Portfolio Home Bias and External Habit Formation

Andreas Stathopoulos*

This version: March 15, 2012

Abstract

There is currently a consensus in the international finance literature that hedging against real exchange rate risk is unable to account for the empirically observed level of equity home bias. As a result, the literature has largely focused on friction-based explanations. This paper aims to challenge that consensus by proposing a novel mechanism that is able to generate significant equity home bias in the absence of any frictions in the markets for goods and assets. I consider a multi-country general equilibrium model which features cross-country heterogeneity in conditional risk aversion, generated by the interaction of home bias in preferences and external habit formation. In equilibrium, each country's consumption share is increasing in its conditional risk aversion and decreasing in all other countries' conditional risk aversion, so financing equilibrium consumption entails hedging against increases in home conditional risk aversion. If preferences are sufficiently home biased, an increase in home risk aversion leads to a relative appreciation of the home equity. As a result, home equity is a better hedge against home risk aversion than foreign equity, inducing portfolio home bias. Furthermore, the presence of external habit formation allows the model to generate realistic asset price dynamics, satisfying a long-standing need of the international asset pricing literature.

JEL classification: G11, G12, G15, F30.

Keywords: Portfolio home bias, habit formation, asset pricing.

*University of Southern California, Marshall School of Business, Los Angeles, CA 90089. Email: astathop@marshall.usc.edu. I am grateful to Geert Bekaert, Wayne Ferson and Tano Santos, as well as seminar participants at USC Marshall and CMU Tepper, for helpful comments.

1 Introduction

In the absence of market frictions, the desire for international risk sharing should lead homogeneous investors across the world to hold the world market portfolio. However, one of the most extensively documented stylized facts in international finance is the overwhelming tendency of investors to hold equity portfolios heavily skewed towards assets of their home country.[1] It has long been recognized that equity home bias is puzzling, as it suggests that investors forego significant international diversification benefits.[2] Theoretical attempts to explain the equity home bias puzzle have been largely unsuccessful and, in some cases, have exacerbated the puzzle, suggesting that optimal equity portfolios should have a foreign bias.[3]

Specifically, the literature has proposed models that relax either the frictionless and complete financial markets assumption or the homogeneous investors assumption, providing one of the following four explanations of equity home bias: i) in frictionless and complete financial markets, it results from hedging against real exchange rate fluctuations, ii) in frictionless, but incomplete, financial markets, it results from hedging against non-traded income shocks, iii) it is due to frictions in international financial markets (such as taxes and trading costs), and iv) it is due to asymmetric information or behavioral biases. Only the first explanation, hedging against adverse real exchange rate shocks, is consistent with the benchmark assumptions of the asset pricing literature, agent rationality and frictionless and complete financial markets. Since real exchange rates are the relative prices of the countries' consumption baskets, deviations from purchasing power parity - and, thus, non-trivial exchange rate behavior - presupposes heterogeneity in the countries' consumption patterns.

The aforementioned cross-country differences in consumption are typically considered to result from either heterogeneous preferences or frictions in the international goods markets. Specifically, it is assumed that either agent have home-biased preferences or that there are finite or infinite deadweight costs in the international goods markets. Both assumptions generate home bias in consumption in equilibrium, consistent with the data. Given that the equilibrium portfolio strategy has to finance equilibrium consumption, hedging against real exchange rate risk generates equity home bias if home equity performs better than foreign equity when the home consumption expenditure is high.

A long literature has explored the conditions that would generate equity home bias under standard CRRA preferences and has shown that hedging against real exchange rate fluctuations is unable to generate sufficient portfolio home bias for reasonable parameter values. In the words of Coeurdacier and Rey (2011) "there is now a consensus that the hedging of real

[1] See French and Poterba (1991).

[2] There is a long literature on the importance and benefits of international diversification; see, for example, Grubel (1968), Levy and Sarnat (1970), Lessard (1973), Solnik (1974a) and Errunza (1983).

[3] Extensive reviews of the literature on the equity home bias puzzle are provided by Lewis (1999), Karolyi and Stulz (2003), Sercu and Vanpee (2007) and Coeurdacier and Rey (2011).

exchange rate risk cannot account empirically for the equity home bias". Furthermore, standard preferences cannot generate realistic asset pricing returns, a severe shortcoming for any model that purports to explain portfolio choice. Lewis (1999) notes that "[A] major problem with reconciling investor home bias with international consumption movements is that the volatility of the implicit intertemporal marginal rate of substitution is not high enough to explain stock price movements."[4] As a result, a large part of the literature has abandoned the quest to generate adequate equity home bias in a frictionless economy populated by rational investors and has focused on the other three explanations.

This paper aims to challenge that consensus by proposing a multi-country general equilibrium model that not only assumes frictionless and complete financial markets, but also frictionless international goods markets. Specifically, I consider a multi-country general equilibrium model, in the tradition of Lucas (1982): the global economy comprises $n+1$ countries, one domestic and n foreign, each populated by a representative agent. There are $n+1$ internationally tradeable goods, one domestic and n foreign, and each agent is endowed with a claim on the entirety of the global endowment of the corresponding good. The key characteristic of the model is the interaction of external habit formation with home bias in preferences.[5] The former generates stochastically varying risk aversion, while the latter ensures that countries do not wish to perfectly pool their consumption, even in the presence of frictionless and complete financial markets. The interaction of those two effects generates cross-sectional heterogeneity in country conditional risk aversion, which, in turn, generates heterogeneity in equilibrium consumption patterns. More specifically, as a result of international risk sharing, each country's equilibrium consumption share is increasing in said country's conditional risk aversion and decreasing in all other countries' conditional risk aversion. As a result, each country's desired portfolio hedges against adverse shocks in its conditional risk aversion. In more detail, to finance her consumption, each agent needs to hold a portfolio that has two components: the tangency portfolio of the instantaneous mean-variance frontier and a portfolio that insures the agent against increases in her conditional risk aversion.

To determine whether the hedging demand can give rise to portfolio home bias, I adopt the simplest and most intuitive specification: the internationally tradeable assets are the claims on the $n+1$ endowments. I show that the price of each endowment claim is determined by two opposing effects: a valuation effect and a cash-flow effect. The latter effect always dominates and is determined by the impact of endowment shocks on goods prices, which, in turn, depends on

[4] More forcefully, van Wincoop and Warnock (2010) show that, under CRRA preferences, home bias in assets cannot be linked to home bias in goods without generating unrealistic results about asset returns and exchange rates. Discussing a wider class of models, Coeurdacier and Rey (2011) note that while certain models "have some success in replicating some feautures of aggregate portfolio data, they cannot replicate realistic moments for asset prices and exchange rates".

[5] Cole and Obstfeld (1991), Zapatero (1995), Kollman (2006b) and Pavlova and Rigobon (2007) also adopt home biased preferences in order to break purchasing power parity and generate non-trivial real exchange rate behavior.

the preference parameters of all countries. Given the variety of potential preference specifications in a multi-country setting, the result allows for a wide range of equilibrium portfolio outcomes. However, I show that, if certain restrictions on the preference parameters that ensure home bias hold, optimal risk sharing implies a very high degree of portfolio home bias: equilibrium portfolios are superbiased towards the home claim, involving a superlong position in the home claim financed by shorting at least one of the foreign claims. The reason is that, given those restrictions, an increase in a country's risk aversion tends to appreciate its endowment claim in relative terms. Given that an increase in risk aversion is associated with an increase in consumption expenditure, each country's equity is a better hedge against adverse risk aversion shocks than foreign equity, generating portfolio home bias. Furthermore, the presence of external habit formation allows the model to generate realistic asset price and exchange rate dynamics, satisfying a long-standing need of the general equilibrium literature in international finance.

This paper is part of the long literature on portfolio home bias and, specifically, of the strand that focuses on hedging against real exchange rate risk. Solnik (1974b), Sercu (1980), Stulz (1981b) and Adler and Dumas (1983) focus on hedging against inflation risk. However, Adler and Dumas (1983) show that inflation hedging portfolios are too small to explain empirical magnitudes, while Cooper and Kaplanis (1994) show that the data strongly reject the claim that portfolio home bias can be explained by hedging for inflation risk. Given the importance of trade costs,[6] Stockman and Dellas (1989) and Baxter, Jermann and King (1998) consider non-tradeable goods and, while they are able to generate portfolio home bias regarding the non-tradeable good claims, they are unable to do so regarding tradeable good claims.[7] Uppal (1993) departs from the assumption of outright non-tradeability and considers finite trade costs, showing that, for realistic levels of risk aversion, the model generates foreign, rather than home, equity bias.[8]

Regarding hedging against non-traded income shocks, Baxter and Jermann (1997) argue that domestic human capital returns are highly correlated with domestic equity returns and, thus, investors should hold foreign biased equity portfolios. However, Julliard (2002) argues that they overstate the advantage of foreign equity over domestic equity for hedging domestic labor income risk, so considering human capital does not necessarily worsen the home bias puzzle. Furthermore, Heathcote and Perri (2008), Coeurdacier and Gourinchas (2009) and Coeurdacier, Kollman and Martin (2010) show that, in the presence of internationally tradeable bonds, equities are indeed valuable as hedges of non-traded income risk. Tesar (1993) and Pesenti and van Wincoop (2002) consider partial equilibrium portfolio choice in economies in which claims on non-tradeable goods are themselves non-tradeable. Tesar (1993) argues that,

[6] See Obstfeld and Rogoff (2000).

[7] In the same vein, Serrat (2001) proposes a model with heterogeneous preferences and non-tradeables, but Kollman (2006a) shows the only aspect of portfolio choice that can be pinned down is that each agent holds an internationally diversified portfolio of claims on tradeable goods.

[8] Coeurdacier (2009) confirms that, for reasonable trade costs, optimal portfolios exhibit foreign bias.

under those assumptions, home bias in the shares of claims on tradeable goods is possible, but Pesenti and van Wincoop (2002) show that the degree of home bias generated in such an economy would be insufficient to account for the bias observed in the data.

Models that examine asset market frictions associated with international portfolio choice typically focus on taxes, trading costs and investment restrictions and argue that frictions may be high enough to wipe out almost all gains from international diversification.[9] However, Tesar and Werner (1995) show that foreign equity investments have much higher turnover rates than domestic equity investments, so international transaction costs are unlikely to explain the lack of sufficient international portfolio diversification. Other models emphasize informational or behavioral explanations: domestic and foreign investors differ in either their information or their beliefs about the distribution of asset returns and those differences are such that tilt their portfolios towards their home assets.[10]

This paper also belongs to the recent literature that embeds complex preferences in general equilibrium open economy models, thus bridging the asset pricing literature with the international finance literature.[11] Inter alia, external habit formation is a feature in the models of Verdelhan (2010) and Stathopoulos (2011). The former focuses on the uncovered interest rate parity puzzle, while the latter shows that a two-country, two-good version of the model featured in this paper can address the Brandt, Cochrane and Santa-Clara (2006) international risk sharing puzzle. However, the recent international asset pricing literature has not focused on equilibrium international portfolios, with the exception of Shore and White (2006), which address the portfolio home bias puzzle with a model that incorporates external habit formation. In their model, portfolio home bias results from the attempt of unconstrained investors to mimic, in a "catching up with the Joneses" spirit, the portfolio behavior of small entrepreneurs, who are forced to hold domestic equity for agency reasons.

The rest of the paper is organized as follows. Section 2 presents the setup of the model, while Section 3 describes the properties of equilibrium prices and quantities. Section 4 examines the connection between hedging against changes in conditional risk aversion and portfolio home bias. Section 5 concludes. The Appendix contains the proofs and all supplementary material not included in the main body of the paper.

