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Abstract

The international trade literature finds strong links between firm growth and export decisions. In spite of this, the literature analyzing cross-country differences in firm growth commonly abstracts from trade. We develop a tractable, dynamic model to understand the consequences of this abstraction. We find that the closed economy (i) under-estimates domestic (firm) growth barriers, potentially modifying the rankings across countries; and (ii) over-predicts the effects of counterfactuals. To asses the quantitative relevance of these findings, we calibrate the model to a set of European countries. The model successfully captures differences in value added per worker, accounting for between 54 and 87% of the differences across countries. We find that a closed economy alters the ranking of countries according to the size of these barriers and over-predicts the effects of counterfactuals on welfare by between 31 and 64% relative to the open economy. Thus, trade is essential for measuring barriers to firm growth and their counterfactuals in open economies.

1 Introduction

An important concern in the macro-development literature is how different policies affect firm growth. Since the effect of these policies is hard to measure directly, one approach to answer this question is to infer differences in policies across countries from differences in the observed size distributions of firms. Using an appropriate model with endogenous firm sizes, researchers can back out the different barriers to growth. Typically, and for tractability

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reasons, most work in this area has been under the closed economy assumption,[1] in spite of the strong interdependence of international trade with firm size. In this paper, we evaluate the importance of the closed economy assumption.

We develop a tractable, open economy, dynamic framework with endogenous firm size distributions. Firm productivity determines firm size, and firms can increase their productivity by incurring a fixed cost (we call this innovation, following Costantini and Melitz, 2008). Therefore, larger innovation costs reduce the size of firms. On the other hand, the gains from innovating are given by the size of the market, and larger trade barriers reduce this size. Therefore, larger trade costs reduce the incentives to innovate. Thus, a country may have on average small firms because of large innovation costs, large trade costs, or both. Therefore, to quantify innovation costs, one cannot abstract from international trade.

Our model is a continuous time model version of Melitz (2003). Any firm may innovate by incurring a convex cost, as in Atkeson and Burstein (2010) and Guadalupe et al. (2012). Since more productive firms are larger, innovation endogenizes the size distribution of firms. Additionally, firms can become exporters by incurring a sunk cost. In equilibrium, firms are born non exporters, grow by innovating, and export after reaching a productivity threshold.

In equilibrium, exporters grow at a constant rate. As in Gabaix (2011), this implies that the upper tail of the size distribution of firms follows a Pareto distribution. Non exporters grow at a rate that is increasing in size, and equals the rate of growth of exporters at the export threshold. Intuitively, the closer they are to becoming exporters, the higher the probability they will succeed in doing so, and the greater the returns to innovation.

We show analytically two main drawbacks of the closed economy assumption. First, the estimates of innovation costs are biased downwards and the bias greatly differs across countries, potentially altering the true ranking of innovation costs. Second, the closed economy overpredicts the effects of changing innovation costs on welfare and aggregate productivity.

The downward bias is due to the fact that the possibility of trade increases the incentives to innovate, so to match the same target, the costs must be larger in the open economy. We analytically decompose the relative bias between two countries into three components: the actual difference in innovation costs; the different export incentives faced by exporters; and the different export incentives faced by non exporters.

The reason why counterfactuals overreact in the closed economy is that exporters are less

[1] Two examples of this approach are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

exposed to an increase in domestic costs. An increase in innovation costs reduces profits, and consequently income and demand, which further reduces profits and innovation. In the open economy, firms shift output towards exports, thus milding down the losses and their effects on welfare. We show theoretically that, under some conditions, domestic sales fall more than exports when innovation costs increase.

We next study the quantitative importance of this problem. We calibrate the model economy to a set of European countries using the EFIGE database. This contains detailed, comparable information on manufacturing firms with more than nine employees. We focus on five countries: Germany, U.K., Italy, France and Spain.[2] This database also contains information on employees in R&D, which we use for evaluating out of sample moments.

The difference between the open and closed assumption is very large. Especially regarding counterfactuals: welfare, in units of consumption and across steady states, reacts by between 31 percent (Italy) and 64 percent (France) more in the closed economy.

The difference in the estimation of the innovation costs is also quantitatively large. First, When an economy is exceptionally good at exporting, the closed economy underestimates innovation costs. In Italy, for example, where trade costs are the lowest, innovation costs are 33 percent larger than Germany's. Under closed economy, this drops to only 12 percent. Low export costs (8 percent lower than Germany) mask huge domestic distortions. Only an open economy model can unmask them.

Second, the closed economy changes the ranking of countries by innovation costs. In the U.K. under closed economy, innovation costs 5 percent lower than Germany's. The open economy estimates them 2 percent *larger*. The high ability of the UK firms to export makes them appear more efficient at innovating than they really are.

Third, we find results that seem counterintuitive at first sight. In Spain, the open economy shows the largest export costs[3], which is the main cause of the slow growth of Spanish firms. Accordingly, one would expect the closed economy to produce large innovation costs to generate the slow growth. However, the innovation costs, relative to Germany, are similar under the closed and open economies. The reason is that the closed economy assumption is not too far off: in Germany, because of its large domestic market (reducing the importance of trade) and in Spain because of large trade costs.

[2]We drop Austria and Hungary because the samples contain less than one million employees.

[3]Waugh (2010) finds similar results.

The fact that we solve most of the model analytically makes the task of identifying the differences across countries relatively easy. We directly pin down parameters to match the targeted moments. For instance, there is a direct (unique) mapping between the slope of the tail of the distribution and the growth rates of firms. Hence, the slopes tell the firm growth rates in equilibrium. Further, the full characterization of the firm's dynamics provides a direct mapping between growth rates and each of the frictions. Thus, we reverse-engineer the innovation cost that generate such growth rates. The only stage in which we must rely on numerical solutions is when solving for the equilibrium wages. But this just involves finding the solution of a standard non-linear system of equations.

Our model provides a direct link between average firm's growth and average size: the smaller the firm's growth the smaller the average size. In turn, smaller average size translates into lower output per worker (our measure of productivity). Luttmer (2007, 2010) and Acemoglu and Cao (2010) derive similar conclusions for the closed economy. This is consistent with Tybout (2000)'s survey of the literature on firm distributions.

An unexpected result is our finding that lower trade barriers increases incentives for non exporters to innovate. This is in line with Yan-Aw et al. (2011), who find that a reduction in export costs increases R&D of both exporters and non exporters. In spite of this, typically, trade researchers assume that all the gains from trade accrue to exporters, and that non exporters are not affected. This has driven papers such as Van Biesebroeck (2005) and De Loecker (2007) to use non exporters as controls during trade liberalizations, assuming they are not affected. Our findings suggest that these firms cannot be used as controls. Furthermore, Bernard and Jensen (1999) conclude that trade is not likely to have an impact on firm productivity because productivity grows *before* firms start to export. Our model suggests that firms increase their productivity *because* they expect to become exporters.

Lastly, we evaluate the performance of the model along a series of non targeted dimensions, including expenditures in R&D, wages, and value added per worker. First, both model and data show a similar share of R&D workers (proxy for innovation) to total workers (relative to Germany), except the U.K. Griffith et al. (2006) points out that most U.K. firms perform their R&D activities abroad, mainly in the U.S. Second, the model successfully captures the differences in wages among countries. Third, the model accounts for between 54 and 87 percent of the differences in value added per worker.

The strong interdepence of trade and innovation has been found both empirically and theoretically. Caselli and Coleman (2001) find this interdependence empirically in the case of computer adoption in a number of countries, and Bustos (2011) finds it for Argentine firms during a trade liberalization. Trefler (2004) shows that Canadian productivity increased when tariffs dropped, and Rubini (2011) shows that a model with trade and innovation can easily account for this, but standard models without cannot. Kambourov (2009) finds that large innovation costs reduce the gains from trade. Guadalupe et al. (2012) find that foreign ownership increases productivity but "the higher levels of innovation by foreign subsidiaries are, in large part, driven by firms that export through a foreign parent." Pavcnik (2002), Goldberg et al. (2009) and Goldberg et al. (2010) find strong effects of trade liberalization on firm productivity in import competing sectors, which we abstract from in this paper.

We are also related to Bhattacharya et al. (2011), who use a model with endogenous innovation to identify resource misallocation in a closed economy framework. Impulitti et. al. (*forthcoming*) is related to us in the sense that they develop a continuous time version of Melitz, where firms also choose to become exporters after reaching a productivity threshold. The difference is that firm growth is exogenous and random in their case.

2 The Model

The model builds on Melitz (2003).[4] Time is continuous. There are J countries that produce a continuum of differentiated goods that can be traded. Each of these countries is assumed to be a small open economy that trade with the rest of the world.[5] Each good can only be produced in one country.

[4]Melitz (2003), as opposed to Eaton and Kortum (2002) has firm profits, which leaves room for innovation. An alternative would be Bernard et al. (2003), although this model is somewhat less tractable.

[5]The reason for assuming a small open is twofold. First, we do not have data to calibrate all the countries in the world. Second, we believe each individual country is small enough, so changes at the domestic level will have negligible effects in other countries. We also solved the model assuming these are the only countries in the world. Qualitatively, the results follow through.

Preferences. There is an infinitively lived representative consumer. The ulitity of a consumer in country j is:

$$U_j\left(q_i(\omega,t)\right) = \int_0^\infty e^{-\rho t}\ln Q_j(t)dt, \text{ where}$$
$$Q_j(t) = \left[\int_{\Omega_i(t)} q_{jj}(\omega,t)^{\frac{\sigma-1}{\sigma}}d\omega + \int_{\Omega^*(t)} q_j^*(\omega,t)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

where ω is the name of the good, $\Omega_j(t), \Omega^*(t)$ is the set of goods produced in country j at time t and the rest of the world and $q_j(\omega), q^*(\omega)$ denote consumption. $\sigma > 1$ is the elasticity of substitution between goods. $\rho > 0$ is the discount factor.

Technologies. There are incumbent firms each period that make production, innovation, and exporting decisions. Firms die each period with an exogenous probability δ. A pool of potential entrants that can enter by paying an entry cost κ_e.

Each instant, there is a continuum of incumbent firms that produce the goods. Firms are owned by the domestic consumer. Each firm is a monopolist producing each good. Given a productivity level z and labor services n, the firm producing good ω has access to the following technology:

$$y(\omega; z, n) = z^{\frac{1}{\sigma-1}}n$$

Note that there is a preference parameter in the technology. [6]. This is simply a normalization that simplifies the algebra. The counterpart is that changing σ would change both preferences and technologies, so a change in this parameter would be hard to interpret.

A firm can make innovation expenses to increase its productivity level z. We choose a functional form for the innovation cost that guarantees that in equilibrium Gibrat's law emerges for exporters (large firms in equilibrium). That is, in equilibrium, the exporter growth rate is independent of firm size. Increasing productivity by $\dot{z}$ costs, in labor units,

$$c_j(z,\dot{z}) = \frac{\kappa_{Ij}z}{2}\left(\frac{\dot{z}}{z}\right)^2$$

To increase productivity by a certain proportion, a firm must incur a cost proportional to that

[6]This is standard, see for instance Atkeson and Burstein (2010)

proportion squared. Additionally, if a very productive firm wants to increase its productivity by 10%, it must incur a cost that is greater than what a low productivity firm would need to incur to increase its productivity by 10%. κ_{Ij} determines how costly innovation is, and it may differ across countries.

A firm can export by incurring a sunk export cost equal to κ_{xj} units of labor, and it may depend on the country. Once a firm becomes an exporter, it remains an exporter until it dies, without the need of paying additional export costs.

There is a large pool of potential entrants that can enter anytime by incurring an entry cost equal to κ_e units of labor. After paying the entry cost, entrants start producing with productivity $z = 1$.

Exports are subject to iceberg trade costs. Transport depletes a proportion τ of the good. So if a consumer consumes an amount q of a good, the exporter in country j exporting to country i must ship an amount $(1 + \tau_{x,ji})q$.

The labor market clearing condition closes the model. Let $M_j(t)$ be the measure of entrants in country j at time t and L_j the total number of workers. The labor market clearing condition is

$$L_j = \int_{\Omega_i(t)} \left[n_j(\omega, t) + c_j(z(\omega, t), \dot{z}(\omega, t)) + \kappa_{xj}\mathcal{I}(\omega, t) \right] d\omega + M_j(t)\kappa_e \tag{1}$$

where $\bar{c}(\omega, t)$ is the labor demand for innovation of firm ω at time t, and $\mathcal{I}$ is the indicator function, which equals 1 if a firm producing good ω becomes an exporter in t, 0 otherwise.

Taxes. Labor and profit taxes τ_{lj} and $\tau_{\pi j}$[7], rebated lump sum to domestic consumers.

Trade Balance. We close the model with a trade balance condition. The exact specification of the trade balance depends on the assumption of who trade with whom. In the next section, we clarify how trade balance works.

