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Networks of relations and Word-of-Mouth Communication

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ABSTRACT

We study networks of relations – groups of agents linked by several cooperative relationships – exploring equilibrium conditions under different network configurations and information structures. Relationships are the links through which soft information can flow, and the value of a network lies in its ability to enforce agreements that could not be sustained without the information and sanctioning power provided by other network members. The model explains why network closure is important; why stable subnetworks may inhibit more valuable larger networks; and why information flows and action choices cannot be analyzed separately. Contagion strategies are suboptimal here, as they inhibit information transmission, delaying punishments.

1. Introduction

Cooperative relationships sustained by "the shadow of the future" are a fundamental governance mechanism for most forms of economic and social interaction. When several long-term relationships link different agents in a group, these agents and their relationships form *networks of relations*. This paper is a step in the economic analysis of these common

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social structures with a focus on their simultaneous and interdependent roles as *multilateral enforcement mechanisms* and *informal communication channels*. It defines and studies networks of relations, identifies equilibrium conditions for different network architectures and informational regimes, and determines whether there are key players affecting cooperation within the network.

Like social interactions, economic transactions are often part of a long-term relationship, episodes in a history of exchanges (Macaulay, 1963; MacLeod, 2007). Economic and social relationships are embedded in a wider network of other economic and social relationships that may be crucial to their governance: sociologists have long argued that by ignoring the networks of social relationships in which economic transactions are "embedded," economists fail to grasp crucial features of the economic process (Coleman, 1988, 1990; Granovetter, 1985).[1] Multilateral "community enforcement" mechanisms based on social or ethnic ties have indeed been the main governance instrument in many historical periods (North, 1991; Greif, 1993; Dixit, 2004). They are still crucial in developing countries where formal enforcement institutions are weak (McMillan and Woodruff, 1999), but are also crucial in advanced economies for complex sectors where contracting is necessarily highly incomplete. In the fast-changing world of high-tech products, firms often form networks of cooperative agreements to share the high risks and returns from their activities (Powell et al., 1996). In the financial industry, where asymmetric information is pervasive, the interbank market constitutes a fundamental network of relationships that in normal times allows intermediaries to smoothly exchange liquidity and maximize financing (Leitner, 2005; Allen and Babus, 2009); a network so important that if severed may lead to a collapse of the financial system, as experienced during the winter of 2008–2009. Within the Internet, reciprocal peering agreements between Internet service providers (ISPs) form a network of long-term cooperative relationships (Shin and Weiss, 2004; Lippert and Spagnolo, 2008). Of course, cooperation is not always for the good of society: networks of relations are an obvious governance instrument for corruption, cartels and many other forms of illegal exchanges (Calvó-Armengol and Zenou, 2004).

1.1. Model and results

Most of the conflicts governed by long-term relationships – from hold-up situations in exchanges with noncontractible dimensions to opportunistic defaults on loans to cheating on cartel prices or public-good contributions – have the strategic features of a Prisoners' Dilemma game. Our basic model is thus a repeated game in which each agent repeatedly interacts in generic, asymmetric bilateral Prisoners' Dilemma games with a subset of other agents whom he is connected to. In contrast with most previous work, we interpret cooperative relationships as the links of the "relational network," as they are the channels ensuring that private information successfully and truthfully flows from an agent to the others.

In this setup, we first consider the two benchmark cases of *Public Information*, where each agent observes the histories of play of all agents in the network, and *Private Information without Communication*, where each agent only observes his own history of play and no information can be exchanged across agents. Our main focus, however, is on the third – core – case of *Word-of-Mouth Communication* (WoM), where agents, while meeting to cooperate/transact, can choose whether to exchange or pass on observed or received private information on the history of play, and whether to do it truthfully or lie. In this core case we allow for different speeds of information circulation within the network. This generates a space–time-neighborhood structure for agents in the network that determines the benefit from belonging to it and that appears not to have been tackled before in economics.

Under WoM, we find that the possibility of transmitting soft information about privately observed defections to other agents in a closed network of relations can indeed be used to sustain it in equilibrium. We also find that Multilateral Grim Trigger strategies (MGs), which are optimal (in the sense of Abreu, 1988) in the bilateral repeated Prisoner's Dilemma and under public information and that correspond to the contagion strategies widely studied in random matching games, are no longer optimal when we endogenize information transmission by explicitly taking into account the agent's incentives to communicate truthfully. When cooperation in the network is disciplined by such strategies, Word-of-Mouth Communication is never used in equilibrium, as an agent that reverts to noncooperative play forever after observing a defection triggers a contagious process that eliminates all prospects of future cooperation in the network, thereby removing incentives and possibilities to truthfully communicate. When forgiving Multilateral Repentance strategies (MRs) are used instead, such that after a defection, nondefecting agents continue cooperating and spread information on the deviation until (only) the initial deviator is punished by a neighbor's profiting from such punishment, agents do have incentives to transmit and pass on information truthfully to avoid the collapse of cooperation. As information transmission within the network speeds up punishment phases, the forgiving MRs strictly dominate the contagious MGs.

Another central finding of our analysis is that, with asymmetric underlying games, networks of relations display a rather general *end-network effect* that occurs under any informational assumption and that resembles the end-game effects of finitely repeated games: Network structures such as trees or stars are never sustainable because agents with only an outgoing link cannot be sanctioned if they defect. We show that this end-network effect is a special case of gatekeeping and characterize those gatekeepers as key players to cooperation in the network. Circular networks overcome this problem, ensuring that all defections can be met with punishment and that networks of relations are sustainable in equilibrium. These

[1] Granovetter writes: "The embeddedness argument stresses instead the role of concrete personal relations and structures (or "networks") of such relations in generating trust and discouraging malfeasance" (1985, p. 490). See Spagnolo (1999a) for a model of embeddedness in the workplace.

results provide a rigorous and intuitive explanation for the importance of the "closure" and "density" of social networks as stressed by sociologists like Coleman.[2]

We then find that with private information (with or without communication) and MGs, bilaterally enforceable relations between some agents may hinder the stability of larger networks, as these agents may not be willing to sacrifice their bilateral relation to perform their part in the multilateral punishment mechanism that could sustain the larger network. This argument extends from bilateral relations to larger sustainable subnetworks and generalizes results from international trade theory showing that bilateral or regional agreements may undermine multilateral ones by softening third-party punishments (e.g., Bagwell and Staiger, 1997, 1999). However, we also show that this problem can be overcome by forgiving MRs, which may explain why in reality – where unforgiving MGs are typically not used – the World Trade Organization trade agreements tend to start from a small group of countries and then extends to the rest (e.g., Horn and Mavroidis, 2001).

