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NOISE BUBBLES*

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We introduce imperfect information in stock prices determination. Agents, whose expectations are not assumed to be rational, receive a noisy signal about the structural shock driving future dividend variations. Equilibrium stock prices are decomposed into a fundamental component and a transitory 'noise bubble' which can be responsible for boom and bust episodes unrelated to economic fundamentals. We propose a non-standard VAR procedure to estimate the effects of noise shocks as well as bubble episodes. Noise explains a large fraction of US stock prices. In particular the dot-com bubble is almost entirely explained by noise.

Stock markets react to news about events whose actual consequences on economic fundamentals are often highly uncertain. An international crisis may be resolved peacefully or escalate into war; inventions may take a lot of time, or even fail, to produce important technological improvements; a sovereign debt crisis may be solved by sound policy measures or end up with a ruinous default. On the one hand, there is news which anticipates major changes of future dividends; on the other hand, there is news whose potential effects never materialise. Typically, when a piece of news arrives, investors do not know which of the two types the news belongs to but they have to take a decision immediately. Since such decisions affect prices, part of stock price fluctuations can be driven by news unrelated to economic fundamentals.

In this article, we introduce noisy information in the determination of stock prices. Dividends are driven by a structural economic shock, let us say the 'dividend' shock. The effects of such shock are delayed, so that traders cannot see it by looking at current dividends. Agents have some information about the current shock, in that they see a signal, given by the sum of the dividend shock and a 'noise' shock, not affecting fundamentals.[1] On impact, investors react to both the dividend shock and the noise shock in just the same way, being unable to distinguish between them. As time goes on, however, agents learn about the true nature of past dividend shocks by looking at realised dividends and adjust their initial response. Thus a noise shock announcing good news leads to a kind of 'undue exuberance': dividends are expected to rise and stock prices go up. But in the end, agents realise that the shock was in fact noise and the bubble bursts. Hence the noise shock generates transitory boom and bust episodes, the 'noise bubbles', unrelated to the intrinsic value of equities.

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[1] In Adam *et al.* (2007), agents are assumed to have limited information about model parameters. By contrast, here information is limited in that the shocks are not observable, as in the recent news-noise business cycle literature (Beaudry and Portier, 2004, 2006; Christiano *et al.*, 2008; Lorenzoni, 2009; Angeletos and La'O, 2010; Forni *et al.*, 2013*b*).

This result is surprisingly general, in that it does not require a specific economic model for stock prices. In particular, we do not assume rational behaviour: our assumptions are consistent with, but not restricted to, set-ups in which agents form expectations rationally.[2] This is notable, in light of the influential literature documenting irrational behaviour in the stock market (Shiller (2000) and the references therein) and the prominent theoretical contributions assuming existence of both rational and irrational traders (Abreu and Brunnermeier, 2003).

We assume that:

(*i*) agents receive a noisy signal about future dividends;
(*ii*) stock prices are affected by agents' expectations about future fundamentals; and
(*iii*) the difference between the log of prices and the log of dividends is stationary.

Both (*ii*) and (*iii*) are common features of existing models: the novelty is assumption (*i*). The reason why (*i*)–(*iii*) are sufficient to obtain the noise bubble is simple. Noise shocks are news which are perceived by agents as potentially anticipating future changes in dividends (this is a consequence of (*i*)). Hence they affect prices in the short run by assumption (*ii*). But in fact dividends will not change, by the very definition of noise shocks. Since prices follow dividends in the long run because of assumption (*iii*), the effect of noise must be transitory.

The point of arrival of our theoretical construction is then a decomposition of stock prices into a 'fundamental' component, related to dividend shocks as well as structural shocks affecting interest rates and risk premia, and a stationary noise bubble, orthogonal to the fundamental component at all leads and lags. Such decomposition is completely different from the standard rational bubble decomposition[3], since noise bubbles, unlike rational bubbles, are part of the stable equilibrium and have nothing to do with self-fulfilling expectations. Here we rule out standard rational bubbles by assuming a single stationary equilibrium for stock price changes.

Noisy information has dramatic implications for empirical analysis: if agents do not see the structural shocks, standard structural VAR methods fail. This is because economic data reflect agents' behaviour, which in turn depends on their information. If agents observe current shocks, the econometrician can in principle infer them from existing data; but if agents do not distinguish the shocks, present and past values of observable variables cannot embed the relevant information (Blanchard *et al.*, 2013).

Despite this, in our theoretical setting, structural VAR methods can still be used successfully, provided that identification is generalised to include dynamic transformations of the VAR residuals. The reason is that, as time goes by, realised dividends reveal whether past signals were true dividend shocks or noise. Hence, current dividend and noise shocks, while not being combinations of current VAR residuals, are combinations of future values of such residuals.[4]

[2] The present value model is used in subsection 2.1 for illustrative purposes only.

[3] See for example Samuelson (1958), Tirole (1985), Santos and Woodford (1997) and, more recently, Martin and Ventura (2012).

[4] This feature is not shared by the business cycle models of Barsky and Sims (2012) and Blanchard *et al.* (2013) where agents never learn completely the true nature of past structural shocks.

A general treatment of dynamic structural VAR identification is found in Lippi and Reichlin (1994). An application to fiscal policy is shown in Mertens and Ravn (2010). Here we propose a specific identification scheme to recover the noise shocks, the related impulse response functions and the noise bubble.[5] The scheme imposes zero effects of noise on dividends, both on impact and in the long run, while leaving unrestricted the effects of all shocks on prices.

The role of economic theory in the econometric procedure is essentially limited to the definition of the noise shock and the characterisation of agents' information set. In this respect, our procedure can be regarded as a statistical method, with a minimum of economic assumptions in the background, to estimate the deviation of stock prices from their fundamental value.[6] The basic difference with respect to the literature aimed at identifying bubble episodes by means of regime-switching models (Al-Anaswah and Wilfling (2011) and the references therein) is that here bubble episodes, which can be both positive and negative, are generated by misinterpreted news rather than non-linearities.

A key condition for the validity of our procedure is that the VAR specification is rich enough to capture adequately other sources of variation, different from noise, which however, just like noise, do not affect dividends. We are thinking in particular of price variations arising from interest rate shocks and shocks to risk premia. If the VAR specification is deficient in this respect, such variations could be erroneously included in the noise bubble and produce an overstatement of its importance.

In the empirical Section, we apply our structural VAR identification technique to US stock market and dividend data. We identify three sources of stock price volatility:

(*i*) dividend shocks;
(*ii*) shocks related to variations of interest rates and risk premia; and
(*iii*) noise shocks.

The fundamental component is driven by sources (*i*) and (*ii*), whereas the bubble component arises from source (*iii*). We find that dividend shocks have a limited impact in the short run but have permanent effects and explain a good deal of stock market fluctuations in the long run. Price variations related to interest rates are transitory but are important at short and medium-run horizons. Consistently with the theory, noise shocks have essentially no effects on dividends and do not affect prices in the long run. They explain a huge fraction (about one half) of stock price volatility in the short-medium run.

The estimated bubble provides a measure of the percentage deviation of prices from their fundamental value. A dating of past bubble episodes can then be obtained by setting a threshold for the percentage deviation of stock prices from their fundamental value. Using 20% as a threshold, we identify four positive bubbles and four negative bubbles. Not surprisingly, the largest positive bubble of the last half century was the

[5] See also the companion paper Forni *et al.* (2013*b*) where a similar news-noise setting is applied to business cycle issues.

[6] Unfortunately, both the shock and the bubble can be estimated reliably only for past events, since, as observed above, they involve future values of the VAR residuals.

dot-com episode, starting in 1997:Q3 and ending in 2002:Q1; the peak was reached in the second quarter of 2000, when prices deviated from their intrinsic value by 56%. The boom peaking in 2007 was not a bubble. The stock market crisis of 2008 was exacerbated by the largest negative bubble our sample, spanning from 2008:Q1 to 2009:Q4 and culminating at −42% in 2009:Q1.

The reminder of the article is organised as follows. In Section 1 we present the model. Section 2 discusses the econometric implications and presents our dynamic, structural VAR identification scheme. Section 3 presents our empirical results. Section 4 concludes. A few simulation exercises are reported in Appendix B.

1. Economics

The idea that stock prices are affected by news about economic and political happenings is largely accepted. Figure 1 depicts the growth rate of the S&P 500 index as well as vertical lines in coincidence with news about major economic and political events. In many of these episodes, the index displays large drops and peaks. For instance the index dropped by about 20% in coincidence of the Franklin National Bank collapse and the Worldcom bankruptcy and increased by around 10% the quarter before the official end of the Vietnam war.

An obvious interpretation is that stock prices change because agents expect future dividends to change in consequence of these events. But this predicted change does not necessarily occur.

Figure 2 plots the quarterly series of log-dividends and log-prices after four major episodes: the Watergate scandal and the Franklin National Bank collapse, the end of the Vietnam War, the Worldcom bankruptcy and the Lehman Brothers bankruptcy. The series are normalised to zero in period 0 which is the period before the event occurs. The vertical line coincides with the event. The drop in stock prices following the Lehman Brothers bankruptcy (lower-right panel) anticipates by about two quarters a similar large decline of future dividends. This happens, though to a lesser extent and with a longer delay, also in two other episodes, namely the Vietnam War (with a reversed sign) and the Watergate scandal. By contrast, dividends did not reduce at all during the year after the fall in prices associated to the Worldcom bankruptcy (lower-left panel).

Traders' expectations were completely wrong in this case. A possible explanation is that, at the time the news arrived, agents were simply unable to predict its effects on future dividends, because they did not know whether the shock leading Worldcom to bankruptcy was a bad financial shock with disastrous consequences on the financial system or a temporary and isolated episode with no further consequences on the economy. Below we develop formally a model based on this idea.

1.1. *An Illustrative Example with the Present Value Model*

In this subsection we use a stripped-down model to clarify the intuition underlying our idea. Let us assume that stock prices follow the present value model proposed by Campbell and Shiller (1988), where the log of stock prices is determined by the expected discounted sum of future log dividends. For the sake of simplicity, we assume

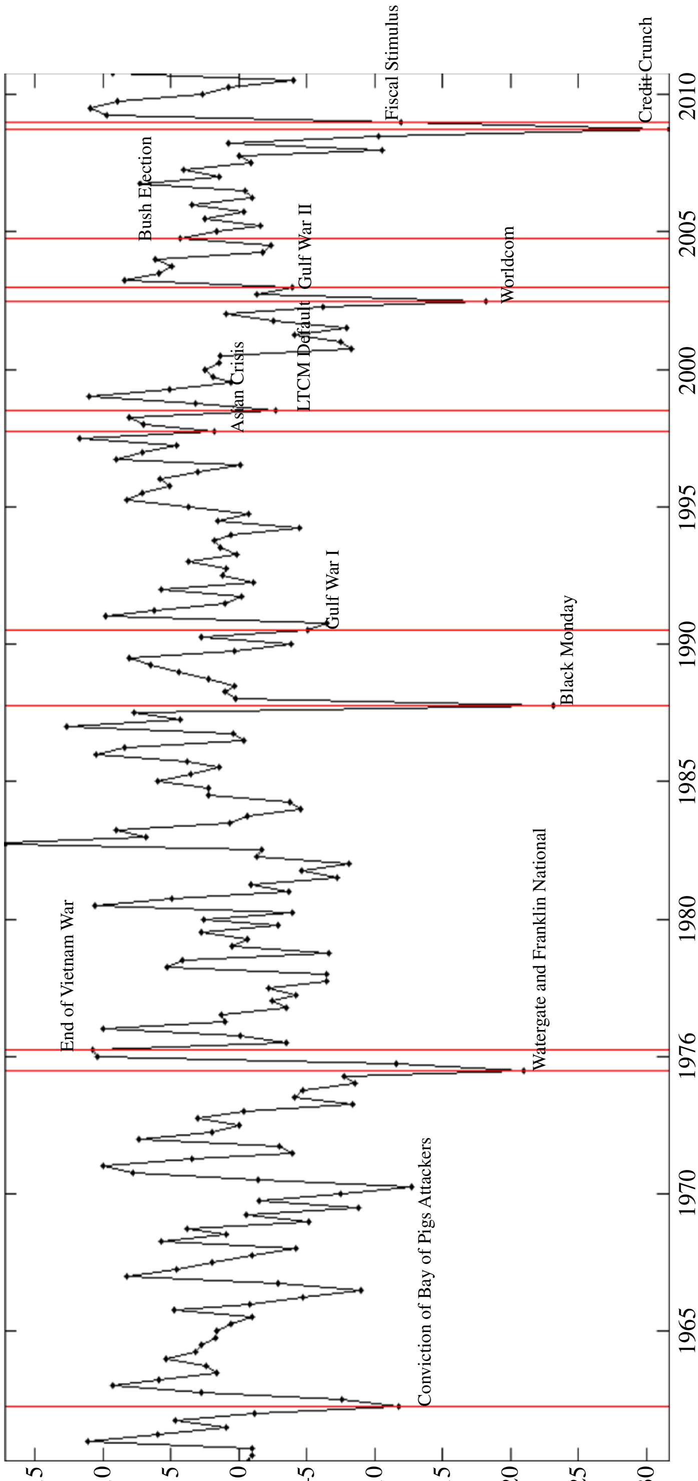

Fig. 1. *Log of the Real S&P 500 Index with Vertical Lines Corresponding to Economic and Political Events*

