**Optimal Managerial Compensation, Earnings Manipulation, and Manager Ownership**

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**Abstract**

The optimal management contract is derived in an environment in which a manager can influence the distribution of earnings through an unobservable effort decision, only the manager observes actual earnings, and the manager can engage in costly reported earnings manipulation. The manager's compensation is required to satisfy an ex ante participation constraint that reflects the cost of effort and an interim participation constraint that guarantees the manager non-negative rent (gross of ex ante effort costs) conditional on the manager having observed the firm's actual earnings. The optimal contract is shown to have several features: a manager who observes earnings in a range at the bottom of the distribution will earn zero rent (gross of the cost of effort), a manager who observes earnings above this range will earn positive net rents, and both under- and over-reporting is induced with over-reporting always corresponding to high earnings. We then study how endowing the manager with shares in the firm affects both the optimal contract and the expected profit of the owner. A general condition is derived that determines when giving the manager additional shares in the firm increases expected owner profit. We find that the optimal managerial compensation contract will contain both stock options whose value depends on the future performance of the firm, as well as bonuses paid on the current (and manipulated) reported firm earnings.

## 1. Introduction.

We consider the problem of an owner of a firm who wishes to hire a manager to run the business. Delegating authority to a manager creates a number of incentive issues. Moral hazard is the classic incentive problem but in recent years the press has reported on the role of earnings manipulation. Studies dating back to Holmström (1979) have shown that to moderate the incentive to shirk when manager effort is unobservable, it is important to offer the manager compensation that varies with firm earnings. However, in many firms actual earnings are observable only by the manager and, at some cost, the manager can manipulate the firm's accounting systems in order to misreport these earnings. In fact, the very incentives needed to induce high effort exacerbate the incentive for a manager to over-report earnings. The purpose of this paper is to derive the optimal manager compensation contract in an environment in which manager effort and firm earnings are unobservable to the owner but observable to the manager and the manager can falsify reported earnings at a cost. In contrast with previous studies, we assume that the manager has no financial resources with which to purchase an interest in the firm, and that the manager is able to quit the firm whenever she wishes to do so.

Our work is most closely related to Crocker and Slemrod (*RAND*, 2007), who derive the optimal ex ante individually rational contract under costly state (earnings) falsification and moral hazard. In that environment, an optimal contract entails bonuses paid to the manager which are increasing in the size of the earnings report, and the structure of the bonuses reflects an efficiency tradeoff between the effect such bonuses have on inducing higher levels of effort by the manager, on the one hand, and the incentives the bonuses generate for the falsification of earnings reports, on the other. While the ex ante individual rationality constraint permits full extraction of the manager surplus by the owner through the use of a lump sum transfer, one feature of the optimal contract is that, for some realized earnings, the manager may prefer to quit rather than continue with the firm. As in Crocker and Slemrod, we will consider the optimal contract under costly earnings falsification and moral hazard where the contract must be ex ante individually rational with respect to the manager's effort choice, but we will also require that the contract be interim individually rational with respect to the manager's earnings report. This latter requirement introduces a surplus extraction role for the optimal bonus arrangement, which substantially changes the nature of the optimal contracting problem.

The efficient balancing of moral hazard and adverse selection in the presence of interim individual rationality creates countervailing incentives of the type examined by Maggi and Rodriguez-Clare (1995). Moderating the hidden action problem requires the owner to share firm profit with the manager, which gives the manager the incentive to over-report earnings, while the extraction of managerial surplus in the presence of hidden information and interim individual rationality requires the owner to engage in differential rent extraction, which gives the manager the incentive to under-report earnings. We show that the optimal contract exploits these competing effects.

In the case where the manager is given no ownership stake in the firm, the optimal contract results in truthful earnings reports and zero manager gross rent (not accounting for the manager's effort cost) for actual earnings levels below a derived earnings threshold. Above this threshold, the manager over-reports earnings and earns positive gross rent. Alternatively, endowing the manager with ownership shares in the firm partially alleviates the moral hazard problem through the usual internalization channel, but it also increases the marginal rent the manager must earn to satisfy the incentive compatibility constraints and exacerbates the surplus extraction role of the bonuses in the optimal contract. With such partial ownership, the optimal contract exhibits the under-reporting of earnings and zero gross managerial rent below a certain earnings threshold. Above this threshold, the manager will continue to under-report earnings but will now earn a positive gross rent. Finally, there will be a second earnings threshold above which the manager over-reports earnings and earns gross rents that are increasing in the reported earnings. Thus, increasing the manager's ownership share reduces the extent of over-reporting induced by the optimal contract, and may actually encourage the under-reporting of earnings by the manager.

The key question is how expected owner profit varies with the manager's ownership share. We will derive a simple test to determine the relationship between expected owner profit and the manager's ownership share. It turns out that as long as the expected earnings distortion (the expected difference between reported earnings and actual earnings) is positive, the owner's expected profit is increasing in the manager's share. Since with no shares in the firm, the optimal contract induces either correct reporting or over-reporting of earnings, our analysis shows that it is optimal for the owner to endow a manager with an ownership stake. While this ownership stake reduces the incentive for over-reporting, it does not eliminate it. Contrary then to conventional wisdom, giving managers a stake in the firm does not create the incentive to over-report earnings but, rather, it actually *reduces* the incentive to over-report earnings.

**2. The Model.**

In this model there are two people who make decisions for a firm: an owner and a manager. The owner is responsible for setting managerial incentives and the manager is responsible for running the firm, which requires both an effort and the reporting of the firm's earnings to the owner. Conditional on managerial effort, $a$, the distribution of the firm's gross earnings, $x$, is given by the distribution $F(x|a)$ with strictly positive density $f(x|a)$ and support [0,1]. We assume that $F_a < 0$, so higher levels of manager effort shifts the distribution of earnings to the right in the sense of first order stochastic dominance.

Effort is costly to the manager and unobservable by the owner, and so that $a$ is a *hidden action.* Let $h(a)$ denote the manager's effort cost, and we assume that $h$ is strictly increasing, strictly convex, and that $h(0) = h^{/}(0) = 0$. The conditions on $h(0)$ and $h^{/}(0)$ imply that the manager incurs no fixed costs of effort, nor has a strictly positive initial marginal cost of effort that could result in the manager exerting zero effort in response to a range of positive incentive levels.

To induce the manager to choose a positive level of effort, the owner must offer an incentive contract which can compensate the manager in two ways: performance-based compensation, and endowing the manager with shares in the firm. To reflect the reality of most large corporations, we assume that only the manager observes the firm's true earnings, so that the value taken by $x$ is *hidden information*. This means the owner cannot contract on the earnings, $x$, directly but only on the earnings reported by the manager, which we denote as $R$.

Let $B(R)$ denote the manager's performance-based compensation and let $\alpha$ denote the share of the firm given to the manager. Reporting earnings, $R$, that differs from actual earnings, $x$, imposes falsification costs $g(R\text{-}x)$ on the manager as it requires the manager to devote resources to managing the accounting to make such a report credible. In general, one would expect the falsification costs to be strictly convex in $R\text{-}x$, strictly increasing in $|R\text{-}x|$, and minimized at 0 such that $g(0)=0$. These properties imply the manager incurs no cost to issuing a truthful earnings report, and that under-reporting and over-reporting earnings are costly. To simplify the analysis and to allow us to focus more directly on the features of the optimal contract, we assume quadratic falsification costs, so that $g(R-x) = (R-x)^2/2$.

The manager's utility given any value of α can be written as

$$\hat{V}(x,R,a;\alpha) = \alpha x + B(R) - g(R-x) - h(a). \tag{1}$$

Two comments are in order before proceeding. First, the risk neutrality of manager utility in the payments $\alpha x$ and $B$ would seem to imply that the first-best solution would be for the owner is to sell the firm to the manager by setting $\alpha = 1$, since doing so would internalize the effect of the manager's effort and earnings report choices on firm profits. Indeed, this is precisely the result in the analysis of Crocker and Slemrod, since ex ante individual rationality permits the manager to pay for the shares of the firm through a lump sum transfer. In our setting, however, such a contract would violate the interim individual rationality constraint because it would result in negative gross profit for the manager when earnings outcomes were low. Any payment for the ownership stake $\alpha$ must be extracted from the manager after earnings are reported, and must respect interim individual rationality.

Second, the fact that the shares are valued at $\alpha x$ implies that the earnings report does not fool the stock market. Investors are able to invert the manager's reporting strategy for valuation purposes even if they are unable to detect the actual falsification. Many firms include clawback provisions in their contracts that, if exercised, would require the manager to return ill-gotten compensation. Empirical evidence suggests that these clawback provisions are rarely invoked. One possible interpretation of the infrequency with which clawback provisions are used is that most investors are making the correct inferences about the value of the firm from the reported earnings. Alternatively, one can interpret the assumption that the manager's shares are valued at $\alpha x$ as meaning the manager is unable to liquidate the shares until some time in the future at which the true value of the firm is realized but it is too late for clawback provisions to be used.

The owner will present the manager with a compensation package ($\alpha$, $B(R)$). If the manager accepts the package, then the manager will choose an effort level, $a$. Following the effort investment by the manager, earnings will be realized, the manager will issue an earnings report, the owner will pay the manager $B(R)$, and the owner will earn profit of

$$\Pi(x,R,\alpha) = (1-\alpha)x - B(R). \quad (2)$$

This sequence of events is illustrated in Figure 1.

