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The Inflation-Output Trade-Off with Downward Wage Rigidities

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The macroeconomic implications of downward nominal wage rigidities are analyzed via a dynamic stochastic general equilibrium model featuring aggregate and idiosyncratic shocks. A closed-form solution for a long-run Phillips curve relates average output gap to average wage inflation: it is virtually vertical at high inflation and flattens at low inflation. Macroeconomic volatility shifts the curve outwards and reduces output. The results imply that stabilization policies play an important role, and that optimal inflation may be positive and differ across countries with different macroeconomic volatility. Results are robust to relaxing the wage constraint, for example, when large idiosyncratic shocks arise. (*JEL* E23, E24, E31, E63)

This paper investigates the macroeconomic implications of downward nominal wage rigidities in a low inflation environment via a dynamic stochastic general equilibrium (*DSGE*) model where forward-looking agents optimally set their wages in an economy hit by aggregate and idiosyncratic shocks. A closed-form solution for the long-run Phillips curve is derived. The inflation-output trade-off is virtually vertical at high inflation and flattens at low inflation, implying progressively larger output costs of reducing inflation. Macroeconomic volatility (among other factors) shifts the curve outward, generating output and employment costs, and thus suggesting the need for stabilization policies. The results are robust (only lessened in magnitude) to partially relaxing the downward wage constraint, for example, allowing wages to freely adjust in the face of large idiosyncratic shocks.

The conventional view argues against the presence of a long-run trade-off and in favor of price stability: the attempt to take advantage of the short-run trade-off would generate costly inflation only in the long run, so that price stability should be

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the objective of central banks.[1] Recent monetary models exhibit a long-run relationship between inflation and real activity, mainly due to symmetric nominal rigidities and asynchronized time-dependent price-setting behavior in an intertemporal setup (see, among others, Goodfriend and Robert G. King 1997; Michael Woodford 2003).[2] Nonetheless, this literature indicates that the optimal long-run inflation rate should be close to zero and unemployment at the natural rate.[3] However, virtually no central bank adopts a policy of zero inflation, and the two traditional reasons relate to the zero nominal interest bound and the presence of downward nominal rigidities.

This paper emphasizes the role of downward nominal rigidities, quite a novel feature in recent monetary models.[4] The traditional view suggests that a lower bound on wages and prices keeps them from falling: a negative demand shock would just reduce inflation if inflation remains positive, but would reduce output and employment if prices needed to fall. Price stability could be achieved only at substantial costs in terms of output and employment, thus entailing significant benefits from "greasing" the labor market via inflation. An extensive discussion is offered by George A. Akerlof, William T. Dickens, and George L. Perry (1996), who derive a trade-off between unemployment and inflation via a static model with downward wage rigidities.

There is now a body of microeconometric evidence suggesting the presence of downward wage rigidities across a wide spectrum of countries, often even at low inflation.[5] Recent studies, some based on cross-country evidence, find that downward wage rigidities have a negative impact on employment (Julián Messina et al. 2008; Christoph Knoppik and Thomas Beissinger 2003). Indeed, while David Card and Dean Hyslop (1997) find only weak evidence in favor of a "grease" effect of inflation for the United States, Ana Maria Loboguerrero and Ugo Panizza (2006) find that the grease effect of inflation is more relevant in countries with highly regulated labor markets, in line with the fact that wage rigidities are stronger in countries with heavier labor market distortions (Holden 2004; Dickens et al. 2007). The effect of downward rigidities is potentially a contributor to recent US labor market developments: data from the US Bureau of Labor Statistics (*BLS*) shows that annual

[1] After the seminal contributions of Alban W. Phillips (1958) and Paul A. Samuelson and Robert M. Solow (1960), various authors have cast serious doubts on the validity of the Phillips curve (Milton Friedman 1968; Robert E. Lucas Jr. 1973). The empirical controversy has yet to settle down (see Laurence Ball, N. Gregory Mankiw, and David Romer 1988).

[2] State-dependent pricing (see Mikhail Golosov and Lucas 2007) would weaken the long-run relationship, as price rigidities would no longer be binding at high inflation.

[3] Otherwise, inflation would induce firms to set a high markup to protect future profits, and would create costly price dispersion (see Aubhik Khan, Robert G. King, and Alexander L. Wolman 2003; Stephanie Schmitt-Grohe and Martin Uribe 2009). See Charles Wyplosz (2001) for an empirical analysis on this topic.

[4] Since Keynes, numerous authors (for example, James Tobin 1972 and Akerlof 2007) stressed the importance of such rigidities for the existence of a trade-off between inflation and unemployment.

[5] See, for example, David E. Lebow, Raven E. Saks, and Beth Anne Wilson (2003), Dickens et al. (2007), and numerous references cited by Akerlof (2007) and by Steiner Holden (2004). Peter Gottschalk (2005) and Alessandro Barattieri, Susanto Basu, and Gottschalk (2009) find that measurement errors in wages reported in surveys can often lead to substantial underestimation of the extent of downward wage rigidities. Several explanations have been put forward for the existence of such rigidities, such as fairness, social norms, and labor market institutions (see, for example, Truman F. Bewley 1999 and Holden 2004). Several authors (see Ball and Mankiw 1994, and the comments to Akerlof, Dickens, and Perry 1996) have conjectured that downward wage rigidities may vanish in a low-inflation environment. However, recent evidence shows that even at low inflation such rigidities are binding (Jonas Agell and Per Lundborg 2003, for Sweden; Ernst Fehr and Lorenz Götte 2005, for Switzerland). Regarding goods prices instead, evidence of downward wage rigidities is weaker and less conclusive (see Alvarez et al. 2006 and Blinder et al. 1998, chap. 18).

growth rate in private industry total compensation declined from only about 3 percent in the first quarter of 2008 to about 1.5 percent in the last quarter of 2009, while unemployment rose from about 5 to 10 percent over the same period.

In this paper, we introduce downward wage rigidities in an otherwise DSGE model with forward-looking optimizing agents who enjoy consumption of goods and experience disutility from labor when working for profit-maximizing firms. Labor markets are characterized by monopolistic competition, goods markets are perfectly competitive, and goods prices are fully flexible. The economy is subject to both idiosyncratic sectoral shocks and aggregate shocks (to productivity and nominal spending), which generate the need for both intratemporal (as in the traditional discussion of the Phillips curve) and intertemporal price adjustments. Extensions to the benchmark model relax and endogenize the downward rigidity constraint, in part to address concerns about the empirical relevance of wage rigidities at low inflation. Indeed, even if we allow the degree of downward rigidities to vary across agents, or with inflation and macroeconomic volatility, or with the size of shocks, the inflation-output trade-off remains sizable for reasonable parametrizations of the model.

The most important novelties of our contribution are: the introduction of idiosyncratic shocks in a DSGE model with downward wage rigidities, the derivation of a closed-form solution for a positive nonlinear relationship between the long-run averages of wage inflation and of output gap (the long-run Phillips curve), and the innovative results related to how such a curve would shift outward with macroeconomic volatility.[6] The output-inflation trade-off flattens at low inflation, a result that suggests that the flattening of the Phillips curve observed in several industrial countries in recent years may not need to be ascribed to globalization (see International Monetary Fund 2006; Claudio E. V. Borio and Andrew Filardo 2006), but may simply be due to the decline in inflation. The results also suggest that the possible end of the Great Moderation, coupled with low inflation, may inflict a compounded negative effect on the economy, which would substantially reduce output and employment unless offset by more wage flexibility, stronger stabilization policies, or higher productivity growth.

The policy implications are quite different from those offered by standard monetary models. First, substantial economic costs at low inflation may imply that the optimal level of inflation may not be negative (as suggested by Friedman 1968), nor close to zero (as recently argued; see, for example, Jinill Kim and Francisco J. Ruge-Murcia 2009; Schmitt-Grohe and Uribe 2009).[7] Moderate inflation may help grease intratemporal and intertemporal relative wage adjustments, especially in countries with substantial macroeconomic volatility. Second, not every country should target the same inflation rate, but those experiencing larger volatility or lower productivity growth may find it desirable to target a higher inflation rate. Third, as volatility or productivity growth change persistently over time, the inflation target may need

[6] A similar formulation in terms of inflation and unemployment is offered in Benigno and Ricci (2008).

[7] There are other arguments in favor of positive inflation rates. The main one is based on a zero nominal interest rate floor, which has recently received a lot of attention (see Olivier Blanchard, Giovanni Dell'Ariccia, and Paolo Mauro 2010). The discussion of this argument has focused primarily on how its relevance depends on the frequency of hitting such a floor; the effectiveness of alternative policies (quantitative measures, or fiscal and exchange rate policies); and the ability of credibly committing to higher future inflation. By emphasizing the role of volatility, our paper offers additional insights for the relevance of the zero interest rate floor, as countries with higher volatility would be more likely to hit the floor and would need, ceteris paribus, a higher level of inflation.

to be adjusted. Fourth, policymakers can improve the output-inflation trade-off via stabilization policies aimed at reducing macroeconomic volatility, thus lowering the output and employment costs of maintaining low inflation or reducing it. This result contrasts with the view that the gains from stabilization policies are negligible (as in Lucas 1987, 2003). Simulations based on the model presented in this paper indicate that an advanced economy enjoying a low macroeconomic volatility (say 2 percent) and low wage inflation (say 2 percent) might face a long-run output gap of minus 1.2 percent. The end of the Great Moderation (say bringing macroeconomic volatility to 5 percent) might widen this estimate to minus 1.6 percent.

Beside the work of Akerlof, Dickens, and Perry (1996), our paper is related to a few recent contributions. Michael W. L. Elsby (2009) offers a partial equilibrium model where downward nominal rigidities arise from negative effects of wage cuts on firms' productivity, and highlights the endogenous tendency for upward rigidity of wages in a dynamic model. Kim and Ruge-Murcia (2009), Stephan Fahr and Frank Smets (2008), and Gabriel Fagan and Julián Messina (2009) present DSGE models with asymmetric costs to wage adjustments, but do not derive a closed-form solution for the long-run Phillips curve, and do not account for idiosyncratic shocks. In particular, Kim and Ruge-Murcia (2009) calibrate their model to the United States and find an optimal inflation rate of about 0.5 percent; however, the absence of idiosyncratic shocks (central to the traditional argument, and present in our framework) and the locally approximated solution are likely to induce substantial underestimation of such an optimal rate.[8]

The paper is organized as follows. Section I describes the model. Sections II and III present the solutions under flexible and downward-rigid wages, respectively. Section IV solves for the long-run Phillips curve. Section V relaxes the degree of wage rigidities. Section VI draws conclusions.

