Monetary Policies and Low-Frequency Manifestations of the Quantity Theory*

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Abstract

To detect the quantity theory of money, we follow Lucas (1980) by looking at scatter plots of filtered time series of inflation and money growth rates and interest rates and money growth rates. Like Whiteman (1984), we relate those scatter plots to sums of two-sided distributed lag coefficients constructed from fixed-coefficient and time-varying VARs for U.S. data from 1900-2005. We interpret outcomes in terms of population values of those sums of coefficients implied by two DSGE models. The DSGE models make the sums of coefficients depend on the monetary policy rule via cross-equation restrictions of a type that Lucas (1972) and Sargent (1971) emphasized in the context of testing the natural unemployment rate hypothesis. When the U.S. data are extended beyond Lucas's 1955-1975 period, the scatter plots mutate in ways that we attribute to prevailing monetary policy rules.

JEL CLASSIFICATION: E4, E5, N1

KEY WORDS: quantity theory, policy regimes, time-varying VAR

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1 Introduction

Robert E. Lucas, Jr., (1980) described low-frequency ramifications of the quantity theory of money that he took to hold across a class of models capable of describing outcomes in the post WWII U.S. data, possibly including ones having very different transient dynamics. He focused on low frequencies because he did not want faulty estimates of transient dynamics to obscure the quantity theory. He verified that the low-frequency characterizations approximated post WWII U.S. data from 1955-1975.

> The virtue of relatively atheoretical tests ... is that they correspond to our theoretically based intuition that the quantity theoretic laws are consistent with a wide variety of possible structures. If so, it would be desirable to test them independently and then, if confirmed, to *impose* them in constructing particular structural models rather than to proceed in the reverse direction. Lucas (1980, p. 1007)

Lucas's quantity theoretic connections can be cast as unit restrictions on sums of coefficients in two-sided distributed lag regressions of an inflation rate and a nominal interest rate on money growth rates.[1] In most DSGE models, population values of these sums of weights depend on all of the structural objects that govern transient dynamics, including the monetary policy rule. In interpreting his empirical findings "as a measure of the extent to which the inflation and interest rate experience of the postwar period can be understood in terms of purely classical monetary forces," Lucas (1980, p. 1005) trusts that a monetary policy rule prevailed that, via the cross-equation restrictions emphasized by Lucas (1972) and Sargent (1971, 1981), makes the quantity theory reveal itself with a unit sum of distributed lag weights. Implicit in Lucas's calculation is an assumption that monetary policy allowed the money supply to vary in ways that unleashed the quantity theory.

In this paper, we do three things. (1) We study whether Lucas's low-frequency findings extend beyond his 1955-1975 period to a much longer 1900-2005 period that arguably witnessed alternative monetary rules. (2) In the context of two DSGE models, one with flexible prices, the other with sticky prices, we study mappings

[1] See Whiteman (1984) and section 2.3 below. Lucas (1972) and Sargent (1971) had warned against using a closely related object to test the natural rate of unemployment theory. A point of Sargent (1972, 1973a) is that empirical manifestations of the natural unemployment rate hypothesis and the Fisher equation are two sides of the same coin. In the context of the Great Moderation, Benati and Surico (2008) show an example in which changes in reduced-form statistics are difficult to interpret because they can be explained either by changes in predictable parts of shocks processes and decision rules, including those for monetary policy, or by changes in variances of shocks.

from key parameters of monetary rules to the sums of distributed lag coefficients associated with the two quantity theoretic propositions. (3) We invert the mappings in part 2 to infer what our estimated sums of distributed lag coefficients imply about prevailing monetary policies.

2 Revisiting Lucas's method and findings

For U.S. data over 1955-1975, Lucas (1980) plotted moving averages of inflation and a nominal interest rate on the y axis against the same moving average of money growth on the x axis in order to pursue

> ... the hunch that identifying long-run with "very low frequency" might isolate those movements in postwar inflation and interest rates which can be accounted for on purely quantity-theoretic grounds. Lucas (1980, p. 1013)

Lucas chose a moving average that isolates low-frequency components. We present outcomes from Lucas's filter in our figure 1, which uses M2, the GDP deflator, and the Federal Funds rate instead of M1, the CPI, and the treasury bill rate used by Lucas. (In section 3.1, we describe our data, which differ from Lucas's in ways that allow us to study a longer time period.) The figure contains scatter plots of our raw data in the top panels and moving averages of the raw data in the bottom panels. Following Lucas, we plot only second quarter data. The bottom panel shows the 45 degree line as well as two simple regression lines through the filtered data, one running 'y on x', the other 'x on y'.[2] Lucas regards low-frequency versions of two quantity-theoretic propositions as asserting that both scatter plots should approximate a 45 degree line. Those assertions are more or less borne out by our filtered data, which seem to wander around lines parallel and below the 45 degree line. For comparison, we report analogous plots for Lucas's measures of inflation and money growth in figure 27 in appendix A.

To appreciate what inspired Lucas to cast the quantity theory in this way, we describe some mechanical features of Lucas's filter and, following Whiteman (1984), how Lucas's scatter plots relate to the sum of weights in a two-sided distributed lag.

[2]These regression lines use all of the data, not just second-quarter data.

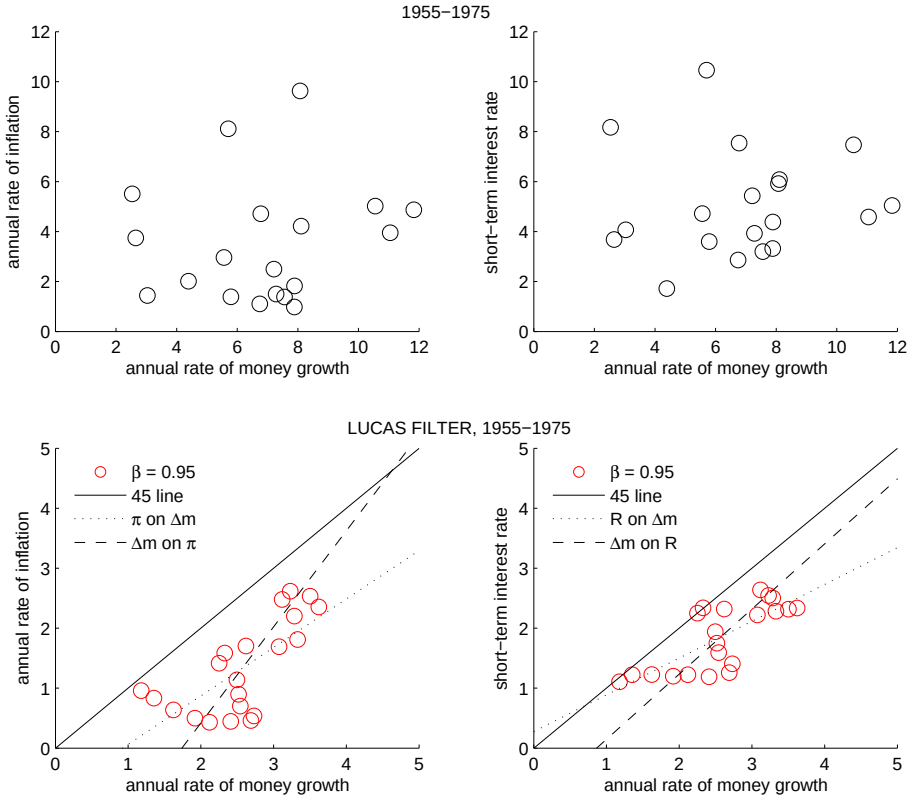

Figure 1: Lucas' filter over his sample, M2 and GDP deflator

2.1 Lucas's low-pass filter

For a scalar series x_t and $\beta \in [0,1)$, Lucas (1980) constructed moving averages

$$x_t(\beta) = \alpha \sum_{k=-n}^{n} \beta^{|k|} x_{t+k} \qquad \text{with} \qquad \alpha = \frac{(1-\beta)^2}{1-\beta^2-2\beta^{n+1}(1-\beta)}. \tag{1}$$

Choosing α according to this formula makes the sum of weights equal one. The Fourier transform of a sequence $\{f_k\}$ is $f(\omega) = \sum_{k=-\infty}^{\infty} f_k e^{-i\omega k}$. The squared Fourier transform of the two-sided sequence $\{\alpha\beta^{|k|}\}_{k=-n}^{n}$ is

$$|f(\omega)|^2 = \frac{(1-\beta)^4 \left(1-\beta^2-2\,\beta^{n+1}\cos\left((n+1)\,\omega\right)+2\,\beta^{n+2}\cos\left(nw\right)\right)^2}{\left(1-\beta^2-2\,\beta^{n+1}\left(1-\beta\right)\right)^2\left(1+\beta^2-2\,\beta\,\cos\left(\omega\right)\right)^2}.$$

Using the value $\beta = .95$ featured in Lucas's graphs that best confirm the quantity theory, figure 2 plots $|f(\omega)|^2$ for $n = 8, 16$, and 100. Because the spectral density of the filtered variable is $|f(\omega)|^2$ times the spectral density of the original variable, application of Lucas's moving average filter with $\beta = .95$ achieves his intention of focusing on low-frequency variations.[3]

2.2 Cross-equation restrictions in a plain vanilla model

To illustrate mappings from structural parameters to slopes of scatter plots, consider the following simple macroeconomic model:[4]

$$\begin{aligned} \pi_t &= (1-\lambda)\mu_t + \lambda E_t \pi_{t+1} + \sigma_\pi \epsilon_t \\ \mu_{t+1} &= (1-\rho)\phi + \rho\mu_t + \sigma_\mu \epsilon_{t+1} \\ R_t &= r + E_t \pi_{t+1} + \sigma_R \epsilon_t, \end{aligned}$$

where π_t is inflation, μ_t is money growth, $r + \sigma_R \epsilon_t$ is the one-period real interest rate, R_t is a one-period nominal interest rate, and ϵ_t is an i.i.d. 3×1 random vector. The first equation is Sargent's (1977) rational expectations version of Phillip Cagan's (1955) demand function for money with $\lambda \in (0,1)$ parameterizing the response of

[3] For a presentation of the classical filtering theory used in this paper, see Sargent (1987, ch XI).

[4] Though the particular example differs, the message of this subsection was also asserted by Lucas (1972), Sargent (1971), and King and Watson (1994). Sargent (1973b) and Barsky (1987) have applied versions of our plain vanilla model to interpret the Gibson paradox. Our plain vanilla model conveys the message of the three time-invariant examples of cross-equation restrictions in the second part of Lucas (1976). The first part of Lucas (1976) is about random coefficients models that account for instabilities over time.

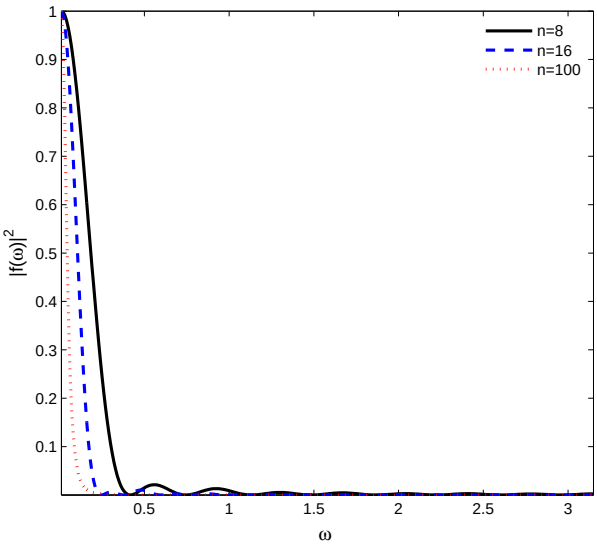

Figure 2: Squared Fourier transform of Lucas's filter with $\beta = .95$ for $n = 8, 16$, and 100.

the demand for real balances to the nominal interest rate. The second equation is an exogenous law of motion for money growth. The third equation is the Fisher equation. A rational expectations equilibrium has representation

$$\begin{aligned} \pi_t &= \phi + \left(\frac{1-\lambda}{1-\lambda\rho}\right)(\mu_t - \phi) + \sigma_\pi \epsilon_t \\ R_t &= r + \phi + \rho\left(\frac{1-\lambda}{1-\lambda\rho}\right)(\mu_t - \phi) + \sigma_R \epsilon_{t+1}, \end{aligned}$$

two equations that are linear least squares projections of π_t and R_t, respectively, on μ_t.

The equilibrium displays two sharp quantity theory predictions: cross-economy variations in the uncondition mean of money growth ϕ show up one-for-one in cross-economy variations in unconditional means of π and R. However, these say nothing about the slopes of scatter diagrams of filtered data.

Because the equilibrium representation expresses π_t and R_t directly as linear least squares regressions on contemporaneous π_t, it immediately follows that for this model the slopes of Lucas's scatter plots on filtered data are, for any filter f, just the slopes of these regressions, namely, $\frac{1-\lambda}{1-\lambda\rho}$ for π on μ and $\rho\left(\frac{1-\lambda}{1-\lambda\rho}\right)$ for R on μ. The π on μ

slope is unity if $\lambda = 0$ (no interest elasticity and a Cagan money demand function with no response to expected inflation) or if $\rho = 1$ (money growth takes a random walk). The R on μ slope is 1 if $\rho = 1$.

If we had specified the evolution equation for μ_t to be a higher order univariate autoregression or some rule feeding back on R and π, we would have to work harder to find the population values of the slopes of Lucas's scatter plots. We do that in section 5, where we show that the message of this section comes through in two DSGE models: the slopes of Lucas's scatter plots are in general functions of structural parameters, prominently including ones that describe the evolution of money growth. If one succeeds in injecting into these DSGE models a highly persistent and highly volatile money growth process, the neutralities that are built into them mean that effects of money growth variations should surface mostly in variations in inflation and interest rates, while letting real variables live lives of their own. We shall confirm this hunch in subsection 5.1.5 by watching how measures of volatility and persistence vary with parameters of the monetary policy rules.

2.3 An equivalent distributed lag procedure

Whiteman (1984) observed that fitting straight lines through scatter plots of moving averages is an informal way of computing sums of weights in long two-sided distributed lag regressions. In this subsection, we shall follow a somewhat different route to Whiteman's result but will return to his argument at the end.

Let $\{y_t, z_t\}$ be a bivariate jointly covariance stationary process with unconditional means of zero and consider the two-sided infinite least-squares projection of y_t on past, present, and future z's:

$$y_t = \sum_{j=-\infty}^{\infty} h_j z_{t-j} + \epsilon_t \tag{2}$$

where ϵ_t is a random process that satisfies the population orthogonality conditions

$$E\epsilon_t z_{t-j} = 0 \quad \forall j.$$

Let the spectral densities of y and z be denoted $S_y(\omega)$ and $S_z(\omega)$, respectively, and let the cross-spectral density be denoted $S_{yz}(\omega)$. Let the Fourier transform of $\{h_j\}$ be $\tilde{h}(\omega) = \sum_{j=-\infty}^{\infty} h_j e^{-i\omega j}$. Then

$$\tilde{h}(\omega) = \frac{S_{yz}(\omega)}{S_z(\omega)} \tag{3}$$

and the sum of the distributed lag regression coefficients is

$$\sum_{j=-\infty}^{\infty} h_j = \tilde{h}(0) = \frac{S_{yz}(0)}{S_z(0)}. \tag{4}$$

Where $\bar{y}_t = \sum_{j=-\infty}^{\infty} f_j y_{t-j}$ and $\bar{z}_t = \sum_{j=-\infty}^{\infty} f_j z_{t-j}$, the regression coefficient b_f of $\bar{y}_t$ on $\bar{z}_t$ is

$$b_f = \frac{\text{cov}(\bar{y}_t, \bar{z}_t)}{\text{var}(\bar{z}_t)} = \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(\omega)|^2 S_{yz}(\omega) d\omega}{\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(\omega)|^2 S_z(\omega) d\omega}. \tag{5}$$

Evidently, (5) implies that for $\bar{y}_t, \bar{z}_t$ constructed by applying a filter $f(\omega)$ that puts most power near zero frequency and for a $\frac{S_{yz}(\omega)}{S_z(\omega)}$ that is sufficiently smooth near $\omega = 0$,[5]

$$b_f \approx \frac{S_{yz}(0)}{S_z(0)}. \tag{6}$$

Remark 1. *Comparing formula* (6) *to formula* (3) *evaluated at* $\omega = 0$ *shows that* $b_f \approx \sum_{j=-\infty}^{\infty} h_j$.

The population R^2 of a regression of $\bar{y}$ on $\bar{z}$ is

$$R^2 = \frac{\text{cov}(\bar{y}_t, \bar{z}_t)^2}{\text{var}(\bar{z}_t)\text{var}(\bar{y}_t)} \tag{7}$$

which, with a filter $f(\omega)$ that puts most power near zero frequency and a $\frac{S_{yz}(\omega)}{S_z(\omega)}$ that is sufficiently smooth near $\omega = 0$, implies

$$R^2 \approx \frac{S_{yz}(0)^2}{S_z(0)S_y(0)}. \tag{8}$$

The low-frequency relationship between inflation and money growth is better identified when there is more variation in the low frequency components of money growth. Government policies that influence the variance of filtered money growth thus affect an econometrician's ability to detect Lucas's low-frequency manifestations of the quantity theory.

Whiteman's (1984) used a different argument than we have to show that the slope of the line drawn between moving averages of y and z can be regarded as an estimator of the sum of distributed lag coefficients $\sum_{j=-\infty}^{\infty} h_j$. In particular,

[5]Appendix E evaluates the quality of approximations (6) in the context of $\frac{S_{yz}(\omega)}{S_z(\omega)}$'s associated with two DSGE models.

appealing to Sims's (1972a) approximation formula enabled Whiteman to point out that Lucas's low-frequency regression coefficient is an estimator of $\sum_{j=\infty}^{\infty} h_j$ that is robust to misspecification of lag lengths in the projection equation (2).

Formula (5) allows us to formalize Lucas's low-frequency characterizations of the two quantity theoretic propositions by investigating how the parameters of a DSGE model, including the monetary policy rule, influence the sum of weights in (2).

2.4 Mappings from VAR and DSGE models to $\tilde{h}(0)$

We construct estimates of sums of coefficients $\sum_{j=-\infty}^{\infty} h_j$ by estimating vector autoregressions (VARs), then interpret them in terms of two log-linear DSGE models.[6] Whether the sums of coefficients reveal Lucas's frequency-domain expressions of the two quantity-theoretic propositions depends on the prevailing monetary policy.

Time-invariant versions of our VARs and of our log-linear DSGE models can both be represented in terms of the state space system

$$\begin{aligned} X_{t+1} &= AX_t + BW_{t+1} \\ Y_{t+1} &= CX_t + DW_{t+1} \end{aligned} \tag{9}$$

where X_t is an $n_X \times 1$ state vector, W_{t+1} is an $n_W \times 1$ Gaussian random vector with mean zero and unit covariance matrix and that is distributed identically and independently across time, Y_t is an $n_Y \times 1$ vector of observables, and A, B, C, D are matrices, with the eigenvalues of A being bounded strictly above by unity (A can be said to be a 'stable' matrix). Elements of the matrices A, B, C, D can be (nonlinear) functions of a vector of structural parameters η. Let y_t, z_t be two scalar components of Y_t and consider the two-sided infinite regression (2). As noted above, the Fourier transform of the population regression coefficients is $\tilde{h}(\omega) = \sum_{j=-\infty}^{\infty} h_j e^{-i\omega j}$ and the sum of coefficients is evidently $\tilde{h}(0)$. We seek a mapping to $\tilde{h}(0)$ from the structural parameters η underneath $A(\eta), B(\eta), C(\eta), D(\eta)$.

