Sophisticated Intermediation and Aggregate Volatility

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Abstract

Increasing sophistication in risk management has made it possible to hedge and allocate risks in the economy more efficiently. On the other side, new financial securities are often thought to be a source of risk for the economy and, thus, a reason for stricter regulation. In this paper, I consider an economy where consumers/investors delegate their portfolio/investment decisions to financial institutions who choose across multiple investment opportunities that may feature different levels of idiosyncratic risk as well as different correlation with the rest of the economy. Investors solve an optimal contracting problem to incentivize financial institutions to reduce the aggregate risk of their investment. I then study how investment decisions are affected when financial securities that allow agents to trade their risks are introduced. Investors do not have the necessary information to understand these securities, but create incentives for financial institutions to hedge certain risks. I show that hedging idiosyncratic risks ameliorates the agency problem between consumers and managers and, thus, reduces aggregate volatility. The opposite is true when aggregate risk can be traded. Finally, I show that the equilibrium may be inefficient and government intervention is required to regulate financial markets.

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1 Introduction

The last two decades have witnessed an enormous expansion of markets of financial securities. Derivative instruments, often customized to the specific needs of their users, have become very popular and have enabled firms and financial institutions to manage their risks more efficiently. As a consequence, the over-the-counter (OTC) derivatives market, where contracts are traded directly between two parties (without going through an exchange), has become the largest market for derivatives.

Figure 1 shows the evolution over the last decade of the notional amount outstanding of OTC derivatives, which by the end of 2010 is estimated to be around 600 trillion dollars. Of course, notional amounts are misleading as they include double counting of positions and hide net exposures[1]. Even with this caveat, the magnitude of these numbers is quite impressive. In particular, com-

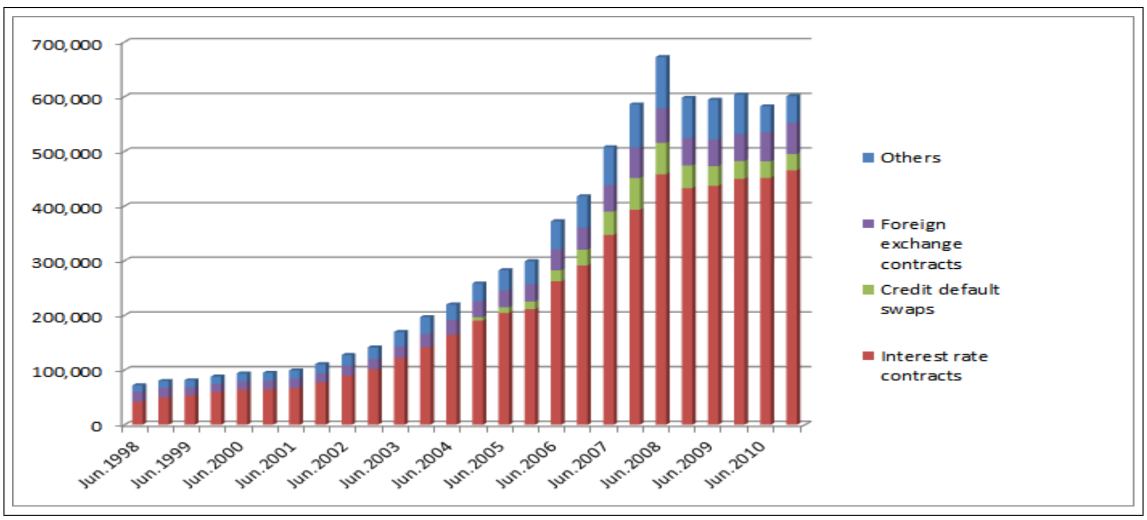


Figure 1. BIS, Quarterly Review.

mercial banks are important participants in derivatives markets as shown in Figure 2. The typical derivatives are swaps (such as, interese rate swaps or exchange rate swaps) with credit derivatives gaining a bigger share over time.

In a perfect world, financial markets allow households and firms to share risks more efficiently. Idiosyncratic risks can be pooled together and eliminated with great benefits for risk-averse agents. Even aggregate risks – which cannot be eliminated – can nonetheless be transferred to agents that are better equipped to bear them.

[1]Note, however, that even if two offsetting positions gives the same net exposure as having no positions at all, these two strategies are quite different when there is a risk that a counterparty can fail to deliver on its promise.

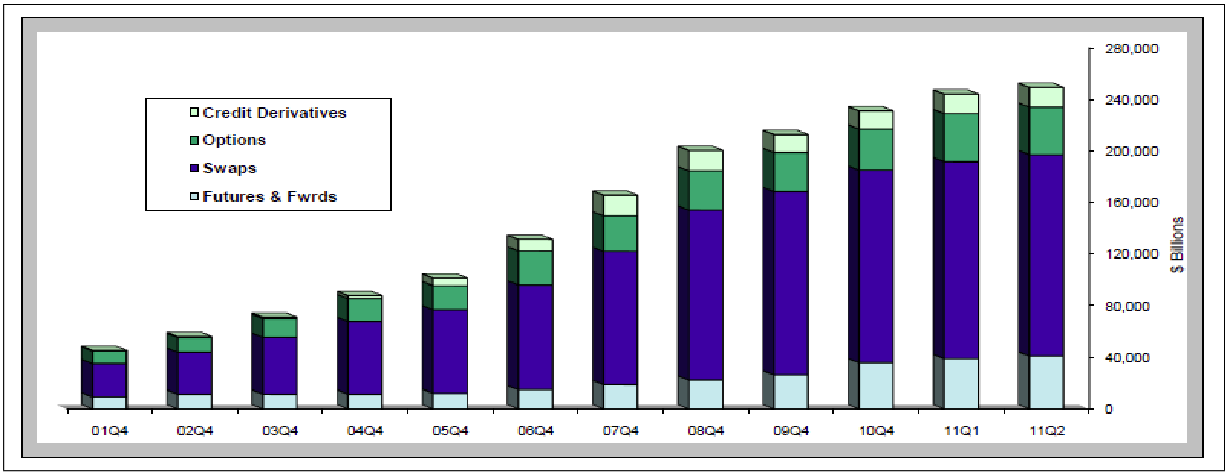


Figure 2. OCC Quarterly Report.

However, following the dramatic events of recent years, markets for derivatives have come under pressure. These products, together with those who failed to regulate them, have taken part of the blame for the turmoil in financial markets and the economy in general. Many people now fear the the "opaqueness" of derivatives markets and the "complexity" of the positions held on and off the balance sheets of big financial institutions. Complex financial securities, the argument goes, may actually pose a threat to the system by increasing and concentrating the risks of some institutions.

Unfortunately, the mechanisms through which complex securities and opaque markets affect the economy are still not clear. But if their effects on the economy are unknown, then we cannot give recommendations on whether and how the government should regulate these markets.

Several contributions in the literature have proposed different perspectives to understand these issues. Investors neglect certain unlikely events (Gennaioli and Vishny (2011)), complex cross-exposures make banks susceptible to contagion (Caballero and Simsek (2011)), complex securities can amplify the costs of asymmetric information (Arora et al. (2011)).

This paper focuses on a different and important aspect of complex securities. The starting point is that investors have to rely on financial institutions to make their investment decisions. Financial institutions, such as banks, have the expertise to select the best firms and to monitor their activities. However, this expertise comes with agency problems: investors have to design the right incentives for these institutions to finance the best projects in the economy. For a risk-averse investor, a project is desirable if its average return is high or if its correlation with the other projects in the economy is low[2]. Alternatively, a good project is characterized by a higher average return

[2] Such preferences also characterize several asset pricing models, such as CAPM, CCAPM, etc.

and a lower sensitivity to the aggregate risk of the economy.

Financial institutions are often also active traders of complex financial securities. The focus of this paper is then to derive the implications of the potential interaction between the agency problem of the banks and their trading of complex securities.

The first contribution of this paper is to solve for the optimal contract of the agency problem between investors and financial institutions when no securities are traded. I show how investors expose banks to both idiosyncratic and aggregate risks. In particular, they punish financial institutions for generating profits that are very correlated with the rest of the economy. This result differs from the standard result in Holmstrom (1979) which states that any observable, common source of risk should not be used in the optimal contract, since it exposes the agent to more risk without affecting his incentives. However, as this paper shows, this result is no longer true when the correlation of the investments, and thus the aggregate risk in the economy, is endogenously chosen by financial institutions. Even when optimal incentives are in place, the agency problem is never fully resolved and investors are exposed to more aggregate volatility.

What happens when complex securities are traded? The second main result of this paper is to show how complex securities interact with the agency problem. Complexity limits the ability of investors to understand what risks are hedged through these securities. Investors cannot fully control the trading activity of the banks, but they can design incentives that take this trading activity into account. I show that the overall effect on equilibrium volatility and welfare depends on whether these securities are used to hedge idiosyncratic or aggregate risks. In particular, securities on idiosyncratic risks mitigate the agency problem and lower aggregate volatility. The opposite is true when aggregate risk can be traded. In summary, the *positive* effects of complex securities are ambiguous and depend on the relative importance of the two types of risks. The key implication is that complexity by itself does not necessarily lead to worse economic outcomes. Instead, in some cases complex securities may even reduce volatility in the economy and, thus, it is not necessarily optimal to shut down these markets.

The third contribution of this paper is to derive the *normative* implications of complex securities. I first show that, when securities are not allowed, the equilibrium with the agency problem is constrained efficient. Thus, the higher exposure to aggregate risk (relative to the first-best) generated by the agency problem does not by itself open the door for government invervention.

Things change substantially when securities can be traded. In particular, while securities can have ambiguous positive effects, the policy implications are always unambiguous. More specifically, I show that government intervention is desirable as long as it is possible to trade securities that hedge aggregate risk. Inefficiencies originate from the inability of investors to understand how the activity of issuing securities interacts with the agency problem of financial institutions. Investors suffer of a coordination failure since they cannot internalize the interaction between the optimal contracts they design for the financial institutions and the activity of the issuers of securities. Therefore, in equilibrium it is too easy for financial institutions to trade aggregate risk. The regulation enacted by the government is a way to fix this coordination failure and restore efficiency. The government can reduce aggregate volatility (and increase welfare) in differet ways. The most effective policy tool is regulation of the financial institutions that issue aggregate risk securities. Another, less effective possibility is to tax transactions in financial markets.

2 Related Literature

The core of this paper is a principal-agent model where the principal delegates an investment choice to the agent. The principal provides incentives by exposing the agent to some risk. The seminal contribution of Holmstrom (1979) shows under what conditions more information should be incorporated in the contract. He studies a moral hazard problem with many agents and correlated signals and derives the general principle that observable, correlated signals which are not affected by the agent's effort should not be included in the optimal contract. Contrary to Holmstrom (1979), in this paper the effort of the agents determines the correlation of the projects in the economy and, thus, the optimal contract exposes the agent to the common noise.

The seminal contribution to the literature on delegated portfolio management is Bhattacharya and Pfleiderer (1985) who propose a model where an informed agent has to reveal his information to the principal. The agency problem in this paper arises because managers have access to better information than investors, but they have to be incentivized to collect this information. This is similar to the model of delegated expertise developed by Demski and Sappington (1987) (see also Allen (1990)) and to the delegated portfolio problem with hidden actions (Admati and Pfleiderer (1997), Stoughton (1993)).

The principal-agent model can also be interpreted as a two-tier incentive problem whereby investors lend money to managers who then monitor entrepreneurs who run the projects and choose in what type of risk to invest. The classical paper on delegated management is Diamond (1984) who shows that it is optimal for banks to fully diversify their portfolios. However, Diamond (1984) considers a model where there is no trade-off between different types of risks as in this model.

In the basic version of the model, the opacity of securities is modelled by assuming that trades are unobservable (Allen (1985), Arnott and Stiglitz (1993), Hellwig (1983), Bisin and Guaitoli (2004), Cole and Kocherlakota (2001), Bizer and DeMarzo (1999)). Unobservable trades limit risk-sharing in Jacklin (1987) who shows that financial markets can reduce welfare. Farhi et al. (2009) show how regulation can correct the externality generated by the unobservable trades (see also Allen and Gale (2004) and Golosov (2007)).

Similarly, in this paper some types of securities (those conditional on the aggregate state) will reduce welfare by limiting the incentives that investors can provide to portfolio managers. In corporate finance, several papers have focused on how hedging opportunities affect incentives when the effort of the managers increase the expected return of the firm (Li (2002), Garvey and Milbourn (2003), Ozerturk (2006), Bisin et al. (2008)). An important difference is Acharya and Bisin (2009) who study a model where firms make investment decisions and can choose the loading on the aggregate state of the economy. They also allow the manager to transfer (aggregate) risk. They focus on the optimal ownership share of the manager and show that a manager who is too risk-averse should own a smaller part of the firm's capital. Importantly, Acharya and Bisin (2009) do not make the distinction between different types of securities, which is central in this paper, and do not allow investors to write the optimal contract to managers. Also, they study a partial equilibrium model and, thus, they don't consider policy implications[3].

In general, the unobservable trades in the basic version of the model are different from the literature on agency problems with side-trades. In those models, agents trade on their own account and undo the incentives provided by the principals. In this paper, instead, managers do not trade on their own account, but make portfolio decisions that affect the balance sheets of the financial institution they manage. I find this assumption more realistic since complex securities are often held on the balance sheets of financial institutions.

A more recent literature studies how the complexity of financial securities and opacity of OTC markets can pose threats to the financial system. Caballero and Simsek (2011) show how complexity (modelled as limited information about the network of counterparties) can potentially cause a cascade of bank failures. Dang et al. (2009) study how some securities, such as debt, that are usually informational insensitive can lose much of their value in bad states of the world because of asymmetric information. Brunnermeier and Oehmke (2011) focus on the definition of complexity when agents are boundedly rational and observe that disclosing more information can lead to

[3] See Acharya (2009) for a model where firms strategically coordinate their actions and increase the *systemic risk* in the economy.

information overload, which has important implications for designing disclosure requirements and consumer protection. Their reason for regulation is not driven by the agency problem combined with the general equilibrium effects as in this paper.

Finally, in the extended model in section 6, securities contingent on the different types of risks are endogenously created by intermediaries as in the literature on general equilibrium with endogenous financial markets. I follow Pesendorfer (1995) and assume that there is a fixed cost to market a new security to a manager (see Allen and Gale (1988), Allen and Gale (1991), Bisin (1998) for alternative assumptions).

The paper is organized as follows. Section 3 introduces the model and defines the equilibrium. Section 4 solves the model for the special case where securities markets are shut down. The solution of the model with securities is derived in section 5, where I consider the different types of securities separately. In section 6, I allow agents to trade both securities and extend the model to include trading costs. The efficiency properties of the equilibrium are studied in section 7 and some optimal policy prescriptions are discussed. Finally, section 8 discusses alternative assumptions and provides some empirical evidence.

3 The model

In this section, I introduce the elements of the basic model, later I will consider some special cases that illustrate how the full model works. In sections 6 and 7, I will extend this model to derive some comparative static results and policy implications.

The economy lasts for two periods, $t = 0, 1$ and there is only one consumption good. There are three types of agents: investors (the principals), managers (the agents), intermediaries. Investors form a continuum of measure 1, are born with an endowment of one unit of the consumption good which they can invest. Managers also form a continuum of measure 1, are indexed by $i \in [0, 1]$, and have no endowment. They borrow money from investors and select and run projects based on their information. Finally, there are N intermediaries which are firms that maximize profits by issuing securities and selling them. Consumers and managers value consumption only in period 1 according to the utility functions $v(\cdot)$ and $u(\cdot)$, which are assumed to be differentiable, increasing and concave.

The economy is characterized by a continuum of "sectors", denoted by $j \in [0, 1]$. At time 0, each manager i is randomly matched to a sector j, his area of expertise. I assume that different sectors are associated to different managers. Let $\mathcal{F}$ the set of all possible realizations of this matching technology. Thus, each element $F \in \mathcal{F}$ is a description of how each manager is matched to each sector. In addition, the economy is hit by a continuum of "shocks" which I denote by ω, ε_j, and u_j, $j \in [0,1]$ and describe below.

Thus, a state of nature $s \in S$ contains the realization of the shocks in the economy together with an element $F \in \mathcal{F}$.

The investment technology is modelled to capture the idea that a manager can choose not only the specific project he wants to run, but also the correlation of his investment to the projects of the other managers.