2.1 Steady State Equilibrium

We solve the model in steady state, and therefore drop the argument t. Let w_j be the wage rate in country j. Let $p_j(\omega)$ be the price of good ω produced in country j. Since in equilibrium a producer will charge the same price no matter the market in which it sells, we

[7]We assume innovation expenses cannot be deducted from profits for tax purposes.

do not introduce additional notation for country of destination. for prices. This price is set by the monopolist to maximize profits subject to the demand for its product. This demand function comes from the consumer maximization problem. Consumers choose how much to consume of each good taking each price as given. Each instant, consumers solve

$$\begin{aligned}
&\max \ln Q_j \\
&s.t. \\
&Q_j = \left[\int_{\Omega_j} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega^*} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \\
&\int_{\Omega} p(\omega)q(\omega)d\omega + (1+\tau_x^*)\int_{\Omega^*} p(\omega)q(\omega)d\omega = 1 + \int_{\Omega} \pi(\omega)d\omega + R
\end{aligned}$$

The last line is the budget constraint. $\pi(\omega)$ is profits of a firm ω. R is tax revenue. A * denotes rest of the world variables. Let the right hand side be equal to I (for income). The demand of a particular good is

$$q(\omega; p, P, I, \tau_x^*) = \begin{cases} p^{-\sigma} P^{\sigma-1} I \text{ if } \omega \in \Omega \\ ((1+\tau_x^*)p)^{-\sigma} P^{\sigma-1} I \text{ if } \omega \in \Omega^* \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

P_j is the Dixit-Stiglitz aggregate price in country j,

$$P_j = \left[\int_{\Omega_j} p_j(\omega)^{1-\sigma} d\omega + (1+\tau_x^*)^{1-\sigma} \int_{\Omega^*} p^*(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \tag{3}$$

Firms solve two kinds of problems, a static problem and a dynamic problem. The static problem is how much to produce and the price given their current productivity, and the dynamic is how much to innovate and, for non exporters, whether to become exporters. The static problem depends on whether the customer is domestic or foreign. For domestic customers, this problem is, given $z(\omega)$,

$$\max_{p,q,n}(1-\tau_{\pi j})(pq - w_j(1+\tau_{lj})n)$$
$$s.t.$$
$$q = z(\omega)^{\frac{1}{\sigma-1}}n = p^{-\sigma}P_j^{\sigma-1}I_j$$

If the customer is foreign, the problem is

$$\max_{p,q,n}(1-\tau_{\pi j})((1+\tau_{xj})pq - w_j(1+\tau_{lj})n)$$
$$s.t.$$
$$q = \frac{z(\omega)^{\frac{1}{\sigma-1}}n}{(1+\tau_{xj})} = (1+\tau_{xj}p)^{-\sigma}P_j^{\sigma-1}I_j$$

The solution to these problems is the mark-up rule

$$p(\omega) = \frac{\sigma}{\sigma-1}\frac{1}{z(\omega)^{\frac{1}{\sigma-1}}}$$

Let $\pi_d(P, I, z)$ be the variable profits for a non exporter (profits before paying innovation or exporting costs). These are

$$\pi_{dj}(z(\omega), P_j, I_j) = \sigma^{-1}I_jP_j^{\sigma-1}z(\omega) = \pi_{dj}z(\omega) \tag{4}$$

and for exporters

$$\pi_{xj}(z(\omega), P_j, I_j, \tau_{xj}) = \pi_d(z(\omega), P_j, I_j) + (1+\tau_{xj})^{1-\sigma}\pi_d(z(\omega), P^*, I^*) = \pi_{xj}z(\omega) \tag{5}$$

Next we describe the dynamic problem of the firms. Before we do that, note that, as in Dixit and Stiglitz (1977), we can drop out the name of the good ω, since all that matters for profits is z. This saves on notation. Firms decide how much to innovate each period, and non exporters choose whether to become exporters. We start by solving the problem of exporters. Their Hamilton-Jacobi-Bellman equation is

$$(\rho+\delta)V_{xj}(z) = \max_{\dot{z}}(1-\tau_{\pi j})\pi_{xj}z - \frac{w_j(1+\tau_{lj})\kappa_{Ij}z}{2}\left(\frac{\dot{z}}{z}\right)^2 + V'_{xj}(z)\dot{z} \tag{6}$$

For non exporters, the dynamic problem consists on when to become exporters and how much to innovate[8]. Their problem is a stopping time problem. They need to choose when to become exporters, and how much to grow while being non exporters. Let z_{xj} be the optimal size at which firms choose to become exporters. The problem of non exporters is, for $z \in [1, z_{xj}]$

$$(\rho+\delta)V_{dj}(z) = \max_{\dot{z}}(1-\tau_{\pi j})\pi_{dj}z - \frac{w_j(1+\tau_{lj})\kappa_{Ij}z}{2}\left(\frac{\dot{z}}{z}\right)^2 + V'_{dj}(z)\dot{z} \tag{7}$$

$$s.t.$$

$$V'_{dj}(z_{xj}) = V'_{xj}(z_{xj}) \tag{8}$$

$$V_{dj}(z_{xj}) = V_{xj}(z_{xj}) - w_j(1+\tau_{lj})\kappa_{xj} \tag{9}$$

Equation (8) is the smooth pasting condition. It imposes that the change in value at the point of switch in status is equal before and after switching. Equation (9) imposes that the value of the firm must be the same before and after switching.

New firms enter the economy whenever their expected profits exceed the entry cost. That is, in equilibrium, the free entry condition is

$$w_j(1+\tau_{lj})\kappa_e = V_{dj}(1) \tag{10}$$

2.2 Characterizing the Steady State

To solve the exporter problem, we guess and verify that $V_x(z)$ is homogeneous of degree 1. The solution is the productivity of exporters grows at a constant rate, and is therefore

[8] It is straightforward to show that a non exporter will always choose to become an exporter if it survives long enough. Simply calculate its value given that it never exports and show that, for a sufficiently large z, the value of becoming an exporter exceeds the value of continuing as a non exporter.

independent of firm size. Thus, Gibrat's law holds. This rate of growth is

$$g_{xj} = (\rho+\delta)\left(1-\sqrt{1-h_{xj}}\right)$$
$$h_{xj} = \frac{2\pi_{xj}}{(\rho+\delta)^2\kappa_{Ij}}$$

The rate of growth is increasing in exporter profits and decreasing in innovation costs.

The closed form solution for this value function is

$$V_{xj}(z) = w_j(1+\tau_{lj})\kappa_{Ij}g_{xj}z \tag{11}$$

The first order condition to the non exporter problem is

$$g_{dj} = \frac{V'_{dj}(z)}{w_j(1+\tau_{lj})\kappa_{Ij}}$$

Introducing the solution in the Bellman equation

$$(\rho+\delta)V_{dj}(z) = \left[(1-\tau_{\pi j})\pi_{dj} + \frac{V'_{dj}(z)^2}{2w_j(1+\tau_{lj})\kappa_{Ij}}\right]z, \quad \forall z \in [1, z_{xj}] \tag{12}$$

Equation (12) defines a first order differential equation that pins down the non exporter value function. From the first order condition, this pins down also the non exporter growth rate. The border condition is given by the value matching condition and the smooth pasting condition. Together, these imply the following

$$g_{dj}(z_{xj}) = g_{xj}$$
$$z_{xj} = \frac{(\rho+\delta)\kappa_{xj}}{(\rho+\delta)\kappa_{Ij}g_{xj} - \frac{(1-\tau_{\pi j})\pi_{dj}}{w_j(1+\tau_{l,j})} - \frac{\kappa_{Ij}}{2}g_{xj}^2}$$

Equation 12 is a first order differential equation that cannot be solved in closed form. However, the next proposition shows that it is strictly increasing in z and depends negatively on trade costs.

Proposition 1 *The non exporter growth rate is (i) increasing in z, (ii) decreasing in τ_{xj} and κ_{xj} and (iii) weakly smaller than the exporter growth rate.*

Proof: We omit the country subindices for the proof. Notice that (i) and the previously derived condition that $g_d(z_x) = g_x$ implies (iii). So we only need to prove (i) and (ii). To see (ii), first notice that $g_d(z) = \frac{V_d'(z)}{w(1+\tau_l)\kappa_I}$ from the first order condition. Thus, we can rewrite equation (12) as

$$\kappa_I g_d(z) = \sqrt{w(1+\tau_l)\kappa_I}\sqrt{(\rho+\delta)V_d(z) - (1-\tau_\pi)\pi_d} \tag{13}$$

The proof works by showing $\partial V_d(z)/\partial \tau_x < 0$ and $\partial V_d(z)/\partial \kappa_x < 0$. Using equation (13), this implies $\partial g_d(z)/\partial \tau_x < 0$ and $\partial g_d(z)/\partial \kappa_x < 0$. Write the value function in its time dependent form:

$$\begin{aligned} V_d(z) = \max_{T(z), g(t)} & \int_0^{T(z)} e^{-(\rho+\delta)t}\left[\pi_d z(t) - \frac{\kappa_I}{2} z(t) g(t)^2\right] dt + \\ & \int_{T(z)}^{\infty} e^{-(\rho+\delta)t}\left[\pi_x z(t) - \frac{\kappa_I}{2} z(t) g(t)^2\right] dt - e^{-(\rho+\delta)T(z)}\kappa_x \\ s.t. & \\ & \dot{z}(t) = z(t)g(t), z(0) = z \end{aligned}$$

Using the envelope theorem shows the result (notice that $\partial \pi_x/\partial \tau_x < 0$).

To see point (i), insert the first order condition into the Bellman equation for non exporters to obtain

$$(\rho+\delta)w(1+\tau_l)\kappa_I g_d(z) = \left[(1-\tau_\pi)\pi_d + w(1+\tau_l)\frac{\kappa_I}{2} g_d(z)^2\right] z$$

Differentiating both sides and rearranging,

$$\frac{g_d(z)}{(\rho+\delta)g_d(z) - \frac{(1-\tau_\pi)\pi_d}{w(1+\tau_l)\kappa_I} - \frac{g_d^2(z)}{2}} g_d'(z) = \frac{1}{z} \tag{14}$$

We show $g_d'(z) > 0$ by showing the denominator in the left hand side is positive. This denominator is a polynomial, and as such can be written as a function of its roots:

$$(\rho+\delta)g_d(z) - \frac{(1-\tau_\pi)\pi_d}{w(1+\tau_l)\kappa_I} - \frac{g_d^2(z)}{2} = (g_d(z) - g_1)(g_2 - g_d(z))$$

where

$$g_1 = (\rho + \delta)(1 - \sqrt{1-h}) \tag{15}$$
$$g_2 = (\rho + \delta)(1 + \sqrt{1-h})$$
$$h = \frac{2\pi_d}{(\rho+\delta)^2 \kappa_I}$$

This holds if $g_1 < g_d(z) < g_2$. To see this, notice that if π_d was replaced by π_x, g_1 would be equal to g_x. In fact, g_1 is the growth rate of a firm that expects to make profits π_d forever, or, in other words, if $\tau_x(\kappa_x) \to \infty$. Since we showed already $\partial g_d(z)/\partial \tau_x < 0 \Rightarrow g_1 < g_d(z)$ for all $z \in [1, z_x]$. Also notice that $g_2 > g_x > g_d(z)$. This implies that the denominator in the left hand side of equation (14) is positive, and thus $g_d'(z) > 0$. □

A problem with the solution of equation (12) is that it has no closed form solution. We need this to derive the distribution of firms. We work around this by solving it numerically and then approximating the solution by the following functional form:

$$g_{dj}(z) = (a_j + b_j z + c_j z^2 + d_j z^3)^{-1} \tag{16}$$

where a, b, c, and d are parameters to be determined in equilibrium. This functional form allows for a closed form distribution of firms in equilibrium. In the quantitative section, we show that the fit is very good.

We next describe the steady state distribution. The details of its characterization are in Appendix B.

$$\mu_j(z) = \begin{cases} M_j \exp\left[\delta(b_j(1-z) + c_j/2(1-z^2) + d_j/3(1-z^3))\right] z^{-a_j\delta}, & \text{if } z < z_{xj} \\ A_j z^{-\frac{\delta}{g_{xj}}}, & \text{if } z > z_{xj} \end{cases}$$

where $A_j = z_{xj}^{(\delta/g_{xj} - a_j\delta)} \exp(b_j(1-z_{xj}) + c_j/2(1-z_{xj}^2) + d_j/3(1-z_{xj}^3))$.

It is straightforward to see that this satisfies Zipf's law. This law is that the upper tail of the distribution of firms according to employees (or sales) follows a Pareto distribution. The upper tail is completely populated by exporters. The distribution of exporters is Pareto in z. Since employees (and sales) are linearly proportional to z, this satisfies Zipf's law.

Given this distribution, we solve for the equilibrium in each country by solving a system of three equations and three unknowns. The unknowns are π_{dj}, M_j and w_j. The equations are free entry (10), labor market clearing (1), and trade balance:

$$\int_{z_{xj}}^{\infty}(1+\tau_{xj})p_j(z)q_{j,*}(z)\mu_j(dz) = \int_{z_x^*}^{\infty}(1+\tau_x^*)p^*(z)q_{*,j}(z)\mu_j(dz) \tag{17}$$

It is convenient to rewrite labor market clearing and trade balance in terms of the unknowns. First derive the following relations

$$\begin{aligned} q_j(z) &= (\sigma-1)w_j(1+\tau_{lj})\pi_{dj}z \\ q_{j,*}(z) &= (\sigma-1)(1+\tau_x^*)^{1-\sigma}(w_j(1+\tau_{lj}))^{-\sigma}(w^*(1+\tau_l^*))^{(\sigma-1)}\pi_d^*z \\ q_{*,j}(z) &= (\sigma-1)(1+\tau_{xj})^{1-\sigma}(w^*(1+\tau_l^*))^{-\sigma}(w_j(1+\tau_{lj}))^{(\sigma-1)}\pi_{dj}z \end{aligned}$$

This determines as well labor used in production. For domestically sold goods, $n_j(z) = q_j(z)/z^{1/(\sigma-1)}$. For exported goods, $n_{j,*}(z) = q_{j,*}(z)/z^{1/(\sigma-1)}$ and $n_{*,j}(z) = q_{*,j}(z)/z^{1/(\sigma-1)}$.