The spectral density matrix of Y is

$$S_Y(\omega) = C(I - Ae^{-i\omega})^{-1}BB'(I - A'e^{i\omega})^{-1}C' + DD'. \tag{10}$$

[6]Rather than estimating $\tilde{h}(1)$ by first estimating a VAR as we do, another worthwhile strategy would be to apply the dynamic ordinary least squares or the dynamic generalized least squares estimator of Stock and Watson (1993) to estimate $\tilde{h}(1)$ as the simple regression coefficient of $\bar{y}_t$ on $\bar{z}_t$. We have yet to compare the sampling distribution of such estimators to the posterior distributions that we describe below. Procedures of Phillips (1991) can also be applied to estimate $\tilde{h}(1)$ viewed as a regression coefficient.

The spectral density matrix is the Fourier transform of the sequence of autocovariance matrices $EY_tY'_{t-j}, j = -\infty, \ldots, -1, 0, 1, \ldots, +\infty$ whose typical element can be recovered from $S_Y(\omega)$ via the inversion formula

$$EY_tY'_{t-j} = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_Y(\omega)e^{i\omega j}d\omega. \tag{11}$$

The Fourier transform of the population regression coefficients $\tilde{h}(\omega)$ can be computed from formula (3) where $S_{yz}(\omega)$, the cross spectrum between y and z, and $S_z(\omega)$, the spectrum of z, are the appropriate elements of $S_Y(\omega)$.

2.5 Measures of volatility and persistence

In section 5, we shall see that within two examples of DSGE models, Lucas's frequency-domain expressions of the two quantity-theoretic propositions require that monetary policy put sufficient volatility and persistence into money growth, inflation, and the nominal interest rate. As a measure of persistence in a univariate time series y, we follow Cogley and Sargent (2001) in using the normalized spectrum at zero:

$$\text{persist}_y \equiv \frac{S_y(0)}{\frac{1}{2\pi}\int_{-\pi}^{\pi} S_y(\omega)d\omega}, \tag{12}$$

where the denominator is the unconditional variance of y. For a first-order univariate autoregression $y_{t+1} = \rho y_t + \epsilon_{t+1}$, where $\{\epsilon_{t+1}\}$ is i.i.d. with mean zero and finite variance

$$\text{persist}_y = \frac{1+\rho}{1-\rho}$$

which we plot in figure 3 for $\rho \in [0, .95]$.

3 A picture show (then some regressions)

In this section, we present the data, report Lucas' representation of the low frequency relationships between money growth and inflation, and money growth and the nominal interest rate. Then we compute sums of distributed lag coefficients by applying formulas (3) and (10) to bi-variate and multi-variate VARs.

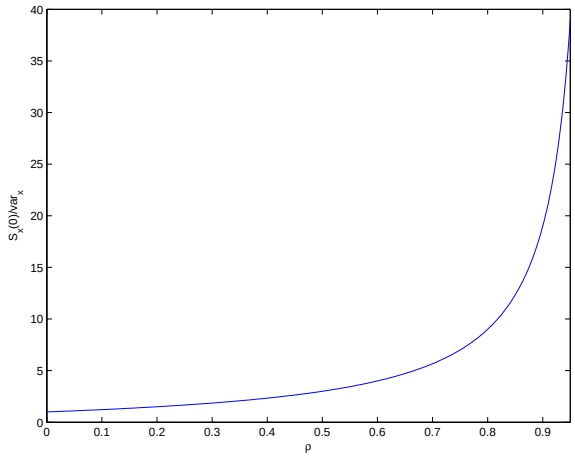

Figure 3: Persistence of a first-order a.r. process with a.r. parameter ρ, as measured by normalized spectrum at zero frequency.

3.1 Data

We use quarterly U.S. data. Real and nominal GDP (M2 stock) are available from the FRED database since 1947Q1 (1959Q1). Prior to that, we apply backward the growth rates on the real GNP and M2 series constructed by Balke and Gordon (1986).[7] As for the nominal short-term interest rate, the Federal funds rate is available from the FRED database since 1954Q3. Prior to that, we apply backward the growth rates on the Commercial Paper rate 6 month constructed by Balke and Gordon (1986). Figure 4 displays year-on-year first differences of logs of raw variables. Figure 5 reports moving averages of the raw data using Lucas's $\beta = .95$ filter. The shaded regions in these two filters isolate the 1955-1975 period that Lucas focused on.

These figures reveal some striking patterns.

- Figure 4 reveals that for money growth, inflation, and output growth, but not for the interest rate, volatility decreased markedly after 1950.
- The filtered data in figure 5 indicates that the shaded period that Lucas studied exhibit persistent increases in money growth, inflation, and the interest rate. These features let Lucas's two quantity-theoretic propositions leap off the page.
- For the filtered data, the shaded area observations are atypical.

[7] As for M2, Balke and Gordon (1986) build upon Friedman and Schwartz (1963).

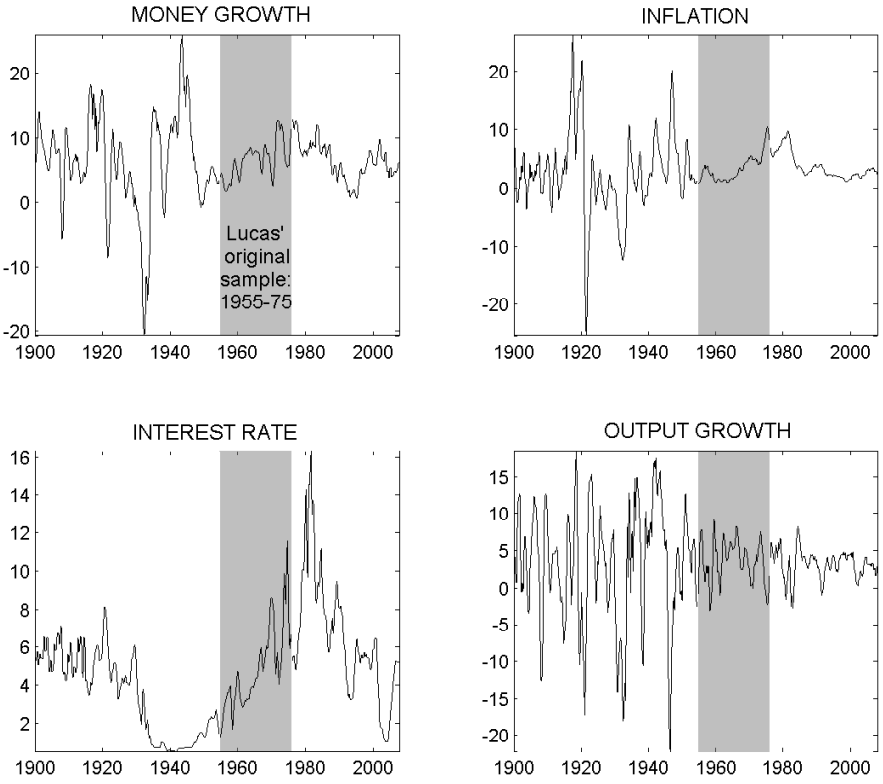

Figure 4: Money growth, inflation, short-term interest rate and output growth.

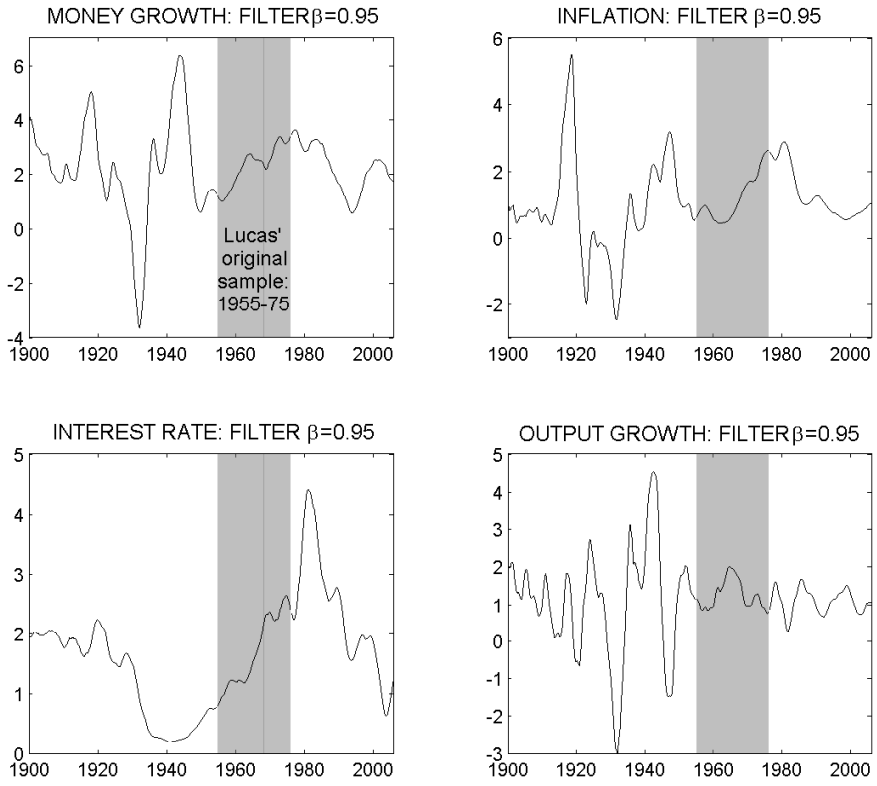

Figure 5: β=.95-filtered Money growth, inflation, short-term interest rate and output growth.

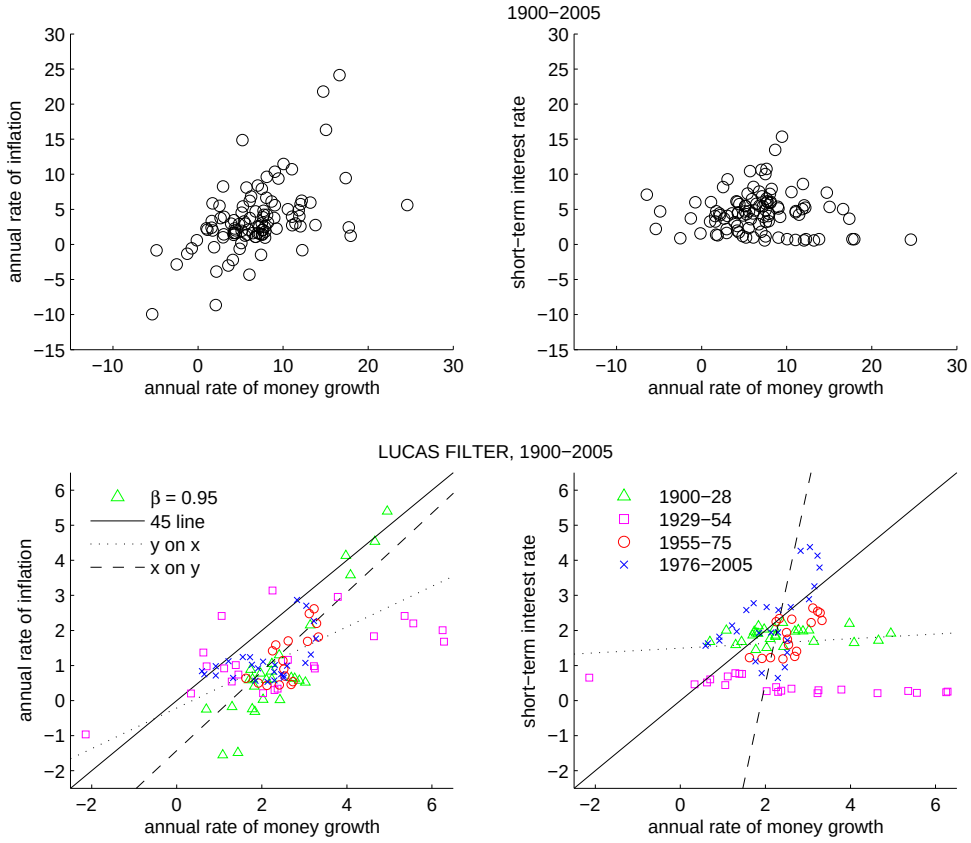

Figure 6: Lucas' filter over the full sample, 2nd quarter

3.2 More scatter plots

Figure 6, which is best viewed in color, shows scatter plots of 2nd quarter observations of filtered series over the entire period of our data sample from 1900-2005. Different colors indicate subperiods 1900-1928, 1929-1954, Lucas's subperiod of 1955-1975, and 1976-2005. Figures 7, 8, and 9 show scatters for subsamples alone from 1900-1928, 1929-1954, and 1976-2005. These are to be compared with figure 1 for Lucas's period 1955-1975.

These graphs reveal the following patterns in our eyes. The scatters of points can be said to align broadly with the two quantity propositions in the 1955-75 and

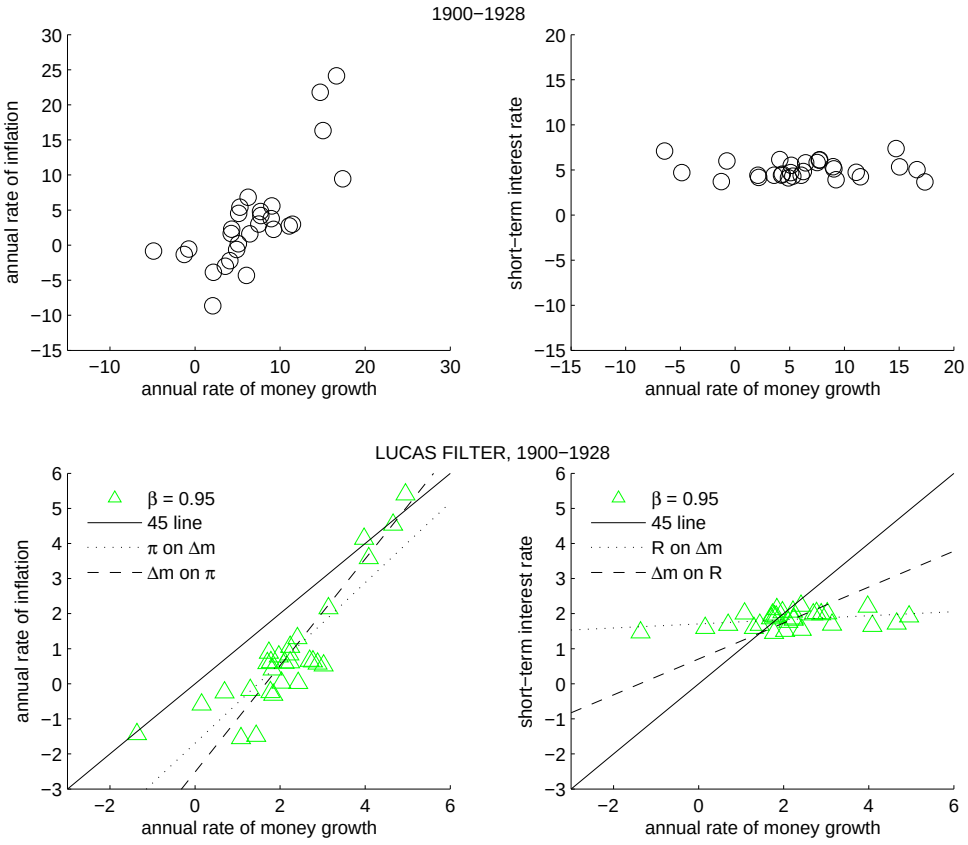

Figure 7: Lucas' filter over the sub-sample 1900-1928, 2nd quarter

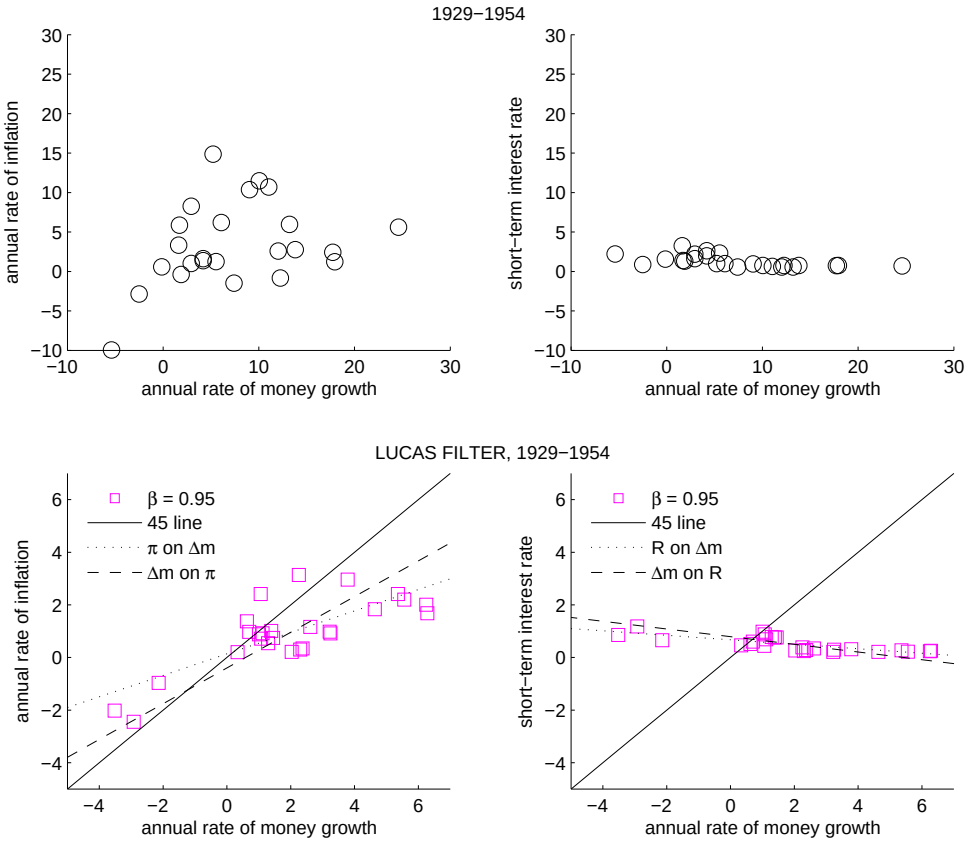

Figure 8: Lucas' filter over the sub-sample 1929-1954, 2nd quarter

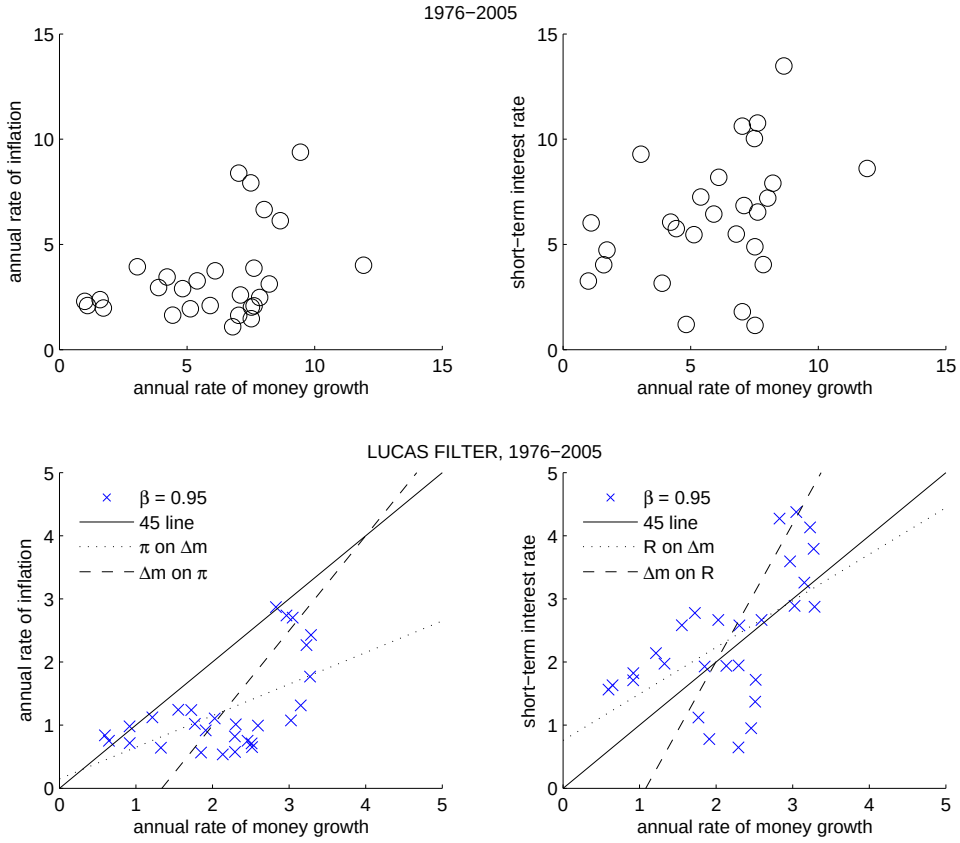

Figure 9: Lucas' filter over the sub-sample 1976-2005.