1.2. Related literature

Our paper brings together several strands of the economics literature. It contributes to the recent literature on the emergence and stability of networks as surveyed in Jackson (2004, 2008) and Goyal (2007). Most models in this literature focus on agents' decisions about whether to build and maintain costly links, while the underlying game and enforceability problems are typically left out of consideration.[3] Our approach is orthogonal and complementary: we do not deal with network formation and evolution, instead digging deeply in terms of microstructure and the sustainability of cooperation in differently shaped network structures, allowing for asymmetries, different punishment strategies, and the choice of whether to transmit information truthfully or to distort or conceal it. Within the networks literature, the studies closest to us are Raub and Weesie (1990), Bloch et al. (2008), Haag and Lagunoff (2006) and Vega-Redondo (2006).[4] Bloch, Genicot and Ray are closest in that they study bilateral self-enforcing informal insurance agreements in fixed network structures with information flowing through the network. As in our paper, they identify bottlenecks as key players in their insurance networks, those likely to defect from the bilateral insurance schemes because the threat of multilateral punishment may be insufficient for some income realizations.[5] Contrary to our paper, they focus on grim-type strategies and an exogenously given information transmission technology and thus do not characterize how strategies and information transmission interact. Raub and Weesie (1990) model cooperation in a network not only with (what we call) public information and private information without communication, but also with public information with delay. However, their underlying game is less general and, contrary to our paper, Raub and Weesie only study exogenous information transmission. Hence, they cannot address the linkages between the information and action spaces, which our paper identifies. Haag and Lagunoff characterize the socially optimal neighborhood structures (network geographies) that a central planner should choose to maximize cooperation among a group of agents with stochastically different discount factors who interact bilaterally with neighbors in identical and symmetric infinitely repeated Prisoner's Dilemma games, focusing on multilateral grim trigger type strategies without communication. Vega Redondo analyzes the dynamic process that leads to the emergence of equilibrium neighborhood structures for agents playing idiosyncratic symmetric infinitely repeated Prisoner's Dilemma games with random payoffs. He assumes grim trigger-type strategies and the exogenous diffusion of information along the network and studies the dynamic process of adaptation and the search for new cooperation opportunities along the network while the quality of existing cooperative relations is subject to stochastic depreciation. We are close to these two papers because we also look at sustainable cooperation within groups of agents playing infinitely repeated bilateral games with neighbors in different network structures. We differ considerably, however, both in focus and because we allow for generic payoff structures, that is, both idiosyncratic and asymmetric payoffs (payoff asymmetry being core to our results without communication), for other than grim trigger-type strategies, and for endogenous communication and soft information transmission.

Our work is also related to the literature on social norms and "community enforcement" in random matching games.[6] Close studies within this literature are Kandori's (1992) paper, which does not limit the analysis to contagion strategies but also considers the effects of a central information transmission institution; Harrington's (1995) paper, which analyzes a model in which agents play a repeated game on a network like ours, though without communication; and Ahn and Suominen (2001).[7] This latter study is closest to ours, as it analyzes how WoM between successive buyers helps to enforce

[2] To be precise, Coleman's formulation of closure suggests high density and thus integrates the two concepts that we treat separately in our model.

[3] In a footnote to their introduction, Belleflamme and Bloch (2004) write: "In this paper, our focus is on the stability of market sharing agreements, and we assume that these agreements are enforceable. The issue of enforceability of market sharing agreements is an important one, which cannot be answered in traditional models of repeated oligopoly interaction. We leave it for further study." Our work can be seen as a first part of this further study.

[4] Also related are Bloch et al.'s (2007) paper, in which they address the relationship between coalitional stability and the value of subgroups, which is closely related to the relationship between self-sustaining subnetworks and the stability of the network in our paper; and Ali and Miller (2010), discussed below.

[5] Ballester et al. (2006) characterize individual equilibrium behavior in a network for agents with linear-quadratic interdependent payoffs. They identify key players to target if a central planner wants to change the aggregate group outcome of the activity chosen by the individuals in the network as a function of their intercentrality measures. Formulating the underlying game, we provide an intuition for why there are players who are key to sustaining the network.

[6] Started by Kandori (1992) and developed, among others, by Ellison (1994), Okuno-Fujiwara and Postlewaite (1995), and Dixit (2004) which also extended the approach to third-party enforcement.

[7] See also Balmaceda (2005), for a model where agents first form costly links and then repeatedly and randomly interact with other agents. The role of the links is to allow information to flow, and agents are assumed to transmit information truthfully and only on directly observed history.

a seller's honest behavior in a random matching environment where agents have private information about part of the seller's history and can send signals to other agents that help sustain cooperation (seller's honest behavior). Differently from our paper, Ahn and Suominen's messages are "public announcements," in the sense that they directly and immediately reach the agent who is supposed to act on that information, that is, the buyer who is in charge in that period. Therefore, relations between agents and the shape of the network play no role.[8]

Several other articles model Word-of-Mouth Communication as the ability to observe other agents' past actions rather than as an agent's deliberate choice of whether to truthfully transmit private information.[9] We, on the other hand, analyze WoM as part of the agents' strategies and show when agents have incentives to pass on their information truthfully in equilibrium. This is closer to what Ben-Porath and Kahneman (1996) do in a classic repeated-games framework with private monitoring. However, in their paper there is no relational network structure and communication is public – messages are observed by all players – so that even if there were a network structure, it would be irrelevant with regard to communication.

Because we model the transmission of information about privately observed histories, our paper is also relevant to the literature on reputation, which typically assumes either that the reputation bearer's behavior is publicly observed by potential future partners or that privately observed behavior is truthfully communicated from one market participant to the others.[10] However, recent empirical research on eBay and other electronic platforms shows that such information may often not be transmitted or that it may be distorted, manipulated or even fabricated, pointing to the importance of explicitly studying agents' incentives to pass on soft information truthfully (Dellarocas, 2006).

Finally, our work is related to the theoretical literature on multimarket contact and collusion (e.g., Bernheim and Whinston, 1990; Spagnolo, 1999b) and to its application to self-enforcing international agreements. The closest paper to ours within these strands of literature is Maggi's (1999), which extends the multimarket contact framework by modeling multilateral self-enforcing international trade agreements, and Bloch's (2004), which looks at the stability of market-sharing agreements as a network. Our paper contributes to these literatures, showing how agents can exploit *indirect* multimarket contact to collude/cooperate in generic strategic situations (payoff functions, number of agents and relations) with imperfect information and endogenous information transmission.

We proceed with the definition of a network of relations in Section 2. In Section 3, we derive results for sustainable networks in Public Information and Private Information without Communication environments. We study the effects of introducing the agents' abilities to engage in Word-of-Mouth Communication in Section 4. Section 5 contains a discussion of related research questions and of how our contribution relates to important concepts in the literature on Social Capital. All proofs are in the Appendixes.

2. Set up

2.1. Interaction

Let there be a set $N = \{1, \dots, n\}$ of infinitely lived agents indexed by $i \in N$ that interact in pairs according to a predetermined link structure $\mathcal{C}$ of two-element subsets of N, where $ij \in \mathcal{C}$, $i, j \in N$, if they are linked.[11] We assume all agents to be fully cognizant of $\mathcal{C}$.

In each period t, linked agents interact in social dilemmas with the strategic features of Prisoner's Dilemma games with idiosyncratic payoffs constant in time and represented by the following normal form:

		agent j	
		C^{ji}	D^{ji}
agent i	C^{ij}	c^{ij}, c^{ji}	l^{ij}, w^{ji}
	D^{ij}	w^{ij}, l^{ji}	d^{ij}, d^{ji}

,

where $l^{ij} < d^{ij} < c^{ij} < w^{ij}$ and $l^{ij} + w^{ij} < c^{ij} + c^{ji}$, $\forall i, j \in N$, $i \neq j$. Time is discrete, and all agents share a discount factor $\delta < 1$.

Agent i can either cooperate with j, choosing C^{ij}, or she can choose a selfish action D^{ij}. The generic (asymmetric) Prisoner's Dilemma structure captures the essential strategic features of most examples discussed in the introduction: one could interpret C^{ij} as *Comply* with the terms of a relational supply or labor contract or as *Contribute* to a local public good,

[8] In fact, there are neither relationships nor a shape of the network in their model.

[9] Among others, Banerjee (1993), Ellison and Fudenberg (1995), and Foster and Rosenzweig (1995).

[10] See Bar-Isaac and Tadelis (2008) and Mailath and Samuelson (2006) for nice surveys of this literature. Fishman and Rob (2005) present a reputation model where WoM is partly endogenized but truthful by assumption.

[11] This underlying link structure can be thought of as geographical, such as in the case of farmers cooperating in water management or blocking soil erosion on the border of their neighboring fields. This underlying geography is not essential for any of our results – in previous versions of the paper (for example, Lippert and Spagnolo, 2005), we did without it – but it appears to facilitate the understanding of the strategic situation we have in mind.