[9] See Black (1974), Stulz (1981a) and Cooper and Kaplanis (1994).

[10] Notable papers include French and Poterba (1991), Kang and Stulz (1997), Brennan and Cao (1997), van Nieuwerburgh and Veldkamp (2008) and Dumas, Lewis and Osambela (2011).

[11] See, for example, Bekaert (1996), Lustig and Verdelhan (2007), Moore and Roche (2006), Aydemir (2008), Farhi and Gabaix (2008), Bansal and Shaliastovich (2010), Heyerdahl-Larsen (2010 and Colacito and Croce (2011a, 2011b).

2 The model

2.1 Endowments

The world economy comprises $n+1$ countries, indexed by i: the Domestic country ($i=0$) and n foreign countries ($i=1,...,n$), each of which is populated by a single risk-averse representative agent. There are $n+1$ distinct perishable goods in the world economy, indexed by j: the domestic good ($j=0$) and n foreign goods ($j=1,...,n$). Uncertainty in the economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, P)$, where $\mathbf{F}$=$\{\mathcal{F}_t\}$ is the filtration generated by the standard m-dimensional Brownian motion $\mathbf{B}_t$, $t \in [0, \infty)$, augmented by the null sets. All the stochastic processes introduced in the remainder of this paper are assumed to be progressively measurable with respect to $\mathbf{F}$ and to satisfy all the necessary regularity conditions for them to be well-defined. All (in)equalities that involve random variables hold P-almost surely.

Each agent i is initially endowed with a claim on the entirety of the world endowment of the corresponding good ($j=i$). The endowment stream of good j is denoted by $\{\widetilde{X}_t^j\}$; all endowment processes are Itô processes satisfying:

$$d \log \widetilde{X}_t^j = \mu_t^{j,X} dt + \boldsymbol{\sigma}_t^{j,X\prime} \mathbf{dB}_t, \; j = 0, 1, ..., n$$

with $\boldsymbol{\sigma}_t^{j,X} \neq \mathbf{0}$ for all j. All goods are frictionlessly traded internationally, so the price of each good, in units of the numeraire, is the same in all countries. Without loss of generality, the global numeraire good is the domestic consumption basket, as defined later. The numeraire price of each foreign good is denoted by Q_t^j.

2.2 Assets

All agents' endowments lie on the asset span, so the world economy is a securities market economy. Agents can frictionlessly trade $m+1$ non-redundant securities; of those, m are risky and 1 is locally riskless in terms of the numeraire. The risky asset returns, in units of the numeraire, are given by the m-dimensional process

$$\mathbf{dR}_t = \boldsymbol{\mu}_t dt + \boldsymbol{\sigma}_t \mathbf{dB}_t$$

where $\boldsymbol{\mu}_t$ is the $m \times 1$ vector of expected returns and $\boldsymbol{\sigma}_t$ is the $m \times m$ asset return diffusion matrix. Since the assets are non-redundant, $\boldsymbol{\sigma}_t$ is a non-singular matrix and securities markets are dynamically complete. Some of the risky assets may be in zero net supply, with the rest in positive net supply. Since the world economy is a securities market economy, the aggregate dividend of the risky assets equals the world endowment for each good and each period.

The locally riskless asset is in zero net supply and its price, in units of the numeraire, is denoted by D_t and satisfies

$$dD_t = r_t^f D_t dt$$

where r_t^f is the continuously compounded numeraire riskless rate. Then, we can define excess returns as

$$\mathbf{dR}_t^e = \boldsymbol{\mu}_t^e dt + \boldsymbol{\sigma}_t \mathbf{dB}_t$$

where $\boldsymbol{\mu}_t^e = \boldsymbol{\mu}_t - r_t^f \mathbf{1}$, with $\mathbf{1}$ representing a $m \times 1$ vector of ones.

2.3 Preferences

Representative agent i has expected discounted utility

$$E_0 \left[\int_0^\infty e^{-\rho t} u^i(X_t^{i,0}, X_t^{i,1}, ..., X_t^{i,n}) dt \right]$$

where $\rho > 0$ is her subjective discount rate, and her instantaneous utility function is

$$u^i(X_t^{i,0}, X_t^{i,1}, ..., X_t^{i,n}) = \log \left(\left(\prod_{j=0}^{n} \left(X_t^{i,j} \right)^{a^{i,j}} \right) - H_t^i \right) = \log(C_t^i - H_t^i)$$

where $X_t^{i,j}$ is the quantity of good j that agent i consumes at time t, $C^i \equiv \left(\prod_{j=0}^{n} \left(X^{i,j} \right)^{a^{i,j}} \right)$ is the domestic consumption basket and H^i is the habit level associated with that consumption basket. Notably, preferences are not symmetric regarding goods; the preferences of agent i towards the $n+1$ goods are described by the vector of preference parameters $\mathbf{a}^i = [a^{i,0}, a^{i,1}, ..., a^{1,n}]$, such that $\sum_{j=0}^{n} a^{i,j} = 1$ and $a^{i,j} > 0$ for all i and j. We also define the $(n+1) \times (n+1)$ preference matrix $\mathbf{A} = [a_{i,j}] = a^{i-1,j-1}$. By construction, $\mathbf{A}$ is a row stochastic matrix.

The external habit is of the Menzly, Santos and Veronesi (2004) form. Specifically, the inverse surplus consumption ratio $G^i = \left(\frac{C^i - H^i}{C^i} \right)^{-1}$ solves the stochastic differential equation

$$dG_t^i = \varphi \left(\bar{G} - G_t^i \right) dt - \delta \left(G_t^i - l \right) \left(\frac{dC_t^i}{C_t^i} - E_t \left(\frac{dC_t^i}{C_t^i} \right) \right) \tag{1}$$

The inverse surplus consumption ratio is a mean-reverting process, reverting to its long-run mean of $\bar{G}$ at speed φ and is driven by consumption growth shocks. The parameter $\delta > 0$ scales the impact of a consumption growth shock and the parameter $l \geq 1$ is the lower bound of the inverse surplus ratio G_t. Obviously, $\bar{G} > l$. The local curvature of the utility function is $-\frac{u_{CC}(C_t^i, H_t^i)}{u_C(C_t^i, H_t^i)} C_t^i = G_t^i$; for that reason, in a slight abuse of terminology, in the remainder of this article G^i will be sometimes referred to as the risk aversion of country i.[12] Importantly, note that country heterogeneity in the composition of the consumption basket (i.e. differences in the rows of $\mathbf{A}$) induce heterogeneity in the consumption shocks and, thus, in the level of conditional risk aversion across countries.

Lastly, we adopt the notation $\boldsymbol{\sigma}_t^{i,G}$ for the diffusion term of $\frac{dG_t^i}{G_t^i}$:

$$\boldsymbol{\sigma}_t^{i,G\prime}\mathbf{dB}_t \equiv -\delta\left(\frac{G_t^i - l}{G_t^i}\right)\left(\frac{dC_t^i}{C_t^i} - E_t\left(\frac{dC_t^i}{C_t^i}\right)\right)$$

2.4 Prices and real exchange rates

The time t price of country i consumption basket C^i, in units of the numeraire good, is denoted by P^i and satisfies

$$P_t^i = \prod_{j=0}^{n}\left(\frac{Q_t^j}{a^{i,j}}\right)^{a^{i,j}} \tag{2}$$

and is defined as the minimum expenditure required to buy a unit of the consumption basket C^i. As mentioned earlier, the domestic consumption basket is set as the global numeraire, so $P_t^0 \equiv 1, \forall t \in [0, \infty)$.

The real exchange rate of foreign country i against the domestic country, which expresses the price of a unit of the foreign consumption basket in units of the domestic consumption basket, is:

$$E_t^i = \frac{P_t^i}{P_t^0} = \frac{\prod_{j=0}^{n}\left(\frac{Q_t^j}{a^{i,j}}\right)^{a^{i,j}}}{\prod_{j=0}^{n}\left(\frac{Q_t^j}{a^{0,j}}\right)^{a^{0,j}}} = \prod_{j=0}^{n}\left(\frac{\left(a^{0,j}\right)^{0.j}}{\left(a^{i,j}\right)^{i,j}}\right)\prod_{j=0}^{n}\left(Q_t^j\right)^{a^{i,j}-a^{0,j}} \tag{3}$$

The real exchange rate of country i is constant and equal to 1 (purchasing power parity holds) only if the two countries' preferences are identical ($a^{i,j} = a^{0,j}$ for all goods j), so that the two consumption baskets have the same composition. In the case of differences in preferences, purchasing power parity is violated, so the two agents face different investment opportunity sets in real terms.

2.5 The agents' problem

Let the $mx1$ vector $\boldsymbol{\pi}_t^i$ describe the investment decision of agent i, which each element of the vector describing the amount, in units of the numeraire, that agent i invests in each risky asset in period t. Thus, agent i chooses consumption shares $X_t^{i,j}$ and amounts $\boldsymbol{\pi}_t^i$ so as to maximize

[12] In constrast to Menzly, Santos and Veronesi (2004), Campbell and Cochrane (1999) model the surplus consumption ratio, while Buraschi and Jiltsov (2007), Santos and Veronesi (2006) and Bekaert, Engstrom and Xing (2009) also model the inverse surplus consumption ratio.

her expected discounted utility

$$\max_{\{X_t^{i,j},\boldsymbol{\pi}_t^i\}} E_0\left[\int_0^\infty e^{-\rho t}\log\left(C_t^i - H_t^i\right)dt\right]$$

subject to the intertemporal flow budget constraint:

$$dW_t^i = \boldsymbol{\pi}_t^{i\prime}\left(\boldsymbol{\mu}_t^e dt + \boldsymbol{\sigma}_t \mathbf{dB}_t\right) + W_t^i r_t^f dt - C_t^i P_t^i dt$$

where W_t^i is the period t wealth of agent i in units of the numeraire; consequently, the investment of agent i in the locally riskless asset is $W_t^i - \boldsymbol{\pi}_t^{i\prime}\mathbf{1}$. If $W_t^i \neq 0$, we can also define the country i portfolio weight vector $\mathbf{x}_t^i$; it denotes the share of each risky asset in the risky portfolio of country i and is given by $\mathbf{x}_t^i = \frac{1}{W_t^i}\boldsymbol{\pi}_t^i$. The portfolio weight of the riskless asset is, thus, $1 - \mathbf{x}_t^{i\prime}\mathbf{1}$.

3 Equilibrium

Under the assumption of effective market completeness, there is a unique numeraire state-price density, Λ, in the world economy. The intertemporal budget constraint of agent i can be written in static form as follows:

$$E_0\left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0} C_t^i P_t^i dt\right] \leq E_0\left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0}\widetilde{X}_t^i Q_t^i dt\right]$$

This is nothing but the familiar restriction that the present value of domestic consumption cannot exceed the present value of the domestic endowment. The global numeraire state-price density Λ has to price all the available assets, so it has satisfy the SDE

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t^f dt - \boldsymbol{\eta}_t^\prime \mathbf{dB}_t \tag{4}$$

where $\boldsymbol{\eta}_t = \boldsymbol{\sigma}_t^{-1}\boldsymbol{\mu}_t^e$ is the market price of risk process. After replacing each agent's intertemporal dynamic budget constraint with her static budget constraint, we can solve for the competitive equilibrium; the solution details can be found in the Appendix.

