Mandatory Disclosure and Financial Contagion*

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Abstract

The paper analyzes the welfare implications of a policy of mandatory disclosure of information on the value of directly held investments by banks. It is based on a model of payments in a network where the value of a bank's equity depends on the value of directly held investment of its trading partners, i.e. there is "contagion". Additionally banks have a profitable investment project for which they need funding from outsiders, but due to agency problems they can only obtain financing if their equity is large enough. Furthermore, banks can, at a cost, disclose the information of their directly held investments. We find that there are equilibrium with no disclosure and no funding for new investments projects. Yet when the disclosure costs are not too large, mandatory disclosure increases ex-ante welfare of banks and outside investors if and only if contagion is severe. The difference for the social and private benefits is that an individual bank's disclosure of the value of its directly held investment can be uninformative due to counterparty default risk. Instead, mandatory disclosure of information by all banks allows outside investors to assess the financial architecture of the system and direct their funding to the solvent banks.

JEL Classification Numbers:

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1 Introduction

In trying to understand how the decline in U.S. house prices evolved into a financial crisis in which trade between financial intermediaries nearly ground to a halt, one important contributing factor that has been singled out is the prevailing uncertainty at the time regarding which entities incurred the bulk of the losses associated with the housing market. For instance, Gorton's (2008) provides an early analysis of the crisis in which he argues

> "The ongoing Panic of 2007 is due to a loss of information about the location and size of risks of loss due to default on a number of interlinked securities, special purpose vehicles, and derivatives, all related to subprime mortgages... The introduction of the ABX index revealed that the values of subprime bonds (of the 2006 and 2007 vintage) were falling rapidly in value. But, it was not possible to know where the risk resided and without this information market participants rationally worried about the solvency of their trading counterparties. This led to a general freeze of intra-bank markets, write-downs, and a spiral downwards of the prices of structured products as banks were forced to dump assets."

Market participants, using more colorful language, emphasized the same phenomenon as the crisis was unfolding. Back in February 24, 2007, the *Wall Street Journal* attributed the following to Lewis Ranieri, one of the originators of mortgage securitization:

> "The problem ... is that in the past few years the business has changed so much that if the U.S. housing market takes another lurch downward, no one will know where all the bodies are buried. 'I don't know how to understand the ripple effects through the system today,' he said during a recent seminar."

In line with this view, some have argued that an important force towards the eventual stabilization of financial markets was the Fed's implementation of bank stress tests. These tests required banks to report to Fed examiners how their respective portfolios would fare under various stress scenarios, thus revealing potential losses each bank was vulnerable to. In contrast to the traditional confidentiality accorded to bank examination results, the results of these stress tests were publicly released. Bernanke (2013) summarizes the view that this public disclosure played an important role in stabilizing financial markets:

> "In retrospect, the SCAP [Supervisory Capital Assessment Program] stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors' public disclosure of the stress test results helped

restore confidence in the banking system and enabled its successful recapitalization."

In this paper, we seek to understand whether uncertainty regarding the identity of the banks that incurred losses - that is, uncertainty as to where the bad apples are located - can lead to market freezes, and under what circumstances if any mandatory disclosure that reveals which banks incurred losses and which did not could represent a welfare-improving intervention. The feature that turns out to be critical for this type of uncertainty to matter is financial contagion, by which we mean a situation in which the losses of banks that are directly hit by some shock trigger losses at other banks that are not themselves hit by the same shock. In the context of the financial crisis, an example of contagion would be if the losses at banks directly exposed to the housing market somehow lead to losses at banks that hold very little housing-related assets in their portfolios.

In what follows, we consider a model of financial contagion that operates through balance sheet effects, i.e. where banks that are hit by shocks default on their obligations to banks that are not hit by shocks, potentially rendering the latter insolvent. We modify this model of financial contagion in two ways. First, we allow banks to raise additional funds from outside investors. However, we introduce an agency problem that makes it profitable for investors to only invest in banks that have sufficient equity. When investors are uncertain about which banks incurred losses, they may be leery about investing in a bank whose equity position is uncertain. Contagion exacerbates this problem, since investors worry not only that the banks they invest in were hit by shocks that wiped out their equity, but that these banks may be indirectly exposed to such shocks because they have financial obligations from banks that were directly hit. When linkages between banks are such that the potential for contagion is greater, market freezes are more likely to occur.

Second, we allow banks to disclose whether they were hit by shocks. An important question for whether mandatory stress-tests whose results will be publicly disseminated are desirable is why banks don't perform and release their own stress tests, especially banks whose stress tests are likely to be favorable. Here, we show that contagion once again plays an important role. Our main result is that when contagion is small, it will not be possible for mandatory disclosure to be Pareto improving relative to a non-disclosure equilibrium, even when non-disclosure is associated with a market freeze in which no bank can raise funds from outside investors. But when contagion is large, there will exist a non-disclosure equilibrium that can be improved upon through mandatory disclosure provided that the cost of disclosure is low. Intuitively, contagion implies that information on the financial health of one bank is relevant for assessing the financial health of other banks. Since banks fail to internalize these informational spillovers, too little information will be revealed, creating a role for mandatory disclosure as a welfare improving intervention. Absent these spillovers, banks that are not hit by shocks internalize the benefits of disclosure, and so if they choose not to disclose in equilibrium it must be because the cost of stress-tests exceed the benefits. In that case, forcing them to disclose will not be desirable. Compared to the existing literature on disclosure, our result provides a new justification for mandatory disclosure that has not been previously identified.

Since we build our model in several stages to highlight the role of each component, it will help to provide a preliminary overview of the model as a whole. There is set of banks arranged along a network that reflects the initial obligations among banks. Some of these banks will be hit with shocks that prevent them from paying their obligations to other banks in full. Consequently, banks that are not hit with shocks can still have their equity wiped out. Regardless of whether a bank is hit by a shock or not, each bank has access to profitable projects that it can undertake if it can raise funds from outside investors. However, because of an agency problem that is present at each bank, outside investors will only want to invest in banks that have enough equity. We allow each bank trying to raise funds to disclose to outsiders at some cost whether it was hit by a shock. This disclosure must be made before the bank knows which other banks were hit with shocks, and thus before it knows its own equity value. Outside investors see all the information that is disclosed and decide which banks to invest in and under what terms. Finally, banks learn their equity. Banks that raised funds and realize their equity has been wiped out will take actions that inflict losses on any outside investors who invested with them. This framework allows us to explore how features of the underlying financial network that govern the extent of financial contagion affect whether banks can raise funds in equilibrium as well as the desirability of mandatory disclosure. For example, we find that banks are highly levered with funds borrowed from other banks on the network, a shock that increases the losses at banks directly hit by shocks will cause markets to freeze and may create a role for mandatory disclosure. However, when banks are not very highly levered against other banks on the network, the same shock will have no effect on the ability of banks to raise outside funding.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 develops the model of contagion we use in our analysis. Section 4 we modify our model so that banks can raise additional funds, and we introduce an agency problem that makes investors leery of investing in banks with little equity. In Section 5, we introduce a disclosure decision. We then examine whether non-disclosure can be an equilibrium outcome, and if so whether mandatory disclosure can be welfare improving relative to that equilibrium. Section 6 considers more general network structures. Section 7 concludes.

2 Literature Review

Our paper is related to several different literatures, specifically work on i) financial networks and contagion, ii) disclosure, iii) market freezes, and iv) stress tests.

Turning first to the literature on financial networks and contagion, various channels have been proposed for why contagion can occur. For a survey of this literature, see Allen and Babus (2009). We focus on models of contagion based on balance sheet effects, i.e. where a bank that is hit by shocks is unable to pay on its obligations, making it difficult for other banks to meet their obligations. Examples of papers that explore this channel include Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Caballero and Simsek (2012), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013). In most of our discussion, we follow Caballero and Simsek (2012) in focusing on a particular network structure known as a ring network or a circular network. However, we generalize their setup to allow losses at multiple banks rather than at just one bank as they assume. This turns out to be important, since the fraction of banks that are hit with shocks plays a role in our results. Our generalization to multiple banks uncovers a connection between the problem of contagion in a circular network and a geometric problem in applied probability known as the circle-covering problem.[1] This connection may be useful for analysis that uses circular networks. Although we focus much of our discussion on circular networks, we subsequently show that our analysis extends to more general class of networks, which includes several of the symmetric network structures that have been discussed in this literature, e.g. many of the special cases described in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013).

In terms of how our result relates to existing work on disclosure, we note that there is a vast literature on disclosure that precedes our work. Two good surveys of this literature include Verrecchia (2001) and Beyer et al. (2010). A key result in this literature, first established by Milgrom (1981) and Grossman (1981), is an "unravelling principle" which holds that all private information will be disclosed because agents with better information want to avoid being pooled together with inferior types and receive worse terms of trade. Beyer et al. (2010) summarize the various conditions subsequent research has established that are necessary for this unravelling result to hold: (1) disclosure must be costless; (2) outsiders know the firm has private information; (3) all outsiders interpret disclosure in the same way, i.e. outsiders have no private information (4) information can be credibly disclosed, i.e. the information disclosed is verifiable; and (5) agents cannot commit to a disclosure policy ex-ante before observing the relevant information. Violating any one of these conditions can result in equilibria where not all relevant information is conveyed. We show that non-disclosure

[1] The same connection is made in Barlevy and Haikady (2013).

can be an equilibrium outcome in our model even when all of these conditions are satisfied. We thus highlight a distinct reason for the failure of the unravelling principle that is due to informational spillovers: In order to know whether a bank in our model has equity and is thus safe to invest in, outside investors need to know not just the bank's own balance sheet, but also the balance sheets of other banks as well.

Ours is certainly not the first paper to explore disclosure in the presence of informational spillovers. A particularly important predecessor is Admati and Pfleiderer (2000), who explicitly model informational spillovers. Like in our model, their setup allows for non-disclosure equilibria. However, these equilibria rely crucially on disclosure being costly; when the cost of disclosure is zero in their model, some information will be disclosed. The reason our framework allows for non-disclosure even when disclosure is costless is because it allows for strong informational complementarities that are not present in their model. In particular, unilateral disclosure in our model is not enough to ascertain whether a firm has positive equity, since this requires knowing information on other banks in the network. This feature, which has no analog in their model, is why non-disclosure equilibria can arise in our framework despite satisfying all of the conditions listed above. However, Admati and Pfleiderer (2000) are similar to us in showing that mandatory disclosure may be welfare-improving because of informational spillovers.[2] That said, there are other important differences between our model and theirs. First, they study an environment where agents must choose whether to commit to disclose information before they learn it. By contrast, we are interested in the case where banks know what losses if any they incurred and must then decide whether to disclose it. Second, our setup allows us to consider comparative static exercises with regards to contagion that cannot be deduced from their setup.

Another literature that our paper is related to is concerned with explaining the phenomenon of market freezes. As in our model, this literature has emphasized the importance of informational frictions as a source of reduced trade. Some of these papers emphasize the role of private information, where agents are reluctant to trade with others in fear of being take advantage by others who are more informed than them. Examples include Rocheteau (2011), Camargo and Lester (2011), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2012), and Kurlat (2013). Other papers have focused on uncertainty concerning each agent's own need for liquidity and the liquidity needs of others which discourages trade. Examples include Caballero and Krishnamurthy (2008) and Gale and Yorulmazer (2013). An important difference between our framework and these papers, aside from various model-specific features, concerns the source of informational frictions. Since in our framework the

[2] Earlier work by Foster (1980) and Easterbrook and Fischel (1984) also argues that spillovers may justify mandatory disclosure, although these papers do not develop a formal model to establish this.

uncertainty concerns information that can in principle be revealed, namely whether a bank incurred losses on a particular asset class, it naturally focuses attention on the possibility that the information agents are uncertain about will be revealed. By contrast, previous papers have focused on private information on individual assets or information that no agents are privy to and thus cannot be disclosed.

Finally, there is an emerging literature on the use of stress tests. In contrast to our paper, this literature is largely empirical. Some of the questions explored in this literature are related to issues we study. For example, Peristian, Morgan, and Savino (2010) find evidence that the release of stress test results in the US revealed relevant information to the market, as evidenced by changes in the stock market values of the banks concerned. Bischof and Daske (2012) study the 2011 stress test conducted on more than 60 banks in Europe and argue that disclosure served a positive role. While these papers examine the positive effects of disclosure, they have little to say on the normative question of whether mandatory disclosure is desirable. We thus view our analysis as complementary to theirs.

3 A Model of Contagion

We begin with a stripped down version of our model in which banks make no decisions. This allows us to highlight how contagion works in the model and what features determine the extent of contagion, as well as to motivate the particular measure of contagion we use for our analysis.

Our approach to modelling financial contagion follows Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Caballero and Simsek (2012), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in focusing on balance sheet effects as the source of contagion. Formally, there are n banks indexed by $j \in \{0, ..., n-1\}$. Each bank i is endowed with a set of financial obligations $\Lambda_{ij} \geq 0$ to each bank $j \neq i$. Following Eisenberg and Noe (2001), we take these obligations as given without modelling where they come from. For much of our analysis, we follow Caballero and Simsek (2012) in restricting attention to the special case in which

$$\Lambda_{ij} = \begin{cases} \lambda & \text{if } j = (i+1) \pmod{n} \\ 0 & \text{else} \end{cases} \tag{1}$$

This case is known as a ring network or a circular network, since we can depict these obligations graphically as if the n banks are located along a circle, and each bank owes λ units of resources to the bank that sits clockwise from it.In Section 6, we show that our analysis can be extended to a larger class of networks than the circular network. However, since the circular network is expositionally convenient, we prefer to focus on this network initially.

In addition to the financial obligations Λ_{ij}, each bank is endowed with some valuable assets that can be liquidated if needed. We do not explicitly model the value of assets, and simply set their value fixed at some value π.

A fixed positive number of banks b, which we shall refer to as "bad" banks, are hit with a negative net worth shocks, where $1 \leq b \leq n-1$. This generalizes Caballero and Simsek (2012), who only consider the case where $b = 1$. Each bad bank incurs a loss ϕ, where ϕ represents a claim on the bank by an outside sector, i.e. by an entity that does not include any of the remaining banks in the network. The obligation ϕ is senior to any obligation to other banks in the network. That is, all of the bank's available resources must first be used to pay the outside sector, and only then can bank j make payments to bank $j+1$ from any remaining funds. For example, ϕ could represent a margin call against the bank by some outside party because the value of some asset the bank used as collateral has fallen in value. We shall refer to all remaining banks as "good."

Let $S_j = 1$ if j is a bad bank and 0 otherwise. The vector $S = (S_0, ..., S_{n-1})$ denotes the state of the banking network. By construction, $\sum_{j=0}^{n-1} S_j = b$. Shocks are equally like to hit any bank, i.e. each of the $\binom{n}{b}$ possible locations of the bad banks within the network are equally likely. In particular, $\Pr(S_j = 1) = \dfrac{b}{n}$ for any bank j.

We now analyze the financial position of banks assuming that payments are made in accordance with our seniority rules. Banks can be insolvent – meaning they are unable to fully repay their obligation λ to another bank – or solvent but with varying degrees of equity. The feature we wish to draw attention to in this section is that our model exhibits contagion effects whereby good banks may end up with a low equity value because of direct or indirect exposure to bad banks.

Let x_j denote the payment bank j makes to bank $j+1$, and let y_j denote the payment bank j makes to the outside sector. Bank j has $x_{j-1} + \pi$ resources it can draw on to meet any of its obligations. Given our restrictions on the seniority, it must first pay the outside sector. Let $\Phi_j \equiv \phi S_j$ denote the obligation to the outsider sector. Then the payment y_j must satisfy

$$y_j = \min\{x_{j-1} + \pi, \Phi_j\} \tag{2}$$

Bank j can then use any remaining resources to pay bank $j+1$, to which it owes λ. Hence, the payment bank j makes to bank $j+1$ is given by

$$x_j = \min\{x_{j-1} + \pi - y_j, \lambda\} \tag{3}$$

Substituting in for y_j yields a system of equations involving only the payments between banks, $\{x_j\}_{j=0}^{n-1}$, that characterizes these payments:

$$x_j = \max\{0, \min(x_{j-1} + \pi - \Phi_j, \lambda)\}, \qquad j = 0, ..., n-1 \tag{4}$$

The system (4) involves n equations and n unknowns. Given a set of payments $\{x_j\}_{j=0}^{n-1}$ that solves (4), we can define the equity of bank j as any residual resources then bank can lay claim to after settling all payments, i.e.

$$e_j = \max\{0, \pi - \Phi_j + x_{j-1} - x_j\} \tag{5}$$

Although the variable e_j is redundant given the payments x_j, equity will turn out to play an important role later on when we expand the model to allow banks to make decisions. Although payments x_j and equity values e_j depend on the state of the network, i.e. $x_j = x_j(S)$ and $e_j = e_j(S)$, we shall omit the explicit reference to S when this dependence does not play an essential role.

We first show that the system of equations (4) has a generically unique solution.[3]

Proposition 1: Given the state of the network S, the system (4) has a unique solution $\{x_j^*\}_{j=0}^{n-1}$ whenever $\phi \neq \frac{n}{b}\pi$.

In the knife-edge case where the total losses across all bad banks, $b\phi$, are just equal to the aggregate value of the asset endowments of banks, $n\pi$, there can be multiple solutions if the obligations λ that banks owe one another are sufficiently large. However, these solutions are equivalent to one another in a particular sense. That is, across all such solutions, the outside sector is paid in full, meaning $w_j = \Phi_j$ for all j, and the equity values $\{e_j\}_{j=0}^{n-1}$ of all banks are the same, namely $e_j = 0$ for all j. The only difference across the solutions are the notional amounts by which banks default on to other banks in the network.

In analyzing the circular network, we restrict attention to the case where $\phi < \frac{n}{b}\pi$, i.e. where the total total losses incurred by bad banks $b\phi$ cannot be so large that they exceed the total resources of the banking system, $n\pi$. Although Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) show that explicitly allowing the case where losses are large can yield important insights on the nature of contagion, for our purposes allowing for large shocks yields few important insights. In particular, when $\phi > \frac{n}{b}\pi$, there are two possible outcomes depending on the value of λ. When λ is small, the distribution of equity values $\{e_j\}_{j=0}^{n-1}$ will be independent of ϕ, and so the implications of this case can be understood even if we restrict $\phi < \frac{n}{b}\pi$.