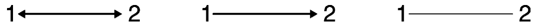

Fig. 1. From left: mutual relation, unilateral relation, bilaterally deficient relation.

or as *Collude* in a cartel or in other forms of organized crime. In each of these examples, the D^{ij} could be interpreted as "*Don't*" collude, contribute, comply, or cooperate.

2.2. Definitions and graphical representation

Let σ be a strategy profile in the n-player supergame. Then two linked agents i and j have a *relation* if and only if they play C^{ij}, C^{ji} in every period on the realization path in σ. To simplify the exposition, we will restrict the focus to equilibria with stationary realization/outcome paths in terms of agents' choice so that two interacting agents either share a relation and cooperate all the time or do not and never cooperate.[12]

Let ij denote pairs of agents in N and let $R \subseteq \mathcal{C}$ denote the set of pairs of agents in N who share a relation. Let $R_i = \{j \mid ij \in R\}$ be the set of agents with whom agent i shares a relation.

Denote with g^{ij} agent i's net expected discounted gains from the relation with agent j in the 2-player supergame (short: i's net gain from cooperating with j), that is, the difference between the discounted payoff from playing (C^{ij}, C^{ji}) forever and those from defecting and playing the static Nash equilibrium (D^{ij}, D^{ji}) forever after

$$g^{ij} \equiv c^{i,j} - (1-\delta)w^{i,j} - \delta d^{i,j}.$$

In a standard *bilateral* repeated game setting, a necessary condition for a cooperative relation to be sustainable in equilibrium is that both agents have a nonnegative net gain from cooperating with each other as, in the repeated Prisoner's Dilemma, Friedman's (1971) Grim Trigger (or "unrelenting Nash reversion") strategies are an optimal punishment in the sense of Abreu (1988).

We call a link between agent i and agent j *deficient* for agent i if and only if i's net gain from cooperation in ij is strictly negative and *nondeficient* for agent i if and only if i's net gain is nonnegative. The link ij is *mutual* if and only if both agents i and j have a nonnegative net gain from cooperation with each other and is *unilateral* if and only if one of them has a strictly negative and the other one has a nonnegative net gain. Finally, it is *bilaterally deficient* if and only if both have a strictly negative net gain from cooperation.

A simple way to represent relations is graphical, where a line or an arrow is drawn between agents i and j if and only if i and j share a relation. We depict a relation on a link ij that is *nondeficient* for i by an *incoming arrow*[13] to agent i. A relation on a unilateral link (short: a *unilateral relation*) is thus depicted by an arc originating from the agent for whom the link is deficient. A relation on a mutual link (short: a *mutual relation*) is depicted by an incoming arc to both agents. A relation on a bilaterally deficient link (short: a *bilaterally deficient relation*) is just a line. (See Fig. 1.)

We define a *network of relations* $\mathcal{N}^S = (N, R)$ to be one consisting of a set of agents N and a set of relations R. We call it *sustainable* if and only if the strategy profile σ in the n-player supergame prescribing the relations in R constitutes a sequential equilibrium. Note that any mutual relation is a sustainable network consisting of two nodes and one relation. Finally, we call a sustainable relational network $N^S = (N, R)$ strategically stable if it fulfills Kohlberg and Mertens' (1986) stability criteria.

A relational *network*, $\mathcal{N}^S = (N, R)$, is *mutual* if it consists only of mutual relations; it is *nonmutual* if it contains no mutual relations; and it is *mixed* if it consists of both mutual and other relations.

Given $\mathcal{N}^S = (N, R)$, the number of agents in N is called the *order* of $\mathcal{N}^S$, and the number of relations in R is called the *size* of $\mathcal{N}^S$. The *degree* of vertex i is the number of edges of agent i, denoted $\deg i$. An agent of degree 1 is called the end vertex. A network is called an $i - j$ *path* if it consists of a finite alternating sequence of agents and links that begins with agent i and ends with agent j, in which each link in the sequence joins the agent that precedes it in the sequence to the agent that follows in the sequence, in which no agent is repeated. An $i - j$ path is called a *cycle* if $i = j$. A relational network satisfies the property of *closure* if each player in the network is part of a cycle. A cycle of size c is called a *c-cycle*. A disconnecting set of edges is a set $F \subseteq R$ such that $\mathcal{N}^S(N, R \setminus F)$ has more than one component (connected subgraph of $\mathcal{N}^S$ that is not contained in any other connected subgraph of $\mathcal{N}^S$).

2.3. Information structures

We will consider three informational assumptions.

(I1) *Public Information*: Every agent observes the history of all bilateral Prisoner's Dilemmas.

[12] We see no reason to believe that our results should not extend to any other less simple and intuitive cooperative equilibria once they are defined as "relationships."

[13] Note that our graphical representation of relational networks departs from the conventional graphical representation in the literature on network formation. There, an arrow going out from a vertex i usually depicts a link sponsored or formed by vertex i. In our graphical representation, on the other hand, the presence of arrows conveys information on the sustainability of relations with optimal *bilateral* punishments, more specifically on each agent's net discounted gains from defecting from a bilateral relation.

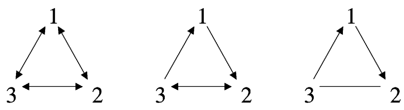

Fig. 2. From left: mutual network, mixed network, nonmutual network.

(I2) *Private Information without Communication*: Every agent only observes the history of the bilateral Prisoner's Dilemmas he plays, that is, the past actions chosen by his neighbors in their interactions with him (and not the actions his neighbors chose in their interactions with other players); and he cannot transmit this information to other agents.

(I3) *Word-of-Mouth Communication*: (i) Every agent observes only the history of the bilateral Prisoner's Dilemmas he plays, as in (I2); (ii) at the end of each period, after the period's actions have been observed by the players of each respective Prisoner's Dilemma, every agent has the choice of whether to transmit information on the history of his interactions (including this period's actions) to other neighboring agents he shares a relation with and whether to do it truthfully or to lie; (iii) at the end of each period, every agent also has the choice of whether to pass on received information to other neighbor agents he shares a relation with and whether to do it truthfully or to lie. A message can be passed on $v \geqslant 1$ times per period; therefore, v defines the maximum number of nonneighbor agents to which information on a neighbor's choice of action can be passed on at the end of each period in a given environment.

(I1) can be thought of as a situation where an institution processes and distributes information centrally and without delay, as in Kandori (1992) and Maggi (1999). All agents are immediately informed about deviations and can adapt their actions in the following period.

(I2), on the other hand, describes the opposite situation: The past actions of neighbors and nonneighbors with respect to third agents cannot be observed, nor can agents report/communicate the observed history in their own bilateral interactions to third parties. For example, in Fig. 2, besides knowing which actions she took herself, player 1 observes C^{21} or D^{21} and C^{31} or D^{31}, but she does not observe nor does she receive information on whether 2 played C^{23} or D^{23} and whether 3 played C^{32} or D^{32}.

(I3), *Word-of-Mouth Communication* requires a more extended discussion. In (I3), the past actions of neighbors and nonneighbors with respect to third agents cannot be observed; however, agents can report/communicate the observed history in their own bilateral interactions to third parties whom they have relations with, and they can pass on these reports/communications. In (I3), to communicate private information on neighbors' past actions is a choice. Each agent i can choose to send an agent j with whom he is cooperating messages of the following kind: "Agent h defected from the network equilibrium strategies (relationship) against agent k at time t." The message can either be based on direct observation (then $k = i$) or be obtained from a partner in another relationship. Agents can also choose to send false messages or not to pass on truthful or false ones received from others. We assume that partners can exchange this information only if they meet to cooperate. They only meet if they cooperate, for example, to help maintain a local common property, and they cannot exchange information if they choose not to cooperate even if they desire to do so, because there is no meeting as a result.

