Experiments On The Lucas Asset Pricing Model

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Abstract

This paper reports on experimental tests of the Lucas asset pricing model with heterogeneous agents and time-varying endowment streams. In order to emulate key features of the model (perishability of consumption, stationarity, infinite horizon), a novel experimental design was required. The experimental evidence provides broad support for cross-sectional and intertemporal pricing restrictions. However, asset prices are significantly more volatile than fundamentals and returns are less predictable than theory suggests. Despite this, allocations are nearly Pareto optimal. The paper argues that this is the result of participants expectations about future prices, which are at odds with the predictions of the Lucas model but are nonetheless almost self-fulfilling. These findings suggest that excessive volatility of prices may not be indicative of welfare losses.[1]

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1 Introduction

For over thirty years, the Lucas asset pricing model (Lucas, 1978) has served as the basic platform for research on dynamic asset pricing and business cycles. The Lucas model provides both cross-sectional and time-series predictions and links the two. The central cross-sectional prediction is consistent with the central predictions of static models: only aggregate risk is priced. For instance, in CAPM (the Capital Asset Pricing Model), the quintessential static model, aggregate risk is measured by the return on the market portfolio, and the price of an asset is a decreasing function (equivalently, the return on the asset is an increasing function) of the covariance of the return on the asset with the return on the market portfolio; i.e., with the "beta" of the asset. In the Lucas model, aggregate risk is a function of aggregate consumption, and prices decrease with "consumption beta." The central time-series predictions of the Lucas model are that asset price changes are correlated with economic fundamentals (aggregate consumption growth) and that there is a strong connection between the volatility of asset prices and the volatility of economic fundamentals. The most important consequence of this prediction is that asset prices need *not* follow a martingale (with respect to the true probabilities) and the price of an asset need *not* be the discounted present value of its expected future dividends (with respect to the true probabilities). These contradict the strictest interpretation of the Efficient Markets Hypothesis (Samuelson, 1973; Malkiel, 1999; Fama, 1991).[2]

The most familiar version of the Lucas model assumes a representative agent, whose holdings consist of the aggregate endowment of securities and whose consumption is the aggregate flow of the (perishable) dividends. Asset prices are constructed as shadow prices with respect to which the representative agent would have no incentive to trade. A crucial characteristic of the Lucas model is that it assumes the representative agent has rational expectations, and so correctly forecasts both future prices and his own future decisions. The multi-agent version of the Lucas model that we study here assumes that all agents have rational expectations, and so correctly forecast both future prices and their own future decisions, and that prices and allocations form an equilibrium, and in particular that allocations are Pareto optimal so that each agents optimally smooths consumption over time and states of nature. Without question, the empirical relevance of the Lucas model hinges on the viability of these underlying assumptions.

[2]Because prices do not admit arbitrage, the Fundamental Theorem of Asset Pricing implies the existence of a probability measure typically different from the true probability measure with respect to which prices *do* follow a martingale.

This paper reports on experimental laboratory tests of the Lucas model. The nature of the Lucas model presents a number of unusual challenges for the laboratory environment: an infinite time horizon, stationary primitives, perishable goods and consumption smoothing. Our experiments address these challenges in novel ways. We emulate an infinite horizon through a random ending time and induce stationarity by using an ending protocol that makes asset payoffs in the last period equal to continuation values of these assets. We emulate perishability by imposing forfeiture of participants' cash holdings (the consumption good) at the end of every non-terminal period: cash held at the end of the terminal period – and only then – is "consumed" and so constitutes the subject's payoff from the session. The desire to smooth consumption is a consequence of this perishability and the risk aversion that subjects bring to the laboratory.[3]

In our laboratory economy, there were two securities: a (consol) "Bond" whose dividend each period is fixed and a "Tree" whose dividend is stochastic, taking one of two values, according to a known process. Our economy is populated by agents whose initial asset endowments and (time-varying) income streams differ. Given these parameters, the asset market is potentially dynamically complete; in particular, it is possible for the economy to achieve Pareto optimal final allocations,

We find experimental evidence that provides broad support for both the theoretical cross-sectional and intertemporal pricing predictions, but with notable differences. On the one hand, as theory predicts, asset prices co-move with economic fundamentals and this co-movement is stronger when cross-sectional price differences are greater (holding consumption beta constant). On the other hand, asset prices are significantly more volatile than fundamentals and returns are less predictable than theory suggests. Indeed, for the consol bond, the noise in the price data is so great that we cannot reject the null that price changes are random. Despite this divergence from the theoretical predictions, we find a great deal of consumption smoothing, suggesting that allocations are nearly Pareto optimal. We suggest that this divergence arises from subjects' forecasts about future asset prices, which appear to be vastly at odds with the predictions of the Lucas model, yet *almost self-fulfilling*. Of course asset price forecasts that are *exactly* self-fulfilling must coincide with the prices predicted by the Lucas model — this is just the definition of equilibrium in the model. Surprisingly, however, asset price forecasts can be almost self-fulfilling and yet far from the predictions of the Lucas model. Among other things, these findings suggest that excessive volatility of prices

[3]That experimental subjects display substantial risk aversion, even for the relatively small stakes of laboratory experiments, is well-documented; see Bossaerts and Zame (2008) for example.

may not be indicative of welfare losses.

The remainder of this paper is organized as follows. The next section elaborates on the difference between experimental tests of the Lucas model and traditional econometric tests on historical field data, and highlights the difficulties when studying the model in the laboratory. Section 3 presents the Lucas model within the framework of the laboratory economy we created. Section 4 provides details of the experimental setup. Results are provided in Section 5. Section 6 discusses potential causes behind the excessive volatility of asset prices observed in the laboratory markets. Section 7 concludes.

2 Test Of The Lucas Model In Experiments Vs. On Historical Field Data

Analysis of the Lucas model, both empirical and theoretical, has traditionally focused on the "stochastic Euler equations" that deliver the equilibrium pricing restrictions (Cochrane, 2001). These equations derive from the first-order conditions of the consumption/investment optimization problem of the representative agent in the economy. Empirical tests of the stochastic Euler equations on historical field data have been disappointing. Starting with Mehra and Prescott (1985), the fit has generally been considered to be poor. Attempts to improve model fit have concentrated on the auxiliary assumptions rather than on its primitives. Some authors have altered the original preference specification (time-separable expected utility) to allow for, among others, time-nonseparable utility (Epstein and Zin, 1991), loss aversion (Barberis et al., 2001), or utility functions that assign an explicit role to an important component of human behavior, namely, emotions (such as disappointment; Routledge and Zin (2011)). Others have looked at measurement problems, extending the scope of aggregate consumption series in the early empirical analysis (Hansen and Singleton, 1983), to include nondurable goods (Dunn and Singleton, 1986), or acknowledging the dual role of certain goods as providing consumption as well as collateral services (Lustig and Nieuwerburgh, 2005). Included in this category should be the long-run risks model of (Bansal and Yaron, 2004) because it is based on difficulty in recovering an alleged low-frequency component in consumption (growth).

Here, we investigate the Lucas model experimentally. We focus on the primitives of the model, rather than merely trying to find an instantiation of the stochastic Euler equations that best fits a given series of price (and aggregate consumption) data. An experimental study of the Lucas model introduces new challenges, however. Foremost, the model assumes the existence of a representative agent. Short of assuming that everyone is identical (which is not only counterfactual, it would also preclude the very trade on which the success of financial markets experiments builds), one needs to structure the economy in the right way, so as to facilitate Pareto improvements. (When allocations are Pareto optimal, the existence of a representative agent is assured.)

One way to facilitate Pareto efficiency would be to organize a complete set of markets, by creating as many securities (with independent payoffs) as there are states. (Allocations in the resulting walrasian equilibrium are guaranteed to be Pareto optimal.) The empirical issue would then be limited to whether the walrasian equilibrium emerges. But in any realistic setting – and in the experiments that we report on here – one cannot have a complete set of markets; there are just too many possible states. The alternative is to organize markets in a way that they could be complete merely in the *dynamic sense* (Duffie and Huang, 1985). This is what we opted for. It should be emphasized that emergence of optimal allocations is not trivial even if markets are dynamically complete. Participants would have to resort to the complex investment policies that exhibit the *hedging* feature at the core of the modern theory of derivatives analysis (Black and Scholes, 1973; Merton, 1973a) and dynamic asset pricing (Merton, 1973b). Furthermore, investors would need to have correct anticipation of (equilibrium) price processes, so that a Radner perfect foresight equilibrium (Radner, 1972) becomes possible.

In the laboratory, we will be able to verify all aspects of the (Radner) equilibrium, and not just whether prices satisfy some set of stochastic Euler equation. It would be hard to do so with historical data from the field. The problem is that in the field we lack crucial structural information, such as aggregate supplies of securities, beliefs about dividend processes, or private income flows. (This is related to the Roll critique (Roll, 1977).) By contrast, laboratory experiments provide control over and knowledge of all the important variables.

Additional challenges need to be addressed before one can test the Lucas model in the laboratory. Specifically, the model assumes that the world is stationary, that it continues forever, and that investment demands are driven primarily by the desire to smooth consumption. The infinite horizon is easy to deal with: as in Camerer and Weigelt (1996), one could introduce a stochastic ending time. The finite experiment duration, however, makes stationarity particularly difficult to induce, as beliefs would necessarily change when time approaches the officially announced termination of the experiment. Likewise, it is difficult to imagine that participants care about the timing of their consumption (earnings) across periods during the course of an experiment, and hence merely spreading payment over time does not induce demand for smoothing. We introduced novel features to the standard design of an intertemporal asset pricing experiment to overcome these challenges.