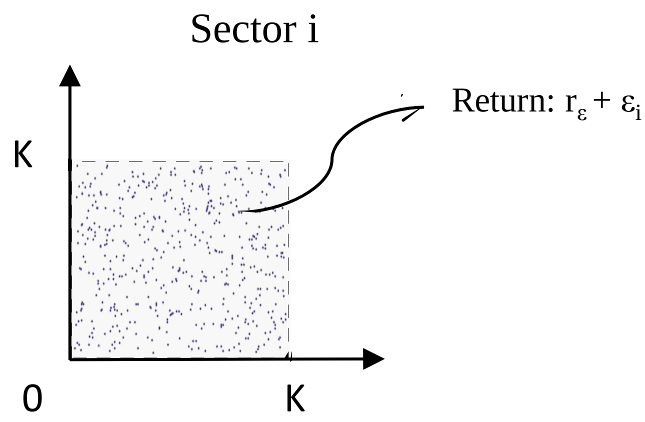
Each project requires 1 unit of capital at time 0 and produces a (random) return at time 1. In each sector there is a two-dimensional continuum of projects indexed in $\mathbb{R}^2_+$. Each project can be of two types. The first type delivers a random return R which has mean $\bar{R}$ and is perfectly correlated with ω, that is, $R = \bar{R} + \omega$. Most of the projects in a sector are of the first type. In fact, I am going to assume that they form a subset of measure 1 in $\mathbb{R}^2_+$. Thus, I will refer to ω as the "aggregate state" of the economy.

Projects of the second type (which I refer to as "specialized" projects) in sector i have higher mean return, zero correlation with ω, and are correlated with ε_i[4]. Formally, they deliver a random return $r_i = \bar{r} + \varepsilon_i$, with $\bar{r} > \bar{R}$. Here, ε_i is the idiosyncratic shock shared by all specialized projects of sector i. These projects form a subset of measure 0 and are uniformly distributed on $\mathbb{R}^2_+$.

Every manager is an expert of a particular sector, that is, he has access to information and can screen projects in that sector. Thus, I identify each manager with the index $i \in [0,1]$ of his sector of expertise. More formally, I assume that, if manager i screens a project in sector i, he receives a signal which is fully informative about the type of the project. So, a manager who wants to invest an amount K in projects of the second type has to screen K^2 projects and then select only those that send a positive signal. Alternatively, the manager can choose projects randomly from different sectors and generate a return $R = \bar{R} + \omega$[5]. The following diagram describes the investment technology in sector i.

[4] The assumption that projetcs of the second type have both higher mean and zero beta is made to simplify the algebra of the model. In a richer envirnment, some projects would have higher mean while others would have zero beta. In an even richer model the manager would have to exert two types of effort: an effort to screen the zero beta projects and an effort to increase the overall return of the investment.

[5] If the manager invests in different sectors, the sectoral shock washes away, so I dropped the superscript i from the notation.



Screening projects is costly for the manager. In particular, a manager that screens K^2 projects incurs in a non-monetary cost $C(K)$, which is assumed to be differentiable, increasing and convex. The function $C(\cdot)$ is a convenient shortcut to capture all the costs incurred by a manager who invests in a project which is uncorrelated with the rest of the economy and can be justified on the grounds that exploring different investment opportunities has bigger costs. These costs can be thought to represent resources required to screen more innovative ideas[6]. Since I assume that investing in the specialized projects is costly for a manager, investors will have to set up the right incentives for a manager to run this project.

In summary, a manager i that receives one unit of capital from investors and invest a fraction k_i in the second type projects of sector i and the remaining $1 - k_i$ in random projects generates a payoff π_i equal to

$$\begin{aligned} \pi_i &= r_i \; k_i + R \; (1 - k_i) + u_i \\ &= \bar{R} + (r + \varepsilon_i) \, k_i + \omega \, (1 - k_i) + u_i, \end{aligned} \tag{1}$$

where $r \equiv \bar{r} - \bar{R} > 0$ where u_i is an extra, per unit of capital invested, idiosyncratic noise which the manager will not be able to insure. Importantly, I assume that the return π_i from the investment of manager i is proportional to the amount of capital invested. A higher k_i in (1) implies that the final payoff is more sensitive to the idiosyncratic component and less to the aggregate state. In the limit when $k_i = 1$, the final payoff is uncorrelated with the rest of the economy. For simplicity

[6] Alternatevely, these costs may represent the fact that managers are concerned about their reputations in the labor market and, hence, they are unwilling to make investment decisions that differ from those of their competitors (Scharfstein and Stein (1990)). This alternative interpretation would generate the same results in this model.

Another possibility is that bailouts may induce banks to fail in the same state of the world (Farhi and Tirole (2011)).

and to obtain neat expressions, I assume that ω has cdf F_ω, with mean 0 and variance σ_ω^2, and the other random variables are Gaussian, $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$, $u_i \sim N\left(0, \sigma_u^2\right)$, $\forall i$. All random variables are assumed to be independent of each other. Let Φ_ε and Φ_u denote the cdf of a Gaussian distribution with mean 0 and variances σ_ε^2 and σ_u^2, respectively.

At time 0, after the contract is signed, the manager has access to a Walrasian market where he can trade securities which can be contingent on all the sources of risk in the economy. Securities are issued by firms, which I call intermediaries, that make money by selling them to managers or investors[7]. There are N intermediaries in the economy, each of them indexed by $\ell \in \{1, ..., N\}$. In the basic version of the model, I assume that intermediaries can issue and trade securities without paying any cost. This not only simplifies the task of finding an equilibrium in the market for securities, but also the notation used to define securities. As it will become clear in section 6, the absence of trading costs implies that it is enough to define only Arrow securities.

Denote by $z_{j,\hat{\varepsilon}}$ the Arrow security that pays off one unit of consumption at time 1 when the realization of the idiosyncratic shock ε_j is $\hat{\varepsilon}$. Similarly, $z_{\hat{\omega}}$ denotes the security that pays off one unit of consumption at time 1 when the realized aggregate state is $\hat{\omega}$. Let $\mathcal{Z}^\varepsilon$ and $\mathcal{Z}^\omega$ be the space of Arrow securities contingent on ε-risk and ω-risk, respectively, and let $\mathcal{Z} = \mathcal{Z}^\varepsilon \cup \mathcal{Z}^\omega$. Let $p : \mathcal{Z} \to \mathbb{R}_+$ be the price schedule of these securities. In the following sections, I will show that, when trading costs are absent, in equilibrium $p(\cdot)$ is a linear function over $\mathcal{Z}$. This will no longer be true when trading costs are introduced. Denote by $d_i : \mathcal{Z} \times \mathbb{R}_+ \to \mathbb{R}$ the demand of Arrow securities $z \in \mathcal{Z}$ at price p_z by agent i. When solving the model, I will make an important distinction between the Arrow securities in $\mathcal{Z}^\varepsilon$ and those in $\mathcal{Z}^\omega$. It is then convenient to denote by d_i^ε and d_i^ω the demand of manager i when the space of Arrow securities is restricted to $\mathcal{Z}^\varepsilon$ and $\mathcal{Z}^\omega$, respectively.

Payoffs. In equilibrium, intermediaries compete with each other and offer Arrow securities at price $p(\cdot)$. Managers decide the investment fraction k_i to invest in the specialized projects and the quantities of Arrow securities to trade.

The final profits generated by manager i who has a demand d_i of Arrow securities are

$$\Pi_i^m = \pi_i + \int \left(z_{j,\hat{\varepsilon}} - p_{j,\hat{\varepsilon}}\right) d_{i,j,\hat{\varepsilon}} \; d\hat{\varepsilon} \; dj + \int \left(z_{\hat{\omega}} - p_{\hat{\omega}}\right) d_{i,\hat{\omega}} \; d\hat{\omega} \tag{2}$$

These profits are delivered to investors who then make a payment ξ_i to manager i. This payment depends on the what investors can observe as stated in Assumption 1. Each manager chooses an

[7] In equilibrium, investors will not want to participate in the securities market.

investment fraction k_i and a demand schedule d_i so as to maximize the expected utility

$$\max_{k_i,d_i} \mathbb{E}\left[u\left(\xi_i\right)\right],$$

where the expectation is taken over the realizations of ξ_i.

Let $y_\ell : \mathcal{Z} \times \mathbb{R}_+ \to \mathbb{R}$ be the quantity of security z supplied by ℓ at price p_z. Intermediary ℓ makes profits Π_ℓ^I by selling Arrow securities to managers:

$$\Pi_\ell^I = \int \left(p_{j,\hat{\varepsilon}} - z_{j,\hat{\varepsilon}}\right) y_{\ell,j,\hat{\varepsilon}} \; d\hat{\varepsilon} \; dj + \int \left(p_{\hat{\omega}} - z_{\hat{\omega}}\right) y_{\ell,\hat{\omega}} \; d\hat{\omega}$$

Every investor owns an equal share of each intermediary and there is no agency problem between them. Thus, if we denote by $m\left(\omega\right)$ the marginal utility of consumption of the representative investor[8], intermediary ℓ solves:

$$\max_{y_\ell} \mathbb{E}\left[m\left(\omega\right)\Pi_\ell^I\right] \tag{3}$$

Finally, investors receive profits from managers and intermediaries and make payments ξ_i to each manager. Therefore, in period 1 their consumption is $c\left(\omega\right) = \int \left(\Pi_i^m - \xi_i\left(\pi_i,\omega\right)\right) di + \sum_\ell \Pi_\ell^I$ and their marginal utility of consumption in state ω is $m\left(\omega\right) \equiv v'\left(c\left(\omega\right)\right)$. In equilibrium, investors will fully diversify across managers and the economy will admit a representative investor. Diversification across managers also implies that the representative investor will be able to write a contract with each manager separately. Formally, the representative investor solves:

$$\max_{\xi_i} \mathbb{E}\left[m\left(\omega\right)\left(\Pi_i^m - \xi_i\right)\right].$$

Information. To complete the description of the model, I need to make assumptions on the information sets of the different agents.

Assumption 1 *(a) The investment fraction k_i is observed only by manager i.*

(b) The random variables ε_i and u_i, $\forall i$, and ω are realized at time 1 *and observed by* every agent*.*

(c) Investors do not observe the realization of the matching technology $F \in \mathcal{F}$.

(d) Investors do not observe the quantities of Arrow securities z traded between managers and intermediaries.

[8] In equilibrium, $m\left(\omega\right)$ will depend only on the aggregate state ω.

Part (a) represents the key moral hazard problem: without the right incentives, a manager will avoid paying the non-monetary cost by investing all the capital in projects correlated to the aggregate state. Part (b) implies that investors can potentially condition the payment to the manager on the realizations of all the shocks in the economy. However, by part (c), they will want to condition the payment only on the realization of ω (and, of course, the profits of the manager). Intuitively, this assumption means that investors do not know the specific risk each manager is exposed to. Without this assumption, investors would be able to perfectly infer k_i and the agency problem would disappear. If we were to relax this assumption and, say, allow investors to observe ε_i but not u_i, the agency problem would still remain, but the contract would look differently. This raises some interesting questions on how the incentives provided in the optimal contract depend on whether both parties or only managers have access to information about ε_i. In section 5.1, I discuss some implications of different assumptions on the observability of ε_i.

Finally, part (d) is a stark way to capture the idea that securities to hedge risks are traded in opaque markets and are often complex for non-specialists. Under this assumption, investors cannot condition their incentives on the trades made by the managers who trade different types of securities to hedge the risks arising from the incentive schemes. This assumption will be responsible for the ambiguous effects that securities markets have on the quantity of aggregate risk and welfare in the economy. Anticipating some results, if investors could observe trading activities, they would always prefer to forbid the trades of securities contingent on the aggregate state. These trades distort the incentives of the managers away from the desired solution and act as a constraint on the incentives that can be provided to managers.

In section 6, I replace part (d) with the assumption that investors can observe the quantity of securities traded by managers, but not the *type* of securities. This alternative assumption is motivated by the idea that, while investors can often observe whether financial institutions are trading securities, they don't have the expertise to understand these often complex securities (Brunnermeier and Oehmke (2011)).

I make the following assumptions on the utility and cost functions.

Assumption 2

(a) *The utility function $u(\cdot)$ is such that $\tilde{u}(x) \equiv (u')^{-1}(1/x)$ is increasing and concave.*

(b) *The utility $u(\cdot)$ and cost $C(\cdot)$ functions are such that $u(\lambda x) - C(\lambda x) = h(\lambda)(\tilde{u}(x) - C(x))$, for some positive function $h(\cdot)$.*

Part (a) of assumption 2 is basically an assumption on the curvature of the utility function of the agent which is discussed in Jewitt (1988)[9]. To characterize the optimal contract I will use the First Order Approach (FOA). There is also another important point which will be apparent once securities markets are introduced. If under the optimal contract the problem of the agent is concave, then he will want to buy insurance when given the possibility. In other words, the agent will participate in the securities markets to insure the risks that he is exposed to by the optimal contract.

Part (b) is a homogeneity property that serves an important purpose. This assumption and the fact that the profits (1) are proportional to the capital invested imply that the contracting problem for each manager will be invariant to the quantity of capital invested. In other words, under this assumption, incentives are invariant on how managers distribute capital across managers. Thus, in equilibrium investors will lend the same amount of capital to each manager and fully diversify their investment. Finally, if investors are fully diversified across managers, I can simplify the problem to that of a represenative investor who designs the optimal contract for each manager separately.

Managers and intermediaries meet in a Walrasian market to trade Arrow securities. The assumption of a Walrasian market deserves some comments. The complexity and the degree of customization of these financial securities often require trading to occur on OTC markets where the seller and the buyer directly trade in a decentralized fashion. Indeed, one of the main goals of this paper is to study the implications for the investment decisions of introducing opaque markets where outside investors have limited ability to monitor and control trades of securities. In this sense, the choice of a Walrasian environment is not very realistic and there is a growing literature that dispenses with the Walrasian assumption and focuses on decentralized markets (Duffie et al. (2005)). However, while models of decentralized trading would describe the functioning of OTC markets more realistically, they would also greatly complicate the analysis without changing the main message of the model. The focus of this paper is on the effects of complex securities on investment choices and not on the specific features of the market where these securities are exchanged. Also, the conclusions of this paper are likely to hold under different trading arrangements as long as the different types of risks are hedgeable and some trades cannot be observed.

Finally, the intermediation role of financial institutions is only implicit: managers borrow money from investors and run the projects themselves. There is, however, an alternative interpretation which leads to similar conclusions. Investors lend money to financial institutions which then channel

[9] See also Rogerson (1985), Sinclair-Desgagne (1994) and Conlon (2009) for different conditions on the validity of the FOA.

this money to entrepreneurs who can select and run projects. In this more general setting there is room for two layers of moral hazard. Financial intermediaries have the expertise to monitor the entrepreneurs (Diamond (1984)) who, in turn, need incentives to make the right investment choice (that is, projects with higher return and lower correlation)[10].

Equilibrium

As it is standard in principal-agent models with trades of securities, the equilibrium of the model is a combination of a standard Walrasian equilibrium and an optimal contracting problem between principals and agents.

Since the problem of every agent is perfectly symmetric, I restrict attention to a symmetric equilibrium where all the investors and managers make the same choice. Also, under assumption 2, in equilibrium investors will fully diversify their investments by lending an equal share of their endowment to each manager. Thus, investors will care only about the mean return and the aggregate risk of their portfolio. Thus, in equilibrium each investor will consume $c(\omega)$ which is a function of only the aggregate state.

The fact that the stochastic discount factor $m(\omega)$ depends only on the aggregate state and that I focus on a symmetric equilibrium greatly simplify the analysis since I can now focus on the optimal contracting problem between a representative investor and a single manager separately.

Definition 1 (Contract) *Given a price schedule $p(\cdot)$, a contract between a principal and a manager i is a tuple (k_i, d_i, ξ_i) where k_i is the suggested level of investment in the specialized projects, d_i is the suggested demand schedule of the different securities, and $\xi_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is the payment made to the manager when (π_i, ω) is observed.*

To simplify notation, I omit the dependence of (k_i, d_i, ξ_i) on $p(\cdot)$. An agent who behaves as specified by a contract (k_i, x_i, ξ_i) receives utility

$$\int u\left(\xi_i\left(\pi_i, \omega\right)\right) dF_{\pi_i,\omega}\left(\pi_i, \omega | k_i, d_i, p(\cdot)\right) - C\left(k_i\right),$$

where $F_{\pi_i,\omega}\left(\pi_i, \omega | k_i, d_i, p(\cdot)\right)$ is the cdf of the joint distribution of (π_i, ω) when the specified allocations are (k_i, d_i) and securities are priced according to $p(\cdot)$. Denote by $\left(k_i^{\bar{u}}, d_i^{\bar{u}}, \xi_i^{\bar{u}}\right)$ a contract that delivers utility at least $\bar{u}$ to agent i, that is,

$$\int u\left(\xi_i^{\bar{u}}\left(\pi_i, \omega\right)\right) dF_{\pi_i,\omega}\left(\pi_i, \omega | k_i^{\bar{u}}, d_i^{\bar{u}}, p(\cdot)\right) - C\left(k_i^{\bar{u}}\right) \geq \bar{u}.$$

[10] See, for example, Holmstrom and Tirole (1997) and Brunnermeier and Sannikov (2011).