Information exchanged is "soft," in the sense that the truthfulness of sent messages cannot be verified by partners or third parties. It is therefore possible, either on purpose or erroneously, for each agent to falsely state that the first known defector was k, even though it was j (a partner may not have told the truth about a previous defection). We assume that agents lie only if they *strictly* prefer to do so. We will show that under this natural assumption with Multilateral Repentance strategies (defined below) agents will choose to truthfully transmit information in equilibrium.

Messages can travel $v \geqslant 1$ relationships per period. Therefore, for example, in a linear network with seven agents and the six relations $\{12, 23, 34, 45, 56, 67\}$ when $v = 2$, the actions of agent 1 in period τ are known to agent 2 and can be transmitted to agents 3 and 4 before they choose their actions in $\tau + 1$; can be passed on to agents 5 and 6 before they choose their actions in $\tau + 2$; and transmitted to agent 7 before she chooses her action in $\tau + 3$. If $v = 3$, then agents 3, 4 and 5 can be informed by agent 2 before they choose their actions in $\tau + 1$, and agents 6 and 7 can be informed before they choose their actions in $\tau + 2$.

3. Public information and private information without communication

For the two types of multilateral punishment strategies, we will study under which conditions different types of relational networks are sustainable, if any. We will do so in this section for the first benchmark case, *Public Information* (I1), and for the second benchmark, *Private Information without Communication* (I2). In the next section, we will focus on the core information structure of this paper, *Word-of-Mouth Communication* (I3). Before giving results specific to information structures, we will first give some general insights on the forces at play in networks of relations.

3.1. End-network effect, gatekeepers, and closure

A straightforward generalization of the sustainability condition for a bilateral relational contract is that for each agent in the network, the net gain from cooperating with other network members has to be nonnegative. Furthermore, if cooperation

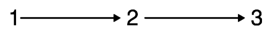

Fig. 3. This network is not sustainable.

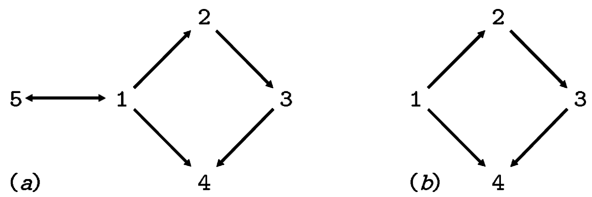

Fig. 4. Gatekeeping.

is to be sustained with multilateral strategies, it is necessary to condition the strategies of each designated punisher on the actions of the relative defector, either through direct observation, through contagion, or through truthfully transmitted information. This leads to the following first observation.

Proposition 1 *(End-network effect and gatekeeping).* (i) *There exists no sustainable network of relations that contains an agent with only deficient relations.*

(ii) *Under* I2 *and* I3 (*private information with or without communication*) *there exists no sustainable network of relations containing an agent without whose relations the network would have two or more disjoint connected subnetworks and who faces negative net gains from cooperation in all of his relations with agents in at least one of these subnetworks.*

Part (i) of Proposition 1 gives rise to the mentioned "end-network effect": As long as none of its relations is mutual, a network with end-vertices is not sustainable because there will always be at least an agent with negative net gains from cooperation who will defect. Fig. 3 illustrates this for a simple linear network: Agent 1 always has an incentive to deviate. This end-network effect is different but brings to mind the so-called "end-game effect" of finitely repeated games and is similarly general. An example is given in Fig. 3.

Part (ii) of Proposition 1 gives rise to a gatekeeping effect. Consider the network in Fig. 4(a). There is a disconnecting set of relations of agent 1, his relations with 2 and 4, for which $g^{12} < 0$ and $g^{14} < 0$. The only way to discipline 1 in these relations would be by retaliation in his relation with 5, but without public information, 5 cannot be aware of a deviation by 1. It is therefore impossible to construct a strategy that relies on punishment from a nonconnected part of the network.

One important way to ensure that a nonmutual network is sustainable is to *close* the network. To capture the effect of *closure* in our model (i.e., that each player in the network is part of a cycle), let us define the network version of Friedman's (1971) "Grim Trigger" strategies, the mentioned "Multilateral Grim Trigger" strategies (MGs), which correspond to the "contagion strategies" introduced by Kandori (1992) in random matching games. Under different informational assumptions, MGs differ slightly, so they will be denoted by (S1) for Public Information (I1) and by (S2) for Private Information without Communication (I2).

Strategy profile (S1): Every agent $i \in \mathcal{N}^S$

1. starts playing C^{ij} $\forall j \in R_i$,
2. continues playing C^{ij} $\forall j \in R_i$ as long as he observes C^{mn} $\forall m, n \in \mathcal{N}^S$, and
3. reverts to D^{ij} $\forall j \in R_i$ forever otherwise.

Strategy and belief profile (S2): Every agent $i \in \mathcal{N}^S$

1. starts playing C^{ij} $\forall j \in R_i$,
2. believes that every agent played C with all neighbors and goes on playing C^{ij} $\forall j \in R_i$ as long as he observes (C^{ij}, C^{ji}) $\forall j \in R_i$ and
3. if a (optimal or suboptimal) defection is observed, forms a consistent belief about the history of play and reverts to D^{ij} $\forall j \in R_i$ forever.

In (S1), agents choose the cooperative action (C) with every neighbor in the network at the beginning of the game and continue doing so as long as everybody in the network has always chosen the cooperative action in the history of the game. They choose the noncooperative action (D) forever after one agent in the network has chosen the noncooperative action. In (S2), agents choose the cooperative action (C) with each of their neighbors in the beginning of the game. As long as throughout the history of the game (i) they always chose the cooperative action with each of their neighbors and (ii) each of their neighbors has always chosen the cooperative action with regard to them, they believe that every agent in the network has done so and continue to choose the cooperative action. As soon as they observe that an agent has chosen the

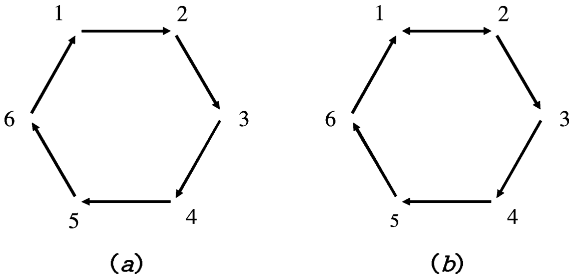

Fig. 5. Nonmutual network on a 6-cycle (a) and mixed network on a 6-cycle (b).

noncooperative action (D), they form consistent beliefs on the history of the game and choose the noncooperative action with all of their neighbors forever after.[14]

Using strategies (S1) for (I1) and (S2) for (I2) we obtain the following:

Proposition 2 *(Closure). A* closed *network of relations is sustainable*

1. *under* (I1) *if and only if, for each agent in the network, the sum of net gains from cooperation in all his relations is nonnegative*: $\forall i \in \mathcal{N}^S$, $\sum_{j \in R_i} g^{ij} \geqslant 0$;
2. *under* (I2) *if and only if, for each agent in the network, the sum of net gains from cooperation in all his relations, discounted for the delay in indirect punishment, is nonnegative*: *Denoting the shortest path from an agent in a set of agents J_i to agent k that does not pass through i by $l(J_i, k)$, $\forall i, \forall J_i \subseteq R_i$, $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta^{l(J_i,k)} g^{ik} \geqslant 0$; and beliefs are such that after any observed deviation, the immediate playing of the noncooperative action with every neighbor is sequentially rational.*

Part 1 of this result implies that with closure, MGs can sustain relational networks that do not contain *any* relation that is bilaterally sustainable in the absence of a network, as in Fig. 5(a). Part 2 requires, for example, for a nonmutual cycle of length c, where $\forall i$, $g^{i,i+1} < 0$ and $g^{i,i-1} \geqslant 0$, that $\forall i \in \mathcal{N}^S$, $\delta^{c-2} g^{i,i-1} + g^{i,i+1} \geqslant 0$.[15] Community enforcement makes relations sustainable that would not be so bilaterally. However, community enforcement relies on each agent's willingness to punish. With MGs, this entails giving up cooperation with all neighbors forever. To see what is required in terms of beliefs under (I2), consider the following two examples:

Example 1. Consider an increase in the cooperation payoff c^{12} in Fig. 5(a), so that the relation 12 becomes mutual as in Fig. 5(b). Because 12 is a mutual relation, agent 1 would want to enter a punishment phase with 2 as late as possible. He will only immediately enter into a punishment phase with 2 if he believes that 2 enters the punishment phase with a high probability, which is the case if 2 is the original defecting agent (and deviated sub-optimally) or if 2 enters the punishment phase in the period thereafter, which is the case if 3 is the original defector (and deviated optimally).