3.1 Macroeconomic quantities and prices

The equilibrium consumption allocation is

$$X_t^{i,j} = \frac{a^{i,j}\lambda^i G_t^i}{\sum_{k=0}^n a^{k,j}\lambda^k G_t^k}\widetilde{X}_t^j$$

or, introducing the *share functions*

$$\omega_t^{i,j} : \omega_t^{i,j} = \frac{a^{i,j}\lambda^i G_t^i}{g_t^j}$$

with $\omega_t^{i,j}$ being the proportion of good j consumed in country i and g_t^j being the *effective conditional risk aversion of the demand for good* j, defined as

$$g_t^j \equiv \sum_{k=0}^{n} a^{k,j}\lambda^k G_t^k$$

Note that g_t^j is a linear combination of all countries' conditional risk aversion, weighted by the strength of each country's preference for good j, given by the product of its preference parameter for good j ($a^{k,j}$) and the welfare weight of that country (λ^k). We can also define the unconditional mean of the effective risk aversion for good j, $\bar{g}^j$, as:

$$\bar{g}^j \equiv \sum_{k=0}^{n} a^{k,j}\lambda^k \bar{G}$$

Consumption shares reflect preferences in two ways. First, consumption shares reflect the preference parameter matrix $\mathbf{A}$. Specifically, the consumption share that country i receives of good j ($\omega_t^{i,j}$) depends on the strength of the preference of country i for good j relative to the preference of all other countries for good j: $\omega_t^{i,j}$ is increasing in $a^{i,j}$ and decreasing in $a^{i',j}$ for $i' \neq i$. In the steady-state ($G_t^k = \bar{G}$ for all k), equilibrium consumption shares are identical to the ones that would prevail in the absence of external habit formation. However, there is also a second effect that operates if there is cross-country heterogeneity in conditional risk aversion: each country's share of all goods' endowment is increasing in its own risk aversion and decreasing in all other countries' risk aversion. Therefore, when a country's conditional risk aversion is high relative to its trading partners, it needs to consume more, as its marginal utility of consumption is higher. Given complete financial markets, countries share risk by insuring each other for periods of abnormally high conditional risk aversion.

It is important to note that cross-sectional heterogeneity in conditional risk aversion results from the interaction of heterogeneous preferences and external habit formation. Specifically, heterogeneous preferences generate heterogeneous equilibrium consumption patterns, so countries are exposed to heterogeneous consumption shocks. Given that conditional risk aversion is driven by past consumption growth shocks, the aforementioned heterogeneity translates into heterogeneity in conditional risk aversion. In the absence of preference heterogeneity, consumption growth shocks would be perfectly correlated across countries, resulting in identical conditional risk aversion worldwide, assuming identical initial conditional risk aversion. In the particular case that preference heterogeneity has the form of preference home bias, then equilibrium consumption is also home-biased, as happens with CRRA preferences. Furthermore, consumption growth shocks are home-biased averages of endowment growth shocks, implying that home conditional risk aversion increases are associated with home endowment declines.

Relative goods prices satisfy

$$\frac{Q_t^j}{Q_t^{j'}} = \frac{g_t^j}{g_t^{j'}} \frac{\widetilde{X}_t^{j'}}{\widetilde{X}_t^j} \tag{5}$$

and depend on the relative scarcity and the relative effective risk aversion of the demand of the two goods, with the former term reflecting a relative supply effect and the latter term a relative demand effect. While the relative supply effect is well-understood, the relative demand effect is novel. Intuitively, a relative increase in the conditional risk aversion of countries that consume more of good j than good j' induces increased relative demand for good j through the cross-country insurance mechanism, increasing its relative price. The relative demand effect can either reinforce or offset the relative supply effect, depending on the particular form of the preference matrix $\mathbf{A}$.

The aforementioned relative supply and demand effects also determine the numeraire price of the domestic good

$$Q_t^0 = \prod_{k=0}^{n} \left(a^{0,k}\right)^{a^{0,k}} \prod_{k=1}^{n} \left(\frac{g_t^0}{g_t^k} \frac{\widetilde{X}_t^k}{\widetilde{X}_t^0}\right)^{a^{0,k}} \tag{6}$$

and the numeraire price of foreign good j

$$Q_t^j = \prod_{k=0}^{n} \left(a^{0,k}\right)^{a^{0,k}} \left(\frac{g_t^j}{\widetilde{X}_t^j}\right)^{1-a^{0,j}} \prod_{k\neq j}^{n} \left(\frac{\widetilde{X}_t^k}{g_t^k}\right)^{a^{0,k}} \tag{7}$$

Since the numeraire, the domestic consumption basket, is a domestic-biased weighted geometric average of all prices and, thus, reflects a weighted average of relative supply and demand effects, Q^0 increases when the domestic good is relatively scarcer than, and in greater demand than, foreign goods. The intuition for Q^j is similar.

The share of global consumption expenditure that corresponds to country i, $\omega_t^{C,i}$:

$$\omega_t^{C,i} \equiv \frac{C_t^i P_t^i}{\sum_{k=0}^{n} C_t^k P_t^k} = \frac{\lambda^i G_t^i}{\sum_{k=0}^{n} \lambda^k G_t^k}$$

The consumption expenditure share $\omega_t^{C,i}$ is increasing in country i risk aversion and decreasing in all other countries' risk aversion. This is due to international risk sharing: since all countries have access to complete financial markets, they are able to risk share by, at each period, transferring consumption from the countries that need them the least (i.e. low risk aversion, low marginal utility countries) to countries that need them the most (high risk aversion, high marginal utility countries).

To finance its equilibrium consumption, country i needs to adopt a trading strategy that pays the necessary resources in each state of the world: since country i wealth W_t^i needs to finance future consumption, it holds that:

$$W_t^i = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} C_s^i P_s^i ds\right]$$

An increase in current risk aversion G_t^i increases the current consumption expenditure share $\omega_t^{C,i}$ and, due to the persistence of the risk aversion process, also increases future expenditure shares $\omega_s^{C,i}$, $s > t$. However, since G_t^i is mean-reverting, the increase in each of the future consumption expenditure shares is a decreasing function of the intervening time $s - t$. The higher the mean reversion speed of the risk aversion process (higher φ), the less the wealth share needs to adjust to changes in the consumption expenditure share. Ultimately, the share of global wealth that country i needs to hold, $\omega_t^{W,i}$, is:

$$\omega_t^{W,i} \equiv \frac{W_t^i}{\sum_{k=0}^{n} W_t^k} = \frac{\lambda^i\left(\rho G_t^i + \varphi \bar{G}\right)}{\sum_{k=0}^{n} \lambda^k\left(\rho G_t^k + \varphi \bar{G}\right)}$$

Similarly to the consumption expenditure share $\omega_t^{C,i}$, the wealth share $\omega_t^{W,i}$ is increasing in G_t^i and decreasing in all $G_t^{i'}, i' \neq i$. As explained above, the wealth share $\omega_t^{W,i}$ is less sensitive than the consumption expenditure share $\omega_t^{C,i}$ to risk aversion fluctuations. As a result, the wealth-consumption ratio is decreasing in G_t^i:

$$\frac{W_t^i}{C_t^i P_t^i} = \frac{1}{\rho}\frac{\rho G_t^i + \varphi \bar{G}}{\left(\rho + \varphi\right) G_t^i}$$

Its steady-state value is $\frac{1}{\rho}$, as in the no habit case, and its limit when G_t^i is approaching infinity is $\frac{1}{\rho+\varphi} < \frac{1}{\rho}$.

3.2 Portfolios

Equilibrium trading strategies have to be consistent with the aforementioned consumption and wealth allocations. The following proposition shows that each agent's equilibrium portfolio has two components: a conditionally mean-variance efficient portfolio, common for both agents, and a hedging portfolio, which differs across countries due to their differing preferences.

Proposition 1 *The equilibrium portfolio of country i satisfies*

$$\mathbf{x}_t^i = \left(\boldsymbol{\sigma}_t \boldsymbol{\sigma}_t'\right)^{-1} \boldsymbol{\mu}_t^e + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi \bar{G}}\right) \left(\boldsymbol{\sigma}_t'\right)^{-1} \left(\boldsymbol{\sigma}_t^{i,G}\right)$$

The first, common, component of equilibrium country portfolios is the tangency portfolio of the instantaneous mean-variance frontier, $\left(\boldsymbol{\sigma}_t \boldsymbol{\sigma}_t'\right)^{-1} \boldsymbol{\mu}_t^e$. On the other hand, hedging portfolios are country-specific: the hedging portfolio of country i is the projection of country i conditional risk aversion shocks to the asset span and, thus, is maximally conditionally correlated with shocks in conditional risk aversion G_t^i, providing the maximum possible insurance against changes in G_t^i. Thus, cross-country heterogeneity in conditional risk aversion generates insurance effects in equilibrium consumption allocations which, in turn, produce cross-country heterogeneity in equilibrium portfolios.

Note that the weight of the hedging portfolio in the optimal portfolio is

$$\frac{\frac{\rho G_t^i}{\rho G_t^i + \varphi \bar{G}}}{1 + \frac{\rho G_t^i}{\rho G_t^i + \varphi \bar{G}}}$$

and, thus, is increasing in risk aversion G_t^i: the more risk averse an agent is, the more important the hedging motive becomes.

In the absence of external habit formation, there is no cross-country heterogeneity in conditional risk aversion and, thus, no insurance effects; each country consumes a fixed proportion of each good. Therefore, consumption expenditure is perfectly correlated across countries, so consumption expenditure shares and wealth shares are constant and equal to the proportional welfare weight of each country:

$$\omega_t^{C,i} = \omega_t^{W,i} = \frac{\lambda^i}{\sum_{k=0}^{n} \lambda^k}$$

As a result, hedging demands are absent and each country holds only the conditional mean-variance optimal portfolio. Thus, although preference home bias is able to generate consumption home bias, it is unable to generate portfolio home bias.

3.3 The pricing of risk

The existence of hedging demands indicates that the standard International CAPM does not hold. The following proposition expresses the risk premium of any asset in the form of an Intertemporal CAPM.

Proposition 2 *The expected excess return of any asset s satisfies*

$$E_t\left(dR_t^{s,e}\right) = cov_t\left(dR_t^{s,e}, \frac{dW_t^A}{W_t^A}\right) - \sum_{i=0}^{n}\left(\frac{\lambda^i \rho G_t^i}{\sum_{k=0}^{n}\lambda^k\left(\rho G_t^k + \varphi \bar{G}\right)}\right) cov_t\left(dR_t^{s,e}, \frac{dG_t^i}{G_t^i}\right)$$

where $W_t^A \equiv \sum_{k=0}^{n} W_t^k$ *is aggregate global wealth.*

Cross-sectional heterogeneity in country risk aversions introduces $n+1$ additional state variables. Risk premiums are compensation for covariance risk not only with aggregate global wealth (as in the standard International CAPM), but also with country conditional risk aversions G^i, $i = 0, 1, ..., n$. Note that positive covariance with any country risk aversion G^i generates a negative risk premium, as it indicates that the asset has hedging value against increases in G^i.

4 Risk aversion hedging and portfolio home bias

The previous section demonstrated that, even given market completeness, country equilibrium portfolios may differ due to differing risk aversion hedging demands. In this section, we assume that there are $m = n+1$ independent sources of risk in the global economy and $n+1$ traded risky assets, the claims to country endowments (which we will also call equity claims in the remainder of the paper). Securities markets are dynamically complete if the $n+1$ equity claims are non-redundant assets, so that the asset return diffusion matrix $\boldsymbol{\sigma}$ is invertible. To satisfy that condition, in the remainder of the paper it will be assumed the preference matrix $\mathbf{A}$ is non-singular; for this condition to be satisfied, no two countries can have identical preferences.