[3] Our result is a special case of Theorem 2 in Eisenberg and Noe (2001) and of Proposition 1 in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013). The latter result establishes uniqueness for a generic network Λ_{ij} but does not provide exact conditions for non-uniqueness as we do. Our finding that equity values are unique is a special case of Theorem 1 in Eisenberg and Noe (2001).

When λ is large, all n banks will be insolvent and have no equity when $\phi > \frac{n}{b}\pi$. Since we are interested in the decisions banks make when are unsure about their equity, the case where banks know their equity to be zero is of little interest.

At the same time, we don't want the loss ϕ to be too small. As the next proposition shows, when $\phi \leq \pi$ even bad banks will be solvent, and so their losses will not affect the equity of other banks in the network.

Proposition 2: If $\phi \leq \pi$, then $x_j^* = \lambda$ for all j.

We therefore restrict the size of losses ϕ to a range of intermediate values:

Assumption A1: Losses at bad banks ϕ are restricted to the following range:

$$\pi < \phi < \frac{n}{b}\pi \tag{6}$$

The fact that $\phi > \pi$ ensures bad banks will be insolvent, since even if these banks receive the full amount λ from the bank that is indebted to them, they will have less than λ resources to pay the bank to which they are obligated. Consequently, the equity of each bad bank will be 0.

To understand the nature of contagion from in this economy, it will help to begin with the case where there is exactly one bad bank, i.e. $b = 1$, as in Caballero and Simsek (2012). Without loss of generality, let bank $j = 0$ be the bad bank. Given that bank 0 receives x_{n-1} from bank $n-1$, the total amount of resources bank 0 can give to bank 1 is $\min\{x_{n-1} + \pi - \phi, 0\}$. Below we show in Proposition 3 that Assumption A1 ensures there is at least one bank in the network will be solvent and pay its obligation λ in full. From this, it follows that bank $n-1$ must be solvent. This is because if any bank $j \in \{1, ..., n-2\}$ were solvent, it would pay its debt to bank $j+1$ in full, who in turn will pay its debt to bank $j+2$ in full, and so on, until we reach bank $n-1$. Hence, $x_{n-1} = \lambda$.

Given that $x_{n-1} = \lambda$, deriving the equity positions of the remaining banks is straightforward. Since $\phi > \pi$, the bad bank will fall short on its obligation to bank 1 by the amount

$$\Delta_0 = \min\{\phi - \pi, \lambda\}.$$

Since bank 1 is endowed with $\pi > 0$ resources, it can use them to make up some of the shortfall it inherits when it pays its obligation to bank 2. If the shortfall $\Delta_0 > \pi$, bank 1 will also be insolvent, although its shortfall will be π less than shortfall it receives. The first bank that inherits a shortfall that is less than or equal to π will be solvent, with an equity position that is at least 0 but strictly less than π. Hence, we can classify banks into three groups: (1) Insolvent banks with zero equity, which includes both the bad bank and potentially other good banks; (2) Solvent banks whose equity is $0 \leq e_j < \pi$. When $b = 1$, there will be exactly one such bank; (3) Solvent banks that are sufficiently far from the bad bank and have equity equal to π.

Since equity will figure prominently in our analysis below, it will be convenient to work with the case where e_j can take on exactly two values, 0 or π. For $b = 1$, this requires only that $\Delta_0 = \min\{\phi - \pi, \lambda\}$ be an integer multiple of π, so either ϕ or λ must be integer multiples of π. For general values of b, we will need to impose that both ϕ and λ are integer multiples of π:

Assumption A2: ϕ and λ are both integer multiples of π.

For the case where $b = 1$, Assumption A2 implies that the one solvent bank with equity less than π will have equity equal to 0, and so the number of good banks with zero equity will be given by

$$k = \min\left\{\frac{\phi}{\pi} - 1, \frac{\lambda}{\pi}\right\} \tag{7}$$

Caballero and Simsek (2012) refer to k as the size of the "domino effect" of a bad bank. When there is only one bad bank, k can be viewed as a measure of contagion. When there can be more than one bad bank, the variable k still captures the *potential* for contagion of any given bad bank: It tells us the maximal amount of good banks that would have their equity wiped out as a result of a shock to a single bad bank. This suggests that as many as bk good banks can have their equity wiped out. But for reasons that will become apparent when we turn to the case where $b > 1$, the actual number of good banks whose equity falls because of their exposure to bad banks can fall below this amount. As such, we will need to introduce a different metric by which to measure contagion for general values of b. This metric will be a function of k, but will depend on other parameters as well.

Two conditions are required for the value of k in (7) to be large. First, a large k requires losses ϕ to be large. Intuitively, when ϕ is small, a bad bank will still be able to pay back a large share of its obligation λ, and so fewer banks will ultimately be affected by the loss. Second, a large k requires the obligation λ be large. Intuitively, when λ is small, banks are not very indebted to one another, and in the limit as $\lambda \to 0$, there will be no contagion to good banks regardless of how large losses ϕ at bad banks are. As λ rises, losses are shifted from the outside sector, which bears all losses when $\lambda = 0$, to other banks in the network. Essentially, higher gross flows between banks imply that the resources of any give bank travel further along the network, including to bad banks where these resources will be captured by the outside sector. The further resources flow along the network, the more they will tend to ultimately end up in the hands of the outside sector. In fact, as we show in Proposition 4 below, for sufficiently large λ the outside sector will incur no losses, and all losses will instead be borne by the banks within the network.

Armed with this intuition, we can now move to the general case of an arbitrary number of banks, i.e. $1 \leq b \leq n-1$. We begin with a preliminary result that under Assumption A1, at least one bank will be solvent and can pay its obligation in full.

Proposition 3: If $\phi < \frac{n}{b}\pi$, then there exists at least one bank j which is solvent, i.e. $x_j = \lambda$. Moreover, among all solvent banks there exists at least one bank j with positive equity, i.e. $e_j > 0$.

As in the case with $b = 1$, there will be three types of banks when $b > 1$: (1) Insolvent banks with zero equity; (2) Solvent banks whose equity is $0 \leq e_j < \pi$; (3) Solvent banks that are sufficiently far away from a bad bank whose equity $e_j = \pi$. Since we know there is at least one solvent bank j, we can start with this bank and move to bank $j+1$. If bank $j+1$ is good, it too will be solvent and its equity will be $e_{j+1} = \pi$. We can continue this way until we eventually reach a bad bank. Without loss of generality, we refer to this bad bank as bank 0. By the same argument as in the case where $b = 1$, Assumption A2 implies that banks $1, ..., k$ will have zero equity, where k is given by (7): Even if all of these banks are good, each will inherit a shortfall of at least π and will have to sell off its π assets. If any of these banks are bad themselves, the shortfall banks to them will inhereit will be even larger, and so equity at the first k banks remains zero.

If bad banks are sufficiently spread out across the network, i.e. if there are at least k banks between any two bad banks, the total number of banks with zero equity would equal $bk + b$, i.e. bk good banks whose equity is wiped out because of their exposure to bad banks and b bad banks. However, in general a bank that is exposed to a bad bank may be bad itself. Because of this, the number of banks with zero equity need not equal $bk + b$. Whether it does or not depends on the size of the obligations across banks, as measured by λ. We now show that when λ is large, the location of the bad banks within the network will not matter, and exactly $bk + b$ banks will have zero equity regardless of whether bad banks are spaced out or not. But when λ is small, the number of banks with zero equity can be smaller than $bk + b$ and will depend on how close bad banks are located to one another.

We begin by showing that for sufficiently large λ, all banks will be able to make some payment to the bank they are obligated to regardless of where the bad banks are located, i.e. regardless of the state of the banking network $S = (S_0, ..., S_{n-1})$.

Proposition 4: Under Assumption A1, payments $x_j > 0$ for all $j = 0, ..., n-1$ and all states S if and only if $\lambda > b(\phi - \pi)$. When $\lambda \leq b(\phi - \pi)$, there exist realizations of S for which $x_j = 0$ for at least one j.

If each bank j can pay some positive amount to bank $j+1$, then each bank j must pay the outside sector in full given whose claims are senior to all other claims. Hence, Proposition 4 implies that regardless of the state of the network S, the outside sector will be paid in full. This in turn implies that the the total amount of resources left within the banking network is the same regardless of where bad banks are located. Since Assumption A2 implies banks can have equity equal to either 0 or π and total equity is the same for all S, it follows that the number of banks with zero equity is the same for all S whenever $\lambda > b(\phi - \pi)$. Formally:

Proposition 5: Under Assumptions A1 and A2, if $\lambda > b(\phi - \pi)$, the number of banks with zero equity is equal to $bk + b$ regardless of the state of the banking network S.

Next, consider the case where λ is small. In this case, resources do not travel far along the network, and a bad bank may end up defaulting on the outside sector. Since defaulting on the outside sector allows the network to retain more of the resources its member banks own, the more banks default on their obligations to the outsider sector, the larger the number of banks who maintain their equity. Proposition 4 suggests that when $\lambda \leq b(\phi - \pi)$, there will be states of the world in which some banks default on their obligation to the outside sector.

For sufficiently low values of λ, specifically when $\lambda < \phi - \pi$, we can explicitly characterize the distribution of the number of banks with zero equity. At such low values of λ, we are at the other extreme where even if bad banks were paid in full by the banks that owe them resources, they would still have to default on the outside sector and pay nothing to the bank they themselves are obligated to. Thus, regardless of where the bad banks located, a bad bank would wipe out the equity of the next $k = \frac{\lambda}{\pi}$ banks and no more. Denote the number of banks with zero equity by ζ. The number of banks with zero equity ζ is now a random variable, with a support that ranges from $b + k$ when all bad banks are located next to each other to $bk + b$ when there are at least k good banks between any two bad banks. By contrast, for $\lambda > b(\phi - \pi)$, the number of banks with zero equity ζ has a degenerate distribution with all of its mass at $bk + b$.

To obtain an exact distribution for ζ for $\lambda < \phi - \pi$, we exploit the fact that when $\lambda < \phi - \pi$, our model corresponds to a discrete version of a well-studied geometric problem in applied probability known as the circle-covering problem that was first introduced by Stevens (1939). In this problem, a fixed number of points are drawn from random locations along a circle of length 1, and then arcs of a fixed length less than 1 are drawn starting from each of these points and proceeding clockwise. The circle-covering problem seeks to determine the probability that the circle is fully covered by the arcs and the distribution of the length that is uncovered given the number of points and the length of each arc. In our setting, the number of bad banks is analogous to the number of points sampled from the circle, while the potential for contagion k, expressed relative to the total number of banks in the network, corresponds to the length of the arc. The region of the circle covered by arcs is analogous to the fraction of all banks that have zero equity. The discrete version of this circle-covering problem has been analyzed in Holst (1985), Ivchenko (1994), and **?**. As first noted by Holst (1985), the discrete version can be analyzed using results on Bose-Einstein statistics. This insight can be used to obtain an exact expression for the distribution of ζ. However, for our purposes only the expected value of $E[\zeta]$ matters, which can be obtained using results in Ivchenko (1994) and **?**. This expectation is summarized in the next lemma.

Lemma 1: Under Assumptions A1 and A2, the expected number of banks with zero equity ζ is given by

$$E[\zeta] = n - \frac{(n-b)!\,(n-k-1)!}{(n-1)!\,(n-b-k-1)!}$$

where $k = \frac{\lambda}{\pi}$ is integer-valued.

Finally, for intermediate values of λ between $\phi - \pi$ and $b(\phi - \pi)$, the number of banks with zero equity ζ will again be random, with support ranging between $b + \frac{\lambda}{\pi}$ and $b\frac{\phi}{\pi} = bk + b$. Thus, the support of ζ depends not just on k but also on $\frac{\lambda}{\pi}$ which is different from k. For these intermediate values of λ, the distribution of banks with zero equity is analogous to a circle covering problem in which the length of the arcs is not fixed but rather depends on the location of the points drawn at random. As far as we know, this variation of the circle-covering problem case has yet to be studied. However, in Proposition 6 below we establish some comparative static results for $E[\zeta]$ for this case.

To recap, as long as the potential for contagion k in (7) is positive, at least one good bank will end up with zero equity because of exposure to bad banks. When $b > 1$, how many good banks will be affected this way depends on more than just k. One way to summarize this, which will turn out to be convenient in our subsequent analysis, is to consider what happens if we chose a good bank at random. The extent to which good banks are exposed to losses will be captured by the probability that this bank will have equity equal to π, i.e. it will be unaffected by losses at bad banks. The smaller the probability that the equity value is equal to π, the more good banks that tend to have equity below π, and thus the greater the extent of contagion. Formally, define p_g as the probability that a good bank retains all of its equity, i.e.,

$$p_g = \Pr(e_j = \pi | S_j = 0) \; . \tag{8}$$

We will use p_g as our measure of contagion. As we show later on, even in more general networks where e_j can take on more than just two values, this definition for p_g turns out to be useful. If we continue to define k as in (7), p_g can be computed as follows:

$$\begin{aligned} p_g &= \sum_{z=b+k}^{bk+b} \Pr\left(e_j = \pi | S_j = 0, \zeta = z\right) \Pr\left(\zeta = z\right) \\ &= \sum_{z=b+k}^{bk+b} \frac{n-z}{n-b} \Pr\left(\zeta = z\right) = \frac{n - E\left[\zeta\right]}{n-b} \, . \end{aligned}$$

Intuitively, the expected number of banks with positive equity is $n - E[\zeta]$. Since only good banks can have positive equity, and there are always $n-b$ good banks, the fraction of good banks with equity equal to π is just the ratio of the two. The next proposition summarizes how p_g varies in our model depending on the underlying parameters:

Proposition 6. Under Assumptions A1 and A2,

$$p_g = \begin{cases} \prod_{i=1}^{\lambda/\pi} \left(\frac{n-b-i}{n-i}\right) & \text{if } 0 < \lambda < \phi - \pi \\ \Psi\left(b, n, \frac{\phi}{\pi}, \frac{\lambda}{\pi}\right) & \text{if } \phi - \pi \leq \lambda \leq b\left(\phi - \pi\right) \\ 1 - \frac{b}{n-b}\left(\frac{\phi}{\pi} - 1\right) & \text{if } b\left(\phi - \pi\right) < \lambda \end{cases} \tag{9}$$

where the function Ψ is weakly decreasing in ϕ/π and in λ/π.

Proposition 6 reveals that p_g generally depends on the magnitude of the losses at bad banks relative to their assets, $\frac{\phi}{\pi}$, the depth of financial ties relative to assets, $\frac{\lambda}{\pi}$, the number of bad banks b, and the total number of banks n. One feature we wish to point out now and that we will revisit below is that the effect of bank losses ϕ on p_g depends on λ. For small values of λ, specifically for $\lambda < \phi - \pi$, changes in ϕ have no effect on contagion. That is, a shock that results in bigger losses at bad banks only affects the outside sector, but has no effect on banks within the network. For large values of λ, though, increasing ϕ will lower p_g. That is, when banks are more intensely interconnected, a shock that results in bigger losses at bad banks will wipe out the equity of a larger number of good banks. Essentially, high values of λ allow losses at bad banks to affect more good banks. While it will be useful to keep this result in mind, in much of our analysis we can take p_g as fixed given the underlying network and losses at bad banks.

Remark: When $\lambda < b(\phi - \pi)$, the fraction of banks with zero equity, $\frac{\zeta}{n}$, is a random variable – even though the number of bad banks b and the losses per bank ϕ are deterministic. For some applications, it will be more convenient to work with a model where the fraction of banks with zero equity is also deterministic. One way to achieve this is to increase the number of banks n and exploit the law of large numbers. In particular, suppose we hold the potential for contagion k in (7) fixed and keep the fraction of bad banks $\frac{b}{n}$ constant at some value θ, but let $n \to \infty$. Let ζ_n denote the (random) number of banks with zero equity when there are n banks in the network. When $\lambda < \phi - \pi$, we can appeal to Theorem 4.2 in Holst (1985) to establish that the random variable $\frac{\zeta_n}{n}$ converges to a constant as $n \to \infty$. Likewise, the realized fraction of good banks that have zero equity, $\frac{n-\zeta_n}{n-b}$, converges to a constant. This constant will be the same as p_g, which recall is just the *expected* fraction of good banks with zero equity. Taking the limit for the expression in (9) for the case where $\lambda < \phi - \pi$ reveals that p_g converges to a particularly simple expression:

$$\lim_{n \to \infty} p_g = (1-\theta)^k \tag{10}$$

Intuitively, a good bank will only have positive equity if each of the k banks located clockwise from him are good. As $n \to \infty$, the probability that any one bank is bad converges to θ independently of what happens to any finite collection of banks around it. Hence, the probability that the relevant k neighbor banks are all good is $(1-\theta)^k$. Although the location of banks with zero equity remains random even when the size of the network becomes large, the fraction of all good banks with positive equity $\frac{n-\zeta_n}{n-b}$ will exhibit no randomness in the limit and will equal the expected fraction p_g. For any given θ, the limiting value of p_g can range between 0 and 1 as k varies from 0 to arbitrarily large integer values. Note that since $k = \min\left\{\frac{\lambda}{\pi}, \frac{\phi}{\pi} - 1\right\}$, values of k that exceed $\frac{1}{\theta} - 1$ will violate the second inequality in Assumption A1, which requires that $\frac{\phi}{\pi}$ be less than $\frac{n}{b} = \frac{1}{\theta}$. However, this restriction can essentially be dispensed with for large values of n, since the probability that equity is wiped out at all banks becomes exceedingly small even without this assumption. The limiting case as $n \to \infty$ is thus useful not only for eliminating uncertainty regarding the extent of contagion, but also for demonstrating that the contagion measure p_g in a circular network can assume the full range of possible values, from nearly no contagion ($p_g \to 1$) to nearly full contagion ($p_g \to 0$).