Next, define $\hat{\mu}_j = \mu_j/M_j$ ($\mu(z)$ is linear in M). Labor market clearing is

$$\frac{L_j}{M_j} = \int_1^{\infty} n_j(z)\hat{\mu}_j(dz) + (1+\tau_{xj})\int_{z_{xj}}^{\infty} n_{j,*}(z)\hat{\mu}_j(dz) + \int_1^{z_{xj}} \frac{z\kappa_{Ij}}{2}g_{dj}^2(z)\hat{\mu}_j dz + \hat{\mu}_j(z_{xj}) + \kappa_e$$

and trade balance is

$$(\pi_{xj} - \pi_{dj})\int_{z_{xj}}^{\infty} z\mu_j(dz) = (\pi_x^* - \pi_d^*)\int_{z_x^*}^{\infty} z\mu^*(dz)$$

Given w_j, we pin down prices $p(z)$. With π_{dj}, we pin down the quantities $q(z)$, the labor used in production per firm, their innovation rates and the distribution of firms up to a scalar M_j.

Finally, we derive a measure of productivity similar to Atkeson and Burstein (2010). This is output per production workers, where output is defined as the CES aggregate of each individual good as defined in the preference specification. That is,

$$Q_j = \left[\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega + \int_{\Omega^*} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

Given this definition of output, we show in Appendix D that the following holds

$$Q_j = Z_j N_{pj}$$

where N_{pj} is labor used in production and Z_j is a constant, which is our measure of productivity. This is

$$Z_j^{\sigma-1} = \int_1^{\infty} z\mu_j(dz) + (1+\tau_x^*)^{1-\sigma} \left(\frac{w^*}{w_j}\right)^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz) \tag{18}$$

Notice that this includes two terms. $\int_1^{\infty} z\mu_j(dz)$ is a measure of the average productivity of the domestic firms, and $\int_{z_x^*}^{\infty} z\mu^*(dz)$ is a measure of the average productivity of imports.

2.3 The Closed Economy

In the closed economy, there is only one type of firm, and their maximization problem is similar to the problem of exporters. For simplicity, we abstract from taxes, but the analysis could be easily extended to include taxes.

Static profits are given by $\pi_j(z) = \pi_j z$, and the value function is $V_j(z) = \kappa_{Ij} g_j z$, where $g_j = (\rho+\delta)\left(1 - \sqrt{1-h_j}\right)$ where $h_j = 2\pi_j/((\rho+\delta)^2\kappa_{Ij})$. The free entry condition pins down the rate of growth of firms in the economy by setting $\kappa_e = \kappa_{Ij} g_j = V_j(1)$. The distribution of firms is given by $\mu_j(z) = M_j z^{-\delta/g_j}$.

2.4 Estimates of Innovation Costs: Open vs. Closed Economies

In this section we compare the estimates of the innovation costs under the open and closed economy assumptions. The calibration target to estimate these costs is the slope of the upper tail of the distribution of firms. In other words, since this slope is given by the growth rate of large firms, the objective is to set the innovation costs such that the exporter growth rate in the open economy model equals the growth rate of all firms in the closed economy model. For clarity, we abstract from taxes.

Proposition 2 shows that the estimated innovation costs under the closed economy assumption are always smaller than the real innovation costs. Further, in a Corollary we show that in addition the differences among countries appear smaller than they actually are, with the possibility of altering the efficiency ranking among countries.

Proposition 2 *Assume that the tail of distribution in the closed and open economies in country j is g_{xj}. Let κ_j^c be the estimated innovation cost under the closed economy assumption. Then, $\kappa_j^c < \kappa_{Ij}$ for all j.*

Proof: Using equation (12) evaluated at $z = 1$ and using the free entry condition,

$$\rho w_j \kappa_e = \pi_{jj} + \frac{\kappa_{I,j} g_{dj}(1)^2}{2}$$

Introducing the above equation in the definition of profits for exporters we obtain

$$\pi_{xj} = \rho w_j \kappa_e - \frac{\kappa_{Ij} g_{dj}(1)^2}{2} + (1+\tau_{xj})^{1-\sigma} \left(\frac{w_j}{w^*}\right)^{1-\sigma} \pi_d^*$$

Define $D_j = 1 + \tau_{xj}$,

$$\frac{2\pi_{xj}}{(\rho+\delta)^2 \kappa_{Ij} w_j} = \frac{2\kappa_e}{\kappa_{Ij}\rho} - \frac{g_{dj}(1)^2}{w_j \rho^2} + \frac{2\pi_d^*}{\kappa_{Ij} w_j \rho^2} \left(D_j \frac{w_j}{w^*}\right)^{1-\sigma}$$

Because of the exporters optimal innovation policy we:

$$1 - \left(1 - \frac{g_{xj}}{\rho}\right)^2 = \frac{2\kappa_e}{\kappa_{Ij}\rho} - \frac{g_{dj}(1)^2}{(\rho+\delta)^2} + \frac{2\pi_d^*}{\kappa_{Ij} w_j (\rho+\delta)^2} \left(D_j \frac{w_j}{w^*}\right)^{1-\sigma} \tag{19}$$

The implied innovation cost in the closed economy, κ_j^c, must satisfy:

$$1 - \left(1 - \frac{\kappa_e}{\rho \kappa^d}\right)^2 = 1 - \left(1 - \frac{g_{x1}}{\rho}\right)^2$$

Introducing the last in (19) and simplifying we obtain

$$2\frac{\kappa_e}{\rho+\delta}\left[\frac{1}{\kappa_j^c} - \frac{1}{\kappa_{Ij}}\right] = \left(\frac{\kappa_e}{\rho \kappa_j^*}\right)^2 - \frac{g_{dj}(1)^2}{(\rho+\delta)^2} + \frac{2\pi_d^*}{\kappa_{Ij} w_j (\rho+\delta)^2} \left(D_j \frac{w_j}{w^*}\right)^{1-\sigma}$$

Recall that free entry in the closed economy case implies $g_{xj} = \frac{\kappa_e}{\kappa_j^c}$, thus

$$2\kappa_e(\rho+\delta)\left[\frac{1}{\kappa_j^c} - \frac{1}{\kappa_{Ij}}\right] = g_{xj}^2 - g_{dj}(1)^2 + \frac{2\pi_d^*}{\kappa_{Ij} w_j} \left(D_j \frac{w_j}{w^*}\right)^{1-\sigma} \tag{20}$$

Since profits are always positive and by Proposition 1, $g_{xj} > g_{dj}(1)$ the right hand side of the above equation is positive. Thus, it must be the case that $\frac{1}{\kappa_j^c} > \frac{1}{\kappa_{Ij}}$, or $\kappa_j^c < \kappa_{Ij}$. $\square$

Thus, the closed economy underestimates innovation costs. The intuition for this is that introducing trade adds an incentive for firms to innovate. If incentives are larger, the calibration requires larger costs to generate the same slope of the distribution of firms. If one has the intuition that larger trade costs in a country translate into larger innovation costs when one assumes that the economy is closed, that intuition is incorrect. The closed economy assumption generates estimates of the domestic innovation costs that are always biased downwards. We show in the quantitative section this effect is actually sizeable.

What we can say about the differences between two countries? Suppose we know that $\kappa_{I2} < \kappa_{I1}$ What is the implied difference $\kappa^{d2} - \kappa^{d1}$? We have the following corollary

Corollary 3 *The difference in innovation costs between countries is given by*

$$\kappa_j^c - \kappa_i^c = K_1(\kappa_{Ij} - \kappa_{Ii}) + K_2[(g_{xi} - \tilde{g}_i) - (g_{xj} - \tilde{g}_j)] + \frac{K_2}{2(\rho+\delta)}[(g_{dj}(1)^2 - \tilde{g}_j^2) - (g_{di}(1)^2 - \tilde{g}_i^2)]$$

where $0 < K_1 = \frac{\kappa_i^c \kappa_j^c}{\kappa_{Ii}\kappa_{Ij}} < 1$, $K_2 = \frac{\kappa_i^c \kappa_i^c}{\kappa_e} > 0$ *and* $\tilde{g}_i$ *is given by equation (15) for all* i.

Proof: Appendix C.

The Corollary shows that the estimated difference in innovation costs under the closed economy assumption depends on three things. The first component is the actual difference in innovation costs, under the open economy assumption. The larger this difference, the larger the difference in the estimates under closed economy, since $\alpha > 0$.

The second component says that the estimates differ more when the incentives from trade for exporters differ more. $\tilde{g}$ is the growth rate of a firm that never expects to export. Thus, the difference $g_{xi} - \tilde{g}_i$ is proportional to the additional incentives of an exporter relative to a firm that never expects to export.

The third component relates to the incentives for non exporters. The difference $g_i(1) - \tilde{g}_i$ denotes the additional incentives of an entrant that expects to export in the future relative to a firm that would never do so. The fact that $g_i(1) > \tilde{g}_i$ implies that firms invest to export.[9]

An interesting result that follows from the Corollary is that when ranking a number of countries according to their innovation cost, this ranking may change depending on whether we model an open or closed economy. In other words, for a country pair i, j, we could have

[9]Notice that we require that in equilibrium $g(1) > \tilde{g}_i$. This holds in the quantitative section.

$\kappa_j^c - \kappa_i^c > 0$ and $\kappa_{Ij} - \kappa_{Ii} < 0$. We show in the quantitative section that this actually happens for certain country pairs.

2.5 Counterfactuals of Innovation Costs: Open vs. Closed Economies

In this section we show that, for a special case, the closed economy over predicts the effects of counterfactuals on productivity and welfare. This special case assumes that $\kappa_x = 0$ for all countries, so all firms export. This simplifies the algebra considerably, and allows us to make strong theoretical statements about the effects of changing innovation costs.

The intuition for this result is the following. Consider an increase in the domestic κ_I. On the one hand, this drives firms to reduce innovation, reducing aggregate productivity and welfare. On the other, associated to the increase in κ_I there is an aggregate negative income effect, which reduces profits, and thus innovation, productivity and welfare fall even more. In the open economy, faced with the reduction in domestic demand, firms shift their output toward the export market, therefore reducing their losses. Thus, productivity and welfare do not fall as much.

We start by showing some closed form solutions for key variables in equilibrium, and then use these forms to prove the main proposition in this section, the over predictions of counterfactuals in the closed economy.

An argument similar to that in section 2.2 shows that $g_x(z) = g_x$ for all z. That is, as before, all exporters grow at the same, constant rate, independently of size. Also, it is easy to show that the value function in equilibrium is

$$V_x(z) = \kappa_I g_x z$$

Evaluating at $z = 1$ and adding the free entry condition shows that in equilibrium, the growth rate of exporters is $g_x = \frac{\kappa_e}{\kappa_I}$. This shows clearly the effect of a change in innovation costs on firm growth rates, that is,

$$\frac{\partial g_x}{\partial \kappa_I} = -\frac{\kappa_e}{\kappa_I^2}$$

This allows us to derive closed form solutions for the distribution of firms. Recall that the distribution of firms is $\mu(z) = M z^{\frac{-\delta}{g_x}}$, so we need a closed form solution for M. The equation that pins down M is market clearing:

$$\frac{L}{M} = \left[\frac{\pi_x}{w} + \frac{\kappa_I g_x^2}{2}\right] \int_1^{\infty} z^{1-\frac{\delta}{g_x}} + \kappa_e$$

Next normalize $L = 1$ and notice that we can rewrite the value function for an entrant as

$$V_x(1) = \frac{\pi_x}{w} + \frac{\kappa_I g_x^2}{2} = \kappa_e$$

Thus,

$$\frac{1}{M} = \kappa_e \left(\frac{1}{\frac{\delta}{g_x} - 2} + 1\right)$$

Rearranging terms and replacing g_x by its value in equilibrium shows

$$\begin{aligned} M =& \frac{\delta\kappa_I - 2\kappa_e}{\delta\kappa_I\kappa_e - \kappa_e^2} \\ \mu(z) =& \frac{\delta\kappa_I - 2\kappa_e}{\delta\kappa_I\kappa_e - \kappa_e^2} z^{-\frac{\delta\kappa_I}{\kappa_e}} \end{aligned}$$

Next define Z_x as the productivity in the open economy (to the power $\frac{1}{\sigma-1}$) and Z_c as the analogous under the closed economy assumption. The next proposition states the relationship between these two.

Proposition 4 *Aggregate productivity in the open economy exceeds aggregate productivity in the closed economy by a factor proportional to the fraction of output exported. In equations,*

$$Z_x = Z_c \frac{\pi_x}{\pi_d}$$

where $Z_x = Z^{\frac{1}{\sigma-1}}$ *as defined in equation (18) and* $Z_c = \int_1^{\infty} z\mu(z)dz$ *(the analogous to* Z_x *in the closed economy).*

Proof: Start with the definition of Z_x and Z_c.

$$Z_x = \int_1^\infty z\mu_x(dz) + w^{\sigma-1}X^*$$
$$Z_c = \int_1^\infty z\mu_c(dz)$$

where $X^* = (1+\tau_x^*)^{1-\sigma} w^{*1-\sigma} \int_1^\infty z\mu^*(z)dz$.