The owner's objective is to choose the indirect compensation ($\alpha$,$B(R)$) to maximize the expected value of $\Pi(x,R,\alpha)$ subject to several incentive constraints. (The term "indirect" refers to the fact that the performance-based term $B(\cdot)$ depends indirectly on the firm's actual earnings through the manager's earnings report.) Because the manager's ownership share is set before the manager chooses her effort and hence before earnings are realized, we can treat $\alpha$ as a parameter and derive the optimal compensation contract $B(R)$ for each value of $\alpha$. We will determine the optimal value of $\alpha$ in a later section. We refer to the contract which solves the owner's problem for each value of $\alpha$ as the optimal *conditional contract*. For each value of $\alpha$, a conditional contract induces an allocation that can be described by three components: an effort level, *a*, the level of manager utility, $\hat{V}$, and an earnings report, *R*, where the latter two depend on the firm's realized earnings, *x*.

We solve the owner's problem by invoking the Revelation Principle, which is a solution technique in which we recast the owner's problem as one in which the owner chooses a direct conditional contract instead of the indirect conditional contract, $B(R)$. Formally, for each value of $\alpha$, a direct conditional contract consists of three components that mirror the allocation structure of this problem: a level of managerial effort the owner would like the manager to choose, *a*, an earnings report, $R(\theta)$, and a compensation schedule, $B(\theta)$, where $\theta$ is the manager's report of his type, *x*.[1] To the extent that the optimal indirect contract consists of a compensation schedule $B(R)$ that induces earnings manipulation, it will show up in the optimal direct contract through the value of $R - x$.

By the Revelation Principle, we will restrict attention to direct contracts that induce the manager to choose the desired level of effort, *a*, and to report truthfully, so that $\theta = x$.[2]

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[1]To apply the Revelation Principle correctly in this model, the earnings report must be part of the contract. Because the earnings report has a direct effect on the manager's utility through the cost term, $g(\cdot)$, the correct application of the Revelation Principle does not allow one to restrict attention to truthful earnings reports. Instead, we need to distinguish between the manager's private information, *x*, and the earnings report made by the manager that is used to determine managerial compensation, $R(x)$. This last point was first made by Dye(1988) and then again by Gresik and Nelson (1994). Dye's paper has been misinterpreted within the accounting literature to imply that one cannot invoke the Revelation Principle in private information problems of this sort. As shown in Gresik and Nelson (1994), in the context of a multinational tax problem, what Dye's analysis implies is that one must include the earnings report as part of the contract. This is the approach taken in this paper, and the one taken by Crocker and Slemrod.

[2]Myerson (1982) refers to such mechanisms as "honest and obedient."

Therefore, a direct conditional contract can be described by an effort level, $a(\alpha)$, a reporting function, $R(x;\alpha)$, and an indirect utility level for the manager, $V(x,a;\alpha)$. Given these values, equation (1) may be used to recover the optimal transfer function $B(x,\alpha)$ associated with the direct conditional contract. While here we have noted explicitly the reliance of the contract on $\alpha$, we will for notational convenience drop explicit reference to $\alpha$ in these contract terms except where the clarification is helpful.

Because our model includes both moral hazard and adverse selection effects, any direct contract must satisfy several incentive compatibility and individual rationality constraints. Incentive compatibility generates four constraints, two of which apply to the manager's choice of $a$, and two of which apply to the manager's selection of $\theta$. The ex ante choice of effort means the manager will choose $a$ to maximize $E\hat{V}$, whereas the choice of a type report is made after learning $x$, so that the manager chooses $\theta$ to maximize $\hat{V}(x,R(\theta),a;\alpha)$.

Let $V(x,a;\alpha)$ denote the conditional indirect utility of the manager who optimally issues a truthful type report. That is,

$$V(x,a;\alpha) \equiv \hat{V}(x,R(x;\alpha),a;\alpha). \tag{3}$$

Truthful type reporting by the manager ($\theta = x$) requires that $V$ satisfy two constraints:

$$V_x = \alpha + g'(R(x)-x) \tag{4}$$

and

$$R'(x) \geq 0. \tag{5}$$

In order for $\theta = x$ to be optimal for the manager, the first order condition $\hat{V}_R \cdot R' = 0$ must be satisfied at $\theta = x$ for all $x \in (0,1)$. This implies that equation (4) follows from an application of the Envelope Theorem to (1), and that the manager will earn a marginal rent that covers the change in the value of her shares plus the change in her manipulation costs.

Inequality (5) is a second-order incentive compatibility condition. Totally differentiating $\hat{V}_R(x,R(x),a;\alpha) = 0$ with respect to $x$ implies $\hat{V}_{RR} \cdot R' + \hat{V}_{Rx} = 0$. Since $\hat{V}_{Rx} = g''(R - x) > 0$, the earnings report function, $R$, must be non-decreasing. Thus, an incentive compatible contract will associate higher earnings, $x$, with higher earnings reports, $R$.

The last two incentive constraints deal with the manager's choice of productive effort, $a$. The manager will choose to invest $a$ units of effort as long as

$$\partial EV(x,a,\alpha)/\partial a = 0 \tag{6}$$

and

$$\partial^2 EV(x,a;\alpha)/\partial a^2 \leq 0. \tag{7}$$

Equation (6) is the first order condition for the manager's choice of $a$, and inequality (7) is the associated second order condition.

The manager's contract will also need to satisfy two individual rationality constraints. As in Crocker and Slemrod (2007), any contract must satisfy ex ante individual rationality, so that

$$EV(x,a;\alpha) \geq 0. \tag{8}$$

No manager would accept a contract that violates (8). In addition to (8), we add the interim individual rationality constraint,

$$V(x,a;\alpha) + h(a) \geq 0. \tag{9}$$

This constraint captures the ability of the manager to quit after observing actual earnings, $x$, but before issuing an earnings report, $R$. Since the manager has already chosen $a$ prior to observing $x$, the effort cost, $h(a)$, is a sunk cost. It is the addition of this constraint that differentiates our model from that in Crocker and Slemrod (2007).

To highlight the role of (9), define the manager's gross (of effort costs) indirect utility as

$$W(x;\alpha) \equiv V(x,a,\alpha) + h(a). \tag{10}$$

Note that $W_x = V_x$ and that $W$ does not depend directly on $a$ since at the earnings report stage the effort choice is sunk. The effort choice, $a$, will affect $EW$ through the distribution $F$..

By using $V$ to substitute $B$ out of $\Pi$ and $W$ to substitute out $V$, the owner's problem can be written as choosing $(a,W(x),R(x))$ to

$$\max \mathrm{E}(x - g - W) \text{ s.t. } \begin{array}{ll} \text{a. } W_x = \alpha + g' \\ \text{b. } \partial \mathrm{E}W/\partial a - h'(a) \leq 0 \\ \text{c. } W \geq 0 \\ \text{d. } EW - h(a) \geq 0 \\ \text{e. } R'(\cdot) \geq 0 \\ \text{f. } \partial^2 EW/\partial a^2 - h''(a) \leq 0. \end{array} \tag{11}$$

Constraints (a) and (e) are incentive compatibility constraints that ensure truthtelling ($\theta = x$) by the manager in the direct revelation contract. Constraint (b) is the manager's first-order condition for the choice of effort, $a$, and constraint (f) is the manager's second-order condition. Constraint (c) is the interim individual rationality constraint, and (d) is the ex ante individual rationality constraint.

Constraint (11a) implies

$$W(x;\alpha) = W(0) + \int_{t=0}^{x} [\alpha + g'(R(t)-t)]dt \tag{12}$$

which, after integrating by parts yields

$$EW(x;\alpha) = W(0) + \int_{t=0}^{1} [\alpha + g'(R(t)-t)](1 - F(t|a)))dt, \tag{13}$$

$$\partial EW(x;\alpha)/\partial a = -\int_{t=0}^{1} [\alpha + g'(R(t)-t)]F_a(t|a))dt, \tag{14}$$

and

$$\partial^2 EW(x;\alpha)/\partial a^2 = -\int_{t=0}^{1} [\alpha + g'(R(t)-t)]F_{aa}(t|a))dt. \tag{15}$$

Note that, if $F(x|a)$ were convex in $a$, then $EW$ would be concave in $a$ for all contracts that did not induce under-reported earnings ($R(x) < x$), which is the situation encountered in Crocker and Slemrod. For contracts that may induce under-reported earnings, however, $EW$ need not be globally concave unless effort costs are sufficiently convex.