1976-2005 subperiods: the points adhere to lines that at least seem to be parallel to the 45 degree line. But for the other two subperiods there are deviations. The inflation on money growth scatter is steeper than 45 degrees during 1900-1928 and flatter during 1929-1954; while the interest on money growth scatter is flatter than the 45 degree line during 1900-1928 and negatively sloped during 1929-1954.[8] We enter these impressions in the appropriate places in table 1 and move on to other entries in the table.

[8]We obtain similar results using the band-pass filter proposed by Christiano and Fitzgerald (2002) and also employed by Benati (2005), with frequency above either eight or twenty years.

Table 1: Regressions on $\beta = .95$-filtered data, 1900-2005

Data - m: M2; p: GNP/GDP deflator; R: 6-month Commercial paper rate/federal funds rate

	π on Δm			R on Δm		
	Lucas graph		median $\tilde{h}(0)$ from $VARs$	Lucas graph		median $\tilde{h}(0)$ from $VARs$
	slope	OLS	(2-, 4-variate)	*slope*	OLS	(2-, 4-variate)
full sample	<1	.58	(.58, .56)	∼0	.07	(.28, .23)
1900-28	>1	1.13	(1.31, 1.21)	∼0	.06	(.06, .05)
1929-54	<1	.39	(.43, .41)	<0	-.08	(-.05, -.06)
1955-75	∼1	.86	(1.02, .90)	∼1	.62	(.70, .78)
1976-05	<1	.48	(.75, .55)	∼1	.75	(1.05, .73)

3.3 Regressions on filtered data

Table 2 reports regression coefficients of y on x and x and y for filtered data using different values of β. We want to focus mainly on the $\beta = .95$ outcomes that contribute entries to table 1.

3.4 Estimates of $\tilde{h}(0)$ from time-invariant VARs

In this section, we report three sets of *fixed coefficient* Bayesian VARs (BVARs) over the full sample as well as for our four sub-samples. The three families of BVARs are:

1. a *bivariate* BVAR in money growth and inflation
2. a *bivariate* BVAR in money growth and the nominal interest rate
3. a BVAR in money growth, inflation, nominal interest rate and output growth.

Following the procedure developed by Litterman (1986) and extended by Kadiyala and Karlsson (1997), we assume that the parameters of a VAR of order p are distributed as a Normal inverse Wishart, centered around the least square estimates of the VAR augmented with dummy observations for the priors.[9]

[9]The prior on the autoregressive parameters is set to zero with tightness $1/p^2$ for the coefficient on the first (own) lag of each variable i and $\hat{\sigma}_i/(\hat{\sigma}_j p^2)$ with $j \neq i$ for all the others. The scale factor

Table 2: Regressions on filtered data, 1900-2005

Data - m: M2; p: GNP/GDP deflator; R: 6-month Commercial paper rate/federal funds rate

	π on Δm				Δm on π				R on Δm				Δm on R			
β	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*
full sample	.58	.57	.56	.54	.89	.84	.73	.64	.07	.05	.02	.01	.18	.15	.09	.04
1900-28	1.13	1.18	1.21	1.15	.67	.65	.61	.57	.06	.04	.00	-.01	2.16	**1.27**	-.01	-.41
1929-54	.39	.39	.37	.34	1.48	1.33	**1.02**	**.84**	-.08	-.07	-.06	-.06	-6.8	-7.1	-7.3	-7.2
1955-75	**.86**	.69	.36	.22	.61	.56	.41	.31	.62	.45	.13	.00	**.89**	.71	.24	.01
1976-05	.48	.45	.38	.32	.65	.59	.50	.46	.75	.74	.66	.56	.45	.43	.37	.32

Note: numbers in bold are not statistically different from one at the 10% significance level, HAC covariance matrix

We use 80000 Gibbs sample replications, discard the first 60000 as burn-in, and then retain one every ten to minimize the autocorrelation across retained draws. For the sake of comparison with the results from the time-varying VAR below, we set $p = 2$ and retain those draws for which the roots of the associated VAR polynomial are not inside the unit circle.

For each BVAR, we compute $\tilde{h}(0)$ using formulas (10) and (4). Posterior distributions of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ for the full sample 1900-2005 are plotted in Figure 10. The positive (negative) ordinate values report the posterior distribution from the bivariate (multivariate) VAR. The posteriors indicate substantial uncertainty about the $\tilde{h}(0)$'s. The probability mass associated with $\tilde{h}_{\pi,\Delta m}(0) = 1$ is zero according to the bivariate VAR, whose median estimate is 0.58. The median values for the money growth-interest rate $\tilde{h}(0)$ are around 0.25 in both VARs with the central 68% (90%) mass of the distribution within the band $[0, 0.55]$ ($[-0.2, 0.84]$).

The sub-sample results for the sum of distributed lag coefficients $\tilde{h}(0)$ between money growth and inflation (money growth and the nominal interest rate) are reported in Figure 11 (Figure 12). The $\tilde{h}(0)$'s estimated using the multivariate VARs are typically characterized by less uncertainty than the bivariate VAR counterparts. In Figure 11, the value of one is inside the 68% posterior bands for the samples 1900-28 and 1955-75, and, only for the bivariate VAR, for the period 1976-2005 too. The distributions for the later two sub-periods, however, have fatter tails than the distributions for the earlier sub-periods.

$\hat{\sigma}_i$ is equal to the sample variance of the residuals from a univariate autoregressive model of order p for the variable i (see Sims and Zha, 1998). The prior on the intercept is diffuse. In appendix B, we report the implied prior distributions and the posterior distributions for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$.

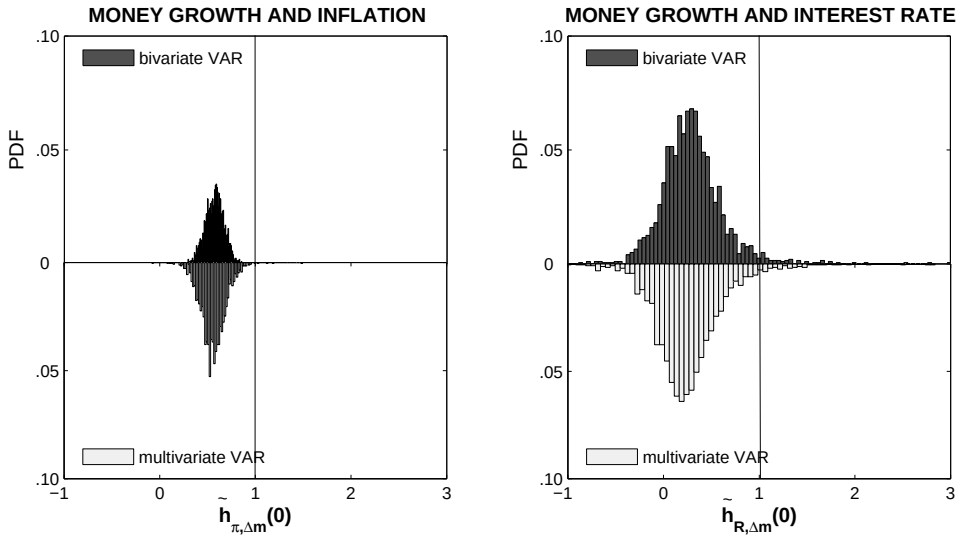

Figure 10: Full sample, 1900-2005. Posterior distributions of the long-run coefficients: multivariate vs. bivariate VAR

As for the sums of coefficients $\tilde{h}(0)$ in the two-sided distributed lag of the nominal interest rate on money growth, figure 12 shows a striking difference between the pre- and post-1955 periods. In the sample 1900-1928, for instance, the value of zero is inside the 68% posterior bands. During the years between 1929 and 1955, the probability mass associated with negative values of $\tilde{h}_{R,\Delta m}(0)$ is 98%. In contrast, the median values for the period 1955-75 (1976-2005) are 0.78 (0.73) for the multivariate VAR and 0.70 (1.05) for the bivariate VAR and a value of one is always inside the 68% interval.

4 Evidence from a time-varying VAR

In this section, we use a time-varying VAR with stochastic volatility to construct 'temporary' estimates of $\tilde{h}(0)$ that vary over time. There are at least two good reasons to allow for such time variation. First, the dynamics of money growth, inflation, nominal interest rate and output growth have exhibited substantial instabilities. Second, our long sample arguably transcends several monetary regimes, starting with a Gold Standard and ending with the fiat standard supported by a dual mandate to

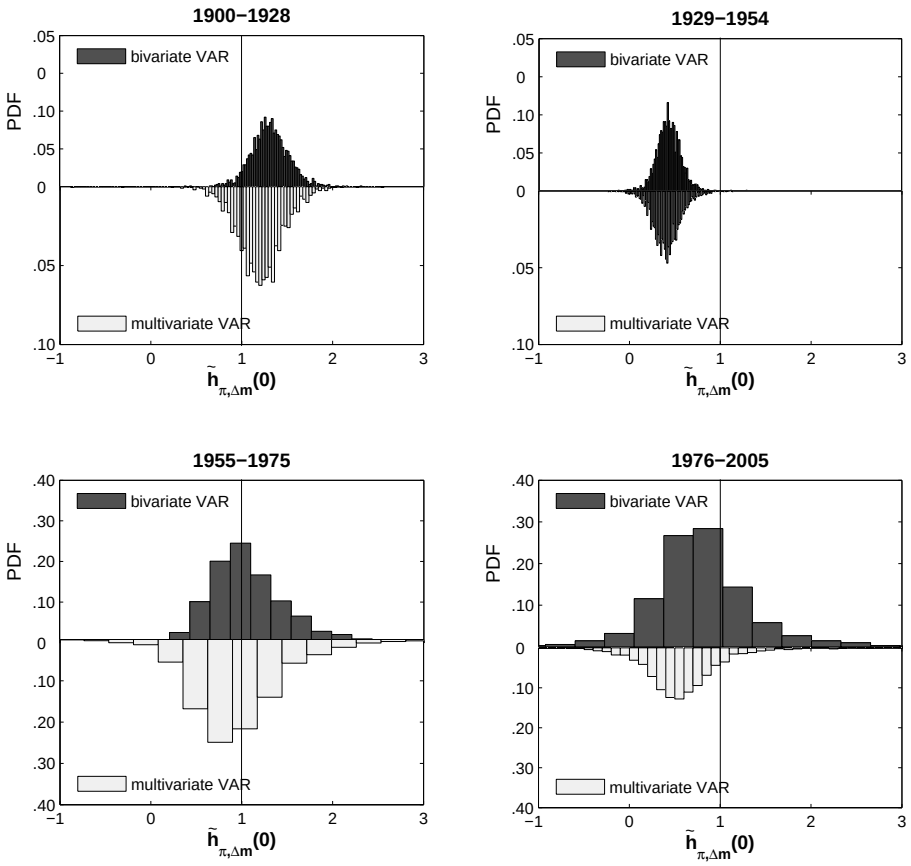

Figure 11: Posterior distributions of the $\tilde{h}_{\pi,\Delta m}(0)$ coefficient between money growth and inflation: multivariate vs. bivariate VAR

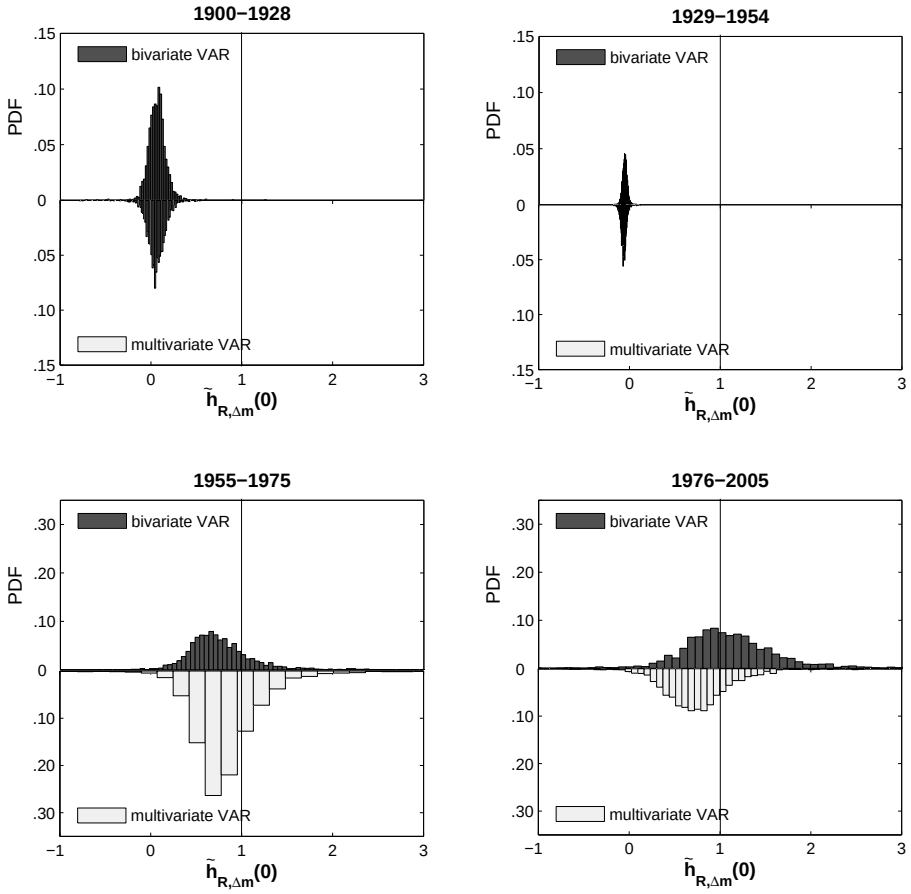

Figure 12: Posterior distributions of the $\tilde{h}_{R,\Delta m}(0)$ coefficient between money growth and the nominal interest rate: multivariate vs. bivariate VAR

promote high employment and stable prices that succeeded Bretton Woods. Before presenting details of the statistical model in subsection 4.2, we hurry to state the punch line.

4.1 Time-variation in sums of coefficients

In Figure 13, we report as red solid lines the central 68% posterior bands of the following object constructed from our time-varying VAR:

$$\tilde{h}_{yx,t|T}(0) = \frac{S_{yx,t|T}(0)}{S_{x,t|T}(0)} \tag{13}$$

namely, the temporary cross-spectrum divided by the temporary spectrum at t, using the smoothed estimates of the time-varying VAR conditioned on the data set $1, \ldots, T$. We compute the temporary spectral objects by applying formulas (10) and (4) to the (t, T) versions of A, B, C, D.

We view equation (13) as a local-to-date t approximation of equation (4). Ideally, when extracting the low-frequency relationships, we should also account for the fact that the parameters drift going forward from date t. But this is computationally challenging because it requires integrating a high-dimensional predictive density across all possible paths of future parameters. Adhering to a practice in the learning literature (referred to as 'anticipated-utility' by Kreps, 1998), we instead update the elements of θ_t, H_t and A_t period-by-period and then treat the updated values as if they would remain constant going forward in time.

For comparison, we also report as blue dotted (solid) lines the 68% posterior bands (median values) based on the estimates from a fixed-coefficient 4-variate VAR for money growth, inflation, the nominal interest rate, and output growth over the full sample.

The medians of the distributions of the $\tilde{h}(0)$s display large amounts of time variation, especially for the money growth and the nominal interest rate. The posteriors reveal substantial uncertainty about the $\tilde{h}(0)$s, however, and in some episodes like the 1970s, $\tilde{h}(0)$ values of zero and one are simultaneously inside the posterior bands for both panels. The most recent twenty years as well as the 1940s are characterized by the lowest values of the median estimates and the smallest uncertainty. The 1970s, in contrast, are associated with the highest values and the largest uncertainty.

It is worth noting that the median estimates of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ based on the fixed coefficient multivariate BVAR for the *full sample* are 0.55 and 0.25 respectively. These are probably similar to the values that one would obtain by

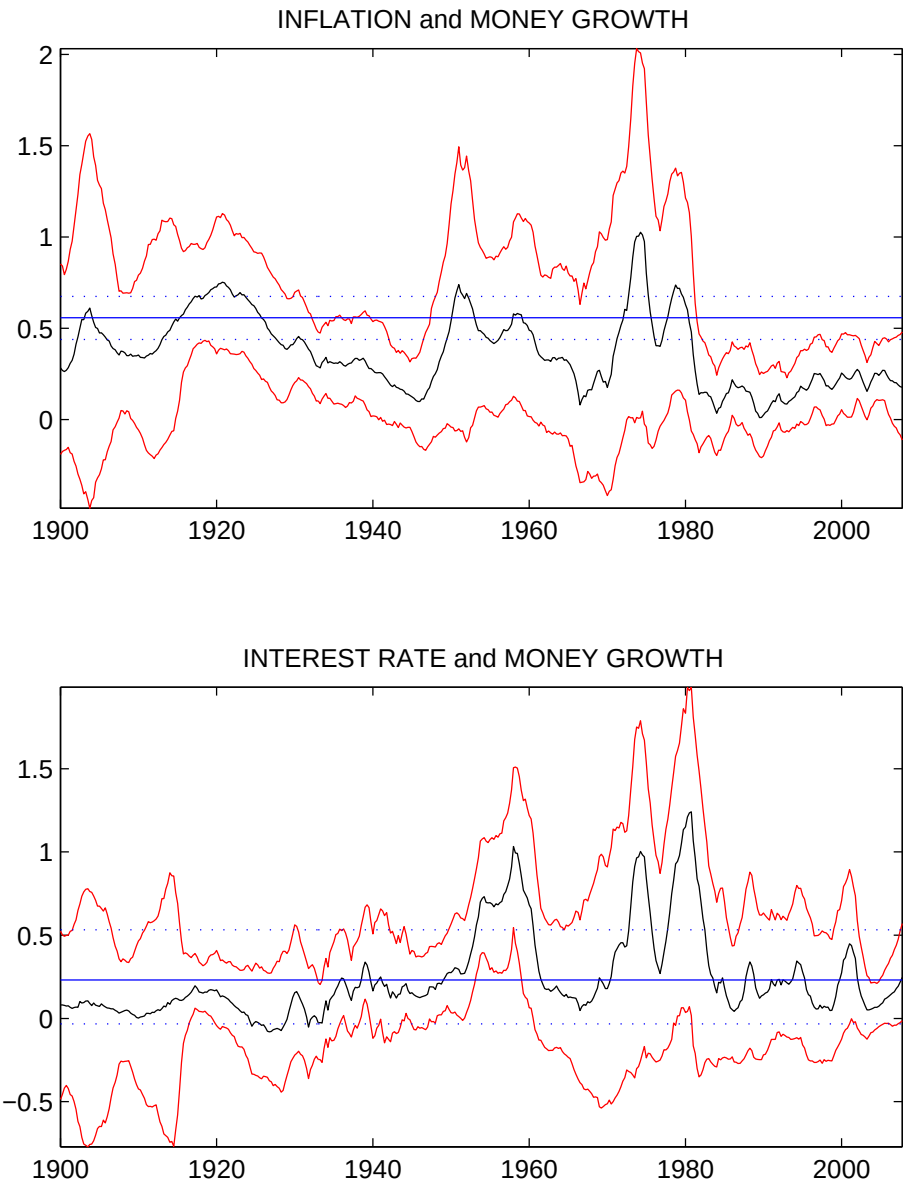

Figure 13: Median and 68% central posterior bands for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ based on a fixed-coefficient VAR over the full samples and a VAR with time-varying coefficient and stochastic volatility.

averaging the time-varying $\tilde{h}_{yx,t|T}(0)$'s over the full sample as well as across Gibbs-sampling repetitions.

