Non-Neutrality of Open-Market Operations*

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Abstract

We analyze the effects on inflation and output of unconventional open-market operations due to the possible income losses on the central bank's balance sheet. We first state a general Neutrality Property, and characterize the theoretical conditions supporting it. We then discuss three non-neutrality cases. First, with no treasury's support, sizeable (current or expected) balance-sheet losses can undermine central bank's solvency and should be resolved through an increase in inflation. Second, a central bank might also engineer higher inflation in the case it wants to limit or reduce losses because of political constraints or to seek more financial independence. Third, if the treasury is unable or unwilling to tax households to cover central bank's losses, the wealth transfer to the private sector also leads to higher inflation.

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1 Introduction

The recent financial crisis has shown an unprecedented intervention of central banks around the world in an attempt to mitigate the adverse effects on the economy through the purchases of long-term risky securities. The Bank of England, the Bank of Japan, the European Central Bank, the Federal Reserve System and the Swedish Central Bank have all enlarged at various stages, and with different speed and composition, their asset holdings to include long-term private securities and government debt of different maturities and credit worthiness.

All these policies have raised worries about the possible stress that the central bank's balance sheet could suffer in terms of income losses and declining net worth.[1] In this paper, we take a general-equilibrium perspective in order to understand under which conditions equilibrium prices and output respond to unconventional open-market operations because of the possible income losses that they imply on the central bank's balance sheet.

To this end, we have to challenge an important property, discussed first by Wallace (1981), affirming the irrelevance of standard open-market operations for equilibrium prices and quantities. We extend Wallace's result to any unconventional composition of central bank's assets in a model in which, among other features, central bank and treasury have separate budget constraints and where the central bank can issue both money and reserves.

Deviations from neutrality can arise, in our analysis, depending on the specifications of the transfer policies, unlike a recent literature that has emphasized frictions in financial markets.[2] Since we separate the budget constraints of treasury and central bank, the transfer policy has two dimensions: 1) the lump-sum taxes levied by the treasury on the private sector and 2) the remittance policy between central bank and treasury. In this context, we define active or passive transfer policies generalizing similar definitions in the literature, where instead they typically apply to models with a consolidated government budget constraint and focus only on one dimension, that of treasury's tax policy.

Our first contribution is to provide examples in which open-market operations are neutral. In general, neutrality arises when the ultimate allocation of risk in the economy does not change as a result of a reallocation of long-term assets holdings between private and public sectors. When the central bank purchases risky assets from the private sector, the equilibrium allocation and price system do not change if the realization of that risk is ultimately borne

[1]A recent literature has evaluated these risks for the U.S. economy based on some projection analysis and concluded that they can be in general of minor importance (see Carpenter et al., 2015, Christensen et al., 2015, Greenlaw et al., 2013).

[2]Sargent and Smith (1987, p.91), who provide a neutrality result in a model where money is dominated in return, also argue that "irrelevance requires that fiscal policy be held constant in a precise sense". In our work we give a thorough understanding on what "held constant" means. See also the discussion of Sargent (2011). For a recent literature that breaks neutrality on the basis of frictions in financial markets see Curdia and Woodford (2011).

by the private sector. This can be the result of appropriate transfer policies: combinations of lump-sum taxes and remittances which rebate any gains or losses on the central bank's balance sheet to the private sector.[3] Interestingly, these transfer policies do not need to be passive.

Conversely, unconventional open-market operations are non neutral if some risk stays in the hands of the central bank. We provide three non-neutrality results.

The first arises when a passive fiscal policy is associated to an active remittance policy – the latter depends on lack of treasury support: the treasury never recapitalizes the central bank. Sizeable balance-sheet losses – as those implied by large credit events – can undermine central bank's solvency and therefore require a change in the monetary policy stance that tilts equilibrium prices and output. The value of money should change – i.e. inflation rises – up to the point in which private agents are forced to hold more currency, so that the seigniorage earnings of the central bank can increase and profitability be restored. Importantly, in order to have non neutrality, such sizeable losses do not need to actually materialize, insofar as they are expected with positive probability.

The second non-neutrality case assumes, in principle, transfer policies that can be consistent with neutrality. However, non-neutrality arises because of the policy actions of the central bank that shape appropriately the path of remittances. We discuss two different scenarios within the canonical example of an economy that is dragged into a liquidity trap by a negative natural interest rate. When the latter unexpectedly turns back positive, the implied change in interest rates produces a fall in the price of long-term bonds and hence losses for the central bank holding them. In the first scenario, by retaining earnings and stopping remittances to the treasury, neutrality can arise. However, if political-economy considerations induce the central bank to reduce the duration of zero remittances, its conventional monetary policy needs to change, hence implying a non-neutrality result. In the second scenario, the remittance policy allows central bank's losses to be covered by the treasury. However, a central bank seeking complete financial independence from the treasury by avoiding its support could change its conventional monetary policy, thus producing, again, non-neutrality. Compared with the familiar optimal-policy response to a liquidity trap of Eggertsson and Woodford (2003), we find that an unconventional open market operation can signal a gradual

[3]This neutrality result goes straight to the heart of a long-lasting debate on how central banks should control the value of money in connection with the assets that they hold in their balance sheet. Indeed, under unconventional asset holdings, it is not gold, nor reserves, nor "real bills" that help to back the value of money. Taxpayers do. Before unconventional monetary policy took place, what seemed to be the prevailing view shares common traits with the "real bills" doctrine (Smith, 1776), according to which central banks should issue money backed by short-term securities free of risk. In a system of this kind, it is understood that the central bank can control the value of money by setting the interest rate on the safe assets held in its portfolio. Sargent (2011) discusses Wallace's irrelevance result in light of the "real bills" doctrine.

exit of the interest rate from the zero-lower bound (if the central bank aims at reducing the duration of zero remittances) or alternatively a delayed but sudden exit (if instead it seeks complete financial independence).

The last non-neutrality result is derived by assuming an active fiscal policy, in which the treasury is unable or unwilling to tax the private sector to cover central bank's losses. Following central bank's purchases of risky securities, the materialization of risk remains in the hands of the whole government and represents a positive transfer of wealth to the private sector. Whoever unloaded the risky securities to the central bank experiences a positive wealth gain. Demand will surge and so will inflation. The value of money will fall.[4]

In all cases, non neutrality implies higher inflation. Asset purchases, therefore, can be inflationary not because the central bank "prints money" or increases the size of its balance sheet, but because a higher inflation is either the desirable response to sizeable income losses in order to regain central bank's profitability or the outcome of a policy geared toward limiting – or avoiding – financial losses or, in general, a consequence of an indirect (or direct) wealth transfer from the government to the private sector.

Our work first contributes to the tradition of the irrelevance results of Wallace (1981), Chamley and Polemarchakis (1984), Sargent and Smith (1987), Sargent (1987) and Eggertsson and Woodford (2003).[5] In this direction, we state a general Neutrality Property and characterize the theoretical conditions supporting it, in an economy where treasury and central bank have separate budget constraints and the central bank issues both non-interest and interest bearing liabilities (money and reserves, respectively). In particular, distinguishing the budget constraint of the central bank from that of the treasury requires to detail the transfer policy that each institution should follow in order to obtain neutrality. Moreover, we enlarge the set of irrelevance results even to cases in which the central bank increases the size of the balance sheet and interest rates are positive by allowing the central bank to issue interest-bearing reserves in line with what has been called a new central banking style (see Hall and Reis, 2015).

In addition, we provide a meaningful departure from that tradition by identifying and discussing several non-neutrality cases of practical interest, focusing in particular on transfer policies between central bank and treasury. In this direction, our paper is inspired by the seminal works of Sims (2000, 2005) who has first emphasized in theoretical models the im-

[4]This monetary/fiscal policy regime represents one of the alternative ways to implement the so-called "helicopter money".

[5]The framework of Wallace (1981) and Chamley and Polemarchakis (1984) is extended to economies where money is dominated in return by Sargent and Smith (1987) and Sargent (1987). Eggertsson and Woodford (2003) generalize the neutrality result to a context in which the central bank's balance sheet includes unconventional asset purchases and the only liability is money. However, they consolidate the budget constraints of treasury and central bank when specifying the transfer policy that delivers neutrality.

portance for policy analysis of separating the budget constraint of the treasury from that of the central bank and by the more recent Bassetto and Messer (2013), Del Negro and Sims (2015), Hall and Reis (2015).[6] Unlike our analysis, all this literature is not concerned about stating a general Neutrality Property or about characterizing the theoretical conditions for non-neutrality. However, Sims (2000, 2005) and Del Negro and Sims (2015) underline that when there is lack of treasury's support the central bank may no longer be able to maintain control of inflation when committing to a certain Taylor's rule because this would lead to insolvency. This is consistent with one of the non-neutrality results that we discuss in which we further emphasize the additional implications of a stochastic environment. Bassetto and Messer (2015), instead, mainly focus on the fiscal consequences of alternative compositions of central bank's assets emphasizing the different accounting procedures and remittance policies between treasury and central bank.[7] Reis (2013, 2015) and Hall and Reis (2015) take instead equilibrium inflation as given and are interested in analyzing the consequences of central bank's insolvency for financial stability, i.e. a non-exploding path of central bank's reserves.

Our case of financially independent central bank, which implies non-neutrality of balance-sheet policies, shares some similarities with Bhattarai et al. (2015), with two main, important differences. First, in our model a financially independent central bank penalizes only negative remittances to the treasury, while they assume a quadratic penalty for non-zero remittances. Accordingly, their environment features non-neutral effects of any balance-sheet policy, while in our case only those that imply potential losses on risky securities. Second, we analyze the optimal allocation under full commitment, while they only consider the case of discretion.

Berriel and Bhattarai (2009) and Park (2015) also consider a separate budget constraints of treasury and central bank, but they analyze the case in which the central bank holds only short-term assets and therefore losses are not possible, unlike our model. In particular, Berriel and Bhattarai (2009) show how optimal policy changes considering different central-bank remittance rule. Park (2015) investigates the determinacy of the equilibrium under alternative remittance and fiscal policies. Our study, instead, focuses on the equilibrium consequences of alternative balance-sheet policies holding "constant" the specification of remittance and fiscal policy, which is the relevant comparison to make in order to evaluate neutrality.[8]

[6]Drawing from the experience of several central banks, Stella (1997, 2005) has also provided evidence for the relationship between central-bank financial strength and monetary policy. See also the recent work of Adler et al. (2012).

[7]There is a substantial literature which has analyzed the different central-bank accounting procedures and remittance policies as, among others, Stella (1997, 2005) and Archer and Moser-Boehm (2013).

