A Central Bank Theory of Price Level Determination*

Pierpaolo Benigno
LUISS and EIEF

June 13, 2019

Abstract

This paper develops a theory in which the central bank can control the price level without fiscal backing. To this end, the remittances' policy and the balance sheet of the central bank are important elements to specify. A central bank appropriately capitalized can succeed in controlling prices by setting the interest rate on reserves, holding short-term assets and rebating its income to the treasury – from which it has to maintain financial independence. If the central bank undertakes unconventional open-market operations, either it has to give up its financial independence or leaves the economy exposed to self-fulfilling inflationary spirals or chronic liquidity traps.

*I am grateful to Gianluca Benigno, John Cochrane, Marco Del Negro, Salvatore Nisticò and Michael Woodford for helpful comments and discussions and to the participants to the seminars at University of Bonn, Universitá Milano Bicocca, University of Copenaghen, Banque de France, DeNederlandscheBank, at the "8th Annual CIGS Conference on Macroeconomic Theory and Policy", "Deutsche Bundesbank Spring Conference 2017: Frontiers in Central Banking", "3rd Oxford-Federal Reserve Bank of New York Conference on Monetary Economics", "Barcelona GSE Summer Forum: Monetary Policy and Central Banking". This paper was previously circulated under the title "Designing Central Banks for Inflation Stability". Pasquale Filiani has provided excellent research assistance. Financial support from the ERC Consolidator Grant No. 614879 (MONPMOD) is gratefully acknowledged.

1 Introduction

The determination of prices has been at the center of the economic debate since the existence of monetary systems with discussions ranging from the policies that monetary institutions should follow to the assets that they should hold to back the value of money.

A recent literature spurred by the works of Sims (2000, 2005) and further developed by Bassetto and Messer (2013), Benigno and Nisticò (2015), Berriel and Bhattarai (2009), Del Negro and Sims (2015), Hall and Reis (2015), Park (2015), Reis (2015) has underlined the importance of separating the budget constraint of the central bank from that of the treasury in contrast with the conventional view that integrates them in the so-called consolidated government budget constraint.

The separation between central bank and treasury balance sheets is an important one. The literature has emphasized both the fiscal and inflationary consequences of alternative central-bank remittances' policy under unconventional open-market operations.

This paper uncovers an additional and important role for the remittances of the central bank that has been neglected so far. It shows that full control of the price level can be achieved by the central bank through an appropriate specification of the remittances' policy. This finding arises from a theory of price level determination in which the central bank can play a solo role in the control of the price level accomplishing naturally its primary goal – what can be labelled a 'central bank theory of the price level'. This theory complements, and in some direction contrasts, the view of the fiscal theory of the price level that "fiscal policy can be a determinant, or even the sole determinant, of the price level" (Sims, 2013).

The results of this paper are relevant for at least two cases of practical interest which cannot be dealt by the fiscal theory of the price level. In the first case, the possible evolution of monetary systems towards forms of private currency requires one to think about how these private entities can control the value of their currency without relying on fiscal backing or control. Moreover, even in more traditional monetary systems like the European Monetary Union, the conventional view of a consolidated government budget constraint seems a poor description of reality – which is instead characterized by significant borders between the central bank and the many national tax authorities. The literature has been so far silent on these two cases and actually criticisms have arisen concerning the architecture of the European

Monetary Union as grounded on precarious foundations for the control of the price level.[1]

The central bank theory of the price level presented in this work relies on some salient features, among them the remittances policy. At its inception, the central bank receives an appropriate capitalization from the treasury (or private sector) and borrows additional resources by issuing interest bearing securities (reserves). Its portfolio of assets consists only of short-term bonds. After that, central bank's profits are entirely remitted to the treasury, and then to the private sector. Moreover, the central bank has to be financially independent from the treasury preventing, therefore, any possible raid on its capital. Finally, monetary policy is specified by setting the interest rate on reserves that actively reacts to the deviations of prices (or inflation) from the target.

All of the above features, which to a certain extent are not far from current central-banking practices, are sufficient to allow the central bank to control the price level ruling out deflationary and inflationary spirals.[2]

The reason for why monetary policy alone can control the price level depends on two important observations. First, in a fiat monetary system, the central bank's liabilities have a special role since they define the 'unit of account', and by this virtue they are always repaid.[3] Differently from any other agent in the economy, the central bank is not subject to a solvency condition or exposed to run. This property gives the central bank special powers, in particular when ruling out inflationary spirals. Second, any monetary policy action has 'fiscal' consequences thereby implying transfers to the treasury and/or to the private sector. The central bank can issue its liabilities at will regardless of solvency issues but the composition of the balance sheet and the remittances' policies become instead important in determining the value of those liabilities in terms of goods – the inverse of the price level.

To rule out deflations or liquidity traps, it is sufficient that the remittances' policy is designed to keep the value of central bank's nominal net worth constant. In this way, the central bank retains in its balance sheet an amount of real resources that, in a deflation, does not vanish over time

[1] See Sims (1999, 2016).

[2] In the case of inflationary solutions, the remittances' policy should switch (or at least threaten to switch) to a real transfer policy that anchors the value of central bank's net worth at the target price level.

[3] See Woodford (2000, 2001a). I am indebted to Michael Woodford for insightful conversations on this point.

and actually grows. This is exactly the reason why deflations cannot arise: the goods market does not clear at deflationary prices – there is an excess supply of goods over demand – unless those central bank's resources are fully expropriated and rebated to the private sector to close the shortfall in goods demand. In this connection, central bank's financial independence is key to shield its new worth from any third-party raid. Finally, a further critical element to rule out deflations is the commitment of the central bank to eventually ease the policy rate until it reaches the zero lower bound. If the central bank stops the easing above the zero-lower bound then a deflationary spiral can form. On the contrary, to rule out deflations, the fiscal theory of the price level relies on the cooperation between the treasury, acting through a fiscal stimulus, and the central bank, financing the stimulus by an unbounded growth of central bank's reserves. Instead, in the central bank theory of the price level, the help of the treasury (or private sector) occurs only at the inception through the injection of capital while, in general, fiscal policy is 'passive' or 'Ricardian', using the terminology of the fiscal theory of the price level.

To rule out inflationary spirals, the central bank theory of the price level requires the central bank to be ready to make a real transfer to the private sector, which can be financed by an unbounded grow of reserves. This is possible given the special power that central bank's reserves have as 'unit of account' since they can be increased at will without any solvency problem. On the contrary, the fiscal theory of the price level rules out inflationary spiral through the ability of the treasury to tax the private sector. Instead, in the central bank theory of the price level, the treasury plays a minor role since it is treated as any other borrower that needs to 'passively' adjust its real primary surplus or deficit to meet its obligations or, otherwise, has to default.

The main message of this work is that price determination can be achieved by relying only on central bank's instruments whereas the treasury can follow 'passive' or 'Ricardian' policies. At the end, the analysis shows that monetary systems of private currency without fiscal backing do not create price control issues. As well, the architecture of the European Monetary Union, with many tax authorities constrained by budget rules and no single authority directly behind the central bank, does not jeopardize the control of the price level by the European Central Bank.

All the features described above are important for the results. If the central bank does not receive initial capital, while other elements of the model remain unchanged, inflationary and deflationary solutions can develop. Similarly, if the central bank purchases risky securities it has either to give up financial independence or lose full control of the price level.

This paper is related to an important literature that has discussed the issue of price determination in general equilibrium monetary economies ranging from the fiscal theory of the price level as in Cochrane (2011), Leeper (1991), Sims (1994, 2000, 2013), Woodford (1995, 2001) to theories of price determination through active interest rate rules supported by fiscal backing as in Benhabib et al. (2001, 2002), Sims (2013), Woodford (2003).[4] Cochrane (2011) provides an extensive and critical discussion of results of determinacy achieved through Taylor's rules.

With respect to all this literature, the contribution of this work is to emphasize that the determination of the price level can be left to the central bank without any fiscal backing or support resting on an appropriate design of how central banks should operate starting from their policy rule, capital, composition of assets and in particular their remittances' policy.

There are some works in the literature that share the same absence of fiscal 'activism' in the determination of the price level. Obstfeld and Rogoff (1983) have shown that deflationary solutions can be ruled out by targeting the growth of money supply, see also Sims (1994) and Woodford (1995). This is equivalent to a commitment taken by the central bank to transfer a certain amount of real resources in a deflation which is the same kind of commitment that, instead, is used in this work to counteract an inflationary spiral. On the opposite, the theory expounded in this work eliminates deflationary traps through the initial capitalization of the central bank and the commitment to maintain nominal net worth constant by using an appropriate remittances' policy. In this case, the real net-asset position of the central bank explodes in a deflationary spiral as opposed to what happens in Obstfeld and Rogoff (1983), and in the fiscal theory, where central bank's liabilities grow unboundedly. To rule out inflationary spiral, Obstfeld and Rogoff (1983) advocates for fiscal backing which allows the central bank to redeem money in exchange of goods.[5] In a recent work, Gaballo and Mengus (2018) have refined this solution obtaining it through a time-consistent equilibrium of a benevolent government sufficiently endowed with real resources.

[4]For an empirical evaluation see among others Canzoneri et al. (2001) and Bianchi and Ilut (2017).

[5]See also Obstfeld and Rogoff (2017).

To eliminate inflationary spirals, instead, this work relies on the threat to switch to a real remittances policy financed by an unbounded increase in central bank's reserves. On top of the above differences, the central bank of this work sets its policy in terms of the nominal interest rate and the economy is cashless while in Obstfeld and Rogoff (1983) the central bank controls money supply.

To rule out inflationary solutions, Woodford (2001b, 2003 ch. 2.4) proposes an interest rate rule which implies an infinite reaction at a positive inflation rate. Similarly, but through a different mechanism, one of the solutions of this paper to prevent inflationary spirals implies a threat to blow up inflation. Bassetto (2004) shows that the central bank can disallow deflationary solutions by imposing negative nominal interest rates as a way to reflate the economy by creating arbitrage opportunities.

Atkeson et al. (2010) and Christiano and Takahashi (2018) are closely related works since they also assume commitment and show that the central bank alone can determine the price level by positing an interest rate rule with reversion to a money rule in the case inflationary or deflationary paths develop. This paper instead always assumes a commitment to an interest rate rule and uses balance-sheet policies to rule out divergent solutions. These results point toward saying that the set of instruments through which the central bank can control the price level is broader than just interest rate or money supply rules, therefore providing an appealing alternative to the proposal of reversion to money rules.

Hall and Reis (2016), in an independent work, have suggested that the central bank should control the price level by committing to a real payment policy on reserves even in normal times in place of using a standard policy rule on the nominal interest rate. Instead, the central bank theory of the price level maintains the use of a nominal interest rate policy and it relies on a specific central-bank remittances policy to determine the price level. As an important difference, the switch to a real remittances policy under the proposal of this work is only a threat that can be used contingently on the development of inflationary spirals – something which is not observed in equilibrium – and not necessary during "normal times".

Park (2015) is an important contribution that illustrates the relevance of the central bank's balance sheet and remittance policy for inflation. However, he analyzes only local equilibria through a log-linear approximation of the model equilibrium condition and therefore he does not investigate how remittances and other elements of central bank's balance sheet should be specified in order to obtain uniqueness of the equilibrium prices in the non-linear model, which is instead the main focus of this work.

The structure of the paper is the following. Section 2 presents a simple monetary model. Section 3 studies the equilibrium implications emphasizing the difference between consolidating and separating the budget constraints of treasury and central bank. Section 4 presents the central bank theory of the price level. Section 5 concludes.

2 Model

I present a simple endowment monetary economy featuring three agents: the consumer, the treasury and the central bank.[6]

The monetary economy is characterized by a currency, let's say dollars, that serves as a 'unit of account' and 'store of value'. Both properties are important for the analysis that follows. Let me first focus on what a 'unit of account' means and its implications. On the one side, a 'unit of account' is the unit of measure to value goods and securities, the *numéraire*. In this simple monetary economy there is only one 'unit of account' and the price of all goods and securities are quoted in that 'unit of account'. On the other side, a fiat 'unit of account' is the liability of an agent (and only of one agent) which in the model is the central bank. By virtue of this, the price of one unit of central bank's liability is just one dollar because that unit of liability exactly defines what a dollar is – a concept extensively discussed by Woodford (2000, 2001a). Therefore, one dollar claim at the central bank is risk-free regardless of the resources that the central bank has and its balance-sheet composition.[7]

I am going to assume that the central bank can issue its liabilities in two different ways: i) money, i.e. banknotes or coins which are physical means of payments and ii) reserves, which are one-period short-term securities. Moreover, the central bank sets its monetary policy by paying an interest rate, i_t^R, on reserves. Since reserves define the 'unit of account', the central bank

[6] The model is similar to Cochrane (2011) except for the breakdown between the balance sheets of the central bank and treasury.

