standardized) pre-announcement average value of households' NFA in the country in column (2). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

C Computational details

The model of Section 4 is solved in six steps.

Step 1 We guess the date $T \leq t_\beta$ when households and firms first learn about R^*. The initial guess is $T = t_\beta$.

Step 2 We guess the price level at $t = T-1$, $p_{T-1} = \hat{p}$.

Step 3 For $R^* \in \{R^l, R^h\}$, we construct the households' continuation value function $V(\omega_{xt}; R^*)$ in (34) and firms' continuation value $W(p_{it-1}; R^*)$ in (46) by solving for the equilibrium of the economy at $t \geq T$ given a value of R^*. We denote the equilibrium quantities when $R^* = R^l$ and when $R^* = R^h$ using the superscript $z = l$ and $z = h$, respectively.

Step 3.1 For each $z = l, h$, we guess an output path, $\{\hat{Y}_t^z\}_{t \geq T}$.

Step 3.2 Given $\{\hat{Y}_t^z\}_{t \geq T}$, (11) together with the labor market clearing condition yields wages, $\{\hat{w}_t^z\}_{t \geq T}$. Then (3) and (37) jointly determine inflation $\{\hat{\Pi}_t^z\}_{t \geq T}$ and nominal interest rates $\{\hat{R}_t^z\}_{t \geq T}$, which in turn determine the path of interest rates $\hat{r}_t^z = \hat{R}_t^z / \hat{\Pi}_t^z$. Dividends $\{\hat{D}_t^z\}_{t \geq T}$ are obtained by using (38); prices $\{\hat{p}_t^z\}_{t \geq T}$ by combining $\hat{p}$ with $\{\hat{\Pi}_t^z\}_{t \geq T}$; taxes ς_t by using $\varsigma_t^z = r_t^z\, A + \nu_t\, B$. Aggregate consumption $\{\hat{C}_t^z\}_{t \geq T}$ is then obtained using (40).

Step 3.3 If the resulting paths of inflation $\{\hat{\Pi}_t^z\}_{t \geq T}$ and aggregate consumption $\{\hat{C}_t^z\}_{t \geq T}$ satisfy (39) at the initial guess for output $\{\hat{Y}_t^z\}_{t \geq T}$, the guess is verified and we move to Step 3.4, otherwise we update the guess for $\{\hat{Y}_t^z\}_{t \geq T}$ and go back to Step 3.1.

Step 3.4 Given equilibrium aggregate quantities $\forall z = l, h$ we construct the functions $V(\boldsymbol{\omega}_{xT}; R^l)$, $V(\boldsymbol{\omega}_{xT}; R^h)$, $W(p_{iT-1}; R^l)$, and $W(p_{it-1}; R^h)$ using splines, by solving the household's problem and the firm's problem at $t = T$ under perfect foresight for $R^* = R^l$ and $R^* = R^h$ and different initial conditions.

Step 4 We solve for the equilibrium of the economy at $t < T$.

Step 4.1 We guess the sequence $\{w_t, q_t^b, q_t^e\}_{t=0}^{t=T-1}$.

Step 4.2 Given w_t, we use (11) to determine output Y_t $\forall t < T$. Under our calibration, the function $W(p_{it-1}; R^*)$ is decreasing in R^*, so the firm problem in (46) implies that firms behave as if $R^* = R^h$ $\forall t < T$. Given $\{w_t, q_t^b, q_t^e\}_{t=0}^{t=T-1}$, we then use (37) and (43) to solve for π_t, R_t, p_t and r_t $\forall t < T$.