[8]Zhu (2004) also distinguishes between the budget constraint of the central bank and that of the treasury but he focuses on how the properties of equilibrium determinacy change when the interest-rate rule followed by the central bank reacts also to variations in its net worth with respect to a target. Jeanne and Svensson (2007) discuss the importance of balance-sheet considerations as a credible device to exit from a liquidity

Our work is also related to a more extensive literature which has studied the monetary policy consequences of alternative assumptions on fiscal policy (see among others Sargent and Wallace, 1981, Sargent, 1982, Leeper, 1991, Sims, 1994, 2013, and Woodford, 1994, 2001) but which, on the contrary, has disregarded the distinction between the balance sheets of treasury and central bank. This separation is key in our analysis.

The plan of this work is the following. Section 2 presents a simple monetary model and Section 3 states the general Neutrality Property; Section 4 then studies neutrality results, while Section 5 discusses deviations from neutrality; Section 6 concludes.

2 Model

We present our analysis in a simple infinite-horizon monetary economy, along the lines of Bassetto and Messer (2013), featuring three sets of agents: households, the treasury and the central bank. A key assumption of our analysis, as mentioned in the introduction, is the separation between the balance sheets of treasury and central bank. There is a financial friction in the model since money is the only asset that can be used to buy goods. This friction is not important at all for our results. It is only useful to capture features of current economies in which money has a relevant role for transactions and for partly financing central bank's assets.[9] Our model economy is perturbed by three stochastic disturbances. We allow for a credit shock and a preference shock, to capture credit and interest-rate risk, respectively. These are the two most relevant risks in thinking about the consequences of recent asset purchases by central banks in advanced economies. Finally, the third stochastic disturbance is the endowment of the only traded good in the economy. In the Appendix, we generalize this model along several dimensions, including endogenous production, nominal rigidities and additional shocks perturbing the economy.

2.1 Households

Households have an intertemporal utility of the form:

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t U(C_t)\right\} \tag{1}$$

trap; however, their focus is on the balance-sheet losses possibly arising because of the effect of exchange-rate movements on the value of reserves.

[9] See Lucas (1984) for further discussion of the usefulness of this class of models for monetary theory and more recently Sargent (2014).

where E_t denotes the standard conditional expectation operator, β is the intertemporal discount factor with $0 < \beta < 1$, ξ is a stochastic disturbance, which affects the intertemporal preferences of the consumer and is assumed to follow a Markov process, with transition density $\pi_\xi(\xi_{t+1}|\xi_t)$ and initial distribution f_ξ. We assume that (π_ξ, f_ξ) is such that $\xi \in [\xi_{\min}, \xi_{\max}]$.[10] C is a consumption good and $U(\cdot)$ is a concave function.

The timing of markets' opening follows that of Lucas and Stokey (1987). In a generic period t, the asset market opens first, followed by the goods market. There is a financial friction since only money can be used to purchase goods and only in the goods market. In the asset market, households can adjust their portfolio according to

$$M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \\ \leq B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + P_{t-1} Y_{t-1} - T_t^F + (M_{t-1} - P_{t-1} C_{t-1}). \quad (2)$$

Households invest their financial wealth in money, M_t – a non-interest-bearing asset issued by the central bank which provides liquidity services – in central bank's reserves, X_t, which carry a risk-free nominal return i_t. Finally they can lend or borrow using short-term, B_t, and long-term, D_t, securities at a price $1/(1 + i_t)$ and Q_t, respectively. In the case of long-term debt, the security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta, \delta^2, \delta^3 ...$ in the following periods and in the case of no default.[11] Finally, T_t^F denotes lump-sum taxes (net of transfers).

In the case of long-term lending or borrowing, the stochastic disturbance $\varkappa_t$ on the right-hand side of (2) captures the possibility that long-term securities can be partially seized by exogenous default; in particular $\varkappa_t$ follows a Markov process with transition density $\pi_\varkappa(\varkappa_{t+1}|\varkappa_t)$ and initial distribution $f_\varkappa$. We assume that $(\pi_\varkappa, f_\varkappa)$ is such that $\varkappa \in [0, 1)$. To shorten the writing of (2), we are including only those securities, among the ones traded, that households can exchange externally with the treasury and central bank. In addition, in each period, households can trade with each other in a set of state-contingent nominal securities spanning all states of nature which they face in the next period. It is assumed that the payoffs of these securities are enough to "complete" the financial markets.

In the budget constraint (2), Y_{t-1} is the time $t - 1$ endowment of the only good traded which is the third stochastic disturbance that also follows a Markov process with transition density $\pi_y(Y_{t+1}|Y_t)$ and initial distribution f_y. We assume that (π_y, f_y) is such that $Y \in [Y_{\min}, Y_{\max}]$. In the budget constraint (2), $P_{t-1} Y_{t-1}$ are the revenues the household obtains by

[10] We restrict our analysis to exogenous stochastic processes with finite state space.

[11] The stock of long-term asset (or debt) follows the law of motion $D_t = Z_t + (1 - \delta) D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, or borrowing, if negative, supplied at time t. See among others Woodford (2001).

selling the endowment in $t-1$ which are deposited in the financial account only in period t; T_t^F are lump-sum taxes levied by the treasury. Unspent money in the previous-period goods market is deposited in the financial account.

When asset market closes, goods market opens and households can use money to purchase goods according to

$$M_t \geq P_t C_t. \tag{3}$$

The households' problem is subject to initial conditions $B_{t_0-1}, X_{t_0-1}, D_{t_0-1}, M_{t_0-1}$ and a borrowing limit of the form[12]

$$\lim_{T\to\infty} E_t \left[R_{t,T}^n \mathcal{W}_T \right] \geq 0, \tag{4}$$

looking forward from each time t where $R_{t,T}^n$ is the nominal stochastic discount factor that is used to evaluate nominal wealth $\mathcal{W}_T$ in a generic contingency at time T with respect to nominal wealth at time t, with $T > t$. Nominal wealth $\mathcal{W}_t$ is given by

$$\mathcal{W}_t = \tilde{\mathcal{W}}_t + M_t + \frac{B_t + X_t}{1+i_t} + Q_t D_t$$

which includes the nominal values of the portfolio of state-contingent securities $\tilde{\mathcal{W}}_t$. It is also required for the existence of an intertemporal budget constraint that

$$E_t \left\{ \sum_{T=t}^{\infty} R_{t,T}^n \left[P_{T-1} C_{T-1} + \frac{i_T}{1+i_T} M_T \right] \right\} < \infty \tag{5}$$

looking forward from any date t, since there is no limit to the ability of households to borrow against future income.[13]

Households choose consumption, and asset allocations to maximize utility (1) under the constraints (2), (3), (4) and (5), given the initial conditions. The optimal choice with respect to consumption, assuming an interior solution, requires that

$$\xi_t U_c(C_t) = (\varphi_t + \beta E_t \lambda_{t+1}) P_t, \tag{6}$$

where λ_t and φ_t are the non-negative Lagrange multipliers associated with constraints (2) and (3), respectively. The first-order condition with respect to money holdings

$$\lambda_t - \varphi_t = \beta E_t \lambda_{t+1}, \tag{7}$$

[12] There are also initial conditions on $P_{t_0-1} Y_{t_0-1}$ and $P_{t_0-1} C_{t_0-1}$ but we assume that they sum to zero as in equilibrium.

[13] It is important to note that the expression in the curly bracket of (5) is never negative since consumption, money holdings, prices and interest rates are all non-negative.

implies in (6) that the marginal utility of nominal wealth is simply given by $\lambda_t = \xi_t U_c(C_t)/P_t$, which is positive.

The optimality conditions with respect to the holdings of short-term treasury bills or central bank's reserves determine the nominal interest rate according to

$$\frac{1}{(1+i_t)} = E_t R^n_{t,t+1}, \tag{8}$$

where the equilibrium nominal stochastic discount factor $R^n_{t,t+1}$ is implied by the optimality conditions with respect to the state-contingent securities and given by

$$R^n_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}. \tag{9}$$

Combining (6)–(9) it follows

$$\varphi_t = \frac{i_t}{1+i_t}\lambda_t \tag{10}$$

from which $i_t \geq 0$, since $\varphi_t \geq 0$ and $\lambda_t > 0$. The complementary slackness condition on the constraint (3) can be written as

$$\varphi_t(M_t - P_t C_t) = 0.$$

The first-order condition with respect to lending or borrowing using long-term fixed-rate securities implies that the price Q_t follows

$$Q_t = E_t[R^n_{t,t+1}(1-\varkappa_{t+1})(1+\delta Q_{t+1})]. \tag{11}$$

To conclude the characterization of the household's problem, a transversality condition applies and therefore (4) holds with equality, given the equilibrium nominal stochastic discount factor $R^n_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$.

2.2 Treasury

The treasury raises lump-sum taxes T^F_t (net of transfers) from the private sector and receives remittances T^C (when T^C is positive) or makes transfers to the central bank (when T^C is negative). The treasury can finance its deficit through short-term (B^F) and long-term (D^F) debt, at the prices $1/(1+i_t)$ and Q_t respectively, facing the following flow budget constraint

$$Q_t D^F_t + \frac{B^F_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D^F_{t-1} + B^F_{t-1} - T^F_t - T^C_t$$

given initial conditions $D^F_{t_0-1}$, $B^F_{t_0-1}$.[14]

2.3 Central bank

The central bank issues non-interest-bearing-liabilities, money M^C_t, and interest-bearing liability, reserves X^C_t, to finance a portfolio of assets including short-term and long-term fixed-rate securities, B^C_t and D^C_t respectively. Central bank's net worth, N^C_t – the difference between the market value of assets and liabilities – is given by

$$N^C_t \equiv Q_t D^C_t + \frac{B^C_t}{1+i_t} - M^C_t - \frac{X^C_t}{1+i_t}, \tag{12}$$

while its law of motion depends on the profits that are not distributed to the treasury:

$$N^C_t = N^C_{t-1} + \Psi^C_t - T^C_t \tag{13}$$

where Ψ^C_t are central bank's profits, which depend on the composition of its balance sheet:[15]

$$\Psi^C_t = \frac{i_{t-1}}{1+i_{t-1}}(B^C_{t-1} - X^C_{t-1}) + \left[(1-\varkappa_t)(1+\delta Q_t) - Q_{t-1}\right] D^C_{t-1}. \tag{14}$$

They can also be written as

$$\Psi^C_t = i_{t-1}(N^C_{t-1} + M^C_{t-1}) + (r_t - i_{t-1})Q_{t-1}D^C_{t-1} \tag{15}$$

having used the definition $(1+r_t) \equiv (1+\delta Q_t)(1-\varkappa_t)/Q_{t-1}$.[16] Central bank's profits depend on two components: the first captures the revenues obtained by issuing non-interest bearing liabilities – net worth is indeed part of the non-interest bearing liabilities; the second component, instead, represents the excess gains or losses of holding long-term securities with respect to a riskless portfolio. Since the realized excess return on these securities can be negative, the latter component may as well be negative – the more so the larger are the holdings of long-term securities – producing income losses for the central bank. Combining

[14] It is worth reminding that to simplify the analysis we have assumed that there is only one long-term security, which is issued either by the private sector or by the treasury. In particular, without losing generality, we assume $D^F_t = 0$ if $D_t < 0$ and $D_t \geq 0$ if $D^F_t > 0$.

