

# Matching, Sorting and Wages

Jeremy Lise\*      Costas Meghir†      Jean-Marc Robin‡

June 30, 2009

Preliminary and Incomplete

## Abstract

We develop an empirical search-matching model with productivity shocks so as to analyze policy interventions in a labour market with heterogeneous agents. To achieve this we develop an equilibrium model of wage determination and employment, which is consistent with key empirical facts. As such our model extends the current literature on equilibrium wage determination with matching and provides a bridge between some of the most prominent macro models and microeconomic research. The model incorporates long-term contracts, on-the-job search and counter-offers, and a vacancy creation and destruction process linked to productivity shocks. Importantly, the model allows for the possibility of assortative matching between workers and jobs, a feature that had been ruled out by assumption in the empirical equilibrium search literature to date.

---

\*University College London and IFS

†University College London and IFS

‡Paris School of Economics and University College London

# 1 Introduction

There has been a growing literature on equilibrium pay determination and employment in the presence of labour market frictions. Such models provide an ideal setting for understanding the impact of various labour market policies such as employment protection legislation and the minimum wage. Indeed, search frictions themselves may provide a rationale as to why such policies may be needed.<sup>1</sup> Moreover, models of wage determination and employment with heterogeneous agents offer the possibility of analysing both the overall welfare and employment effects of policy as well as their distributional impacts, leading to a better understanding of who gains and who pays for such policies. However, existing models do not allow for a number of features that when combined can have important implications and can change our views on the merits and costs of such policies. First, many of these models ignore the role of shocks to productivity; however employment protection legislation is supposed to help shield workers from the impact of such shocks. Second, most of existing models ignore the possibility of complementarities and the potential for sorting in the labour market, with better firms hiring better workers. Policy, might eliminate matches of low quality firms and the cost of this will very much depend on the type of worker likely to be hired by such firms. A model that allows for sorting such as ours can address such issues. Finally, many such models ignore on the job search. This can have very important implications on job acceptance behaviour.

While the potential importance of such elements have been recognised before,<sup>2</sup> bringing them together in an equilibrium model of pay determination and employment has not been done. Achieving this is the first step towards reconciling the rich models of wage and earnings dynamics that have been estimated in the literature<sup>3</sup> with structural models of search that allow for heterogeneous agents providing a realistic setting for understanding employment and wage dynamics.<sup>4</sup>

The empirical work on wage dynamics has offered important insights on how wages evolve over time for individuals. Some of this work has emphasised the stochastic evolution of wages<sup>5</sup>,

---

<sup>1</sup>On the minimum wage with frictions see for example Flinn (2006).

<sup>2</sup>Topel and Ward (1992), Altonji and Shakotko (1987), Altonji and Williams (1997), Adda, Dustmann, Meghir and Robin (2009), Smimer and Smith (2000)

<sup>3</sup>Lillard and Willis (1978), MaCurdy(1982), Abowd and Card (1989), Meghir and Pistaferri (2004).

<sup>4</sup>Postel-Vinay and Robin (2002), Cahuc, Postel-Vinay and Robin (2006), Shimer and Smith (2000), Melo (2008).

<sup>5</sup>See Lillard and Willis (1978), MaCurdy (1982), Abowd and Card (1989), Meghir and Pistaferri (2005)

while other work has considered carefully wage growth over the lifecycle including the role of experience, tenure and job mobility.<sup>6</sup> Most of this work is essentially reduced form, in that there is little explanation of how the stochastic components arise or how wages are determined and the match surplus, if it exists, gets shared between workers and firms. As a result these empirical studies offer rich descriptions of wages, but do not allow us to assess the impact of policies in all but the most restrictive labour market frameworks. Essentially we do not have a theoretical framework that can explain the empirical facts and justify the complex statistical models of wages fitted to the data. In parallel a rich literature has developed on equilibrium wage distributions with some degree of heterogeneity in workers and/or jobs. However, this literature is not capable of accounting for the rich wage dynamics and for a number of key issues such as sorting; it does not provide a framework that is rich enough to explain what we observe about wages. Our aim in this paper is to take the first steps in bridging this gap offering a model of wages with stochastic shocks, which is consistent with equilibrium wage determination, when jobs and workers are heterogeneous and there are search costs.

Thus the key ingredients of our model are: Workers differ from each other according to a productivity relevant characteristic. Firms are also heterogeneous and their productivity is subject to possibly persistent shocks - this will lead to stochastic shocks to wages. The production function allows for complementarity between worker and job characteristics, leading to the possibility of sorting in the labour market. Jobs can decide whether to remain completely idle, or post a vacancy. They hire any worker who leads to a positive surplus - this accounts of course for the option value of keeping the vacancy. Finally, there are search costs: workers receive job offers while unemployed or employed at exogenously given rates (that may differ across these states). We solve this model and derive the implied dynamics of wages and the cross sectional wage distribution, as well as the distribution of matches.

Our model offers both an empirical framework for understanding wage determination and as a result offers a way for evaluating the impact of labour market regulation, such as the minimum wages or restrictions on firing. In a search framework protecting workers from being fired can have ambiguous effects on employment. Our framework will allow this effect to be quantified. But, as important, it allows us to analyse the effect of regulation on the distribution of wages and

---

<sup>6</sup>see Topel (1991), Topel and Ward (1992), Altonji and Shakotko (1987), Altonji and Williams (1997), Abowd, Kramarz and Margolis (1999), Dustmann and Meghir (2005) amongst others.

profits and thus showing who pays and who benefits from such a policy in this non-competitive environment.

Our paper draws from the literature on matching and assignment models (Sattinger, 1995) as well as on the literature on equilibrium wage distributions in a search environment (Mortensen and Pissarides, 1994 and Burdett and Mortensen, 1998). Matching models of the labour market have become standard in the macroeconomic literature since the seminal works of Diamond (1982), Mortensen (1982) and Pissarides (1990). Moreover, it is now well understood that search models can give rise to wage dispersion even if workers are homogeneous (see Burdett and Judd, 1983 and Burdett and Mortensen, 1998). However, matching models with heterogeneous workers and jobs is a relatively new topic of interest fueled by the need to understand dispersion of wages of similar individuals. In general, workers differ by the numbers of years of education and experience, and jobs differ by the type of industry. There is thus an enormous amount of differences between workers and between jobs that are not accounted for by observables in the data. Marriage models with heterogeneous agents in a frictional environment are studied in Sattinger (1995), Lu and McAfee (1996), Shimer and Smith (2000), and Atakan (2006).<sup>7</sup> To the best of our knowledge, there have not yet been any empirical applications of assignment models with transferable utility in a frictional environment with heterogeneous agents.

There is a large body of empirical evidence showing that wages differ across industries, thus indicating that a matching process is at work in the economy (see for example Krueger and Summers, 1988). Static, competitive equilibrium models of sorting (Roy models) have been estimated by Heckman and Sedlacek (1985) and Heckman and Honore (1990), while Moscarini (2001) and Sattinger (2003) explore theoretical extensions of Roy models with search frictions.

How much sorting is there with respect to these unobserved characteristics? Abowd, Kramarz and Margolis (2000; AKM) and Abowd, Kramarz, Lengermann and Roux (2003) use French and U.S. matched employer-employee data to estimate a static, linear log wage equation with employer and worker fixed effects (by OLS). They find a small, and if anything negative, cross-sectional correlation between job and worker fixed effects. Abowd, Kramarz, Lengermann and Perez-Duarte (2004) document the distribution of these correlations calculated within industries.

---

<sup>7</sup>Sattinger develops a framework but does not prove the existence of an equilibrium. Lu and McAfee prove the existence for a particular production function ( $f(x, y) = xy$ ). Shimer and Smith prove the existence of an equilibrium in a more general setup and derive sufficient conditions for assortative matching. Atakan shows that Becker's (1973) complementarity condition for positive sorting is sufficient if there exist explicit search costs.

In the U.S. 90% of these correlations range between -15% and 5%, and in France between -27% and -5%. These negative numbers, although hard to interpret, offer *prima facie* evidence of no positive sorting. However, the evidence based on the log-linear decomposition used by AKM should not be interpreted as evidence that there is no sorting: the person and firm effects which are estimated from the linear log wage equation are complicated transformations of the underlying individual-specific, unobserved productivity-relevant characteristics; A structural model is thus required to recover the true underlying joint distribution of characteristics. Abowd *et al.* present some evidence that a matching model inspired from Shimer (2005) could both generate sorting on unobservables and the sort of empirical regularity that they find. More recently, Melo (2008) has proposed a matching model, extending Lu and McAfee (1996) and Shimer and Smith (2000) to allow for on-the-job search, that also produces the same prediction. Our framework allows us to investigate empirically whether sorting actually is important in practice.

