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**Labor Supply with Job Assignment  
under Balanced Growth**

by

**Claudio Michelacci**

**(EIEF)**

**Joseph Pijoan-Mas**

**(CEMFI and CEPR)**

# Labor Supply with Job Assignment under Balanced Growth\*

Claudio Michelacci  
*EIEF*

Josep Pijoan-Mas  
*CEMFI and CEPR*

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## Abstract

We consider a competitive equilibrium growth model where technological progress is embodied into new jobs which are assigned to workers of different skills. In every period workers decide whether to actively participate in the labor market and if so how many hours to work on the job. Balanced growth requires that the job technology is complementary with the worker's total labor input in the job, which is jointly determined by his skill and his working hours. Since lower skilled workers can supply longer hours, we show that the equilibrium features positive assortative matching (higher skilled workers are assigned to better jobs) only if differences in consumption are small relative to differences in worker skills. When the pace of technological progress accelerates, wage inequality increases and workers participate less often in the labor market but supply longer hours on the job. This mechanism can explain why, as male wage inequality has increased in the US, labor force participation of male workers of different skills has fallen while their working hours have increased.

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*Keywords:* working hours, participation, job heterogeneity, wage inequality

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## 1 Introduction

The idea that new technologies come embodied into a limited supply of new capital vintages dates back at least to Solow (1960). If the production technology requires each worker to be assigned to a specific capital unit, technological progress also leads to heterogeneity in jobs. In the words of Akerlof (1981), good jobs become a scarce resource, which the economy should assign to workers with potentially different skills. This assignment friction has been widely studied, see Sattinger (1993) for a literature review. But existing assignment models have typically abstracted away from labor supply decisions either at the intensive margin (how many hours to work on the job) or at the extensive margin (whether to actively participate in the labor market). This is an interesting issue because, in assignment models, standard income and substitution effects in labor supply lead to a non-trivial allocation problem between the number of hours worked in the job, which determines the output each job produces, and labor force participation, which determines the number and quality of operating jobs. Income and substitution effects also play a non trivial role in determining whether the equilibrium features positive assortative matching—i.e. whether higher skilled workers are assigned to better jobs. This is because the amount of labor input supplied by a worker on the job is determined by his skill as well as by his working hours. So a low skilled worker can supply greater working hours to compensate for his lower skill level, which implies that standard conditions for assortative matching based on capital-skill complementarity (Becker 1973) are directly affected by labor supply.

To study labor supply in an assignment model, we consider a simple neoclassical growth model with perfectly competitive labor markets and vintage capital as in Jovanovic (1998). Technological progress is embodied into new jobs, which are slowly created over time. Hence in equilibrium there is dispersion in job technologies. Workers differ in skills and they can be employed in at most one job. This leads to a simple assignment problem in the spirit of Becker (1973) and Sattinger (1975). But in our framework labor supply is endogenous because in every period each worker decides whether to actively participate in the labor market, which involves a fixed utility cost, and how many hours to work in the job he is assigned to. To guarantee the existence of a balanced growth path, we assume log preferences in consumption (so that in the long run income and substitution effects cancel out) and a production technology in the job that features unitary elasticity of substitution between the job technology and worker's total labor input, which is jointly determined by the worker's skill and his working hours. In equilibrium, the model endogenously generates inequality in jobs, wages, and labor supply, but all workers of the same skill

consume the same amount—which is a natural implication of the permanent income hypothesis. Subject to the assignment friction, the competitive equilibrium is efficient and its allocation coincides with the solution chosen by a social planner who gives (potentially) different Pareto weights to workers of different skills.

When labor supply is exogenous, complementarity between the job technology and worker skill ensures that the equilibrium features positive assortative matching (see for instance Becker (1973)). But in our framework the amount of labor input supplied by a worker in a job is function both of his skill and his working hours. Since working hours depend positively on the job technology (due to the substitution effect) and negatively on the worker's wealth (due to the income effect), the total labor input supplied by a poor low skilled workers assigned to a high technology job could be higher than the analogous amount supplied by a wealthy high skilled worker in the same job. This could make profitable assigning a low skilled worker to a high technology job. We show that positive assortative matching requires that workers consumption differences are small relative to their skill differences. This ensures that a low skilled worker assigned to a high technology job faces a small substitution effect relative to the income effect, which in turn guarantees that his total labor input in the job is smaller than the analogous amount supplied by a high skilled worker in the same job. In the social planner problem consumption differences just reflect differences in Pareto weights. But in the decentralized economy, consumption differences arise endogenously as the result of differences in wage income and non labor income of workers. In the absence of differences in non labor income, the condition for positive assortative matching requires that workers skill differences are large enough compared to differences in job technologies. If this is not the case, positive assortative matching still arises in equilibrium if low skilled workers enjoy a sufficiently large amount of non-labor income.

In the model technological progress is embodied into a limited supply of new jobs. So when technological progress accelerates, newly created jobs become more technologically advanced than old jobs that embody relatively more obsolete technologies. As jobs technological differences widen, working hours in high technology jobs increase, while more technologically obsolete jobs are scrapped earlier. This makes aggregate hours per worker increase, while labor force participation falls. The assignment friction is essential for generating opposite movements in the intensive and extensive margins of labor supply. When workers can operate any amount of capital units, so that the assignment friction is absent, faster technological progress leaves the two margins unaffected, so hours per worker and labor force participation never move in opposite directions.

In principle, this mechanism can explain why in the US since the 70's, as wage in-

equality has increased, labor force participation of male workers has fallen while hours per employed worker have increased. To study the quantitative relevance of the mechanism, we parameterize the model to account for differences in employment rates, hours per worker, labor income and consumption across educational groups in the 1970's. The calibrated model implies that, in the 70's, 75 percent of the hourly wage premium between college graduates and workers with no high school degree was due to skill differences, while the remaining 25 percent was due to differences in job technologies. We then follow Greenwood and Yorokoglu (1997), Greenwood, Hercowitz, and Krusell (1997) and Violante (2002) in arguing that the speed of technological progress embodied in new jobs has increased over the period 1970-2000. In the model, the acceleration in the speed of technological progress accounts for 40% of the observed fall in the labor income of workers with no high school degree relative to college graduates. More importantly, the model generates a fall of 8 percentage points in the participation rate and an increase of 1.2 hours worked per week by an average employed worker. This is in line with the data, which show an 8 percent fall in the aggregate participation rate and an increase of 1.5 weekly hours. Finally, the model accounts reasonably well for the observed variation across educational groups. In particular the fact that highly educated workers have experienced a larger increase in hours per worker and a less pronounced fall in participation rates.

Our findings are related to Elsbey and Shapiro (2012) who argue that the fall in productivity growth in the US since the 70's has caused a decrease in the return to labor market experience, which can explain why male employment rates for different skill groups have fallen. Our model provides a novel alternative mechanism whereby changes in the long run rate of growth affect labor supply in models with balanced growth preferences. According to our model, employment rates have fallen because of an acceleration in technological progress which has exacerbated technological differences across jobs.

The remaining of the article is organized as follows. In Section 2 we set up an economy with identical workers, which we solve in Section 3. In Section 4 we allow for heterogeneous workers and analyze conditions for an assortative matching equilibrium to exist. In Section 5 we discuss how to decentralize this equilibrium allocation. Section 6 discusses our quantitative results. Section 7 discusses the quantitative properties of the model under an alternative specification for the cost of creating new jobs. Section 8 concludes.

## **2 The economy with homogenous workers**

We start characterizing an economy where all workers are homogenous. This is useful to analyze the key trade-off between labor force participation and working longer hours in an

economy with a job assignment problem. In Section 4 we extend the model by allowing workers to have different skills.

## 2.1 Worker preferences

The economy is in continuous time and it is populated by a representative household with subjective discount rate  $\rho$ . The household consists of a measure one of identical infinitely lived workers and by an entrepreneur. The consumption good is the numeraire and we assume log preferences in consumption to guarantee the existence of a balanced growth path with constant aggregate labor supply.<sup>1</sup> The time- $t$  instantaneous worker's disutility from working  $n_t$  hours in the period is equal to

$$v(n) = \begin{cases} \lambda_0 + \lambda_1 \frac{n^{1+\eta}}{1+\eta} & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases} \quad (1)$$

where  $\lambda_0 > 0$  measures the fixed cost of going to work,  $\lambda_1 > 0$  governs the magnitude of the variable component and  $\eta \geq 0$  regulates the Frisch elasticity. To produce consumption units a worker has to be matched with a job. We will think of a job as a machine and we will use the term job and machine interchangeably. We assume that, at a time, workers can not work in more than one job and that a job can not be matched with more than one worker. This is the key friction of our economy, which arises because workers and jobs are indivisible. A job of quality  $k$  when matched with a worker who supplies  $n$  hours of work produces an amount of consumption units given by the homogenous of degree one function  $f(k, n) = k^\alpha n^{1-\alpha}$ , with  $\alpha \in (0, 1)$ .

## 2.2 Job qualities

As in Jovanovic (1998), at every instant in time  $t$ ,  $m < \infty$  new jobs of quality  $e^{qt}$  become available, with  $q > 0$  measuring the speed of embodied technical change. Jobs are in excess supply because the number of potential workers is fixed to one while new jobs of relatively better quality become continuously available. There is heterogeneity in the quality of available jobs and at each point in time we rank jobs by their age  $\tau$ . Let  $\tau^*$  denote the critical age such that all jobs older than  $\tau^*$  are scrapped. Then the age distribution of operating jobs is uniform with support  $[0, \tau^*]$ , which implies a probability density equal to  $1/\tau^*$ . Let  $p$  denote the aggregate *participation rate*, i.e. the fraction

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<sup>1</sup>We could have allowed for more general preferences as in King, Plosser, and Rebelo (1988), but the assumption of separability between consumption utility and disutility of working is convenient for the analysis.

of workers who actively participate to the labor market. We focus the analysis on a balanced growth path equilibrium, where the aggregate participation rate  $p$ , the critical age threshold  $\tau^*$  and the mass of newly created jobs  $m$  are constant over time. For simplicity, we assume  $p$  to be in the open interval  $(0, 1)$ . Since every worker is paired with a job, the number of jobs in operation must be equal to the number of employed workers

$$\int_0^{\tau^*} m d\tau = p$$

which implies that  $\tau^* = \frac{p}{m}$ . This means that the age distribution of operating jobs has support over the interval  $[0, \frac{p}{m}]$  and density  $m/p$ .

Let  $\tilde{k}_t^\tau = e^{q(t-\tau)}$  denote the quality of a job of age  $\tau$  at time  $t$ . Then, the quality of the worst job in operation at time  $t$  can be expressed as  $\tilde{k}_t^* \equiv \tilde{k}_t^{\tau^*} = e^{q(t-\tau^*)}$ . In the rest of the paper we will work with detrended job qualities:

$$k^\tau \equiv \tilde{k}_t^\tau e^{-qt} = e^{-q\tau} \quad (2)$$

This implies that the detrended quality of the best job in operation is equal to  $k^0 = 1$ , while the worst job quality in operation is  $k^* \equiv k^{\tau^*} = e^{-\frac{qp}{m}}$ .