[15] Profits of central bank are defined as the net income derived from the portfolio of assets and liabilities which once fully distributed are such to keep the central bank's nominal net worth constant. This is in line with similar definitions given by Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015). We abstract from the dividends that the central bank gives to the member banks. In the US, this amounts to 6% of capital (see Carpenter et al., 2015).

[16] The definition is only valid for positive asset prices.

(12) and (13), we can write the central bank's flow budget constraint as follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$.

2.4 Equilibrium

Here, we describe in a compact way the equations that characterize the equilibrium allocation.

The Euler equations for short-term and long-term securities, (8) and (11), imply that

$$\frac{1}{1+i_t} = E_t\left\{\beta\frac{\xi_{t+1}U_c(Y_{t+1})}{\xi_t U_c(Y_t)}\frac{P_t}{P_{t+1}}\right\}, \tag{16}$$

and

$$Q_t = E_t\left\{\beta\frac{\xi_{t+1}U_c(Y_{t+1})}{\xi_t U_c(Y_t)}\frac{P_t}{P_{t+1}}(1-\varkappa_{t+1})(1+\delta Q_{t+1})\right\}, \tag{17}$$

respectively, in which we have used the equilibrium value of the Lagrange multiplier $\lambda_t = \xi_t U_c(C_t)/P_t$ and equilibrium in goods market $Y_t = C_t$.

The cash-in-advance constraint (3) together with the equilibrium in the goods market implies

$$M_t \geq P_t Y_t, \tag{18}$$

while the complementary slackness condition can be written as

$$i_t(M_t - P_t Y_t) = 0. \tag{19}$$

The bound (5) in equilibrium is equal to

$$E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}\xi_T U_c(Y_T)\left[Y_{T-1} + \frac{i_T}{1+i_T}Y_T\right]\right\} < \infty,$$

in which we have also used (18) and (19). Note that the above equilibrium condition is always satisfied given the assumption of bounded processes for Y_t and ξ_t.

The transversality condition, with equality, completes the demand side of the model

$$\lim_{T\longrightarrow\infty} E_t\left[\beta^{T-t}\frac{\xi_T U_c(Y_T)}{P_T}\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right] = 0, \tag{20}$$

which is derived from (4), where we have used $R_{t,T}^n = \beta^{T-t}\lambda_T/\lambda_t$, $\lambda_t = \xi_t U_c(C_t)/P_t$, and the goods market equilibrium together with the fact that the state-contingent securities are traded in zero-net supply within the private sector: the transversality condition therefore constrains just the long-run behavior of the "outside" assets held by the households.

The treasury's and central bank's budget constraints are given by

$$Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^F + B_{t-1}^F - T_t^F - T_t^C \tag{21}$$

and

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C \tag{22}$$

respectively, where equilibrium in the asset markets implies that

$$B_t^F = B_t + B_t^C \tag{23}$$

$$D_t^F - D_t = D_t^C. \tag{24}$$

Note moreover that in (22) we have used the equilibrium conditions that money and reserves issued by the central bank are held by households, $M_t = M_t^C$ and $X_t = X_t^C$.[17]

To complete the characterization of the rational expectations equilibrium we need to specify the monetary/fiscal policy regime. First we note that excluding the complementary-slackness condition (19) and the bound (20) there are seven equilibrium conditions for the thirteen unknown stochastic processes $\{P_t, i_t, Q_t, M_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}_{t=t_0}^{\infty}$ implying that the monetary/fiscal policy regime should specify six additional equations. In particular, the monetary/fiscal policy regime specifies six of the stochastic processes $\{i_t, M_t, X_t, B_t^C, B_t^F, D_t^C, D_t^F, T_t^F, T_t^C\}_{t=t_0}^{\infty}$, possibly as functions of some other endogenous and/or exogenous variables.

It is out of the scope of this paper to analyze all possible monetary/fiscal policy regimes. Indeed, we restrict attention to a subset of regimes, which is however quite inclusive and broad enough to encompass all interesting cases. The monetary/fiscal policy regimes under consideration can be described by a combination of *conventional monetary policy, transfer policy* and *balance-sheet policy.*

To simplify notation, in what follows, we define the vector $\mathbf{Z}_t \equiv (P_t, i_t, Q_t, M_t)$ and the vector $\mathbf{K}_t \equiv (X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C)$ while the vectors $\bar{\mathbf{Z}}_t$ and $\bar{\mathbf{K}}_t$ include $\mathbf{Z}_t$

[17] It is important to note that the transversality condition (20) implies an aggregate transversality condition on the net consolidated liabilities of both treasury and central bank which together with the flow budget constraints (21) and (22) entails a consolidated intertemporal budget constraint. We do not explicitly write this constraint since it is already implied by the set of equations written above.

and $\mathbf{K}_t$, respectively, and their own lags.

To understand what we mean by *conventional monetary policy*, consider the equilibrium conditions (16) to (19). Since (19) is a complementary-slackness condition, there are three equations in the vector of four unknown stochastic processes $\{\mathbf{Z}_t\}_{t=t_0}^{\infty}$, given the exogenous state process $\{Y_t\}_{t=t_0}^{\infty}$. Considered alone this set of equations leaves one degree of freedom to specify one of the endogenous stochastic processes to eventually determine all four endogenous variables.

We call *conventional monetary policy* the specification of one of the stochastic processes $\{i_t, M_t\}_{t=t_0}^{\infty}$ as a function of the other endogenous variables P, Q and/or exogenous state variables like Y. A type of rule in this class is setting in each contingency i_t as a function $i_t = \mathcal{I}(\bar{\mathbf{Z}}_t,\, \zeta_t)$ where ζ_t is a generic vector of exogenous stochastic disturbances that may include ξ_t, Y_t and $\varkappa_t$ while $\mathcal{I}(\cdot)$ is non-negative for all the values of its arguments, consistently with the zero-lower bound on the short-term nominal interest rate. Another type of rule in this class involves instead setting M_t in each contingency as $M_t = \mathcal{M}(\bar{\mathbf{Z}}_t,\, \zeta_t)$ where $\mathcal{M}(\cdot)$ is positive for all values of its arguments.

An important restriction in the specification of the *conventional monetary policy* is the requirement that the endogenous variables included in the arguments of $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ are just those belonging to the vector $\mathbf{Z}_t$ and not to $\mathbf{K}_t$. This represents the conventional way to think about determination of prices in this class of models: specify either an interest-rate rule or a money-supply rule and possibly determine the path of prices, interest rate and money using equations (16) and (18). Equation (17) residually determines the asset price Q_t, if the policy rule does not itself react to Q_t. Therefore, for a given *conventional monetary policy* the set of equations (16) to (19) can in principle determine the path of the vector of stochastic processes $\{\mathbf{Z}_t\}_{t=t_0}^{\infty}$.

However, it is key to note that this path, to be an equilibrium, needs also to satisfy the other equilibrium conditions together with the additional restrictions coming from the remaining specification of the monetary/fiscal policy regime. In this respect we specify a *transfer policy* in which both the stochastic processes $\{T_t^F, T_t^C\}$ are functions of the other endogenous and/or exogenous variables. A general *transfer policy* that we assume in this work is the following: *i*) $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ in which the treasury is setting lump-sum taxes as a function, among other variables, of the current and past levels of central bank's remittances and of treasury's outstanding short and long-term liabilities;[18] *ii*) $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$ in which the central bank is setting remittances as a function, among other variables, of the level of its own net worth N_t^C.[19] In what follows we denote compactly the two

[18] Consistently with the notation introduced before: $\bar{\mathbf{T}}_t^C \equiv (T_t^C, T_{t-1}^C, ...)$, $\bar{\mathbf{D}}_t^F \equiv (D_t^F, D_{t-1}^F, ...)$, $\bar{\mathbf{B}}_t^F \equiv (B_t^F, B_{t-1}^F, ...)$ and $\bar{\mathbf{N}}_t^C \equiv (N_t^F, N_{t-1}^F, ...)$.

[19] Note that a reaction of remittances to the variables $\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t$, ζ_t also accounts for a response to current

transfer policies with the two-dimensional vector of functions $\mathcal{T}(\cdot)$. The above policies are not comprehensive of all the possible policies that can be considered but are broad enough to encompass all the relevant cases for our analysis.

We are left with the specification of three of the sequences $\{X_t, B_t^C, B_t^F, D_t^C, D_t^F\}_{t=t_0}^{\infty}$ to complete the characterization of the monetary/fiscal policy regime. In this work we limit our attention to regimes in which three of the sequences $\{B_t^C, B_t^F, D_t^C, D_t^F\}_{t=t_0}^{\infty}$ are specified as functions of the other endogenous and/or exogenous variables. This is what we define a *balance-sheet policy.* Moreover, it should be noted from the flow budget constraint (21) that since the monetary/fiscal policy regime specifies already a *conventional monetary policy* and a *transfer policy*, only one of the stochastic processes $\{B_t^F, D_t^F\}_{t=t_0}^{\infty}$ can be chosen independently. Without losing generality, we assume that $\{D_t^F\}_{t=t_0}^{\infty}$ is specified. In what follows we denote the specification of the *balance-sheet policy* with a three-dimensional vector $\mathbf{B}_t \equiv (D_t^F, B_t^C, D_t^C)$ and with the non-negative functional form $\mathcal{B}(\cdot)$ such that $\mathbf{B}_t = \mathcal{B}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$, capturing the possibility that a *balance-sheet policy* reacts also to current and past macroeconomic conditions. An *unconventional open-market operation* is a balance-sheet policy in which $D_t^C > 0$ in some contingencies.