Finally, in some of our subsequent analysis we will need the unconditional probability that a given bank chosen at random has positive equity. Denote this probability by p_0. Since there are exactly b bad banks and $n-b$ good banks, and since all bad banks have zero equity under Assumption A1, p_0 can be expressed directly in terms of p_g:

$$\begin{aligned} p_0 &= \frac{n-b}{n} p_g + \frac{b}{n} \times 0 \\ &= \left(1 - \frac{b}{n}\right) p_g \end{aligned} \tag{11}$$

4 Outside Investors and Bank Equity

We now build on the model of contagion introduced in the previous section by allowing banks to raise external funds in order to finance productive opportunities. Although all banks can use the funds they raise profitably regardless of their equity position, we introduce a moral hazard problem that implies only banks with enough equity will go ahead and use the funds for productive opportunities. Specifically, we allow banks to divert the funds they raise to achieve private gains, a temptation that is mitigated by the equity a bank would have to give if it diverts the funds it raised. More generally, there are various actions banks can undertake when their equity is low that would be against the interests of outside investors, e.g. investing in riskier projects or gambling for resurrection.

In this section, we focus on the full-information benchmark in which banks and outside investors know precisely which banks are bad, and thus which banks have positive equity. In this case, allowing banks to raise funds has no impact on the contagion that arises in the model. In particular, since outside investors are only willing to finance banks with enough equity, banks that would have had zero equity in the original model will not be able to raise new funds. Letting banks raise funds merely accentuates the inequality between banks with zero and positive equity. While this leads to no new insights regarding contagion, it does introduce a reason for why bank equity can matter for the allocation of resources: Bank equity facilitates gains from trade that would not occur in its absence. When we allow banks to withhold information about whether they were hit by shocks or not, as we do in the next section, policy can potentially affect what agents believe about the equity at any given bank and thus whether trade takes place.

Formally, suppose that outside investors – which can be the same original outsiders that banks are indebted to or a new group of outsiders – can choose whether to invest with any of the n banks in the network. Banks have profitable projects they can undertake, but funding these projects require outside financing; banks cannot use their assets to finance them. For simplicity, we assume that each bank has a finite number of profitable projects it can undertake. We set the capacity of the bank to 1 unit of resources. On their own, outside investors can earn a gross return of $\underline{r}$ per unit of resources. Banks can earn a gross return of R on the projects they undertake, where $R > \underline{r}$. Thus, there is scope for gains from trade.

We restrict banks and outside investors to transact through debt contract that are junior to all of the bank's other obligations. Allowing for equity contracts would not resolve the moral hazard problem we introduce below, and so we invoke this assumption for convenience only. Let r_j^* denote the equilibrium gross interest rate bank j offers outside investors for any new funds they invest in the bank. We assume that the outside sector is large enough that

r_j^* is determined competitively, i.e. the expected gross returns to the outside sector from investing with a bank must equal $\underline{r}$. Hence, $r_j^* \geq \underline{r}$, and the most a bank can earn from raising new funds is $R - \underline{r}$.

After banks raise funds from outsiders, they observe which banks in the network are bad, from which they can deduce their equity position. At this point, a bank can choose to either invest any funds they raised and earn a return R, or divert the funds to a project that accrues a purely private benefit v per unit of resources. These private benefits cannot be seized by outsiders. Outside investors cannot monitor what banks do with the funds they raise and prevent banks from diverting funds. However, if the bank fails to pay the required obligation r_j^*, they can go after any assets the bank owns.

We want v to be large enough to ensure that banks with no equity at stake would divert funds – so the moral hazard problem is binding – but not so large that even a good bank that keeps its π worth of assets will be tempted to divert funds. To satisfy the first condition, we need $v > R - \underline{r}$, i.e. the private benefit v exceeds the most a bank can earn from raising funds and undertaking projects with them. To ensure that a good bank will not be tempted, we need to make sure that the payoff to the bank from undertaking the project, $\pi + R - r_j^*$, exceeds the payoff if it diverts the funds, $v + \max\left\{\pi - r_j^*, 0\right\}$, i.e. the bank would earn v in private benefits but would have to liquidate at least some of its assets to meet the promised obligation of r_j^*. Comparing the two expressions implies we need $v < R - \max\left\{r_j^* - \pi, 0\right\}$. Since a bank that can be entrusted not to divert funds need not offer more than $\underline{r}$ to outsiders, the condition that ensures banks with assets worth π can credibly promise to invest the funds they raise is if

$$v < R - \max\left\{\underline{r} - \pi, 0\right\} \tag{12}$$

In sum, we impose the following restriction on the private benefit term v:

Assumption A3: The private benefits v to a bank from diverting 1 unit of resources it raises from outsiders are neither too high nor too low, specifically

$$R - \underline{r} < v < R - \max\left\{\underline{r} - \pi, 0\right\} \tag{13}$$

Note that the second inequality in Assumption A3 implies that $v < R$. Hence, diversion is never socially optimal.

In the full information benchmark, banks know the entire state of the network S, i.e. they know the location of the bad banks. In Section 3, we showed that when banks were unable to raise new funds, Assumption A2 implies there will be ζ banks with zero equity and $n - \zeta$ with equity equal to π, where ζ is a random variable (with possibly degenerate distribution).

We now show that when banks can raise outside funds, the same ζ banks that would be left with zero equity in the original model would be unable to attract additional funds and will thus remain with zero equity, while the remaining $n-\zeta$ banks that would have had equity equal to π in the original model would be able to raise funds and so their current equity will now be $\pi+R-\underline{r}$. In other words, allowing banks to raise funds when there is full information does not change the pattern of contagion in our original model.

To derive this result, define a new variable $I_j \in [0,1]$ denote the amount outside investors invest in bank j. Since Assumption A3 involves strict inequalities, banks will either divert all of the funds the raise or none. Let $D_j = 1$ if bank j decides to divert the funds and 0 otherwise. Recall that y_j denotes the obligation of bank j to its most senior creditors and x_j the payment bank j makes to bank $j+1$. We introduce a new variable w_j to denote the payment bank j makes to outside investors who invest additional funds with bank j. Then we have

$$\begin{aligned} y_j &= \min\{x_{j-1}+\pi+R(1-D_j)I_j, \Phi_j\} \\ x_j &= \min\{x_{j-1}+\pi+R(1-D_j)I_j-y_j, \lambda\} \\ w_j &= \min\left\{x_{j-1}+\pi+R(1-D_j)I_j-y_j-x_j, r_j^* I_j\right\} \end{aligned}$$

Finally, the equity each bank is given by

$$e_j = \max\{0, x_{j-1}+\pi+R(1-D_j)I_j-y_j-x_j-w_j\}$$

For comparison, let $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$ denote the payments to senior creditors and other banks, respectively, if outside investors did not fund any of the banks, i.e. if we set $I_j = 0$ for all j. Likewise, define $\{\widehat{e}_j\}_{j=1}^n$ as the equity positions given $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$, i.e.

$$\widehat{e}_j = \max\{0, \pi-\Phi_j+\widehat{x}_{j-1}-\widehat{x}_j\}$$

Note that $\widehat{e}_j$ corresponds to the equity positions we solved for in the previous section in the absence of any additional investment in banks. Our claim is that in the full-information benchmark, $e_j = 0$ whenever $\widehat{e}_j = 0$, and $e_j > 0$ whenever $\widehat{e}_j > 0$.

Proposition 7: Given Assumption A1-A3, with full information, $e_j = 0$ for any bank j for which $\widehat{e}_j = 0$, and $e_j > 0$ if $\widehat{e}_j > 0$. Moreover, $I_j = 0$ if and only if $\widehat{e}_j = 0$.

Proposition 7 reveals that even though all banks can raise funds and generate new profits that they can use to repay their obligations, the banks with the greatest resource needs are the same ones that would divert any funds they receive. Hence, when investors can identify which banks were hit with losses, banks that incur losses directly or indirectly will be unable to raise new funds, and the pattern of contagion will continue unabated. Although letting banks raise funds but subjecting them to moral hazard problems implies contagion will be unaffected, modifying our model to include this possibility is important in two respects. First, we can now assign a social cost to contagion, even if there is nothing in our model that allows policymakers to prevent it. In particular, when bank balance sheets are linked, shocks drain more equity away from the banking system and redirect it to senior creditors, reducing the scope for trade. Second, in the full-information benchmark, some banks receive funding and some banks do not. By contrast, if no information about the location of the bad banks became public, investors would view all banks equally. Disclosure can thus affect the allocation of resources, either by generating investment that would not take place otherwise or preventing banks that would divert those funds for private gains from securing funds. We explore whether these changes are desirable in the next section.

5 Disclosure

We can now analyze the implications of mandatory disclosure in our model. To do this, we introduce one final component into our model, namely a decision by banks whether to incur a cost and disclose their financial position before they raise funds. We first provide conditions under which there exists a non-disclosure equilibrium where no bank reveals whether it was hit by a shock. We then examine under what conditions mandatory disclosure that forces all banks to reveal whether they were hit with losses can be Pareto improving relative to a non-disclosure equilibrium.

Our main insight is captured in Theorem 1, which shows that the desirability of mandatory disclosure depend on the degree of financial contagion as captured by p_g. In particular, we show that when the extent of contagion is small, so p_g is close to 1 and few good banks fall victim to losses that hit bad banks, mandatory disclosure cannot be Pareto improving. But when the extent of contagion is large, so p_g is close to 0 and almost all good banks fall victim to losses at bad banks, mandatory disclosure will be Pareto improving as long as disclosure costs are not too large. Intuitively, when there is little contagion, the traditional unravelling result applies: Banks that know they are good benefit more from disclosure than a policymaker expects to gain from forcing a bank whose type is unknown to disclose. Thus, absent contagion, if a policymaker wants to force disclosure, good banks would already be willing to disclose. By contrast, when the extent of contagion is very high, so most banks are likely to have little equity, outsiders would be reluctant to invest in any bank when no information about the bank is known. Unilateral disclosure will not help, since even if the bank reveals it is good, without knowing anything about its neighbor banks it will remain highly likely that the bank will have no equity. Good banks are thus stuck in a coordination trap in which they might all be better off disclosing, but each bank unilaterally disclosing yields no benefits. However, our analysis reveals some interesting results for intermediate values of p_g. For example, mandatory disclosure can be welfare improving even at intermediate valued of p_g when a bank that discloses it is good will be able to raise funds. Thus, the inefficiency of equilibrium is not entirely a matter of coordination failures, but more generally reflects inefficiencies due to informational spillovers.

Formally, we model the process of disclosure as follows. After nature chooses the location of bad banks as summarized by the vector S, each bank observes whether it is good or bad, i.e. each bank j observes S_j, but not whether any other bank is good or bad. At this point, banks may choose to disclose their own situation, at a utility cost $c \geq 0$. The cost of disclosure c captures the fact that conducting and documenting the result of stress-test exercises can be costly, but it can also crudely capture the fact that disclosure may entail revealing information about trading strategies to competitors. The latter feature suggests disclosure may be costly even if the bank is insolvent, which explains why we assume c is a utility cost rather than a resource cost.

Outside investors observe which banks make which announcements, and decide whether to invest in any of the banks, as well as the terms of such contracts, given all announcements. Banks then learn the state of the network S, and only then do they decide whether to invest the funds they may have received from outside investors or divert them to reap private benefits. At the last stage, any potential profits are realized and payments are settled.

Since the cost c is borne regardless of whether the bank is solvent, a bad bank (i.e. a bank with $S_j = 1$) will never choose to announce. As such, we can describe the bank's decision by $a_j \in \{0, 1\}$, where $a_j = 1$ means bank j discloses it is good and $a_j = 0$ means it does not disclose any information. For now, we assume banks choose a_j simultaneously. Once all banks choose whether to disclose, outside investors observe the vector $a = (a_1, ..., a_n)$ and choose whether to allocate funds to any of the banks. Let $I_j(a)$ denote the amount outside investors allocate to bank j, and $r_j^*(a)$ denote the rate bank j is charged for any funds it raises.

5.1 Existence of a Non-Disclosure Equilibrium

We begin by exploring when non-disclosure, meaning $a_j = 0$ for all j, can be part of an equilibrium. For our equilibrium concept, we use the notion of sequential equilibria as introduced by Kreps and Wilson (1982). This means that off-equilibrium beliefs must correspond to the limit of beliefs from a sequence of games in which players choose all strategies with positive probability but the weight on suboptimal policies tends to zero. Effectively, this means that when outsiders observe banks act differently than is expected in equilibrium, they cannot adopt beliefs that imply banks knew something about other banks when they made their disclosure decisions. For example, our restriction implies that if a bank deviates and discloses, outsiders must not believe that the disclosing bank is definitely located next to a bad bank. Such beliefs are consistent with weaker notions of equilibria, but we view them as implausible given the deviating bank remains ignorant about the financial position of other banks in the network when it makes its disclosure decision, and so there is no reason that would justify such pessimism on the part of outside investors.

We now show that the existence of a non-disclosure equilibrium in which $a_j = 0$ for all j depends on two parameters – the cost of disclosure, c, and the extent of contagion p_g. For non-disclosure to be an equilibrium, each good bank must weakly prefer not to disclose, i.e. set $a_j = 0$, when it anticipates all other banks choose not to disclose, i.e. $a_{-j} = 0$. The payoff to disclosure depends on what outside investors do when no bank discloses and when one bank discloses it is good. If no bank discloses, the probability that any bank chosen at random will have positive equity is $p_0 = \left(1 - \frac{b}{n}\right) p_g$ as defined in (11). Under Assumptions A2 and A3, banks that learn they have zero equity would divert funds and leave nothing for investors, while banks with positive equity all have π worth of assets. Whether the latter would choose to invest depends on how much r_j^* they promise outside investors. The next lemma summarizes when banks would divert funds:

Lemma 2: Assume Assumptions A2 and A3 hold. For any bank j where pre-investment equity would be positive, i.e. $e_j > 0$ it is optimal not to divert funds, i.e to set $D_j = 0$, if and only if

$$r_j^*(a) \leq \overline{r} \equiv \pi + R - v. \tag{14}$$

In other words, if outside investors charge a rate above some threshold $\overline{r}$, banks will divert funds they raise regardless of their equity. In principle, outside investors might still be willing to fund banks at a rate above $\overline{r}$, since they can count on grabbing part of the equity π of viable. However, it turns out that the equilibrium interest rate charged to any bank will never exceed $\overline{r}$:

Lemma 3: Assume Assumptions A2 and A3 hold. In any equilibrium, $r_j^*(a) \leq \overline{r}$ for any bank j that receives funding, i.e. for which $I_j(a) = 1$.

Assumption A3 ensures that the maximum pledgeable amount $\overline{r}$ is bigger than the outside option of outside investors $\underline{r}$.[4] We now argue that if p_0 is sufficiently low, specifically if it

[4] To see this, consider the two cases $\underline{r} > \pi$ and $\underline{r} \leq \pi$. If $\underline{r} > \pi$, the second inequality in Assumption A3 implies $\underline{r} < R + \pi - v \equiv \overline{r}$. If $\underline{r} \leq \pi$, the second inequality in Assumption A3 implies $v < R$, and hence $\overline{r} = \pi + R - v > \pi \geq \underline{r}$.

falls below $\underline{r}/\overline{r} < 1$, then the only possible non-disclosure equilibrium is where $I_j = 0$. The reason is that Lemma 3 implies there is an upper bound on the rate banks can charge in equilibrium. But when no information about banks is available, the rate outside investors must charge banks to make it more profitable than their outside option is equal to $\frac{\underline{r}}{p_0}$. If p_0 is high enough, banks that do not receive funding can offer to pay a rate between $\frac{\underline{r}}{p_0}$ and $\overline{r}$ and receive funding. Hence, the only possible non-disclosure equilibria when $p_0 < \underline{r}/\overline{r}$ would involve $I_j = 1$ for all j. For low values of p_0, the rate outside investors must charge banks to make it worthwhile would exceed $\overline{r}$, which contradicts Lemma 3. Hence, at these values banks will be unable to raise any funds, i.e. any non-disclosure equilibrium would involve $I_j = 0$ for all j. Since p_0 is proportional to p_g as seen in (11), whether a non-disclosure equilibrium involves $I_j = 0$ or $I_j = 1$ can be expressed as a condition on p_g rather than p_0, i.e. how p_g compares with $\frac{n}{n-b}\underline{r}/\overline{r}$. It will turn out to be more natural to think of these conditions in terms of p_g rather than p_0.

We begin with the case where $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$. We just argued that if there is a non-disclosure equilibrium, it will involve $I_j = 1$ for all j. We need to verify there is no incentive for a good bank to disclose it is good if no other good bank discloses. Since each bank must already receive funding in equilibrium, the benefit to disclosure will not be that a bank can attract investment it would not be able to in equilibrium. Instead, the benefit to disclosure is that it allows the bank to pay outside investors less than it would have to otherwise. In particular, disclosure will increase the probability outsiders attach to the bank having positive equity from p_0 to p_g. This would allow a bank to borrow at a lower promised rate. Along the equilibrium path, the equilibrium interest rate all banks charge must equal $\frac{\underline{r}}{p_0}$, since this is the rate that ensures outside investors just earn their outside option in expectation. The implied payoff for a good bank is then

$$p_g\left(\pi + R - \frac{\underline{r}}{p_0}\right) + (1-p_g)\,v \tag{15}$$

Since a good bank knows it is good, the payoff in (15) is computed using the conditional probability p_g, even though outsiders assign probability p_0 that the bank will have positive equity. If the bank opts to disclose it is good, it will be able to still attract funding it offered any rate between $\frac{\underline{r}}{p_g}$ and $\frac{\underline{r}}{p_0}$. Hence, when no other good bank chooses to disclose, good banks will be willing not to disclose its own position if and only if the disclosure cost exceeds the maximal gain from lowering the rate they are charged, i.e.

$$\begin{aligned} c &\geq p_g\left(\frac{\underline{r}}{p_0} - \frac{\underline{r}}{p_g}\right) \\ &= \frac{b\underline{r}}{n-b} \end{aligned}$$

Hence, when $p_g > \frac{n}{n-b} \underline{r}/\overline{r}$, a non-disclosure equilibrium exists if and only if $c > \frac{b\underline{r}}{n-b}$, i.e. when disclosure costs exceed a fixed threshold. In this case, the unique non-disclosure equilibrium is one where all banks receive funding, and investors who end up investing in banks with zero equity will take losses. While this is the unique non-disclosure equilibrium, there may be other equilibria with partial or full disclosure given these values for p_g and c. Since we are interested in whether there is scope for mandating disclosure when disclosure wouldn't occur otherwise, we only provide conditions that ensure non-disclosure equilibria exist rather than characterize the full set of equilibria.