As is commonly done in these settings, we will proceed by solving a modified version of (11) in which constraint (e) is dropped, and then check at the end to make sure that (e) is satisfied. Call this modified problem (11'), which yields the Hamiltonian

$$\mathcal{H} = (x - g(R\text{-}x) - W)f + \phi(\alpha + g'(R\text{-}x))$$

where $\phi$ is the co-state variable, $W$ is the state variable, and $R$ is the control. Using (14) yields the Lagrangian

$$\mathcal{L} = \mathcal{H} + \tau W - \mu[(\alpha + g'(R\text{-}x))F_a + h'(a)f] + \lambda f(W - h) \tag{16}$$

where $\tau$(x) is the non-negative multiplier on the interim individual rationality constraint (11c), $\mu$ is the non-negative multiplier on the effort constraint (11b), and $\lambda$ is the non-negative multiplier on the ex ante participation constraint (11d).[3]

### 3. Informal Discussion of Results

Before proceeding to characterize formally a solution to (16), we will describe the nature of our primary result, how this problem relates to others in the extant literature, and the role of countervailing incentives in the optimal contract. Under a set of regularity conditions pertaining to the distribution function *F* which are specified in the next section, we demonstrate that the optimal reporting function, *R*, satisfies

$$R - x = \frac{(1-\lambda)(F-1)}{f} - \frac{\mu F_a}{f} \tag{17}$$

where λ is the multiplier associated with the ex ante individual rationality constraint (11d), and μ is the multiplier associated with the effort constraint (11b). As long as $\lambda < 1$, the first term on the right hand side is negative, and the second term is positive, so that an optimal reporting function may entail either over- or under-reporting of earnings depending on which effect dominates.

In the special case where $F_a = 0$, manager effort has no effect on firm earnings and the contracting problem reduces to the costly state falsification environment examined by Crocker and Morgan (1998). In this setting, the parties face the standard tradeoff between efficiency and surplus extraction that is commonly observed when contracting in the presence of hidden information. When the parties face only an ex ante participation constraint, the solution to the

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[3]The reason $\mu \geq 0$ is as follows. Replace the right-hand side of (11b) with $\beta \geq 0$. An increase in β increases the marginal cost of inducing any given *a* and hence reduces owner expected profit. If *V* denotes the owner's value function, then standard optimal control procedures imply $\partial V/\partial\beta = -\mu \leq 0$ (with strict inequality if $a > 0$). Thus, μ must be non-negative.

contracting party is to sell the firm to the manager ($\alpha = 1$) for a lump sum payment equal to the firm's expected profit and then pay a bonus that is uniformly zero in reported earnings. Since the ex ante participation constraint permits full extraction of managerial surplus through lump sum transfers without efficiency cost, it is straightforward to show that $\lambda = 1$ and, from (17), $R = x$.

If instead the contracting parties face only an interim individual rationality constraint, then $\lambda = 0$ and (17) reduces to $R - x = (F - 1)/f$. As long as $F$ satisfies the monotone hazard rate property, so that $\frac{d}{dx}\left(\frac{f}{1-F}\right) \geq 0$ as depicted in Figure 2, then earnings are under-reported, the amount of under-reporting is monotonically decreasing in $x$, and $R'(x) \geq 0$. Moreover, as long as $V_x \geq 0$, interim individual rationality is satisfied by setting the bonus schedule so that $V(0) = 0$, which results in the manager earning information rents that are increasing in actual earnings, $x$.[4] Thus, in the presence of interim individual rationality, the optimal contract reflects the tradeoff between efficiency and surplus extraction that is commonly observed in settings with adverse selection, and because of surplus extraction the manager under-reports earnings.

In the case where $F_a \leq 0$, so that an increase in (unobservable) managerial effort shifts the distribution of earnings to the right, the optimal contract now has a moral hazard component. When the contracting parties face only the ex ante participation constraint, we are in the Crocker and Slemrod environment in which the use of lump sum transfers permits the frictionless extraction of managerial surplus, so that $\lambda = 1$. Then (17) reduces to $R - x = -\mu F_a / f$ and, as

[4]Crocker and Morgan ensure the monotonicity of $V$ by assuming that $|g'| < 1$ and restricting their analysis to the case in which $\alpha = 1$. The problems encountered when $V$ is nonmonotonic are discussed below.

depicted in Figure 2, the optimal reporting function entails earnings overstatement by the manager. The optimal contract pays a bonus, *B*, to the manager which is increasing in the reported earnings, *R*. A bonus structure that is more sensitive to higher earnings reports gives the manager the incentive to take higher levels of the (privately) costly action, *a*, but also increases the returns to the overstatement of earnings. Thus, the efficient contract reflects an efficiency tradeoff between the benefits of incentivization and the costs of falsification.

The introduction of interim individual rationality adds a surplus extraction role to the optimal reporting contract and the associated bonus structure. Since the optimal contract in the Crocker and Slemrod setting violates interim individual rationality, it follows that $\lambda < 1$ and the optimal reporting function satisfies (17), which is depicted in Figure 3. The optimal contract results in both under- and over-reporting, depending on the actual level of earnings, reflecting a tradeoff between surplus extraction and efficiency. In addition, there is a technical problem in satisfying the interim individual rationality constraint since $V_x$ (and, hence, $W_x$ ) is necessarily non-monotonic for small values of $\alpha$. In the case of quadratic falsification costs, $W_x = 0$ implies that $R - x = -\alpha$ , as depicted in Figure 3. Thus, for the values of $R$ - $x$ implied by (17), it follows that $W_x$ is necessarily negative for $x < \hat{x}$, and positive for higher earnings values.

As a result, the introduction of interim individual rationality introduces countervailing incentives, which require an application of the approach developed by Maggi and Rodriguez-Clare (1995) to characterize an optimal contract. In a nutshell, the optimal contract will entail all managers with earnings below $\hat{x}$ earning zero profit and under-reporting earnings by the amount $\alpha$. For those managers with earnings between $\hat{x}$ and $x^+$, manager profit is increasing in *x*, and the amount of under-reporting is decreasing. Finally, for earnings levels above $x^+$, manager profit is increasing in actual earnings and managers over-report earnings.

We now turn to a formal derivation of our results.

## 4. The Optimal Conditional Contract: A Formal Characterization

In order to characterize a solution to (16), we must make several regularity assumptions regarding the behavior of the distribution function, *F*.

***Distribution Assumptions:***

a. $F(x|a)$ is strictly decreasing, convex and continuously differentiable in $a$ for all $x$ and for all $a \geq 0$ .

b. There exists M > 0 such that for all $x$ and for all $a$, $f(x|a) < M$ and $f_x(x|a) < M$.

c. $f_x(0|a) > 0$ for all $a \geq 0$.

d. $(F(x|a)-1)/f(x|a)$ is strictly increasing in $x$ for all $a \geq 0$.

e. $(F(x|a)-1)/f(x|a)$ is concave in $x$ for all $a \geq 0$ and $F_a(x|a)/f(x|a)$ is convex in $x$ for all $a \geq 0$.

Assumption (a) implies that higher manager effort induces a first-order stochastic improvement in the distribution of earnings (*F* decreasing in *a*) and results in diminishing marginal returns from effort (*F* convex in *a*). The convexity of *F* with respect to effort will help ensure (but not guarantee) that the first-order approach is valid. Assumptions (b) and (c) are technical assumptions adopted to simplify the several of the proofs. Assumption (b) restricts attention to densities that are bounded and have bounded first derivatives with respect to earnings. Assumption (c) requires that small but positive earnings are relatively more likely than zero earnings. Our proofs will indicate where these assumptions are used.

Assumption (d) is the standard monotone hazard rate assumption found in most adverse selection models, and is used to guarantee manager indifference curves exhibit the single-crossing property. For a family of distributions indexed by *a*, it will be satisfied, for instance, as long as $\partial f/\partial x > 0$ for all *a*, and example 1 (below) provides an example of one such family. In this paper, assumption (d) is sufficient to support single-crossing only at sufficiently low effort levels. Because of the moral hazard component of this problem, the manager's indifference curves can fail to exhibit the single-crossing property at high enough levels of effort. Finally, assumption (e) is a regularity condition that implies the single-crossing property will only be violated for high earnings levels.

Turning to the Lagrangean expression (16), term $\tau W$ is included because constraint (11a) reveals that *W* need not be strictly monotonic in *x* if the manager under-reports earnings (so $g' < 0$). In standard contract design problems when *V* is monotonic, one can replace the continuum of constraints represented by (11c) with a single constraint that sets either *W*(0) or *W*(1) equal to 0. Because the manager's indirect utility, *V*, may not be monotonic in *x*, the manager type that receives zero surplus (*W*) is endogenously determined. Introducing the term $\tau W$ formally accounts for this endogeneity.

The non-monotonicity is due to two countervailing incentives created by the moral hazard and adverse selection effects in the presence of an interim individual rationality constraint. The first incentive (moral hazard) comes through the ownership term, $\alpha$. Increasing $\alpha$ gives the manager a greater share of actual firm earnings and hence induces the manager to invest in higher effort. The second incentive (adverse selection) comes through the earnings report term, $g'$.

When the direct contract reflects incentives to over-report earnings ($R(x) > x$), marginal manipulation costs will be increasing in $x$. This means that the owner can pay the manager a rent either by increasing the manager's ownership share *or* by inducing more over-reporting of earnings. When the direct contract reflects incentives to under-report ($R(x) < x$), marginal manipulation costs will be decreasing in $x$. Now the ownership incentives and the under-reporting incentives work in opposite directions. These countervailing incentives give the owner the ability to combine increases in the manager's ownership share with incentives to under-report (via $R(\cdot)$) that result in zero marginal rent being paid to the manager. We will show that this type of countervailing incentive structure will play a key role in the optimal contract. Formally, the presence of the countervailing incentives means our analysis will employ the same techniques as found in Maggi and Rodriguez-Clare (1995).