Our experiment is related to that of Crockett and Duffy (2010). There are at least two major differences, however. First, we did not induce demand for smoothing by means of nonlinearities in take-home pay as a function of period earnings, but induced it as the result of novel experimental design. The predicted pricing patterns are therefore driven solely by the uncertainty of the dividends of (one of) the assets, exactly as in the original Lucas model, unconfounded by nonlinearities. Second, to avoid endgame effects, and hence, to ensure stationarity, we altered the design in a way that was consistent with the theory. The aims of the two studies were different, though. In Crockett and Duffy (2010), the goal was to show that asset price bubbles did not emerge once Lucas-style consumption smoothing was introduced. The goal here was, as mentioned before, to test the primitives of the Lucas model.

3 The Lucas Asset Pricing Model

We envisage an environment with minimal complexity yet one that generates a rich set of predictions about prices across time and allocations across types of investors. Perhaps most importantly, the environment is such that trading is necessary in *each period*. Following Bossaerts and Zame (2006), we wanted to avoid a situation (as in Judd et al. (2003)), where theory predicts that trade will take place only once. When bringing the setting to the laboratory, it would indeed be rather awkward to give subjects the opportunity to trade every period while the theory predicts that they should not![4]

Our environment generates the original Lucas model, which is stationary in levels (dividends, and hence, prices). This is in contrast to the models that have informed empirical research of historical field data. Starting with Mehra and Prescott (1985), these are stationary in growth. Level stationarity is easier to implement in the laboratory, and thus is preferred for an experiment that already poses many challenges in the absence of growth. While there is no substantive difference between the level and growth versions of the Lucas model (e.g., in both cases, prices move with fundamentals), the reader is cautioned that results are not isomorphic. For instance, when

[4] Crockett and Duffy (2010) confirm that it is crucial to give subjects a reason to trade *every* period in order to avoid bubbles in laboratory studies of dynamic asset pricing.

dividend levels are independently and identically distributed (i.i.d.), dividend growth is *not* (dividend growth is expected to be high when dividends are low, and low when dividends are low).

We consider a stationary, infinite horizon economy in which infinite-lived agents with time-separable expected utility are initially allocated two types of assets: (i) a *Tree* that pays a stochastic dividend of \$1 or \$0 every period, each with 50% chance, independent of past outcomes, and (ii) a (consol) *Bond* that always pays \$0.50.

There is an *equal number* of two types of agents. Type I agents receive income of \$15 in even periods (2, 4, 6,...), while those of Type II receive income of \$15 in odd periods. As such, total (economy-wide) income is constant over time. Before period 1, Type I agents are endowed with 10 Trees and no Bonds; Type II agents start with 0 Trees and 10 Bonds.

Assets pay dividends $d_{k,t}$ ($k \in \{\text{Tree}, \text{Bond}\}$) *before* period t ($t = 1, 2, ...$) starts. At that point, agents also receive their income, $y_{i,t}$ ($i = 1, ..., I$), as prescribed above. As dividends and income are fungible, we refer to them as *cash*, and cash is perishable. In what follows, $c_{i,t}$ denotes the cash available to agent i in period t . Agents have *common* time-separable utility for cash:

$$U_i(\{c_{i,t}\}_{t=1}^{\infty}) = E\left\{\sum_{t=1}^{\infty} \beta^{t-1} u(c_{i,t})\right\}. \tag{1}$$

Markets open and agents can trade their Trees and Bonds for cash, subject to a standard budget constraint. To determine optimal trades, agents take asset prices $p_{k,t}$ ($k \in \{\text{Tree}, \text{Bond}\}$) as given, and correctly anticipate (à la Radner (1972)) that future prices are a time-invariant function of the only variable economic fundamental in the economy, namely, the dividend on the Tree $d_{\text{Tree},t}$. In particular they know that prices are set as follows:

$$p_{k,t} = \beta E[\frac{u'(c_{i,t+1})}{u'(c_{i,t})}(d_{k,t+1} + p_{k,t+1})]. \tag{2}$$

We shall not go into details here, because the derivation of the equilibrium is standard. Instead, here are the main predictions of the resulting (Lucas) equilibrium. For the parametric illustrations, we set $\beta = 5/6$, and we assume constant relative risk aversion; if risk aversion equals 1, agents are endowed with logarithmic utility ($u(c_{i,t}) = \log(c_{i,t})$).

1. **Cross-sectional Restrictions:** Because the return on the Tree has higher covariability (or "beta") with aggregate consumption (which varies only because of the dividend on the Tree), its equilibrium price is lower than that of the Bond, replicating a well-known result from static asset pricing theory. Note that this

Table 1: Equilibrium Prices And Dollar Equity Premium As A Function Of (Constant Relative) Risk Aversion And State (Level Of Dividend On Tree).

Risk Aversion	State	Tree	Bond	(Dollar Equity Premium)
0.1	High	2.50	2.55	($0.05)
	Low	2.40	2.45	($0.05)
	(Difference)	(0.10)	(0.10)	($0)
0.5	High	2.50	2.78	($0.22)
	Low	2.05	2.27	($0.22)
	(Difference)	(0.45)	(0.51)	($0)
1	High	2.50	3.12	($0.62)
	Low	1.67	2.09	($0.42)
	(Difference)	(0.83)	(1.03)	($0.20)

Table 2: Equilibrium Returns And (Percentage) Equity Premium As A Function Of State (Level Of Dividend On Tree), Logarithmic Utility (Risk Aversion = 1).

State	Tree	Bond	(Equity Premium)
High	3.4%	-0.5%	(3.9%)
Low	55%	49%	(6%)

result is far from trivial: returns are determined not only by future dividends, but also future *prices*, and it is not *a priori* clear that prices behave like dividends! With logarithmic utility, the difference between the price of the Tree and that of the Bond is \$0.62 if the dividend on the Tree is high (\$1), and \$0.42 when this dividend is low (\$0). See Table 1. This table also lists prices and corresponding equity premia for risk aversion coefficients equal to 0.5 (square-root utility) and 0.1.[5] We refer to the difference between the Bond and Tree prices as the equity premium. Usually, the equity premium is defined as the difference in *expected returns* (between a risky benchmark and a relatively riskfree security). To avoid confusion, we refer to our version of the equity premium as the *dollar equity premium.* For logarithmic preferences, the results translate into expected (percentage) returns and percentage equity premia as in Table 2.

2. **Intertemporal Restrictions:** Asset prices depend on the dividend of the Tree. As such, prices depend on fundamentals, a key prediction of the Lucas model. The explanation is that when dividends are abundant (the state is High), agents need to be incentivized to consume the (perishable) dividend rather than buying assets. Markets provide the right incentives by pricing the assets dearly. Conversely, in the Low state, agents should be induced to save and invest rather than consume, which is accomplished through low pricing of the assets. Numerically, with logarithmic utility, the Tree price is \$2.50 when the Tree dividend is high, and \$1.67 when it is low; the corresponding Bond prices are \$3.12 and \$2.09. See Table 1. Such prices induce significant predictability in the asset returns: when the dividend of the Tree is high, the expected return on the Tree is only 3.4% (equal to $(0.5. * (2.50 + 1) + 0.5 * 1.67)/2.5 - 1$) while it equals 55% when the dividend on the Tree is low! See Table 2. This predictability contrasts with simple formulations of EMH (Fama, 1991) which posit that expected returns are constant. Time-varying expected returns obtain despite the fact that the dividends are i.i.d. (in levels). Notice that the equity premium (difference in expected return on the Tree and the Bond) is *countercyclical.* Again, this incentivizes agents correctly. When dividends are low, the equity premium is high, enticing agents to take risk and invest in Trees, keeping them from consuming the (scarce) dividends. When dividends are high, the equity premium is low, keeping agents from taking too much risk and investing in Trees, thus incentivizing them to consume

[5]The equilibrium prices are unique; in particular, they do not depend on the State outcome in Period 1 (State = dividend on tree).

the abundant dividends.[6]

3. **Linking Cross-sectional and Intertemporal Restrictions**: as risk tolerance increases, the (cross-sectional) difference between the prices of the Tree and the Bond diminishes, as does the (time-series) dependence of prices on economic fundamentals. Table 1 shows how the difference in prices of an asset decreases with risk aversion (the Tree price difference decreases from 0.83 to 0.45 and 0.10 as one moves from logarithmic utility down to risk aversion equal to 0.5 and 0.1) while at the same time the dollar equity premium (averaged across states) drops from 0.52 to 0.22 to 0.05. In the extreme case of risk neutrality, both the Tree and Bond are priced at a constant $2.50. For the range of risk aversion coefficients between 0 (risk neutrality) and 1 (logarithmic utility), the correlation between the difference in prices across states and the dollar equity premium (averaged across states) equals 0.99 for the Tree and 1.00 for the Bond![7]

4. **Equilibrium Consumption:** In equilibrium, consumption across types is perfectly rank-correlated, a key property of Pareto optimal allocations with time-separable expected utility. With only two (dividend) states, this means that consumption for both types is high in the high state and low in the low state. If we assume that agents have identical preferences, they should consume a constant fraction of the aggregate cash flow (the total of dividends and incomes). Thus, agents fully offset their income fluctuations and as a result obtain smooth consumption. Pareto optimal allocations obtain as if we had a complete set of state securities. But we don't. We only have two securities (a Tree and a Bond). Still, conditional on investors implementing sophisticated dynamic trading strategies (more on those below), two securities suffice. Markets are said to be *dynamically complete*. Our model, therefore, is an instantiation of the general proposition that complete-markets Pareto optimal allocations can be implemented through trading of a few well chosen securities (Duffie and Huang, 1985). Implementation depends, however, on correct anticipation of future prices. That is, implemen-

[6]From Equation 2, one can derive the (shadow) price of a one-period pure discount bond with principal of $1, and from this price, the one-period risk free rate. In the High state, the rate equals -4%, while in the low state, it equals 44%. As such, the risk free rate mirrors changes in expected returns on the Tree and Bond. The reader can easily verify that, when defined as the difference between the expected return on the market portfolio (the per-capita average portfolio of Trees and Bonds) and the risk free rate, the equity premium is countercyclical, just like it is when defined as the difference between the expected return on the Tree and on the Bond.