As for the unit coefficients associated with the quantity theory of money, the value of one is outside the posterior bands for most of the sample, with the exceptions typically concentrated in the 1970s. A comparison between the results based on the time-varying VAR and the straight lines from the fixed-coefficient VAR over different sub-samples reveal that the two models can yield very different results. Notice that in each sub-sample, estimates of $\tilde{h}(0)$ based on the fixed-coefficient model (reported in the previous section) appear to give disproportionate weight to the episodes whose $\tilde{h}(0)$'s seem outliers when viewed through the lens of the time-varying estimates.

4.2 A model with drifting coefficients and stochastic volatilities

We now describe the time-varying statistical model underlying the results presented above. The model is a VAR(p) with drifting coefficients and stochastic volatility:

$$Y_t = B_{0,t} + B_{1,t}Y_{t-1} + ... + B_{p,t}Y_{t-p} + \epsilon_t \equiv X_t'\theta_t + \epsilon_t \tag{14}$$

where X_t' collects the first p lags of Y_t, θ_t is a matrix of time-varying parameters, ϵ_t are reduced-form errors and Y_t is defined as $Y_t \equiv [\Delta m_t,\ \pi_t,\ \Delta y_t, R_t]'$. The operator Δ denotes a first log difference; m_t denotes the money, π_t is the inflation rate, the first difference of the log of the GDP deflator, p_t; and y_t is real GDP. The short-term nominal interest rate is R_t. Following Cogley and Sargent (2005), we set the lag order p=2. The time-varying VAR parameters, collected in the vector θ_t, are postulated to evolve according to:

$$p(\theta_t \mid \theta_{t-1},\ Q) = I(\theta_t)\ f(\theta_t \mid \theta_{t-1},\ Q) \tag{15}$$

where $I(\theta_t)$ is an indicator function that takes a value of 0 when the roots of the associated VAR polynomial are inside the unit circle and is equal to 1 otherwise. $f(\theta_t \mid \theta_{t-1},\ Q)$ is given by

$$\theta_t = \theta_{t-1} + \eta_t \tag{16}$$

with $\eta_t \sim N(0,\ Q)$. The VAR reduced-form innovations in (14) are postulated to be zero-mean normally distributed, with time-varying covariance matrix Ω_t that is factored as

$$Var(\epsilon_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \tag{17}$$

The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 \\ 0 & 0 & 0 & h_{4,t} \end{bmatrix} \qquad A_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 \end{bmatrix} \tag{18}$$

with the elements $h_{i,t}$ evolving as geometric random walks:

$$\ln h_{i,t} = \ln h_{i,t-1} + \nu_{i,t} \tag{19}$$

Following Primiceri (2005), we postulate:

$$\alpha_t = \alpha_{t-1} + \tau_t \tag{20}$$

where $\alpha_t \equiv [\alpha_{21,t},\ \alpha_{31,t},\ ..,\ \alpha_{43,t}]'$, and assume that the vector $[u_t',\ \eta_t',\ \tau_t',\ \nu_t']'$ is distributed as

$$\begin{bmatrix} u_t \\ \eta_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N\left(0, V\right), \text{ with } V = \begin{bmatrix} I_4 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & Z \end{bmatrix} \text{ and } Z = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \tag{21}$$

where u_t is such that $\epsilon_t \equiv A_t^{-1} H_t^{\frac{1}{2}} u_t$.

The model (14)-(21) is estimated using Bayesian methods (see Kim and Nelson (2000)). Full descriptions of the algorithm, including the Markov-Chain Monte Carlo (MCMC) used to simulate the posterior distribution of the hyperparameters and the states conditional on the data, are provided in a number of papers (see, for instance, Cogley and Sargent, 2005, and Primiceri, 2005) and will not be repeated here.

Even though one cannot characterize analytically the joint posterior distribution of the model parameters, it is possible to construct a Markov chain whose invariant distribution is the posterior. The MCMC procedure draws from the marginal density of a set of random variables j, conditional on some realizations for another set of random variables i, and then drawing from the marginal distribution of i conditional on the realizations of j in the previous step. Under some assumption, the chain converge to an invariant density that equals the desired posterior density.

To calibrate the priors for the VAR coefficients, we use a training sample of twenty five years, from 1875Q1-1899Q4. The results hereafter, then, refer to the period 1900Q1 to 2007Q4. The elements of S are assumed to follow an inverse-Wishart distribution centered at 10^{-3} times the prior mean(s) of the relevant element(s)

of the vector α_t with the prior degrees of freedom equal to the minimum allowed. The priors for all the other hyperparameters are borrowed from Cogley and Sargent (2005). We use 80000 Gibbs sampling replications, discard the first 60000 as burn-in, and then retain every tenth one to minimize the autocorrelation across retained draws. In Appendix C, we show that the posterior moments vary little across subsets of retained draws, providing some evidence of convergence.

4.3 Macroeconomic volatility

We measure volatilities by computing the temporary variances

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{x,t|T}(\omega)d\omega$$

where as in Cogley and Sargent (2005), $S_{x,t|T}(\omega)$ is the spectral density formed by applying formula (10) with the time t estimates of the state-space matrices formed using all the data from $t = 1, \ldots, T$, which in our case span the period 1900-2007.

The results for the median estimates and the 68% central posterior bands are reported in Figure 14. Money growth and inflation were very volatile towards the end of 1910s. WWI was associated with output volatility and moderate interest rate variation. The volatilities of money growth and output growth exhibited their highest values in the intra-wars sample, which was dominated by the Great Depression and Roosevelt's New Deal. Inflation was volatile too, though not at the levels seen during WWI. After the peaks associated with WWII, all series experienced a significant decline in volatility that lasted until the 1970s.

The years between 1973 and 1984 were characterized by the largest fluctuations since the end of WWII. Unlike the first part of the of the twentieth century, however, the variation in money growth and inflation coincided with the highest sample value for the interest rate volatility. When judged against a broader historical perspective, the so-called Great Moderation in output in recent years seems less impressive. Since the second half of the 1980s, inflation and output growth have been most stable. The volatilities of money growth and interest rate have also been limited by historical standards, with a common local peak in the early 2000s.

4.4 Innovation standard deviation and stochastic volatility

Appendix D reports measures of stochastic volatility constructed from our time-varying VAR. These indicate a significant decline in the variance of forecast errors for money growth, inflation, output growth and the interest rate. The flip-side of

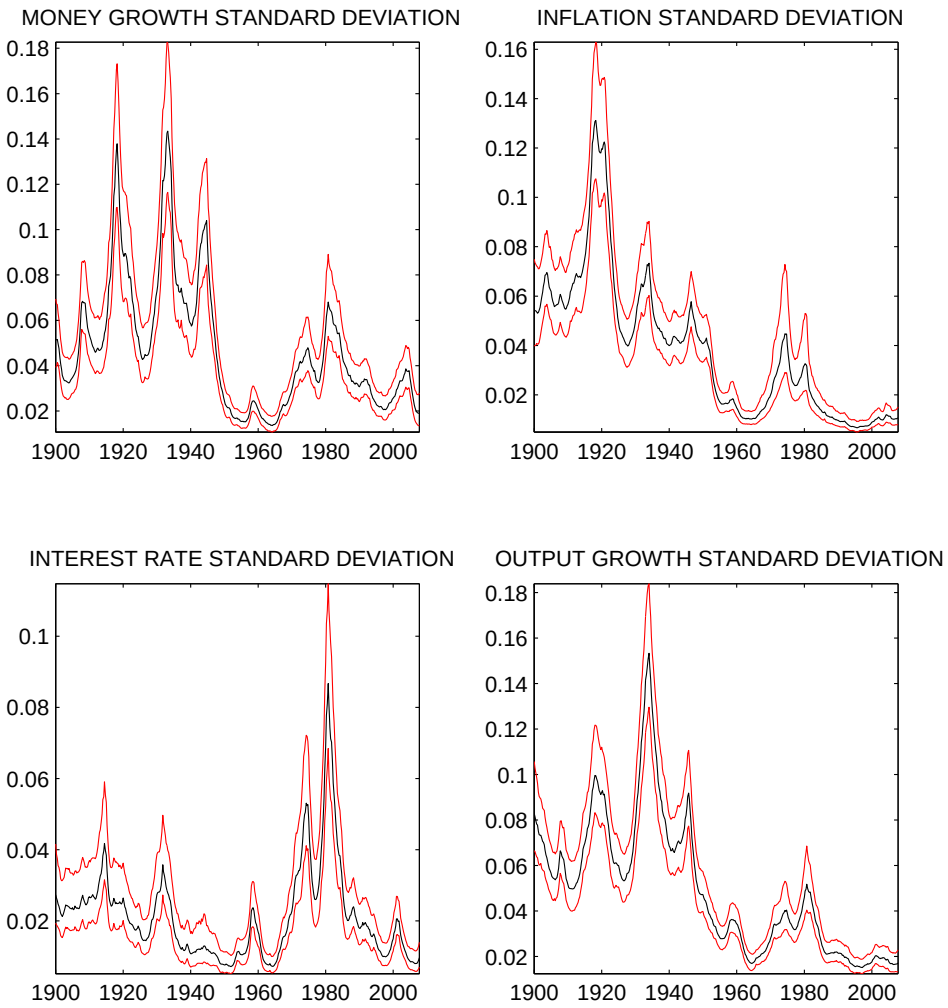

Figure 14: Standard deviations of the variables

the reduction in the innovation variances (but not the flip-side of the reduction in the variances of the series) is that the forecasts based on a naive model such as the unconditional mean have become relatively more accurate than the forecasts based on more sophisticated models such as VARs.[10]

A similar picture emerges from Figure 32, which plots the stochastic volatility of each variable j computed as the square root of $h_{j,t}$.

4.5 Persistence

Figure 15 shows the evolution of persistence for the four variables in the VAR as measured by the temporary normalized spectra

$$\frac{S_{x,t|T}(0)}{\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{x,t|T}(\omega)d\omega}. \tag{22}$$

Four findings stand out. First, there seems to be little variation in the persistence of money growth. Second, inflation persistence experienced a substantial and unprecedented increase during the 1960s and the 1970s. Third, the highest persistence for the nominal interest rate occurred around 1940, which is not surprising after we have observed the behaviour of the series shown in Figure 4. Fourth, the persistence for output growth appears relatively stable, with possible peaks both in our estimates of persistence and in the uncertainty surrounding these estimates towards the end of the 1970s.

5 Two DSGE models

> . . . we have specific theoretical examples exhibiting both quantity-theoretic laws in clear, exact form, and others which suggest possibly important qualifications. This is all we can ever hope for from our theory; some strong clues as to what to look for in the data; some warnings as to potential sources of error in these predictions. Lucas (1980, p. 1006)

This section applies formulas (10) and (4) to study how theoretical values of the sums of coefficients $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ depend on monetary policy rules in two DSGE models. The first model has completely flexible prices while the second has sticky prices. If monetary policies are conducted in particular ways, it is possible for

[10] See D'Agostino, Giannone and Surico (2006) for a discussion of the link between (the breakdown in) predictability and the Great Moderation.

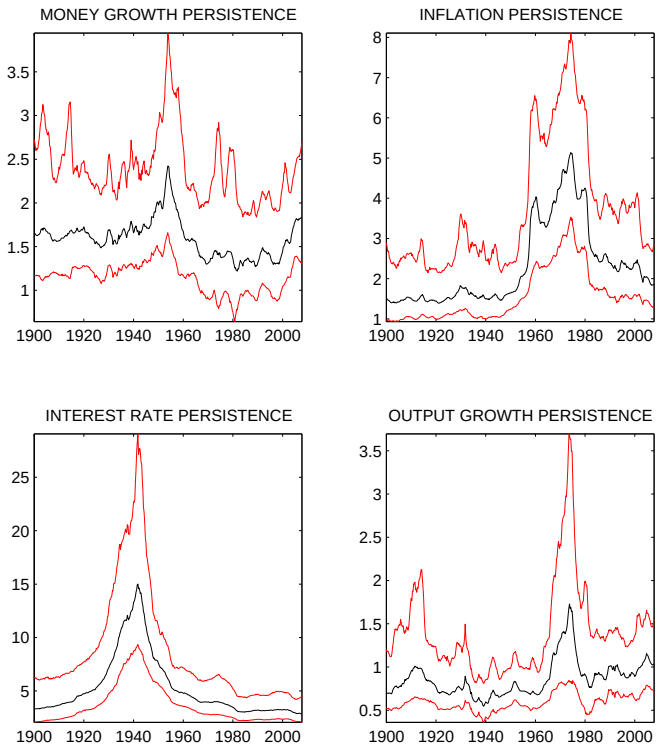

Figure 15: Persistence

Lucas's low frequency characterizations of the two quantity theory propositions to come through in both models. But if policies are conducted in other ways, Lucas's characterization does not prevail.

We posit more general monetary policies than did Lucas and Whiteman, both of whom assumed that money growth is econometrically exogenous in the sense of Sims (1972b). We consider two types of monetary policy rules, each of which, depending on parameter values, allows extensive feedback from endogenous variables to money growth.[11] The first is a money growth rule according to which the central bank sets the growth rate of money in response to movements in inflation and output growth. The second is a Taylor rule according to which the central bank sets the short-term nominal interest rate in response to movements in inflation and output growth.

5.1 A neoclassical model

The competitive equilibrium of Lucas's (1975) monetary business cycle model can be expressed in the state-space form (9). A parameter vector η implies a 4-tuple of matrices $A(\eta), B(\eta), C(\eta), D(\eta)$. We are interested in how monetary policies affect population values of the sums of distributed lag coefficients of inflation on money growth and the short term interest rate on money growth.

5.1.1 The structure

The structural equations of Lucas's model are:

$$
\begin{aligned}
r_t &= -\delta_k k_t && (23)\\
k_{t+1} &= \theta_r E_t r_{t+1} + \theta_\pi E_t \pi_{t+1} + \theta_k k_t + \varepsilon_{kt} && (24)\\
\Delta m_t &= \pi_t + z_t - \tau_r E_t \Delta r_{t+1} - \tau_\pi E_t \Delta \pi_{t+1} + \tau_k \Delta k_t + \varepsilon_{\chi t} && (25)\\
y_t &= \alpha_k k_t + \ln(Z_t), \quad \Delta y_t = \alpha_k \Delta k_t + z_t && (26)\\
R_t &= r_t + E_t \pi_{t+1} && (27)
\end{aligned}
$$

where π_t, k_t, Δm_t, r_t and R_t are inflation, the capital stock, nominal money growth, the real and the nominal short-term interest rates, respectively. The rate of technological progress is $z_t \equiv \Delta \ln (Z_t)$ and the output growth is Δy_t. The mathematical expectation operator conditional on information available at time t is denoted E_t.

Equation (23) is a marginal productivity condition for capital, (24) is a portfolio balance equation that expresses the behavior of owners of capital, while (25) is the

[11] For us, depending on monetary policy rule parameter values, other variables can Granger cause money growth rates (see Granger (1969) and Sims (1972b)).

demand for money, and (26) is a production function. The Fisher equation (27) asserts that the nominal interest rate is the sum of the real rate and the expected rate of inflation. The structural shocks are *iid* and normally distributed with variances σ_k^2, σ_χ^2, and σ_z^2, respectively. All variables are expressed in log deviations from their steady state values.

5.1.2 Monetary policy

We study two types of policies.

A money supply rule. A rule adjusts the growth rate of money smoothly in response to movements in inflation and output growth and a shock ε_{mt}.

$$\Delta m_t = \rho_m \Delta m_{t-1} + (1-\rho_m)\left(\phi_\pi \pi_t + \phi_{\Delta y}\Delta y_t\right) + \varepsilon_{mt}, \quad \varepsilon_{mt} \sim N(0, \sigma_m^2) \tag{28}$$

A Taylor rule. A Taylor rule adjusts the short-term nominal interest rate smoothly in response to movements in inflation and output growth and a monetary policy shock ε_{Rt}.

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left(\psi_\pi \pi_t + \psi_{\Delta y}\Delta y_t\right) + \varepsilon_{Rt} \quad \text{with } \varepsilon_{Rt} \sim N(0, \sigma_R^2) \tag{29}$$

5.1.3 Parameter values

We set parameter values in Table 3. These respect the theoretical restrictions $\theta_r > \theta_\pi \geq 0$, $\tau_\pi > \tau_r > 0$ and $\theta_k, \tau_k \in (0,1)$.

Fixing the other structural parameters at their table 3 values, we solve the model for alternative values of the monetary policy rule parameters, deduce the associated A, B, C, D matrices, then use formulas (10) and (4) to compute the theoretical values of sums of distributed lag coefficients $\tilde{h}(0)$. Under the configurations that imply indeterminacy in the Taylor rule regime 2, we apply the orthogonality solution method developed by Lubik and Schorfheide (2004). Here we set the standard deviation of the sunspot shock, σ_{ss}, to 0.2, their estimated value. Under the money supply rule regime 1, the configurations of policy parameters always imply determinacy.

Table 3: Parameter values

economy		*shocks*		*policy rules*	
δ_k	0.05	σ_k	0.5	ψ_π	$[0, 3]$
θ_r	0.2	σ_χ	0.4	$\psi_{\Delta y}$	$[0, 1]$
θ_π	0.1	σ_z	0.5	ρ_r	0.7
θ_k	0.97	σ_{ss}	0.2	σ_r	0.4
τ_r	0.1			ϕ_π	$[-2, 1]$
τ_π	0.2			$\phi_{\Delta y}$	$[-1, 0]$
τ_k	0.2			ρ_m	0.7
α_k	0.3			σ_m	0.4

5.1.4 Sums of weights $\tilde{h}(0)$ across monetary regimes

Figures 16 and 17 record the results of applying formulas (10) and (4) to our numerical version of Lucas's model.

A more anti-inflationary stance, as exemplified by lower values of ϕ_π in figure 16, is associated with monotonically smaller values of $\tilde{h}(0)$, which reach their minima around 0.4 at $\phi_\pi = -2$. The explanation for this outcome is that the more successfully monetary policy stabilizes inflation, the less persistent is inflation and therefore also the interest rate, with the consequence that, as encoded in $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$, the low frequency associations between these variables and money growth become attenuated.

However, weaker policy responses of money growth to inflation (i.e. a ϕ_π that approaches one) generate one-to-one low frequency comovements between money growth and inflation and money growth and the nominal interest rate as reflected in the $\tilde{h}(0)$'s.[12]

Moving to outcomes with a Taylor rule, under a passive monetary policy, (i.e. one with a less than proportional response of the interest rate to inflation), we note high values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$. Money growth and inflation (the nominal interest rate) display the highest sums of distributed lag coefficients 1 (0.9) for monetary policies in the neighborhood of $\psi_\pi = 1$, largely independently from the policy response $\psi_{\Delta y}$ to output growth. Within the active policy regime, outcomes for the interest rate rule are mirror images of those for the money growth rule.

[12]Results are robust to halving the transmission mechanism parameters.