From trade balance,

$$w^{\sigma-1}X^* = \frac{\pi_x - \pi_d}{\pi_d} \int_1^\infty z\mu(z)dz$$

Thus

$$Z_x = \left(1 + \frac{\pi_x - \pi_d}{\pi_d}\right) \int_1^\infty z\mu(z)dz = \frac{\pi_x}{\pi_d} \int_1^\infty z\mu(z)dz$$

Next let $\mu_c(z)$ be the distribution in the closed economy. Since the growth rate of firms must be the same in the open and closed economy to match the same distribution of firms in equilibrium, it follows that $\mu_c(z) = \mu(z)$ for all z, and $\int_1^\infty z\mu(z)dz = \int_1^\infty z\mu_c(z)dz = Z_c$.$\square$

Notice the intuition behind this proposition. Productivity in the open economy is productivity in the closed economy times the ratio π_x/π_d, which is larger than one. More importantly, a change in innovation costs will affect productivity in the open economy via two ways: the direct effect on the distribution of firms, which operates exactly as in the closed economy, and the effect on firms' exposure to trade. This leads to the main proposition in this section

Proposition 5 *If $\sigma < 3/2$[10], a change in κ_I has a larger effect in the closed economy than in the open economy.*

Proof: For the full proof, see Appendix D. Intuitively, it works as follows. First, we show

$$Z_x = Z_c \frac{\pi_x}{\pi_d}$$

[10] $\sigma < 3/2$ is a sufficient condition which we need to prove the proposition, but it is far from necessary. We found quantitatively this statement is true for many values of σ using numerical methods. The reason why a small σ is needed is that this increases the returns to scale in the economy, and the larger the gains from trade. In the extreme case where $\sigma = \infty$, there are no gains from trade.

The proposition shows that when κ_I increases, Z_c falls, Z_x falls, but π_x/π_d increases, so the change in Z_c is larger than the change in Z_x.

3 Data and Calibration

3.1 Data

We use the *European Firms In a Global Economy* (EFIGE) database, which contains detailed manufacturing firm level information in seven European countries: Austria, France, Germany, Hungary, Italy, Spain, and U.K. We do not include Austria and Hungary in the analysis, since these samples are too small. A policy report for the Bruegel Institute, Rubini et al. (2012) performs a similar analysis to this paper including all seven countries.

The database contains around 15,000 firms. We exclude firms that do not export but maintain some kind of international activity, such as importing, being part of a multinational, or investing abroad since these activities are not modeled in this paper.[11]

We first document large differences in employee-size distributions across the European countries. France, Germany, and U.K. have relatively larger firms than Italy and Spain. The latter countries have the lowest productivity in the sample according to several definitions of productivity[12], an observation that is consistent with Tybout (2000), who surveyed the literature studying firm size distributions and noted that countries with relatively smaller firms have lower GDP per capita.

Figure 1 shows these distributions. It includes firms with more than 30 employees, and excludes firms with more than 10,000 employees. The x-axis plots the log of employees, and the y-axis the log of the share of firms with more than x employees. The slope of this figure shows the "speed" at which the mass of given sizes decreases. That is, a steeper slope implies relatively higher number of small firms. The difference is robust to a number of control variables and to a one digit level industry (unfortunately, higher digit levels implies very few firms in some industries). Also, the estimation is robust to different minimum employee thresholds.

One determinant that is highly relevant is export status. Figure 2 shows the distribution

[11] Figures 1 and 2 would hardly change by including these firms. Figure 3 would, if it includes firms that belong to a multinational organization.

[12] For example, figure 9 shows how these countries compare in manufacturing value added per worker.

excluding non exporters. At first sight, the picture looks the same as figure 1. In contrast, figure 3 shows only non exporters. Here we can appreciate important differences. While Italy still has the steepest distribution, Spain now is mixed with France and U.K. Germany has the flattest distribution. This suggests that trade costs are important in accounting for the difference in distributions.

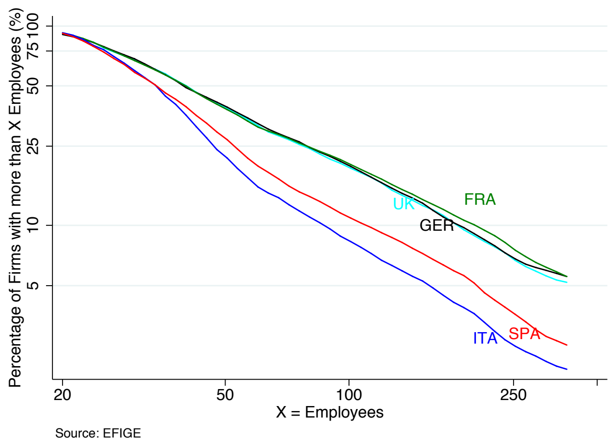


Figure 1: Firm Size Distributions

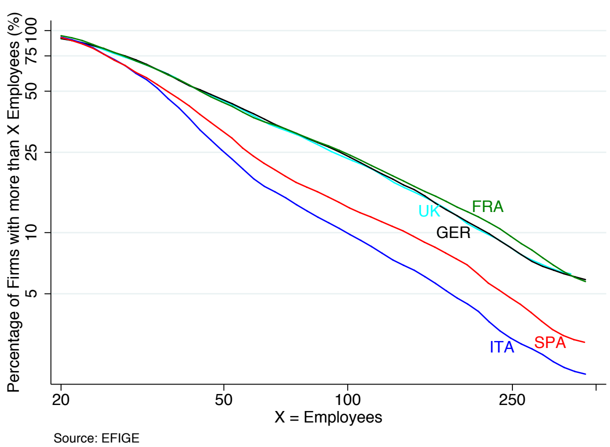


Figure 2: Exporters' Distributions

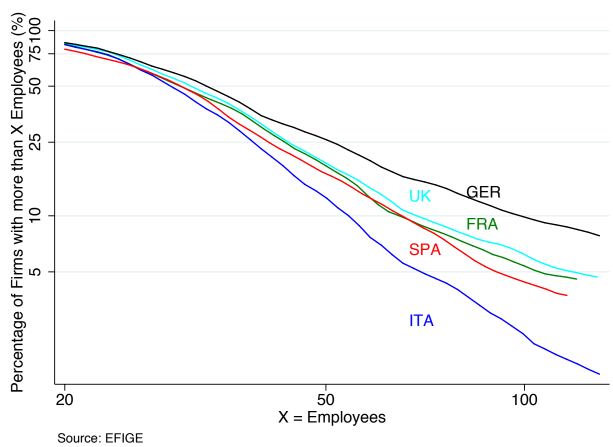


Figure 3: Non-exporters' Distributions

3.2 Calibration

We set the total number of countries equal to 5 to represent France, Germany, Italy, Spain and U.K. This leaves out Austria and Hungary, also in the database, on the basis of there being less than 1 million workers in those countries in the database. The numeraire is the wage rate in Germany, which we set to 1. We set $\rho = 0.04, \delta = 0.06$ and $\sigma = 5$, following Atkeson and Burstein (2010). We obtain labor taxes from McDaniel (2007) using the 2007-2009 average and profit taxes from the *Doing Business* report for 2012 (the only year with data).

A problem is how to calibrate the rest of the world. As it turns out, if we normalize the iceberg trade cost for one country, we do not need to calibrate the rest of the world. Thus, our calibrated iceberg costs are subject to the normalization that we choose. We normalize $\tau_{x,GER} = 0$. This does not affect the ratio $\frac{1+\tau_{xj}}{1+\tau_{x,GER}}$. We next describe this normalization.

Intuitively, the argument is as follows. We use independent targets to pin down π_{dj} and π_{xj}. These contain information about the demand for goods produced in country j in the rest of the world. However, this depends also on the iceberg export costs in country j. To pin these down, we normalize this cost in Germany, which determines the foreign demand in Germany. Given the foreign demand in Germany we identify the demand in the remaining countries, and use this to pin down the iceberg costs.

Start with trade balance. We can simplify equation (17) to

$$(\pi_{xj} - \pi_{dj}) \int_{z_{xj}}^{\infty} z\mu_j(dz) = \frac{\pi_{dj}}{((1+\tau_{lj})w_j)^{1-\sigma}} (1+\tau_x^*)^{1-\sigma}((1+\tau_l^*)w^*)^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz)$$

Let $X^* = (1+\tau_x^*)^{1-\sigma}((1+\tau_l^*)w^*)^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz)$. This determines the supply of good from the rest of the world. The assumption of small open economy implies that we can take this as a constant. Next divide this equation by the analogous for Germany:

$$\frac{(\pi_{xj} - \pi_{dj}) \int_{z_{xj}}^{\infty} z\mu_j(dz)}{(\pi_{x,GER} - \pi_{d,GER}) \int_{z_{x,GER}}^{\infty} z\mu_{GER}(dz)} = \frac{\frac{\pi_{dj}}{((1+\tau_{lj})w_j)^{1-\sigma}}}{\frac{\pi_{d,GER}}{((1+\tau_{l,GER}))^{1-\sigma}}} \tag{21}$$

Given $\pi_{dj}, \pi_{xj}, \pi_{d,GER}$ and $\pi_{x,GER}$ (we explain later how we identify these), we can determine $z_{xj}, z_{x,GER}$ and $\mu_j(z), \mu_{GER}(z)$, and therefore the only unknown is w_j.

To determine the iceberg costs in each country, we first need the demand from the rest of the world. The equation that determines export profits is

$$\pi_{xj} - \pi_{dj} = (1 + \tau_{xj})^{1-\sigma}(w_j(1 + \tau_{lj}))^{1-\sigma} \frac{\pi_d^*}{(w^*(1 + \tau_l^*))^{1-\sigma}}$$

Again, divide by the same equation for Germany:

$$\frac{\pi_{xj} - \pi_{dj}}{\pi_{x,GER} - \pi_{d,GER}} = \frac{(1 + \tau_{xj})^{1-\sigma}(w_j(1 + \tau_{lj}))^{1-\sigma}}{(1 + \tau_{x,GER})^{1-\sigma}(1 + \tau_{lj})^{1-\sigma}} \tag{22}$$

Normalizing $\tau_{x,GER} = 0$, this equation determines the iceberg cost in all countries relative to Germany.

We use the EFIGE database to calibrate the size of each economy L_i, the innovation cost κ_{Ii}, the fixed export cost κ_{xi} and the variable export cost τ_{xi}. These are four parameters per country. We use four targets from the EFIGE database. We clean the database by eliminating firms that do not export but have some foreign operations, such as importing and investing abroad. The targets are

- The number of workers in each country
- The slope of the distribution of exporters. We calculate the slope by focusing on firms with more than 29 employees since we are mostly interested in the upper tail.
- The share of firms that export
- The value of exports relative to the value of production. This is problematic since firms do not report their sales. They do report the number of employees and the share of output exported. Our measure of trade volume in country i is the sum of employees in country i times the export ratio divided by the sum of employees in country i.

The calibration strategy is especially clean and direct. It works as follows. From the data, we know the exporter growth rates g_x. These generate the slope of the upper tail as in the data. These growth rates pin down the exporter profits in each country π_x. We then calculate what the export threshold z_x and the non exporter profit π_d should be to match the share of exporting firms and the ratio of exports to total sales in each country. We set the parameters κ_{Ii}, κ_{xi} and τ_{xi} consistent with these equilibrium variables. The details are as follows:

Table 1: Calibration Targets and Relevant Parameter

Country	Employment	Exp Vol	Exp Firms	Slope	Profit Tax	Labor Tax	
France	2,903,820	27%	71%	1.11	8%	10%	
Germany	5,739,365	20%	65%	1.16	19%	10%	
Italy	3,555,052	33%	77%	1.42	23%	14%	
Spain	2,010,424	21%	68%	1.27	1%	9%	
U.K.	3,768,663	26%	73%	1.06	23%	15%	
Parameter	L	τ_x	κ_x	κ_I	τ_π	τ_l	δ

1. We first obtain π_{xi}. The slope of the distribution of exporters in the data identifies the exporter growth rate, given the death rate. This relationship is $slope_i = 1 - \frac{\delta}{g_{xi}}$. Knowing g_{xi} we also know π_{xi}.

2. Given g_{xi}, we identify κ_{Ii}, κ_{xi} and π_{di} with the free entry condition, the share of exporters, and the export volume. To do so, we find κ_{xi} and κ_{Ii} as a function of g_{di}, and then solve a non linear equation in g_{di}.

3. The last remaining parameter is τ_{xj}. Given π_{xi}, π_{di}, equation (21) determines w_j. We use this in equation (22) to determine the iceberg costs.

Table 1 shows the calibration targets and the parameter that is most affected by each target. We should mention that a key step in this calibration is the approximation of the non exporter growth rates. In Appendix F we show the values for the fitted parameters and the goodness of fit. The goodness of fit essentially plots the numerical solution together with the approximation for the growth rates. We also plot the numerically obtained value function and the one derived from our approximation. Figures 12 through 16 show that, for most cases, the approximation is indistinguishable from the numerical solution.

4 Results

Table 2 shows the values for the calibration of the key parameters and the implied exporter growth rates. We normalize costs so that they equal 1 in Germany.

Table 2: Growth Rates and Parameter Values

Country	g_x	κ_I	κ_x	$1+\tau_x$
France	2.84%	1.08	1.48	1.00
Germany	2.78%	1.00	1.00	1.00
Italy	2.48%	1.33	1.67	0.92
Spain	2.64%	1.07	1.02	1.43
U.K.	2.91%	1.02	1.24	0.92

Consider Italy and Spain, the countries with the flattest distributions. This flattness is consistent with the low rates of growth we identify. Italy's low growth rates are mainly because innovation is expensive: 33% more expensive than in Germany. Spain, on the other hand, has a hard time exporting: it costs 43% more than in Germany.