### 2.3 Job creation

Jobs are created by entrepreneurs. Creating a job has a cost  $\kappa$  in utility terms. This guarantees the existence of a balanced growth with constant job creation, since both the cost and the value of a newly created job, once evaluated in utils, remain constant over time. The entrepreneur and the workers belong to the same representative household with time- $t$  instantaneous utility equal to

$$u(\tilde{c}_t, m_t) = \ln \tilde{c}_t - \int_0^1 v(n_t(j)) dj - \kappa m_t \quad (3)$$

where  $\tilde{c}_t$  is per capita consumption in the household,  $\kappa m_t$  is aggregate investment measured in utils, while  $n_t(j)$  denotes the hours worked by worker  $j \in [0, 1]$  in the family. There are alternative ways of modeling the cost of job creation. For example we could assume that the cost of creating a new job at time  $t$  is in consumption units and equal to  $\bar{\kappa} e^{\alpha qt}$ . Since the cost grows exogenously at the same rate as consumption, this formulation also implies that the utility cost of a newly created job is constant over time, which guarantees the existence of a balanced growth path with constant job creation. This alternative formulation would have the advantage of mimicking more closely what

is typically assumed in the neoclassical growth model, but at the cost of complicating substantially the theoretical analysis. This is because changes in the job creation rate  $m$  would affect aggregate consumption via the aggregate resource constraint, which thereby affects labor supply through income effects. We study this alternative formulation in the quantitative analysis of Section 7, where we find that these additional income effects are relatively unimportant. So for the sake of exposition, we start focusing the analysis on the specification where jobs are created by entrepreneurs.

### 3 Solving the economy with homogenous workers

We characterize the social planner problem and postpone the discussion on how to decentralize the planner allocation to Section 5. As a term of reference, we start characterizing the economy where workers can operate any amount of machines, so that no assignment friction is present. We then study the properties of our economy with the assignment friction. The analysis focuses on the effects of technological progress on labor supply under balanced growth, which is the topic of the quantitative exercise in Section 6.

#### 3.1 The frictionless economy

In the corresponding frictionless economy aggregate consumption is obtained by combining aggregate capital  $K$  with aggregate labor  $L$  according to the Cobb-Douglas production function  $K^\alpha L^{1-\alpha}$ . Detrended capital is equal to

$$K = m \int_0^\infty k^\tau d\tau = m \int_0^\infty e^{-q\tau} d\tau = \frac{m}{q} \quad (4)$$

which means that all capital available in the economy is used in production. The aggregate supply of labour is equal to  $L = np$ , where  $n$  denotes average hours worked by an employed worker and  $p$  is the participation rate. Clearly detrended consumption  $c = \tilde{c}_t e^{-\alpha q t}$  should be equal to the total amount of consumption units produced in the economy:

$$c = K^\alpha (np)^{1-\alpha}. \quad (5)$$

The social planner cares equally for all individuals and chooses to give them the same level of (detrended) consumption  $c$ . The problem of choosing consumption  $c$ , labor force participation  $p$ , and hours worked  $n$  is intrinsically static and it amounts to maximizing

the sum of the instantaneous utility of all individuals in the economy:

$$\max_{c,p,n} \{\log c - p v(n) - \kappa m\} \quad (6)$$

subject to the aggregate resource constraint in (5). Taking the first order conditions with respect to  $n$  and  $p$  we obtain the two conditions:

$$\frac{1 - \alpha}{n} = p v'(n) \quad \text{and} \quad \frac{1 - \alpha}{p} = v(n)$$

which implies that  $n$  and  $p$  are independent of  $q$ . This follows from consumption log-preferences that make income and substitution effects cancel out exactly. We then have:

**Proposition 1** *In the absence of a job assignment problem, labor force participation and average hours worked on the job are unaffected by the pace of technological progress  $q$ .*

The optimal choice for the number of newly created machines  $m$  is determined by equalizing the cost of a machine  $\kappa$  to its value. Let  $U_t$  denote the value in utils of one additional unit of capital at time  $t$ . This value solves the asset type equation

$$\rho U_t = \frac{1}{\tilde{c}_t} \alpha \frac{\tilde{c}_t}{K_t} + \dot{U}_t$$

where  $K_t \equiv e^{qt} K$  and  $\tilde{c}_t \equiv e^{\alpha qt} c$ . The first term in the right hand side measures the instantaneous value in utils of one additional unit of capital at time  $t$ —equal to the product of the marginal utility of consumption and the marginal productivity of capital—, while the second term measures capital gains due to the change of time. After using (4), it is easy to check that  $U_t = U e^{-qt}$  with

$$U = \frac{\alpha q}{(\rho + q) m}. \quad (7)$$

Since the creation of a new job at time  $t$  involves the creation of  $e^{qt}$  units of capital, we have that the social planner will choose  $m$  so that  $\kappa = U$  which, given (7), implies that

$$m = \frac{\alpha q}{(\rho + q) \kappa}.$$

This means that the rate of job creation  $m$  is increasing in  $q$  unless  $\rho = 0$ , when  $m$  is independent of  $q$ . Intuitively, with higher  $q$  the stock of detrended capital is lower (see equation (4)) and its marginal product is larger, which increase the incentive of creating new machines. But a higher  $q$  also increases the obsolescence rate of detrended capital,

which makes new machines less valuable. With  $\rho > 0$  the former effect dominates, with  $\rho = 0$  the two effects cancel out exactly and the rate of creation of new machines  $m$  becomes independent of  $q$ . Also notice that with  $\rho = 0$ , the optimal value of  $m$  is set to maximize the steady state utility of the representative household in (6). But when  $\rho$  is positive, the planner chooses a lower rate of job creation because increasing steady state utility is costly today. This is the traditional distinction between the golden rule proposed by Phelps (1961), and the modified golden rule emphasized in the neoclassical growth model by Ramsey (1928), Cass (1965) and Koopmans (1965).

### 3.2 The planner problem with job assignment

We now go back to the economy where different jobs have to be assigned to workers. In this economy the social planner equalizes detrended consumption  $c = \tilde{c}_t e^{-\alpha q t}$  across all individuals, as in (6). In every period, the planner also chooses how many workers should actively participate in the labor market, and if so the job they should be matched with as well as their working hours in the match. As workers are homogenous, the exact identity of workers is irrelevant and thereby indeterminate, therefore we index them by the age of the machine they are paired with. We start characterizing the economy for constant job creation rate  $m$ , but we later show that the results of the analysis generally survive when  $m$  is endogenized. For given rate of job creation, the planner's problem is intrinsically static and consists in maximizing the sum of instantaneous utilities

$$\max_{c, p, n^\tau} \left\{ \log c - m \int_0^{\frac{p}{m}} v(n^\tau) d\tau - \kappa m \right\} \quad (8)$$

subject to the aggregate resource constraint for detrended consumption:

$$c = m \int_0^{\frac{p}{m}} f(e^{-q\tau}, n^\tau) d\tau \quad (9)$$

Here  $n^\tau$  denotes the working hours of a worker matched with a machine of age  $\tau$ .

Let  $\mu$  denote the Lagrange multiplier of the resource constraint in (9). Then, by maximizing (8) with respect to  $c$  we immediately obtain that  $\mu$  is equal to the marginal utility of consumption:  $\mu = 1/c$ . By writing the first order conditions with respect to  $p$  and  $n^\tau$  we obtain:

$$v(n^*) = \mu f(k^*, n^*) \quad (10)$$

$$v'(n^\tau) = \mu f_2(k^\tau, n^\tau) \quad (11)$$

where  $n^* \equiv n^\tau$  denotes hours worked in the marginal job. Equation (10) implicitly determines the participation rate by equating the disutility of sending the marginal individual to work to the value of output in the marginal job. Equation (11) determines working hours in jobs of age  $\tau$ ,  $n^\tau$ , by equating the marginal disutility of a working hour to the marginal value of hours in production. This condition determines  $n^\tau$  as a function of job age  $\tau$  and the marginal value of income  $\mu$ :

$$n^\tau = \psi(\tau, \mu) = \left( \frac{1 - \alpha}{\lambda_1} \right)^{\frac{1}{\eta + \alpha}} e^{-\frac{\alpha q}{\eta + \alpha} \tau} \mu^{\frac{1}{\eta + \alpha}} \quad (12)$$

This implies that hours are increasing in job quality so decreasing in job age,  $\psi_1 < 0$ , which characterizes the substitution effect. Hours are also increasing in the marginal value of income  $\mu$ ,  $\psi_2 > 0$ , which characterizes the income effect. The amount of hours in the marginal job can be characterized by evaluating (11) at  $k^\tau = k^*$  and  $n^\tau = n^*$ , and then dividing the resulting expression side by side by (10). After rearranging this yields

$$\frac{n^* v'(n^*)}{v(n^*)} = 1 - \alpha \quad (13)$$

which determines  $n^*$  just as a function of preferences and the output elasticity to labor, which is constant under a Cobb-Douglas production function. Overall we have proved that hours worked have the following properties:

**Lemma 1** *Hours worked are decreasing in the age of the job the worker is matched with and increasing in the marginal utility of consumption. Hours worked in the marginal job  $n^*$  depend just on preferences and the output elasticity to labor.*

### 3.3 An increase in the speed of embodied technical change

Clearly an acceleration in the pace of technological progress (an increase in  $q$ ) leads to an increase in welfare. But, when focusing on detrended quantities, the increase in  $q$  is equivalent to an increase in the depreciation rate of capital. To see this notice that after detrending, the quality of a newly created job is always equal to one,  $k^0 = 1$ , while the quality of a job of any age  $\tau$ ,  $k^\tau = e^{-q\tau}$ , falls with  $q$ . This makes detrended output and consumption  $c$  fall, while differences in job technologies, as measured by the ratio between the quality of a newly created job and a marginal job, equal to  $1/k^* = e^{\frac{q\tau}{m}}$ , increase. Also notice that the fall in detrended consumption together with (12) evaluated at  $\tau = 0$  implies that hours worked in newly created jobs  $n^0$  increase. From (13) it instead follows

that hours worked  $n^*$  in the marginal job  $k^*$  remain unchanged when  $q$  increases. Overall these considerations lead to the following Lemma:

**Lemma 2** *For given rate of job creation  $m$ , when technological progress accelerates ( $q$  goes up), we have that: (a) Detrended consumption  $c$  falls; (b) The ratio of the quality between the top and the marginal job,  $\frac{1}{k^*}$  increases; (c) Hours worked in newly created jobs  $n^0$  increase; (d) Hours worked in the marginal job  $n^*$  remain unchanged.*

**Proof of Lemma 2.** Only points (a) and (b) were not formally proved by the considerations above. To prove (a), we first use (12) to totally differentiate (9):

$$\left(1 + \frac{m \int_0^{\frac{p}{m}} f_2 \psi_2 d\tau}{c^2}\right) dc = f(k^*, n^*) dp - \left[ \int_0^{\frac{p}{m}} m\tau \left( e^{-q\tau} f_1 + \frac{\alpha}{\eta + \alpha} f_2 \psi \right) d\tau \right] dq \quad (14)$$