[7]The relationship is slightly nonlinear, which explains why the correlation is not a perfect 1.

Table 3: Type I Agent Equilibrium Holdings and Trading As A Function Of (Constant Relative) Risk Aversion And Period (Odd; Even).

Risk Aversion	Period	Tree	Bond	(Total)
0.1	Odd	5.17	2.97	(8.14)
	Even	4.63	6.23	(10.86)
	(Trade in Odd)	(+0.50)	(-3.26)	
0.5	Odd	6.32	1.96	(8.28)
	Even	3.48	7.24	(10.72)
	(Trade in Odd)	(+2.84)	(-5.28)	
1	Odd	7.57	0.62	(8.19)
	Even	2.03	7.78	(9.81)
	(Trade in Odd)	(+5.54)	(-7.16)	

tation is through a Radner equilibrium (Radner, 1972). This contrasts with a complete-markets version of our model, which would generate Pareto optimality through a simple Walrasian equilibrium. In the complete-markets Walrasian equilibrium, there is no need to formulate beliefs about future prices, because all state securities, and hence, all prices are available from the beginning. See Bossaerts et al. (2008) for an experimental iuxtaposition of the two cases.

5. **Trading for Consumption Smoothing:** Agents obtain equilibrium consumption smoothing mostly through exploiting the price differential between Trees and Bonds: when they receive no income, they sell Bonds and buy Trees, and since the Tree is always cheaper, they generate cash; conversely, in periods when they do receive income, they buy (back) Bonds and sell Trees, depleting their cash because Bonds are more expensive. See Table 3.[8] To see why agents obtain consumption smoothing mostly through the difference in prices of Trees and Bonds rather than by simpling selling any security when in need of cash, one needs to consider price risk, which we do next.

6. **Trading to Hedge Price Risk:** Because prices move with economic funda-

[8]Equilibrium holdings and trade do *not* depend on the state (dividend of the Tree). However, they do depend on the state in Period 1. Here, we assume that the state in Period 1 is high (i.e., the Tree pays a dividend of \$1). When the state in Period 1 is low, there is a technical problem for risk aversion of 0.5 or higher: in Odd periods, agents need to short sell Bonds. In the experiment, short sales were not allowed.

mentals, and economic fundamentals are risky (because the dividend on the Tree is), there is price risk. When they sell assets to cover an income shortfall, agents need to insure against the risk that prices might change by the time they are ready to buy back the assets. In equilibrium, prices increase with the dividend on the Tree, and agents correctly anticipate this. Since the Tree pays a dividend when prices are high, it is the perfect asset to hedge price risk. Consequently (but maybe counter-intuitively!), agents *buy* Trees in periods with income shortfall and they *sell* when their income is high. See Table 3, which shows, for instance, that a Type I agent with logarithmic preferences will *purchase* more than 5 Trees in periods when they have no income (Odd periods), subsequently selling them (in Even periods) in order to buy back Bonds. Hedging is usually associated with Merton's intertemporal asset pricing model (Merton, 1973b) and is the core of modern derivatives analysis (Black and Scholes, 1973; Merton, 1973a). Here, it forms an integral part of the trading predictions of the Lucas model.

In summary, our implementation of the Lucas model predicts that securities prices differ cross-sectionally depending on consumption betas (the Tree has the higher beta), while intertemporally, securities prices move with fundamentals (dividends of the Tree). The two predictions reinforce each other: the bigger the difference in prices across securities, the larger the intertemporal movements. Investment choices should be such that consumption (cash holdings at the end of a period) across states becomes perfectly rank-correlated between agent types (or even perfectly correlated, if agents have the same preferences). Likewise, consumption should be smoothed across periods with and without income. Investment choices are sophisticated: they require, among others, that agents hedge price risk, by buying Trees when experiencing income shortfalls (and selling Bonds to cover the shortfalls), and selling Trees in periods of high income (while buying back Bonds). In the experiment, we tested these six, inter-related predictions.

4 Implementing the Lucas Model

When planning to implement the above Lucas economy in the laboratory, three difficulties remain.

a. There is no natural demand for consumption smoothing in the laboratory. Because actual consumption is not feasible until after an experimental session concludes, it would not make much of a difference if we were to pay subjects' earnings gradually, over several periods.

b. The Lucas economy has an infinite horizon, but an experimental session has to end in finite time.

c. The Lucas economy is stationary.

In our experiment, we used the standard solution to resolve issue (b), which is to randomly determine if a period is terminal (Camerer and Weigelt, 1996). This ending procedure also introduces discounting: the discount factor will be proportional to the probability of continuing the session. We set the termination probability equal to 1/6, which means that we induced a discount factor of $\beta = 5/6$ (the number used in the theoretical calculations in the previous section). In particular, after the markets in period t close, we rolled a twelve-sided die. If it came up either 7 or 8, we terminated; otherwise we moved on to a new period.

To resolve issue (a), Crockett and Duffy (2010) resorted to nonlinearities in payoff-earnings relationships: period payoffs are transformed into final experiment earnings through a nonlinear transformation. This way, it mattered that subjects spread payoffs across periods, and hence, demand for smoothing was induced. Ideally, however, one would like to avoid this, because nonlinearities are not part of the original Lucas model. Instead, risk sharing is what drives pricing in the model.

Our solution was to make end-of-period individual cash holdings *disappear* in each period that was not terminal; only securities holdings carried over to the next period. If a period *was* terminal, however, securities holdings perished. Participants' earnings were then determined entirely by the cash they held at the end of this terminal period. As such, if participants have expected utility preferences, their preferences will automatically become of the time-separable type that Lucas used in his model, albeit with an adjusted discount factor: the period-t discount factor becomes $(1-\beta)\beta^{t-1}$.[9] It is straightforward to show that all results (prices; allocations) remain the same, simply because the new utility function to be maximized is proportional to the old one [Eqn. (1)] with constant of proportionality $(1-\beta)$.

As such, the task for the subjects was to trade off cash against securities. Cash is needed because it constituted experiment earnings if a period ended up to be terminal.

[9]Starting with Epstein and Zin (1991), it has become standard in research on the Lucas model with historical field data to use time-nonseparable preferences, in order to allow risk aversion and intertemporal consumption smoothing to affect pricing differentially. Because of our experimental design, we cannot appeal to time-nonseparable preferences if we need to explain pricing anomalies. Indeed, time separability is a natural consequence of expected utility. We consider this to be a strength of our experiment: we have tighter control over preferences. This is addition to our control of beliefs: we make sure that subjects understand how dividends are generated, and how termination is determined.

Securities, in contrast, generated cash in future periods, for in case a current period was not terminal. It was easy for subjects to grasp the essence of the task. The simplicity allowed us to make instructions short. See Appendix for sample instructions.

It is far less obvious how to resolve problem (c). In principle, the constant termination probability would do the trick: any period is equally likely to be terminal. This does imply, however, that the chance of termination does not depend on how long the experiment has been going, and therefore, the experiment could go on forever, or at least, take much longer than a typical experimental session. Our own pilots confirmed that subjects' beliefs were very much affected as the session reached the 3 hour limit.

Here, we propose a simple solution, exploiting essential features of the Lucas model. It works as follows. We announced that the experimental session would last until a pre-specified time and there would be as many replications of the (Lucas) economy as could be fit within this time frame. If a replication finished at least 10 minutes before the announced end time, a new replication started. Otherwise, the experimental session was over. *If a replication was still running by the closing time, we announced before trade started that the current period was either the last one (if our die turned up 7 or 8) or the penultimate one (for all other values of the die).* In the latter case, we moved to the next period and this one became the terminal one with certainty. This meant that subjects would keep the cash they received through dividends and income for that period. (There will be no trade because assets perish at the end, but we always checked to see whether subjects correctly understood the situation.) In the Appendix, we re-produce the time line plot that we used alongside the Instructions to facilitate comprehension.

It is straightforward to show that the equilibrium prices remain the same whether the new termination protocol is applied or if termination is perpetually determined with the roll of a die. In the former case, the pricing formula is:[10]

$$p_{k,t} = \frac{\beta}{1-\beta} E[\frac{u'(c_{i,t+1})}{u'(c_{i,t})} d_{k,t+1}]. \tag{3}$$

To see that the above is the same as the formula in Eqn. (2), apply the assumption of i.i.d. dividends and the consequent stationary investment rules (which generate i.i.d. consumption flows) to re-write Eqn. (2) as follows:

$$\begin{aligned} p_{k,t} &= \sum_{\tau=0}^{\infty} \beta^{\tau+1} E[\frac{u'(c_{i,t+\tau+1})}{u'(c_{i,t+\tau})} d_{k,t+\tau+1}] \\ &= \beta E[\frac{u'(c_{i,t+1})}{u'(c_{i,t})} d_{k,t+1}] \sum_{\tau=0}^{\infty} \beta^{\tau} \\ &= \frac{\beta}{1-\beta} E[\frac{u'(c_{i,t+1})}{u'(c_{i,t})} d_{k,t+1}], \end{aligned}$$

which is the same as Eqn. (3).