Step 4.3 Given aggregate prices and the functions $V(\boldsymbol{\omega}_{xT}; R^l)$ and $V(\boldsymbol{\omega}_{xT}; R^h)$ determined in Step 3, we solve the problem in (44) for all households $x \in [0, 1]$. Household x could be (i) skeptical, (ii) trusting, or (iii) Zen. Household x is is skeptical and behaves as if $R^* = R^h$ if $V(\boldsymbol{\omega}_{xT}; R^h) < V(\boldsymbol{\omega}_{xT}; R^l)$; she is trusting and behaves as if $R^* = R^l$ if $V(\boldsymbol{\omega}_{xT}; R^h) > V(\boldsymbol{\omega}_{xT}; R^l)$; she is Zen if $V(\boldsymbol{\omega}_{xT}; R^l) = V(\boldsymbol{\omega}_{xT}; R^h)$ meaning that, for given b_{xT} and e_{xT}, short term assets should be equal to $a^*(b_{xT}, e_{xT})$ which satisfies

$$V(a^*(b_{xT}, e_{xT}), b_{xT}, e_{xT}; R^l) = V(a^*(b_{xT}, e_{xT}), b_{xT}, e_{xT}; R^h).$$

Step 4.3.1 The first order conditions for a_{xt+1}, b_{xt+1}, and e_{xt+1} when household x is skeptical ($R^* = R^h$) or trusting ($R^* = R^l$) are given by

$$c_{xt} = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + (\beta_t\, r^z_{t+1})^{-\frac{1}{\sigma}} \left[c_{xt+1} - \frac{\psi_0}{1+\psi} \left(Y^z_{t+1}\right)^{1+\psi} \right], \tag{90}$$

$$\overline{\chi}_b\, \Delta b_{xt+1} = \left(\frac{q^{bz}_{t+1} + \nu^z_{t+1}}{r^z_{t+1}} - q^{bz}_t \right) sgn(b_{xt}) + \frac{\overline{\chi}_b\, \Delta b_{xt+2}\, \left(1 + \frac{1}{2}\, \Delta b_{xt+2}\right)}{r^z_{t+1}}, \tag{91}$$

$$\overline{\chi}_e\, \Delta e_{xt+1} = \frac{q^{ez}_{t+1} + d^z_{t+1}}{r^z_{t+1}} - q^{ez}_t + \frac{\overline{\chi}_e\, \Delta e_{xt+2}\, \left(1 + \frac{1}{2}\, \Delta e_{xt+2}\right)}{r^z_{t+1}}, \tag{92}$$

where $sgn(j)$ is the sign function—equal to one if $j > 0$, equal to minus one if $j < 0$—while $\Delta b_{xt+1} = (b_{xt+1} - b_{xt})/b_{xt}$ and $\Delta e_{xt+1} = (e_{xt+1} - e_{xt})/e_{xt}$ denote percentage changes. In (90)-(92) for $t \geq T$, household x uses the equilibrium quantities from Step 3 corresponding to $z = h$ when $R^* = R^h$, or to $z = l$ when $R^* = R^l$, while for $t < T$ we have that aggregate quantities are independent of household x's beliefs so that $R^l_t = r^h_t = r_t$, $Y^l_t = Y^h_t = Y_t$, $m^l_{ts} = m^h_t = m_{ts}$, $d^l_t = d^h_t = d_t$, $\nu^l_t = \nu^h_t = \nu_t$, $\varsigma^l_t = \varsigma^h_t = \varsigma_t$, $q^{bl}_t = q^{bh}_t = q^b_t$, and $q^{el}_t = q^{eh}_t = q^e_t$. The consumption of a skeptical or a trusting household is then obtained by combining (90)-(92) with the budget constraint in (29) to obtain

$$c_{xt} = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \frac{r^z_t a_{xt} + \sum_{s=t}^{\infty} m^z_{ts} \left[\frac{\psi_0 \psi}{1+\psi} (Y^z_s)^{1+\psi} + d^z_s e_{xs} + \nu^z_s b_{xs} - \chi_{xs} - \varsigma^z_s \right]}{\sum_{s=t}^{\infty} \beta_{ts}^{\frac{1}{\sigma}} \left(m^z_{ts}\right)^{1-\frac{1}{\sigma}}} \tag{93}$$

where

$$\chi_{xs} \equiv \chi(b_{xs+1}, b_{xs}, e_{xs+1}, e_{xs}) + q^b_j (b_{xs+1} - b_{xs}) + q^e_j (e_{xs+1} - e_{xs}).$$

Given (29) and (91)-(93), and $\forall z = l, h$ we determine household x wealth at T, $\boldsymbol{\omega}^z_{xT}$, which allows to verify whether household x is trusting or skeptical.