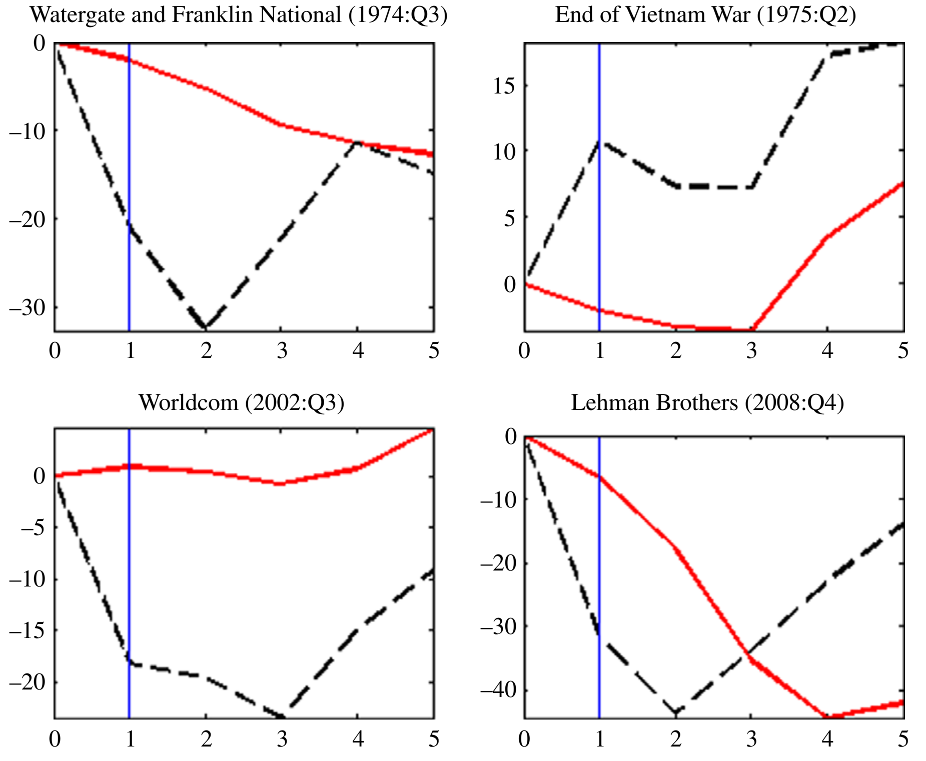

Fig. 2. *Real S&P 500 (Dotted Line) and Net Corporate Dividends (Solid Line) Both in Logs*
Notes. The x axis shows quarters. The events occurs in quarter 1.

that the 'discount factors' are constant. Formally, the log of prices p_t is given by:

$$p_t = \frac{k}{1-\rho} + \frac{1-\rho}{\rho} \sum_{j=1}^{\infty} \rho^j \mathrm{E}_t d_{t+j}, \tag{1}$$

where d_t are dividends, expressed in logs, E_t denotes expected value, conditional to information available at time t, $\rho = 1/(1 + e^{\mu})$, where $\mu = \mathrm{E}(d_t - p_t)$ and $k = -\log(1 + r) - \log \rho + (1 - \rho) \log(1/\rho - 1)$, r being the constant rate of return on equities. Observe that, in the above equation, speculative bubbles, as defined in standard textbook models, are ruled out and stock prices are simply given by what is usually referred to as the 'fundamental' value.[7]

Let us assume here that dividends are driven by a structural shock whose effects are delayed. We assume that d_t follows the equation:

$$d_t = d_{t-1} + a_{t-1}. \tag{2}$$

where a_t is the structural dividend shock, a Gaussian white noise with variance σ_a^2. Notice that a_t does not affect d_t on impact. This is a feature typical of the so-called 'news' shocks, i.e. shocks which may change agents' expectation before affecting

[7] Equation (1) is derived in Appendix A.

economic fundamentals.[8] The basic novelty of our model is that agents have incomplete information, so that our news is noisy. To be precise, agents do not see the current dividend shock but only observe the signal:

$$s_t = a_t + e_t, \tag{3}$$

where e_t (the 'noise') is a Gaussian white noise, orthogonal to a_t at all leads and lags. The signal sometimes conveys relevant information about the future (when e_t is small), sometimes is essentially misleading (when e_t is large).

Finally, we assume that economic agents observe d_t at time t, so that agents' information set at time t, say Ω_t, is given by the linear space spanned by present and past values of dividends and the signal s_t. Below we compare results for Ω_t with what obtained with complete information, i.e. the information set Φ_t, spanned by present and past values of a_t and e_t.

From (1)–(3),

$$\mathrm{E}_t d_{t+1} = \mathrm{E}_t d_t + \mathrm{E}_t a_t = d_t + \mathrm{E}_t a_t.$$

Moreover, $\mathrm{E}_t d_{t+2} = \mathrm{E}_t d_{t+1} + \mathrm{E}_t a_{t+1}$. Since a_{t+1} is unpredictable, we have $\mathrm{E}_t d_{t+2} = \mathrm{E}_t d_{t+1}$. Proceeding recursively we obtain:

$$\mathrm{E}_t d_{t+j} = \mathrm{E}_t d_{t+1} = d_t + \mathrm{E}_t a_t \quad \text{for } j \geq 1.$$

Applying (1), we get:

$$p_t = \frac{k}{1-\rho} + d_t + \mathrm{E}_t a_t. \tag{4}$$

Now let us consider the expectation of a_t. For the sake of comparison, we begin by deriving the stock price equation under the assumption that a_t is observable, i.e. the information set is Φ_t. Denoting by E_t^{Φ} the expectation conditional to Φ_t, we have $\mathrm{E}_t^{\Phi} a_t = a_t$. Using (2) and (4), we obtain:

$$\Delta p_t^{\Phi} = a_t. \tag{5}$$

When a positive shock arrives, the market reacts immediately by raising prices by the amount a_t and the noise shock has no effect on prices.

Coming to the present setting, a_t is not observed. The information set of the agents is given by Ω_t. By (2), d_t reveals the past of a_t but is completely uninformative about the present. Similarly, past values of s_t do not say anything about a_t. Hence, $\mathrm{E}_t^{\Omega} a_t$ is simply the projection of a_t on s_t, that is:

$$\mathrm{E}_t^{\Omega} a_t = (\sigma_a^2/\sigma_s^2) s_t = (\sigma_a^2/\sigma_s^2) a_t + (\sigma_a^2/\sigma_s^2) e_t. \tag{6}$$

[8] The fact that the variable on the left-hand side only depends on past values of a shock, is unusual in the literature and may seem unconvincing. Were a_t be observed at time t, d_{t+1} would be known without error at time t, which of course is highly implausible. However, in our theoretical context, a_t cannot be observed at time t, so that d_{t+1} is not perfectly predictable. The equation for dividends should be read as simply meaning that a_t have some effect on agents' information at time t, before affecting dividends. And this, we think, is perfectly plausible.

Substituting in (4), taking the first difference and rearranging terms gives:

$$\Delta p_t = \frac{\sigma_a^2}{\sigma_s^2}\left(a_t + \frac{\sigma_e^2}{\sigma_a^2}a_{t-1}\right) + \frac{\sigma_a^2}{\sigma_s^2}(e_t - e_{t-1}). \tag{7}$$

To interpret the above equation, let us assume that, at time t, the signal is perfectly correct, i.e. the noise is zero and $s_t = a_t$. Agents do not see this, so that they are too cautious and under-react on impact, the coefficient being σ_a^2/σ_s^2, which is less than the perfect information response of one. After one period, however, by observing d_t, agents realise that the signal was indeed correct and adjust their behaviour to get a cumulated response equal to $\sigma_a^2/\sigma_s^2 + \sigma_e^2/\sigma_s^2 = 1$.

At the opposite extreme, when the signal is completely false, i.e. the structural economic shock is zero in t and $s_t = e_t$, agents are too optimistic, if e_t is positive, or pessimistic, if e_t is negative, and over-react on impact, with coefficient σ_a^2/σ_s^2, which of course is greater that the 'correct' response of zero. Notice that the impact response to e_t is the same of a_t, since people cannot distinguish false news from true news on impact. Again, after one period, this kind of 'rational exuberance' disappears and prices go back to the previous level.

According to (7) and (2), the noise shock affects stock prices, though dividends are noise free. Price changes are driven by two components, let us say the 'fundamental component', driven by the present and past values of a_t, and the 'noise component', driven by the present and past values of e_t. The latter is a bubble in that it is not related to fundamentals. Noticeably, the effect of false news is transitory, the cumulated response being zero (whereas the effect of true news is permanent, the cumulated response being one). Hence, the noise bubble is fated to burst.[9]

Our simple present value model can be useful to show why standard structural VAR techniques fail but the structural shocks can be recovered as linear combinations of present and future values of the VAR innovations. Consider (2), (5) and (6). Abstracting from constants, the joint model for Δd_t and $p_t - d_t$ is:

$$\begin{pmatrix} \Delta d_t \\ p_t - d_t \end{pmatrix} = \begin{pmatrix} L & 0 \\ \psi & \psi \end{pmatrix}\begin{pmatrix} a_t \\ e_t \end{pmatrix},$$

where $\psi = \sigma_a^2/\sigma_s^2$ is the coefficient of the projection of a_t on s_t. The determinant of the MA matrix above id ψL, which vanishes for $L = 0$. Hence the above representation is not invertible. As a consequence, the structural shocks cannot be found as linear

[9] Two interesting limit cases are $\sigma_e^2 = 0$, i.e. there is no noise at any t, and $\sigma_e^2 \to \infty$, i.e. false news is largely predominant. When $\sigma_e^2 = 0$, the signal s_t is equal to a_t, so that agents can see the true economic shock. Obviously in this case the noise bubble is not there and (7) reduces to (5).

Somewhat surprisingly, the noise bubble disappears even in the opposite case, when σ_e^2 goes to infinity. For, the variance of the noise component is $2\sigma_a^4\sigma_e^2/\sigma_s^4$, which vanishes for $\sigma_e^2 \to \infty$. The economic intuition is that, when e_t is very large, the signal is not reliable, so that the stock market does not react to it. Equation (7) reduces to $p_t = p_{t-1} + a_{t-1}$, reflecting the fact that, s_t being not informative, agents see only a_{t-1} and therefore respond to the structural shock with delay.

The noise bubble is large when dividend and noise shocks have approximately the same size. To see this, let us compute the ratio of the variance of the noise component to the variance of Δp_t. The structural component in (7) has variance $\sigma_a^2(\sigma_a^4 + \sigma_e^4)/\sigma_s^4$, whereas the variance of the noise component is $2\sigma_a^4\sigma_e^2/\sigma_s^4$. Summing the two variances gives the variance of Δp_t, i. e. σ_a^2. The ratio of the variance of the noise component to the total variance is then $2\sigma_e^2\sigma_a^2/\sigma_s^4$. Such ratio is zero for both $\sigma_e^2 = 0$ and $\sigma_e^2 \to \infty$, as observed above, and reaches its maximum $1/2$ for $\sigma_e^2 = \sigma_a^2$.

combinations of present and past values of the variables appearing on the left-hand side, so that standard VAR techniques cannot be used successfully.

Now let us set $u_{1t} = (1-\psi)a_{t-1} - \psi e_{t-1}$ and $u_{2t} = \psi a_t + \psi e_t$. With these shocks, we have the following MA representation:

$$\begin{pmatrix} \Delta d_t \\ p_t - d_t \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}.$$

This representation is invertible, since the determinant is 1. Moreover, it is the Cholesky representation of the variables, since the upper-right element of the matrix above is zero for $L = 0$. Hence u_{1t} and u_{2t} are the Cholesky innovations and can be estimated by means of a standard recursive VAR identification. The relation linking the structural shocks to the Cholesky innovations is:

$$\begin{pmatrix} a_t \\ e_t \end{pmatrix} = \begin{pmatrix} L^{-1} & 1 \\ -L^{-1} & 1 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}.$$

Hence the structural shocks are linear combinations of present and future values of the VAR residuals.

1.2. *The General Two-shock Model: Dividends and Information Sets*

Let us now come to our general model. For the dividend process we assume:

$$\Delta d_t = c(L)a_t, \tag{8}$$

where $c(L)$ is a rational function in L and the dividend shock a_t is a white noise process. At time t, agents get some (noisy) information about a_t, since they see the signal s_t, which is still given by (3). We retain the assumption that the shock a_t is a news shock which does not affect d_t on impact, i.e. $c(0) = 0$. Economic agents can observe Δd_t at time t. Hence agents' information set is $\Omega_t = \text{span}(\Delta d_{t-k},\ s_{t-k},\ k \geq 0)$.

Since a basic feature of our model is that the information set of the agents, Ω_t, does not coincide with the information set spanned by the structural shocks, Φ_t, we start by studying the relation between these information sets. The difference between Ω_t and Φ_t is characterised by the relation linking the variables $(\Delta d_t \quad s_t)'$, which agents can observe, on the one hand, and the shocks $(a_t \quad e_t)'$, which agents cannot observe, on the other hand. From (8) and (3), we have:

$$\begin{pmatrix} \Delta d_t \\ s_t \end{pmatrix} = \begin{pmatrix} c(L) & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ e_t \end{pmatrix}. \tag{9}$$

This relation is not invertible, since the determinant of the MA matrix is $c(L)$, which by assumption vanishes for $L = 0$, which is less than 1 in modulus. Non-invertibility implies that we do not have a VAR representation for Δd_t and s_t in the structural shocks, and that present and past values of the observed variables Δd_t and s_t contain strictly less information than present and past values of a_t and e_t.[10]

[10] Notice that, if the representation were invertible, such a VAR would exist, so that the structural shocks could be written as a linear combination of present and past values of observable variables, and the information sets Φ_t and Ω_t would be equal, contrary to the assumption that the dividend shock does not belong to the information set of the agents.

Representation (9) is not the only MA representation of Δd_t and s_t. In particular, there is a 'fundamental' representation, i.e. an MA representation in the innovations of Ω_t.[11] Let r_j, $j = 1, \ldots, n$, be the roots of $c(L)$ which are smaller than one in modulus and:

$$b(L) = \prod_{j=1}^{n} \frac{L - r_j}{1 - \bar{r}_j L}, \tag{10}$$

where $\bar{r}_j$ is the complex conjugate of r_j. Note that $b(L)^{-1} = b(F)$, F being the forward operator such that $Fx_t = x_{t+1}$. Consider the innovation representation:

$$\begin{pmatrix} \Delta d_t \\ s_t \end{pmatrix} = \begin{pmatrix} \dfrac{c(L)}{b(L)} & \dfrac{c(L)\sigma_a^2}{\sigma_s^2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}, \tag{11}$$

where

$$\begin{pmatrix} u_t \\ s_t \end{pmatrix} = \begin{pmatrix} b(L)\dfrac{\sigma_e^2}{\sigma_s^2} & -b(L)\dfrac{\sigma_a^2}{\sigma_s^2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ e_t \end{pmatrix}. \tag{12}$$

It is easily verified that (11) and (12) imply (9). Moreover, u_t and s_t are jointly white noise and orthogonal.[12] Finally, the determinant of the matrix in (11), i.e. $c(L)/b(L)$, vanishes only for $|L| \geq 1$ because of the definition of $b(L)$. It follows that u_t and s_t are orthogonal innovations for Ω_t, i.e. $\Omega_t = \text{span}(u_{t-k}, s_{t-k}, k \geq 0)$.