[7] This property is not shared by any other agent in the economy since their liabilities are denominated in the 'unit of account', but do not define the 'unit of account'. A dollar debt issued by these agents is therefore priced at the market value.

can set their interest rate independently of the quantity issued.[8] By setting i_t^R the central bank is also determining the short-term interest rate, i_t, on any other short-term debt security issued in the economy and repaid with certainty. Absence of arbitrage opportunities implies that $i_t = i_t^R$. In what follows, I am going to simply use the notation i_t in a interchangeable way to denote either the interest rate on reserves or that on other short-term securities.

Let me now focus on the property of currency as a 'store of value' and its implications. For its physical characteristics, money serves as a store of value. The existence of money implies that the interest rate on reserves cannot be negative, otherwise arbitrage opportunities would arise. In the simple model of this section, I am assuming that money and reserves provide the same payment services. Therefore the demand of money is going to be zero whenever the interest rate on reserves is positive. When, instead, the interest rate on reserves is zero, money and reserves are perfect substitute. Without losing generality, I am setting the demand of money to zero even in this case. Therefore, as in other papers, I am modelling a completely cashless economy. On the other hand and in contrast with some of the literature which has emphasized the importance of money for price determination (see among others Obstfeld and Rogoff, 1983, Sims, 1994, and Woodford, 1995), in the framework expounded in this paper the control of money is not going to play any role for price determination. Instead, I am going to assume that the central bank sets interest-rate policies and show that there is no problem for price determination consistent with that policy. There are, however, other elements which are going to be key for the results among which central-bank balance-sheet policy and the issuance of central-bank interest-bearing securities, like reserves.

I am going to assume that the monetary system starts at time t_0, which implies that the economy does not inherit any security denominated in dollars from period $t_0 - 1$. This environment serves the purpose of studying whether it is possible to design from scratch institutions that can control the price level, without any inheritance from the past.[9]

[8] See again Woodford (2000, 2001a).

[9] The assumption of zero liabilities at time t_0 is done without losing any generality. All the results of the paper hold in the more general context.

2.1 Consumers

Consumers are maximizing an intertemporal utility, starting from period t_0, which is separable with a discount factor given by $0 < \beta < 1$ and utility flow $u(c_t)$, where $u(\cdot)$ is a concave function and c_t is consumption of the single consumption good present in the economy:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t).$$

They face the following budget constraint:

$$\frac{B_t + X_t}{1 + i_t} = B_{t-1} + X_{t-1} + P_t(y - c_t) - T_t, \tag{1}$$

since they can lend or borrow using short-term securities, B_t, at the interest rate i_t (a positive value of B_t indicates assets, a negative value debt); they can hold central-bank issued reserves, X_t;[10] T_t denotes lump-sum transfers handed over to the other agents in the economy, if positive, or received, if negative.[11] Consumer's transfers have as a counterparty either the treasury or the central bank or both. y denotes the endowment of goods and P_t is the price of goods in the unit of account.

The consumer's problem is subject to a natural borrowing limit at each time $t \geq t_0$

$$\frac{B_t + X_t}{P_{t+1}} \geq -\sum_{j=0}^{\infty} R_{t+1,t+1+j}(y - \tau_{t+1+j}) > -\infty \tag{2}$$

saying that the debt to be paid at a generic time $t+1$, and contracted at time t, cannot exceed in real term the present-discounted value of real net income, where $R_{t+1,t+j}$ is the appropriate market discount factor to evaluate a unit of good at time $t+j$ with respect to time $t+1$, with $R_{t+1,t+1} \equiv 1$, and τ_t are the transfers in real terms, i.e. $\tau_t = T_t/P_t$. The optimization problem of the consumer implies that the Euler equation holds

$$u_c(c_t) = \beta(1 + i_t)\frac{P_t}{P_{t+1}} u_c(c_{t+1}) \tag{3}$$

[10] In practice, banks, and not consumers, hold central-bank reserves and issue deposit, held instead by consumers. In a model in which intermediaries are owned by consumers, (1) represents the consolidated budget constraint of the private sector.

[11] Given that the economy is cashless, as I have already explained, I am completely abstracting from money in the writing of the budget constraint.

in an interior solution for each $t \geq t_0$, in which $u_c(\cdot)$ is the marginal utility of consumption, and that the consumer exhausts his intertemporal budget constraint:

$$\sum_{t=t_0}^{\infty} R_{t_0,t} c_t = \sum_{t=t_0}^{\infty} R_{t_0,t}(y - \tau_t). \tag{4}$$

The present-discounted value of consumption should be equal to the present-discounted value of net income. In the above intertemporal budget constraint there is no financial wealth carried from period $t_0 - 1$, since I have assumed that the monetary system starts at period t_0 and therefore $B_{t_0-1} = X_{t_0-1} = 0$.

The mirror image of the exhaustion of the intertemporal budget constraint of the consumer is the transversality condition

$$\lim_{t\to\infty} \left\{ R_{t_0,t} \frac{B_t + X_t}{P_t(1 + i_t)} \right\} = 0, \tag{5}$$

that constraints the long-run behavior of the assets (or debt) held by the consumer.[12]

2.2 Treasury

The budget constraint of the treasury is:

$$\frac{B_t^F}{1 + i_t} = B_{t-1}^F - T_t^F - T_t^C \tag{6}$$

where B_t^F is now treasury's debt with initial condition $B_{t_0-1}^F = 0$, T_t^F are lump-sum taxes and T_t^C are the nominal remittances received from the central bank, when positive, or transfers made to the central bank, when negative.

2.3 Central bank

Central bank's flow budget constraint is instead

$$\frac{B_t^C - X_t^C}{1 + i_t} = B_{t-1}^C - X_{t-1}^C - \tilde{T}_t^C - T_t^C, \tag{7}$$

where B_t^C are central bank's holdings of short-term assets while X_t^C is the supply of central bank's reserves, with initial conditions $B_{t_0-1}^C = X_{t-1}^C = 0$,

[12] Equations (4) and (5) are equivalent equilibrium conditions taking into account (1) and the initial condition $B_{t_0-1} = X_{t_0-1} = 0$.

and $\tilde{T}_t^C$ are the remittances that the central bank delivers directly to the private sector, if positive, or receives, if negative.

2.4 Equilibrium

Equilibrium in asset markets requires that

$$B_t + B_t^C = B_t^F$$

and

$$X_t = X_t^C.$$

Needless to say, the market for central-bank issued reserves clears in a separate way from that of short-term bonds, although both securities have same interest rate in equilibrium.

Equilibrium in the goods market implies that consumption is equal to output:

$$c_t = y.$$

It follows that the discount factor $R_{t_0,t}$ is equal to $R_{t_0,t} = \beta^{t-t_0}$ and that the Euler equation (3) implies a relationship between the nominal interest rate and the inflation rate, through the real rate,

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}. \tag{8}$$

Note that (4) or (5) imply that the following intertemporal constraint holds:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0}\tau_t = 0, \tag{9}$$

saying that the present discounted value of real transfers should sum to zero. Taxes and transfers from and to the private sector originate either from the taxes levied by the treasury or from the remittances the central bank gives directly to the private sector, therefore

$$T_t = T_t^F - \tilde{T}_t^C. \tag{10}$$

An equilibrium is a set of sequences $\{P_t, i_t, T_t, B_t^F, T_t^F, T_t^C, \tilde{T}_t^C, B_t^C, X_t^C\}_{t=t_0}^{\infty}$ that solve equations (6), (7), (8), (9) (in which $\tau_t = T_t/P_t$), and (10), given the specification of the monetary/fiscal policy regime and initial conditions.

Note that the restriction (9) is just an intertemporal one and it does not constrain the variables at each point in time. Therefore there are nine unknowns and four restrictions leaving five degrees of freedom to specify the monetary/fiscal policy regime. I first specify interest-rate policy. The central bank sets the nominal interest rate on reserves to follow the simple rule

$$1 + i_t = \max\left\{ \frac{1}{\beta} \left(\frac{P_t}{P^*} \right)^{\phi}, 1 \right\} \tag{11}$$

where ϕ is a non-negative parameter; P^* is positive. I am assuming that at its inception the central bank receives a mandate in terms of the target price level, P^*.[13] When $\phi > 0$ the instrument of policy reacts directly to the deviation of the actual price level from the target. When $\phi = 0$, the nominal interest rate is pegged to a constant value, the real rate, but P^* is still the objective of policy.[14]

Given the interest-rate policy, I am left with four additional restrictions to specify policy. Looking at the flow budget constraint of the central bank (7), it is possible to specify the path of remittances to the treasury and to the private sector, i.e. the sequences $\{T_t^C, \tilde{T}_t^C\}_{t=t_0}^{\infty}$. Given that the interest-rate path is specified by (11), the additional degree of freedom in equation (7) can be filled by setting the path of reserves $\{X_t^C\}_{t=t_0}^{\infty}$. Note that the central bank can set at the same time the path of reserves and its price because its liabilities define the unit of account of the monetary system. Looking at the flow budget constraint of the treasury (6), it is possible to specify the path of taxes $\{T_t\}_{t=t_0}^{\infty}$ to complete the characterization of the monetary/fiscal policy regime.

Definition 1 *An equilibrium is a set of sequences* $\{P_t, i_t, T_t, T_t^F, T_t^C, \tilde{T}_t^C, B_t^F,$ $B_t^C,\ X_t^C\}_{t=t_0}^{\infty}$ *with* $P_t, i_t \geq 0$ *that solve equations (8), (9), (6), (7), (10) given the specification of the monetary/fiscal policy regime which sets the sequences* $\{i_t,\ T_t^F,\ T_t^C,\ \tilde{T}_t^C,\ X_t\}_{t=t_0}^{\infty}$*, and given initial conditions* $B_{t_0-1}^F = B_{t_0-1}^C = X_{t_0-1}^C = 0$ *and the definition* $\tau_t = T_t/P_t$*.*

[13] This target can be also self-imposed by the central bank.

[14] By assuming the policy rule (11), I am departing from the conventional assumption of the literature according to which the nominal interest rate reacts to the deviations of current inflation rate with respect to a target. Were this the case, indeed, price determination would inherit an initial condition, namely the price level at time $t_0 - 1$ which is not defined in my framework since the monetary system starts at time t_0. Although I could overcome the problem by arbitrarily fixing P_{t_0-1} at any fictitious positive number, by assuming the rule (11) I completely avoid the issue without losing any generality at all.

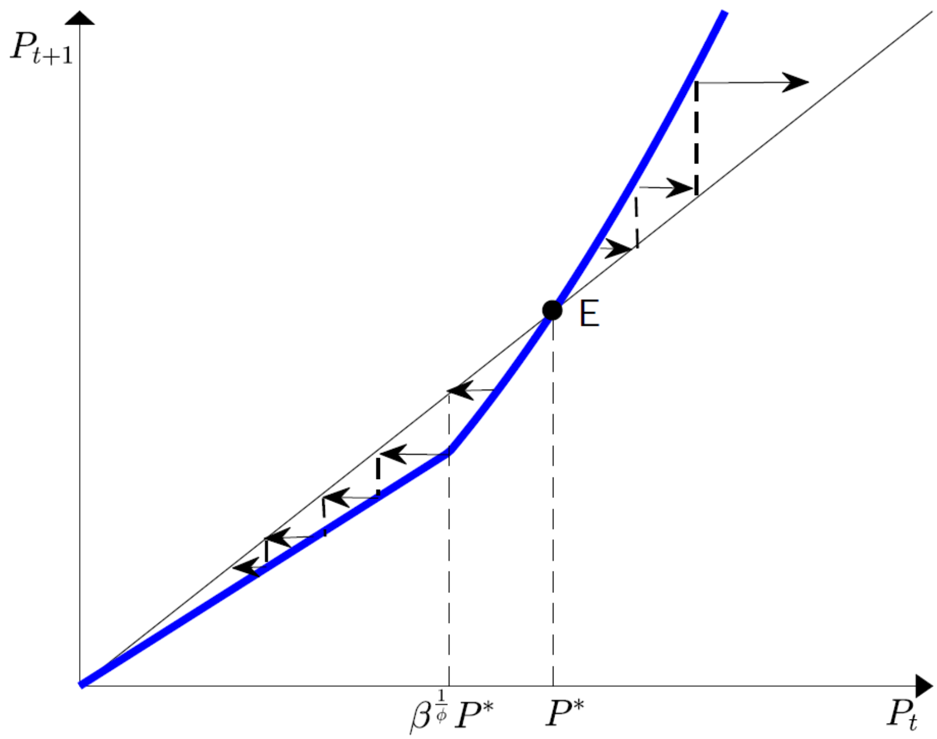

Figure 1: Plot of the difference equation (12) in which $\phi > 0$. The point E is the stationary solution $P_t = P^*$ at each date t. If $P_t \leq \beta^{\frac{1}{\phi}} P^*$ the rate of deflation of the price level, P, is β, with $0 < \beta < 1$.