This section illustrates that equilibrium portfolios are determined by the preference matrix $\mathbf{A}$, as the preference parameters regulate the impact of changes in countries' conditional risk aversion on the price of the endowment claims. Furthermore, it is shown that, given sufficient home bias in preferences, each country's endowment claim is a better hedge against changes in its domestic risk aversion. As a result, equity portfolios are home biased; in fact, it will be shown that equity portfolios are superbiased, in the Bennett and Young (1999) sense.

4.1 Asset Prices

The equilibrium price of the country i endowment claim satisfies

$$V_t^i = \frac{1}{\rho}\frac{\rho g_t^i + \varphi \bar{g}^i}{(\rho+\varphi)\, g_t^i}\left(\widetilde{X}_t^i Q_t^i\right) \tag{8}$$

Note that the price-dividend ratio of claim i depends solely on g^i, the effective conditional risk aversion for the demand of good i: an increase in the conditional risk aversion of any country decreases the valuation ratio, with changes in the conditional risk aversion of the countries that consume a lot of good i having a greater impact than changes in the conditional risk aversion of countries that do not consume large quantities of good i. When g^i is higher (lower) than its steady-state value, $\bar{g}^i$, the price-dividend ratio is higher (lower) than its steady-state value, $\frac{1}{\rho}$.

To understand equilibrium portfolio dynamics, we are interested in relative asset prices. Specifically, consider the relative price of claims i and i':

$$\frac{V_t^i}{V_t^{i'}} = \frac{\rho g_t^i + \varphi \bar{g}^i}{\rho g_t^{i'} + \varphi \bar{g}^{i'}}$$

It is useful to decompose the relative price into the product of the relative valuation ratio

$$\frac{\frac{V_t^i}{\widetilde{X}_t^i Q_t^i}}{\frac{V_t^{i'}}{\widetilde{X}_t^{i'} Q_t^{i'}}} = \frac{\rho g_t^i + \varphi \bar{g}^i}{\rho g_t^{i'} + \varphi \bar{g}^{i'}} \frac{g_t^{i'}}{g_t^i}$$

and the relative cash-flow ratio

$$\frac{\widetilde{X}_t^i Q_t^i}{\widetilde{X}_t^{i'} Q_t^{i'}} = \frac{g_t^i}{g_t^{i'}}$$

The two relative effects have opposing directions: the relative valuation ratio is decreasing in g^i and decreasing in $g^{i'}$, while the relative cash-flow ratio is increasing in g^i and decreasing in $g^{i'}$. However, the relative cash-flow effect always dominates, so the relative price is increasing in g^i and decreasing in $g^{i'}$. Thus, the impact of each country's conditional risk aversion on relative claim prices is determined by the preference matrix $\mathbf{A}$. In the following section, we will see that, given restrictions in $\mathbf{A}$ that ensure preference home bias, claim i will have superior hedging properties for agent i, leading to portfolio home bias.

As mentioned earlier, agent i desires to hedge against increases in conditional risk aversion G^i. The net financial assets of country i, NFA^i, are defined as the difference between the country's foreign assets and foreign liabilities. Therefore, NFA^i equal the difference between the financial wealth of country i and the capitalized value of its future endowment:

$$\begin{aligned} NFA_t^i &= W_t^i - V_t^i \\ &= \frac{1}{\rho}\left(\rho\left(\lambda^i G_t^i - \sum_{k=0}^{n} a^{k,i}\lambda^k G_t^k\right) + \varphi \bar{G}\left(\lambda^i - \sum_{k=0}^{n} a^{k,i}\lambda^k\right)\right)\frac{\widetilde{X}_t^0}{(\rho+\varphi)\, g_t^0} \end{aligned}$$

It follows that NFA^i are increasing in G^i and decreasing in the risk aversion of all other countries (G^k, $k \neq i$): country i hedges its exposure to the increase in its risk aversion by investing in a portfolio that generates a positive net financial position with respect to the rest of the world when G^i increases and finances its elevated future consumption needs - which involve net imports for a number of subsequent years - by progressively drawing down its savings.

4.2 Equilibrium portfolios

The remaining task is to determine the equilibrium country portfolios. The following proposition, proven in the Appendix, provides the answer.

Proposition 3 *In equilibrium, each country i holds a buy-and-hold risky portfolio that contains $\theta^{i,j}$ shares of the claim on the endowment of country j. The risky portfolio matrix $\mathbf{\Theta}$, defined as $[\theta_{i,j}] = \theta^{i-1,j-1}$, satisfies $\mathbf{\Theta} = \left(\mathbf{A}^{-1}\right)^{\top}$. Furthermore, no country holds any of the riskless asset. As a result, the portfolio weights of country i are:*

$$x_t^{i,j} = \frac{\theta^{i,j} V_t^j}{W_t^i} = \theta^{i,j} \frac{\rho g_t^j + \varphi \bar{g}^j}{\lambda^i \left(\rho G_t^i + \varphi \bar{G}\right)}$$

Since no country holds the riskless asset, all equilibrium portfolios are equity-only portfolios. Furthermore, each country adopts a buy-and-hold strategy: it chooses a portfolio at $t = 0$ and never reallocates. Although the number of shares in each country's portfolio is constant, the portfolio weight vector $\mathbf{x}_t^i$ is time-varying, due to time variation in the price of the claims. Furthermore, there is also time variation in the degree of portfolio home bias measure of country i, defined as

$$HB_t^i = x_t^{i,i} - b_t^i$$

where b_t^i is the benchmark weight of asset i, defined as the weight of the asset i in the value-weighted global asset portfolio:

$$b_t^i \equiv \frac{V_t^i}{\sum_{k=0}^{n} V_t^k} = \frac{\rho g_t^i + \varphi \bar{g}^i}{\sum_{k=0}^{n} \left(\rho g_t^k + \varphi \bar{g}^k\right)}$$

since both $x^{i,i}$ and the benchmark weight b_t^i are time-varying, the latter due to the time variation to the relative prices of the $n+1$ claims.

Proposition 3 establishes a correspondence between the preference matrix and the portfolio matrix. Given the range of potential preference specifications, a wide array of equilibrium portfolios can be generated. The following proposition establishes sufficient conditions under which portolio home bias arises in equilibrium.

Proposition 4 *Given that the risky portfolio matrix $\mathbf{\Theta}$ satisfies $\mathbf{\Theta} = \left(\mathbf{A}^{-1}\right)^{\top}$, it holds that:*

(a) If $a^{i,i} - a^{i',i} > \sum_{s \neq i,i'} \left| a^{i,s} - a^{i',s} \right|$ (condition HB1) for all $i' \neq i$, country i holds a home superbiased portfolio: $\theta^{i,i} > 1$ and $\theta^{i,j} < 0$ for at least one $j \neq i$.

(b) If $\frac{a^{i,i}}{a^{i,j}} - \frac{a^{i',i}}{a^{i',j}} > \sum_{s \neq i,i',j} \left| \frac{a^{i,s}}{a^{i,j}} - \frac{a^{i',s}}{a^{i',j}} \right|$ *(condition HB2) for all distinct* i*,* i' *and* j*, country* i *holds a home hyperbiased portfolio:* $\theta^{i,i} > 1$ *and* $\theta^{i,j} < 0$ *for all* $j \neq i$*.*

If preferences satisfy ($HB1$), as described above, each country holds a superbiased portfolio: each country has a superlong position in its own claim, financed by short-selling at least one other country's claim. If preferences satisfy (*HB2*), equilibrium country portfolios are hyperbiased: each country is superlong on its claim and short on the claims of all the other countries in the world.

It is useful to contrast our economy with a benchmark economy that is characterized by absence of external habit formation, but is otherwise identical to our economy. In that case, as mentioned in a previous section, each country invests in the mean-variance optimal portfolio. However, it can be shown that if the available assets are the $n+1$ endowment claims, equilibrium asset prices are such that all claims have perfectly correlated returns in terms of the numeraire. Therefore, equilibrium country portfolios are indeterminate: the competitive equilibrium allocation can be achieved by any buy-and-hold portfolio $\boldsymbol{\theta}^i$ that satisfies

$$\sum_{j=0}^{n} \left(\sum_{k=0}^{n} a^{k,j} \lambda^k \right) \theta^{i,j} = \lambda^i$$

The aforementioned set of portfolios includes both the external habit equilibrium portfolio and the autarky portfolio. Since the autarky portfolio can generate the optimal allocation, asset trading is not necessary: countries can achieve optimality by trading only in the spot market for goods.

4.2.1 Intuition: the two-country case

To understand how portfolio home bias arises in equilibrium, we can focus on the two-country case. In this setting, portfolio superbias also implies hyperbias. Condition ($HB1$) is satisfied if $a^{0,0} > a^{1,0}$. For convenience, I consider the familiar symmetric home bias case, with the home bias parameter being $a > 0.5$; the preference matrix is

$$\mathbf{A} = \begin{bmatrix} a & 1-a \\ 1-a & a \end{bmatrix}$$

and satisfies condition *HB1*.Thus, the equilibrium portfolio matrix is

$$\boldsymbol{\Theta} = \left(\mathbf{A}^{-1}\right)^{\top} = \frac{1}{2a-1} \begin{bmatrix} a & -(1-a) \\ -(1-a) & a \end{bmatrix} \tag{9}$$

so the domestic country portfolio comprises $\theta^{0,0} = \frac{a}{2a-1} > 1$ shares of the domestic asset and $\theta^{0,1} = -\frac{1-a}{2a-1} < 0$ shares of the foreign asset.

Consider the effects of an one standard deviation negative domestic endowment shock for the next 120 quarters (Figures 1, 2 and 3).[13] Figure 1 presents the effects on endowment and consumption. Since the negative domestic endowment shock (panel (a)) is a permanent shock, it entails a reduction of the steady-state level of domestic endowment (panel (b)). As a result of risk sharing, consumption growth experiences a negative shock in both countries (panel (c)). However, due to home bias in preferences and, hence, consumption, international risk sharing is imperfect: the domestic consumption growth shock is greater in magnitude and the steady-state level of domestic consumption is lower than that of foreign consumption (panel (d)). The adjustment to the new steady-state consumption levels does not happen instantaneously, but occurs gradually.

Figure 2 illustrates the adjustment process. To understand why the adjustment is gradual and not instantaneous, we need to consider conditional risk aversion. Since conditional risk aversion is perfectly negatively correlated with consumption growth, both countries' risk aversion increases, but domestic risk aversion increases more; furthermore, in the absence of any future endowment (and, thus, consumption) shocks, both countries' risk aversion reverts to its steady-state level (panel (a)). Since domestic conditional risk aversion increases more than foreign risk aversion, the effective conditional risk aversion of the demand for the domestic good, g^0, increases more than the effective risk aversion of the demand for the foreign good, g^1. As a result, the domestic good appreciates, while the foreign good depreciates (panel (b)); thus, the relative price of the foreign good declines (panel (c)): the relative demand effect reinforces the relative supply effect. Since the risk aversion shock is a temporary one, the two countries stagger their transition to their new steady-state consumption levels in order to minimize utility losses: during the adjustment period, domestic consumption is supported by net imports from the foreign country (panel (d)).

This international insurance effect is implemented by holding portfolios that finance those international trade flows. Specifically, the proportion of global wealth held by the domestic country increases (panel (e)) and domestic assets exceed foreign liabilities: the domestic country has positive net foreign assets (panel (f)). Thus, the stream of domestic net imports during the adjustment period is financed by gradually drawing down domestic net foreign assets. The adjustment period ends when the domestic country has exhausted its net foreign assets, in which time the trade balance between the two countries is zero and consumption achieves its steady-state level in both countries. As we have seen, this decrease in country i net savings does not involve decreasing the holdings of the number of shares in country i portfolio; rather, country i net imports are financed by its portfolio dividends.