In many ways, our model is similar to Mortensen and Pissarides's (1994) model. Workers and jobs meet at random at a frequency that depends on some matching function and productivity shocks are responsible for endogenous lay-offs. As in equilibrium search models, on-the-job search generates both job-to-job mobility and wage dispersion, and we follow Postel-Vinay and Turon's (2009) extension of Postel-Vinay and Robin's (2002) sequential auction model to model offers and counter offers and contract renegotiation upon productivity shocks.

## 2 An Overview of the Model

In the economy there is a fixed number of individuals and a fixed number of jobs or production lines. Individuals may be matched with a job and thus working, or they may be unemployed job seekers. Jobs on the other hand may be in three different states. First they may be matched with a worker, in which case output is produced. Second, they may be vacant and waiting for a suitable worker to turn up. Finally jobs may be inactive, and thus potential entrants in the labour market. Individuals all have different levels of human capital, indexed by  $x$ . Jobs on the other hand also differ from each other according to some productivity relevant characteristic  $y$ . Output depends on the characteristics of both sides with possible complementarities. Crucially though, productivity follows a first order Markov process, which leads to the value of the match changing,

with consequences for wage dynamics, worker mobility, job creation, and job destruction that are at the centre of our model.

Individuals maximise their discounted income over an exponential lifetime; jobs maximise profits. When a job and a worker meet and the total match surplus is positive, the worker is hired and thus the match is formed. At this point the worker is paid a wage consistent with the reservation value of the best alternative option, *plus* a share of the excess match surplus. This process is discussed in detail in the next section.<sup>8</sup> A further important feature is that a shock to job productivity may trigger a wage renegotiation. This will happen if under then new productivity the match surplus remains positive but the wage is too high under the new conditions. On the other hand there is no incentive to renegotiate when there is a shock increasing the surplus, but as we shall show this will increase the value of being employed in this job because of the prospect of future wage increases. Finally we close the model by a free entry condition: all production lines, whether active or not have a productivity relevant parameter, which they know. This determines whether they will want to enter the market and post a vacancy. The marginal job has zero surplus from entering the market and posting a vacancy.

### 3 The Formal Description of the Model

#### 3.1 Setup

Each individual worker is characterised by a permanent productivity relevant characteristic which we denote by  $x$ . The agents all observe  $x$ . We assume that  $x$  has continuous and bounded support defined by  $[\underline{x}, \bar{x}]$ .<sup>9</sup> The measure (number) of workers with productivity  $x$  in the population is  $\ell(x)$ . There are  $\bar{L}$  individuals of which  $\bar{U}$  are unemployed. We also denote by  $u(x)$  the (endogenous) measure of  $x$  among the unemployed.

Jobs are characterised by a productivity parameter  $y$  with continuous and bounded support  $[\underline{y}, \bar{y}]$ . The (stationary) measure of job productivity in the population of jobs, whether vacant, matched, or inactive is denoted by  $n(y)$ . There are  $\bar{N}$  jobs in the economy and the (endogenous) measure of vacant posts is  $v(y)$ . The number of vacancies is denoted by  $\bar{V}$ . The number of

---

<sup>8</sup>In parallel work Lentz (2008) considers a model of on-the-job search with endogenous search intensity, where all workers match with any job when transiting from unemployment and sorting is the result of differing returns to search effort by worker type.

<sup>9</sup>Continuity and boundedness are not important assumptions.

inactive posts, i.e. potential posts for which jobs have not advertised a vacancy, is  $\bar{I}$ . The endogenous measure of  $y$  among these posts is  $i(y)$ .

In a given job,  $y$  fluctuates according to a jump process.  $\delta$  is the instantaneous arrival rate of jumps and  $q(y'|y)$  is the (Markov) transition probability for  $y$ .

A match between a worker  $x$  and a job  $y$  produces a flow of output  $f(x, y)$ ; this allows for the possibility that  $x$  and  $y$  are complementary in production, implying that sorting will increase total output.

We denote the measure of existing matches by  $h(x, y)$ . We can relate the density of individual productivities to the density of active matches as well as the density of productivities for the unemployed by

$$\int h(x, y) dy + u(x) = \ell(x). \quad (1)$$

Similarly we can write an equivalent relationship between the distribution of job productivities, active matches, unfilled vacancies and inactive jobs

$$\int h(x, y) dx + v(y) + i(y) = n(y). \quad (2)$$

In both cases the relationship is essentially an accounting identity. Finally, matches can end both endogenously, as we characterise later, and exogenously. We denote by  $\xi$  the rate at which workers retire.

We now discuss the process by which workers get to know about vacant jobs. We assume that the unemployed workers search for work at a fixed intensity  $s_0$ . The search intensity for an employed worker is  $s_1$ . The process of search leads to a total number of meetings, that as usual depends on the number of posted vacancies as well as on the number of total searchers in the economy, weighted by their search intensities. This matching function is denoted by  $M(s_0\bar{U} + s_1(\bar{L} - \bar{U}), \bar{V})$ . We define the equilibrium parameter

$$\kappa = \frac{M(s_0\bar{U} + s_1(\bar{L} - \bar{U}), \bar{V})}{[s_0\bar{U} + s_1(\bar{L} - \bar{U})]\bar{V}}. \quad (3)$$

Then  $s_0\kappa v(y)$  and  $s_1\kappa v(y)$  are the rates at which unemployed and employed workers of any type contact vacancies of type  $y$ . Symmetrically,  $s_0\kappa u(x)$  and  $s_1\kappa h(x, y')$  are the rates at which a job of any type contacts a worker of type  $x$ , either unemployed or currently employed at a job

of type  $y'$ .

### 3.2 Match formation and rent sharing

Define  $P(x, y)$  as the present value of all that the worker and the firm will produce together or separately earn in the future. For the time being, we assume that this value only depends on the partners' characteristics  $x$  and  $y$ . We shall later verify that this is indeed the case. Let  $W_0(x)$  denote the present value of unemployment for a worker with characteristic  $x$ , and let  $\Pi_0(y)$  denote the present value of a vacancy. We can define the "surplus" of an  $(x, y)$  match as

$$S(x, y) = P(x, y) - \Pi_0(y) - W_0(x). \quad (4)$$

Feasible matches  $(x, y)$  are such that  $S(x, y) \geq 0$ . By convention, production is better than inactivity.

When an unemployed worker  $x$  finds a vacant job  $y$  a match is thus formed if and only if  $S(x, y) \geq 0$ , and the surplus is split according to Nash bargaining:

$$W_1^*(x, y|0) - W_0(x) = \beta S(x, y). \quad (5)$$

We assume that incumbent employers match outside offers. A negotiation game is then played between the worker and both jobs as in Cahuc, Postel-Vinay and Robin (2006). If a worker  $x$ , currently paired to a job  $y$  such that  $S(x, y) \geq 0$ , finds an alternative job  $y'$  such that  $S(x, y') > S(x, y)$ , the worker moves to the alternative job and the new employer signs with the worker a contract that is worth the value of the total surplus of the previous  $(x, y)$  match plus a share  $\beta$  of the quasi-rent at the new match:

$$\begin{aligned} W_1^*(x, y'|y) - W_0(x) &= S(x, y) + \beta [S(x, y') - S(x, y)] \\ &= \beta S(x, y') + (1 - \beta) S(x, y) \end{aligned} \quad (6)$$

Next, consider the case where  $W_1 - W_0(x) < S(x, y') \leq S(x, y)$ , where  $W_1 > W_0(x)$  is the value to the worker of the current wage contract that  $x$  and  $y$  have agreed upon. The worker uses the external offer to obtain a wage rise, increasing the value of being employed at this job



(over and above the unemployment value) to  $W_1^*(x, y|y') - W_0(x)$ , where  $W_1^*(x, y|y') - W_0(x)$  is as defined in equation (??). Finally, if  $S(x, y') \leq W_1 - W_0(x)$ , the worker has nothing to gain from the competition between  $y$  and  $y'$  and the wage does not change.

Note that the present value of the new wage contract  $W_1^*(x, y'|y)$  does not depend on the last wage contracted with the incumbent employer, but only on the total surplus of the previous match. The continuation value for workers when the match is destroyed, by the worker moving to unemployment or an alternative job, is not a function of the last negotiated contract. This is the fundamental reason why the total output  $P(x, y)$  and the total surplus  $S(x, y)$  are only functions on  $x$  and  $y$ . Therefore, a simple rent splitting mechanism applies. This follows from the Bertrand competition that is engendered by employees' search on the job which disconnects the poached employees' outside option from both the value of unemployment and their current wage contract. Shimer (2005) provides an analysis where incumbent employers do not match outside offers and where the current wage determines the new contract.