The shock u_t, let us call it the surprise shock, is the deviation of realised dividends from agents' expectation, i.e. agents' new information resulting from the observation of Δd_t. The contemporaneous value of Δd_t conveys information concerning the past dividend and noise shocks (there is no information about the present, since, by the definition of $b(L)$, the condition $c(0) = 0$ implies that $b(0) = 0$).

In the long run, the observation of economic fundamentals completely unveils whether past signals were true or not: as time passes, agents learn whether the shock was a dividend shock or a noise shock. To make this point clear, consider that the roots of the determinant of the matrix in (12), $b(L)$, are smaller than one in the modulus by the definition. Hence representation (12), though not invertible toward the past, can be inverted toward the future:

$$\begin{pmatrix} a_t \\ e_t \end{pmatrix} = \begin{pmatrix} b(F) & \dfrac{\sigma_a^2}{\sigma_s^2} \\ -b(F) & \dfrac{\sigma_e^2}{\sigma_s^2} \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}. \tag{13}$$

The above equation generalises the last equation of the previous subsection. It shows that the dividend shock and the noise shock are linear combinations of future and

[11] 'Fundamental' in the present context is a term of time series theory, which has nothing to do with the 'fundamental' value of a security or economic 'fundamentals'.

[12] Let us first observe that $u_t = b(L)(\sigma_e^2 a_t - \sigma_a^2 e_t)$ is white noise process. To see this, consider that $\sigma_e^2 a_t - \sigma_a^2 e_t$ is white noise (being the sum of two white noise processes, orthogonal at all leads and lags) and $b(L)$ is a so called 'Blaschke' factor, such that $b(L)b(L^{-1}) = 1$. Hence the covariance generating function is $\sigma_u^2 b(L)b(L^{-1}) = \sigma_u^2$, so that all lagged covariances are zero. Obviously, $s_t = a_t + e_t$ is white noise as well. In addition, u_t is orthogonal to s_t at all leads and lags, since $\sigma_e^2 a_t - \sigma_a^2 e_t$ is orthogonal to $a_t + e_t$.

present values of the surprise shock u_t and the signal shock s_t. This point is crucial for the identification of the econometric model, as shown in Section 2.

1.3. *The General Two-shock Model: Stock Prices*

Coming to stock prices, we abandon the present value model. We simply assume that:

(*i*) expectations about future dividends play a role in the determination of prices, so that investors react to new information, i.e. to news and surprise shocks, i.e.

$$\Delta p_t = m(L)u_t + d(L)s_t, \tag{14}$$

where $m(L)$ and $d(L)$ are rational functions. For identification purposes, we need prices to react on impact to the signal, i.e. $d(0) \neq 0$.
In addition, we assume that

(*ii*) the difference between log prices and log dividends, $p_t - d_t$, is stationary, as implied by standard models and in line with empirical evidence.[13]

Two observations are in order. First, we do not assume rational expectations, even if, of course, our assumptions are compatible with rational expectations. Second, since stock price differences are defined as an MA process, there is a unique equilibrium: this rules out standard rational bubbles.

Stationarity of $p_t - d_t$ entails restrictions on $m(L)$ and $d(L)$: namely, each one of the shocks must have the same long-run effect on both p_t and d_t. In particular, from (14) and the first line of (11), we get:

$$m(1) = c(1)/b(1), \tag{15}$$

$$d(1) = c(1)\sigma_a^2/\sigma_s^2. \tag{16}$$

Now let us derive the structural representation for prices. Using (14) and (12) we obtain:

$$\begin{aligned}
\Delta p_t &= \begin{pmatrix} m(L) & d(L) \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix} = \begin{pmatrix} m(L) & d(L) \end{pmatrix} \begin{pmatrix} b(L)\sigma_e^2/\sigma_s^2 & -b(L)\sigma_a^2/\sigma_s^2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ e_t \end{pmatrix} \\
&= [m(L)b(L)\sigma_e^2/\sigma_s^2 + d(L)]a_t + [d(L) - m(L)b(L)\sigma_a^2/\sigma_s^2]e_t \\
&= \alpha(L)a_t + \beta(L)e_t.
\end{aligned} \tag{17}$$

Using restrictions (15) and (16) it is seen that $\beta(1) = 0$, so that $\beta(L)$ can be factorised as $(1 - L)\tilde{\beta}(L)$ and we can write:

$$\Delta p_t = \alpha(L)a_t + (1 - L)\tilde{\beta}(L)e_t. \tag{18}$$

Equation (18) generalises (7). Prices depend on e_t, even if dividends do not. The effect of noise is transitory, as in (7), but now the bubble (the second term on the right-hand side) may last more than one period, depending on both the responses $d(L)$ and $m(L)$, as well as the 'learning' factor $b(L)$. The intuition for this result is simple. Noise shocks are perceived by agents as potentially anticipating future changes in dividends. Hence

[13] Cochrane (2008) shows that d_t and p_t are indeed cointegrated.

they affect prices in the short run by assumption (*i*). But in fact, dividends will not change, by the very definition of noise shocks. Since prices follow dividends in the long run because of assumption (*ii*), the effect of noise must be transitory.

1.4. *Additional Shocks*

The model specified in subsection 1.2 can be easily generalised to include additional shocks, say v_t, provided that such shocks are observed by economic agents. This generalisation is very important, in view of the vast literature documenting the dependence of stock prices on interest rates and risk premia (Cochrane, 1994). The equation for dividends becomes:

$$\Delta d_t = c(L)a_t + h(L)v_t, \tag{19}$$

where $h(L)$ is a row vector of rational functions in the lag operator L and v_t is an orthonormal white noise vector of structural shocks, orthogonal to a_t at all leads and lags. If agents can observe both Δd_t and v_t, they can observe $\Delta d_t^* = c(L)a_t$ either. The relation between agents' information set and the information set spanned by the structural shocks is still characterised by the equations in subsection 1.2, with Δd_t^* in place of Δd_t.

Prices will potentially react to shocks in v_t, according to:

$$\Delta p_t = m(L)u_t + d(L)s_t + n(L)v_t, \tag{20}$$

where the entries of $n(L)$ are again rational functions. Assuming stationarity of $p_t - d_t$, the structural equation for prices is now:

$$\begin{aligned} p_t &= f_t + b_t \\ \Delta f_t &= \alpha(L)a_t + n(L)v_t \\ \Delta b_t &= (1-L)\tilde{\beta}(L)e_t, \end{aligned} \tag{21}$$

where $\alpha(L)$ and $\tilde{\beta}(L)$ are as before. The above equation is the final product of our theoretical construction. Stock prices are decomposed into a 'fundamental' component f_t, which is related to the dividend shock and the structural shocks in v_t, and a bubble component $b_t = \tilde{\beta}(L)e_t$, which is stationary and orthogonal to economic fundamentals a_t and v_t (and therefore f_t), at all leads and lags.[14]

2. Econometrics

In the present Section, we analyse the fundamentalness problem, which is the basic econometric problem related to the estimation of the structural shocks a_t and e_t. To begin, we focus on the bivariate representation of Δd_t and Δp_t, assuming $h(L) = n(L) = 0$. The multivariate generalisation including the additional shocks in the

[14] An interesting special case is when dividends are not affected by v_t in the long run, i.e. $h(1) = 0$ (or are not affected by v_t at all, $h(L) = 0$). In this case, it is easily seen, in virtue of cointegration, that $n(1) = 0$, i.e. the effect of v_t on stock prices is temporary. The fundamental component f_t is then decomposed into a long-run component, related to dividend variations and a short run component, related to v_t, which can include interest rates shocks and shocks to risk premia, as in the present value model with variable discount factors (Campbell and Shiller, 1988).

vector v_t is relatively simple and is considered in subsection 3.4. Our final goal is to estimate decomposition (21), along with the related shocks and impulse response functions.

2.1. *Non-invertibility in Models With Noisy Shocks*

From (11) and (17), it is seen that the structural representation of Δd_t and Δp_t can be written as:

$$\begin{pmatrix} \Delta d_t \\ \Delta p_t \end{pmatrix} = \mathbf{C}(L)\begin{pmatrix} a_t^* \\ e_t^* \end{pmatrix} = \begin{pmatrix} c(L)\sigma_a & 0 \\ \alpha(L)\sigma_a & \beta(L)\sigma_e \end{pmatrix}\begin{pmatrix} a_t/\sigma_a \\ e_t/\sigma_e \end{pmatrix}, \tag{22}$$

where $\beta(1) = 0$ and the shocks are normalised to have unit variance, as usual in structural VAR analysis. Just like in representation (9), however, the determinant of the MA matrix vanishes for $L = 0$, since $c(0) = 0$. It follows that the representation is non-fundamental.

The problem of non-invertibility, or 'non-fundamentalness' is a debated issue in the structural VAR literature. Early references are Hansen and Sargent (1991) and Lippi and Reichlin (1993, 1994); more recent contributions include Giannone and Reichlin (2006), Fernandez-Villaverde *et al.* (2007), Chari *et al.* (2008), Mertens and Ravn (2010), Forni and Gambetti (2014), Forni *et al.* (2014). In essence, the problem is that standard SVAR methods assume that the structural shocks are linear combinations of the residuals obtained by estimating a VAR. If the structural MA representation of the variables included in the VAR is non-fundamental, the structural shocks are not linear combinations of such residuals, so that the method fails.[15]

In most of the economic literature, the structural shocks are elements of agents' information set and non-fundamentalness may arise if the econometrician uses less information than the agents. In this case, non-fundamentalness can in principle be solved by enlarging the information set used by the econometrician (Forni *et al.*, 2009, 2014; Forni and Gambetti, 2014). In the present setting, non-fundamentalness stems from agents' ignorance and cannot be solved by adding variables to the VAR (Blanchard *et al.*, 2013). The economic intuition is that agents' behaviour cannot reveal information that agents do not have. Stock prices or other variables which are the outcome of agents' decisions do not add anything to the information already contained in d_t and s_t. More generally, in models in which agents cannot see the structural shocks, the structural representation is non-fundamental for whatever set of observable variables. For, if it were, agents could infer the shocks from the variables themselves, contrary to the assumption (unless we assume that there are variables that are observable for the econometrician but not for the agents).

[15] An MA representation is fundamental if and only if its associated matrix is non-singular for all L with modulus less than one (Rozanov, 1967, ch. 2). This condition is slightly different from invertibility, since invertibility requires non-singularity also when L is unit modulus. Hence non-fundamentalness implies non-invertibility, whereas the converse is not true. When the variables are cointegrated, for instance, the MA representation of the first differences is not invertible, but, nonetheless, can be fundamental. In such a case, non-invertibility can be easily circumvented by resorting to structural ECM or level VAR estimation. Non-fundamentalness is a kind of non-invertibility which cannot be solved in this way.

In our theoretical framework, if identification is generalised to include dynamic unitary transformations (i.e. Blaschke matrices), structural VAR estimation may still be successful. Dynamic unitary transformations are rotations which may involve, besides current values, past and future values of the VAR residuals. In fact, we have already seen in (13), that the dividend and noise shocks can be written as linear combinations of the current signal shock and future values of the surprise shock, which in principle can be found with a standard VAR procedure.

A general treatment of dynamic identification in structural VARs can be found in Lippi and Reichlin (1994). When considering the more general class of dynamic rotations, identification is more demanding than in the standard, contemporaneous rotation setting, because it requires stronger theoretical restrictions (Mertens and Ravn, 2010). A contribution of the present article (and the companion paper Forni *et al.*, 2013*b*) is to show that in models with noisy signals the restrictions arising naturally from the theory are sufficient to identify the structural shocks. Below we explain in detail how to find the structural and the noise shock, as well as the corresponding impulse response functions.[16]

2.2. *Dynamic Identification of the Bivariate VAR*

In this subsection, we present our identification and estimation strategy for the bivariate case; the target is then estimation of representation (22).

From (11) and (14), we get the innovation representation for the normalised shocks:

$$\begin{pmatrix} \Delta d_t \\ \Delta p_t \end{pmatrix} = \mathbf{A}(L)\begin{pmatrix} u_t^* \\ s_t^* \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix}\begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix}, \tag{23}$$

where[17]

$$\mathbf{A}(L) = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix} = \begin{pmatrix} \dfrac{c(L)\sigma_u}{b(L)} & \dfrac{\sigma_a^2 c(L)}{\sigma_s} \\ m(L)\sigma_u & d(L)\sigma_s \end{pmatrix}. \tag{24}$$

Moreover, from (12), we get the mapping between the normalised innovations and the normalised structural shocks:

$$\begin{pmatrix} u_t^* \\ s_t^* \end{pmatrix} = \mathbf{B}(L)\begin{pmatrix} a_t^* \\ e_t^* \end{pmatrix} = \begin{pmatrix} b(L)\dfrac{\sigma_e}{\sigma_s} & -b(L)\dfrac{\sigma_a}{\sigma_s} \\ \dfrac{\sigma_a}{\sigma_s} & \dfrac{\sigma_e}{\sigma_s} \end{pmatrix}\begin{pmatrix} a_t/\sigma_a \\ e_t/\sigma_e \end{pmatrix}. \tag{25}$$

Comparing (22), (23) and (25) it is seen that the matrix of the structural impulse response functions in (22) can be factored as $\mathbf{C}(L) = \mathbf{A}(L)\mathbf{B}(L)$, i.e.

[16] Barsky and Sims (2012) and Blanchard *et al.* (2013) present news-noise models where agents never learn the true nature of past shocks. The basic difference with respect to our model in this respect is that in both papers there are three structural shocks, whereas agents see just two dynamically independent sources of information. Since the dynamic dimension of the structural shocks is larger than the dynamic dimension of agents' information space, there is no way for the agents to see such shocks, even when assuming known the future values of the observable series. For the same reason, the econometrician cannot recover the shocks and the impulse response functions by means of a structural VAR, even by resorting to dynamic transformations of the VAR residuals.

[17] Recall that $\sigma_u = (\sigma_e\sigma_a)/(\sigma_s)$.

$$\begin{pmatrix} c(L)\sigma_a & 0 \\ \alpha(L)\sigma_a & \beta(L)\sigma_e \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix} \begin{pmatrix} b(L)\dfrac{\sigma_e}{\sigma_s} & -b(L)\dfrac{\sigma_a}{\sigma_s} \\ \dfrac{\sigma_a}{\sigma_s} & \dfrac{\sigma_e}{\sigma_s} \end{pmatrix}. \tag{26}$$

Our basic idea is to estimate $\mathbf{C}(L)$ by estimating $\mathbf{A}(L)$ and $\mathbf{B}(L)$. Let us discuss these two steps in turn.