3 Equilibrium implications and price determination

Combining (8) and (11), the price level follows a non-linear difference equation:

$$\frac{P_{t+1}}{P_t} = \max\left\{\left(\frac{P_t}{P^*}\right)^{\phi}, \beta\right\}. \tag{12}$$

Equation (12) has infinite solutions irrespective of the value $\phi \geq 0$. Consider first the case $\phi > 0$ which is shown in Figure 1. There is a stationary solution, with $P_t > 0$, if and only if $P_{t_0} = P^*$. If instead $P_{t_0} > P^*$, the solution will be monotone increasing, an inflationary solution. On the other side, if $P_{t_0} < P^*$, the solution will be monotone decreasing, a deflationary solution, and in particular when $P_t \leq \beta^{1/\phi} P^*$ the rate of deflation is β. Note, moreover, that solutions associated with different P_{t_0} never cross along the time dimension. Therefore when $\phi > 0$, there are infinite solutions; these can be simply indexed by the value taken by the initial price level, P_{t_0}, in the interval $(0, \infty)$.

Consider now the case $\phi = 0$. Infinite solutions are also possible, and these can also be indexed by the value taken by the initial price level P_{t_0} in the range $(0, \infty)$. However, all these solutions are stationary.

In this simple framework, the problem of price level determination is that of nailing down the price level at time t_0 possibly at the target P^*.

To proceed, I will now follow the key insight of the fiscal theory of the price level, i.e. that other equilibrium conditions should be exploited to uniquely determine the price level. As a matter of fact, I have only characterized the solutions of (12) but not equilibria.

Recall that the present-discounted value of consumers' real transfers to and from the other agents in the economy should be equal to zero in equilibrium, equation (9). This condition is key to determine the price level because it holds for equilibrium price sequences and not necessarily for all price sequences that solve (12). Specifically, price sequences solving (12) can be ruled out as equilibria if they imply violations of (9). The specification of the transfer policy is therefore critical for the determination of the price level.

There are three cases of interest that can be analyzed in the above framework. The first can be labelled as the "consolidated government budget constraint" case according to which central bank and treasury are pooled together. This is the setting expounded by the fiscal theory of the price level. In this case, the important element for price determination combined with the interest rate policy (11) is the specification of the path of taxes T_t^F. The second and third cases, which are the focus of this paper, rely on the separation between balance sheets of treasury and central bank, and lead to a central bank theory of the price level. Key for price determination is the remittances policy of the central bank. In the second case, the central bank is still part of the government and makes or receives remittances to and from the treasury. Here, what is relevant is the path of remittances T_t^C. In the third case, the central bank is a private institution and makes or receives remittances to and from the private sector. In this case, the relevant variable to get price determination is $\tilde{T}_t^C$.

3.1 Consolidated budget constraint

Let me first focus on the standard case treated in the literature. The central bank and the treasury have consolidated balance sheets, and the central bank is not making any direct transfer to the private sector, i.e. $\tilde{T}_t^C = 0$. Therefore, equation (10) implies that $T_t = T_t^F$. Aggregating (6) and (7), I get

$$\frac{B_t^F - B_t^C + X_t^C}{1 + i_t} = B_{t-1}^F - B_{t-1}^C + X_{t-1} - T_t^F. \tag{13}$$

In the fiscal theory, price determination occurs through an appropriate specification of the path of real taxes T_t^F / P_t combined with the interest rate rule (11). I leave to the Appendix an extensive discussion of the fiscal theory since it is well know in the literature. However, in what follows, I am going to underline some of its main features in comparison with the proposal of this work. An important consequence of consolidating budget constraints is that the remittances that the central bank delivers to the treasury, T_t^C, cancel out and therefore they are irrelevant for the equilibrium determination whereas they play a crucial role under the central bank theory of the price level, as next Section is going to show.

3.2 Separated budget constraints: central bank is part of the government

I now consider the case in which balance sheets of treasury and central bank are separated but in which the central bank is part of the government and makes direct remittances to the treasury. In this case, $\tilde{T}_t^C = 0$, and therefore equation (10) implies that $T_t = T_t^F$. Given the policy rule (11), what is left in the specification of the monetary/fiscal policy regime of Definition 1 are the sequences $\{T_t^F, T_t^C, X_t\}_{t=t_0}^{\infty}$. In this context, I am going to show that the central bank's remittances policy T_t^C plays a key role in determining the price level combined with the interest rate rule (11), while instead T_t^F adjusts to ensure solvency of treasury's liabilities in and off equilibrium through the following condition

$$\frac{B_{t-1}^F}{P_t} = \sum_{t=t_0}^{\infty} R_{t_0,t} \left(\frac{T_t^F}{P_t} + \frac{T_t^C}{P_t} \right). \tag{14}$$

When the treasury is separated from the central bank, its liabilities do not share the 'risk-free' properties of central-bank issued reserves and, therefore, taxes should adjust to ensure solvency, in one case, or debt should be defaulted in the other case.[15]

Equation (14) cannot be used to determine the price level for two reasons. If debt is fully repaid, taxes T_t^F should adjust to back the nominal value of debt for any feasible path of the price level. As a consequence there cannot be any price determination through the above equation. Alternatively, if the treasury is not solvent, debt should be appropriately defaulted to meet the resources available and this will be reflected by adjusting the left-hand side of the above expression through the recovery rate, in which case there is again no price determination using equation (14).

Notwithstanding fiscal policy is 'passive' or 'Ricardian', next section shows that price determination can be still obtained by an appropriate specification of the path of central bank's remittances T_t^C and other features of the central bank's balance sheet.

3.3 Separated budget constraints: central bank is a private institution

Price determination can be also achieved when currency is privately issued and therefore the central bank and treasury are completely disjoint institutions. Remittances to the treasury are in this case zero, $T_t^C = 0$. Similar to the previous case the treasury is not different from any other institutions regarding debt solvency: either its debt is repaid with certainty through an appropriate backing of resources or it is defaulted on. Prices can be determined by appropriately specifying the central bank's remittances policy to the private sector, i.e. $\tilde{T}_t^C$.

4 Central bank theory of the price level

This section presents the main result of price determination through the central bank theory of the price level focusing on the case in which the central bank is part of the government. All the results apply to the case of

[15] The European Monetary Union is an example in this framework since the central bank and the several treasuries are linked together through transfer mechanisms but the debt issued by the national treasuries is not explicitly considered as a liability of the central bank.

a private central bank with appropriate qualifications: namely $\tilde{T}_t^C$ replaces T_t^C in any instance, while T_t^C is set equal to zero.

4.1 Ruling out deflationary traps

I will now show how it is possible to rule out deflationary spirals through the following monetary/fiscal policy regime.

Definition 2 *Define a monetary/fiscal policy regime setting the sequences* $\{i_t, T_t^F, T_t^C, X_t\}_{t=t_0}^{\infty}$ *: i) the policy rule (11), with* $\phi > 0$*, specifies* i_t*, ii)* $T_t^F = -T_t^C$ *at each date* $t \geq t_0$*, iii)* $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ *for some* $n_{t_0}^C > 0$ *and* $T_t^C = \Psi_t^C$ *at each date* $t > t_0$*; iv)* $X_t > 0$ *for each* $t \geq t_0$*.*

There are four main ingredients defining the monetary/fiscal policy regime that will defeat deflationary spirals. First, the central bank should set the policy rule as in (11), with $\phi > 0$. The requirement $\phi > 0$ is needed to allow, and discuss, deflationary and inflationary spirals in the analysis. Second, the treasury should passively transfer the remittances received from the central bank to the private sector, $T_t^F = -T_t^C$ at each date $t \geq t_0$. Note that an implication of this assumption, given the initial condition $B_{t_0-1}^F = 0$, is that treasury's debt is zero at all times, $B_t^F = 0$ for each $t \geq t_0$. The third element is the remittances' rule followed by the central bank. At time t_0, the central bank receives an initial capitalization from the private sector, through the treasury, $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ in which $n_{t_0}^C$ is real capital. This specifies the transfer policy at time t_0. Then, in the following periods, for each $t > t_0$, it commits to transfer all profits to the private sector, $T_t^C = \Psi_t^C$, again through the treasury. Finally, the central bank should issue a positive amount of reserves.

Proposition 3 *Given the monetary/fiscal policy regime of Definition 2 it follows that* $P_t \geq P^*$ *in equilibrium for each* $t \geq t_0$*.*

Proof. Given the assumption $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ and $X_{t_0} > 0$, the central bank balance sheet at time t_0 is

$$\frac{B_{t_0}^C - X_{t_0}}{(1 + i_{t_0})} = P_{t_0} n_{t_0}^C. \tag{15}$$

Note that profits at a generic time t are given by

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C). \tag{16}$$

Define net worth as

$$N_t^C \equiv \frac{B_t^C - X_t^C}{(1+i_t)}$$

and note that

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C, \tag{17}$$

where I have used the definition of net worth and equation (16) into (7). Considering the remittances rule $T_t^C = \Psi_t^C$ at each date $t > t_0$, net worth is time-invariant $N_t^C = N_{t-1}^C = ... = P_{t_0} n_{t_0}^C > 0$. Furthermore, note the following equivalence:

$$\begin{aligned} \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_t}\frac{B_t+X_t}{(1+i_t)}\right\} &= \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_t}\frac{B_t^C-X_t^C}{(1+i_t)}\right\} \\ &= -\lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_t}N_t^C\right\}, \end{aligned} \tag{18}$$

where in the first line I have used the equilibrium in the asset markets and in the second line the definition of net worth. Note that $P_t < P^*$ if and only if $P_{t_0} < P^*$. Moreover if $P_{t_0} < P^*$ there will be a finite time $\tilde{t} \geq t_0$ such that $P_{t+1}/P_t > \beta$ for each $t_0 \leq t \leq \tilde{t}$ and $P_{t+1}/P_t = \beta$ for each $t > \tilde{t}$. Time $\tilde{t}$ coincides with t_0 if and only if $P_{t_0} \leq \beta^{\frac{1}{\phi}} P^*$. It follows that for any path in which $P_{t_0} < P^*$

$$\begin{aligned} \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_t}N_t^C\right\} &= P_{t_0}n_{t_0}^C \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_{\tilde{t}}}\frac{P_{\tilde{t}}}{P_t}\right\} = P_{t_0}n_{t_0}^C \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_{\tilde{t}}}\frac{1}{\beta^{t-\tilde{t}}}\right\} \\ &= \beta^{\tilde{t}-t_0}\frac{P_{t_0}}{P_{\tilde{t}}}P_{t_0}n_{t_0}^C > 0 \end{aligned} \tag{19}$$

where in the first equality I have used the result that nominal net worth is constant and in the second the fact that when $P_{t_0} < P^*$ the rate of deflation is β after period $\tilde{t}$. Given (18), the result in (19) implies violation of the transversality condition (5). Therefore prices in which $P_t < P^*$ at some point in time cannot be equilibria. ■

I will now describe each element of the proposal in details and discuss the consequences of relaxing each assumption in turn.

The first element of Definition 2 is the policy rule (11). A critical assumption in the policy rule is that the central bank's limit to its policy easing should coincide with the zero lower bound.[16] Replace now the rule (11) with

$$1 + i_t = \max\left\{\frac{1}{\beta}\left(\frac{P_t}{P^*}\right)^{\phi}, 1 + \bar{\imath}\right\},$$

in which $\bar{\imath}$, with $0 \leq \bar{\imath} < \beta^{-1} - 1$, denotes the lower bound on the nominal interest rate at which the central bank stops to further ease its policy. The benchmark case presented above is nested when $\bar{\imath} = 0$. Combining this interest-rate policy into (8), I get that whenever $P_t \leq [\beta(1+\bar{\imath})]^{1/\phi}P^*$, prices continue to fall at the rate of deflation $\beta(1+\bar{\imath})$ which is higher than β when $\bar{\imath} > 0$. Therefore, preventing the nominal interest rate to go to the zero floor does not rule out deflationary traps as equilibria.[17] Indeed, the transversality condition is not violated when $\bar{\imath} > 0$

$$\begin{aligned}\lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_t}N_t^C\right\} &= P_{t_0}n_{t_0}^C \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{P_{t_0}}{P_{\tilde{t}}}\frac{P_{\tilde{t}}}{P_t}\right\} \\ &= P_{t_0}n_{t_0}^C\beta^{\tilde{t}-t_0}\frac{P_{t_0}}{P_{\tilde{t}}}\lim_{t\to\infty}\left\{\frac{1}{(1+\bar{\imath})^{t-\tilde{t}}}\right\} = 0.\end{aligned}$$

The second element of Definition 2 is the tax policy, $T_t^F = -T_t^C$ at each date $t \geq t_0$, which implies in (6) that $B_t^F = 0$ at all times, given the initial condition $B_{t_0-1}^F = 0$. It is important to underline that this tax rule does not hide any need of cooperation between treasury and central bank. A balanced-budget policy is in the call of 'passive' or 'Ricardian' policies, given zero initial debt. What really matters in the specification of the tax policy is the solvency of the treasury at any price path implied by (12). With a positive supply of treasury debt, the results of the paper are unchanged by assuming a 'passive' or 'Ricardian' fiscal policy according to the definition of Benigno and Nisticò (2015) and in line with Leeper (1991).