Definition 1 *A* ***conventional monetary policy*** *specifies either the stochastic process* $\{M_t\}_{t=t_0}^{\infty}$ *as* $\mathcal{M}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ *where* $\mathcal{M}(\cdot)$ *is positive for all values of its arguments or* $\{i_t\}_{t=t_0}^{\infty}$ *as* $i_t = \mathcal{I}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ *where* $\mathcal{I}(\cdot)$ *is non-negative for all the values of its arguments. A* ***transfer policy*** *specifies the stochastic processes* $\{T_t^F, T_t^C\}_{t=t_0}^{\infty}$ *as* $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ *and* $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$. *A* ***balance-sheet policy*** *specifies the vector of stochastic processes* $\{\mathbf{B}_t\}_{t=t_0}^{\infty} \equiv \{B_t^C, D_t^C, D_t^F\}_{t=t_0}^{\infty}$ *as* $\mathbf{B}_t = \mathcal{B}\ (\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ *where* $\mathcal{B}(\cdot)$ *is non-negative for all the values of its arguments.*

Given this premise we now introduce the definition of rational expectations equilibrium in which $\mathbf{w}_{t_0-1}$ is a vector that includes $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$ and other initial conditions that could be specified by the monetary/fiscal policy regime.[20]

Definition 2 *Given a* ***conventional monetary policy****, a* ***transfer policy*** *and a* ***balance-sheet policy*** *a rational expectations equilibrium is a collection of stochastic processes* $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}_{t=t_0}^{\infty}$ *such that* $i_t^* \geq 0$, $P_t^* > 0$, $Q_t^* > 0$, $X_t^* \geq 0$ *and* $B_t^{F*} \geq 0$ *at each time* $t \geq t_0$ *(and in each contingency at* t*) and such that: i)* $\{\mathbf{Z}_t^*\}_{t=t_0}^{\infty}$ *satisfies each of the conditions in equations (16) to (19) at each time* $t \geq t_0$ *(and in each contingency at* t*) and the specification of the* ***conventional monetary policy****, given the stochastic processes for the exogenous disturbances* $\{\zeta_t\}$ *and the initial conditions* $\mathbf{w}_{t_0-1}$*; ii)* $\{\mathbf{K}_t^*\}_{t=t_0}^{\infty}$ *satisfies each of the conditions in*

and past profits, given the definition (14).

[20]We assume that initial conditions are such that $N_{t_0-1} = \bar{N} > 0$.

equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) and the specification of the ***transfer policy*** *and* ***balance-sheet policy****, given the vector of stochastic processes* $\left\{\mathbf{Z}_t^*\right\}_{t=t_0}^{\infty}$ *of part i), the stochastic processes for the exogenous disturbances* $\{\zeta_t\}$ *and initial conditions* $\mathbf{w}_{t_0-1}$.

In the definition, the time-t component of the endogenous stochastic process is meant to be a function of the history of shocks, $\mathbf{s}^t \equiv (\mathbf{s}_t, \mathbf{s}_{t-1}, \mathbf{s}_{t-2}, ..., \mathbf{s}_{t_0})$ and the initial conditions $\mathbf{w}_{t_0-1}$. Therefore $\mathbf{Z}_t^* \equiv \mathbf{Z}^*(\mathbf{s}^t, \mathbf{w}_{t_0-1})$ and $\mathbf{K}_t^* \equiv \mathbf{K}^*(\mathbf{s}^t, \mathbf{w}_{t_0-1})$. The state $\mathbf{s}_t$ is a vector including ξ_t, $\varkappa_t$ and Y_t and other exogenous state variables, which could be specified by the monetary/fiscal policy regime. It can also include sunspot disturbances.[21] In what follows, to simplify notation, we just use $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\}$ in place of $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\}_{t=t_0}^{\infty}$.

3 The Neutrality Property

Taking as a starting point an equilibrium, the main objective of our analysis is to study whether alternative compositions of the central bank's balance sheet can influence equilibrium variables such as prices and interest rates. In particular, Definition 2 allows to make the right comparison in order to rule out a causal relationship between *balance-sheet policies* and prices, when instead something else in the specification of the monetary/fiscal policy regime has also changed, and is actually responsible for the variation in prices observed in equilibrium.

Consider a rational expectations equilibrium $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\}$ and the associated *conventional monetary policy,* $\mathcal{I}(\cdot)$ *or* $\mathcal{M}(\cdot)$*, transfer policy* $\mathcal{T}(\cdot)$, and *balance-sheet policy* $\mathcal{B}(\cdot)$. In particular let us first focus on an equilibrium in which the nominal interest rate is always above the zero-lower bound, $i_t^* > 0$. Next, change the *balance-sheet policy* from $\mathcal{B}(\cdot)$ to $\tilde{\mathcal{B}}(\cdot)$. The alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ is said to be neutral if there is an equilibrium $\left\{\tilde{\mathbf{Z}}_t, \tilde{\mathbf{K}}_t\right\}$ with $\tilde{\mathbf{Z}}_t = \mathbf{Z}_t^*$ associated with the same *conventional monetary policy* and *transfer policy* and the new *balance-sheet policy.* The vector $\mathbf{Z}_t^*$ is therefore invariant to the change in *balance-sheet policy* while keeping the same *conventional monetary policy* and *transfer policy.*

More generally, a Neutrality Property applies if the above result holds for *any* appropriately-bounded *balance-sheet policy*. Indeed, it might very well be possible that only some balance-sheet policies are neutral, but not all. For example, temporary purchases of long-term bonds may be neutral while permanent ones may not. In this case, the Neutrality Property does not apply.

[21] Our analysis is not concerned about the uniqueness (local or global) of the rational expectations equilibrium. See Bassetto (2005) on how to implement desired equilibria through certain strategies followed by the policymakers.

The invariance of $\mathbf{Z}_t^*$ following the alternative *balance-sheet policies* captures the defining feature of the neutrality result: that the new *balance-sheet policy* does not induce any wealth effect on the households – at the initial prices – so that no change is implied in either aggregate demand or equilibrium prices.

The case in which the nominal interest rate is not above the zero-lower bound in every contingency deserves special treatment. When $i_t = 0$ money becomes a perfect substitute of reserves. In this case, it can be possible that a *balance-sheet policy* that leads to an increase in the supply of money M_t could deliver a neutrality result since at the zero-lower bound households are willing to absorb any additional supply of money without changing their portfolio choices and their consumption decisions. Therefore, in the case of equilibria in which the nominal interest rate stays even occasionally at the zero-lower bound, for a Neutrality Property to hold, we should require that, in the contingencies in which $i_t^* = 0$, only the stochastic processes $\{P_t^*, Q_t^*, i_t^*\}$ – rather than the whole vector $\{\mathbf{Z}_t^*\}$ – do not vary under the alternative *balance-sheet policy*, while in the same contingencies, and following the alternative *balance-sheet policy*, M_t can instead take any value $\tilde{M}_t \geq P_t^* Y_t$.[22]

Definition 3 *(Neutrality Property) Consider the set $\mathcal{N}$ of rational expectations equilibria associated with a given **transfer policy** $\mathcal{T}(\cdot)$ and a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{N}$ associated with a **conventional monetary policy**, $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$, and a **balance-sheet policy** $\mathcal{B}(\cdot)$. Consider an alternative, appropriately bounded, **balance-sheet policy** $\tilde{\mathcal{B}}(\cdot)$. The **balance-sheet policy** $\tilde{\mathcal{B}}(\cdot)$ is neutral with respect to $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ if there exists a rational expectations equilibrium $\{\tilde{\mathbf{Z}}_t, \tilde{\mathbf{K}}_t\} \in \mathcal{N}$ associated with the same **conventional monetary policy** $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and **transfer policy** $\mathcal{T}(\cdot)$ and with the **balance-sheet policy** $\tilde{\mathcal{B}}(\cdot)$ where:*

1. *$\tilde{P}_t = P_t^*$, $\tilde{\imath}_t = i_t^*$, and $\tilde{Q}_t = Q_t^*$ in each contingency,*

2. *$\tilde{M}_t = M_t^*$ in each contingency in which $i_t^* > 0$ while $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$.*

[22] It is instead key for a proper definition of neutrality that M_t^* is invariant when the nominal interest rate is positive. See also Eggertsson and Woodford (2003). Auerbach and Obstfeld (2005) show that policies raising the money supply at the zero-lower bound consistently with $M_t > P_t Y_t$ can have an effect on current price level since they affect the price level once the economy exits the zero-lower bound. They can also influence the duration of the trap. However, these effects rely on a change in policy (what we called *conventional monetary policy*) that lasts after the trap ends, i.e. M is changed also after the end of the trap. Therefore, in this case, the change in prices observed in equilibrium is due to the change in *conventional monetary policy.* Instead, the neutrality result holds if the *conventional monetary policy* is kept unchanged after the end of the liquidity trap (see also Robatto, 2014). Buiter (2014) shows that an expansion in the stock of base money can have permanent wealth effects even in a permanent liquidity trap provided money is not seen as a liability by the central bank.

In the contingencies in which $i_t^* = 0$ (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in ***conventional monetary policy*** *if and only if the latter is specified as $M_t = \mathcal{M}(\bar{\mathbf{Z}}_t, \zeta_t)$. The Neutrality Property holds if the neutrality result applies for any appropriately bounded* ***balance-sheet policy*** *$\tilde{\mathcal{B}}(\cdot)$ and for each equilibrium $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\} \in \mathcal{N}$.*

A key feature of the Neutrality Property is that the specification of the functional forms of the *transfer policy,* $\mathcal{T}(\cdot)$, and of the *conventional monetary policy* – either $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ – is not changed across the comparison (with the caveat mentioned in Definition 3) while what is varied is the functional form of the *balance-sheet policy* $\mathcal{B}(\cdot)$.

What we are going to show in the next section is that the Neutrality Property holds in our model only conditional on some specifications of the *transfer policy.* Indeed, Definition 3 characterizes neutrality starting from a set of equilibria identified by a certain *transfer policy*. This is not surprising: Wallace (1981) proved his irrelevance result of open-market operations in an overlapping-generation monetary model provided only "that lump-sum taxes are adjusted in an appropriate way" where "appropriate means, among other things, that fiscal policy is held constant." Wallace (1981) considers an environment in which fiat currency is not dominated in return. Sargent and Smith (1987) extend the irrelevance theorems to include cases of return dominance. Both Wallace (1981) and Sargent and Smith (1987) consider models in which open-market operations involve only risk-free short-term securities and in which there is a consolidated government's balance sheet, pooling together treasury and central bank.

Eggertsson and Woodford (2003) instead extend the irrelevance result to a model in which the central bank engages in unconventional open-market operations. However, they limit their analysis to the case in which central bank's net worth is zero and its only liability is money. This assumption constrains the set of balance-sheet policies that can be consistent with neutrality. Indeed, policies that enlarge the size of the central bank's balance sheet are neutral in their economy only when the nominal interest is at the zero-lower bound. We will instead show that this kind of policies can be neutral even when the nominal interest rate is positive, provided that the central bank issues interest-bearing reserves.[23] The reason is that the central bank can adjust reserves without necessarily varying *conventional monetary policy.*[24]

[23] In this respect, our analysis is instead in line with the more recent literature on the role of reserves in dealing with the expansion of the central bank's balance sheet (see Bassetto and Messer, 2013, Del Negro and Sims, 2015, Hall and Reis, 2015).