Step 4.3.2 If household x is neither trusting nor skeptical, household x is Zen and first order conditions do not hold at $t = T-1$ because the value function V has a kink. A Zen household should have wealth at time T equal to $\boldsymbol{\omega}^*_{xT} = (a^*(b_{xT}, e_{xT}), b_{xT}, e_{xT})$, so for any terminal value of b_{xT} and e_{xT} we evaluate

$$V^*_0(\boldsymbol{\omega}_{x0}, b_{xT}, e_{xT}) = \sum_{t=0}^{T-1} \beta_t\, U(c_{xt}, Y_t) + \beta_T\, V(a^*(b_{xT}, e_{xT}), b_{xT}, e_{xT}; R^l). \tag{94}$$

where c_{xt} is obtained using the initial and terminal conditions of wealth $\boldsymbol{\omega}_{x0}$ and $\boldsymbol{\omega}^*_{xT}$, the budget constraint (29), and the first order conditions (90)-(92), which for a Zen household holds $\forall t < T-1$, but fails to hold at $t = T-1$. The consumption and wealth profiles of the Zen household is obtained by maximizing (94) with respect to b_{xT} and e_{xT}.

Step 4.4 We aggregate choices of all households in the economy to check whether $\forall t < T$ we have $|\int_0^1 c_{xt} dx + \int_0^1 \chi_{xt} dx + \kappa(\pi_t, Y_t) + \zeta - Y_t| < 10^{-5}$, $|\int_0^1 b_{xt} dx - B| < 10^{-5}$ and

$|\int_0^1 e_{xt}dx - 1| < 10^{-5}$. If one of these conditions fail, we go back to Step 4.1 after updating our guess for $\{w_t, q_t^b, q_t^e\}_{t=0}^{t=T-1}$; otherwise we move to Step 5.

Step 5 We compute price p_{T-1} implied by Step 4 and compare it to the guess $\hat{p}$ in Step 2. If $|\hat{p} - p_{T-1}| < 10^{-5}$ we move to Step 6; otherwise we go back to Step 2 after using p_{T-1} to update $\hat{p}$.

Step 6 We verify our guess for T in Step 1, which requires $\hat{R}_t = 0, \forall t < T$ and $\hat{R}_t > 0$ at $t = T$. If instead $\hat{R}_t > 0$ at $t < T$, we update our guess for T and move back to Step 1.

D Derivation of equations

Derivation of (15) To have $a' = 0$ it should that the household does not want to borrow, which requires

$$\left(N_0 + \bar{R}\,a_0\right)^{-\sigma} < \frac{\bar{N}^{-\sigma}}{\min\{1, \varepsilon\}}, \tag{95}$$

and does not want to lend, which requires

$$\left(N_0 + \bar{R}\,a_0\right)^{-\sigma} > \frac{\bar{N}^{-\sigma}}{\max\{1, \varepsilon\}} \tag{96}$$

By manipulating (95) and (96) we obtain that $a' = 0$ requires that

$$\min\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}\,a_0 \;<\; N_0 \tag{97}$$

$$\max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}\,a_0 \;>\; N_0 \tag{98}$$

Derivation of (93) From the intertemporal-Euler we can express c_{xt+j} as a function of c_{xt}:

$$c_{xt+j} = \psi_0 \frac{Y_{t+j}^{1+\psi}}{1+\psi} + \left[\prod_{s=0}^{j} (\beta_{t+s} r_{t+1+s})^{\frac{1}{\sigma}}\right] \left(c_{xt} - \psi_0 \frac{Y_t^{1+\psi}}{1+\psi}\right) \tag{99}$$