The third element of the proposal is the initial injection of real capital, $n_{t_0}^C$, through taxes levied on the private sector, i.e. $T_{t_0}^C = -P_{t_0}n_{t_0}^C$ for some $n_{t_0}^C > 0$, and the remittances policy rebating all profits to the treasury, i.e. $T_t^C = \Psi_t^C$ at each date $t > t_0$. The initial capitalization is key, as shown in

[16] I am grateful to Michael Woodford for pointing out this result.

[17] Note that absence of arbitrage opportunities implies that $\bar{\imath} \geq 0$ and therefore $\bar{\imath}$ and interest rates cannot be negative.

(19), otherwise there will not be any violation of the transversality condition at deflationary prices. The remittances policy is also important to keep nominal net worth constant, and at a positive level, which is also key to lead to violation of the transversality condition at deflationary prices.

The fourth element of the proposal is just the specification of the path of reserves, to be positive. A positive value of reserve is needed to equalize interest rate on reserve to any other short-term rate closing arbitrage opportunities.

Another important element in the above specification is the short-term composition of the central bank's assets. In the Appendix, I am going to discuss how results change when the central bank purchases risky assets.

I will now provide more intuition on why deflationary paths are ruled out. In the analysis of this paper, the government is committed to follow the monetary/fiscal policy regime of Definition 2 irrespective of what is going to be the equilibrium path of prices. The "unconditionality" of the commitment is indeed important to determine the price level. A deflationary solution is disallowed as an equilibrium because the transversality condition is violated by the fact that the central bank retains a positive value of resources in its balance sheet. What makes this possible is the initial capitalization and the commitment to follow the remittances policy of Definition 2. The important requirement behind this commitment is the financial independence of the central bank from the treasury meaning, in particular, that the treasury does not make any raid on central bank's capital or ask for extraordinary dividends.[18] Otherwise, a deflationary path can develop as an equilibrium.

I now compare the results of this section with those of the fiscal theory of the price level. As already emphasized, the key difference to rule out deflationary spirals is in the policy instrument used to determine the price level. The fiscal theory relies on the path of real taxes T_t^F/P_t; the central bank theory of the price level instead on the path of nominal remittances T_t^C. The second difference is on how deflationary spirals are ruled out. In the central bank theory of the price level, I have shown that the violation of

[18]Financial independence is a two-sided symmetric concept, as discussed by Buiter (2009), which requires, on the one side, the treasury not to deplete the financial resources of the central bank by taxing it or asking for extraordinary dividends and, on the other side, the central bank not to rely on further external support beyond the initial capitalization. This is the case in the analysis because profits are never negative. Indeed, $\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) = i_{t-1}N_{t-1} = i_{t-1}P_{t_0}n_{t_0}^C \geq 0$, which are zero when the nominal interest rate is zero.

the transversality condition in a deflationary spiral occurs because the central bank's net *asset* position is growing unboundedly. The key features for this to happen are the initial capitalization of the central bank, its financial independence and the remittances policy. In the fiscal theory of the price level, instead, the overall *liabilities* of the government grow unboundedly in a deflationary spiral (see among others Benhabib et al., 2002). The violation of the transversality condition is on the opposite side. What can make government's liabilities to grow unboundedly is the fact that central bank's reserves are part of them. Since solvency of the central bank is not an issue, central bank's reserves are the only securities that can grow unboundedly and be paid with certainty. As discussed in the Appendix, the fiscal theory of the price level is not purely 'fiscal' since central bank's liabilities play a key role to combat deflations.

Another proposal discussed in the literature (see among others Sims, 1994, Woodford, 1995 and 2003) to rule out deflations, argues that money supply should be kept constant, which means that it should grow unboundedly in real terms in the case of a deflationary spiral. This solution is similar to that proposed by the fiscal theory. Money, as part of the *liabilities* of the government, is allowed to grow unboundedly, which is feasible since money defines the unit of account of the monetary system. The counterpart of this policy is the financing of a transfer policy from the central bank to the treasury and then to the private sector that rebates more real resources than what could be feasible for an agent subject to a 'standard' solvency condition.

A final remark is needed to counteract a possible criticism of the theory proposed in this work, namely that there is still an important role for the treasury because it provides the initial capitalization of the central bank. First, one should consider that the initial injection of capital can be very small, but not zero. Second, the initial capitalization does not imply any 'active' role for treasury's taxes in contrast with what required by the fiscal theory of the price level. Fiscal policy should be 'passive' or 'Ricardian', as already emphasized. Finally, in the case the central bank is a private institution, there is no role for the treasury since capitalization is done by the private sector.

4.2 Ruling out inflationary spirals

I now rule out inflationary spirals with a simple amendment to the remittances' policy.

Definition 4 *Define a monetary/fiscal policy regime setting the sequences* $\{i_t, T_t^F, T_t^C, X_t\}_{t=t_0}^{\infty}$ *: i) the policy rule (11), with* $\phi > 0$*, specifies* i_t*, ii)* $T_t^F = -T_t^C$ *at each date* $t \geq t_0$*, iii)* $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ *for some* $n_{t_0}^C > 0$*,* $T_t^C = \Psi_t^C$ *for* $t_0 < t < \tilde{t}$ *and* $\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta}\frac{P_{t_0}}{P^*} n_{t_0}^C$ *for each* $t \geq \tilde{t}$*; iv)* $X_t > 0$ *for each* $t \geq t_0$*.*

The difference with respect to the policy of Definition 2 is in item *iii*). I am still assuming that the central bank rebates all its profits to the treasury following the rule $T_t^C = \Psi_t^C$, but only up to time $\tilde{t}$. This implies that

$$\frac{T_t^C}{P_t} = i_{t-1}\frac{N_{t-1}}{P_t} = i_{t-1}\frac{P_{t_0}}{P_t} n_{t_0}^C, \tag{20}$$

for each $t_0 < t < \tilde{t}$. The additional assumption is that the central bank commits to switch to the following constant real remittances' policy after and including time $\tilde{t}$

$$\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta}\frac{P_{t_0}}{P^*} n_{t_0}^C. \tag{21}$$

Proposition 5 *Given the monetary/fiscal policy regime of Definition 4 it follows that* $P_t = P^*$ *in equilibrium for each* $t \geq t_0$*.*

Proof. Consider the transversality condition (5) evaluated at the equilibrium discount factor $R_{t_0,t} = \beta^{t-t_0}$ and equilibrium in the asset markets, i.e. $X_t^C = X_t$ and $B_t = -B_t^C$ given that $T_t^F = -T_t^C$ at each date $t \geq t_0$ and $B_{t_0-1}^F = 0$ imply $B_t^F = 0$ at each date $t \geq t_0$. Condition (5) implies that, in equilibrium, central bank's real net worth should be appropriately bounded

$$\lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{B_t^C - X_t}{P_t(1+i_t)}\right\} = \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{N_t}{P_t}\right\} = 0, \tag{22}$$

where I have used the definition of central bank's net worth. Using the above condition and the central bank's flow budget constraint (7), I can write that

$$\frac{N_t^C}{P_t} = \sum_{T=t+1}^{\infty}\beta^{T-t}\frac{T_T^C}{P_T}, \tag{23}$$

for each $t \geq t_0$. Consider (23) at time $\tilde{t}-1$. Since $T_t^C = \Psi_t^C$ for $t_0 < t \leq \tilde{t}-1$, the law of motion of net worth (17) implies $N_{\tilde{t}-1}^C = P_{t_0} n_{t_0}^C > 0$ and therefore in (23) that

$$\frac{P_{t_0} n_{t_0}^C}{P_{\tilde{t}-1}} = \sum_{T=\tilde{t}}^{\infty}\beta^{T+1-\tilde{t}}\frac{T_T^C}{P_T}.$$

Now, substitute into the right-hand side of the above equation the path of real remittances (21) for each $t \geq \tilde{t}$ to obtain

$$\frac{P_{t_0}}{P_{\tilde{t}-1}} n_{t_0}^C = \frac{P_{t_0}}{P^*} n_{t_0}^C. \tag{24}$$

The above equation determines $P_{\tilde{t}-1} = P^*$ if and only if $n_{t_0}^C \neq 0$. Therefore, equation (12) implies that the only equilibrium is one in which prices are forever at the target P^*. ■

Inflationary spirals are ruled out by the central bank's commitment to back the real value of its net worth at the desired level P^*. This commitment can be also viewed as a threat since, although the policy rule does change, the remittances that the central bank delivers to the private sector do not vary in equilibrium as it can be seen by comparing (20), evaluated at $P_t = P^*$ and $1 + i_t = 1/\beta$, with (21). It is indeed possible to achieve the same result of Proposition 5 by conditioning the switch to a real remittances rule to the contingency of an inflationary path – an off-equilibrium path.

The assumption that the remittances policy switch to a real transfer as in item iii) of Definition 4 is necessary to exclude an inflationary path given the other elements of the monetary/fiscal policy regime. However, as shown in (24), without the initial capitalization of the central bank, $n_{t_0}^C > 0$, prices are not determined. Moreover, equation (24) shows that the remittances rule $T_t^C = \Psi_t^C$ up to time $\tilde{t}$, which ensures a constant nominal net worth, is also a necessary condition given the other elements of Definition 4.

I now discuss another way to understand the commitment to rule out inflationary paths by analyzing what is implicitly required in the case an inflationary path develops. If $P_{\tilde{t}-1} > P^*$, the central bank is committed from time $\tilde{t}$ onwards to transfer real resources by an amount that exceeds the real value of its net worth at current prices, i.e.

$$\sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T_T^C}{P_T} = \frac{N_{\tilde{t}-1}^C}{P^*} > \frac{N_{\tilde{t}-1}^C}{P_{\tilde{t}-1}}. \tag{25}$$

The equality in equation (25) follows after plugging the real remittances policy from time $\tilde{t}$ onwards and noting that $N_{\tilde{t}-1}^C = P_{t_0} n_{t_0}^C$. How is such a commitment possible? The central bank can generate a stream of real resources equal to (21) for each t since it can issue an increasing amount of its reserves – which are paid with certainty – in a way that their real value grows unboundedly at a rate equal or higher than $1/\beta$. It can indeed violate the transversality condition (22) by setting

$$\lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{X_t}{P_t(1+i_t)}\right\}>0, \tag{26}$$

which is the mirror image of the commitment (25).

Note that (25) implicitly defines an excess demand of goods, which pushes prices to infinite in a finite period of time, if P_t is above P^*. However, the equilibrium in which currency has zero value is dominated by the equilibrium with price stability since some real resources, $n_{t_0}^C$, are wasted at time 0 for the commitment to capitalize the central bank. Consumption will be below output in that period and welfare lower than under the price stability allocation.

I now turn to compare the above proposal with that of the fiscal theory of the price level. Similarly to this theory, the equilibrium condition (23) is a valuation equation which can be used to determine the price level. However, what matters in the central bank theory is not treasury's debt nor its primary surplus, but the level of nominal net worth of the central bank and its remittances' policy. An important difference is in the instrument through which price determination occurs. In the fiscal theory, the treasury levies real taxes in an amount to exceed what is needed to make its debt solvent. In the central bank theory, instead, the central bank is committed to make a real transfer to the treasury and then to the private sector. In this respect, the proposal to eliminate inflationary spirals is similar to that of the literature that requires the central bank to keep the money supply fixed in a deflation. This is equivalent to a commitment to make a real transfer to the private sector. Indeed, in real terms, the violation of the transversality condition as shown in (26) is exactly of the same sign and type. As discussed earlier and in line with the proposal of this work, what is important in both cases is that the liabilities that unboundedly increase in real terms are those defining the unit of account of the monetary system.

I compare now my proposal with that of Obstfeld and Rogoff (1983) who have also shown that the central bank has the power to eliminate inflationary spirals. However, as discussed by Obstfeld and Rogoff (2017), their proposal is in the same spirit as the fiscal theory since it requires fiscal backing: to redeem money at a fixed value the treasury has to levy lump-sum taxes. In the central bank theory of the price level, instead, there is no need of any fiscal support since the central bank can commit to make real transfers by printing reserves unboundedly.

In the Appendix, I present an alternative way to achieve price determination in which the central bank raises additional resources by the means of financial repression, i.e. by taxing the private sector imposing a reserve requirement at a below-market interest rate. In this case, if an inflationary spiral develops the central bank is committed to transfer to the private sector less resources than it has on its balance sheet. What entitles the central bank of the power to tax the financial sector through reserve requirement is again the special characteristics of its liabilities that define the 'unit of account' and are risk-free by definition. The financial sector can also manufacture risk-free securities but can be subject to run due to the possible illiquidity of the resources used to back them. The central bank is the only institution that can credibly be the lender of last resort in the 'unit of account' and can therefore solve illiquidity problems. By this virtue, it can exert a taxation power on the financial sector by forcing them to hold reserves at a rate below the market value.