Because income and dividends, and hence, cash, fluctuated across periods, and cash were taken away as long as a period was not terminal, subjects had to constantly trade. As we shall see, trading volume was indeed uniformly high. In line with Crockett and Duffy (2010), we think that this kept serious pricing anomalies such as bubbles from emerging. Trading took place through an anonymous, electronic continous open book system. The trading screen, part of software called Flex-E-Markets,[11] was intuitive, requiring little instruction. Rather, subjects quickly familiarized themselves with key aspects of trading in the open-book mechanism (bids, asked, cancelations, transaction determination protocol, etc.) through one mock replication of our economy during the instructional phase of the experiment. A snapshot of the trading screen is re-produced in Figure 1.

[10]To derive the formula, consider agent i's optimization problem in period t, which is terminal with probability $1-\beta$, and penultimate with probability β, namely: max $(1-\beta)u(c_{i,t}) + \beta E[u(c_{i,t+1})]$, subject to a standard budget constraint. The first-order conditions are, for asset k:

$$(1-\beta)\frac{\partial u(c_{i,t})}{\partial c} p_{k,t} = \beta E[\frac{\partial u(c_{i,t+1})}{\partial c} d_{k,t+1}].$$

The left-hand side captures expected marginal utility from keeping cash worth one unit of the security; the right-hand side captures expected marginal utility from buying the unit; for optimality, the two expected marginal utilities have to be the same. Formula (3) obtains by re-arrangement of the above equation. Under risk neutrality, and with $\beta = 5/6$, $p_{k,t} = 2.5$ for $k \in \{\text{Tree}, \text{Bond}\}$

[11]Flex-E-Markets is documented at http://www.flexemarkets.com/site; the software is freely available to academics upon request.

Shortsales were not allowed because of an obvious problem with ensuring subject solvency. Indeed, human subject protection rules do not allow us to charge subjects in case they finish with negative experiment earnings, which they could very well end up with if we had allowed shortsales. This is also why, contrary to Lucas' original model, the Bond is in positive net supply. This way, more risk tolerant subjects could merely reduce their holdings of Bonds rather than having to sell short (which was not permitted). Allowing for a second asset in positive supply only affects the equilibrium quantitatively, not qualitatively.[12]

All accounting and trading was done in U.S. dollars. Thus, subjects did not have to convert from imaginary experiment money to real-life currency.

We ran as many replications as possible within the time allotted to the experimental session. In order to avoid wealth effects on subject preferences, we paid for only a fixed number (say, 2) of the replications, randomly chosen after conclusion of the experiment. (If we ran less replications than this fixed number, we paid multiples of some or all of the replications.)

5 Results

We conducted six experimental sessions, with the participant number ranging between 12 and 30. Three sessions were conducted at Caltech, two at UCLA, and one at the University of Utah. This generated 80 periods in total, spread over 15 replications. Table 4 provides specifics. Our novel termination protocol was applied in all sessions. The starred sessions ended with a period in which participants knew for sure that it was the last one, and hence, generated no trade.

We first discuss volume, and then look at prices and choices.

Volume. Table 5 lists average trading volume per period (excluding periods in which should be no trade). Consistent with theoretical predictions, trading volume in Periods 1 and 2 is significantly higher; it reflects trading needed for agents to move to their steady-state holdings. In the theory, subsequent trade takes place only to smooth consumption across odd and even periods. Volume in the Bond is significantly lower in Periods 1 and 2. This is an artefact of the few replications when the state in Period 1 was low. It deprived Type I participants of cash (Type I participants start with 10 Trees and no income). In principle, they should have been able to sell enough Trees to buy Bonds, but evidently they did not manage to complete all the necessary trades in

[12]Because both assets are in positive supply, our economy is an example of a Lucas *orchard* economy (Martin, 2011).

Table 4: Summary data, all experimental sessions.

Session	Place	Replication Number	Periods (Total, Min, Max)	Subject Count
1	Caltech*	4	(14, 1, 7)	16
2	Caltech	2	(13, 4, 9)	12
3	UCLA*	3	(12, 3, 6)	30
4	UCLA*	2	(14, 6, 8)	24
5	Caltech*	2	(12, 2, 10)	20
6	Utah*	2	(15, 6, 9)	24
(Overall)		15	(80, 1, 10)	

the alotted time (four minutes). Across all periods, 23 Trees and 17 Bonds were traded on average. With an average supply of 210 securities of each type, this means that roughly 10% of available securities was turned over each period.[13] Overall, the sizeable volume is therefore consistent with theoretical predictions. To put this differently: we designed the experiment such that it would be in the best interest for subjects to trade every period, and subjects evidently did trade a lot.

Cross-Sectional Price Differences. Table 6 displays average period transaction prices as well as the period's state ("High" if the dividend of the Tree was \$1; "Low" if it was \$0). Consistent with the Lucas model, the Bond is priced *above* the Tree, with the price differential (the dollar equity premium) of about \$0.50. When checking against Table 1, this reflects a (constant relative) risk aversion aversion coefficient of 1 (i.e., logarithmic utility).

Prices Over Time. Figure 2 shows a plot of the evolution of (average) prices over time, arranged chronologically by experimental sessions (numbered as in Table 4); replications within a session are concatenated. The plot reveals that prices are volatile. In theory, prices should move only because of variability in economic fundamentals, which in this case amounts to changes in the dividend of the Tree. Specifically, prices should be high in High states, and low in Low states. In reality, much more is going on; prices are *credpb excessively volatile.* In particular, contrary to the Lucas model, price drift can be detected. Still, the direction of the drift is not obvious; the drift appears to be stochastic.

[13] Since trading lasted on average 210 seconds each period, one transaction occurred approximately every 5 seconds.

Table 5: Trading volume.

Periods	Tree Trade Volume	Bond Trade Volume
All		
Mean	**23**	**17**
St. Dev.	12	11
Min	3	2
Max	59	58
1 and 2		
Mean	**30**	**21**
St. Dev.	15	14
Min	5	4
Max	59	58
≥ 3		
Mean	**19**	**15**
St. Dev.	8	9
Min	3	2
Max	36	41

Table 6: Period-average transaction prices and corresponding 'equity premium'.

	Tree Price	Bond Price	'Equity Premium'
Mean	2.75	3.25	0.50
St. Dev.	0.41	0.49	0.40
Min	1.86	2.29	-0.20
Max	3.70	4.32	1.79

Table 7: Mean period-average transaction prices and corresponding dollar equity premium, as a function of state.

State	Tree Price	Bond Price	Equity Premium (Dollar)
High	2.91	3.34	0.43
Low	2.66	3.20	0.54
Difference	0.24	0.14	-0.11

Nevertheless, behind the excessive volatility, evidence in favor of the Lucas model emerges. As Table 7 shows, prices in the high state are on average 0.24 (Tree) and 0.14 (Bond) above those in the low state. That is, prices do appear to move with fundamentals (dividends). The table does not display statistical information because (average) transaction prices are not i.i.d., so that we cannot rely on standard t tests to determine significance. We will provide formal statistical evidence later on, taking into account the stochastic drift evident from Figure 2.[14]

Cross-Sectional And Time Series Price Properties Together. While prices in High states are above those in Low ones, the differential is small compared to the size of the dollar equity premium. The average equity premium of \$0.50 corresponds to a coefficient of relative risk aversion of 1, as mentioned before. This level of risk aversion would imply a price differential across states of \$0.83 and \$1.03 for the Tree and Bond, respectively. See Table 1. In the data, the price differentials amount to only \$0.24 and \$0.14. In other words, the co-movement between prices and fundamentals is lower than implied by the cross-sectional differences in prices between securities.

Still, the theory also states that the differential in prices between High and Low states should increase with the dollar equity premium. Table 8 shows that this is true in the experiments. The observed correlation is not perfect (unlike in the theory), but marginally significant for the Tree; it is insignificant for the Bond.

Prices: Formal Statistics. To enable formal statistical statements about the price differences across states, we ran a regression of period transaction price levels

[14]Table 7 also shows that the dollar equity premium is higher in periods when the state is Low than when it is High. This is inconsistent with the theory. The average level of the dollar equity premium reveals logarithmic utility, and for this type of preferences, the equity premium should be *lower* in bad periods; see Table 1. This prediction is true for other levels of risk aversion too, but for lower levels of risk aversion, the difference in dollar equity premium across states is hardly detectible.

Table 8: Correlation between dollar equity premium (average across periods) and price differential of tree and bond across High and Low states.

	Tree	Bond
Correlation	0.80	0.52
(St. Err.)	(0.40)	(0.40)

onto the state (=1 if high; 0 if low). To adjust for time series dependence evident in Figure 2, we added session dummies and a time trend (Period number). In addition, to gauge the effect of our session termination protocol, we added a dummy for periods when we announce that the session is about to come to a close, and hence, the period is either the penultimate or last one, depending on the draw of the die. Lastly, we add a dummy for even periods. Table 9 displays the results.

We confirm the positive effect of the state on price levels. Moving from a Low to a High state increases the price of the Tree by \$0.24, while the Bond price increases by \$0.11. The former is the same number as in Table 7; the latter is a bit lower. The price increase is significant ($p = 0.05$) for the Tree, but *not* for the Bond.