This allows us to define matching sets, i.e. sets of acceptable job/productivity levels that make a match preferable to the current state. Specifically:

- $\mathcal{M}_0(x)$  is the set of jobs  $y$  such that match  $(x, y)$  is feasible:

$$\mathcal{M}_0(x) = \{y \mid S(x, y) \geq 0\}. \quad (7)$$

- $\mathcal{M}_1(x, y)$  is the set of jobs  $y'$  such that match  $(x, y')$  can be formed and is preferred to the current match  $(x, y)$ :

$$\mathcal{M}_1(x, y) = \{y' \in \mathcal{M}_0(x) \mid S(x, y') > S(x, y)\}. \quad (8)$$

- $\mathcal{M}_2(w, x, y)$  is the set of  $y'$  such that match  $(x, y')$  does not produce a higher surplus than  $(x, y)$  but the competition for the worker yields a wage increase:

$$\mathcal{M}_2(w, x, y) = \{y' \in \mathcal{M}_0(x) \mid S(x, y) > S(x, y') > W_1(w, x, y) - W_0(x)\}, \quad (9)$$

where  $W_1(w, x, y)$  denotes the present value of wage contract  $w$ .

In what follows we will denote by  $\bar{A}$  the complement of a set  $A$ .

### 3.3 Renegotiation

Wages can only be renegotiated by mutual agreement. This will happen either when a suitable outside offer is made, or because a productivity shock reduced the value of the surplus sufficiently. Specifically, a productivity shock changes  $y$  to  $y'$ . If  $y'$  is such that  $S(x, y') < 0$ , the match is endogenously destroyed. The worker becomes unemployed and the job will either post a vacancy, or perhaps become idle and not seek to fill the position again. Suppose now that  $S(x, y') \geq 0$ . The value of the current wage contract becomes  $W_1(w, x, y')$ . If  $W_1(w, x, y') - W_0(x) \in [0, S(x, y')]$ , neither the worker nor the job has a credible threat to force renegotiation because both are better off with the current wage  $w$  being paid to the worker than walking away from the match. In this case there will be no renegotiation. If, however,  $W_1(w, x, y') - W_0(x) < 0$  or  $W_1(w, x, y') - W_0(x) > S(x, y')$  (with  $S(x, y') \geq 0$ ) then either the worker has a credible threat to quit or the job has a credible threat to fire the employee. In this case a new wage contract is negotiated. To define how the renegotiation takes place and what is the possible outcome we use a setup similar to that considered by MacLeod and Malcolmson (1993) and Postel-Vinay and Turon (2009). The new wage contract is such that it moves the current wage the smallest amount necessary to put it back in the bargaining set. Thus, a new wage  $w'$  is set such that  $W_1(w', x, y') - W_0(x) = 0$  if at the old contract  $W_1(w, x, y') - W_0(x) < 0$  and  $W_1(w', x, y') - W_0(x) = S(x, y')$  if at the old contract  $W_1(w, x, y') - W_0(x) > S(x, y')$ .

It is convenient for notational reasons to define the sets of  $y$  that are consistent with a particular wage contract  $x$ , giving a positive share to both parties, and the two cases where the resulting share of the job or the worker respectively is negative. Thus define:

- $\mathcal{C}(w, x)$  is the set of productivities  $y$  such that contract  $w$  is sustainable and thus no renegotiation takes place:

$$\mathcal{C}(w, x) = \{y \in \mathcal{M}_0(x) \mid 0 \leq W_1(w, x, y) - W_0(x) \leq S(x, y)\}.$$

- $\mathcal{C}^+(w, x)$  is the set of jobs  $y$  such that contract  $w$  gives the worker more than the whole surplus :

$$\mathcal{C}^+(w, x) = \{y \in \mathcal{M}_0(x) \mid S(x, y) < W_1(w, x, y) - W_0(x)\}.$$

- $\mathcal{C}^-(w, x)$  is the set of jobs  $y$  such that contract  $w$  gives the job more than the whole

surplus:

$$\mathcal{C}^-(w, x) = \{y \in \mathcal{M}_0(x) \mid W_1(w, x, y) - W_0(x) < 0\}.$$

### 3.4 Value Functions

The value functions of the agents have been kept implicit up to now. The next step in solving the model is thus to characterise the value functions of workers and jobs. These define the decision rules for each agent. Proceed by assuming that time is discrete. Within a period the timing of events is as follows: a worker may receive a retirement shock and a job may receive a productivity shock or a worker may contact a vacancy. The discount rate is denoted by  $r$ .

**Unemployed workers.** Unemployed workers are always assumed to be available for work at a suitable wage rate. Thus the present value of unemployment to a worker of type  $x$  is  $W_0(x)$ , which satisfies the option value equation:

$$\begin{aligned} (1+r)W_0(x) &= b(x) + (1-\xi)s_0\kappa \int_{\mathcal{M}_0(x)} W_1^*(x, y|0)v(y)dy + (1-\xi)(1-s_0\kappa V(\mathcal{M}_0(x)))W_0(x) \\ &= b(x) + (1-\xi)s_0\kappa\beta \int_{\mathcal{M}_0(x)} S(x, y)v(y)dy + (1-\xi)W_0(x). \end{aligned} \quad (10)$$

The second equality follows from our assumption that a worker, when first employed, has a wage determined by Nash bargaining, which means that whatever job they meet  $W_1^*(x, y|0) - W_0(x) = \beta S(x, y)$  (see equation ??). The integration is over all  $y$  that lead to feasible (positive surplus) matches for someone with human capital  $x$ .

**Vacant jobs.** Vacancies can be open or idle depending on whether the expected profit is greater or less than the cost of posting the vacancy. Define the present value of profits for an unmatched job meeting a worker with human capital  $x$ . Similarly  $\Pi_1^*(x, y|y')$  is the present value of profits for a job matched with a worker  $x$  who was poached from a firm of type  $y'$ . Based on

this notation, the present value of an open vacancy for a job with productivity  $y$  is

$$\begin{aligned}\Pi_0^{open}(y) &= -\frac{c}{1+r} + \delta \int \frac{\Pi_0(y') - \Pi_0(y)}{1+r} q(y'|y) dy' \\ &\quad + (1-\delta) s_0 \kappa \int_{\mathcal{M}_0^{-1}(y)} \frac{\Pi_1^*(x, y|0) - \Pi_0(y)}{1+r} u(x) dx \\ &\quad + (1-\delta) s_1 \kappa \iint_{\mathcal{M}_1(x, y)} \frac{\Pi_1^*(x, y|y') - \Pi_0(y)}{1+r} h(x, y') dx dy' + \frac{\Pi_0(y)}{1+r} \quad (11)\end{aligned}$$

where  $c$  is a per-period cost of keeping a vacancy open. In (??) the second term reflects the impact of a change in productivity from  $y$  to  $y'$ . The third term is the flow of benefits from matching with a previously unemployed worker; the integration is over the entire set of  $x$  that would lead to a positive surplus with a  $y$ -type job that is currently vacant. The fourth term is the flow of benefits from poaching a worker who is already matched with another job; the integration is over all possible  $y'$  that are less attractive to worker type  $x$  and would thus move to the job with type  $y$ . In the future, the firm behaves optimally and the value of a vacancy is  $\Pi_0(y) = \max\{\Pi_0^{open}(y), \Pi_0^{idle}(y)\}$ , i.e. the max of the value of an opened and an idle vacancy.

The job keeps a share  $(1-\beta)$  of the surplus in a new match and has to pay workers it poaches the surplus in the job they were poached from plus a share of the quazi rent in the new match. This gives

$$\begin{aligned}\Pi_1^*(x, y|0) - \Pi_0(y) &= (1-\beta) S(x, y) \geq 0, \\ \Pi_1^*(x, y|y') - \Pi_0(y) &= (1-\beta) [S(x, y) - S(x, y')] \geq 0,\end{aligned}$$

which after substitution in (??) we obtain:

$$\begin{aligned}(1+r)\Pi_0^{open}(y) &= -c + \delta \int [\Pi_0(y') - \Pi_0(y)] q(y'|y) dy' \\ &\quad + (1-\delta) s_0 \kappa (1-\beta) \int_{\mathcal{M}_0^{-1}(y)} S(x, y) u(x) dx \\ &\quad + (1-\delta) s_1 \kappa (1-\beta) \iint_{\mathcal{M}_1(x, y)} [S(x, y) - S(x, y')] h(x, y') dx dy' + \Pi_0(y) \quad (12)\end{aligned}$$

If the job decides to remain inactive, then it does not pay the cost of posting vacancies and has no chance of meeting a worker. Its present value only depends on future productivity draws,

which may lead them into posting a vacancy:

$$(1+r)\Pi_0^{idle}(y) = \delta \int \Pi_0(y')q(y'|y)dy' + (1-\delta)\Pi_0(y).$$

Note that a match  $(x, y)$  may yield a positive output  $f(x, y)$  but the cost of a vacancy exceeds the expected profit. Thus it is possible that a job which loses a worker to another job, just “closes down” rather than posting a new vacancy, until its potential productivity  $y$  increases. Combining the two ways in which a job may be unmatched, the present value of an unmatched job of type  $y$  is obtained by:

$$\begin{aligned} (1+r)\Pi_0(y) &= \max \left\{ (1+r)\Pi_0^{idle}(y), (1+r)\Pi_0^{open}(y) \right\} \\ &= \max \{0, -c + \underline{c}(y)\} + \delta \int \Pi_0(y')q(y'|y)dy' + (1-\delta)\Pi_0(y). \end{aligned} \quad (13)$$

with

$$\begin{aligned} \underline{c}(y) &\equiv (1-\delta)s_0\kappa(1-\beta) \int_{\mathcal{M}_0^{-1}(y)} S(x, y)u(x)dx \\ &\quad + (1-\delta)s_1\kappa(1-\beta) \iint_{\mathcal{M}_1(x, y)} [S(x, y) - S(x, y')]h(x, y')dxdy'. \end{aligned} \quad (14)$$

and a job  $y$  is inactive whenever

$$\Pi_0^{idle}(y) > \Pi_0^{open}(y)$$

or

$$c > \underline{c}(y). \quad (15)$$

This condition is like the free entry condition of a standard search-matching model with ex-ante homogeneous jobs; with heterogeneous firms it defines the marginal  $y$  opening vacancies.