5 Conclusion

I have described a monetary/fiscal policy regime that can uniquely determine prices in a simple endowment monetary economy. The important feature of the regime is that once the central bank is appropriately designed with an initial level of capital, a specified remittances' policy and the requirement of holding only short-term securities, and maintains financial independence from third parties, then it is equipped with all the relevant tools to defeat deflationary and inflationary spirals without the need of fiscal support.[19] The elements underlined are not new compared with the evidence on how central banks are designed and some of them are consistent with what economists have been arguing for hundreds years.[20] What is new is that they can determine uniquely a stable price level, once combined with a Taylor's rule, something that the related literature has hardly ever managed to achieve without treasury 'activism'. This work confirms the tendency of the last

[19] The Appendix shows that in the case the central bank holds long-term securities inflationary spirals or deflationary traps can develop in equilibrium.

[20] Discussions on the composition of the assets of the central bank go back to the 'real bills' doctrine proposed by John Law.

twenty years to establish central banks that are more and more independent from the treasuries and other third-party interferences.

References

[1] Atkeson, Andrew, V.V. Chari and Patrick Kehoe. "Sophisticated Monetary Policies." *The Quarterly Journal of Economics* 125 (1): 47-89.

[2] Bassetto, Marco. 2002. "A Game-Theoretic View of the Fiscal Theory of the Price Level." *Econometrica* 70 (6): 2167-2195.

[3] Bassetto, Marco. 2004. "Negative Nominal Interest Rates." *American Economic Review Papers and Proceedings* 94 (2): 104-108.

[4] Bassetto, Marco and Todd Messer. 2013. "Fiscal Consequence of Paying Interest on Reserves." Federal Reserve Bank of Chiacago, w.p. No. 2013-04.

[5] Benigno, Pierpaolo and Salvatore Nisticò. 2015. "Non-Neutrality of Open-Market Operations." CEPR Discussion Paper No. 10594.

[6] Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe. 2001. "Monetary Policy and Multiple Equilibria." *American Economic Review* 91: 167-186.

[7] Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe. 2002. "Avoiding Liquidity Traps." *Journal of Political Economy* 110: 535-563.

[8] Berriel, Tiago C. and Saroj Bhattarai. 2009. "Monetary Policy and Central Bank Balance Sheet Concerns." *The B.E. Journal of Macroeconomics*, Vol. 9(1), Contributions, Article 1.

[9] Bianchi, Francesco and Cosmin Ilut. 2017. "Monetary/Fiscal Policy Mix and Agent's Beliefs." *Review of Economic Dynamics* 26: 113-139.

[10] Buiter, Willem. 2009. "What's Left of Central Bank Independence?" *Financial Times,* Willem Buiter's Meverecon Blog.

[11] Buiter, Willem. 2017. "The Fallacy of the Fiscal Theory of the Price Level – Once More" CEPR Discussion Paper No. 11941.

[12] Canzoneri, Matthew, Robert Cumby and Behzad Diba 2011. "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review* 91(5): 1221-1238.

[13] Christiano, Lawrence and Yuta Takahashi (2018). "Discouraging Deviant Behavior in Monetary Economics." Unpublished manuscript, Northwestern University.

[14] Cochrane, John H. 2011. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy* 119 (3): 565-615.

[15] Del Negro, Marco and Christopher A. Sims. 2015. "When Does a Central Bank's Balance Sheet Require Fiscal Support?" *Journal of Monetary Economics* 73: 1-19.

[16] Hall, Robert and Ricardo Reis. 2015. "Maintaining Central-Bank Solvency Under New-Style Central Banking." NBER Working Paper No. 21173.

[17] Hall, Robert and Ricardo Reis. 2016. "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves." NBER Working Paper No. 22761.

[18] Gaballo, Gaetano and Eric Mengus. 2018. "Implicit Fiscal Guarantee for Monetary Stability." Unpublished manuscript, Banque de France and HEC Paris.

[19] Leeper, Eric. 1991. "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27: 129-147.

[20] Niepelt, Dirk. 2004. "The Fiscal Mith of the Price Level." *The Quarterly Journal of Economics* 119 (1): 277-300.

[21] Park, Seok Gil. 2015. "Central Banks' Quasi-Fiscal Policies and Inflation." *International Journal of Central Banking,* March 2015.

[22] Obstfeld, Maurice, and Kenneth Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91: 675-687.

[23] Obstfeld, Maurice, and Kenneth Rogoff. 2017. "Revisiting Speculative Hyperinflations in Monetary Models." CEPR Discussion Paper No. 12051.

[24] Reis, Ricardo. 2015. Comment on: "When does a central bank's balance sheet require fiscal support?" by Marco Del Negro and Christopher A. Sims, *Journal of Monetary Economics* 73: 20-25.

[25] Sims, Christopher. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4: 381-399.

[26] Sims, Christopher. 1999. "The Precarious Fiscal Foundations of EMU." *De Economist* 147: 415-436.

[27] Sims, Christopher. 2000. "Fiscal Aspects of Central Bank Independence." Unpublished manuscript, Princeton University.

[28] Sims, Christopher. 2005. "Limits to Inflation Targeting." In Ben S. Bernanke and Michael Woodford (Eds.) *National Bureau of Economic Research Studies in Business Cycles,* vol. 32, ch. 7, Chicago: University of Chicago Press, pp 283-310.

[29] Sims, Christopher. 2013. "Paper Money." *American Economic Review* 103(2): 563-584.

[30] Sims, Christopher. 2016. "Fiscal Policy, Monetary Policy and Central Bank Independence." Unpublished manuscript, Princeton University.

[31] Woodford, Michael. 1995. "Price Level Determinacy without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy* 43: 1-46.

[32] Woodford, Michael. 2000. "Monetary Policy in a World without Money." *International Finance,* 2(3): 229-260.

[33] Woodford, Michael. 2001a. "Monetary Policy in the Information Economy." In *Economic Policy for the Information Economy.* Kansas City: Federal Reserve Bank of Kansas City: 297-370.

[34] Woodford, Michael. 2001b. "Fiscal Requirements for Price Stability." *Journal of Money, Credit and Banking* 33 (1): 669-728.

[35] Woodford, Michael. 2003. *Interest and Prices.* Princeton, NJ: Princeton University Press.

A Fiscal theory of the price level

In this Appendix, I discuss price level determination according to the fiscal theory of the price level using the model of Sections 2 and 3. To determine equilibrium prices, consider the following specification of the path of real primary surpluses $\{\tau_t\}_{t=t_0}^{\infty}$. Let the government run a deficit at time t_0 in real terms, $\tau_{t_0} = \tau_{t_0}^* < 0$, and instead set the path of future real primary surpluses $\{\tau_t\}_{t=t_0+1}^{\infty}$ at the level $\tau_t = \tau_t^*$ under the following restriction

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}\tau_t^* = \frac{B_{t_0}^G}{P^*}, \tag{A.1}$$

in which I have defined the overall government liabilities as

$$B_t^G \equiv X_t^C + B_t$$

comprising central bank's reserves and treasury's debt held by the private sector.

The discounted path of real primary surpluses, as of time t_0+1, is independent of the price level at the same time but directly related to the outstanding nominal liabilities, $B_{t_0}^G$, that the government has to pay at time t_0+1. Use (A.1) and $\tau_{t_0} = \tau_{t_0}^*$ into (9) to obtain

$$\beta\frac{B_{t_0}^G}{P^*} + \tau_{t_0}^* = 0. \tag{A.2}$$

Consider now the government's flow budget constraint

$$\frac{B_t^G}{1+i_t} = B_{t-1}^G - P_t\tau_t, \tag{A.3}$$

with $B_{t_0-1}^G = 0$. In equilibrium $B_t^G = B_t + X_t$. Since $\tau_{t_0} = \tau_{t_0}^*$, the budget constraint (A.3) implies that the government has to issue debt in the amount $B_{t_0}^G = -(1+i_{t_0})P_{t_0}\tau_{t_0}^*$ which in turn implies that $\tau_{t_0}^* = -\beta B_{t_0}^G/P_{t_0+1}$, having used the Fisher equation (8).

After substituting $\tau_{t_0}^* = -\beta B_{t_0}^G/P_{t_0+1}$ into (A.2), I obtain

$$\beta\left(\frac{B_{t_0}^G}{P^*} - \frac{B_{t_0}^G}{P_{t_0+1}}\right) = 0. \tag{A.4}$$

The above equation is satisfied if and only $P_{t_0+1} = P^*$. Using this result into (12), it follows that $P_{t_0} = P^*$. Therefore, if the government commits to $\{\tau_t^*\}_{t=t_0}^{\infty}$, there is only one equilibrium path of prices: $P_t = P^*$ for each $t \geq t_0$.[21]

I am now going to underline the importance of a key feature of the notion of competitive equilibrium used, i.e. that the sequence $\{\tau_t\}_{t=t_0}^{\infty}$ is taken as given by the consumers when maximizing utility under the constraints (1) and (2). In particular, consumers understand that the sequence $\{\tau_t\}_{t=t_0+1}^{\infty}$ satisfies (A.1) without questioning the strength of the commitment. This is coherent with the notion of competitive equilibrium since, indeed, $\{\tau_t^*\}_{t=t_0}^{\infty}$ is what observed in equilibrium. However, the beliefs of the consumers on the path followed by fiscal policy are critical to rule out deviations of P_{t_0} from P^* already at time t_0. To be clear, if the price level at time t_0 is lower than P^*, the adjustment mechanism operating through the excess demand of goods, which pushes up the price level, relies on the consumer belief that the tax policy (A.1) is going to be implemented in such circumstances. If the tax policy is unfeasible or not going to be implemented in these conditions, for reasons that I will explain, then prices below P^* are equilibrium prices. Therefore it is important to investigate which kind of commitment the government is taking off equilibrium.[22]

In what follows, whenever I am analyzing deflationary or inflationary solutions, I am implicitly focusing on the case in which $\phi > 0$ in the policy rule (11). But the analysis applies also to the case $\phi = 0$ with appropriate amendments.

Suppose that a deflationary path develops and that $P_t < P^*$ at a generic time t. The government reaches period t with outstanding real debt B_{t-1}^G/P_t but the commitment (A.1) promises a path of real primary surplus that is below the outstanding level of obligations that the government would face at that time

$$\sum_{T=t}^{\infty} \beta^{T-t} \tau_T^* = \frac{B_{t-1}^G}{P^*} < \frac{B_{t-1}^G}{P_t}. \tag{A.5}$$

This is indeed consistent with the proposal of Benhabib et al. (2001), namely that, in a deflation, the government should commit to reduce taxes in order

[21] Note, however, that a barter economy is always an equilibrium if $\phi > 0$.

[22] Bassetto (2002) has analyzed the equilibrium that would result as an outcome of a strategic game between government and private sector.

to inflate the economy. However, as shown by the above equation, the commitment leaves part of the outstanding real obligations unbacked.

To understand what is behind this commitment, I have to go back to the notion of a 'unit of account' that I have explained in the introduction. Is B_{t-1}^G just short-term treasury's debt denominated in the 'unit of account' or includes also the liabilities that define the 'unit of account'? And in the former case, is there a connection, even at some future point in time, with the liabilities that define the 'unit of account'? Recall the definition $B_t^G \equiv X_t^C + B_t$.

To clarify these questions, I provide a simple example. Consider the European Monetary Union, in which the 'unit of account' is defined in terms of the liabilities of the European Central Bank. As I explained, the ECB can set its policy by fixing the interest rate on reserves even if it stands ready to supply a negligible amount of reserves. Suppose this is the case. Therefore, in this example, B_t^G denotes mostly sovereign debt, denominated in euro, of a group of countries belonging to the union, i.e. $B_t^G \simeq B_t$. The above inequality shows that, in a deflation, the real value of this debt is less than the resources the countries commit to pay. Three things can happen: i) the debt is fully repaid; ii) it is defaulted on; iii) it is fully backed by central bank's reserves. In the first case, countries have to increase their real primary surpluses to back all the outstanding real obligations. It follows that deflations are going to be equilibria since the commitment to lower taxes in a deflation is not feasible. In the second case, taxes are not adjusted therefore debt should be seized and its market price adjusts along the path. Even in this case, the deflation cannot be ruled out as an equilibrium. In the third case, it is tacit that at some point in time the ECB is going to buy the countries' debt and proportionally issues units of account in the form of reserves. In this case, the deflation is disallowed as an equilibrium if and only if it is understood that along the deflationary path the supply of reserves is let to growth unboundedly in real terms at a rate higher than $1/\beta$.[23] Indeed, by iterating forward the budget constraint of the government (A.3), I get that

$$\frac{B_{t-1}^G}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} \tau_T + \lim_{T \to \infty} \left\{ \beta^T \frac{B_T^G}{P_T(1+i_T)} \right\}$$

[23] This is also the case if, to start with, B_t denotes central bank's reserves.

which implies, by using the inequality (A.5), that

$$\lim_{T\to\infty}\left\{\beta^T\frac{B_T^G}{P_T(1+i_T)}\right\}>0. \tag{A.6}$$

The key distinction between the third case and the first two is that the central bank is the only agent in the economy which is not subject to a solvency condition since its liabilities are always repaid in the 'unit of account' regardless of the path of the price level and of central bank's resources.[24] Therefore it is the only agent that can lead to violations of the transversality condition on off-equilibrium paths. Deflations are ruled out not on the basis of a single action of the fiscal authority but on the coordination between fiscal policy, through the reduction of primary surpluses, and monetary policy, through the expansion of its liabilities. The success of the combination of policies necessarily relies on the power of central bank liabilities

The above argument does not apply in a symmetric way to the case in which an inflationary solution develops. Now the commitment (A.1) requires the present-discounted value of the real primary surplus to exceed the level of outstanding real obligations if $P_t > P^*$:

$$\sum_{T=t}^{\infty}\beta^{T-t}\tau_T^*=\frac{B_{t-1}^G}{P^*}>\frac{B_{t-1}^G}{P_t}.$$

Whether B^G denotes sovereign debt or central bank's reserves, it does not really matter since in any case this debt is going to be free of risk at the off-equilibrium price $P_t > P^*$.