***Proposition 1.*** *Given Assumptions a-c, and assuming quadratic falsification costs, if a conditional contract (a,W,R) satisfies*

$$\mathrm{R} - x = (\phi - \mu F_a)/f, \tag{18}$$

$$-(1-\lambda)f + \tau = -\phi' \text{ (almost everywhere)}, \tag{19}$$

$$a[\int_{t=0}^{1} (\alpha + g'(R(t)-t))F_a(t|a)dt + h'(a)] = 0, \tag{20}$$

$$\lambda(EW - h(a)) = 0 \text{ and } \lambda \geq 0, \tag{21}$$

$$\phi(0) \leq 0,\ \phi(1) \geq 0,\ \phi(0)W(0) = \phi(1)W(1) = 0,\ \tau \geq 0, \text{ and } \tau(x)W(x) = 0, \text{ and} \tag{22}$$

$$R'(x) \geq 0, \tag{23}$$

*then it is an optimal conditional contract.*

Proposition 1 is a translation of Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter

(1987) to the specifics of (11') with constraint (11e) added for completeness. Eq. (18) is the Euler equation and defines the optimal reporting function. The sign of the term $(\phi-\mu F_a)/f$ determines for which earnings levels the contract induces over-reporting and for which earnings levels the contract induces under-reporting. The co-state variable, $\phi$, will capture both the ownership and manipulation distortions in the contract. To determine the manipulation incentive (captured by $R - x$), one must subtract out the ownership effect, measured by the term $\mu F_a$. Thus, contracts that create strong incentives for the manager to invest in a larger amount of effort than she would otherwise correspond to a high value of $\mu$ and for a given value of $\phi$, a small manipulation incentive. Eq. (20) is the manager's first-order condition with respect to effort. Condition (21) represents the complementary slackness conditions with regard to the ex ante individuality constraint. The conditions in (22) are the transversality conditions that will help determine which actual earnings level correspond to zero manager rents.

The countervailing incentives allow for the possibility that the manager earns zero marginal rent over a range of earnings. To determine if such an outcome can be the result of an optimal contract, suppose the contract implies zero marginal rent for the manager on a non-degenerate interval or earnings, i.e., $W_x = 0$. Then (11a) implies $\alpha + g'(R\text{-}x) = 0$ or $R(x) - x = -\alpha \leq 0$ for all $x$ in this interval. For an incentive compatible contract to result in zero marginal manager rents, it must induce the manager to under-report earnings. Only in the case in which the manager owns no shares in the firm will incentive compatibility and zero marginal rents imply truthful reporting. Let $\hat{\phi}(x)$ denote the value of the co-state variable in this case when the countervailing ownership and manipulation incentives exactly offset each other. Thus, (18) implies

$$\hat{\phi}(x) = \mu F_a(x|a) - \alpha f(x|a). \tag{24}$$

For all $\alpha$, $\hat{\phi}(x) < 0$ for all $x \in (0,1)$. Eq. (24) defines a feasible co-state variable as long as it also satisfies (19) and (22). With $\tau \geq 0$, (19) implies that $\phi' \leq (1-\lambda)f$ which, in conjunction with (22) implies that[5]

$$(1-\lambda)(F - 1) \leq \phi(x) \leq (1-\lambda)F. \tag{25}$$

As long as $\hat{\phi}$ falls within this range defined by (25), an optimal conditional contract can induce zero marginal rents for a range of earnings. In general, $\hat{\phi}$ will satisfy (25) for earnings below a level we denote by $\hat{x}$. For earnings above $\hat{x}$, $\hat{\phi}$ will fall below $(1-\lambda)(F-1)$. Exploiting the countervailing incentives by setting $\phi = \hat{\phi}$ for $x \leq \hat{x}$ and setting $\phi=(1-\lambda)(F-1)$ for $x > \hat{x}$ results in the reporting function from (18) of

$$R(x,a;\alpha) = \begin{cases} x - \alpha & \text{if } x \leq \hat{x} \\ x + \dfrac{(1-\lambda)(F(x|a) - 1) - \mu F_a(x|a)}{f(x|a)} & \text{if } x > \hat{x} \end{cases} \tag{26}$$

where the value of $\hat{x}$ is endogenous and calculated as part of the optimal contract.

For $\alpha = 0$, (26) gives the manager the incentive to report low earnings truthfully and over-report high earnings. For $\alpha > 0$, (26) gives the manager the incentive to under-report low earnings and over-report high earnings. Truthful reporting when $\alpha > 0$ will only occur at $x = 1$ and at one other earnings level greater than $\hat{x}$. In addition for all $\alpha$, the manager earns zero rent

[5] Integrating both sides from 0 to $x$, and noting that $\phi(0) \leq 0$ from (22), yields the left inequality, while integration of both sides from $x$ to 1 and noting that $\phi(1) \geq 0$ from (22) yields the right hand inequality.

($W$ = 0) and not just zero marginal rent ($W_x$ = 0) when $x < \hat{x}$ and positive rent when $x > \hat{x}$. Incentives that induce under-reporting are attractive to the owner because they reduce the manager's information rent but they also reduce the manager's marginal effort incentive. By adjusting α, the owner can control the balance between these countervailing effects.

*Example.* Let $F(x|a) = (1-a)x + ax^2$, $h(a) = a^3/3$, α = 0, and λ = 0.[6] Figure 4 plots $F$, $F$ - 1, and $\hat{\phi}$. For all distributions, $F$ and $F$ - 1 are increasing functions while $\hat{\phi}$ must be decreasing near $x$ = 0 and increasing near $x$ = 1. For this specific distribution, the convexity of all three curves ensures that $\hat{\phi}$ and $F$ - 1 intersect once on (0,1). The point of intersection of $\hat{\phi}$ and $F$-1 is $\hat{x}$. Using (26) to define the reporting function, the optimal conditional contract for α = 0 induces an effort level of .049 and results in the earnings manipulation shown in top curve in Figure 5. Increasing α has the effect of shifting the $\hat{\phi}$ curve down thus reducing the range of earnings over which the manager earns zero rent. Now zero manager rent is associated with under-reported earnings as illustrated by the lower curve in Figure 2. In addition, under-reporting persists above $\hat{x}$ even while the manager starts to earn positive rent. Over-reported earnings arise only for the highest earnings levels but notice that the magnitude of the earnings manipulation is reduced. Optimal effort rises to .055. *End of example.*

The Example highlights three interesting properties of an optimal contract: zero manager rent at low earnings levels induced by exploiting countervailing ownership and manipulation

[6]More precisely, we solved for an optimal contract in the example ignoring the ex ante participation constraint, and then checked to ensure that the solution identifed satisfied (11.d).

incentives, incentives for both under-reporting and over-reporting earnings, and a compensation schedule that incorporates both insurance and options features. We now prove that these are general properties of optimal conditional contracts.

***Proposition 2.*** *Assume the distributional assumptions are satisfied. The optimal conditional contract induces a strictly positive level of effort and there exists earnings $x^+$ such that for all $x^+ < x < 1$ the manager over-report earnings ($R > x$) and earns positive rent ($W > 0$). If the level of effort induced by the optimal conditional contract is sufficiently small, then there exists an earnings level, $\hat{x} > 0$, such that for all $x < \hat{x}$, the manager weakly under-reports earnings ($R \leq x$) and earns zero rent ($W = 0$) (with strict under-reporting for $\alpha > 0$) while for earnings sufficiently close to 1, the manager over-reports earnings and earns positive rent.*

Proposition 2 establishes that over-reporting of high earnings is a robust feature of an optimal contract. Moreover, if the targeted level of effort is not too costly for the owner to induce ($\mu$ is small), then the optimal contract will also induce under-reporting of low earnings and a range of earnings over which the manager earns zero rent.

The fact that an optimal conditional contract can result in under-reported earnings may seem surprising to some as it does not appear to be a feature of contracts used in practice. To address this issue, note that (1) and (10) imply that

$$B(x) = W(x) + g(R(x)-x) - \alpha x. \tag{27}$$

For $x \leq \hat{x}$, $W(x) = 0$ so $B(x) = -\alpha x + g(-\alpha) = -\alpha\hat{x} + \alpha(\hat{x} - x) + g(-\alpha)$. This compensation schedule effectively allows the manager to pay for her shares in the firm after the fact at a price equal to the realized earnings of the firm by netting this cost from the rents she earns and the reimbursed manipulation cost. Note that the manager's compensation is minimized at $x^+$. For earnings below $\hat{x}$, the manager receives a refund that is decreasing in *x* of $g(-\alpha) + \alpha(\hat{x} - x)$. For earnings above $\hat{x}$, the manager earns an increasing rent but, for $\hat{x} < x < x^+$, the level of compensation, *B*, is still decreasing. To see this, note that for $x > \hat{x}$, we may write

$W(x) = \int_{\hat{x}}^{x} W_x(t)dt$ , from which it follows

$$B(x) = g(R(x) - x) + \int_{\hat{x}}^{x} g'(R(t) - t)dt - \alpha\hat{x} \quad ,$$

so that $B'(x) = g'(R - x)R'$ , which is negative when the manager under-reports earnings and positive for over-reporting.