Example 2. Assume that player 1 in Fig. 5(a) observes D^{21} and C^{61}. Then, (S2) requires player 1 to play D^{16}, thus giving up a nondeficient relation, and D^{12} from the next period on. Again, giving up the nondeficient relation is sequentially rational only if player 1 assigns a sufficiently high probability that 2 will enter the punishment phase within the next two periods. These beliefs imply that either player 2 deviated suboptimally or more than one deviation occurred.

In both examples, beliefs are consistent. However, they can imply suboptimal deviations or several deviations from the equilibrium path. The restrictions on beliefs necessary to sustain an equilibrium are more stringent for agents who are part of a higher number of sustainable subnetworks. Agents who benefit (too) strongly from relations with everyone they are connected to can hurt cooperation between other agents because they could be unwilling to do their part in a multilateral punishment phase. A consistent belief of agent i that makes it rational for i to enter into Nash reversion with all neighbors is that, in the period of an observed deviation by one or more of his neighbors, all actions in the network that i cannot observe were noncooperative actions D.

With public information, every agent observes the complete history of the network in every period. This leads to a destabilizing effect of subnetworks that are sustainable in the absence of the rest of the network.

Proposition 3 *(Destabilizing subnetworks). Under* (I1)*, if a relational network is sustained by MGs and can be divided into subnetworks that could be sustained without the rest of the network and a subnetwork that could not, then it is not strategically stable.*

[14] Note that in a sustainable network, there is no information for why any agent should have been the first to deviate. Therefore, any belief, for example, the belief that everybody who has not been observed played the noncooperative action in the last period, is consistent.

[15] We give more examples for these sustainability conditions and compare them with those with Word-of-Mouth Communication in Section 4.2 below.

The intuition for this result is that by suboptimally deviating only in the part of the network that is not sustainable without the rest, a member of the sustainable subnetwork may “send a message” to the other agents in the sustainable subnetwork indicating that he wants to go on cooperating in that part of the network. This indication may lead the agents in the sustainable subnetwork not to implement the costly punishment phase. Again, agents who benefit (too) strongly from relations with everyone they are connected to can hurt cooperation between other agents because they could be unwilling to do their part in a multilateral punishment phase.

4. Word-of-Mouth Communication

Let us now turn to the central informational assumption of this paper, *Word-of-Mouth Communication* (I3), which allows agents that sustain a relation to communicate and transmit their private information if they so choose. It is worth noting at the outset that because agents have the option to lie, transmitted messages under (I3) are valuable only if all transmitters in the chain have no incentive to lie or conceal information. Indeed, agents would be better off lying if, after a deviation of one of their neighbors, the multilateral mechanism would not provide enough incentives for the agents to go on cooperating with the other neighbors.

4.1. Word-of-Mouth Communication and Multilateral Grim Trigger

Before discussing equilibria that make use of information transmission, we first note that agents can always decide not to transmit/pass on information. Indeed, if agents adopt Multilateral Grim Trigger strategies, Word-of-Mouth Communication will not be used in equilibrium. We can state the following:

Proposition 4 *(WoM with MGs). If agents adopt MGs, then agents do not make use of Word-of-Mouth Communication, and all of the results obtained under* (I2) *apply unchanged under* (I3)*.*

Entering a punishment phase under MGs implies a permanent breakdown of the network. Therefore, every agent who observes a defection of a neighbor and transmits this information to another neighbor will deprive himself of the benefits from defecting in his other relations without the prospect of future cooperation. Therefore, information is not transmitted, and a social network is sustainable under the conditions of Proposition 2, part 2.

4.2. Word-of-Mouth Communication and Multilateral Repentance

In this subsection, we will construct equilibria that use communication. A crucial role will be played by noncontagious Multilateral Repentance strategies (MRs), which are formally defined below. MRs are an adaptation of the repentance strategies discussed by van Damme (1989) for the standard bilateral infinitely repeated Prisoner’s Dilemma. In that setting repentance strategies are optimal punishment strategies, as are grim trigger strategies. We will show that under WoM, in equilibria relying on MRs, information is passed on truthfully through the network, there are no dangerous subnetworks, and networks are sustainable at lower discount factors than with MGs, implying that if WoM is available in networks of relations, MGs are not optimal punishment strategies.

Multilateral Repentance strategies (MRs)

1. Each agent i starts by playing C^{ij} $\forall j \in R_i$.
2. Each agent i goes on playing C^{ij} $\forall j \in R_i$ as long as he does not observe D^{ji} for any $j \in R_i$ and as long as he does not receive a message containing D^{jn} for some $j \in R_i$.
3. If agent i observes D^{ji} for any $j \in R_i$ and received no message about an earlier defection by j, she
   (a) sends a message about the deviation to her other neighbors and goes on playing C with them,
   (b) plays D^{ij} until j and i have played (D^{ij}, C^{ji}) for T_{ji} periods,
   (c) sends her other neighbors a message about the end of the punishment phase for agent j, and
   (d) goes back to 2, thereafter.
4. If a neighbor k of j receives a message about j’s initial deviation, she
   (a) plays D^{kj} until she receives the message that D^{ij}, C^{ji} has been played for T_{ji} periods and until D^{kj}, C^{jk} has been played for T_{jk} periods, and
   (b) returns to 2, thereafter.
5. If agent j played D^{ji}, she
   (a) plays C^{ji} for the next T_{ji} periods, D^{jk} in the period when k receives the information on her initial deviation and C^{jk} for the next T_{jk} periods, and
   (b) returns to 2, thereafter.
6. Each agent truthfully passes on the messages.
7. If some agent deviates from the actions in 3–5, punishment commences against this agent.

Under MRs, every player who is part of the network of relations starts cooperating with every neighbor in the network and continues to do so as long as no deviation is observed or reported. If a player i observes a deviation from neighbor j in the network, he sends a message describing the deviation to his other neighbors and goes on playing cooperatively with them, while players i and j play Defect (D^{ij}) and Cooperate (C^{ji}), respectively, for a specified number of periods. Thereafter, i sends a message about the completion of the punishment of j to other network neighbors and resumes playing cooperatively with j. If another neighbor k of j receives the message about j's defection, players k and j start playing Defect (D^{kj}) and Cooperate (C^{jk}), respectively, for a specified number of periods. After that period and after having received a message from the original sender of the message, i, about the completion of his punishment by i, k also goes back to cooperation and sends the message that his part of the punishment was also completed. Finally, deviants from this punishment phase are subject to the same punishment as the first defecting agent j.

Note that forgiving MRs are natural punishment strategies in this setting. Strategies targeted to punish only the guilty agent generally cannot last forever, as an innocent agent defecting for a long time against a defecting guilty agent would lose incentives to cooperate with other neighbors.[16]

We can now state the following:

Lemma 1. *With Word-of-Mouth Communication and MRs no player has an incentive to conceal truthful information or pass on false information.*

The intuition for this result is that with MRs, transmitting messages only affects the payoffs of other agents and, as a result, no agent has an incentive to transmit false information. Given that truthful information is passed on, MRs make use of the network's ability to implement the targeted punishment of cheaters. We will now show that Multilateral Repentance strategies, when combined with Word-of-Mouth Communication – that is, the passing on of soft information – (1) are superior to Private Information without Communication, (2) do not entail dangerous subnetworks, and (3) constitute optimal punishment strategies.