To understand the commitment I can pose the following two questions. First, has the treasury enough resources to back debt at a higher real value? Second, has the treasury the willingness to provide such an anchor?

Let me answer the first question. Suppose that there is an upper limit $\bar{d}_t$ on how many real resources the treasury is able to raise at any point in time

[24] Several works in the literature, among which Obstfeld and Rogoff (1983) and Sims (1994), have shown that deflationary solutions can be defeated by setting a target for the supply of money. However, in these analyses, the central bank sets its policy in terms of the supply of money rather than an interest-rate rule. Woodford (1995, 1999, 2001b., 2003 ch. 2) instead assumes an interest-rate policy and shows that deflations can be ruled out by targeting the growth of the overall nominal liabilities of the government (including both treasury and central bank). This is in line with my analysis with the caveat that the floor should be necessarily put on the path of central bank's liabilities. See also Buiter (2017).

so that

$$\sum_{T=t}^{\infty} \beta^{T-t} \tau_T \leq \bar{d}_t.$$

If $\bar{d}_t$ is less than B^G_{t-1}/P^* for any finite level of debt B^G_{t-1} reached in an inflationary path, the treasury does not have enough resources to disallow all the inflationary solutions.[25] This means that a policy rule (11) with $\phi > 0$ is at the same time consistent with a stable price level P^* and inflationary equilibria. If the commitment to (11) is irrevocable, it is not even possible for the fiscal authority to backstop inflation by promising to repay debt at a price P^{**} greater then P^*, unless the central bank changes simultaneously (11) using a target P^{**} rather than P^*. Therefore, for countries that have weak fiscal ability, the feasibility of the commitment (A.1) and, at the same time, of (11) can be questioned. In monetary unions with several fiscal authorities, like the EMU, coordination problems can further weaken the overall fiscal capacity, as Sims (1999) has emphasized.

Consider now that there is no upper limit $\bar{d}_t$ on the resources that the treasury can raise. This is not sufficient to disallow inflationary solutions. At the end, on off-equilibrium paths, fiscal policy could passively accommodate an inflationary spiral and save on taxes following own – here unmodeled – incentives. It could also set primary surpluses to target a higher $P^{**} > P^*$, save on taxes, but conflict with the interest-rate policy of the central bank. In this case, either the treasury or the central bank should give up on their policy.

The only possibility for inflationary solutions to be ruled out is that the treasury internalizes the objective of the central bank – to keep prices at P^*– and, thereby, provides a large enough fiscal adjustment in any possible upward deviation. Without this fiscal anchor, the central bank is helpless to counteract inflationary spirals. Though, as shown before, it plays an important role in eliminating deflationary spirals.[26]

[25] Note that the level of nominal debt B^G_{t-1} reached in an inflationary path is higher than that under the constant price P^*, for the same path of real primary surpluses followed until that point in time.

[26] The only case in which the central bank has no role in the fiscal theory of the price level is that in which treasury's liabilities define the 'unit of account'. But, in this case, there is no need of a central bank at all.

B Addendum to Section 4.2

In Appendix, I present an alternative to the theory of Section 4.2 through which inflationary solutions are ruled out by the means of transfers made by the central bank that raises additional resources through financial repression, i.e. by taxing the private sector imposing a reserve requirement at a below-market interest rate. In this case, if an inflationary spiral develops the central bank is committed to transfer to the private sector less resources than it has on its balance sheet.

Starting from the same time $\tilde{t}$ defined in Section 4.2, the central bank could impose a reserve requirement on the debt issued by the private sector, while maintaining all the other features specified in the Section. This is possible since the private sector, as of time $\tilde{t}-1$, is net debtor with respect to the central bank, indeed $N^C_{\tilde{t}-1} > 0$ implies that $B_{\tilde{t}-1} + X_{\tilde{t}-1} < 0$. Denote with X^r_t the required reserves which are remunerated at i^r_t, below the market rate. After time $\tilde{t}$, central bank's net worth is given by:

$$N^C_t = \frac{B^C_t - X^C_t}{(1+i_t)} - \frac{X^r_t}{(1+i^r_t)}.$$

Taking into account the transversality condition (22), I can replace the equilibrium condition (23), at time $\tilde{t}-1$, with

$$\frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}} + \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \left(\frac{i_T - i^r_T}{1+i_T}\right) \frac{X^r_T}{P_T} = \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T^C_T}{P_T}. \tag{B.7}$$

The central bank has now two additional intertwined instruments, X^r_t and i^r_t that can be set for each period $t \geq \tilde{t}$. One possibility is to assume that they are implicitly defined by the following condition

$$\sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \left(\frac{i_T - i^r_T}{1+i_T}\right) \frac{X^r_T}{P_T} = (1+\epsilon) \left(\frac{N^C_{\tilde{t}-1}}{P^*} - \frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}}\right) \tag{B.8}$$

for any finite $P_{\tilde{t}-1}$ and for some positive ϵ, which can be considered small enough. Note that the left-hand side of (B.8) is positive, given that $X^r \geq 0$ and $i \geq i^r$, if and only if the central bank net worth $N^C_{\tilde{t}-1}$ at time $\tilde{t}-1$ is positive.

Substituting (B.8) into the left-hand side of (B.7), I obtain

$$\frac{N^C_{\tilde{t}-1}}{P^*} + \epsilon \left(\frac{N^C_{\tilde{t}-1}}{P^*} - \frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}}\right) = \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T^C_T}{P_T}.$$

It is again the case that by following threat (21) after time $\tilde{t} - 1$, the central bank can determine uniquely the price level at P^* forever. To get the result, substitute (21) for the sequence of real taxes, T^C/P, into the above equation. The important difference with respect to the solution of Section 4.2 is that now the central bank is committed to rebate less resources to the private sector than what it has available on its balance sheet if an inflationary spiral would ever develop. The mechanism that rules out inflationary solutions acts now through a contraction in aggregate demand, because of the resources withheld by the central bank, until the target price level P^* is reached. Differently from the solution given in Section 4.2, prices are not pushed to infinite but converge back to P^*.

C Unconventional open-market operations

In this Appendix, I extend the model of Section 2 to allow for long-term securities in order to analyze how results change when the central bank engages in unconventional open-market operations and holds risky securities in its balance sheet.

C.1 Consumers

Consumers have preferences:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t) \tag{C.9}$$

where β is the intertemporal discount factor with $0 < \beta < 1$, c is a consumption good and $u(\cdot)$ is a concave function, twice continuously differentiable, increasing in c.

The consumers' budget constraint is:

$$\frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + P_t(y - c_t) - T_t^F. \tag{C.10}$$

Consumers can invest their financial wealth in interest-bearing reserves, X_t, issued by the central bank at the risk-free nominal interest rate i_t and can lend or borrow using short-term securities, B_t, at the same interest rate i_t. D_t indicates holdings of long-term securities issued at a price Q_t. The security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta,\ \delta^2, \delta^3...$ in the following periods and in the case of no default.[27] The variable $\varkappa_t$ on the right-hand side of (C.10) captures the possibility that long-term securities can be partially seized by exogenous default. y is a constant endowment of the only good traded; T_t^F are lump-sum taxes levied by the treasury. There are no financial markets before time t_0, therefore $B_{t_0-1},\ X_{t_0-1}, D_{t_0-1}$ are all equal to zero.

The consumers' problem is subject to a borrowing limit of the form

$$\lim_{T\to\infty}\left\{R_{t_0,T}\left(\frac{B_T+X_T}{1+i_T}+Q_TD_T\right)\right\}\geq 0 \tag{C.11}$$

and to the bound

$$\sum_{T=t_0}^{\infty} R_{t_0,T}P_Tc_T<\infty \tag{C.12}$$

since there is no limit to the ability of households to borrow against future income.

Households choose consumption, and asset allocations to maximize utility (C.9) under constraints (C.10), (C.11), (C.12) given the initial conditions. The set of first-order conditions imply the Euler equation

$$\frac{u_c(c_t)}{P_t}=\beta(1+i_t)\frac{u_c(c_{t+1})}{P_{t+1}} \tag{C.13}$$

at each time $t\geq t_0$ assuming interior solution.

Absence of arbitrage opportunities implies that

$$Q_t=\beta\frac{u_c(c_{t+1})}{u_c(c_t)}\frac{P_t}{P_{t+1}}(1-\varkappa_{t+1})(1+\delta Q_{t+1}) \tag{C.14}$$

from which a "fundamental" solution for long-term bond prices follows:

$$Q_t=\sum_{T=t}^{\infty}\delta^{T-t}\beta^{T+1-t}\frac{u_c(c_{T+1})}{u_c(c_t)}\left(\frac{P_t}{P_{T+1}}\right)\prod_{j=t+1}^{T+1}(1-\varkappa_j),$$

[27] The stock of long-term asset follows the law of motion $D_t = Z_t+(1-\delta)D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, supplied at time t. See among others Woodford (2001).

at each time $t \geq t_0$.

In a perfect-foresight equilibrium the return on long-term bonds is also equal to the short-term interest rate as shown by combining (C.13) and (C.14)

$$r_{t+1} = i_t \tag{C.15}$$

with the return on long-term bonds defined by $r_{t+1} \equiv (1-\varkappa_{t+1})(1+\delta Q_{t+1})/Q_t - 1$.

To conclude the characterization of the consumer's problem, a transversality condition applies and therefore (C.11) holds with equality, given the equilibrium nominal stochastic discount factor

$$R_{t_0,T} = \beta^{T-t_0} \frac{u_c(c_T)}{u_c(c_{t_0})} \frac{P_{t_0}}{P_T}.$$

C.2 Treasury

The treasury raises lump-sum taxes T_t^F (net of transfers) from the private sector and receives remittances T^C (when T^C is positive) or makes transfers to the central bank (when T^C is negative). The treasury can finance its deficit through short-term debt (B^F) at the price $1/(1+i_t)$, facing the following flow budget constraint

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t^F - T_t^C$$

given initial condition $B_{t_0-1}^F = 0$. To simplify the analysis, I assume that the treasury does not issue long-term securities.

C.3 Central Bank

The central bank can invest in short and long-term securities, B_t^C and D_t^C, by issuing reserves X_t^C. Net worth, N_t^C is defined as

$$N_t^C \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - \frac{X_t^C}{1+i_t}, \tag{C.16}$$

with law of motion given by:

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C \tag{C.17}$$

where Ψ_t^C are central bank's profits:

$$\Psi_t^C = i_{t-1} N_{t-1}^C + (r_t - i_{t-1}) Q_{t-1} D_{t-1}^C. \tag{C.18}$$

Combining (C.16), (C.17) and (C.18), the central bank's flow budget constraint follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C$, all equal to zero.

C.4 Equilibrium

Equilibrium in the goods market implies that

$$c_t = y,$$

at each time $t \geq t_0$ while equilibrium in the asset markets that

$$B_t + B_t^C = B_t^F,$$

$$X_t = X_t^C,$$

$$D_t + D_t^C = 0.$$

C.5 Equilibrium conditions

I now characterize in a compact way the equilibrium conditions of the model.

The Fisher's equation follows from the Euler equation (C.13) using equilibrium in the goods market

$$1 + i_t = \frac{1}{\beta} \frac{P_{t+1}}{P_t}, \tag{C.19}$$

while the equilibrium price of long-term securities is:

$$Q_t = \sum_{T=t}^{\infty} \delta^{T-t} \beta^{T+1-t} \left(\frac{P_t}{P_{T+1}} \right) \prod_{j=t+1}^{T+1} (1 - \varkappa_j). \tag{C.20}$$

The household's transversality condition can be simplified to

$$\lim_{T\to\infty}\left\{\beta^{T-t_0}\left(\frac{P_{t_0}}{P_T}\right)\left(\frac{B_T+X_T}{1+i_T}+Q_TD_T\right)\right\}=0, \tag{C.21}$$

while the bound (C.12) can be written as

$$\sum_{T=t_0}^{\infty}\beta^{T-t_0}y<\infty$$

which is naturally satisfied.