This non-monotonic compensation function helps induce the desired level of effort as it effectively insures the manager against very low earnings levels ($x < \hat{x}$) and rewards the manager for earnings above $x^+$. This discussion thus leads to the following proposition.

***Proposition 3.*** *If the optimal contract induces under-reporting of earnings below $x^+$, the optimal compensation schedule will be decreasing in earnings up to $x^+$ and increasing in earnings above $x^+$.*

One way of viewing this compensation schedule is to think of the manager’s ownership stake as being an option to purchase $\alpha$ of the firm at the share price $\hat{x}$. For $x < \hat{x}$, the option is out of the money, so the manager does not exercise the option and receives as compensation the wage $g(-\alpha)$. When $x > \hat{x}$, the option is in the money, and the manager exercises the option at the price $\alpha\hat{x}$. Finally, for $x > x^{+}$, the manager continues to exercise the stock option, and in addition is paid a bonus that is increasing in the level of reported earnings.

4. Optimal Firm Ownership by the Manager

The previous section demonstrated that the optimal contract conditional on the percent of the firm owned by the manager has certain features that are robust to the manager's stake in the firm. However, changes in α can be expected to effect not only the manager's behavior under the contract but also the owner's expected profit. In this section, we study the effect of changes in α on both the manager and the owner.

First, consider the effect of an increase in α on the manager. The range of earnings that imply $W = 0$ can either increase or decrease with α. To see why, let $M(a,\alpha) \equiv -\int_{t=0}^{1}[\alpha + g'(R(t)-t)]F_a(t)dt - h'(a)$ denote the derivative of the manager's expected profit with respect to effort evaluated at the conditionally optimal value of μ and with $R$ defined by (26). Thus, $M(a,\alpha) \equiv 0$ for $a > 0$. **[NOTE: We are assuming λ=0 in what follows]** Differentiating $M(a,\alpha)$ with respect to α yields

$$\int_{t=\hat{x}}^{1}[1 - F_a g''\mu_\alpha/f]F_a dt = 0. \tag{28}$$

Since $F_a(\cdot) \leq 0$, $\mu_\alpha$ must be negative. This means α has two effects on $\hat{x}$. First, an increase in α requires greater under-reporting to offset the rent benefits of greater ownership. This directly reduces $\hat{\phi}$ holding μ constant (see (24)) and hence decreases $\hat{x}$. Second, an increase in α reduces the cost of inducing a given level of effort (μ decreases) and requires the owner to pay the manager fewer rents. This corresponds to an increase in $\hat{\phi}$ and hence an increase in $\hat{x}$. Under the conditions of the Example, $\hat{x}$ will increase at low effort levels and decrease at high effort levels.

Second, consider the effect of α on expected owner profit. From (11), the indirect expected profit of the owner can be written as

$$E\pi(\alpha) = E(x - g - W).$$

Given the optimal reporting function in (26), the indirect expected owner profit equals

$$\begin{aligned} E\pi(\alpha) &= \int_{x=0}^{\hat{x}} (x - \alpha^2/2) f(x)dx + \int_{x=\hat{x}}^{1} [x - (R^+(x) - x)^2/2 - \int_{t=\hat{x}}^{x} (\alpha + R^+(t) - t)dt] f(x)dx \\ &= \int_{x=0}^{\hat{x}} (x - \alpha^2/2) f(x)dx + \int_{x=\hat{x}}^{1} [(x - (R^+(x) - x)^2/2) f(x) - (1 - F(x))(\alpha + R^+(x) - x)]dx \end{aligned} \tag{29}$$

where $R^+(x) = x + ((1-\lambda)(F(x)-1) - \mu F_a(x))/f(x)$, and the last term follows from (13). Since $R^+(\hat{x}) = x - \alpha$ by the definition of $\hat{x}$, the Envelope Theorem implies

$$\begin{aligned} dE\pi(\alpha)/d\alpha &= -\alpha F(\hat{x}) - \int_{x=\hat{x}}^{1} [(R^+(x) - x) R_\alpha^+(x) f(x) + (1 - F(x))(1 + R_\alpha^+(x))]dx \\ &= -\alpha F(\hat{x}) + \int_{x=\hat{x}}^{1} [(R^+(x) - x) f(x) + \mu F_a(x)(1 + R_\alpha^+(x))]dx \end{aligned} \tag{30}$$

where the second line is derived by using the definition of $R^+(\cdot)$ to substitute out 1-$F$ in the integrand in line 1 and $R_\alpha^+$ denotes the effect of α on $R^+$ holding $a$ constant.

To derive the term $R_\alpha^+$ note that the value of μ is set to induce the manager to choose the effort level the owner wishes her to take. Denote this optimal value of μ by μ($a$,α). It is implicitly defined by the manager's first-order condition

$$M(a,\alpha) \equiv -\int_{x=\hat{x}(a,\mu(a,\alpha),\alpha)}^{1} [\alpha + (F(x,a) - 1 - \mu(a,\alpha)F(x,a))/f(x,a)]F_a(x,a)dx - h'(a) \equiv 0. \quad (31)$$

Since (31) is a tautology for all $a$ and all α, differentiating (31) with respect to α implies

$$\partial M(a,\alpha)/\partial\alpha \equiv -\int_{x=\hat{x}(a,\mu(a,\alpha),\alpha)}^{1} [1 - \mu_\alpha(a,\alpha)F(x,a))/f(x,a)]F_a(x,a)dx \equiv 0. \quad (32)$$

(Because the integrand in (31) evaluated at $\hat{x}$ is zero by construction, the effect of α on the lower limit of integration in (31) is zero.) In particular, if $a$*(α) denotes the optimal effort the owner wishes the manager to take, then (32) holds for $a = a$*(α). Returning to the definition of $R^+(\cdot)$, the optimal reporting function for earnings above $\hat{x}$ will be

$$R^+(x,a^*(\alpha),\mu(a^*(\alpha),\alpha),\alpha) = x + ((1-\lambda)(F(x,a^*) - 1) - \mu(a^*,\alpha)F_a(x,a^*))/f(x,a^*) \quad (33)$$

and

$$R_\alpha^+(x,a^*(\alpha),\mu(a^*(\alpha),\alpha),\alpha) = -\mu_\alpha(a^*,\alpha)F_a(x,a^*))/f(x,a^*). \quad (34)$$

Substituting (34) into (32) implies that for all $a$ and for all α,

$$\int_{x=\hat{x}(a,\mu(a,\alpha),\alpha)}^{1} [1 + R_\alpha^+(x,a,\mu(a,\alpha),\alpha)]F_a(x,a)dx \equiv 0. \quad (35)$$

Consequently, (30) simplifies to

$$dE\pi(\alpha)/d\alpha = -\alpha F(\hat{x}) + \int_{x=\hat{x}}^{1} (R^{+}(x) - x) f(x) dx. \tag{36}$$

At $\alpha = 0$, the sign of $dE\pi(\alpha)/d\alpha$ depends only on the average misstatement of earning which by (26) is strictly positive.

***Proposition 4***. *Given the Distribution Assumptions, the owner will want to endow the manager with a strictly positive share of the firm.*

Eq. (36) clearly reflects the trade-offs associated with making the manager a shareholder. The cost, $-\alpha F$, reflects the misreporting costs the manager would incur from under-reporting earnings. The benefit to the owner of increasing $\alpha$ is the savings from paying lower marginal rents through overstated earnings. Thus, the optimal percentage of shares for the manager trades off the cost of earnings management at low earnings versus lower marginal rents paid to the manager at high earnings.

## 5. Conclusions

We have in this paper characterized an optimal compensation contract for a manager who must take a hidden action which impacts on the probability distribution of firm profits. These profits, when realized, are themselves hidden information to the manager, who my engage in earnings management by making earnings reports which differ from the actual level of profits. In contrast with previous work, we assume that the manager has no financial resources with which to purchase a stake in the firm, and that the manager may leave the firm whenever it is in her best interest to do so.

In this setting, the optimal compensation arrangement consists of (i) an option to purchase shares of the firm, which the manager can always afford to exercise out of her ex-post compensation, as well as (ii) bonuses which are increasing in reported earnings once those reports exceed a well-defined earnings threshold. We also find that the optimal compensation arrangement results in managers under-reporting earnings for small levels of profit, and over-reporting earnings when actual profits are higher.

Appendix

***General information for the proofs.***

Problem (11') is solved in two steps. In step 1, $a$ is fixed and we use (16) to solve for the optimal earnings report function, $R$, and the optimal indirect manager utility function, $W$, as a function of $a$. In step 2, the optimal value of $a$ is derived. We may write the owner's expected profit solely as a function of the conditionally optimal earnings report function, $R$. Noting that $\Pi = x - g - W$ and substituting from (13), in step 2 we need only choose $a$ to maximize

$$E[x - g(R(x;a) - x) - W] = E[x - g(R(x;a) - x) + \frac{F(x|a) - 1}{f(x|a)}(\alpha + g'(R(x;a) - x)] - W(0). \qquad \text{(A.1)}$$

Step 1 is completed by using Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter (1987). Because of the pure state constraint, $W \geq 0$, it is possible for $\phi(x)$ to be discontinuous. Seierstad and Sydsæter (p. 319) allow for this possibility but indicate that with the optimal contract $\phi(x)$ can only jump down at a finite set of points. At all other points, $\phi$ must be continuous. Given $\alpha$, sufficient conditions for an optimal conditional contract are (18)-(23).