When this inequality is satisfied, I will say that a contract is $\bar{u}$-individually rational ($\text{IR}^{\bar{u}}$).

A contract is incentive compatible (IC) if the agent doesn't want to deviate by choosing different quantities $\left(\hat{k}_i, \hat{d}_i\right)$, formally

$$(k_i, d_i) \in \arg\max_{\left(\hat{k}_i, \hat{d}_i\right)} \int u\left(\xi_i\left(\pi_i, \omega\right)\right) dF_{\pi_i,\omega}\left(\pi_i, \omega | \hat{k}_i, \hat{d}_i, p\left(\cdot\right)\right) - C\left(\hat{k}_i\right).$$

Denote by $\mathcal{C}^{\bar{u}}\left(p\left(\cdot\right)\right)$ the set of contracts which are $\text{IR}^{\bar{u}}$ and IC when the equilibrium price schedule is $p\left(\cdot\right)$. To simplify notation, in the rest of the paper I will drop the superscript $\bar{u}$ from the contract.

Definition 2 (Equilibrium) *An equilibrium is a price schedule $p\left(\cdot\right)$, contracts $(k_i, d_i, \xi_i) \in \mathcal{C}\left(p\left(\cdot\right)\right)$, $\forall i$, and supply schedules y_ℓ, $\forall \ell$, such that:*
(a) Given prices $p\left(\cdot\right)$, (k_i, d_i, ξ_i) is optimal for the investors;
(b) Given prices $p\left(\cdot\right)$, intermediaries supply y_ℓ so as to maximize $\mathbb{E}\left[m\left(\omega\right)\Pi_\ell^I\right]$, $\forall \ell$;
(c) Prices $p\left(\cdot\right)$ are such that securities markets clear: $\int d_i di = \sum_\ell y_\ell$, $\forall z$.

The definition of equilibrium essentially requires that every agent optimizes by taking the pricing function $p\left(\cdot\right)$ as given and markets clear. It is common in problems with endogenous financial innovation that many securities are not created/traded in equilibrium. The issue is then to price these (latent) securities so that an equilibrium exists. This problem doesn't arise here because there is no cost of issuing securities and so all securities are created in equilibrium.

In the next sections I consider some special cases of the general model which will clarify how the full model works. In these sections, the definition of equilibrium comes directly from definition 2 and, therefore, I will not repeat it.

4 No contingent securities

This section focuses on the important special case where markets for securities are shut down. The goal of this section is twofold: it helps gain intuition before solving the full model and it represents an important benchmark for the full model's solution. Also, I show that the optimal contract in this special case displays some interesting features which are novel in the literature.

With no trades of securities, the only frictions in the economy are the fact that only managers observe the fraction of wealth invested in the idiosyncratic risk (and, of course, that managers have the technology to screen the projects). I show that the agency problem reduces the level of investment in the specialized projects that it is optimal to implement in equilibrium. In turn, this increases aggregate volatility in the economy and lowers welfare. The agency problem, therefore, exacerbates volatility in the economy. Nevertheless, this is not a reason for policy intervention: the equilibrium when without trading of securities is constrained efficient.

Of course, when trades of securities are forbidden, there is no need for intermediaries or prices. Thus, in what follows I will drop quantities d_i, y_ℓ and the price schedule $p(\cdot)$ from all the equilibrium objects.

Assumption 2 implies that investors will fully diversify across managers and I can focus on the contracting problem between a representative investor and a single manager, which greatly simplifies the analysis.

Let k_i be the investment fractions that the principal wants to implement in equilibrium. It turns out that it is convenient to rewrite the problem by considering the following transformation of π_i

$$x_i = \frac{\pi_i - \bar{R} - r\ k_i - \omega\,(1-k_i)}{\sigma_x},$$

where $\sigma_x = \sqrt{k_i^2\sigma_\varepsilon^2 + \sigma_u^2}$ is the idiosyncratic volatility of π_i if the manager chooses exactly k_i. Suppose now that the investor recommends an investment fraction k_i to manager i, but the latter deviates to a fraction $\hat{k}_i \neq k_i$. Then, the distribution of x_i will in general depend on both k_i and $\hat{k}_i$. Also, when $\hat{k}_i = k_i$, x_i is the linear projection of π_i on the space othogonal to ω and, in fact, in equilibrium we have that x_i is uncorrelated with ω. Moreover, by the Gaussian assumption, x_i turns out to be the best predictor of the idiosyncratic component of π_i, that is, $k_i\varepsilon_i + u_i$.

The distribution of x_i conditional on ω when the recommended fraction is k_i but the manager deviates to $\hat{k}_i$ is also Gaussian with with mean and variance given by

$$\mu_{x|\omega} = \frac{r-\omega}{\sigma_x}\left(\hat{k}_i - k_i\right), \qquad \sigma_{x|\omega}^2 = \frac{1}{\sigma_x^2}\left(\hat{k}_i\sigma_\varepsilon^2 + \sigma_u^2\right), \tag{4}$$

and, in equilibrium where $\hat{k}_i = k_i$, we have $\mu_{x|\omega} = 0$ and $\sigma_{x|\omega}^2 = 1$. This is the reason why this transformation makes it easier to solve the problem.
Thus, if we denote this distribution by $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right)$, we have that in equilibrium where $\hat{k}_i = k_i$, $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right)$ doesn't depend on ω nor on k_i and $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right) = \Phi(x)$, where $\Phi(x)$ is the cdf of a standard Gaussian distribution.

With this transformation, the contracting problem with no securities solves

$$\max_{\xi,k} \int m(\omega) \left(x + \bar{R} + r \ k + (1-k)\,\omega - \xi(x,\omega) \right) d\Phi(x)\, dF_\omega(\omega) \tag{P(NS)}$$

subject to:

$$k \in \arg\max_{\hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}\right) dF_\omega(\omega) - C\left(\hat{k}\right), \tag{IC}$$

$$\int u(\xi(x,\omega))\, d\Phi(x)\, dF_\omega(\omega) - C(k) \geq \bar{u}. \tag{IR}$$

The investor maximizes the final payoff of the investment weighted by his marginal utility of consumption subject to two constraints. The first constraint requires that the compensation scheme and the recommended efforts are such that the manager finds it optimal to comply with the recommendation. The second constraint is the usual IR constraint.

The common strategy in the moral hazard lietarature is to relax problem P(NS) by replacing the IC constraint with its first-order condition. In the appendix, I show the conditions under which the relaxed problem has the same solution as the original problem, that is, the FOA is valid. Formally, I can replace IC by

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}\right) dF_\omega(\omega) \bigg|_{\hat{k}=k} - C'(k) = 0. \tag{IC'}$$

The big advantage is that we can now use Lagrangian methods and solve P(NS) by taking first-order conditions. Let λ and μ be the Lagrange multipliers on the IR and IC' constraints, respectively. The following proposition characterizes the optimal contract.

Proposition 1 *Let k be the investment fraction that the principal wants to implement. The optimal contract for the model with no contingent securities solves*

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda \left[\frac{1}{\sigma_x}(r-\omega)\,x + \frac{k\ \sigma_\varepsilon^2}{\sigma_x^2}\left(x^2 - 1\right) \right]. \tag{5}$$

To gain some intuition on the optimal contract (5) we can compare it to the case with no agency problem where investors are allowed to observe also k_i (I refer to this case as the "first-best"). If investors can observe k_i, they will severily punish the manager who doesn't comply with the recommendation. Assuming that the punishment can be made severe enough that we can drop the IC constraint from the problem we have that the first-best contract will be given by (5) with

$\lambda = 0$[11]. Thus, the first-best optimal contract simply allocates aggregate risk between the two risk-averse agents and the optimal payment schedule does not depend on the realization of π (Stoughton (1993)). Importantly, since the manager incurs in the cost $C(k)$, even in the first-best the optimal choice of k may be different from 1 and there may be some aggregate risk in the economy.

The shape and the interpretation of (5) is made easy by the assumption of Gaussian random variables. The contract has two main components. First, the usual risk-sharing component given by the left-hand side of (5). This term determines how aggregate risk is shared between the investor and the manager depending on the curvature of their utility functions. This term was the only piece in the first-best contract which completely insulated the agent from the idiosyncratic risk. However, to provide the manager with the right incentives to comply with the contract, the payment schedule has to be also a function of the new variable x. Incentives are provided through the right-hand side of (5).

The optimal contract has the same structure as that obtained by Holmstrom (1979), who shows that the best way to incentivize the agent is to make his payment conditional on the likelihood ratio of his action. The Gaussian assumption for ε_i and u_i delivers this simple expression for the likelihood ratio, which is given by the the term that multiplies λ in (5).

From (4) we know that, if the agent invests in the specialized projects a fraction $\hat{k}_i$ that is slightly lower than the suggested k_i, this will have three effects on the distribution of x_i. First, the mean of x_i will be lower. Thus, when observing a lower realization of x_i the principal should infer a deviation by the agent and punish him accordingly. This explains the term $r\ x_i$ in the right-hand side of (5). Secondly, when a lower $\hat{k}_i$ is selected, the distribution of x_i will be correlated with ω. Thus, a comovement between x_i and ω is a signal of a possible deviation and, thus, the optimal contract punishes the agent (this explains the term $\omega\ x$ in the contract). Finally, a lower choice of $\hat{k}_i$ also reduces the volatility of x_i and so the contract rewards the agent when realizations of x_i which are far from its mean are observed. This is the reason why the convex term x^2 enters the contract. From (4) we know that in equilibrium the variance of x is 1, hence the optimal contract rewards the agent for realizations of x^2 relative to this value. Of course, in equilibrium the term that multiplies λ in (5) has mean 0 (this is a general property of likelihood ratios).

The optimal contract uses the aggregate state ω to provide the agent with incentives. In equilibrium, the investor conditions the payment of each manager to the average performance of the other managers in the economy. This type of benchmarking, however, is different from the result stressed in the moral hazard literature with common noise following Holmstrom (1979). The latter

[11] Note that the first-best Lagrange multiplier μ_{FB} will differ from μ in (5).

also considers a principal-agent problem with multiple agents and correlated risk. Importantly, he assumes that $\pi_i = (r + \varepsilon_i) k_i + \omega + u_i$, that is, the choice of the agent doens't affect the amount of aggregate risk in the project. With this payoff structure, the model of this paper would reproduce the classical result that when the aggregate state is known, the optimal contract should not be conditioned upon it. Intuitively, more risk that is not related to the agent's effort only makes it harder to incentivize a risk-averse agent[12].

The agency problem reduces make is more expensive for the principal to implement a certain value of k. Thus, it is natural to expect a lower value of k to be implemented in equilibrium.

Proposition 2 *When the choice of k is not observable, equilibrium k is lower (and aggregate volatility is higher).*

The agency problem, therefore, cause the volatility of the economy to increase. Is it inefficient? It turns out that a social planner with the same information as the investors (that is, the planner also faces the same agency problem) cannot improve on the equilibrium.

Proposition 3 *The equilibrium outcome of the economy is efficient.*

The agency problem causes the economy to be more volatile and yet there is no room for policy intervention. This conclusion will change radically when agents will be allowed to trade securities.

5 Trades of Securities

In this section, I consider the full model where managers can trade securities contingent on the different risks in the economy. The assumptions on the distributions of the shocks guarantee that managers will find it optimal to trade contingent securities and hedge their risks. One of the main conclusions of this model is that, under certain conditions, trading of securities has dramatically different implications for aggregate volatility and welfare depending on whether idiosyncratic or aggregate risk is traded. More specifically, I show that investors are better off when managers pool

[12] We can see this in my model by observing that with this new definition of π_i (5) becomes:

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda \left[\frac{1}{\sigma_x} r \ x + \frac{k \ \sigma_\varepsilon^2}{\sigma_x^2} \left(x^2 - 1 \right) \right],$$

and the aggregate state disappears from the incentives component of the contract (ω appears only through the risk-sharing component). As expected, the principal doesn't uses aggregate risk to incentivize the agent. If in addition the principal was risk-neutral, then he would completely insulate the agent from aggregate risk.

and eliminate their exposure to idiosyncratic risks. Importantly, idiosyncratic risk can be eliminated without concentrating risk on the balance sheets of the intermediaries and, hence, without the need for investors to bear extra risk through the profits of the intermediaries.

These conclusions change substantially when aggregate risk is considered. By definition, aggregate risk cannot be pooled and eliminated, but some agents have to ultimately bear it. Also, in the symmetric equilibrium that I consider in this paper, managers receive the same contract and, thus, bear the same amount of aggregate risk. Thus, the only gains from trading securities on aggregate risk are possible only if investors paricipate in the market. The fact that investors are willing to take some aggregate risk back, however, might seem unreasonable. After all, investors are those who design the contract that exposes managers to some aggregate risk to incentivize them. Indeed, if investors could affect the amount of ω-securities that are issued by intermediaries and traded by managers, they would prevent from doing so. However, by assumption 1, investors do not observe and, hence, cannot contract upon the trades investors make. Even if investors cannot observe the trades of securities, they are sophisticated enough to change the optimal contract so that managers are not willing to trade ω-securities in equilibrium. Thus, in this model trading of ω-securities will only happen off equilibrium and it will act as a constraint on the optimal contracting problem. This is in contrast with the ε-securities, which are traded on equilibrium.

The amount of insurance bought by managers depends on its equilibrium price. In the case of ε-securities, the possibility of eliminating risks by pooling them together allows for insurance to trade at an actuarially fair price[13]. If actuarially fair insurance is at least possible for securities contingent on idiosyncratic risk, this is no longer true when aggregate risk is traded. Intuitively, this risk has to shared between investors and managers and the price of insurance will depend, among other things, on their marginal utility of consumption. Trading costs and market frictions contribute to make insurance of aggregate risk more expensive.

Since they have potentially different effects on the princiapl-agent problem, it is helpful to first analyze ε-securities and ω-securities separately.

5.1 Securities on idiosyncratic risk

I first focus on securities contingent on idiosyncratic risk and forbid trades of securities contingent on the aggregate state. Assumption 1 tells us that investors cannot observe the trades made by managers and, thus, they can't condition the payment schedule on this information. I start with

[13] Of course, in the presence of a cost to trade securities (as in sections 6 and 7), market power, or other frictions, the price of insurance would deviate from the the actuarially fair price.

the characterization of the optimal contract for given prices and then solve for the equilibrium price schedule which clears the securities market. In this paper, markets are assumed to be competitive and all the agents take the price schedule $p(\cdot)$ as given. Also, in the basic model presented in this section, there is no cost of trading securities.

When ω-securities are not allowed, the final payoff (2) of manager i who invests a fraction k_i in the specialized projects and buys a quantity $d_{i,j,\hat{\varepsilon}}$ of security $z_{j,\hat{\varepsilon}}$ at price $p_{j,\hat{\varepsilon}}$ is

$$\Pi_i^m = \pi_i + \int (z_{j,\hat{\varepsilon}} - p_{j,\hat{\varepsilon}}) \; d_{i,j,\hat{\varepsilon}} \; d\hat{\varepsilon} \; dj,$$

here, I use d_i^{ε} since the space of securities is restricted to Z^{ε}. Let $F_{\Pi_i,\omega}(\Pi_i, \omega | k_i, d_i, p(\cdot))$ be the distribution of the pair (Π_i, ω) for given choice of k_i, demand schedule d_i, and price schedule $p(\cdot)$.