Similarly by taking logs in (10), and then totally differentiating, after remembering that (13) implies that  $n^*$  is independent of  $q$ , we obtain:

$$\frac{dc}{c} = -\frac{\alpha}{m} (pdq + qdp). \quad (15)$$

After solving for  $dp$  in (15) we obtain:

$$dp = -\frac{m}{\alpha qc} dc - \frac{p}{q} dq$$

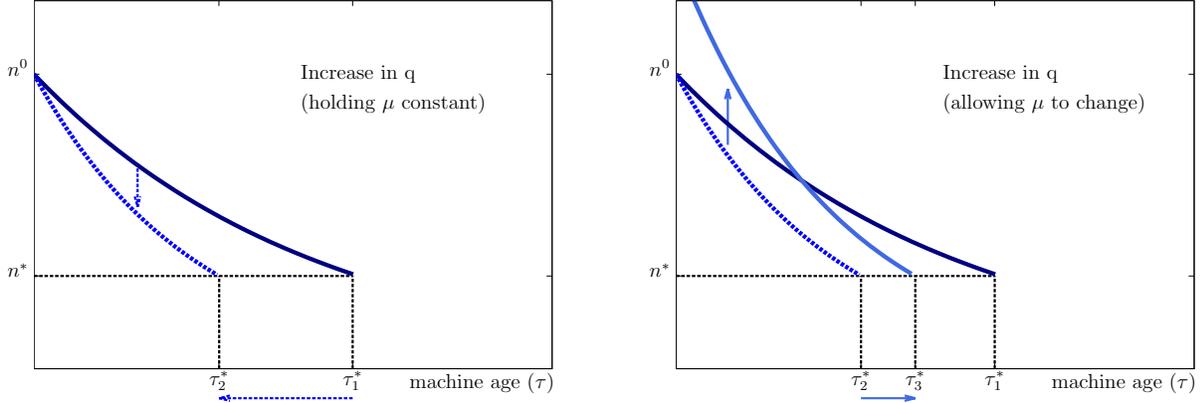
which substituted into (14), and after some rearranging, leads to:

$$\frac{dc}{dq} = -\frac{f(k^*, n^*) \alpha cp + \alpha qcm \int_0^{\frac{p}{m}} \tau \left( e^{-q\tau} f_1 + \frac{\alpha}{\eta + \alpha} f_2 \psi \right) d\tau}{\alpha qc + \frac{\alpha qm}{c} \int_0^{\frac{p}{m}} f_2 \psi_2 d\tau + mf(k^*, n^*)} < 0$$

To prove (b) just notice that (10), for given  $n^*$ , implies that when  $\mu$  goes up (which happens when  $c$  falls),  $k^*$  falls. ■

Figure 1 provides a graphical representation of the response of working hours to the change in  $q$ . The dark blue solid line in the left panel corresponds to the profile of working hours in jobs of different ages  $\tau$ —i.e. to the function  $n^\tau$  defined in (12). Hours are maximum at  $\tau = 0$  and fall exponentially with age until reaching the age of the marginal job  $\tau^*$ , which is equal to  $\frac{p}{m}$ . From (12) and (13) it follows that an increase in  $q$ , while holding consumption constant, leads to a fall in hours in all jobs except in new jobs  $\tau = 0$  and in the marginal job, whose age falls from  $\tau_1^*$  to  $\tau_2^*$  (see dashed blue line). The light blue solid line in the right panel characterizes the total effect on hours of the

Figure 1: Hours worked and machine age



increase in  $q$  by also incorporating the effects of the increase in  $\mu$ , which makes  $n^\tau$  shift upward. Relative to the case with constant  $\mu$ , hours increase in all machines and the age of the marginal job increases from  $\tau_2^*$  to  $\tau_3^*$ .

Lemma 2 implies that when technological change accelerates ( $q$  goes up) technological differences across jobs widen, so working hours in high technology jobs increase, while old more technologically obsolete jobs are scrapped earlier. As a result average hours per employed worker, which are equal to

$$\bar{n} = \int_0^{\frac{p}{m}} \psi(\tau, \mu) \frac{m}{p} d\tau \quad (16)$$

increase, while the participation rate  $p$  falls. This leads to the following Proposition:

**Proposition 2** *For given rate of job creation  $m$ , when technological progress accelerates ( $q$  increases) the participation rate  $p$  falls, while average hours per employed worker  $\bar{n}$  increase.*

**Proof of Proposition 2.** The resource constraint in (9) can be written as

$$c = \left( \mu \frac{1 - \alpha}{\lambda_1} \right)^{\frac{1 - \alpha}{\eta + \alpha}} \int_0^{\frac{p}{m}} m e^{-\frac{(1 + \eta)\alpha q}{\eta + \alpha} \tau} d\tau$$

After solving the integral, remembering that  $\mu = 1/c$ , and some rearranging we obtain

$$c^{\frac{1 + \eta}{\eta + \alpha}} = \left( \frac{1 - \alpha}{\lambda_1} \right)^{\frac{1 - \alpha}{\eta + \alpha}} \frac{(\eta + \alpha) m}{(1 + \eta) \alpha q} \left[ 1 - e^{-\frac{(1 + \eta)\alpha q p}{(\eta + \alpha) m}} \right]. \quad (17)$$

By rewriting (10) and then solving for consumption we obtain

$$c = e^{-\frac{\alpha qp}{m}} \frac{(n^*)^{1-\alpha}}{v(n^*)}, \quad (18)$$

which can be used to replace  $c$  in (17) to yield

$$\left[ \frac{(n^*)^{1-\alpha}}{v(n^*)} \right]^{\frac{1+\eta}{\eta+\alpha}} = \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} \frac{(\eta+\alpha)m}{(1+\eta)\alpha} \cdot \frac{e^{\frac{(1+\eta)\alpha qp}{(\eta+\alpha)m}} - 1}{q}.$$

The left hand side is independent of  $q$  by point (c) in Lemma 2. The right hand side is increasing both in  $p$  and in  $q$  since the Appendix shows that the function  $\frac{e^{\gamma_0 x} - 1}{x}$  is increasing in  $x$  when  $\gamma_0 > 0$ . This proves that  $\frac{dp}{dq} < 0$ .

Integrating (16) after using (12) yields

$$\bar{n} = \frac{\eta + \alpha}{\alpha} \left( \frac{1 - \alpha}{\lambda_1 c} \right)^{\frac{1}{\eta + \alpha}} \frac{1 - e^{-\frac{\alpha}{\eta + \alpha} \cdot \frac{qp}{m}}}{\frac{qp}{m}}$$

After using (18) to replace consumption we finally obtain

$$\bar{n} = \frac{\eta + \alpha}{\alpha} \left[ \frac{(1 - \alpha)v(n^*)}{\lambda_1 (n^*)^{1-\alpha}} \right]^{\frac{1}{\eta + \alpha}} \frac{e^{\frac{\alpha}{\eta + \alpha} \cdot \frac{qp}{m}} - 1}{\frac{qp}{m}}, \quad (19)$$

which implies that average hours per worker  $\bar{n}$  are increasing in  $qp$ , due again to the properties of the function  $\frac{e^{\gamma_0 x} - 1}{x}$ . This concludes the proof, since point (b) in Lemma 2 states that  $k^* = e^{-\frac{qp}{m}}$  is decreasing in  $q$ . ■

By comparing this result with Proposition 1, we conclude that the assignment friction is essential for generating opposite movements in the intensive and the extensive margin of labor supply in response to an increase in  $q$ . With the assignment friction, as technology differences widen, working longer hours in new technologically advanced jobs is more valuable (due to the increase in  $\mu$ ), while hours worked in the marginal job are unchanged (see (13)). As a result average hours per worker  $\bar{n}$  increase. Moreover faster technological progress makes old jobs more technologically obsolete. So they are scrapped earlier, which, for given  $m$ , implies a fall in the participation rate.

### 3.4 Job creation

So far we have analyzed the properties of the economy for a given rate of job creation  $m$ . We now show that Proposition 2 holds true also when  $m$  is endogenized, as long as the

subjective discount rate of agents  $\rho$  is small enough, which in the quantitative exercise of Section 6 appears to be the relevant case.

Let  $s(t, \tau)$  denote the current net flow value in utils of a job of age  $\tau \in [0, \tau^*)$  at time  $t$ . This is equal to the difference between the utility value of the job output and the disutility cost of working in the job:

$$s(t, \tau) = \frac{1}{\tilde{c}_t} f(e^{qt} k^\tau, n^\tau) - v(n^\tau)$$

where  $\tilde{c}_t = e^{\alpha qt} c$  denotes consumption at time  $t$ , so  $1/\tilde{c}_t$  denotes the corresponding value of income. After using (12) to replace  $n^\tau$  we obtain that  $s(t, \tau)$  is independent of  $t$  and equal to

$$s(t, \tau) = s(\tau) = \bar{s} \mu^{\frac{1+\eta}{\eta+\alpha}} e^{-\frac{(1+\eta)\alpha q}{\eta+\alpha} \tau} - \lambda_0$$

where  $\mu$  is the steady state value of detrended income while  $\bar{s}$  is a constant equal to

$$\bar{s} = \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} \frac{\eta + \alpha}{1 + \eta}.$$

The value of a newly created job is equal to the discounted sum of all the  $s(\tau)$ 's generated during its production life:

$$V = \int_0^{\tau^*} e^{-\rho\tau} s(\tau) d\tau = \frac{\bar{s} \mu^{\frac{1+\eta}{\eta+\alpha}}}{\frac{(1+\eta)\alpha q}{\eta+\alpha} + \rho} \left\{ 1 - e^{-\left[\frac{(1+\eta)\alpha q}{\eta+\alpha} + \rho\right] \frac{p}{m}} \right\} - \frac{\lambda_0}{\rho} \left( 1 - e^{-\frac{\rho p}{m}} \right). \quad (20)$$

The optimal level of job creation satisfies the condition

$$\kappa = V, \quad (21)$$

which says that jobs are created up to the exhaustion of any surplus from job creation. The following Proposition proved in the Appendix shows that the results in Proposition 2 hold also when  $m$  is endogenized, provided that the subjective discount rate  $\rho$  is small enough:

**Proposition 3** *If  $\rho$  is close enough to zero, when technological progress accelerates ( $q$  increases) we have that: (a) The rate of job creation  $m$  increases; (b) The participation rate  $p$  falls; (c) Average hours per employed worker  $\bar{n}$  increase.*

We were not able to prove the result for an arbitrary value of  $\rho$ . But, for plausible calibrations of the model we have always found that in response to an increase in  $q$  the

participation rate  $p$  falls and average hours per employed worker  $\bar{n}$  increase. We further discuss this issue in Section 6 and 7.

## 4 The model with worker heterogeneity

We now extend the model to allow workers to have different skills and consumption levels. After characterizing the economy we discuss under which conditions the optimal allocation features positive assortative matching. We then solve the social planner problem and characterize optimal choices for consumption, working hours, and labor market participation.

### 4.1 Assumptions

There are  $N$  types of workers with skill level  $h_i > h_{i+1} > 0$  for  $i = 1, 2, \dots, N - 1$ . The mass of type  $i$  workers is  $z_i \in (0, 1)$  and  $\sum_{i=1}^N z_i = 1$ . We assume that a worker with human capital  $h_i$  working  $n$  hours supplies

$$e = h_i^{1-\theta} n^\theta \tag{22}$$

efficiency units of labor, which are combined with job quality  $k$  to produce output according to:

$$f(k, h_i, n) = k^\alpha (h_i^{1-\theta} n^\theta)^{1-\alpha}.$$

This specification allows the existence of a balanced growth path with constant growth. To allow consumption levels to vary by worker's type, we assume that the social planner gives different Pareto weights  $\nu_i$  to workers of different type. We impose  $\nu_i \geq \nu_{i+1}$  for all  $i = 1, 2, \dots, N - 1$  with  $\sum_{i=1}^N \nu_i z_i = 1$ . Strict equality implies equality of consumption across workers skill types. Strict inequality means that more skilled workers enjoy higher consumption, which is the empirically relevant case. This justifies why we disregard the case  $\nu_i < \nu_{i+1}$ . We allow job quality to depreciate at a constant rate  $\delta$ , which will be important for the quantitative fit of the model in Section 6 and 7. This implies that, the (detrended) quality of a job of age  $\tau$  is now equal to  $k^\tau = e^{-(q+\delta)\tau}$  and since  $\tau^* = p/m$ , the quality of the marginal job is  $k^* = e^{-\frac{(q+\delta)p}{m}}$ .