[24] In the case in which $X_t = 0$, the first important difference is that the monetary/fiscal policy regime should specify five instead of six additional restrictions. If we maintain the same definitions of *conventional monetary policy* and *transfer policy* as in Definition 1, then *balance-sheet policies* can only specify two of the sequences $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ as functions of the other endogenous variables and/or of exogenous state variables. This implies that a model without interest-bearing reserves limits substantially the kind of balance-sheet policies that can be considered independently of the specification of the *conventional monetary policy* and *transfer policy.*

As discussed in the introduction, this is an important and distinct feature of our framework, which builds on the recent literature that has defined a "new central-banking style" on the basis of the importance of interest-bearing reserves in the conduction of monetary policy.[25]

Another important difference between our result of neutrality and that of Eggertsson and Woodford (2003) is related to the transfer policy that ensures neutrality. In our case, it consists of two elements, as in Definition 1: *i*) the transfer policy between central bank and treasury and *ii*) that between treasury and the private sector. In their analysis, the key transfer policy to ensure neutrality is only that between the treasury and the private sector. The transfer policy between the central bank and the treasury cancels out in the consolidated budget constraint pooling together treasury and central bank. We instead keep separate budget constraints in the neutrality analysis and this is also key to address departure from non-neutrality, which is another novelty of our contribution with respect to all of the above literature.[26]

Finally, it is worth emphasizing that the Neutrality Property is specified for generic balance-sheet policies regarding $\left\{D_t^C\right\}$, without spelling out whether the issuer is the treasury or the private sector. Indeed, the only feature of the issuer that matters is its credit worthiness, which may determine the magnitude of the wealth effects affecting households when the central bank holds long-term assets experiencing income losses.

4 Neutrality property holds

We start with the case mostly studied in the literature, in which the Neutrality Property holds conditional on certain *transfer policies*. The literature usually proceeds to make assumptions about the "consolidated" behavior of government, including central bank and treasury.[27] It is instead a key distinction of our analysis to keep the two institutions independent of each other to characterize departures from neutrality. We start by defining a regime in which the fiscal policy and the remittance policy are both *passive*. These definitions are similar to those of the literature but, with the important caveat, that in our framework they apply to both institutions while in the literature, given the consolidation of budget constraints, they apply only to the path of taxes since central bank's remittances cancel out in the consolidation.

Definition 4 *Under a passive fiscal policy the stochastic path of taxes* $\left\{T_t^F\right\}$ *is specified to*

[25] See Hall and Reis (2015).

[26] Another difference with Eggertsson and Woodford (2003) is that they assume a more general composition of the assets portfolio of the central bank, while we focus only on two securities. This difference has no consequence for the generality of our results.

[27] This is the case of Wallace (1981), Sargent and Smith (1987).

ensure that the following limiting condition

$$\lim_{T \longrightarrow \infty} E_t \left\{ R_{t,T} \left(Q_T \frac{D_T^F}{P_T} + \frac{1}{1+i_T} \frac{B_T^F}{P_T} \right) \right\} = 0 \tag{25}$$

is satisfied looking forward from each date $t \geq t_0$ (and in each contingency at t) together with the sequence of equilibrium conditions (21) for any finite D_{t-1}^F, B_{t-1}^F, any appropriately bounded stochastic process $\left\{T_T^C\right\}_{T=t}^{\infty}$ and for any collection of stochastic processes $\left\{\mathbf{Z}_t^*\right\}$ satisfying the conditions of part i) of Definition 2, consistently each with a specified conventional monetary policy.

According to Definition 4, lump-sum taxes are set in a way that the expected present discounted real value of treasury liabilities converges to zero for any vector of stochastic processes $\left\{\mathbf{Z}_t^*\right\}$ satisfying the equilibrium conditions (16) to (19) given a (and for any) *conventional monetary policy*. In equation (25), we have defined the real stochastic discount factor as $R_{t,T} \equiv \beta^{T-t} \xi_T Y_T^{-\rho} / \xi_t Y_t^{-\rho}$ which is only driven by exogenous processes. An example of fiscal rule in the class of passive fiscal policy is the following:

$$\frac{T_t^F}{P_t} = \bar{T}^F - \gamma_f \frac{T_t^C}{P_t} + \phi_f \left[\frac{(1+r_t) Q_{t-1} D_{t-1}^F + B_{t-1}^F}{P_t} \right] \tag{26}$$

under the conditions $\gamma_f = 1$ and $0 < \phi_f < 2$.[28]

In (26), recall that we have defined the gross nominal return on long-term debt as $(1+r_t) \equiv (1+\delta Q_t)(1-\varkappa_t)/Q_{t-1}$. According to the fiscal rule (26), an increase in the outstanding real market value of treasury debt – of whatever maturity – signals an adjustment in the path of real taxes needed to repay it given the requirement that the parameter ϕ_f should be positive and in the range $0 < \phi_f < 2$.[29] Furthermore, the rule is such that the treasury does not have to rely on central bank's remittances to repay its obligations, since the parameter γ_f should be equal to one. Therefore, if the central bank is reducing payments to the treasury, the latter should immediately raise lump-sum taxes on the private sector to "support" the same equilibrium allocation for prices and interest rate.

A passive fiscal policy has direct implications for the equilibrium path of central bank's net worth. Indeed, equation (25), together with the equilibrium condition (20) implies that the expected present discounted value of the central bank's real net worth converge to zero

[28] See the Appendix for a derivation of the necessary and sufficient conditions for (26) to be in the class of passive fiscal policies.

[29] Indeed, $0 < \phi_f < 2$ ensures appropriate boundedness of the process of treasury's liabilities, which may follow an oscillatory path if $1 < \phi_f < 2$.

in equilibrium:

$$\lim_{T\longrightarrow\infty} E_t\left\{R_{t,T}\frac{N_T^C}{P_T}\right\}=0,$$

which together with the flow budget constraint (22) and (16)-(17) now implies the following central bank's intertemporal budget constraint

$$\frac{X_{t-1}}{P_t}+\frac{M_{t-1}}{P_t}-\frac{B_{t-1}^C}{P_t}-(1+r_t)\frac{Q_{t-1}D_{t-1}^C}{P_t}=E_t\sum_{T=t}^{\infty}R_{t,T}\left[\frac{i_T}{1+i_T}\frac{M_T}{P_T}-\frac{T_T^C}{P_T}\right]. \tag{27}$$

The real market value of the outstanding net liabilities of the central bank at a generic time t, which corresponds to the left-hand side of (27), should be backed by the present discounted value of the revenues obtained by issuing money net of the transfers between the central bank and the treasury. Interestingly, there could be rational expectations equilibria in which the left-hand side of (27) is positive (net worth is negative), provided that the incoming seigniorage net of transfers is enough to back the net liabilities of the central bank. However, the key observation is that, in general, (27) can restrict the path of prices, interest rates and other endogenous variables in a way that the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ satisfying the equilibrium conditions (16) to (19) consistently with some *conventional monetary policy* is not part of an equilibrium, unless additional assumptions are added to which we now turn our attention. We define a *passive policy of central bank's remittances*, in a similar way to the above definition of *passive fiscal policy* and irrespective of the latter specification.

Definition 5 *Under a passive policy of central bank's remittances the stochastic path of remittances* $\{T_t^C\}$ *is specified to ensure that*

$$\lim_{T\longrightarrow\infty} E_t\left\{R_{t,T}\frac{N_T}{P_T}\right\}=0 \tag{28}$$

is satisfied looking forward from each date $t\geq t_0$ *(and in each contingency at* t*) together with the sequence of equilibrium conditions (22) for any finite* $X_{t-1}, B_{t-1}^C, D_{t-1}^C$ *and for any sequence of stochastic processes* $\{\mathbf{Z}_t^*\}$ *satisfying the conditions of part* i*) of Definition 2, consistently each with a specified conventional monetary policy.*

Under a passive remittance policy, a worsening of the market value of the central bank's net liability position signals an increase in seigniorage revenues or in transfers from the treasury. According to Definition 5, we could design many remittance policies that can satisfy the definition. One possibility is the following:

$$\frac{T_t^C}{P_t}=\bar{T}^C+\gamma_c\frac{\Psi_t^C}{P_t}+\phi_c\frac{N_{t-1}^C}{P_t} \tag{29}$$

if and only if $0 < \gamma_c < 2$ and $0 < \phi_c < 2$.[30]

The above rule shows a positive relationship between the remittances to the treasury and both central bank's profits and past level of net worth. The central bank should avoid that net worth diverges and therefore prevent any wealth effect on households at the equilibrium prices. The reaction to both profits and past net worth ensures the boundedness of net worth which then satisfies condition (28) at any equilibrium prices. It is worth noting that a reaction to current profits implicitly builds also a reaction to the past level of net worth as shown in (15).

Given the above two definitions, we now discuss four cases in which the neutrality property holds. The first case considers both a passive fiscal policy and a passive remittance policy. Neutrality follows in a straightforward way given the Definitions 4 and 5. The other three cases analyzed show the interesting result that to have neutrality it is not even necessary that both policies are passive. Building on these four examples, we then provide some general intuition on when neutrality can hold.

4.1 Passive fiscal and remittance policies

The first neutrality result that we discuss holds under a combined regime given by the two passive policies defined above.

Proposition 1 *Under a combined regime of passive fiscal policy and passive policy of central-bank remittances the Neutrality Property holds.*

Proof. In the Appendix. ■

Under the conditions stated in Proposition 1 whether or not the central bank purchases long-term risky securities, eventually recording losses on these operations, is irrelevant for the equilibrium allocation of prices, interest rates and asset prices. The intuition for this result follows directly from Definitions 4 and 5. Passive transfer policies make irrelevant any alternative balance-sheet policy for equilibrium prices.