**The match output and joint surplus** We can define the value of production from an  $(x, y)$  match as follows

$$\begin{aligned}
(1+r)P(x, y) &= f(x, y) + \xi(1-\delta)\Pi_0(y) + \xi\delta \int_{y'} \Pi_0(y') q(y'|y) dy' \\
&+ (1-\xi)\delta \int_{\mathcal{M}_0(x)} P(x, y') q(y'|y) dy' + (1-\xi)\delta \int_{\overline{\mathcal{M}}_0(x)} [W_0(x) + \Pi_0(y')] q(y'|y) dy' \\
&+ (1-\xi)(1-\delta)s_1\kappa \int_{\mathcal{M}_1(x, y)} [\Pi_0(y) + W_1^*(x, y'|y)] v(y') dy' \\
&+ (1-\xi)(1-\delta)(1-s_1\kappa V(\mathcal{M}_1(x, y))) \max\{P(x, y), W_0(x) + \Pi_0(y)\}. \quad (16)
\end{aligned}$$

The timing of events is as follows. At the end of a period the match produces  $f(x, y)$ . Then, several events can occur. The worker retires with probability  $\xi$ , leaving the job vacant. With probability  $\delta$ , the firm receives a productivity shock. If the surplus remains positive after this shock the value of production changes to  $P(x, y')$ , if the surplus becomes negative, the match is destroyed and the worker and job become unemployed and vacant respectively. If the worker doesn't retire and the job doesn't receive a productivity shock, the worker meets an outside firm with probability  $s_1\kappa V$ . If the worker meets a job with a higher surplus, ( $S(x, y') > S(x, y)$ ) she leaves the current match, obtain value  $W_1^*(x, y'|y)$ , and the job becomes vacant. If none of these event occur, and the current match value is non-negative, the match continues producing  $P(x, y) = S(x, y) + W_0(x) + \Pi_0(y)$ . If the current surplus is negative,  $P(x, y) < W_0(x) + \Pi_0(y)$ , the match splits at the end of the current period; the worker becomes unemployed and the job becomes vacant.

Using the expression,

$$W_1^*(x, y'|y) = W_0(x) + S(x, y)^+ + \beta [S(x, y') - S(x, y)^+]$$

where  $S(x, y)^+ = \max\{S(x, y), 0\}$ , and combining equations (??), (??), and (??) we can write

the surplus of any  $(x, y)$  match as the fixed point defined by

$$\begin{aligned}
(1+r)S(x, y) &= f(x, y) - b(x) - (1-\xi)s_0\kappa\beta \int_{\mathcal{M}_0(x)} S(x, y') v(y') dy' \\
&\quad - \max\{0, -c + \underline{c}(y)\} + (1-\xi)\delta \int_{\mathcal{M}_0(x)} S(x, y') q(y'|y) dy' \\
&\quad + (1-\xi)(1-\delta)s_1\kappa\beta \int_{\mathcal{M}_1(x, y)} [S(x, y') - S(x, y)^+] v(y') dy' \\
&\quad + (1-\xi)(1-\delta)S(x, y)^+. \quad (17)
\end{aligned}$$

The important point to note from this expression is that the surplus of an  $(x, y)$  match never depends on the wage; the Bertrand competition between the two jobs for the worker ensures this. As a result the Pareto possibility set for the value of the worker and the job is convex in all cases, implying that the conditions for a Nash bargain are satisfied. This contrasts with Shimer's (2006) model, where jobs do not respond to outside offers and where the actual value of the wage determines employment duration in a particular job.

**Employed workers** Let  $(x, y)$  characterise a viable match with  $S(x, y) \geq 0$ . Let  $W_1(w, x, y)$  denote the present value to the worker of a wage contract  $w$  for this match. In order to determine the wage which solves  $W_1(w, x, y) = W_1^*(x, y|y')$  we need to determine  $W_1(w, x, y)$  for any  $w$ . Note that there is no need to define  $W_1(w, x, y)$  for the case  $S(x, y) < 0$ . If a productivity shock moves  $y$  to  $y'$  such that  $S(x, y') < 0$  then the worker and the firm separate irrespective of the wage.

The timing of events is assumed to be as follows: after the wage  $w$  is paid at the end of the period. Then one of the three events may happen: the worker may die with probability  $\xi$ , or a productivity shock arrives, with probability  $(1-\xi)\delta$ , changing  $y$  into  $y'$  according to the probability density function  $q(y'|y)$ , or the worker may receive an offer from an alternative employer. The productivity shock may result in either an endogenous match destruction (think of it as a quit) or a wage renegotiation if the match is still viable, i.e. has positive surplus at  $(x, y')$ . A contact with an alternative employer that results in a job transition or a wage renegotiation occurs with probability  $(1-\xi)(1-\delta)s_1\kappa V(\mathcal{M}_1(x, y) \cup \mathcal{M}_2(w, x, y))$ , where, as defined above,  $s_1$  is the employed worker's search intensity, and  $\kappa V(\mathcal{M}_1(x, y) \cup \mathcal{M}_2(w, x, y))$  is the probability that the contacting job has productivity such that it can induce a wage

renegotiation or poach the worker. If none of these events happen, which occurs with probability  $(1 - \xi)(1 - \delta)(1 - s_1\kappa V(\mathcal{M}_1(x, y) \cup \mathcal{M}_2(w, x, y)))$ , it is optimal to renegotiate the wage if  $y \in \mathcal{C}^+(w, x) \cup \mathcal{C}^-(w, x)$ . Otherwise the match continues with the same wage  $w$ . The above can be formalised in the value function

$$\begin{aligned}
W_1(w, x, y) &= \frac{w}{1+r} + (1-\xi)\delta Q(\overline{\mathcal{M}}_0(x)|y) \frac{W_0(x)}{1+r} + (1-\xi)\delta \int_{\mathcal{C}(w,x)} \frac{W_1(w, x, y')}{1+r} q(y'|y) dy' \\
&+ (1-\xi)\delta \int_{\mathcal{C}^-(w,x)} \frac{W_0(x)}{1+r} q(y'|y) dy' + (1-\xi)\delta \int_{\mathcal{C}^+(w,x)} \frac{S(x, y') + W_0(x)}{1+r} q(y'|y) dy' \\
&+ (1-\xi)(1-\delta) s_1\kappa \int_{\mathcal{M}_2(w,x,y)} \frac{W_1^*(x, y|y')}{1+r} v(y') dy' + (1-\xi)(1-\delta) s_1\kappa \int_{\mathcal{M}_1(x,y)} \frac{W_1^*(x, y|y')}{1+r} v(y') dy' \\
&+ (1-\xi)(1-\delta)(1 - s_1\kappa V(\mathcal{M}_1(x, y) \cup \mathcal{M}_2(w, x, y))) \\
&\times \left[ \mathbf{1}_{\mathcal{C}(w,x)}(y) \frac{W_1(w, x, y)}{1+r} + \mathbf{1}_{\mathcal{C}^-(w,x)}(y) \frac{W_0(x)}{1+r} + \mathbf{1}_{\mathcal{C}^+(w,x)}(y) \frac{S(x, y) + W_0(x)}{1+r} \right].
\end{aligned}$$

where  $\mathbf{1}_A$  is the indicator function. After simplification, this becomes

$$\begin{aligned}
W_1(w, x, y) - W_0(x) &= \frac{w - (r + \xi)W_0(x)}{1+r} \\
&+ (1-\xi)\delta \int_{\mathcal{C}(w,x)} \frac{W_1(w, x, y') - W_0(x)}{1+r} q(y'|y) dy' + (1-\xi)\delta \int_{\mathcal{C}^+(w,x)} \frac{S(x, y')}{1+r} q(y'|y) dy' \\
&+ (1-\xi)(1-\delta) s_1\kappa \int_{\mathcal{M}_1(x,y)} \frac{\beta S(x, y') + (1-\beta)S(x, y)}{1+r} v(y') dy \\
&+ (1-\xi)(1-\delta) s_1\kappa \int_{\mathcal{M}_2(w,x,y)} \frac{\beta S(x, y) + (1-\beta)S(x, y')}{1+r} v(y') dy' \\
&+ (1-\xi)(1-\delta)(1 - s_1\kappa V(\mathcal{M}_1(x, y) \cup \mathcal{M}_2(w, x, y))) \\
&\times \left[ \mathbf{1}_{\mathcal{C}(w,x)}(y) \frac{W_1(w, x, y) - W_0(x)}{1+r} + \mathbf{1}_{\mathcal{C}^+(w,x)}(y) \frac{S(x, y)}{1+r} \right] \quad (18)
\end{aligned}$$