### 4.2 Assortative matching

Let  $p_i$  denote the participation rate of workers of type  $i$ . Let  $c_i$  denote their (detrended) consumption and let  $n_i^\tau$  denote their working hours when matched with a machine of

age  $\tau$ . Finally let  $\mu$  denote the Lagrange multiplier of the aggregate resource constraint, which measures the marginal value of income to the social planner. Then the utility flow value of matching a job of age  $\tau$  with a worker of type  $i$  is equal to:

$$\tilde{s}_i(\tau) = \max(0, s_i(\tau)) \quad (23)$$

where  $s_i(\tau)$  measures the flow value when supplying positive working hours in the job:

$$s_i(\tau) = \max_{n>0} \{ \mu f(e^{-(q+\delta)\tau}, h_i, n) - \nu_i v(n) \}. \quad (24)$$

This is equal to the difference between the value of the income the worker produces in the job and the disutility cost of working to the social planner. Notice that this expression is independent of  $t$ . The zero value in (23) simply reflects the option value of staying out of the labor market. For given participation rates  $p_i$ 's, it is optimal to assign higher skilled workers to higher quality jobs, if and only if higher skilled workers are relatively more valuable in newer than in older jobs, which is equivalent to requiring that  $\frac{\partial[s_i(\tau) - s_{i+1}(\tau)]}{\partial\tau} \leq 0$ . By solving for  $n$  in (24) we immediately obtain that a worker of type  $i$  in a job of age  $\tau$  should supply an amount of hours equal to

$$n_i^\tau = \left[ \frac{(1-\alpha)\theta h_i^{(1-\alpha)(1-\theta)}}{\lambda_1 \nu_i} \right]^{\frac{A}{1+\eta}} e^{-\frac{\alpha(q+\delta)A}{1+\eta}\tau} \mu^{\frac{A}{1+\eta}}, \quad (25)$$

where  $A = \frac{(1+\eta)}{1+\eta-(1-\alpha)\theta} > 1$ . By applying the envelope theorem in (24) it also follows that

$$\frac{\partial s_i(\tau)}{\partial\tau} = -\mu(q+\delta)\alpha f(e^{-(q+\delta)\tau}, h_i, n_i^\tau) < 0.$$

This implies that  $\frac{\partial[s_i(\tau) - s_{i+1}(\tau)]}{\partial\tau} \leq 0$  holds, if and only if job output is increasing in worker's type,  $f(e^{-(q+\delta)\tau}, h_i, n_i^\tau) \geq f(e^{-(q+\delta)\tau}, h_{i+1}, n_{i+1}^\tau)$ , which, given (25) and the definition of output, happens if and only if the following condition holds:

$$\frac{h_i}{h_{i+1}} \geq \left( \frac{\nu_i}{\nu_{i+1}} \right)^{\frac{\theta}{(1-\theta)(1+\eta)}}, \quad \forall i < N. \quad (A1)$$

This immediately leads to the following Proposition:

**Proposition 4** *The optimal allocation features positive assortative matching if and only if job output is increasing in worker's type which requires that condition A1 holds true.*

Condition A1 states that human capital differences are large relative to Pareto weights. The condition is more likely to hold when  $\theta$  is low, which implies that hours matter less for the total labor input supplied in the job or when  $\eta$  is large, which means that substitutions effects have small impact on working hours in the job.

### 4.3 The planner problem with worker heterogeneity

We now write the social planner problem under Assumption A1, so that the optimal allocation features positive assortative matching. The mass of jobs assigned to workers of type  $i$  is given by  $p_i z_i$ . Let's define  $\tau_0^* = p_0 z_0 = 0$ . Then the minimal age of the jobs where workers of type  $i \geq 1$  are employed is  $\tau_{i-1}^*$  while the maximal age is  $\tau_i^*$  where

$$\tau_i^* = \frac{\sum_{j=0}^i p_j z_j}{m} = \tau_{i-1}^* + \frac{p_i z_i}{m}. \quad (26)$$

The aggregate resource constraint is given by

$$\sum_{i=1}^N z_i c_i = C \equiv \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} f(e^{-(q+\delta)\tau}, h_i, n_i^\tau) m d\tau. \quad (27)$$

For given job creation  $m$ , the social planner then solves the following (static) problem

$$\max_{c_i, p_i, n_i^\tau} \left\{ \sum_{i=1}^N \nu_i \left[ z_i \log c_i - \int_{\tau_{i-1}^*}^{\tau_i^*} v(n_i^\tau) m d\tau \right] - \kappa m \right\} \quad (28)$$

subject to the resource constraint in (27) with associated Lagrange multiplier  $\mu$ .

### 4.4 Solving the model with worker heterogeneity

We now characterize choices for consumption  $c_i$ , participation rates  $p_i$ , and hours worked  $n_i^\tau$  in the problem (28). The first order conditions for consumption  $c_i$  leads to

$$\mu = \frac{\nu_i}{c_i} \quad (29)$$

which implies that the relative consumption of different worker types is equal to their relative Pareto weights. We can now multiply equation (29) by  $z_i$ , and then add up over all  $i$ 's. After remembering that  $\sum_{i=1}^N \nu_i z_i = 1$ , we obtain that the Lagrange multiplier of

the aggregate resource constraint satisfies

$$\mu = \frac{1}{C}.$$

To write the first order condition with respect to  $p_i$ , notice that (26) implies that  $\frac{d\tau_j^*}{dp_i} = \frac{z_i}{m}$ ,  $\forall j \geq i$  and zero otherwise. This is because, as the participation of type  $i$  workers increases, all workers of lower type,  $j > i$ , are displaced to marginally older jobs, while workers of higher types,  $j < i$ , are left unaffected. Let's start assuming for simplicity that  $p_i \in (0, 1)$ ,  $\forall i$ . Then the first order condition with respect to  $p_N$  immediately leads to

$$s_N(\tau_N^*) = 0, \quad (30)$$

which means that the worst job operated by the lowest skill workers must have zero value to the social planner. The analogous condition for  $p_i$   $i < N$  can be expressed as

$$s_i(\tau_i^*) - \sum_{j=i+1}^N [s_j(\tau_{j-1}^*) - s_j(\tau_j^*)] = 0, \quad (31)$$

which emphasizes that assigning a job to a worker of type  $i$  has an opportunity cost, because the same job can not be operated by other workers. So when we employ one more worker of type  $i$ , this worker would operate a job of age  $\tau_i^*$  that has value  $s_i(\tau_i^*)$  to the social planner. But since this job was already operated by a type  $i + 1$  worker, the net increase in social value is smaller than  $s_i(\tau_i^*)$ . This fall in value is measured by the second term in (31), which takes into account that, as the mass of type  $i$  workers used in production increase, all employed workers of type  $j > i$  are displaced to marginally older jobs. Condition (31) can be solved recursively using (30) and starting from  $i = N - 1$  to obtain

$$s_i(\tau_i^*) = s_{i+1}(\tau_i^*), \quad \forall i < N \quad (32)$$

which simply says that at the critical age threshold  $\tau_i^*$  the planner is indifferent between using a type  $i$  or a type  $i + 1$  worker. This again emphasizes the opportunity cost of assigning a job to a type  $i$  worker rather than to a type  $i + 1$  worker.

Finally the first order condition for working hours  $n_i^r$  immediately leads to (25), which determines working hours of workers of type  $i$  as a function of job age  $\tau$  and the marginal value of income  $\mu$ . Exactly as in the one-type model, hours worked decrease with  $\tau$  and increase with  $\mu$ . For given  $\tau$  and  $\mu$ , hours worked are increasing in the worker's skill  $h_i$  and decreasing in the worker's Pareto weight  $\nu_i$ , which, given (29) determine worker's

consumption. Since higher skilled workers have both higher skill  $h_i$  and higher Pareto weight  $\nu_i$ , it is generally unclear whether working hours are increasing in workers skills. By evaluating (25) for workers of different type  $i$ , we can characterize the conditions under which  $n_i^\tau$  decrease with  $i$ :

**Lemma 3** *For given marginal value of income  $\mu$  and job quality  $k^\tau$ , working hours are increasing in the skill type of workers if and only if*

$$\frac{h_i}{h_{i+1}} > \left( \frac{\nu_i}{\nu_{i+1}} \right)^{\frac{1}{(1-\alpha)(1-\theta)}} \quad \forall i < N \quad (33)$$

Basically this condition says that working hours are increasing in the skill type of workers when the skill advantage of better workers is large relative to their consumption premium, which simply means that the substitution effect dominates the income effect. Of course, if (33) holds, also output in a job is increasing in skill type but the converse is not necessarily true. For output to be increasing in the skill type of a worker it has to be that A1 holds, which is less restrictive than (33).

A particular case arises when Pareto weights are independent of worker skills,  $\nu_i = \nu \forall i$ . In this case consumption is equalized across workers, see (29), and, since A1 is satisfied, the optimal allocation features positive assortative matching. By Lemma 3 we also have that working hours are increasing in the skill type of workers. Finally, since the value of labor market participation is higher for higher skilled workers at all machine ages,  $s_i(\tau) > s_j(\tau)$ ,  $\forall i < j, \forall \tau$ , condition (32) can never hold as an equality, leading to corner solutions in participation rates. In particular, there will be an  $i^*$  such that  $p_i = 1, \forall i < i^*$  and  $p_i = 0, \forall i > i^*$ , which implies that higher skilled workers participate more in the labor market.

Finally the planner chooses the optimal job creation rate  $m$  so as to equate the cost of a newly created job to its present value  $V$ , which is equal to the discounted value of the utility flow values generated by all workers who produce in the job over its entire production life. So that in the optimal allocation it must be that

$$\kappa = V = \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} s_i(\tau) d\tau. \quad (34)$$

## 5 Decentralization

We now discuss how the social planner allocation can be decentralized through prices using employment lotteries in the same spirit as Hansen (1985) and Rogerson (1988). The labor

market is characterized by a wage function  $w_{it}(n)$ , that specifies the (detrended) income paid to workers of type  $i$  when supplying  $n > 0$  hours in a job, and by an assignment function  $\varphi_{it}(\tau)$  that specifies the probability density at which a worker of type  $i$  actively participating in the labor market in the period is assigned to a job of age  $\tau$ . In steady state, both the functions  $\varphi_{it}$  and  $w_{it}$  are constant over time, so we have  $w_{it}(n) = w_i(n)$  and  $\varphi_{it}(\tau) = \varphi_i(\tau) \forall t$ . Given the functions  $w_i$  and  $\varphi_i$ , jobs can freely choose their demand for working hours while workers choose their labor supply, i.e. whether to actively participate in the labor market and how many hours to supply. Stable assignment requires that no job and no worker should find optimal to deviate from the allocation prescribed by the assignment function  $\varphi_i(\tau)$ . We conjecture and later verify that the equilibrium features positive assortative matching.