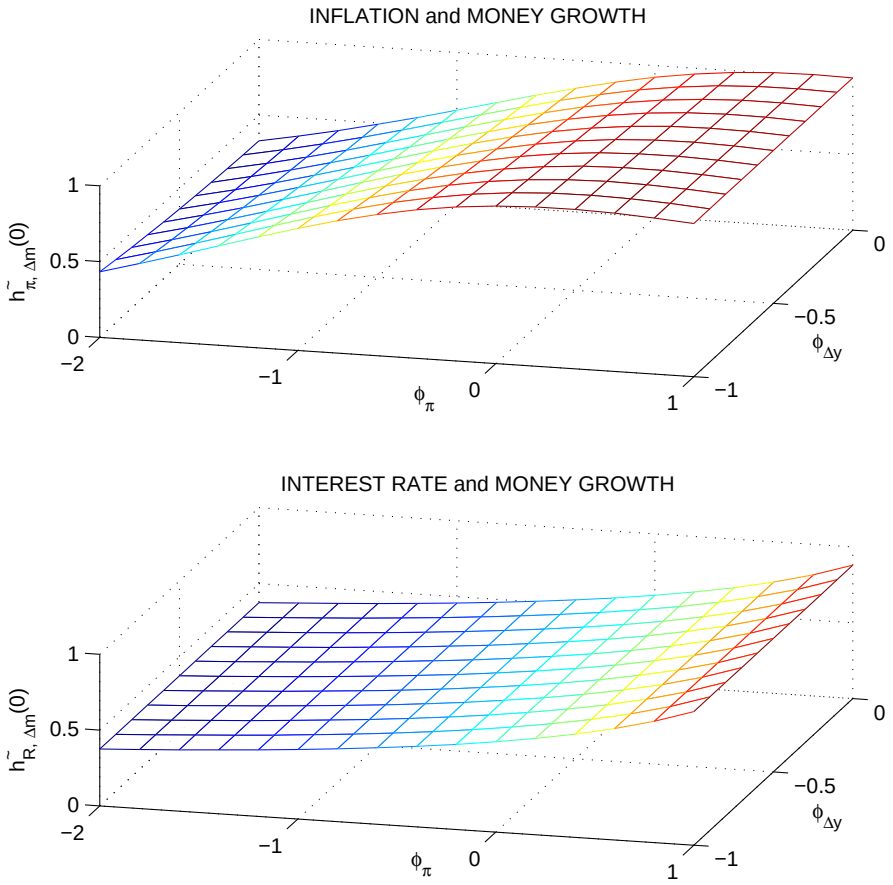

Figure 16: Sums of weights $\tilde{h}(0)$ in Lucas model under money supply rule.

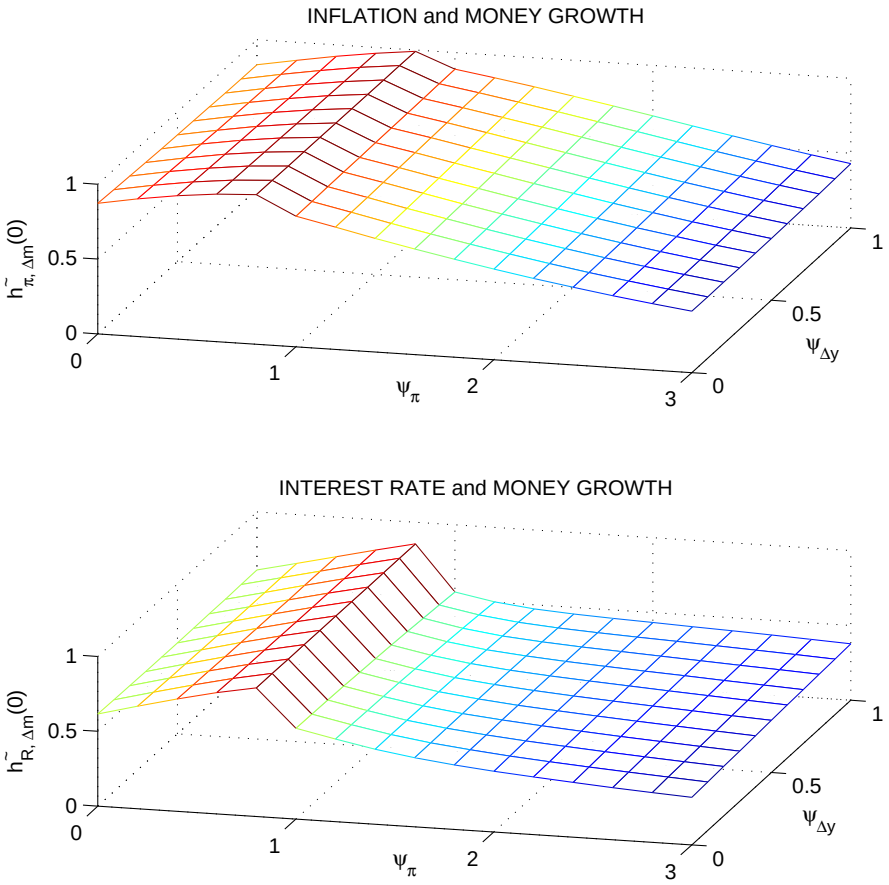

Figure 17: Sums of weights $\tilde{h}(0)$ in Lucas model under Taylor rule.

5.1.5 Volatility and persistence

To highlight a force that drives these outcomes, Figure 18 plots the persistence of money growth, as measured by the normalized spectrum at zero frequency defined in equation (12), and the volatility, as measured by the unconditional variance of money growth. We plot these under both a money growth rule and a Taylor rule. A more aggressive policy response to inflation (lower values of ϕ_π in the money rule and higher values of ψ_π in the Taylor rule) diminishes both the persistence and the volatility of the money supply within the determinacy region.

The shapes of persistence and volatility as functions of the policy parameters resemble the shapes of sums of distributed lag coefficients $\tilde{h}(0)$'s as functions of the same parameters, depicted in Figures 16 and 17. This pattern suggests that the amounts of variability and persistence of money growth are key features that intermediate how the $\tilde{h}(0)$s depend on policy.[13] Furthermore, the fact that high volatility and high persistence are associated with $\tilde{h}(0)$s near one confirms the hunch articulated in subsection 2.2 about the sources of variation in the data that could allow the low frequency connections featured by Lucas (1980) to emerge from his plots of one filtered data series against another.

5.1.6 Mundell-Tobin effect

Figures 16 and 17 lock the Mundell-Tobin effect parameter θ_π at the value of .1 reported in table 3. Figure 19 shows the consequences of setting this parameter first to eradicate the Mundell-Tobin effect ($\theta_\pi = 0$) and then to strengthen it (θ_π equal to .5 or 1).[14] The figure is constructed for money growth rules and we intend it to be compared with figure 16. Outcomes confirm Lucas's assertions about how the Mundell-Tobin effect should affect the $\tilde{h}(0)$ sums of distributed lag coefficients for two-sided distributed lag regressions of interest on money supply growth and how it should not affect that for inflation on money supply growth.

It is notable that, with the parameterization in table 3, the model requires a significant Mundell-Tobin effect to be able to match the $\tilde{h}_{R,\Delta m}(0)$ estimated for the sub-samples at the beginnings of both the twentieth and the twenty-first centuries. Similar results, not reported but available upon request, are obtained using a Taylor rule for monetary policy.

[13] See King and Watson (1994, 1997) for a discussion of related forces that affect particular tests of the natural unemployment rate hypothesis.

[14] To conform to the inequality $\theta_r > \theta_\pi$, in the second and third columns of figure 19 we have set θ_r equal to .6 and 1.1, respectively. Similar results, however, are obtained keeping θ_r to .2.

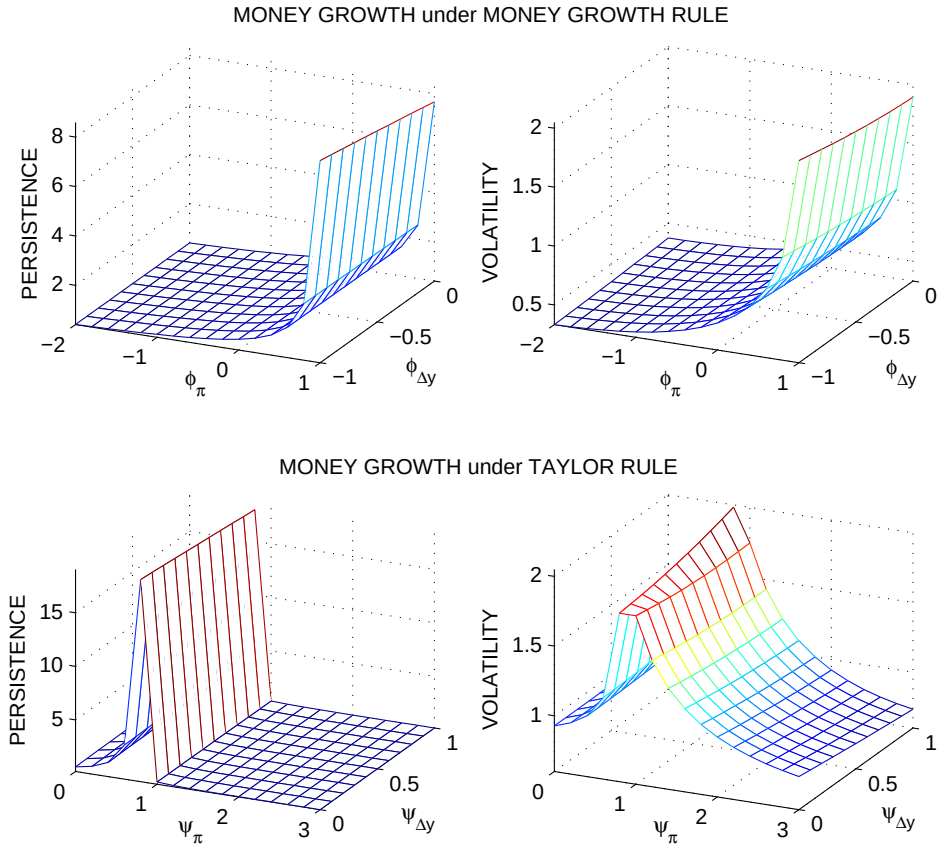

Figure 18: Properties of money growth in the Lucas model

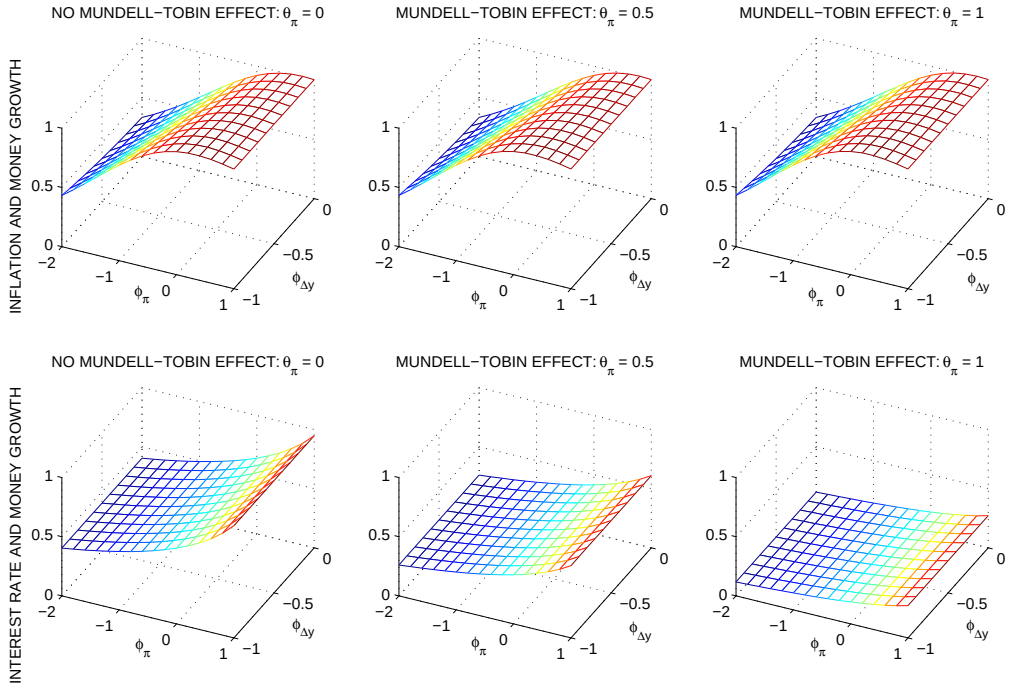

Figure 19: Mundell-Tobin effects, as measured by sums of coefficients $\tilde{h}(0)$ on the y axes, under alternative money supply rules.

5.1.7 The variance of the monetary policy shock

The results in Figure 16 are based on a parameterization in which the standard deviation of the shocks to monetary policy is as large as the standard deviations of the shocks to technology and the process for capital accumulation. Another way that monetary policy may change, however, is through the frequency and the size of the deviations from its systematic behaviour. In Figure 20, we explore the consequences for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ of halving the standard deviation of the monetary policy shock, σ_m, from the baseline value of 0.4 to 0.2. For expositional convenience, the left column reports the two panels of figure 16.

Two findings are worth noting. First, $\tilde{h}_{R,\Delta m}(0)$ is virtually unaffected by the change in σ_m. Second, the model can now generate low (and even slightly negative) values of $\tilde{h}_{\pi,\Delta m}(0)$, when the policy response to inflation is sufficiently aggressive, (i.e. $\phi_\pi \leq -0.5$). This is important for the ability of the model to replicate the estimated values of $\tilde{h}_{\pi,\Delta m}(0)$ over the most recent period reported in figure 13. Halving the standard deviation of the money demand shock, in contrast, has little impact on the $\tilde{h}(0)$'s.[15] Interestingly, low values of the variance of the monetary policy shocks appear important to generate low values of $\tilde{h}_{\pi,\Delta m}(0)$ (with virtually no impact on $\tilde{h}_{R,\Delta m}(0)$) while high values of the Mundell-Tobin effect appear important to generate low values of $\tilde{h}_{R,\Delta m}(0)$ (with virtually no impact on $\tilde{h}_{\pi,\Delta m}(0)$).

5.1.8 Policy smoothing

In the plain vanilla model of section 2.2, high values of the policy smoothing parameter ρ are associated with high values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$. In this section, we will confirm that result in the context of the Lucas (1975) model by showing how the $\tilde{h}(0)$'s vary with the policy smoothing parameter ρ_m in the money growth rule. To this end, figure 21 shows the $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ for three different values of the smoothing coefficient: 0, .4 and .95. These results should be compared with the findings for $\rho_m = .7$ in figure 16.

A money growth rule with no smoothing is associated with values for $\tilde{h}_{\pi,\Delta m}(0)$ between -.5 and 1.1, spanning a larger interval than in figure 16. A weaker anti-inflationary stance (i.e. higher values of ϕ_π) implies a stronger low-frequency relationship between inflation and money growth. The relationship becomes less non-linear in the middle column for $\rho_m = .4$. A policy smoothing parameter of .95, in the last column, makes the $\tilde{h}_{\pi,\Delta m}(0)$'s very close to one, with little impact from the other policy coefficients. This is in line with the results from the plain vanilla model

[15] We obtain similar results halving the variance of the monetary policy shock in the Taylor rule.

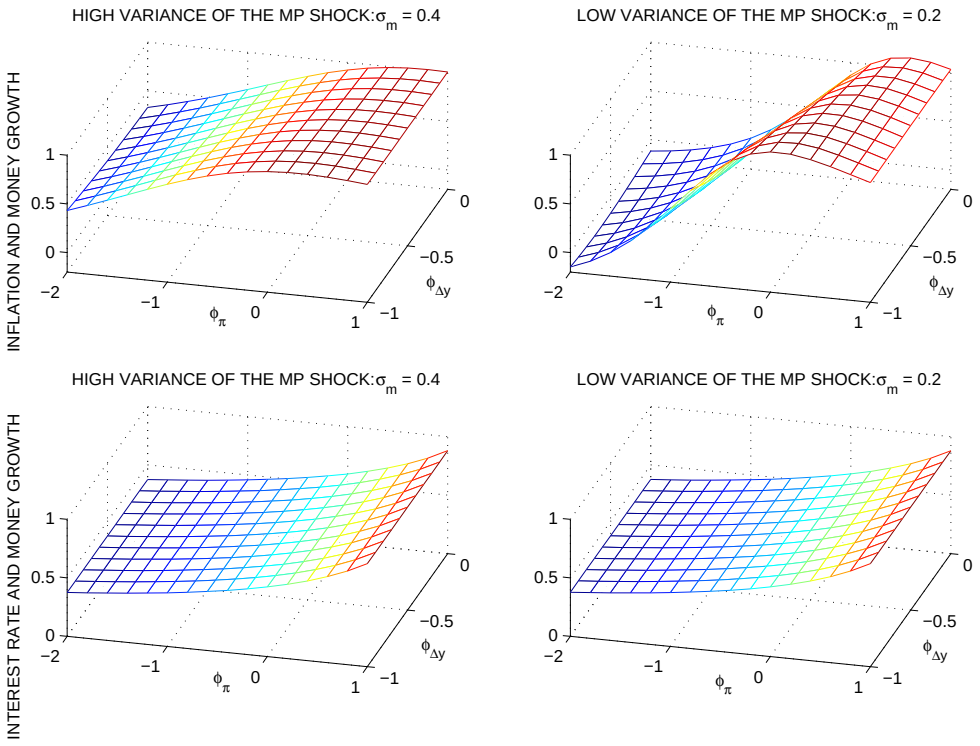

Figure 20: The effects of the variance of the monetary policy shock on the sums of coefficients $\tilde{h}(0)$ on the y axes, under alternative money supply rules.

of section 2.2.

As for the low-frequency relationship between the nominal interest rate and money growth, the bottom row of figure 21 shows that this is zero in the absence of policy smoothing. The $\tilde{h}_{R,\Delta m}(0)$'s increase monotonically with ρ_m, and in the last column, a very high value for the smoothing coefficient implies high values of the sums of distributed lags. The impact of the other policy parameters, ϕ_π and $\phi_{\Delta x}$, is larger than in the case of $\tilde{h}_{\pi,\Delta m}(0)$, in line with the predictions of the plain vanilla model.

Similar results for the inflation money growth relationship prevail under a Taylor rule. As for relationship between the nominal interest rate and money growth, $\rho_R = .95$ is associated with values of $\tilde{h}_{R,\Delta m}(0)$ close to zero. The reason for this finding is that a higher value of the policy smoothing coefficient in the Taylor rule implies a smaller value of ΔR_t. According to the money demand equation (25), smaller values of ΔR_t weaken the link between the nominal interest rate and money growth.

5.2 A new neoclassical model

In this section, we execute calculations like those described in section 5.1 but for a DSGE model with sticky prices, separability between consumption and real money balances, habit formation in households' preferences, price indexation by firms, and a unit root in technology. This type of model is said by Goodfriend and King (1997) to represent a New Neoclassical Synthesis. Related models have been studied extensively by Woodford (2003).

We continue to assume that the central bank uses either a money-growth rule or a Taylor interest-rate rule. However, now the policy instrument will respond to the output gap rather than to output growth, as well as to inflation.