4.2 Passive fiscal policy and full treasury's support

We now use an active remittance policy in place of the passive, while maintaining the passive fiscal policy. The motivation is that, unlike rule (29), remittances commonly used in central banks' practices are not much related to past levels of net worth but instead they rebate

[30] See the Appendix for the derivation of the necessary and sufficient conditions for the rule (29) to be in the class of passive remittance policy.

profits as they mature. One interesting example is the case of *full treasury's support*, which we define as transfers from central bank equal to profits at each point in time:

$$T_t^C = \Psi_t^C. \tag{30}$$

The central bank remits positive profits to the treasury and, specularly, the treasury is ready to immediately cover central bank's losses when they occur.[31]

As shown by rule (29), a regime of full treasury's support is not in the class of passive policies, since $\phi_c = 0$. This means that some stochastic processes $\{\mathbf{Z}_t\}$ satisfying the equilibrium conditions (16) to (19) are ruled out as equilibrium allocations under a regime of full treasury's support. In particular, as it is shown in the Appendix, we can exclude equilibria in which the short-term nominal interest rate remains at zero for an infinite period of time.[32] Following the notation of Definition 3, the set $\mathcal{N}$ of rational expectations equilibria associated with a passive fiscal policy and *full treasury's support* is smaller than that associated with both *passive* transfer policies. However, even within this smaller set, the Neutrality Property holds.

Proposition 2 *Under a passive fiscal policy and full treasury's support the Neutrality Property holds.*

Proof. In the Appendix. ■

The fact that an active remittance policy can be consistent with a neutrality result is somewhat surprising and might suggest that other types of remittance policy could share a similar property. We do not provide a general result – althought we are going to show other active rules that can work as well – but we give a simple intuition for why a policy of full treasury's support delivers same results as passive remittances policies – an intuition that can apply also to the other cases discussed next.

For central bank's long-term asset purchases to have an effect, there should be some change in total (financial and human) wealth of households to induce them to vary their consumption choices. The consequent change in aggregate demand, given an exogenous stream of output, would then result in a variation of equilibrium prices. However, rules (26) and (30) make instead sure that there is no such a change in households' total wealth. There are only

[31] An example is that of the Bank of England which in January 2009 established a wholly-owned subsidiary called Bank of England Asset Purchase Facility Fund Limited with the responsibility of buying private and public long-term securities through funds of the same Bank of England raised through increases in reserves (see Bank of England, 2013). The created company is fully indemnified by the Treasury since any financial losses as a result of the asset purchases are borne by the Treasury and any gains are owed to the Treasury.

[32] This is, however, an interesting result since it implies that permanent liquidity traps are not equilibria if the treasury follows a passive fiscal policy and at the same time central bank's profits are fully transferred to the treasury or losses are fully covered by the treasury.

offsetting adjustments of human and financial wealth. Indeed, if the central bank purchases some risky securities from the hands of the private sector, that risk does not remain in the hands of the central bank since rule (30) ensures that the treasury immediately transfers resources to the central bank in the case the risk materializes in negative profits, while rule (26) ensures that the treasury gets these resources from the private sector through higher lump-sum taxes. At the end, the materialization of risk falls back on the shoulder of the private sector, whose total wealth does not change when evaluated at the initial equilibrium prices, because the increase in financial wealth is completely offset by a fall in human wealth: equilibrium prices therefore do not need to change. Alternative transfer policies that break these linkages can challenge the result of neutrality.

The proof of the above Proposition sheds also light on the key role played by interest-bearing reserves for neutrality cases. Indeed, under Propositions 1 and 2, Neutrality Property holds even if the central bank increases the size of its balance sheet when nominal interest rate is positive. Eggertsson and Woodford (2003) instead obtain neutrality only at the zero-lower bound. The role of reserves is indeed critical to explain the different result and to obtain that the non-neutrality property holds even when nominal interest rate is positive.

A constant nominal net worth, as implied by a regime of full treasury's support, requires that in equilibrium

$$Q_t^* \tilde{D}_t^C + \frac{\tilde{B}_t^C}{1+i_t^*} - M_t^* - \frac{\tilde{X}_t}{1+i_t^*} = \bar{N}.$$

This equation shows that alternative balance-sheet policies $\tilde{\mathcal{B}}(\cdot)$ implying different paths for the asset composition of the central bank $\left\{\tilde{B}_t^C, \tilde{D}_t^C\right\}$ can be accommodated by variations in central-bank reserves $\left\{\tilde{X}_t\right\}$ at the equilibrium prices $\left\{i_t^*, Q_t^*\right\}$ without changing the equilibrium path of money $\left\{M_t^*\right\}$.[33] Without interest-bearing reserves, the specification of the *balance-sheet policy* loses one degree of freedom and can only set one of the stochastic processes $\left\{B_t^C, D_t^C\right\}$ while the other is endogenously determined by the equilibrium conditions and the remaining specification of the monetary/fiscal policy regime. The central bank can, for example, increase the stock of long-term securities held in its portfolio. When $i_t^* > 0$ and under full *treasury's support* and *passive fiscal policy*, these purchases are neutral because they are followed by a drop in the holdings of short-term securities that keeps invariant the total value of the assets of the central bank. Increases in the total value of the assets, though, are possible but they have to be matched by a higher level of M_t and therefore, given (18), are consistent with a different, and higher, equilibrium price level – a non-neutrality result.

[33] As detailed in the Appendix, it is also clear that alternative *balance-sheet policies* should be appropriately bounded to satisfy the non-negative requirement on central-bank reserves. In the case of positive nominal interest rates, the value of the total assets purchased by the central bank should be bounded below by the value of non interest-bearing liabilities, *i.e.* $Q_t^* \tilde{D}_t^C + \tilde{B}_t^C/(1+i_t^*) \geq M_t^* + \bar{N}$.

Instead, when we allow the central bank to issue interest-bearing reserves, balance-sheet policies that raise the value of the assets of the central bank are neutral even when the nominal interest rate is positive, if the conditions of Propositions 1 and 2 are met.[34]

A regime of full treasury support has two special features: first, nominal net worth is constant; second, deflationary solutions are excluded as possible equilibria. For these reasons, it could be argued that the neutrality result is a special one. We now show, instead, that neutrality property holds even if there are stronger restrictions on the set of possibile equilibria and even if net worth varies over time.

4.3 Passive fiscal policy and full treasury's support with real transfer

Consider – as a twist of the previous policy – a remittance policy of the following form

$$\frac{T_t^C}{P_t} = \bar{T}_t^C + \frac{i_{t-1}M_{t-1}^C + (r_t - i_{t-1})Q_{t-1}D_{t-1}^C}{P_t} \tag{31}$$

which is now including a real transfer, captured by the term $\bar{T}_t^C$, and, on top, is transferring part of the central bank's profits by excluding the component derived from issuing equity.[35] The real transfer $\bar{T}_t^C$ is a Markov process, with transition density $\pi_C(\bar{T}_{t+1}^C|\bar{T}_t^C)$ and initial distribution f_C; we assume that (π_C, f_C) is such that $\bar{T}_t^C \in [\bar{T}_{\min}^C, \bar{T}_{\max}^C]$. One key aspect of the above remittance policy is that it is transferring to the treasury all the gains or losses from holding long-term assets and all the revenues obtained by issuing non-interest-bearing liabilities. Therefore, we can still consider this policy as belonging to the class of full-treasury-support policies.

Proposition 3 *Under a passive fiscal policy and the remittance policy (31) the Neutrality Property holds.*

The result can be proved by first noting that rule (31) implies a path of net worth which is independent of the composition of the balance sheet of the central bank

$$N_t^C = (1 + i_{t-1})N_{t-1}^C - P_t\bar{T}_t^C \tag{32}$$

[34]When $i_t = 0$ and $X_t = 0$, an increase in the holdings of long-term securities, and therefore an enlargement of the balance-sheet size, can be instead accommodated through an increase in money supply (or reserves), without the need of an offsetting fall in the holdings of short-term securities. The higher supply of money can be absorbed by households without affecting their consumption choices since the opportunity cost of money is zero. Under a *passive fiscal policy* and *treasury's support*, therefore, we obtain a neutrality result similar to that discussed by Eggertsson and Woodford (2003), using however a different *transfer policy*.

[35]See the definition of profits in equation (15)

having used (13) and (15), given the initial condition $N^C_{t_0-1} = \bar{N}^C$. Moreover, under a passive fiscal policy, the intertemporal budget constraint (27) applies and, after plugging inside (31) we get

$$\frac{(1+i_{t-1})N^C_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T}(\bar{T}^C_T). \tag{33}$$

which holds at each time $t \geq t_0$ and each contingency at time t given the initial condition $N^C_{t_0-1}$. Therefore, an equilibrium path of prices and interest rates $\{P^*_t, i^*_t, R^*_{t,T}\}$ satisfying (32) and (33) still remains an equilibrium path even if there is a change in the central bank's balance sheet policy. Another interesting feature of the remittance policy (31), as opposed to the regime of full treasury support, is that it can uniquely determine the price level through the equilibrium condition (33) evaluated at time t_0, given the initial condition $N^C_{t_0-1}$. Combining it with an appropriate interest-rate policy, it can also determine the path of prices at each point in time.[36]

The intuition for why the remittances policy (31) is consistent with the neutrality property is that it features similar characteristics to a regime of full treasury support in transferring gains or losses on central bank's balance sheet to the treasury, which appropriately rebates them to the private sector. Therefore, as previously discussed, any reallocation of long-term asset between the central bank and the private sector is neutral since it does not produce any wealth effect on private consumption.

4.4 Active fiscal policy and full treasury's support

We now show that the neutrality property can be also consistent with an active fiscal policy, or with a case in which both transfer policies are active, provided that the balance-sheet policy does not involve any change in long-term debt issued by the treasury. This last restriction is perfectly in line with the focus of our analysis since we are analyzing the efficacy of a swap in long-term assets between the central bank and the private sector. In what follows, we assume that the central bank sets the active remittance policy (31) and the treasury an active fiscal policy of the type

$$\frac{T^F_t}{P_t} = \bar{T}^F_t - \frac{T^C_t}{P_t} \tag{34}$$

in which $\bar{T}^F_t$ is a Markov process, with transition density $\pi_F(\bar{T}^F_{t+1}|\bar{T}^F_t)$ and initial distribution f_F.[37] We assume that (π_F, f_F) is such that $\bar{T}^F_t \in [\bar{T}^F_{\min}, \bar{T}^F_{\max}]$. The transfer policy (34) is

[36] Benigno (2017) gives other examples of price determination through the central-bank balance sheet by proposing a central-bank theory of the price level.

[37] This specification of tax policy belongs to the class of fiscal policies considered by Wallace (1981) to prove irrelevance of open-market operations.

not in the class of passive transfer policies because the parameter ϕ_f in (26) is equal to zero. Therefore both transfer policies are now active.