I. The Model

The closed-economy model is populated by a continuum of infinitely lived households and sectors (both in a $[0,1]$ interval). Each household derives utility from the consumption of a continuum of goods and disutility from supplying a continuum of varieties of labor, which are specific to the households and to the sector in which they are employed. The model assumes the presence of downward nominal rigidities: wages are chosen by optimizing households under the constraint that they cannot fall (this assumption will be relaxed in Section V). In each sector, firms operate in a competitive market to produce one of the continuum of consumption goods. The economy is subject to two aggregate shocks: a productivity and a nominal spending shock. The productivity shock is denoted by A_t, whose logarithmic a_t is distributed as a Brownian motion with drift g and variance σ_a^2:

$$da_t = gdt + \sigma_a dB_{a,t}, \tag{1}$$

[8] Other related works are Torben M. Andersen (2001), presenting a static model which can be solved in a closed form, and V. Bhaskar (2002), offering a framework that endogenizes downward price rigidities. Our model also shares similarities with the literature on irreversible investment (see, among others, Giuseppe Bertola and Ricardo J. Caballero 1994).

where $dB_{a,t}$ denotes a standard Brownian motion with zero drift and unit variance. The nominal spending shock is denoted by $\tilde{Y}_t$, whose logarithmic $\tilde{y}_t$ is also distributed as a Brownian motion, now with drift θ and variance σ_y^2:

$$d\tilde{y}_t = \theta\,dt + \sigma_y dB_{y,t}, \tag{2}$$

where $dB_{y,t}$ is a standard Brownian motion with zero drift and unit variance that might be correlated with $dB_{a,t}$.

The economy is also subject to a continuum of idiosyncratic preference shocks that affect directly the disutility of supplying the varieties of labor among the different sectors i. The logarithmic value of each shock $\xi_t(i)$, with i belonging to the $[0,1]$ interval, is distributed as a Brownian motion with zero drift and variance $\sigma_\xi^2(i)$:

$$d\ln \xi_t(i) = \sigma_\xi(i)\,dB_{\xi,t}(i), \tag{3}$$

where $dB_{\xi,t}(i)$ is a standard Brownian motion with zero drift and unit variance that is correlated across i and is uncorrelated with $dB_{y,t}$ and $dB_{a,t}$. We assume that idiosyncratic shocks cancel out at the aggregate level, i.e.,

$$\int_0^1 \ln \xi_t(i) = 0. \tag{4}$$

Household j has preferences over time given by

$$E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\ln C_t^j - \int_0^1 \frac{[\xi_t(i)\,l_t(j,i)]^{1+\eta}}{1+\eta}\,di\right)dt\right], \tag{5}$$

where the expectation operator $E_{t_0}(\cdot)$ is defined by the shock processes (1), (2), and (3), and $\rho > 0$ is the rate of time preference. Current utility is logarithmic in the consumption aggregate of the continuum of goods i produced in the respective sector

$$C_t^j \equiv e^{\int_0^1 \ln c_t^j(i)\,di}, \tag{6}$$

where $c_t^j(i)$ is household j's consumption of good i. An appropriate consumption-based price index is defined as

$$P_t \equiv e^{\int_0^1 \ln p_t(i)\,di},$$

where $p_t(i)$ is the price of the single good i.

Utility declines with labor efforts. Given (5), each household j supplies a continuum of varieties of labor, each specific to a sector i of the economy. Hence, $l_t(j,i)$ is the variety of labor supplied by household j to sector i. The disutility of exerting labor efforts is separable across the different varieties i and isoelastic, with $\eta \geq 0$ measuring the inverse of the Frisch elasticity of labor supply; the idiosyncratic shock

$\xi_t(i)$ affects in a multiplicative way the disutility that household j faces when supplying $l_t(j,i)$ to sector i.[9] Household j's intertemporal budget constraint is given by

$$E_{t_0}\left\{\int_{t_0}^{\infty} Q_t P_t C_t^j dt\right\} \leq E_{t_0}\left\{\int_{t_0}^{\infty} Q_t\left[\int_0^1 w_t(j,i)\, l_t(j,i)\, di + \Pi_t^j\right] dt\right\}, \tag{7}$$

where Q_t is the stochastic nominal discount factor in capital markets where claims to monetary units are traded; $w_t(j,i)$ is the nominal wage for labor of variety (j,i) offered by household j; and Π_t^j is the profit income that household j derives from the ownership of the firms operating in the economy. (In equilibrium, profits will be zero.)

Starting with the consumption decisions, household j chooses goods demand, $\{c_t^j(i)\}$, to maximize (5) under the intertemporal budget constraint (7), taking prices as given. The first-order conditions for consumption choices imply

$$e^{-\rho(t-t_0)} C_t^{-1} = \chi Q_t P_t, \tag{8}$$

$$\frac{c_t(i)}{C_t} = \left(\frac{p_t(i)}{P_t}\right)^{-1}, \tag{9}$$

where the multiplier χ does not vary over time. The index j is omitted from the consumption's first-order conditions, because we are assuming perfect consumption risk-sharing through a set of state-contingent claims to monetary units. The optimality condition (9) implies the equalization of the consumption expenditure among the different goods.

Before we turn to the labor supply decision, we analyze the firms' problem. In each sector i, firms produce goods in a competitive market using the varieties of labor i supplied by the continuum of household j. However, each household j has a monopoly power in supplying the variety (j,i) of labor. Labor used to produce each good i is a CES aggregate, $L(i)$, of the continuum of individual types of labor of variety i defined by

$$L_t(i) \equiv \left[\int_0^1 l_t^d(j,i)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$

with an elasticity of substitution $\theta > 1$. Here $l_t^d(j,i)$ is the demand for labor of type i supplied by household j. As the production function of each sector i exhibits "love for variety" in types of labor j, every household sells labor to every sector. Given that each differentiated type of labor is supplied in a monopolistic-competitive market, the demand for labor of type (j,i) on the part of wage-taking firms is given by

$$l_t^d(j,i) = \left(\frac{w_t(j,i)}{W_t(i)}\right)^{-\theta} L_t(i), \tag{10}$$

[9] Preferences are consistent with a balanced-growth path since we are assuming a drift in technology.

where $W_t(i)$ is the Dixit-Stiglitz aggregate wage index

$$W_t(i) \equiv \left[\int_0^1 w_t(j,i)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \tag{11}$$

We assume a common linear technology for the production of all goods:[10]

$$y_t(i) = A_t L_t(i).$$

Profits of a generic firm in sector i, $\Pi_t(i)$, are given by

$$\Pi_t(i) = p_t(i) y_t(i) - W_t(i) L_t(i).$$

Perfect competition implies that prices are equal to marginal costs:

$$p_t(i) = \frac{W_t(i)}{A_t}. \tag{12}$$

Since in equilibrium $y(i) = c(i)$, the conditions (9) and (12) imply the following equalities:

$$\tilde{Y}_t = P_t C_t = p_t(i) y_t(i) = W_t(i) L_t(i), \tag{13}$$

where $\tilde{Y}_t$ denotes nominal spending whose logarithmic follows the process (2).

Given firms' demand (10), a household of type j chooses labor supply of variety (j, i) in a monopolistic-competitive market to maximize (5) under the intertemporal budget constraint (7), taking as given prices $\{Q_t\}$, $\{P_t\}$ and the other relevant aggregate variables. The optimization problem is equivalent to maximizing the following objective function:

$$E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\lambda_t w_t(j,i)\, l_t(j,i) - \frac{[\xi_t(i) l_t(j,i)]^{1+\eta}}{1+\eta}\right) dt\right],$$

where λ_t is the marginal utility of nominal income, which is common across households because of the complete market assumption and given by $\lambda_t = (P_t C_t)^{-1} = \tilde{Y}_t^{-1}$. An equivalent formulation of the labor choice is the maximization of the following objective:

$$E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi\left(w_t(j,i), W_t(i), \tilde{Y}_t(i)\right) dt\right], \tag{14}$$

[10] Sector-specific productivities would leave results unchanged, because of the assumption of flexible prices.

by choosing $\{w_t(j,i)\}_{t=t_0}^{\infty}$, where

$$\pi\left(w_t(j,i), W_t(i), \tilde{Y}_t(i)\right) \equiv \left(\frac{w_t(j,i)}{W_t(i)}\right)^{1-\theta} - \frac{1}{1+\eta}\left(\frac{w_t(j,i)}{W_t(i)}\right)^{-(1+\eta)\theta}\left(\frac{\tilde{Y}_t(i)}{W_t(i)}\right)^{1+\eta}.$$

Households would then supply as much labor as demanded by firms in (10) at the chosen wages. In deriving $\pi(\cdot)$, we have used (8), (10), and (13).[11] Note that $\pi(\cdot)$ is homogeneous of degree zero in $(w_t(j,i), W_t(i), \tilde{Y}_t(i))$, and that $\tilde{Y}_t(i)$ is the product of the nominal spending shock and the sectoral idiosyncratic shock $(\tilde{Y}_t(i) \equiv \tilde{Y}_t \xi_t(i))$.

II. Flexible Wages

We first analyze the case in which wages are set without any friction, so that they can be moved freely and fall if necessary. With flexible wages, maximization of (14) corresponds to per-period maximization and implies the following optimality condition:

$$\pi_w\left(w_t(j,i), W_t(i), \tilde{Y}_t(i)\right) = 0,$$

where $\pi_w(\cdot)$ is the derivative of $\pi(\cdot)$ with respect to the first argument. Since all wage setters in sector i face the same problem, the equilibrium is symmetric, $w_t(j,i) = W_t(i)$ for each j. Given our preference specification, nominal wages in sector i, denoted by $W_t^f(i)$, are proportional to the combination of the aggregate nominal spending shock and the idiosyncratic shock

$$W_t^f(i) = \mu^{\frac{1}{1+\eta}} \tilde{Y}_t \xi_t(i), \tag{15}$$

where the factor of proportionality is given by the wage markup, defined by $\mu \equiv \theta/(\theta - 1)$, and by the elasticity of labor supply. We can also obtain the flexible-wage equilibrium level of labor in sector i, $L^f(i)$, using (13) and (15),

$$L^f(i) = (\mu)^{-\frac{1}{1+\eta}} \xi_t(i)^{-1},$$

which depends on the wage markup as well as on the labor elasticity, and is negatively related to the idiosyncratic shock $\xi_t(i)$. Aggregate labor, L^f, defined by

$$L^f \equiv e^{\int_0^1 \ln L^f(i) di},$$

is therefore constant at

$$L^f = (\mu)^{-\frac{1}{1+\eta}},$$

[11] The productivity shock, A_t, does not enter the objective function because of three assumptions: (i) the log utility in consumption, which is compatible with a balanced-growth path; (ii) the flexibility of prices, which allows us to isolate the effect of the downward rigidity constraint in wages; and (iii) the exogeneity of the process of nominal spending (notice that assumptions (i) and (iii) are also in Golosov and Lucas 2007). Productivity would, of course, affect the optimization problem insofar as it influences nominal spending growth. Adding menu-cost pricing would enrich the model and would open the way for an additional effect of productivity.

because of assumption (4). Note that aggregate labor does not depend on the productivity shock, because of the log utility, and does not depend on the idiosyncratic shocks, which instead shift wages and employment across sectors:

$$\frac{L^f(i)}{L^f(i')} = \frac{\xi_t(i')}{\xi_t(i)} = \frac{W_t(i')}{W_t(i)}.$$

Consumption and output follow from the production function and, in particular, the flexible level of output is given by

$$Y_t^f = A_t L^f,$$

which moves proportionally to the productivity shock. With flexible wages and prices, output is always at potential and the Phillips curve is vertical.