From the discussion in the text, (19) and (22) imply (25), hence $\lambda \leq 1$ (since $\lambda > 1$ would lead to a contradiction). Recall also from the text that if $W_x = 0$ on a non-degenerate interval, then

$$\hat{\phi}(x) = \mu F_a(x|a) - \alpha f(x|a). \qquad \text{(A.2)}$$

If $\alpha = 0$, then $\hat{\phi}(0) = \hat{\phi}(1) = 0$ and for $x \in (0,1)$, $\hat{\phi}(x) < 0$. If $\alpha > 0$, $\hat{\phi}(x) < 0$ for all $x$. Thus, $\hat{\phi}(1) \leq 0 = (1-\lambda)(F(1|a) - 1)$ and, since $f$ is bounded by Assumption (b), it follows that, for $\alpha$ sufficiently small and for each $\lambda < 1, \hat{\phi}(0) \geq \lambda - 1 = (1-\lambda)(F(0|a) - 1)$. Then by continuity there must exist an actual earnings $\tilde{x} > 0$ such that for all $x < \tilde{x}$, $\hat{\phi}(x) > (1-\lambda)(F - 1)$. However, for $\alpha$ and $\lambda$ sufficiently large, it is possible that $\hat{\phi}(0) \leq \lambda - 1 = (1-\lambda)(F(0|a) - 1)$. Holding $\lambda$, $\mu$, and $a$

fixed, this possibility suggests two cases.

Case 1. $\hat{\phi}(0) \leq \lambda - 1$.

There are two possibilities in this situation. First, suppose that $\hat{\phi}$ lies below (1-$\lambda$)($F$-1) for every $x$. Since (25) requires that the optimal value of the costate variable satisfy $\phi(x) \geq$(1-$\lambda$)($F$($x$|$a$)-1), and we know that $\phi(\cdot)$ cannot jump up, the only feasible co-state function is $\phi(x) = (1-\lambda)(F(x|a) - 1)$. Alternatively, suppose that $\hat{\phi}$ crosses (1-$\lambda$)($F$-1). Then the optimal value of the costate variable cannot be $\hat{\phi}$ until $\hat{\phi}$ lies above (1-$\lambda$)($F$-1), by the argument above. And, we also know that, since $\tau \geq 0$, (19) and (22) require $\phi'(x) \leq (1-\lambda)f(x|a)$ and $\phi'(x) = (1-\lambda)f(x|a)$ whenever $W(x) > 0$, which implies that the slope of the costate variable cannot be greater than that of (1-$\lambda$)($F$-1). Thus, for the costate variable to be $\hat{\phi}$ would require that the costate variable jump up, which is not permitted. So, the only feasible co-state function is (again) $\phi(x) = (1-\lambda)(F(x|a) - 1)$. Result in Case 1 is that $W(0)$ will equal zero and $W(x) > 0$ for all $x > 0$. $R(x)$ will be strictly greater than $x$ on [0,1).

Case 2. $\hat{\phi}(0) > \lambda - 1$.

Define $\tilde{x} = \min\{x|\hat{\phi}(x) = (1-\lambda)(F(x|a)-1)$. Since both $\hat{\phi}$ and $F$ are continuous and $\hat{\phi}(1) \leq 0$, $\tilde{x}$ is well-defined. . Also, define $\bar{x} = \inf\{x|\hat{\phi}'(x) > (1-\lambda)f(x|a)\}$. If $\hat{\phi}'(x) \leq (1-\lambda)f(x|a)$ for all $x$, define $\bar{x} = 1$. Finally, define $\hat{x} = \min\{\tilde{x}, \bar{x}\}$.

From Assumption (c), $\hat{\phi}'(0) < 0$ so $\bar{x}$ must be strictly greater than zero. In this case, define the costate variable as $\phi(x) = \hat{\phi}(x)$ for $x \leq \hat{x}$, and define $\phi(x) = (1-\lambda)(F(x|a)-1)$ for $x > \hat{x}$. The case in which $\tilde{x} < \bar{x}$ is depicted in Figure 4 of the paper. Note that if $\hat{x} = \bar{x} < \tilde{x}$, then $\phi$ will jump down at $\hat{x}$. This is depicted in Figure A.2, and in Figure A.1 for the case in which $\tilde{x}$ =1. $W(x) = 0$ for all $x \leq \hat{x}$ and $W(x) > 0$ for all $x > \hat{x}$.

(Note: Without Assumption (c), it is possible for $\hat{x} = 0$. In this situation, $\phi$ can neither equal $\hat{\phi}$ nor $(1-\lambda)(F-1)$ for $x$ close to zero. Then there will exist earnings $0 < x_0 < x_1$ such that for $x < x_0$, $\phi(x) = (1-\lambda)(F-1) + k$ where $k$ is a positive constant and $W(x)$ will be positive. For $x > x_1$, $\phi(x) = (1-\lambda)(F-1)$. For some $x > x_1$, $W(x) = 0$.)

***Proof of Proposition 2.***

Based on the above analysis, for any $a > 0$ and for any $\lambda \leq 1$, (12) and (26) define a conditional contract that satisfies (18), (19), and (22) and hence also satisfies the properties of optimal conditional contracts described in Proposition 2 for Cases 1 and 2. (For Case 1, $\hat{x} = 0$.)

For $x \leq \hat{x}$, $R'(x) = 1$ and for $x > \hat{x}$,

$$R'(x) = 1 + \partial((F-1)/f)/\partial x - \mu\cdot\partial(F_a/f)/\partial x. \tag{A.3}$$

By Assumption (d), the second term in (A.3) is strictly positive. The third term can be either positive or negative. For $a$ sufficiently close to zero, $\mu$ will be small enough to ensure that $R'(x)>0$. The distribution in example 1 can be used to show that for $a$ sufficiently large, (26) implies $R'(x) < 0$ for some (high) $x$, which would violate the second order condition (23). Thus, this proof must also show that the solution to (11) when (23) binds for some $x$ has the same features as contracts based on (26). A solution in this case will require the use of "ironing" techniques (Fudenberg and Tirole, 1999) to characterize an optimal contract.

The more general formulation of the optimal control problem associated with (11) has the Hamiltonian

$$\mathcal{H} = (x - g - W)f + \phi_W(\alpha + g') + \phi_R R' \tag{A.4}$$

and the Lagrangean

$$\mathfrak{L} = \mathcal{H} + \tau W - \mu[(\alpha + g')F_a + h'f] + \lambda_1 f(W-h) + \lambda_2 R'$$

where $W$ and $R$ are now treated as state variables and $R'$ is the control. $\phi_W$ and $\phi_R$ are the new co-state variables and $\lambda_2 \geq 0$ is the multiplier for constraint (23). Sufficient conditions analogous to those in Proposition 1 (again following Theorems 1 and 2 from chapter 6 in Seierstad and Sydsæter) are (20), (23),

$$R - x = \frac{\phi_W - \mu F_a}{f} + \phi_R', \tag{A.5}$$

$$-(1-\lambda_1)f + \tau = \phi_W', \tag{A.6}$$

$$\phi_R + \lambda_2 = 0, \tag{A.7}$$

$$\lambda_1(EW - h(a)) = 0 \text{ and } \lambda_1 \geq 0, \tag{A.8}$$

$$\phi_W(0) \leq 0,\ \phi_W(1) \geq 0,\ \phi_W(0)W(0) = \phi_W(1)W(1) = 0,\ \tau(x) \geq 0,\ \tau(x)W(x) = 0, \tag{A.9}$$

$$\phi_R(0) = \phi_R(1) = 0, \text{ and} \tag{A.10}$$

$$\lambda_2(x) \geq 0 \text{ and } \lambda_2(x)R'(x) = 0. \tag{A.11}$$

As with Proposition 1, $\phi_W$ and $\phi_R$ can jump down but not up at a countable number of values of $x$. Note that if the solution to (11') implies (23), then $\phi_R(x) \equiv 0$ and (A.5)-(A.11) plus (20) and (23) collapses down to (18)-(23).

Note that by (A.7), $\phi_R \neq 0$ only if $R' = 0$ as then $\lambda_2 > 0$. By assumption (e), solutions to (11') that violate (23) must do so on an interval from some $x'$ to 1. And, since $R(1) = 1$by (26) ,it follows that, on $(x',1)$, we must have $R(x) > 1$. The "ironed" solution to (11) will induce $R = \bar{R} > 1$ on some interval $\underline{x}$ to 1 where $\underline{x} < x'$.

The optimal conditional contract is constructed as follows. First, define $\hat{x}$ and $\hat{\phi}$ as above. $\hat{\phi}$ now represents the value of the co-state variable, $\phi_W$, for which $W_x = 0$ when $\phi_R' = 0$. Then define $\phi_W = \hat{\phi}$ on $[0,\hat{x}]$ and define $\phi_W = (1-\lambda)(F-1)$ on $(\hat{x},1]$. This is identical to the definition of $\phi$ above.