Proposition 4 *Under the active fiscal policy (34) and the active remittance policy (31) the Neutrality Property holds.*

To see that these policies are consistent with neutrality, consider that the set of equilibrium conditions (20) to (24) implies a consolidated intertemporal budget constraint

$$\frac{X_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[\frac{i_T}{1+i_T} \frac{M_T}{P_T} + \frac{T_t^F}{P_T} \right] \tag{35}$$

showing that the overall liabilities of the whole government should be backed by the expected present discounted value of seigniorage revenues and primary surpluses. Using the transfer policies (31) and (34) into it, we can obtain

$$\frac{B_{t-1}^F}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}^F}{P_t} - \frac{N_{t-1}^C}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} (\bar{T}_T^F + \bar{T}_T^C), \tag{36}$$

Note again that given the remittance policy (31), the path of net worth is independent of the central bank's balance-sheet policy. Consider now an equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ under the two transfer policies, given a *conventional monetary policy* and a *balance-sheet policy*. Assume an alternative *balance-sheet policy* that just changes the long-term securities held by the central bank but not those of the treasury. We are thinking here at a swap between central bank and private sector in the holdings of long-term securities. In this case, nothing changes in (36) and therefore the stochastic sequences $\{\mathbf{Z}_t^*\}$ are still part of an equilibrium since they are consistent with (36) under the new *balance-sheet policy*.

There are some lessons that can be learnt from the examples of this Section. Passive transfer policies are a stronger requirement to obtain neutrality results. They work since they make sure that the intertemporal budget constraints of both treasury and central bank hold at any equilibrium prices and interest rates regardless the *balance-sheet policy*. In particular, they make sure that any reallocation of wealth due to balance sheet policies is neutralized by an appropriate transfer of gains or losses even if these take place over time. However, neutrality holds even if transfers occur each period, as losses or gains are realized, and even if transfer policies are active, meaning that the intertemporal budget constraint of treasury, or central bank, or consolidated government, impose some restrictions on equilibrium prices. In particular, the last two examples have shown that neutrality results hold even when the equilibrium is unique. Therefore there is no one-to-one correspondence between economies in which equilibrium is indeterminate and economies in which the neutrality properties hold.

The final important message is that, althought passive transfer policies are not necessary to imply neutrality results, it is instead necessary to have at least one of the two transfer policies to be active in order to find a non-neutrality result. We turn to this analysis next.

5 Neutrality property does not hold

In this section, we discuss three cases for which the Neutrality Property does not hold. In the first two cases, we maintain the assumption that fiscal policy is passive and instead elaborate more on the kind of remittance policies and central bank's actions that can lead to violations of the Neutrality Property. In the third case, we discuss the implications of an active fiscal policy regime.

We label the first case as one of "financial independence" since we assume that the treasury does not make any transfer to the central bank, i.e. $T_t^C \geq 0$. We will show that large losses on central bank's balance sheet can deliver non-neutrality result. In the second case, instead, we show that the source of non-neutrality is not in the transfer policy but in the action of the central bank that tries to achieve a certain path of remittances. This case reflects the internalization of political constraints or the willingness of the central bank to seek financial independence.

The results of these two examples show an interesting monetary economics trilemma arising among choosing freely the *conventional monetary policy* and the *balance-sheet policy* while maintaining *financial independence*. Propositions 1, 2 and 3 have shown one leg of this trilemma: a central bank which engages in any arbitrary *balance-sheet policy* and is committed to a certain *conventional monetary policy* may need some support from the treasury and therefore cannot be *financially independent*: transfers from the central bank to the treasury should be negative under some conditions, i.e. $T_t^C < 0$ for some t. As well, a central bank committed to a certain *conventional monetary policy* that always wants to maintain *financial independence* has to restrict the type of *balance-sheet policies.* For example, it should limit purchases only to riskless short-term securities as in the conventional open-market operations. Finally, a central bank that chooses an arbitrary *balance-sheet policy* and is (or aims at remaining) *financially independent* cannot freely choose the *conventional monetary policy*. The examples of the next two subsections are consistent with this leg of the trilemma. Moreover, the third subsection considers an active rathen than a passive fiscal policy and derives another neutrality result.

5.1 Financial independence: absence of treasury's support

In this subsection, we limit the type of remittance policy between central bank and treasury by assuming that the treasury never transfers resource to the central bank, i.e. $T_t^C \geq 0$: the central bank lacks treasury's support. To intuit the implications of this restriction, consider an allocation $\{\mathbf{Z}_t^*\}$ satisfying the equilibrium conditions (16) to (19) at each point in time and contingency given a *conventional monetary policy*. We evaluate whether this allocation can be an equilibrium considering a passive fiscal policy, a non-negative remittances policy and a generic *balance-sheet policy* $\mathcal{B}(\cdot)$. Given that fiscal policy is still passive, equation (25) together with (20) implies that (28) must hold in equilibrium.

The allocation $\{\mathbf{Z}_t^*\}$ is part of a rational expectations equilibrium, given the specification of the monetary/fiscal policy regime and in particular without treasury's support, if there are stochastic processes $\{X_t^*, T_t^{C*}\}$, with $X_t^*, T_t^{C*} \geq 0$, such that

$$\frac{X_{t-1}^*}{P_t^*} - \frac{B_{t-1}^{C*}}{P_t^*} + \frac{M_{t-1}^*}{P_t^*} - (1+r_t^*)\frac{Q_{t-1}^* D_{t-1}^{C*}}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} - \frac{T_T^{C*}}{P_T^*} \right] \tag{37}$$

holds at all times and in each contingency considering that the stochastic path of $\{B_t^{C*}, D_t^{C*}\}$ is implied by the *balance-sheet policy*.[38] The Neutrality Property holds if the above condition is satisfied for any appropriately bounded *balance-sheet policy* $\mathcal{B}(\cdot)$ at each time t and contingency at t.

In this section, we focus on a particular remittance policy which we call *"deferred-asset" regime*. This is meant to capture the case of the U.S. Federal Reserve, which does not receive full support from the treasury and, in the case of negative profits, stops making remittances and issues a "deferred asset" that can be paid back by retaining future earnings. Only once the "deferred asset" is paid in full, the central bank resumes remitting positive profits to the treasury.[39]

Definition 6 *A "deferred-asset" policy of central-bank remittances is defined as* $T_t^C = \Psi_t^C$ *if* $N_{t-1}^C \geq \bar{N} > 0$ *and* $\Psi_t^C \geq 0$ *otherwise* $T_t^C = 0$.[40]

[38] Note that $R_{t,T}$ is function of exogenous states only.

[39] See Carpenter et al. (2015). In our analysis we are abstracting from operating costs and standard dividends to member banks subscribing the capital of the central bank.

[40] Writing explicitly a "deferred asset" in the problem like a negative liability, as it is done in practice to avoid that the accounting value of net worth declines (see also Hall and Reis, 2015), does not really matter for the analysis since values of net worth below the threshold $\bar{N}$ would correspond to periods in which the deferred asset is positive. In both cases, positive income will be retained by the central bank either to increase net worth or to pay the "deferred asset". Note that we are adopting a nominal mark-to-market dividend rule according to the definition given by Hall and Reis (2015).

Under a *"deferred-asset" policy* whether or not the Neutrality Property is violated depends on the actual size of the central bank's income losses.

Proposition 5 *Under a passive fiscal policy and a "deferred-asset" policy of central bank's remittances the Neutrality Property holds if and only if*

$$\frac{\tilde{N}_t^C}{P_t^*} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{38}$$

in equilibrium at each time t (and in each contingency at t).[41]

Proof. In the Appendix. ■

We leave the proof to the Appendix but here we provide some intuition for condition (38). Note that an alternative way to write (37), using the definitions of central bank's net worth and profits, is:

$$\frac{N_t^C}{P_t^*} + E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) = E_t \sum_{T=t+1}^{\infty} R_{t,T} \left(\frac{T_T^C}{P_T^*} \right). \tag{39}$$

The left hand side of the above equation is the value of the central bank (in real terms) given by the sum of its real net worth and the value of current and future resources that can be obtained from the monopoly power of issuing money. In equilibrium, given a *passive fiscal policy*, the value of the central bank should be equal to the expected present discounted value of real transfers to and from the treasury.

In the absence of treasury's support, the right-hand side of (39) cannot be negative, and thus imposes a lower bound on the level that net worth can reach (equation 38) consistently with the allocation $\{\mathbf{Z}_t^*\}$, which can be in principle violated if the central bank purchases long-term risky securities.[42]

Indeed, income losses translate directly into declining net worth

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C < N_{t-1}^C$$

which in turn can be inconsistent with (38) or (39). This inconsistency implies that $\{\mathbf{Z}_t^*\}$ is not part of a rational expectations equilibrium given the *balance-sheet policies* undertaken,

[41] Note that (38) should be evaluated at $\{\mathbf{Z}_t^*\}$ of the "starting" equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ of Definition 3 where $\tilde{N}_t^C$ instead is the value of net worth reached under the alternative balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$.

[42] The solvency condition (39) has been already emphasized in the works of Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015). In particular Del Negro and Sims (2015) have discussed that violations of the solvency conditions without treasury support could lead to a change in the interest-rate policy while Hall and Reis (2015) have instead focused on the possible non-stationarity of the path of reserves, since they consider equilibrium prices as given.

meaning that $\{\mathbf{Z}_t\}$ should change in a way that the central bank remains solvent at the level of net worth N_t^C reached. In practice, this can be accomplished by an increase in the present discounted value of seigniorage revenues and/or an increase in the current price level. This result is in line with the third leg of the trilemma enounced above: a financially-independent central bank pursuing any arbitrary balance-sheet policies needs to change its conventional policy in a way to satisfy the solvency condition (39). Otherwise, if it wants to maintain its conventional monetary policy, it can only choose the balance-sheet policy within the smaller subset that makes such a conventional policy consistent with (39).

We now investigate an important result implied by Proposition 5 and peculiar to the stochastic structure of the model. There can be a non-neutrality result even if there is just the possibility of a future loss, sizeable enough to produce a violation of condition (38). This result arises even if that future contingency does not realize. To support this result, we provide a numerical example using a calibrated version of the model, extended to allow for nominal rigidities.[43] Figure 1 illustrates it. We consider that at time 0 there is a probability that in the next periods there is going to be a credit event on long-term assets implying a capital loss big enough to violate condition (38). The figure shows the optimal response – under full commitment – of selected endogenous variables.[44] In the case of full treasury support, there is no change in output, inflation and interest rate no matter what is the composition of the central bank's balance sheet. Under a deferred-asset regime and if the central bank holds a risky portfolio of assets (i.e. $D^C > 0$), the probability of a large loss in the future has non-neutral effects on the equilibrium path of several endogenous variables.