III. Downward Nominal Wage Rigidity

When nominal wages cannot fall below the level reached in the previous period, an additional condition needs to be taken into account: the constraint that $dw_t(j, i)$ should be nonnegative (Section V explores less stringent constraints).[12] The households' objective is then to maximize (14) under

$$dw_t(j, i) \geq 0, \tag{16}$$

with $w_{t_0}(j, i) > 0$. In other words, agents choose a nondecreasing positive nominal wage path to maximize (14). Let us define the value function $V(\cdot)$ for this problem as

$$V\big(w_t(j, i), W_t(i), \tilde{Y}_t(i)\big) = \max_{\{w_\tau(j,i)\} \in \mathcal{W}} E_t\left\{\int_t^\infty e^{-\rho(\tau - t)} \pi\big(w_\tau(j, i), W_\tau(i), \tilde{Y}_\tau(i)\big) d\tau\right\},$$

where $\mathcal{W}$ is the set of nondecreasing positive sequences $\{w_\tau(j, i)\}_t^\infty$. Optimality conditions require

$$V_w\big(w_t(j, i), W_t(i), \tilde{Y}_t(i)\big) = 0 \qquad \text{if } dw_t(j, i) > 0,$$

$$V_w\big(w_t(j, i), W_t(i), \tilde{Y}_t(i)\big) \leq 0 \qquad \text{if } dw_t(j, i) = 0,$$

where $V_w(\cdot)$ is the derivative of $V(\cdot)$ with respect to the first argument. Moreover, the maximization problem is concave and the conditions above are also sufficient to characterize a global optimum as shown in the Appendix. It follows that all wage

[12] The downward-rigidity constraint is purely exogenous in this model and could be rationalized by considering every worker as associated with a union that does not allow the wage to decline for reasons related to fairness and social norms (Bewley 1999; Akerlof 2007).

setters in sector i are going to set the same wage, $w_t(j, i) = W_t(i)$ for all j. As we further show in the Appendix, the solution to this problem corresponds to finding a function $W(\tilde{Y}_t(i))$ that satisfies appropriate boundary conditions and represents the current *desired* wage, taking into account future downward-rigidity constraints but not the current constraint (i.e., if agents were free to choose the current wage, even below the past wage, but considering that future wages cannot fall).

The downward constraint implies that if attempting to set $W_t(i) = W(\tilde{Y}_t(i))$ would entail $dW_t(i) < 0$, then the wage would remain unchanged, so that $dW_t(i) = 0$ and $W_t(i) > W(\tilde{Y}_t(i))$. Otherwise, agents will set $W_t(i) = W(\tilde{Y}_t(i))$, so that $dW_t(i) \geq 0$. Hence, *actual* wages, $W_t(i)$, are the maximum of past wages and current *desired* wages $W(\tilde{Y}_t(i))$. It follows that actual wages cannot fall below current desired wages, i.e., $W_t \geq W(\tilde{Y}_t(i))$: actual wages are either above the desired level, when the downward-rigidity constraint is binding, or equal to the desired level, when a wage adjustment occurs.

The desired wage is always lower than the flexible-equilibrium wage by a factor $c_i(\cdot)$, as shown in the Appendix:[13]

$$(17) \qquad \begin{aligned} W(\tilde{Y}_t(i)) &= c(\theta, \sigma^2(i), \eta, \rho) \cdot \mu^{\frac{1}{1+\eta}} \tilde{Y}_t \xi_t(i) \\ &= c(\theta, \sigma^2(i), \eta, \rho) \cdot W_t^f(i), \end{aligned}$$

where $\sigma^2(i)$ (a crucial parameter in our model) is defined as the sum of the variances of the aggregate nominal spending shocks and the idiosyncratic shocks, $\sigma^2(i) \equiv \sigma_y^2 + \sigma_\xi^2(i)$, and $c_i(\cdot)$ is a nonnegative function of the model's parameters as follows:

$$(18) \quad c(\theta, \sigma^2(i), \eta, \rho) \equiv \left(\frac{\theta + \frac{1}{2}\gamma(\theta, \sigma^2(i), \rho) \cdot \sigma^2(i)}{\theta + \frac{1}{2}(\gamma(\theta, \sigma^2(i), \rho) + \eta + 1) \cdot \sigma^2(i)} \right)^{\frac{1}{1+\eta}} \leq 1,$$

with $\gamma(\cdot)$ being the following nonnegative function:

$$\gamma(\theta, \sigma^2(i), \rho) = \frac{-\theta + \sqrt{\theta^2 + 2\rho\sigma^2(i)}}{\sigma^2(i)},$$

as derived in the Appendix.[14]

Hence, agents' optimizing behavior in the presence of exogenous downward wage rigidities implies an endogenous tendency for *upward* wage rigidities, as indicated by $c_i(\cdot)$ being ≤ 1. Indeed, optimizing wage setters try to offset the inefficiencies of downward wage inflexibility, as they are worried about being stuck with an excessively high wage should future unfavorable shocks require a wage decline or a

[13] We use interchangeably $c_i(\cdot)$ for $c(\theta, \sigma^2(i), \eta, \rho)$.

[14] It is possible that the desired wage, $W(\tilde{Y}_t(i))$, falls below the one associated with full employment. While temporary overemployment is not unrealistic, in Benigno and Ricci (2008) we also solve the model with the additional constraint $l_t(j, i) \leq 1$ for each j, with similar results.

fall in employment. As a consequence, they refrain from excessive wage increases when favorable shocks require upward adjustment, thus keeping the current wage low and pushing current employment above the flexible-case level.[15]

Note that actual wages (unlike desired wages) are not necessarily below the flexible-equilibrium wage. Indeed, when the downward-rigidity constraint is binding, actual wages are higher than desired wages and are likely to be higher (and employment lower) than with flexible wages. As we will see in the next section, in the long run, the average output gap is negative, and a lower $c_i(\cdot)$ would help reduce its size.

The desired wage level is a lower fraction of the flexible-equilibrium wage (i.e., $c_i(\cdot)$ is low) when the variances of nominal expenditure growth and/or of the idiosyncratic shocks are high ($\sigma^2(i)$ large), as it is more likely that negative shocks would force wages to hit the lower bound; when the mean of nominal expenditure growth is small (θ small), as it is more likely that even small shocks would push wages to hit the lower bound;[16] when agents discount less the future (ρ low), as they are more concerned with the future negative consequences of current wage decisions; and when the elasticity of labor is higher (η low), as agents are willing to accept larger fluctuations in hours worked in order to ensure a higher average employment.

In Figure 1 we plot $c_i(\cdot)$ as a function of the mean of the log of nominal spending growth, θ, with different assumptions on the overall standard deviation of the shocks, $\sigma(i)$, ranging from 0 to 20 percent at annual rates. The parameters' calibration is based on a discretized quarterly model. In particular, the rate of time preference ρ is equal to 0.01 as standard in the literature, implying a 4 percent real interest rate at annual rates; and the Frisch elasticity of labor supply is set equal to 0.4, as it is done in several studies, therefore implying $\eta = 2.5$.[17] When $\sigma(i) = 0$ percent, $c_i(\cdot) = 1$. With positive standard deviations, $c_i(\cdot)$ decreases as θ decreases (i.e., the gap between desired wages and flexible-equilibrium wages widens when inflation is lower). The decline in $c_i(\cdot)$ is larger the higher is the standard deviation of the nominal spending shock and/or of the idiosyncratic shock, as previously discussed.

IV. The Phillips Curve

We can now solve for the equilibrium level of output and characterize the long-run inflation-output trade-off in the presence of downward nominal wage rigidities. We define the output gap as the difference between output under downward wage rigidity and output under flexible wages and prices, which is equal to the difference between the corresponding employment levels. In logs terms we can write

$$y_t - y_t^f = \ln L_t - \ln L^f = \int_0^1 \ln L_t(i)\, di - \ln L^f. \tag{19}$$

[15] This result is consistent with the theoretical argument and empirical evidence offered by Elsby (2009). While he emphasizes the importance of idiosyncratic shocks, we also stress the importance of macroeconomic volatility.

[16] When the drift in nominal spending growth becomes very large, it is unlikely that downward wage inflexibility is going to be binding, so that $c_i(\cdot)$ gets close to one and the flexible-wage level of employment will be achieved most of the time.

[17] See, for example, Frank Smets and Raf Wouters (2003).

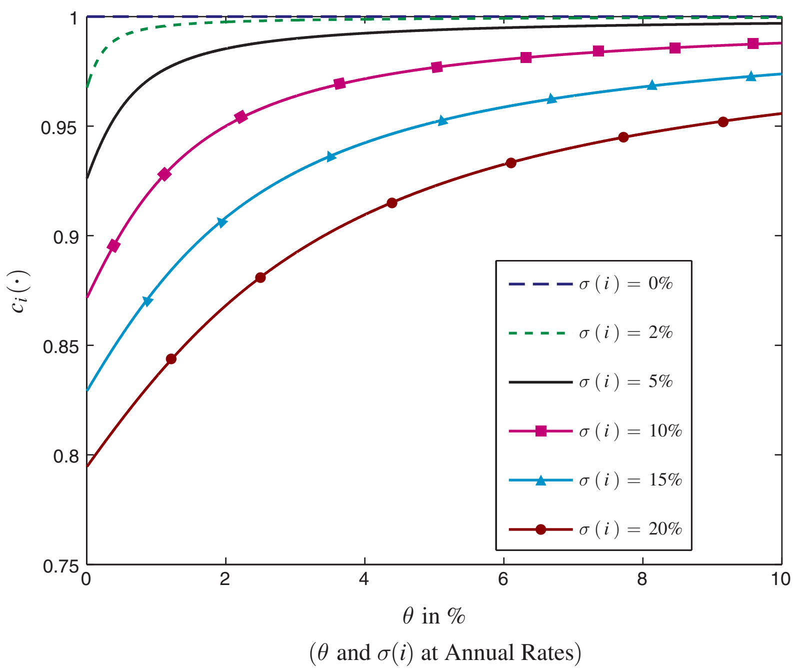


Figure 1. Plot of the Function $c_i(\cdot)$ Defined in (18)