The Bellman equation defines  $W_1(w, x, y) - W_0(x)$  as a fixed point of a contracting operator. A simple iterative algorithm can be used to approximate the fixed point. Let  $W_1^0$  be an initial guess of  $W_1$ . Values  $\mathcal{C}(w, x)$ ,  $\mathcal{C}^+(w, x)$ ,  $\mathcal{M}_1(x, y)$ , and  $\mathcal{M}_2(w, x, y)$  follow from the initial value  $W_1^0$ . Then calculate an update of  $W_1(w, x, y) - W_0(x)$  using the previous equations. Knowledge of the value function  $W_1(w, x, y)$  will eventually allow us to compute the optimal wage contract  $w$  given  $(x, y, y')$ .



### 3.5 Steady-state flow equations.

To solve for equilibrium we need to define the steady state flow equations.

The total number of matches in the economy will be

$$\bar{L} - \bar{U} = \bar{N} - \bar{V} - \bar{I} = \iint h(x, y) dx dy. \quad (19)$$

Existing matches, characterised by the pair  $(x, y)$ , can be destroyed for a number of reasons. First, there is exogenous job destruction resulting from worker death, at rate  $\xi$ ; second, with probability  $\delta$ , the job component of match productivity changes to some value  $y'$  different from  $y$ , and the worker may move to unemployment or may keep the job; third, the worker may change job, with probability  $s_1 \kappa V(\mathcal{M}_1(x, y))$  – i.e., a job offer has to be made (at rate  $s_1 \kappa \bar{V}$ ) and has to be acceptable ( $y' \in \mathcal{M}_1(x, y)$ ). On the inflow side, new  $(x, y)$  matches are formed when some unemployed or employed workers of type  $x$  match with vacant jobs  $y$ , or when  $(x, y')$  matches are hit with a productivity shock and exogenously change from  $(x, y')$  to  $(x, y)$ . In a steady state all these must balance leaving the match distribution unchanged. Thus formally we have for all  $(x, y)$  such that the match is acceptable, i.e.  $y \in \mathcal{M}_0(x)$  or  $S(x, y) > 0$ :

$$\begin{aligned} [\xi + (1 - \xi) \delta + (1 - \xi) (1 - \delta) s_1 \kappa V(\mathcal{M}_1(x, y))] h(x, y) &= (1 - \xi) \delta \int q(y|y') h(x, y') dy' \\ &+ (1 - \xi) (1 - \delta) \left[ s_0 u(x) + s_1 \int_{\mathcal{M}_1(x, y)} h(x, y') dy' \right] \kappa v(y). \end{aligned} \quad (20)$$

This equation defines the steady-state equilibrium, together with the accounting equations for the workers:

$$u(x) = \ell(x) - \int h(x, y) dy, \quad (21)$$

and the jobs

$$v(y) + i(y) = n(y) - \int h(x, y) dx, \quad (22)$$

Noting that vacancies are posted only if costs are low enough (given productivity) we get that the density of vacancies is

$$v(y) = \begin{cases} n(y) - \int h(x, y) dx & \text{if } c \leq \underline{c}(y) \\ 0 & \text{if } c > \underline{c}(y), \end{cases} \quad (23)$$

and the density of inactive jobs is given by

$$i(y) = \begin{cases} 0 & \text{if } c \leq \underline{c}(y) \\ n(y) - \int h(x, y) dx & \text{if } c > \underline{c}(y), \end{cases} \quad (24)$$

The total number of vacancies  $\bar{V}$  is thus obtained as

$$\bar{V} = \int_{c \leq \underline{c}(y)} \left[ n(y) - \int h(x, y) dx \right] dy. \quad (25)$$

### 3.6 Equilibrium

In equilibrium all agents follow their optimal strategy and the steady state flow equations defined above hold. The exogenous parameters of the model are the number of workers and jobs  $\bar{L}/\bar{N}$ , the distribution of worker types and job productivities  $l(x)$  and  $n(y)$  respectively, the transition function for productivity dynamics  $q(y'|y)$ , the matching function  $M(s_0U + s_1(L - U), V)$  as well as the arrival rate of shocks,  $\delta$ , the retirement rate  $\xi$ , the search intensities for the unemployed,  $s_0$  and employed workers,  $s_1$ , the discount rate,  $r$ , the value of leisure  $b$ , the cost of posting a vacancy  $c$  bargaining power  $\beta$ , and the production function  $f(x, y)$ . The equilibrium is characterized by knowledge of the number of vacancies,  $\bar{V}$ , the joint distribution of active matches,  $h(x, y)$  and the surplus function  $S(x, y)$  obtained by solving simultaneously equations (??), (??) and (??). In these equations, we substitute for  $\bar{U}$  using equation (??),  $\kappa$  using equation (??),  $u(x)$  using equation (??),  $\underline{c}(y)$  using equation (??), and  $v(y)$  using equation (??).

### 3.7 Policy instruments

Our model provides a way of evaluating the employment and distributional impact of labour market regulation. It is ideally suited for understanding who pays and who benefits from such policies. Labor market regulation can consist of payments for the unemployed, increasing  $b(x)$ , which is already in the model. We also introduce three further policy instruments: experience rating, minimum wages, and severance pay. We model experience rating as a tax on endogenous separations. This can be accomplished by subtracting the term  $\delta Q(\bar{\mathcal{M}}_0(x) | y) \tau$  from equations (??) and (??).

Incorporating a minimum wage puts a constraint on the ability of workers and jobs to make

transfers. As a result, the condition for match feasibility will depend on the match surplus being high enough to cover a minimum wage contract, and still provide positive surplus to the job. Given this constraint, an  $(x, y)$  match is feasible if and only if  $S(x, y) \geq 0$  and  $\Pi_1(\underline{w}, x, y) \geq \Pi_0(y)$ . This requires defining the value to a filled job of an  $(x, y)$  match paying a wage  $\underline{w}$ . The matching sets defined in (??), (??) and (??) modified to incorporate the constraint become

$$\begin{aligned}\mathcal{M}_0^c(x) &= \{y \mid S(x, y) \geq 0 \text{ and } \Pi_1(\underline{w}, x, y) \geq \Pi_0(y)\}, \\ \mathcal{M}_1^c(x, y) &= \{y' \in \mathcal{M}_0^c(x) \mid S(x, y') \geq S(x, y)\}, \\ \mathcal{M}_2^c(w, x, y) &= \{y' \in \mathcal{M}_0^c(x) \mid S(x, y) \geq S(x, y') > W_1(w, x, y) - W_0(x)\}.\end{aligned}\tag{26}$$

The key practical difficulty is that the matching set for the unemployed  $\mathcal{M}_0^c(x)$  depends both on whether the surplus is positive and on the value of filling the vacancy at the minimum wage. Before  $S(x, y) > 0$  was sufficient to determine the feasible matches. For all matches that are feasible subject to the minimum wage constraint, the wage is determined as

$$w = \max\{\underline{w}, w^*\},$$

where  $w^*$  solves (??) with either (??) or (??) on the left hand side depending on whether the worker is hired away from another job or hired from unemployment.

Severance payments are modelled as a transfer from the job to the worker at the time of endogenous job destruction. This involves adding  $\delta Q(\overline{\mathcal{M}}_0(x) | y) s$  to equation (??) and subtracting  $\delta Q(\overline{\mathcal{M}}_0(x) | y) s$  from the expression for  $(1 + r) \Pi_1(\underline{w}, x, y)$ . The severance payment does not change the surplus of an  $(x, y)$  match. The effect it has on feasible matches occurs indirectly through the effect on  $\Pi_1(\underline{w}, x, y)$  which affects the constrained matching set  $\mathcal{M}_0^c(x)$ .