The flow budget constraints of treasury and central bank are respectively

$$\frac{B_t^F}{1+i_t}=B_{t-1}^F-T_t^F-T_t^C, \tag{C.22}$$

$$Q_tD_t^C+\frac{B_t^C}{1+i_t}-\frac{X_t^C}{1+i_t}=(1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C+B_{t-1}^C-X_{t-1}^C-T_t^C, \tag{C.23}$$

while equilibrium in the securities market closes the model

$$B_t+B_t^C=B_t^F, \tag{C.24}$$

$$X_t=X_t^C, \tag{C.25}$$

$$D_t+D_t^C=0. \tag{C.26}$$

A rational-expectations equilibrium is a collection of processes $\{P_t,\, i_t,\, Q_t,\, T_t^F,\, T_t^C,\, B_t,\, B_t^C,\, B_t^F,\, D_t^C,\, X_t\}_{t=t_0}^{\infty}$ that satisfy (C.19)-(C.24) at each date $t\geq t_0$ given (C.25)-(C.26) and initial conditions $B_{t_0-1}^C$, $B_{t_0-1}^F$,D_{t_0-1},X_{t_0-1} all equal to zero. Since (C.21) is a bound, there are five degrees of freedom to specify the monetary/fiscal policy regime. Note equation (C.21) can be replaced by (9) which is an equivalent condition using equations (C.22)-(C.26).

I can now define equilibrium in this more general model.

Definition 6 *An equilibrium is a set of sequences* $\{P_t,\, i_t,\, T_t,\, T_t^F,\, T_t^C,\, B_t^F,\, B_t^C,\, D_t^C,\, X_t^C\}_{t=t_0}^{\infty}$ *with* $P_t, i_t\geq 0$ *that solve equations (8), (9), (6),* $T_t=T_t^F$*, and (C.23), given the specification of the monetary/fiscal policy regime which sets the sequences* $\{i_t, T_t^F, T_t^C, X_t, D_t^C\}_{t=t_0}^{\infty}$*, given initial conditions* $B_{t_0-1}^F=B_{t_0-1}^C=D_{t_0-1}^C=X_{t_0-1}^C=0$ *and the definition* $\tau_t=T_t/P_t$*.*

There are now four restrictions for the nine unknowns leaving five degrees of freedom to specify the monetary/fiscal policy regime. On top of the specifications given in Definitions 2 and 4, I need to set the sequence $\{D_t^C\}_{t=t_0}^{\infty}$ of long-term asset purchases by the central bank.

C.6 Results with treasury's support

By holding long-term bonds, the central bank can be subject to income losses due to unforeseen shocks. However, the results of Section 4.1 on how to trim deflationary spirals hold even in this more general framework, as shown in the following Proposition.

Proposition 7 *Given the monetary/fiscal policy regime of Definition 2 and a non-negative sequence* $\{D_t^C\}_{t=t_0}^{\infty}$*, it follows that* $P_t \geq P^*$ *in equilibrium for each* $t \geq t_0$*.*

Proof. As in Proposition 3. ■

The key element for extending the result of Section 4.1 is to interpret the transfer rule $T_t^C = \Psi_t^C$ in a symmetric way. In particular, the rule implies that the treasury is committed to transfer resources to the central bank in the case of negative profits.[28] Given the initial capitalization and the commitment to $T_t^C = \Psi_t^C$, central bank's nominal net worth remains constant and all the discussion of Section 4.1 to eliminate deflationary spirals applies to this more general context.

However, in this case, the central bank is no longer financially independent from the treasury which brings about the risk that it could be asked to remit additional dividends at the treasury's will. What is going to be weakened, in this case, is the strength of the commitment that rules out deflationary paths, as discussed in Section 4.1. If this weakness is understood by the private sector then deflations can develop unraveling the uniqueness of equilibrium.

The result of this section can be consistent with the story of a central bank that undertakes unconventional open-market operations with a deflation going on. In this environment, it is also possible that the central bank derives profits from its holdings of risky assets, as a consequence of unexpected deflationary shocks, and can therefore rebate income to the treasury. However, it is understood that in the case of losses – following perhaps a future exit from a policy of zero nominal interest rates – the treasury stands ready to support the central bank. This implicit support might be enough to undermine the financial independence of the central bank during the zero

[28] An interesting example in the recent financial crisis of explicit treasury's support is that of the Bank of England which in January 2009 established a wholly-owned subsidiary with the responsibility of buying private and public long-term securities. The company is fully indemnified by the Treasury since any financial losses are borne by the Treasury and any gains are owed to the Treasury.

interest-rate policy because it suggests that it could be in the treasury's ability to expropriate central bank's net worth. Although these raids are not in the observation period, the expectation that they will occur is sufficient to validate the deflationary path.

The results of Section 4.2 to eliminate inflationary spirals extend as well to the case in which the central bank holds long-term securities.

Proposition 8 *Given the monetary/fiscal policy regime of Definition 4 and a non-negative sequence* $\{D_t^C\}_{t=t_0}^{\infty}$*, it follows that* $P_t = P^*$ *in equilibrium for each* $t \geq t_0$*.*

Proof. As in Proposition 5. ■

Even in this case, nothing changes because the treasury is covering central bank's losses and therefore central bank's net worth is kept at the initial value.

In the next section, I am going to analyze the case in which the central bank retains financial independence by refusing any treasury's support beyond the initial capitalization. I am going to show that by purchasing risky securities it can lose control of the price level.

C.7 Results without treasury's support

I still maintain the assumption that at time t_0 the treasury provides the initial capital through which the central bank starts its operations. However, after time t_0, remittances are assumed to be non-negative, $T_t^C \geq 0$, excluding any possible support from the treasury. In particular, I assume that the central bank transfers all its income to the treasury provided nominal net worth is not below the initial level $\bar{N}$.[29] But, as nominal net worth falls below $\bar{N}$ because of negative profits, the central bank rebuilds it by retaining earnings up to the point in which the initial level $\bar{N}$ is recovered. Therefore for each $t > t_0$ $T_t^C = \max(\Psi_t^C, 0)$ whenever $N_{t-1}^C \geq \bar{N}$ and $T_t^C = 0$ if $N_{t-1}^C < \bar{N}$. This remittances' policy has a real-world counterpart in the deferred-asset regime currently used by the Federal Reserve System for which, whenever capital falls, the central bank stops making remittances and accounts for a deferred asset in its balance sheet paid later by retained earnings. Only once the deferred asset is paid in full, the central bank returns to rebate profits to the treasury.

[29] I define $\bar{N} \equiv P_{t_0} n_{t_0}^C$.

I now define the two monetary/fiscal policy regimes which are the closest counterparts to those of Definitions 2 and 4, with the appropriate qualifications. The following will be used for deflationary spirals.

Definition 9 *Define the following monetary/fiscal policy regime setting the sequences* $\{i_t, T_t^F, T_t^C, X_t, D_t^C\}_{t=t_0}^{\infty}$ *: i) the policy rule (11), with* $\phi > 0$*, specifies* i_t*, ii)* $T_t^F = -T_t^C$ *at each date* $t \geq t_0$*, iii)* $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ *for some* $n_{t_0}^C > 0$*,* $T_t^C = \max(\Psi_t^C, 0)$ *if* $N_{t-1}^C \geq \bar{N}$ *and* $T_t^C = 0$ *if* $N_{t-1}^C < \bar{N}$ *for* $t > t_0$*; iv)* $X_t > 0$ *and v)* $D_t^C > 0$ *for each* $t \geq t_0$*.*

The only difference with respect to Definition 2 is in the specification *iii)* which excludes transfers from the treasury to the central bank. The following monetary/fiscal policy regime instead replaces that of Definition 4 to trim inflationary spirals.

Definition 10 *Define the following monetary/fiscal policy regime setting the sequences* $\{i_t, T_t^F, T_t^C, X_t, D_t^C\}_{t=t_0}^{\infty}$ *: i) the policy rule (11), with* $\phi > 0$*, specifies* i_t*, ii)* $T_t^F = -T_t^C$ *at each date* $t \geq t_0$*, iii)* $T_{t_0}^C = -P_{t_0} n_{t_0}^C$ *for some* $n_{t_0}^C > 0$ *and* $T_t^C = \max(\Psi_t^C, 0)$ *if* $N_{t-1}^C \geq \bar{N}$ *and* $T_t^C = 0$ *if* $N_{t-1}^C < \bar{N}$ *and for* $t_0 < t < \tilde{t}$ *while* $\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta} \frac{P_{t_0}}{P^*} n_{t_0}^C$ *for each* $t \geq \tilde{t}$ *and for some* $\tilde{t} > t_0$ *provided* $N_{\tilde{t}-1}^C \geq \bar{N}$*; iv)* $X_t > 0$ *and v)* $D_t^C > 0$ *for each* $t \geq t_0$*.*

In Definition 10 one important aspect to underline is that the switch to a real remittances rule (see item iii) is triggered at a generic time $\tilde{t} > t_0$ only if the previous-period net worth is at the initial threshold $\bar{N}$. I am also going to assume that time $\tilde{t}$ can be postponed until this occurs.

C.7.1 Interest-rate shock

This Section shows that in the case of an unforeseen interest-rate shock there can be multiple solutions: the price stability equilibrium coexists with inflationary spirals, whereas deflationary solutions are ruled out.

Before presenting Propositions and Proofs, let me discuss some ingredients that will be relevant for the analysis. First, note that using (12) into (C.20), the price of long-term bonds Q_t can be expressed as a function of P_t, that is $Q_t = Q(P_t)$, which has an upper-bound value of $\bar{Q} = 1/(1-\delta)$ and is decreasing in P_t.[30] As a consequence, I can also write the return on

[30] In this subsection, I am assuming that $\varkappa_t = 0$ at all times.

long-term securities and central bank's profits at time t as a function of P_t, that is $r(P_t)$ and $\Psi^C(P_t)$ respectively, where profits are given by

$$\Psi^C(P_t) = i_{t-1}N^C_{t-1} + (r(P_t) - i_{t-1})Q_{t-1}D^C_{t-1}.$$

Given its dependence on r_t and therefore on Q_t, the level reached by central bank's net worth at time t is also a function of P_t

$$\begin{aligned} N^C(P_t) &= N^C_{t-1} + \Psi^C(P_t) - \max(\Psi^C_t(P_t), 0) \qquad \text{if} \quad N^C_{t-1} \geq \bar{N} \\ N^C(P_t) &= N^C_{t-1} + \Psi^C(P_t) \qquad \text{if} \quad N^C_{t-1} < \bar{N} \end{aligned}$$

which follows from (17) where I have used the remittances' rule of Definitions 9 and 10.

First, I show that deflationary paths are ruled out extending Proposition 3.

Proposition 11 *Given the monetary/fiscal policy regime of Definition 9 it follows that $P_t \geq P^*$ in equilibrium for each $t \geq t_0$.*

Proof. First I show that solution paths in which $P_t < P^*$ at time t_0 are ruled out. Since the deflationary path is foreseen at time t_0, profits are always positive and therefore Proposition 3 applies. I now consider the case in which prices are first foreseen to be constant, $P_t = P^*$, for each time $t \geq t_0$ then, unexpectedly, they fall, $P_{\hat{t}} < P^*$, at a generic time $\hat{t}$ (with $\hat{t} > t_0$) and from that time they follow a deflationary path according to (12). When unexpectedly $P_t < P^*$ at $\hat{t} > t_0$, the return on long-term bonds $r(P_{\hat{t}})$ exceeds $i_{\hat{t}-1}$, and profits remain positive. Therefore the proof of Proposition 3 still applies. ■

The intuition for why deflationary solutions are eliminated even when the central bank holds long-term bonds depends on the fact that an unexpected fall in the price level produces a rise in the return on long-term bonds and therefore profits remain positive and net worth constant, given the remittances rule. A constant nominal net worth rules out deflationary solutions as the proof of Proposition 3 shows.

I will now discuss the existence of inflationary equilibria.

Proposition 12 *Given the monetary/fiscal policy regime of Definition 10 it follows that there are multiple equilibria. There is an equilibrium in which $P_t = P^*$ for each $t \geq t_0$ and an equilibrium in which $P_t = P^*$ for each*

$t_0 \le t < \hat{t}$ and unexpectedly $P_t > P^*$ at time $\hat{t}$, with $t_0 < \hat{t} < \tilde{t} - 1$, and in which prices follow an inflationary spiral afterward according to (12) and net worth is zero for each $t \ge \hat{t}$.