Equation (13) implies that

$$L_t(i) = \frac{\tilde{Y}_t}{W_t(i)}.$$

To compute the equilibrium output gap, it is convenient to define the variable $X_t(i)$ such that $X_t(i) \equiv \xi_t(i) L_t(i)$, from which it follows

$$\int_0^1 \ln X_t(i)\, di = \int_0^1 \ln L_t(i)\, di \tag{20}$$

because of assumption (4). Moreover,

$$X_t(i) = \frac{\tilde{Y}_t(i)}{W_t(i)}.$$

Since we have shown that $W_t(i) \geq c_i(\cdot)\mu^{1/(1+\eta)}\tilde{Y}_t(i)$, it is the case that $0 \leq X_t(i) \leq L^f/c_i(\cdot)$. The existence of downward wage rigidities endogenously adds an upward barrier on the variable $X_t(i)$. Since $\tilde{y}_t(i) \equiv \ln \tilde{Y}_t(i)$ follows a Brownian motion with drift θ and variance $\sigma^2(i)$, $x_t(i) = \ln X_t(i)$ will also follow a Brownian motion with the same properties, when $dW_t(i) = 0$, but with a regulating barrier at $\ln(L^f/c_i(\cdot))$. The probability distribution function for such a process can be computed at each point in time.[18] When the drift of the process $\tilde{y}_t(i)$ is positive, i.e., $\theta > 0$, this probability distribution converges to an equilibrium distribution for $t \to \infty$, a result that allows the characterization of the long-run probability distribution for employment, and thus the output gap.[19] In this case, it can be shown that the long-run cumulative distribution of $x_t(i)$, denoted with $P(\cdot)$, is given by

$$P(x_\infty(i) \leq z) = e^{\frac{2\theta}{\sigma^2(i)}[z-(\ln L^f - \ln c_i)]}$$

for $-\infty \leq z \leq \ln(L^f/c_i(\cdot))$, where $x_\infty(i)$ denotes the long-run equilibrium level of the variable $x_t(i)$. We can also evaluate the long-run mean of $x_t(i)$:

$$E[x_\infty(i)] = \ln L^f - \ln c_i(\cdot) - \frac{\sigma^2(i)}{2\theta}. \tag{21}$$

Integrating across the sectors i, we obtain

$$\int_0^1 E[x_\infty(i)]\,di = \ln L^f - \int_0^1 \ln c_i(\cdot)\,di - \int_0^1 \frac{\sigma^2(i)}{2\theta}\,di. \tag{22}$$

We can then substitute (22) into (19) using (20) to obtain

$$E(y_\infty - y_\infty^f) = -\int_0^1 \ln c(\theta, \sigma^2(i), \eta, \rho)\,di - \int_0^1 \frac{\sigma^2(i)}{2\theta}\,di.$$

To construct the long-run Phillips curve, a relationship between average wage inflation and output gap, we need to solve for the long-run equilibrium level of wage inflation.[20] From the equilibrium condition (13), we note that

$$\tilde{y}_t = \int_0^1 x_t(i)\,di + \int_0^1 \ln W_t(i)\,di,$$

from which it follows that

$$d\tilde{y}_t = \int_0^1 dx_t(i)\,di + \pi_t^w\,dt,$$

[18] See David R. Cox and Hilton D. Miller (1990, 223–25) for a detailed derivation.

[19] When $\theta \leq 0$, the probability distribution collapses to zero everywhere, with a spike of one at zero employment. However, a negative average nominal spending growth, θ, is not realistic.

[20] While the original formulation of the Phillips curve was in terms of unemployment and wage inflation (Phillips 1958), this paper defines it as the trade-off between the output gap and wage inflation. The output gap has indeed been widely used in modern macro models as a measure of slack. Benigno and Ricci (2008) present the equivalent formulation in terms of unemployment-inflation trade-off.

where π^w is the rate of aggregate wage inflation in the economy. Since $E(d\tilde{y}_t) = \theta\, dt$ and $dx_t(i)$ converges to an equilibrium distribution for each i, implying $E(dx_\infty(i)) = 0$, the long-run mean wage inflation rate is given by

$$E[\pi^w_\infty] = \theta. \tag{23}$$

Substituting (23) into (21), we obtain the following closed-form solution for the long-run Phillips curve:

$$E(y_\infty - y^f_\infty) = -\int_0^1 \ln c(E[\pi^w_\infty], \sigma^2(i), \eta, \rho)\, di - \int_0^1 \frac{\sigma^2(i)}{2E[\pi^w_\infty]}\, di, \tag{24}$$

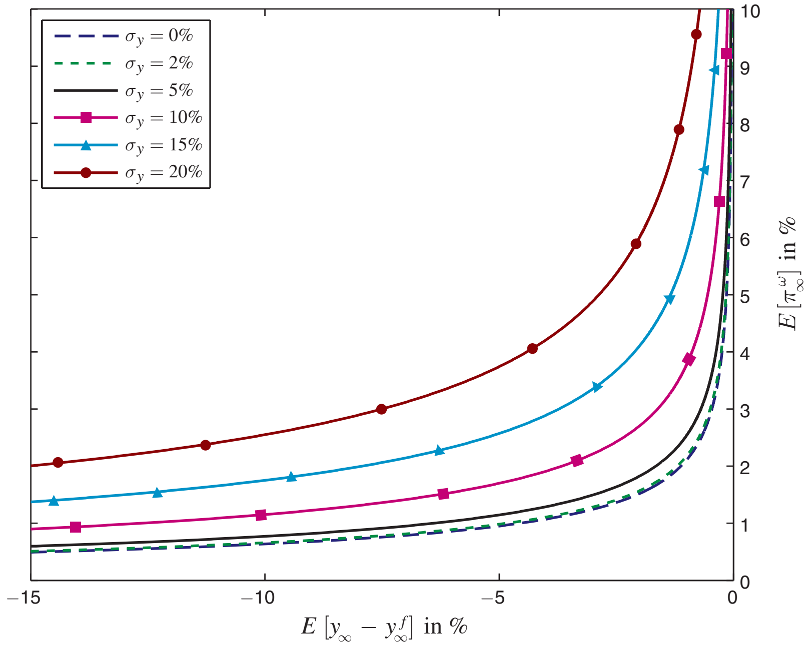
a relation between mean output gap and mean wage inflation rate.

The long-run Phillips curve is no longer vertical and the "natural" rate of output is not unique, but depends on the mean inflation rate. There are two components (influenced by the parameters of the model η, ρ, and $\sigma^2(i)$) which explain the long-run Phillips curve and act on opposite directions. The first integral on the right-hand side captures the forward-looking reaction of wage setters to the presence of downward wage rigidities, which induces them to set a wage lower than the flexible one when adjusting their wage (as captured by $c_i(\cdot) \leq 1$), and hence generates a positive output gap. Such a gap would be larger the lower is $c_i(\cdot)$. The second integral depends on the variance-to-mean ratio and captures the cost of downward wage rigidities in the presence of a need for relative price adjustments, which is the standard argument supporting the presence of a Phillips curve.[21]

The resulting output gap is always nonpositive in the long run (i.e., $E(y_\infty - y^f_\infty) \leq 0$), because the second component dominates, since $-\ln c_i(\cdot) \leq \sigma^2(i)/(2E[\pi^w_\infty])$.[22] Also, the output gap is larger when the volatility is higher and when the mean of inflation is low, because the downward wage constraint is more likely to be binding and more costly in terms of lower employment. Indeed, when the mean wage inflation rate becomes very high, the average output gap converges to zero, as the two components of the gap get close to zero: $c_i(\cdot)$ becomes close to one, and the costs of downward rigidities become small. Hence, at high inflation rates, the Phillips curve is almost vertical, and there is virtually no long-run trade-off between inflation and output gap. When, instead, wage inflation is low, a trade-off emerges (the Phillips curve is flatter) and depends heavily on the volatility of the economy. If there is no uncertainty, $\sigma^2(i) = 0$ and $c_i(\cdot) = 1$, then the long-run output gap is zero. In the stochastic case, the higher the variance of nominal-spending growth and of the idiosyncratic shocks ($\sigma^2(i)$), the more a fall in the inflation rate would worsen the average output gap (generating a more negative gap), and flatten

[21] Note that our dynamic framework introduces not only the need for intratemporal relative wage adjustments, due to $\sigma^2_\xi(i)$, as in Akerlof, Dickens, and Perry(1996), but also the need for intertemporal relative wage adjustment, arising from σ^2_y.

[22] Benigno and Ricci (2008) show that in the short run the Phillips curve may imply a positive (rather than negative) output gap.



($E[\pi_\infty^\omega]$ and $\sigma_y(i)$ at Annual Rates)

Figure 2. Long-Run Phillips Curve

the Phillips curve.[23] These patterns are evident in Figure 2, which plots the long-run Phillips curve for different levels of volatility.[24]

As an illustrative example, the model would suggest (on the basis of the parametrization underlying Figure 2) that a country that is subject to low macroeconomic volatility (say a standard deviation of nominal GDP growth equal to 2 percent) may experience a worsening of the output gap equal to 0.4 percent of flexible-wage GDP when average wage inflation declines from 6 to 3 percent, and equal to 4.6 percent when wage inflation goes from 4 to 1 percent (see Table 1). However, a country with a significant macroeconomic volatility (say 10 percent) may face much larger costs (about −1.2 percent and −11.8 percent, respectively).[25]

Our model therefore suggests that a reduction in the macroeconomic volatility as a consequence of better stabilization policies can have important first-order effects, unlike the arguments of Lucas (1987, 2003), and substantially improve the output gap, especially at low wage inflation (as shown in Table 2). At a wage inflation rate

[23] In Lucas (1973), higher volatility reduces the information content of relative price dispersion. Introducing such an effect would steepen the Phillips curve.

[24] The standard deviation of the idiosyncratic shocks ($\sigma_\xi(i)$) is set at 10 percent for all sectors, such that (for a σ_y on the order of 5 to 10 percent) the overall $\sigma^2(i)$ would roughly imply the standard deviation of annualized changes in wages observed in microstudies (Barattieri, Basu, and Gottschalk 2009; Card and Hyslop 1997). The other parameters are as in Figure 1.

[25] In reality, macroeconomic volatility of nominal GDP growth is likely to decline as inflation comes down, which would imply a steeper Phillips curve. However, the decline should be less than proportional (mainly because of the real GDP component; see Benigno and Ricci 2008, for simple supporting evidence), so that volatility would persist even at zero inflation. Moreover, the volatility of idiosyncratic shocks is likely to be affected even less than the aggregate one when inflation declines.

TABLE 1—WORSENING IN OUTPUT GAP DUE TO REDUCTION IN WAGE INFLATION

	σ_y					
$\Delta E[y_\infty - y^f_\infty]$	0 percent	2 percent	5 percent	10 percent	15 percent	20 percent
Reduction in $E[\pi^w_\infty]$ from:						
4 percent to 1 percent	−4.3	−4.6	−6.0	−11.8	−22.4	−38.1
5 percent to 2 percent	−1.0	−1.0	−1.4	−3.1	−6.6	−12.2
6 percent to 3 percent	−0.4	−0.4	−0.5	−1.2	−2.8	−5.5

Notes: Change in long-run mean output gap, $\Delta E[y^f_\infty]$, due to a reduction in long-run mean wage inflation rate, $\Delta E[\pi^w_\infty]$, for different standard deviation of nominal spending growth, σ_y. $E[y_\infty - y^f_\infty]$ is in percent, $E[\pi^w_\infty]$ and $\sigma_y(i)$ in percent and at annual rates; $\eta = 2.5$, $\rho = 0.01$, $\sigma_\xi(i) = \sigma_\xi = 10$ percent.