Second, define $x_1 = \min\{x | x + ((1-\lambda)(F-1) - \mu F_a)/f = 1\}$. $x_1$ represents the first earnings level at which the reporting function which solves (11') first equals 1. Since (26) implies $R(1) = 1$, $x_1$ always exists. If $x_1 = 1$, then (26) will define an increasing reporting function on [0,1) and $\phi_R \equiv 0$. If $x_1 < 1$, then (26) must define a reporting function that is decreasing for $x$ close to 1.

Third, choose $\bar{R}$ from the interval $(1, \max_x\ x + ((1-\lambda)(F-1) - \mu F_a)/f)$. For each $\bar{R}$, there will be two solutions to the equation: $x + ((1-\lambda)(F-1) - \mu F_a)/f) = \bar{R}$ on $(x',1)$. Denote these two solutions by $\bar{x}_1$ and $\bar{x}_2$ such that $\bar{x}_1 < \bar{x}_2$. The various values of $x$ that have been identified are noted in Figure A3.

Define $\phi_R = 0$ for $x \le \bar{x}_1$ and for $x > \bar{x}_1$ solve (A.5) for $\phi'_R$ when $R = \bar{R}$. This implies $\phi'_R(x) = \bar{R} - x - ((1-\lambda)(F-1) - \mu F_a)/f$. $\phi'_R(x)$ will be negative on $(\bar{x}_1, \bar{x}_2)$ and it will be positive on $(\bar{x}_2, 1]$. As a result,

$$\phi_R(x) = \int_{t=\bar{x}_1}^{x} [\bar{R} - t - ((1-\lambda)(F-1) - \mu F_a)/f] dt \tag{A.12}$$

on $(\bar{x}_1, 1]$. $\bar{R}$ must be chosen so that $\phi_R(\bar{x}_1) = \phi_R(1) = 0$. Using (A.12), $\bar{R} = 1$ implies $\phi_R(1) < 0$ and $\bar{R} = \max_x\ x + ((1-\lambda)(F-1) - \mu F_a)/f)$ implies $\phi_R(1) > 0$. Thus, there must exist a value of $\bar{R}$ for which $\phi_R(1) = 0$. Thus, ensuring that (23) is satisfied does not alter the quantitative properties of optimal conditional contracts given $a$ and $\mu$.

To check (20), the manager's first-order condition for effort, note that (11b) is strictly negative for $\mu = 0$ and strictly positive for $\mu = \infty$. Thus, for any $a > 0$, there exists a $\mu$ that satisfies (20).

To summarize, for any $a > 0$, there exists $\mu > 0$ such that the optimal conditional contract must fall into one of four categories: Case1 without ironing at the top, Case 1 with ironing at the top, Case 2 without ironing at the top, and Case 2 with ironing at the top. The transition between these four cases is continuous and hence expected owner profit is a continuous function of $a$. If $\bar{a}$ denotes the first-best level of effort, then the optimal level of effort will be less than or equal to $\bar{a}$. This means the owner's choice of effort to induce exists since it maximizes a continuous function on $[0,\bar{a}]$. Since $h'(0) = 0$, the optimal value of $a$ must be strictly positive. ***Q.E.D.***

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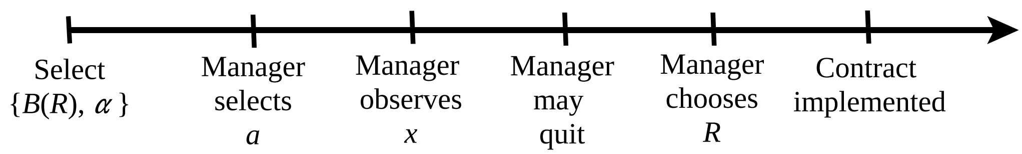