We can learn also from the behavior of the remaining countries. Notice that France has higher costs than Germany, particularly higher sunk export costs, and still their exporters grow faster. This is intuitive. The larger sunk export costs acts as a barrier to entry in the export market, which reduces competition, and so insiders enjoy larger profits and thus innovate more, growing faster. U.K. exporters grow fast because the sunk export costs are larger, and the variable trade costs are lower than in Germany. This more than compensates a slightly larger cost of innovation, resulting in a larger growth rate.

Before moving any further, we try to understand the magnitude of these estimates. We focus on innovation costs and iceberg costs. Innovation costs have to do with how easy it is for firms to grow. Our estimates suggest that it is much costlier to grow in Italy than in any other country in the sample. The World Bank estimates how easy it is to do business in each country, based on a number of costs such as dealing with construction permits, registering property, getting credit, and enforcing contracts. Based on these (and more) categories, they prepare a general ranking. Table 3 reports the ranking of each country in the sample, together with our estimate for innovation costs.

The order of both the World Bank ranking and our estimates are quite similar. The only exception is the U.K., where it is easier to do business than in Germany according to the report, but not according to our estimates of innovation costs.

Next we turn to iceberg costs. Waugh (2010) estimates trade costs for many countries. His list includes all the countries we have except Germany. Table 4 shows his estimates of

Table 3: Doing Business Report Ranking(2011)

Country	Ease of Doing Business Rank	κ_I
France	26	1.08
Germany	19	1.00
Italy	83	1.33
Spain	45	1.07
United Kingdom	6	1.02

Source: http://www.doingbusiness.org

trade costs relative to France for the four countries we have in common, and ours. In both estimates Spanish trade costs are the highest, although our differences are larger than his.

Table 4: Waugh's Trade Cost Estimates

Country	Waugh's	$1 + \tau_x$
UK	0.85	0.92
Italy	1.04	0.92
France	1.00	1.00
Spain	1.18	1.43

Source: Waugh (2010)

Another characteristic that may contribute to Spain's large export costs is the fact that their customers are farther away than the other countries', and this naturally requires larger transport costs. The EFIGE dataset includes information on where the products are shipped. Table 5 reports the share of exports within Europe, to North America, and to South and Central America. The farthest place is South and Central America, and the country exporting the most to South and Central America is Spain.

Finally, the World Bank reports measures of export costs. This measures are of reference when we compare countries that choose similar export methods. This is the case of Italy and Spain, two Peninsulas, with a natural advantage in maritime exports. The World Bank reports two big differences that increase export costs in Spain relative to Italy: (i) Spanish exports require 50% more paperwork than Italian (an average of 4 documents in Italy vs. 6 in Spain); and (ii) Spanish goods take 50% more days from the time they leave the factory until they reach the port of departure (2.6 in Italy vs. 4 in Spain). This last point may well

Table 5: Export Destination

Country	Europe	North America	South and Central America
France	76%	6%	2%
Germany	81%	6%	2%
Italy	79%	6%	3%
Spain	73%	4%	8%
UK	67%	12%	1%

Source: EFIGE

be due to geography. While both countries are Peninsulas, Italy is a relatively thin area of land surrounded by water, while Spain is not so thin. This translates into many more ports in Italy than Spain: 212 vs. 105. These are the total number of ports, but only the largest are used to export goods. In Spain there is only one large port in Barcelona. Italy has five major ports, in Genoa, La Spezia, Livorno, Venice, and Napoli.[13]

5 Open vs. Closed Economies

In this section, we compare our results with a closed economy model. This is the type of model most of the literature focuses on, so our results can provide guidance as to how the predictions of such models would be affected by adding international trade. Examples of such models are Luttmer (2007), Luttmer (2010) and Acemoglu and Cao (2010), who develop closed economy models in which firms decide how much to grow by making innovation investments.

5.1 Estimates of Innovation Costs

Proposition 2 states that the closed economy underestimates the innovation costs. Table 6 shows quantitatively the bias introduced by assuming the economy is closed. In the first column, we normalize the innovation cost in the open economy in Germany equal to one, and express every other cost in terms of it. The second column shows, in absolute terms, the biased implied by assuming the economy is closed. In the third column, we express the innovation costs in the closed economy relative to Germany, to see clearly how the countries

[13] http://www.worldportsource.com.

costs rank and compare them with the open economy.

Table 6: Innovation Cost: Closed vs. Open Economies

	Open (GER = 1)	$\frac{\text{Closed}}{\text{Open}}$	Closed (Closed GER =1)
France	1.07	0.71	0.98
Germany	1.00	0.78	1.00
Italy	1.33	0.65	1.12
Spain	1.07	0.76	1.05
U.K	1.02	0.73	0.95

The quantitative analysis reveals that the bias introduced by the closed economy assumption is of between 22 percent and 35 percent. The country with the largest distortion, both in terms of the actual cost and relative to the other countries, is Italy. Interestingly, the order of the innovation costs changes when we assume the economies are closed. In particular, both France and the U.K. have innovation costs that are larger than Germany's when modelling the open economy, but they appear to be smaller than Germany under the closed economy assumption.

Corollary 3 gives some intuition as to why the bias might be different across countries. Mainly, Corollary 3 states that the bias will be larger when (i) the difference in actual costs is large; (ii) when the difference in exporter growth rates is large; and (iii) when the difference in the growth rates of entrants is large. Comparing Italy with Germany, we already concluded that (i) and (ii) are true, accounting for the large bias. In addition, Table **??** shows that the difference in entrant growth rates, and the variable g_1 is also smaller in Italy relative to the other countries. Thus, in Italy, working with an open economy framework is important.

On the other extreme, in Spain, the bias in innovation costs is similar to Germany's. While the exporter growth rates are different in these countries, Table 7 shows that the entrant growth rate is actually larger. This compensates for the difference in exporter growth rate and results in the bias being similar in both countries. Intuitively, the reason why entrant growth rates are so high in Spain is that innovation costs are relatively low and there is not much competition due to the high export costs.[14] However, these large trade costs reduce

[14]Through trade balance, if a country exports little, it must also import little.

Table 7: Entrant Growth Rates and Variable g_1

	$g_d(1)$	g_1
France	2.23	1.95
Germany	2.23	2.11
Italy	2.02	1.54
Spain	2.41	1.98
U.K	2.33	2.04

the incentives to innovate, and firm innovation does not grow too much.

Table 7 delivers an interesting result. Recall that g_1 is the growth rate a firm would maintain if it were never to export. The fact that $g_{di}(1) > g_{1i}$ for all i reveals that entrants grow faster than they would if they expected never to export, which shows that firms invest to export.

In fact, using Corollary 3 we can decompose the biases into different categories. Recall the equation determining the relative bias between two countries:

$$\kappa_j^c - \kappa_i^c = \underbrace{K_1(\kappa_{Ij} - \kappa_{Ii})}_{\text{actual costs}} + \underbrace{K_2[(g_{xi} - \tilde{g}_i) - (g_{xj} - \tilde{g}_j)]}_{\text{incentives to exporters}} + \underbrace{\frac{K_2}{2(\rho+\delta)}[(g_{dj}(1)^2 - \tilde{g}_j^2) - (g_{di}(1)^2 - \tilde{g}_i^2)]}_{\text{incentives to non exporters}}$$

5.2 Counterfactuals: Innovation Costs

Proposition 5 showed that counterfactuals react more in the closed economy than under the open economy. We needed to make some assumptions to prove the theoretical results, and these are not met in the quantitative section (for example, no fixed export costs). In this section, we show that in the calibrated model, the closed economy also amplifies the response to counterfactuals.

Table 8 shows the elasticity of productivity with respect to a change in innovation costs for each country, and compares the closed and open economy predictions. We increase innovation costs by 0.5% and report elasticities. The closed economy estimates an elasticity between 43 and 73 percent larger than the open economy. These differences are similar for welfare, as we show in Table 9.

Table 8: Elasticity of Productivity to Innovation Costs

Country	Open Economy	Closed Economy	Ratio Closed to Open
France	-0.62	-1.07	1.73
Germany	-0.62	-0.91	1.47
Italy	-0.48	-0.69	1.44
Spain	-0.63	-0.90	1.43
U.K.	-0.64	-1.01	1.58

Table 9: Elasticity of Welfare to Innovation Costs

Country	Open Economy	Closed Economy	Ratio Closed to Open
France	-3.12	-5.13	1.64
Germany	-3.09	-4.37	1.42
Italy	-2.52	-3.29	1.31
Spain	-3.13	-4.31	1.38
U.K.	-3.25	-4.83	1.49

Our measure of welfare is the aggregate consumption good C as a measure of welfare. For country j, this is

$$C_j = \left[\sum_{i=1}^{5} \int_{z_{xi}}^{\infty} q_{ij}(z)^{\frac{\sigma-1}{\sigma}} + \int_{1}^{z_{xj}} q_{jj}(z)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

From the derivation of productivity, $C_j = Z_j L_{pj}$, where L_{pj} is labor used in production.

As proposition 5 suggests, the reason why productivity in the closed economy model reacts more than in the open economy is that increases in innovation costs bring about losses, and exporters can hedge against these losses by focusing more intensively on the foreign market. Table 10 shows the share of exports to total sales per exporter in each country under the calibrated innovation costs, and when these costs increase by five percent. The results show that exporters shift their sales toward the export market.

5.3 Counterfactuals: Iceberg Trade Costs

The tractability of our model allows us to characterize very precisely the reaction of firms to a change in trade costs, both exporters and non exporters. Proposition 1 states that a

Table 10: Share of Exports to Total Sales by Exporters

	Calibrated κ_I	5% increase in κ_I
France	29.59%	27.85%
Germany	22.33%	20.97%
Italy	36.19%	34.73%
Spain	23.75%	22.39%
U.K	28.09%	26.34%

reduction in trade costs would trigger an increase in the growth rate of non exporters, in addition to the increase by exporters. While the proposition is for partial equilibrium, we show in this section that it extends to general equilibrium. Figures 6 through 8 show the behavior of non exporter growth rates, before and after a 10 percent reduction in iceberg trade costs. The x-axis shows the productivity of the firm, and the y-axis the growth rate. The numbers in the x-axis show the export thresholds and the numbers in the y-axis the exporter growth rates.

As proposition 1 suggests, the rate of growth is increasing in z, and equals the growth rate of exporters at the switching threshold z_x. A reduction in trade costs increases the growth rate for everyone, exporters and non exporters. It also increases the ratio of exporting firms by reducing the threshold z_x. This holds for every country.

Figure 4: French Growth Rates

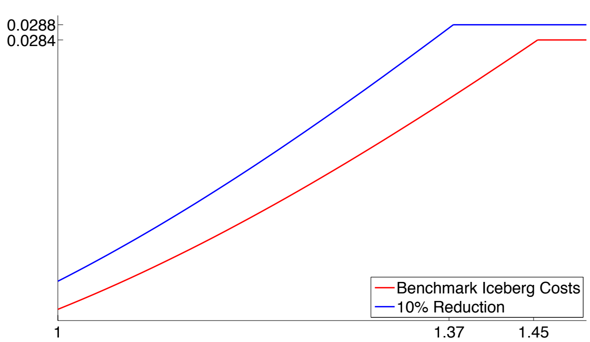
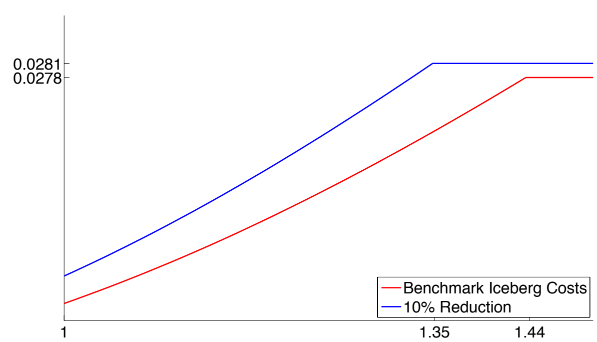


Figure 5: German Growth Rates



The fact that non exporter behavior changes in response to a change in trade costs has,

Figure 6: Italian Growth Rates

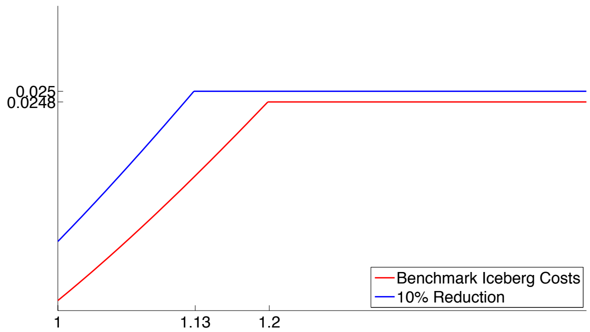


Figure 7: Spanish Growth Rates

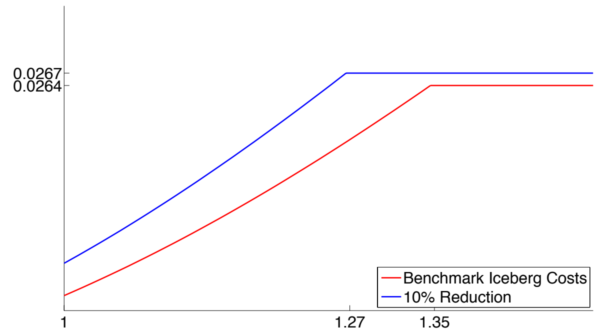
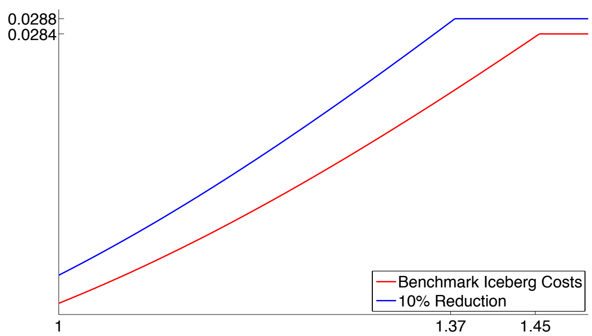


Figure 8: UK Growth Rates



to the best of our knowledge, not been highlighted so far. This is an important contribution to the empirical trade literature. Many papers study the gains from trade by observing the behavior of exporters relative to a control group of non exporters after a trade liberalization episode. Our results show that, if the non exporter used as a control has the potential of becoming an exporter in the future, its productivity should increase, and as such their behavior cannot be taken as exogenous. This implies that the gains from trade are larger than what these papers find.