5.2.1 The economy

The structure is:

$$\pi_t = \beta\left(1-\alpha_\pi\right)E_t\pi_{t+1} + \beta\alpha_\pi\pi_{t-1} + \kappa x_t - \frac{1}{\tau}e_t \tag{30}$$

$$x_t = \left(1-\alpha_x\right)E_t x_{t+1} + \alpha_x x_{t-1} - \sigma(R_t - E_t\pi_{t+1}) + \sigma\left(1-\xi\right)\left(1-\rho_a\right)a_t \tag{31}$$

$$\Delta m_t = \pi_t + z_t + \frac{1}{\sigma\gamma}\Delta x_t - \frac{1}{\gamma}\Delta R_t + \frac{1}{\gamma}\left(\Delta\chi_t - \Delta a_t\right) \tag{32}$$

$$\tilde{y}_t = x_t + \xi a_t, \quad \Delta y_t = \tilde{y}_t - \tilde{y}_{t-1} + z_t \tag{33}$$

where π_t, x_t, Δm_t and R_t are inflation, the output gap, nominal money growth and the short-term interest rate, respectively. The level of de-trended output is $\tilde{y}_t$ and

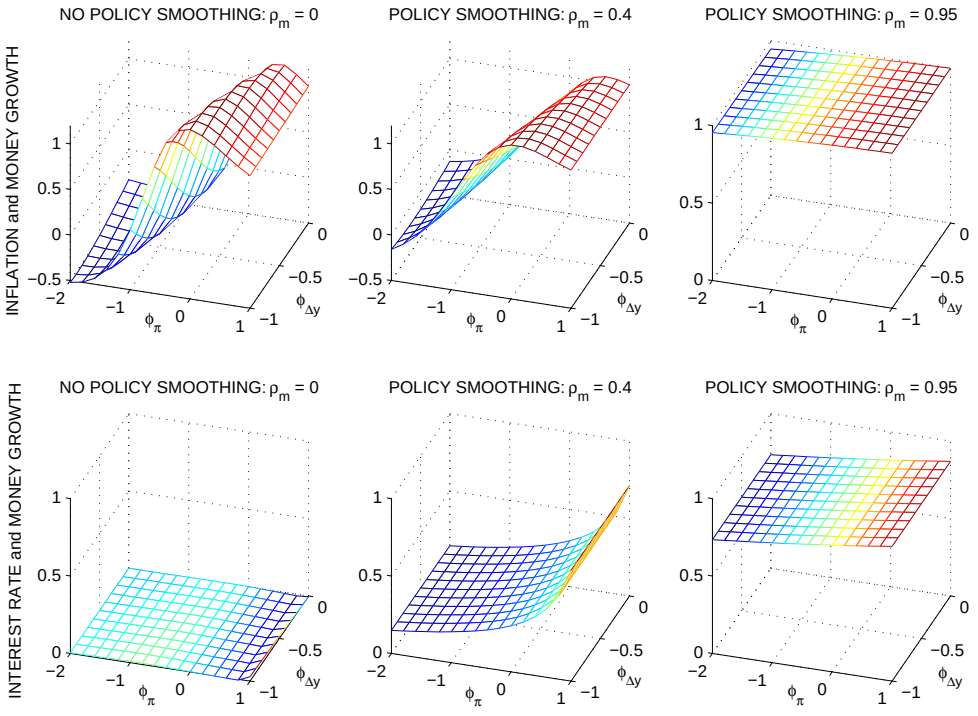

Figure 21: The effects of policy smoothing on the sums of coefficients $\tilde{h}(0)$ on the y axes, under alternative money supply rules.

Δy_t refers to output growth. The rate of technological progress is z_t. Equation (30) is an example of a new Keynesian Phillips curve, while (31) is the so-called new Keynesian IS curve. Equation (32) is a money demand equation of the type derived by McCallum and Nelson (1999) and Ireland (2003).

The discount factor is β, the parameter α_π is price setters' extent of indexation to past inflation, α_x captures the extent of habit formation. The coefficients κ and σ are the slope of the Phillips curve and the elasticity of intertemporal substitution in consumption. The price adjustment cost parameter in Rotemberg's (1982) quadratic function is τ, while ξ represents the inverse of the labor supply elasticity. The inverse of the interest elasticity of money demand is captured by γ.

The economy is exposed to four non-policy disturbances: a markup shock e_t, a demand shock a_t, a money demand shock χ_t, and a technology shock Z_t that evolve as

$$\begin{aligned} e_t &= \rho_e e_{t-1} + \varepsilon_{et}, \text{ with } \varepsilon_{et} \sim N(0, \sigma_e^2) \\ a_t &= \rho_a a_{t-1} + \varepsilon_{at}, \text{ with } \varepsilon_{at} \sim N(0, \sigma_a^2) \\ \chi_t &= \rho_\chi \chi_{t-1} + \varepsilon_{\chi t}, \text{ with } \varepsilon_{\chi t} \sim N(0, \sigma_\chi^2) \\ \Delta \ln(Z_t) &\equiv z_t = \varepsilon_{zt}, \text{ with } \varepsilon_{zt} \sim N(0, \sigma_z^2) \end{aligned}$$

All variables are expressed in log deviations from their steady state values. More details about the specification are to be found in Ireland (2004).

Unlike the model of section 5.1, there is no capital or capital accumulation here. The model generates persistence through its specification of the processes of the shocks and the backward looking dynamics appended to the Phillips curve and the IS curve.

5.2.2 Monetary policy

There are two types of monetary regime.

A money supply rule. Money growth adjusts smoothly in response to movements in inflation and the output gap. Unlike (28), money growth depends on the output gap rather than output growth.

$$\Delta m_t = \rho_m \Delta m_{t-1} + (1 - \rho_m)(\phi_\pi \pi_t + \phi_x x_t) + \varepsilon_{mt}, \varepsilon_{mt} \sim N(0, \sigma_m^2) \tag{34}$$

A Taylor rule. The short-term nominal interest rate is adjusted smoothly in response to movements in inflation and the output gap.

$$R_t = \rho_r R_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_x x_t) + \varepsilon_{Rt}, \quad \varepsilon_{rt} \sim N(0, \sigma_R^2) \tag{35}$$

Table 4: Parameter values

economy		*shocks*		*policy rules*	
β	0.99	ρ_e	0.5	ψ_π	$[0, 3]$
α_π	0.5	ρ_a	0.5	ψ_x	$[0, 1]$
α_π	0.5	ρ_χ	0.7	ρ_r	0.7
κ	0.1	σ_e	0.5	σ_r	0.4
τ	6	σ_a	0.5	ϕ_π	$[-2, 1]$
σ	0.1	σ_χ	0.4	ϕ_x	$[-1, 0]$
ξ	0.15	σ_z	0.5	ρ_m	0.7
γ^{-1}	0.15	σ_{ss}	0.2	σ_m	0.4

5.2.3 Parameters

We report parameters in table 4. We fix them so as to fall roughly in the middle of the ranges of available estimates. For most of the parameter space associated with $\psi_\pi < 1$ under the Taylor rule, the model implies equilibrium indeterminacy. Under the money supply rule, the configurations of policy parameters always imply determinacy.

5.2.4 The quantity theory across monetary regimes

Figures 22 and 23 report results that are broadly similar to those obtained using the Lucas model of section 5.1. But three differences are worth noting. First, the variation in the coefficients describing monetary policy is such that the model can attain the entire [0,1] interval for $\tilde{h}_{R,\Delta m}(0)$ under both policy rules. Second, a larger policy response to the output gap is associated with significantly larger $\tilde{h}(0)$ values. Third, the move from indeterminacy to determinacy in figure 23 is associated with a somehow more abrupt change in the $\tilde{h}_{\pi,\Delta m}(0)$ values across the boundary.

Notice that small values of ϕ_x are associated with low values of $\tilde{h}(0)$ for the nominal interest rate. This outcome emerges because under a money growth rule, the nominal interest rate is pinned down by the money demand equation (32). In the new neoclassical model money balances depend upon x_t, and therefore a policy that does not stabilize the output gap induces weaker comovements between money and the nominal interest rate.

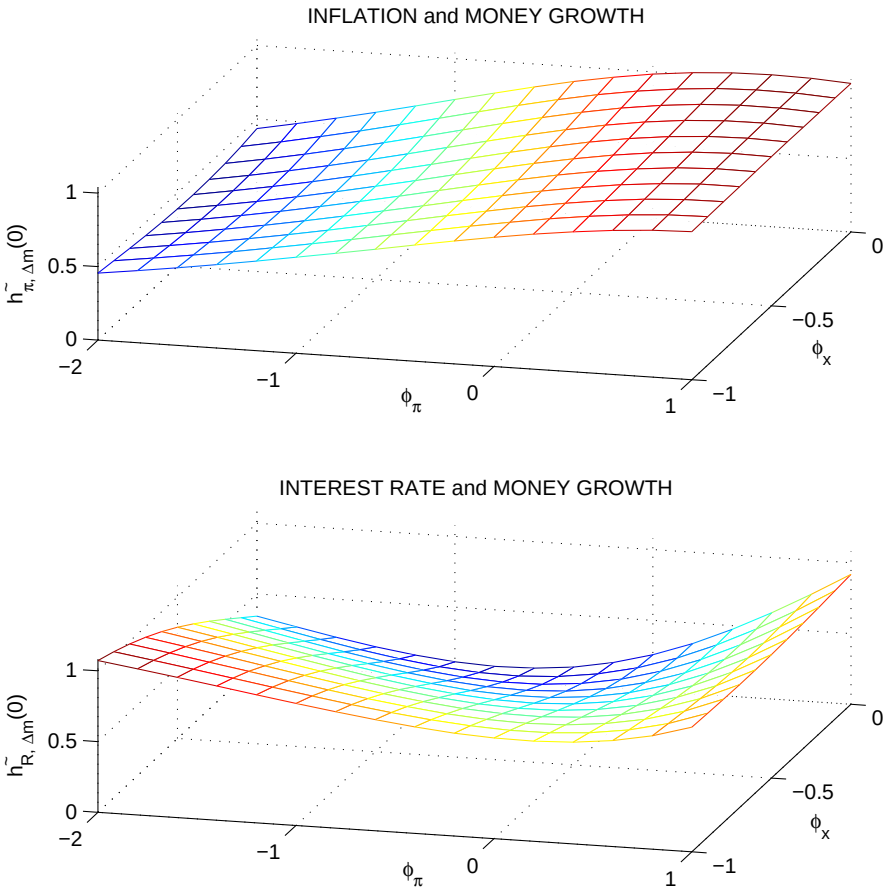

Figure 22: Sums of weights $\tilde{h}(0)$ in new neoclassical model under money supply rule.

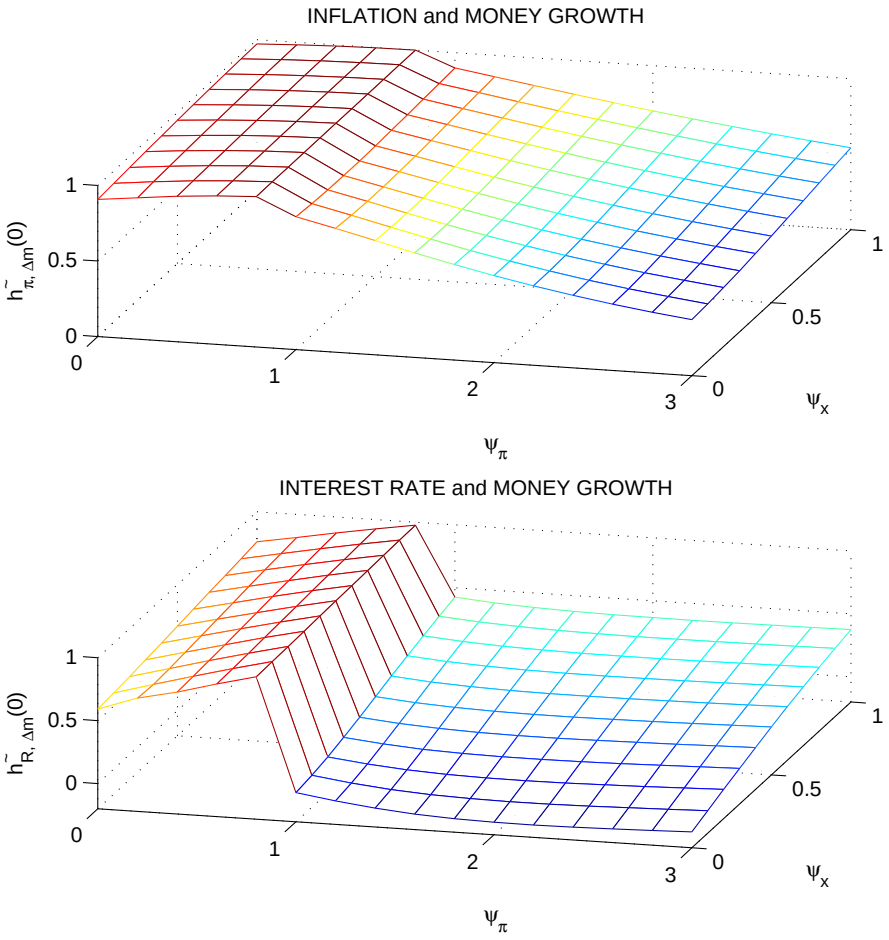

Figure 23: Sums of weights $\tilde{h}(0)$ in new neoclassical model under a Taylor rule.

As for the outcomes for persistence and volatility of money growth within the determinacy region, the findings for the new neoclassical model are qualitatively and quantitatively similar to the section 5.1 findings for the Lucas (1975) model: low values of the long-run response coefficients in figures 22 and 23 are associated with low persistence and low volatility. In the presence of equilibrium indeterminacy, which occurs under a Taylor rule only for $\psi_\pi < 1$, the persistence and volatility of money growth in the new neoclassical model are larger than the persistence and volatility in the Lucas model for values of ψ_π close to but below 1.

5.2.5 The roles of non-policy shocks

In this section, we explore whether, under a Taylor rule, alterations in the process for the non-policy shocks in the new neoclassical model are capable of generating time profiles for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ like those that emerge in the U.S. data.[16] To this end, we study the effects of changing parameters that govern the degrees of persistence and the variances for all shocks. We report outcomes only for those alterations that we find to be associated with substantial changes in the low-frequency relationships between inflation and money growth and between the nominal interest rate and money growth.

In figure 25, we move the autoregressive parameters in the process for the supply shock, ρ_e, from 0.5 to 0.9 while keeping all other coefficients to the values in table 4. A comparison with the plots in figure 23 reveals that more persistent supply shocks are typically associated with higher values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$. It should be noted, however, that high values of ρ_e are neither necessary nor sufficient to generate high values of the $\tilde{h}(0)$'s. In fact, an activist monetary policy stance that assigns a sufficiently large weight to inflation (i.e., ψ_π above 1.5) and little or no weight to the output gap response (i.e., ψ_x close to zero) is capable of generating values for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ that are substantially lower than one.

A similar finding emerges from figure 26, where we increase the standard deviation of the supply shocks, σ_e, from 0.5 to 2, while keeping values for all other parameters unchanged. The low-frequency relationships now seem less influenced by monetary policy relative to figure 23, with the notable exception of the policy rules associated with low values of ψ_x and $\psi_\pi > 1$. Our findings suggest that while a change in the process for the supply shocks (in the form of higher persistence and/or higher variance) may have helped to account for the high values of the sums of distributed lags observed in U.S. data during the original period studied by Lucas (1980), a monetary policy response that placed sufficient weight on inflation relative to the

[16] Under a money growth rule, we obtained results similar to those reported in this section.

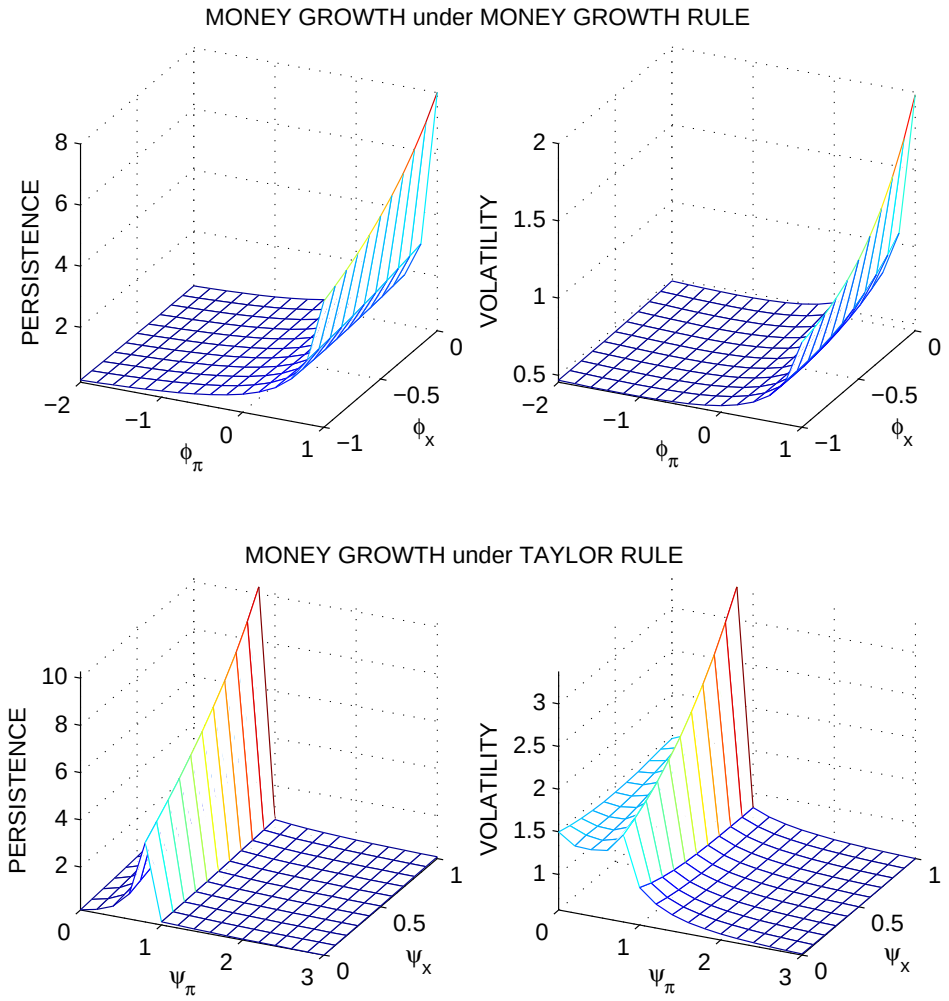

Figure 24: Volatility and persistence of money growth in the new neoclassical model

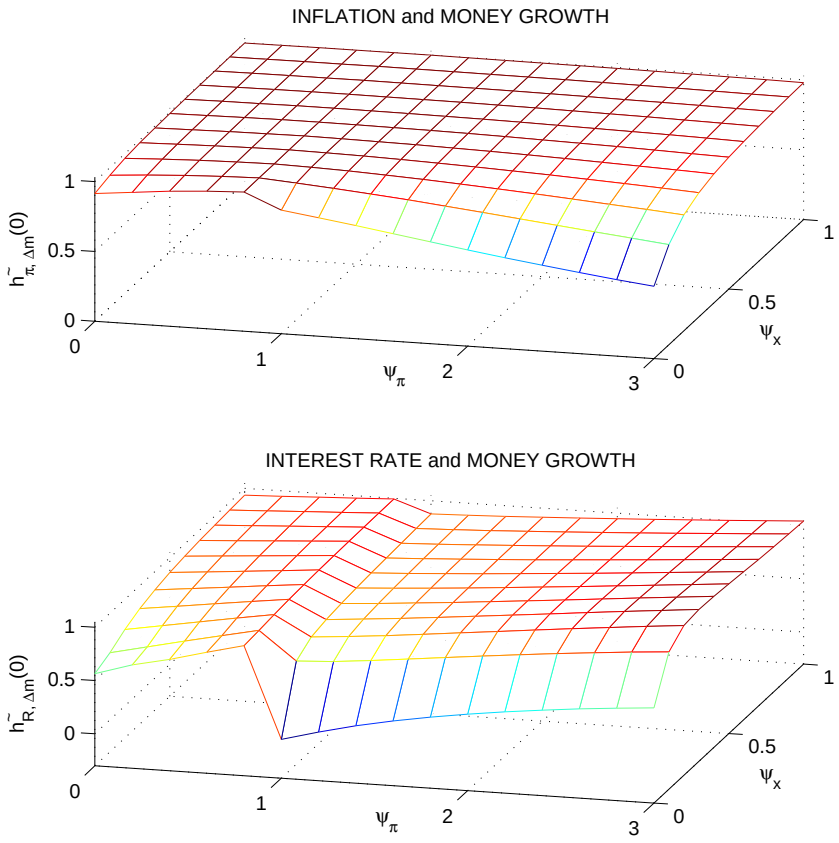

Figure 25: Sums of weights $\tilde{h}(0)$ in new neoclassical model under a Taylor rule with highly persistent supply shocks.

output gap could have prevented the U.S. from attaining realizations of these large values for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$.