Step 1. An estimate of $\mathbf{A}(L)$, $\hat{\mathbf{A}}(L)$, along with $\hat{u}_t^*$ and $\hat{s}_t^*$, is obtained by estimating a standard structural (unrestricted) VAR. Identification is obtained by imposing $\hat{a}_{12}(0) = 0$, which corresponds to the condition $c(0) = 0$, which is derived by the theory. The theory imposes further restrictions on the entries of the MA matrix appearing in (26). We do not use such restrictions for estimation, since we want to use them for testing purposes (see below).

Step 2. Let us come now to estimation of $\mathbf{B}(L)$. First, we need an estimate of $b(L)$, which is given by the roots of $c(L)$ that are smaller than 1 in modulus (see (10)). Such roots are revealed by our estimate $\hat{a}_{12}(L)$, which, being proportional to $c(L)$ (see (24)), has the same roots (of course, one out of these roots will be zero because of the identification constraint $\hat{a}_{12}(0) = 0$). This is the crucial step of our procedure. The proportionality of the reaction of dividends to the dividend shock, on the one hand, and the signal shock, on the other hand, is due to the assumption that noise shocks do not affect dividends at any lag – an assumption which is essential, from a theoretical point of view, to distinguish the dividend shock from the noise shock.[18]

Next, we need an estimate of σ_a/σ_s and σ_e/σ_s. Since $b(1) = 1$, an estimate of σ_a/σ_e can be obtained as:

$$\widehat{\sigma_a/\sigma_e} = \frac{\hat{a}_{12}(1)}{a_{11}\hat{(1)}}.$$

Considering that $\sigma_a^2/\sigma_s^2 + \sigma_e^2/\sigma_s^2 = 1$, it is seen that σ_a/σ_s and σ_e/σ_s are the sine and the cosine, respectively, of the angle whose tangent is σ_a/σ_e. Hence $\widehat{\sigma_a/\sigma_s}$ and $\widehat{\sigma_e/\sigma_s}$ can be obtained as $\sin(\arctan(\widehat{\sigma_a/\sigma_e}))$ and $\cos(\arctan(\widehat{\sigma_a/\sigma_e}))$, respectively.[19] From $\widehat{\sigma_a/\sigma_s}$, $\widehat{\sigma_e/\sigma_s}$ and $\hat{b}(L)$ we get an estimate of $B(L)$ and can estimate $\mathbf{C}(L)$ as $\hat{\mathbf{C}}(L) = \hat{\mathbf{A}}(L)\hat{\mathbf{B}}(L)$. This concludes our procedure for the estimation of the impulse response functions.

Finally, we want an estimate for the normalised structural shocks, a_t^* and e_t^*, and the bubble component b_t appearing in decomposition (21). By inverting (25) we get:

$$\begin{pmatrix} a_t^* \\ e_t^* \end{pmatrix} = \mathbf{B}(L)^{-1} \begin{pmatrix} u_t^* \\ s_t^* \end{pmatrix} = \frac{1}{\sigma_s} \begin{pmatrix} b(F)\sigma_e & \sigma_a \\ -b(F)\sigma_a & \sigma_e \end{pmatrix} \begin{pmatrix} u_t^* \\ s_t^* \end{pmatrix}. \tag{27}$$

[18] In the context of a general equilibrium model the noise shock may affect the real economy and therefore may indirectly affect dividends, invalidating our assumption. We assume here that the feedback is negligible. This can be true even if noise has sizable effects on profits, provided that firms smooth dividends over time, as argued in Cochrane (1994).

[19] From basic trigonometry, recall that $\sin^2(x) + \cos^2(x) = 1$, $\tan(x) = \sin(x)/\cos(x)$ and $\arctan(\tan(x)) = x$.

From the first step above we have $\hat{u}_t^*$ and $\hat{s}_t^*$. From the second step we have $\hat{\mathbf{B}}(L)^{-1}$. The desired estimates are obtained as $(\hat{a}_t^*\ \hat{e}_t^*)' = \hat{B}(L)^{-1}(\hat{u}_t^*\ \hat{s}_t^*)'$.

As for the bubble component of stock prices, we need an estimate of $\tilde{\beta}(L)$ (see (21)), which is given by the cumulated sum of $\hat{c}_{22}(L)$ (see (22)), let us write $\hat{c}_{22}(L)(1-L)^{-1}$. We estimate b_t as $\hat{b}_t = \hat{c}_{22}(L)(1-L)^{-1}\hat{e}_t^*$. The fundamental component can then be estimated as $\hat{f}_t = p_t - \hat{b}_t$.

Relation (27) involves future values of u_t and s_t, so that the structural shocks and the noise bubble cannot be estimated consistently at the end of the sample. This is perfectly in line with the assumption that neither the agents, nor the econometrician can see the current values of the structural shocks. However, in the middle of the sample future is known and relation (27) can provide reliable estimates.

Summing up, our estimation strategy is the following.

Step 1. Estimate an (unrestricted) VAR for d_t and p_t[20] and identify by imposing $\hat{a}_{12}(0) = 0$. In such a way we get $\hat{\mathbf{A}}(L)$, $\hat{u}_t^*$ and $\hat{s}_t^*$ ((23)).
Step 2. Get the estimate $\hat{b}(L)$ by computing the roots of $\hat{a}_{12}(L)$, selecting those which are smaller than one in modulus and using (10).
Step 3. Estimate σ_a/σ_e as the ratio $\hat{a}_{12}(1)/\hat{a}_{11}(1)$.[21] Then get $\widehat{\sigma_a/\sigma_s}$ and $\widehat{\sigma_e/\sigma_s}$ as $\sin(\arctan(\widehat{\sigma_a/\sigma_e}))$ and $\cos(\arctan(\widehat{\sigma_a/\sigma_e}))$, respectively. Steps 2 and 3 provide $\hat{B}(L)$ ((25)).
Step 4. Estimate the structural impulse response functions as $\hat{\mathbf{C}}(L) = \hat{\mathbf{A}}(L)\hat{\mathbf{B}}(L)$ ((26)).
Step 5. Estimate the structural shocks as $(\hat{a}_t^*\ \hat{e}_t^*)' = \hat{\mathbf{B}}(L)^{-1}(\hat{u}_t^*\ \hat{s}_t^*)'$ ((27)).
Step 6. Estimate the noise bubble as $\hat{b}_t = \hat{c}_{22}(L)(1-L)^{-1}\hat{e}_t^*$ ((22)) and the fundamental component of stock prices as $\hat{f}_t = p_t - \hat{b}_t$ ((21)).

2.3. *Testing and Additional Estimation Issues*

As already noticed, the restrictions appearing in representation (23) which are not used for identification can be used for testing. In particular, we do not impose the cointegration restriction so that we can test the theoretical implication that e_t has temporary effects on prices. Moreover, $\hat{a}_{11}(L)\hat{b}(L)\widehat{\sigma_a/\sigma_s}$ should be equal to $\hat{a}_{12}(L)\widehat{\sigma_e/\sigma_s}$.[22] Such a condition implies that in the structural representation (22) the upper-right response function is zero, which can be tested by verifying whether the confidence bands include the x axis for all lags.

Let us now go back to the first step of our estimation procedure, i.e. estimation of (23). As assumed above, d_t and p_t are cointegrated. Hence we estimate a VAR in the levels of the variables, see Section 3, rather than the first differences.

Moreover, a bivariate VAR does not necessarily include enough information to estimate representation (23). A simple check for informational sufficiency, which can be used in this case, has been proposed by Forni and Gambetti (2014). The test consists in verifying whether the estimated shocks are orthogonal to past values of the

[20] We estimate the VAR in levels for reasons which will be clarified below.
[21] In practice we compute the cumulated long-run effects as the effects at forty quarters.
[22] Our identification conditions imply that such relation is satisfied both on impact and in the long run. At intermediate lags, the relation can be violated.

principal components of a large data set of macroeconomic series. In the empirical application below we replace the principal components with a set of selected control variables. If orthogonality is rejected, the shocks cannot be innovations with respect to available information, and the VAR should be amended by adding variables reflecting agents' information.

2.4. *Higher-dimensional Specifications*

The multivariate generalisation of the bivariate model above is straightforward. Let the vector v_t have dimension $(n-2)$ and let Δy_t be an $(n-2)$-dimensional vector of additional variables driven by v_t and, possibly, by s_t and u_t. The innovation representation for the vector $(\Delta y_t\ \Delta d_t\ \Delta p_t)'$ is:

$$\begin{pmatrix} \Delta y_t \\ \Delta d_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} \mathbf{N}(L) & f(L) & g(L) \\ h(L) & a_{11}(L) & a_{12}(L) \\ n(L) & a_{21}(L) & a_{22}(L) \end{pmatrix} \begin{pmatrix} v_t \\ u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix}, \tag{28}$$

where $a_{ij}(L)$, $i = 1, 2$, $j = 1, 2$, is as in (29) and $f(L)$, $g(L)$ and the entries of the $(n-2) \times (n-2)$ matrix $\mathbf{N}(L)$ are rational functions in L.

Within the multivariate framework, the condition that the dividend shock does not affect d_t on impact is no longer sufficient, by itself, to identify the model. In the empirical analysis we impose a Cholesky triangularisation with y_t ordered first, d_t ordered second, and p_t ordered third, i.e. $f(0) = g(0) = a_{12}(0) = 0$ and $\mathbf{N}(0)$ lower triangular. The reason for this ordering is that we want to allow for a contemporary effect of v_t on dividends and stock prices; in particular, we want to allow for contemporary effects of interest rates and risk premia on prices.

The corresponding structural representation is obtained by post-multiplying the above matrix by the multivariate extension of the matrix that maps innovations in structural shocks in (25), i.e.

$$\begin{pmatrix} \mathbf{I}_{n-2} & 0 & 0 \\ 0' & b(L)\sigma_e/\sigma_s & -b(L)\sigma_a/\sigma_s \\ 0' & \sigma_a/\sigma_s & \sigma_e/\sigma_s \end{pmatrix}, \tag{29}$$

where 0 denotes the $(n-2)$-dimensional null column vector.

To conclude this Section, let us summarise our econometric assumptions. The main ingredients are:

(*i*) a_t is a news shock;
(*ii*) the noise shock does not affect dividends at any lag;
(*iii*) the signal shock is the sum of dividend and noise shocks. Moreover, we impose that:
(*iv*) the additional shocks v_t affecting dividends are observed; and
(*v*) the signal and surprise shocks do not affect on impact the variables in y_t.

The equalities $a_{12}(L) = \sigma_a^2 c(L)/\sigma_s$ (24) and $\hat{a}_{12}(1)/\hat{a}_{11}(1) = \sigma_a/\sigma_e$, which are sufficient to identify the Blaschke matrix $\mathbf{B}(L)$, stem from (*i*) to (*iv*). Assumptions (*i*)–(*iii*) also imply that the signal shock does not affect dividends on impact; this, along with (*v*), is sufficient to identify the matrix $\mathbf{A}(L)$.

These five assumptions are all what is needed (despite non-fundamentalness) for the econometric procedure. Of course, as in Section 2, we do not assume any economic model for stock prices. Moreover, unlike Section 2, we do not impose here that prices react to the signal and that p_t and d_t are cointegrated. Finally, we do not assume that the effects of noise on prices is temporary. The effects of all shocks, including the noise shock, on prices is left unrestricted at all horizons. The role of economic theory in the econometric procedure is therefore limited to (*i*)–(*iii*), i.e. the very definition of the noise shock and the characterisation of agents' information set.

3. Empirics

In this Section, we present our empirical analysis. Our benchmark specification is a four-variable VAR with dividends, stock prices and two interest rates. We find that, in line with the theory, noise shocks do not affect dividends and have transitory effects on stock prices. Despite this, noise explains a large fraction of stock market fluctuations at short and medium run horizons and is responsible for large deviations of stock prices from the intrinsic value of equities. Then we estimate of the noise bubble component of stock prices. Noise explains most of the information technology bubble, as well as other boom-bust episodes, including a sizable fraction of the stock market crash of 2008–9.

3.1. *The Data*

We use US quarterly series covering the period 1960:Q1–2010:Q4. The stock price series is the monthly average of the Standard & Poor's Index of 500 Common Stocks reported by Datastream (code US500STK). We converted the series in quarterly figures by taking simple averages and dividing the resulting series by the GDP implicit price deflator in order to express it in real terms.[23] We do not divide prices by population since we are interested in the historical decomposition of the stock price series and the dating of the bubbles (see subsection 3.4). Using per capita figures to this end would be somewhat unnatural.[24] Dividends are NIPA Net Corporate Dividends, divided by the GDP implicit price deflator and population aged 16 years or more (the BLS Civilian Non-institutional Population, converted to quarterly frequency by taking monthly averages). Both dividends and stock prices are taken in log-levels rather than differences to avoid estimation problems related to cointegration. The interest rates included in our baseline specification are the 3-Month Treasury Bill, Secondary Market Rate and the Moody's Seasoned AAA Corporate Bond Yield. We take the monthly averages of business days (original source: Board of Governors of the Federal Reserve System) and converted the monthly series to quarterly figures by taking simple averages. Interest rates are taken in levels.

To test for sufficient information, we use an additional interest rate, the 10-Year Treasury Constant Maturity Rate, the inflation rate and two leading indexes. The

[23] We use average values because end-of-period values are likely affected by very short-run fluctuations which are of no interest for our purposes.

[24] We verified the robustness of our approach to the use of per capita values: results, not shown here, are very much similar.

interest rate is treated as the interest rates described above. The inflation rate is the NIPA GDP Implicit Price Deflator, taken in first differences of the logs. The leading indexes are the Conference Board Leading Economic Indicators Index (Datastram code USCYLEAD) and the Michigan University Survey of Consumers Expected Index.

Stock prices and the Conference Board leading index are taken from Datastream, the consumer confidence index is taken from the website of the Michigan University, whereas all other series are downloaded from the FRED data base.

3.2. *The Effects of Dividend and Noise Shocks*

Our VAR specification includes four variables:

(*i*) dividends;
(*ii*) the S&P500;
(*iii*) the 3-Month Treasury Bill, Secondary Market Rate; and
(*iv*) the Moody's Seasoned Aaa Corporate Bond Yield.