### 5.1 Representative households

All workers of the same type  $i$  are endowed with the same initial level of wealth  $b_{i0}$ . In steady state, consumption, wealth and income all grow at the constant rate  $\alpha q$ . Workers are infinitely lived, they can freely borrow and save at the equilibrium steady state interest rate  $r$ , and there is no aggregate uncertainty. Therefore workers can achieve perfect consumption smoothing, which guarantees the existence of a representative household for workers of the same type  $i$ . The representative household will give the same consumption level to all workers of the same type and ensure that the present value of the disutility cost of working is equalized across all workers in the household. The representative household of type  $i$  chooses, in each period  $t$ , the probability  $p_{it}$  with which each of its members goes to work, and if so how many hours to supply in the job he is assigned to  $n_{it}^\tau$ . This yields per capita labor income to the household equal to  $e^{\alpha q t} W_{it}$  where  $W_{it}$  is detrended labor income which satisfies

$$W_{it} = p_{it} \int_{R^+} w_i(n_{it}^\tau) \varphi_i(\tau) d\tau. \quad (35)$$

The size of the household of type  $i$  is  $z_i$ . To guarantee that (34) holds in the decentralized equilibrium, we assume that entrepreneurs are randomly assigned across households, with a mass  $z_i$  of entrepreneurs being part of the household of type  $i$ . Creating a new job requires the effort of all entrepreneurs in the economy, with each of them incurring a utility cost  $\kappa$ . In each period, the household chooses per capita consumption  $\tilde{c}_{it}$  and assets  $\tilde{b}_{it}$ , labor supply  $p_{it}$  and  $n_{it}^\tau$ , and entrepreneurial effort  $m_t$  so as to maximize

$$\max_{\tilde{c}_{it}, \tilde{b}_{it}, p_{it}, n_{it}^\tau, m_t} \int_0^\infty e^{-\rho t} z_i \left[ \log \tilde{c}_{it} - p_{it} \int_{R^+} v(n_{it}^\tau) \varphi_i(\tau) d\tau - \kappa m_t \right] dt$$

subject to the sequence of budget constraints:

$$\dot{\tilde{b}}_{it} = e^{\alpha qt} W_{it} + e^{\alpha qt} \phi_i m_t - \tilde{c}_{it} + r_t \tilde{b}_{it}, \quad (36)$$

where a dot denotes a time derivative and  $e^{\alpha qt} \phi_i$  denotes the compensation that each entrepreneur in the household receives for creating a new job. By solving the problem we obtain the standard Euler equation for per capita consumption:

$$\frac{\dot{c}_{it}}{c_{it}} = r - \rho - \alpha q \quad (37)$$

where  $c_{it} = e^{-\alpha qt} \tilde{c}_{it}$  is detrended per capita consumption of workers of type  $i$ . This condition can be used to integrate forward (36), which, together with the transversality condition, yields:

$$\int_0^\infty e^{-(r-\alpha q)t} c_{it} dt = b_{i0} + \int_0^\infty e^{-(r-\alpha q)t} (W_{it} + \phi_i m_t) dt. \quad (38)$$

By solving (38) and after using the fact that (37) implies that in steady state

$$r = \rho + \alpha q \quad (39)$$

we obtain that

$$c_i = W_i + \rho b_{i0} + \phi_i m \quad (40)$$

which says that per capita consumption of type  $i$  household is equal to permanent income.

The first order condition for  $n_{it}^\tau$  reads as

$$v'(n_{it}^\tau) = \frac{1}{c_{it}} w'_i(n_{it}^\tau). \quad (41)$$

We can also write the first order condition for labor market participation  $p_{it}$ , which for simplicity we assume holds as an equality,  $p_{it} \in (0, 1)$ .<sup>2</sup> This yields

$$v(n_{it}^\tau) = \frac{1}{c_{it}} w_i(n_{it}^\tau). \quad (42)$$

Finally by taking the first order condition with respect to  $m_t$ , we obtain that an en-

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<sup>2</sup>If instead participation rates are at a corner,  $p_i \in \{0, 1\}$ , we should have that

$$[w_{it}(n_{it}^\tau) - c_{it}v(n_{it}^\tau)](1 - 2p_i) \leq 0$$

which says that the value of participating in the labor market is negative (positive) if  $p_i = 0$  ( $p_i = 1$ ).

trepreneur in the household, for each job he contribute creating, receives a compensation

$$\phi_i = c_{it}\kappa. \quad (43)$$

## 5.2 Jobs

Jobs of age  $\tau$  (detrended quality  $k^\tau = e^{-(q+\delta)\tau}$ ) matched with a worker of type  $i$  choose hours to maximize  $e^{\alpha qt}\pi_i(\tau)$  where

$$\pi_i(\tau) = \max_{n_{it}^\tau} \{f(k^\tau, h_i, n_{it}^\tau) - w_i(n_{it}^\tau)\} \quad (44)$$

denotes detrended profits, which in a steady state will be constant over time. By maximizing we then obtain

$$f_3(k^\tau, h_i, n_{it}^\tau) = w_i'(n_{it}^\tau), \quad (45)$$

which implicitly defines the demand of a job of quality  $k^\tau$  for the hours of a type  $i$  worker.

## 5.3 Matching

In a steady state equilibrium with positive assortative matching there are job age thresholds  $\tau_i^*$  that satisfy (26) as in the social planner problem, with  $\tau_0^* = 0$ . This implies that the assignment function is such that, for given  $i$ ,  $\varphi_i(\tau)$  is equal to zero for any  $\tau$  outside the interval  $[\tau_{i-1}^*, \tau_i^*]$ . So we have

$$\varphi_i(\tau) = \begin{cases} \frac{1}{\tau_i^* - \tau_{i-1}^*}, & \text{if } \tau \in [\tau_{i-1}^*, \tau_i^*] \\ 0 & \text{otherwise} \end{cases} \quad (46)$$

which integrates to one over the support  $[\tau_{i-1}^*, \tau_i^*]$ .

## 5.4 Stable assignment

Since there is an excess supply of jobs, it must be that at the critical technological gap  $\tau_N^*$ , the job makes zero profits:

$$\pi_N(\tau_N^*) = 0. \quad (47)$$

Moreover a stable matching between workers and jobs require that  $\forall \tau \in [\tau_{i-1}^*, \tau_i^*]$  a job prefers to hire workers of type  $i$  rather than any other worker type

$$\pi_i(\tau) = \max_j \pi_j(\tau) \geq 0, \quad \forall \tau \in [\tau_{i-1}^*, \tau_i^*], \quad \forall i \geq 1. \quad (48)$$

This guarantees that no job finds optimal to deviate and hire a worker of a type different from that prescribed by the assignment function  $\varphi_i(\tau)$  in (46). The positive constraint simply states that a job should at least break even. Notice that, at the age threshold  $\tau_i^*$ , (48) should hold both for  $i$  and  $i + 1$ . So it must be that

$$\pi_i(\tau_i^*) = \pi_{i+1}(\tau_i^*), \quad \forall i < N, \quad (49)$$

which means that at the marginal job, higher skilled workers capture all rents.

## 5.5 Job creation

The present value of a newly created job at time  $t$  is equal to  $e^{\alpha qt} P_t$  where  $P_t$  is equal to the (detrended) present value of the profit flows generated by all workers employed by the job over its entire production life:

$$P_t = \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} \pi_i(\tau) d\tau, \quad (50)$$

which is constant in steady state. In writing the expression we used (39) which implies that  $\rho = r - \alpha q$ . The creation of a new job requires the simultaneous effort of all entrepreneurs in the economy, each of them receiving a compensation given by (43). The total cost of creating a job is then equal to  $e^{\alpha qt} C_t \kappa$  where  $C_t = \sum z_i c_{it}$  is detrended aggregate consumption. Jobs will be created up to the point where the cost and the value of a newly created job are equalized, so that

$$\kappa = \frac{P_t}{C_t}. \quad (51)$$

## 5.6 Market clearing

Market clearing in the goods market implies that aggregate consumption is equal to the total amount of consumption units produced in the economy so that

$$\sum_{i=1}^N z_i c_i = \sum_{i=1}^N z_i W_i + \Pi \quad (52)$$

where  $\Pi$  denotes steady state profits equal to

$$\Pi = \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} \pi_i(\tau) m d\tau, \quad (53)$$

which, given (40), is also equal to the total non labor income earned by all households in the economy. Non labor income of a type  $i$  household can then be written as a constant fraction  $\sigma_i$  of aggregate profits

$$\rho b_{i0} + \phi_i m = \sigma_i \Pi, \quad (54)$$

where  $\sum_{i=1}^N z_i \sigma_i = 1$ . This means that differences in non-labor income across households' types can be parameterized indifferently in terms of either  $\sigma_i$ 's or  $b_{i0}$ 's.

## 5.7 Balanced growth path equilibrium

In a balanced growth path equilibrium de-trended consumption  $c_i$ , assets  $b_i$ , aggregate profits  $\Pi$ , wages schedules  $w_i(n)$ , and profits schedules  $\pi_i(\tau)$  as well as the assignment function  $\varphi_i(\tau)$ , participation rates  $p_i$ , working hours  $n_i^\tau$ , entrepreneurial compensations  $\phi_i$ , the flow of news machines  $m$ , the interest rate  $r$ , and the value of a newly created job  $P$  remain constant through time. After defining the tuple

$$x = [c_i, b_i, \Pi, w_i(n), \pi_i(\tau), \varphi_i(\tau), p_i, n_i^\tau, \phi_i, m, r, P]$$

we can then state the following definition for a balanced growth equilibrium:

**Definition** A balanced growth equilibrium is a tuple  $x$  such that (i) each representative household solves her optimization problem, so that (37)-(43) hold; (ii) jobs maximize profits, so (45) holds; (iii) the conditions for stable assignment (47)-(48) are satisfied; (iv) the free-entry condition for job creation in (51) holds; (v) the labor market clears, so (26) and (46) hold; (vi) the goods market clears so (52)-(54) are satisfied; (vii) the equilibrium interest rate is given by (39).

## 5.8 Decentralized equilibrium

We conjecture that, if  $\forall i p_i \in (0, 1)$ , the equilibrium features the wage function

$$w_i(n) = \begin{cases} c_i \lambda_0 + c_i \lambda_1 \frac{n^{1+\eta}}{1+\eta}, & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases} \quad (55)$$

which implies that (41) and (42) in the household problem hold as an identity.<sup>3</sup> As in Prescott, Rogerson, and Wallenius (2009), this means that, in every period, households

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<sup>3</sup>If some  $p_i$ 's are at a corner, then the fixed terms in the wage compensation schedule  $w_i(n)$  in (55), call it  $a_{0i}$ , will have to be modified slightly. Generally the  $a_{0i}$ 's are pinned down by the conditions (47) and (49) leading to  $a_{0i} < c_i \lambda_0$  if  $p_i = 0$ , to  $a_{0i} > c_i \lambda_0$  if  $p_i = 1$ , and to  $a_{0i} = c_i \lambda_0$  if  $p_i \in (0, 1)$ .

are just indifferent about whether to participate in the labor market and about how many hours to supply in the job. In equilibrium the aggregate use of labor is determined by firms demand for labor. By comparing (24) with (44), we also have that, under (55), the value of a job to the social planner is equal to the firm's private value,  $Cs_i(\tau) = \pi_i(\tau)$ ,  $\forall \tau \geq 0$  and  $i \geq 1$ , if and only if type  $i$  households consume the same in the two economies. Given (29), this requires that the share of type  $i$  household on aggregate non labor income,  $\sigma_i$ , is such that the consumption  $c_i$  that solves (40) is equal to a fraction  $\nu_i$  of the aggregate consumption units  $C$  produced in the economy:

$$c_i = \nu_i C, \quad \forall i \geq 1. \quad (56)$$

When (56) holds, it can be easily checked that the equilibrium conditions of the decentralized economy are identical to the conditions that characterize the solution of the social planner problem. For example by comparing (30) and (32) with (47) and (49), we immediately see that the critical age thresholds  $\tau_i^*$ 's are equal, while by comparing (25) with (45) we obtain the same working hours decisions  $n_i^r$ . We can also notice that  $V = \frac{P}{C}$ , where  $V$  is given in (20) and  $P$  in (50). After comparing (34) with (51) this implies that the job creation rates  $m$  are also equal. To analyze under which conditions the decentralized equilibrium features positive assortative matching we can use (48) and apply the same logic that allowed us to prove Proposition 4. All this leads to the following Proposition:

**Proposition 5** *If the shares of type  $i$  households on aggregate non labor income,  $\sigma_i$ 's, are such that*

$$\frac{h_i}{h_{i+1}} \geq \left( \frac{c_i}{c_{i+1}} \right)^{\frac{\theta}{(1-\theta)(1+\eta)}}, \quad \forall i < N, \quad (A2)$$

*then the equilibrium of the decentralized economy features positive assortative matching and its allocation solves the social planner problem in (28) with the set of Pareto weights  $\nu_i$  which satisfy (56).*

It is interesting to clarify the difference between Assumption A1 and A2: in the decentralized allocation, consumption differences are an equilibrium outcome, while in the social planner problem they just reflect differences in Pareto weights. Generally A2 requires that skill differences are large relative to consumption differences. A2 is more likely to hold when technological differences across jobs are small: due to the capital skill complementarity induced by the assignment friction, small differences in technologies compress the return to skill and thereby reduce differences in consumption. It is also easy to prove that A2 is more likely to hold when the share of non labor income on total income of low skilled workers is greater.