The results in this section are similar to findings that Woodford (2007) and Benati (2007) obtained by using versions of the new neoclassical model that differ from ours. Woodford (2007), for instance, showed that in a model where the low-frequency variation in money growth is mostly driven by trend inflation (defined as a unit root process for the central bank's inflation target), the slow-moving components of inflation and money growth tend to be highly correlated.[17]

We conclude that the sources of variation in the process for money growth, as exemplified by shocks in a money demand equation like (32), are crucial for identifying and interpreting the low-frequency associations between nominal variables. In particular, if the variances of the determinants of the low-frequency components of inflation are sufficiently larger than the variances of the determinants of the low-frequency components of output growth and the nominal interest rate, then an econometrician would get higher values for the sum of distributed lag coefficients for inflation on money growth. We have shown that within two DSGE models monetary policy can strongly influence the relative variances of the slow-moving components of inflation, output growth, and the nominal interest rates, and through those avenues it can strongly influence the slow-moving components of money growth. Using a historical sample similar to ours, Ireland (2008) provides a further example in which instability in the long-run money demand can be used to infer the stance of monetary policy.

6 Inferring the monetary policy stance from $\tilde{h}(0)$

Section 5 described how low-frequency manifestations of the quantity theory depend on the stance of monetary policy. In this section, we surrender to the temptation to invert the mapping from policy rule parameters to sums of weights and draw some inferences about prevailing policy rules from our estimates of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$.

We select two years, 1973 and 2005. In figure 13, the median estimates of the sums of the distributed lag coefficients from the time-varying VAR are approximately 0.9 for both $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ in 1973, but they are around 0.2 in 2005. A comparison with figure 16 (17) reveals that, according to Lucas's (1975) neoclassical model, the values for 1973 can have only been generated by weak policy responses to

[17] In the presence of *both* highly persistent *and* highly volatile supply shocks, of the magnitude considered in this section, the sums of weights $\tilde{h}(0)$ are close to one, virtually independently of monetary policy parameters. The stability that the low-frequency relationships would display across time under this scenario, however, is at variance with the *instability* of $\tilde{h}(0)$ in U.S. data documented in sections 3 and 4.

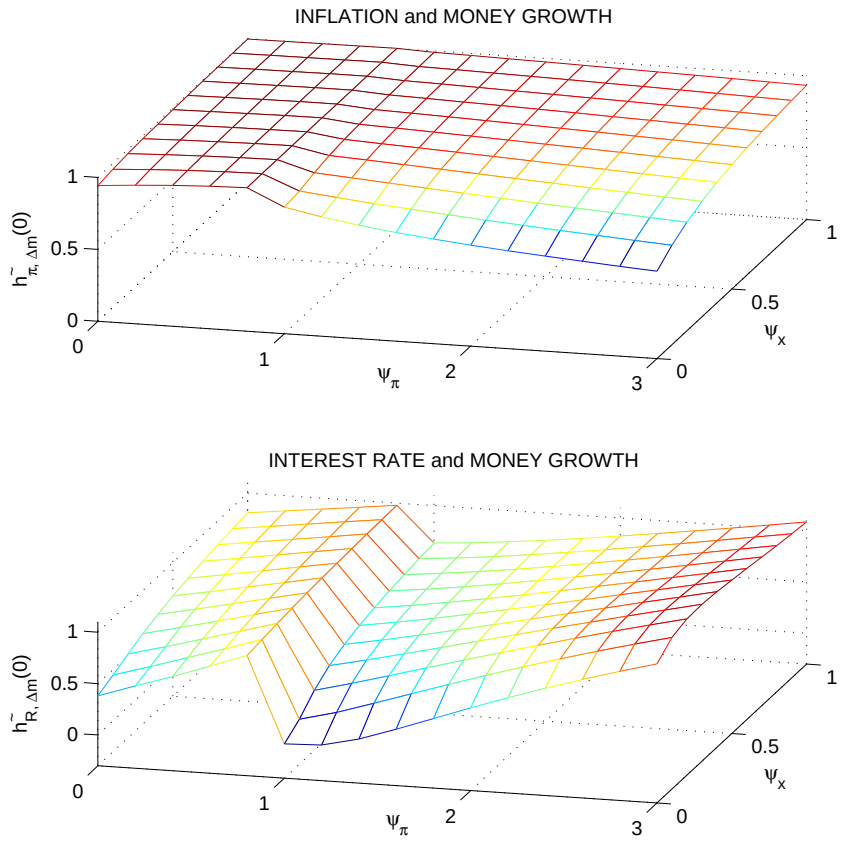

Figure 26: Sums of weights $\tilde{h}(0)$ in new neoclassical model under a Taylor rule with highly volatile supply shocks

inflation, as measured for instance by values of ϕ_π (ψ_π) close to 1 (0.8) in the money supply rule (interest rate rule). Very similar values for ϕ_π and ψ_π can be backed out using the results for the new neoclassical model in figures 22 and 23.

As for 2005, values of 0.2 for both low frequency relationships can be generated in the neoclassical model by a strong anti-inflationary monetary policy stance (i.e. ϕ_π close to -2), but only in the presence of large Mundell-Tobin effects for $\tilde{h}_{R,\Delta m}(0)$ in figure 19 and small values of the variance of the monetary policy shock for $\tilde{h}_{\pi,\Delta m}(0)$ in figure 20. In the new-neoclassical model parameterized according to table 4, estimates of the sum of the distributed lags coefficients around 0.2 require configurations of the policy rule parameters that attach large weight to the inflation response (i.e. ϕ_π close to -2 and ψ_π close to 2) as well as small or no weight to the output response (i.e. $\phi_{\Delta y}$ and ψ_x close to 0) in figures 22 and 23.

We view these results as tantalizing invitations to extend this study by bringing to bear evidence from all frequencies to estimate the evolution of monetary policy rules. We leave this work to a sequel to this already long paper.

7 Concluding remarks

A long-standing, but flawed, tradition in macroeconomics has regarded low-frequency quantity theory relationships as policy-invariant features of macroeconomic models that embody long-run neutrality propositions. We say 'flawed' for reasons that Lucas (1972), Sargent (1971), and King and Watson (1994, 1997) described in the context of econometric tests of the natural unemployment rate hypothesis and that Whiteman (1984) analyzed in the context of the quantity theory of money: low-frequency properties of two-sided infinite projections are themselves functionals of government policies.[18]

To study how Lucas's (1980) low-frequency manifestations of the quantity theory have evolved, we have estimated time-invariant and time-varying VARs for U.S. data spanning 1900-2005. We computed equilibria of two DSGE models for different monetary policies to study how the low-frequency relationships between inflation and money growth and the short-term interest rate and money growth should vary with monetary policy. Our results show how the low-frequency co-movements between nominal variables that Lucas featured convey information about the stance of monetary policy. In particular, Lucas's low-frequency manifestations of the quantity theory are (more) less likely to emerge when the monetary authorities respond (in)sufficiently to inflationary pressures.

[18] Also see Sargent (1987, ch. XI).

A CPI and M1 data

In this appendix, we reproduce the calculations in Lucas (1980) using his favourite measures of money (M1) and prices (CPI), over the sample 1955-2005, and the subsample 1975-2005.[19] In table A, we report the full set of low frequency relationships for different values of β in (1).

Table A: Regressions on filtered data, Lucas' measures of money and prices

Data as in Lucas (1980) - m: M1; p: consumer price index. R: federal funds rate

	π on Δm				Δm on π				R on Δm				Δm on R			
β	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*	*.95*	*.8*	*.5*	*0*
full sample	.44	.36	.21	.13	.57	.49	.34	.24	.58	.45	.24	.14	.51	.43	.27	.17
1955-75	1.39	1.22	.74	.46	.54	.49	.36	.28	1.15	**1.00**	.58	.30	.74	.66	.41	.24
1976-05	.30	.24	.13	.08	.58	.48	.31	.20	.46	.33	.17	.09	.47	.38	.22	.13

Note: numbers in bold are not statistically different from one at the 10% significance level, HAC covariance matrix

[19] The official definition of M1 was broaden in 1980 to include nonbank checkable deposits.

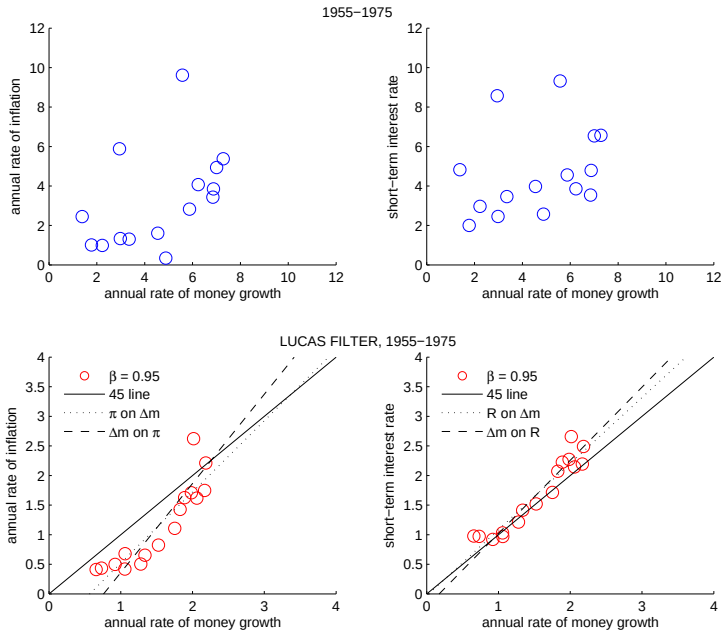

Figure 27: Lucas' filter over his sample, 1955-1975, using M1 and CPI.

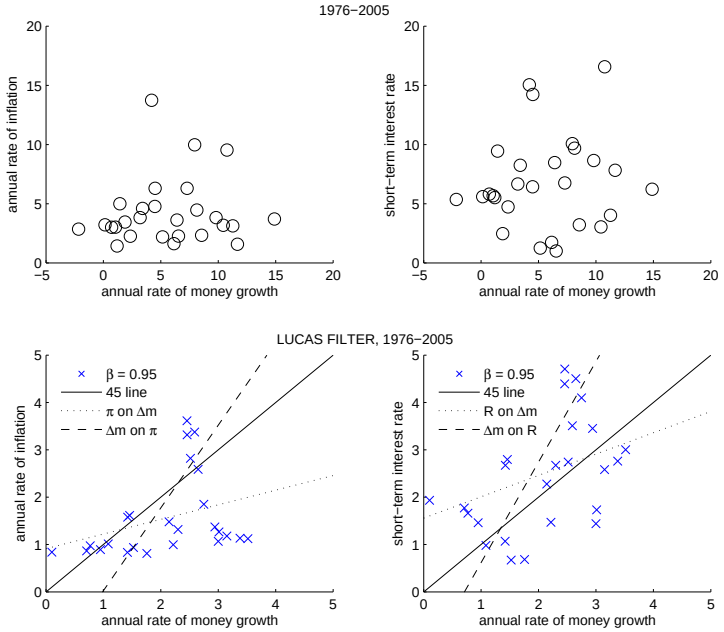

Figure 28: Lucas' filter over 1976-2005, using M1 and CPI.

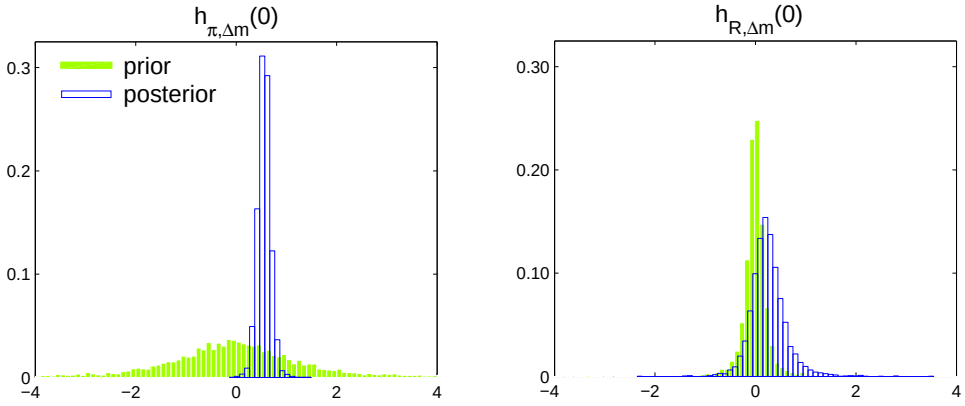

Figure 29: Prior and posterior distributions of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ based on the multivariate time-invariant VAR, 1900-2005.

B Priors on the $\tilde{h}(0)$'s from the time-invariant VAR

In specifying our priors over the VAR coefficients and innovation variances, our intention was to make them relatively uninformative about $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$. In this appendix, we map the priors for the coefficients and innovation covariances of the time-invariant VAR into implied prior distributions for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$. In particular, we (i) draw from the prior distributions of the VAR parameters, (ii) compute the low-frequency slope coefficients by applying formula (4) to the realized values of the VAR coefficients, and (iii) repeat steps (i) and (ii) 4000 times. The resulting prior distributions for the $\tilde{h}(0)$'s are reported in figure 29 together with the posterior distributions. Evidently, the posteriors for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ differ markedly from the priors, indicating fulfillment of our intention not to let a prior dominate a likelihood in its influence over a posterior. Note that both posteriors are shifted to the right relative to the priors, and that while the posterior for $\tilde{h}_{\pi,\Delta m}(0)$ is considerably tighter than the prior, the posterior for $\tilde{h}_{R,\Delta m}(0)$ is more diffuse. The prior (posterior) median for the $\tilde{h}_{\pi,\Delta m}(0)$ distribution is 0.03 (0.56), while the 16^{th} and 84^{th} percentiles are -1.35 (0.44) and 1.35 (0.67) respectively. As for $\tilde{h}_{R,\Delta m}(0)$, the prior (posterior) median is 0.00 (0.23), while the 16^{th} and 84^{th} percentiles are -0.17 (-0.03) and 0.16 (0.53).

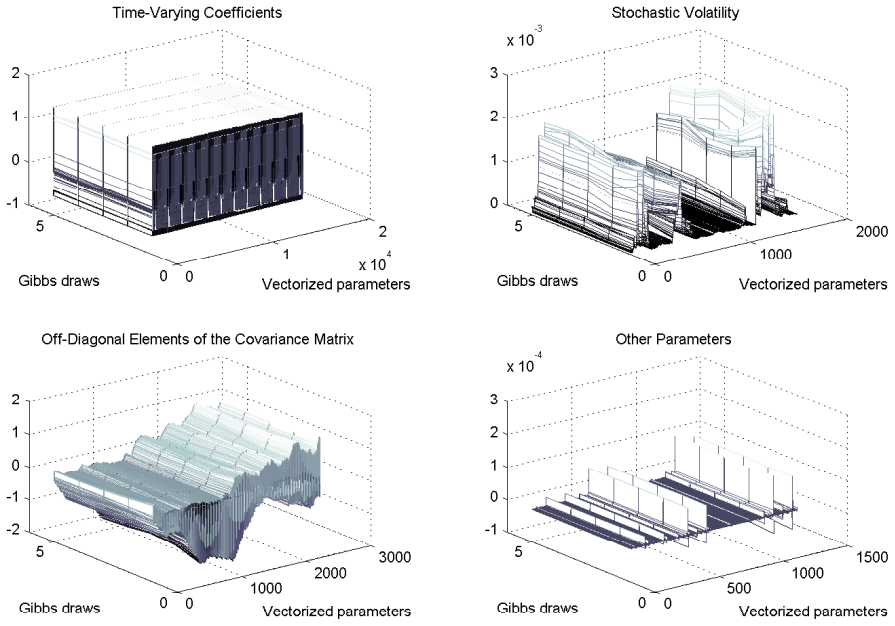

Figure 30: Posterior means of key parameters of the time-varying VAR

C Convergence

In figure 30, we plot the posterior means of key model parameters. These statistics are computed recursively as the average for every 20th draw of the retained repetitions of the Gibbs sampler. The figure reveals that the fluctuations in the posterior means are modest, thereby providing informal evidence in favour of convergence.

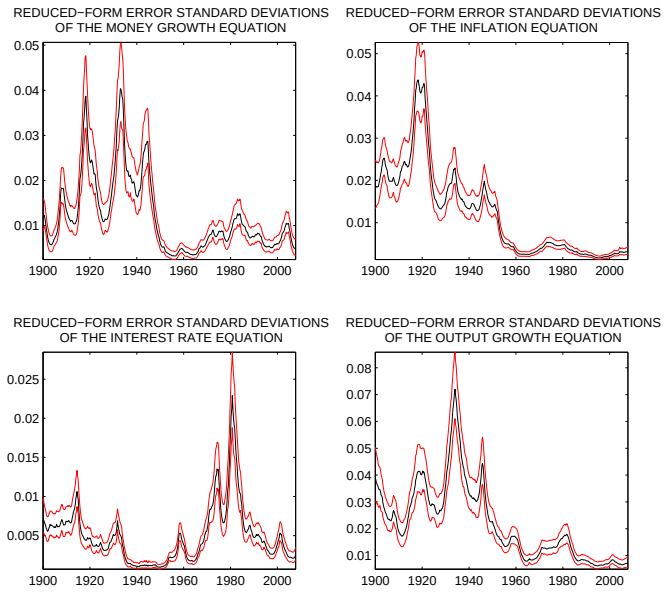

Figure 31: Standard deviations of the VAR reduced-form errors

D Volatility statistics from time-varying VAR

In figure 31, we report the evolution of the standard deviations of the VAR innovations computed as the square root of the elements in (17). A comparison of the time profiles in figures 14 and 31 reveal that in the late 1970s and early 1980s money growth, inflation, output growth and the interest rate displayed a significant surge in volatility whereas their innovations were relatively more stable. It should be noted, however, that during this episode, the volatility of the variables were eight (ten, four and three) times larger for money growth (inflation, the interest rate and output) than the volatility of the reduced-form errors. During the most volatile episodes of the first part of last century, in contrast, the ratios between variable and innovation volatilities were always below four. This implies that, during the second half of the sample, it has become more difficult for a statistical model such a VAR to produce forecasts for money growth, inflation, output growth and the interest rate which are more accurate than the forecasts produced by a naive model such as the unconditional mean. A similar picture emerges from Figure 32, which plots the stochastic volatility of each variable j computed as the square root of $h_{j,t}$.