We use 4 lags according to the AIC criterion and identify by imposing a Cholesky scheme with the interest rates ordered first as explained in subsection 2.4. There are two main reasons behind the choice of the specification. First, there is a large literature arguing that stock prices are affected by transitory variations arising from changes of interest rates and risk premia (Cochrane, 1994). By including two interest rates, one risky and one essentially risk free, we aim at capturing such sources of variation. Were the VAR specification to be deficient in this respect, such variations could be erroneously ascribed to noise and produce an overstatement of the noise bubble. Second, the shocks estimated in a bivariate VAR with stock prices and dividends are predicted by the control variables described above (particularly the interest rates), meaning that it is not informationally sufficient (see Table 1).[25] By contrast, the baseline specification including interest rates is not rejected by the sufficient information test (see Table 2).[26]

Figure 3 shows the impulse response functions of dividends and stock prices to signal and surprise shocks. The dark grey and the light grey areas show the 68% and the 90% confidence bands, respectively, obtained by performing 2,000 bootstrap replications using the Kilian (1998) method. A positive signal, anticipating future dividend growth, has large and significant contemporaneous effects on stock prices. On the other hand, the stock market reacts more cautiously and gradually to a positive surprise shock, which has large contemporaneous and permanent effects on dividends.

[25] We estimated the two-variable VAR with dividends and stock prices (4 lags) and identified the signal, surprise, dividend and noise shocks. Then we tested for informational sufficiency as explained in subsection 2.3, by regressing the estimated shocks onto 2 and 4 lags of the 3-Month Treasury Bill, the Aaa Corporate Bond Yield, and the four control variables described in the previous subsection, one at a time. Dividend and noise shocks were truncated at time $T-4$ since the filter obtained by inverting (A.1) involves the leads of the signal and the surprise shocks, producing an end-of-sample bias. The p-values of the F-statistic of these regressions are reported in Table 1. The null hypothesis that the signal is orthogonal to the past of the regressors is rejected at the 5% level for all interest rates and the inflation rate. A similar result holds for the noise shock.

[26] We repeated the orthogonality test procedure for the four-variable VAR and tested for orthogonality of the dividend and noise shocks with respect to the four remaining control variables. As shown in Table 2, orthogonality cannot be rejected, even at the 10% level, for all regressions.

Table 1

Results of the Fundamentalness Test in the Bivariate VAR

		Regressors					
Shock	Lags	(1)	(2)	(3)	(4)	(5)	(6)
Surprise	2	0.66	0.82	0.98	0.92	0.66	0.93
	4	0.07	0.09	0.42	0.64	0.41	0.88
Signal	2	0.03	0.00	0.00	0.02	0.81	0.05
	4	0.10	0.01	0.01	0.05	0.91	0.05
Dividend	2	0.21	0.39	0.73	0.86	0.51	0.21
	4	0.39	0.48	0.78	0.97	0.62	0.41
Noise	2	0.01	0.00	0.00	0.02	0.63	0.03
	4	0.03	0.00	0.01	0.05	0.86	0.05

Notes. The Table reports the p-values of the F-test in the regressions of the estimated shocks on 2 and 4 lags of the regressors (1)–(6). Dividend and noise shocks are truncated at time $T-4$ since end-of-sample estimates are inaccurate. Regressors: (1) 3-Month Treasury Bill: Secondary Market Rate; (2) 10-Year Treasury Constant Maturity Rate; (3) Moody's Seasoned Aaa Corporate Bond Yield; (4) GDP Implicit Price Deflator; (5) The Conference Board Leading Economic Indicators Index; (6) Michigan University Consumer Confidence Expected Index.

Table 2

Results of the Fundamentalness Test in the 4-variable VAR

		Regressors					
Shock	Lags	(1)	(2)	(3)	(4)	(5)	(6)
Surprise	2	1.00	0.91	1.00	0.79	0.82	0.60
	4	1.00	0.95	1.00	0.84	0.98	0.70
Signal	2	1.00	0.98	1.00	0.43	0.95	0.53
	4	1.00	0.97	1.00	0.59	0.98	0.27
Dividend	2	0.91	0.90	0.98	0.80	0.89	0.22
	4	0.96	0.72	0.97	0.91	0.98	0.51
Noise	2	0.95	0.96	0.93	0.31	0.73	0.20
	4	0.99	0.99	0.99	0.58	0.93	0.14

Notes. The Table reports the p-values of the F-test in the regressions of the estimated shocks on 2 and 4 lags of the regressors (1)–(6). Dividend and noise shocks are truncated at time $T-4$ since end-of-sample estimates are inaccurate. Regressors: (1) 3-Month Treasury Bill: Secondary Market Rate; (2) 10-Year Treasury Constant Maturity Rate; (3) Moody's Seasoned Aaa Corporate Bond Yield; (4) GDP Implicit Price Deflator; (5) The Conference Board Leading Economic Indicators Index; (6) Michigan University Consumer Confidence Expected Index.

Let us now consider the structural representation. We begin the analysis by examining the series of the estimated shocks. First of all notice that the point estimate of σ_a/σ_s is 0.44 (standard error 0.3), which entails a large noise, i.e. $\sigma_e/\sigma_s = 0.90$ (standard error 0.15). Figure 4 plots the two shocks. The vertical lines report events, most of them exogenous, coinciding with peaks and troughs in the estimated series of the signal. All of the events coincide with peaks or troughs of the noise shock. For instance the largest negative noise shock is observed in 1987:Q4 and corresponds to Black Monday (October 1987). Other negative shocks are registered in coincidence with the collapse of the Franklin National Bank, the First Gulf War and the bankruptcy of the Lehman Brothers. Positive noise shocks are found in coincidence of the Bush re-election and the 2009 fiscal stimulus. Coincidently with some of these events, the dividend shock has the same sign as the noise shock although is somehow

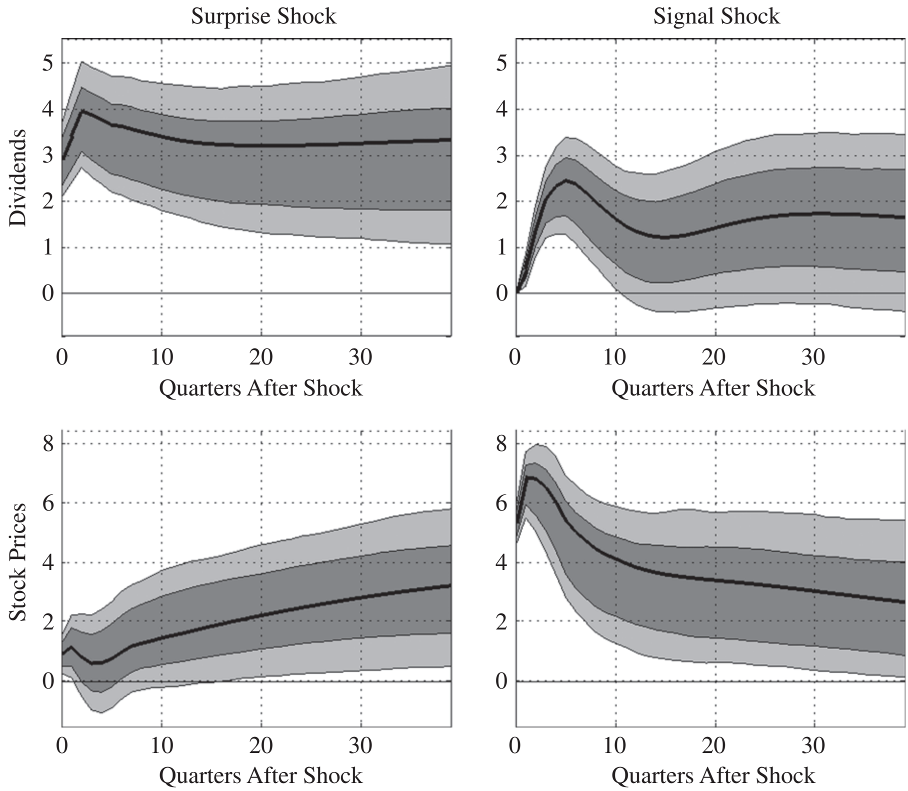

Fig. 3. *Impulse Response Functions of Dividends and Stock Prices to Surprise and Signal Shocks in the 4-variable VAR*

Notes. Solid line: point estimates. Dark grey area: 68% confidence bands. Light grey area: 90% confidence bands.

smaller in terms of magnitudes. A notable difference between the dividend shock and the noise shock is observed in 2004:Q4, in coincidence with the Bush re-election, where the two shocks have opposite sign. According to our estimates, the Bush re-election is an episode with large negative effects on economic fundamentals, accompanied by a large positive noise shock responsible for the under-reaction of the stock market.

Figure 5 shows the impulse response functions of dividend and stock prices to dividend and noise shocks. Positive dividend shocks are followed by an increase in dividends, which reach their new long-run level after three quarters. Stock prices react immediately by a similar percentage amount and then remain approximately stable at the new level. In line with the theory, the effect of the noise shock on dividends is small and not significant at all horizons, even considering the tighter confidence region. By contrast, the effect of noise on the stock market, large and strongly significant on impact, declines sharply after a few quarters and approaches zero in the long run, confirming the temporary effect predicted by the model.

It can be shown that the reaction of prices to both dividends and noise shocks reported in the Figure is too large with respect to what is predicted by the present value

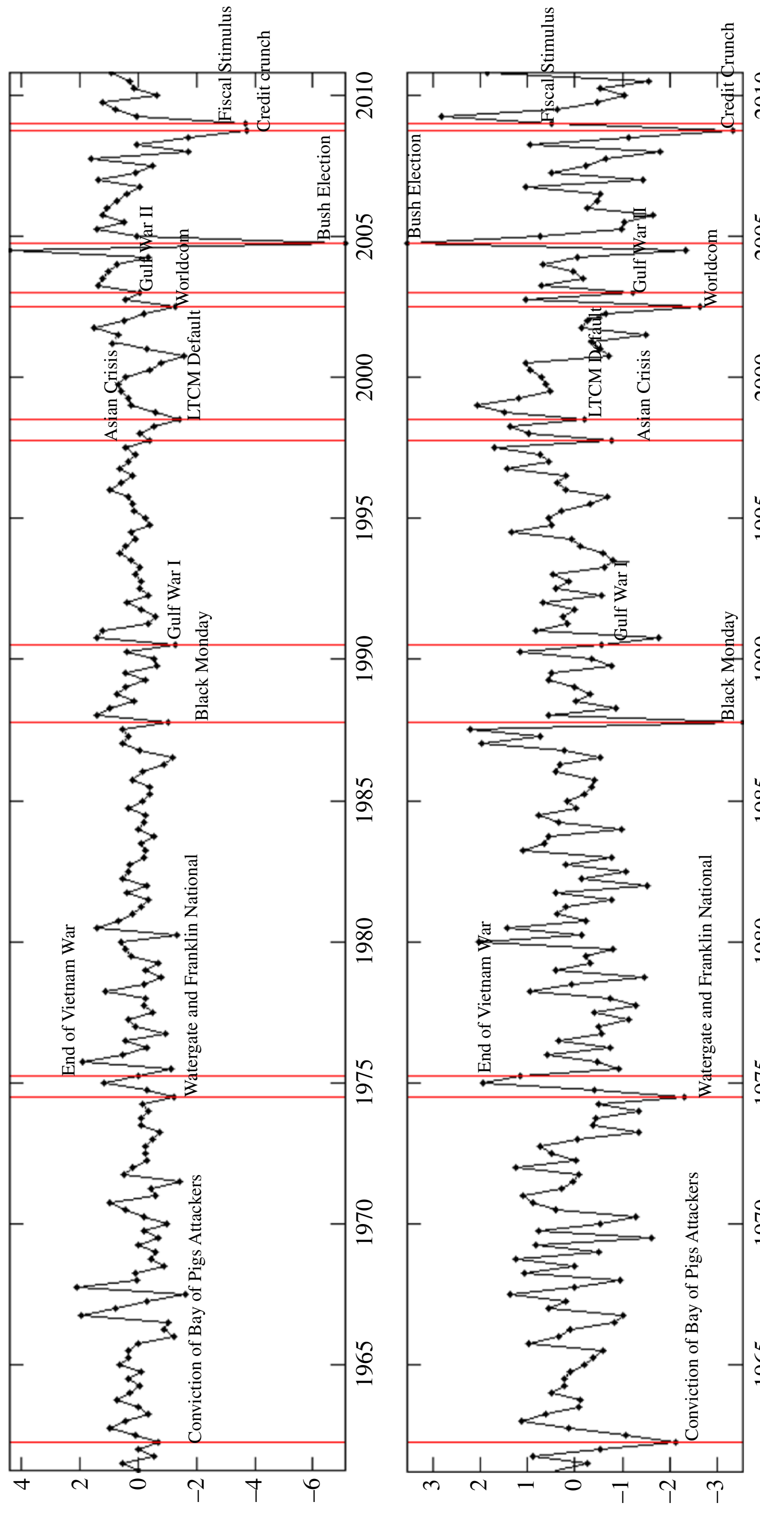

Fig. 4. *The Estimated Dividend Shock (Upper Panel) and Noise Shock (Lower Panel) with Vertical Lines Corresponding to Economic and Political Events*

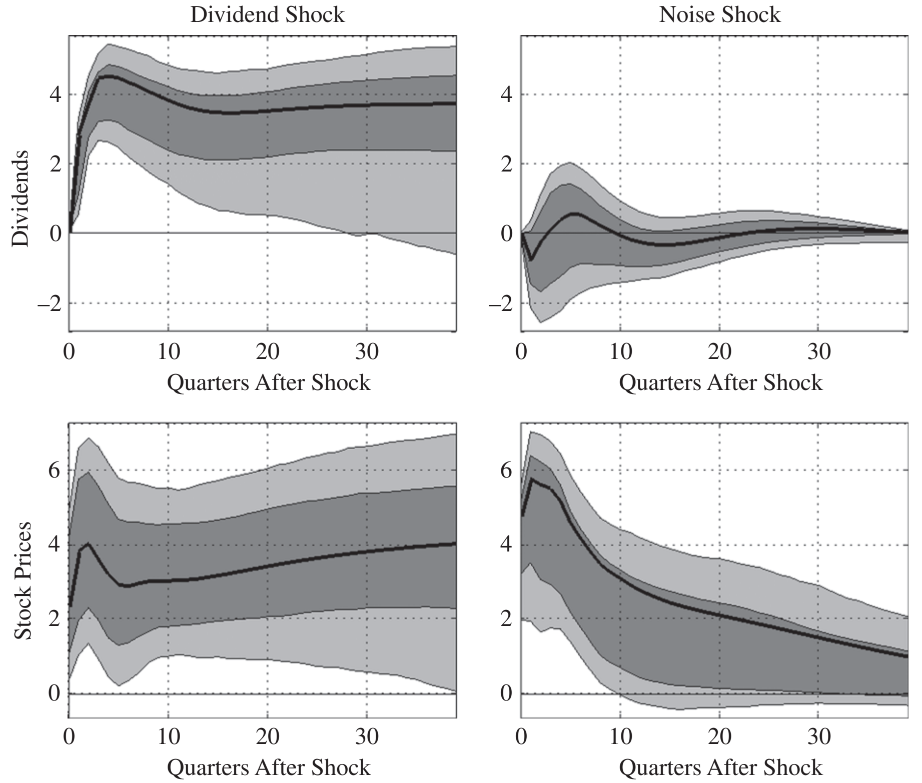

Fig. 5. *Impulse Response Functions of Dividends and Stock Prices to Dividend (First Column) and Noise Shocks (Second Column) in the 4-variable VAR*

Notes. Solid line: point estimates. Dark grey area: 68% confidence bands. Light grey area: 90% confidence bands.

theory.[27] A similar result holds for the reactions to signal shocks. Agents react too much to news (no matter if news reflects true dividend shocks or noise) with respect to what is justified by rational forecasts of future dividends (even when taking into account price variations related to interest rates and risk premia). This finding is in line with Shiller (1981) and West (1988). It gives increased importance to the fact that our decomposition is more general than the present value set-up and is valid even in presence of irrational traders.[28]

Table 3 reports the estimated decomposition of the forecast error variance at different horizons. The signal explains about 20% of dividend variation at medium and long-run horizons (two years or more), while the bulk of dividend volatility is captured by the surprise shock. As for stock prices, the role of the signal and the surprise shocks

[27] In a previous version of the present article, we derive the theoretical reaction of stock prices to noise shock within a present value model, when dividends are driven by the general (8), i.e. $\Delta d_t = c(L)a_t$ Forni *et al.* (2013*a*) (17). Such a reaction is much smaller than the one shown in Figure 5.