## 6 A quantitative analysis

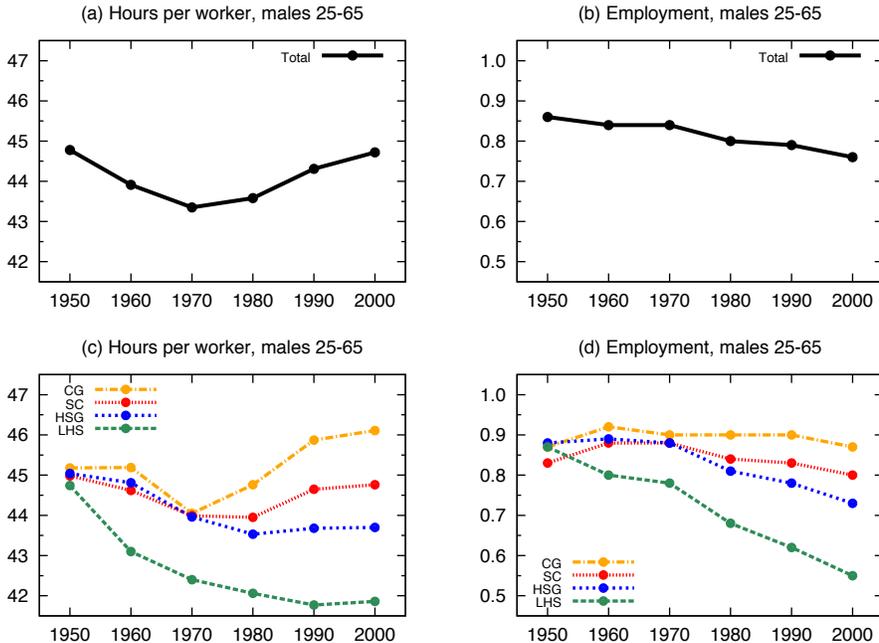
Our theory states that an increase in the speed of embodied technical change rises wage inequality, rises hours per employed worker, and diminishes participation. According to Greenwood and Yorokoglu (1997) and Greenwood, Hercowitz, and Krusell (1997) investment specific technological progress has actually accelerated since the 1970, and it has been argued —see for instance Violante (2002)— that this is the cause of the increase in US wage inequality documented among other by Katz and Autor (1999) and Heathcote, Perri, and Violante (2010). In this Section we study whether the observed change in the pace of investment specific technological progress  $q$  can account for some important changes in male labor supply observed in the US over the 1970-2000 period. Male labor supply has indeed changed substantially in the US between 1970 and 2000. As documented by Juhn (1992), Aaronson, Fallick, Figura, Pingle, and Wascher (2006) and Michelacci and Pijoan-Mas (2012) the participation rate of US male workers has fallen substantially while average hours worked per employed worker have increased. Figure 2 documents these facts using the 1 percent sample of the decennial Census, as provided by the Integrated Public Use Microdata Series (IPUMS) at the University of Minnesota ([www.ipums.org](http://www.ipums.org)). We focus the analysis on a sample of male workers aged between 25 and 64 years old. Panels (a) and (b) describe the evolution of average hours per employed worker and of the employment rate, respectively. It is also well known that these changes have varied depending on the skill level of workers, here identified by their educational level: as shown in Panels (c) and (d), high skilled workers have experienced a larger increase in hours per worker and a smaller fall in employment rates.

To analyze how investment specific technological progress can account for these changes we consider the decentralized economy studied in Section 5, with  $N = 4$  household types corresponding to 4 education groups in the data: college graduates, workers with some college education but no college degree, high school graduates, and workers without a high school degree.<sup>4</sup> We focus on steady state equilibria and the economy is parameterized in terms of non-labor income shares  $\sigma_i$ 's. We calibrate the economy to the 70's and then study the effects of increasing  $q$  so as to match the observed change in the pace of investment specific technological progress.

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<sup>4</sup>Alternatively we could have used a version of the social planner problem studied in Section 4. But in this model, for given Pareto weights, a change in technological progress would have no effects on relative consumption by skill groups, which as shown in Table 3 would be highly counterfactual.

Figure 2: Hours and Participation



Note. Data from US Census for male workers of age 25-65. CG refers to college graduates, SC to high school graduates with some college education, HSD to high school graduates, and NHS to workers with no high school degree.

## 6.1 Calibration

Calibrating a version of the model with  $N = 4$  involves choosing 21 parameters, of which 3 are set using one normalization condition and two add-up constraints. Of the remaining 18 parameters, 6 are set directly while 12 are set by requiring that the model matches 12 moments from the data. Table 1 summarizes the resulting parameter values and the corresponding calibration targets.

We set the annual discount rate  $\rho$  to 4%, and the Frisch labor elasticity parameter  $\eta$  to 2. The depreciation rate  $\delta$  is set to 6%, which is taken from Nadiri and Prucha (1996). The shares of workers of different skill type  $i$ ,  $z_i$ 's, are chosen to match the corresponding values in 1970.

We choose  $\lambda_0$  and  $\lambda_1$  to match the average male employment rate and average hours per employed male worker in 1970, which are equal to 0.84 and 43.3 weekly hours, respectively.<sup>5</sup> The cost of job creation  $\kappa$  determines the mass of newly created machines  $m$ , which affects the average machine age in the model economy, equal to  $\frac{p}{2m}$ . We then use

<sup>5</sup>Hours in the model are calibrated to 43.3/112, where 112 corresponds to the amount of non-sleeping hours in a week available to the worker (7 days a week times 16 hours a day). Hours in the model are then multiplied by 112 to report the results in tables.

as calibration target the average age of capital equipment as reported by the Bureau of Economic Analysis (BEA), which in the mid 60's was equal to 5.6 years.<sup>6</sup>

Table 1: Parameter values and calibration targets, 1970

Symbol	Model		Statistic	Calibration target	Value
	Benchmark	Alternative			
Preferences					
$\rho$	0.04	0.04	—		—
$\eta$	2	2	—		—
$\lambda_0$	0.65	0.92	Average employment to population ratio		0.84
$\lambda_1$	6.97	9.82	Average hours per employed person		43.3
Technology					
$\delta$	0.06	0.06	—		—
$q$ (%)	5.51	5.51	Rate of fall of price of investment goods		0.02
$\kappa, \bar{\kappa}$	3.87	1.39	Average machine age		5.6
$\alpha$	0.64	0.64	Capital share		0.33
$\theta$	0.98	0.98	Labor income ratio between group 4 and 1, $\bar{w}_4/\bar{w}_1$		0.54
Population					
$z_1$	0.15	0.15	Add-up constraint $\sum z_i = 1$		—
$z_2$	0.11	0.11	Population share of group 2		0.11
$z_3$	0.31	0.31	Population share of group 3		0.31
$z_4$	0.43	0.43	Population share of group 4		0.43
$h_1$	1	1	Normalization		—
$h_2$	0.76	0.76	Consumption of group 2 relative to group 1, $c_2/c_1$		0.84
$h_3$	0.64	0.64	Consumption of group 3 relative to group 1, $c_3/c_1$		0.76
$h_4$	0.51	0.51	Consumption of group 4 relative to group 1, $c_4/c_1$		0.68
$\sigma_1$	1.00	-1.12	Add-up constraint, $\sum \sigma_i z_i = 1$		—
$\sigma_2$	0.92	-0.31	Participation rate for group 2, $p_2$		0.88
$\sigma_3$	0.90	0.25	Participation rate for group 3, $p_3$		0.88
$\sigma_4$	1.09	2.62	Participation rate for group 4, $p_4$		0.78

Note. *Benchmark* refers to the benchmark model and *Alternative* refers to the model described in Section 7. Groups 1 to 4 refer to college graduates, high school graduates with some college education, high school graduates and workers with no high school degree, respectively. All statistics are for male workers of age 25-65. Population shares, employment rates, hours per worker and income differences are from the 1970 US Census. Consumption levels are from 1980 CEX.

We set  $\alpha$  to match a labor share of one third which in the model is calculated as

$$\text{Labor share} = \frac{\sum_{i=1}^4 z_i W_i}{C},$$

which follows Cooley and Prescott (1995) in splitting entrepreneurial income between capital and labor income in the same proportion as in the rest of the economy.

<sup>6</sup>See Table 2.10 at <http://www.bea.gov/National/FAweb/AllFATables.asp>

To determine the rate of growth of capital-embodied technical change  $q$  we follow Hornstein, Krusell, and Violante (2007) in reproducing the 2% rate of fall of the relative price of investment goods in the 70's reported by Greenwood, Hercowitz, and Krusell (1997). As in Hornstein, Krusell, and Violante (2007) the rate of decline of the price of a new efficiency unit of capital is equal to  $(1 - \alpha)q$ , which implies  $q = 5.51\%$ .<sup>7</sup>

We normalize  $h_1$  to one. The remaining three values for  $h_i$ , together with the three independent values for  $\sigma_i$  and the value for  $\theta$  are chosen to match the employment rate for the three educational types (the fourth is then matched since the aggregate participation rate is a target), the consumption level of each type of worker relative to the consumption of workers with a college degree, and the average labor income per employed worker of the lowest skill group relative to the highest skill group. Relative consumption comes from CEX in 1980, which is the first wave available. Average labor incomes by skill group are calculated using the 1970 Census.

## 6.2 Properties of the calibrated economy

The calibrated economy satisfies the condition A2 for assortative matching. Column 1 in panel A of Table 2 reports the value of the employment rates and hours worked in 1970 in the data. The corresponding values for the calibrated economy are in column 1 of Panel B. Since the aggregate employment rate, the average hours per worker, and the employment rates by educational groups are calibration targets, the match of the model with the data is perfect.

In the calibration we do not target hours per employed worker by educational group, which in the model corresponds to  $\bar{n}_i \equiv \int_{\tau_{i-1}^*}^{\tau_i^*} n_i^\tau \varphi_i(\tau) d\tau$ . The model rightly predicts that better educated workers work longer hours, although it slightly over-predicts differences by educational group. Columns 1 in Table 3 report the relative labor income and the relative consumption by skill group in the data (in panel A) and in the model (in panel B). Relative consumption patterns are matched by construction and so is the labor income of workers with no high school degree relative to college graduates. But the relative labor income of the two other educational groups was not targeted, still the model matches their value quite accurately.

Finally, we can use the calibrated model to measure how much of the wage return to education in 1970 was due to worker skill differences and how much was due to job differences. To do so, we solve the calibrated model for a very small value of  $\kappa$  such that

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<sup>7</sup>The value of creating a new capital unit is  $e^{\alpha qt} P_t m$  and the capital unit embodies  $e^{qt}$  units of capital. Hence, the relative price of an efficiency unit of capital is  $P_t e^{-(1-\alpha)qt}$ , which falls at rate  $(1 - \alpha)q$ .

