Motives and Consequences of Libor Misreporting: How Much Can We Learn from Banks' Misleading Submissions?[*]

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Abstract

Libor is an estimate of interbank borrowing costs computed daily from quotes submitted by a fixed panel of banks. There is evidence suggesting that several banks manipulated these rates in recent years by misreporting their borrowing costs. In this paper, I use structural econometric methods from the empirical auctions literature to estimate a model of strategic quote submission that identifies banks' borrowing costs as well as their motives to misreport. The structural estimation puts a lower bound on the value that Libor would have taken had banks truthfully reported their borrowing costs. The model is also used to determine to what extent misreporting was motivated by signaling or by banks' portfolio exposure to Libor. The model is identified even when there is unobserved heterogeneity in the form of a common cost component that is known by all banks but unobservable to the econometrician, and that is allowed to be persistent in time, even non-stationary. The only data used for identification are the banks' quotes. Therefore, the paper answers the question of how much information about banks' borrowing costs can be inferred from their misreports, given a model of strategic submission. Overall, I find that the estimated lower bound for the truthful Libor is always above the published Libor, with an average deviation of 23 basis points (bp) at the worst of the financial crisis. Moreover, the estimated bound is close to two other measures of interbank borrowing costs that have been used previously to assess the extent of Libor manipulation. Finally, the estimation results imply that sending signals of credit worthiness may be the main driver of systematic misreporting.

1 Introduction

On April 16th, 2008, the Wall Street Journal published an article suggesting that banks had been underreporting their borrowing costs in the interbank market for unsecured funds, by submitting quotes for the computation of Libor (London interbank offer rate) below estimates of such costs implied by credit default swaps (CDS) spreads. Libor is a measure of average costs of short-term unsecured borrowing in the London interbank market. Before recent regulatory adjustments, it was computed daily by Thomson Reuters, on behalf of the British Banking Authority (BBA), as a trimmed mean of the quotes submitted by a fixed panel of large banks. Several banks have been investigated in recent years for their alleged attempts to manipulate these rates. In May 21, 2015, the Council on Foreign Relations reported on its website that banks had paid more than $9 billion dollars in fines and settlements with regulators globally, due to their involvement in the Libor rigging scandal.[1] According to rough estimates, manipulation of Libor may have cost around $6 billion to issuers in the municipal-bond market,[2] just to cite an example of the disruption it might have caused in financial markets. Even after the manipulation scandal, Libor remains as the primary benchmark for short term interest rates globally. According to a report issued by the Market Participants Group on Reforming Interest Rate Benchmarks (2014), financial contracts with notional values adding up to around 216.5 trillion dollars were pegged to Libor at the time of the report, including 97% of the syndicated loans, 15% of the nonsecuritized residential mortgages, and 65% of the over-the-counter US dollar-denominated interest rate swaps (see Duffie and Stein (2015) for a more detailed account of the crucial role that Libor plays in financial markets around the world).

All the banks submitting Libor quotes hold billions of dollars worth of these loans, credits and derivatives that are indexed to this benchmark. As a result, their returns are exposed to variation in the benchmark rate, for the duration of the corresponding contracts. Also, Libor is used as a measure of perceived credit risk in the banking sector. Therefore, a bank submitting Libor quotes above the average could be seen as facing higher risks than others, which, in turn, might increase its funding costs or even trigger a run, as well documented in Duffie and Stein (2015) and the references

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[1]A more detailed account of the penalties that regulators imposed on banks can be found at: http://www.cfr.org/united-kingdom/understanding-libor-scandal/p28729

[2]See Preston (2012) for a description of the impact of Libor manipulation on the municipal-bond market.

therein. In fact, financial regulators, the academic literature and the press have all recognized that sending signals of creditworthiness to the market, and benefiting from exposure to the benchmark were the two main drivers of Libor misreporting during the financial crisis. Section 2 provides a description of these motives to submit quotes that do not accurately reflect the underlying funding costs.

Throughout the paper, I say a bank misreports its borrowing costs if it submits Libor quotes that do not accurately reflect them. Libor manipulation, on the other hand, refers to the act of distorting the reference rate from the value that it would have had under truthful reporting. There is empirical evidence suggesting that Libor misreporting was not limited to just a few isolated cases. Abrantes-Metz, Villas-Boas, and Judge (2011) find that the US Libor rate violates Benford's Law, a statistical regularity commonly observed in several data sets, suggesting anomalies in the quotes submitted by banks. Abrantes-Metz, Kraten, Metz, and Seow (2012) fail to reject the null hypothesis of no Libor manipulation during the period 8/9/2007 through 4/16/2008, when comparing Libor to other benchmarks of average borrowing costs. However, they do find suspicious clustering patterns in Libor quotes for the same period, as well as inconsistencies between the ordinal rankings of CDS spreads and Libor quotes. Both results suggest that misreporting was a generalized practice, not restricted to specific banks or dates. Cassola, Hortaçsu, and Kastl (2013) estimate the maximum rates that banks are willing to pay for short-term collateralized loans at the liquidity auctions run by the European Central Bank (ECB). They find that Euribor[3] quotes are usually below the estimated willingness to pay for secured loans during 2007 and 2008, even though Euribor quotes correspond to interest rates on unsecured lending, and hence should include a risk premium. In several cases, Euribor quotes are even bellow the banks' bids at the liquidity auctions, despite the fact that banks strategically shade their bids. Similarly, Kuo, Skeie, and Vickery (2012) compare Libor quotes to other measures of individual borrowing costs from 2007 to 2009. They use individual bids for funds at the Federal Reserve term auction facility (TAF), as well as rates inferred from Fedwire payments data, and find evidence of underreporting. In particular, they show that after March 2008 (when Bear Stearns failed), Libor quotes frequently lie below matching TAF bids and term rates inferred from Fedwire, and the frequency increased after the bankruptcy of Lehman Brothers, in September

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[3]Euribor is the equivalent of Libor for the euro interbank market.

2008. They also analyze changes in the spreads between Libor and other estimates of average costs for unsecured funding, specifically, the New York Funding Rate (NYFR) and a Eurodollar deposits rate published by the Federal Reserve, and show that during the period considered the spread is negative and wider than in previous years. This suggests that if Libor was in fact manipulated, the published rate underestimated banks' average borrowing costs during the financial crisis. In Section 3, I describe the behavior of these two spreads, paying special attention to the 2007 - 2010 period.

In this paper, I use structural econometric methods from the empirical auctions literature to estimate a model of strategic quote submission that identifies a set of parameters determining banks incentives to misreport, as well as (the underlying distributions of) their borrowing costs. Based on my estimates, I put a lower bound on the value that Libor would have taken, had banks truthfully reported their borrowing costs. The only data used to estimate this lower bound on the truthful Libor are the banks' quotes. Strikingly, it is above the published Libor during the whole period considered, suggesting that Libor understates interbank borrowing costs in that period. Moreover, it is closer than Libor to both the NYFR and the Eurodollar deposits rate, which is consistent with the results of the aforementioned studies. The model also identifies signaling and portfolio exposures to Libor as distinct motives to misreport. The results suggest that the main driver of misreporting during the financial crisis was an attempt to send signals of creditworthiness to the market.

Section 4 presents a model of quote submission where such process is analyzed as a Bayesian game. Each day, each bank receives a private shock to its borrowing cost in the interbank market and submits a quote to the regulator. The regulator computes a trimmed mean, after dropping roughly the top 25% and bottom 25% of the quotes, and publishes the resulting reference rate, as well as all the quotes submitted together with the identities of the submitting banks. Banks interact strategically, they all potentially benefit from manipulating the reference rate, given their portfolio exposure to it, and from underreporting their private borrowing costs, to signal themselves as creditworthy, adequately liquid financial institutions. But they also face potential costs from misreporting, since there is a risk of being detected by the regulator. The banks make their strategic decisions simultaneously and independently. Explicit cooperation is not allowed in the model, but implicit coordination may arise as long as banks know they share similar incentives. The individual

borrowing costs are affiliated, in the sense of Milgrom and Weber (1982), but I impose a specific type of affiliation that seems natural in the present context. I assume that the borrowing cost of each bank is the sum of a private an a common component. Under this same assumption, Duffie and Dworczak (2014) propose a robust benchmark rate that is based on observed transactions. Unlike them, I assume further that the common component is known by all banks but unobservable to the econometrician (or the regulator computing the benchmark). This introduces unobserved heterogeneity, analogous to the one studied by Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011) in the context of procurement auctions estimation.

Section 5 presents identification results, borrowing methods from these two papers. Roughly, the identification relies on the assumption that, once the unobserved heterogeneity is controlled for, the quotes observed each day correspond to the banks' equilibrium strategies of the same static game, for repeated independent realizations of the private costs. A contribution of this paper is that I allow the realizations of the common cost component to be dependent, that is, to follow a persistent (even non-stationary) process, relaxing an implicit assumption in both Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011). In Section 6, I argue that allowing for persistence is relevant, not just as a natural generalization of their method, but in order to provide a more accurate model of the series involved. In the present context, assuming either independence or stationarity might result in largely inaccurate estimates, given the high persistence of these time-series (at least during September, 2007 to May, 2010), as suggested by an augmented Dickey-Fuller test performed on both Libor and the TED Spread (Libor - Three-month Treasury Bill rate). Consequently, Section 7 proposes an estimation method that is suitable for the case with dependent, highly persistent unobserved common costs. Similarly to a result in Krasnokutskaya (2011), the quotes inherit the additive structure of the borrowing costs. Moreover, quotes can be expressed as the sum of the common cost component and an independent individual term. A necessary step in the estimation procedure consists of removing the persistent common cost component to obtain normalized quotes that are independently drawn from the same distribution each day (although from different distributions for each bank).

The model in Section 4 is based on previous work by Snider and Youle (2012), Chen (2013) and Youle (2014). Snider and Youle (2012) assume complete information and, as a result, banks in their

model know with certainty if their own quotes would be included by the regulator in the computation of the trimmed mean. Their focus is to propose a manipulation test, based on characteristics of the distribution of quotes predicted by the model. They compare the observed distribution of quotes to a "plausible joint distribution of true borrowing costs" and find strong evidence of manipulation driven by the exposure of banks' portfolios to Libor. Their model does not include signaling as a likely motive for manipulation. Chen (2013) assumes symmetric independent private costs and adds signaling to the banks' objective function. In his model, all banks are *ex ante* identical and they draw mutually independent private signals (borrowing costs) from the same distribution. He proves the existence of a Bayesian Nash equilibrium (BNE) in pure non decreasing strategies and analyzes how the mechanism in place provides incentives to rig Libor. In particular, he studies how those incentives change under alternative hypothetical distributions of private costs with different levels of dispersion. However, he does not propose an estimation procedure to recover such costs. A related literature studies alternative mechanisms that could reduce the incentives for misreporting or prevent manipulation. See Duffie and Stein (2015) and the references therein.

Youle (2014) adds *ex ante* heterogeneous banks to the model, allowing them to differ in their incentives to misreport and in the distributions of their borrowing costs. He conducts a structural estimation of the parameters of the model capturing the portfolio based incentives to manipulate Libor. However, his strategy does not identify the signaling motives for misreporting and, thus, he cannot recover the manipulation free Libor. The model I propose in this paper (Section 4) generalizes the one in Chen (2013), and coincides with Youle (2014) in the specification of the potential costs and benefits from submitting quotes different than the borrowing costs (the utility or payoff functions of the banks). Nonetheless, there are crucial differences in the way I model the process followed by borrowing costs, that are necessary for a valid identification and estimation of the model parameters. Specifically, as already mentioned, I assume borrowing costs include a common component that is known by all banks, but not observable to the econometrician. As I show in Section 5, the key for the identification of the quotes distributions is to remove this common component, by applying an extension of the deconvolution method in Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011). Consequently, the identification strategy proposed here differs considerably from the previous work,

which mainly relies on additional data sources (CDS spreads) and the use of instrumental variables. To the best of my knowledge, such differences allow me to be the first to separately identify portfolio exposures and signaling as the two main drivers of misreporting, and to recover a lower bound for the truthful Libor that accounts for both potential sources of manipulation. It is worth emphasizing that I obtain identification from banks' Libor quotes alone, without relying on any other information on their borrowing costs. The fact that this lower bound is closer than the published Libor to the NYFR and the Eurodollar deposits rate, as shown in Section 8, supports the validity of the estimation.

In Section 7, I propose a method to estimate the quotes that banks would have submitted every day, if the common cost was just a constant (a counterfactual scenario with no unobserved heterogeneity), and banks had been subject to the same private costs shocks that they actually experienced during each of the periods for which the estimation is performed. I call these the normalized quotes. Once the common cost component is removed from the quotes, two additional assumptions are sufficient to separately identify the parameters of the model that determine the two drivers of misreporting. First, I assume that, for each bank, the idiosyncratic components of the borrowing costs are drawn from a distribution with the same mean an median, which is weaker than assuming that such distribution is symmetric. In principle, there are no reasons to expect that the private costs of any given bank would lie more frequently either above or below their mean. Second, I assume that the means are the same for all banks in the US Dollar Libor panel (but I allow the higher moments of these distributions to differ across banks, in particular their variance). It is important to keep in mind that, for each bank, the private cost is the daily difference between the interest rate that it pays for an average-size loan from other bank and the mean interbank rate (the common cost). Hence, I am assuming that over longer periods, no bank in the panel significantly deviates on average from the overall panel's mean in terms of the interest rates that it pays on interbank loans. With truthful reporting, the distributions of the normalized quotes would inherit these properties of the private costs, just because, for all banks, these quotes would be equal to their private costs plus a constant common term. However, as shown in Section 8, the estimated distributions of the normalized quotes do not have the same means across banks and, for a given bank, do not generally have the same mean and median. In the model, these alterations of

the normalized quotes distributions could only be explained by banks misreporting their borrowing costs due to signaling and portfolio exposures. Moreover, since there are two characteristics of the quotes distributions to be rationalized for each bank, both parameters in the corresponding pay-off function of each bank are separately identified. In the Appendix, I report an alternative estimation that does not use the assumption that the means of the private costs are equal for all banks, but relies on additional data, besides the quotes, to obtain an approximate measure of the common cost. The results regarding the relative importance of signaling and portfolio exposures as drivers of misreporting are robust to a relaxation of this assumption.