[13] I assume that $a = 0.9$.I further assume that both countries' initial conditional risk aversion is equal to its steady-state value ($G_0^0 = G_0^1 = \bar{G}$).

Figure 3 presents the impact on asset prices and portfolios. Since g^0 increases more than g^1, the price-dividend ratio of the domestic equity decreases more than the price-dividend ratio of the foreign equity (panel (a)). On the other hand, as a result of the relative appreciation of the domestic good, the value of the dividend of the domestic claim decreases less than the value of the dividend of the foreign claim (panel (b)). The cash-flow effect dominates the valuation effect, so V^1, the numeraire price of the foreign equity, decreases more than V^0, the numeraire price of the domestic equity (panel (c)). Therefore, domestic equity is a better hedge against adverse fluctuations of domestic risk aversion than foreign equity, justifying the portfolio home bias. Furthermore, the long-short nature of the domestic portfolio is justified by the fact that there is no purely long position that can hedge against domestic risk aversion increases, as both asset prices decline.

As mentioned above, the degree of home bias is not constant. Indeed, the benchmark weight of the domestic asset, b^0, increases, while the benchmark weight of the foreign asset, b_1, decreases (panel (d)). Furthermore, the weight of domestic equity in the domestic portfolio ($x^{0,0}$) decreases, while the weight of the foreign asset in the foreign portfolio ($x^{1,1}$) decreases (panel (e)). Thus, the domestic agent invests a relatively low proportion of her wealth in the domestic asset, exactly when the domestic asset constitutes a relatively high proportion of the global portfolio. Those two effects reinforce each other, decreasing domestic portfolio home bias. Following the same analysis, foreign portfolio home bias increases (panel (f)).

Notably, (9) suggests that portfolio bias is decreasing in preference home bias: as a increases from 0.5 to 1, the number of domestic (foreign) shares in the domestic portfolio decreases (increases), and vice versa for the foreign portfolio. The reason is that as preference home bias increases (a increases), the two equities' returns become less correlated. Specifically, the disparity between the behavior of the domestic equity price V^0 and the foreign equity price V^1 is generated solely by the disparity in the behavior of g^0 and g^1 and that disparity is increasing in preference home bias. The higher the preference home bias is, the more differently shocks in domestic and foreign risk aversion affect g^0 and g^1 and, thus, V^0 and V^1, so each country needs a less home biased portfolio in order to be able to generate the same hedging outcome.

5 Simulation

To examine the quantitative aspects of the model, I simulate a two-country economy adopting the following simple specification for the two endowment processes:

$$d\log \widetilde{X}_t^0 = \mu dt + \sigma dB_t^0$$

and

$$d\log \widetilde{X}_t^1 = \mu dt + \sigma dB_t^1$$

For both goods, endowment growth has constant drift μ and constant volatility σ; endowment growth is not correlated across countries. The annualized parameter values used are reported in Table 1.[14] I consider the home bias parameter values of $a = 0.51$, 0.7, 0.95 and 0.99. I simulate 1,000 sample paths of the model economy, with each path consisting of 320 quarterly observations (80 years). Of the 320 observations, the first 120 (30 years) are discarded to reduce the dependence on initial conditions.

Tables 2, 3 and 4 present the properties of key moments in this economy. As Table 2 shows, consumption growth volatility and numeraire excess returns have empirically plausible magnitudes for all home bias parameters. As preferences become more home biased, international risk sharing declines: the cross-country correlation of consumption growth rates declines, while real exchange rate volatility increases. As a result, each country bears more risk: inter alia, consumption growth volatility increases. The decline in international risk sharing is reflected in asset prices: equity excess returns increase and become more volatile and less internationally correlated. For all home bias parametrizations, the two numeraire asset returns are very highly correlated, due to the very high correlation in conditional risk aversion.

Table 3 concerns the pricing of risk in the global economy. The first three rows present the means of the conditional prices of the three priced risk factors (aggregate wealth, domestic conditional risk aversion and foreign conditional risk aversion), denoted by λ^{WA}, λ^{G^0} and λ^{G^1}, respectively. The most highly priced risk factor is aggregate wealth: agents require that a unit loading on the aggregate wealth factor be compensated by an annualized risk premium of $3.5 - 4.5\%$, depending on the degree of preference home bias. The compensation for exposure to fluctuations of either domestic or foreign conditional risk aversion is about (minus) 1% per year.[15] It should be stressed that the three factors are not orthogonal to each other; rather, they are very highly correlated. As seen in Table 4, positive innovations in either domestic or foreign risk aversion are associated with negative innovations in aggregate wealth, due to valuation effects, while the innovations in the two countries' risk aversion processes are very highly correlated, as a result of international risk sharing. The last six rows of Table 3 present the unconditional betas of the two country's equity claims with respect to the three risk factors. Importantly, each equity claim hedges its home risk aversion better than the other country's equity: domestic (foreign) equity hedges domestic (foreign) risk aversion fluctuations better than foreign (domestic) equity does.

Tables 5 and 6 report the equilibrium portfolios. I decompose the number of shares each

[14] The system is initialized at the steady state ($G_0^0 = G_0^1 = \bar{G}$) and I adopt the normalization $\widetilde{X}_0^0 = \widetilde{X}_0^1 = 1$. The mean and standard deviation of endowment growth are set equal to the corresponding moments of quarterly US endowment growth from 1975:Q1 to 2007:Q2, with endowment defined as the sum of consumption of non-durables and services and net exports. The habit parameters are those used in Menzly, Santos and Veronesi (2004), with the exception of k, which is set slightly lower.

[15] The price of risk for G^0 and G^1 is negative, as a positive loading on those two factors indicates hedging against increases in risk aversion (and, thus, against increases in marginal utility).

agent holds into two components: the number of shares in the mean-variance portfolio and the number of shares in the agent's hedging portfolio. I also present the unconditional betas of the different portfolios. Notably, the mean-variance portfolio for both countries is highly leveraged, long on domestic equity and short on foreign equity: the two equity returns are very highly correlated, so mean-variance optimality entails holding a very leveraged position in the asset with the marginally higher Sharpe ratio (domestic equity), financed by a short position in the asset with the lower Sharpe ratio (foreign equity). On the other hand, both agents' hedging portfolios are long on the foreign asset and short on the domestic asset. However, the foreign hedging portfolio is more skewed toward the foreign asset than the domestic portfolio: the foreign hedging portfolio is more sensitive to fluctuations of domestic risk aversion and less sensitive to fluctuations of foreign risk aversion than the domestic portfolio, reflecting the difference in the hedging desires of the two agents. The skew towards the foreign asset in the foreign hedging portfolio is enough to reverse the skew towards the domestic asset in the mean-variance portfolio and, thus, the foreign portfolio ends up being home superbiased. On the other hand, the foreign asset skew in the domestic hedging portfolio is not enough to reverse the domestic asset skew of the mean-variance portfolio, so the domestic portfolio is superbiased towards the domestic asset.

6 Conclusion

This paper shows that external habit formation, coupled with home bias in preferences, can generate significant equity home bias, along with realistic asset pricing moments, in a frictionless economy populated by fully rational agents. The mechanism that generates equity portfolio home bias is novel: countries bias their portfolio towards the home asset due to their desire to hedge against adverse movements in their conditional risk aversion. Importantly, the model generates realistic asset pricing moments, satisfying a need that most of the extant international portfolio literature has been unable to address.

The model proposed in this paper challenges the prevailing consensus that portfolio home bias can only be due to frictions. Furthermore, it shows that not only is the model capable of generating portfolio home bias, but that, under certain parameter restrictions that may not be unrealistic, too much portfolio home bias ensues. While portfolio superbias is at variance with the data, the model proposes a new frictionless benchmark that underscores the importance of frictions that tend to decrease portfolio home bias (such as no short-selling restrictions), rather than frictions that tend to increase home bias, which have been the focus of the existing literature. More complex models that build on the success of the external habit formation model and extend it by adopting more realistic features would likely enhance our understanding of portfolio home bias.

References

[1] Adler, M. and B. Dumas, 1983, International Portfolio Choice and Corporation Finance: A Synthesis, *Journal of Finance* 38, 925-984.

[2] Aydemir, A. C., 2008, Risk Sharing and Counter-cyclical Variation in Market Correlations, *Journal of Economic Dynamics and Control* 32, 3084-3112.

[3] Bansal, R. and I. K. Shaliastovich, 2010, A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets, Working paper, Duke University and the University of Pennsylvania.

[4] Baxter, M. and U. J. Jermann, 1997, The International Diversification Puzzle is Worse Than You Think, *American Economic Review* 87, 170-180.

[5] Baxter, M., U. J. Jermann and R. G. King, 1998, Nontraded Goods, Nontraded Factors and International Non-Diversification, *Journal of International Economics* 44, 211-229.

[6] Bekaert, G., 1996, The Time Variation of Risk and Return in Foreign Exchange Markets: A General Equilibrium Perspective, *Review of Financial Studies* 9, 427-470.

[7] Bekaert, G., E. Engstrom and Y. Xing, 2009, Risk, Uncertainty and Asset Prices, *Journal of Financial Economics* 91, 59-82.

[8] Bennett, J. A. and L. Young, 1999, International Stock Market Equilibrium with Heterogeneous Tastes, *American Economic Review* 89, 639-648.

[9] Black, F., 1974, International Capital Market Equilibrium with Investment Barriers, *Journal of Financial Economics* 1, 337-352.

[10] Brandt, M. W., J. H. Cochrane and P. Santa-Clara, 2006, International Risk Sharing Is Better Than You Think, or Exchange Rates are too Smooth, *Journal of Monetary Economics* 53, 671-698.

[11] Brennan, M. and H. Cao, 1997, International Portfolio Investment Flows, *Journal of Finance* 52, 1851-1880.

[12] Buraschi A. and A. Jiltsov, 2007, Habit Formation and Macroeconomic Models of the Term Structure of Interest Rates, *Journal of Finance* 62, 3009-3063.

[13] Campbell, J. Y. and J. H. Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, *Journal of Political Economy* 107, 205-251.

[14] Coeurdacier, N., 2009, Do Trade Costs in Goods Market Lead to Home Bias in Equities?, *Journal of International Economics* 77, 86-100.

[15] Coeurdacier, N. and P. - O. Gourinchas, 2009, When Bonds Matter: Home Bias in Goods and Assets, Working paper, London Business School and UC Berkeley.

[16] Coeurdacier, N., R. Kollman and P. Martin, 2010, International Portfolios, Capital Accumulation and Foreign Assets Dynamics, *Journal of International Economics* 80, 100-112.

[17] Coeurdacier, N. and H. Rey, 2011, Home Bias in Open Economy Financial Macroeconomics, Working paper, Sciences Po and London Business School.

[18] Colacito R. and M. M. Croce, 2011a, Risks for the Long Run and the Real Exchange Rate, *Journal of Political Economy* 119, 153-181.

[19] Colacito R. and M. M. Croce, 2011b, International Asset Pricing with Risk-Sensitive Agents, Working paper, University of North Carolina.

[20] Cole H. L. and M. Obstfeld, 1991, Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?, *Journal of Monetary Economics* 28, 3-24.

[21] Cooper, I. and E. Kaplanis, 1994, Home Bias in Equity Portfolios, Inflation Hedging and International Capital Market Equilibrium, *Review of Financial Studies* 7, 45-60.