## 4 An Illustrative Numerical Example

In order to illustrate the properties and empirical implications of the model we solve and simulate under a particular set of parameters, for several production functions. For the numerical example we set  $\overline{N} = 1.25\overline{L}$  and assume that  $L(x)$  and  $N(y)$  are log-normally distributed. We solve

using two different production functions:  $xy$  and  $x + y$ . These production functions are natural representations of complementarity and substitutability. The process governing the technological evolution is represented by a Gaussian copula with persistence  $\rho = 0.9$  and marginals equal to  $n(y)$ ,  $q(y'|y) = n(y')\mathbb{C}(N(y'), N(y))$ ;  $\delta = 0.083$ ;  $M(s_0\bar{U} + s_1(\bar{L} - \bar{U}), \bar{V}) = \alpha(s_0\bar{U} + s_1(\bar{L} - \bar{U}))^\gamma \bar{V}^{1-\gamma}$ , with  $\alpha = 1$  and  $\gamma = 0.78$ ;  $r = 0.05$  (annualized);  $b(x) = 0$ ; and  $c = 0.1$ . We also assume that  $\beta = 0.5$ . In order to provide some comparability between the simulations, we calibrate  $s_0$ ,  $s_1$ , and  $\xi$  to obtain an unemployment rate of 7 percent, monthly job loss rate of 3 percent, monthly job changing rate of 6 percent and a monthly job finding rate (by the unemployed) of 42 percent. These numbers correspond to high-school educated white men in the US, and were calculated from the 1993 panel of the Survey of Income and Program Participation (SIPP).

The key aspects of the equilibrium surplus function,  $S(x, y)$ , are represented by the matching sets in panels (a) and (b) of Figures ?? and ?. In panel (a) we plot the equilibrium matching set, that is all feasible matches:  $\{(x, y) | S(x, y) \geq 0\}$ . The asymmetry of the matching set is the result of on-the-job search, endogenous match destruction resulting from technology shocks, and  $\bar{L} < \bar{N}$ . By way of comparison, if we set  $s_1 = 0$ ,  $\delta = 0$ ,  $c = 0$ ,  $\bar{N} = \bar{L}$ , and  $l(x) = n(y) \sim U[0, 1]$ , we have the environment studied in Shimer and Smith (2000), and replicate their Figure 1 (for the production function  $xy$ ) here as Figure ?. For production functions displaying complementarity, matches between high and low types do not occur in equilibrium, while with substitutability it is matches among low types that do not occur.

In panel (b) of Figures ?? and ? we illustrate the preferred matches, those that a worker would leave her current match for. The contour lines here mark the contours of the surplus function, with “hotter” colours indicating a higher match surplus. For a given worker  $x$ , and reading horizontally across the job types  $y$ , the process of on-the-job search will cause workers to move to matches with a higher surplus, increasing the degree of sorting above what is induced from the matching set  $\mathcal{M}_0(x)$  alone.

The features of the joint distribution of  $(x, y)$  matches,  $h(x, y)$  are displayed in panels (c) and (d) of the same figures. The process of job accepting; on-the-job search; endogenous and exogenous job destruction; and the decision of which types of jobs to post vacancies for or leave inactive induces the equilibrium distribution of matches. The effect of on-the-job search

is evident in that workers initially accept any job in their matching set, and then switch to jobs with a higher surplus. The vertical line that cuts through the distribution (seen for  $xy$  at  $y = 0.21$ ) illustrates the effect of the decision over which types of unmatched jobs to leave inactive and which to post as vacancies. Matches in which the job component,  $y$ , falls into this region due to a shock produce sufficient surplus that they are not endogenously destroyed. However, if the match was exogenously destroyed, the job would become inactive, rather than posted as a vacancy. The expected flow output associated with such a  $y$  is not sufficient to cover the expected posting costs required to obtain a new worker. In the illustration of  $xy$ , all matches begin with a  $y$  greater than 0.21. In panel (d) we plot the average worker (job) type matched to a given job (worker) type, further illustrating the positive sorting in the cases of  $xy$  and the negative/zero sorting in the case of  $x + y$  production.

In Figure (??) we plot the equilibrium distributions of wages and output. The general observation is that distributions of the observables, wages and output varies markedly across production technologies, which suggests identification of the production function may be feasible.

## 5 What might we learn about sorting from wages?

[Incomplete]

As discussed in the introduction, Abowd *et al* (1999) use a simple empirical measure of sorting that can be obtained by estimating a log-wage equation in which wages are a linear function of a worker fixed effect, a firm fixed effect, and an orthogonal worker-firm effect

$$\log(w_{it}) = z_{it}\beta + \alpha_i + \sum_{j=1}^J d_{it}^j \psi_j + u_{it}, \quad (27)$$

where  $z_{it}$  are time varying observables of workers,  $\alpha_i$  is a worker fixed effect,  $\psi_j$  is a firm fixed effect, and  $u_{it}$  is an orthogonal residual. The correlation between  $\hat{\alpha}_i$  and  $\hat{\psi}_{j(i)}$  in a given match is taken as an estimate of the degree of sorting.

To assess the degree to which the correlation between these estimated fixed effects is informative on the degree of sorting on type, we will conduct this exercise for each of the numerical examples considered in Section ???. Based on the results in Melo (2008) and our own simulations (not reported here) this estimated correlation is not necessarily informative on the degree of

sorting in the model.<sup>10</sup> Indeed, this suggests the need to estimate the production function in order to answer the question regarding the degree of sorting on unobservables.

## 6 Conclusion and further work

Estimation of the model presented here is the subject of current research. The natural type of data to use in the empirical implementation is matched worker and firm data, an avenue we are actively pursuing. One obstacle in this strategy is the need to take a stand on the formation of jobs into firms, and the possibility of interaction between workers within a firm. An additional interesting question is how much we can learn about earnings processes using standard panel data on workers and the restrictions from the model. The model predicts an earning process with lots of heterogeneity, in which the time varying part of earnings is dependent on the permanent component. In addition, job mobility and unemployment durations are dependent on the same underlying permanent component.

With an estimated version of the model in hand we will be well placed to evaluate important policy questions, such as employment protection legislation and minimum wages, within a coherent empirical economic model.

---

<sup>10</sup>In addition to this effect, Postel-Vinay and Robin (2006) note that in terms of asymptotics, OLS estimate of  $\beta$  is consistent as  $i \rightarrow \infty$  for fixed  $T$  and OLS estimates of  $\alpha$  and  $\psi$  are consistent when  $T \rightarrow \infty$  faster than  $I$  and  $J$ . In practice, the data contains millions of workers, tens of thousands of firms, and fewer than ten years. Indeed, empirical estimates of sorting which are based on worker and firm fixed effects introduce a negative bias, which will introduce a spurious negative correlation when calculating the correlation between worker and firm fixed effects. This is illustrated as follows; empirically,  $\beta$  and  $\psi_j$  are estimated from the within transformation

$$\log w_{it} - \overline{\log w_i} = (x_{it} - \bar{x}_i)\beta + \sum_{j=1}^J (d_{it}^j - \bar{d}_i^j)\psi_j + u_{it} - \bar{u}_i.$$

This makes it clear that we need to see workers change firm to identify the firm fixed effects  $\psi_j$ . The worker fixed effects are estimated as

$$\hat{\alpha}_i = \overline{\log w_i} - \bar{x}_i\hat{\beta} - \sum_{j=1}^J \bar{d}_i^j \hat{\psi}_j.$$

Notice, any statistical error affecting the estimate of the firm effect translates directly to the estimate of the worker effect, with a sign reversal. OLS estimates of firm and worker effects are likely to be imprecise and spuriously negatively correlated given short time dimension and limited worker mobility.