Section 8 presents the results of the estimation, using data on the USD three-month Libor from 09/03/2007 to 05/17/2010. To begin with, I establish precisely how much we can learn about banks' incentives and borrowing costs from their quotes alone, that is, without relying on any other data sources. In particular, I find that a lower bound on the common cost component is identified, and thus a bound on the value of Libor under truthful reporting can be inferred from quotes alone, given the model of strategic reporting. The estimated lower bound is always above the published Libor, suggesting that Libor underestimates banks' average borrowing costs during this period, which confirms the results in Kuo, Skeie, and Vickery (2012). In the aftermath of Lehman Brother's bankruptcy, the estimated lower bound is, on average, 23 basis points (bp) higher than the published Libor. The former is also closer than the latter to NYFR and the Eurodollar deposits rate, which suggests that the estimated truthful Libor is a better measure of interbank borrowing costs than the published rate. After providing estimates for all the parameters in the payoff functions of the banks, in particular those that determine whether misreporting is motivated by portfolio exposures to Libor or signaling, I found that most of the deviation between Libor quotes and borrowing costs is explained by banks' attempts to send signals of credit worthiness and adequate liquidity to the market. Thus, previous studies that have focused exclusively on the portfolio channel have ignored the main potential reason of manipulation. A robustness check reported in the Appendix suggests that this result does not depend on the assumption that the means of the private cost components are the same for all banks.

Additionally, the estimated parameters show substantial heterogeneity in banks' portfolio ex-

posures to Libor, both in time and across different banks. For instance, in the period beginning with the bankruptcy of Lehman Brothers, there are two banks in the panel for whom the portfolio incentives seem to imply an average deviation of at least -15bp of their reported quotes from their borrowing costs, while for others there is no statistically significant evidence that they misreported their costs to benefit from their exposure to Libor. The estimates also suggest large changes in the incentives to misreport through time. A plausible explanation (although, just a conjecture) is that the high level of uncertainty characterizing financial markets at the peak of the crisis, facilitated Libor misreporting by decreasing the perceived likelihood, and thus the expected costs, of being detected.

2   Bank Motives to Misreport

Libor in an average of interbank offer rates, computed daily for different currencies and maturities, based entirely on quotes submitted by a fixed panel of banks that participate in the corresponding markets. The published reference rate is a trimmed mean of the quotes submitted by banks, after dropping, roughly, the 25% highest and 25% lowest quotes. The exact number of quotes that are dropped changes with the currency and the size of the panel. Each panel consists of approximately sixteen leading banks trading in London in the corresponding currency.[4] Each day, each bank in the panel submits a quote answering the following question: "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11 am?" Banks are thus expected to truthfully report their own borrowing cost. After receiving all quotes, the regulator publishes the reference rate, and also the set of all quotes submitted by banks with their respective identities.[5]

   At least three general motives for misreporting have been highlighted in the academic literature

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[4]There might be some variation in the number and the identities of the banks in the panel when long periods, comprising several years, are considered. Also, for some currencies the panel is smaller. In particular, there were sixteen banks in the USD Libor panel from September 2007 to May 2010, the period used for the estimation of the model, although there was a change in its composition in January 2009.

[5]From 1985 to 2013, the British Banking Authority (BBA) was responsible for the administration of Libor. As a consequence of the manipulation scandal, in January 31, 2014, administration was handed over to Intercontinental Exchange Benchmark Administration (IBA). Since then, some changes have been implemented following the recommendations contained in Wheatley (2012). In particular, the individual quotes are now published three months after the submission date.

and the financial press (see, among others, Duffie and Stein (2015), Mollenkamp and Whitehouse (2008), Abrantes-Metz, Kraten, Metz, and Seow (2012) and Gandhi, Golez, Jackwerth, and Plazzi (2014)). To begin with, interest rates on short-term loans outside the interbank market are indexed to Libor. As a result, net borrowers in this markets would have a clear incentive to manipulate the published rate downwards and the opposite would hold for net lenders. Also, trillions of dollars in derivatives are pegged to these rates and banks hold such assets in their portfolios, exposing them to variation in the rates. For example, issuers in the municipal bonds market hedge their Auction Rate Securities (ARS) with interest rate swaps pegged to Libor. In those swap contracts, municipalities agree to pay fixed rates to the banks and receive in exchange a floating rate that is a fixed fraction of Libor. The interest rates on ARS are expected to rise with borrowing costs elsewhere, but if Libor is kept artificially low, banks would benefit from the swap. Furthermore, banks might use their Libor quotes to send signals to the market about their creditworthiness, liquidity, and more broadly, their financial soundness, specially during times of general distress. As argued in Duffie and Stein (2015), during the financial crisis, a bank that had seemed less creditworthy than its competitors could have faced even higher funding costs and even risked a run. Correspondingly, the model presented in Section 4 assumes that banks potentially benefit from reporting Libor quotes below their borrowing costs to protect their reputation, but also from manipulating the reference rate published by the regulator in a direction consistent to their portfolio exposure to it. For banks with a long exposure this two incentives counteract, while for banks with a short exposure, they reinforce each other.

3   Descriptive Empirical Evidence

For a period of more than two years overlapping the financial crisis, Libor seems to lie below other measures of borrowing costs for unsecured funds in the interbank market. Following Kuo, Skeie, and Vickery (2012), I compare the three-month USD Libor to two such measures: the New York Funding Rate (NYFR), provided by ICAP (an interbank broker), and a Eurodollar deposits rate (ICAP's Eurodollar), also computed by ICAP and published by the FED on its H.15 report. NYFR is also based on a survey of participants in interbank markets. Contrary to Libor, the individual quotes are not reported and the identity of the contributors answering the survey is unknown. Besides,

contributors are asked to report estimates of funding rates for a representative A1/P1 institution, rather than their own borrowing costs. ICAP's Eurodollar is computed from quotes provided by brokers that serve as intermediaries in the interbank market. Hence, these brokers observe the terms (rates and quantities) of some transactions within a regular day.

Figure (1) shows the spreads between Libor and these two rates. From the left panel, it is clear that Libor had followed ICAP's Eurodollar rare closely since its inception until late 2007, mostly lying slightly above it. However, for the next three years, Libor was considerably lower than ICAP's Eurodollar rate, reaching a striking spread of almost 200bp (two percentage points) at the peak of the crisis. The right panel focuses on the period 2007 - 2012, and adds the Libor - NYFR spread to the comparison. Unfortunately, NYFR was not introduced until June 2, 2008. For a few months before Lehman Brothers filed for bankruptcy protection, the Libor - NYFR spread was, on average, no more than 1.5bp. However, this difference increases drastically after September 15, 2008 and remains large and negative for at least one year. The Libor - ICAP's Eurodollar spread shows some interesting patterns during the financial crisis too. It becomes negative and relatively large in magnitude around August 2007, when BNP Paribas announced that it had suspended redemption in two funds invested in subprime mortgages. Then it jumps drastically with Lehman's collapse, reaching and unprecedented lowest historical value. As the crises evolves, this spread narrows but it remains negative, and large in magnitude compare to its pre-crisis behavior.

The empirical findings presented in this section suggest that banks might have been able to manipulate Libor during the crisis, pushing it below their average borrowing costs. However, this does not constitute a proof of manipulation, and there might be other explanations for why the spreads show this peculiar patterns. Kuo, Skeie, and Vickery (2012) , explore some of these alternatives, all of them related to differences in the way each rate is computed and to institutional aspects of the markets involved. For instance, the trading sessions in London (Libor) and New York (NYFR) do not take place simultaneously and market segmentation prevents funds to flow freely between the two markets. Despite these differences, I show in Section 8 that the estimated lower bound on the counterfactual truthful Libor (absent any misreporting) is closer too NYFR and ICAP's Eurodollar rate, providing additional evidence that the published Libor underestimated average borrowing costs
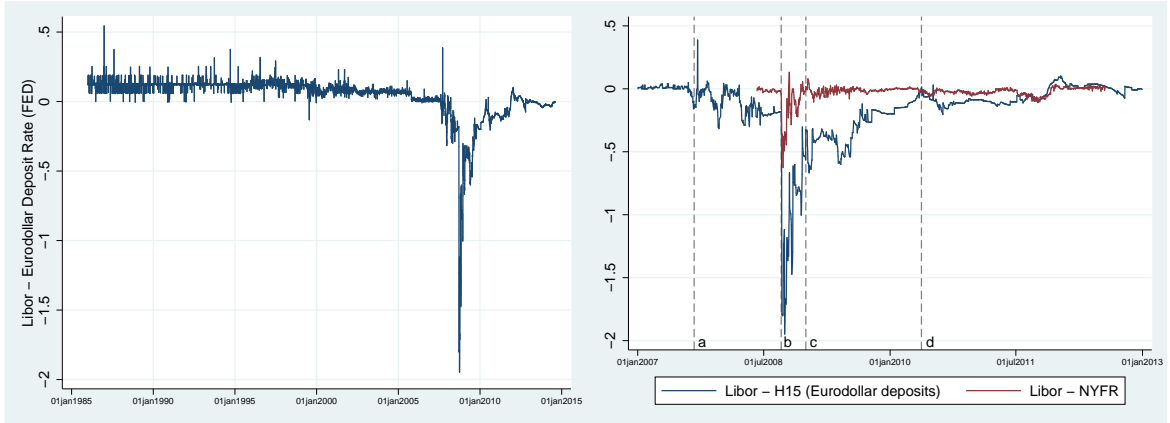
Figure 1: Spreads between Libor and other measures of borrowing costs

H15 is a measure of Eurodollar deposits rates computed from quotes provided by ICAP's brokers that have access to data on actual interbank transactions. The New York Funding Rate (NYFR) is based on a survey of contributors that participate in the market for unsecured funds. Clearly, during the financial crisis Libor lies below these other two measures of borrowing costs for unsecured funds. Four dates are highlighted with vertical lines in the right panel. (a) 09/03/2007: First trading day after BNP Paribas suspended redemption in two funds heavily invested in subprime mortgages. (b) 09/15/2008: Lehman Brothers filed for bankruptcy protection. (c) 12/31/2008: The BBA changed the composition of the Libor panel, Scottish HBOS was replaced by french Societa Generale. (d) 05/17/2010: A somewhat arbitrary date chosen based on an apparent sudden and persistent change in the Libor - ICAP's Eurodollar spread trend.

during the financial crisis. In the next section we review what could have been the incentives for individual banks to misreport their borrowing cost and to attempt manipulation of the reference rate.

4   The Model

There is a fixed set of banks, $\mathcal{N} = \{1, ..., N\}$, that participate in the game. Each day t, bank $i \in \mathcal{N}$ observes its own borrowing costs in the interbank market for unsecured funds, $s_{i,t}$, for a given currency and maturity. The regulator asks the banks to report their cost, and in response each submits a quote $r_{i,t}$. For all $i \neq j$, $s_{i,t}$ and $s_{j,t}$ are assumed to be independent, conditional on a common cost component μ_t that is common knowledge. The distributions of the costs might be different for different banks, and are common knowledge too.

4.1 Banks' Incentives and Costs Affiliation

At a given day t, all banks submit their quotes simultaneously, to maximize their expected utility from participating in the static game, conditional on μ_t. After observing all the quotes, the regulator computes the reference rate $\tilde{r}_t(r_{1,t}, ..., r_{N,t})$, and reveals $\tilde{r}_t$ and the whole vector of individual quotes to the public. Banks could benefit from the game in two different ways: by manipulating the reference rate (in a direction that is consistent to their portfolio exposure), and from sending signals to the market of their creditworthiness. Correspondingly, their utility functions include a linear gain on the level of the reference rate $\tilde{r}_t$ (portfolio), and a linear term on the difference between the reference rate and their quote (signaling). Additionally, a cost component that is quadratic in the difference $s_{i,t} - r_{i,t}$ reflects credibility concerns and the expected costs from the likelihood of being detected submitting quotes different than their costs. Formally, bank i's expected utility from submitting quote $r_{i,t}$ after observing both $s_{i,t}$ and μ_t is:

$$u_i\left(r_{i,t}; s_{i,t}, \mu_t\right) = E\left[\underbrace{\alpha_i \tilde{r}_t\left(r_{i,t}, r_{-i,t}\right)}_{\text{portfolio}} + \underbrace{v_i\left(\tilde{r}_t\left(r_{i,t}, r_{-i,t}\right) - r_{i,t}\right)}_{\text{signaling}} - \underbrace{\gamma_i\left(s_{i,t} - r_{i,t}\right)^2}_{\text{cost}}|\mu_t\right] \qquad (1)$$

where $v_i > 0$ and $\gamma_i > 0$, but α_i is allowed to have an arbitrary sign reflecting the possibility that banks' exposure to the reference rate might be long or short. All these constants are common knowledge, thus the only source of uncertainty are the private components of other banks' costs. The reference rate $\tilde{r}_t$ is a function of the vector of quotes submitted by all banks and is defined by:

$$\tilde{r}_t = \frac{1}{\tilde{N}} \sum_{k=\underline{n}+1}^{\bar{n}-1} r_t^{(k)} \qquad (2)$$

where $r_t^{(k)}$ is the k-th smallest element of the vector of quotes $(r_{1,t}, ..., r_{N,t})$. $\underline{n}$ and $\bar{n}$ are cut-offs set by the regulator, and known by the banks, determining which quotes are left out from the trimmed mean.

Assuming a specific functional form for banks' utility might raise some concerns on how accurately the model captures their incentives. Nevertheless, the linear gain on the portfolio exposure to

the reference rate $\tilde{r}_t$ seems quite reasonable. Suppose, for simplicity, a bank has a single asset that pays periodic coupons of $A_0 \tilde{r}_t$. Clearly, the change in the bank's cash flow due to an increase in $\tilde{r}_t$ of 1bp (0.01 percent points) would be constant, regardless of the level of the reference rate. Besides, a single bank cannot cause the reference rate to deviate largely from the value it would have under truthful reporting, since extreme quotes would not be included in the computation of the reference rate. Regarding the signaling and cost components of the utility function, they can be interpreted as a first and second order approximations to more general specifications of the respective gains and costs. Higher order effects might be plausible, but they should not be the main drivers of banks' quotes.

The borrowing costs are allowed to be affiliated in the sense of Milgrom and Weber (1982), but I assume a specific type of affiliation. The cost of each bank $(s_{i,t})$ is determined by a common component μ_t, known to all banks, and a private shock $\epsilon_{i,t}$. Each day t, $\epsilon_{i,t}$ is drawn independently from the same distribution F_i with mean zero.