The coefficient to the termination dummy is insignificant, suggesting that our termination protocol is neutral, as predicted by the Lucas model. This constitutes comforting evidence that our experimental design was correct.

Closer inspection of the properties of the error term did reveal substantial dependence over time, despite our including dummies to mitigate time series effects. Table 9 shows Durbin-Watson (DW) test statistics with value that correpond to $p < 0.001$.

Proper time series model specification analysis revealed that the best model involved first differencing price changes, effectively confirming the stochastic drift evident in Figure 2 and discussed before. All dummies could be deleted, and the highest R^2 was obtained when explaining (average) price changes as the result of a *change* in the state. See Table 10.[15] For the Tree, the effect of a change in state from Low to High is a significant \$0.19 ($p < 0.05$). The effect of a change in state on the Bond price remains insignificant, however ($p > 0.05$). The autocorrelations of the error terms are now acceptable (marginally above their standard errors).

At 18%, the explained variance of Tree price changes (R^2) is high. In theory,

[15] We deleted observations that straddled two replications. Hence, the results in Table 10 are solely based on intra-replication price behavior. The regression does not include an intercept; average price changes are insignificantly different from zero.

Table 9: OLS regression of period-average transaction price levels on several explanatory variables, including state dummy. (* = significant at p = 0.05; DW = Durbin-Watson statistic of time dependence of the error term.)

Explanatory Variables	Tree Price Estim.	Tree Price (95% Conf. Int.)	Bond Price Estim.	Bond Price (95% Conf. Int.)
Session Dummies:				
1	2.69*	(2.53, 2.84)	3.17*	(2.93, 3.41)
2	2.69*	(2.51, 2.87)	3.31*	(3.04, 3.59)
3	1.91*	(1.75, 2.08)	2.49*	(2.23, 2.74)
4	2.67*	(2.50, 2.84)	2.92*	(2.66, 3.18)
5	2.47*	(2.27, 2.67)	2.86*	(2.56, 3.17)
6	2.23*	(2.05, 2.40)	3.42*	(3.16, 3.69)
Period Number	0.06*	(0.03, 0.08)	0.06*	(0.01, 0.10)
State Dummy (High=1)	0.24*	(0.12, 0.35)	0.11	(-0.07, 0.29)
Initiate Termination	-0.07	(-0.28, 0.14)	-0.01	(-0.33, 0.31)
Dummy Even Periods	-0.00	(-0.11, 0.11)	-0.11	(-0.28, 0.06)
R^2		0.71		0.52
DW		1.05*		0.88*

Table 10: OLS regression of changes in period-average transaction prices. (* = significant at $p = 0.05$.)

Explanatory Variables	Tree Price Change Estim.	Tree Price Change (95% Conf. Int.)	Bond Price Change Estim.	Bond Price Change (95% Conf. Int.)
Change in State Dummy (None=0; High-to-Low=-1, Low-to-High=+1)	0.19*	(0.08, 0.29)	0.10	(-0.03, 0.23)
R^2		0.18		0.04
Autocor. (s.e.=0.13)		0.18		-0.19

Table 11: Average consumption (end-of-period cash holdings) as a function of participant Type and State. Autarky numbers in parentheses.

	Consumption ($)		Consumption Ratio	
Type	High	Low	High	Low
I	14.93 (19.75)	7.64 (4.69)	1.01 (0.52)	1.62 (3.26)
II	15.07 (10.25)	12.36 (15.31)		

one should be able to explain 100% of price variability. But prices are excessively volatile, as already discussed in connection with Figure 2. Overall, the regression in first differences shows that, consistent with the Lucas model, fundamental economic forces are behind price changes, significantly so for the Tree. But at the same time, prices are excessively volatile, with no distinct drift.

Figure 3 displays the evolution of price changes, after chronologically concatenating all replications for all sessions. Like in the data underlying the regression in Table 10, the plot only shows intra-replication price changes. The period state (=1 if Low; 2 if High) is plotted on top.

Consumption Across States. Prediction 4 of the Lucas model states that agents of both types should trade to holdings that generate high consumption in High states, and low consumption in Low states. Assuming identical preferences, they should consume a fixed fraction of total period cash flows, or the ratio of Type I to Type II consumption should be equal in both states. The left-hand panel of Table 11 displays the average amount of cash (consumption) per type in High vs. Low states.[16] In parentheses, we indicate consumption levels assuming that agents do not trade (i.e., under an autarky). The statistics in the table confirm that consumption of *both* types increases with dividend levels. The result is economically significant because consumption is *anti*-correlated under autarky. This is strong evidence in favor of Lucas' model.

Notice that under autarky (numbers in parentheses), consumption is anti-correlated across subject types. Type I subjects consume more in the High than in the Low state, and *vice versa* for Type II subjects. We deliberately picked parameters for our experiment to generate this autarky outcome, to contrast it with the (Pareto-optimal) outcome of the Lucas model, which predicts that consumption would become positively correlated.

[16] To compute these averages, we ignored Periods 1 and 2, to allow subjects time to trade from their initial holdings to steady state positions.

Table 12: Average consumption (end-of-period cash holdings) as a function of participant and period Types.

	Consumption ($)	
Type	Odd	Even
I	7.69 (2.41)	13.91 (20.65)
II	14.72 (20)	11.74 (5)

The right panel of Table 11 displays Type II's consumption as a ratio of Type I's consumption. The difference is substantially reduced from what would obtain under autarky (which is displayed in parentheses). Again, this supports the Lucas model, though the theory would want the consumption ratios to be exactly equal across states if preferences are the same.

Consumption Across Odd And Even Periods. Our fourth prediction is that subjects should be able to perfectly offset income differences across odd and even periods. Table 12 demonstrates that our subjects indeed managed to smooth consumption substantially; the outcomes are far more balanced than under autarky (in parentheses; averaged across High and Low states, excluding Periods 1 and 2).[17]

Price Hedging. The above results suggest that our subjects (on average) managed to move substantially towards (Pareto-optimal) equilibrium consumption patterns in the Lucas model. However, contrary to model prediction, they did not resort to price hedging as a means to ensure those patterns. Table **??** displays average asset holdings across periods for Type I subjects (who receive income in even periods). They are net sellers of assets in periods of income shortfall (see "Total" row), just like the theoretical agents with logarithmic utility (see Table 3). But unlike in the theoretical model, subjects *decrease* Tree holdings in low-income periods and *increase* them in high-income periods (compare to Table 3). As a by-product, Type I subjects generate cash mostly through selling Trees as opposed to exploiting the price differential between the Bond and the Tree. Only in period 9 is there some evidence of price hedging: Type I subjects on average *buy* Trees when they are income-poor (Period 9's holding of Trees is higher than Period 8's).

Altogether, it appears that the findings from our experiments are in line with the

[17] Autarky consumption of Type II subjects is not affected by states, because they are endowed with Bonds which always pay $0.50 in dividends. In contrast, autarky consumption of Type I subjects depends on states. We used the sequence of realized states across all the sessions to compute their autarky consumption.

Table 13: End-Of-Period Asset Holdings Of Three Randomly Chosen Type I Subjects. Initial allocation is listed in column (0), for reference. Data from one replication in the first Caltech session.

Subject	(0)	1	2	3	4	5	6
Trees:							
3	10	4	4	3	4	3	4
5	10	1	1	0	1	1	3
7	10	7	10	13	15	19	20
Bonds:							
3	0	3	5	3	5	3	4
5	0	8	15	14	15	16	17
7	0	2	3	0	4	0	4

predictions of the Lucas model, with two exceptions: (i) prices were excessively volatile; (ii) subjects did not engage in price hedging.

These two anomalies could be related. Subjects may actually have expected volatile prices with no relation to economic fundamentals. If so, they would rationally have perceived no need to hedge price risk. In the short time span of a typical replication, their beliefs could not easily be falsified because prices were indeed very noisy. Still, the beliefs are actually wrong, at least as far as the Tree is concerned – but this we ourselves discovered only after pooling the price behavior from all sessions.

Regarding choices, however, there are significant individual differences, reminiscent of the huge cross-sectional variation in choices in static asset pricing experiments that led to the development of the ε-CAPM (Bossaerts et al., 2007a). Table 13 illustrates how three randomly chosen subjects of the same type (Type I) end up holding vastly different portfolios of Trees and Bonds. Curiously, subject 7 increased his holdings of Trees over time. Significantly, this subject bought Trees even in periods with income shortfall (odd periods), effectively implementing the price hedging strategy of the theory. Subject 3 is almost a perfect agent, diversifying across Trees and Bonds. But Subject 3 does not resort to price hedging, because Tree holdings decrease in odd periods. The message to be drawn from the table should be clear: one ought not to draw conclusions about prices or allocations at the market level from merely observing a single individual. Conversely, it is difficult to find a subject whose choices “explain” prices.

6 Discussion

The experiments demonstrate that our design "delivers" in the sense that it generates meaningful results in line with the Lucas model, with the exception of the excessively volatile prices and the absence of concern for price hedging among subjects.

Here, we discuss how these two anomalies may actually be related.