TABLE 2—GAIN IN OUTPUT GAP DUE TO REDUCTION IN VOLATILITY OF SPENDING

	$E[\pi^w_\infty]$				
$\Delta E[y_\infty - y^f_\infty]$	1 percent	2 percent	3 percent	4 percent	5 percent
Reduction in σ_y from:					
2 percent to 0 percent	0.28	0.08	0.03	0.02	0.01
5 percent to 0 percent	1.84	0.53	0.23	0.12	0.08
10 percent to 5 percent	6.24	2.00	0.92	0.51	0.32
15 percent to 10 percent	18.06	6.27	3.03	1.74	1.11

Notes: Change in long-run mean output gap, $\Delta E[y_\infty - y^f_\infty]$, due to a reduction in the standard deviation of nominal spending growth, $\Delta\sigma_y$, for different long-run mean wage inflation rates, $E[\pi^w_\infty]$. $E[y_\infty - y^f_\infty]$ is in percent, $E[\pi^w_\infty]$ and $\sigma_y(i)$ in percent and at annual rates; $\eta = 2.5$, $\rho = 0.01$, $\sigma_\xi(i) = \sigma_\xi = 10$ percent.

of 2 percent, reducing the macroeconomic volatility from 5 to 0 percent improves the output gap by about 0.5 percent. The improvement is four times larger if the volatility is reduced from 10 to 5 percent (for the same level of wage inflation), while it is more than three times larger if volatility declines from 5 to 0 percent when the wage inflation rate is at 1 percent.

Notice that in the long run wage inflation is equal to the sum of productivity-growth mean and price inflation. Hence, when interpreting the implications of the model (such as in Figure 2 or Table 1 or 2) for price inflation, wage inflation should be correspondingly adjusted by the level of productivity growth. For a given inflation target in terms of good prices, a lower level of productivity would lower wage inflation and increase the cost of downward wage rigidities.

A few additional implications arise from the model.[26] First, the probability that wages remain fixed depends on the level of wage inflation and on the degree of macroeconomic volatility. When wage inflation is very low or the variance of the shocks is high, the probability that wages remain rigid even upward is close to one. The probability declines when inflation increases—in line with the evidence of Card and Hyslop (1997) that the fraction of wages subject to rigidities is higher when wage inflation is low—and it declines faster when macroeconomic volatility is lower.

[26] Benigno and Ricci (2008) provide a more extensive discussion.

Second, a long run trade-off between volatility of wage inflation and volatility of output gap emerges, for given distributions of the idiosyncratic shocks. Indeed, at low inflation there is more adjustment via employment and less via wages, while the opposite emerges at high-wage inflation. Trade-offs of this nature have been generally assumed in monetary policy analysis over the past 30 years (see Finn E. Kydland and Edward C. Prescott 1977; Robert J. Barro and David B. Gordon 1983). Woodford (2003) has provided a microfoundation for these trade-offs and for their link to monetary reaction functions widely employed in inflation-targeting models (although he derives the trade-off as a local approximation, while in our model it is a feature of the global equilibrium).

V. Relaxing the Downward Rigidity Constraint

The benchmark model presented in the previous sections encompasses nominal wage rigidities as a constraint, which is homogenous across agents and is independent of the level of wage inflation, the degree of macroeconomic volatility, and the presence of large shocks. The reality is more nuanced, and this section explores various ways to relax this assumption. First, we consider the case in which the threshold for wage changes at which downward wage rigidities become binding may be negative (rather than zero) and may depend on wage inflation and volatility. This corresponds to the idea that when agents expect the constraint to be more relevant, they could adjust their behavior and set wages more flexibly. Second, we allow for some heterogeneity, by considering the case in which only some agents are subject to the constraint. Finally, we offer a setup in which wage rigidities may not be binding when high-variance shocks occur.

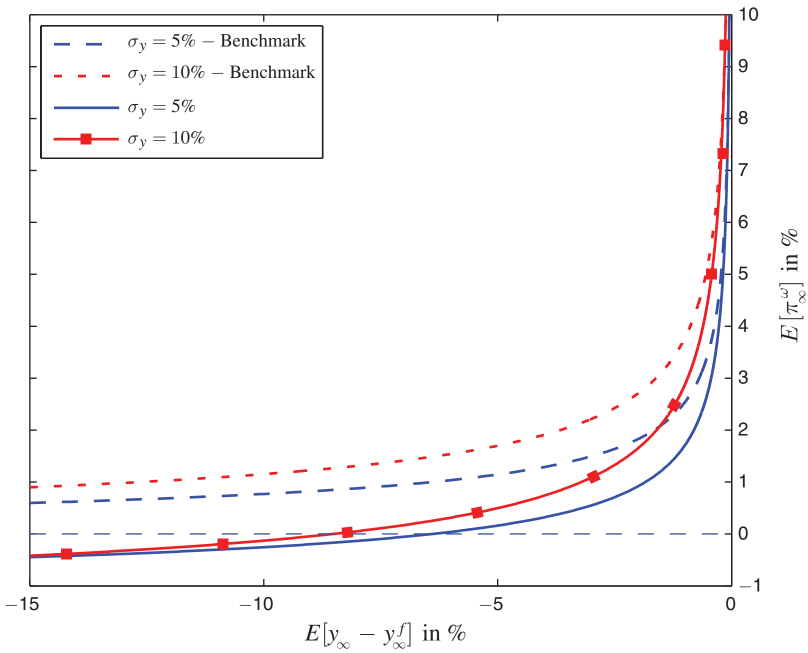
A. *Varying the Degree of Downward Rigidities*

The main criticism of an approach that includes downward wage rigidities is that this inflexibility should disappear as the wage inflation rate declines toward zero (see the comments to Akerlof, Dickens, and Perry 1996; Ball and Mankiw 1994). As we discussed in the introduction, there is now more evidence that downward wage rigidities persist even during low wage inflation periods. Nonetheless, it is valuable to explore the implications of a link between the degree of downward rigidities and wage inflation by replacing the assumption $dw_t(j,i) \geq 0$ with

$$dw_t(j,i) \geq -\kappa(\theta, \sigma^2(i))\, w_t(j,i)\, dt, \tag{25}$$

which nests the previous model. Nominal wages are now allowed to fall, but the percentage decline cannot exceed $\kappa(\theta, \sigma^2(i))$, where $\kappa(\theta, \sigma^2(i))$ is a decreasing function of the mean of nominal-spending growth θ (so that at lower inflation, wages can fall more), and also an increasing function of the variance of the aggregate and idiosyncratic shocks $\sigma^2(i)$ (with higher variance wages can fall more).[27] The solution

[27] The relationship between wage setting and volatility is explored by Jo Anna Gray (1976).



($E[\pi_\infty^\omega]$ and $\sigma_y(i)$ at Annual Rates)

FIGURE 3. LONG-RUN PHILLIPS CURVE: BENCHMARK VERSUS MODEL GIVEN BY CONSTRAINTS (25) AND (26)

of the model is similar to the previous case except that θ should be replaced by $\lambda(\theta,\sigma^2(i)) \equiv \theta + \kappa(\theta,\sigma^2(i))$.[28] In particular, the long-run Phillips curve becomes

$$E(y_\infty - y_\infty^f) = -\int_0^1 \ln c\left(\lambda(E[\pi_\infty^w],\sigma^2(i)),\sigma^2(i),\eta,\rho\right) di - \int_0^1 \frac{\sigma^2(i)}{2\lambda\left(E[\pi_\infty^w],\sigma^2(i)\right)} di,$$

since it is still true that $E[\pi_\infty^w] = \theta$. Obviously the way the rigidities endogenously decline (i.e., the functional form of $\kappa(\theta,\sigma^2(i))$) is crucial in shaping the Phillips curve.

For example, if the percentage decline could not exceed a fixed amount κ_1 (hence, $\kappa(\cdot) = \kappa_1$), then the Phillips curve would simply shift down by κ_1 (when compared to the one presented in Figure 2). Under more general assumptions for $\kappa(\theta,\sigma^2(i))$, the effect of inflation would be to tilt the Phillips curve counterclockwise at low inflation, while an increase in volatility would steepen the curve (as

[28] In this case, the condition ensuring that the probability distributions converge in the long run to a nontrivial distribution becomes $\lambda(\theta,\sigma^2(i)) > 0$. A supplementary Appendix that presents the model solution under this general case is available upon request.

the downward wage rigidities become less binding).[29] For illustrative purposes, Figure 3 shows the Phillips curve resulting from equation (25) and the following function:

$$\kappa\left(\theta, \sigma^2(i)\right) = \sqrt{\sigma^2(i)}(\kappa_1 - \kappa_2\theta) \tag{26}$$

for two different levels of volatility. Compare these curves with the benchmark ones from Figure 2.[30] The cost of low wage inflation in terms of output gap would decline, but would remain nonnegligible. Reducing wage inflation from 5 to 2 percent worsens the output gap by 0.6 percent, when $\sigma_y = 5$ percent and $\sigma_\xi(i) = 10$ percent, compared to the benchmark case in which the reduction was 1.4 percent, and by 1.1 percent, when $\sigma_y = 10$ percent and $\sigma_\xi(i) = 10$ percent, compared to the benchmark case in which the reduction was 3.1 percent.

B. *Heterogeneous Rigidities*

This subsection allows for heterogeneity in the way the rigidity affects agents, in line with recent findings of Barattieri, Basu, and Gottschalk (2009), who suggest the presence of heterogeneity across occupations. To preserve simplicity, we make the assumption that a fraction of sectors i (of measure α, with $0 \leq \alpha \leq 1$) employs wage setters constrained by downward rigidities, while the remaining fraction $1 - \alpha$ enjoys wage flexibility. It can be easily shown that the long-run Phillips curve in this case becomes

$$E(y_\infty - y^f_\infty) = -\int_0^\alpha \ln c(E[\pi^w_\infty], \sigma^2(i), \eta, \rho)\, di - \int_0^\alpha \frac{\sigma^2(i)}{2E[\pi^w_\infty]}\, di,$$

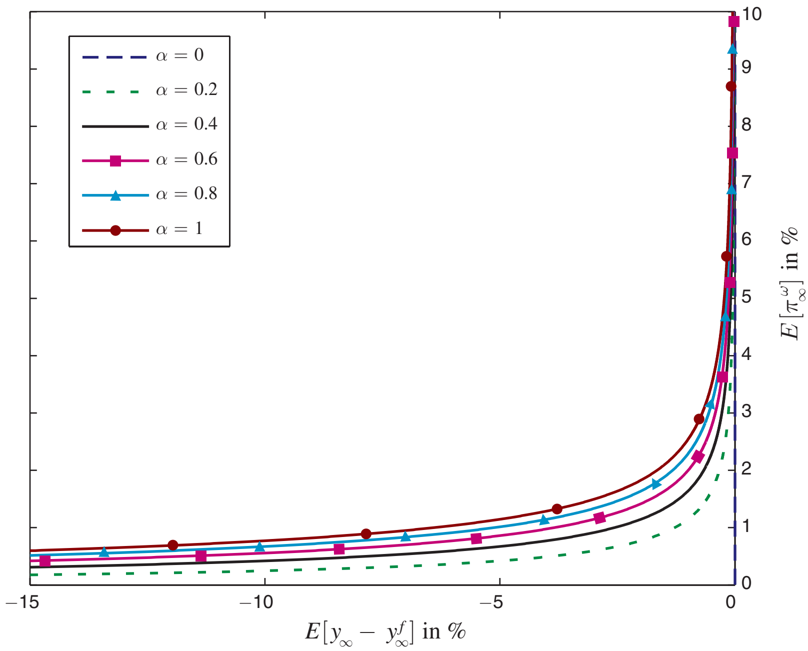
where the only difference is that integrals are taken over a different interval $[0, \alpha)$, i.e., across the sectors affected by downward-wage rigidities. The two boundary values for α nest the models presented in Sections II and III: the flexible case when $\alpha = 0$, and the rigidity constraint case when $\alpha = 1$.