Assumption 1. *For all $i \in \mathcal{N}$ and all t, $s_{i,t} = \mu_t + \epsilon_{i,t}$. μ_t is common knowledge but $\epsilon_{i,t}$ is only privately known.*

In their work on the design of robust interest rates benchmarks, Duffie and Dworczak (2014) also assume that private borrowing costs are the sum of a common component and a zero mean idiosyncratic shock. Unlike them, I also assume that μ_t is common knowledge, but not observed by the econometrician.

Assumption 2. *For a given i, $\epsilon_{i,t} \overset{iid}{\sim} F_i$ with $E[\epsilon_{i,t}] = 0$, and for any t and $j \neq i$, $\epsilon_{i,t}$ and $\epsilon_{j,t}$ are independent. The support of F_i is a closed interval $[\underline{\epsilon}, \bar{\epsilon}]$ on the real line, assumed to be the same for all i. The distributions F_i, for all i, are also common knowledge.*

Notice that the independence of $s_{i,t}$ and $s_{j,t}$, conditional on μ_t, follows directly from Assumptions 1 and 2. Also, from Assumption 2, let $\mathcal{S}(\mu) = [\mu + \underline{\epsilon}, \mu + \bar{\epsilon}]$ be the support of $s_{i,t}$, conditional on μ_t. $\mathcal{S}(\mu)$ is the set of all possible types for each player i, given μ.

Assumption 3. *For all i, the random variable $\epsilon_{i,t}$ is absolutely continuous with respect to the Lebesgue measure.*

The last assumption guarantees that both $\epsilon_{i,t}$ and $s_{i,t}$ have Lebesgue integrable densities on their support.

4.2 Bayes Nash Equilibrium

The focus of this section is to describe the Bayes Nash equilibrium of the static game. Specifically, I will show that there is a BNE equilibrium in pure, strictly increasing, bounded strategies. To do so, let us start by characterizing the best response of any bank i, ρ_i, to a set of bounded strategies $\rho_{-i} = \{\rho_j : j \neq i\}$ followed by all other players. Given ρ_{-i}, bank i's best response is defined by

$$\rho_i(s_i) = \underset{r_i \in \mathbb{R}}{\operatorname{argmax}} E\left[\alpha_i \tilde{r}(r_i, r_{-i}) + v_i(\tilde{r}(r_i, r_{-i}) - r_i) - \gamma_i(s_i - r_i)^2 | \mu\right] \tag{3}$$

for all $s_i \in \mathcal{S}(\mu)$ (here I drop the time sub-index for convenience).

We need to make sure that $\rho_i(s_i)$ is well defined for all $s_i \in \mathcal{S}(\mu)$, that is, that the corresponding maximization problem does have a solution. Given that all other strategies ρ_{-i} are bounded, it follows from equation (2) that $E\left[\tilde{r}(r_i, r_{-i}) | \mu, s_i\right]$ attains a minimum as a function of r_i, say at $\underline{r}$[6]. Then, in the set $(-\infty, \underline{r})$, the derivative of the the utility function u_i in (3) simplifies to $\frac{\partial u_i(r_i; s_i, \mu)}{\partial r_i} = -v_i + 2\gamma_i(s_i - r_i)$. Therefore, $\frac{\partial u_i}{\partial r_i} > 0$ for all r_i such that $r_i < \min\left(s_i - \frac{v_i}{2\gamma_i}, \underline{r}\right)$. It follows that $\rho_i(s_i) \geq \min\left(s_i - \frac{v_i}{2\gamma_i}, \underline{r}\right)$. Similarly, $E\left[\tilde{r}(r_i, r_{-i}) | \mu, s_i\right]$ attains a maximum at some $\bar{r}$ and thus, $\frac{\partial u_i}{\partial r_i} < 0$ for all $r_i > \max\left(s_i - \frac{v_i}{2\gamma_i}, \bar{r}\right)$. Then $\rho_i(s_i) \leq \max\left(s_i - \frac{v_i}{2\gamma_i}, \bar{r}\right)$. It follows that the continuous function $u_i(\cdot; s_{i,t}, \mu_t)$ attains a maximum in the compact set $\left[\min\left(s_i - \frac{v_i}{2\gamma_i}, \underline{r}\right), \max\left(s_i - \frac{v_i}{2\gamma_i}, \bar{r}\right)\right]$. Moreover, since $\mathcal{S}(\mu) = [\mu + \underline{\epsilon}, \mu + \bar{\epsilon}]$, $\min\left(\mu + \underline{\epsilon} - \frac{v_i}{2\gamma_i}, \underline{r}\right) \leq \rho_i(s_i) \leq \max\left(\mu + \bar{\epsilon} - \frac{v_i}{2\gamma_i}, \bar{r}\right)$, for all $s_i \in \mathcal{S}(\mu)$, and hence ρ_i is bounded.

Moreover, when thought of as a function of both the action r_i and the type s_i, the utility function u_i satisfies the single crossing property (SCP) of Milgrom and Shannon (1994), for any set of strategies ρ_{-i}, as can be verified by direct differentiation (in fact, $\frac{\partial^2 u_i}{\partial s_i \partial r_i} = 2\gamma_i > 0$). Therefore, the best response ρ_i is non-decreasing. Notice that we have not ruled out the possibility that there is more than one solution to problem (3), for some s_i. Hence, in principle, ρ_i might be set-valued

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[6]An explicit equation for $E\left[\tilde{r}(r_i, r_{-i}) | \mu, s_i\right]$ in terms of the distributions of all other player's actions can be found in the Appendix.

or, equivalently, a correspondence. By Berge's Theorem of the Maximum, we know ρ_i is nonempty, upper hemicontinous and compact valued. Moreover, in the Appendix I show that, as a correspondence, ρ_i is not decreasing in a very strong sense, i.e, for all $s, s' \in \mathcal{S}(\mu)$, with $s < s'$, $\max \rho_i(s_i) \leq \min \rho_i(s'_i)$.

The utility function of bank i satisfies SCP, in particular, when all other banks use nondecreasing strategies. Therefore, it also satisfies the single crossing condition for games of incomplete information of Athey (2001) and it follows from her Theorem 2 that the game has a Bayes Nash equilibrium in pure non-decreasing strategies. Moreover, since u_i satisfies SCP, regardless of the strategies used by all other players, it follows that in any pure strategy Bayes Nash equilibrium of this game the strategies are non-decreasing.

A set of necessary conditions for $(\rho_1, ..., \rho_N)$ to be a vector of equilibrium strategies can be obtained from the first order conditions of the utility maximization problem of the banks (3). For any $i \in \mathcal{N}$ and $s_i \in S_i$, if $\rho_i(s_i)$ is a solution (best response) to the strategies of all other banks ,ρ_{-i}, then

$$(\alpha_i + v_i) \frac{\partial E\left[\tilde{r}\left(\rho_i(s_i), \rho_{-i}(s_{-i})\right) | \mu, s_i\right]}{\partial r_i} - v_i + 2\gamma_i(s_i - \rho_i(s_i)) = 0 \tag{4}$$

where $\rho_{-i}(s_{-i}) = \{\rho_j(s_j) : j \neq i\}$ and $E[\cdot | \mu, s_i]$ denotes the conditional expectation operator *from the perspective of bank i*, that is, when s_i is known (non-stochastic), but s_j, for all $j \neq i$, is considered a random variable, with known conditional distribution.

As noted by Chen (2013), the derivative $\frac{\partial E[\tilde{r}(r_i, r_{-i}) | \mu, s_i]}{\partial r_i}$ can be expressed in terms of the probability (as perceived by bank i) that its quote would be included in the trimmed mean defining the reference rate, that is, the probability that $r^{(\underline{n})} \leq r_i \leq r^{(\bar{n})}$ (see the Appendix for a proof of this statement). That is,

$$\frac{\partial E\left[\tilde{r}(r_i, r_{-i}) | \mu, s_i\right]}{\partial r_i} = \frac{1}{\tilde{N}} P_i \left\{ r^{(\underline{n})} \leq r_i \leq r^{(\bar{n})} | \mu \right\} \tag{5}$$

Let $\phi_i(r_i | \mu)$ denote such probability. Notice that it depends on the vector of strategies of all other banks and the distributions of their costs. Besides, it is not necessary to condition on s_i given that borrowing costs are independent, conditional on μ.

Since applying affine transformations to the utility functions in equation (1) would not change

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the (equilibrium of) the game, I apply the normalization $\gamma_i = \frac{1}{2}$, without loss of generality.[7] The expression for the necessary conditions in (4) simplifies to:

$$\beta_i \phi_i \left(\rho_i \left(s_i \right) | \mu \right) - v_i + \left(s_i - \rho_i \left(s_i \right) \right) = 0 \tag{6}$$

for all $i \in \mathcal{N}$ and all $s_i \in \mathcal{S}\left(\mu\right)$, where $\beta_i = \frac{\alpha_i + v_i}{\bar{N}}$.

For all bank i, its equilibrium strategy ρ_i satisfies equation (6). It follows that, the corresponding system of functional equations (one for each bank), characterizes any Bayes Nash equilibrium of the game. Unfortunately, for an arbitrary vector of distributions $(F_1, ..., F_N)$, this system cannot be solved analytically for the vector of equilibrium strategies $(\rho_1, ..., \rho_N)$. In other words, we cannot obtain explicit analytical expressions for the equilibrium strategies. Roughly, the reason is that the functions $\phi_i \left(\cdot | \mu \right)$ depend on ρ_{-i} and the distributions$(F_1, ..., F_N)$ in a non-trivial way, in fact, it can be shown that:

$$\phi_i \left(\cdot | \mu \right) = \sum_{k=\underline{n}}^{\bar{n}-2} \sum_{\substack{|M| = k \\ i \notin M}} \left(\prod_{j \in M} F_j \left(\rho_j^{-1} \left(r_i \right) - \mu \right) \prod_{j \in M^c / \{i\}} \left(1 - F_j \left(\rho_j^{-1} \left(r_i \right) - \mu \right) \right) \right) \tag{7}$$

where ρ_j^{-1} denotes the inverse of the function ρ_j, and the inner sum is performed over all sets of banks of size k, not including bank i.

However, each equation in the system of first order conditions can be easily inverted as follows:

$$s_i = r_i - \beta_i \phi_i \left(r_i | \mu \right) + v_i \tag{8}$$

Such inversion is crucial for proving some of the properties of the Bayes Nash Equilibrium. Besides, it also provides the key for the identification of the model, following a strategy similar to Guerre, Perrigne, and Vuong (2000), Li, Perrigne, and Vuong (2000), Li, Perrigne, and Vuong (2002), and all the subsequent literature in structural estimation of auctions, as will be shown below in the section

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[7]Equivalently, since α_i, v_i and γ_i cannot be separately identified, I redefined α_i and v_i to be measured as fractions of $2\gamma_i$.

on identification. In fact, together with Proposition 1, it allows us to write explicit expressions for the inverse equilibrium strategies.

Proposition 1. *For each bank i, the equilibrium strategy ρ_i is strictly increasing at all $s_i \in \mathcal{S}(\mu)$*

The proof of Proposition 1 is straightforward. For all $s_i \in \mathcal{S}(\mu)$, $\rho_i(s_i)$ satisfies equation (6), and thus (8). Moreover, (8) provides an explicit expression for the inverse function of the equilibrium strategy, $s_i = \rho_i^{-1}(r_i)$. Since ρ_i is nondecreasing and invertible, it must be strictly increasing in $\mathcal{S}(\mu)$.

5 Identification Strategy

Ideally, we would want the model in section (4) to identify the distributions of the borrowing costs (at least conditional on μ) and, if possible, for every bank i and every day t, the specific borrowing cost $s_{i,t}$ it faced. If we could recover all these costs, we would be able to compute the daily average borrowing cost of the N banks in the panel, which is precisely the value that the reference rate would have under truthful reporting (the manipulation-free Libor). Unfortunately, for reasons that will be made clear below, this goal is too ambitious to be feasible. However, we can identify a lower bound on the manipulation-free Libor, which is quite useful given the evidence suggesting that Libor was manipulated downwards during the period considered. In fact, as shown in Section 8, the estimated lower bound is always above the published rate during this period, which is consistent with the previous evidence.

Consider, as a benchmark, the first price sealed bid auction with independent private values analyzed in Guerre, Perrigne, and Vuong (2000), where the private values of all bidders at each auction are indeed identified from the set of bids submitted by all bidders. As opposed to their case, here the inverse equilibrium strategies (8) depend on parameters (β_i and v_i), that are unknown to the econometrician. Moreover, the costs are not independent across banks, but only conditionally independent given a common component μ, that is common knowledge for the banks, but unobservable to the econometrician. Our setup is closer to that of Campo (2012), who admits heterogeneous risk aversion, and Krasnokutskaya (2011), who allows for unobserved auction heterogeneity. In the

former, the risk aversion coefficients of all bidders are identified by imposing strong conditions on the distributions of their private values (they are all assumed to have the same marginal distributions). In the latter, the distributions of the private and the common cost components (in a procurement auction) are identified, but the specific realizations of these costs that correspond to the observed bids, are not. A common assumption when estimating static games is that at each repetition of the game, the observable set of actions chosen by the players correspond to the same equilibrium (conditional on the unobserved heterogeneity). Such an assumption is hard to sustain if the game under consideration has multiple equilibria. For the time being, I will implicitly assume uniqueness to obtain the identification results. In the Appendix I present the results of numerical simulations suggesting that the BNE of the game is in fact unique.

The set of inverse equilibrium strategies (8) expresses the private costs as functions of the observable quotes and their conditional distributions. In particular, the probability that bank i's quote is included in the computation of the reference rate when it reports r_i, i.e. $\phi_i\left(r_i|\mu\right) = P_i\left\{r^{(\underline{n})} \leq r_i \leq r^{(\bar{n})}|\mu\right\}$, is clearly a function of r_i and the distributions of all other banks' quotes conditional on μ.

Suppose μ_t was constant for all t in our sample (taking an arbitrary value $\bar{\mu}$). Then banks would be playing the exact same static game at each t, and each observed quote $r_{i,t}$ would be a draw from their equilibrium distribution, implied by the distribution of costs and the equilibrium strategies. In such case, the distribution of the equilibrium quotes could be estimated directly from the observed quotes, for each bank i participating in the game. Moreover, we could also estimate the probabilities $\phi_i\left(r_{i,t}|\bar{\mu}\right)$, from the estimated distributions of the quotes. Additionally, if v_i and β_i were identified, for all $i \in \mathcal{N}$, then we could recover the private costs observed by each bank, at each t, from the inverse equilibrium strategies:

$$s_{i,t} = r_{i,t} - \beta_i\phi_i\left(r_{i,t}|\bar{\mu}\right) + v_i \tag{9}$$

Unfortunately, this conclusion follows from highly unrealistic assumptions. In particular, as I will argue below, the data strongly suggests that the unobserved common cost component μ_t is not constant, its realizations are not drawn independently each day, and it might even follow a non-stationary process.