[28] We rule out standard rational bubbles in our theoretical model. If, contrary to our assumptions, such bubbles were there in the data, they would be included into the noise component of stock prices by our empirical procedure. Our historical decomposition and our dating of the bubbles reported below would still be valid but, of course, our 'noise' interpretation of the bubbles would be partially incorrect.

Table 3

Variance Decomposition in the 4-variable VAR

Variable	Horizon: Impact	1-year	2-year	4-years	10-years
			Surprise		
3-M T. Bill Rate	0.0	0.3	0.5	0.6	1.2
	(0.0)	(1.5)	(2.7)	(3.6)	(5.1)
Aaa C. Bond Yield	0.0	0.2	0.1	0.5	0.8
	(0.0)	(1.3)	(2.0)	(3.7)	(5.9)
Dividends	99.7	81.7	75.2	73.8	74.0
	(10.7)	(9.0)	(11.4)	(12.5)	(15.9)
Stock Prices	2.5	1.3	1.9	4.6	18.4
	(2.1)	(2.7)	(4.7)	(8.9)	(14.6)
			Signal		
3-M T. Bill Rate	0.0	5.4	7.3	8.0	17.0
	(0.0)	(4.2)	(6.1)	(6.4)	(8.8)
Aaa C. Bond Yield	0.0	3.0	3.8	2.7	11.1
	(0.0)	(3.4)	(4.9)	(5.6)	(9.2)
Dividends	0.0	14.7	20.7	17.3	17.8
	(0.0)	(7.0)	(10.1)	(11.6)	(15.4)
Stock Prices	87.5	72.1	67.7	63.1	56.8
	(5.0)	(9.5)	(11.5)	(13.7)	(15.8)
			Dividend shock		
3-M T. Bill Rate	0.0	1.4	2.8	2.6	5.9
	(0.0)	(2.8)	(4.6)	(4.8)	(8.1)
Aaa C. Bond Yield	0.0	0.3	0.4	0.7	1.7
	(0.0)	(1.7)	(2.7)	(3.6)	(7.5)
Dividends	0.0	94.2	94.4	90.0	91.3
	(0.0)	(11.5)	(10.8)	(13.4)	(13.7)
Stock Prices	17.3	21.4	21.5	26.1	45.0
	(21.3)	(18.8)	(18.1)	(18.3)	(19.7)
			Noise		
3-M T. Bill Rate	0.0	4.2	4.9	6.0	12.2
	(0.0)	(3.6)	(4.8)	(6.2)	(8.0)
Aaa C. Bond Yield	0.0	2.8	3.6	2.5	10.2
	(0.0)	(2.9)	(4.3)	(5.3)	(8.8)
Dividends	0.0	1.3	1.1	0.8	0.4
	(0.0)	(9.6)	(7.7)	(6.3)	(4.3)
Stock Prices	72.4	52.0	47.9	41.4	30.0
	(21.5)	(19.1)	(18.1)	(16.7)	(14.1)
			Interest rate shocks (sum of the point estimates)		
3-M T. Bill Rate	100	94.3	92.2	91.4	81.8
Aaa C. Bond Yield	100	96.8	95.1	96.8	88.1
Dividends	0.3	3.6	4.1	8.9	8.2
Stock Prices	10.0	26.6	30.4	32.3	24.8

Notes. The entries are the percentages of forecast error variance explained by the shocks at the specified horizons. Standard errors in parenthesis.

are inverted: the signal explains the bulk of stock price volatility, whereas the surprise shock has a sizable effect only in the long run (about 20%). The surprise and the signal shocks explain together about 90% of stock price variation on impact and about 70% at longer horizons, the remaining 30% being explained by interest rates shocks. The dividend shock explains about 20% of stock price variation on impact and almost one half at the ten year horizon. Noise is very important, in that it explains the bulk of stock price variance on impact (about 70%) and in the short-medium run (about 50% and 40% at the 2-year and the 4-year horizons, respectively).

Figure 6 shows the impulse response functions of the two interest rates to dividend and noise shocks. Both shocks induce a monetary policy tightening; the T-Bill increases significantly for a few quarters according to the narrower bands. Interestingly enough, after about two years the response of the T-Bill rate to the noise shock becomes negative and significant at the 68% confidence level. Given that the noise shock has negligible real effects, the dividends are largely unaffected, the result seems to support the idea that monetary policy, to some extent, responds to fluctuations in stock prices. Nonetheless, the response turns out to be relatively small. In fact, the T-Bill increases up to 0.2% in front of an increase of about 6% of stock prices. Moreover only a small fraction of the interest rate, about 5%, is explained by noise.

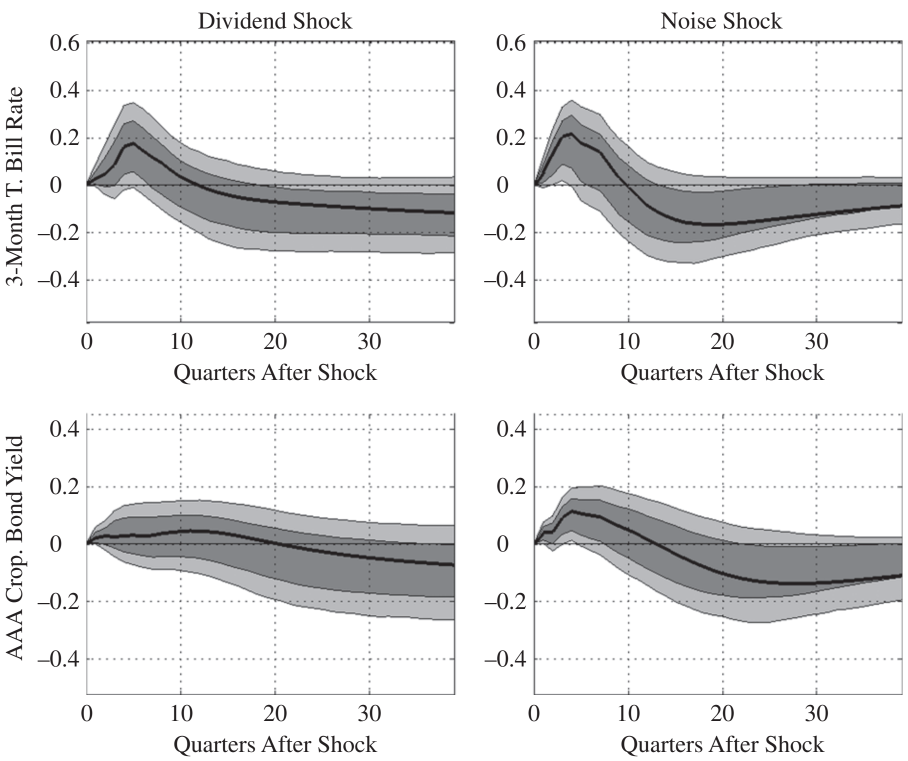

Fig. 6. *Impulse Response Functions of the 3-month T-bill Rate and Aaa Bond Yields to Dividend (First Column) and Noise Shocks (Second Column) in the 4-variable VAR*

Notes. Solid line: point estimates. Dark grey area: 68% confidence bands. Light grey area: 90% confidence bands.

3.3. *The Effects of Interest Rate Shocks*

Cochrane (1994) documents that there is a large transitory component in stock prices, which does not affect dividends. He argues that such a component can be due to a 'discount rate' or 'expected return' shock. As explained above, we included two interest rates in our VAR to capture price variations caused by variations of the interest rates and the risk premia.[29]

Figure 7 plots the response of stock prices and dividends to the two interest rate shocks (the two shocks in the vector v_t appearing in (28)). Both shocks are positive shocks, i.e. they increase the corresponding interest rate on impact (upper panel). Both shocks have significant negative transitory effects on prices and small, insignificant effects on dividends, in line with the time-varying discount rate present value

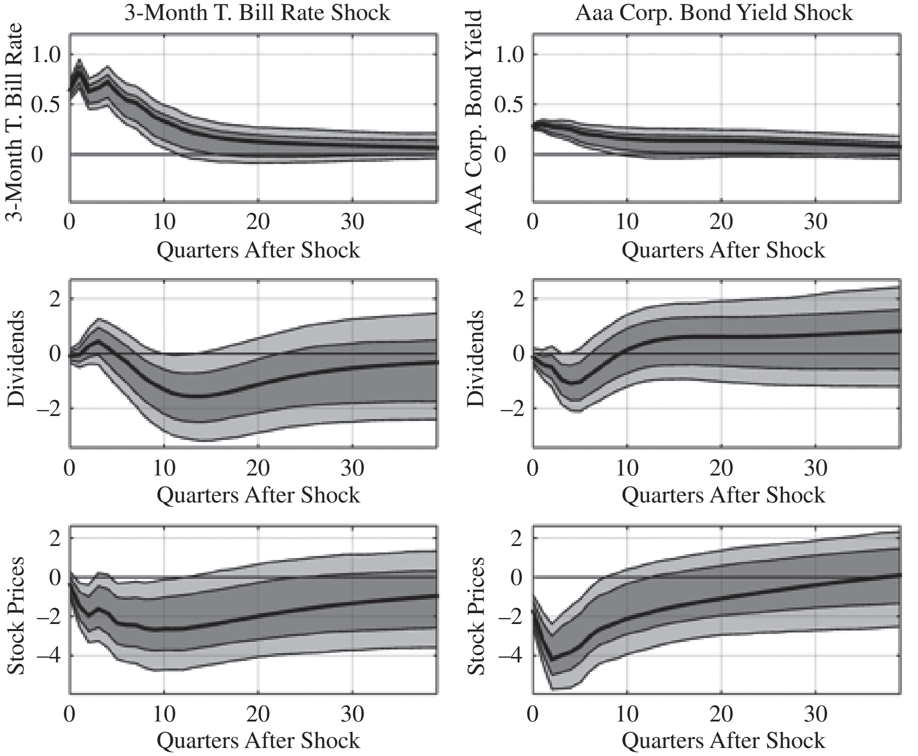

Fig. 7. *Impulse Response Functions to a Positive 3-month T-bill Rate Shock (First Column) and a Positive Aaa Bond Yields Shock (Second Column) in the 4-variable VAR*
Notes. Solid line: point estimates. Dark grey area: 68% confidence bands. Light grey area: 90% confidence bands.

[29] We are aware that these two interest rates (and the implied spread) could be insufficient to capture all of the stock price volatility related to risk premia. Since this problem is not the main focus of our analysis we do not examine it in depth here.

model and Cochrane's arguments. They jointly explain about one third of price variations at the 2-year horizon, as against about 5% of dividend variations (see Table 3). Their effects on dividend and prices are very much similar to the effects of noise, the basic difference being that the noise shock have almost no effects on interest rates, whereas the the interest rate shocks explain more than 90% of interest rates variations. Therefore our strategy seems very effective in discriminating between the transitory component of prices driven by 'discount rate' shocks and the transitory component induced by noise.

Figure 8 plots the historical decomposition of stock prices. The solid line is the interest rate component, obtained by filtering the two estimated interest rate shocks with the respective impulse response functions and summing the resulting series; the dashed line is the stock prices series and the dotted line is the stock prices minus the interest component. There are two periods where the interest rate component seems to be particularly important: in the second half of the 1960s, where prices would have been substantially lower without the interest rate shocks; and during the first half of the 1980s, when prices would have been significantly higher. The second episode is particularly interesting since it coincides with the Volcker disinflation and suggests an important role for monetary policy in shaping asset prices fluctuations. Interestingly, part of the bull market of the late 1990s is explained by interest rates shocks. On the other hand, interest rate shocks play no role in the years immediately before the Great Recession.