The presence of some flexible wages generates a more vertical Phillips curve (see Figure 4 for various degrees of wage flexibility in the case of moderate volatility, $\sigma_y = 5$ percent).[31] Still, the costs are significant even when α is small. For example, when α is just 0.2, meaning that 20 percent of firms are constrained by downward wage rigidities, then lowering wage inflation from 5 percent to 2 percent still produces costs equal to 0.3 percent, which are obviously smaller than the 1.4 percent found in the benchmark case, but not negligible.

[29] Obviously, if $\kappa(\theta, \sigma^2(i))$ were to be very large for any theta, then the Phillips curve would become virtually vertical. However, as discussed extensively in the introduction, there is substantial evidence that downward wage rigidities persist even at low inflation.

[30] We set κ_1 and κ_2 such that $\kappa_1\sigma(i) = 1$ percent at annual rates and $\kappa_2\sigma(i) = 0.1$ under the assumption $\sigma_y = 5$ percent (for comparability, the same κ_1 and κ_2 are maintained when $\sigma_y = 10$ percent). Other parameters as in Figure 2. Note that the various Phillips curves associated with different levels of volatility would now cross, as a change in volatility not only shifts the curve outward, but also steepens it.

[31] All other parameters are as in Figure 2.



($E[\pi_\infty^\omega]$ at Annual Rates; $(1 - \alpha)$ is the Fraction of Flexible-Wage Sectors)

Figure 4. Long-Run Phillips Curve in the Presence of Some Flexible-Wage Sectors

C. *Adjustment under High-Variance Shocks*

This subsection extends the benchmark model to the case in which high-variance shocks warrant a wage adjustment, by introducing two additional features. First, we assume, on top of the aggregate and idiosyncratic shocks, the presence of additional idiosyncratic shocks that hit less frequently but with large variations. When wages are affected by such high-variance idiosyncratic shocks, wage setters can adjust their wages either upward or downward in an optimal way. When, instead, agents do not face these infrequent idiosyncratic shocks, they are subject to the usual downward wage rigidity constraint.[32]

Second, we introduce the probability of switching between the low- and the high-probability regime, which captures the frequency of occurrence of high-variance shocks. This is indeed an important parameter in order to study the relevance of the real effects of monetary policy.[33] If the large shocks were occurring very frequently,

[32] The idea that wages can adjust in a state-contingent way following high-variance shocks is borrowed from the menu-cost literature on firms' pricing (see, in particular, Mark Gertler and John Leahy 2008; Golosov and Lucas 2007). To preserve simplicity, we approximate the implications of an Ss model by introducing a regime-switching model for the idiosyncratic shocks between a low- and a high-volatility regime. The approximation is accurate to the extent to which an Ss model would trigger an adjustment for most of the shocks of the high-volatility regime, which is more likely when the variance of such shocks is high (as is the case in our model for the variance of the additional idiosyncratic shocks).

[33] In the Golosov and Lucas (2007) model, large shocks are very frequent, so the real effects of monetary policy are small. On the contrary, Gertler and Leahy (2008) shows that with infrequent idiosyncratic shocks it is still possible to characterize the response of the economy to aggregate shocks through a Phillips curve.

wages would adjust often and the Phillips curve would be quite vertical. With infrequent large shocks, wages would be more subject to the downward wage rigidity constraint and the Phillips curve would be flatter, as in the benchmark model. We discuss below how micro-data evidence on the frequency of wage adjustments and on the wage distribution can help discriminate between these two views.

To model such probability of switching, we add a process $\{s_t\}$ that follows a two-state Markov chain taking values 1 and 2. These two states are associated, respectively, with the benchmark situation of downward wage rigidities and with the case in which wages can freely adjust. We assume that the process $\{s_t\}$ has matrix of transition probabilities between time t and $t + dt$ given by

$$\begin{bmatrix} 1 - \lambda dt & \lambda dt \\ \phi dt & 1 - \phi dt \end{bmatrix},$$

where λdt is the interperiod probability of switching from state 1 to state 2; $1 - \lambda dt$ is the probability of remaining in state 1; ϕdt is the probability of switching from state 2 to 1; and $1 - \phi dt$ is the probability of remaining in state 2.[34] Given this structure, we assume that the idiosyncratic shock $\xi_t(i)$ is now given by two multiplicative components, $\xi_t(i) = \xi_{v,t}(i)\varepsilon_t(i)$, where $\xi_{v,t}(i)$, as in the benchmark model, exhibits its logarithmic distributed as a Brownian motion with zero drift and variance $\sigma_\xi^2(i)$:

$$d\ln \xi_{v,t}(i) = \sigma_\xi(i)\, dB_{\xi,t}(i),$$

while the additional term is given by the shock $\varepsilon_t(i)$, whose log is distributed as

$$d\ln \varepsilon_t(i) = \sigma_\varepsilon(i,s)\, dB_{v,t}(i),$$

where

$$\sigma_\varepsilon(i, s = 1) = 0,$$

$$\sigma_\varepsilon(i, s = 2) > 0,$$

$dB_{v,t}(i)$ might be correlated with $dB_{\xi,t}(i)$, and both are standard Brownian motion with zero mean and unitary variance. In state 1, the time variation of the shock ε_t is zero so that it does not move; in state 2, instead, its variation follows a Brownian motion with variance $\sigma_\varepsilon^2(i)$.[35]

In light of these two additions to the model, wage setters still maximize the objective function (14) but now they take into account the possibility of freely adjusting wages when state 2 occurs, while in state 1 they continue to face the downward rigidity constraint; moreover, they anticipate the possibility of switching across states. Optimality conditions require that the derivative of the value function with respect to wages in state 2 is equal to zero, i.e., $V_w(w_t(j,i), W_t(i), \tilde{Y}_t(i), s = 2) = 0$, since

[34] It is assumed that $0 \leq \lambda dt \leq 1$ and $0 \leq \phi dt \leq 1$.

[35] The model of this section nests the benchmark model under the assumption that $\lambda dt = 0$ and the flexible-wage model under the assumption that $\phi dt = 0$.

in this state it is possible to relax the downward rigidity constraint (where now we have defined $\tilde{Y}_t(i) \equiv \tilde{Y}_t \xi_{v,t}(i)\, \varepsilon_t(i)$). In state 1, instead, $V_w(w_t(j,i), W_t(i), \tilde{Y}_t(i), s = 1) \times dw_t(j,i) = 0$ and $dw_t(j,i) \geq 0$ as in the benchmark case, since the downward wage rigidity constraint applies.

In state 1, the value function follows the following functional equation:[36]

$$(27) \qquad (\lambda + \rho)\, v_1(\cdot)\, dt = \pi_w(\cdot)\, dt + v_{1,y}(\cdot)\, \theta\, dt + \frac{1}{2} v_{1,yy}(\cdot)\, \sigma^2(i)\, dt,$$

under the appropriate boundary conditions, where we have defined the derivative of the value function with respect to wages as $v_1(\cdot) \equiv V_w(W_t(i), W_t(i), \tilde{Y}_t(i), s = 1)$. By inspection, this is similar to the functional equation characterizing the benchmark model and is associated with the same boundary conditions. The only difference is in the discount factor, which is now higher and given by $(\lambda + \rho)$, because workers internalize the probability of switching to the flexible-wage regime.[37] It follows that in state 1, wages are set at the level $W_t(i) = c(\theta, \sigma^2(i), \eta, \lambda + \rho) \times \mu^{1/(1+\eta)} \tilde{Y}_t \xi_{v,t}(i)\, \varepsilon_{t_1}(i)$ whenever $dW_t(i) \geq 0$, where $\varepsilon_{t_1}(i)$ represents the realization of $\varepsilon_t(i)$ at time t_1 (with $t_1 < t$), which is the last time before t at which state 2 occurred. In other words, desired wages are again proportional to the flexible wages, as they were in the main model with downward wage rigidities (Section III), but with a higher proportional factor: the same function $c_i(\cdot)$ now depends on a higher discount factor $(\lambda + \rho)$.

In state 2, instead, wages can be freely adjusted so that the derivative of the value function with respect to wages is set to zero, $v_2(\cdot) = 0$, and the optimality condition in this state simplifies to[38]

$$(28) \qquad \pi_w(\cdot)\, dt + \phi v_1(\cdot)\, dt = 0.$$

This does not correspond to the optimality condition under fully flexible wages, since in state 2 wage setters take into account the probability of reverting to state 1, given by $\phi\, dt$ (indeed, as $v_1(\cdot) \leq 0$, we obtain $\pi_w(\cdot) \geq 0$). It can be shown that wages in state 2 are set below, and proportionally to, the level that would prevail in the permanently flexible-wage case $(W_t^f(i))$ such that $W_t(i) = \tilde{c}_i(\cdot)\, \mu^{1/(1+\eta)}\, \tilde{Y}_t \xi_t(i) \varepsilon_t(i)$, where $c_i(\cdot) < \tilde{c}_i(\cdot) < 1$.[39]

The results are quite intuitive. In state 1, i.e., when downward rigidities are binding, the desired wage is closer to the flexible-wage case than in the benchmark case of Section III, because agents internalize the positive probability of a readjustment when the state switches. In state 2, i.e., when the downward rigidities are not binding, wage setters will set wages below the flexible-wage level, as they will internalize the fact that with positive probability they will enter state 1 in which the downward wage rigidity constraint is binding.

[36] These are standard optimality conditions associated with models of switching regimes (see Bernt Øksendald and Agnes Sulem 2004, 52–57). See John Driffil, Marzia Raybaudi, and Martin Sola (2003) and Xin Guo, Jianjun Miao, and Erwan Morellec (2005) for related problems in the irreversible investment literature. See the online Appendix at http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.4.1436.

[37] The value function in state 2 does not enter into (27) because wages can be freely adjusted in that state.

[38] See the additional Appendix at http://docenti.luiss.it/benigno/research/publications/. Notice that the composite state variable $\tilde{Y}_t(i)$ is continuous when switching from state 2 to state 1, but jumps from state 1 to state 2.

[39] Proof available upon request.