[22] Dumas, B., K. K. Lewis and E. Osambela, 2011, Differences of Opinion and International Equity Markets, Working paper, INSEAD, University of Pennsylvania and Carnegie Mellon University.

[23] Errunza, V., 1983, Emerging Markets: A New Opportunity for Improving Global Portfolio Performance, *Financial Analysts Journal* 39, 51-58.

[24] Farhi E. and X. Gabaix, 2008, Rare Disasters and Exchange Rates, Working paper, Harvard University and NYU.

[25] French, K. R. and J. M. Poterba, 1991, Investor Diversification and International Equity Markets, *American Economic Review* 81, 222-226.

[26] Grubel, H., 1968, Internationally Diversified Portfolios: Welfare Gains and Capital Flows, *American Economic Review* 58, 1299-1314.

[27] Heyerdahl-Larsen, C., 2010, Asset Prices and Real Exchange Rates with Deep Habits, Working paper, London Business School.

[28] Heathcote, J. and F. Perri, 2008, The International Diversification Puzzle is not as Bad as You Think, Working paper, Federal Reserve Board and University of Minnessota.

[29] Jones, R. W., S. Marjit and T. Mitra, 1993, The Stolper-Samuelson Theorem: Links to Dominant Diagonals, in R. Becker, M. Boldrin, R. Jones and W. Thomson eds., General Equilibrium, Growth and Trade, Vol. 2, New York: Academic Press, 429-441.

[30] Julliard, C., 2002, The International Diversification Puzzle is not Worse Than You Think, Working paper, London School of Economics.

[31] Kang, J. - K. and R. Stulz, 1997, Why is there Home Bias? An Analysis of Foreign Equity Ownership in Japan, *Journal of Financial Economics* 46, 3-28.

[32] Karolyi, G. A. and R. M. Stulz, 2003, Are Assets Priced Locally or Globally?, in Constantinides, G., M. Harris and R. M. Stulz (eds.), The Handbook of the Economics of Finance, North Holland, 2003.

[33] Kollman, R., 2006a, A Dynamic Equilibrium Model of International Portfolio Holdings: Comment, *Econometrica* 74, 269-273.

[34] Kollman, R., 2006b, International Portfolio Equilibrium and the Current Account, CEPR Discussion Paper No. 5512.

[35] Lessard, D., 1973, World, National and Industry Factors in Equity Returns, *Journal of Finance* 29, 379-391.

[36] Levy, H. and M. Sarnat, 1970, International Diversification of Investment Portfolios, *American Economic Review* 60, 668-675.

[37] Lewis, K. K., 1999, Trying to Explain Home Bias in Equities and Consumption, *Journal of Economic Literature* 37, 571-608.

[38] Lucas, R. E., 1982, Interest Rates and Currency Prices in a Two-Country World, *Journal of Monetary Economics* 10, 335-359.

[39] Lustig, H. and A. Verdelhan, 2007, The Cross-Section of Foreign Currency Risk Premia and Consumption Growth Risk, *American Economic Review* 97, 89-117.

[40] Menzly, L., T. Santos and P. Veronesi, 2004, Understanding Predictability, *Journal of Political Economy* 112, 1-47.

[41] Mitra, T. and R. W. Jones, 1992, Factor Shares and the Chipman Condition, Working paper, University of Rochester.

[42] Moore, M. J. and M. J. Roche, 2006, Solving Exchange Rate Puzzles with neither Sticky Prices nor Trade Costs, Working paper, Queen's University Belfast and National University of Ireland.

[43] Obstfeld, M. and K. Rogoff, 2000, The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?", NBER Working Paper No. 7777.

[44] Pavlova, A. and R. Rigobon, 2007, Asset Prices and Exchange Rates, *Review of Financial Studies* 20, 1139-1180.

[45] Pesenti, P. and E. van Wincoop, Can Nontradeables Generate Substantial Home Bias?, *Journal of Money, Credit and Banking* 34, 25-50.

[46] Santos, T. and P. Veronesi, 2006, Habit Formation, the Cross Section of Stock Returns and the Cash-Flow Risk Puzzle, Working paper, Columbia University and University of Chicago.

[47] Sercu, P., 1980, A generalization of the International Asset Pricing Model, *Revue de l'Association Francaise de Finance* 1, 91-135.

[48] Serrat, A., 2001, A Dynamic Equilibrium Model of International Portfolio Holdings, *Econometrica* 69, 1467-1489.

[49] Shore, S. H. and J. S. White, 2006, External Habit Formation and the Home Bias Puzzle, Working paper, University of Pennsylvania and University of Illinois, Urbana-Champaign.

[50] Solnik, B., 1974a, Why not Diversify Internationally Rather Than Domestically?, *Financial Analysts Journal* 30, 48-54.

[51] Solnik, B., 1974b, An Equilibrium Model of the International Capital Market, *Journal of Economic Theory* 8, 500-524.

[52] Stathopoulos, A., 2011, Asset Prices and Risk Sharing in Open Economies, Working paper, University of Southern California.

[53] Stockman, A. C. and H. Dellas, 1989, International Portfolio Nondiversification and Exchange Rate Variability, *Journal of International Economics* 26, 271-289.

[54] Stulz, R., 1981a, On the Effects of Barriers to International Investment, *Journal of Finance* 36, 923-934.

[55] Stulz, R., 1981b, A Model of International Asset Pricing, *Journal of Financial Economics* 9, 383-406.

[56] Tesar, L. L., 1993, International Risk Sharing and Non-traded Goods, *Journal of International Economics* 35, 1803-1832.

[57] Tesar, L. L. and I. M. Werner, 1995, Home Bias and High Turnover, *Journal of International Money and Finance* 14, 467-492.

[58] Uppal, R., 1993, A General Equilibrium Model of International Portfolio Choice, *Journal of Finance* 48, 529-553.

[59] van Nieuwerburgh, S. and L. Veldkamp, 2008, Information Immobility and the Home Bias Puzzle, Working paper, New York University.

[60] van Wincoop, E. and F. E. Warnock, 2010, Can trade costs in goods explain home bias in assets?, Journal of International Money and Finance 29,1108-1123.

[61] Verdelhan, A., 2010, A Habit-Based Explanation of the Exchange Rate Risk Premium, *Journal of Finance* 65, 123-146.

[62] Zapatero, F., 1995, Equilibrium Asset Prices and Exchange Rates, *Journal of Economic Dynamics and Control* 19, 787-811.

A Appendix

A.1 Equilibrium

A.1.1 Quantities and Prices

The first order conditions (FOCs) of agent i are:

$$e^{-\rho t} a^{i,j} \frac{G_t^i}{X_t^{i,j}} = \frac{1}{\lambda^i} \frac{\Lambda_t}{\Lambda_0} Q_t^j, \text{ for all } j \tag{10}$$

where $\frac{1}{\lambda^i}$ is the Lagrange multiplier associated with the budget constraint of agent i holding with equality.

In equilibrium, all goods markets clear:

$$\sum_{k=0}^{n} X_t^{i,j} = \tilde{X}_t^j, \text{ for all } j$$

Combining the FOCs with the market clearing conditions, we get the equilibrium consumption allocation.

Note that (10) can be rearranged as follows:

$$\lambda^i e^{-\rho t} a^{i,j} G_t^i = \frac{\Lambda_t}{\Lambda_0} X_t^{i,j} Q_t^j$$

and, summing over all goods, we have:

$$e^{-\rho t} \lambda^i G_t^i \sum_{j=0}^{n} a^{i,j} = \frac{\Lambda_t}{\Lambda_0} \sum_{j=0}^{n} \left(X_t^{i,j} Q_t^j \right)$$

or

$$e^{-\rho t} \lambda^i G_t^i = \frac{\Lambda_t}{\Lambda_0} C_t^i P_t^i$$

so the numeraire state-price density is:

$$\frac{\Lambda_t}{\Lambda_0} = e^{-\rho t} \lambda^i \frac{G_t^i}{C_t^i P_t^i}$$

The expression above is true for all countries i. Since the home consumption basket is the numeraire, we have $P_t^0 = 1$ for all t, so the numeraire state-price density, Λ, is:

$$\frac{\Lambda_t}{\Lambda_0} = e^{-\rho t} \lambda^0 \frac{G_t^0}{C_t^0} \tag{11}$$

The risk-free rate r^f and the market price of risk $\boldsymbol{\eta}$ satisfy (4), so an application of Itô's lemma on (11) yields

$$r_t^f = \rho + E_t\left(\frac{dC_t^0}{C_t^0}\right) + \varphi\left(\frac{G_t^0 - \bar{G}}{G_t^0}\right) - \left(1 + \delta\left(\frac{G_t^0 - l}{G_t^0}\right)\right)\boldsymbol{\sigma}_t^{0,C\prime}\boldsymbol{\sigma}_t^{0,C}$$

and

$$\boldsymbol{\eta}_t = \left(1 + \delta\left(\frac{G_t^0 - l}{G_t^0}\right)\right)\boldsymbol{\sigma}_t^{0,C}$$

Relative prices are also determined using (10); for any country i, we can use the expressions for good j and good j' and get:

$$\frac{Q_t^j}{Q_t^{j'}} = \frac{a^{i,j}X_t^{i,j'}}{a^{i,j'}X_t^{i,j}}$$

Using the equilibrium sharing rule in the expression above, we derive (5). After imposing the normalization restriction $P_t = 1$ for all t, (5) yields (6) and (7).

To calculate the Lagrange multipliers $\frac{1}{\lambda^i}$, we substitute equilibrium quantities and prices in the static budget constraint of agent i (holding with equality). After some algebra, we get:

$$\lambda^i\left(\varphi\bar{G} + \rho G_0^i\right) = \sum_{k=0}^{n} a^{k,i}\lambda^k\left(\varphi\bar{G} + \rho G_0^k\right)$$

This system of equations has solutions of the form

$$\frac{\lambda^i}{\lambda^0} = b^i\frac{\varphi\bar{G} + \rho G_0^0}{\varphi\bar{G} + \rho G_0^i}$$

where the vector $\mathbf{b} = [b^1, b^2, ..., b^n]'$ is the unique solution of

$$\mathbf{b} = \begin{bmatrix} a^{0,1} & a^{1,1} & ... & a^{n,1} \\ a^{0,2} & a^{1,2} & ... & a^{n,2} \\ ... & ... & ... & ... \\ a^{0,n} & a^{1,n} & ... & a^{n,n} \end{bmatrix}\begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix}$$

The budget constraint determines only the ratios $\frac{\lambda^i}{\lambda^0}$. However, note that, for $t = 0$, (11) gives:

$$\widetilde{X}_0 = \sum_{k=0}^{n} a^{k,0}\lambda^k G_0^k$$

Substituting for the λ^i, we can solve for λ^0 in terms of the known initial conditions $\widetilde{X}_0$ and G_0^i,

$i = 0, 1, ..., n$:

$$\lambda^0 = \frac{\frac{1}{\varphi \bar{G} + \rho G_0^0}}{\frac{a^{0,0} G_0^0}{\varphi \bar{G} + \rho G_0^0} + \sum_{k=1}^{n} \frac{a^{k,0} b^k G_0^k}{\varphi \bar{G} + \rho G_0^k}} \widetilde{X}_0$$