Proof. First, I show that solution paths in which $P_t > P^*$ at time t_0 are ruled out. Since the inflationary path is foreseen at time t_0, profits are always positive and therefore Proposition 3 applies. I will now check for solution paths in which it is first expected that $P_t = P^*$ for each $t \ge t_0$ and then unexpectedly $P_{\hat{t}} > P^*$ at $\hat{t} > t_0$ with an inflationary path following equation (12) afterward. In this case, $r(P_{\hat{t}}) < i_{\hat{t}-1}$ and $r(P_t) = i_{t-1}$ at all other times. Several cases are possible. First note that Proposition 5 applies if the time $\hat{t}$ is larger or equal to $\tilde{t} - 1$, i.e. $\hat{t} \ge \tilde{t} - 1$, given that the remittances rule requires that $\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta} \frac{P_{t_0}}{P^*} n_{t_0}^C$ for each $t \ge \tilde{t}$. Indeed

$$\frac{N^C(P_{\hat{t}})}{P_{\hat{t}}} = \sum_{T=\hat{t}+1}^{\infty} \beta^{T-\hat{t}} \frac{T_T^C}{P_T} = \frac{P_{t_0} n_{t_0}^C}{P^*}.$$

Note that $N^C(P_{\hat{t}})$ is monotone non-increasing with $P_{\hat{t}}$ and, therefore, the left-hand side of the above equation is decreasing with $P_{\hat{t}} > P^*$ and is equal to the right-hand side only when $P_{\hat{t}} = P^*$. Therefore paths in which $P_{\hat{t}} > P^*$ with $\hat{t} \ge \tilde{t} - 1$ are not equilibria. Let us focus on the case in which $P_{\hat{t}} > P^*$ at $\hat{t} < \tilde{t} - 1$. I distinguish several possibilities. First, if $\Psi^C(P_{\hat{t}}) > 0$, $T_t^C = \Psi_t^C$ in all periods and therefore Proposition 5 applies ruling out paths in which $P_{\hat{t}} > P^*$. Second, if $\Psi^C(P_{\hat{t}}) < 0$ and $N^C(P_{\hat{t}}) > 0$, then net worth is increasing after time $\hat{t}$ because profits are going to be positive since the inflationary path is perfectly foreseen after $\hat{t}$. Therefore, net worth increases and it will reach the value $\bar{N}^C$ at time $\tilde{t} - 1$ since indeed I have assumed that time $\tilde{t}$ can be postponed until that occurs. Therefore at that time, the following equivalence holds

$$\frac{\bar{N}^C}{P_{\tilde{t}-1}} = \sum_{T=\tilde{t}}^{\infty} \beta^{T+1-\hat{t}} \frac{T_T^C}{P_T} = \frac{P_{t_0} n_{t_0}^C}{P^*}$$

which can only be satisfied if and only if $P_{\tilde{t}-1} = P^*$ ruling out path in which $P_{\tilde{t}-1} > P^*$ and therefore $P_{\hat{t}} > P^*$ at $\hat{t} < \tilde{t} - 1$. Third, consider the case in which $\Psi^C(P_{\hat{t}}) < 0$ and also net worth is negative $N^C(P_{\hat{t}}) < 0$ then net worth remains negative in all the following periods since profits are negative. Net worth will follow the path $N_t^C = (1 + i_{t-1}) N_{t-1}^C$ since remittances will be always zero and the real remittances policy is not triggered. Given that $1 + i_t = \beta^{-1} P_{t+1}/P_t$, real net worth will decrease at a rate $N_t/P_t = \beta^{-1} N_{t-1}/P_{t-1} < 0$ violating the transversality condition. Therefore, these inflationary paths are not equilibria. Finally consider the case in which $\Psi^C(P_{\hat{t}}) < 0$ and net worth is zero, $N^C(P_{\hat{t}}) = 0$. Then net worth remains zero since profits are zero for each $t > \hat{t}$. Remittances are also going to be zero given the remittances' policy. This is an equilibrium since it will not violate the transversality condition. ■

Finally note that the results of this section are related to Del Negro and Sims (2015) but with an important difference. In their analysis, multiplicity appears as a shift to a different interest-rate rule, since it is P^*, the inflation target in their case, that changes across equilibria. In my analysis, the policy rule remains unchanged and the multiplicity arises along the multiple solutions that (12) implies.

C.7.2 Credit shock

I consider now the consequences of an unexpected realization of a credit event showing that the stationary solution $P_t = P^*$ stops to be an equilibrium when the credit event is sizeable whereas divergent solutions (inflationary and deflationary) might also emerge as equilibria.

Starting from a perfect foresight equilibrium in which $\varkappa_t = 0$ at all times, assume that at a generic time $\hat{t} > t_0$ long-term securities are unexpectedly seized, even partially, at the rate $0 < \varkappa \leq 1$. The time-$\hat{t}$ return on long-term bonds unexpectedly falls which could lead to negative profits and to a fall in net worth.

The profit function at time $\hat{t}$ is now also function of $\varkappa$

$$\Psi^C(\varkappa, P_{\hat{t}}) = i_{\hat{t}-1}(N^C_{\hat{t}-1} + M^C_{\hat{t}-1}) + (r(\varkappa, P_{\hat{t}}) - i_{\hat{t}-1}) Q_{\hat{t}-1} D^C_{\hat{t}-1}$$

given the dependence of the return function $r(\varkappa, P_{\hat{t}})$ on $\varkappa$. Central bank's net worth at time $\hat{t}$ is given by

$$\begin{aligned} N^C(\varkappa, P_{\hat{t}}) &= N^C_{\hat{t}-1} + \Psi^C(\varkappa, P_{\hat{t}}) - \max(\Psi^C(\varkappa, P_{\hat{t}}), 0) \qquad \text{if} \quad N^C_{\hat{t}-1} \geq \bar{N}, \\ N^C(\varkappa, P_{\hat{t}}) &= N^C_{\hat{t}-1} + \Psi^C(\varkappa, P_{\hat{t}}) \qquad \text{if} \quad N^C_{\hat{t}-1} < \bar{N}. \end{aligned}$$

In what follows, I restrict the analysis to equilibria in which the price level can jump in an unexpected way at the time in which the credit shock hits. The following Proposition describes the equilibria in this case.

Proposition 13 *Consider an equilibrium in which* $\varkappa_t = 0$ *at each date and unexpectedly* $\varkappa_t = \varkappa$ *at time* $\hat{t} > t_0$. *Given the monetary/fiscal policy regime of Definition 9 it follows that: i)* $P_t = P^*$ *in equilibrium for each* $t \geq t_0$ *if and only if* $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) \geq 0$ *at time* $\hat{t}$ *and ii) there is an equilibrium in which* $P_t = P^*$ *for each* $t_0 \leq t < \hat{t}$ *and unexpectedly* $P_{\hat{t}} < P^*$ *with a deflationary path afterward which follows (12) if and only if* $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) < 0$ *and* $N^C(\varkappa, P_{\hat{t}}) = 0$ *at time* $\hat{t}$.

Proof. There are different cases to analyze. If profits are still positive, following the $\varkappa-$shock evaluated at the target price P^*, i.e. $\Psi^C(\varkappa, P^*) > 0$, then it is the case that $\Psi^C(\varkappa, P_{\hat{t}}) > \Psi^C(\varkappa, P^*)$ for any unexpected fall in the price level at time $\hat{t} > t_0$, $P_{\hat{t}} < P^*$, and therefore $T^C_t = \Psi^C_t > 0$ for each $t \geq \hat{t}$ implying $N^C_t = \bar{N}$ for each $t \geq \hat{t}$. This result implies that deflationary paths are excluded as shown in Proposition 3. Consider now the case in which $\Psi^C(\varkappa, P^*) < 0$ at time $\hat{t}$ but $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) > 0$. Therefore $N^C(\varkappa, P_{\hat{t}}) > N^C(\varkappa, P^*)$ for any unexpected fall in the price level at time $\hat{t}$, $P_{\hat{t}} < P^*$. Given that net worth is positive, profits will be positive (in a perfect-foresight equilibrium) after time $\hat{t}$ and they reach the threshold $\bar{N}^C$ in a finite period of time. After that period, net worth will be constant at the level $\bar{N}^C$. Therefore Proposition 3 applies and deflationary paths are rule out. Consider now the case in which $\Psi^C(\varkappa, P^*) < 0$ at time $\hat{t}$ and $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) = 0$. Given that net worth is zero, profits are zero after time $\hat{t}$ and therefore also remittances. Net worth remains zero without violating the transversality condition. $P_t = P^*$ is an equilibrium at all times. On the contrary, paths in which $P_t = P^*$ for each $t_0 \leq t < \hat{t}$ and $P_{\hat{t}} < P^*$ at time $\hat{t}$ with deflationary paths in the following periods according to (12) are not equilibria since they lead to violation of the transversality condition. In the case in which $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) < 0$ path in which $P_t = P^*$ at each t cannot be equilibria since net worth is negative, profits are negative and net worth diverges to negative values at a rate violating the transversality condition, i.e. $N_t/P_t = \beta^{-1}N_{t-1}/P_{t-1} < 0$ for each $t \geq \hat{t}$. However, if prices fall at time $\hat{t}$ when the credit shock hits such that $N^C(\varkappa, P_{\hat{t}}) = 0$ then profits will be zero afterward. Zero remittances implies that net worth remains constant at zero without violating the transversality condition. Therefore there can be equilibria with a deflationary path for a sizeable credit shock. ■

The above Proposition shows that depending on the size of the credit shock there can be different types of equilibria. If the credit shock is small, price stability is an equilibrium. On the contrary, for large credit shock, deflationary equilibria arise and they are such that $N^C(\varkappa, P_{\hat{t}}) = 0$ at time $\hat{t}$ for some $P_{\hat{t}}$, with $P_{\hat{t}} < P^*$, and moreover net worth remains zero in all the following periods. The next Proposition shows instead that whenever the size of the shock is small, there are multiple equilibria. If $N^C(\varkappa, P^*) > 0$, the price stability equilibrium coexists with an inflationary equilibrium such that $N^C(\varkappa, P_{\hat{t}}) = 0$ at time $\hat{t}$ for some $P_{\hat{t}}$, with $P_{\hat{t}} > P^*$.

Proposition 14 *Consider an equilibrium in which $\varkappa_t = 0$ at each date and unexpectedly $\varkappa_t = \varkappa$ at time $\hat{t} > t_0$. Given the monetary/fiscal policy regime of Definition 10 , it follows that: i) $P_t = P^*$ in equilibrium for each $t \geq t_0$ if and only if $N^C(\varkappa, P^*) = N^C_{\hat{t}-1} + \Psi^C(\varkappa, P^*) \geq 0$ at time $\hat{t}$ and ii) whenever $N^C(\varkappa, P^*) > 0$ there is also an equilibrium in which $P_t = P^*$ for each $t_0 \leq t < \hat{t}$, with $\hat{t} < \tilde{t} - 1$, and unexpectedly $P_{\hat{t}} > P^*$ with an inflationary path which follows (12) if and only if $N^C(\varkappa, P_{\hat{t}}) = 0$.*

Proof. I will now check whether paths in which $P_t = P^*$ for each $t \geq t_0$ and prices unexpectedly jump at the time of the credit shock and follow an inflationary path in the following periods, i.e. $P_t > P^*$ for each $t \geq \hat{t} > t_0$ according to (12), are equilibria. Recall that the remittances rule requires that $\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta}\frac{P_{t_0}}{P^*} n_{t_0}^C$ for each $t \geq \tilde{t}$ at time $\tilde{t}$ such that $N^C_{\tilde{t}-1} = \bar{N}^C$. Since $N^C(\varkappa, P^*) < \bar{N}^C$ and $N^C(\varkappa, P_{\hat{t}}) < \bar{N}^C$ for any unexpected increase of prices, i.e. $P_{\hat{t}} > P^*$, time $\tilde{t} - 1$ can only occur after time $\hat{t}$. Consider then the case in which $\hat{t} < \tilde{t} - 1$. If $N^C(\varkappa, P_{\hat{t}}) > 0$ for some $P_{\hat{t}} > P^*$ then profits are positive after time $\hat{t}$ and net worth increases. Therefore net worth can reach the threshold $\bar{N}^C$ at a future time $\tilde{t}-1$. The intertemporal budget constraint at time $\tilde{t} - 1$ is given by

$$\frac{\bar{N}^C}{P_{\tilde{t}-1}} = \sum_{T=\tilde{t}}^{\infty} \beta^{T+1-\tilde{t}} \frac{T_T^C}{P_T} = \frac{P_{t_0} n_{t_0}^C}{P^*}$$

which can only be satisfied if and only if $P_{\tilde{t}-1} = P^*$ ruling out paths in which $P_{\tilde{t}-1} > P^*$ and therefore $P_{\hat{t}} > P^*$ at $\hat{t} < \tilde{t} - 1$. If instead $N^C(\varkappa, P^*) > N^C(\varkappa, P_{\hat{t}}) = 0$, profits are zero, remittances are zero and net worth remains zero in all the following periods. The inflationary solution in which prices jump at $\hat{t}$, with $P_{\hat{t}} > P^*$, and follow an inflationary path consistently with (12) is an equilibrium. Finally when $N^C(\varkappa, P_{\hat{t}}) < 0$, net worth follows the path $N_t^C = (1 + i_{t-1})N_{t-1}^C$ and given that $1 + i_t = \beta^{-1}P_{t+1}/P_t$, real net worth will decrease at a rate $N_t/P_t = \beta^{-1}N_{t-1}/P_{t-1} < 0$ violating the transversality condition. As shown in Proposition 13, whenever $N^C(\varkappa, P^*) \geq 0$ price stability, i.e. $P_t = P^*$ at each date t, is an equilibrium. ■