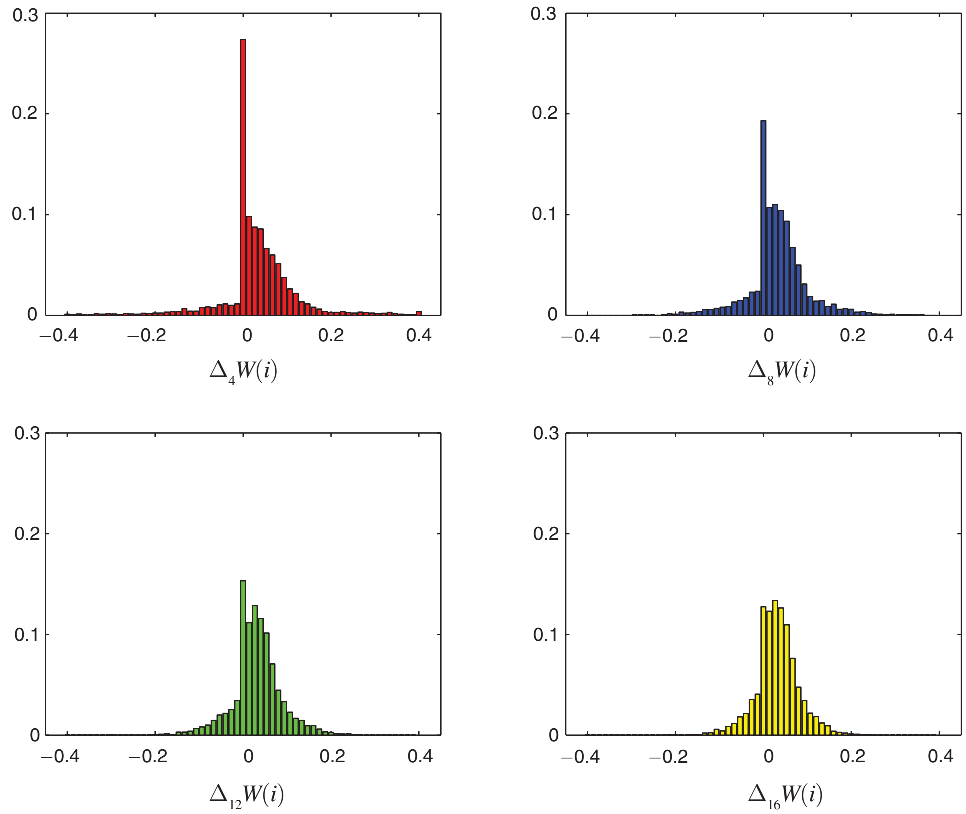
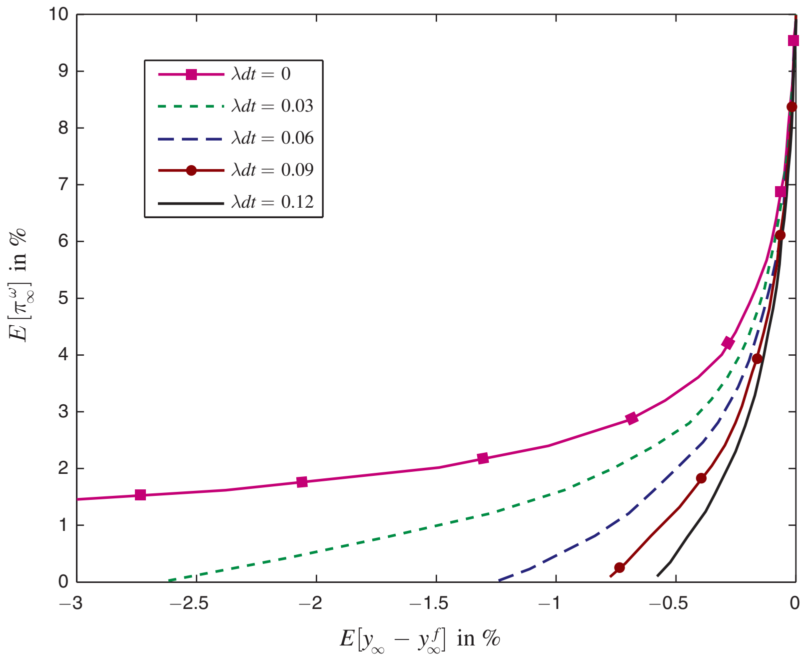


Figure 5. Frequency Distribution Function for Wage Changes over 4- , 8- , 12- , and 16-Quarter Horizons

The implications of this model for the steepness of the Phillips curve and for the output gap-inflation trade-off depend crucially on λdt, i.e., the probability of switching from the normal state where downward wage rigidities are binding to an exceptional state where major shocks warrant wage flexibility. One can expect this parameter to be quite low. For example, $\lambda dt = 0.1$ (coupled with $\phi dt = 1$) would imply that wages become flexible during one quarter out of two and a half years, while $\lambda dt = 0.01$ would imply that wages become flexible during one quarter out of 25 years.

One way to calibrate λdt is to ask the model to match some key empirical patterns uncovered by the micro literature on individual wage setting. For example, Card and Hyslop (1997, table 2) show, for the United States, that in the presence of low inflation the fraction of rigid wages (zero change) at a one-year horizon is around 16 percent. The fraction decreases to 8 percent at a two-year horizon and to 5 percent at a three-year horizon (during the period 1985–1988, when inflation was about 3 percent). Moreover there are negative wage changes. In Figure 5 we show the frequency distribution implied by our model for the wage changes over one-year, two-year, three-year, and four-year horizons, when we adopt the following calibration: $\theta = 4$ percent, $\sigma_y = 5$ percent, $\sigma_\xi = 10$ percent, and $\sigma_\varepsilon(i, s = 2) = 65$ percent,



($E[\pi_\infty^\omega]$ at Annual Rates)

FIGURE 6. LONG-RUN PHILLIPS CURVE FOR DIFFERENT PROBABILITIES (λdt) OF SWITCHING TO A REGIME WITHOUT DOWNWARD RIGIDITY

all at annual rates, $\rho = 0.01$ on a quarterly basis, $\eta = 2.5$, and $\phi dt = 1$ (the last assumption implies that once a high-variance shock occurs then the state switches back immediately to state 1). When we set $\lambda dt = 0.06$, the fraction of zero wage changes implied by the model over the four horizons considered is 16.5 percent, 8.6 percent, 4.9 percent, and 3.8 percent, respectively, which is in line with the evidence presented by Card and Hyslop (1997). Moreover, the fraction of negative wage changes on a year horizon is equal to 11 percent (or less than 3 percent on a quarterly basis), so that our model is consistent with some wage decreases.

In Figure 6, we allow λdt to vary in the $(0.00, 0.12)$ interval, i.e., a range surrounding the value calibrated above, in order to study the implications of this model for the shape of the long-run Phillips curve.[40] When the probability of switching to state 2 increases, wages are, on average, more flexible: the Phillips curve moves inward and becomes more vertical. In the case of $\lambda dt = 0.06$, reducing wage inflation from 5 percent to 2 percent would increase the output gap by about 0.3 percent of GDP, about 1 percentage point less than in the benchmark case, but still by a sizable amount. For countries or during periods in which the high variance shocks are more (less) frequent, hence λdt is higher (lower), the trade-off would be better (worse) and the costs would be lower (higher).

[40] The calibration is based on the same parametrization as in Figure 5, apart from λdt.

VI. Conclusions

This paper offers a theoretical foundation for the long-run Phillips curve, by introducing downward nominal wage rigidities in a DSGE model with forward-looking agents and flexible-goods prices, in the presence of both idiosyncratic and aggregate shocks. Downward nominal rigidities (the main difference with respect to current monetary models) have been advocated for a long time as a justification for the Phillips curve, and have recently received theoretical and empirical support (see discussion in the introduction).

The model, as shown in equation (24), generates a closed-form solution uncovering a highly nonlinear relationship for the long-run trade-off between average wage inflation and output gap: the trade-off is virtually inexistent at high inflation rates, while it becomes relevant in a low inflation environment. The relation shifts with several factors, and in particular with the degree of macroeconomic volatility.[41] In a country with significant macroeconomic stability, the Phillips curve is also virtually vertical at low wage inflation. However, a country with moderate to high volatility may face substantial costs in terms of output and employment if attempting to reach price stability. Higher productivity growth would imply an upward shift along the Phillips curve (where it is steeper), as it would feed into higher nominal spending growth. The Phillips curve would also steepen if the degree of wage rigidities declines. Indeed, the benchmark model is extended to allow for the possibility that downward wage rigidities may be heterogenous across agents, and may be endogenous to inflation, macro-volatility, or the occurrence of large shocks. Nonetheless, for reasonable parameter values, downward rigidities continue to generate a non-negligible long-run trade-off between inflation and the output gap. Further work would be necessary to achieve a deeper understanding of the labor market and of the wage setting behavior, which is crucial to measure and assess more accurately the extent and the implications of downward wage rigidities.

Several important implications arise. First, the optimal inflation rate may not be zero, but positive, as inflation helps the intratemporal and intertemporal relative price adjustments, especially in countries with substantial macroeconomic volatility or low productivity growth. Second, the ideal inflation rate could differ across countries (and in particular it would be higher in countries with larger macroeconomic volatility and lower productivity growth), and may change over time. Third, stabilization policies can play a crucial role, as they can improve the inflation-output trade-off.

Additional theoretical implications arise. First, the overall degree of wage rigidity is endogenously stronger at low inflation rates and disappears at high inflation rates, unlike in time-dependent models of price rigidities where prices remain sticky even in a high-inflation environment. This arises from the endogenous tendency for upward wage rigidities (as in Elsby 2009), resulting from forward-looking agents anticipating the effect of downward rigidities on their future

[41] With respect to the other parameters of the model, the Phillips curve would flatten when labor elasticity is lower and agents heavily discount the future; and it would shift outward if labor and goods market competition weakens. When measured in terms of goods price inflation, the curve would correspondingly be lower by the level of productivity growth.

employment opportunities. Second, this endogenous wage rigidity also introduces a trade-off between the volatility of the output gap and the volatility of inflation, as at low inflation adjustments occur mainly via changes in output and at high inflation via changes in wages. Third, the Phillips curve may arise not only from the need for intratemporal relative price adjustments across sectors in the presence of downward rigidities (as in the traditional view), but also from the need for intertemporal relative price adjustments, which open the way for the important role of macroeconomic stabilization policies discussed above. Fourth, nominal shocks can have high persistent real effects, suggesting that introducing downward wage inflexibility in a menu-cost model à la Golosov and Lucas (2007) would likely change their conclusion that nominal shocks have only transient effects on real activity at any level of inflation.