The agent now chooses both the fraction of specialized investment k_i and the demand schedule d_i for given price schedule $p(\cdot)$. The IC constraint for the contracting problem becomes:

$$(k_i, d_i) \in \arg\max_{\hat{k}, \hat{d}_i} \int u(\xi_i(\Pi_i, \omega))\, dF_{\Pi_i,\omega}\left(\Pi_i, \omega | \hat{k}_i, \hat{d}_i, p(\cdot)\right) - C\left(\hat{k}_i\right) \tag{6}$$

As for the analysis of section (4), it is convenient to consider the linear projection of Π_i^m onto the space orthogonal to ω. Formally, define x_i as follows:

$$x_i = \frac{\Pi_i^m - \bar{R} - r\; k_i - \omega(1 - k_i) + \int p_{j,\hat{\varepsilon}} \; d_{i,j,\hat{\varepsilon}} \; d\hat{\varepsilon} \; dj}{\sigma_{i,x}}, \tag{7}$$

where $\sigma_{i,x}^2 \equiv Var(x_i)$ is the equilibrium variance of x_i (that is, when $\hat{k}_i = k_i$).

As lemma 1 shows, the equilibrium price is such that the idiosyncratic risk is traded at a actuarially fair price.

Lemma 1 *The equilibrium price of an Arrow security $z_{i,\hat{\varepsilon}}$ is $p_{i,\hat{\varepsilon}} = \phi_{\varepsilon}(\hat{\varepsilon})$.*

The question is now whether principals want agents to buy full insurance at the price of lemma 1. The answer is much complicated by the fact that the idiosyncratic shock ε_i multiplies k_i in the final payoff of the manager. Thus, a more volatile ε_i can potentially convey some information about the actual choice of k_i. This reasoning, of course, does not apply to the shocks of all the sectors to which the manager is not matched. If a manager traded securities conditional on the shocks of other sectors, this would only add noise to his profits and worsen the agency problem.

Lemma 2 *If managers trade ε-securities conditional on shocks of sectors to which they have not been matched, equilibrium k and welfare decrease.*

The main complication with the fact that a more volatile ε_i contains information about k_i is that we are allowing for any ε-security to be traded. By trading in the securities market, the manager can buy ε-securities that change the distribution of ε. More formally, the distribution of (7) is not necessarily Gaussian, since the agent can potentially demand any quantity $d_{i,j,\hat{\varepsilon}}$ of any security $z_{j,\hat{\varepsilon}}$. The principal has to incentivize manager i to choose a demand schedule d_i and, thus, a whole distribution $F_{\Pi_i,\omega}(\Pi_i, \omega | k_i, d_i, p(\cdot))$ so that his welfare is maximized. Thus, the quantity of securities demanded by the agent depends on the optimal contract which, in turn, has to be chosen by the principal so that the agent demands the right amoung of securities.

However, when full insurance is optimal for the principal the problem becomes simpler. To see this in a more formal way, note that an agent wants to buy full insurance whenever his payment schedule ξ makes his problem concave in x. Conjecture now that the agent buys full insurance and solve for the optimal ξ. If this payment schedule makes the problem of the agent concave, then the conjecture is verified and we have found the solution to original the problem.

The problem is then to find conditions under which the principal wants the agent to buy full insurance. It is easy to see that full insurance would be optimal in the absence of the error term u_i in the payoff of the manager. In fact, with no u_i in the payoff, full insurance would make the choice of k_i perfectly observable and the agency problem would disappear altogether. This would lead to the first-best solution which, by definition, is the best outcome for the principal. On the other side, suppose that the volatility of ω is close to 0 and so is the mean r. In this case, it is harder for the principal to identify the value of k_i chosen by the agent and a more volatile ε_i can help the principal by making the distribution of Π_i^m more sensitive to k_i.

In the appendix, I derive a condition for full insurance to be optimal. This condition is related to the volatility of the likelihood ratio of the distribution $F_{x_i|\omega}$. Intuitively, the likelihood ratio can be seen as a measure of how informative are the signals about the choice of k_i. In turns out that signals are more informative when the likelihood ratio is more volatile. Therefore, a more volatile likelihood ratio leads to a better outcome for the principal. The condition in the appendix requires that the likelihood ratio is most volatile when the variance of σ_ε^2 is 0, that is, when the agent is fully insured. This condition is more likely to be satisfied for higher values of r, for lower values of σ_u^2 and higher values of σ_ω^2. This is intuitive in light of the discussion above.

The problem would be simpler if the agent was allowed to trade only *linear* securities, that is, securities with a payoff $q\ \varepsilon_i$, for some scalar q. These securities, in fact, preserve the normality of the $F_{x_i|\omega}$ and let me identify easy sufficient conditions on the paramters of the model for which full insurance is optimal.

Lemma 3 *Assume that only linear ε-securities can be traded. If $0 \leq \sigma_u^2 \leq (r-\omega)^2, \forall\omega$, then full insurance is optimal.*

The condition is easy to interpret. Take for example $r = 0$. Then the condition says that full insurance is optimal whenever the realizations of the random variable ω are "big" enough[14]. Also, this sufficient condition may seem restrictive because it holds for any problem (of course, under the assumption of linear securities). For each specific problem, however, it is possible to weaken this assumption (for example, to substitute it with some appropriate average of ω).

Let's conjecture that it is optimal for the agent to buy full insurance. Formally, this means that an agent who invest a fraction k_i in the idiosyncratic project will demand $-\hat{\varepsilon}\ k_i$ units of the Arrow securities $z_{i,\hat{\varepsilon}}$, $\forall\hat{\varepsilon}$, and zero units of all the other securities. Also, from lemma 1, we know that the cost of this insurance is

$$\int p_{i,\hat{\varepsilon}}\ d_{i,j,\hat{\varepsilon}}\ d\hat{\varepsilon} = -k_i \int \hat{\varepsilon}\ d\Phi_\varepsilon(\hat{\varepsilon}) = 0$$

Combining these two results implies that the profits of the agent are given by

$$\Pi_i^m = \pi_i - k_i\varepsilon_i$$

Similarly, (7) becomes

$$x_i = \frac{\Pi_i^m - \bar{R} - r\ k_i - \omega(1-k_i)}{\sigma_u}.$$

As usual, in equilibrium where the agent trades $-k_i\varepsilon_i$ the random variables x_i and ω are again uncorrelated.

With a slight abuse of notation, let $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i, p(\cdot)\right)$ be the cdf of the Gaussian random variable x_i conditional on ω, when the agent chooses $\hat{k}_i$ and trades $-k_i\varepsilon_i$. Importantly, note that when the agent buys full insurance, the distribution $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i, p(\cdot)\right)$ will not depend on $p(\cdot)$, thus I will simply write $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i\right)$. However, out of the equilibrium, if the agent deviates to a different portfolio allocation, then this distribution will depend on the price schedule $p(\cdot)$.

[14] Remember that ω is not restricted to be continuous, but it can be a discrete random variable with mean 0. For example, $\omega_H > 0 > \omega_L = -\omega_H$.

The moments of $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i\right)$ are given by

$$\mu_{x|\omega} = \frac{r-\omega}{\sigma_u}\left(\hat{k}_i - k_i\right), \quad \sigma^2_{x|\omega} = \left(\hat{k}_i - k_i\right)^2 \frac{\sigma^2_\varepsilon}{\sigma^2_u} + 1 \tag{8}$$

In equilibrium, $\hat{k}_i = k_i$ and the agent trades $-k_i\varepsilon_i$, so the moments are $\mu_{x|\omega} = 0$ and $\sigma^2_{x|\omega} = 1$. Thus, once again we have that $F_{x|\omega}\left(x|\omega; k_i, k_i, -k_i\varepsilon_i\right) = \Phi\left(x\right)$.

We are now ready to solve the optimal contracting problem where the agent buys full insurance and both the principal and the agent take the price of ε-securities as given. As shown in (6), the agent now faces two choices. First, he has to decide what fraction k_i to invest in the specialized projects. Second, he has to decide the quantity of Arrow securities to trade with intermediaries.

Formally, the optimal contract solves (omitting subscripts i for convenience):

$$\max_{\xi,k,d^\varepsilon} \int m\left(\omega\right)\left(x + \bar{R} + r\ k + \left(1-k\right)\omega - \xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_\omega\left(\omega\right) \tag{P(ε)}$$

subject to:

$$\left(k, d\right) \in \arg\max_{\hat{k},\hat{d}} \int u\left(\xi\left(x,\omega\right)\right) F_{x|\omega}\left(x|\omega; k, \hat{k}, \hat{d}, p\left(\cdot\right)\right) - C\left(\hat{k}\right), \tag{IC}$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_\omega\left(\omega\right) - C\left(k\right) \geq \bar{u}. \tag{IR}$$

This problem is similar to P(NS) in the case with no securities, except that now the IC constraint takes into account the two choices of the agent. Under the assumptions that make full insurance optimal, we can considerably simplify this problem.

To see this, under the conjecture that the principal wants the agent to buy full insurance, let's relax problem P(ε) by dropping the constraint IC on the choice of d^ε. If the contract that solves the relaxed problem is such that the agent wants to buy full insurance then this contract must also solves the original problem with the full IC constraints. Formally, we look for the values of ξ and k that solve

$$\max_{\xi,k} \int m\left(\omega\right)\left(x + \bar{R} + r\ k + \left(1-k\right)\omega - \xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_\omega\left(\omega\right)$$

subject to:

$$k \in \arg\max_{\hat{k}} \int u\left(\xi\left(x,\omega\right)\right) F_{x|\omega}\left(x|\omega; k, \hat{k}, -k\ \varepsilon, p\left(\cdot\right)\right) - C\left(\hat{k}\right),$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_\omega\left(\omega\right) - C\left(k\right) \geq \bar{u}.$$

Once agan, I conjecture that the FOA is valid for this problem, relax the IC constraint by replacing it with its first-order condition, and then verify that this conjecture is indeed verified at the optimal contract. The FOA allows us to solve for the optimal contract using Lagrangian methods as shown in the the next proposition.

Proposition 4 *Let λ and μ be the Lagrange multipliers on the IC and IR constraints, respectively, and suppose full insurance is optimal. The optimal payment schedule $\xi(x,\omega)$ satisfies:*

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda \frac{1}{\sigma_u}(r-\omega)\ x. \tag{9}$$

Proposition 4 immediately implies that the approach of relaxing the IC constraint and then verify that the agent wants to buy full insurance is valid. This follows from assumption 2 which guarantees that, under the optimal contract (9), the agent's problem is concave. In turn, concavity implies that the agent wants to buy full insurance (at actuarially fair price) when given the possibility. Incidentally, concavity of the agent's problem also implies that the FOA is valid for this problem (see, for example, Jewitt (1988)).

The shape of the new optimal contract is similar to (5) and, not surprisingly, the main difference is the absence of the convex term x^2. In this setting the agent is supposed to fully insure his idiosyncratic risk, thus the principal does not reward him if his profits display high volatility.

To gain more intuition on the contract in lemma 4 and on how ε-securities affect the equilibrium, it is useful to see what would happen if assumption 1 was relaxed so as to allow the manager to observe the identity i of the specific sector where the manager is investing. Of course, to make the problem nontrivial, I will assume that investors do not observe the realization of the noise u_i. In this case the two shocks, ω and ε_i, are perfectly symmetric in the sense that the payment ξ_i can be made conditional on both.

Similarly to the case with ω-securities analyzed in the following section, if the principal can observe the realization of ε_i then it is easy to see that there are no gains from allowing trades of ε-securities. Instead, the presence of ε-securities can only hurt investors to the extent that the latter cannot limit these trades. However, the scope of this exercise is to compare the equilibrium of the model when the principal has to go through the market to get insurance for the agent to case where the principal can provide this insurance directly if he wishes to do so. Thus, other relaxing assumption 1, I will also shut down the securities market.

Formally, the contracting problem is similar to P(NS) of section 4 with the difference that now

ξ can also be a function of ε. I can then define x as follows:

$$x_i = \frac{\pi_i - \bar{R} - r\ k_i - \omega\left(1 - k_i\right) - \varepsilon_i k_i}{\sigma_u},$$

so that, in equilibrium where $\hat{k}_i = k_i$, I have that $\pi_i = u_i/\sigma_u$. The next lemma describes the optimal contract under these new assumptions.

Lemma 4 *Let λ and μ be the Lagrange multipliers on the IC and IR constraints, respectively, and suppose full insurance is optimal. The optimal payment schedule $\xi\left(x,\omega\right)$ satisfies:*

$$\frac{m\left(\omega\right)}{u'\left(\xi\left(x,\omega,\varepsilon\right)\right)} = \mu + \lambda\frac{1}{\sigma_u}\left(r - \omega + \varepsilon\right)\ x. \tag{10}$$

As expected, in the contract (10) the two shocks, ω and ε_i, are treated symmetrically. The agent is punished if x_i displays correlation with the aggregate state ω and is *rewarded* if x_i comoves with the idiosyncratic shock ε_i. In fact, a correlation between x_i and ε_i is a sign that a higher than the agent selected a fraction $\hat{k}_i > k_i$. Lemma 4 clearly shows that it is not the same for the principal to have to rely on the securities markets to incentivize the agent. Importantly, the principal does not fully insure the manager against the idiosyncratic risk, but exposes him to some ε-risk. Moreover, since the outcome of lemma 4 is feasible under the assumptions of lemma 4, it follows that welfare has to be lower when i is not observable and ε-securities are available.

Lemma 4 is interesting also from another point of view. Suppose that the principal has access to some information about ε_i. For example, suppose that the principal receives a partially informative signal about the identity i of the specialized investment. How will the principal use this information? Lemma 4 suggests that the principal will expose the agent to some ε-risk by conditioning the optimal contract to his signal and the optimal payment will resemble (10).

I can now state the main results of this section. The key question is what happens to the aggregate volatility of the economy and to welfare when securities contingent on ε-risk are traded in the market. Since in every problem managers face a binding IR constraint, welfare here is simply the utility of the representative investor.

It turns out that, under the conditions that make full insurance of the ε-risk optimal for the principal, these securities reduce aggregate volatility and increase welfare.

Proposition 5 *Securities contingent on ε-risk increase equilibrium k (and, thus, lower aggregate volatility) and increase welfare in the economy.*

As discussed above, securities on ε-risk make it easier for a principal observing the profits of the agent to identify whether the agent has deviated or not. An easier problem for the principal translates into a lower cost of implementing higher values of k and, thus, into lower aggregate volatility. Similarly, a principal who can implement higher values of k more cheaply will also enjoy higher welfare.

5.2 Securities on aggregate risk

In this section, I only allow trades of securities contingent on the aggregate state ω. This case is very different from the previous section where only the idiosyncratic risk was hedgeable. In the symmetric equilibrium considered in this paper, all the managers are perfectly symmetric and, hence, share the same quantity of aggregate risk. Thus, the only way to hedge the aggregate risk is to transfer it back to investors.

From a mathematical point of view, the main difference between these two types of risks is that investors can always condition the payment schedule on aggregate risk if they find it optimal to do so. Thus, the fact that managers can trade away some of their aggregate risk should only act as a constraint on the contracting problem. I show that the possibility for managers to trade away some of the aggregate risk through ω-securities weakens the incentives that investors can provide in equilibrium. Therefore, contrary to the case where only idiosyncratic risk can be hedged, the fact that managers can transfer some risk back to investors, makes the agency problem worse. A direct implication is that the existence of ω-securities reduces welfare. Again, the intuition for this result is very simple: if transfering the risk was optimal for the principal, the optimal contract would already take this into account. The key friction is, therefore, that trades are not observable. Intuitively, if investors could contract upon the quantities of ω-securities purchased by managers, they would provide incentives for the latter to stay out of this market.

The environment with only ω-securities follows a logic similar to that with ε-securities. There are, however, two main differences. First, remember that investors can observe ω and, thus, the payment schedule ξ_i can be made conditional on ω. The principal can always condition the payment to the aggregate state and, intuitively, can replicate the outcome of the trading mechanism if he wishes to do so. This is an important difference which implies the sharp prediction for welfare of trading ω-securities. Secondly, aggregate risk cannot be eliminated by pooling, so some (risk-averse) agent in the economy has to ultimately bear it. In turn, this implies that the price to hedge aggregate risk cannot be actuarially fair as it was for ε-securities. In equilibrium, aggregate risk will be transferred back to the principals who are risk-averse and, thus, demand a compensation to take this risk.