**Figure 1: Sequence of Events**

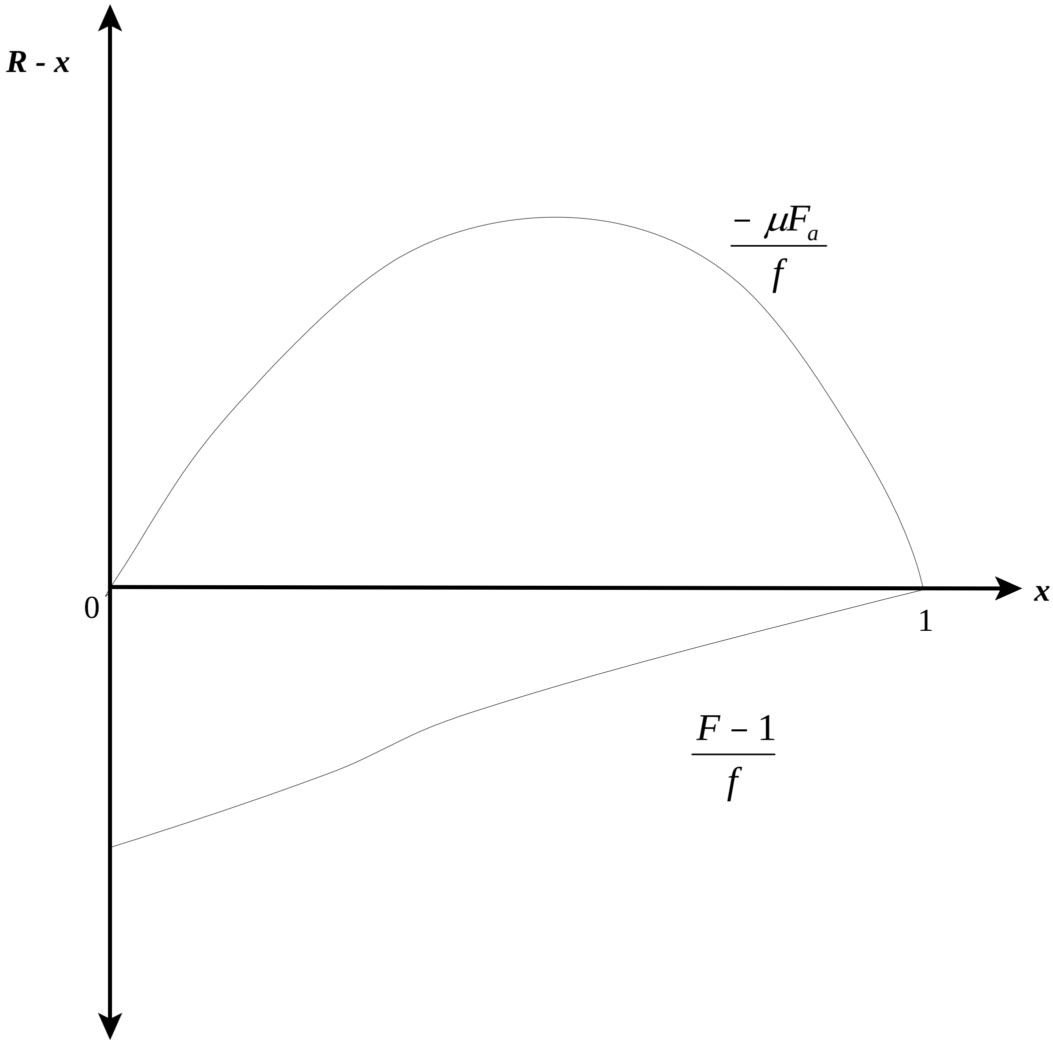


**Figure 2**

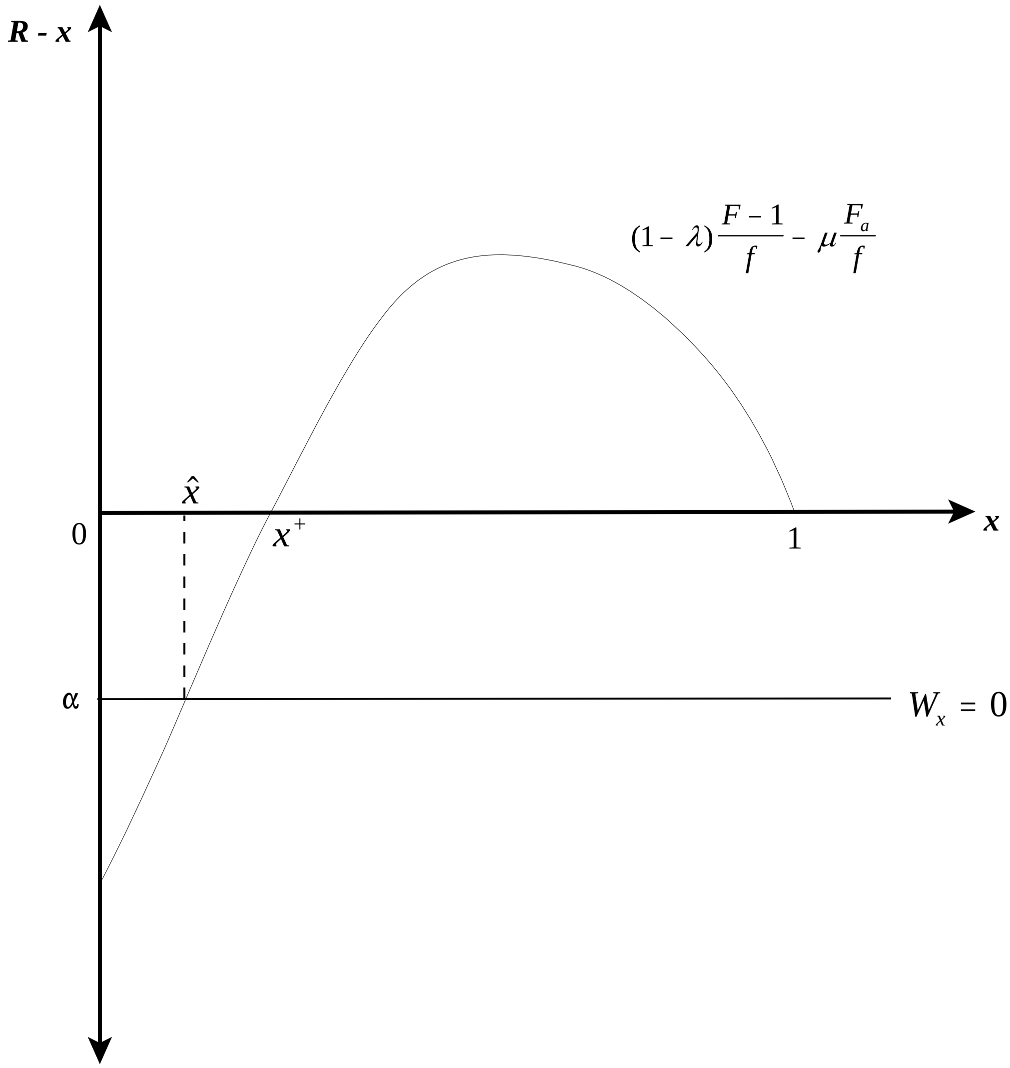


**Figure 3**

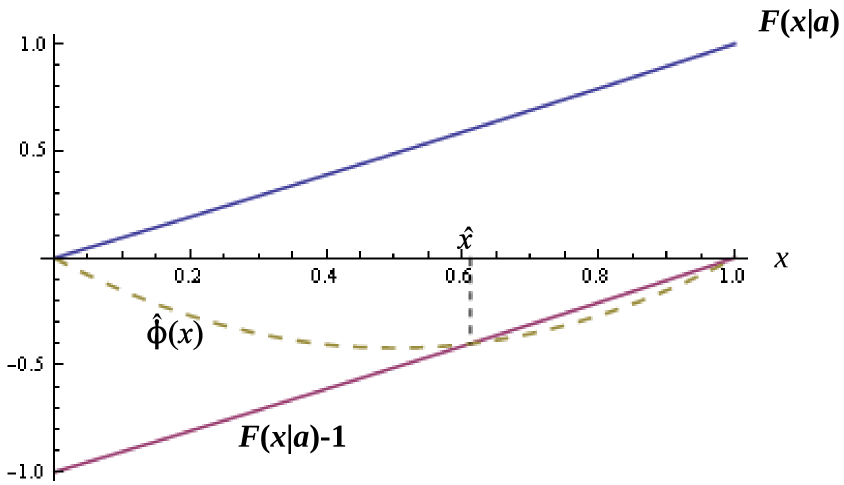


**Figure 4: The Example**

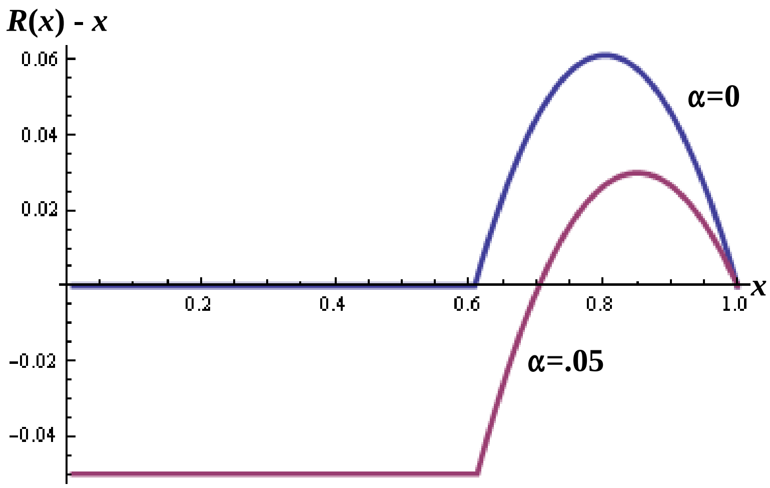


**Figure 5: Optimal Earnings Manipulation**

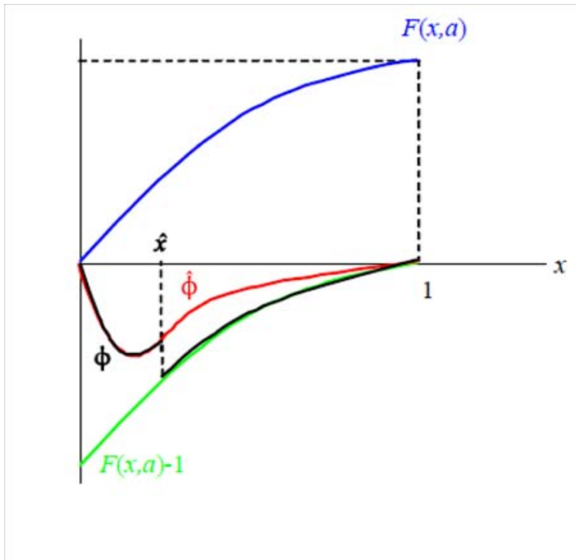


**Figure A1**

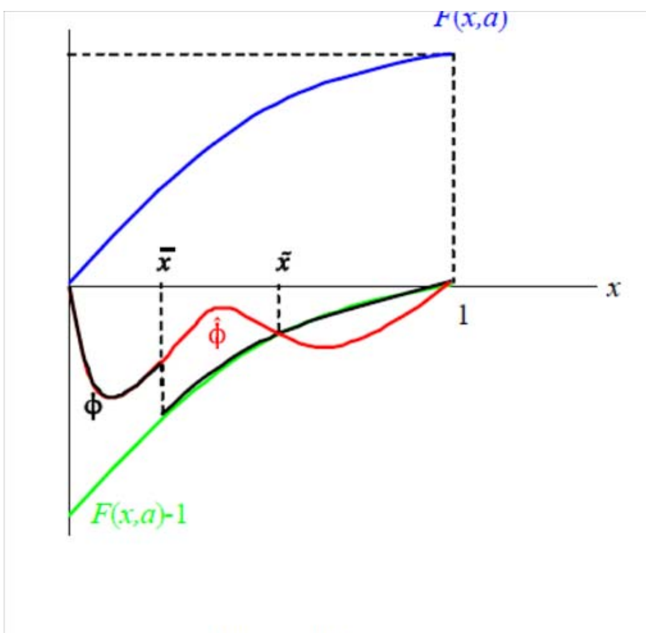


**Figure A2**

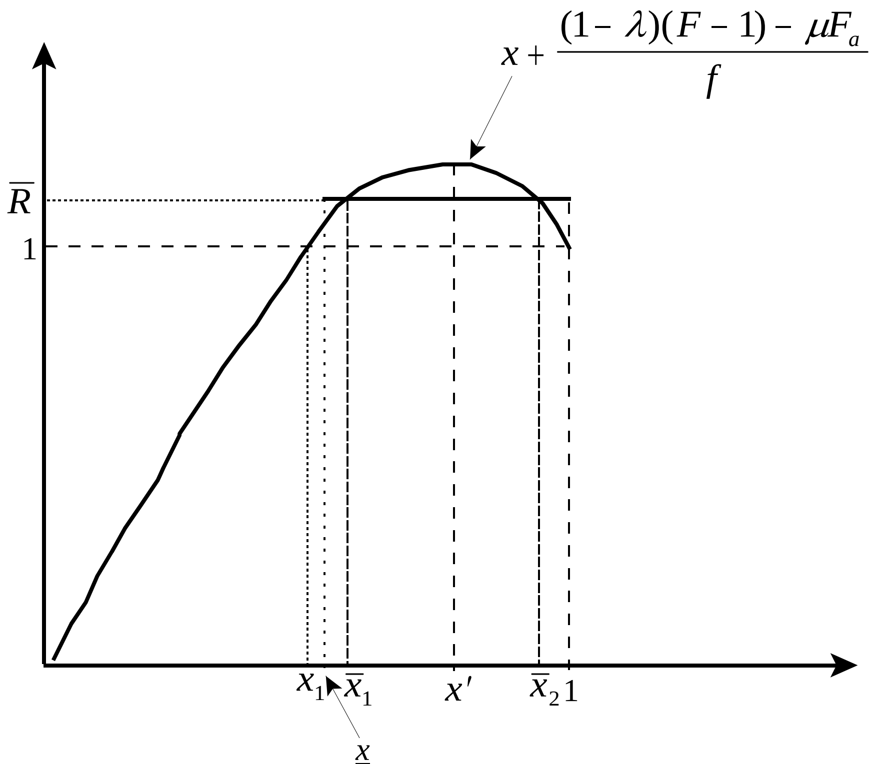
$x + \frac{(1-\lambda)(F-1)-\mu F_a}{f}$
$\overline{R}$
1
$x_1$
$\overline{x}_1$
$x'$
$\overline{x}_2$
1
$\underline{x}$

**Figure A3**