To see how, remember that in our setting, for the Lucas pricing results to obtain, agents need to have perfect foresight of the equilibrium relationship between prices and dividends. That is, the Lucas pricing results are the outcome of a Radner equilibrium (Radner, 1972). An interesting question is whether small deviations from perfect foresight would have noticeable effects on equilibrium prices and allocations. As we shall see, the effect on prices is substantial; while the effect on allocations is minimal. The effect on prices was independently discovered in Adam et al. (2012), and proves useful to explain excess volatility in historical field data, just like we can use it to make sense of our experimental data.[18]

The absence of demand for (price) hedging may indeed reflect subjects' belief that prices do not co-move with economic fundamentals. It seems that subjects expected prices to be a martingale, as if subjects believed in the naive version of the EMH that the Lucas model discredits. The belief is wrong, as we pointed out, but not readily falsifiable within the short time of an experiment, and even after 80 observations (80 periods), not falsifiable for Bond prices (see Table 10). That is, the belief is a credible working hypothesis.

To determine the equilibrium effects of these beliefs, imagine that agents always expect past prices to be best predictions of future prices, irrespective of future economic fundamentals. Given these beliefs, agents correctly solve their dynamic investment-consumption problem, send corresponding demands to markets, where prices are such that there is equilibrium each period. How would this equilibrium evolve over time? Simulations which we performed reveal that prices behave very much like in the experiment. They exhibit stochastic drift. While they do co-move with dividends, they are very noisy, and hence, excessively volatile. Figure 4 plots the evolution of the prices of

[18]Most analyses of belief mistakes in the context of the Lucas model have looked at false beliefs about the (exogenous) economic fundamentals. See, e.g., Hassan and Mertens (2010). Adam et al. (2012) and ours are the only studies that focus exclusively on the *endogenous* aspects of a Lucas economy, namely, the mapping of states to prices. We claim that agents would have a much harder time learning about endogenous random variables – prices – than about exogenous random variables – dividends. In fact, in our experimental setting, subjects were told what the process of exogenous random variables was, while we left it to them to correctly anticipate future prices.

the Tree and Bond over the first 100 periods in one simulation. Plotted also are the state realizations (in red). The similarity with prices in our experiments (see Figure 2) is striking; the similarity was confirmed in formal statistical tests.

Importantly, just like in the experiments, excessive volatility makes it hard for one to reject the null that prices are unrelated to fundamentals. At conventional p levels (5%), it almost always takes 10 periods before detecting significant correlation between prices and fundamentals, and even then, the effect is only marginal economically, meaning that one may not want to implement the elaborate price hedging strategies that are required to generate the Lucas equilibrium.

Overall, then, beliefs are not far off the mark. Agents expect past prices to be best predictors of future prices, and these beliefs are almost confirmed in the resulting equilibrium. This is "almost" a Radner equilibrium (beliefs about prices are almost correct), yet prices are vastly different from the real Radner equilibrium (i.e., the Lucas model). Our exercise teaches that the Lucas model is not robust to slight mistakes in expectations about prices. As mentioned before, Adam et al. (2012) derived an analogous result, and showed that it provides a good rationale for excessive volatility of historical real-world stock market prices.

Interestingly, allocations in our "near" Radner equilibrium are close to Pareto optimal, just like in the experiments. This is because agents invest correctly given their beliefs, and their beliefs are "near" correct. As such, "near" Radner equilibria may look vastly different from the Lucas outcome in terms of pricing, but generate nearly the same allocations. The message is clear: if welfare is the goal, excessive volatility is of little concern, as long as beliefs are nearly fulfilled, and agents correctly act on them.[19]

7 Conclusion

Over the last thirty years, the Lucas model has become *the* core theoretical model through which scholars of macroeconomics and finance view the real world, advise investments in general and retirement savings in particular, prescribe economic and financial policy and induce confidence in financial markets. Despite this, little is known about the true relevance of the Lucas model. The recent turmoil in financial markets

[19]Notice that our conclusion is diametrically opposed to that in Hassan and Mertens (2010), but this is because the latter paper assumes that agents have incorrect beliefs about economic fundamentals. Instead, our agents have the right beliefs about fundamentals, and only slightly incorrect beliefs about equilibrium prices.

and the effects it had on the real economy has severely shaken the belief that the Lucas model has anything to say about financial markets. Calls are being made to return to pre-Lucas macroeconomics, based on reduced-form Keynesian thinking. This paper was prompted by the belief that proper understanding of whether the Lucas model (and the Neoclassical thinking underlying it) is or is not appropriate would be enormously advanced if we could see whether the model did or did not work in the laboratory.

Of course, it is a long way from the laboratory to the real world, but it should be kept in mind that no one has ever seen convincing evidence of the Lucas model "at work" – just as no one had seen convincing evidence of another key model of finance (the Capital Asset Pricing Model or CAPM) at work until the authors (and their collaborators) generated this evidence in the laboratory (Asparouhova et al., 2003; Bossaerts and Plott, 2004; Bossaerts et al., 2007a). The research provides absolutely crucial – albeit modest – evidence concerning the scientific validity of the core asset pricing model underlying formal macroeconomic and financial thinking.

Specifically, despite their complexity, our experimental financial markets exhibited many features that are characteristic of the Lucas model, such as the co-existence of a significant equity premium and (albeit reduced) co-movement of prices and economic fundamentals. Consistent with the model, the co-movement increased with the magnitude of the equity premium. And subjects managed to smooth consumption over time and across states. Smoothing was not perfect, but sufficient for consumption to become positively correlated across subject types, consistent with Pareto optimality, and in sharp contrast with consumption under autarky, which was negatively correlated. Prices were excessively volatile though (not unlike in the real world, incidentally). And we did not observe price hedging, perhaps because subjects believed that the best predictor for future prices were past prices (reminiscent of a naive version of EMH?). Still, such beliefs were not irrational: within the time frame of a single replication, there was insufficient evidence to the contrary, because the excess volatility made it hard to determine to what extent prices really reacted to fundamentals.

Overall, we view our experiments as a success for the Lucas model. Note that this model is only a reduced-form version of a general equilibrium. It assumes that markets somehow manage to reach Pareto optimal allocations, but is silent about how to get there. In our experiment, markets were incomplete, which makes attainment of Pareto optimality all the more challenging – markets had to be dynamically complete, and sophisticated trading strategies were required. Our design was in part mandated by experimental considerations: with incomplete markets, subjects had to trade every period. Crockett and Duffy (2010) have demonstrated that subjects need a serious reason to trade in all periods, otherwise pricing anomalies (bubbles) emerge. But we would also argue that realistic markets are generically incomplete, and as such, our experiments provide an ecologically relevant test of the Lucas model.

Real-world financial markets are thought to be excessively volatile, and policy makers have long been worried about this. On the policy side, our experimental results lead to a provocative conclusion: *if welfare is the goal, excessive volatility is of little concern.* This conclusion holds as long as beliefs are nearly fulfilled, and agents correctly act on them. The possibility that financial markets may be able to generate Pareto optimal outcomes in spite of excessive volatility has so far escaped the attention of empiricists and theorists, because evaluation of Pareto optimality cannot be performed on field data. Pricing had to be focused on, taking allocations as *given*. Experiments like ours, in contrast, allow one to study the optimality of allocations, and the interplay between prices and allocations.

Our experimental results also illustrate that it is dangerous to extrapolate from the individual to the market. As in our static experiments (Bossaerts et al., 2007b), we find substantial heterogeneity in choices across subjects; most individual choices have little or no explanatory power for market prices, or even for choices averaged across subjects of the same type (same endowments). Overall, the system (market) behaves as in the theory (modulo excessive price volatility), but the theory is hardly reflected in individual choices. As such, we would caution against developing asset pricing theories where the system is a mirror image of (one of) its parts. For instance, it is doubtful that prices in financial markets would reflect, say, prospect theoretic preferences (Barberis et al., 2001), merely because many humans exhibit such preferences (leaving aside the problem that these preferences do not easily aggregate). The "laws" of the (financial) system are different from those of its parts.

The next step in our experimental analysis should be to reduce the termination probability, thereby generating longer time series. This way, one could study to what extent markets eventually converge to the Lucas equilibrium. The reader may wonder why we have not done so already. One reason is experimental. We wanted to make sure that subjects understood that any period, including the first one, could be terminal. To be credible, we needed to generate a few cases where termination occurred early on (one of the replications in the first session terminated after one period, and we did not fail to mention this during the instruction phase of the subsequent sessions). This required a high termination probability. The second reason is based on personal opinion. We do not believe that the real world is stationary. Parameters change before full convergence to the Lucas equilibrium. As such, it is irrelevant to ask what happens in Lucas economies of long duration. Despite the short horizon, we find it remarkable that our experimental markets manage to generate results that are very much in line with general equilibrium theory (suitably modified to explain the noise prices). Nevertheless, eventual convergence to the Lucas equilibrium is an interesting theoretical possibility, and here again, laboratory experiments could be informative.

Appendix: Instructions (Type I Only)

Web Address: filagora.caltech.edu/fm/

User name:

Password:

INSTRUCTIONS

1. Situation

One session of the experiment consists of a number of replications of the same situation, referred to as ***periods***.

You will be allocated ***securities*** that you can carry through all periods. You will also be given ***cash***, but cash will not carry over from one period to another.

Every period, markets open and you will be free to ***trade*** your securities. You buy securities with cash and you get cash if you sell securities.

Cash is not carried over across periods, but there will be two sources of fresh cash in a new period. First, the securities you are holding at the end of the previous period may pay ***dividends***. These dividends become cash for the subsequent period. Second, before the start of specific periods, you may be given ***income***. This income becomes cash for the period. It will be known beforehand in which periods you receive income.

Each period lasts 5 minutes. The total number of periods is not known beforehand. Instead, at the end of a period, we determine whether the experiment continues, as follows. We throw a twelve-sided die. If the outcome is 7 or 8, we terminate the session. Otherwise we continue and advance to the next period. Notice: the termination chance is time-invariant; it does not depend on how long the experiment has been going.