A.1.2 Equilibrium consumption processes

Since the vector of equilibrium consumption $C = [C^0, ..., C^n]'$ is a function of family of the state variables G^i, which, in turn, are driven by consumption growth shocks, we need to solve for the corresponding fixed point that satisfies both the equilibrium consumption allocations and the law of motion for G^i, $i = 0, ..., n$. By definition, we have:

$$C^i \equiv \left(\prod_{j=0}^{n} \left(X^{i,j} \right)^{a^{i,j}} \right)$$

so, applying Itô's lemma and equating the diffusion terms, we get after some algebra:

$$\psi_t^{i,i} \boldsymbol{\sigma}_t^{i,C} + \sum_{i' \neq i} \psi_t^{i,i'} \boldsymbol{\sigma}_t^{i',C} = \sum_{j=0}^{n} a^{i,j} \boldsymbol{\sigma}_t^{j,X}$$

where

$$\psi_t^{i,i} \equiv 1 + \left(1 - \sum_{j=0}^{n} \frac{a^{i,j} a^{i,j} \lambda^i G_t^i}{g_t^j} \right) \delta \left(\frac{G_t^i - l}{G_t^i} \right)$$

$$\psi_t^{i,i'} \equiv - \left(\sum_{j=0}^{n} \frac{a^{i,j} a^{i',j} \lambda^{i'} G_t^{i'}}{g_t^j} \right) \delta \left(\frac{G_t^{i'} - l}{G_t^{i'}} \right)$$

This is a system of $m(n+1)$ equations and an equal number of unknowns. We can rewrite this in compact form as:

$$\Psi_t \boldsymbol{\sigma}_t^C = \mathbf{A} \boldsymbol{\sigma}_t^X$$

where

$$\Psi_t = [\psi_{i,j}] = \psi_t^{i-1,j-1}$$

$\boldsymbol{\sigma}_t^C$ is the $(n+1)xm$ matrix

$$\boldsymbol{\sigma}_t^C = \begin{bmatrix} \boldsymbol{\sigma}_t^{0,C\prime} \\ ... \\ \boldsymbol{\sigma}_t^{n,C\prime} \end{bmatrix}$$

and $\boldsymbol{\sigma}_t^X$ is the $(n+1)xm$ matrix

$$\boldsymbol{\sigma}_t^X = \begin{bmatrix} \boldsymbol{\sigma}_t^{0,X\prime} \\ ... \\ \boldsymbol{\sigma}_t^{n,X\prime} \end{bmatrix}$$

It follows that the consumption growth conditional volatilities satisfy:

$$\boldsymbol{\sigma}_t^C = \left(\Psi_t^{-1}\mathbf{A}\right)\boldsymbol{\sigma}_t^X$$

We can similarly derive expected consumption growth $E_t\left(\frac{dC_t^i}{C_t^i}\right)$ for $i = 0, ..., n$ by matching the drift terms.

A.2 Proofs

Proof of Proposition 1

The wealth of the agent i satisfies:

$$W_t^i = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} C_s^i P_s^i ds\right] = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t}\left(\lambda^i G_s^i \frac{e^{-\rho s}}{\frac{\Lambda_s}{\Lambda_0}}\right) ds\right] = \lambda^i \frac{\rho G_t^i + \varphi \bar{G}}{\rho(\rho+\varphi)} e^{-\rho t}\frac{\Lambda_0}{\Lambda_t}$$

so, applying Itô's lemma, we get

$$\frac{dW_t^i}{W_t^i} = [\cdot]dt + \left(\boldsymbol{\eta}_t + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi\bar{G}}\right)\boldsymbol{\sigma}_t^{i,G}\right)' \mathbf{dB}_t$$

Considering the intertemporal budget constraint of the domestic agent:

$$\frac{dW_t^i}{W_t^i} = \mathbf{x}_t^{i\prime}\left(\boldsymbol{\mu}_t^e dt + \boldsymbol{\sigma}_t \mathbf{dB}_t\right) + r_t^f dt - \frac{C_t^i P_t^i}{W_t^i} dt$$

and matching diffusions, we get

$$\mathbf{x}_t^{i\prime}\boldsymbol{\sigma}_t = \left(\boldsymbol{\eta}_t + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi\bar{G}}\right)\boldsymbol{\sigma}_t^{i,G}\right)'$$

Solving for the portfolio weights $\mathbf{x}_t$, we get:

$$\mathbf{x}_t^i = \left(\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\eta}_t + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi\bar{G}}\right)\left(\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\sigma}_t^{i,G}$$

Recall that $\boldsymbol{\eta}_t = \boldsymbol{\sigma}_t^{-1}\boldsymbol{\mu}_t^e$, so

$$\mathbf{x}_t^i = \left(\boldsymbol{\sigma}_t\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\mu}_t^e + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi\bar{G}}\right)\left(\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\sigma}_t^{i,G}$$

Proof of Proposition 2

Aggregate global wealth W_t^A is defined as the discounted present value of the aggregate global endowment

$$W_t^A = E_t \left[\int_t^{\infty} \frac{\Lambda_s}{\Lambda_t} \sum_{k=0}^{n} \left(\widetilde{X}_t^k Q_t^k \right) ds \right]$$

and, in equilibrium, is equal to the sum of the wealth of all individual countries

$$W_t^A = E_t \left[\int_t^{\infty} \frac{\Lambda_s}{\Lambda_t} \sum_{k=0}^{n} \left(C_s^k P_s^k \right) ds \right] = \sum_{k=0}^{n} W_t^k$$

due to the market clearing condition

$$\sum_{k=0}^{n} \widetilde{X}_t^k Q_t^k = \sum_{k=0}^{n} C_s^k P_s^k$$

Solving, we get:

$$W_t^A = e^{-\rho t} \frac{\Lambda_0}{\Lambda_t} \frac{1}{\rho(\rho+\varphi)} \sum_{k=0}^{n} \lambda^k \left(\rho G_t^k + \varphi \bar{G} \right)$$

so, applying Itô's lemma, we have:

$$\frac{dW_t^A}{W_t^A} = [\cdot]dt - \frac{d\Lambda_t}{\Lambda_t} + \sum_{i=0}^{n} \left(\frac{\rho \lambda^i G_t^i}{\sum_{k=0}^{n} \lambda^k \left(\rho G_t^k + \varphi \bar{G} \right)} \right) \frac{dG_t^i}{G_t^i}$$

or

$$\frac{d\Lambda_t}{\Lambda_t} = [\cdot]dt - \frac{dW_t^A}{W_t^A} + \sum_{i=0}^{n} \left(\frac{\rho \lambda^i G_t^i}{\sum_{k=0}^{n} \lambda^k \left(\rho G_t^k + \varphi \bar{G} \right)} \right) \frac{dG_t^i}{G_t^i}$$

The risk premium for any asset s is given by:

$$E_t\left(dR_t^{s,e}\right) = -E_t\left(dR_t^{s,e} \frac{d\Lambda_t}{\Lambda_t} \right)$$

so, substituting for $\frac{d\Lambda_t}{\Lambda_t}$, we get the ICAPM reported in Proposition 2.

Proof of Proposition 3

The price of claim i, V^i, satisfies:

$$V_t^i = E_t\left[\int_t^{\infty}\frac{\Lambda_s}{\Lambda_t}\left(\tilde{X}_s^i Q_s^i\right) ds\right] = \frac{1}{\rho}\frac{\rho g_t^i + \varphi\bar{g}^i}{(\rho+\varphi)\, g_t^i}\tilde{X}_t^i Q_t^i = \frac{1}{\rho}\frac{\rho g_t^i + \varphi\bar{g}^i}{(\rho+\varphi)}e^{-\rho t}\frac{\Lambda_0}{\Lambda_t}$$

Therefore, the numeraire return of asset i has diffusion process

$$\boldsymbol{\sigma}_t^{i,R} = \rho\sum_{k=0}^{n} a^{k,i}\lambda^k G_t^k \boldsymbol{\sigma}_t^{k,G} + \boldsymbol{\eta}_t$$

and the asset diffusion matrix is

$$\boldsymbol{\sigma}_t = \begin{bmatrix} \boldsymbol{\sigma}_t^{0,R\prime} \\ \dots \\ \boldsymbol{\sigma}_t^{n,R\prime} \end{bmatrix}$$

If $\mathbf{A}$ is invertible, $\boldsymbol{\sigma}_t$ is also invertible, so the equilibrium portfolio of agent i is unique and given by

$$\mathbf{x}_t^i = \left(\boldsymbol{\sigma}_t\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\mu}_t^e + \left(\frac{\rho G_t^i}{\rho G_t^i + \varphi\bar{G}}\right)\left(\boldsymbol{\sigma}_t'\right)^{-1}\boldsymbol{\sigma}_t^{i,G}$$

where

$$\boldsymbol{\mu}_t^e = \boldsymbol{\sigma}_t\boldsymbol{\eta}_t$$

I guess that the solution is a fixed portfolio that comprises $\theta^{i,j}$ shares of asset j. If every agent has a fixed portfolio, there is no asset trade at any $t > 0$, so the equilibrium portfolio of agent i should finance her consumption expenditure at all times and states. Therefore, it must hold that:

$$\sum_{j=0}^{n}\theta^{i,j}\widetilde{X}_t^j Q_t^j = C_t^i P_t^i$$

or, after some algebra:

$$\sum_{j=0}^{n}\sum_{k=0}^{n}\theta^{i,j}a^{k,j}\lambda^k G_t^k = \lambda^i G_t^i$$

Since each country's conditional risk aversion is stochastic, the expression above holds almost surely if and only if

$$\sum_{j=0}^{n}\theta^{i,j}a^{i,j} = 1 \text{ and } \sum_{j=0}^{n}\theta^{i,j}a^{k,j} = 0 \text{ for all } k \neq i \tag{12}$$

or, using matrix notation:

$$\mathbf{A}\boldsymbol{\Theta}^{\top} = \mathbf{I}_{n+1}$$

so, given that $\mathbf{A}$ is invertible, the fixed portfolios satisfy

$$\mathbf{\Theta} = \left(\mathbf{A}^{\top}\right)^{-1} = \left(\mathbf{A}^{-1}\right)^{\top}$$

Since the problem has a unique solution, the fixed portfolio solution is the only solution.

Proof of Proposition 4

Following Bennett and Young (1999), we can recognize that (12) describes a problem with a Stolper-Samuelson structure. Thus, the results (a) and (b) in Proposition 4 follow directly from results (b) and (d) of Proposition 3 in Bennett and Young (1999), which, in turn, follow from Jones, Marjit and Mitra (1993) and Mitra and Jones (1992), respectively.

Table 1

Calibration Parameters (annualized)

Endowment parameter	Symbol	Value
Endowment growth rate	μ	0.015
Endowment growth volatility	σ	0.015
Preference parameter	Symbol	Value
Subjective rate of time preference	ρ	0.04
Speed of G mean reversion	φ	0.12
G sensitivity to consumption growth shocks	δ	79.39
Lower bound of G	l	20
Steady-state value of G	$\bar{G}$	34

Table 2

Simulated moments: prices and quantities

Moment	$a = 0.51$	$a = 0.7$	$a = 0.95$	$a = 0.99$
Domestic consumption growth st. dev.	1.06%	1.07%	1.15%	1.31%
Foreign consumption growth st. dev.	1.06%	1.07%	1.16%	1.31%
Consumption growth corr.	1.000	0.954	0.709	0.399
Exchange rate change st. dev.	0.04%	0.99%	8.40%	26.36%
Conditional risk aversion correlation	1.000	0.999	0.921	0.535
Domestic excess return mean	4.77%	4.77%	4.79%	5.33%
Domestic excess return st. dev.	18.61%	18.61%	18.71%	19.85%
Domestic Sharpe ratio	0.269	0.268	0.268	0.280
Foreign excess return mean	4.77%	4.77%	4.85%	6.13%
Foreign excess return st. dev.	18.61%	18.61%	19.07%	23.53%
Foreign Sharpe ratio	0.269	0.268	0.266	0.274
Excess return corr.	1.000	1.000	0.995	0.949

Simulated annualized moments. I simulate 1,000 sample paths of the model economy, with each path consisting of 320 quarterly observations. Of the 320 observations, the first 120 (30 years) are discarded to reduce the dependence on initial conditions. For each of the moments of interest, Table 3 presents the sample average across the 1,000 simulations.