The fact that the principal can always condition the payment to the aggregate state and replicate any portfolio of ω-securities chosen by the agent implies that in equilibrium I can focus on the case where there is no trading of ω-securities. To see this, suppose that manager i demands a quantity $d_{i,\hat{\omega}}$[15] of security $z_{\hat{\omega}}$ at price $p_{\hat{\omega}}$, generates profits equal to $\Pi_i^m = \pi_i + \int (z_{\hat{\omega}} - p_{\hat{\omega}})\, d_{i,\hat{\omega}}\, d\hat{\omega}$ and obtains a payment $\xi_i(\Pi_i^m, \omega)$. The principal can always define a new payment $\tilde{\xi}_i$ such that $\tilde{\xi}_i(\pi_i, \omega) = \xi_i(\Pi_i^m, \omega)$, $\forall \pi_i,\ \omega,\ \Pi_i^m$, such that the agent finds it optimal not to trade ω-securities. Clearly, this is suboptimal for the principal who had chosen ξ_i over $\tilde{\xi}_i$ in the first place.

Proposition 6 *The optimal contract is such that on the equilibrium path the manager does not trade ω-securities, that is, $d_{i,\hat{\omega}} = 0$, $\forall \hat{\omega}$, i.*

Thus, there are no trades in the securities market when only the ω-risk can be traded. However, even in the absence of trades, prices are still determined by the assumption of competition and trading costs. Remember that intermediaries are owned by investors and, thus, share their stochastic discount factor, $m(\omega)$. Intermediaries are then willing to trade a positive amount of ω-security (in fact, an infinite amount of them) whenever the price of these securities is above the marginal value of the investors. Therefore, in equilibrium the price of these securities has to be such that intermediaries are indifferent on the quantity of securities to sell.

Lemma 5 *The equilibrium price of an Arrow security $z_{\hat{\omega}}$ is $p_{\hat{\omega}} = m(\hat{\omega})\, f_\omega(\hat{\omega}) / \mathbb{E}[m(\omega)]$.*

The price of insurance against state $\hat{\omega}$, $m(\hat{\omega})\, f_\omega(\hat{\omega}) / \mathbb{E}[m(\omega)]$, is a combination of the probability density that $\hat{\omega}$ is realized (this is the same as in the equilibrium with ε-securities) and the principal's marginal utility of consumption. The more valuable for the principal is consumption in the state of the world $\hat{\omega}$, that is, the higher is $m(\hat{\omega}) / \mathbb{E}[m(\omega)]$, the higher will be the price of a security that pays off in that state.

The contracting problem with ω-securities is different from P(ε) in an important way. Contrary to the case with ε-securities, here we cannot conjecture that the agent doesn't want to trade the ω-risk and then verify this conjecture. In fact, if we were to relax the problem by assuming no trades of ω-securities and derive the optimal contract, this contract would not satisfy the initial conjecture: an agent receiving the payment schedule that solves the relaxed problem has an incentive to deviate if he can trade in the securities markets. This implies that we have to explicitly incorporate the portfolio choice of the agent into the optimal contracting problem.

[15] Similarly to the case with ε-securities, in this section I am using d_i^ω since the space of securities is restricted to be Z^ω.

In the case with only ω-securities, the profits of a manager are

$$\Pi_i^m = \pi_i + \int (z_{\hat{\omega}} - p_{\hat{\omega}})\, d_{i,\hat{\omega}} d\omega,$$

and, in equilibrium where no securities are traded, we have $\Pi_i^m = \pi_i$. Similarly, (7) becomes

$$x_i = \frac{\Pi_i^m - \bar{R} - r\ k_i - \omega\,(1-k_i)}{\sigma_x}.$$

Let $F_{x_i|\omega}\left(x|\omega; k_i, \hat{k}_i, \hat{d}_i, p\,(\cdot)\right)$ be the conditional distribution of x_i when the agent chooses $\hat{k}_i$ and instead of k_i and demands $\hat{d}_i$. In equilibrium, the principal wants the agent to choose $\hat{d}_{i,\hat{\omega}} = 0$, $\forall \hat{\omega}$, and of course $\hat{k}_i = k_i$. Again, if the agent follows the optimal contract, the equilibrium distribution becomes $F_{x_i|\omega}\left(x|\omega; k_i, \hat{k}_i, \hat{d}_i, p\,(\cdot)\right) = \Phi\,(x)$.

Let $\bar{U}_\omega\,(k_i, p\,(\cdot))$ be the value of the best deviation available to the manager:

$$\bar{U}_\omega\,(k_i, p\,(\cdot)) = \max_{\hat{k},\ \hat{d}} \int u\,(\xi\,(x,\omega))\, dF_{x_i|\omega}\left(x|\omega; k_i, \hat{k}, \hat{d}, p\,(\cdot)\right) dF_\omega\,(\omega) - C\left(\hat{k}\right)$$

So $\bar{U}_\omega\,(k_i, p\,(\cdot))$ is the value for a manager of deviating to a different $\hat{k}$ and a different demand schedule $\hat{d}$. Thus, to prevent the manager from deviating, the optimal contract has to be such that

$$\int u\,(\xi\,(x,\omega))\, d\Phi\,(x)\, dF_\omega\,(\omega) \geq \bar{U}_\omega\,(k_i, p\,(\cdot))\,. \tag{11}$$

Assume that the FOA is valid, the optimal contracting problem P(ω) is then given by (omitting subscripts i for convenience):

$$\max_{\xi,k} \int m\,(\omega)\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi\,(x,\omega)\right) d\Phi\,(x)\, dF_\omega\,(\omega) \tag{P(ω)}$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\,(\xi\,(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, d_i = 0, p\,(\cdot)\right) dF_\omega\,(\omega)\bigg|_{\hat{k}=k} - C'\,(k) = 0, \tag{IC$_k$}$$

$$\frac{\partial}{\partial \hat{d}} \int u\,(\xi\,(x,\omega))\, dF_{x|\omega}\,(x|\omega; k, k, d_i = 0, p\,(\cdot))\, dF_\omega\,(\omega)\bigg|_{\hat{d}=0} = 0, \tag{IC$_\omega$}$$

$$\int u\,(\xi\,(x,\omega))\, d\Phi\,(x)\, dF_\omega\,(\omega) - C\,(k) \geq \bar{u}. \tag{IR}$$

Problem P(ω) clearly shows how the presence of ω-securities acts as an extra constraint on the problem (the IC$_\omega$ constraint). The next lemma characterizes the optimal contract with ω-securities.

Lemma 6 *Let λ, ν_ω, and μ be the Lagrange multipliers on the IC$_k$, IC$_\omega$, and IR constraints, respectively. The optimal payment schedule $\xi(x,\omega)$ satisfies:*

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda \frac{1}{\sigma_x}(r - h(\omega))\; x + \lambda \frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2 - 1\right) \tag{12}$$

where $h(\omega) = \omega - \nu_\omega(1 - \mathbb{E}[m(\omega)])$ and $\nu_\omega \geq 0$.

Finally, while according to proposition 6 the principal never finds it optimal for the agent to trade ω-securities, problem P(ω) shows that the presence of these securities acts as an extra constraint on the contracting problem. Intuitively, this should lead to lower welfare. Relatedly, the presence of ω-securities makes it more costly for the principal to implement higher levels of k and, thus, aggregate volatility in the economy increases. The proof of these results, however, is not immediate since the marginal utility of the principals, $m(\omega)$, is endogenous. The feedback between the contracting problem with each manager (which takes $m(\omega)$ as given) and the marginal utility of consumption of the representative investor may upset our original intuition.

The next proposition confirms our original intuition and represents the main result of this section.

Proposition 7 *Securities contingent on ω-risk decrease equilibrium k (and, thus, increase aggregate volatility) and reduce welfare in the economy.*

6 Full model

In this section I allow both types of securities to be traded. The previous analysis showed that the two types of securities tend to have opposite effects on aggregate volatility and welfare. It is natural to expect that when we introduce both types of securities the overall effect will be ambiguous. More specifically, we can expect welfare to increase when it is possible to hedge the ε-risk and the opposite result to hold for the ω-risk.

To derive further results, I generalize the model in two ways. First, I assume that there is a cost for trading securities. In general, the portfolio problem with ε-securities can become very hard to solve with most assumptions on costs of trading securities. An easy departure from the basic model of the previous sections is to assume that, every time an intermediary sells a security to a manager, the former has to pay a fixed cost[16]. This assumption has the advantage to allow for some flexibility in the cost of securities without making the model intractable. Naturally, in equilibrium, prices of securities will reflect the presence of these costs. The key feature of having a fixed cost per trade is that equilibrium prices will resemble a two-part tariff, that is, the price of a security will be given by a fee (which is independent of the specific security and is high enough to cover the fixed cost) plus a term which is the same as those in lemmas 1 and 5[17].

Secondly, I assume that transactions in the financial market are observable, that is, I relax part (d) of assumption 1. As I show later, the presence of the fixed cost implies that each manager will trade at most once and with only one intermediary. Thus, a transaction has to be intrepreted as any trade between a manager and an intermediary, independently of how many securities are exchanged.

The fact that now managers have to pay a fixed fee per trade to buy securities implies that it is optimal for them to trade with only one intermediary (and, thus, pay the fee only once). For example, a manager who in the previous sections was buying two Arrow securities from two intermediaries (or even the same intermediary), now will prefer to combine the two Arrow securities and make only one trade. Thus, I define *insurance contracts* as general functions of the underlying Arrow securites. A manager will find it optimal to buy this insurance contracts instead of the Arrow securities to save on the trading costs.

Every insurance contract can be contingent on idiosyncratic risks or aggregate risk. Let $J^{\varepsilon} = \left\{ s : \mathbb{R}^{[0,1]} \to \mathbb{R} \text{ such that } \int \int s\left(\{\varepsilon_i\}\right) d\Phi_{\varepsilon}\left(\varepsilon_i\right) di = 0 \right\}$, where $\mathbb{R}^{[0,1]}$ is the set of functions from the unit interval to $\mathbb{R}$. Similarly, for the case of ω-securities, let

$J^{\omega} = \left\{ s \in \mathbb{R} \to \mathbb{R} \text{ such that } \int s\left(\omega\right) dF_{\omega}\left(\omega\right) = 0 \right\}$. Finally, let $J = J^{\varepsilon} \cup J^{\omega}$ be the space of all securities.

Let $s^{\varepsilon}_{\ell,m} \in J^{\varepsilon}$ be the m-th insurance contract issued by intermediary ℓ that is in principle contingent on all the idiosyncratic shocks of the economy. Similarly, $s^{\omega}_{\ell,m} \in J^{\omega}$ the corresponding insurance contract contingent on aggregate risk. Each contract can be represented as a function of the Arrow securities defined above. Let $p : J \to \mathbb{R}_{+}$ be the price schedule of these contracts. As it will be clear after introducing the costs of trading securities, in equilibrium $p\left(\cdot\right)$ is not a linear function over J.

To denote that agent i is not participating in the market for ε-risk (ω-risk) I will simply write $s^{\varepsilon}_{i} = \varnothing$ ($s^{\omega}_{i} = \varnothing$).

[16] This assumption in the context of a model with endogenous creation of securities was first proposed by Pesendorfer (1995).

[17] Makowski (1979) first derives the result that the price schedule in a competitive equilibrium with fixed costs of trading can be represented as a two-part tariffs.

The non-negativity constraint prohibits short-sales of securities. This is a natural restriction in the presence of fixed costs of trading new securities (which I introduce below). If short-selling were allowed, it would be equivalent to selling securities without having to incur in the fixed cost of trading. Thus, managers would have an advantage over intermediaries in selling new securities which, in turn, would lead to no securities being traded in equilibrium. The only way for an agent to sell an insurance contract s short is to buy $-s$ from an intermediary and then sell it on the market. Of course, this strategy is not profitable.

Marketing insurance contracts to potential buyers is a costly process. I assume that an intermediary who sells an insurance contract conditional on ε-risk (ω-risk) to a manager has to pay a fixed cost $c_\varepsilon > 0$ ($c_\omega > 0$).

For simplicity, the trading costs are common across intermediaries and do not depend on the state where the security pays-off nor on the identity i of the project on which they are contingent. In other words, costs will differ only on whether the securities depend on idiosyncratic or aggregate risk. As it will be clear in following sections, an interesting comparative static exercise will be to vary the costs c_ε and c_ω and study the implications for the managers' investment decisions. This analysis will be central in section (7), where I introduce taxes to fix the inefficiency of the equilibrium. Indeed, in this model taxes rise the costs of issuing and selling securities and, thus, are isomorphic to a particular increase of the trading costs.

Payoffs. The fixed cost for every trade immediately implies that each manager will buy at most one security of each type (idiosyncratic or aggregate) from at most one intermediary. This also implies that we can indentify each security with the index of the manager i who buys it. Thus, $s^\varepsilon_{\ell,i}$ and $s^\omega_{\ell,i}$ will be the ε-security and ω-security created by intermediary ℓ and customized to manager i, respectively. I will simply write $s_{\ell,i}$ to denote any security, idiosyncratic or aggregate, sold by intermediary ℓ to manager i. Given that in equilibrium intermediaries make zero profits, it is without loss of generality assume that each intermediary will face an equal mass of measure $1/N$ of agents.

The final profits generated by manager i who buys securities $s^\varepsilon_{\ell,i}$ and $s^\omega_{\ell,i}$ from intermediary ℓ are

$$\Pi^m_i = \pi_i + s^\varepsilon_{\ell,i} - p\left(s^\varepsilon_{\ell,i}\right) + s^\omega_{\ell,i} - p\left(s^\omega_{\ell,i}\right). \tag{13}$$

Similarly, the profits of intermediary ℓ[18] are

$$\Pi_\ell^I = \int_{(\ell-1)/N}^{\ell/N} \left(p\left(s_{\ell,i}^\varepsilon\right) - s_{\ell,i}^\varepsilon\right) di - \frac{c_\varepsilon}{N}$$

As already discussed above, the equilibrium price of a security will now contain a fee to cover the fixed cost. Since ω-securities are not traded on equilibrium, all prices at which intermediaries do not want to trade ω-securities can be consistent with the equilibrium. Here, I am going to select the equilibrium price that leaves intermediaries indifferent between trading or not.

Lemma 7 *The equilibrium price of an insurance contract $s_{\ell,i}^\varepsilon$ ($s_{\ell,i}^\omega$) is given by a two-part tariff: $p^\varepsilon + \int\int s_{\ell,i}^\varepsilon\left(\{\hat{\varepsilon}_j\}\right) d\Phi_\varepsilon\left(\hat{\varepsilon}_j\right) dj$ ($p^\omega + \int s_{\ell,i}^\omega\left(\hat{\omega}\right) m\left(\hat{\omega}\right) dF_\omega\left(\hat{\omega}\right) / \mathbb{E}\left[m\left(\omega\right)\right]$), where $p^\varepsilon = c_\varepsilon$ and $p^\omega = c_\omega$.*

Except for the costs of trading contracts, the model of this section resembles the particular cases studied in sections 5.1 and 5.2. In particular, it is still true that the principal wants the manager to stay away from ω-securities. Finally, I am going to assume that the cost of trading ε-securities is small enough that it is optimal for the principal to pay the fee p^ε and have the agent trade ε-securities (and buy full insurance).

Assume now that trading costs are zero (so that $c_\varepsilon = c_\omega = 0$) and, as we did in section 5.1, and assume that in equilibrium the agent buys full insurance. The principal wants the agent to trade only the ε-risk and observes the transactions made by the agent. However, the principal does not observe the type of security that the agent is trading, that is, whether the security is contingent on ε-risk or ω-risk. The agent is constrained by the principal to make only one transaction, thus, the only feasible deviation is to stop trading ε-securities and trade only ω-securities. This double-deviation cannot be detected by the principal who will still observe that only one transaction has occurred.

Formally, let $\bar{U}_{\varepsilon\omega}\left(k, p\left(\cdot\right)\right)$ be the value of the double-deviation for the agent, that is,

$$\bar{U}_{\varepsilon\omega}\left(k, p\left(\cdot\right)\right) = \max_{\hat{k}, s^\omega} \int u\left(\xi\left(x, \omega\right)\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}, s^\varepsilon = \varnothing, s^\omega \neq \varnothing, p\left(\cdot\right)\right) dF_\omega\left(\omega\right),$$

where $F_{x|\omega}\left(x|\omega; k, \hat{k}, s^\varepsilon = \varnothing, s^\omega \neq \varnothing, p\left(\cdot\right)\right)$ is the conditional distribution of x when the agent trades only ω-securities.

[18] To simplify notation, I am writing profits of intermediaries using the fact that ω-securities will not be traded in equilibrium.

