Tests of hypotheses on the α relations

A graduate course in the Cointegrated VAR model: Special topics in Rome

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November 2011

Formulating hypotheses as restrictions on the alpha vectors

Two types of interesting hypotheses:

1. Tests of a zero row in α : long-run exogeneity of a variable
2. Tests of a known (unit vector) in α : a variable is purely adjusting (is endogenous)

The two type of tests can identify (some of) the pushing and pulling forces of the system.

Tests of long-run exogeneity (weak exogeneity)

We test the following hypothesis on α:

$$\mathcal{H}_{\boldsymbol{\alpha}}(r) : \alpha = H\alpha^c \tag{1}$$

where α is $p \times r$, H is a $p \times s$ matrix, α^c is a $s \times r$ matrix of nonzero α-coefficients and $s \geq r$. (Compare this formulation with the hypothesis of the same restriction on all β, i.e. $\beta = H\varphi$.) As with tests on β we can express the restriction (1) in the equivalent form:

$$\mathcal{H}_{\alpha}(r) : R'\alpha = 0 \tag{2}$$

where $R = H_{\perp}$.

Testing weak exogeneity of the bond rate

$$\begin{bmatrix} \Delta m_t^r \\ \Delta y_t^r \\ \Delta^2 p_t \\ \Delta R_{m,t} \\ \Delta R_{b,t} \end{bmatrix} = \ldots + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ . & . & . \\ . & . & . \\ . & . & . \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \end{bmatrix} \begin{bmatrix} \beta_1' x_{t-1} \\ \beta_2' x_{t-1} \\ \beta_3' x_{t-1} \end{bmatrix} + \ldots$$

$$\Delta x_t = \ldots \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{11}^c & \alpha_{12}^c & \alpha_{13}^c \\ . & . & . \\ . & . & . \\ \alpha_{41}^c & \alpha_{42}^c & \alpha_{43}^c \end{bmatrix} \begin{bmatrix} \beta_1' x_{t-1} \\ \beta_2' x_{t-1} \\ \beta_3' x_{t-1} \end{bmatrix} + \ldots$$

$$\begin{bmatrix} \Delta m_t^r \\ \Delta y_t^r \\ \Delta^2 p_t \\ \Delta R_{m,t} \\ \Delta R_{b,t} \end{bmatrix} = \ldots \begin{bmatrix} \alpha_{11}^c & \alpha_{12}^c & \alpha_{13}^c \\ . & . & . \\ . & . & . \\ \alpha_{41}^c & \alpha_{42}^c & \alpha_{43}^c \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_1' x_{t-1} \\ \beta_2' x_{t-1} \\ \beta_3' x_{t-1} \end{bmatrix} + \ldots$$

The test statistic, distributed as $\chi^2(3)$, is 4.64 and the weak exogeneity of

Exogeneity tests of all variables and for all choices of rank

Table: Tests of long-run weak exogeneity

r	v	$\chi^2(v)$	m^r	y^r	Δp	R_m	R_b
1	1	3.84	3.48 [0.06]	0.07 [0.79]	11.49 [0.00]	0.62 [0.43]	0.63 [0.43]
2	2	5.99	8.22 [0.02]	**2.24** [0.33]	16.25 [0.00]	**1.94** [0.38]	**0.78** [0.68]
$\longrightarrow$ 3 $\longleftarrow$	3	7.81	17.56 [0.00]	**6.84** [0.08]	26.65 [0.00]	8.88 [0.03]	**4.64** [0.20]
4	4	9.49	21.14 [0.00]	13.44 [0.01]	27.12 [0.00]	10.92 [0.03]	10.25 [0.04]

Tests of joint exogeneity of the bond rate and the real income variable

The design matrix for the joint test is specified as:

$$R' = \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{array} \right],$$

and the test statistic, distributed as $\chi^2(6)$, became 15.05 with a p-value of 0.02 and we reject that both variables are jointly weakly exogenous in this system.

Tests of a known (unit vector) in alpha

The hypotesis is formulated as:

$$\mathcal{H}_0 : \alpha = \{a, \tau\}$$

where a is a $p \times n_k$ known matrix and $n_k \leq r$ and τ is a $p \times (r - n_k)$ matrix of unrestricted adjustment coefficients. In most cases $n_k = 1$, so that a is a vector in α.

Illustration

The hypothesis that real money stock is purely adjusting is formulated as a unit vector in α:

$$\alpha^c = \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & * & * \\ 0 & * & * \\ 0 & * & * \end{bmatrix} \rightarrow \alpha^c_\perp = \begin{bmatrix} 0 & 0 \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix}.$$

The zero row in $\alpha^c_\perp$ will produce a zero column in the matrix C

Under the assumption that $r = 3$, the table below reports the tests of a known unit vector in α, where the unit vector corresponds to each of the variables in turn.

Table: Tests of known vector in alpha

r	v	$\chi^2(v)$	m^r	y^r	Δp	R_m	R_b
3	2	5.99	**3.81**	10.32	**3.96**	**5.09**	6.88
			[0.15]	[0.01]	[0.14]	[0.08]	[0.03]

Testing several unit vectors in alpha jointly

The joint hypothesis was of money stock, inflation and the short-term interest rate was rejected with a p-value of 0.02 based on a test value of 15.05 distributed as $\chi^2(6)$, i.e. exactly the test value of the joint weak exogeneity hypothesis. Thus, it is only a different way of testing the joint weak exogeneity of the bond rate and real income.
The two exogeneity restrictions (if jointly accepted) would have determined the common trends in our empirical model (recalling that $p - r = 2$). Consequently, the correponding two rows of the Π matrix would have been zero in this case and the remaining three rows would have been represented as:

$$\Pi = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \alpha_{32} & 0 \\ 0 & 0 & \alpha_{43} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_1' x_{t-1} \\ \beta_2' x_{t-1} \\ \beta_3' x_{t-1} \end{bmatrix}.$$

Thus, when there are exactly $p - r$ weakly exogenous variables, the α