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5.1 Unobserved Game Heterogeneity

A difficulty when controlling for μ_t in the estimation of the model is not only that it is unobservable to the econometrician, but also that it changes the game that banks are playing each day, since their payoffs and the distributions of their costs are all functions of μ_t. In a few words, μ_t introduces unobserved game heterogeneity. Still, following an argument analogous to that in Haile, Hong, and Shum (2003) and Krasnokutskaya (2011), Proposition 2 states that the equilibrium strategies (when thought of as functions of both the purely private costs $\epsilon_{i,t}$ and the common cost μ_t), are additively separable into a common component and a private quote component.

Proposition 2. *Let $\rho_j\left(\epsilon_i; 0\right)$ be the equilibrium strategies of all banks $j \in \mathcal{N}$, when $\mu = 0$. If the strategy of each bank $j \neq i$, when $\mu \neq 0$, is*

$$\rho_j\left(\epsilon_j; \mu\right) = \mu + \rho_j\left(\epsilon_j; 0\right) \tag{10}$$

then i's best response is also $\rho_i\left(\epsilon_i; \mu\right) = \mu + \rho_i\left(\epsilon_i; 0\right)$.

The proof of Proposition 2 is left for the Appendix. It consists of directly verifying that $\mu + \rho_i\left(\epsilon_i; 0\right)$ maximizes i's expected utility, when all other players are using the strategies in (10). An immediate corollary of Proposition 2 is that, if $\left(\rho_1^0, ..., \rho_N^0\right)$ is a BNE of the game with $\mu = 0$, then $\left(\rho_1, ..., \rho_N\right)$, with $\rho_i = \rho_i^0 + \mu$ for all $i \in \mathcal{N}$, is a BNE of the game with $\mu \neq 0$. Again, analogous results for the first price sealed bid auction are proved in Haile, Hong, and Shum (2003) and Krasnokutskaya (2011).[8]

Let $\epsilon_{i,t}$ be the private cost component of bank i at time t, and $q_{i,t} = \rho_i\left(\epsilon_{i,t}; 0\right)$ be the normalized equilibrium quote that bank i would submit, when its purely private cost is $\epsilon_{i,t}$, and $\mu_t = 0$. It follows that , for all $i \in \mathcal{N}$,

$$r_{i,t} = \mu_t + q_{i,t} \tag{11}$$

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[8]The normalization $\mu = 0$ and the fact that $E\left[\epsilon_{i,t}\right] = 0$, for all $i \in \mathcal{N}$, imply that we are allowing for negative borrowing costs, which seems unrealistic. However, such normalization is completely arbitrary and I chose zero just for mathematical convenience. I could have chosen any other fixed value $\bar{\mu}$ such that $\bar{\mu} + \underline{\epsilon} > 0$, to avoid negative borrowing costs, and all the identification results would remain the same.

In particular, for any two banks $i_1, i_2 \in \mathcal{N}$,

$$r_{i_1,t} = \mu_t + q_{i_1,t}$$

$$r_{i_2,t} = \mu_t + q_{i_2,t}$$

Notice that $q_{i_1,t}$ and $q_{i_2,t}$ are each a function of $\epsilon_{i_1,t}$ and $\epsilon_{i_2,t}$, respectively, and hence they are independent. Moreover, from the point of view of the econometrician, the random variable μ_t is also independent of $q_{i_1,t}$ and $q_{i_2,t}$. The goal now is to recover the unknown distributions of μ_t, $q_{i_1,t}$ and $q_{i_2,t}$ from the observable distributions of $r_{i_1,t}$ and $r_{i_2,t}$. Under the additional assumption that μ_t is independently drawn from the same distribution at all t, this is the same deconvolution problem introduced by Li and Vuong (1998), in the context of measurement error with multiple indicators, and later studied by Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011) for the estimation of procurement auctions with a common cost component. All of them base their results on a Lemma proved by Kotlarski (1966).

An additional difficulty here is that we want to allow μ_t to be drawn from a different distribution at each t and we do not want to assume that these draws are independent across time, hence the assumptions of Kotlarski's Lemma no longer hold. In particular, Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011) provide identification results for the case with i.i.d. μ_t, that require that at least two bids and the identities of the corresponding bidders are observable. I will not reproduce their results here. Instead, I will show identification in the case where μ_t is not necessarily stationary and the draws are not necessarily independent. The additional assumption necessary for identification is that at least three quotes, with the identities of the banks, are observable.

Proposition 3. *If the characteristic functions of the normalized quotes $q_{i_1,t}$, $q_{i_2,t}$ and $q_{i_3,t}$ are nonvanishing everywhere, their distributions are identified, up to an additive constant, from the joint distribution of $(r_{i_1,t} - r_{i_2,t}, r_{i_3,t} - r_{i_2,t})$.*

Proof. Notice that (11) implies

$$r_{i_1,t} - r_{i_2,t} = -q_{i_2,t} + q_{i_1,t}$$

$$r_{i_3,t} - r_{i_2,t} = -q_{i_2,t} + q_{i_3,t}$$

where $q_{i_1,t}$, $q_{i_2,t}$ and $q_{i_3,t}$ are mutually independent and, thus, these two equations satisfy the assumptions of Kotlarski's Lemma. Let Ψ denote the joint characteristic function of $(r_{i_1,t} - r_{i_2,t}, r_{i_3,t} - r_{i_2,t})$ and Ψ_1, its partial derivative with respect to its first argument. Also, let $\Phi_{q_{i_1}}$, $\Phi_{-q_{i_2}}$ and $\Phi_{q_{i_3}}$ denote the characteristic functions of $q_{i_1,t}$, $-q_{i_2,t}$ and $q_{i_3,t}$, respectively. Then,

$$
\begin{aligned}
\Phi_{-q_{i_2}}(s) &= \exp\left(\int_0^s \frac{\Psi_1(0,u)}{\Psi(0,u)} du - isE[q_{i_1}]\right) \\
\Phi_{q_{i_1}}(s) &= \frac{\Psi(s,0)}{\Phi_{-q_{i_2}}(s)} \\
\Phi_{q_{i_3}}(s) &= \frac{\Psi(0,s)}{\Phi_{-q_{i_2}}(s)}
\end{aligned}
\tag{12}
$$

Since the characteristic function of a random variable uniquely determines its distribution, it follows that the distributions of $q_{i_1} - E[q_{i_1}]$, $q_{i_2} - E[q_{i_1}]$ and $q_{i_3} - E[q_{i_1}]$ are all identified. $\qquad\square$

It is worth noticing that the constant term $E[q_{i_1}]$ in the proof of Proposition 3 is not identified. That is precisely why we only achieve identification of the normalized quotes up to an additive constant. Still, Proposition 3 is useful for the identification of the model, because it removes the unobserved heterogeneity from the game and, most importantly, from the equilibrium quotes.

Furthermore, if μ_t was i.i.d. we could directly apply the results in Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011) to show that the distributions of the observable quotes (up to a constant), but also the distribution of μ_t, are all identified. In Section 6 I show some empirical evidence suggesting that μ_t is highly persistent and might not even be stationary.

5.2 Identified Parameters

Besides unobserved game heterogeneity, an additional challenge for the identification of the distributions of each bank's borrowing costs are the unknown preference parameters β_i and v_i in the inverse

equilibrium strategies (8). I will show now that, under an additional assumption on the distribution of the purely private costs $\epsilon_{i,t}$, we can identify β_i, v_i and the distribution of $\epsilon_{i,t}$, for all $i \in \mathcal{N}$.

Consider a hypothetical game with $\mu = -E\left[q_{i_1}\right]$.[9] Substituting terms in (8), we obtain

$$-E\left[q_{i_1}\right] + \epsilon_i = q_i - E\left[q_{i_1}\right] - \beta_i \phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right) + v_i$$

for all $i \in \mathcal{N}$. From Proposition 3, we know that the distribution of $q_i - E\left[q_{i_1}\right]$ is identified for all $i \in \mathcal{N}$, since we observe the quotes and identities of all banks. Hence, the probabilities $\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right)$ are also identified for all values of $q_i - E\left[q_{i_1}\right]$ in its support. Moreover, taking expectations,

$$-E\left[q_{i_1}\right] = E\left[q_i\right] - E\left[q_{i_1}\right] - \beta_i E\left[\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right)\right] + v_i \tag{13}$$

and subtracting the latter equation from the former,

$$
\begin{aligned}
\epsilon_i \;=\;& q_i - E\left[q_{i_1}\right] - \left(E\left[q_i\right] - E\left[q_{i_1}\right]\right) \\
& -\beta_i \left(\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right) - E\left[\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right)\right]\right)
\end{aligned}
\tag{14}
$$

Again, the distributions of the random variables at the right hand side of equation (14), that is, $q_i - E\left[q_{i_1}\right]$ and $\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right)$, are identified and, thus, their means, $E\left[q_i\right] - E\left[q_{i_1}\right]$ and $E\left[\phi_i \left(q_i - E\left[q_{i_1}\right] \middle| -E\left[q_{i_1}\right]\right)\right]$, are identified as well. Equation (14) is the key for the identification of β_i and the distribution of ϵ_i, F_i. However, an additional restriction must be imposed on F_i to achieve identification.

Assumption 4. *For all $i \in \mathcal{N}$, the median of ϵ_i is zero, which is denoted by $Med\left(\epsilon_i\right) = 0$.*

By Assumption 2, $E\left[\epsilon_i\right] = 0$, then Assumption 4 only requires some symmetry of the distributions of the purely idiosyncratic components of the borrowing costs. Under such assumption, the parameters β_i and v_i, as well as the distribution of ϵ_i are identified, as stated in the next two

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[9]Again, the mathematical possibility of having negative borrowing costs is just a consequence of the particular normalization chosen, but all the results hold under a different normalization that implies a zero probability of having negative rates (see footnote 8).

propositions.

Proposition 4. *Under Assumptions 1-4, the preference parameter β_i is identified, for all $i \in \mathcal{N}$, from the distributions of the normalized quotes. Moreover,*

$$\beta_i = \frac{Med\left(q_i - E\left[q_{i_1}\right]\right) - \left(E\left[q_i\right] - E\left[q_{i_1}\right]\right)}{\phi_i\left(Med\left(q_i - E\left[q_{i_1}\right]\right)\mid - E\left[q_{i_1}\right]\right) - E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right]} \tag{15}$$

The proof of Proposition 4 is left for the Appendix. It relies on the fact that equation (14) is a translation of the inverse equilibrium strategy of bank i in the game with $\mu = -E\left[q_{i_1}\right]$ and, thus, it is strictly monotone as a function of q_i. The existence of a unique q_i^* such that $\epsilon_i = 0$ in equation (14) guarantees the existence of a unique β_i such that $Med\left(\epsilon_i\right) = 0$.

Proposition 5. *For all $i \in \mathcal{N}$, the distribution of ϵ_i, F_i, and the deviation of the private signaling parameters from their mean, $v_i - \bar{v}$, are identified.*

Identification of F_i follows in a straightforward way from Proposition 4 and equation (14), since all the distributions and the constants on the right hand side of this equation are identified. Regarding $v_i - \bar{v}$, we can take the intraday means in equation (13) and subtract the result from (13) again to obtain:

$$-E\left[q_{i_1}\right] = E\left[\bar{q}\right] - E\left[q_{i_1}\right] - E\left[\bar{\beta}\phi\right] + \bar{v}$$

$$-E\left[q_{i_1}\right] = E\left[q_i\right] - E\left[q_{i_1}\right] - \beta_i E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right] + v_i$$

$$v_i - \bar{v} = E\left[\bar{q}\right] - E\left[q_i\right] + \beta_i E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right] - E\left[\bar{\beta}\phi\right] \tag{16}$$

where

$$E\left[q_i\right] - E\left[\bar{q}\right] = E\left[q_i - E\left[q_{i_1}\right]\right] - \frac{1}{N}\sum_{i\in\mathcal{N}} E\left[q_i - E\left[q_{i_1}\right]\right]$$

and $\bar{\beta}\phi = \frac{1}{N}\sum_{i\in\mathcal{N}}\left(\beta_i\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right)$. Hence, all the terms in the right hand side are identified from the distributions of $q_i - E\left[q_{i_1}\right]$.

Unfortunately $\bar{v}$ is not identified, but we can still obtain conservative estimates of the individual signaling parameters by introducing an additional assumption.

Assumption 5. $\min\left\{v_i : i \in \mathcal{N}\right\} = 0$

Suppose a given bank, say k, has the lowest signaling incentive to underreport its borrowing cost or, equivalently, the highest perceived cost of doing so. Under 5, the only reason why this bank would misreport is due to its portfolio exposure to the reference rate. Moreover, since $v_i - \bar{v}$ is identified for all banks, and $v_k = 0$, by assumption, it follows that $\bar{v}$ and v_i are also identified, for all $i \in \mathcal{N}$. Notice that if v_k was any positive constant c, then we would be underestimating every v_i by c. Thus, after imposing 5, we identify lower bounds for all the signaling parameters v_i, as the next proposition states.

Proposition 6. *For all $i \in \mathcal{N}$, a lower bound on its private signaling parameter v_i is identified.*

Notice that I have not provided any argument showing that the daily realizations of the common cost component μ_t are identified. In fact, the identification strategy described in this section consists of removing the game heterogeneity due to μ_t from each instance of the game, so they can be considered as independent occurrences of the same game. That allows us to identify the distributions of the normalized quotes, but the corresponding normalization implies that the results are invariant to additive constants affecting all costs equally. Nevertheless, in Section 7, I propose a method to recover (with low-variance noise, though) the whole set of purely private costs components, $\epsilon_{i,t}$, observed by all banks, for each day in the sample. Furthermore, combining such noisy estimates of the idiosyncratic shocks with estimates of the parameters shown to be identified in this section, I propose a (noisy) estimator of a lower bound of the common cost component μ_t. In Section 8, I show that the estimated lower bound always lays above the reported Libor, and mostly in-between two other measures of average borrowing costs already described in Section 3. The estimation provides further evidence that if banks succeeded in manipulating Libor during the period considered, they set it below their true average borrowing costs.