The contracting problem becomes:

$$\max_{\xi,k} \int m(\omega)\left(x+\bar{R}+r\ k+(1-k)\,\omega-p^{\varepsilon}-\xi(x,\omega)\right)d\Phi(x)\,dF_{\omega}(\omega) \tag{P(full)}$$

subject to:

$$\frac{\partial}{\partial\hat{k}}\int u(\xi(x,\omega))\,dF_{x|\omega}\left(x|\omega;k,\hat{k},s^{\varepsilon}=-k\ \varepsilon,s^{\omega}=\varnothing\right)dF_{\omega}(\omega)\bigg|_{\hat{k}=k}-C'(k)=0, \tag{IC$_k$}$$

$$\bar{U}_{\varepsilon\omega}(k,p(\cdot))\leq\bar{u}, \tag{IC$_{\varepsilon\omega}$}$$

$$\int u(\xi(x,\omega))\,d\Phi(x)\,dF_{\omega}(\omega)-C(k)=\bar{u}. \tag{IR}$$

Here, $F_{x|\omega}\left(x|\omega;k,\hat{k},s^{\varepsilon}=-k\varepsilon,s^{\omega}=\varnothing\right)$ is the conditional distrbution of x when the agent buys full insurance against the ε-risk and doesn't trade ω-securities. The optimal contract is a combination of 9 and 12, so I will not repeat it here.

The next proposition contains the effects on equilibrium k and welfare of changing the trading costs of the two types of insurance contracts.

Proposition 8 *For low enough prices c_{ε} and c_{ω}, when both types of insurance contracts are traded, equilibrium k and welfare decrease with the cost of ε-securities, c_{ε}, and increase with the cost of ω-securities, c_{ω}.*

Proposition 8 shows that changing the price of the two types of securities has opposite effects on equilibrium volatility and welfare. These comparative static results extend the conclusions derived separately in sections 5.1 and 5.2 to the case with fixed costs of trading. Importantly, these effects will be the source of the trade-off faced by the social planner, which I consider in section 7, who has the power to tax transactions in the securities markets.

7 Efficiency and optimal policy

7.1 Taxation

This section studies the efficiency properties of the equilibrium derived in section 6. Of course, whether the equilibrium is socially optimal will also depend on the powers we grant to the social planner. In particular, different conclusions on the efficiency of the equilibrium – and, thus, different policy prescriptions – follow from different assumptions on the information available to the social planner. A stark way to see this is by going back to the intuition behind the welfare implications of cheaper ω-risk insurance in section 5.2. There, I proved that, since the principal could always replicate the market allocation, he could only suffer from cheaper insurance of the ω-risk. It follows immediately that a social planner, who maximizes the welfare of investors and who can observe the trades of the different securities, could easily improve on the equilibrium allocation by forbidding the trades of ω-securities. For this reason, in what follows I will restrict the social planner's information set by assuming that he doesn't have access to more information than the representative investors.

Investors fail to coordinate the contract they design for the managers with the incentives faced by the intermediaries. This coordination failure is related to the conclusions in agency problems with multiple agents and common principal (Holmstrom and Milgrom (1990), Itoh (1993), Mookherjee (1984)). In most of these models, both agents face an agency problem and can potentially interact with each other. The principal has to design the contract by taking into account this interaction. In this model, the two agents are the managers and the intermediaries who interact through the securities market. Intermediaries are owned by the principal and they don't face any agency problem. However, the principal fails to understand how their activity affects prices and, thus, the incentives of the managers. The planner, then, can restore efficiency by fixing this coordination problem.

These sections contain two main results. First, I show that if the planner can observe the total number of transactions in the economy and investors cannot observe the trading activity of intermediaries, then there is an optimla positive tax that increases welfare in the economy. This tax makes it harder for managers to deviate and trade ω-securities. Secondly, I show that when investors can observe the transactions made by each manager (as in section 6), then they can do better than the social planner and the transaction tax is redundant.

Suppose for now that the planner cannot observe the total quantity of transactions, but the investors do not observe any trading activity made by the managers (as in the basic model of section 3).

Let $e(\tau)$ be the equilibrium for a given value of the tax τ[19] and $\mathcal{W}(e(\tau))$ the equilibrium welfare of the investors. It is immediate to derive that $e(\tau)$ resembles the equilibrium derived in section 5, except that the insurance fees are now $p^{\varepsilon}+\tau$ and $p^{\omega}+\tau$, $\tau>0$. The social planner chooses τ so as to maximize welfare for investors subject to allocations and prices being an equlibrium. Formally, the social panner solves

$$\max_{\tau} \mathcal{W}(e(\tau))$$

[19] If we choose the price of ω-securities so that the fee p^{ω} equals the trading cost, the equilibrium is unique.

For given τ, this is exactly the same contracting problem that we explored in section 6, so I will not repeat the characterization of the contract here.

Lemma 8 *A small enough positive tax τ increases welfare in the economy.*

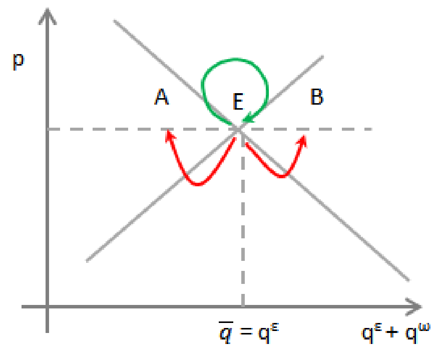
When choosing τ, the social planner optimally weighs the benefits and the costs of changing insurance prices p^ε and p^ω. A higher p^ε makes it more profitable for the agent to deviate by refusing to trade in the ε-risk market[20]. On the other hand, a higher p^ω has the effect of lowering the value for the agent of trading ω-securities and, hence, relaxes the cotracting problem. When τ is small enough, the latter effect dominates since the former effect is only second order.

Suppose now that investors can observe the number of transactions made by the manager in the securities market as in section 6. The question is whether the planner can improve on equilibrium welfare by using only the transaction tax τ.

To gain some intuition, suppose for a moment that only linear securities can be traded and there is a constant marginal cost for each unit of security. This case is easier to analyze since the choice variables are continuous. Suppose that the principal can observe the total number of units bought by the manager, call it $\bar{q}$, but not the type of security traded. The principal can then use the extra choice variable $\bar{q}$ to control the trades of the managers. As usual, the principal will design a contract so that in equilibrium the manager will buy a quantity q^ε of ε-securities and a quantity $q^\omega = 0$ of ω-securities. Thus, the principal sets $\bar{q} = q^\varepsilon$. Now, when $\bar{q}$ is observable, at the margin the only deviation available to the manager is to reduce q^ε by $d\, q^\varepsilon$ and increase $d\, q^\omega = d\, q^\varepsilon$ so as to leave the total quantity $\bar{q}$ unchanged. On the contrary, with a transaction tax, the manager has still three possible deviations: decrease q^ε, increase q^ω, and do both. In particular, in the third case a tax that increases the marginal cost by τ, would have no effect on the deviation ($\bar{q}$ doesn't change). The following diagram illustrates the different possibilities: the first two deviations lead to points A and B, respectively, while the double-deviation leaves the total quantity of trades unaffected at

[20] Note that the tax raises revenues from the transactions of ε-securities (these securities are traded in equilibrium), but these revenues are rebated to investors.

point E.



Therefore, intuitively the cap on the quantity traded is at least weakly preferred to a tax on the transactions in the setting with linear securities. In ongoing work, I am exploring the consequences of restricting the space of securities to linear securities, but to allow investors to observe a signal on the total amount of resources that managers invest in the trading activity. However, managers can still benefit from a deviation that leaves the value of trades unaffected. The interpretation is that investors have access to the balance sheets of financial institutions and can infer the amount of resources spent in trading activities. However, they don't have the expertise to understand the type of securities that are being traded.

A similar result holds in the case considered here. A transaction is a trade between a manager and an intermediary and there can we know that the principal wants the agent to trade only ε-securities. The manager can deviate to buying no securities, buying both ω-securities, and buying only ω-securities (double-deviation). Again, in the third case a tax that increases the trading cost by an amoung τ would not affect the value of this deviation directly. The following proposition confirms this intuition. Formally, the principal designs the optimal contract with only the constraint on the double deviation (and, of course, the usual IC and IR constraints).

Proposition 9 *When transactions are observable, the transaction tax cannot improve on equilibrium welfare.*

7.2 Regulation

When transactions are observable welfare is higher and the planner cannot help investors by simplying taxing them. However, managers have still access to a potential double-deviation that allows them to trade ω-securities. Thus, even this deviation is less profitable, ω-securities can still be traded off the equilibrium.

Taxing transactions in derivatives markets is not the only possible way to increase welfare in this economy. It turns out that even maintaining the assumption that the social planner has no superior information over the other agents, the social planner can do much better by *regulating* the issuers of financial securities. By regulation I mean giving the planner the power to write a contract that incentivizes intermediaries to maximize welfare in the economy. Regulation goes to the heart of the coordination problem of the investors: the planner takes the place of investors and realizes that incentives for managers have to be coordinated with incentives for intermediaries.

To see this, remember that trades of securities contingent on the aggregate risk can only undo the incentives set up by investors and these securities are not taded in equilibrium. Hence, intermediaries trade only ε-securities and make constant (zero) profits in equilibrium. In contrast, off the equilibrium, intermediaries sell ω-securities to managers and, thus, take some aggregate risk on their balance sheets. As shown in the analysis of section 5.2, the presence of intermediaries selling ω-securities matters to the extent that it constrains the contracts space of the principal. In other words, ω-securities matter only as they can represent a profitable deviation for the agents.

Profits of intermediaries are assumed to be observable, hence, a social planner can always increase welfare in the economy by punishing any volatility of these profits. The optimal regulation in this model is to forbid intermediaries to ever take aggregate risk on their balance-sheets and to punish them in case of deviation. This policy would limit (and, in the extreme, eliminate) the incentives to trade aggregate risk out of the equilibrium and, therefore, it would relax the investors' problem.

Formally, I am going to assume that the social planner can regulate the trading activity of financial intermediaries by choosing a function $\eta_\ell : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that maps pairs $\left(\Pi_\ell^I, \omega\right)$ into a payment to each intermediary.

Proposition 10 *Let Π_ℓ^{I*} be the equilibrium profits of intermediary ℓ (Π_ℓ^{I*} is zero since ω-securities are not traded in equilibrium). Then, the optimal policy η_ℓ^* is given by*

$$\eta_\ell^* \left(\Pi_\ell^{I*}, \omega\right) = \Pi_\ell^{I*} \text{ and } \eta_\ell^* \left(\Pi_\ell^{I}, \omega\right) = -\infty \text{ if } \Pi_\ell^{I} \neq \Pi_\ell^{I*}.$$

If this policy is implemented then equilibrium welfare coincides with that of proposition 5.

The intuition for proposition 10 was already discussed. The result on welfare is also quite intuitive. If the optimal policy η_ℓ^* is implemented, then intermediaries will never find it optimal to sell ω-securities. Thus, the IC constraint that precludes deviations with ω-securities drops from the contracting problem and we are back to the case of section 5.1.

The reason why regulation is so effective is not related to the fact that ω-securities are not traded in equilibrium and profits of intermediaries are constant. If there was some trading of aggregate risk in equilibrium (say, due to some unmodelled demand for hedging), then profits would vary with the aggregate state of the economy. However, as long as the planner can determine the right amount of aggregate risk that intermediaries can hold on their balance sheets, then the optimal policy would still have the same power as in proposition 10. Of course, this is possible because the social planner is assumed to observe the aggregate state ω. The optimal regulation would then resemble a limit on how risky the balance sheets of the issuers of financial securities can be.

The optimal policy still requires a great deal of information to be implemented since the social planner has to understand what is the optimal amount of aggregate risk that should be traded. In particular, the planner has to realize what part of the ω-risk is traded by those portfolio managers who determine the quantity of risk in the economy through their investment decisions. While this is easy in this abstract model, it may be less so in real financial markets.

8 Discussion

Unlike the securities of this model – which are contingent either on idiosyncratic or aggregate risk – derivative contracts traded in financial markets are often contingent on many different risks, both aggregate and idiosyncratic. Thus, while the model highlights a fundamental difference between securities contingent on idiosyncratic and aggregate risks, this distinction is much less clear in real financial markets.

Nonetheless, we can argue that some financial securities are more sensitive to idiosyncratic risks while others are used to hedge risks that are more aggregate. For example, a Credit Default Swap (CDS)[21] that insures the buyer against the default of a firm that is independent from the rest of the economy is a derivative that is relatevely more sensitive to idiosyncratic risks. On the contrary, a CDS written on a bond issued by a big firm (say, GE or Walmart) is likely to be relatively more sensitive to the aggregate state of the economy.

Another example of a security contingent on aggregate risk is an Interest Rate Swap that banks use to hedge the interest rate risk of their loan portfolios.

[21] A CDS is a credit derivative which obliges the seller to compensate the buyer in the event of a loan default. The buyer pays a premium to the seller for this insurance.

Finally, a more involved example is given by a tranche of a Funded Synthetic CDO[22]. This derivative allows an investors to take a position on the credit risk of a portfolio of loans and, as the number of loans in the underlying portfolio increases, idiosyncratic risks wash away and the CDO will be relatevely more sensitive to common risks.

Some empirical evidence. Recent work in empirical finance focuses on how trading in derivatives markets affects the risk of financial institutions. Ideally, to see whether the predictions of the model are consistent with the data, we would need to make a distinction between securities contingent on idiosyncratic or aggregate risks. In the former case, the model predicts that banks' balance sheets become less correlated with each other while latter case leads to the opposite conclusion. This ideal experiment assumes that we can distinguish financial securities depending on the type of risk they hedge. In practice, however, this distinction is not as sharp.

A less demanding exercise would be to ask what happens to a financial institution's balance sheet after it start trading in the derivatives markets. In a recent work, Nijskens and Wagner (2011) study two separate datasets of banks which include information on various types of securitization around the world. In particular, one dataset contains data on Credit Default Swaps (CDS) and the other on Collateralized Loan Obligation (CLO)[23].

Both datasets allow Nijskens and Wagner (2011) to observe the date on which each bank start trading each of these financial products. They then look at the effect on the returns of each bank after the date of the first trade of CDS or CLO. They find that a bank that trades CDS or CLO experiences a permanent *increase* in its beta, which is a measure of the systematic risk of a bank. Also, the magnitude of such effect is bigger in the case of CLO. Remember that in our model the profits of a bank are given by

$$\pi_i = \bar{R} + (r + \varepsilon_i)\, k_i + \omega\,(1 - k_i) + u_i.$$

If we take the average across different sectors, $\int \pi_i \; di$, then the market return is given by $\bar{R} + \omega\,(1-k)$ (where k is the equilibrium choice of all managers), If we let σ_i be the standard deviation of the return of bank i, the correlation of bank i with the market is $\rho_i = (1-k)\,\sigma_\omega/\sigma_i$. Nijskens and Wagner (2011) find that, after the first CLO or CDS trade, the value of ρ_i increases while the relative volatility $\sigma_i/\,(1-k)\,\sigma_\omega$ decreases. This is consistent with the prediction of this model that

[22] This is a derivative contract that allows investors with different appetite for risk buy tranches of a Special Purpose Vehicle (SPV). The SPV then buys Treasuries and sells a portfolio of CDS. The buyers of the CDS pay a periodic premium to the SPV which transfers it to the investors. However, if a loan in the portfolio defaults, then the Treasuries are sold to pay the buyer of the protection.

[23] A CLO is a form of securitization through which banks transfer pools of loans to the buyers of these securities. The payoff of this derivative resembles, to a first approximation, the payoff of a funded synthetic CDO.

derivatives tend to decrease the value of k_i. It is also tempting to speculate that the bigger effect of CLO trades on the beta of the bank relative to CDS trades is related to the fact that CLOs, which are pools of loans, are more similar to the ω-securities of this paper.

Similar evidence is found by Haensel and Krahnen (2007). They use a dataset of CDOs issued by European financial institutions. They also find that banks engaging in these transactions tend to increase their exposure to the market. Of course, while these results are consistent with the conclusions of this paper, they are certainly not conclusive evidence. Tthere may be many reasons for why banks increase their systematic risk after trading some types of derivatives. In particular, the agency problem may be abset and investors may find it optimal to let the bank trade these derivatives.