Proposition 5 *(Optimal punishments with WoM). Under Word-of-Mouth Communication, MRs are optimal punishment strategies in networks with closure, whereas MGs are not.*

The intuition behind this result is that the delay of the punishment can be reduced compared with MGs. Given that the expected payoff stream when the punishment starts for a defector minimaxes the defector both under MRs and MGs, punishment under MRs is both as strong and as fast as possible. Therefore, under (I3) MRs constitute optimal punishment strategies in our network, while MGs do not.

Note that sustainable subnetworks do not affect the sustainability or the strategic stability of the network. There are two reasons for this. First, as reliable Word-of-Mouth Communication informs players about the relevant part of the history of play in the network, entering a punishment phase does not hinge on beliefs. Second, because agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate within a sustainable subnetwork.

Proposition 6 *(Sustainable subnetworks with MRs and WoM). With Word-of-Mouth Communication and MRs, sustainable subnetworks do not affect the sustainability or the strategic stability of a larger network containing them.*

MRs also imply that, if information about past actions can be transmitted, it is not necessary to have a complete and indefinite breakdown of cooperation in the network in the case of a deviation. The equilibrium is consequently more robust (against, for example, mistakes or trembles), and it increases welfare during punishment phases as compared with equilibria relying on MGs.

Compare sustainability conditions for different network structures.

Example 3. Consider first *complete* networks of relations where each player shares a relation with each other player. With MGs (or without WoM), contagion implies community punishment that begins only one period later. Therefore, any deviation of an agent i vis-à-vis agents $k \in R_i$ in period t triggers punishment by agents k starting in period $t+1$ and by agents $m \in R_i \setminus k$ in period $t+2$. Therefore, each player i's optimal deviation is to play D^{ik} with all neighbors $k \in R_i$ for which $g^{ik} < 0$ and C^{im} with all neighbors $m \in R_i \setminus k$ in t and D^{im} in period $t+1$. The reason is that for $m \in R_i \setminus k$, $g^{ik} > 0 \Leftrightarrow c - (1-\delta) - \delta d > 0 \Leftrightarrow c + \delta w > w + \delta d$. Denote the set of neighbors k of agent i for which $g^{ik} < 0$ with R_i^-, and denote the set of neighbors m of agent i for which $g^{im} \geqslant 0$ with R_i^+. Then, for MGs to constitute a sequential equilibrium, payoffs and discount factors have to satisfy

$$\sum_{k \in R_i^-} g^{ik} + \sum_{m \in R_i^+} \delta g^{im} \geqslant 0 \quad \forall i \in \mathcal{N}^S.$$

[16] We are grateful to David Miller for pointing this out.

Under WoM and MRs instead *any* deviation of an agent i in period t is punished by *all* neighbors of i from period $t+1$ onward. Therefore, a player's optimal deviation is to play D with all neighbors in the same period. Therefore, MRs constitute a sequential equilibrium as long as

$$\sum_{j \in R_i} g^{ij} \geqslant 0 \quad \forall i \in \mathcal{N}^S.$$

Therefore, MRs implement cooperation for a larger set of discount factors than MGs.

Example 4. Consider now a *nonmutual cycle* of length c, where $\forall i$, $g^{i,i+1} < 0$ and $g^{i,i-1} \geqslant 0$. Then with MGs (or without WoM) the sustainability condition for agent i is

$$\delta^{c-2} g^{i,i-1} + g^{i,i+1} \geqslant 0 \quad \forall i \in \mathcal{N}^S.$$

Defining $\theta(c, v) = \max\{0, int(\frac{c-2}{v}) - 1\}$, with MRs and WoM the sustainability condition is instead

$$\delta^{\theta(c,v)} g^{i,i-1} + g^{i,i+1} \geqslant 0 \quad \forall i \in \mathcal{N}^S.$$

Since $\theta(c, v) < c - 2$ for any v, MRs implement cooperation for a larger set of discount factors than MGs.

Example 5. Consider a *cycle* of six agents as in Fig. 5(a). Add a relation between agents 6 and 3. Then the sustainability conditions with MG strategies are

$$\begin{aligned} &g^{6,1} + \delta^2 g^{6,3} + \delta^4 g^{6,5} \geqslant 0, \\ &g^{3,4} + \delta^2 g^{3,6} + \delta^4 g^{3,2} \geqslant 0, \text{ and} \\ &g^{i,i+1} + \delta^2 g^{i,i-1} \geqslant 0 \quad \forall i \in \{1, 2, 4, 5\}. \end{aligned}$$

Furthermore, if any of the subnetworks are sustainable without the rest of the network, entering a punishment phase requires breaking a sustainable subnetwork. Beliefs have to be such that this is rational. In the same network, assume again MRs, WoM and $v = 2$. Therefore, the sustainability conditions are

$$\begin{aligned} &g^{6,1} + g^{6,3} + \delta g^{6,5} \geqslant 0, \\ &g^{3,4} + g^{3,6} + \delta g^{3,2} \geqslant 0, \text{ and} \\ &g^{i,i+1} + g^{i,i-1} \geqslant 0 \quad \forall i \in \{1, 2, 4, 5\}. \end{aligned}$$

Again, MRs implement cooperation for a larger set of discount factors than MGs. Furthermore, repentance ensures that even if some subnetworks are sustainable, all players have the incentives to carry out punishments and transmit information truthfully.

The extent to which MRs are better than MGs can be measured by the set of discount factors for which a given network, which is sustainable with MRs, is not sustainable with MGs. The size of this set is a function of the length of paths between any two neighbors of any given agent that do not go through that agent[17] as well as the speed of information transmission. With increases in the network's density – the ratio of the number of adjacencies that are present divided by the number of pairs of actors – these paths shorten. Shortening these paths has two effects on the set of discount factors for which a given network, which is sustainable with MRs, is not sustainable with MGs. First, each unit of saved delay becomes less important because the time it takes for third-party punishment to arrive increases as the network becomes less dense. Second, for high speeds of information transmission, v, as the paths lengthen, the difference in the delay with which community punishment sets in with MRs and MGs increases. These two effects work in opposite directions, and the net effect depends on the speed of information transmission.

Proposition 7 *(Impact of network density). For a given network, let the length of the shortest path between any two neighbors of any agent that does not go through this agent weakly increase, with a strict increase for at least one pair of neighbors of one agent. Then there exists a threshold speed of information transmission under Word-of-Mouth Communication, $\underline{v}$, such that,*

1. *for $v > \underline{v}$, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs increases*; *and*
2. *for $v < \underline{v}$, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs decreases.*

[17] For example, in Fig. 5(a), the path between the two neighbors of agent 1 that does not go through agent 1 has a length of 4.

5. Discussion and conclusion

5.1. Other questions

At this point, one could also ask other questions, such as what is the top Pareto equilibrium or the largest network of relations that can be sustained in a given exogenous connection structure, $\mathcal{C}$, as is done in a different environment by Haag and Lagunoff (2006). We believe that our model as it stands is not well-suited to ask these other questions both because it is too general (Prisoner's Dilemma payoffs are idiosyncratic and generic/asymmetric across players and interactions) and because it does not incorporate a cost of sustaining relations to balance their benefits so that a more cohesive structure with maximal connection tends to be optimal by definition, at least under the most important and novel assumptions of MRs and WoM. More links and a denser network of relations speed up punishments besides increasing payoffs; as a result, the greater number of relations that can be sustained leads to more intense cooperation that can be sustained in each relation. The only constraint is the discount factor (besides coordination). Recent work (e.g., Ali and Miller, 2010) focuses on these other questions in a similar but simpler environment relative to relations – they assume symmetric payoffs, contagion, and no communication – which is also enriched by the introduction of costs of establishing and maintaining each link and by a network formation stage preceding the cooperation phase. We therefore leave these other questions to their (and others') future research.