6 Non-Stationary Unobserved Game Heterogeneity

Section 5 contains identification results that do not require the common borrowing cost μ_t to be i.i.d., either the distribution might change or the draws could be dependent across time. Correspondingly, Section 7 will present an estimation procedure that is valid even if μ_t is not i.i.d. To the best of

my knowledge, the particular case with non-stationary heterogeneity has not been considered before in the empirical auctions literature, although it is just a natural extension of the model with i.id. heterogeneity, as already shown. In this section, I argue that allowing for non-stationary μ_t is not just a generalization that is worth considering for theoretical completeness. Indeed, it might actually better capture a key feature of the game under scrutiny. Thus, assuming a stationary or i.i.d. μ_t might compromise the precision of the resulting estimates.

Let us recall, from equation (11), that the quotes are additively separable in a common and a private component. Moreover, the private component is indeed stationary. That is, from the point of view of the econometrician each quote $r_{i,t} = \mu_t + q_{i,t}$, where μ_t is the unobservable common cost component and $q_{i,t}$ is drawn independently from the same distribution, at each day t. Therefore, Libor ($\tilde{r}_t$) can be also decomposed as $\tilde{r}_t = \mu_t + \tilde{q}_t$, where $\tilde{q}_t = \frac{1}{N} \sum_{k=\underline{n}+1}^{\bar{n}-1} q_t^{(k)}$. Notice that $\tilde{q}_t$ is also stationary, since it is just a function of the vector of normalized quotes, and it is also independent of μ_t. It follows that μ_t is stationary if and only if Libor is stationary. However, some empirical evidence suggests that Libor might not be stationary, or at least is highly persistent, when the period used for the estimation of the model is considered.

A simple eyeball analysis of the three-month USD Libor time series from 09/03/2007 to 17/05/2010 (Figure 2) suggests very high persistence. Moreover, an augmented Dickey-Fuller test does not reject the null hypothesis that Libor has a unit root. Similarly, Rose (1988) fails to reject the null hypothesis that nominal interest rates have unit roots, using quarterly data from several countries, and monthly data on Treasury bills (T-bills) returns for the US. However, such results have been disputed by other authors (e.g., Garcia and Perron (1996) argue that regime shifts might explain why the unit root hypothesis cannot be rejected when long periods including the shifts are considered).

A related question would be if the risk premium implicit in Libor also seems to exhibit non-stationary, or highly persistent, behavior. Let us then consider the TED spread, which is defined as the difference between the three-month USD Libor and the interest rate on three-month T-bills. Since the T-bills are considered risk-free, TED is a measure of credit risk. If TED were stationary, we could simply express the whole model (in particular, the individual quotes) in deviations from the three-month T-bill interest rate, and we would get rid of the possibility of a unit root in the

Figure 2: Three_Month USD LIBOR: 09/03/2007 - 17/05/2010
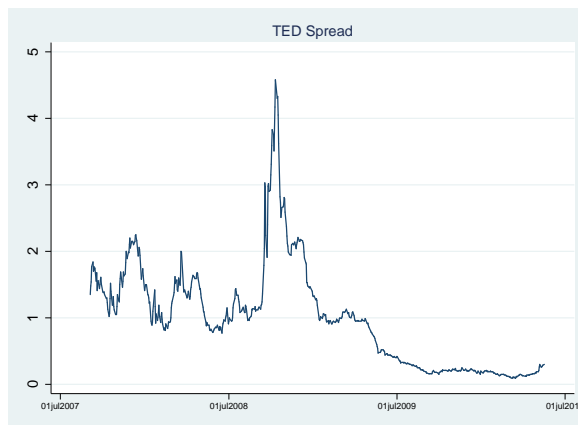
Figure 3: TED Spread: 3M USD Libor - 3M T-BILL Interest Rate

There is statistical evidence that the TED spread behaves as a non-stationary process during the financial crisis. An augmented Dickey-Fuller test (without drift, no trend term included and one lag) does not reject the null hypothesis that TED follows a random walk. The test statistic is 0.553 and the 10% critical value for rejection of the null is -2.57 (for a one-sided test). The result is robust to other specifications of the test.

common cost component. Nevertheless, once again an augmented Dickey-Fuller test fails to reject the null hypothesis that TED has a unit root (when conducted using daily data from the 09/03/2007 - 17/05/2010 period). Figure (3) shows the corresponding TED spread time series and also includes the results of the test. The test statistic is 0.553 and the 10% critical value for rejection of the null is -2.57.

Despite the evidence presented in this Section, the question of whether interest rates or risk premia have unit roots its out of the scope of this paper. My purpose here is just to argue that assuming independence or stationarity of the unobserved common cost component might result in

highly inaccurate estimates, given that both Libor and the TED spread exhibit very high persistence during the period considered.

7   Estimation

The model can be estimated using a procedure based on the deconvolution result presented in Section 5. Very similar estimation algorithms are proposed in Li and Vuong (1998), Li, Perrigne, and Vuong (2000) and Krasnokutskaya (2011). Roughly, the idea would be to use the empirical characteristic function of $(r_{i_1,t} - r_{i_2,t}, r_{i_3,t} - r_{i_2,t})$ to estimate Ψ and its derivative Ψ_1. Then, the formulas in (12) can be used to obtain estimates of the characteristic functions $\Phi_{q_{i_1}}$, $\Phi_{-q_{i_2}}$ and $\Phi_{q_{i_3}}$. Subsequently, the densities of q_{i_1}, q_{i_2} and q_{i_3} can be estimated by applying the inverse Fourier transform to the estimates of their characteristic functions. As a result, we would obtain estimated densities (distributions) of the normalized quotes $q_{i_1} - E[q_{i_1}]$, $q_{i_2} - E[q_{i_2}]$ and $q_{i_3} - E[q_{i_3}]$. Given that we observe the quotes of all banks, with their identities, this procedure could be used to estimate the distributions of the normalized quotes for all banks in the panel. From there, the estimation of the identified parameters is straightforward, since Section 5 contains explicit expressions for β_i and $v_i - \bar{v}$, as functions of the distributions of the normalized quotes.

Li, Perrigne, and Vuong (2000) establish conditions under which the estimators of the densities just described are uniformly consistent. However, this type of deconvolution estimators are known to have slow convergence rates, as shown by Li and Vuong (1998), even under strong assumptions about the smoothness of the distributions involved. Besides, the empirical characteristic function must be smoothed to guarantee the existence of the integral in the inverse Fourier transform, which requires the econometrician to make some choices regarding smoothing functions and parameters.[10] Broadly speaking, the moral is that for short samples (say, less than 500 observations) the density estimators would have non-negligible noise and would be highly sensitive to the choice of a particular smoothing method.

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[10] See Ch. 5 in Horowitz (2009) for a general description of the deconvolution estimator and a discussion on the slow rate of convergence of the corresponding density estimator.

7.1 Alternative Estimation Procedure

As an alternative, I propose an estimation method that has a noise term that does not vanish asymptotically, but still has low variance relative to the variance of the quotes, as I show below. From Section 5, under maintained assumptions, the quotes are additively separable in a common component and a purely private component. That is, $r_{i,t} = \mu_t + q_{i,t}$, for all $i \in \mathcal{N}$ and all t. Therefore, $\bar{r}_t = \mu_t + \bar{q}_t$, where $\bar{r}_t = \frac{1}{N} \sum_{i \in \mathcal{N}} r_{i,t}$ and $\bar{q}_t$ is analogously defined. Now, let $\xi_t = \bar{q}_t - E[\bar{q}]$, be the deviation of the intraday average of the normalized quotes from its (long run) mean. It follows that

$$r_{i,t} - \bar{r}_t = q_{i,t} - E[\bar{q}] - \xi_t$$

where, by construction, $E[\xi_t] = 0$ and $Var[\xi_t] = Var[\bar{q}_t] = \frac{1}{N^2} \sum_{i \in \mathcal{N}} Var[q_{i,t}]$. The idea is to use the observable deviation of the individual quotes from their intraday average, $r_{i,t} - \bar{r}_t$, as (moderately) noisy measures of the normalized quotes $q_{i,t} - E[\bar{q}]$.[11]

By subtracting the intraday mean from the observed quotes we are removing the unobserved heterogeneity almost entirely, except for the remaining mean-zero error term ξ_t, that has a variance one order of magnitude lower than the variance of the normalized quotes. For each day t in the sample, and each bank i in the panel, the observed $r_{i,t} - \bar{r}_t$ is an independent draw from the distribution of the normalized quote $q_i - E[\bar{q}]$ (plus noise $-\xi_t$). Thus, the entire sample $\{r_{i,t} - \bar{r}_t\}_{t=1}^{T}$ can be used to estimate the (empirical) distribution of $q_i - E[\bar{q}]$, for each $i \in \mathcal{N}$. Section 5 shows that the distributions of $q_i - E[q_{i_1}]$ are identified. Equivalently, the distributions of $q_i - E[\bar{q}]$ are identified, since $q_i - E[\bar{q}] = q_i - E[q_{i_1}] - \frac{1}{N} \sum_{i \in \mathcal{N}} E[q_i - E[q_{i_1}]]$. Thus, all the results concerning (the distributions of) $q_i - E[q_{i_1}]$ also hold for $q_i - E[\bar{q}]$, after substituting $E[q_{i_1}]$ for $E[\bar{q}]$ everywhere. In other words, in Section 5.2, we could have considered the hypothetical game with $\mu = -E[\bar{q}]$ (instead of $\mu = -E[q_{i_1}]$) and all the results would hold.

An advantage of this alternative estimation procedure, compared to the one based on deconvolution, is that it identifies (with low-variance mean-zero error) the actual realizations of the normalized quotes for each t, and not just their distributions. That is, suppose that at day t, bank i observes

---

[11]Even though $E[\xi_t] = 0$, it should be noted that ξ_t cannot be interpreted as classical measurement error, since it is correlated with $q_{i,t}$.

the cost $s_{i,t} = \mu_t + \epsilon_{i,t}$, for a specific realization of $\epsilon_{i,t}$, and correspondingly submits a quote $r_{i,t}$. Then, $r_{i,t} - \bar{r}_t$ identifies (with low-variance noise) the quote that bank i would have submitted in a counterfactual game with $\mu_t = -E[\bar{q}]$, if it had faced the cost $s_{i,t} = -E[\bar{q}] + \epsilon_{i,t}$ (for the same realization of $\epsilon_{i,t}$). Thus, for each t, $(r_{1,t} - \bar{r}_t, ..., r_{N,t} - \bar{r}_t)$ is an estimate of the vector of realized normalized quotes, corresponding to the actual purely private costs observed by banks, $(\epsilon_{1,t}, ..., \epsilon_{N,t})$, under the normalization $\mu_t = -E[\bar{q}]$.

Let $\tilde{q}_{i,t} = r_{1,t} - \bar{r}_t$, it follows from the previous argument that we have the whole sample $\{(\tilde{q}_{1,t}, ..., \tilde{q}_{N,t})\}_{t=1}^{T}$ of realized normalized quotes, available for the estimation of the model. In fact, an estimator of $\phi_i(r_{i,t}|\mu_t)$, the probability that bank i's quote is included in the computation of the reference rate, can be easily obtain from it. Intuitively, these probabilities do not really depend on the common cost component μ_t (despite the notation), due to the additive separability of the quotes, only on the distributions of the purely private components, and thus can be computed from the normalized quotes. Formally,

$$
\begin{aligned}
\phi_i(r_{i,t}|\mu_t) &= P_i\left\{r_t^{(n)} \leq r_{i,t} \leq r_t^{(\bar{n})}|\mu_t\right\} \\
&= P_i\left\{r_t^{(n)} - \mu_t \leq r_{i,t} - \mu_t \leq r_t^{(\bar{n})} - \mu_t\right\} \\
&= P_i\left\{q_t^{(n)} \leq q_{i,t} \leq q_t^{(\bar{n})}\right\} \\
&= P_i\left\{q_t^{(n)} - E[\bar{q}] \leq q_{i,t} - E[\bar{q}] \leq q_t^{(\bar{n})} - E[\bar{q}]\right\}
\end{aligned}
$$

where P_i denotes the probability from the point of view of bank i, that is, when μ_t is observed and the quotes of all other banks are regarded as random variables with known distributions. $r_t^{(n)}$ and $r_t^{(\bar{n})}$ ($q_t^{(n)}$ and $q_t^{(\bar{n})}$) are order statistics of the vector of all (normalized) quotes. Hence, a natural estimator of $\phi_i(r_{i,t}|\mu_t)$ would be:

$$
\hat{\phi}_i(r_{i,t}|\mu_t) = \frac{1}{T}\sum_{\tau=1}^{T} \mathbf{1}\left(\tilde{q}_\tau^{(n)} \leq \tilde{q}_{i,t} \leq \tilde{q}_\tau^{(\bar{n})}\right) \tag{17}
$$

However, to increase the efficiency of $\hat{\phi}_i(r_{i,t}|\mu_t)$, I use a resampling method based on Hortaçsu (2000) and Hortaçsu and McAdams (2010). The procedure works as follows. Fix any bank i, and

for each other bank $j \neq i$, take T_s draws with replacement from the sample $\{\tilde{q}_{j,t}\}_{t=1}^T$ of normalized quotes. Let $q_{j,\tau}^s$ denote the τ-th of such draws. Then, build T_s possible scenarios for bank i, that is T_s vectors of normalized quotes, $\left\{ q_{-i,\tau}^s \right\}_{\tau=1}^{T_s}$ that the other banks could have submitted, given the distribution of their normalized quotes, which is known by bank i in equilibrium. Finally, for each $r_{i,t}$ estimate the probability $\phi_i\left(r_{i,t}|\mu_t\right)$ by

$$\hat{\phi}_i\left(r_{i,t}|\mu_t\right) = \frac{1}{T^s} \sum_{\tau=1}^{T^s} \mathbf{1}\left(q_\tau^{s(\underline{n})} \leq \tilde{q}_{i,t} \leq q_\tau^{s(\bar{n})}\right) \tag{18}$$