Let $m_{tj} = \prod_{u=t+1}^{j} (r_u)^{-1}$, with $m_{tt} = 1$, and $\beta_{tj} = \prod_{u=t}^{j-1} \beta_u$, with $\beta_{tt} = 1$. Iterating forward the budget constraint we have

$$\sum_{j=t}^{\infty} m_{tj}\;(c_{xj} - w_j L_j - d_j\, e_{xj} - \nu_j b_{xj} + \chi_{xj} + \zeta_j) = r_t a_{xt}$$

where

$$\chi_{xj} \equiv \chi(b_{xj+1}, b_{xj}, e_{xj+1}, e_{xj}) + q_j^b(b_{xj+1} - b_{xj}) + q_j^e(e_{xj+1} - e_{xj}).$$

By manipulating this last expression we obtain

$$c_{xt} - w_t L_t - d_t\, e_{xt} - \nu_t b_{xt} + \chi_{xt} + \zeta_t + \sum_{j=t+1}^{\infty} m_{tj}\, c_{xj}$$
$$+ \sum_{j=t+1}^{\infty} m_{tj}\, \left(-w_j L_j - d_j\, e_{xj} - \nu_j b_{xj} + \chi_{xj} + \zeta_j\right) = r_t a_{xt},$$

which, after using (99), can be written as follows:

$$c_{xt} + \sum_{j=t+1}^{\infty} m_{tj}\, \psi_0 \frac{Y_j^{1+\psi}}{1+\psi} + \left(c_{xt} - \psi_0 \frac{Y_t^{1+\psi}}{1+\psi}\right) \sum_{j=t+1}^{\infty} m_{tj}\, m_{tj}^{-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}$$
$$+ \sum_{j=t+1}^{\infty} m_{tj}\, \left(-w_j L_j - d_j\, e_{xj} - \nu_j b_{xj} + \chi_{xj} + \zeta_j\right)$$
$$= r_t a_{xt} + w_t L_t + d_t\, e_{xt} + \nu_t b_{xt} - \chi_{xt} - \zeta_t,$$

which can be written as follows:

$$c_{xt} \left(1 + \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}\right) - \psi_0 \frac{Y_t^{1+\psi}}{1+\psi} \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}} +$$
$$+ \sum_{j=t+1}^{\infty} m_{tj}\, \left(\psi_0 \frac{Y_j^{1+\psi}}{1+\psi} - w_j L_j - d_j\, e_{xj} - \nu_j b_{xj} + \chi_{xj} + \zeta_j\right)$$
$$= r_t a_{xt} + w_t L_t + d_t\, e_{xt} + \nu_t b_{xt} - \chi_{xt} - \zeta_t,$$

Using $w_j L_j = w_j Y_j = \psi_0 Y_j^{1+\psi}$ we have

$$c_{xt} = \frac{r_t a_{xt} + w_t L_t + d_t\, e_{xt} + \nu_t b_{xt} - \chi_{xt} - \zeta_t}{1 + \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}} + \frac{\sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}}{1 + \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}} \psi_0 \frac{Y_t^{1+\psi}}{1+\psi}$$
$$+ \frac{\sum_{j=t+1}^{\infty} m_{tj}\, \left(\psi_0 \psi \frac{Y_j^{1+\psi}}{1+\psi} + d_j\, e_{xj} + \nu_j b_{xj} - \chi_{xj} - \zeta_j\right)}{1 + \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}},$$

After using again the fact that $w_t L_t = \psi_0 Y_t^{1+\psi}$ once again we obtain

$$c_{xt} = \psi_0 \frac{Y_t^{1+\psi}}{1+\psi} + \frac{r_t a_{xt} + \sum_{j=t}^{\infty} m_{tj}\, \left(\psi_0 \psi \frac{Y_j^{1+\psi}}{1+\psi} + d_j\, e_{xj} + \nu_j b_{xj} - \chi_{xj} - \zeta_j\right)}{1 + \sum_{j=t+1}^{\infty} m_{tj}^{1-\frac{1}{\sigma}}\, \beta_{tj}^{\frac{1}{\sigma}}},$$

which corresponds to (93).