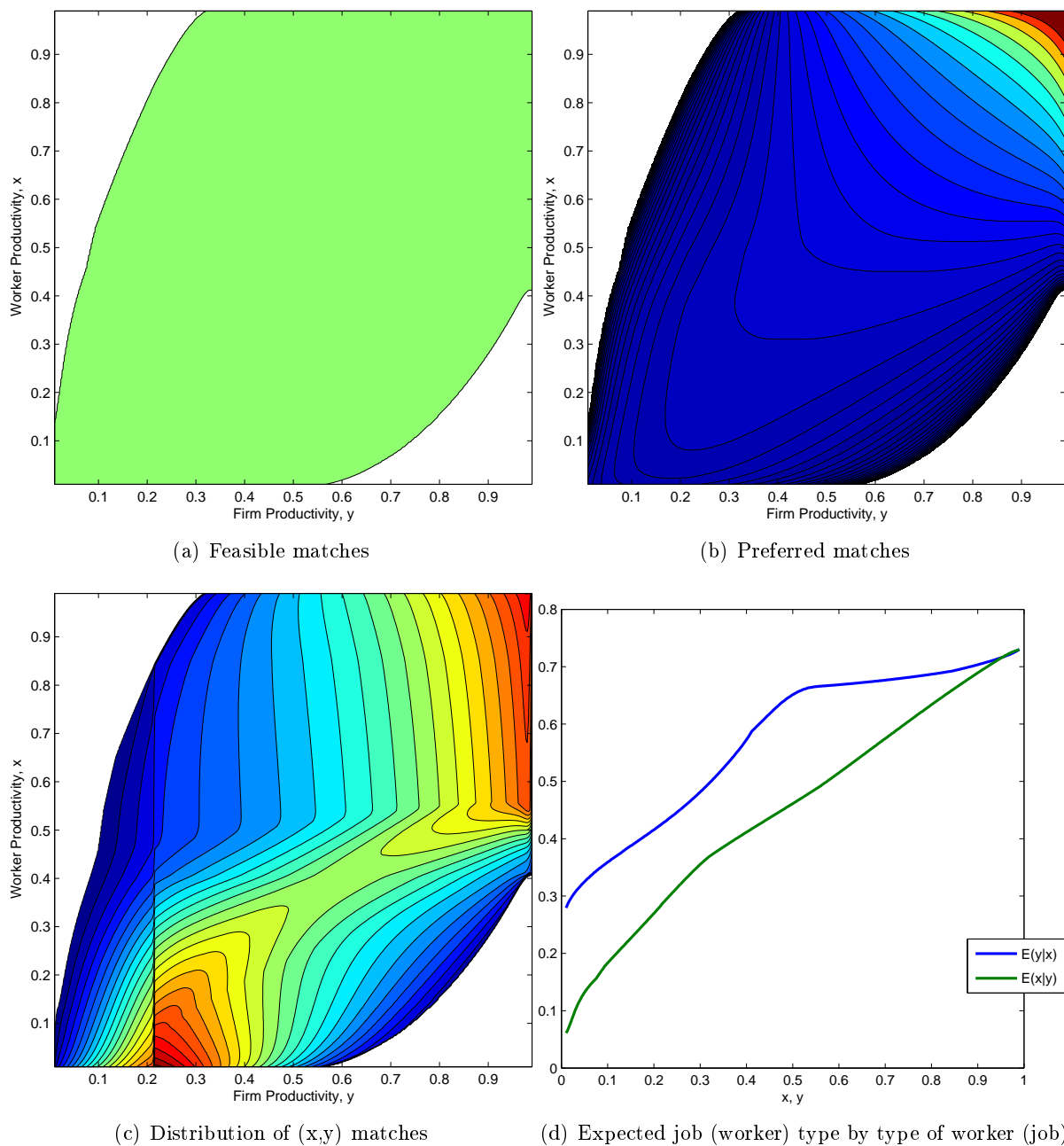


Figure 1: The production function is  $xy$ . The green area in panel (a) represent all the feasible matches, that is all pairs of  $(x, y)$  such that  $S(x, y) \geq 0$ . In panel (b) we plot the contour lines of  $S(x, y)$ . The “hotter” colours represent higher values of  $S(x, y)$ . A worker of type  $x$  will leave an  $(x, y)$  to form an  $(x, y')$  match whenever she is contacted by a  $y'$  and  $S(x, y') > S(x, y)$ . In panel (c) we plot the joint distribution of matches,  $h(x, y)$ , with “hotter” colours indicating more matches. In panel (d) we plot the average type of job (worker) that a worker (job) of a given type matches with.

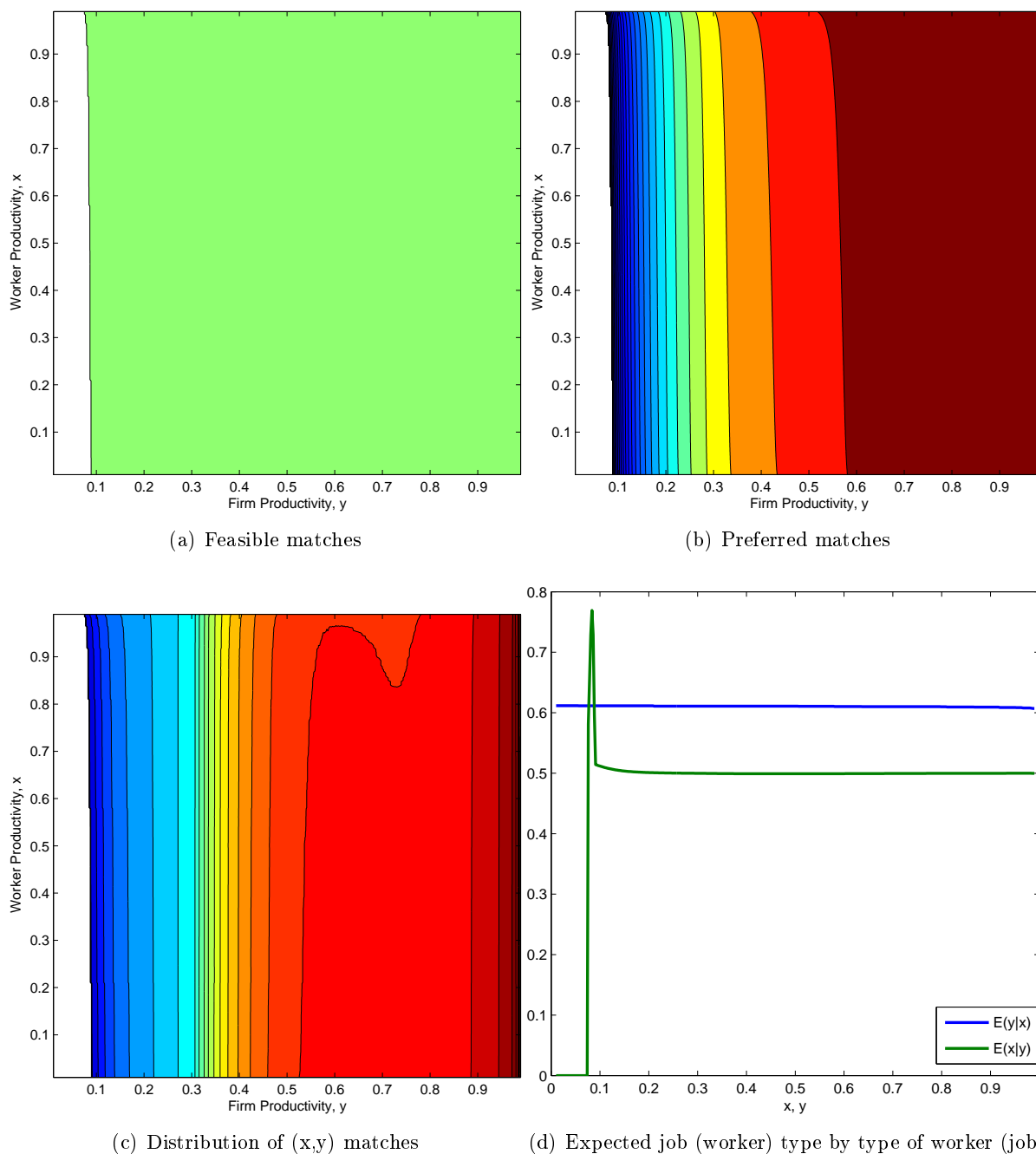


Figure 2: The production function is  $x + y$ . The green area in panel (a) represent all the feasible matches, that is all pairs of  $(x, y)$  such that  $S(x, y) \geq 0$ . In panel (b) we plot the contour lines of  $S(x, y)$ . The “hotter” colours represent higher values of  $S(x, y)$ . A worker of type  $x$  will leave an  $(x, y)$  to form an  $(x, y')$  match whenever she is contacted by a  $y'$  and  $S(x, y') > S(x, y)$ . In panel (c) we plot the joint distribution of matches,  $h(x, y)$ , with “hotter” colours indicating more matches. In panel (d) we plot the average type of job (worker) that a worker (job) of a given type matches with.