With an estimate of $\phi_i\left(r_{i,t}|\mu_t\right)$ at hand, the preference parameter β_i could then be estimated using the sample counterpart of the expression for β_i in Proposition 4. However, below I propose a different estimator that is bounded by the inequalities in (19) in order to guarantee that the estimated equilibrium strategies are, in fact, well defined functions of the costs. Suppose, for now, that we have such an estimate $\hat{\beta}_i$, for all $i \in \mathcal{N}$. Then, the private cost component observed by each bank i at each date t in the sample, $\epsilon_{i,t}$ can be estimated by

$$\hat{\epsilon}_{i,t} = \tilde{q}_{i,t} - \frac{1}{T}\sum_{t=1}^T \tilde{q}_{i,t} - \hat{\beta}_i\left(\hat{\phi}_i\left(\tilde{q}_{i,t}| - E\left[\bar{q}\right]\right) - \frac{1}{T}\sum_{t=1}^T \hat{\phi}_i\left(\tilde{q}_{i,t}| - E\left[\bar{q}\right]\right)\right)$$

which is the empirical counterpart of equation (14). Notice that $\hat{\phi}_i\left(\tilde{q}_{i,t}| - E\left[\bar{q}\right]\right) = \hat{\phi}_i\left(r_{i,t}|\mu_t\right)$, as defined in equation (17). Similarly, an estimate of $v_i - \bar{v}$, the signaling parameter of bank i's utility function, is given by

$$\hat{v}_i = -\frac{1}{T}\sum_{t=1}^T \tilde{q}_{i,t} + \hat{\beta}_i\frac{1}{T}\sum_{t=1}^T\left(\hat{\phi}_i\left(\tilde{q}_{i,t}| - E\left[\bar{q}\right]\right) - \frac{1}{N}\sum_{i\in\mathcal{N}}\hat{\beta}_i\hat{\phi}_i\left(\tilde{q}_{i,t}| - E\left[\bar{q}\right]\right)\right)$$

the empirical counterpart of equation (16). Moreover, lower bounds on each v_i can be obtained from 5 by letting the lowest one of all v_i's be equal to zero or, equivalently, by setting:

$$\underline{\hat{v}}_i = \hat{v}_i - \min\left\{\hat{v}_i : i \in \mathcal{N}\right\}$$

With such lower bounds at hand, we can even estimate a lower bound for the whole process

$\{\mu_t\}_{t=0}^{T}$ of the common cost components, since $\mu_t = r_{i,t} - \beta_i \phi_i \left(r_{i,t} | \bar{\mu} \right) + v_i - \epsilon_{i,t}$, from 8, and we either observe or can estimate all the terms at the right hand side of this expression. Clearly, an estimate for this lower bound is given by

$$\bar{\mu}_t = \frac{1}{N} \sum_{i=1}^{N} \left(r_{i,t} - \hat{\beta}_i \hat{\phi}_i \left(\tilde{q}_{i,t} | - E\left[\bar{q} \right] \right) + \hat{\underline{v}}_i - \hat{\epsilon}_{i,t} \right)$$

7.2 Estimation of β_i

The identification of β_i relies on the assumption $\mathrm{Med}\left(\epsilon_i \right) = 0$. In fact, Section 5 shows that under such assumption, β_i can be explicitly expressed as a function of the distributions of the normalized quotes. Therefore, we should be able to define a point estimator of β_i that is a function of the estimated distributions of the normalized quotes, regardless of the method used to obtain the latter. I propose such an estimator in this section, but before I derive a result of the model that will prove to be useful in the estimation of β_i, since it put bounds on the values of β_i that are consistent with the optimality of the observed quotes $r_{i,t}$.

As previously shown, for all $s_i \in \mathcal{S}\left(\mu \right)$, the optimal quote $\rho_i \left(s_i \right)$ satisfies

$$s_i = \rho_i \left(s_i \right) - \beta_i \phi_i \left(\rho_i \left(s_i \right) | \bar{\mu} \right) + v_i$$

Assuming further that the best response ρ_i is differentiable[12], it follows that, for all $s_i \in \mathcal{S}\left(\mu \right)$

$$\frac{d\rho_i \left(s_i \right)}{ds_i} = \left(1 - \beta_i \frac{d\phi_i \left(\rho_i \left(s_i \right) | \mu \right)}{dr_i} \right)^{-1}$$

Since ρ_i is strictly increasing in $\mathcal{S}\left(\mu \right)$, $\frac{d\rho_i(s_i)}{ds_i} > 0$ and thus $1 - \beta_i \frac{d\phi_i(\rho_i(s_i)|\mu)}{dr_i} > 0$, for all $s_i \in \mathcal{S}\left(\mu \right)$. Therefore, for all optimal r_i, in particular, for any equilibrium quote r_i,

$$1 - \beta_i \frac{d\phi_i \left(r_i | \mu \right)}{dr_i} > 0 \tag{19}$$

---

[12]I assume differentiability of the best response functions only for notational convenience, but the bounds on β_i can also be derived from weaker assumptions using only one-sided derivatives.

or, equivalently,

$$
\begin{aligned}
\beta_i &< 1/\frac{d\phi_i\left(r_i|\mu\right)}{dr_i} \text{ if } \frac{d\phi_i\left(r_i|\mu\right)}{dr_i} > 0 \\
\beta_i &> 1/\frac{d\phi_i\left(r_i|\mu\right)}{dr_i} \text{ if } \frac{d\phi_i\left(r_i|\mu\right)}{dr_i} < 0
\end{aligned}
\tag{20}
$$

Now let $\mu = 0$, and define two sets of normalize quotes Q_i^+ and Q_i^- by,

$$
Q_i^+ = \left\{ q_i \in \left(\underline{q}_i, \bar{q}_i\right) : \frac{d\phi_i\left(q_i\right)}{dq_i} > 0 \right\}
$$

and

$$
Q_i^- = \left\{ q_i \in \left(\underline{q}_i, \bar{q}_i\right) : \frac{d\phi_i\left(q_i\right)}{dq_i} < 0 \right\}
$$

It follows that,

$$
1/\inf_{q_i \in Q_i^-}\left(\frac{d\phi_i\left(q_i\right)}{dq_i}\right) \leq \beta_i \leq 1/\sup_{q_i \in Q_i^+}\left(\frac{d\phi_i\left(q_i\right)}{dq_i}\right)
\tag{21}
$$

These inequalities put bounds on the values of β_i that rationalize the observed (normalized) quotes. It is possible, though, that the set $Q_i^-(Q_i^+)$ is empty for some banks. That is, it might be the case that some banks never choose a quote such that the derivative of the probability that such quote is included in the computation of the reference rate is negative (positive). In such case the lower (upper) bound is just $-\infty$ (∞).

We have shown so far, then, that under Assumptions 1-4, a set of distributions of the normalized quotes is rationalized by the model in Section 4 only if β_i lies within the bounds in (21), for all $i \in \mathcal{N}$, and, more strongly, only if β_i satisfies equation (15) in Proposition 4. Clearly, any β_i that satisfies (15) also meets the inequalities defining these bounds. However, despite the point identification result, the bounds are still useful for estimation because, in a finite sample, the corresponding point estimate could lie outside the interval defined by the estimated bounds. In principle, we could define a criterion function for the estimation of β_i that penalizes both deviations from the sample counterpart of (15) as well as some measure of distance to the interval defined by the bounds. However, any violation of the bounds would imply that some of the observed quotes are not optimal, which contradicts the assumptions of the model. Therefore, to obtain estimates that are consistent with

the assumption that all observed quotes correspond to the BNE of the game, I define an estimator of β_i that always lies within (estimators of) the bounds in (21).

Consequently, we first need some estimators of the bounds. For the estimation of $\frac{d\phi_i(q_i)}{dq_i}$, I apply a standard two-point finite difference formula to the estimate $\hat{\phi}_i(\tilde{q}_{i,t}| - E[\bar{q}])$ at each observed normalized quote $\tilde{q}_{i,t}$. For brevity, let me denote this estimate $\hat{\phi}'_i(\tilde{q}_{i,t})$. Then I estimate the sets Q_i^+ and Q_i^- as

$$\hat{Q}_i^+ = \left\{\tilde{q}_{i,t} \in \left[\underline{\hat{q}}_i, \bar{\hat{q}}_i\right] : \hat{\phi}'_i(\tilde{q}_{i,t}) > 0\right\}$$

and

$$\hat{Q}_i^- = \left\{\tilde{q}_{i,t} \in \left[\underline{\hat{q}}_i, \bar{\hat{q}}_i\right] : \hat{\phi}'_i(\tilde{q}_{i,t}) < 0\right\}$$

where $\underline{\hat{q}}_i = \min\{\tilde{q}_{i,t} : t = 1, ..., T\}$ and $\bar{\hat{q}}_i = \max\{\tilde{q}_{i,t} : t = 1, ..., T\}$ are just the minimum and the maximum normalized quotes observed in the sample. The corresponding estimators of the lower and upper bounds, denoted $\hat{\beta}_i^{lb}$ and $\hat{\beta}_i^{ub}$, respectively, are:

$$\hat{\beta}_i^{lb} = \begin{cases} 1/\min_{q_i \in \hat{Q}_i^-}\{\hat{\phi}'_i(\tilde{q}_{i,t})\} & \text{if } Q_i^- \neq \emptyset \\ b_T^{lb} & \text{if } Q_i^- = \emptyset \end{cases}$$

and

$$\hat{\beta}_i^{ub} = \begin{cases} 1/\max_{q_i \in \hat{Q}_i^+}\{\hat{\phi}'_i(\tilde{q}_{i,t})\} & \text{if } Q_i^+ \neq \emptyset \\ b_T^{ub} & \text{if } Q_i^+ = \emptyset \end{cases}$$

for some constants $b_T^{lb} < 0$ and $b_T^{lb} > 0$ that grow unboundedly (in absolute value) with the sample size T.

Again, since $\text{Med}(\epsilon_i) = 0$ is the key assumption for the identification of β_i, the idea is to find a value $\hat{\beta}_i$ in the interval $\left[\hat{\beta}_i^{lb}, \hat{\beta}_i^{ub}\right]$ that minimizes the squared median of the sample of private shocks to the borrowing costs implied by the inverse equilibrium strategies, and the observed distributions of the normalized quotes.

For any $b \in \left[\hat{\beta}_i^{lb}, \hat{\beta}_i^{ub} \right]$, let

$$\epsilon_{i,t}(b) = \tilde{q}_{i,t} - \frac{1}{T} \sum_{t=1}^{T} \tilde{q}_{i,t} - b \left(\hat{\phi}_i \left(\tilde{q}_{i,t} \middle| - E[\bar{q}] \right) - \frac{1}{T} \sum_{t=1}^{T} \hat{\phi}_i \left(\tilde{q}_{i,t} \middle| - E[\bar{q}] \right) \right)$$

The estimator just proposed can be formally defined as

$$\hat{\beta}_i = \underset{b \in \left[\hat{\beta}_i^{lb}, \hat{\beta}_i^{ub} \right]}{\operatorname{argmin}} \left(\operatorname{Med} \left\{ \epsilon_{i,t}(b) : t = 1, ..., T \right\} \right)^2 \tag{22}$$

which is a minimum distance estimator.

Instead of relying on its asymptotic distribution, we use bootstrapping to estimate its variance and to perform inference, as described in the Appendix.

8 Results

The model in Section 4 is estimated using only three-month USD Libor quotes, for three different sample periods covering the financial crisis: (i) 09/03/2007 - 09/14/2008, (ii) 09/15/2008 - 12/31/2008 and (iii) 02/09/2009 - 05/17/2010. (i) corresponds to the period in between BNP Paribas' announcement and Lehman's bankruptcy. (ii) Starts the day Lehman filed for bankruptcy and goes until the end of 2008, when the BBA made a change in the composition of the USD Libor panel.[13] Finally, (iii) extends until May 2010, when there seems to be a sudden and persistent change in the behavior of Libor, relative to other measures of average borrowing costs in interbank markets. The main criterion to choose these periods is that at the reference dates there is an apparent structural change in borrowing costs, or there are reasons to expect changes in banks' incentives to misreport (see Figure 1). Within each period, we estimate a lower bound for each realization of the common cost component, the daily idiosyncratic cost shocks faced by each bank, and all the parameters in the utility function of the banks. It is worth emphasizing that the common cost component is precisely what Libor is meant to capture. Thus, we recover a lower bound on the

---

[13]Since banks are ex-ante heterogeneous, and all the parameters in their utility functions are common knowledge, a change in the Libor panel implies a new game structure. Therefore, I separately estimate the model before and after this change.

truthful Libor, that is, the value that Libor should have taken if all banks had truthfully reported their borrowing costs. In order to estimate the corresponding parameters, the potential benefits for the banks from manipulating Libor or misreporting their borrowing costs, and the expected costs from untruthful reporting, are held constant for each period. Hence, the corresponding estimates are, roughly, averages of these possibly time varying quantities. In principle, the model could be estimated separately with data from more, but shorter periods, to capture more subtle changes in those parameters, at the cost of decreasing the precision of the estimators. Instead, I choose as reference dates for splitting the sample, two events that are well known for having caused large disruptions in interbank markets (BNP Paribas and Lehman) and another one that changed a main feature of the game (i.e., the set of banks in the panel).

8.1 Estimation of the Borrowing Costs

The estimates of the common cost component μ_t reported in this section provide an estimated lower bound for the value that the Libor should have had, if all banks had truthfully reported their borrowing cost. Since most of the previous literature have found that if banks succeeded in manipulating Libor during the period considered here, they actually pushed it downwards, most likely the following results are conservative estimates of the extent of such manipulation. Figure 4 illustrates these results. The left panel shows the daily estimated spread between Libor and the common cost component. According to these estimates, Libor always lies below the common cost, confirming the results of other studies, with an average spread of -23bp at the worst of the financial crisis, reaching a maximum absolute deviation of 32bp on 09/24/2008, only seven days after Lehman's bankruptcy. However, there is no strong evidence of manipulation in the period before Lehman, when the average lower bound of the spread is above -2bp. Finally, for the period starting on 02/09/2009, the average spread is -6pb, and it displays very low daily variation. The right panel compares the estimated common cost to two other measures of average borrowing costs in the interbank market, already described in Section 3. On 82% of the days after 06/02/2008, when ICAP started publishing the NYFR, our estimated truthful Libor lies between the other two measures, and is consistently closer to the NYFR. This suggests that, after Lehman's bankruptcy,
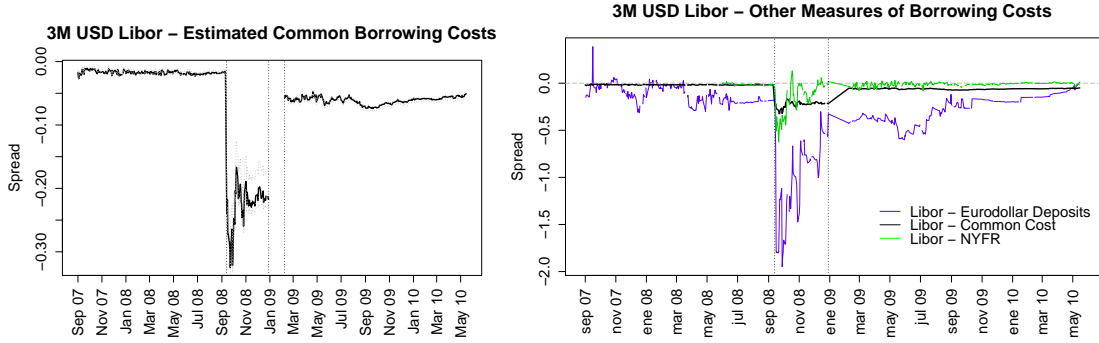
Figure 4: Spreads between Libor and the Estimated Average Borrowing Costs

The left panel shows the spread between Libor and the estimated lower bound for the common cost component μ_t (our measure of the counterfactual truthful Libor). The two highlighted dates are 09/15/2008, when Lehman Brothers filed for bankruptcy protection, and 12/31/2008, when the BBA changed the composition of the Libor panel. I have no data from 01/01/2009 to 02/09/2009. At the worst of the crisis, Libor was below the estimated average borrowing costs of the banks in the USD Libor panel by more than 30 bp. 95% confidence intervals are shown in gray dotted lines.