Your experiment earnings are determined by the cash you are holding at the end of the period in which the session ends.

So, if you end a period without cash, and we terminate the session at that point, you will not earn any money for the session. This does not mean, however, that you should ensure that you always end with only cash and no securities. For in that case, if we continue the experiment, you will not receive dividends, and hence, you start the subsequent period without cash (and no securities) unless this is a period when you receive income.

We will run as many sessions as can be fit in the allotted time of two hours for the experiment. If the last session we run has not been terminated before the scheduled end of the experiment we will terminate the session and you will earn the cash you are holding at that point.

You will be paid the earnings of two randomly chosen sessions. If we manage to run only one session during the allotted time for the experiment, you will be paid double the earnings for that session.

During the experiment, accounting is done in real dollars.

2. Data

There will be two types of securities, called **tree** and **bond**. One unit of the tree pays a random *dividend of zero or one dollar*, with equal chance; past dividends have no influence on this chance. (The actual draw is obtained using the standard pseudo-random number generator in the program "matlab"). One unit of the bond *always pays fifty cents*. You will receive the dividends on your holdings of trees and bonds in cash **before** a new period starts. As such, you will receive dividends on your initial allocation of trees and bonds before the first period starts.

You will start this session with **10 trees** and **0 bonds**. Others may start with different initial allocations.

In addition, you will receive income every alternate period. In **odd** periods (1,3…) you will receive nothing, and in **even** periods (2,4,…) you will receive 15 dollars. This income is added to your cash at the beginning of a new period. Others may have a different income flow.

Because cash is taken away at the end of a period when the session does not terminate, the dividend payments you receive, together with your income, are the sole sources of cash for a new period.

3. Examples (for illustration only)

Tables 1 and 2 give two sample examples of outcomes in a session. It is assumed that the session ends after the 6^{th} period. Table 1 shows the asset holdings, dividend and cash each period if the states are as per row 2 and the individual sticks to the initial allocation throughout. The final take-away cash/earning is 25 dollars as the session terminated after 6^{th} period. It would have been $0 if it had terminated in period 5.

Table 2 shows the case where the individual trades as follows:

- In period 1, to an allocation of 5 trees and 5 bonds,
- And subsequently, selling to acquire more cash if dividends and income are deemed too low,
- Or buying more assets when dividends and income are high.

Since there is a 1/6 chance that the session ends in the period when a security is bought, its expected value equals ($\frac{5/6}{1-5/6}$) times the expected dividend (which is equal for both the tree and the bond), or (5) * (0.5) = 2.50. Trade is assumed to take place at 2.50. Note, however, that the actual trading prices may be different, and that they may even change over time, depending on, e.g., the dividend on the tree. The final take-away cash in this case is $15.00. It would have been $13.00 if the session had terminated in period 5.

Table 1.

PERIOD	1	2	3	4	5	6
State	H	L	L	H	L	H
Initial Holdings						
Tree	10	10	10	10	10	10
Bond	0	0	0	0	0	0
Dividends						
Tree	$1*10=10	$0*10=0	$0*10=0	$1*10=10	$0*10=0	$1*10=10
Bond	$0.5*0=0	$0.5*0=0	$0.5*0=0	$0.5*0=0	$0.5*0=0	$0.5*0=0
Income	0	15	0	15	0	15
Initial Cash	$10 (=10+0+0)	$15 (=0+0+15)	$0 (=0+0+0)	$25 (=10+0+15)	$0 (=0+0+0)	$25 (=10+0+15)
Trade						
Tree	0	0	0	0	0	0
Bond	0	0	0	0	0	0
Cash Change	$0	$0	$0	$0	$0	$0
Final Holdings						
Tree	10	10	10	10	10	10
Bond	0	0	0	0	0	0
CASH	$ 10.00	$ 15.00	$ 0.00	$ 25.00	$ 0.00	$ 25.00

Table 2.

PERIOD	1	2	3	4	5	6
State	H	L	L	H	L	H
Initial Holdings						
Tree	10	5	6	4	5	3
Bond	0	5	6	4	6	4
Dividends						
Tree	$1*10=10	$0*5=0	$0*6=0	$1*4=4	$0*5=0	$1*3=3
Bond	$0.5*0=0	$0.5*5=2.5	$0.5*6=3	$0.5*4=2	$0.5*6=3	$0.5*4=2
Income	$0	$15	$0	$15	$0	$15
Initial Cash	$10 (=10+0+0)	$17.5 (=0+2.5+15)	$3 (=0+3+0)	$21 (=4+2+15)	$3 (=0+3+0)	$20 (=3+2+15)
Trade						
Tree	-5	+1	-2	+1	-2	+1
Bond	+5	+1	-2	+2	-2	+1
Cash Change	$0	-$5	+$10	-$7.5	+$10	-$5
Final Holdings						
Tree	5	6	4	5	3	4
Bond	5	6	4	6	4	5
CASH	$ 10.00	$ 12.50	$ 13.00	$ 13.50	$ 13.00	$ 15.00

Appendix: Time Line Plot To Complement Instructions

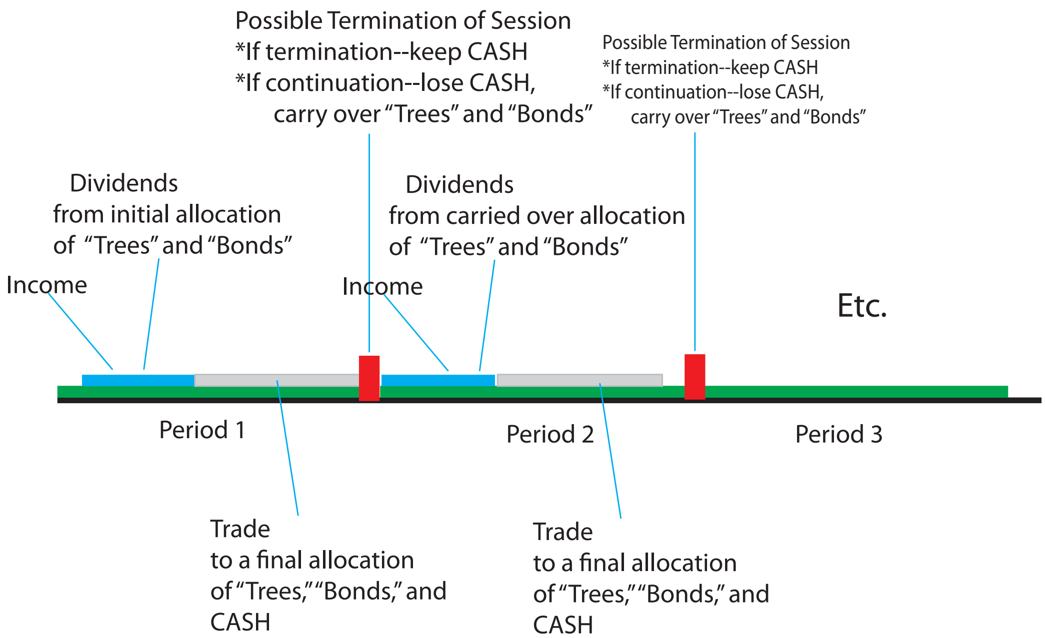

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Figure 1: Snapshot of the trading interface. Two bars graphically represent the book of the market in Trees (left) and in Bonds (right). Red tags indicate standing asks; blue tags indicate standing bids. Detailed information about standing orders is provided by clicking along either of the bars (here, the Tree bar is clicked, at a price level of \$3.66). At the same time, this populates the order form to the left, through which subjects could submit or cancel orders. Asset holdings are indicated next to the name of the market, and cash balances are given in the top right corner of the interface. The remaining functionality in the trading interface is useful but non-essential.