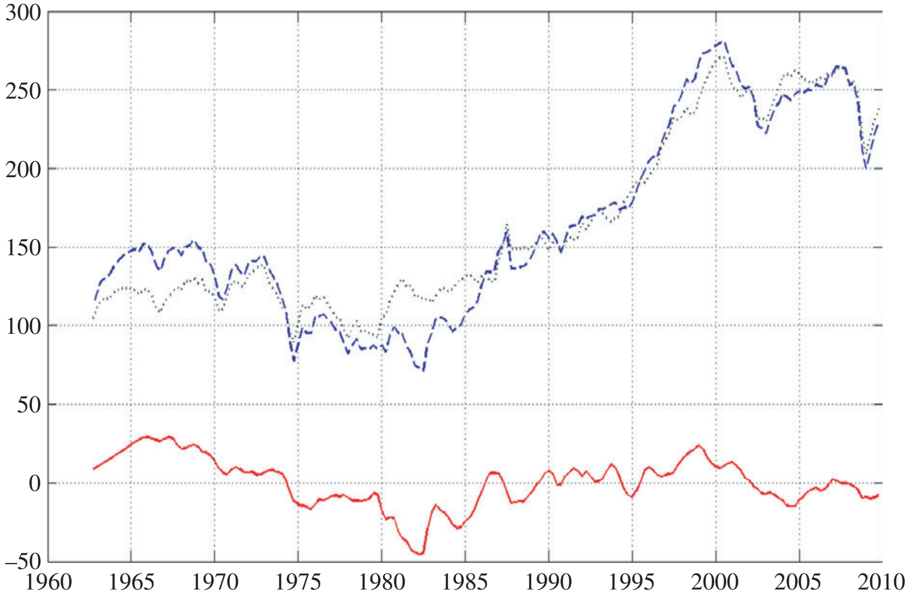

Fig. 8. *Historical Decomposition in the 4 Variables VAR*
Notes. Dashed line: log of the real S&P 500 stock price index. Solid line: two interest rate components of the stock price index. Dotted line: difference between the stock price index and the two interest rate components. The decomposition is truncated at time $T-4$ since end-of-sample estimates are inaccurate.

3.4. *Measuring Historical Boom-bust Episodes*

We estimated the bubble and the fundamental component of stock prices as explained in point 6 of our estimation procedure (subsection 2.2). Since stock prices are expressed in logs, the bubble component must be interpreted as a percentage deviation. Let lower-case letters represent logs and upper-case letters represent non transformed values, so that $p_t = \log P_t$ and $f_t = \log F_t$. From decomposition (21) we get:

$$b_t = \log(P_t/F_t).$$

Hence b_t measures the percentage deviation of current prices from their fundamental value.

Figure 9 shows the bubble (solid line). For the sake of comparison, the Figure also reports the stock price series (dashed line) and the fundamental component (dotted line). The estimates show several episodes of prolonged and sustained deviations from the fundamental price. Unlike most of the bubble literature and newspaper articles, which focus on positive deviations from fundamental values, we have both positive and

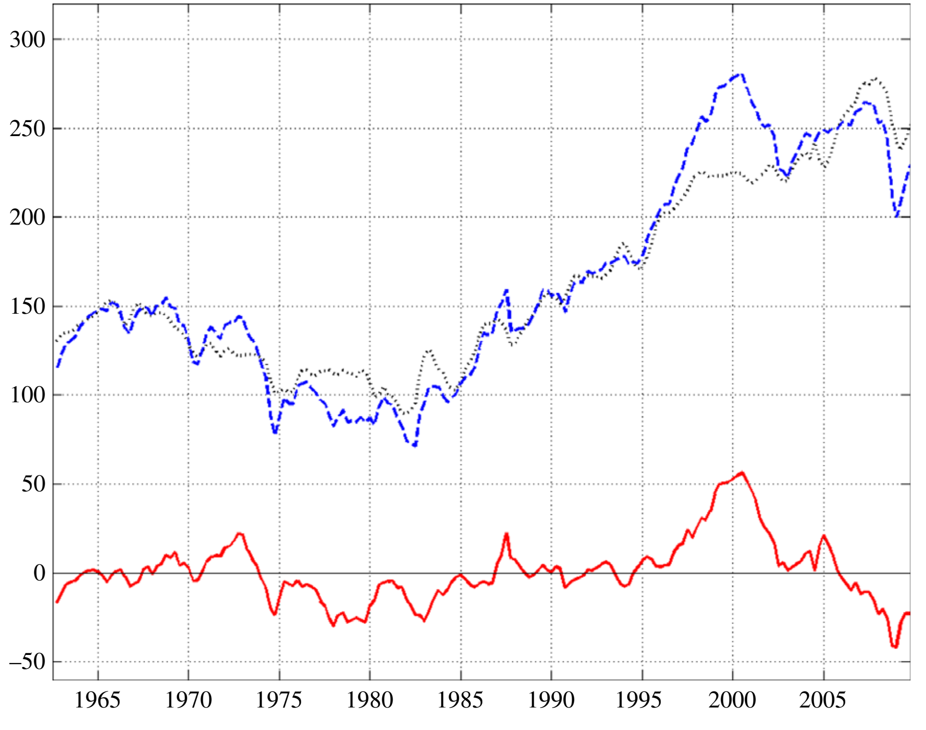

Fig. 9. *Historical Decomposition in the 4 Variables VAR*
Notes. Dashed line: log of the real S&P 500 stock price index. Solid line: noise component of the stock price index. Dotted line: difference between the stock price index and the noise component. The decomposition is truncated at time $T - 4$ since end-of-sample estimates are inaccurate.

negative deviations. Here we limit our attention to those episodes in which deviations are 20% or higher.[30] We find eight such episodes: four positive and four negative bubbles. The positive bubbles episodes are:

(*i*) First half of the 1970s (span: 1972:Q3–1973:Q1, max: 22.1% in 1972:Q3)
(*ii*) Second half of the 1980s (span: 1987:Q3; max: 22.5% in 1987:Q3)
(*iii*) Dot-com (span: 1997:Q3–2002:Q1; max: 56.4% in 2000:Q2)
(*iv*) Mid 2000 (span: 2005:Q1; max: 21.1% in 2005:Q1)

The negative bubbles are

(*v*) 1974 stock market crash (span: 1974:Q4; min: −23.7% in 1974:Q4)
(*vi*) Second half of the 1970s (span: 1977:Q4–1979.Q4; min: −29.8% in 1978:Q1)
(*vii*) First half of the 1980s (span: 1982:Q3–1983.Q2; min: −27.1% in 1983:Q1)
(*viii*) Great Recession (span: 2008:Q1–2009:Q4; min: −41.7% in 2009:Q1)

We have a positive bubble in the first half of the 1970s and one in the third quarter of 1987, just before the Black Monday crash. The two episodes are considered as bull markets by many commentators. The results support the idea that the episodes were driven by factors completely disconnected from economic fundamentals. Commentators at that time were conscious that to some extent the price was over-evaluated, see for instance *Wall Street Journal* (1987). It is interesting to notice that, excluding the 1987 episode, there is a long span of time free of noise-driven stock prices movements and the period largely coincides with the Great Moderation. Note also that the bull market of the second half of the 1960s is not a bubble, being driven mainly by interest rates shocks, as observed above.

The dot-com bubble represents, by far, the episode with the largest and longest-lasting deviations. Between 1997 and 2002 prices have been over-valued on average by 40% with a peak of 56% in 2000:Q2. From the figures it emerges clearly that the bulk of fluctuations in prices around these years is attributable to news having no effect on future fundamentals, which were largely interpreted as genuine good news.

Notice that while our estimates point to a relatively large noise component in 2005, they show that the peak in 2007:Q2 was not a bubble. On the contrary, stock prices were under-valued by about 10% compared to their fundamental value in that quarter.

The first negative bubble is the peak of the 1973–4 bear market. The crash follows the fall of the Bretton Woods system and the US dollar devaluation occurred between 1971 and 1973. The second and the third episodes are known bear market periods.[31] The noise component accounts to a large extent for the stock market crash of 2008–9, which is the largest negative bubble episode, troughing at −41.7% in 2009:Q1. The fall in stock prices due to the fundamental component during the Great Recession has been substantially exacerbated by a wave of over-pessimism about future figures of the economy.

[30] Adalid and Detken (2007) identifies boom-bust episodes for a number of industrial countries. For the US they find two episodes, the 1986–7 and the dot-com bubble, which appear also in our list.
[31] See e.g. the list of bear market periods reported in the website of Yardeni Research.

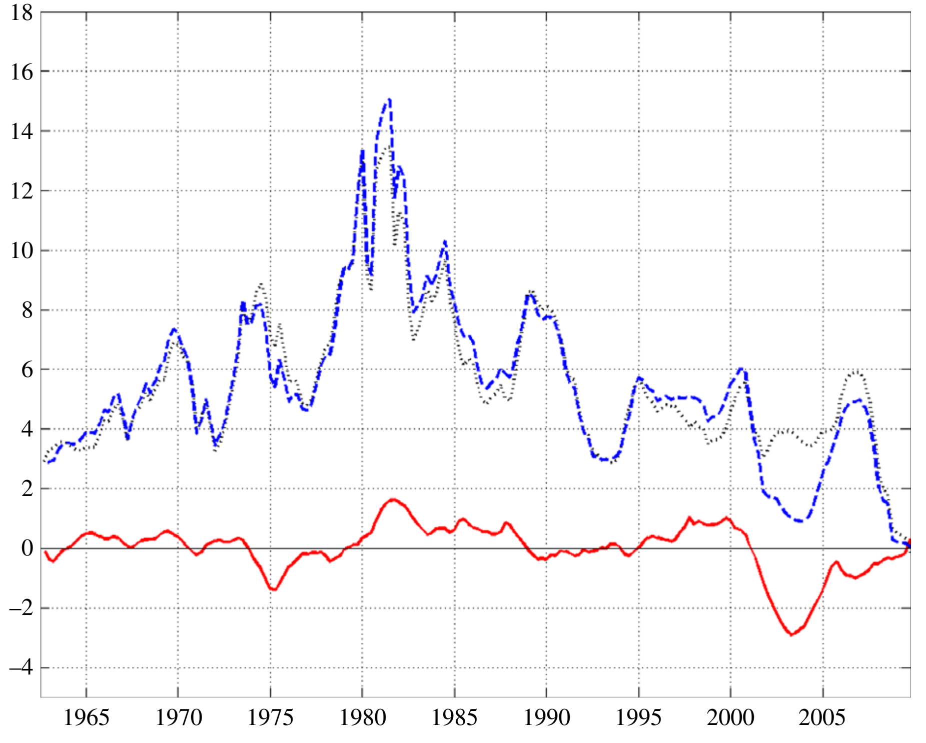

Fig. 10. *Historical Decomposition in the 4 Variables VAR*
Notes. Dashed line: 3-Month T-Bill rate. Solid line: noise component of the 3-Month T-Bill rate. Dotted line: difference between the 3-Month T-Bill rate and the noise component. The decomposition is truncated at time $T - 4$ since end-of-sample estimates are inaccurate.

We conclude this Section with an historical digression on the conduct of monetary policy in response to noise shocks. Figure 10 plots the 3M T-Bill rate (dashed line), the noise component (solid line) and the difference between the variable and the noise component (dotted line). Figure 11 plots the noise components of the 3M T-Bill rate and the stock prices together. As observed above, noise shocks have little effects on interest rates. However, fluctuations in the interest rate driven by noise have become larger since the late 1990s. More specifically, at the onset of the dot-com bubble, monetary policy responded to the increase in prices by increasing the interest rate by about 1%. However, around 1997, while prices kept rapidly growing, the interest rate stalled. The stock prices bust was followed by a huge drop in the interest rate, by around 3%. Actually during the first half of the 2000, without the bubble, the interest rate would have been much higher, around 4%, than the observed value of 1.5%. Given that the real effects of the noise shock are relatively limited, the huge fall of the interest rate supports the idea that monetary policy has reacted quite strongly to the burst of the bubble and that the low levels of the interest rate observed until 2005 were driven by factors disconnected from economic fundamentals.

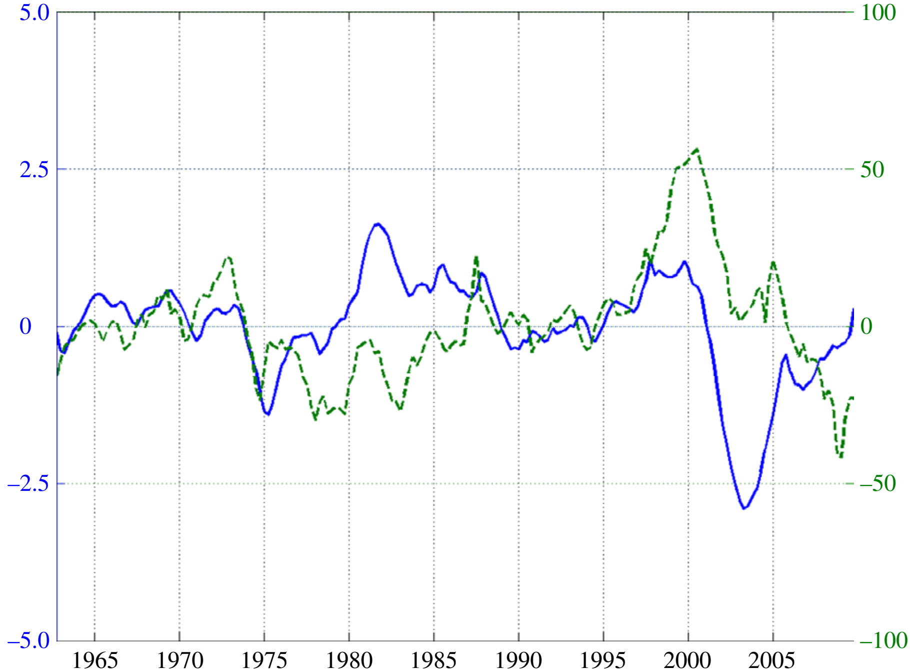

Fig. 11. *Historical Decomposition in the 4 Variables VAR*
Notes. Solid line: noise component of the 3-Month T-Bill rate (left axis). Dashed line: noise component of the log of the real S&P 500 stock price index (right axis). The decomposition is truncated at time $T-4$ since end-of-sample estimates are inaccurate.

4. Conclusions

In this article, we have study an environment in which agents, who are not necessarily rational, receive noisy signals about future economic fundamentals. We show that the resulting stock price equilibrium includes a transitory component which can be responsible for boom and bust episodes unrelated to fluctuations of economic fundamentals – the 'noise bubbles'. This result is obtained under very mild theoretical conditions about stock prices: we simply assume that traders react immediately to news about future dividends and that dividends and stock prices are cointegrated.

We show that, in our theoretical framework, the structural shocks – 'dividend' and 'noise' shocks – can be estimated by using a non-standard structural VAR procedure, where identification is obtained by imposing a 'dynamic' rotation of the VAR residuals, involving their future values.