Next, we turn to the case where p_g falls below $\frac{n}{n-b} \underline{r}/\overline{r}$. In this case, we argued above that the only possible non-disclosure equilibrium in one in which outside investors refrain from investing in any bank, i.e. $I_j = 0$ for all j. We need to verify that no good bank would wish to disclose its position given no other bank discloses. Since there is no investment in equilibrium, the only way a bank could benefit from disclosure is if revealing it has equity will induce outsiders to fund it. Hence, non-disclosure will be an equilibrium if either unilateral disclosure will not induce outsiders to invest in that bank, or if unilateral disclosure will induce investment but the cost of disclosure is too high relative to the gains from attracting investment.

Given our restriction to sequential equilibria, a good bank that unilaterally discloses it is good should expect outside investors to assign probability p_g that the bank has positive equity. Hence, outsiders will demand at least $\frac{\underline{r}}{p_g}$ to lend to the bank, since a bank that learns it has zero equity will divert funds for sure. From Lemma 2, we know that if $\frac{\underline{r}}{p_g} > \overline{r}$, a bank will not be able to both pay enough to outsiders and credibly commit not to divert funds. Hence, if $p_g < \underline{r}/\overline{r}$, a good bank will not be able to attract investment if it discloses unilaterally. In this case, non-disclosure will be an equilibrium for any $c \geq 0$. The fact that non-disclosure is an equilibrium even when $c = 0$ is of particular interest, since it reveals that our model gives rise to non-disclosure equilibria in cases not already captured by previous work on disclosure as summarized in Beyer et al. (2010). What drives our result is that our model exhibits a strong informational spillover in which information from more than one agent is required to piece together whether a bank has sufficient equity to be worth investing in. This feature is absent from previous work on disclosure that allowed for certain types of informational spillovers, e.g. Admati and Pfleiderer (2000). In the latter case, a firm can disclose all of the relevant information on its own when other firms fail to disclose, and their model only yields non-disclosure equilibria when disclosure is costly.[5]

[5] Okuno-Fujiwara, Postlewaite, and Suzumura (1990) obtain a result that is closer in spirit to our finding. They provide several examples where non-disclosure can be an equilibrium. In one of these (Example 4), a firm can disclose information it has about another firm, and so piecing information about one agent requires disclosure from others. In their setup, a firm does not benefit from disclosing unfavorable information about

The only remaining case is the one where $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. Under these parameter restrictions, $p_0 < \underline{r}/\overline{r} < p_g$. This means that when no bank discloses, investors will be too worried about default to invest in any one bank, but once a single bank reveals it is good it will be able to attract investment from outsiders. In particular, since $p_g > \underline{r}/\overline{r}$, a bank that discloses can offer a rate below $\overline{r}$ that remains competitive with the return $\underline{r}$ outsiders can earn. By disclosing and attracting investment, the bank will change its utility by

$$p_g \left(R - \underline{r}/p_g\right) + \left(1 - p_g\right) v - c$$

Hence, for non-disclosure to be an equilibrium, the cost c must be sufficiently large to make disclosure unprofitable, i.e.

$$p_g \left(R - v\right) + v - \underline{r} < c$$

As in the case with $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$, a non-disclosure equilibrium exists only if the cost of disclosure c is sufficiently high.

We collect the analysis of the three cases above into the following proposition:

Proposition 8. Assume that Assumptions A2 and A3 hold. Then

1. A non-disclosure equilibrium *with no investment* can only exist if $p_g \leq \min\left(1, \frac{n}{n-b}\underline{r}/\overline{r}\right)$. Such an equilibrium exists if either

   (i) $p_g \leq \underline{r}/\overline{r}$; or
   (ii) $\underline{r}/\overline{r} < p_g \leq \frac{n}{n-b}\underline{r}/\overline{r}$ and $c \geq p_g(R - v) + v - \underline{r}$

2. A non-disclosure equilibrium *with investment* can exist only if $p_g \geq \frac{n}{n-b}(\underline{r}/\overline{r})$. Such an equilibrium exists if

   (i) $\frac{b}{n} \leq 1 - \underline{r}/\overline{r}$; and
   (ii) $p_g \geq \frac{n}{n-b}\underline{r}/\overline{r}$ and $c \geq \frac{b}{n-b}\underline{r}$

Figure **??** shows the same results graphically. The shaded region corresponds to the set of parameters for which a non-disclosure equilibria exists. Since the thresholds for c are not generally comparable for $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$ and $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$, we drew the figures for these two cases separately.

While we characterize our results in terms of the contagion parameter p_g, recall from Proposition 6 that this measure in turn depends on primitives that describe the financial

its competitor because in the absence of disclosure the firm's competitor is already at a corner and would act the same way if the firm disclosed unfavorable information about it. In this case, the information doesn't matter. By contrast, in our case disclosure matters – outside investors are willing to charge the bank a lower rate after it discloses, but the information isn't enough to make an impact without disclosure by others.

network for banks, e.g. the magnitude of losses ϕ and the magnitude of obligations λ across banks. Acknowledging this connection reveals some interesting comparative statics. For example, consider the case where ϕ is small. In that case, p_g will be close to 1, and so if there is a non-disclosure equilibrium it will be one in which all banks are able to raise funds. In response to news that losses at banks are now much bigger, so ϕ is higher, the implications for equilibrium depend on λ. When λ is small, p_g will not change much. Indeed, recall from Section 3 that when $\lambda < \phi$, a change in ϕ will have no effect on p_g. Thus, the economy can remain in the same equilibrium. But if λ is large, p_g will fall with ϕ. If p_g falls sufficiently, then if non-disclosure persists, the only possible equilibrium is one in which no banks are able to attract funds. In this sense, the model suggests that large degrees of leverage against other banks as measured by λ allow shocks to give rise to market freezes that would not occur when λ is smaller.

5.2 Mandatory Disclosure and Welfare

We now turn to the question of whether if a non-disclosure equilibrium exists, a policy of mandatory disclosure can Pareto improve upon this equilibrium. From Proposition 8, we know that there are two types of non-disclosure equilibria, those where p_g is low and outsiders refrain from investing in any of the banks, and those where p_g is high and outsiders invest in all banks even when no information is disclosed.

We begin with non-disclosure equilibria where there is no investment, i.e. when $p_g < \frac{n}{n-b} {}^{\underline{r}}\!/_{\overline{r}}$. In this case, mandatory disclosure induces a shift from a situation in which no bank receives funding to one in which all banks that have positive equity before raising any funds can attract funds they subsequently invest as a gross return of R. The expected number of banks that will be able to attract investment under mandatory disclosure is equal to $(n-b)\, p_g$. Each of these can create a surplus of $R - \underline{r}$, i.e. resources that are unavailable when no disclosure and no investment takes place. Since each bank will have to incur a cost c to disclose its information, the cost of revealing that information that lets banks with positive equity raise funds is equal to cn. Hence, starting from a situation in which no banks receive funding because no bank discloses, mandatory disclosure will be Pareto improving if and only if

$$(n-b)\, p_g\, (R - \underline{r}) - cn > 0. \tag{16}$$

To determine whether this condition is compatible with the existence of a non-disclosure equilibrium with no investment in the first place, we need to compare (16) and the conditions in Proposition 8 for the existence of such an equilibrium. There are two cases to consider.

First, when $p_g < \underline{r}/\overline{r}$, such an equilibrium always exists regardless of c. This is because for very low values of p_g, a bank that unilaterally discloses it is good will not be able to attract investment. By contrast, (16) implies that forcing all firms to disclose in order to identify banks with positive equity will only be valuable if the cost of disclosure c is not too large. The next proposition summarizes the values of c that imply mandatory disclosure will be Pareto improving for a given value of $p_g \leq \underline{r}/\overline{r}$:

Proposition 9. Assume that Assumptions A2 and A3 hold. If $0 < p_g \leq \underline{r}/\overline{r}$ and $c \leq (R - \underline{r}) \frac{n-b}{n} p_g$, mandatory disclosure will Pareto dominate the non-disclosure equilibrium.

Next, we consider the case where $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. For these values of p_g, non-disclosure equilibria only exist if c is large enough. This is because for these values, a bank that announces it is good will be able to attract investment, and so it will internalize the benefits from disclosing that are inherent in determining when mandatory disclosure is beneficial. However, the private benefit from unilateral disclosure can deviate from the social benefit from mandatory disclosure. Mandatory disclosure would lead to a situation in which only banks that have enough equity will get funding. Since no bank ends up diverting funds, the value v does not appear in (16). By contrast, when a good bank is contemplating disclosing its financial position unilaterally, it realizes that it may end up with too little equity because it was exposed to losses at other banks, in which case it will divert funds and earn v. Thus, in deciding whether to unilaterally disclose, a bank will care about the value of v. Hence, the condition that ensures a non-disclosure equilibrium with no investment exists need not coincide with the condition that ensures mandatory disclosure will be preferable to a non-disclosure equilibrium with no investment. Hence, even for this range of values for p_g, there may be scope for mandatory disclosure to be Pareto improving.

The conditions for when a non-disclosure equilibrium exists for $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$ but which can nonetheless be improved upon are summarized in Proposition 10 below. Essentially, there are two necessary conditions. First, we need $v < \underline{r}$, i.e. diversion of funds is socially inefficient, since the private benefits it generates are below what outsiders could earn on their own. Without this condition, it must be the case that whenever it is socially optimal to force mandatory disclosure, the private gains from unilateral disclosure will be even higher given a bank expects to profit considerably even if the bank that owes it money defaults. But if v is sufficiently low, banks may opt not to disclose even when it will be socially beneficial to do so. The second condition is that the fraction of bad banks $\frac{b}{n}$ cannot be too large. Intuitively, for a bank considering disclosing unilaterally, the cost of communicating to investors that it is good is c. But for a policymaker who does know in advance which banks are good, the cost of disclosure per good bank is $\frac{n}{n-b}c$ since mandatory disclosure of all banks implies getting not just good banks to disclose. This implicitly higher cost of disclosure can make mandatory disclosure undesirable, and so for mandatory disclosure to be welfare improving we need the fraction of bad banks to be small. Formally, the conditions for when mandatory disclosure can impove upon a non-dislcosure equilibrium when $p_g \in \left(\underline{r}/\overline{r}, \frac{n}{n-b}\underline{r}/\overline{r}\right]$ can be summarized as follows:

Proposition 10. Assume that Assumptions A2 and A3 hold. Suppose $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\,\underline{r}/\overline{r}$. Then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot lead to a Pareto improvement upon this equilibrium.

2. If $v < \underline{r}$, then

   (a) If $\frac{b}{n} > \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, there exists no non-disclosure equilibrium with no investment which can be Pareto improved via mandatory disclosure.

   (b) If $\frac{b}{n} \leq \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, a non-disclosure equilibrium with no investment exists but is Pareto dominated by mandatory disclosure whenever

      i. $\underline{r}/\overline{r} < p_g < \min\left\{\frac{n}{n-b}\underline{r}/\overline{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\}$, and

      ii. $(R-v)p_g + (v-\underline{r}) \leq c \leq \frac{n-b}{n}p_g\left(R-\underline{r}\right)$.

      Since $\min\left\{\frac{n}{n-b}\underline{r}/\overline{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\} < 1$, condition (i) only holds for $p_g < 1$.

Note that the upper bound on p_g in part (2.b.i) of Proposition 10 is strictly below 1. Hence, using mandatory disclosure to induce investment when no investment would occur otherwise will only be desirable when there is sufficiently high contagion, i.e. when p_g is sufficiently below 1.

Finally, we turn to the case where $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$. Recall from Proposition 8 that in this case, if no firm disclosed, outside investors will invest in all banks. This does not mean that banks have no reason to disclose: A bank that reveals it is good will be able to offer a lower return to outside investors who invest with it. This represents a purely private gain: The bank is able to grab more of the surplus from outside investors, but disclosing it is good yields no additional surplus given the bank was receiving funds already. As Jovanovic (1982) points out, when disclosure decisions are driven by purely private gains, mandating disclosure is typically undesirable: It represents a costly activity that yields no social gains. Fishman and Hagerty (1989) also show that when disclosure is driven by rent-seeking, forcing more disclosure than occurs in equilibrium is not necessarily desirable. By contrast, since our model exhibits informational spillovers, disclosure may remain desirable even though each bank's decision to disclose is entirely driven by rent-seeking. To see this, observe that the expected amount of available resources is given by

$$(n-b)\,p_g\,(\pi+R)+[n-(n-b)\,p_g]\,v \tag{17}$$

where $(n-b)\,p_g$ represents the expected number of banks with zero equity. While this represents the wealth of the economy, these resources cannot be freely allocated since v are private benefits and are only available to banks. Nevertheless, we can compare this amount to the total available resources when all information is disclosed and investors know to avoid banks with zero equity. In this case, the expected resources are given by

$$(n-b)\,p_g\,(\pi+R)+[n-(n-b)\,p_g]\,\underline{r} \tag{18}$$

When $v<\underline{r}$, the private benefits banks obtain are less than the returns outsiders can earn and diversion is socially wasteful. As long as the benefits from disclosure exceed the costs, i.e. if $[n-(n-b)\,p_g]\,(\underline{r}-v)$ exceeds cn, or alternatively if $c<\left(1-\frac{n-b}{n}p_g\right)(\underline{r}-v)$, mandatory disclosure could be socially beneficial. Comparing with the condtion for the existence of a non-disclosure equilibrium implies that for a given $p_g>\frac{n}{n-b}\underline{r}/\overline{r}$ reveals that mandatory disclosure can be welfare improving relative to a non-disclosure equilibrium when the cost of disclosure c satsifies

$$\frac{b\underline{r}}{n-b}<c<\left(1-\frac{n-b}{n}p_g\right)(\underline{r}-v) \tag{19}$$

Once again, for this set of values to be non-empty, two conditions must be satisfied. First, $v<\underline{r}$. This is because when investment already takes place in the absence of any disclosure, the value of disclosure can only come from preventing socially wasteful diversion. Second, the fraction of bad banks $\frac{b}{n}$ cannot be too large. Again, a larger fraction of bad banks raises the effective cost of mandatory disclosure relative to the considerations that determine whether an individual bank would like to disclose. Formally, the conditions for when mandatory disclosure can impove upon a non-dislcosure equilibrium when $p_g>\frac{n}{n-b}\underline{r}/\overline{r}$ can be summarized as follows:

Proposition 11. Assume that Assumptions A2 and A3 hold. Suppose $p_g\geq\frac{n}{n-b}\,\underline{r}/\overline{r}$. Then

1. If $v\geq\underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot lead to a Pareto improvement upon this equilibrium.

2. If $v<\underline{r}$, then

    (a) If $\frac{b}{n}>\frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\overline{r})+\underline{r}}\,(1-\underline{r}/\overline{r})$, there exists no non-disclosure equilibrium with investment which can be Pareto improved via mandatory disclosure.

(b) If $\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\bar{r})+\underline{r}}(1-\underline{r}/\bar{r})$, a non-disclosure equilibrium with investment exists but is Pareto dominated by mandatory disclosure whenever

i. $\frac{n}{n-b}\underline{r}/\bar{r} \leq p_g \leq \frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)$, and

ii. $\frac{b}{n-b}\,\underline{r} \leq c \leq (1-\frac{n-b}{n}p_g)(\underline{r}-v)\underline{r}/\bar{r}$.

Since $\frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right) < 1$, condition (i) only holds for $p_g < 1$.

Note the parallel with Proposition 10. Once again, the upper bound on p_g in part (2.b.i) of Proposition 11 is strictly below 1. Hence, using mandatory disclosure to avoid wasteful diversion of funds to low return private gains can only be beneficial when there is sufficiently high contagion. Although banks who contemplate disclosure care are motivated by private rents, when diversion is socially costly it will drive up how much banks have to pay to borrow when outside investors do not know if they are good. Thus, banks will internalize the benefits from disclosure, and in the absence of contagion will already be motivated to disclose whenever mandatory disclosure is socially desirable.

Summarizing Propositions 9-11 yields the following result regarding the desirability of mandatory disclosure as a function of the extreme values p_g can assume:

Theorem 1. Assume that Assumptions A2 and A3 hold. For p_g sufficiently close to 1, mandatory disclosure cannot Pareto improve upon a non-disclosure equilibrium. Conversely, for p_g sufficiently close to but not equal to 0, if the c is low, the non-disclosure equilibrium is Pareto-improveable.

While there are disclosure cost for which a Pareto improvement on the non-disclosure equilibrium exists when p_g is small but positive, this will not be true when $p_g = 0$. In that case, there are no banks worth investing in, and so disclosure serves no role. More generally, for $p_g < \frac{n}{n-b}\underline{r}/\bar{r}$, the maximal social gains from mandatory disclosure are increasing in p_g, since a higher value of p_g implies a larger fraction of banks can profitably invest if they could raise funding. This illustrates an important tension inherent in our model: More contagion makes it more likely that mandatory disclosure can be Pareto improving, but it also makes the gains from such intervention smaller. When $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$, the maximal gains from mandatory disclosure are instead decreasing in p_g. This is because for these values, investors finance all banks and the benefit of mandatory disclosure instead from avoiding socially wasteful diversion. A higher p_g implies more such diversion will take place. In this case, more contagion makes it both more likely that mandatory dislcosure can be Pareto improving and increases the gains from such intervention. But now there is a different tension: Although more contagion makes the case for mandatory disclosure stronger, it also makes non-disclosure equilibria in which investors agree to invest when no information is disclosed less likely.

Finally, returning to the question of what drives p_g, we can relate our observation on the implications of λ for contagion to the desirability of requiring banks to disclose their potential losses. Recall that a shock which increases the losses ϕ at bad banks will have no effect on the degree of contagion when λ is small but will increase contagion when λ is large. Thus, a shock that would not have led to market freezes or made mandatory disclosure desirable when banks were not strongly connected can both lead to market freezes and make mandatory disclosure desirable when banks are more strongly connected.