The right panel compares the estimated common cost component to two other measures of average borrowing costs already described in Section 3 and Figure 1. On 82% of the days after 06/02/2008, when the NYFR became available, our estimated truthful Libor lies between NYFR and H15 (Eurodollar Deposits Rate).

NYFR was a more accurate measure of average costs in the interbank market than the reported USD 3M Libor. Moreover, it is an indication that the Libor - NYFR, and Libor - H15 Eurodollar deposits rate spreads could be interpreted as providing evidence of manipulation, despite the caveats put in place by Kuo, Skeie, and Vickery (2012).

8.2   Incentives to Misreport

As previously shown, we recover the whole time series of the common cost component μ_t (daily lower bounds) and all the realizations of the daily idiosyncratic shocks $\epsilon_{i,t}$. As a result, we can also obtain lower bounds for the true daily borrowing cost of each bank, $s_{i,t} = \mu_t + \epsilon_{i,t}$, as well as for the deviations of their Libor quotes from these borrowing costs, $r_{it} - s_{i,t}$. Furthermore, the difference between a bank's quote and its funding cost, at time t, can be separated into two components, one that captures the deviation due to the bank's portfolio exposure to Libor and another that measures misreporting as motivated by signaling. In fact,

$$r_{it} - s_{i,t} = \underbrace{\frac{\alpha_i}{\tilde{N}} \phi_i\left(r_{i,t} | \mu_t\right)}_{\text{Portfolio}} + \underbrace{\frac{v_i}{\tilde{N}} \phi_i\left(r_{i,t} | \mu_t\right) - v_i}_{\text{Signaling}} \tag{23}$$

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Therefore, after identifying v_i and α_i, we can estimate to what extent misreporting of borrowing costs corresponded to each of these two types of incentives. Table 1 contains estimates of the respective terms in equation (23), for each bank. Even though we can estimate the whole time series of these two expressions, I only report their averages for each of the three periods in which the sample is divided. The estimates show substantial heterogeneity among banks. Again, it seems reasonable to obtain such heterogeneity in banks' portfolio exposure to Libor, given the large range of derivatives that are pegged to the rate. In fact, Gandhi, Golez, Jackwerth, and Plazzi (2014) find considerable variation in such exposure, both among banks and through time, as measured by equity return sensitivity to Libor changes. Considering that the parameters v_i and α_i measure the benefits from misreporting relative to the expected costs of being sanctioned by the regulator, a possible explanation for the heterogeneity in signaling are differences in the perceived costs of misreporting (which depend on the expected probability of being detected) and changes on this perception through time.

For most banks signaling seems to be the main driver of systematic misreporting, in all three periods. Clearly, period (ii) displays the largest deviations of bank's quotes from their borrowing costs. According to these estimates, in period (ii) signaling alone is responsible for differences of approximately 20bp, on average, and this number is only a lower bound. In contrast, during the same period, for most of the banks, the differences due to portfolio exposures are lower than 5bp (in magnitude). Notable exceptions are Citibank (-16bp) and Lloyds Bank (-15bp). For the other two periods, the indications are less clear. In contrast, for period (i) there is no strong statistical evidence of misreporting. In this period, the largest estimated deviation due to signaling is -3bp (JPM), and the differences attributable to portfolio exposures are all below 1bp. Finally, in period (iii) the portfolio incentives remain low, but there is a persistent effect of the signaling incentives from period (ii), with a deviation of at least 5bp, on average, between banks' borrowing costs and their reported quotes.

In the Appendix, I report a robustness check based on an alternative estimation of the model that do not imposes the assumption that the private costs distributions have the same mean for all banks. Instead, I estimate these possibly different means, but I restrict the parameter v associated with signaling to be the same for all banks. In this version of the model, an additional measure of the

	(i): 09/032007 - 09/14/2008		(ii): 09/14/2008 - 12/31/2008		(iii): 02/09/2009 - 05/17/2010	
	Signaling	Portfolio	Signaling	Portfolio	Signaling	Portfolio
BOA	-2.7*	0.2	-29.5*	1.8*	-6.9*	0.7*
BTMU	-1.3*	0.5*	-13.9*	2.4*	-0.9*	0.5*
Barclays	0.0	0.5*	0.0	3.7*	-4.0*	-2.1*
Citibank	-2.1*	-1.0*	-15.9*	-15.6*	-7.8*	0.1
Credit Suisse	-0.4	-0.7*	-13.2*	1.2	-1.7*	-2.8*
Deutsche	-2.1*	0.1	-19.0*	-0.7	-8.5*	-1.0*
HBOS	-0.6	-0.5*	-7.4*	-5.1*	-7.8*	-1.3
HSBC	-2.4*	-0.6*	-30.8*	1.8*	-10.3*	-0.1
JPM	-2.8*	0.1	-47.7*	-0.9*	-7.2*	1.2*
Lloyds	-2.3*	0.2*	-16.3*	-14.8*	0.0	-0.1
Norinchukin	-0.6*	0.3*	-19.4*	3.1*	-0.8*	-3.0*
RBC	-1.6*	-0.3	-25.2*	1.6	-0.6	-0.8*
RBS	-2.2*	-0.1	-11.5*	-1.8*	-8.5*	0.6*
Rabobank	-2.4*	-1.0*	-34.7*	-4.0*	-7.2*	0.7*
UBS	-2.4*	0.0	-26.0*	3.1*	-3.3*	-1.9*
WestLB	-1.0*	-0.0	-24.9*	2.9*	-0.7*	0.7*

Table 1: Signaling vs Portfolio Incentives

This table presents estimates of two separate components of the average differences $s_i - r_i$, that depend on whether incentives to misreport are driven by signaling or portfolio exposures to Libor.

*Signicant at the 5% level (Inference is performed using the bootstrap, as described in the Appendix).

common cost component μ_t is necessary to separately identify the signaling and the portfolio drivers of misreporting. I use the NYFR and the Eurodollar deposits rate as plausible approximations to the common cost. In both cases, signaling seems to be the main driver of misreporting, which confirms the results in Table 1. Thus, the former are robust to a relaxation of the assumption about the means of the private costs.

9   Conclusion

This paper serves as an illustration that econometric methods from the empirical auctions literature can be applied to a broader class of Bayesian games, to identify and estimate parameters of the payoff functions and the distributions of the players' types. Despite evidence that Libor quotes did not accurately represent interbank rates in recent years, in the specific application studied, a structural analysis of the strategic interaction between banks helps to identify their incentives to misreport, and to estimate a more precise measure of their borrowing costs. In particular, the identification

strategy proposed here allows for an estimation of a lower bound on the "truthful Libor", defined as the value that the published Libor would have had if banks had truthfully reported their borrowing costs. It is worth emphasizing that the aforementioned result relies solely on the banks' quotes and the model of strategic interaction that rationalizes those quotes, no additional data is needed for identification. When compared to other available measures of average borrowing costs, the estimated bound is generally closer than the published Libor, which validates the estimation results. Looking forward, this paper might contribute to evaluate reforms to Libor regulation and to design alternative benchmark interest rates that are not prone to manipulation. Specifically, the results in the last section of the paper suggest that the recent decision to delay the publication of the quotes, until three months after their submission, should increase their reliability substantially, since using quotes as signals of credit worthiness seems to have been the main determinant of misreporting.

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Appendix

Here I include the proofs of several claims and propositions stated in the main body of the paper, as well as a description of the method used to derive confidence intervals for the reported estimates, using the bootstrap. I also present the results of an alternative estimation that allows the means of the private costs components to differ across banks.

Nondecreasing Best Response Correspondences

Let the best response correspondence of bank i be defined by

$$\Gamma_i(s_i) = \underset{r_i \in \mathbb{R}}{\operatorname{argmax}} E\left[\alpha_i \tilde{r}(r_i, r_{-i}) + v_i(\tilde{r}(r_i, r_{-i}) - r_i) - \gamma_i(s_i - r_i)^2\right]$$

We will show now that Γ_i is nondecreasing in the following sense (which is stronger than nondecreasing in the strong set order, as defined in Athey (2001)). For all $s, s' \in S$. with $s < s'$, $\max \Gamma(s_i) \leq \min \Gamma(s'_i)$.

Let $s_i \in \mathcal{S}(\mu)$, and $r_i = \max \Gamma(s_i)$. By definition, for all $r < r_i$

$$E\left[\alpha_i \tilde{r}(r_i, r_{-i}) + v_i(\tilde{r}(r_i, r_{-i}) - r_i) - \gamma(s_i - r_i)^2\right] \quad \geq \quad E\left[\alpha_i \tilde{r}(r, r_{-i}) + v(\tilde{r}(r, r_{-i}) - r) - \gamma(s_i - r)^2\right]$$

$$\Longleftrightarrow$$

$$(\alpha_i + v_i)(E[\tilde{r}(r_i, r_{-i})] - E[\tilde{r}(r, r_{-i})]) \quad \geq \quad v_i(r_i - r) + 2\gamma s_i(r - r_i) + \gamma_i(r_i^2 - r^2)$$

$$\Longleftrightarrow$$

$$2\gamma_i s_i(r_i - r) \quad \geq \quad v_i(r_i - r) + \gamma_i(r_i^2 - r^2) - (\alpha_i + v_i)(E[\tilde{r}(r_i, r_{-i})] - E[\tilde{r}(r, r_{-i})])$$

Moreover, since $\gamma_i > 0$, for any $s'_i > s_i$, $2\gamma_i s'_i(r_i - r) > 2\gamma_i s_i(r_i - r)$, then

$$2\gamma_i s'_i(r_i - r) \quad > \quad v_i(r_i - r) + \gamma(r_i^2 - r^2) - (\alpha_i + v)(E[\tilde{r}(r_i, r_{-i})] - E[\tilde{r}(r, r_{-i})])$$

$$\Longleftrightarrow$$

$$E\left[\alpha_i \tilde{r}(r_i, r_{-i}) + v_i(\tilde{r}(r_i, r_{-i}) - r_i) - \gamma_i(s'_i - r_i)^2\right] \quad > \quad E\left[\alpha_i \tilde{r}(r, r_{-i}) + v_i(\tilde{r}(r, r_{-i}) - r) - \gamma_i(s'_i - r)^2\right]$$

Thus, for all $s'_i > s_i$, r_i yields a strictly higher payoff than any $r < r_i$. Therefore, any element of $\min \Gamma(s'_i)$ is at least as large as r_i, in particular $\max \Gamma(s_i) \leq \min \Gamma(s'_i)$.

Proof of equation (5).

We want to show that $\frac{\partial E_i[\tilde{r}(r_i, r_{-i})|\mu]}{\partial r_i}$ is equal to the probability that r_i is included in the computation of the reference rate $\tilde{r}$. The probability is computed from the perspective of bank i, that is, when the quotes of all other banks are random variables with known probability distributions (conditional on the common cost component μ).

Proof. Let $R_{-i}^{(n)}$ be the n-th order statistic of the vector r_{-i} of quotes submitted by all banks other than i. From the perspective of bank i, $R_{-i}^{(n)}$ is a random variable with known distribution $G_{(n)|\mu}$. I follow the convention $R_{-i}^{(1)} \leq R_{-i}^{(2)} \leq ... \leq R_{-i}^{(N-1)}$. Notice that, regardless of i's quote, r_i, for all $k \in \{\underline{n}+1, ..., \bar{n}-2\}$, $R_{-i}^{(k)}$ is included in the computation of the reference rate $\tilde{r}$. Moreover, whether r_i, $R_{-i}^{(\underline{n})}$ or $R_{-i}^{(\bar{n}-1)}$ is the other quote included, depends on their relative positions. For instance, with probability $1 - G_{(\underline{n})|\mu}(r_i)$, $R_{-i}^{(\underline{n})} > r_i$, and in such case $R_{-i}^{(\underline{n})}$ would be included.

Therefore, the expected value of the reference rate $\tilde{r}$, when i submits quote r_i can be written as:

$$
\begin{aligned}
E\left[\tilde{r}(r_i)|\mu\right] = & \frac{1}{\tilde{N}}\left(\sum_{k=\underline{n}+1}^{\bar{n}-2} E\left[R_{-i}^{(k)}|\mu\right] + E\left[R_{-i}^{(\underline{n})}|R_{-i}^{(\underline{n})} > r_i, \mu\right]\left(1 - G_{(\underline{n})|\mu}(r_i)\right)\right. \\
& + r_i\left(G_{(\underline{n})|\mu}(r_i) - G_{(\bar{n}-1)|\mu}(r_i)\right) \\
& \left. + E\left[R_{-i}^{(\bar{n}-1)}|R_{-i}^{(\bar{n}-1)} \leq r_i, \mu\right]G_{(\bar{n}-1)|\mu}(r_i)\right)
\end{aligned}
$$

Notice that

$$
E\left[R_{-i}^{(\underline{n})}|R_{-i}^{(\underline{n})} > r_i, \mu\right]\left(1 - G_{(\underline{n})|\mu}(r_i)\right) = \int_{r_i}^{\bar{r}} x g_{(\underline{n})|\mu}(x)\, dx
$$

where $g_{(\underline{n})|\mu}$ is the probability density function of $R_{-i}^{(\underline{n})}$. A similar expression can be easily found for $E\left[R_{-i}^{(\bar{n}-1)}|R_{-i}^{(\bar{n}-1)} \leq r_i, \mu\right]G_{(\bar{n}-1)|\mu}(r_i)$ and thus, it follows from Leibniz integral rule that:

$$
\frac{\partial E\left[\tilde{r}(r_i)|\mu\right]}{\partial r_i} = \frac{1}{\tilde{N}}\left(G_{(\underline{n})|\mu}(r_i) - G_{(\bar{n}-1)|\mu}(r_i)\right)
$$

where $G_{(\underline{n})|\mu}(r_i) - G_{(\bar{n}-1)|\mu}(r_i)$ is precisely the probability that r_i is included in the computation of $\tilde{r}$. □

Proof of Proposition 1

Proposition. *Let $\mu \neq 0$, if the strategy of all bank $j \neq i$ is*

$$\rho_j \left(\mu + \delta_j + \epsilon_j; \mu \right) = \mu + \rho_j \left(\delta_j + \epsilon_j; 0 \right)$$

Then i's best response is $\rho_i \left(\mu + \delta_i + \epsilon_i; \mu \right) = \mu + \rho_i \left(\delta_i + \epsilon_i; 0 \right)$.