Next we consider aggregate effects of changing trade costs. Atkeson and Burstein (2010) show that adding innovation into a model of trade does not change considerably the gains from reducing iceberg trade costs. In this section we confirm their results using our model. We do so in the same way as they do. We simulate a small reduction in trade costs in our model, and then report the percentage change in productivity per percentage change in iceberg costs.

We compare these numbers with those obtained with a model with no innovation, in which all firms export. In this economy, we can obtain the change in productivity in closed form solution. We recalibrate this economy so that the trade volumes are as in the benchmark model. We report these numbers in Table 11.

Table 11: Percentage Change in Productivity per Percentage Change in Iceberg Costs

Country	Benchmark Model	No innovation, all firms export
France	-0.30	-0.27
Germany	-0.22	-0.20
Italy	-0.24	-0.33
Spain	-0.22	-0.27
U.K.	-0.25	-0.21

The gains in the model with no innovation are only the direct gains, that is, the gains from exporting being more efficient (less is lost in transit). The fact that in the model with innovation and an extensive margin of exporters the gains are the same implies that the indirect effects cancel each other. That is, any gain in innovation by exporters will be offset by a corresponding reduction in innovation by non exporters and a reduction in the measure of entrants in the economy.

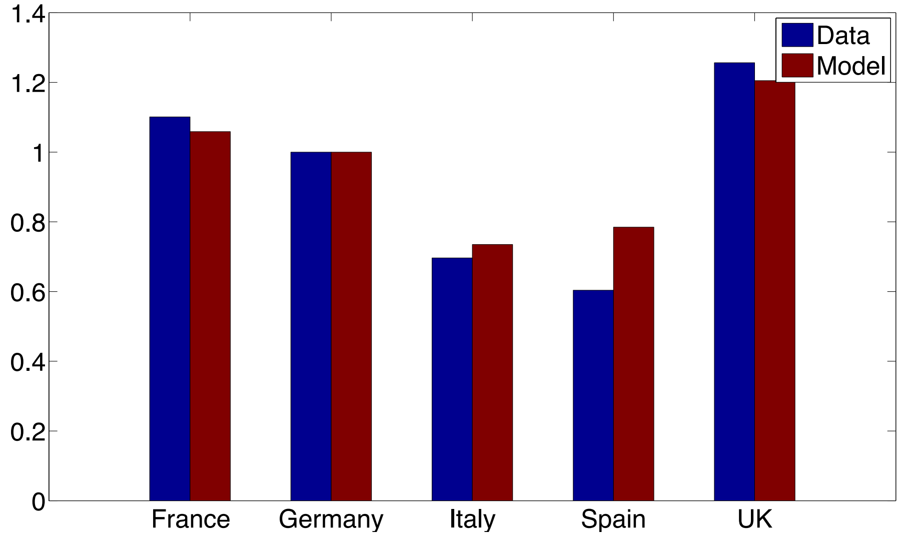
6 The Model Along Non Targeted Dimensions

This section explores the fit of the model along dimensions that were not targeted in the calibration. We compare innovation rates, wage rates and value added per worker in the model with their data counterparts.

6.1 Value Added Per Worker: Model vs. Data

Value added per worker in the data is from Eurostat, averaging years 2004 through 2010. The model performs exceptionally well. Figure 9 shows this comparison. The model can account for a large fraction of the differences in value added per worker in the manufacturing sector in these economies. Table 12 compares the value added per worker in the data with the model. The model can account for between 54 and 87 percent of the differences in value added per

Figure 9: Manufacturing Value Added per Worker (Germany = 1)



worker. It is worth mentioning that the accounting of the closed economy model would be very similar, except that the closed economy would put the blame entirely on innovation cost differences, ignoring the effect of trade barriers.

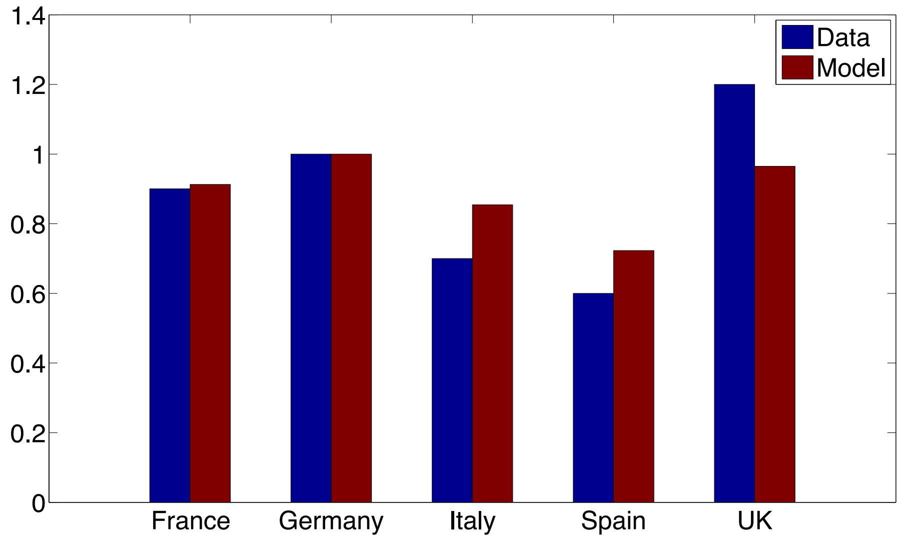
Table 12: Manufacturing Value Added per Worker Relative to Germany

Country	Data	Model	Model accounting
France	1.10	1.06	58%
Italy	0.70	0.74	87%
Spain	0.60	0.78	54%
U.K.	1.26	1.21	80%

6.2 Wages: Model vs. Data

Next we turn to comparing wages in the model and the data. We use data from Eurostat, for the year 2006, since the year 2008 did not include data for U.K. We compare the wages in the model with the mean hourly wage relative to Germany in the data. Figure 10 shows this comparison. The model does a good job at generating the wage differences we observe

Figure 10: Mean Hourly Wages (Germany = 1)



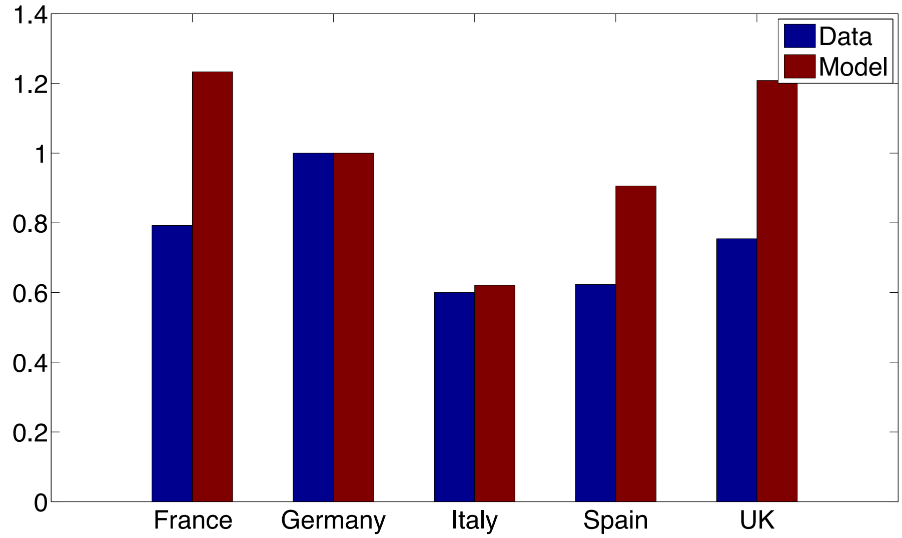
in the data.

6.3 Innovation: Model vs. Data

Identifying innovation rates in the data is challenging, since it is an abstract concept and as such not well defined. In the model, it is any expense that increases productivity. Probably the most obvious expense of this sort in the data is R&D. The EFIGE database has information on the fraction of employees devoted to R&D in each firm. Thus, we use this number to compare with innovation employees in the model. However, since R&D is only part of innovation, we cannot compare the two numbers directly. Instead, we assume that the share of R&D to total innovation is constant in all firms, normalize everything so that the data and model are the same in Germany, and compare the relative levels for the other countries. There is a problem of missing data: some firms do not report their R&D employees. We eliminate these observations for our comparison.

The model performs particularly well for Italy. In Spain, France and U.K. it predicts too many employees will go into R&D.

Figure 11: Share of Employees in R&D (Germany = 1)



7 Conclusion

The large availability of firm level data allows economists to analyze the dstribution of firms in a country and derive conclusions based on its shape. Typically, models that focus on these distributions work under the assumption of closed economy. We have argued that this abstraction is very costly in countries that are open, such as European countries, both qualitatively and quantitatively.

In particular, we find that when analyzing the distribution of firms in Europe, a model of closed economy will wrongly conclude that innovation costs are lower in Italy than what the open economy model would conclude. Also, the closed economy model would predict that changes that affect domestic innovation costs have too much of an effect on domestic macro aggregates, such as productivity. This is because in the open economy model, only some of the firms that count for domestic macro aggregates are affected by the change in innovation costs: the domestic firms.

Finally, we deliver a key message for the empirical estimates on gains from trade. Many trade econometricians estimate the effects of a change in trade policy by comparing the performance of exporters versus non exporters, under the assumption that non exporters are not affected by the change in policy. We find that their behaviour is not exogenous: both exporters and non exporters react to a change in trade costs.

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Appendix A The Problem of Non Exporters

The value function for the non exporters is

$$(\rho+\delta)V_d(z) = \max_{g_d}\{\Pi_d z - \frac{\kappa_I}{2}g_d^2 z + V_d'(z)g_d z\}, \quad \forall z \in [1, z_x]$$

Subject to

$$V_d(z_x) = \kappa_I g_x z_x - \kappa_x \quad \text{Value matching}$$

$$V_d'(z_x) = \kappa_I g_x \qquad \text{Smooth pasting}$$

The first order condition yields

$$g_d = \frac{V_d'(z)}{\kappa_I}$$

Notice first that by comparing the smooth pasting condition and the first order condition, it follows that $g_d(z_x) = g_x$. That is, the instant non exporters become exporters, their growth rate was that of exporters'. Introducing the solution in the Bellman equation

$$(\rho+\delta)V(z) = [\Pi_d + \frac{1}{2\kappa_I}V_z(z)^2]z, \quad \forall z \in [1, z_x]$$

Rearranging, we obtain the differential equation that, given z_x, solves for the non exporter growth rate

$$V_d'(z) = \sqrt{2\kappa_I\left((\rho+\delta)\frac{V_d(z)}{z} - \pi_d\right)} \tag{23}$$

where $g_d(z) = \frac{V_d'(z)}{\kappa_I}$. To solve this differential equation, in addition to z_x, we need an initial condition, which we derived before: $g_d(z_x) = g_x$.

Equation (23) cannot be solved in closed form. This closed form would be useful to obtain later the distribution of non exporters, and solve the general equilibrium model. Therefore, we approximate this solution with the following parameterization:

$$g_d(z) \approx (a + bz + cz^2 + dz^3)^{-1}$$

The approximation works as follows. We first obtain a numerical solution to equation (23), then compute numerically the non exporter growth rates, and then pin down the parameters a, b, c, and d to minimise the distance between the numerical and the approximated solution. We use a Matlab built in function (*ode45*) for the numerical solution of the differential equation, and another Matlab built in function (*fit*) for the approximation. We show the fitted values and the goodness of fit in Appendix F.