6 Alternative Network Structures

So far, our model of financial contagion relied on a very particular network structure. Recall that a network is generally defined by a set of financial obligations Λ_{ij} that represent the liabilities of each bank $i \in \{0, ..., n-1\}$ to all other banks $j \neq i$. We considered the special case where $\Lambda_{ij} = \lambda$ for $j = i + 1 \pmod{n}$. We now argue that our key result on welfare extends to a larger class of networks.

Under the most general specification, the liabilities across banks can be summarized by an $n \times n$ matrix Λ with zeros along the diagonal. We want to preserve the feature that in the absence of any shocks, the equity at any given bank will equal π, the value of the assets it is endowed with. This requires that each bank has a zero net position with the remaining banks in the network, i.e. its obligations to other banks are equal to the obligations it has from other banks:

$$\sum_{j \neq i} \Lambda_{ij} = \sum_{j \neq i} \Lambda_{ji} \tag{20}$$

Condition (20) is often described in the network literature as a statement that the weighted network Λ is regular. See, for example, Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013).

As in the case of the circular network, we let the financial network be hit by a shock process governed by two parameters: b, the number of bad banks, and ϕ, the losses at each bad bank. We continue to assume that each of the $\binom{n}{b}$ possible locations of the bad banks within the network are equally likely.

Rather than just n payments $\{x_j\}_{j=0}^{n-1}$ we now have payments between all pairs of banks, $\{x_{ij}\}_{i \neq j}$. Since banks that are hit with shocks will be unable to meet all of their obligations, we need to specify a rule for how available resources should be divided. We follow Eisenberg and Noe (2001) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in assuming that an insolvent bank pays the same pro-rata share to each of the banks it owes resources. That is, define

$$\Lambda_i = \sum\nolimits_{j=0}^{n-1} \Lambda_{ij} \tag{21}$$

as the total obligations of bank i. If bank i is insolvent, it will pay each bank j it is obligated to a fraction $\frac{\Lambda_{ij}}{\Lambda_i}$ of the resources is does have. This implies that the set of payments x_{ij} solve the system of equations

$$x_{ij} = \frac{\Lambda_{ij}}{\Lambda_i} \max \left\{ \min \left\{ \Lambda_i \, , \, \pi - S_i \phi + \sum\nolimits_{j=0}^{n-1} x_{ji} \right\} \, , \; 0 \right\} \text{ for all } i \neq j \tag{22}$$

where recall $S_i = 1$ if bank i is bad. We can then define the equity of bank i as

$$e_i = \pi + \sum\nolimits_{j=0}^{n-1} x_{ji} - S_j \phi - \sum\nolimits_{j=0}^{n-1} x_{ij} \tag{23}$$

An important feature of the circular network that makes it analytically tractable is that it implies a particular symmetry: Each good bank is equally exposed to contagion regarldess of its location along the network. This allows us to summarize contagion with a single statistic, p_g, the probability that a bank will be unaffected by contagion and will be able to keep all of its initial endowment π. We shall now argue that for networks that exhibit a similar type of symmetry, our key insight continues to hold regarding the connection between the degree of contagion and whether mandatory disclosure can be welfare improving when a non-disclosure equilibrium exists. Formally, we have

Definition: A financial network with payment obligations Λ is *symmetrically vulnerable to contagion* given the shock process $\{b, \phi\}$ if with b bad banks that are each hit with a shock of size ϕ, the distribution of equity for a good bank across all possible realizations of S does not depend on its location in the network, i.e.

$$\Pr\left(e_j = x | S_j = 0\right) \text{ for all } x \in [0, \pi] \tag{24}$$

is independent of j.

The circular network we have focused on so far clearly satisfies this condition. However, there are other networks that are symmetrically vulnerable to contagion. One example is the class of circulant networks, i.e. networks in which the payment obligation Λ_{ij} from bank i to bank j can be expressed as a function of $i - j \pmod n$, i.e. the distance between banks i and j. In other words, the matrix of obligations Λ is a circulant matrix:

$$\Lambda = \begin{bmatrix} 0 & \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} \\ \lambda_{n-1} & 0 & \lambda_1 & \cdots & \lambda_{n-2} \\ \lambda_{n-2} & \lambda_{n-1} & 0 & \cdots & \lambda_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1 & \lambda_2 & \lambda_3 & \cdots & 0 \end{bmatrix} \tag{25}$$

The next lemma confirms that circulant networks exhibit symmetric vulnerability to contagion:

Lemma 4: Suppose Λ is a circulant matrix. Then the financial network with payment obligations Λ is symmetrically vulnerable to contagion.

Note that the circular network is a special case in which $\lambda_1 = \lambda$ and $\lambda_j = 0$ for all $j \neq 1$. But the class of ciruclant networks encompasses other network structures that have been discussed in the literature, including complete financial networks where banks maintain equal liabilities with all other banks, i.e. $\lambda_j = \lambda$ for all $j \neq 0$, partially complete networks where banks have liabilities with more than one bank, e.g. the interconnected ring network in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and symmetric isolated networks such as isolated pairs.

Remark: Note that the symmetry property we impose concern the symmetry of an outcome along the network rather than about the network itself. Circulant networks are symmetric networks, in the sense that the nodes and links in the network are exchangeable, which makes it easy to confirm symmetric vulnerability to contagion. Indeed, for any symmetric network, the same argument behind Lemma 4 can be used to show that the network is symmetrically vulnerable to contagion. However, symmetric vulnerability to contagion can also arise with asymmetric networks in which no nodes or links are exchangeable. In the Appendix we give an example of an asymmetric network Λ that nevertheless exhibits symmetric vulnerability to contagion, at least for particular values of b and ϕ.

For any network that is symmetrically vulnerable to contagion, we can continue to define p_g as $\Pr(e_j = \pi | S_j = 0)$ for any j. That is, p_g represents the probability that a good bank will be unaffected by contagion, meaning it will not have to sell off any of its initial resources. However, in contrast with the case of the circular network under Assumption A2, p_g must no longer represent the probability that a good bank will divert funds if it learns that it is bad. This is because when banks have obligations to multiple banks, the distribution for e_j at any good bank j will assign positive probability to more than 0 and π. At intermediate values, a bank affected by contagion might still have enough equity that it will prefer to invest its funds. Hence, the conditions that determine whether a non-disclosure equilibrium exists we derived for the circular network under Assumption A2 do not apply in this more general case, since the probability agents assign to a good bank investing the funds need not equal p_g. Nevertheless, we still obtain an analogous result to Theorem 1 on the connection between p_g and the improveability of non-disclosure equilibria:

Theorem 2. Suppose Λ is regular, i.e. it satisfies (20), and is symmetrically vulnerable to contagion. Suppose that $\pi < \phi$ and that Assumption A3 holds. For p_g sufficiently close to 1, mandatory disclosure cannot Pareto improve upon a non-disclosure equilibrium. Conversely, for p_g sufficiently close to but not equal to 0, if the c is low, the non-disclosure equilibrium is Pareto-improveable.

Note that Theorem 2 implies Theorem 1 and thus strictly generalizes our results for the circular network. However, it would be incorrect to conclude that the structure of the network is irrelevant. In fact, there are two reasons why network structure matters for the desirability of mandatory disclosure.

First, for values of p_g that are sufficiently below 1 that a non-disclosure equilibrium can be Pareto improving, the values of c for which mandatory disclosure can be Pareto improving will depend on the network structure. Interestingly, we find that for a given value p_g, the range of values of c for which a non-disclosure equilibrium exists but can be improved upon by mandating disclosure is weakly larger than what we derived for the circular network under Assumption A2. Intuitively, when a firm discloses it is good, it would reduce the risk of investing with the banks that are exposed to that bank because of balance sheet effects. That would allow those banks to borrow at lower rates. At a lower rate, the threshold level of equity at which a firm would be willing to invest the funds it raised rather than divert them rises. Since diversion is socially wasteful under Assumption A3 given $v < R$, this is an implicit benefit of disclosure that banks do not take into account. In the circular network under Assumption A2, this margin was missing because equity could only assume two values, 0 and π.

Second, the structure of the network determines the contagion probability p_g. As such, the structure of the network will certainly matter for whether mandatory disclosure can be Pareto improving over a non-disclosure equilibrium, even if our result concerning what would happen at extreme values of p_g are unchanged. As an example, consider the complete network in which $\lambda_j = \lambda$ for all $j \neq 0$. In this case, the exact location of bad banks is irrelevant, since the equity of any good bank will be the same regardless of which banks are bad given the uniform exposure. Intuitively, mandatory disclosure can serve no positive role in this case. Consistent with this, for this network p_g can take on only two values, 0 and 1, and so p_g never falls in the region where Theorem 2 tells us mandatory disclosure can be Pareto improving.

This is in contrast to the circular network, where recall p_g can assume any value between 0 and 1, at least for large values of n.

As another example, suppose n is even, and consider the case where there are $n/2$ isolated pairs indebted to one another. If ϕ and λ are both large, a good bank will have equity above $\underline{e}$ only if the other bank it is paired with is good. Since there are b bad banks and $n-1$ locations, the probability that a given good bank is paired with another good bank, or p_g, is equal to $1-\frac{b}{n-1}$. In this case, the degree of contagion will be large if b is sufficiently large. Hence, mandatory disclosure can be Pareto improving. However, for intermediate values p_g, mandatory disclosure can only be beneficial if $\frac{b}{n}$ is small, just as in the case with a circular network under Assumption A2. But since high contagion requires relatively high values of b, the two conditions may be incompatible.

7 Current Limitations and Future Work

The main result in this paper that in the when contagion is substantial and disclosure cost are not too high a policy of mandatory disclosure results in a Pareto improvement relative to an equilibrium without disclosure. The model is very simple, which makes the arguments, we hope, transparent. Yet it leaves out many features which we briefly mention here. We first comment on features that are mainly related to the modeling of the network of banks, and second on those that are mainly related to the game played by banks and outside investors.

The simplicity of our game between banks and outside investors relies, in part, on the symmetry of the network model of banks. In particular, in our set-up all banks that learn the value of the shock to the directly held assets but that have no other information, i.e. that known $S_j \in \{0,1\}$, are alike. This simplifies the type of information that banks will have at the time of their disclosure decision in an equilibrium with no disclosure. The restrictions on the type of network we analyze, in particular that the obligations Λ are known to all, that Λ is a circulant matrix, that the number of total bad banks is known, and that the size of the negative shock is the same for all bad banks. This combination of assumptions excludes several interesting cases. First, our set up excludes the case where some banks are more centrally located with others, such as the core-periphery networks analyzed by Babus and Kondor (2013). This type of heterogeneity is both realistic for financial institutions, and may have interesting implications for the disclosure policy. For instance, it may be the case that mandatory disclosure of the core-banks is enough to improve on an equilibrium without disclosure, i.e. it may be just enough to require information disclosure for systemically important institutions. Related to this, notice that stress tests are done on large banks only. Second, we assume that banks and outside investors know the structure of the network.

Relaxing this will be challenging, but interesting since it captures the idea of complexity and opacity often associated with trading in derivatives. Third, we assume that in every realization , i.e. for every S, the number of bad banks and the severity of the shocks to each of them is the same. Our setup allows uncertainty on the total value of banks' equity in the networks across different realizations. For instance, as analyzed in the case of the circle covering problem, the total equity across banks depend on whether the realization of the b bad banks are spread out in the network (in which case the total equity in the banking system is low) or bunch up, in which case the total equity in the banking system is higher. Yet a more general specification, may better capture actual investors' uncertainty about banks in the case of stress tests.

Next we comment on features that our model left out and that are primarily related to the game between banks and outside investors. First, in our model we use a simultaneous move game for disclosure, which highlights the possibility of coordination failure and information spillovers. We could, instead, assume that the information disclosure by banks is sequential. This alternative set up allows for informational cascades and herding, where information gets "trapped". Second, our information structure and the modeling of disclosure is binary: either banks completely and credibly reveal whether they had a negative shock on the value of their directly held assets, or they don't provide any information. Alternatively, banks can face a continuous information disclosure decision. We conjecture that this will make the cost for which there is an equilibrium without information disclosure vary continuously with the degree of contagion. Third, we assume that banks can disclose an incontrovertible proof of their state. A more realistic model will be that banks can give an informative, yet imperfect signal. This open new possibilities which are relevant for the difference between the social and private value of information disclosure. In particular in the presence of noisy but correlated information -which is implied by the interconnection between banks- it may be easier to find equilibrium without disclosure that can be Pareto dominated by mandated disclosure. Forth, we have considered a policy in which all banks are forced to disclose their state. But there is no presumption that such a policy is optimal subject to the informational constraints and the contracting frictions. For instance, some form of partial forced disclosure, where only some institutions are forced to disclose, may be better.

Finally, there are two features of our model which differ from even a stylized description of stress tests. One is that our model the uncertainty is about realized losses of banks. Instead, stress test emphasize the exposure to unrealized tail risks. Second, in our model the negative outcome is zero equity: banks which either receive bad shocks or which are owned by those with bad shocks go bankrupt. For banks subject stress tests, the negative outcome is low equity, which may either mandate as a matter of policy, or require as a market outcome, recapitalization.

The result in our paper implies that stress test, at least insofar as they include mandatory disclosure of information clauses, are socially beneficial provided that there is enough dependence on their counterpart risk or enough "contagion" , a condition which several observers of the crises regard as realistic for banks. Stress tests have other features too which are not captures by our simple model. Our model also has implications for the recommendation of migration of trade of derivatives from over-the-counter to centralized exchanges.[6] One of the reasons for this recommendation is the fragility that chain of indirect exposure of counterpart risk, which we argue it is captured in our model. We do not model the equivalent to migrating to an exchange, which can be considered to have a different network for trade, but presumably there are cost and benefits of such different network. Instead we view the policy of mandatory disclosure of information as a substitute to the migration to exchanges, i.e. we view it as a policy that can address some of the shortcomings of over-the-counter trade which motivate the policy of migration to centralized exchanges.

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[6] For a discussion see, for example, Duffie and Zhu (2011) and Duffie, Li, and Lubke (2010) and the references therein.

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A Proofs

Proof of Proposition 1: We can rewrite the system of equation as

$$x_j = T_j\left(x_{j-1}\right) \equiv \max\left\{0, \min\left(x_{j-1} + \pi - \Phi_j, \lambda\right)\right\}$$

By repeated substitution, we have that the system of equations $\left\{x_j = T_j\left(x_{j-1}\right)\right\}_{j=1}^{n}$ can be reduced to a single equation

$$x_0 = T^*\left(x_0\right)$$

where

$$T^*\left(x_0\right) \equiv T_n \circ T_{n-1} \circ \cdots \circ T_1\left(x_0\right)$$

The mapping T^* is continuous, monotone, bounded. Moreover, for any x and y in $[0, \lambda]$, we have $|T^*(x) - T^*(y)| \leq |x - y|$. Let

$$\begin{aligned} \underline{x} &= \lim_{m \to \infty} (T^*)^m (0) \\ \overline{x} &= \lim_{m \to \infty} (T^*)^m (\lambda) \end{aligned}$$

These limits exist given T is monotone and bounded. By continuity, $\underline{x}$ and $\overline{x}$ are solutions, i.e.

$$\underline{x} = T(\underline{x}) \text{ and } \overline{x} = T(\overline{x})$$

Moreover, by monotonicity, $(T^*)^m (0) \leq (T^*)^m (\lambda)$ for any m. Taking the limit, $\underline{x} \leq \overline{x}$.

Suppose $\underline{x} < \overline{x}$. Then for any $\mu \in (0, 1)$, the value $x_\mu = \mu \underline{x} + (1 - \mu) \overline{x}$ must also be a solution, i.e.

$$x_\mu = T(x_\mu)$$

For suppose

$$x_\mu > (T^*)(x_\mu)$$

In this case, we have

$$\begin{aligned} x_\mu - \underline{x} &> T^*(x_\mu) - \underline{x} \\ &= T^*(x_\mu) - T^*(\underline{x}) \geq 0 \end{aligned}$$

But this counterfactually implies

$$|x_\mu - \underline{x}| > |T^*(x_\mu) - T^*(\underline{x})|$$

Likewise, if

$$x_\mu < (T^*)(x_\mu)$$

then we can show that

$$\begin{aligned} \overline{x} - x_\mu &> \overline{x} - T^*(x_\mu) \\ &= T^*(\overline{x}) - T^*(x_\mu) \geq 0 \end{aligned}$$

which again counterfactually implies

$$|x_\mu - \underline{x}| > |T^*(x_\mu) - T^*(\underline{x})|$$

We conclude that $T^*(x) = x$ for all $x \in [\overline{x}, \overline{x}]$. This in turn implies that for all $x \in [\underline{x}, \overline{x}]$ and all $j \in \{1, ..., n\}$,

$$T_j \circ \cdots \circ T_1(x) = x + \pi - \Phi_j$$

This condition would be violated if for some $x \in [\underline{x}, \overline{x}]$, one of these two conditions must hold:

$$\begin{array}{ll} \text{(i)} & T_{j-1}(x) + \pi - \Phi_j > \lambda \\ \text{(ii)} & T_{j-1}(x) + \pi - \Phi_j < 0 \end{array}$$

But this implies there exist two values $x' \neq x''$ from$[\underline{x}, \overline{x}]$ such that

$$T_j(x') = T_j(x'')$$

and hence $T^*(x') = T^*(x'')$, which requires $x' = x''$, a contradiction. It follows that

$$T^*(x) = x + \sum_{j=1}^{n} (\pi - \Phi_j)$$

for all $x \in [\underline{x}, \overline{x}]$. But since $T^*(x)$ must equal x in this interval, we must have

$$\sum_{j=1}^{n} (\pi - \Phi_j) = 0$$

This implies that $\underline{x} = \overline{x}$, i.e. there is a unique solution, whenever

$$\sum_{j=1}^{n} (\pi - \Phi_j) \neq 0$$

This completes the proof for the case where $n\pi \neq b\phi$. ■

Proof of Proposition 2: Since $\phi \leq \pi < \dfrac{n}{b}\phi$, we know the solution is unique based on Proposition 1. It will suffice to guess and verify that $x_j = \lambda$ is a solution. For any $j \in \{1, ..., n\}$, we have

$$x_j = \max\{0, \min(\lambda + \pi - \Phi_j, \lambda)\}$$

Since $\pi - \Phi_j \geq 0$ regardless of Φ_j, $x_j = \lambda$ represents a proper solution. ■

Proof of Proposition 3: By construction, $e_j \geq \pi - \Phi_j + x_{j-1} - x_j$. Summing up over all j yields

$$\begin{aligned}\sum_{j=1}^{n} e_j &\geq \sum_{j=1}^{n} (\pi - \Phi_j + x_{j-1} - x_j) \\ &= n\pi - b\phi \\ &> 0\end{aligned}$$

This contradicts the fact that $e_j = 0$ for all j. Hence, there must exist at least one j for which $x_j = \lambda$.