5.2. Full cognizance of the game structure

We have assumed that all players are fully cognizant of the game structure. This includes full cognizance of the set of players, N, of the links in the underlying connection structure, $\mathcal{C}$, of the players who are part of a multilateral agreement that forms the relational network, and of the payoffs of those players. Reliable Word-of-Mouth Communication (and a limited number of agents) should enable each agent to get to know over time which agents are cooperating and which are not and thus make it possible to lift the assumptions of full cognizance. Following this line of research would imply answering the question of how cooperation comes into existence. Because the main questions in this essay are how much cooperation is possible within a relational network under different strategies and information regimes and what the best way of enforcing cooperation for each of them is, this would imply a considerable shift in focus, which is not within the scope of this paper.[18] We therefore leave also this to future research.

5.3. Social capital

The model provides a micro-foundation for Granovetter's (1985) idea of "embeddedness," while the "end-network effect" provides a clear explanation for why the "closure" of social networks is so important for social capital, as argued by Coleman (1988, 1990). Immediate applications of our framework include the organization of interfirm relations in industrial districts, the enforcement of collusive behavior in business networks, interbank relations and the effects of "social capital" on the governance of economic and social interactions. In her widely acclaimed book, Saxenian (1994) attributes a large part of Silicon Valley's success to a special culture of cooperation in that industrial district stemming from a common background of the early workforce in that area. We believe that our model offers a complementary explanation for how social networks may facilitate information circulation in a community.

In our model, networks of relations generate "slack enforcement power" for some agents, and we have shown how this allows them to sustain cooperation in additional deficient relations, including *one-shot* Prisoner's Dilemma interactions. This use of networks of relations as cooperation-enforcement/governance devices for new social dilemmas can be interpreted as a game-theoretic definition of *social capital* very close to that proposed by Bourdieu (1986) and Coleman (1990).[19] In particular, our model allows the definition of this individual social capital as follows: *"The individual social capital agents i and k can draw on by being part of a social network is the slack enforcement power of the network available to enforce cooperation–compliance in other interactions in need of governance."* With complete information, this is only an agent-pair-specific definition.

[18] In their contributions to inductive game theory, Kaneko and Matsui (1999) and Kaneko and Kline (2008) model games where players are not at all cognizant of the game structure. Inductive game theory consists of four stages (see Kaneko and Mitra, 2009): "(i) experimentation and transformation of short-term memories into long-term memories; (ii) inductive derivation of a personal view from the long-term memories; (iii) use of a personal view for his own decision making; and (iv) bringing his decision from (iii) back to (i)." Using their approach would not likely lead to significant new results addressing the main questions of this paper. It would, however, complicate the analysis significantly. For this reason, we choose not to follow this approach.

[19] For example, Bourdieu writes: "Social capital is the aggregate of the actual or potential resources which are linked to possession of a durable network of more or less institutionalized relationships of mutual acquaintance and recognition – or in other words, to membership in a group – which provides each of its members with the backing of the collectivity-owned capital, a 'credential' which entitles them to credit, in the various senses of the word. [...] The volume of the social capital possessed by a given agent thus depends on the size of the network of connections he can effectively mobilize and on the volume of the capital (economic, cultural or symbolic) possessed in his own right by each of those to whom he is connected," whereas Coleman's definition is less precise but, as does Bourdieu's, characterizes social capital as an attribute of individuals: "Social capital is defined by its function. It is not a single entity, but a variety of different entities having two characteristics in common: They all consist of some aspect of social structures, and they facilitate certain actions of individuals who are within that structure. [...] Unlike other forms of capital, *social capital inheres the structure of relations between persons and among persons*. It is lodged neither in individuals nor in physical implements of production." [Our Italics] Both represent micro-perspectives on social capital.

For any agent, the sum of his net gains from cooperation in all his social relations determines the social capital available to enforce cooperation in one-shot interactions with other agents. Instead, for the other informational regimes, the extent to which existing relations in a social network can facilitate "the achievement of certain ends" for an agent does not depend only on his net gains from cooperation, that is, how much he has to lose in his social relations.

Because the delay with which an eventual punishment sets in matters, it also depends on *partners' locations* in the network. Reviewing "(game-)theoretical questions stimulated by a reflection on social capital," Sobel (2002) identifies two ways in which Coleman's (1990) network closure or – put differently – "dense social networks make enforcement of group cooperative behavior more effective." This is accomplished first by creating "common knowledge of information" and second by increasing "the quality and reliability of third-party monitoring needed to enforce cooperative dynamic equilibria." In this paper, we offer an additional explanation for why closure is important for the enforcement of cooperative behavior, namely, the pooling of payoff asymmetries.

For Robert Putnam (1995), social capital "refers to the collective value of all 'social networks' and the inclinations that arise from these networks to do things for each other." Taking this macro-perspective on social capital, one can also define aggregate social capital in a sustainable social network as the average individual social capital in that society. Our model thus also connects the two main measures of social capital used in empirical studies: the expectation of cooperative outcomes in (one-shot) collective action problems, or *Trust* (as in Knack and Keefer, 1997, or La Porta et al., 1997), and the social structure that can lead to it, *Social Networks* (as in Narayan and Pritchett, 1999 or Temple and Johnson, 1998). Our model also highlights that it is the collective norm to enforce business cooperation in the social sphere, which generates trust and which allows social networks to serve as a proxy for social capital, that is, as in Lin (2001), to measure the return to social networks in the business sphere.

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Appendix A. Proof of Proposition 1

Proof. Part 1: Immediate. Part 2: Denote the set of neighbors k of agent i for which $g^{ik} < 0$ with R_i^- and the set of neighbors m of agent i for which $g^{im} \geqslant 0$ with R_i^+. Cooperation in relations of an agent i with agents $m \in R_i^-$ have to be enforced by a retaliation threat from agents in $m \in R_i^+$. Let F_i be a set of relations of agent i with neighbors $k \in R_i^-$ such that $\mathcal{N}^S(N, R \setminus F_i)$ is a disconnected graph. Because F_i is a disconnecting set of relations of agent i, if there is no public information, actions in the other relations of i cannot be a function of the history of i's interactions in F_i. Therefore, agent i does not have to fear retaliation in his other relations for deviating from relations in F_i. □

Appendix B. Proof of Proposition 2

Proof. Part (1): Immediate. Part (2): *Sufficiency*: Consider an optimal deviation from (S2). This deviation will not be profitable if the sum of the net gains from cooperation, corrected for the delay of indirect punishment by contagion, is positive. Furthermore, assumptions regarding the beliefs make the immediate playing of the noncooperative action with every neighbor rational. *Necessity*: During the infinite punishment phase, agents play their minmax strategy. This is the strongest punishment available. As there is no possibility of transmitting information on past behavior, it is also not possible to enter a punishment phase with a cheater at an earlier time. For these two reasons, (S2) are optimal punishments, which shows necessity. □