Appendix B Deriving the Endogenous Distribution of Firms

Define $\mathcal{Z} = [z_1, z_2]$

$$\hat{\mu}(t + dt, \mathcal{Z}) = \int_{\mathcal{Z}} \hat{\mu}(t, z - \dot{z}dt)e^{-\delta dt}dz$$

Taking limits as $z_1 \to z_2 \to z$

$$\hat{\mu}(t + dt, z) = \hat{\mu}(t, z - \dot{z}dt)e^{-\delta dt}$$

For small dt, the following holds:

$$\begin{aligned}
\hat{\mu}(t + dt, z) &\approx \hat{\mu}(t, z) + \hat{\mu}_1(t, z)dt \\
\hat{\mu}(t, z - \dot{z}dt) &\approx \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt \\
e^{-\delta dt} &\approx (1 - \delta dt)
\end{aligned}$$

Thus,

$$\hat{\mu}(t, z) + \hat{\mu}_1(t, z)dt = \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt - \delta dt \left(\hat{\mu}(t, z) + \hat{\mu}_2(t, z)\dot{z}dt\right)$$

Note that in steady state $\hat{\mu}_1(t, z) = 0$. Putting all together,

$$\hat{\mu}(t, z) = \hat{\mu}(t, z) - \hat{\mu}_2(t, z)\dot{z}dt - \delta dt \left(\hat{\mu}(t, z) + \hat{\mu}_2(t, z)\dot{z}dt\right)$$

Eliminating all the terms with dt elevated to a power larger than 1,

$$\hat{\mu}(t,z) = \hat{\mu}(t,z) - \hat{\mu}_2(t,z)\dot{z}dt - \delta\hat{\mu}(t,z)dt$$

Cancelling terms and dividing by dt,

$$\delta\hat{\mu}(t,z) = -\hat{\mu}_2(t,z)\dot{z}$$

Define the steady state distribution as $\mu(z) = \hat{\mu}(t,z)$ for all t. For non exporters, the distribution is

$$\delta\mu(z) = -\mu'(z)g_d(z)z$$

To solve, use the border condition $\mu(1) = M$. For exporters

$$\delta\mu(z) = -\mu'(z)g_x z$$

To solve, use the border condition $\mu(z_x) = \mu_d(z_x)$, where $\mu_d(z_x)$ is the measure of non exporters that reach the export threshold.

The solution to these distributions works as follows. Start with the exporter distribution. The differential equation can be written as

$$-\frac{\mu'(z)}{\mu(z)} = \frac{\delta}{g(z)z} \tag{24}$$

where $g(z) = g_x$ for exporters and $g_d(z)$ for non exporters. For exporters, integrating on both sides,

$$\log(\mu(z)) = \log(z^{-\delta/g_x}) + C_x$$

where C_x is the constant of integration, and is determined using the border condition. Taking exponentials yields the distribution of exporters.

For non exporters, we can only integrate both sides of (24) given our guess for the growth rates. The equation becomes

$$-\frac{\mu'(z)}{\mu(z)} = \delta(a/z + b + cz + dz^2)$$

Integrating on both sides,

$$\log(\mu(z)) = \delta(a\log(z) + bz + \frac{cz^2}{2} + \frac{dz^3}{3}) + C_d$$

where C_d is the constant of integration and is determined using the border condition. Taking exponentials yields the distribution of non exporters.

Appendix C Proof of Corollary

Taking the difference of (20) for the two countries and reorganizing:

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \frac{\chi}{2\rho}[g_{x1}^2 - g_{x1}^2 + g_{d2}(1)^2 - g_{d1}(1)^2] + \frac{\chi}{2\rho}\left[\frac{2\pi_2}{\kappa_{I,1}w_1}\left(D_1\frac{w_1}{w_2}\right)^{1-\sigma} - \frac{2\pi_1}{\kappa_{I,2}w_2}\left(D_2\frac{w_2}{w_1}\right)^{1-\sigma}\right]$$

where $0 < \alpha = \frac{\kappa^{d2}\kappa^{d1}}{\kappa_{I2}\kappa_{I1}} < 1$ and $\chi = \frac{\kappa^{d2}\kappa^{d1}}{\kappa_e} > 0$.

Because of the definition of exporters's profits

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \frac{\chi}{2\rho}[g_{x1}^2 - g_{x1}^2 + g_{d2}(1)^2 - g_{d1}(1)^2] + \frac{\chi}{2\rho}\left[\frac{2(\pi_{x1} - \pi_1)}{\kappa_{I,1}w_1} - \frac{2(\pi_{x2} - \pi_2)}{\kappa_{I,2}w_2}\right]$$

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \frac{\chi}{2\rho}[g_{x1}^2 - g_{x1}^2 + g_{d2}(1)^2 - g_{d1}(1)^2] + \frac{\rho\chi}{2}\left[h_{x1} - h_{d1} - (h_{x2} - h_{d2})\right]$$

Using the optimal innovation policy of the exporter

$$h_{x1} - h_{x2} = \left(1 - \frac{g_{x2}}{\rho}\right)^2 - \left(1 - \frac{g_{x2}}{\rho}\right)^2 = \left(\frac{g_{x2}}{\rho}\right)^2 - \left(\frac{g_{x1}}{\rho}\right)^2 + \frac{2}{\rho}(g_{x1} - g_{x2})$$

Therefore:

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \chi[g_{x1} - g_{x2}] + \frac{\chi}{2\rho}[g_{d2}(1)^2 - g_{d1}(1)^2] + \frac{\rho\chi}{2}[h_{d2} - h_{d1}]$$

Similarly,

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \chi[g_{x1} - g_{x2}] + \frac{\chi}{2\rho}[g_{d2}(1)^2 - g_{1,2}^2 - g_{d1}(1)^2 + g_{,11}^2] + \chi[g_{1,2} - g_{1,1}]$$

where $g_{1,i}$ are given by equation (15) for all i.

$$\kappa^{d2} - \kappa^{d1} = \alpha(\kappa_{I2} - \kappa_{I1}) + \chi[g_{x1} - g_{x2} + g_{1,2} - g_{1,1}] + \frac{\chi}{2\rho}[g_{d2}(1)^2 - g_{1,2}^2 - g_{d1}(1)^2 + g_{1,1}^2]$$

where $g_{di}(1) \geq g_i = \rho\left(1 - \sqrt{1 - h_{di}}\right)$

Appendix D Productivity

The goal is to derive the reduced form for aggregate output

$$Q_j = Z_j N_{pj}$$

where $Q_j = \left[\int_1^\infty q_j(z)^{\frac{\sigma-1}{\sigma}}\mu_j(dz) + \int_{z_x^*}^\infty q_j^*(z)^{\frac{\sigma-1}{\sigma}}\mu^*(dz)\right]^{\frac{\sigma}{\sigma-1}}$ and N_{pj} is labor used for production. Let $n_{dj}(z)$ denote labor for production of units sold domestically and $n_{j,*}(z)$ for exports. With some algebra, we find

$$n_{dj}(z) = (\sigma - 1)\frac{\pi_{dj}}{w_j(1 + \tau_{lj})}z$$
$$n_{j,*}(z) = (\sigma - 1)\frac{\pi_{xj} - \pi_{dj}}{w_j(1 + \tau_{lj})}z$$

Labor used in production is

$$N_{pj} = (\sigma - 1)\left[\frac{\pi_{dj}}{w_j(1 + \tau_{lj})}\int_1^\infty z\mu_j(dz) + \frac{\pi_{xj} - \pi_{dj}}{w_j(1 + \tau_{lj})}\int_{z_{xj}}^\infty z\mu_j(dz)\right]$$

From trade balance,

$$\sigma(\pi_{xj} - \pi_{dj}) \int_{z_{xj}}^{\infty} z\mu(dz) = \sigma \frac{\pi_{dj}}{(w_j(1+\tau_{lj}))^{1-\sigma}} X^* \tag{25}$$

where $X^* = (1+\tau_x^*)^{1-\sigma}(w^*(1+\tau_l^*))^{1-\sigma} \int_{z_x^*}^{\infty} z\mu^*(dz)$. The left hand side of equation (25) is exports and the right hand side is imports. X^* is supply of foreign goods, which we take as given following the small open economy assumption. We can rewrite total production labor as

$$N_{pj} = (\sigma - 1)\frac{\pi_{dj}}{(w_j(1+\tau_{lj}))} \left[\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^* \right]$$

Since $\pi_{dj} = Q_j P_j^{\sigma} (w_j(1+\tau_{lj}))^{1-\sigma} \sigma^{-\sigma} (\sigma-1)^{\sigma-1}$.

$$\begin{aligned}
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma} \frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}} \left[\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^* \right] \\
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma} \frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}} \tilde{Z}_j
\end{aligned}$$

where $\tilde{Z}_j = \int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^*$.

Next consider the price P_j. By definition,

$$\begin{aligned}
P_j^{1-\sigma} &= \int_1^{\infty} p_j(z)^{1-\sigma} + (1+\tau_x^*)^{1-\sigma} \int_{z_x^*}^{\infty} p^*(z)^{1-\sigma} \\
&= \left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\right)^{1-\sigma} \left(\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^* \right) \\
&= \left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\right)^{1-\sigma} \tilde{Z}_j
\end{aligned}$$

Thus,

$$
\begin{aligned}
N_{pj} &= \left(\frac{(\sigma-1)}{\sigma}\right)^{\sigma} \frac{Q_j P_j^{\sigma}}{(w_j(1+\tau_{lj}))^{\sigma}} P^{1-\sigma} \left(\frac{\sigma}{\sigma-1}(w_j(1+\tau_l j))\right)^{\sigma-1} \\
&= \frac{(\sigma-1)}{\sigma} \frac{Q_j P_j}{w_j(1+\tau_{lj})} \\
&= \frac{(\sigma-1)}{\sigma} \frac{Q_j}{w_j(1+\tau_{lj})} \frac{\sigma}{\sigma-1}(w_j(1+\tau_{lj}))\tilde{Z}_j^{\frac{1}{1-\sigma}} \\
&= Q_j \tilde{Z}_j^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Rearranging,

$$
Q_j = Z_j N_{pj}
$$

where

$$
Z_j = \left[\int_1^{\infty} z\mu_j(dz) + (w_j(1+\tau_{lj}))^{\sigma-1} X^*\right]^{\frac{1}{\sigma-1}}
$$

Appendix E Proof of Proposition 5

When $\kappa_x = 0$, given the closed form solution for the variables in equilibrium derived in the main section of the paper, and using trade balance, wages are

$$
\begin{aligned}
w^{\sigma-1} &= \frac{\int z d\mu(z)}{X^*} \frac{\pi_x - \pi_d}{\pi_d} = \frac{\int z d\mu(z)}{X^*} \frac{w^{1-\sigma}\tilde{\pi}}{\pi_d} \Rightarrow w^{2(\sigma-1)} = \frac{\kappa_e}{\delta\kappa_I\kappa_e - \kappa_e^2} \frac{\tilde{\pi}}{X^*} \frac{1}{\pi_d} \\
w &= \left(\frac{\kappa_e}{\delta\kappa_I\kappa_e - \kappa_e^2} \frac{\tilde{\pi}}{X^*} \frac{1}{\pi_d}\right)^{\frac{1}{2(\sigma-1)}}
\end{aligned}
$$

To solve this, we need to know the value of π_d. Notice that

$$
\frac{\pi_x}{w} = \kappa_e \left(1 - \frac{\kappa_e}{2\kappa_I}\right) = \frac{\pi_d + w^{1-\sigma}\tilde{\pi}}{w}
$$

where $\tilde{\pi} = (1+\tau_x)^{1-\sigma}\frac{\pi^*}{w^{*1-\sigma}}$.

Introducing in this expression the value for w defines the following implicit function

$$\pi_d^{\frac{2\sigma-1}{2(\sigma-1)}}\left(\frac{(\delta\kappa_I-\kappa_e)X^*}{\tilde{\pi}}\right)^{\frac{1}{2(\sigma-1)}}+\pi_d^{\frac{\sigma}{2(\sigma-1)}}\left(\frac{(\delta\kappa_I-\kappa_e)X^*}{\tilde{\pi}}\right)^{\frac{\sigma}{2(\sigma-1)}}\tilde{\pi} = \kappa_e\left(1-\frac{\kappa_e}{2\kappa_I}\right)$$

Next we build towards showing that $\frac{\partial Z_x}{\partial \kappa_I} > \frac{\partial Z_c}{\partial \kappa_I}$. We cannot show it generally, but we can find a sufficient condition for this to happen. This condition is that $\sigma < 3/2$.

We first show that $\frac{\partial \frac{\pi_x}{\pi_d}}{\partial \kappa_I} < 0$, which, by proposition 4, implies that $\frac{\partial Z_x}{\partial \kappa_I} > \frac{\partial Z_c}{\partial \kappa_I}$. To show $\frac{\partial \frac{\pi_x}{\pi_d}}{\partial \kappa_I} < 0$ we proceed by contradiction. Thus, we show that if $\frac{\partial \frac{\pi_x}{\pi_d}}{\partial \kappa_I} \geq 0$, then it must be the case that $\frac{\partial \pi_d}{\partial \kappa_I} \geq 0$. But we also show that under our sufficient condition this cannot happen. We start by showing this last result, and then the main proposition.

Lemma 6 *If* $\sigma < 3/2$

$$\partial\pi_d/\partial\kappa_I < 0$$

Proof: Using the implicit function theorem, we show that if $\sigma < 3/2$ then $\partial\pi_d/\partial\kappa_I < 0$. Notice, this is a *sufficient* condition, but it will help us prove that $\partial\pi_x/\pi_d/\partial\kappa_I > 0$.