Next, we argue that the fact that $e_j > 0$ for some j implies $x_j = \lambda$ for some j. For suppose not. Since $x_j = \max\{0, \min\{x_{j-1} + \pi - \Phi_j, \lambda\}\}$, it follows that $x_{j-1} + \pi - \Phi_j < \lambda$ for all j. Hence, $x_j = \max\{0, x_{j-1} + \pi - \Phi_j\}$. From this, it follows that $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = 0$, since either $\pi - \Phi_j + x_{j-1} < 0$ in which case $x_j = 0$ and e_j is the maximum of a negative expression and 0, and thus equal to 0, or else $x_j = x_{j-1} + \pi - \Phi_j$ and so $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = \max\{0, 0\} = 0$. ■

Proof of Proposition 4: Define S^* as the state of the world in banks $n-b+1$ through n, are the bad banks. That is, $S^*_j = 1$ for $j \in \{n-b+1, ..., n\}$. This correspond to the case where the location of the bad banks is concentrated in the sense that all of the bad banks are adjacent to one another on the network.

Result 1: Suppose $\lambda > b(\phi - \pi)$. Then if $S = S^*$, we have $x_j > 0$ for all j.

Proof of Result 1: Suppose $x_n = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n-b\}$. Since $n\pi > b\phi$, then

$$(n-b)\pi > b(\phi - \pi)$$

Set $\lambda = b(\phi - \pi) + \varepsilon$ where $\varepsilon > 0$. Choose ε sufficiently small so that

$$(n-b)\pi > b(\phi - \pi) + \varepsilon$$

Then $x_{n-b} = \lambda = b(\phi - \pi) + \varepsilon$. Since the next b banks are bad, it follows that

$$\begin{aligned} x_n &= \min\{0, x_{n-b} - b(\phi - \pi)\} \\ &= \varepsilon \end{aligned}$$

Therefore, $x_n > 0$, a contradiction. Since since T^* is weakly increasing in λ, then if $T^*(0) > 0$ for $\lambda = b(\phi - \pi) + \varepsilon$, then $T^*(0) > 0$ for any $\lambda' > b(\phi - \pi) + \varepsilon$.

Let $T_j(x; S)$ denote the operator T_j for a particular state of the network S. Likewise, let $T^*(x; S)$ denote $T^*(x; S) = T_n(\cdot; S) \circ \cdots \circ T_1(x; S)$ for a particular S. Result 3 implies that $T^*(0; S^*) > 0$ whenever $\lambda > b(\phi - \pi)$.

Result 2: $T^*(0; S) \geq T^*(0; S^*)$ for all S.

Proof: Take any S. Starting from a state of the banking network S^*, we can reach any new state $S \neq S^*$ with a finite number of steps where in each step we find a pair of adjacent banks, one good bank with a lower index and one bad bank with a higher index, and swap them so that the bad bank has the lower index and the good bank has the higher index. Formally, there exists a sequence of vectors of the state of the banking network $S^0, S^1, ..., S^Q$ such that $S^0 = S^*$, $S^Q = S$, and for each q, we have

$$S^{q+1}_l = \begin{cases} S^q_l & \text{if } l \notin \{j_q - 1, j_q\} \\ 1 - S^q_l & \text{if } l \in \{j_q - 1, j_q\} \end{cases}$$

for some j_q. It is easy to construct one such sequence, but cumbersome to describe it formally, so we omit the details.

By construction, starting with a given x_n, define

$$\begin{aligned} x^q_j &= T_j(\cdot; S^q) \circ \cdots \circ T_1(x_n; S^q) \\ x^{q+1}_j &= T_j\left(\cdot; S^{q+1}\right) \circ \cdots \circ T_1\left(x_n; S^{q+1}\right) \end{aligned}$$

That is, x^q_j is the payment bank j makes when the network is in state S^q given bank n makes a payment x^n, and likewise for x^{q+1}_j. By construction, $S^q_j = S^{q+1}_j$ for $j \leq j_q - 2$, which implies $x^q_{j_q-2} = x^{q+1}_{j_q-2}$.

Let $G(\xi)$ denote the payment a good bank will make if it receives a payment ξ from its neighboring bank, and let $B(\xi)$ denote the payment a bad bank will make. Then

$$\begin{aligned} G(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi\}\} \\ B(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi - \phi\}\} \end{aligned}$$

Note that by definition

$$B(\xi) = G(\xi - \phi) \tag{26}$$

Note that $G(\xi)$ is weakly increasing in ξ with slope bounded above by 1. We can now characterize the payment made by bank j_q when $S = S^q$ and $S = S^{q+1}$ using $G(\cdot)$ and $B(\cdot)$ as follows

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q-2}^q\right)\right) \\ x_{j_q}^{q+1} &= G\left(B\left(x_{j_q-2}^{q+1}\right)\right) \end{aligned}$$

For any real number ξ, (26) implies

$$\begin{aligned} G(B(\xi)) &= G(G(\xi - \phi)) \\ B(G(\xi)) &= G(G(\xi) - \phi) \end{aligned}$$

Since $G(\cdot)$ has a slope bounded above by 1, then since $\phi > 0$,

$$G(\xi - \phi) \geq G(\xi) - \phi$$

Applying $G(\cdot)$ to both sides and using the fact that $G(\cdot)$ is monotone yields

$$G(G(\xi - \phi)) \geq G(G(\xi) - \phi)$$

or alternatively

$$G(B(\xi)) \geq B(G(\xi))$$

Setting $\xi = x_{j_q-2}^q = x_{j_q-2}^{q+1}$, we have

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q-2}^q\right)\right) \\ &\leq G\left(B\left(x_{j_q-2}^q\right)\right) \\ &= G\left(B\left(x_{j_q-2}^{q+1}\right)\right) = x_{j_q}^{q+1} \end{aligned}$$

In other words, the state of the network that minimizes the resources bank n has at its disposal is when bank n is bad as are the $b-1$ banks that precede it in the chain of obligations across banks.

Results 1 and 2 together imply that for any S, there cannot be any bank pays 0 to the bank to which it owes λ: Even in the worst case scenario, since λ is large enough, a bank that pays nothing must have positive resources with which it can pay.

Result 3: If $\lambda = b(\phi - \pi)$, then there exists an S such that $x_j = 0$ for some j. In particular, $S = S^*$.

Proof of Result 3: The proof is by construction for the case where $\lambda = b(\phi - \pi)$. Suppose $S = S^*$, and consider $x_n = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n-b\}$. Since $n\pi > b\phi$, then

$$(n-b)\pi > b(\phi - \pi)$$

Hence, $x_{n-b} = \lambda = b(\phi - \pi)$. Since the next b banks are bad, it follows that

$$\begin{aligned} x_n &= \min\{0, x_{n-b} - b(\phi - \pi)\} \\ &= 0 \end{aligned}$$

Hence, the solution has $x_j = 0$ for at least one bank j. ■

Proof of Proposition 5: From Proposition 4, we know that $x_j > 0$ for all $j \in \{1, ..., n\}$. Hence,

$$x_j = \min\{\lambda, x_{j-1} + \pi - \Phi_j\}$$

Given the state of the banking network S, equity is given by

$$e_j(S) = \max\{0, x_{j-1}(S) + \pi - \Phi_j(S) - x_j(S)\}$$

We consider the possible cases for the second term on the right hand side. If $x_j(S) = x_{j-1}(S) + \pi - \Phi_j(S)$, then

$$e_j(S) = 0 = x_{j-1}(S) + \pi - \Phi_j(S) - x_j(S)$$

If $x_j(S) = \lambda$, then $x_{j-1}(S) + \pi - \Phi_j(S) \geq \lambda$ and so

$$e_j(S) = \max\{0, x_{j-1}(S) + \pi - \Phi_j(S) - \lambda\} = x_{j-1}(S) + \pi - \Phi_j(S) - \lambda$$

Hence, the fact that $x_j(S) > 0$ implies

$$e_j(S) = x_{j-1}(S) + \pi - \Phi_j(S) - x_j(S)$$

Summing up yields

$$\sum_{j=1}^{n} e_j(S) = n\pi - b\phi$$

That is, the sum of equity values is the same regardless of S. Assumption A2 implies $e_j \in \{0, \pi\}$. But this implies the cardinality of the set $\{j : e_j = 0\}$ is the same for all S. Let $\zeta \equiv \#\{j : e_j = 0\}$. Hence,

$$\sum_{j=1}^{n} e_j(S) = (n - \zeta)\pi = n\pi - b\phi$$

Since $\lambda > b(\phi - \pi)$, then $\min\{\phi - \pi, \lambda\} = \phi - \pi$, and so $\phi - \pi$ is an integer. Recall we previously defined

$$k \equiv \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\phi - \pi}{\pi}$$

From this, we get $\phi = (k+1)\pi$. It follows that

$$(n - \zeta)\pi = n\pi - b(k+1)\pi$$

which gives

$$\zeta = b(k+1)$$

as claimed. ■

Proof of the Proposition 6: For $0 < \lambda < \phi - \pi$, using Lemma 1 implies

$$\begin{aligned} p_g &= \frac{n - E[\zeta]}{n - b} \\ &= \frac{(n-b)!\,(n-k-1)!}{(n-b)\,(n-1)!\,(n-b-k-1)!} \\ &= \prod_{i=1}^{k}\left(\frac{n-b-i}{n-i}\right) \end{aligned}$$

From (7), we know that $\lambda < \phi - \pi$ implies $k = \lambda/\pi$, and so

$$p_g = \prod_{i=1}^{\lambda/\pi}\left(\frac{n-b-i}{n-i}\right)$$

For $\lambda > b(\phi - \pi)$, Proposition 5 implies $\zeta = bk + b$ with probability 1. Hence,

$$\begin{aligned} p_g &= \frac{n - bk - b}{n - b} \\ &= 1 - \frac{bk}{n-b} \end{aligned}$$

Since $b \geq 1$, from (7), $\lambda > b(\phi - \pi)$ implies $\lambda > \phi - \pi$, and so $k = \frac{\phi}{\pi} - 1$, and so

$$p_g = 1 - \frac{b}{n-b}\left(\frac{\phi}{\pi} - 1\right)$$

Finally, for $\phi - \pi \leq \lambda \leq b(\phi - \pi)$, we have

$$p_g = \frac{n - E[\zeta]}{n - b}$$

Hence, the derivative of p_g with respect to any parameter has the opposite sign as the derivative of $E[\zeta]$ with respect to that parameter.

Let $\zeta(\omega)$ denote the number of banks with zero equity when the state of the network $S=\omega$. Let Ω denote the set of all possible values s can take. Then

$$\Omega=\left\{x\in\{0,1\}^n:\sum_{j=1}^{n}x_j=b\right\}$$

The expectation $E[\zeta]$ is given by

$$E[\zeta]=\sum_{\omega\in\Omega}\Pr(S=\omega)\times\zeta(\omega)$$

Note that while $\zeta(\omega)$ also depends on λ, ϕ, and π, the probability $\Pr(S=\omega)$ only depends on n and b. It will therefeore suffice to show that $\zeta(\omega)$ is weakly increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ for all $\omega\in\Omega$.

Define $d_j(\omega)=\lambda-x_j(\omega)$ as the amount bank j is in default to bank $j+1$, i.e. the amount $x_j(\omega)$ falls short of the promised obligation λ. For bank j to have zero equity, two scenarios can happen. First, if $d_j(\omega)>0$, then since the bank is not meeting its obligation to bank $j+1$, its equity must be zero given the priority of payments. The other scenario is if $d_j(\omega)=0$ and $d_{j-1}(\omega)$ is exactly equal to π. In this case, even though bank j pays his obligation to bank $j+1$ in full, it has to make up a shortfall of π to meet its obligation, which exhausts its profits. For any other feasible configuation of shortfalls, equity must be positive. Formally, the number of banks with zero equity $\zeta(\omega)$ when $S=\omega$ can be expressed as follows:

$$\zeta(\omega)=\sum_{j=0}^{n-1}1\{d_j(\omega)>0\cup(d_j(\omega)=0\cap d_{j-1}(\omega)=\pi)\}$$

We now show that for a given ω, the vector $\{d_j(\omega)\}_{j=0}^{n-1}$ is weakly increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$. Inspection of the two cases associated with $e_j=0$ confirms that $\zeta(\omega)$ must be increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ as well, since increasing $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ can only increase the value of the indicator function.

From Proposition 3, under Assumption A1, for every ω, there exists at least one bank j for which $x_j(\omega)=\lambda$, and so $d_j(\omega)=0$. Since $x_j(\omega)$ is continuous in ϕ and λ, so is $d_j(\omega)$. Given that the number of banks is finite, for every λ, there exists an $\varepsilon>0$ such that for all $\lambda'\in[\lambda,\lambda+\varepsilon)$, there exists some $j\in\{0,...,n-1\}$ such that $d_j(\omega)=0$ for all values of λ'. Simliarly, for every ϕ, there exists an $\varepsilon>0$ such that for all $\phi'\in[\phi,\phi+\varepsilon)$, there exists some $j\in\{0,...,n-1\}$ such that $d_j(\omega)=0$ for all values of ϕ'. Without loss of generality, we label this bank as $j=0$, and so $d_0(\omega)=0$ over the interval $[\lambda,\lambda+\varepsilon)$ or $[\phi,\phi+\varepsilon)$.

Next, using (4), we have

$$d_{j+1}(\omega)=\begin{cases}\max\{d_j(\omega)-\pi,0\} & \text{if } \omega_{j+1}=0\\ \min\{d_j(\omega)+\phi-\pi,\lambda\} & \text{if } \omega_{j+1}=1\end{cases}\quad\text{for } j=0,...,n-2 \tag{27}$$

Observe that the solution to the system of equations given by (27) and $d_0(\omega)=0$ is homogeneous of degree 1 in (ϕ,λ,π). Hence, $\zeta(\omega)$ must be homogeneous of degree 0 in (ϕ,λ,π), and so $\zeta(\omega)$ can be written as a function of the ratios $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ alone. From (27), it is clear that the solution $\{d_j(\omega)\}_{j=0}^{n-1}$ is weakly increasing in both π and ϕ, and so $\zeta(\omega)$ is weakly increasing. ■

Proof of Proposition 7: Our proof is by construction. We know from Proposition 3 that there exists at least one bank for which $\widehat{e}_j > 0$. Start with this bank and move to bank $j+1$, continuing on until reaching the first bad bad bank. Without loss of generality, we can refer to this as bank 1. Moreover, we know that $\widehat{x}_n = \lambda$, i.e. if outsiders did not invest in any of the banks, then bank n would be able to pay its obligation to bank 1 in full.

First, we argue that $x_n = \lambda$, i.e. when banks can raise outside funds, it will still be the case that bank n will be able to pay its debt obligation to bank 1 in full. To see this, define

$$\begin{aligned} T_j(x) &= \max\{0, \min\{x + \pi + R(1 - D_j) I_j - \Phi_j, \lambda\}\} \\ &\geq \max\{0, \min\{x + \pi - \Phi_j, \lambda\}\} \equiv \widehat{T}_j(x) \end{aligned}$$

As before, the payment x_n must solve the fixed point

$$x_n = T^*(x_n) = T_n \circ \cdots \circ T_1(x_n)$$

But we have

$$x_n \geq \widehat{T}_n \circ \cdots \circ \widehat{T}_1(x_n)$$

Suppose there was a value $\overline{x} < \lambda$ such that $\overline{x} = T^*(\overline{x}) \geq \widehat{T}^*(\overline{x})$. Then since $T(0) < 0$, it follows from continuity that we can find an $x_n \in (0, \overline{x})$ s.t.

$$x_n = T^*(x_n)$$

But this contradicts Proposition 1 which implies $\widehat{x}_n = \lambda$ is the unique solution to $x_n = T^*(x_n)$.

Now, suppose bank 1 was able to raise funding, i.e. $I_1 = 1$. If bank 1 diverted the funds it obtained, its expected payoff would be v. If it invested the funds, it would get to keep

$$\max\{\lambda + \pi + (R - r) - y_1 - x_1, 0\}$$

where

$$\begin{aligned} y_1 &= \min\{\phi, \lambda + \pi + (R - r_1)\} \\ x_1 &= \min\{\lambda + \pi + R - y_1, \lambda\} \end{aligned}$$

If $y_1 = \lambda + \pi + (R - r_1)$, then the bank would get to keep 0, which is less than v. If $y_1 = \phi$, the bank would get to keep

$$\max\{\lambda + \pi + (R - r_1) - \phi - x_1, 0\}$$

which is 0 if $x_1 = \lambda + \pi + R - y_1$ and $\pi + (R - r_1) - \phi$ if $x_1 = \lambda$. Since $\phi > \pi$ under Assumption A1, this is less than $R - r_1$. Moreover, since $r_1 \geq \underline{r}$ in any equilibrium, $R - r_1 \leq R - \underline{r} < v$, where the last inequality follows from Assumption A3. Thus, bank 1 will not be able to raise outside funds, i.e. $I_1 = 0$. From this we can conclude that $e_1 = 0$, since its resources $\lambda + \pi - \phi$ are less than its obligation to bank 2.

Next, suppose $e_1 = \cdots = e_{j-1} = 0$ and $I_1 = \cdots = I_{j-1} = 0$ Under Assumption A2, there are two possible cases to consider: $\widehat{e}_j = 0$ and $\widehat{e}_j = \pi$.