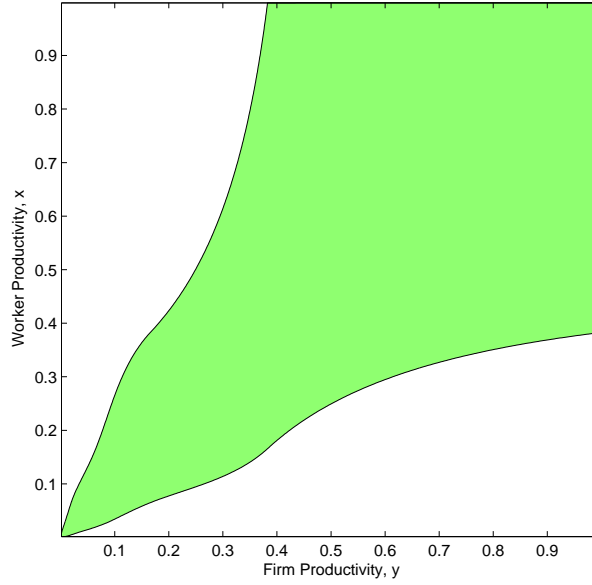


Figure 3: Feasible matches with  $xy$  production, without on-the-job search, without endogenous job destruction, without vacancy costs, and with an exogenous number of firms set equal to the number of workers. ( $s_1 = 0$ ,  $\delta = 0$ ,  $c = 0$ , and  $\bar{N} = \bar{L}$ )

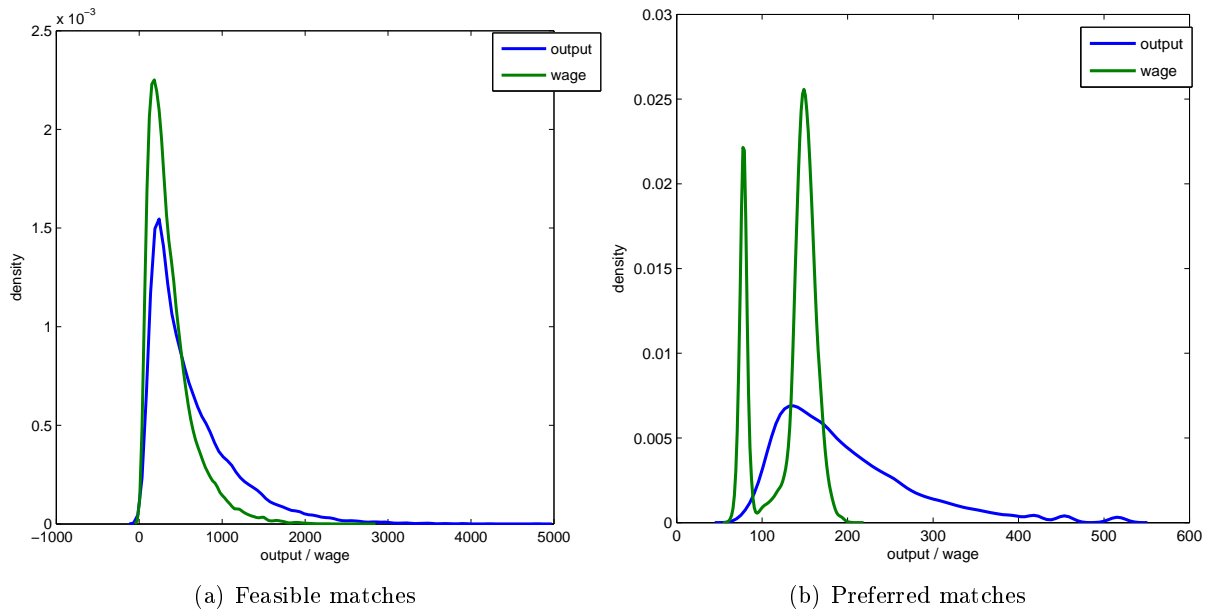


Figure 4: Output and wage distributions

## References

- [1] ABOWD, J. and CARD, D. (1989): "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 57: 411–445.
- [2] ABOWD, J. M., KRAMARZ, F., LENGERMANN, P., and S. PEREZ-DUARTE (2005): "Persistent Inter-Industry Wage Differences: Rent Sharing and Opportunity Costs"
- [3] ABOWD, J. M., KRAMARZ, F., LENGERMANN, P., and ROUX, S. (2003): "Interindustry and firm-size wage differentials in the United States and France," Cornell University, mimeo.
- [4] ABOWD, J. M., F. KRAMARZ and D. N. MARGOLIS (1999) "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.
- [5] ADDA, J., DUSTMAN, C., MEGHIR, C., and ROBIN, J-M. (2009): "Career Progression and Formal versus On-the-Job Training," mimeo.
- [6] ALTONJI, J. and SHAKOTKO, R. (1987): "Do Wages Rise with Job Seniority?" *Review of Economic Studies*, 54: 437–439.
- [7] ALTONJI, J. and WILLIAMS, N. (1997): "Do Wages Rise with Job Seniority? A Reassessment," NBER Working Paper Series #6010.
- [8] ATAKAN, A. (2006) "Assortative Matching with Explicit Search Costs," *Econometrica*, 74, 667–680.
- [9] BECKER, G. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81, 813–846.
- [10] BURDETT, K. and JUDD, K. (1983): "Equilibrium Price Dispersion," *Econometrica*, 51: 955–969.
- [11] BURDETT, K. and MORTENSEN, D. (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39: 257–273.
- [12] CAHUC, P., POSTEL-VINAY, F. and ROBIN, J-M. (2006): "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 74: 323–364.

- [13] DIAMOND, P. (1982): "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90, 881–894.
- [14] DUSTMANN, C. and MEGHIR, C. (2005): "Wages, Experience and Seniority," *Review of Economic Studies*, 72: 77–108.
- [15] FLINN, C. (2006): "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining, and Endogenous Contact Rates," *Econometrica*, 74: 1013–1062.
- [16] HECKMAN, J. and G. SEDLACEK (1985): "Heterogeneity, Aggregation and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, 93(6), 1077–1125.
- [17] HECKMAN, J. and B. HONORE (1990): "The Empirical Content of the Roy Model," *Econometrica*, 58(5), 1121–1149.
- [18] KRUEGER, A. and L. SUMMERS (1988): "Efficiency Wages and the Inter-industry Wage Structure," *Econometrica*, 56(2): 259–93.
- [19] LU, X., and R. P. McAFFEE (1996): "Matching and Expectations in a Market with Heterogeneous Agents," in *Advances in Applied Microeconomics*, Vol. 6, ed. by M. Baye, Greenwich, CT: JAI Press, pp. 121–156.
- [20] LENTZ, R. (2008): "Sorting in a General Equilibrium On-th-Job Search Model," Wisconsin-Madison, mimeo.
- [21] LILARD, L. and WILLIS, R. (1978): "Dynamic Aspects of Earning Mobility," *Econometrica*, 46: 985–1012.
- [22] MACLEOD, W. and MALCOMSON, J. (1993), "Investments, Holdup, and the Form of Market Contracts," *American Economic Review*, 83: 811–37.
- [23] MACURDY, T. (1982): "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," *Journal of Econometrics*, 18: 82–114.
- [24] MEGHIR, C. and PISTAFERRI, L. (2004): "Income Variance Dynamics and Heterogeneity," *Econometrica*, 72: 1–32.

- [23] MELO, R. (2008): "Sorting in the Labor Market: Theory and Measurement," Yale, mimeo.
- [25] MORTENSEN, D. (1982): "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 72, 968–979.
- [26] MORTENSEN, D. and PISSARIDES, C. (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61: 397–415.
- [27] MOSCARINI, G. (2001): "Excess Worker Reallocation." *The Review of Economic Studies*, 68, No. 3, pp. 593–612
- [28] PISSARIDES, C. (1990): *Equilibrium Unemployment Theory*. Oxford: Blackwell.
- [29] POSTEL-VINAY, F. and ROBIN, J-M. (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, No. 6, pp. 2295–2350.
- [30] POSTEL-VINAY, F. and ROBIN, J-M. (2006): "Microeconomic Search-Matching Models and Matched Employer-Employee Data," *Advances in Econometrics, Eighth World Congress*, eds. Blundell, Newey and Persson.
- [31] POSTEL-VINAY, F. and TURON, H. (2009): "On-the-Job Search, Productivity Shocks, and the Individual Earnings Process," *International Economic Review*, forthcoming.
- [32] SATTINGER . M. (1995): "Search and the Efficient Assignment of Workers to Jobs," *International Economic Review*, 36, 283–302.
- [33] SATTINGER . M. (2003): "A Search Version of the Roy Model." SUNY Albany, mimeo.
- [34] SHIMER, R. (2005): "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113(5): 996–1025.
- [35] SHIMER, R. (2006): "On-the-job Search and Strategic Bargaining," *European Economic Review*, 50(4): 811–830. Also published in *Structural Models of Wage and Employment Dynamics*, conference volume in honor of Dale Mortensen, edited by Henning Bunzel, Bent J. Christensen, George R. Neumann, and Jean-Marc Robin, Elsevier, 2006.
- [36] SHIMER, R. and SMITH, L. (2000): "Assortative Matching and Search," *Econometrica*, 68: 343–369.

- [37] TEULINGS, C. and GAUTIER, P. (2004): “The Right Man for the Job,” *Review of Economic Studies*, 71: 553–580.
- [38] TOPEL, R. (1991): “Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority,” *Journal of Political Economy*, 99: 145–176.
- [39] TOPEL, R. and WARD, M. (1992): “Job Mobility and the Careers of Young Men,” *Quarterly Journal of Economics*, 107: 439–4.