Define

$$\hat{\pi}_d = \pi_d^{\frac{1}{2(\sigma-1)}}$$

Then the equation that defines $\hat{\pi}_d$ is

$$F = \hat{\pi}_d^{2\sigma-1}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{1}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{1}{2(\sigma-1)}}+\hat{\pi}_d^{\sigma}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{\sigma}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{\sigma}{2(\sigma-1)}}\tilde{\pi}-\kappa_e+\frac{\kappa_e^2}{2\kappa_I} = 0 \tag{26}$$

The implicit function theorem says

$$\frac{\partial\hat{\pi}_d}{\partial\kappa_I} = -\frac{\frac{\partial F}{\partial\kappa_I}}{\frac{\partial F}{\partial\hat{\pi}_d}}$$

It is easy to check that $\frac{\partial F}{\partial \hat{\pi}_d} > 0$. So we need to check that $\frac{\partial F}{\partial \kappa_I} > 0$.

$$\frac{\partial F}{\partial \kappa_I} = \frac{\delta\left[\hat{\pi}_d^{2\sigma-1}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{1}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{1}{2(\sigma-1)}} + \sigma\hat{\pi}_d^{\sigma}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{\sigma}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{\sigma}{2(\sigma-1)}}\tilde{\pi}\right]}{2(\sigma-1)(\delta\kappa_I-\kappa_e)} - \frac{\kappa_e^2}{2\kappa_I^2} >$$

$$\frac{\delta\left[\hat{\pi}_d^{2\sigma-1}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{1}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{1}{2(\sigma-1)}} + \hat{\pi}_d^{\sigma}\left(\frac{X^*}{\tilde{\pi}}\right)^{\frac{\sigma}{2(\sigma-1)}}(\delta\kappa_I-\kappa_e)^{\frac{\sigma}{2(\sigma-1)}}\tilde{\pi}\right]}{2(\sigma-1)(\delta\kappa_I-\kappa_e)} - \frac{\kappa_e^2}{2\kappa_I^2} =$$

$$\frac{\delta\left[\kappa_e(1-\frac{\kappa_e}{2\kappa_I})\right]}{2(\sigma-1)(\delta\kappa_I-\kappa_e)} - \frac{\kappa_e^2}{2\kappa_I^2} = \frac{\delta}{2(\sigma-1)(\delta\kappa_I-\kappa_e)}\frac{\pi_x}{w} - \frac{g_x^2}{2}$$

The first term on the third line comes from rearranging the expression F. The second term comes from the expressions derived previously for π_x/w and the equilibrium value for g_x.

Multiplying the equation by κ_I gives

$$\frac{\delta\kappa_I}{2(\sigma-1)(\delta\kappa_I-\kappa_e)}\frac{\pi_x}{w} - \frac{\kappa_I g_x^2}{2} = \frac{\pi_x}{w}\left(\frac{\delta\kappa_I}{2(\sigma-1)(\delta\kappa_I-\kappa_e)} - 1\right) > \frac{\pi_x}{w}\left(\frac{1}{2(\sigma-1)} - 1\right) > 0$$

$\square$

We use the lemma for the proof of the proposition. The proposition says

$$\frac{\partial Z_x}{\partial \kappa_I} > \frac{\partial Z_c}{\partial \kappa_I}$$

To prove it, we proceed by contradiction. So suppose this is not true. Then if $\frac{\partial Z_x}{\partial \kappa_I} \leq \frac{\partial Z_c}{\partial \kappa_I}$ it must be the case that

$$\frac{\partial \frac{\pi_x}{\pi_d}}{\partial \kappa_I} \leq 0$$

From the definition of π_x,

$$\frac{\pi_x}{\pi_d} = 1 + \frac{w^{1-\sigma}}{\pi_d}\tilde{\pi} \Rightarrow \frac{\partial \frac{w^{1-\sigma}}{\pi_d}}{\partial \kappa_I} \leq 0$$

From trade balance,

$$\frac{Exports}{Imports} = \frac{\pi_x - \pi_d}{\pi_d}\frac{Z_c}{X^*}w^{1-\sigma} = 1$$

We know that $\frac{\partial \frac{\pi_x - \pi_d}{\pi_d}}{\partial \kappa_I} \leq 0$ and $\frac{\frac{Z_c}{X^*}}{\partial \kappa_I} < 0$. Then we must have $\frac{\partial w^{1-\sigma}}{\partial \kappa_I} > 0$. Since $\partial \frac{\frac{w^{1-\sigma}}{\pi_d}}{\partial \kappa_I} < 0$, this implies that $\frac{\partial \pi_d}{\partial \kappa_I} > 0$, which is a contradiction.

Appendix F Fit of the Approximation

Recall that our solution for the non exporter growth rate involves a differential equation with no closed form solution. Since we need a closed form to derive the distribution of firms, we approximate the non exporter with the following functional form

$$g_{di}(z) = (a_i + b_i z + c_i z^2 + d_i z^3)^{-1}$$

In this section, we discuss the goodness of this fit. Table 13 shows the values we compute for the variables a, b, c, and d for each country. Figures 12 through 16 show how good this approximation is for the growth rates and the non exporter value function.

Table 13: Fitted Values

Country	a	b	c	d
France	39.93	44.58	-54.27	14.69
Germany	23.39	66.24	-62.75	15.83
Italy	-20.83	275.59	-296.97	91.45
Spain	36.33	50.34	-56.99	15.17
U.K.	42.65	31.97	-47.01	13.80

Figure 12: France

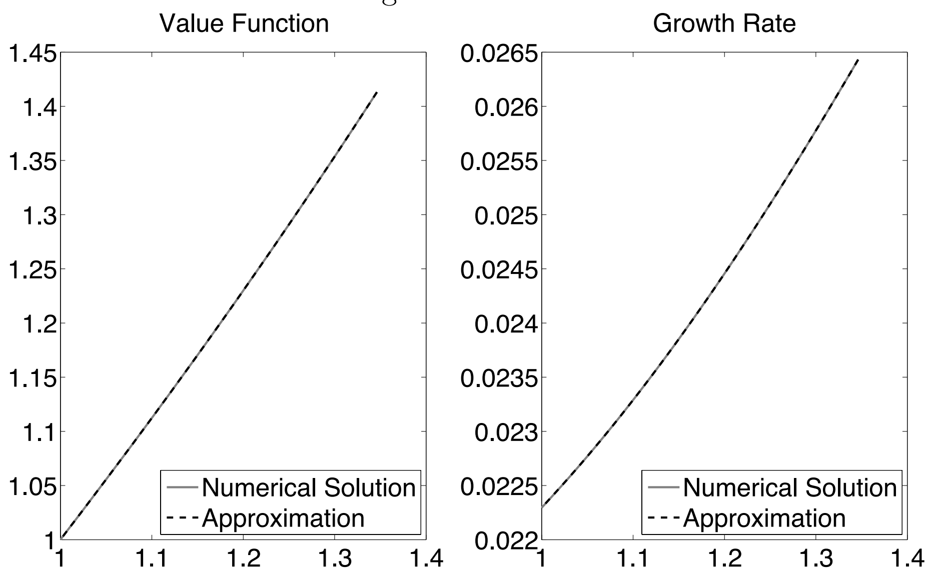


Figure 13: Germany

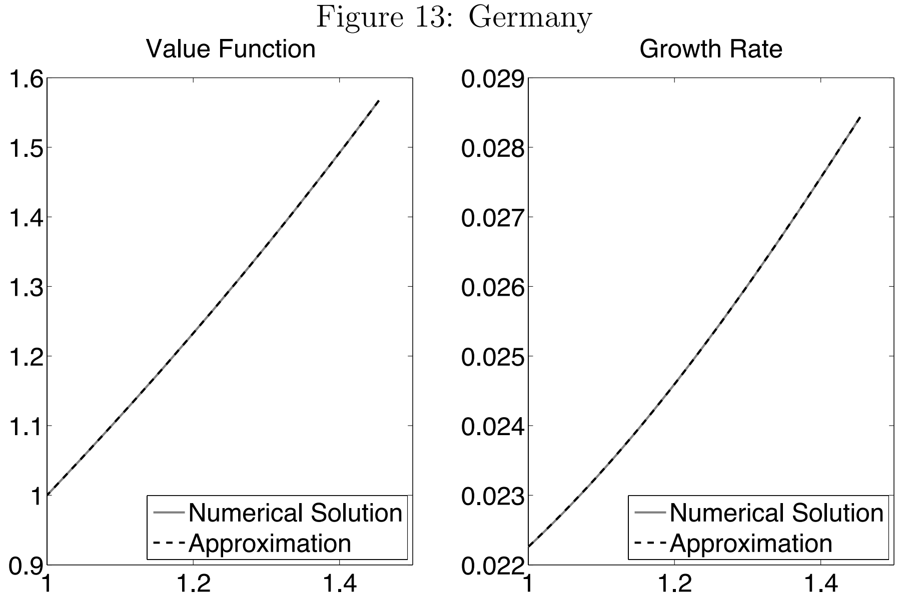


Figure 14: Italy

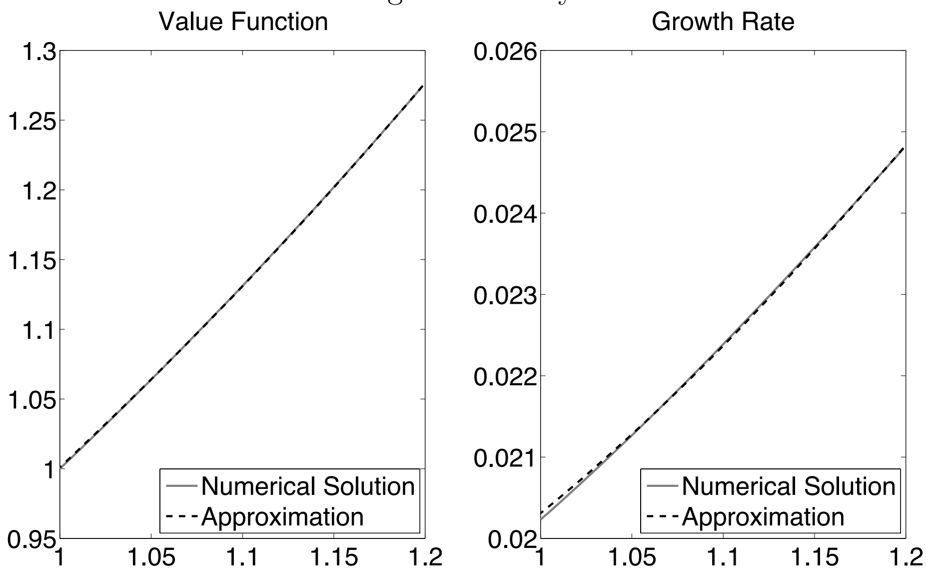


Figure 15: Spain

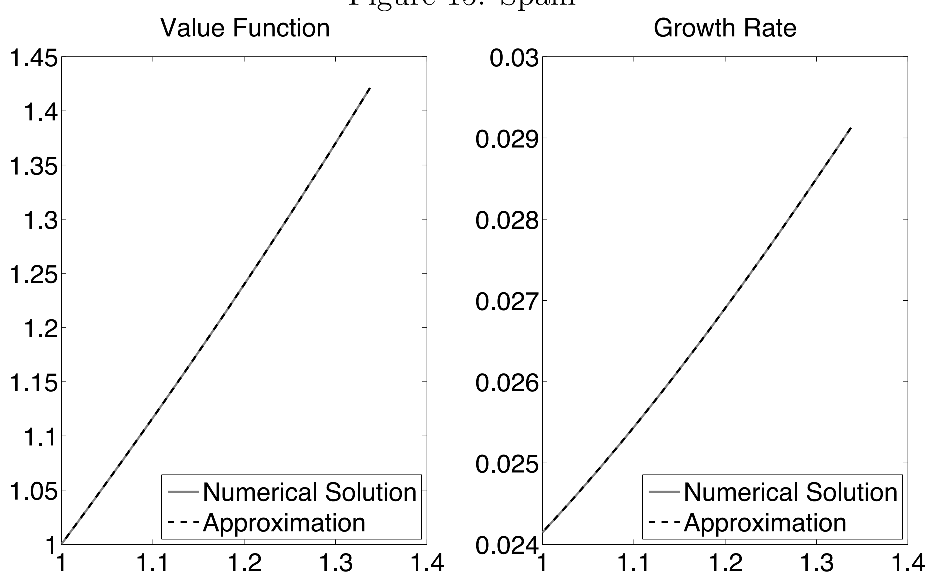
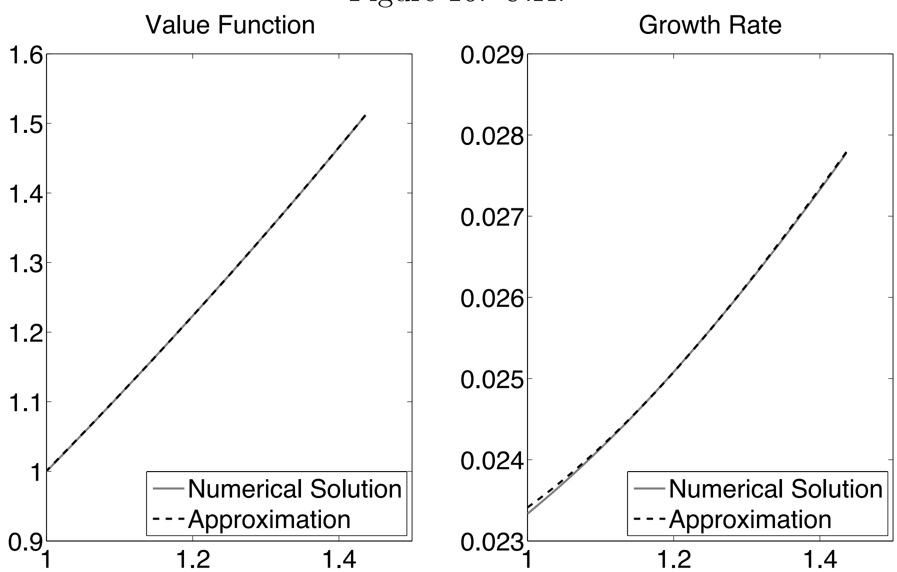


Figure 16: U.K.



Value Function
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
1
1.2
1.4
Numerical Solution
Approximation
Growth Rate
0.029
0.028
0.027
0.026
0.025
0.024
0.023
1
1.2
1.4
Numerical Solution
Approximation