For simplicity, in this model I have assumed that agents and intermediaries trade in a Walrasian market by paying a fixed cost per trade. These costs can be interpreted as a reduced-form way to capture the effects of imperfect competition in the securities market or, more importantly, the liquidity of these markets. Remember that the financial market in this model is an abstraction of OTC markets where typically market makers provide liquidity by posting a price and trading securities at that price. The creation of new financial products and the growth of OTC markets have stimulated important research on decentralized markets. These papers explore the main features of these markets like price determination, liquidity and diffusion of information. Duffie et al. (2005), for example, provide a theory of asset pricing in decentralized markets[24]. The focus of this paper, however, is not about the specific trading environment, but on how complex securities can affect the portfolio choice of investors and, thus, the aggregate volatility of the economy.

A Walrasian market for securities is also the typical assumption in the literature on markets with endogenous securities creation (Allen and Gale (1991), Pesendorfer (1995), Bisin (1998)). These papers depart from the standard assumption that traded securities are exogenously given and, instead, assume that they are issued by optimizing intermediaries. Also, a common assumption of these papers is that issuing securities is a costly activity and issuers incur in both fixed and variable costs. Fixed costs, in particular, are more appealing (Tufano (1989)) for securities markets, but introduce non-convexities which considerably complicate the problem. One way to guarantee the existence of the solution in the presence of fixed costs is to assume that intermediaries have market power (Allen and Gale (1988) and Bisin (1998)) or to consider non-linear pricing functions (Allen and Gale (1991)). In the extended model of sections 6 and 7, I follow the latter approach and assume perfect competition with non-linear pricing. The assumption of market power is probably more

[24] Other important contributions are Duffie et al. (2005), Duffie et al. (2007), Duffie and Manso (2007), Lagos (2010), Lagos and Rocheteau (2009), Lagos et al. (2007), Vayanos (1998), Vayanos and Weill (2008), and Weill (2008).

realistic but would considerably complicate the analysis as I would need to solve a complicated problem where intermediaries choose prices by taking into account the demand schedule of the agents. This is beyond the scope of this paper. However, the social planner's problem shows that, by increasing the price of ω-securities, some market power may actually be beneficial for welfare. Similarly, if we interpret the trading cost as the liquidity of these markets, then it may be that case that *less liquid* markets are beneficial for welfare.

The stark conclusion about the welfare effects of ω-securities depend on some strong assumptions of the model. First, the assumption of symmetric preferences, technology, and equilibrium eliminates any gains from trading aggregate risk among managers. Also, I have assumed that investors can perfectly condition their contracts on aggregate states, but they cannot do the same for idiosyncratic states. Financial markets help allocate aggregate risk to the agents who are better prepared to hold it. However, as long as investors cannot fully control the risks traded by their managers, then trading of ω-risk has the potential to reduce welfare. Also, the assumptions of this model help me isolate this particular mechanism and analyze its (negative) implications. In a more general model, different effects of aggregate risk trading would coexist and the optimal policy would be characterized by a richer set of actions.

On the information side, this model implies that an easy way to improve welfare is by requiring more information disclosure. Formally, this would be equivalent to modify part (d) of Assumption 1 and assume perfect observability of trades and types of security. Once investors have the ability to contract on the different securities traded by managers, they will forbid trading of ω-risk (and allow trading of ε-risk). In fact, we can conjecture that the equilibrium when part (d) of assumption 1 is removed will resemble that of section 5.1.

While information disclosure is a strong and interesting implication of this model, it derives from the mathematical way I chose to model complex securities and OTC markets. In general, it is realistic to assume that even if big financial institutions were required to disclose all their trading activities to outside investors, it would probably be a daunting task for many investors to process this information (Brunnermeier and Oehmke (2011)). To make things more complicated, in reality most financial securities are likely to be a combination of the ε-securities and ω-securities considered in this model.

Appendix

A Proofs and additional lemmas

Proof of proposition 1. Assume for now that the FOA is valid, the maximization problem P(NS) is

$$\max_{\xi,k}\int m\left(\omega\right)\left(x+\bar{R}+rk+\left(1-k\right)\omega-\xi\left(x,\omega\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)$$

subject to:

$$\frac{\partial}{\partial\hat{k}}\int u\left(\xi\left(x,\omega\right)\right)dF_{x|\omega}\left(x|\omega;k,\hat{k}\right)dF_{\omega}\left(\omega\right)\bigg|_{\hat{k}=k}-C'\left(k\right)=0,$$

$$\int u\left(\xi\left(x,\omega\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)-C\left(k\right)\geq0.$$

By using the properties of Gaussian distributions together with the moments (4), I can rewrite the IC constraint as:

$$\int u\left(\xi\left(x,\omega\right)\right)\left(\frac{1}{\sigma_{x}}\left(r-\omega\right)x+\frac{k\ \sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}\left(x^{2}-1\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)-C'\left(k\right)=0.$$

Here, I have used the fact that in equilibrium x and ω are independent by construction. Let λ and μ be the Lagrange multipliers on the two constraints, respectively, and define the Lagrangian:

$$\begin{aligned}\Lambda\ =\ & \int m\left(\omega\right)\left(x+\bar{R}+rk+\left(1-k\right)\omega-\xi\left(x,\omega\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)\\ & +\lambda\left(\int u\left(\xi\left(x,\omega\right)\right)\left(\frac{1}{\sigma_{x}}\left(r-\omega\right)x+\frac{k\ \sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}\left(x^{2}-1\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)-C'\left(k\right)\right)\\ & +\mu\left(\int u\left(\xi\left(x,\omega\right)\right)d\Phi\left(x\right)dF_{\omega}\left(\omega\right)-C\left(k\right)\right)\end{aligned}$$

If we differentiate Λ pointwise w.r.t. $\xi\left(x,\omega\right)$, we get that the optimal contract solves:

$$\frac{m\left(\omega\right)}{V'\left(\xi\left(x,\omega\right)\right)}=\mu+\lambda\left[\frac{1}{\sigma_{x}}\left(r-\omega\right)x+\frac{k\ \sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}\left(x^{2}-1\right)\right]$$

which is the expression in proposition 1. ∎

Proof of proposition 3. To see that this equilibrium is efficient we can set up the social planner's problem. The only difference between the planner's problem and the equilibrium is that the former takes into account how aggregate consumption $c\left(\omega\right)$ depends on the profits of all the managers. Formally, the planner solves:

$$\max_{\xi,k,c} \int v\left(c\left(\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi\left(x,\omega\right)\right) dF_{x|\omega}\left(x|\omega;k,\hat{k}\right) dF_{\omega}\left(\omega\right)\bigg|_{\hat{k}=k} - C'\left(k\right) = 0,$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq 0,$$

$$c\left(\omega\right) + \int \xi\left(x,\omega\right) d\Phi\left(x\right) = \int \left[x + \bar{R} + r\ k + \left(1-k\right)\omega\right] d\Phi\left(x\right),\ \forall\omega.$$

The last constraint is the resource constraint of the economy. Note that only this constraint depends on $c\left(\omega\right)$. So, if we let $\varphi\left(\omega\right) f_{\omega}\left(\omega\right)$ denote the Lagrange multilpier on the resource constraint, we can separate the problem by first solving for c:

$$\max_{c} \int v\left(c\left(\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right)$$

subject to:

$$c\left(\omega\right) + \int \xi\left(x,\omega\right) d\Phi\left(x\right) = \int \left[x + \bar{R} + r\ k + \left(1-k\right)\omega\right] d\Phi\left(x\right),\ \forall\omega.$$

Setting up the Lagrangian and taking the first-order condition w.r.t. $c\left(\omega\right)$ gives:

$$m\left(\omega\right) \equiv v'\left(c\left(\omega\right)\right) = \varphi\left(\omega\right).$$

Now, for given $\varphi\left(\omega\right)$, we can solve the dual problem of maximizing total resources, that is,

$$\max_{\xi,k} \int m\left(\omega\right)\left[x + \bar{R} + r\ k + \left(1-k\right)\omega - \xi\left(x,\omega\right)\right] d\Phi\left(x\right) dF_{\omega}\left(\omega\right),$$

subject to the IR and IC constraint. This is the same problem as P(NS). This proves that the equilibrium is a solution to the planner's problem. ■

A.1 Only ε-securities

Proof of lemma 1. Consider the problem of intermediary ℓ:

$$\max_{y_\ell} \mathbb{E}\left[m\left(\omega\right)\Pi_{\ell}^{I}\right],$$

where

$$\Pi_\ell^I = \int (p_{i,\hat{\varepsilon}} - z_{i,\hat{\varepsilon}})\, y_{\ell,i,\hat{\varepsilon}}\; d\hat{\varepsilon}\; di.$$

Take the first-order condition pointwise w.r.t. y_ℓ

$$p_{i,\hat{\varepsilon}} - \phi_\varepsilon(\hat{\varepsilon}) \geq 0.$$

If $p_{i,\hat{\varepsilon}}$ was different from $\phi_\varepsilon(\hat{\varepsilon})$, then an intermediary would buy or sell an infinite amount of this security. This cannot be an equilibrium, so $p_{i,\hat{\varepsilon}} = \phi_\varepsilon(\hat{\varepsilon})$. ■

Proof of lemma 3. The optimality of full insurance can be proved using the results in Kim (1995) and Jewitt (2007).

These papers show how to rank different information systems in the principal-agent framework. They consider a principal-agent model in which the principal can choose among different set of signals about the agent's action. An information system will be preferred to another if the former can implement every action at a lower cost for the principal. This criterion is more general than the informativeness criterion in Holmstrom (1979) who considers only information systems which are inclusive in the sense that one system contains more signals than the other (Holmstrom (1979) defines the notion of informativeness in terms of a sufficiency criterion).

This is, however, restrictive in our context since systems are not inclusive. Kim (1995) and Jewitt (2007) show how to rank information systems which are not inclusive in terms of their likelihood ratios. More specifically, they show that, given two information systems, one of them will implement any action at a lower cost for the principal if and only if its likelihood ratio is a mean preserving spread of the other.

The result in Kim (1995) requires the principal to be risk-neutral, but in this paper there is the stochastic discount factor $m(\omega)$. However, the argment easily generalizes to our case if we consider the *conditional* likelihood ratio. Thus, I will use the result that given two signals $\hat{x}$ and $\tilde{x}$, it is cheaper for a principal to implement a given action k with $\hat{x}$ if the conditional likelihood ratio

$$L_{\hat{x}|\omega}(x|\omega; k, d^\varepsilon, p(\cdot)) = \frac{\partial f_{\hat{x}|\omega}(x|\omega; k, k, d^\varepsilon, p(\cdot))/\partial \hat{k}}{f_{\hat{x}|\omega}(x|\omega; k, k, d^\varepsilon, p(\cdot))}$$

is riskier than the likelihood ratio obtained with $\tilde{x}$. Intuitively, since different actions lead to different realizations of the signal with higher probability, the higher the volatility of the likelihood ratio, the more informative the signal.

Also, when $\hat{k} = k$ the distribution $F_{x|\omega}$ does not depend on ω, so I will simply write F_x. The mean of $L_{x|\omega}$ is given by:

$$\begin{aligned}
& \int \left(L_{x|\omega}\left(x|\omega; k, d^{\varepsilon}, p\left(\cdot\right)\right)\right) dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) dF_{\omega}\left(\omega\right) \\
= & \int \frac{\partial}{\partial \hat{k}} \int f_{x|\omega,\varepsilon}\left(x|\omega; k, d^{\varepsilon}, p\left(\cdot\right)\right) \; d\Phi_{\varepsilon}\left(\varepsilon\right) \; dx \\
= & \int\int \left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_u^2}\right) f_{x|\omega,\varepsilon}\left(x|\omega,\varepsilon; k, d^{\varepsilon}, p\left(\cdot\right)\right) \; d\Phi_{\varepsilon}\left(\varepsilon\right) \; dx \\
= & \int\int \left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_u^2}\right) dF_{\varepsilon|\omega,x}\left(\varepsilon|\omega, x; k, d^{\varepsilon}, p\left(\cdot\right)\right) \; dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) \\
= & \frac{1}{\sigma_u^2} \int \left(\left(r-\omega+\mathbb{E}\left[\varepsilon|\omega, x\right]\right) x - k\mathbb{E}\left[\left(r-\omega+\varepsilon\right)\varepsilon|\omega, x\right]\right) dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) \\
= & \frac{1}{\left(\sigma_u^2\right)^2} \int \left(\left(r-\omega\right)\mathbb{E}\left[u|x\right] + \mathbb{E}\left[\varepsilon\ u|x\right]\right) dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) = 0,
\end{aligned}$$

since the distribution of x does not depend on ω.

Thus, the variance of $L_{x|\omega}$ is:

$$\begin{aligned}
& \int \left(L_{x|\omega}\left(x|\omega; k, d^{\varepsilon}, p\left(\cdot\right)\right)\right)^2 dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) dF_{\omega}\left(\omega\right) \\
= & \int \frac{1}{f_x\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right)} \left(\frac{\partial}{\partial \hat{k}} \int f_{x|\omega,\varepsilon}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) d\Phi_{\varepsilon}\left(\varepsilon\right)\right)^2 dx \\
= & \int \frac{1}{f_x\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right)} \left(\int \left[\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_u^2}\right] f_{x|\omega,\varepsilon}\left(x|\omega,\varepsilon; k, d^{\varepsilon}, p\left(\cdot\right)\right) d\Phi_{\varepsilon}\left(\varepsilon\right)\right)^2 dx \\
= & \int \left(\int \left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_u^2}\right) dF_{\varepsilon|\omega,x}\left(\varepsilon|\omega, x; k, d^{\varepsilon}, p\left(\cdot\right)\right)\right)^2 dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) \\
= & \frac{1}{\left(\sigma_u^2\right)^2} \int \left(\left(r-\omega+\mathbb{E}\left[\varepsilon|\omega, x\right]\right) x - k\mathbb{E}\left[\left(r-\omega+\varepsilon\right)\varepsilon|\omega, x\right]\right)^2 dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right) \\
= & \frac{1}{\left(\sigma_u^2\right)^2} \int \left(\left(r-\omega\right)\mathbb{E}\left[u|x\right] + \mathbb{E}\left[\varepsilon\ u|x\right]\right)^2 dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right)
\end{aligned}$$

Now, for given distribution for ε, if the variance σ_ε^2 of this distribution increases, then $\mathbb{E}\left[u|x\right] \to 0$ $\forall x$ (x becomes a worse predictor for u). On the contrary, as σ_ε^2 increases, the cross moment $\mathbb{E}\left[\varepsilon\ u|x\right]$ also increases.

The condition for full insurance to be optimal is that the first effect dominates the second. Formally, the function

$$G\left(\omega, d^{\varepsilon}, k\right) = \frac{1}{\left(\sigma_u^2\right)^2} \int \left(\left(r-\omega\right)\mathbb{E}\left[u|x\right] + \mathbb{E}\left[\varepsilon\ u|x\right]\right)^2 dF_{x}\left(x; k, d^{\varepsilon}, p\left(\cdot\right)\right)$$

is maximized at $d^{\varepsilon} = -k\ \varepsilon$ for any value of ω and k. This condition is implicit because different choices of d^{ε} affect the distribution of x.

However, if we restrict attention to linear securities, then x follows a Gaussian distribution and, in particular,

$$\mathbb{E}\left[\varepsilon|x\right] = \frac{k\sigma_\varepsilon^2}{\sigma_x^2}x,$$

and

$$\mathbb{E}\left[\varepsilon^2|x\right] = \frac{k\sigma_\varepsilon^2}{\sigma_x^2}x = \frac{\sigma_u^2\sigma_\varepsilon^2}{\sigma_x^2} + \left(\frac{k\sigma_\varepsilon^2}{\sigma_x^2}\right)^2 x^2.$$

Therefore,

$$\int \left(L_{x|\omega}\left(x|\omega;k,d^\varepsilon\right)\right)^2 dF_x\left(x;k,d^\varepsilon\right) = \frac{\left(r-\omega\right)^2}{\sigma_x^2} + \frac{k^2\left(\sigma_\varepsilon^2\right)^2}{\left(\sigma_x^2\right)^2}.$$

A sufficient condition for this expression to be maximized at $\sigma_\varepsilon^2 = 0$ for every value of ω and k is: $0 \le \sigma_u^2 \le \left(r-\omega\right)^2, \forall\omega$. This is the condition in lemma 3. ■

Proof of proposition 9. Assume that he conditions for full insurance to be optimal are met. Under the assumption that the FOA is valid, we can rewrite $\mathrm{P}(\varepsilon)$ assuming that the distribution of ε is degenerate at 0. By taking the first-order condition w.r.t. ξ, we obtain:

$$\frac{m\left(\omega\right)}{V'\left(\xi\left(x,\omega\right)\right)} = \mu + \lambda\frac{1}{\sigma_x}\left(r-\omega\right)x.$$

With this contract the problem of the manager is concave in x. This implies two things. First, the agent will buy full insurance at the actuarially fair price of lemma 1. In turn, this means that this contract is optimal for the principal. Secondly, the concavity of the problem implies that the FOA is valid (Jewitt (1988)). ■

Proof of proposition 5.