Consider first the case where $\widehat{e}_j = 0$. We argue that $I_j = 0$. For suppose not. Given $I_1 = \cdots = I_{j-1} = 0$, we have that

$$x_{j-1} = \widehat{x}_{j-1}$$

Since $\widehat{e}_j = 0$, we know that under Assumptions A1 and A2, $x_{j-1} = \widehat{x}_{j-1} \leq \lambda - \pi$. Suppose bank j were able to raise funds. Then if bank j diverts the funds it obtains, its payoff would be v. That is, since

$$\begin{aligned} y_j &= \min\{\Phi_j, x_{j-1} + \pi\} = \widehat{y}_j \\ x_j &= \max\{0, \min\{x_{j-1} + \pi - y_1, \lambda\}\} = \widehat{x}_j \end{aligned}$$

and since

$$\widehat{e}_j = \max\{0, \widehat{x}_{j-1} + \pi - \widehat{y}_j - \widehat{x}_j\} = 0$$

then even before paying back outside investors w_j, the bank would have no resources left. By contrast, if the bank invested, then since $\widehat{x}_{j-1} \leq \lambda - \pi$, its payoff will be at most $R - r_j^* \leq R - \underline{r} < v$. Hence, $I_j = 0$ as claimed. Since $I_j = 0$ implies $x_j = \widehat{x}_j$, it follows that $e_j = \widehat{e}_j = 0$.

Next, suppose $\widehat{e}_j = \pi$. Note that this implies $S_j = 0$, i.e. j must be a good bank.We argue that $I_j = 1$ and $x_j = \lambda$. To see this, observe that $\widehat{e}_j = \pi$ implies $x_{j-1} = \widehat{x}_{j-1} = \lambda$. Hence, we have

$$\begin{aligned} y_j &= \min\{\Phi_j, \lambda + \pi\} = 0 \\ x_j &= \max\{0, \min\{\lambda + \pi + R(1 - D_j) I_j, \lambda\}\} = \lambda \end{aligned}$$

If the bank obtained funds from outside investors, i.e. $I_j = 1$, and did not divert funds, its payoff would equal $\pi + R - r$. If it chose to divert funds, it would receive $v + \min\{\pi - r, 0\}$. At $r = \underline{r}$, Assumption A3 ensures that the bank would prefer to invest than to divert the funds. It follows that the unique equilibrium is one where $r_j^* = \underline{r}$ and $I_j = 1$.

So far, we have established that starting from bank 1, continuing through all the consecutive banks for which $\widehat{e}_j = 0$ implies $I_j = 0$. The first bank for which $\widehat{e}_j = \pi$, since $x_j = \lambda$, we can keep going until we reach the next bad bank. Since this bank receives λ, the analysis would be the same as for bank 1. The claim then follows. ■

Proof of Lemma 2: If bank j has positive equity in equilibrium, it must be that $x_{j-1} = \lambda$, i.e. bank j is paid in full. This is because Assumptions A1 and A2 imply that if $x_{j-1} < \lambda$, then $\widehat{e}_j = 0$, i.e. such a bank would have no equity prior to raising any funds from outside investors. But we know from Assumption A3 that such a bank would divert funds, i.e. $D_j = 1$, and so such a bank would have no equity. Given this, a bank that receives outside funding would choose to invest the funds rather than divert them if and only if

$$v + \max\left\{\pi - r_j^*, 0\right\} < \pi + R - r_j^* \tag{28}$$

Suppose $r_j^* < \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = \pi - r_j^*$. But then Assumption A3 tells us that (28) must hold, since it implies $v < R$. Next, suppose $r_j^* \geq \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = 0$. In that case, (28) only holds if $\pi \leq r_j^* \leq \pi + R - v$. Since $v < R$, this bound exceeds π. It follows that $D_j = 0$ if and only $r_j^* \leq \pi + R - v$. ■

Proof of Lemma 3: From Lemma 2, the only scenario we have to explore is whether there exists an equilibrium with $r_j > \overline{r}$ in which a bank with positive equity chooses to divert, i.e. $D_j = 1$. Let p_j denote the probability that bank j has positive equity along the equilibrium path. Then the expected payoff to a bank is given by $p_j\left(r_j^*\left(1 - D_j\right) + \min\left\{\pi, r_j^*\right\} D_j\right) = p_j \pi$ given that $D_j = 1$ requires $r_j^* > \overline{r} > \pi$. But suppose a lender were to charge $r_j = \pi + \varepsilon$ where ε is sufficiently small so ensure that $r_j < \overline{r}$. In that case, the bank would be strictly better off since it is charged a lower rate. Moreover, since $\pi + \varepsilon < \overline{r}$, the bank will invest and pay $r_j = \pi + \varepsilon$ in full, so the bank that charges this amount will be better off. But then the original outcome with $r_j^* > \overline{r}$ could not have been an equilibrium. ■

Proof of Proposition 10: First, suppose $v \geq \underline{r}$. Then for any $p_g \in (0,1)$, we have

$$\begin{aligned}
(R - v)\, p_g + (v - \underline{r}) &= p_g\left(R - \underline{r}\right) + \left(1 - p_g\right)\left(v - \underline{r}\right) \\
&\geq p_g\left(R - \underline{r}\right) \\
&> p_g \tfrac{n-b}{n}\left(R - \underline{r}\right)
\end{aligned}$$

Mandatory disclosure is preferable to no investment if

$$c < \left(R - \underline{r}\right)\tfrac{n-b}{n} p_g$$

But from above it follows that

$$c < (R - v) p_g + (v - \underline{r})$$

Since $p_g > \underline{r}/\overline{r}$ implies a good bank that unilaterally discloses will be able to raise funds, while the above inequality implies the benefits from attracting funds exceed the disclosure cost, it follows that non-disclosure cannot be an equilibrium whenever mandatory disclosure is preferable to no investment.

Next, suppose $v < \underline{r}$. For any $p_g > \frac{\underline{r}}{\overline{r}}$, a non-disclosure equilibrium with no investment will exist if

$$p_g \leq \frac{n}{n-b}\underline{r}/\overline{r} \text{ and } c \geq (R - v)p_g + (v - \underline{r})$$

and mandatory disclosure will be preferable to no investment if

$$c \leq p_g \frac{n-b}{n}\left(R - \underline{r}\right)$$

The only way both inequalities involving c can be satisfied is if

$$(R - v)p_g + (v - \underline{r}) \leq p_g \frac{n-b}{n}\left(R - \underline{r}\right)$$

Rearranging, improveability on a non-disclosure equilibirum with no investment is possible only if

$$p_g \leq \frac{\underline{r} - v}{(R - v) - \frac{n-b}{n}(R - \underline{r})}$$

For this bound to exceed $\underline{r}/\overline{r}$ requires

$$\frac{(\underline{r} - v)}{(R - v) - (1 - b/n)(R - \underline{r})} \leq \underline{r}/\overline{r}$$

which, rearranging, implies

$$\frac{b}{n} \leq \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r} - v}{R - \underline{r}}$$

Finally, from A3,

$$\begin{aligned}\frac{\underline{r} - v}{(R - v) - (1 - b/n)(R - \underline{r})} &= \frac{\underline{r} - v}{(\underline{r} - v) + \frac{b}{n}(R - \underline{r})} \\ &< 1\end{aligned}$$

which completes the proof. ■

Proof of Proposition 11: First, suppose $v \geq \underline{r}$. The expected amount banks pay to investors is $\underline{r}$ both when there is no disclosure and when there is mandatory disclosure. For a good bank, then, the expected payoff under the non-dislcosure equilibrium with investment is $p_g R + (1 - p_g) v - \underline{r}$. Under mandatory disclosure, the expected payoff for a good bank is $p_g (R - \underline{r})$, which is strictly lower. This confirms some party will be made worse off with mandatory dislcosure.

Next, suppose $v < \underline{r}$. A non-disclosure equilibrium with investment can only exist if $c > \frac{b\underline{r}}{n-b}$. At the same time, mandatory disclosure will be Pareto improving relative to an equilibrium where outsiders invest in all banks only if $c < \left(1 - \frac{n-b}{n}p_g\right)(\underline{r} - v)$. For mandatory disclosure to be Pareto improving and for there to exist a non-disclosure equilibrium with investment, we need

$$\frac{b\underline{r}}{n - b} < \left(1 - \frac{n - b}{n}p_g\right)(\underline{r} - v)$$

or, rearranging, if

$$p_g \leq \frac{n}{n - b}\left(1 - \frac{b}{n - b}\frac{r}{\underline{r} - v}\right)$$

If this inequality is violated at $p_g = \frac{n}{n-b}\frac{r}{\underline{r}}$, then it will be violated for all $p_g \geq \frac{n}{n-b}(\underline{r}/\overline{r})$. Hence, a necessary condition for the existence of a Pareto-improveable non-disclosure equilibrium is for

$$\frac{n}{n - b}\left(1 - \frac{b}{n - b}\frac{r}{\underline{r} - v}\right) \geq \frac{n}{n - b}\underline{r}/\overline{r}$$

Rearranging, we have the condition

$$\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)\,(1-\underline{r}/\overline{r})+\underline{r}}\left(1-\frac{\underline{r}}{\overline{r}}\right)$$

Hence, without this condition, there exists no Pareto-improveable non-disclosure equilibrium with investment. With this condition, the interval $\left[\frac{n}{n-b}\underline{r}/\overline{r},\, \frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)\right]$ will be non-empty. For any p_g in this interval, and so the as long as $c \in \left[\frac{b}{n-b}\underline{r},\,(1-\frac{n-b}{n}p_g)(\underline{r}-v)\underline{r}/\overline{r}\right]$, which is necessarily non-empty given the restriction on $\frac{b}{n}$, a non-dislcosure equilibrium with investment is Pareto-improveable. Finally, observe that since $v < \underline{r}$, then

$$\frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{r}{r-v}\right) < \frac{n}{n-b}\left(1-\frac{b}{n-b}\right)$$

But then we have

$$\begin{aligned}
\frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{r}{r-v}\right) &< \frac{n}{n-b}\left(\frac{n-2b}{n-b}\right) \\
&= \frac{n^2-2nb}{n^2-2nb+b^2} \\
&< 1\ .
\end{aligned}$$

Proof of Lemma 4. We want to show that distribution of equity e_j for the bank j conditional on this bank being good ($S_j = 0$) is the same as the distribution of banks' k equity e_k conditional on bank k being good ($S_k = 0$), for any pair k, j. First, notice that, by assumption, $\Pr\ \{S'\} = \Pr\ \{S\}$ for any two S, S'. Second, we will show that, using that Λ is circulant, for any S for which $e_j(S) = e'$ then there is a $\hat{S}$ for which $e_k(\hat{S}) = e'$.

For the second property we will first show a similar condition for the payments. Take, to simply notation, $0 \leq j < k$ and $j < k \leq n-1$. Take a set of payments solving the system 22 for S. We argue that we for the state of the network $\hat{S}$ satisfying:

$$S_i = \hat{S}_{i+u} \ \text{ for } \ i = 0, ..., n-1-u \ \text{ and } \ S_{n-1-u+i} = \hat{S}_i \ \text{ for } i = 0, ..., u \tag{29}$$

where $u = k - j$, then the payments:

$$\hat{x}_{rk}(\hat{S}) = x_{ij}(S) \ \text{ where } r-k = j-i \pmod{n} \tag{30}$$

solve the system of equations 22 for the state of the network $\hat{S}$. To see this note that for a circulant matrix Λ we can write and network S we can write for any $i = j + p$ with $p > 0$:

$$x_{ij}(S) = (\lambda_p/\bar{\lambda})\ \max\left\{\min\left\{\bar{\lambda}\,,\, \pi - S_j\phi + \sum_{r\neq j} x_{jr}(S)\right\}\,,\ 0\right\} \tag{31}$$

where $\bar{\lambda} \equiv \sum_{s=0}^{n-1} \lambda_s$. Likewise for state of the network $\hat{S}$ and any $i = k + p$:

$$\hat{x}_{ik}(\hat{S}) = (\lambda_p/\bar{\lambda}) \max \left\{ \min \left\{ \bar{\lambda} \, , \, \pi - \hat{S}_k \phi + \sum_{r \neq k} \hat{x}_{kr}(\hat{S}) \right\} \, , \, 0 \right\} \tag{32}$$

Inspection of (32) and (31) when $\hat{S}$ is given by (29) verifies (30), i.e. they are the same system of equations once they are relabeled. Using this result we can obtain the required equality:

$$e_j(S) = \pi + \sum_{r \neq j} x_{jr}(S) - S_j \phi - \sum_{i \neq j} x_{ij}(S) = e_k(\hat{S}) = \pi + \sum_{r \neq k} \hat{x}_{kr}(\hat{S}) - \hat{S}_k \phi - \sum_{i \neq k} \hat{x}_{ik}(\hat{S}) \, .$$

Now we show that $p_0 = (1 - b/n) p_g$. For this we just need to argue that any bank with $S_j = 1$, and given the assumption $\pi < \phi$ each bad bank equity will be wiped out, i.e. bank's j equity $e_j(S) = 0 < \underline{e}$. Note that since $x_{jr} \leq \Lambda_{jr}$. Take an i for which $\Lambda_{ij} > 0$ we have: then

$$\begin{aligned} x_{ij}(S) &= (\lambda_{ij}/\bar{\lambda}) \max \left\{ \min \left\{ \bar{\lambda} \, , \, \pi - \phi + \sum_{r \neq j} x_{jr}(S) \right\} \, , \, 0 \right\} \\ &\leq (\lambda_{ij}/\bar{\lambda}) \max \left\{ \min \left\{ \bar{\lambda} \, , \, \pi - \phi + \bar{\lambda} \right\} \, , \, 0 \right\} = 0 \end{aligned}$$

and hence $e_j(S) = 0$ for any with $S_j = 1$. Hence: $\Pr \left\{ e_j(S) > \underline{e} \mid S_j = 1 \right\} = 0$ since $\underline{e} > 0$. Then we have:

$$\begin{aligned} p_0 &= \Pr \left\{ e_j(S) > \underline{e} \right\} = \Pr \left\{ e_j(S) > \underline{e} \mid S_j = 0 \right\} \Pr \left\{ S_j = 0 \right\} \\ &+ \Pr \left\{ e_j(S) > \underline{e} \mid S_j = 0 \right\} \Pr \left\{ S_j = 1 \right\} = p_g \left(1 - \frac{b}{n} \right) \, . \end{aligned}$$

■

The next lemma will be used to prove Theorem 2.

Lemma 5: Suppose Assumption A3 holds. Suppose a bank where charged r_g to borrow. Define $e^* = v + R - r_g$. Then a bank with $e_j > e^*$ will choose not to divert funds, i.e. $D_j = 0$, while a bank with $e_j < e^*$ will choose to divert, i.e. $D_j = 1$.

Proof of Lemma 5: Note that under assumption A3, $v < R$. Hence, if we define $e^* = v - R + r_g$, then $e^* < r_g$. The payoff to a bank with equity e_j from investing the funds it raises is $e_j + R - r_g$. The payoff from diverting the funds it raises is given by $v + \max\{e_j - r_g, 0\}$. A bank will prefer to divert the funds if

$$e_j + R - r_g \leq v + \max\{e_j - r_g, 0\} \tag{33}$$

and to invest the funds if

$$e_j + R - r_g \geq v + \max\{e_j - r_g, 0\} \tag{34}$$

Consider first the case where $e_j < e^*$. Since $\max\{e_j - r_g, 0\} = 0$, it follows that

$$\begin{aligned} e_j + R - r_g &< v \\ &= v + \max\{e_j - r_g, 0\} \end{aligned}$$

and so from (33) the bank would choose to divert. Next, suppose $e^* < e_j \leq r_g$. In that case, $\max\{e_j - r_g, 0\} = 0$, and we have

$$\begin{aligned} e_j + R - r^* &> v \\ &= v + \max\{e_j - r_g, 0\} \end{aligned}$$

and so the bank will prefer to invest from (34). Finally, suppose $e_j > r_g > e^*$. In that case, $\max\{e_j - r_g, 0\} = e_j - r_g$. Since $v < R$ under Assumption A3, we have

$$\begin{aligned} R + e_j - r_g &> v + e_j - r_g \\ &= v + \max\{e_j - r_g, 0\} \end{aligned}$$

and so the bank will prefer to invest from (34).

Finally, we show that under (13), $0 < e^*(r) \leq \pi$ for all $r \in [\underline{r}, \overline{r}]$. Using the first inequality in (13) we have that for any $r \geq \underline{r}$,

$$\begin{aligned} e^*(r) &= v + r - R \geq v + \underline{r} - R \\ &> (R - \underline{r}) + \underline{r} - R \\ &= 0 \end{aligned}$$

In the other direction, since $\overline{r} = \pi + R - v$, for any $r \leq \overline{r}$ we have

$$\begin{aligned} e^*(r) &= v + r - R \leq v + \overline{r} - R \\ &= \pi \end{aligned}$$

■

Proof of Theorem 2

For a the general symmetric network structure, Assumption A2 would no longer guarantee that e_j can only assume two values even if we imposed this assumption. To analyze this case, we define a threshold equity $e^*(r)$ that can be expressed as a function of the interest rate r that outside investors charge a given bank:

$$e^*(r) = v + r - R$$

Lemma 4 below establishes that $e^*(r)$ is the level of equity at which a bank is indifferent between diverting and not diverting if it were charged r to borrow, and that under Assumption A3, $0 < e^*(r) \leq \pi$ for all $r \in [\underline{r}, \overline{r}]$.

Consider $e^*(\underline{r})$, i.e. the threshold when banks are charged the risk-free rate $\underline{r}$. With full-disclosure, $\underline{r}$ is the natural benchmark for the interest rates banks will be charged by outside investors. This is because with full disclosure, any bank with enough equity that they will not divert when borrowing at $\underline{r}$ will receive funding at rate $\underline{r}$ since they expose outside investors to no risk. This would suggest that the analog to the number of banks with zero equity ζ in the ring network under Assumption A2 is the number of banks with pre-investment equity below $e^*(\underline{r})$. This suggests defining a new variable $\zeta(r)$ as the number of banks with pre-investment equity below $e^*(r)$.