In the empirical Section, we apply our procedure to US data. We find that, consistent with the theory, the noise shock has transitory effects on stock prices, whereas the dividend shock has permanent effects. Moreover, noise is very important, in that it explains the bulk of stock price fluctuations at short and medium-run horizons. Finally, the historical decomposition shows that the component of stock prices driven by the noise shock is responsible for the information technology bubble; the boom peaking in

2007 was entirely driven by genuine news, whereas the following stock market crisis is largely accountable to a negative noise bubble.

Appendix A. The Log-linear Present Value Model

To obtain formula (1), let us start from the accounting identity:

$$P_t = \frac{1}{1 + r_{t+1}}(P_{t+1} + D_{t+1}),$$

where P_t is the price of equities, D_t is dividends and r_t is the rate of return on equities. Setting $r_t = r$ and taking logs we get:

$$p_t = -\log(1+r) + \log\left(e^{p_{t+1}} + e^{d_{t+1}}\right) = -\log(1+r) + p_{t+1} + \log\left(1 + e^{d_{t+1} - p_{t+1}}\right), \tag{A.1}$$

where $p_t = \log P_t$ and $d_t = \log D_t$. Now let us set $w_t = d_t - p_t$ and linearise $\log(1 + e^{w_{t+1}})$ with respect to w_{t+1} around $\mu = \mathrm{E}w_t$. We obtain:

$$\log(1 + e^{w_{t+1}}) \approx \log(1 + e^{\mu}) + \frac{e^{\mu}}{1 + e^{\mu}}(w_{t+1} - \mu) = -\log\rho - \mu(1-\rho) + (1-\rho)(d_{t+1} - p_{t+1}),$$

where $\rho = (1 + e^{\mu})^{-1}$. Replacing in (B.5) we get the approximate accounting identity:

$$p_t = k + \rho p_{t+1} + (1-\rho)d_{t+1}, \tag{A.2}$$

where $k = -\log(1+r) - \log\rho - \mu(1-\rho) = -\log(1+r) - \log\rho + (1-\rho)\log(1/\rho - 1)$. Equation (1) is obtained by solving forward, taking expectations at time t on both sides and imposing a transversality condition.

Appendix B. MonteCarlo Simulations

In this Appendix, we run a Monte Carlo experiment to show that:

(*i*) our identification method is able to recover impulse responses to noise and news shocks;
(*ii*) when an unobserved contemporaneous shock v_t affects dividends contemporaneously, the estimated impulse responses do not change significantly when σ_v^2 is small; and
(*iii*) when the contemporaneous shock v_t is observed, the multivariate VAR model discussed in subsection 3.4 recovers the impulse responses.

B.1. *No Contemporaneous Shock to Dividends*

In this part, we use the simple present value model discussed in subsection 2.1:

$$d_t = d_{t-1} + a_{t-1}, \tag{B.1}$$

$$p_t = d_t + \frac{\sigma_a^2}{\sigma_s^2}s_t, \tag{B.2}$$

$$s_t = a_t + e_t. \tag{B.3}$$

We assume $\sigma_a^2 = 0.2, \sigma_e^2 = 0.8$, in line with the empirical evidence in Section 4. We generate 1000 time series of dimension $T = 500$, and we apply our identification strategy on a VAR for $[d_t\ p_t]'$ with 6 lags. Figure B1 shows that the identification strategy is able to recover the impulse responses to news and noise shocks.

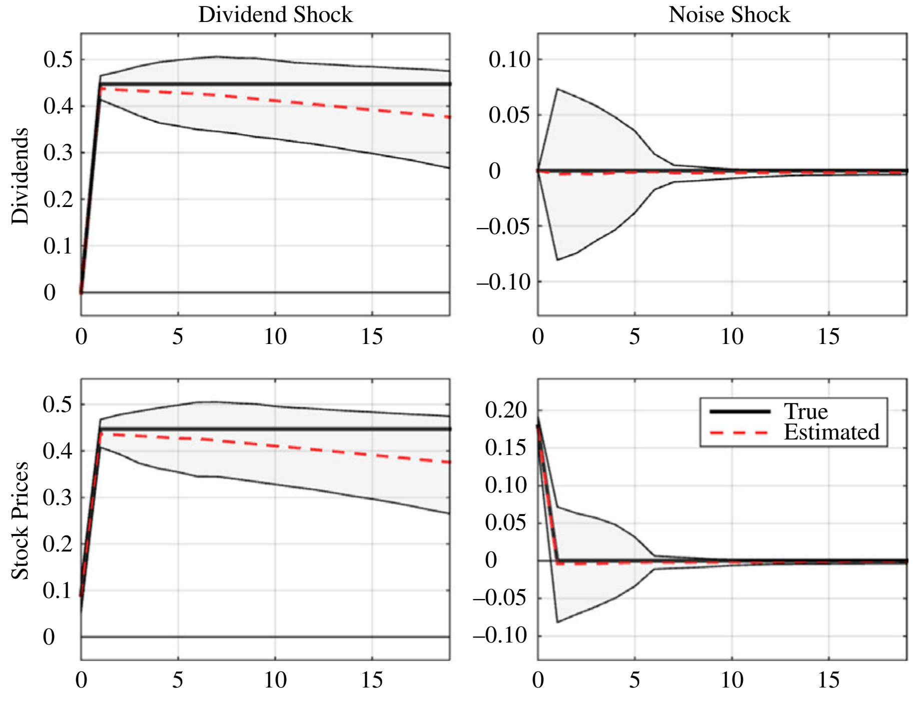

Fig. B1. *Impulse Responses from Model Equations (B.1)–(B.3)*
Notes. Line: true impulse response. Dashed: mean impulse response from a VAR for $[d_t\ p_t]'$ with 6 lags. Grey areas denote a 90% confidence interval.

B.2. *Contemporaneous Shock to Dividends Unobserved by Agents*

In this Section, we assume that a contemporaneous shock v_t affects dividends at time t. We also assume that the shock v_t is not observed by the agents. The data generating process is therefore:

$$d_t = d_{t-1} + a_{t-1} + v_t, \tag{B.4}$$

$$p_t = d_t + \frac{\sigma_a^2}{\sigma_s^2} s_t, \tag{B.5}$$

$$s_t = a_t + e_t. \tag{B.6}$$

We assume $\sigma_a^2 = 0.2, \sigma_e^2 = 0.8$ and $\sigma_v^2 = \alpha\sigma_a^2$, with $\alpha = 0$, 0.25, 0.5, 0.75, 1, 1.25. For each value of α, we generate 1,000 time series of dimension $T = 500$ and apply our identification strategy on a VAR for $[d_t\ p_t]'$ with 6 lags.

Figure B2 shows the true impulse responses, the mean impulse responses obtained for each value of σ_v^2 and the confidence bands for $\sigma_v^2 = 0$ derived in the previous Section. As σ_v^2 is relatively small, impulse responses are very similar to those obtained when $\sigma_v^2 = 0$.

Figure B3 shows the true impulse responses, the mean impulse responses obtained for each value of σ_v^2 and the confidence bands for $\sigma_v^2 = 1.25\sigma_a^2$. In this case, the noise shock has significant effects on dividends, violating the assumptions of the model.

In sum, when an unobserved contemporaneous shock is added to dividends we can observe two things. If the shock v_t is small, impulse responses change mildly and it is virtually impossible

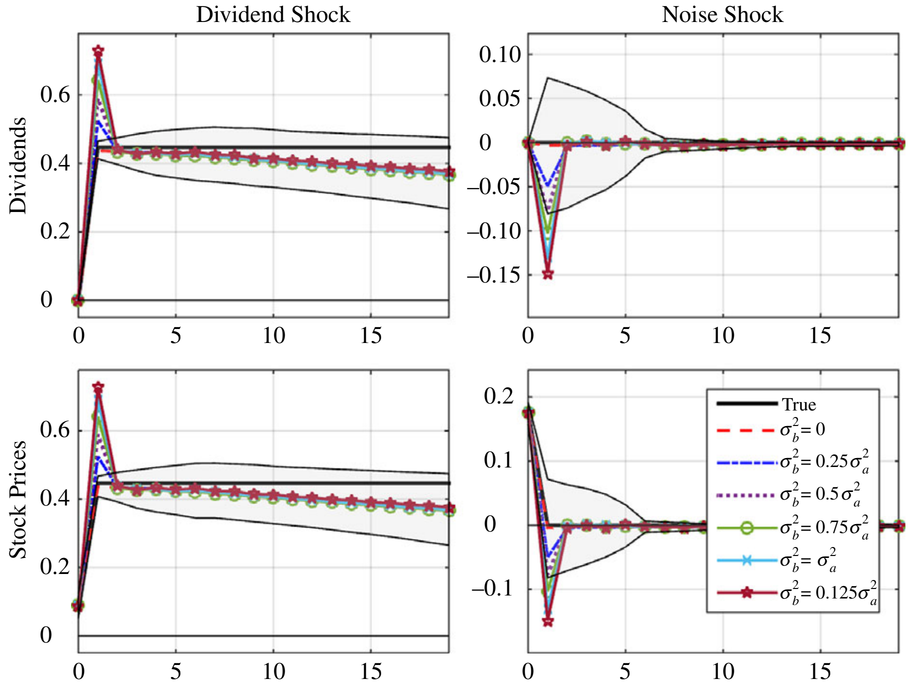

Fig. B2. *Impulse Responses from Model Equations (B.4)–(B.6)*

Notes. Line: true impulse response for dividend shock a_t (first column) and noise shock e_t (second column). Dashed lines and lines with markers are obtained by using our multivariate identification scheme on estimated VARs for $[d_t \; p_t]'$ with 6 lags for different values of σ_b^2 (see legend). Grey areas denote a 90% confidence interval in the case $\sigma_b^2 = 0$ (confidence intervals are the same as in Figure B1).

to identify the effect of the v_t shock. If the shock v_t is big, impulse responses change significantly and do not satisfy the model's restriction that noise shock do not affect dividends.

B.3. *Contemporaneous Shock to Dividends Observed by Agents*

In the third experiment, we assume that the contemporaneous v_t shock is observed by agents. We assume that there is an additional variable y_t, satisfying the contemporaneous restrictions imposed by the Choleski decomposition in (26):

$$y_t = y_{t-1} + v_t + a_{t-1} + e_{t-1}, \tag{B.7}$$

$$d_t = d_{t-1} + a_{t-1} + v_t, \tag{B.8}$$

$$p_t = d_t + \frac{\sigma_a^2}{\sigma_s^2} s_t, \tag{B.9}$$

$$s_t = a_t + e_t. \tag{B.10}$$

We assume $\sigma_b^2 = 0.25\sigma_a^2$ and generate 1,000 time series of dimension $T = 500$, and apply the multivariate identification strategy explained in Section 3 on a VAR for $[y_t \; d_t \; p_t]'$ with 6 lags. Figure B4 show that it is possible to uncover structural impulse responses using a multivariate model.

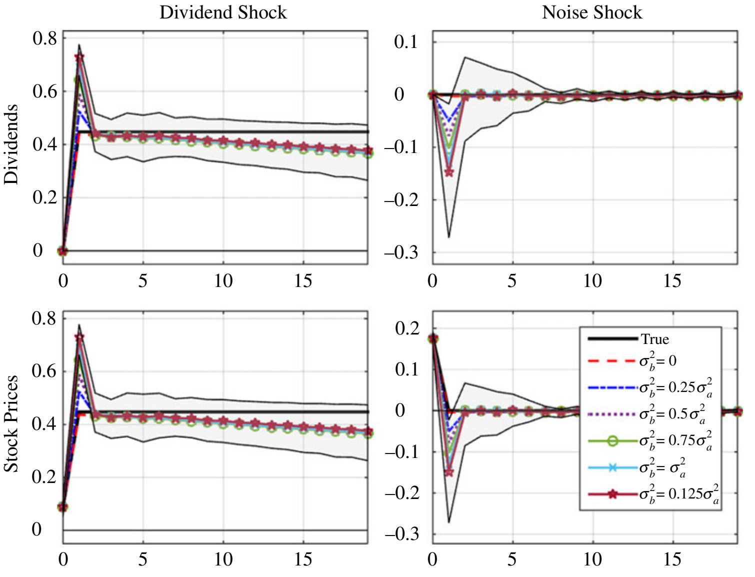

Fig. B3. *Impulse Responses from Model Equations (B.4)–(B.6)*

Notes. Line: true impulse response for dividend shock a_t (first column) and noise shock e_t (second column). Dashed lines and lines with markers are obtained by using our multivariate identification scheme on estimated VARs for $[d_t\ p_t]'$ with 6 lags for different values of σ_b^2 (see legend). Grey areas denote a 90% confidence interval for $\sigma_b^2 = 1.25\sigma_a^2$.

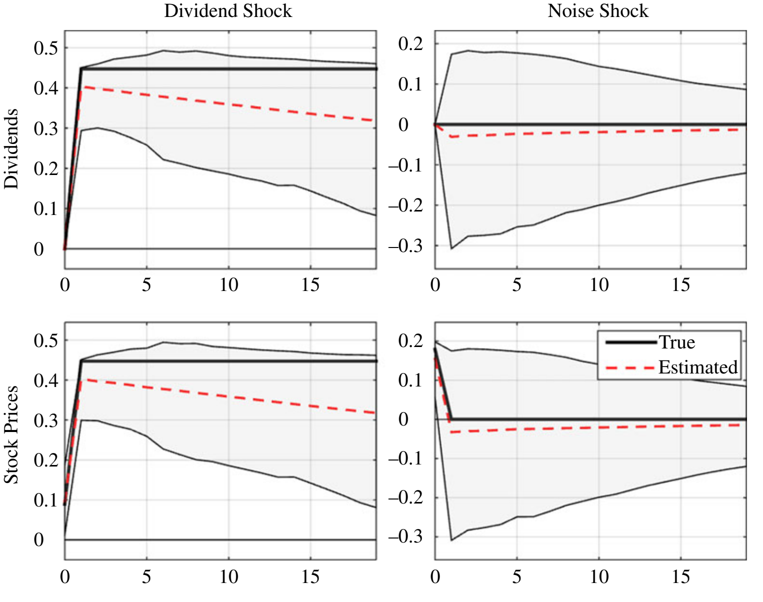

Fig. B4. *Impulse Responses from Model Equations (B.7)–(B.9)*

Notes. Line: true impulse response for dividend shock a_t (first column) and noise shock e_t (second column). Dashed lines are mean impulse responses obtained by using our multivariate identification scheme on estimated VARs for $[y_t\ d_t\ p_t]'$ with 6 lags. Grey areas denote a 90% confidence interval.

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Additional Supporting Information may be found in the online version of this article:

Data S1.

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