Table 2: Labor supply

Statistic	(A) Data		(B) Benchmark			(C) Same $\sigma_i$		(D) Alternative		
	1970 (1)	$\Delta_{00-70}$ (2)	1970 (1)	$\Delta q, \bar{m}$ (2)	$\Delta q$ (3)	1970 (1)	$\Delta q$ (2)	1970 (1)	$\Delta q, \bar{m}$ (2)	$\Delta q$ (3)
PARTICIPATION RATE										
Average	0.84	-0.08	0.84	-0.14	-0.08	0.84	-0.08	0.84	-0.06	-0.04
CG	0.90	-0.03	0.90	-0.11	-0.07	0.90	-0.07	0.90	-0.05	-0.04
SC	0.88	-0.08	0.88	-0.12	-0.07	0.85	-0.08	0.88	-0.05	-0.04
HSG	0.88	-0.15	0.88	-0.12	-0.08	0.83	-0.08	0.88	-0.04	-0.03
NHS	0.78	-0.23	0.78	-0.16	-0.10	0.82	-0.09	0.78	-0.08	-0.04
HOURS PER WORKER										
Average	43.4	1.5	43.3	2.0	1.2	43.3	1.2	43.3	3.0	2.2
CG	44.1	2.5	47.3	3.6	2.1	47.3	2.1	47.3	5.2	3.8
SC	44.0	1.0	46.6	3.3	2.0	46.6	2.0	46.6	4.9	3.6
HSG	44.0	-0.2	44.2	2.3	1.4	44.4	1.4	44.2	3.4	2.6
NHS	42.3	-0.5	40.0	0.7	0.4	40.1	0.5	40.0	1.1	0.9

Note. Panel (B) refers to the benchmark model; Panel (C) refers to the same model with the same parameters, but where we impose  $\sigma_i = 1, \forall i$ ; Panel (D) refers to the model described in Section 7. Employment rates and hours per worker in the data are for male workers of age 25-65 from the 1970 US Census. Changes in columns 2 and 3 of each panel are differences in participation rates and weekly hours, respectively.

in equilibrium  $m$  is very large and hence all jobs are identical. We find that 75% of the hourly wage ratio between college workers and workers with no high school degree is still present in the low  $\kappa$  economy. This implies that, in 1970, 3/4 of the college premium was due to differences in worker skills with the remaining 1/4 was due to job differences.

### 6.3 The experiment

We now increase the value of  $q$  from 5.51% to 12.39%. This matches the evidence in Greenwood, Hercowitz, and Krusell (1997) that the rate of fall in the relative price of investment goods has increased from 2% to 4.5%. All other parameters are left unchanged.

Column 2 of Panel B in Table 2 reports the implied changes in the economy when we keep  $m$  constant. The model predicts a fall in the aggregate participation rate of 14 percentage points, which is larger than the 8 percentage fall observed in the data. The model also predicts an increase of 2 weekly hours, in line with the observed increase of 1.5 hours. Finally, the model is also consistent with the increase in the dispersion of employment rates and hours per worker across education groups. In particular, the drop

Table 3: Labor income and consumption

Statistic	(A) Data		(B) Benchmark		
	1980 (1)	2000 (2)	1980 (1)	$\Delta q, \bar{m}$ (2)	$\Delta q$ (3)
AVERAGE LABOR INCOME					
CG	1.00	1.00	1.00	1.00	1.00
SC	0.79	0.66	0.81	0.77	0.79
HSG	0.69	0.54	0.71	0.65	0.68
NHS	0.54	0.39	0.54	0.44	0.48
AVERAGE CONSUMPTION					
CG	1.00	1.00	1.00	1.00	1.00
SC	0.84	0.77	0.84	0.81	0.83
HSG	0.76	0.68	0.76	0.72	0.74
NHS	0.68	0.54	0.68	0.64	0.66

Note. Statistics are for male workers of age 25-65. Labor income comes from 1970 Census; Consumption from 1980 CEX.

in participation rates is larger for the less educated, while the increase in hours per worker is larger for the more educated. But when  $q$  increases the value of new jobs increases. In equilibrium this leads to an increase in  $m$ , which reduces the increase in the dispersion in job technologies due to the acceleration in the pace of technological progress. As shown in column 3 of Panel B, this leads to slightly more muted responses in the labor market: the aggregate employment rate now declines by 8 percentage points while weekly hours increase by 1.2 hours. These numbers are well in line with the actual changes observed in the data. Regarding the differences by education groups, they are qualitatively identical to the model with fixed  $m$  but quantitatively slightly less pronounced. In the model  $m$  increases by 20%, from 0.075 to 0.090, and the average age of a machine falls by 1.4 years. In the data, the average age of capital equipment as reported by the Bureau of Economic Analysis (BEA) fell only by 0.7 years. This might suggest that the model tends to slightly overpredict the response of job creation in the data.

Finally, in columns 2 and 3 of Panel B in Table 3 we report the model implications for relative labor income and relative consumption across educational groups for constant  $m$  and once allowing  $m$  to respond. With constant  $m$ , the increase in  $q$  generates a substantial increase in labor income inequality: the labor income of workers with no high school degree relative to college graduates falls from 0.54 to 0.44 in the model. This is around two thirds of the fall observed in the data (see Column 2 in Panel A). The consumption of workers with no high school degree relative to college graduates falls from

0.68 to 0.64. Once allowing  $m$  to respond, the increase in  $q$  makes the labor income of workers with no high school degree relative to college graduates fall from 0.54 to 0.48 and the consumption of workers with no high school degree relative to college graduates fall from 0.68 to 0.66 (see Column 5). Overall the differences between the two specifications are quantitatively small.

As shown in Table 1, differences in the shares of non labor income by educational group  $\sigma_i$  are small: the share is 9% higher than average for workers with no high school degree ( $i = 4$ ), 10% lower for high school graduates ( $i = 3$ ), 8% lower for high school graduates with some college ( $i = 2$ ), and less than 1% lower for college graduates ( $i = 1$ ). As a result, forcing the shares  $\sigma_i$ 's to be identical across household types changes little the quantitative results, see Panel C in Table 2. We have also checked that the condition A2 for positive assortative matching is satisfied in the economy with equal  $\sigma_i$ 's.

Overall this implies that differences in non labor income across educational groups matter little for the quantitative results. As shown in Table 4, this is because the ratio of labor income to total income for the baseline calibration (Row 1) and for the calibration with  $\sigma_i = 1 \forall i$  (Row 2) have a similar profile, which is increasing in workers skill.<sup>8</sup>

Table 4: Labor income shares

	Total	CG	SC	HSG	NHS
Benchmark model	0.67	0.74	0.72	0.70	0.59
Model with $\sigma_i = 1$	0.67	0.75	0.70	0.67	0.63

## 7 Job creation costs in consumption units

So far we have assumed that jobs are created by entrepreneurs who incur a utility cost  $\kappa$  for each newly created job. We now assume that job creation involves a cost in terms of consumption units equal to  $\bar{\kappa}e^{\alpha qt}$ . This cost is paid by a representative firm that decides how many jobs to create to maximize its profits. At the end of the period the firm rebates back to households the amount  $(\Pi - \bar{\kappa}m)e^{\alpha qt}$  as dividend payments. In this new set-up, all the equilibrium equations of the model remain unchanged except for the determination of type  $i$  consumption in (40) and the optimal job creation condition in (51). Type  $i$  (detrended) consumption per capita is now given by

$$c_i = W_i + \sigma_i (\Pi - \bar{\kappa}m), \quad (57)$$

<sup>8</sup>This feature of the model is consistent with the US evidence by Budría, Díaz-Giménez, Quadrini, and Ríos-Rull (2002) who report that in the Survey of Consumer Finances of 1998 the share of labor income in total income is 67.2% for college graduates and 64.1% for workers with no high school degree.

where the key difference relative to the utility model is that now job creation costs affect the non-labor income transferred to households, so changes in the job creation rate  $m$  will affect labor supply through income effects. The optimal choice for  $m$  is now governed by

$$\bar{\kappa} = P, \tag{58}$$

which equalizes the cost and the value of a new job, both measured in consumption units.

We calibrate this economy to the same targets as before and we change  $q$  accordingly. In Table 1 we report the parameters for this economy. In comparing this economy with the baseline economy, we see that there are two groups of parameters that change substantially: the preference parameters in the utility function  $\lambda_0$  and  $\lambda_1$ , and the households' shares in aggregate non labor income  $\sigma_i$ . These differences can be explained by comparing (40) with (57): for given  $m$ , in the specification with creation costs in consumption units, less non labor income is transferred to households, so consumption is lower, which makes workers more willing to work. To compensate for this the utility cost parameters  $\lambda_0$  and  $\lambda_1$  should be increased. Similarly, the lower non-labor income implies that the dispersion in the shares  $\sigma_i$ 's has to increase to match the same dispersion in participation rates.

Panel C in Table 2 (which corresponds to panel B for the baseline economy) shows that the properties of this economy are qualitatively similar to those of the benchmark model. The acceleration in the pace of technological progress leads to a fall in the aggregate participation rate and an increase in weekly hours. But now the participation rate falls less while weekly hours increase more. For example, when holding  $m$  constant, the participation rate falls by 6 percentage points, compared with 14 percentage points in the baseline model, while hours increase by 3.0 hours per week, compared with the increase of 2.0 hours obtained in the benchmark model. Differences in the response of hours by educational groups are also now more pronounced than in the baseline economy. When allowing  $m$  to increase, changes in participation and in hours are smaller but the quantitative effect of endogenizing  $m$  is less important than in the benchmark model.

## 8 Conclusions

We have studied labor supply decisions in an assignment model with balanced growth. In the model, technological progress is embodied into new jobs which are slowly created over time. Hence there is dispersion in job technologies. Workers differ in skills and they can be employed in at most one job. This leads to a simple assignment problem in the spirit of Becker (1973) and Sattinger (1975). But in our framework labor supply is endogenous

because in every period each worker decides whether to actively participate in the labor market, and how many hours to work in the job he is assigned to. Since lower skilled workers can supply longer hours, we have shown that the equilibrium features positive assortative matching (higher skilled workers are assigned to better jobs) only if differences in consumption are small relative to differences in workers skills, which guarantees that low skilled workers do not compensate their lower skill level with much greater working hours. In equilibrium, the model endogenously generates inequality in jobs, wages, and labor supply, but all workers of the same skill consume the same amount. When the pace of technological progress accelerates, differences in job technologies widen, wage inequality increases and workers participate less often in the labor market but supply longer hours on the job. We have shown quantitatively that this mechanism can explain why, as male wage inequality has increased in the US, labor force participation of male workers of different skills has fallen while their working hours have increased. The model also matches reasonably well the observed variation by skill groups.

Our analysis could be extended along several dimensions. In particular, in our model skill differences are perfectly observable, constant over time, and exogenously given. This simplifies the analysis, but it neglects some important features of the labor market, such as worker types learning, as in Eeckhout and Weng (2011) and Groes, Kircher, and Manovskii (2010), or human capital accumulation as in Eeckhout and Jovanovic (2002), Imai and Keane (2004), and Michelacci and Pijoan-Mas (2012). Introducing dynamic elements into the analysis would make the return to labor supply intertemporal, which would affect the incentive to participate in the labor market, working hours decisions and the value of being matched to a specific job. Following Jovanovic (1998) we have also assumed that different jobs produce perfectly substitutable goods. But as emphasized by Costinot and Vogel (2010), different vintages could produce different goods and it would be worth characterizing how the elasticity of substitution across these goods affects the conditions under which the equilibrium features positive assortative matching as well as the response of labor supply to changes in the pace of technological progress. Additionally, in our model machines and workers are combined in a fixed proportion which is exogenously given. As in Eeckhout and Kircher (2011), it would be interesting to have a richer theory of the firm where not only the skill level but also the number of workers matched with each machine is endogenously determined.