Regarding the empirical implications, the long-run output gap with respect to the flexible-wage output is not zero in our model, but depends, among other things, on the extent of inflation and volatility of the economy. This implies that standard empirical methods deriving an estimate of the output gap as a deviation from filtered series may be misleading, as such a measure would, by construction, average out to about zero in the long run. Indeed, in our model, the long-run output gap should simply be a mirror image of the gap between the unemployment rate and the frictionless unemployment rate, which would persist in the long run. Moreover, empirical studies of the Phillips curve might prove inaccurate unless they properly account for macroeconomic volatility, especially in a low inflation environment. For example, the "Great Moderation" experienced by the United States until recently may have significantly steepened the Phillips curve over the past two decades, thus potentially strengthening the empirical case for the conventional view of a vertical long-run curve in this country. However, this does not need to apply to periods where volatility becomes persistently higher or to countries with generally higher macroeconomic instability.

Appendix

In this Appendix, we derive conditions (17) in the text. Let $\mathcal{W}$ represent the space of nondecreasing nonnegative stochastic processes $\{w_t(j,i)\}$. This is the space of processes that satisfy the constraint (16). First we show that the objective function is concave over a convex set. To show that the set is convex, note that if $\mathbf{x} \in \mathcal{W}$ and $\mathbf{y} \in \mathcal{W}$, then $\tau\,\mathbf{x} + (1-\tau)\,\mathbf{y} \in \mathcal{W}$ for each $\tau \in [0,1]$. Since the objective function is

$$E_{t_0}\left\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\pi\left(w_t(j,i), W_t(i), \tilde{Y}_t(i)\right)dt\right\}$$

and $\pi(\cdot)$ is concave in the first argument, the objective function is concave in $\{w_t(j,i)\}$ since it is the integral of concave functions.

Let $\{w_t^*(j,i)\}$ be a process belonging to $\mathcal{W}$ that maximizes (14) and $V(\cdot)$ be the associated value function defined by

$$V\left(w_t(j,i), W_t(i), \tilde{Y}_t(i)\right) = \max_{\{w_\tau(j,i)\}\in\mathcal{W}} E_t\left\{\int_t^{\infty} e^{-\rho(\tau-t)}\pi\left(w_\tau(j,i), W_\tau(i), \tilde{Y}_\tau(i)\right)d\tau\right\}.$$

We now characterize the properties of the optimal process $\{w_t^*(j,i)\}$. The Bellman equation for the wage-setter problem can be written as

$$\text{(A1)} \qquad \rho V\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\, dt = \max_{dw_t(j,i)} \pi\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\, dt + E_t\big\{dV\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\big\}$$

subject to

$$\text{(A2)} \qquad dw_t(j,i) \geq 0.$$

At optimum we search for a process $\{w_t^*(j,i)\}$ that satisfies

$$V_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big) = 0 \quad \text{if } dw_t(j,i) > 0,$$

$$V_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big) \leq 0 \quad \text{if } dw_t(j,i) = 0.$$

Differentiating (A1) with respect to $w_t(j,i)$ we get

$$\text{(A3)} \qquad \rho V_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\, dt = \pi_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\, dt + E_t\big\{dV_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big)\big\},$$

where

$$\pi_w\big(w_t(j,i), W_t(i), \tilde{Y}_t(i)\big) = k_w\left[\left(\frac{w_t(j,i)}{W_t(i)}\right)^{1-\theta}\frac{1}{w_t(j,i)} - \mu\left(\frac{w_t(j,i)}{W_t(i)}\right)^{-(1+\eta)\theta}\left(\frac{\tilde{Y}_t(i)}{W_t(i)}\right)^{1+\eta}\frac{1}{w_t(j,i)}\right],$$

with $k_w \equiv 1 - \theta$. Since the objective is concave and the set of constraints is convex and each household j faces the same problem in supplying variety i, the optimal choice for $w_t(j,i)$ is unique. It follows that $w_t(j,i) = W_t(i)$ for each j. We can then write (A3) as

$$\text{(A4)} \qquad \rho v\big(W_t(i), \tilde{Y}_t(i)\big)\, dt = \pi_w\big(W_t(i), \tilde{Y}_t(i)\big)\, dt + E_t\big\{dv\big(W_t(i), \tilde{Y}_t(i)\big)\big\},$$

where

$$\pi_w\big(W_t(i), \tilde{Y}_t(i)\big) \equiv k_w\left[\frac{1}{W_t(i)} - \mu\left(\frac{\tilde{Y}_t(i)}{W_t(i)}\right)^{1+\eta}\frac{1}{W_t(i)}\right],$$

and we have defined $v(W_t, \tilde{Y}_t) \equiv V_w(W_t, W_t, \tilde{Y}_t)$. Using Ito's Lemma we can write

$$E_t\{dv(W_t(i), \tilde{Y}_t(i))\} = v_w(W_t(i), \tilde{Y}_t(i))\, dW_t(i) + v_y(W_t(i), \tilde{Y}_t(i))\, \tilde{Y}_t(i)\, \theta'(i)\, dt$$
$$+ \frac{1}{2} v_{yy}(W_t(i), \tilde{Y}_t(i))\, \tilde{Y}_t^2(i)\, \sigma^2(i)\, dt$$
$$= v_y(W_t(i), \tilde{Y}_t(i))\, \tilde{Y}_t(i)\, \theta'(i)\, dt + \frac{1}{2} v_{yy}(W_t(i), \tilde{Y}_t(i))\, \tilde{Y}_t^2(i)\, \sigma^2(i)\, dt$$

since $dw_t(j,i)$ has finite variation, as does $dW_t(i)$, implying $(dW_t(i))^2 = dW_t(i) d\tilde{Y}_t = 0$. We have defined $\theta'(i) \equiv \theta + ½\sigma^2(i)$ and $\sigma^2(i) \equiv \sigma_y^2 + \sigma_\xi^2(i)$. From the first to the second line, we have used the super-contact conditions requiring

$$v_w(W_t(i), \tilde{Y}_t(i))\, dW_t(i) = 0.$$

It follows that we can write (A4) as

$$\text{(A5)} \quad \rho \tilde{v}(\tilde{Y}_{w,t}(i)) = \tilde{\pi}_w(\tilde{Y}_{w,t}(i)) + \tilde{v}_y(\tilde{Y}_{w,t}(i)) \tilde{Y}_{w,t}(i) \theta' + \frac{1}{2} \tilde{v}_{yy}(\tilde{Y}_{w,t}(i)) \tilde{Y}_{w,t}^2(i) \sigma_y^2(i),$$

since we have noticed that $v(W_t(i), \tilde{Y}_t(i)) = \tilde{v}(\tilde{Y}_{w,t}(i))/W_t$ with $\tilde{Y}_{w,t}(i) \equiv \tilde{Y}_t(i)/W_t$ and $\pi_w(W_t(i), \tilde{Y}_t(i)) = \tilde{\pi}_w(\tilde{Y}_{w,t}(i))/W_t$, where

$$\tilde{\pi}_w(\tilde{Y}_{w,t}(i)) \equiv k_w \left[1 - \mu (\tilde{Y}_{w,t}(i))^{1+\eta}\right].$$

The problem boils down to looking for a function $\tilde{v}(\tilde{Y}_{w,t}(i))$ and a regulating barrier $\hat{c}(i)$ such that $\tilde{v}(\tilde{Y}_{w,t}(i)) \le 0$ and

$$\text{(A6)} \quad \tilde{v}(1/\hat{c}(i)) = 0,$$

$$\text{(A7)} \quad \tilde{v}_y(1/\hat{c}(i)) = 0.$$

A particular solution to (A5) is given by

$$\tilde{v}^p(\tilde{Y}_{w,t}(i)) = \frac{k_w}{\rho} - \frac{k_w}{\rho - \theta'(1 + \eta) - \frac{1}{2}(1 + \eta)\eta\sigma^2(i)} \mu (\tilde{Y}_{w,t}(i))^{1+\eta},$$

while in this case the complementary solution has the form

$$v^c(\tilde{Y}_{w,t}(i)) = \tilde{Y}_{w,t}^{\gamma(i)}(i),$$

where $\gamma(i)$ is a root that satisfies the following characteristic equation:

$$\text{(A8)} \quad \frac{1}{2}\gamma^2(i)\sigma^2(i) + \gamma(i)\theta - \rho = 0,$$

i.e.,

$$\gamma(i) = \frac{-\theta + \sqrt{\theta^2 + 2\rho\sigma^2(i)}}{\sigma^2(i)}.$$

When $\tilde{Y}_{w,t}(i) \to 0$, the length of time until the next wage adjustment can be made arbitrarily long with probability arbitrarily close to one. Then it should be the case that

$$\lim_{\tilde{Y}_{w,t}(i) \to 0} \left[\tilde{v}\left(\tilde{Y}_{w,t}(i)\right) - \tilde{v}^{p}\left(\tilde{Y}_{w,t}(i)\right)\right] = 0,$$

which requires that $\gamma(i)$ should be positive. The general solution is then given by the sum of the particular and the complementary solution:

$$\text{(A9)} \quad \tilde{v}\left(\tilde{Y}_{w,t}(i)\right) = \frac{k_w}{\rho} - \frac{k_w}{\rho - \theta'(i)(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma^2(i)}\,\mu\left(\tilde{Y}_{w,t}(i)\right)^{1+\eta} + k(i)\,\tilde{Y}_{w,t}^{\gamma(i)}(i),$$

for a constant $k(i)$ to be determined. Moreover,

$$\text{(A10)} \quad \tilde{v}_y\left(\tilde{Y}_{w,t}(i)\right) = -\frac{k_w(1+\eta)}{\rho - \theta'(i)(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma^2(i)}\,\mu\left(\tilde{Y}_{w,t}(i)\right)^{\eta+1} + \gamma k(i)\,\tilde{Y}_{w,t}^{\gamma(i)}(i).$$

The boundary conditions (A6)–(A7) imply

$$\text{(A11)} \quad \frac{k_w}{\rho} - \frac{k_w}{\rho - \theta'(i)(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma^2(i)}\mu\hat{c}(i)^{-(1+\eta)} + k_i\,\hat{c}(i)^{-\gamma(i)} = 0,$$

$$\text{(A12)} \quad -k_w\frac{1+\eta}{\rho - \theta'(i)(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma^2(i)}\mu\hat{c}(i)^{-(1+\eta)} + \gamma k(i)\hat{c}(i)^{-\gamma(i)} = 0.$$

From the last two conditions we can determine $k(i)$ and $\hat{c}(i)$. In particular, $\hat{c}(i) = (\mu)^{1/(1+\eta)}c(i)$, where $c(i)$ is given by

$$c_i(\cdot) \equiv \left(\frac{\gamma - \eta - 1}{\gamma}\,\frac{\rho}{\rho - \theta'(i)(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma^2(i)}\right)^{\frac{1}{1+\eta}}.$$

Using (A8), we can write

$$c\left(\theta, \sigma^2(i), \eta, \rho\right) = \left(\frac{\theta + \frac{1}{2}\gamma(\theta, \sigma^2(i), \rho)\sigma^2(i)}{\theta + \frac{1}{2}(\gamma(\theta, \sigma^2(i), \rho) + \eta + 1)\sigma^2(i)}\right)^{\frac{1}{1+\eta}},$$

which shows that $0 < c\left(\theta, \sigma^2(i), \eta, \rho\right) \le 1$.

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