Table 3

Simulated moments: determinants of asset returns

Moment	$a = 0.51$	$a = 0.7$	$a = 0.95$	$a = 0.99$
λ^{WA} mean	3.47%	3.47%	3.56%	4.48%
λ^{G^0} mean	-1.16%	-1.16%	-1.17%	-1.38%
λ^{G^1} mean	-1.16%	-1.16%	-1.17%	-1.36%
Domestic β^{WA} mean	0.98	0.98	0.98	0.91
Foreign β^{WA} mean	0.98	0.98	0.99	1.08
Domestic β^{G^0} mean	-0.74	-0.74	-0.74	-0.74
Foreign β^{G^0} mean	-0.74	-0.74	-0.76	-0.84
Domestic β^{G^1} mean	-0.74	-0.74	-0.70	-0.45
Foreign β^{G^1} mean	-0.74	-0.74	-0.69	-0.36

All moments are annualized.

Table 4

Correlation matrix of the innovations of priced factors

$a = 0.51$

Factor	W^A	G^0	G^1
W^A	1.00	-0.98	-0.98
G^0		1.00	1.00
G^1			1.00

$a = 0.7$

Factor	W^A	G^0	G^1
W^A	1.00	-0.98	-0.98
G^0		1.00	1.00
G^1			1.00

$a = 0.95$

Factor	W^A	G^0	G^1
W^A	1.00	-0.98	-0.91
G^0		1.00	0.95
G^1			1.00

$a = 0.99$

Factor	W^A	G^0	G^1
W^A	1.00	-0.97	-0.47
G^0		1.00	0.61
G^1			1.00

Table 5

Simulated moments: domestic portfolio

Moment	$a = 0.51$	$a = 0.7$	$a = 0.95$	$a = 0.99$
Total portfolio				
θ_1	25.50	1.75	1.06	1.01
θ_2	-24.50	-0.75	-0.06	-0.01
β^{WA}	0.98	0.98	0.97	0.91
β^{G^0}	-0.74	-0.74	-0.74	-0.74
β^{G^1}	-0.74	-0.74	-0.70	-0.45
Mean-variance portfolio				
θ_1	32.80	2.25	1.36	1.30
θ_2	-31.51	-0.96	-0.07	-0.01
β^{WA}	0.98	0.98	0.96	0.77
β^{G^0}	-0.74	-0.74	-0.74	-0.65
β^{G^1}	-0.74	-0.74	-0.70	-0.43
Hedging portfolio				
θ_1	-7.30	-0.50	-0.30	-0.29
θ_2	7.01	0.21	0.02	0.00
β^{WA}	0.98	0.98	1.01	1.21
β^{G^0}	-0.74	-0.74	-0.76	-0.94
β^{G^1}	-0.74	-0.74	-0.69	-0.38

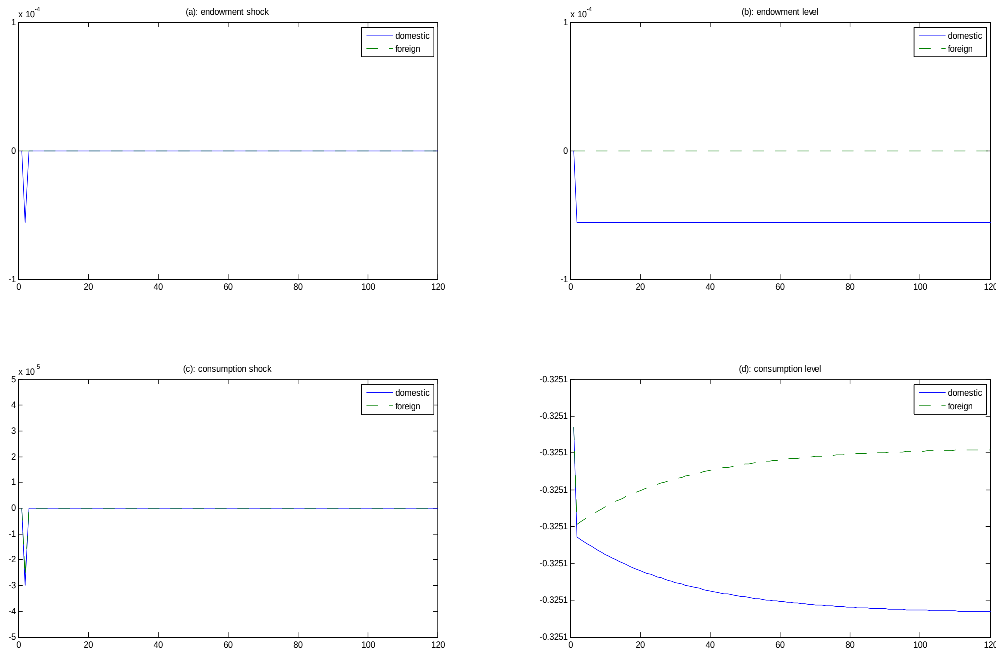
Table 6

Simulated moments: foreign portfolio

Moments	$a = 0.51$	$a = 0.7$	$a = 0.95$	$a = 0.99$
Total portfolio				
θ_1	-24.50	-0.75	-0.06	-0.01
θ_2	25.50	1.75	1.06	1.01
β^{WA}	0.98	0.98	1.00	1.08
β^{G^0}	-0.74	-0.74	-0.76	-0.85
β^{G^1}	-0.74	-0.74	-0.69	-0.36
Mean-variance portfolio				
θ_1	32.80	2.25	1.36	1.31
θ_2	-31.51	-0.96	-0.07	-0.01
β^{WA}	0.98	0.98	0.96	0.77
β^{G^0}	-0.74	-0.74	-0.74	-0.65
β^{G^1}	-0.74	-0.74	-0.70	-0.43
Hedging portfolio				
θ_1	-57.30	-3.00	-1.41	-1.32
θ_2	57.01	2.71	1.13	1.02
β^{WA}	0.98	0.98	1.03	1.38
β^{G^0}	-0.74	-0.74	-0.78	-1.04
β^{G^1}	-0.74	-0.74	-0.68	-0.29

Figure 1

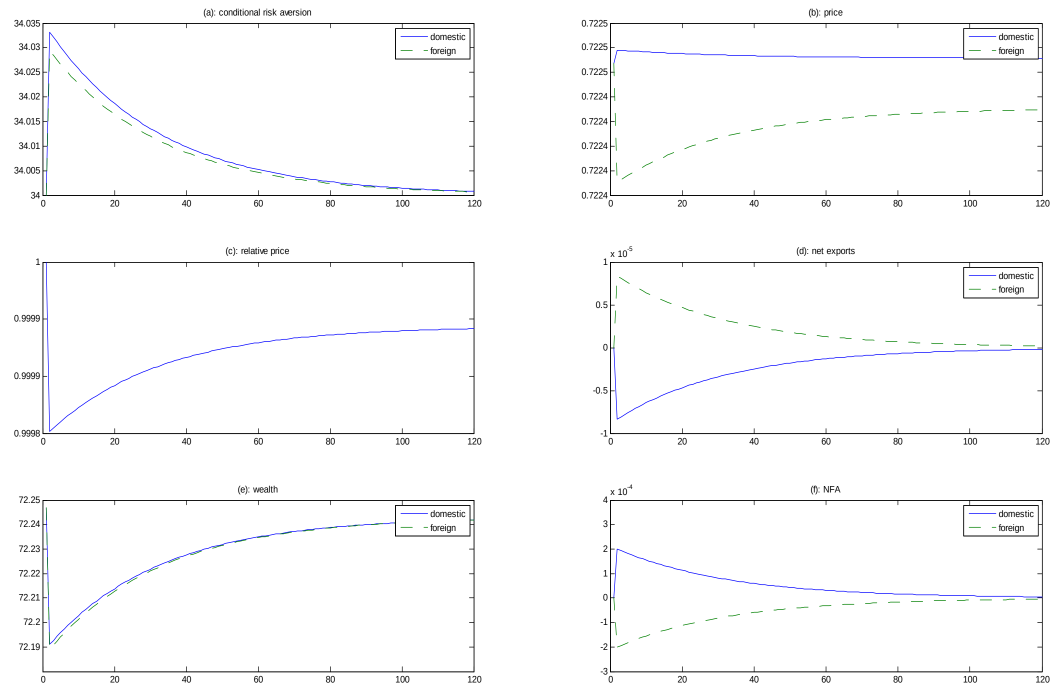
Impulse response functions: endowment and consumption



The horizontal axis measures time (in quarters) and the vertical axis the value of the moment of interest.

Figure 2

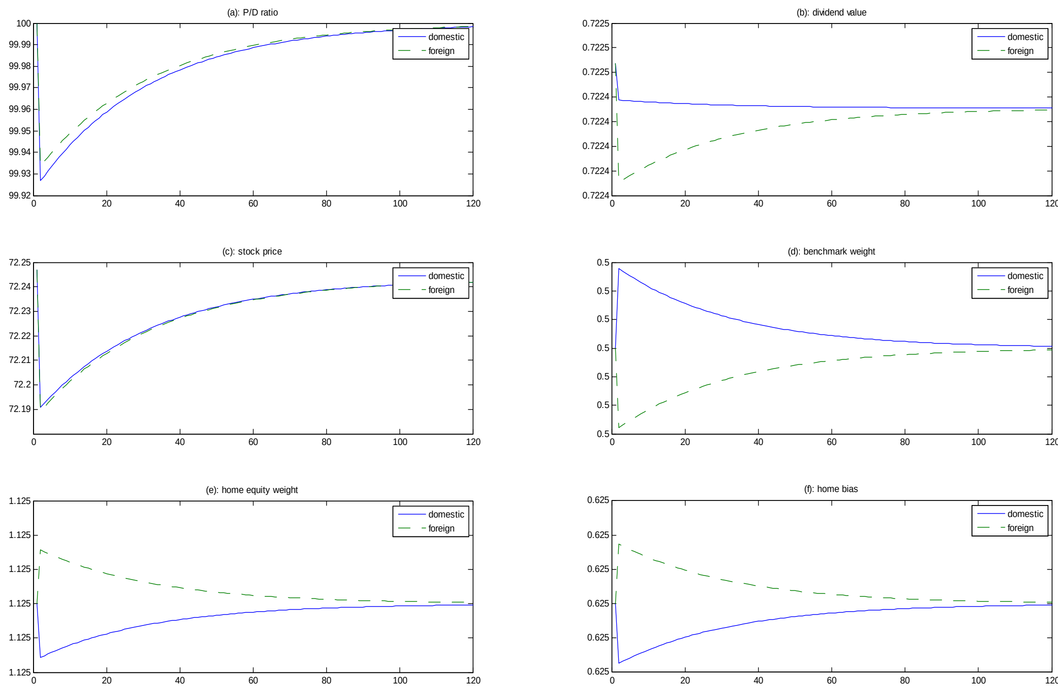
Impulse response functions: prices, wealth and the external sector



The horizontal axis measures time (in quarters) and the vertical axis the value of the moment of interest.

Figure 3

Impulse response functions: asset prices and portfolios



The horizontal axis measures time (in quarters) and the vertical axis the value of the moment of interest.