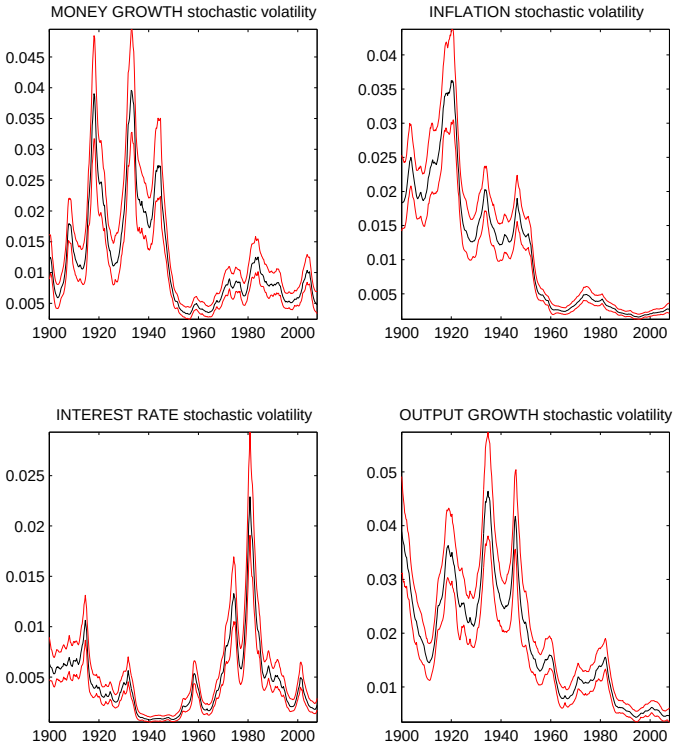

Figure 32: Square roots of the stochastic volatility

E Slopes using Lucas' filter and the sums of distributed lags coefficients

By applying formulas from section 2.3, this appendix evaluates how well b_f approximates $\tilde{h}(0)$. In figure 33, we report estimates of b_f and $\tilde{h}(0)$ obtained in the Lucas model under a Taylor rule. For expositional convenience the first column reproduces the charts in figure 17, which correspond to $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$, respectively. The second (third) column depicts estimates of b_f for a window width of n = 8 (100) quarters in Lucas' filter (see equation 1).[20]

The first row of figure 33 reveals that for inflation and money growth, b_f does a good job of approximating $\tilde{h}(0)$ for both n equal to 8 and n equal to 100, with the approximation being uniformly better for $n = 100$. Interestingly, very similar results for inflation and money growth are obtained using a money growth rule in Lucas model and using either a Taylor rule or a money growth rule in the new neoclassical model.

As for the low-frequency relationship between the interest rate and money growth, the approximation errors typically appear to be larger. A comparison of the bottom left panel with the other two panels in the second row suggests that the gap between b_f and $\tilde{h}(0)$ can be as large as .3 (.1) for n=8 (100) in the Lucas filter when $\psi_\pi > 1$. Under a money growth rule, however, b_f and $\tilde{h}(0)$ become very close again, indicating that, in the Lucas model, the monetary policy rule matters for the quality of the approximation. However, in the new neoclassical model, the maximum distance between b_f and $\tilde{h}(0)$ for the nominal interest rate and money growth is .2, independently on the monetary policy rule in place. The gap is smaller using a window of n=100 in the Lucas filter.

[20]The element $S_{yz}(\omega)$ of b_f in equation (5) is computed as the sum of the squared co-spectrum and the squared quadrature.

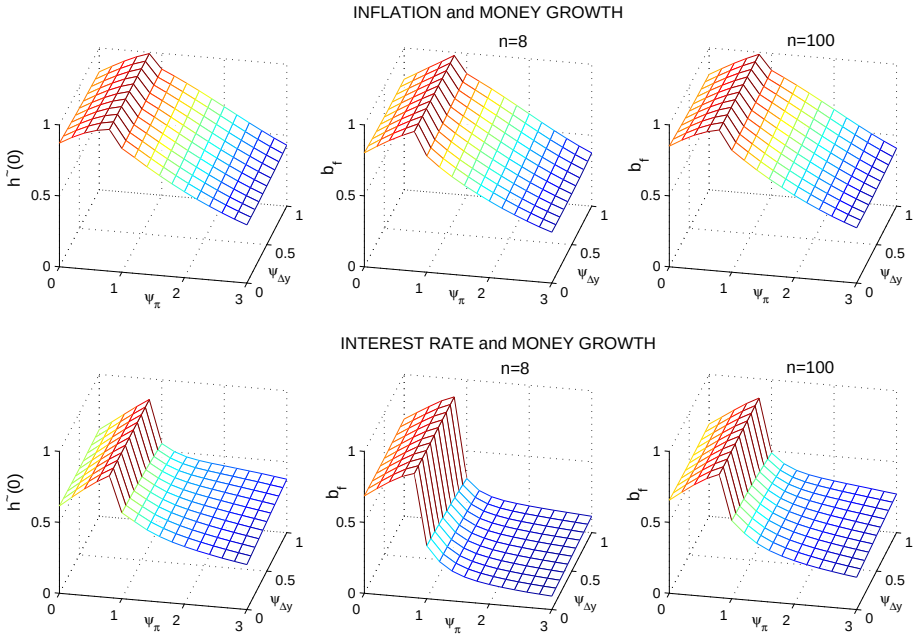

Figure 33: Lucas' slope estimator b_f vs. the sums of weights $\tilde{h}(0)$ in Lucas model under a Taylor rule.

F Other approximation issues

Whiteman indicated how approximation issues raised by Sims (1972a) can mean that low-order distributed lags can produce unreliable estimates of sums of coefficients. Similar issues can plague estimates of these sums constructed by using formula (4) in conjunction with cross-spectra estimated by applying a version of (10) to parameter estimates for a prematurely truncated VAR. To evaluate such approximation issues in the context of Lucas's model and VARs of the sizes that we have used in our empirical work, we have also calculated $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ by simulating the equilibrium of Lucas's model, and then computing VARs and the associated sums of coefficients displayed in figures 16 and 17.

We simulate $5,000$ times a period of 120 observations, which at quarterly frequency correspond to 30 years.[21] It should be noted that 30 years lie at the upper bound of the sample sizes used in the sub-period analysis of Section 3. For each simulation, we run a four-variate VAR in money growth, inflation, the short-term interest rate, and output growth. For each VAR, we compute the sums of distributed lag coefficients reported in figures 16 and 17, and then we take averages across the $5,000$ simulations. We report the deviations of these averages from the analytical $\tilde{h}(0)$ as a function of the coefficients in both policy rules.

In the Lucas model, the estimates of $\tilde{h}_{\pi,\Delta m}(0)$ based on the small sample VARs on simulated data appear to do a good job of approximating their population counterparts under both policy rules.[22] As for $\tilde{h}_{R,\Delta m}(0)$, the approximation errors are small only under a money rule. When monetary policy is conducted according to a Taylor rule, in contrast, the small sample estimates of $\tilde{h}_{R,\Delta m}(0)$ tend to lie above (below) the population values whenever ψ_π is below (above) 1.

To explore the sources of these deviations, in figure 35 we report the approximation errors on $\tilde{h}_{R,\Delta m}(0)$ for six different combinations of lag order of the VAR (i.e. $p = 2, 10, 20$) and sample size (i.e. $T = 400, 600$ observations, which at quarterly frequency correspond to 100 years -roughly the size of our full sample- and 150 years) in the context of the Lucas model under a Taylor rule.

Three results stand out. First, increasing the number of observations to 100 years (first column) and 150 years (second column), within the determinacy region, halves the approximation errors relative to the results from the 30 years simulated sample reported in the bottom right panel of figure 34. Second, increasing the order of the VAR to 10 lags (second row) and 20 lags (third row) further reduces the distance

[21] To reduce dependence from initial conditions, we run a pre-simulation of 100 periods, which we then discard.

[22] A similar result holds for the new neoclassical model

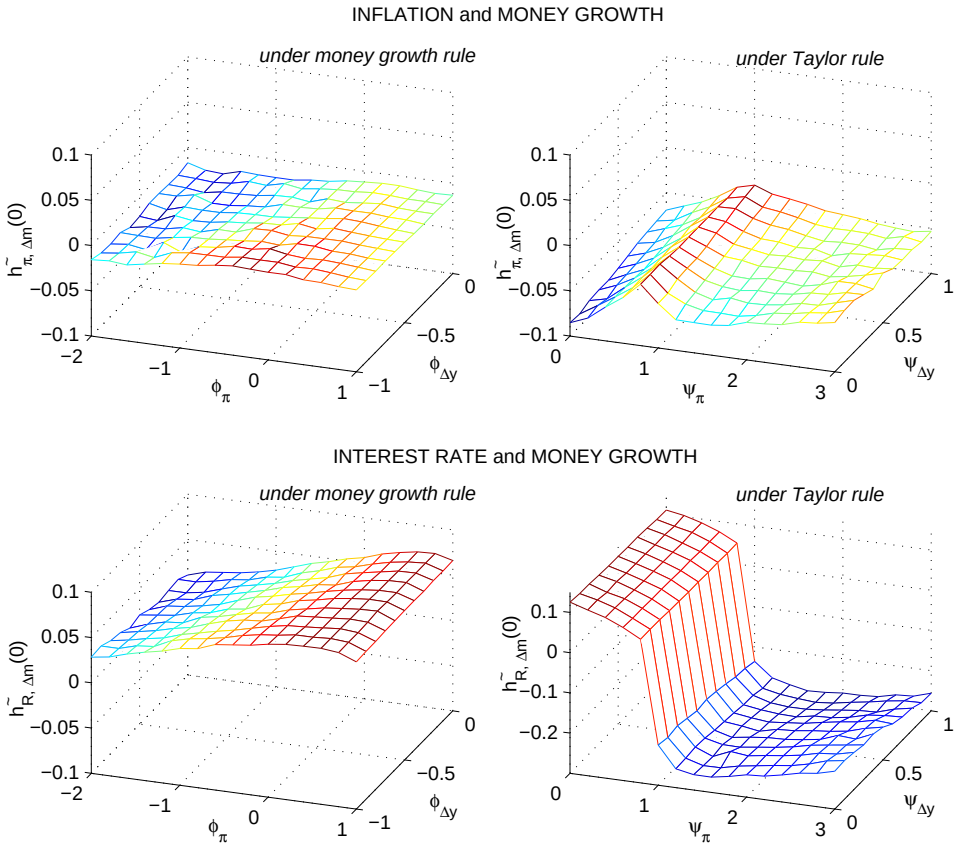

Figure 34: Approximation errors on $\tilde{h}(0)$s in Lucas model.

between estimated and population values of $\tilde{h}_{R,\Delta m}(0)$. Third, the largest accuracy gains from increasing the lag order occur in the indeterminacy region. Consistent with the findings in Benati and Surico (2008) for the new neoclassical model, a possible interpretation of the third result is that indeterminacy introduces a small MA component in the VAR(MA) representation of the DSGE model. Altogether, fitting a VAR of order ten on a sample of about 100 years produces, on average, approximation errors of the order 7e-02.

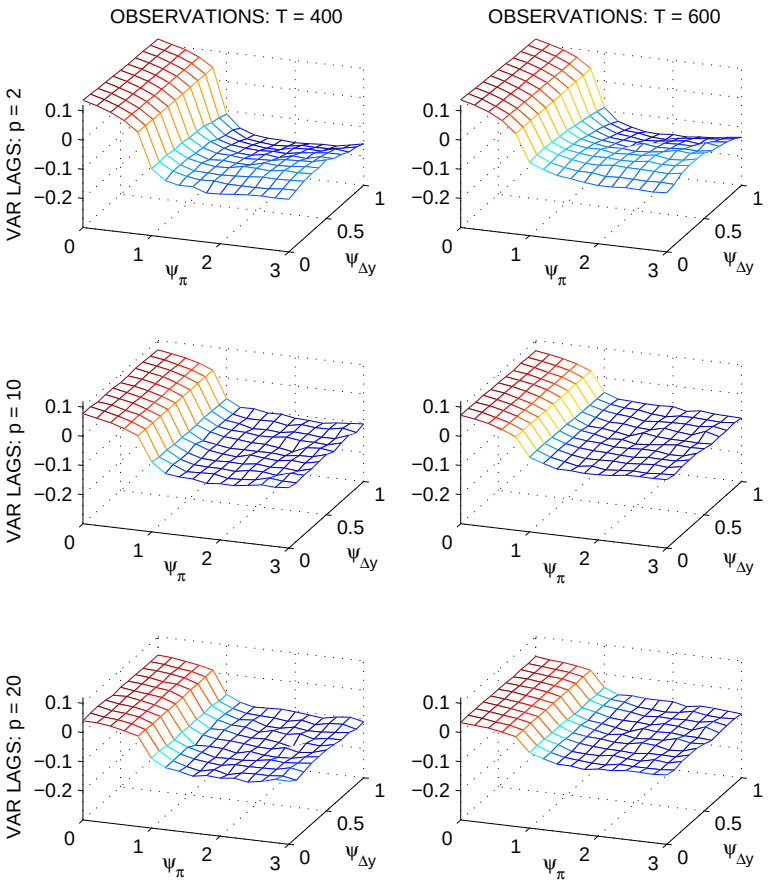

Figure 35: Approximation errors on $\tilde{h}_{R,\Delta m}(0)$ in Lucas model under a Taylor rule.

G Evidence on R^2: data and DSGE models

This appendix compares, on the one hand, the R^2's in equation (8) based on the time-varying and the fixed-coefficient VARs estimated on U.S. data for money growth, inflation, the short-term interest rate and output growth, and, on the other hand, the R^2's based on the Lucas model under both money growth and Taylor rules at parameter values recorded in table 3.

In figure 36, we note that both R^2 statistics computed on actual data seem characterized by an extent of time variation similar to the one of their $\tilde{h}(0)$ counterparts in figure 13. The amount of uncertainty, however, is so large that the probability distributions span most of the R^2 domain. Over the end of the 1970s, for instance, the values of 0.85 and 0.05 are both inside the 68% central posterior bands in the top panel as well as in the bottom panel of figure 36.

In line with the evidence presented in section 4, a fixed coefficient VAR over the full-sample, represented as straight blue lines, delivers estimates that are, in some years, significantly different from the estimates based on the time-varying VAR, especially for inflation and money growth.

Moving to the DSGE models, in figure 37, we vary the parameters of both policy rules in Lucas model to assess the extent of time variation in the R^2's observed on actual data implied by alterations of monetary policy. The patterns uncovered by this exercise resemble the patterns disclosed by figures 16 and 17, and the same arguments used in section 5 carry over to this appendix. We obtain similar results with the new neoclassical model.

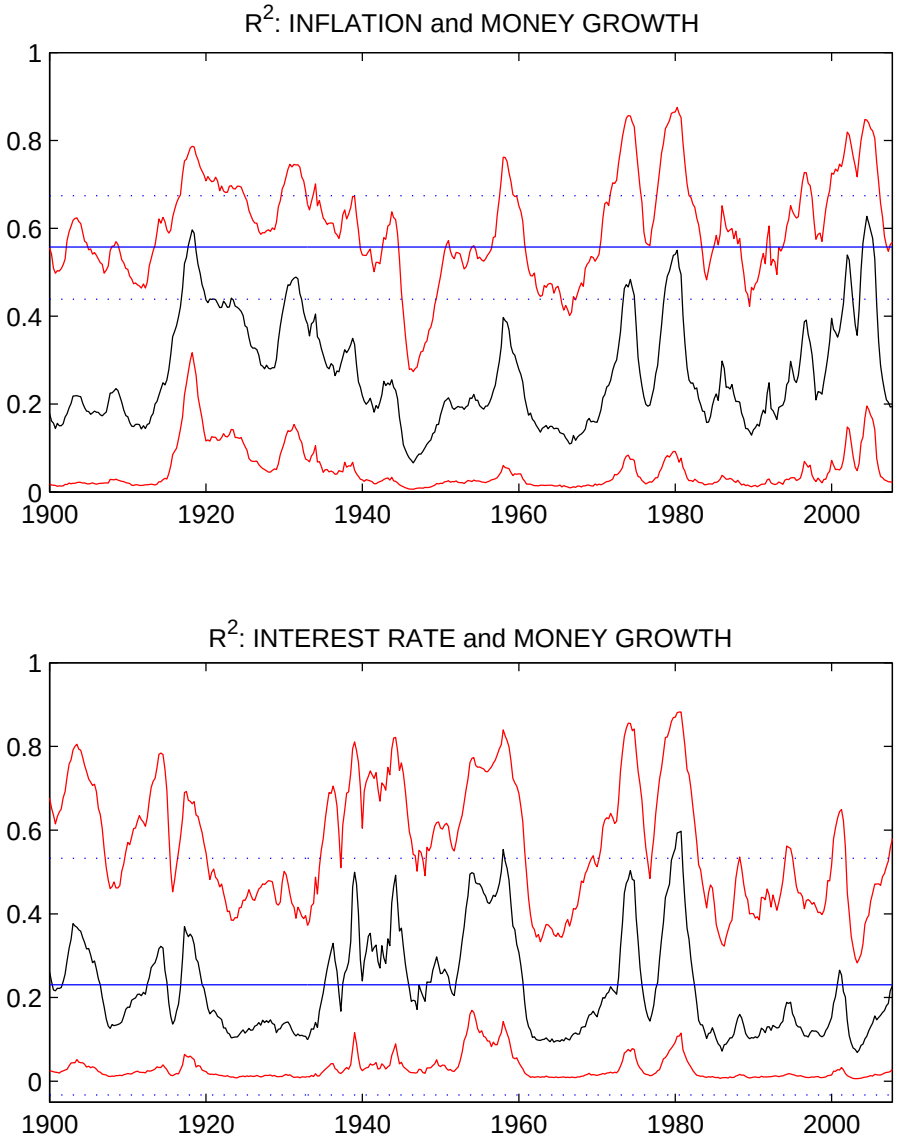

Figure 36: Median and 68% central posterior bands for R^2 based on a fixed-coefficient VAR over the full samples and a VAR with time-varying coefficient and stochastic volatility.

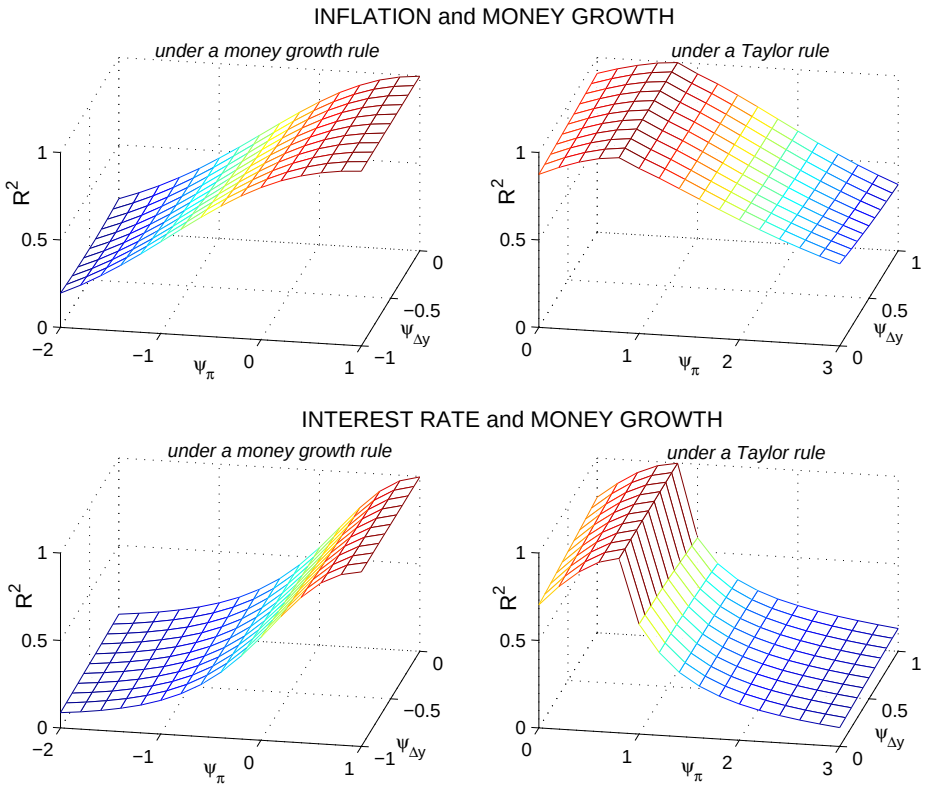

Figure 37: R^2 in Lucas model under money supply and Taylor rules.

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