Proof. Consider the game with $\mu = 0$ and let $q_i = \rho_i \left(\delta_i + \epsilon_i; 0 \right)$. Clearly, for all $\breve{q}_i \in \left[\underline{q}_i, \bar{q}_i \right]$, $u_i \left(q_i, \delta_i + \epsilon_i \right) \geq u_i \left(\breve{q}_i, \delta_i + \epsilon_i \right)$. Then

$$\left(\alpha_i + v \right) E \left[\tilde{r} \left(q_i, q_{-i} \right) | 0 \right] - vq_i - \gamma \left(\delta_i + \epsilon_i - q_i \right)^2 \geq \left(\alpha_i + v \right) E \left[\tilde{r} \left(\breve{q}_i, q_{-i} \right) | 0 \right] - v\breve{q}_i - \gamma \left(\delta_i + \epsilon_i - \breve{q}_i \right)^2$$

where q_{-i} is the vector of actions of all other players $j \neq i$ and $q_j = \rho_j \left(\delta_j + \epsilon_j; 0 \right)$. For any $\mu \neq 0$, let $r_j = \rho_j \left(\mu + \delta_j + \epsilon_j; \mu \right)$ then, by assumption $r_j = \mu + q_j$ for all $j \neq i$.

Intuitively, compared to the game where $\mu = 0$, when $\mu \neq 0$, from the point of view of i, the distributions of all other banks' quotes are the same, except for a change in a location parameter μ. Thus,

$$E \left[\tilde{r} \left(\breve{q}_i + \mu, r_{-i} \right) | \mu \right] = E \left[\tilde{r} \left(\breve{q}_i, q_{-i} \right) | 0 \right] + \mu$$

for all $\breve{q}_i \in \left[\underline{q}_i, \bar{q}_i \right]$, and it follows that

$$
\begin{aligned}
\left(\alpha_i + v \right) E \left[\tilde{r} \left(q_i + \mu, r_{-i} \right) | \mu \right] - v \left(q_i + \mu \right) - \gamma \left(\mu + \delta_i + \epsilon_i - \left(q_i + \mu \right) \right)^2 &= \\
\left(\alpha_i + v \right) \left(E \left[\tilde{r} \left(q_i, q_{-i} \right) | 0 \right] + \mu \right) - v \left(q_i + \mu \right) - \gamma \left(\delta_i + \epsilon_i - q_i \right)^2 &\geq \\
\left(\alpha_i + v \right) \left(E \left[\tilde{r} \left(\breve{q}_i, q_{-i} \right) | 0 \right] + \mu \right) - v \left(\breve{q}_i + \mu \right) - \gamma \left(\delta_i + \epsilon_i - \breve{q}_i \right)^2 &
\end{aligned}
$$

Since every $\breve{r}_i \in \left[\underline{q}_i + \mu, \bar{q}_i + \mu \right]$ can be written as $\breve{r}_i = \breve{q}_i + \mu$ for some $\breve{q}_i \in \left[\underline{q}_i, \bar{q}_i \right]$, it follows that for such $\breve{r}_i$, $u_i \left(q_i + \mu, \mu + \delta_i + \epsilon_i \right) \geq u_i \left(\breve{r}_i, +\mu + \delta_i + \epsilon_i \right)$ and, thus, $r_i = q_i + \mu$ is i's best response when $\mu \neq 0$, all other banks strategies are $\rho_j \left(\mu + \delta_j + \epsilon_j; \mu \right) = \mu + \rho_j \left(\delta_j + \epsilon_j; 0 \right)$ and its cost is $\mu + \delta_i + \epsilon_i$.

That is, in the game with μ, bank i's best response strategy is

$$\rho_i\left(\mu + \delta_i + \epsilon_i; \mu\right) = \mu + \rho_i\left(\delta_i + \epsilon_i; 0\right)$$

$\square$

Proof of Proposition 4

Proposition. *Under Assumptions 1-4, the preference parameter β_i is identified, for all $i \in \mathcal{N}$, from the distributions of the normalized quotes. Moreover,*

$$\beta_i = \frac{Med\left(q_i - E\left[q_{i_1}\right]\right) - \left(E\left[q_i\right] - E\left[q_{i_1}\right]\right)}{\phi_i\left(Med\left(q_i - E\left[q_{i_1}\right]\right) \mid - E\left[q_{i_1}\right]\right) - E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right]}$$

Proof. Notice the equation $\epsilon_i = \hat{q}_i - \beta_i \hat{\phi}_i$ can be interpreted as the inverse equilibrium strategy of bank i, in the game with $\mu = -E\left[q_{i_1}\right]$ (translated by a constant). Since the equilibrium strategies are strictly increasing, the inverse equilibrium strategies are strictly increasing as well, as functions of q_i, and hence, there is a unique q_i^* such that $\epsilon_i = 0$. It follows from equation (14) that only at q_i^*:

$$q_i^* - E\left[q_{i_1}\right] - \left(E\left[q_i\right] - E\left[q_{i_1}\right]\right) - \beta_i\left(\phi_i\left(q_i^* - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right) - E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right]\right) = 0$$

$$(24)$$

Moreover, by assumption, zero is an interior point in the support of ϵ_i. Therefore, since the inverse equilibrium strategy is strictly increasing

$$\hat{q}_i - \beta_i\hat{\phi}_i < 0 \text{ for all } q < q_i^* \quad \text{and} \quad \hat{q}_i - \beta_i\hat{\phi}_i > 0 \text{ for all } q > q_i^*$$

It follows that $P\left\{\hat{q}_i - \beta_i\hat{\phi}_i \leq 0\right\} = P\left\{q \leq q_i^*\right\}$. Since, by assumption $P\left\{\hat{q}_i - \beta_i\hat{\phi}_i \leq 0\right\} = P\left\{\epsilon_i \leq 0\right\} = \frac{1}{2}$, then $P\left\{q_i^* \leq q\right\} = \frac{1}{2}$ and hence q_i^* is the median of the distribution of q_i. Let $Med\left(X\right)$ denote the median of X. We just showed that $Med\left(\epsilon_i\right) = 0$ implies $Med\left(q_i\right) = q_i^*$. Besides, q_i^* satisfies equation (24), which uniquely identifies β_i as

$$\beta_i = \frac{Med\left(q_i - E\left[q_{i_1}\right]\right) - \left(E\left[q_i\right] - E\left[q_{i_1}\right]\right)}{\phi_i\left(Med\left(q_i - E\left[q_{i_1}\right]\right) \mid - E\left[q_{i_1}\right]\right) - E\left[\phi_i\left(q_i - E\left[q_{i_1}\right]\mid - E\left[q_{i_1}\right]\right)\right]}$$

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□

Standard Errors and Confidence Intervals Based on the Bootstrap

The method described in Section 7 requires the estimation of the bounds in equation (21). Such bounds are in the boundary of the support of the distribution of the random variable $\frac{d\phi_i(q_i)}{dq_i}$. However, the bootstrap is inconsistent for parameters in the boundary, as shown by Andrews (2000). To solve this issue, I estimate the bounds using the whole sample, and I keep this estimates fixed at each iteration of the bootstrap. Except for that modification, the procedure used is standard. That is, a random sample is obtained by sampling with replacement T vectors of normalized quotes from the whole sample $\{(q_{1,t}, ..., q_{N,t})\}_{t=1}^T$. K random samples are drawn in this way, and $\left\{\left(q_{1,t}^k, ..., q_{N,t}^k\right)\right\}_{t=1}^T$ denotes the k-th such sample. For each k, the estimation procedure described in Section 7 is applied to $\left\{\left(q_{1,t}^k, ..., q_{N,t}^k\right)\right\}_{t=1}^T$, except for the modification just mentioned, to obtain a bootstrap replication of the vector of estimates $\hat{\theta}$, denoted $\hat{\theta}^k$. For simplicity, we use θ here for the vector of all identified parameters. The K replications are used to compute standard errors for $\hat{\theta}$, and confidence intervals for θ. The confidence intervals are based on the 0.05 and 0.95 quantiles of the distribution of the root $\sqrt{T}\left(\hat{\theta}^k - \hat{\theta}\right)$, under the conjecture that it consistently estimates the distribution of the root $\sqrt{T}\left(\hat{\theta} - \theta\right)$.

Robustness Check: Heterogeneous Private Costs Means

I conduct an alternative estimation where I relax the assumption that the mean of the private costs distribution is the same for all banks. Instead, I estimate these means for all banks (allowing them to differ), but now I restrict the parameter v associated with signaling to be the same for all banks. In this version of the model, an additional measure of the common cost component μ_t is necessary to separately identify the signaling and the portfolio drivers of misreporting. I use the NYFR and the Eurodollar deposits rate (H15) as plausible approximations to the common cost. In both cases, signaling seems to be the main driver of misreporting, which confirms the results of Section 8.2. Thus, the former do not depend on the assumption about the means of the private costs.

Table 2 presents the results of this alternative estimation. There are substantial differences in

magnitude depending on whether H15 or NYFR is used to approximate μ_t, given the large spread between these two rates (see Figure 1). However, regardless of the rate, signaling seems to be the main driver of systematic misreporting, confirming the qualitative results reported in Table 1. According to the estimates in Table 2, in period (ii) signaling alone is responsible for differences of, roughly, 14 to 90bp between bank's borrowing costs and their quotes. In contrast, during the same period, for most of the banks, the differences due to portfolio exposures are lower than 5bp (in magnitude). Notable exceptions are Citibank (15 to 20.5 bp), HBOS (5.7 to 12.6 bp) and Lloyds Bank (14.7 to 19.4 bp) which, once again, is consistent with the results in Section 8.2.

For the other two periods, the deviations from truthful reporting seem much lower. The less conservative estimates, based on H15, suggest misreports due to signaling alone of approximately 11bp and 23bp below borrowing costs, in periods (i) and (iii) respectively. However, if NYFR is used instead to approximate μ_t, the corresponding numbers are closer to zero, and not even statistically significant in period (iii). Only for this last period, and just for the most conservative estimates (NYFR), the deviations due to portfolio incentives seem larger in magnitude than those motivated by signaling, but are still lower than 3bp for all banks. Again, this changes across periods resemble the results in Section 8.2.

(i): 09/032007 - 09/14/2008				(ii): 09/14/2008 - 12/31/2008				(iii): 02/09/2009 - 05/17/2010			
Portfolio		Signaling		Portfolio		Signaling		Portfolio		Signaling	
H15	NYFR	H15	NYFR	H15	NYFR	H15	NYFR	H15	NYFR	H15	NYFR
-0.4*	0.3*	-11.3*	-1*	-3.2*	3.1*	-90.7*	-14.1*	-1.1*	1.5*	-23.2*	-0.1
-0.2	0.5*	-11.3*	-1*	-0.9	2.4*	-94.2*	-14.6*	0	0.6*	-25.3*	-0.1
0.1*	0.5*	-11.6*	-1*	0.9	3.3*	-95.3*	-14.8*	-3.7*	-1.8*	-23.9*	-0.1
-1.6*	-1*	-11.4*	-1*	-20*	-15.5*	-92.7*	-14.4*	-1.3*	0.7*	-23.8*	-0.1
-1.7*	-0.8*	-11.1*	-1*	-2.9*	1.1	-93.3*	-14.5*	-5.2*	-2.7*	-23.3*	-0.1
-0.5*	0.1	-11.3*	-1*	-7.7*	-0.2	-89.3*	-13.9*	-1.5*	-0.7*	-25.1*	-0.1
-1.4*	-0.6*	-11.1*	-1*	-12.6*	-5.7*	-89.9*	-14*	-2.1	-0.9	-24.7*	-0.1
-1.1*	-0.5*	-11.4*	-1*	-3.2	3.2*	-90.6*	-14.1*	-0.2*	0	-25.7*	-0.1
-0.4	0.2	-11.4*	-1*	-1.1*	-0.8*	-97.8*	-15.2*	-1	2.2*	-22.7*	-0.1
-0.5*	0.3*	-11.1*	-1*	-19.4*	-14.7*	-92.5*	-14.4*	-0.5*	-0.1	-25.4*	-0.1
-0.2*	0.2*	-11.5*	-1*	-2	3.4*	-91.7*	-14.2*	-5.4*	-2.9*	-23.3*	-0.1
-1.3*	-0.2	-10.9*	-1*	-5.8*	3*	-87.7*	-13.6*	-1.9*	-0.8*	-24.7*	-0.1
-0.8*	0	-11.2*	-1*	-4.5*	-1.9*	-95*	-14.8*	-0.7*	1.3*	-23.8*	-0.1
-1.4*	-0.9*	-11.4*	-1*	-6*	-3.3*	-94.8*	-14.7*	-0.7	1.3*	-23.7*	-0.1
-0.7*	0.1	-11.2*	-1*	-4.4*	4.6*	-87.4*	-13.6*	-3.7*	-1.7*	-23.8*	-0.1
-0.8*	0	-11.2*	-1*	-4	4.1*	-88.5*	-13.7*	-0.2*	0.7*	-24.9*	-0.1

Table 2: Average differences between Libor Quotes and Borrowing Costs

This table presents the results of an alternative estimation, where the means of the private cross distributions are allow differ across banks. The estimates are based on the assumption that either ICAP's Eurodollar deposits rate (H15) or NYFR are approximate measures of the average borrowing costs μ_t. The results are consistent with those reported in Section 8.2 and thus suggest that the former do not depend on the assumption that the means of the private costs are the same for all banks.