Appendix C. Proof of Proposition 3

Proof. Divide network $\mathcal{N}^S$ into subnetworks A and B so that A is sustainable by *MG* strategies on its own and B is not. As a result, there are three equilibria: cooperation in $\mathcal{N}^S$, cooperation only in A, and no cooperation. Furthermore, there exists an agent i who is an element in A and B and who has negative net gains from cooperation with his neighbors in B. Consider a history such that in period τ, agent i deviated only from his relation(s) in B but not from those with neighbors in A. In period $\tau+1$, all agents $j \in \mathcal{N}^S$ would have to revert to the noncooperative action vis-à-vis all of their neighbors, including those in the sustainable subnetwork A; and i would have been better off also deviating from his neighbors in A. Then, by deviating only from his relation(s) in B but not from those with neighbors in A, preplay communication for the subgame has ended with the following message from player i to all other players in A: "Look, I had the opportunity to get [the continuation payoff from deviating in all my relations] for sure, and nevertheless I decided to play in this subgame, [play C with all my neighbors in A] and my move is already made. And we both know that you can no longer talk to me, because we are in the game, and my move is made. So think well, and make your decision." (adapted from Kohlberg and Mertens, 1986, p. 1013). The equilibrium formed by (S1) violates Kohlberg and Mertens' (1986) requirement that the solution of a game should be invariant under inessential transformations of the tree. The inessential transformation is the deletion of the strategy "play C with neighbors in A and D with neighbors in B and enter into the punishment phase (Nash reversion) with all neighbors the period after" as it is dominated by "play D with neighbors in A and D with neighbors in B and enter into the punishment phase (Nash reversion) with all neighbors the period after." Once this strategy is eliminated, it is clear that, following "C with neighbors in A and D with neighbors in B," the continuation equilibrium should be MG in A and Nash reversion in B. □

Appendix D. Proof of Lemma 1

Proof. First, if a neighbor played C, passing on a message saying D, it will have the effect that this neighbor's neighbors will enter into a punishment phase with the neighbor (who will continue cooperating with the liar). The liar does not gain. Second, if instead a neighbor played D, passing on a message saying C will delay the punishment for that neighbor on the other side but not affect the liar's payoff (it is not possible to extend the punishment phase for oneself, as this would mean opening a punishment phase against oneself). The liar does not gain. Third, changing a message that a neighbor's neighbor played C into D will have the effect that the neighbor's neighbors will enter into a punishment phase with the neighbor who will go on cooperating with his neighbor. The liar does not gain. Finally, changing a message that a neighbor played D into C will again delay the punishment for the cheater but will not affect the liar's payoff. □

Appendix E. Proof of Proposition 5

Proof. The following constraints have to hold in equilibrium. First, no agent can have an incentive to deviate in a cooperation phase; denote this constraint by (IC^{CI}). Second, noncheaters must not have an incentive to deviate in a punishment phase, which entails (a) that they continue to cooperate with noncheaters and send a message about the deviation; denote this constraint by (IC^{CII}); and (b) that they punish cheaters; denote this constraint by (IC^P). Third, cheaters must not have an incentive to deviate in a punishment phase; denote this constraint by (IC^{LP}). First note that (IC^{CI}) $\Rightarrow$ (IC^{CII}): if the noncheater in a punishment phase is a neighbor of a cheater and a neighbor to noncheaters, continuation payoffs are the same in relations with noncheaters as in a cooperation phase, and continuation payoffs with a cheater are larger than in a cooperation phase as $w^{ij} > c^{ij}$. Second, note that (IC^P) always holds because $w^{ij} > c^{ij}$. Third, because $d^{ij} > l^{ij}$, (IC^{LP}) implies that for each neighbor j of each agent i, the maximum length of the repentance is such that the continuation payoff for the cheater equals the infinite stream of her minmax payoff in that interaction. Let $i, k \in R_j$ and denote L_{ik} the length of the shortest path from i to k that does not go through j. Given the closure of the network and Word-of-Mouth Communication, actions of agent $i \in R_j$ in his interaction with j in period τ can be a function of the actions j took vis-à-vis any neighbor $k \in R_j$ in periods $t \leqslant \tau - \frac{L_{ik}}{v} - 1$. With the use of Word-of-Mouth Communication, this delay is as short as possible. Given that punishment is as strong as possible and comes as early as possible, MR strategies are optimal punishment strategies.

Because there is no information transmission with MGs, actions of agent $i \in R_j$ in his interaction with j in period τ can only be a function of the actions j took vis-à-vis any neighbor $k \in R_j$ in periods $t \leqslant \tau - L_{ik}$. Therefore, the minimum discount factor necessary to sustain a given relational network with Word-of-Mouth Communication and MR strategies, (S3), is strictly less than the minimum discount factor necessary to sustain the same relational network under MG strategies, (S2). This implies that MG strategies, (S2), are not optimal punishment strategies. □

Appendix F. Proof of Proposition 6

Proof. Agents are being rewarded for punishing their neighbor. Therefore, they always have an incentive to carry out a punishment during a punishment phase, even if they are able to cooperate within a sustainable subnetwork. Therefore, there is no continuation equilibrium "play MG in A and Nash reversion in B" and the strategies satisfy Kohlberg and Mertens' (1986) criteria for strategic stability. □

Appendix G. Proof of Proposition 7

Proof. Denoting the shortest path from an agent in a set of agents J_i to agent k that does not pass through i by $l(J_i, k)$, for MGs, the sustainability condition is $\forall i, \forall J_i \subseteq R_i$, $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta^{l(J_i,k)} g^{ik} \geqslant 0$, and for MRs, the sustainability condition is $\forall i, \forall J_i \subseteq R_i$, $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta^{\theta(l(J_i,k)+2, v)} g^{ik} \geqslant 0$.

Define δ_{MG} to be the minimum discount factor for which a given network is sustainable with MGs. Then there is an agent i with $J_i \subseteq R_i$, such that $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta_{MG}^{l(J_i,k)} g^{ik} = 0$. Ceteris paribus, increase the delay in punishment by agent k against agent i's defection from j. Then, using the implicit function theorem, we derive $\frac{d\delta_{MG}}{dl(J_i,k)} = \frac{-\delta_{MG}^{l(J_i,k)} \ln(\delta_{MG}) g^{ik}}{\frac{\partial g^{ij}}{\partial \delta_{MG}} + l(J_i,k)\delta_{MG}^{l(J_i,k)-1} g^{ik} + \delta_{MG}^{l(J_i,k)} \frac{\partial g^{ik}}{\partial \delta_{MG}}} > 0$. Define δ_{MR} to be the minimum discount factor for which a given network is sustainable with MRs. Then, for agent i, $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta_{MR}^{\theta(l(J_i,k)+2,v)} g^{ik} = 0$. Assume $v = 1$. Then δ_{MR} is such that $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta_{MR}^{l(J_i,k)-1} g^{ik} = 0$, whereas δ_{MG} is such that $\sum_{j \in J_i} g^{ij} + \sum_{k \in R_i, k \notin J_i} \delta_{MG}^{l(J_i,k)} g^{ik} = 0$. As $\frac{d^2\delta_{MG}}{dl(J_i,k)^2} = \frac{\delta^{2l(J_i,k)-1} \ln(\delta_{MG})(g^{ik})^2 - \delta^{l(J_i,k)} \ln^2(\delta_{MG}) g^{ik} \frac{\partial g^{ik}}{\partial \delta_{MG}}}{(\frac{\partial g^{ij}}{\partial \delta_{MG}} + l(J_i,k)\delta_{MG}^{l(J_i,k)-1} g^{ik} + \delta_{MG}^{l(J_i,k)} \frac{\partial g^{ik}}{\partial \delta_{MG}})^2} < 0$, we know that $\frac{d\delta_{MR}}{dl(J_i,k)} > \frac{d\delta_{MG}}{dl(J_i,k)} > 0$. Therefore, for a low speed of information transmission, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs is decreasing in l. Assume $v \to \infty$. Then $\frac{d\delta_{MR}}{dl(J_i,k)} \to 0$, whereas $\frac{d\delta_{MG}}{dl(J_i,k)} > 0$. Therefore, for a high speed of information transmission, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs is increasing in l. Therefore, for a given network, there exists a $\underline{v}$ such that, for $v < \underline{v}$, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs is increasing in the length of the shortest indirect path between agents and, for $v > \underline{v}$, the set of discount factors for which a given network that is sustainable with MRs is not sustainable with MGs is decreasing in the length of the shortest indirect path between agents. □

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