## A Proofs

### A.1 Proof that $\frac{e^{\gamma_0 q} - 1}{q}$ is increasing in $q$ when $\gamma_0 > 0$ and $q \geq 0$

Let  $\gamma_0 > 0$ . The derivative of the function

$$z(q) = \frac{e^{\gamma_0 q} - 1}{q} \quad (59)$$

has the same sign as

$$g(q) = \gamma_0 e^{\gamma_0 q} q - e^{\gamma_0 q} + 1,$$

which is positive for  $q \geq 0$ , since  $g(0) = 0$  and  $g(q)$  is increasing in  $q$  for all  $q > 0$  which follows from

$$g'(q) = \gamma_0^2 e^{-\gamma_0 q} q > 0.$$

### A.2 Proof of Proposition 3

**Proof.** We use continuity arguments and prove the result for  $\rho = 0$ . When  $\rho = 0$ , the optimal allocation is characterized by the following system of three equations in the three unknowns  $\mu$ ,  $m$  and  $p$ :

$$\kappa = S \mu^{\frac{1+\eta}{\eta+\alpha}} \frac{1 - e^{-\frac{p}{m} \gamma(q)}}{\gamma(q)} - \lambda_0 \frac{p}{m} \quad (60)$$

$$\mu^{-\frac{1+\eta}{\eta+\alpha}} = \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} m \cdot \frac{1 - e^{-\frac{p}{m} \gamma(q)}}{\gamma(q)} \quad (61)$$

$$\mu = e^{\frac{\alpha p q}{m}} \frac{v(n^*)}{(n^*)^{1-\alpha}} \quad (62)$$

where

$$\gamma(q) = \frac{(1+\eta) \alpha q}{\eta + \alpha}$$

which is linear in  $q$ . Equation (60) is (21) evaluated at  $\rho = 0$ , equation (61) corresponds to (17) after using the definition of  $\gamma(q)$  and the fact that  $c = 1/\mu$ , equation (62) corresponds to (18). Notice that  $n^*$  is constant and determined by (13). We can now substitute equation (62) into both (60) and (61) to obtain the following system in the two unknowns  $p$  and  $m$ :

$$\begin{aligned} \kappa &= S \left[ \frac{v(n^*)}{(n^*)^{1-\alpha}} \right]^{\frac{1+\eta}{\eta+\alpha}} \frac{e^{\frac{p}{m} \gamma(q)} - 1}{\gamma(q)} - \lambda_0 \frac{p}{m} \\ \left[ \frac{(n^*)^{1-\alpha}}{v(n^*)} \right]^{\frac{1+\eta}{\eta+\alpha}} &= \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} m \cdot \frac{e^{\frac{p}{m} \gamma(q)} - 1}{\gamma(q)}. \end{aligned}$$

After substituting the second equation into the first we obtain the following system in  $m$  and  $p$  :

$$m = \frac{S}{\kappa} \left( \frac{\lambda_1}{1-\alpha} \right)^{\frac{1-\alpha}{\eta+\alpha}} - \lambda_0 \frac{p}{\kappa} \quad (63)$$

$$\left[ \frac{(n^*)^{1-\alpha}}{v(n^*)} \right]^{\frac{1+\eta}{\eta+\alpha}} = \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} \cdot \frac{e^{\frac{p\gamma(q)}{m}} - 1}{\frac{\gamma(q)}{m}} \quad (64)$$

The first equation establishes a negative (linear) relation between  $m$  and  $p$ . The right hand side of the second equation is decreasing in  $m$  (again due to the properties of the function  $\frac{e^{\gamma_0 x} - 1}{x}$ , which is increasing in  $x$  when  $\gamma_0 > 0$ ) and increasing in  $p$ . This means that (64) establishes a positive relation between  $p$  and  $m$ . This implies that the system (63) and (64) yields a unique solution for  $m$  and  $p$ . We can also notice that (63) is independent of  $q$  while the right hand side of (64) is increasing in  $q$ . So (64) implies that, when  $q$  goes up,  $p$  should fall for given  $m$ . But since (63) establishes a negative relation between  $m$  and  $p$ , we immediately have that an increase in  $q$  leads to an increase in  $m$  ( $dm/dq > 0$ ) and a decrease in  $p$  ( $dp/dq < 0$ ), which proves point (a) and (b) of the Proposition. Now multiply and divide by  $p$  the right hand side of (64) to obtain

$$\left[ \frac{(n^*)^{1-\alpha}}{v(n^*)} \right]^{\frac{1+\eta}{\eta+\alpha}} = \left( \frac{1-\alpha}{\lambda_1} \right)^{\frac{1-\alpha}{\eta+\alpha}} \cdot p \frac{e^{\frac{p\gamma(q)}{m}} - 1}{\frac{p\gamma(q)}{m}}$$

We know that  $p$  falls and that the last fraction in the above expression is increasing in  $\frac{p\gamma(q)}{m}$ . This implies that  $\frac{qp}{m}$  increases when  $q$  goes up. To see that average hours per worker increases (point c), we can then just use (19) to notice that  $\bar{n}$  is an increasing function of  $\frac{qp}{m}$ . ■

## B Numerical solution

To solve the model, we use the conditions for stable assignment (48) and (49), the households intertemporal budget constraints (40), and the job creation condition (51). This yields a system of  $2N + 1$  non-linear equations that we solve for  $c_i$ ,  $p_i$ , and  $m$  with a Gauss-Seidel algorithm that uses a bisection method for each equation. To calibrate the economy we write a system of 12 non-linear equations (the 12 model statistics described in Section 6.1) in the 12 unknown model parameters. This system is solved exactly with the Broyden's method.

In this appendix we start deriving the expressions for firm profits (see expression (66) below), which are needed to write the conditions for stable assignment that form the first set of  $N$  equations. Then we rewrite the household budget constraints, which represent the second set of  $N$  equations (see expression (70) below). Finally we obtain the expression for the optimal job creation condition, which completes the system (see equation (71) below).

## B.1 Firm profits

Firms profits  $\pi_i(\tau)$  can be written as

$$\pi_i(\tau) = f(k^\tau, h_i, n_i(\tau)) - c_i\lambda_0 - c_i\lambda_1 \frac{n_i(\tau)^{1+\eta}}{1+\eta}$$

where

$$n_i(\tau) = \left( \frac{(1-\alpha)\theta}{c_i\lambda_1} f(k^\tau, h_i, n_i(\tau)) \right)^{\frac{1}{1+\eta}},$$

which can be substituted in the expression for profits to obtain

$$\pi_i(\tau) = \frac{1}{A} f(k^\tau, h_i, n_i(\tau)) - c_i\lambda_0. \quad (65)$$

The optimal demand for labor by firms comes from (45) which implies that

$$n_i(\tau) = \left( \frac{(1-\alpha)\theta (k^\tau)^\alpha h_i^{(1-\alpha)(1-\theta)}}{c_i\lambda_1} \right)^{\frac{A}{1+\eta}}.$$

Substituting this expression into output we obtain that

$$f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) = \left( \frac{(1-\alpha)\theta}{c_i\lambda_1} \right)^{A-1} e^{-\alpha A(q+\delta)\tau} h_i^{(1-\alpha)(1-\theta)A}$$

which can be plugged into (65) to obtain

$$\pi_i(\tau) = \frac{1}{A} \left( \frac{(1-\alpha)\theta}{c_i\lambda_1} \right)^{A-1} e^{-\alpha A(q+\delta)\tau} h_i^{(1-\alpha)(1-\theta)A} - c_i\lambda_0. \quad (66)$$

Note that  $\pi_i(\tau)$  is function of model parameters and of the endogenous variable  $c_i$ , and once evaluated at  $\tau_i^*$  it will also be function of  $p_j \forall j \leq i$  and  $m$ . Substituting (66) into the conditions for stable assignment (48) and (49) gives us the first set of  $N$  equations in the  $2N+1$  unknowns.

## B.2 The household budget constraint

The intertemporal budget constraint of household  $i$  implies that

$$c_i = W_i + \sigma_i \Pi \quad (67)$$

where  $\Pi$ , defined in (53), can be written as

$$\Pi = \sum_{i=1}^N z_i \Pi_i$$

and  $W_i$  and  $\Pi_i$  are average labor income and profits generated by workers of type  $i$

$$W_i = \frac{1}{z_i} \int_{\tau_{i-1}^*}^{\tau_i^*} w_i(n_i^\tau) m d\tau \quad \text{and} \quad \Pi_i = \frac{1}{z_i} \int_{\tau_{i-1}^*}^{\tau_i^*} \pi_i(n_i^\tau) m d\tau.$$

Let  $F_i$  denote the average output generated by jobs assigned to workers of type  $i$ :

$$F_i = \frac{1}{z_i} \int_{\tau_{i-1}^*}^{\tau_i^*} f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) m d\tau \quad (68)$$

Using (65) and the fact that  $F_i = W_i + \Pi_i$  we write average profits and average wages as:

$$W_i = \left(1 - \frac{1}{A}\right) F_i + \lambda_0 c_i p_i \quad \text{and} \quad \Pi_i = \frac{1}{A} F_i - \lambda_0 c_i p_i \quad (69)$$

After integrating (68), average output  $F_i$  can be expressed as equal to

$$F_i = \frac{m}{z_i} \left( \frac{(1-\alpha)\theta}{\lambda_1 c_i} \right)^{A-1} h_i^{(1-\alpha)(1-\theta)A} \frac{1}{\alpha A (q+\delta)} e^{-\alpha A (q+\delta) \tau_{i-1}^*} \left[ 1 - e^{-\alpha A (q+\delta) \frac{p_i z_i}{m}} \right]$$

The additional set of  $N$  equations is then obtained by substituting (69) into (67) to obtain

$$c_i = \left(1 - \frac{1}{A}\right) F_i + \lambda_0 c_i p_i + \sigma_i \sum_{i=1}^N z_i \left( \frac{1}{A} F_i - \lambda_0 c_i p_i \right), \quad (70)$$

where  $F_i$  is defined in (68). The equation in (70) again depends on  $c_i$ ,  $p_i$ , and through  $F_i$ , on  $m$ .

### B.3 Job creation

To write the condition (51) for the optimal choice of  $m$ , we need an expression for  $P$ . From (50) we have:

$$P = \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} \pi_i(\tau) d\tau = \sum_{i=1}^N \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} \left( \frac{1}{A} f(k^\tau, h_i, n_i(\tau)) - \lambda_0 c_i \right) d\tau$$

After integrating we obtain

$$\begin{aligned} P &= \frac{1}{A} \frac{[(1-\alpha)\theta]^{A-1}}{[\alpha A (q+\delta) + \rho]} \sum_{i=1}^N \frac{h_i^{(1-\alpha)(1-\theta)A}}{(\lambda_1 c_i)^{A-1}} \left[ e^{-[\alpha A (q+\delta) + \rho] \tau_{i-1}^*} - e^{-[\alpha A (q+\delta) + \rho] \tau_i^*} \right] \\ &\quad - \sum_{i=1}^N \frac{\lambda_0 c_i}{\rho} \left( e^{-\rho \tau_{i-1}^*} - e^{-\rho \tau_i^*} \right) \end{aligned} \quad (71)$$

This is the final equation that completes the system of  $2N + 1$  equations that we solve at the computer.

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