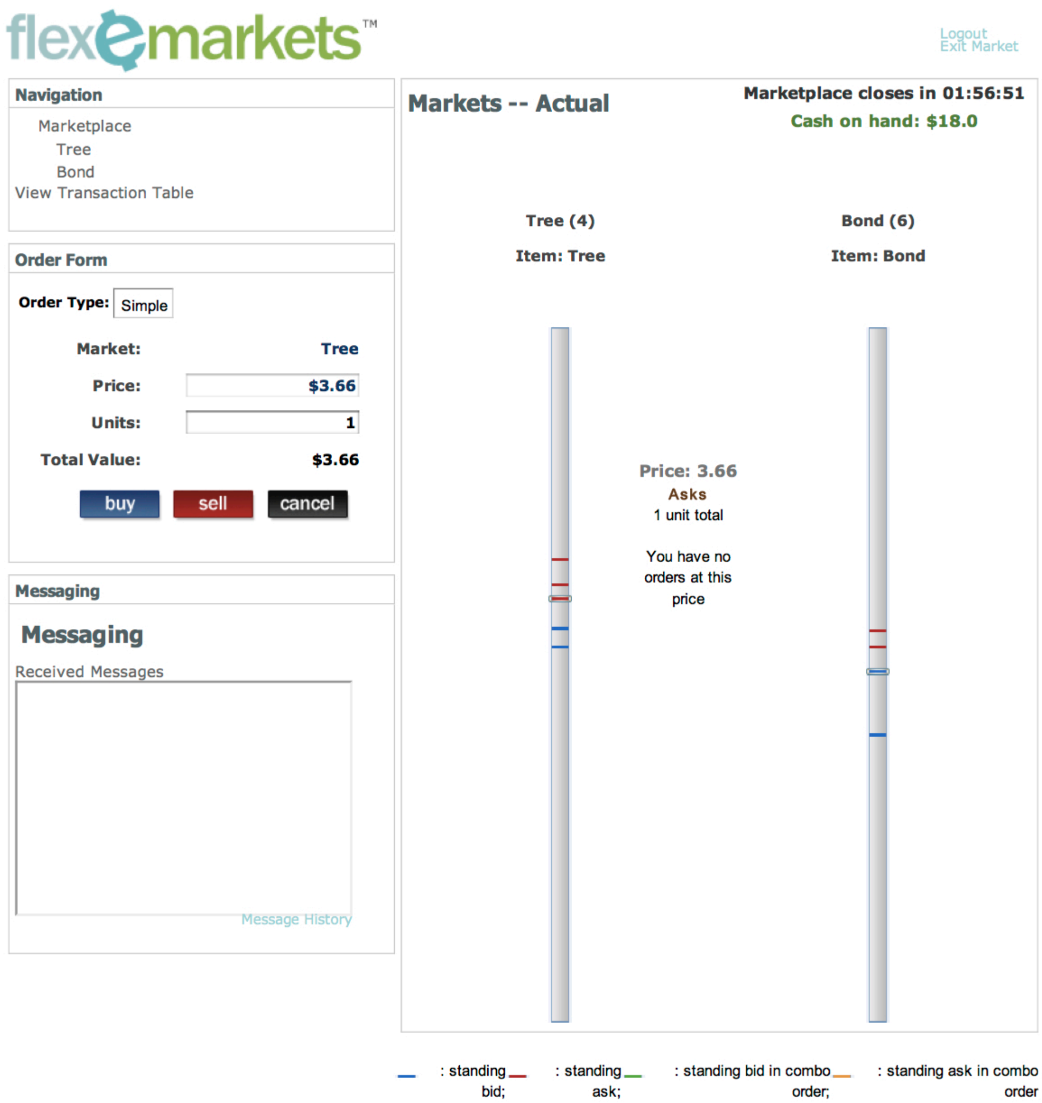

Figure 2: Time series of Tree (solid line) and Bond (dashed line) transaction prices; averages per period. Session numbers underneath line segments refer to Table 4.

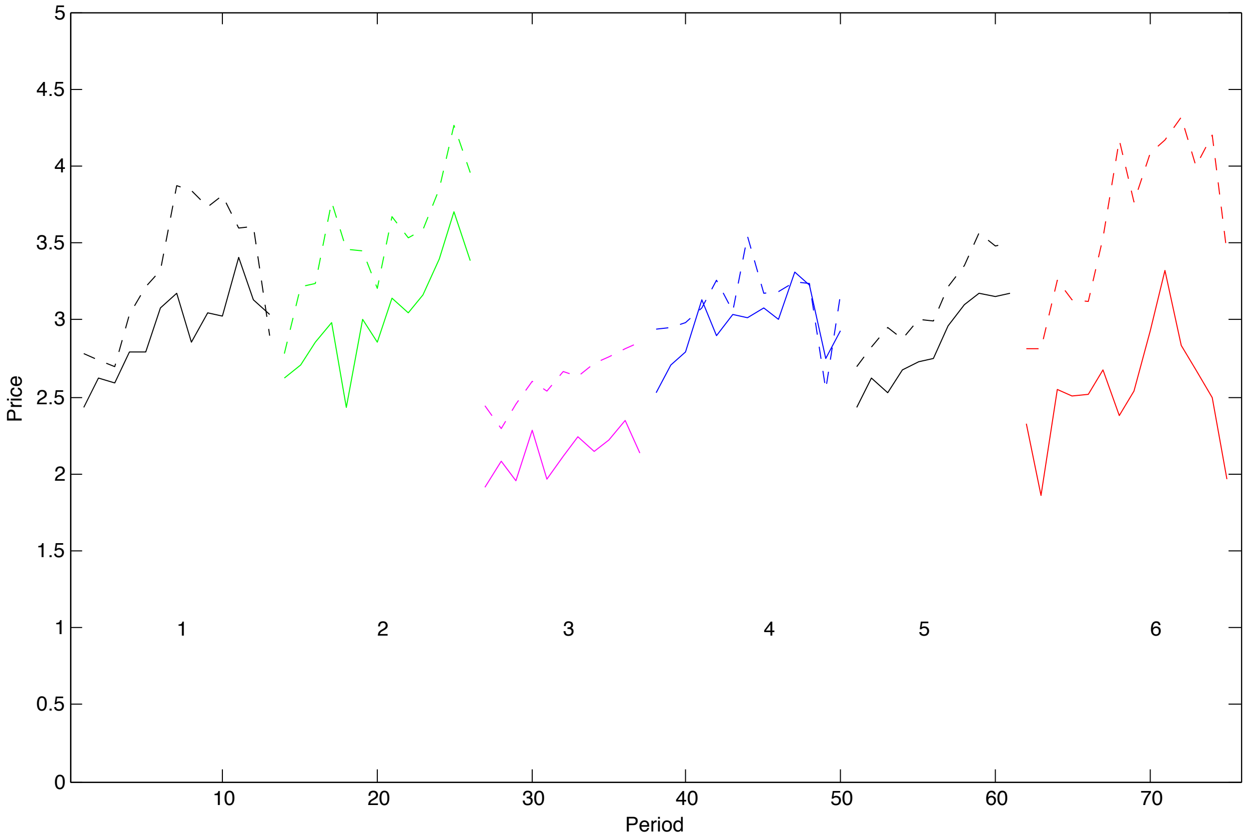

Figure 3: Time series of period-average Tree (red line) and Bond (blue line) transaction price changes. Changes are concatenated across all replications and all sesions, but exclude inter-replication observations. State is indicated by black solid line on top; state = 2 when High (tree dividend equals \$1); state = 0 when Low (tree dividend equals \$0).

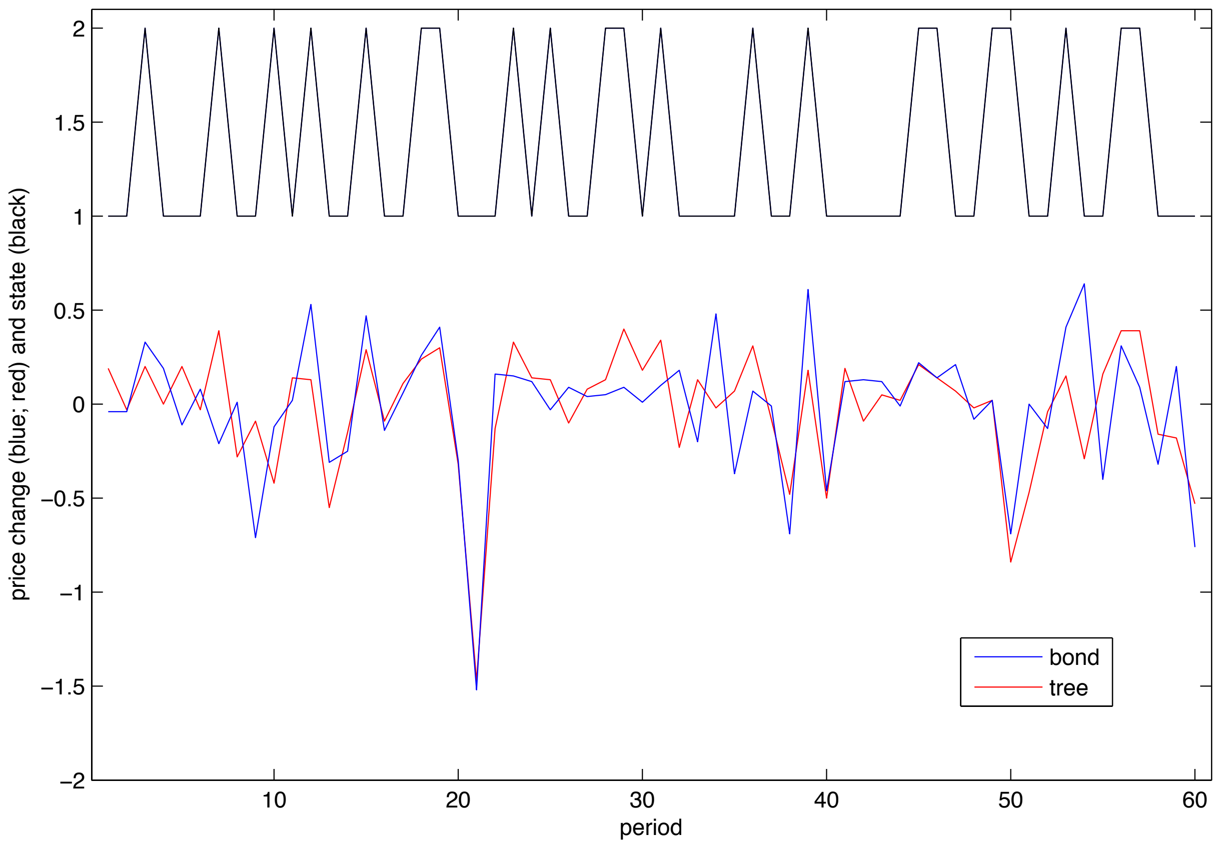

Figure 4: Simulated equilibrium Tree (blue line) and Bond (green line) prices, first 100 periods. State is indicated by red solid line on bottom; state = 1 when High (tree dividend equals \$1); state = 0 when Low (tree dividend equals \$0). Equilibrium is based on (false) agent beliefs that the past prices are the best prediction of future prices.