Let $C^*\left(k\right)$ and $C_\varepsilon^*\left(k, p\left(\cdot\right)\right)$ be the minimum costs of implementing k when no securities are available and when the agent fully hedges his idiosyncratic risk, respectively. I consider the equilibrium where $p\left(\cdot\right)$ is given by lemma 1. To prove that the optimal k increases when the agent fully insures his risk, I can show that $C^*\left(k\right) - C_\varepsilon^*\left(k, p\left(\cdot\right)\right)$ is increasing is k and use a monotone comparative static argument. First, note that under the assumption that full insurance is optimal, $C^*\left(k\right) - C_\varepsilon^*\left(k, p\left(\cdot\right)\right) > 0$.

Remember that in equilibrium the utility of the agent is an average of $\tilde{u}\left(\left(\mu + \lambda L_{x|\omega}\left(x|\omega;k,d^\varepsilon,p\left(\cdot\right)\right)\right)/m\left(\omega\right)\right)$.

By the envelope theorem, differentiating $C^*(k) - C^*_\varepsilon(k, p(\cdot))$ w.r.t. k gives:

$$
\begin{aligned}
& \frac{\partial}{\partial k}\left(C^*(k) - C^*_\varepsilon(k, p(\cdot))\right) \\
= \; & -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) + \hat{\lambda}C''(k) \\
& + \lambda\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, -k\ \varepsilon, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, -k\ \varepsilon, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) - \lambda C''(k) \\
& - \frac{\partial}{\partial k}\hat{\mu}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right) d\Phi(x)\, dF_\omega(\omega) + \hat{\mu}C'(k) \\
& + \frac{\partial}{\partial k}\mu\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, -k\ \varepsilon, p(\cdot))}{m(\omega)}\right) d\Phi(x)\, dF_\omega(\omega) - \mu C'(k),
\end{aligned}
$$

where a "hat" denotes the Lagrange multipliers for the case with no insurance. Therefore,

$$
\begin{aligned}
\frac{\partial}{\partial k}\left(C^*(k) - C^*_\varepsilon(k, p(\cdot))\right) &= -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) \\
& \quad + \left(\hat{\lambda} - \lambda\right)C''(k) + (\hat{\mu} - \mu)\, C'(k) \\
&= -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) + \\
& \quad \underbrace{\left(\hat{\lambda} - \lambda\right)}_{>0}C''(k) + \underbrace{(\hat{\mu} - \mu)}_{>0}C'(k) > 0,
\end{aligned}
$$

where $\hat{\lambda} > \lambda$ and $\hat{\mu} > \mu$ are implied by the assumption that insuring the idiosyncratic risk is optimal for the principal and $\int \tilde{u}\left((\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot)))/m(\omega)\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) < 0$ since the variance of $L_{x|\omega}$ is lower when k is higher.

The proof that welfare is higher when ε-securities are traded follows from similar steps to the proof of proposition 3. I first define a social planner that chooses ξ, k, and c so as to maximize the welfare of investors. Then I show that, for given aggregate consumption, the planner's problem is equivalent to finding the optimal contract that maximizes the value of resources produced by each single manager. Under the assumption that full insurance is optimal, it follows that the value of resources when ε-securities are available is higher. This proves the claim that welfare in the economy is higher. ■

A.2 Only ω-securities

Proof of lemma 5. Again, consider the problem of intermediary ℓ:

$$\max_{y_\ell} \mathbb{E}\left[m(\omega)\,\Pi_\ell^I\right],$$

where

$$\Pi_\ell^I = \int \left(p_{\hat\omega} - z_{\hat\omega}\right) y_{\ell,\hat\omega}\; d\hat\omega,$$

and differentiate the expression pointwise w.r.t. y_ℓ

$$\mathbb{E}\left[m(\omega)\right] p_{\hat\omega} - m(\hat\omega)\, f_\omega(\hat\omega) \geq 0.$$

If $p_{\hat\omega}$ was different from $m(\hat\omega)\, f_\omega(\hat\omega)\,/\mathbb{E}\left[m(\omega)\right]$, then an intermediary would buy or sell an infinite amount of this security. This cannot be an equilibrium, so $p_{\hat\omega} = m(\hat\omega)\, f_\omega(\hat\omega)\,/\mathbb{E}\left[m(\omega)\right]$. ■

Proof of lemma 12. When only ω-securities are traded, the optimal contract solves

$$\max_{\xi,k} \int m(\omega)\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi(x,\omega)\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}, 0, p(\cdot)\right) dF_\omega(\omega)\Big|_{\hat{k}=k} - C'(k) = 0,$$

$$\frac{\partial}{\partial \hat{d}} \int u\left(\xi(x,\omega)\right) dF_{x|\omega}\left(x|\omega; k, k, \hat{d}, p(\cdot)\right) dF_\omega(\omega)\Big|_{\hat{d}=0} = 0,$$

$$\int u\left(\xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega) - C(k) = \bar{u}.$$

This is program P(ω) in the main text. I am assuming here that the FOA is valid. We can now define the Lagrangian and maximize it poitwise w.r.t. ξ. The optimal contract solves:

$$\frac{m(\omega)}{V'\left(\xi(x,\omega)\right)} = \mu + \lambda\frac{1}{\sigma_x}\left(r - h(\omega)\right)\ x + \lambda\frac{k\ \sigma_\varepsilon^2}{\sigma_x^2}\left(x^2 - 1\right)$$

where $h(\omega) = \omega - \nu_\omega\left(1 - \mathbb{E}\left[m(\omega)\right]\right)$ and $\nu_\omega \geq 0$ is the Lagrange multiplier on the IC constraints that determine the choice of $\hat{d}$. This is the expression in proposition 6. ■

Proof of proposition 7. First, I will show that, when ω-securities are available, the agent has a profitable deviation. Take the special case of section 4 and let ξ denote the payment schedule that solves (5). Consider the deviation where the agent sells some risk by buying a portofolio of

ω-securities that pays off $-\kappa\omega/m(\omega)$, for a small $\kappa > 0$. From lemma 5, the price of this security is $-\kappa\mathbb{E}[\omega]/\mathbb{E}[m(\omega)] = 0$. If the agent buys this portfolio, the mean of x will be unaffected, but x becomes less correlated with ω. With a slight abuse of notation, let $F_{x|\omega}(x|\omega; k, k, -\kappa\omega/m(\omega), p(\cdot))$ be the conditional disitribution of x, when the portfolio $-\kappa\omega/m(\omega)$ is selected. Differentiating the agent's utility w.r.t. κ around $\kappa = 0$ (that is, around the point where the agent doesn't deviate) yields

$$\begin{aligned}
& \frac{\partial}{\partial\kappa}\left(\int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, k, -\kappa\frac{\omega}{m(\omega)}, p(\cdot)\right) dF_{\omega}(\omega) - C(k)\right) \\
= & \int u(\xi(x,\omega))\left(\frac{\partial}{\partial\kappa} dF_{x|\omega}\left(x|\omega; k, k, -\kappa\frac{\omega}{m(\omega)}, p(\cdot)\right)\right) dF_{\omega}(\omega) \\
= & \int u(\xi(x,\omega))\left(-\frac{1}{\sigma_x}\frac{\omega x}{m(\omega)}\right) d\Phi(x)\, dF_{\omega}(\omega) > 0.
\end{aligned}$$

The last inequality comes from the optimal contract (5). Thus, $\bar{U}_{\omega}(k, p(\cdot)) \geq \int u(\xi(x,\omega))\, d\Phi(x)\, dF_{\omega}(\omega) - C(k)$ holds with a strict inequality.

I have to prove that ω-securities reduce the equilibrium level of k. A quick way to prove this result is to rewrite the contracting problem without using the transformation x, that is, I let the payment ξ be conditional on (Π, ω) instead of (x, ω). Let

$$L_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right) = \frac{\partial F'_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right)/\partial\hat{k}}{F_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right)}$$

be the likelihood ratio of Π conditional on ω. Note that $L_{\Pi|\omega}$ depends only on the actual choice of the agent, $\hat{k}$, but not on the level suggested by the principal, k. Without restating the problem, from Holmstrom (1979), we know that the optimal payment will be such that the agent's utility is the average of $\tilde{u}\left(\left(\mu + \lambda L_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, 0, p(\cdot)\right)\right)/m(\omega)\right)$. The reason why using (Π, ω) instead of (x, ω) simplifies the proof is that now the outside option

$$\bar{U}_{\omega}(p(\cdot)) = \max_{\hat{k},\, \hat{d}} \int u(\xi(x,\omega))\, dF_{\Pi|\omega}\left(x|\omega; \hat{k}, \hat{d}, p(\cdot)\right) dF_{\omega}(\omega) - C\left(\hat{k}\right)$$

depends on the recommended k only through the contract ξ. In the contract of proposition 5, μ and λ are both increasing functions of k. Therefore, if a deviation is profitable for some k, that is, $\bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}$, then the same deviation must be profitable for a higher k. Formally, $\bar{U}_{\omega}(k', p(\cdot)) \geq \bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}$ for $k' \geq k$.

Finally, we have to prove that welfare decreases when ω-securities are available. The proof is analogous to the proof of proposition 3. The social planner solves the problem of choosing ξ, k, and c so as to maximize welfare in the economy. Formally, the social planner solves:

$$\max_{\xi,k,c} \int v\left(c\left(\omega\right)\right) dF_{\omega}\left(\omega\right)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi\left(x,\omega\right)\right) dF_{x|\omega}\left(x|\omega;k,\hat{k},0,p\left(\cdot\right)\right) dF_{\omega}\left(\omega\right)\Big|_{\hat{k}=k} - C'\left(k\right) = 0,$$

$$\bar{U}_{\omega}\left(k,p\left(\cdot\right)\right) \leq \bar{u}$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) = \bar{u},$$

$$c\left(\omega\right) + \int \xi\left(x,\omega\right) d\Phi\left(x\right) = \int \left[x + \bar{R} + r\ k + \left(1-k\right)\omega\right] d\Phi\left(x\right),\ \forall \omega.$$

Compared to problem P(ω), the social planner has one extra control variable, $c\left(\omega\right)$, but he has to satisfy the resource constraint of the economy. Note that $c\left(\omega\right)$ appears only in the objective function and in the resource constraint. So, we can separate the problem by first choosing $c\left(\omega\right)$ to solve

$$\max_{c} \int v\left(c\left(\omega\right)\right) dF_{\omega}\left(\omega\right)$$

subject to

$$c\left(\omega\right) + \int \xi\left(x,\omega\right) d\Phi\left(x\right) = \int \left[x + \bar{R} + r\ k + \left(1-k\right)\omega\right] d\Phi\left(x\right),\ \forall \omega.$$

Let $\varphi\left(\omega\right)$ be the Lagrange multiplier on the resource constraint. The first-order condition w.r.t. c is

$$m\left(\omega\right) \equiv v'\left(c\left(\omega\right)\right) = \varphi\left(\omega\right).$$

Now, conditional on $\varphi\left(\omega\right)$, we can solve the dual problem of choosing ξ and k to maximize the value of resources in the economy. This problem is the same as P(ω). Thus, the equilibrium of the economy is a solution to the planner's problem and $\bar{U}_{\omega}\left(k,p\left(\cdot\right)\right) \leq \bar{u}$ is a constraint on the planner's problem. Therefore, welfare is lower when ω-securities are available. ■

A.3 Both types of securities

Proof of Lemma 7. The proof of this result is similar to those of lemmas 1 and 5 combined with the analysis in Makowski (1979). ■

Proof of proposition 8. Assume that we are in the environment of section 5.1. The optimal contract is given in proposition 4. Assume now that the double-deviation is possible and its value is given by $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$. Consider first a small increase of c_ε and, thus, p^ε (the case with c_ω is analogous). From proposition 4 we know that the optimal contract is increasing in the mean of x. This implies

$$\frac{\partial}{\partial p^\varepsilon}\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \geq 0,$$

which tightens the constraint $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \leq \bar{u}$. Now, for higher values of k, the Lagrange multipliers μ and λ in 9 increase to satisfy the IC and IR constraint. From

$$\bar{U}_{\varepsilon\omega}(k, p(\cdot)) = \max_{\hat{k}, s^\omega} \int \tilde{u}\left(\frac{\mu + \lambda\ (r-\omega)\ x/\sigma_u}{m(\omega)}\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}, s^\varepsilon = \varnothing, s^\omega \neq \varnothing, p(\cdot)\right) dF_\omega(\omega)$$

we see that if a deviation was profitable for a certain k, $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) > \bar{u}$, then it will profitable also for $k + dk$. Formally,

$$\frac{\partial^2}{\partial k \partial p^\varepsilon}\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \geq 0.$$

The latter proves that higher values of p^ε have a bigger effect on the cost of implementing a certain action when k is higher. In turn, this implies that the optimal level of k decreases with p^ε. ■

Proof of lemma 8. Consider a tax on the transactions in the securities market. A transaction in this context has to be interpreted as a trade between an intermediary and a manager. In other words, I assume that intermediaries and managers will pay the tax any time they trade something, independently of the quantity of securities exchanged.

When transactions are not observable, the principal has to consider three possible deviations: not trading any security, trading both securities, and trading only ω-securities (double-deviation). Formally, let $\bar{U}_\varepsilon(k, p(\cdot))$, $\bar{U}_\omega(k, p(\cdot))$, and $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$ denote the values of the three deviations, respectively. Also, let τ be the tax per transaction, then in equilibrium the fixed cost of transaction increases from c_ε to $c_\varepsilon + \tau$ (and from c_ω to $c_\omega + \tau$).

The social planner's problem is:

$$\max_{\tau} \max_{\xi, k} \int v'(c(\omega))\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega) + p^\varepsilon\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, d^\varepsilon = -k\varepsilon, d_i = \varnothing\right) dF_\omega(\omega)\bigg|_{\hat{k}=k} - C'(k) = 0,$$

$$\bar{U}_\varepsilon(k, p(\cdot)) \leq \bar{u},$$

$$\bar{U}_{\omega}\left(k, p\left(\cdot\right)\right) \leq \bar{u},$$

$$\bar{U}_{\varepsilon\omega}\left(k, p\left(\cdot\right)\right) \leq \bar{u},$$

$$\int u\left(\xi\left(x, \omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq \bar{u}.$$

Here, the tax τ affects prices both directly (the tax is imposed on each transaction) and through its effect on $c(\omega)$ and, thus, $m(\omega)$. However, note that in equilibrium the proceeds from tax are rebated to the representative investor. Thus, aggregate consumption $c(\omega)$ is not affected directly by τ, but only through the effect on $p(\cdot)$. Now, if we relax the problem by dropping the contraints on the three deviaitions, the problem is the same as P(ε). In section 5.1, I proved that the optimal contract of this proble is such that the agent wants to buy full insurance. Thus, at $\tau = 0$, we have that $\bar{U}_{\varepsilon}(k, p(\cdot)) = \bar{u}$. However, when $\tau = 0$, ω-securities represent a profitable deviation for the agent, that is, $\bar{U}_{\omega}(k, p(\cdot)) < \bar{u}$. This implies that a small positive $\tau > 0$ has a second order effect on $\bar{U}_{\varepsilon}(k, p(\cdot))$ but a first order effect on $\bar{U}_{\omega}(k, p(\cdot))$. Thus, a small positive $\tau > 0$ increase welfare in the economy. ■

Proof of proposition 9. The proof of this result follows from the intuition given in the text. When transactions are observable, then the principal can limit the manager to make only one transactions by punishing him (by setting $\xi_i = -\infty$) for deviating. Thus, the manger will always trade one and only one security. Now, except for choosing a different investment fraction k, the only other possible deviation is the double-deviation of trading only ω-securities. Thus, the only constraint on the optimal contracting problem is $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \leq \bar{u}$. Also, note that the value of τ doesn't directly affect $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$ since the agent is trading only one security. In turn, this implies that this problem is a relaxed version of the problem of lemma 8 and the result follows. ■

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