Before we proceed, one issue that deserves to be mentioned is that since we have a discrete distribution, it can matter whether we define $\zeta(r)$ as the number of banks with equity strictly below $e^*(\underline{r})$ or the number of banks with equality less than *or equal to* $e^*(\underline{r})$. However, this distinction may not matter generically. Moreover, in principle we can impose symmetry in the way we break indifference across all allocations, so the issue is more of consistency than of how to properly define $\zeta(r)$. To be concrete, let us suppose that when indifferent between diverting funds and not, banks choose not to divert. Hence, define $\zeta(r)$ as the number of banks with equity strictly less than $e^*(r)$. Under this definition, for any $r \in [\underline{r}, \overline{r}]$, which are the only interest rates that will ever be charged in equilibrium, the ring network will imply that $\zeta(r) = \zeta$, i.e. the number of banks with zero equity. As in the ring network, we can define $p_g(r)$ as the probability that a good bank will have equity above $e^*(r)$, which is given by

$$p_g(r) = \Pr(e_j \geq e^*(r) \,|S_j = 0)$$

Since $e^*(r)$ is increasing in r, it follows that $p_g(r)$ is decreasing in r. Note that r here simply determines a threshold level of equity that we are interested in and does not need to be interpreted as an interest rate.

To determine whether mandatory disclosure will be welfare improving over an allocation in which no outside investor agrees to finance banks, observe that after full-disclosure, there will be $n - E[\zeta(\underline{r})]$ banks on average with equity at or above the treshhold $e^*(\underline{r})$. Each of these banks will be able to trade with outside investors and create $R - \underline{r}$ worth of surplus. Hence, full-disclosure is desirable ex-ante if this expected surplus exceeds the cost of disclosure, i.e. if

$$(n - E[\zeta(\underline{r})])(R - \underline{r}) > cn$$

Note that we can rewrite this condition as

$$(R - \underline{r})\frac{n-b}{n}p_g(\underline{r}) > c$$

where

$$p_g(\underline{r}) = \Pr(e_j \geq e^*(\underline{r}) \,|S_j = 0) = \frac{n - E[\zeta(\underline{r})]}{n-b}$$

analogously to the condition in Proposition 9. Note that here we make use of the fact that $\phi > \pi$ to ensure that $e_j < 0$ if bank j is bad.

Next, we consider the condition for existence of a non-disclosure equilibrium. Starting with a situation of no disclosure, if a bank unilaterally discloses, it will either be able to borrow from outsiders at some rate r_g that is uniquely determined given our restriction to sequential equilibria, or else that it will not be able to receive funds at all.

Consider first the case where the bank anticipates it will not be able to borrow if it unilaterally discloses. This implies that even when the bank were charged the highest possible rate $\overline{r}$, outside investors would still expect to receive less than $\underline{r}$ in expected terms. If charged $\overline{r}$, the bank's equity threshold will be $e^*(\overline{r}) = v+R-\overline{r}$. For this to be profitable, it follows that we obtain the condition that $\Pr(e_j \geq e^*(\overline{r}) \,|S_j = 0) < \underline{r}/\bar{r}$. This condition can be rewritten as

$$p_g(\overline{r}) < \underline{r}/\bar{r}$$

Hence, if $p_g(\overline{r}) < \underline{r}/\bar{r}$, a non-disclosure equilibrium with no investment exists for all values of c. In addition, there exists a range of values for c related to $p_g(\underline{r})$ rather than to $p_g(\overline{r})$ under which mandatory disclosure will be a Pareto improvement over no investment in any bank. Since $p_g(\underline{r}) > p_g(\overline{r})$, the region is non-empty whenever $p_g(\overline{r}) > 0$. Thus, the relevant analog to Theorem 1 is to let $p_g(\overline{r})$ be arbitrarily close to 0, which directly implies that $p_g(\bar{r}) < \underline{r}/\bar{r}$. But, since definition of $\bar{r}$, see equation (14), we have that for $p_g(\bar{r}) = \Pr\{e_j = \pi \mid S_j = 0\}$, the conclusion that an equilibrium with non disclosure can be improved by mandatory disclosure is identical to the one in Theorem 1.

Next, consider the case where the bank anticipates it will be able to borrow at rate r_g if it is the only bank that discloses. For the original non-dislcosure equilibrium to involve on investment, it must be the case that even when banks charge $\overline{r}$ when no bank discloses, expected profits would be negative. This means

$$\frac{n-b}{n} p_g(\bar{r})\, \overline{r} + \frac{b}{n} \times 0 \leq \underline{r}$$

or, rearranging, that

$$p_g(\overline{r}) < \frac{n}{n-b} \underline{r}/\bar{r}$$

Hence, the only non-disclosure equilibrium when $p_g(\overline{r}) < \frac{n}{n-b}\underline{r}/\bar{r}$ is one with no investment. Conversely, for $p_g(\overline{r}) > \frac{n}{n-b}\underline{r}/\bar{r}$, the only possible non-disclosure equilibrium is one where outsiders invest in all banks.

For non-disclosure to be an equilibrium when a bank anticipates it can borrow from outsiders at rate r_g if it is the only bank that discloses, the cost of discLosure c must be large enough to ensure that the expected gain from attracting funds is exceeded by the disclosure cost, i.e.

$$p_g(r_g)(R - r_g) + [1 - p_g(r_g)]\, v < c$$

In equilibrium, $r_g = \underline{r}/p_g(r_g)$, and so we have

$$p_g(r_g)(R - \underline{r}) + [1 - p_g(r_g)](v - \underline{r}) < c$$

or alternatively

$$p_g(r_g)(R - v) + v - \underline{r} < c$$

Hence, when $\underline{r}/\bar{r} < p_g(\overline{r}) < \frac{n}{n-b}\underline{r}/\bar{r}$, a non-disclosure equilibrium exists and can be Pareto improved upon whenever c falls within a particular region

$$p_g(r_g)(R - v) + v - \underline{r} < c < (R - \underline{r}) \frac{n-b}{n} p_g(\underline{r}) \tag{35}$$

where $p_g(r_g)$ is given by a solution to $r_g\, p_g(r_g) = \underline{r}$. We need to determine when this region is non-empty. In Proposition 10, we showed that if $\frac{b}{n} < \left(\frac{\overline{r}}{\underline{r}} - 1\right) \frac{r-v}{R-\underline{r}}$, then the region was non-empty whenever $p_g\left(r_g\right) = p_g\left(\underline{r}\right)$. Since

$$\left(R - \underline{r}\right) \frac{n-b}{n} p_g\left(\underline{r}\right) > \left(R - \underline{r}\right) \frac{n-b}{n} p_g\left(r_g\right)$$

it follows that there exists a non-empty region in this case as well. Note that there is a "bias" towards a (weakly)larger region of inefficient non-disclosure in this more general case.

Consider what happens when we take the limit as $p_g\left(\overline{r}\right) \to 1$. Since $p_g\left(\overline{r}\right) \leq p_g\left(r\right)$ for all $r \in [\underline{r}, \overline{r})$, it follows that $p_g\left(\underline{r}\right) \to 1$ and $p_g\left(r_g\right) \to 1$. In this case, the lower bound on c that admits a non-disclosure equilibrium with no investment to be Pareo-dominated by mandatory disclosure limits to $R - \underline{r}$ while the upper bound limits to $\frac{n-b}{n}\left(R - \underline{r}\right)$ which is strictly smaller. Hence, a non-disclosure equilibrium with no investment cannot be improved upon when $p_g\left(\overline{r}\right) \to 1$. Since $e^*\left(\overline{r}\right) = v + \overline{r} - R$ and $\overline{r} = \pi + R - v$, then $e^*\left(r\right) = \pi$ and we have

$$p_g\left(\overline{r}\right) = \Pr\left(e_j \geq \pi | S_j = 0\right)$$

Since $e_j \leq \pi$, it follows that

$$p_g\left(\overline{r}\right) = \Pr\left(e_j = \pi | S_j = 0\right)$$

which is the same expression as in the ring network in Theorem 1.

It only reminds to consider the case where $p_g\left(\overline{r}\right) > \frac{n}{n-b} \underline{r}/\overline{r}$, in which case a non-disclosure equilibrium must involve outsiders investing in all banks...[THIS CASE SHOULD BE EVEN CLOSER TO THM 1, TO BE COMPLETED]

■

Positive equity and zero shortfall for some bank if $b\phi < n\pi$, general symmetric case.

Proposition $3'$**.** If $\phi < \frac{n}{b}\,\pi$ then for all S there is a bank j for which $x_{ij}(S) = \Lambda_{ij}$ for all $i \in \{0, ..., n-1\}$, so bank's j equity $e_j(S) > 0$.

Proof of Proposition $3'$**.** To see this, fix S an add all the payments of to firm j and all the payments, including to its owner (its equity e_j) and the initial investors (y_j), of firm j. We get

$$\sum_i x_{ij}(S) + e_j(S) + y_j(S) = \pi + \sum_r x_{jr}(S)$$

Adding across the n banks:

$$\sum_{i,j} x_{ij}(S) + \sum_j e_j(S) + \sum_j y_j(S) = n\pi + \sum_{r,j} x_{jr}(S)$$

Using the identity $\sum_{i,j} x_{ij}(S) = \sum_{r,j} x_{jr}(S)$, we get

$$\sum_j e_j(S) = n\pi - \sum_j y_j(S) \geq n\pi - b\phi > 0$$

and since $0 \le y_j(S) \le \phi$, and equity is non-negative, then $e_j(S)$ must be positive for some j. For that j we have $x_{ij} = \Lambda_{ij}$ for all $i = 0, ..., n-1$.

$\square$

Comparative static w.r.t μ and ϕ, general symmetric case

We start with a definition of shortfall.

Definition of shortfall

We define the shortfall as the difference between what a bank owes to another bank less what the bank actually paid, so $D_{ij} = \Lambda_{ij} - x_{ij}$ for all $i, j \in \{0, ..., n-1\}$ when the state of the network is S. We suppress the state of the network from the notation whenever seems clear. We define the shortfall operator as $F : \mathcal{D} \to \mathcal{D}$ where $\mathcal{D} \subset \mathbb{R}^n_+$ is given by

$$\mathcal{D} = \{D_{ij} \in [0\ ,\ \Lambda_{ij}]\ ,\ i, j \in \{0, ..., n-1\}\}$$

as the following function

$$(F)(D)_{ij} \quad = \quad \left(\frac{\Lambda_{ij}}{\bar{\lambda}}\right) \max\left\{\min\left\{\bar{\lambda}\ ,\ \sum_r D_{jr} - \pi + S_j\phi\right\}\ ,\ 0\right\} \tag{36}$$

The explanation of this mapping is as follows. Bank j receives a short fall of $\sum_r D_{jr}$ from other banks, which is reduced by adding all its profits π and increased by losses ϕ, if they occur. Thus the contribution to the total shortfall of bank j is $\sum_r D_{rj} - \pi + \phi S_j$. The most the total shortfall can be is $\bar{\lambda}$, when bank j pays nothing, so we take the minimum of the total shortfall and the total debt $\bar{\lambda}$. Shortfall cannot be negative, since at most the bank will pay all its debt, so we take the maximum of this quantity and zero. Thus the total shortfall is

$$\text{Total shortfall of bank } j = \max\left\{\min\left\{\bar{\lambda}\ ,\ \sum_r D_{jr} - \pi + S_j\phi\right\}\ ,\ 0\right\}$$

We then take this total shortfall and we subtract it from the total debt of j, which equals $\bar{\lambda}$, to get the total payments that bank j will make, or

$$\text{Total payments of } j \text{ to other banks} = \bar{\lambda} - \max\left\{\min\left\{\bar{\lambda}\ ,\ \sum_r D_{jr} - \pi + S_j\phi\right\}\ ,\ 0\right\}$$

Multiplying it by $\Lambda_{ij}/\bar{\lambda}$ we obtain the payments to be made to bank i.

$$x_{ij} = \frac{\Lambda_{ij}}{\bar{\lambda}}\left(\bar{\lambda} - \max\left\{\min\left\{\bar{\lambda}\ ,\ \sum_r D_{jr} - \pi + S_j\phi\right\}\ ,\ 0\right\}\right) \tag{37}$$

Finally we can subtract this payment to be made from the amount owed, so to get the shortfall of bank j with bank i, i.e.

$$D_{ij} = \ \Lambda_{ij} - \frac{\Lambda_{ij}}{\bar{\lambda}}\left(\bar{\lambda} - \max\left\{\min\left\{\bar{\lambda}\ ,\ \sum_r D_{jr} - \pi + S_j\phi\right\}\ ,\ 0\right\}\right)$$

Simplifying this equation, we obtain the expression for F in equation (36). Alternatively, we can just show that x_{ij} as given in (22) equals (37) using that $D_{jr} = x_{jr} - \Lambda_{jr}$ and that $\sum_r \Lambda_{jr} = \bar{\lambda}$. Indeed using these two equalities we can see that

$$\begin{aligned} \bar{\lambda} &\geq \sum_r D_{jr} - \pi + S_j \phi \iff \pi - S_j \phi + \sum_r x_{jr} \geq 0 \text{ and} \\ 0 &\leq \sum_r D_{jr} - \pi + S_j \phi \iff \pi - S_j \phi + \sum_r x_{jr} \leq \bar{\lambda} \,. \end{aligned}$$

Then it is immediate that (22) is equivalent to (37).

For the next proposition we write the circulant vector λ describing the matrix Λ as the product of a scalar $\bar{\lambda}$ times a vector on the $n-1$ simples $\hat{\lambda}$. We will use this to make a comparative static with respect to $\hat{\lambda}$. In particular we let:

$$\begin{aligned} \lambda_s &= \bar{\lambda}\, \hat{\lambda}_s \text{ for } s \in \{0, 1, ..., n-1\} \text{ where,} \\ 1 &= \sum_{j=0}^{n-1} \hat{\lambda}_j \,, \text{ and } \hat{\lambda}_j \geq 0 \text{ for } j \in \{0, 1, ..., n-1\} \,. \end{aligned} \tag{38}$$

For fix values of π, the scalar $\hat{\lambda}$ can be taken as a measure of the leverage of banks with respect to other banks in the network.

Proposition $6'$**.** Let $\{\pi, \phi, n, b, \Lambda\}$ be a network where the matrix Λ is given by a circulant vector λ indexed by $\hat{\lambda}$ as in equation (38). Assume that $\phi > \pi$. The probability p_g is weakly decreasing in ϕ and $\bar{\lambda}$.

Proof of Proposition $6'$. The proof proceeds as follows. First we show that for each S the shortfall $D(S)$ are weakly increasing in ϕ and in $\bar{\lambda}$. Next we argue that this implies that the distribution of equity is stochastically decreasing with ϕ and in $\bar{\lambda}$. Then the result follows from the relationship between p_0 and p_g.

A fixed point of the shortfall is given by $D^* \in \mathcal{D}$ satisfying $D^* = F(D^*)$. It is easy to see that the operator is monotone on D in that $F(D') \geq F(D)$ if $D' \geq D$, where the comparison is component by component. It is easy to show that $F(0) \geq 0$ and that $(F)(\Lambda)_{ij} \geq \Lambda_{ij}$.

Let D^* be a fixed point. If $D_0 \leq D^*$ then $F(D_0) \geq D$. Recall that successive iterations converge to a fixed point, i.e. $D^n \equiv F^n(D_0)$ will converge to a fixed point D^*.

Let $\hat{D}$ be a fixed point of $\hat{F}$ and consider a larger value of ϕ and/or $\bar{\lambda}$, but the same ratios $\Lambda_{ij}/\bar{\lambda}$ (so this corresponds to multiply the circulant vector λ by a scalar larger than one). Let F the operator that uses the larger values of ϕ and/or $\bar{\lambda}$. Then $F(D') \geq D'$ and $D^n = F^n(D')$ will converge to a fixed point $D^* \geq D'$.

This implies that for each S the set of fixed points are weakly increasing in $\bar{\lambda}/\pi$ and on ϕ/π. While the operator may have multiple fixed points D, the corresponding vector of equity values is unique, by Theorem 1 in Eisenberg and Noe (2001).

Next we link the shortfall with equity. Note that

$$
\begin{aligned}
e_j(S) &= \max \left\{ 0,\ \pi - \phi S_j - \sum_i \Lambda_{ij} + \sum_r x_{jr}(S) \right\} & (39)\\
&= \max \left\{ 0,\ \pi - \phi S_j - \bar{\lambda} - \sum_r D_{jr}(S) + \bar{\lambda} \right\} & (40)\\
&= \max \left\{ 0, \pi - \phi S_j - \sum_r D_{jr}(S) \right\} & (41)
\end{aligned}
$$

so the equity of bank j when the state of the network is S is weakly decreasing in D. Thus, if $D(S)$ is weakly higher, each of the $e_j(S)$ are weakly lower. Since the probabilities of each S do not depend on $\bar{\lambda}$ or ϕ, and the right hand side of equation (39) is decreasing in ϕ (but $\bar{\lambda}$ is not an argument of it), we have that the distribution of e_j is stochastically lower if the $D(S)$ are weakly higher for each S. Thus, p_0 is weakly decreasing in $\bar{\lambda}$ and ϕ.

Finally, we have shown that as ϕ and π increase, the unconditional probabilities of positive equity $e_j = \pi$ decreases. But the unconditional probability of a bank being good (i.e. having $S_j = 0$) is independent of ϕ and π so

$$
p_0 \equiv \Pr\ \{e_j \geq \pi\} = \Pr\ \{e_j \geq \pi \mid S_j = 0\} \times \Pr\ \{S_j = 0\} = p_g \times \left(1 - \frac{b}{n}\right)
$$

where we use that, due to the assumptions that $\phi > \pi$ and that $\pi > 0$ that $\Pr\ \{e_j \geq \pi \mid S_j = 1\} = 0$. Thus p_g is weakly decreasing in ϕ and $\bar{\lambda}$.

$\square$

Example of a non-circulant network that is symmetrically vulnerable to contagion (the cuboctahedral network):

$$
\Lambda = \begin{bmatrix}
0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & \lambda & 0 & 0\\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & \lambda\\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda\\
\lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$