In the contingency in which the loss materializes, indeed, the conventional monetary policy would need to change in order to support an increase in the price level that can restore the central bank's profitability, as discussed above. Once this effect is accounted for into the private sector's expectations at time 0, prices rise in advance and the nominal interest rate increases to sterilize the effect of inflationary expectations on current inflation, and the output gap thereby falls. The expected capital loss, moreover, reduce the current level of long-term return, inducing a fall also in the remittances to the treasury, as well as in the market value of the long-term portfolio held by the central bank, and thereby the central bank's reserves.

[43]Please refer to the appendix for details on the extended model, calibration and numerical simulations.

[44]In particular, the probability that such a credit event occurs next period increases from zero to 1.5%, and gradually reverts to zero with a half-life of about one year.

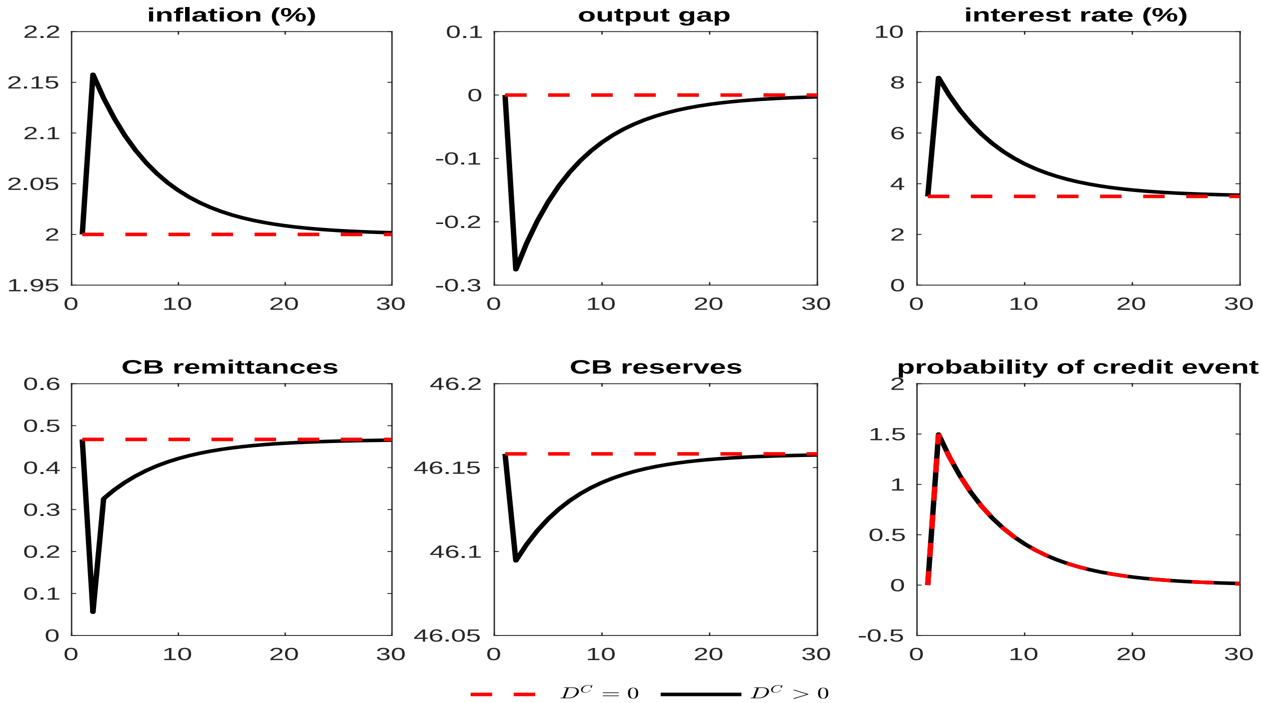

Figure 1: Equilibrium response of selected variables under optimal monetary policy under commitment to an increase in the probability of a credit event. The probability temporarily increases from 0 to 1.5%, with half life of one year. Red dashed line: central bank holds only short-term assets. Black solid line: central bank holds also long-term assets. Inflation and the nominal interest rate are annual percentage points, the output gap and long-term asset price are percentage deviations from steady state, remittances and reserves are percentages of steady-state central bank's balance sheet. X-axis displays quarters.

5.2 Seeking financial independence or political constraints

In this subsection, we discuss cases in which it is the action of the central bank in shaping a desired path of remittances that produces non-neutrality results rather than the remittance policy itself. We analyze two cases.

In the first, we use the deferred-asset regime of the previous section and start with a scenario in which neutrality holds because the central bank can absorb losses on long-term assets by retaining future profits – thus delivering zero remittances to the treasury – until net worth returns back to normal levels. The non-neutrality result arises, instead, when the central bank aims at reducing the duration of zero remittances because of political constraints. Indeed a too long duration can shed doubts on the operating procedures of the central bank and undermine its independence.

In the second case, instead, we analyze a transfer policy of full treasury support, $T_t^C = \Psi_t^C$, which taken alone is consistent with the neutrality property. But, a non-neutrality result arises when the central bank wants to avoid treasury support and, therefore, seeks *financial independence* by keeping profits non-negative, $\Psi_t^C \geq 0$.

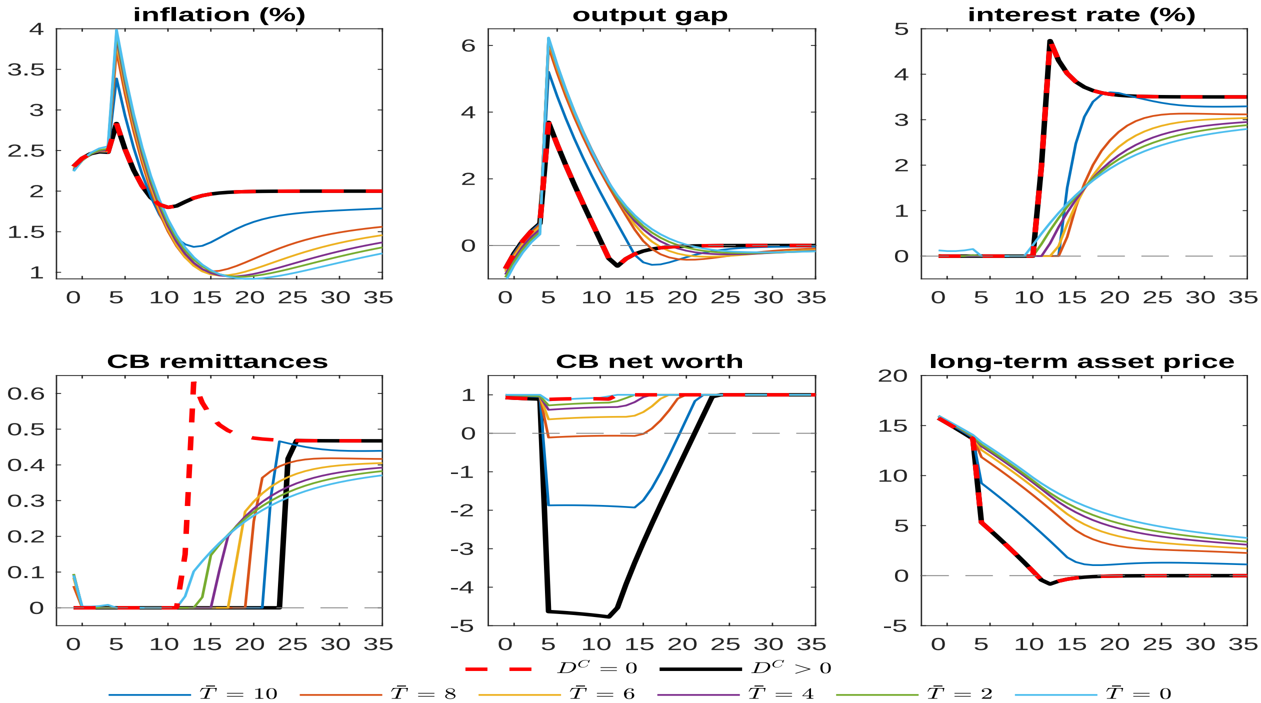

Figure 2: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk: "deferred asset" regime. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns unexpectedly and permanently to steady state after one year. Red dashed line: central bank holds only short-term assets. Black solid line: central bank holds also long-term assets. Inflation and the nominal interest rate are annual percentage points, the output gap and long-term asset price are percentage deviations from steady state, remittances and net worth are percentages of steady-state central bank's balance sheet. X-axis displays quarters.

In both cases, fiscal policy is assumed to be passive. We analyze an economy in a liquidity trap because of a negative natural rate of interest. As this rate unexpectedly turns positive, the drop in the price of long-term assets can cause losses on central bank's balance sheet. Under the two remittances policy discussed above and with no further action of the central bank in shaping remittances, whether the central bank holds or not long-term assets will be irrelevant for the path of output, inflation and interest rates. Indeed, these paths will follow the same optimal monetary policy discussed by Eggertsson and Woodford (2003). Conversely, non-neutrality results arise under political constraints or when the central bank seeks financial independence.

We discuss first the case of the deferred-asset regime, but in which the central bank puts limit to the duration of zero remittances. Suppose the economy at time $t_0 - 1$ is already in a liquidity trap, because of a fall in the natural rate of interest that occurred sometime in the past. At time $t_0 + 4$ an unexpected preference shock hits, and brings the natural interest rate permanently back to its positive steady-state level. Figure 2 displays the optimal response of the economy to such upward shift in the natural interest rate. We know from Eggertsson and

Woodford (2003) that the optimal response under commitment to this unexpected upward shift in the natural rate is to hold the nominal interest rate at the zero-lower bound for some more time (6 more quarters in the simulation), in order to limit the deflationary and recessionary effect of the liquidity trap. When the nominal interest rate is lifted, moreover, it jumps and overshoots its steady-state level, to which it quickly reverts from above, ensuring a fast convergence of inflation and output to their targets. The red dashed line in Figure 2 shows this benchmark case, arising also when the central bank holds only riskless assets ($D^C = 0$). The bottom panels, in particular, show that when the positive shock hits, the path of current and future short-term rates changes, producing an unexpected fall in the price of long-term securities and therefore implying income losses for whoever holds those securities. If the central bank does not hold long-term assets ($D^C = 0$), the dynamics of long-term asset prices are irrelevant for the central bank's balance sheet, so that remittances to the treasury only depend on the return on short-term assets. Therefore, they turn back positive as soon as the short-term nominal interest rate is lifted off, while nominal net worth does not move.

If, however, the central bank holds long-term assets ($D^C > 0$, black solid line) it incurs a financial loss on its portfolio when their prices fall. Under a "deferred asset" regime, this loss implies a fall in net worth, and the central bank retains future profits until the latter is back at its steady-state level: remittances to the treasury stay at zero for an additional 12 quarters. In the absence of any other shock after time $t_0 + 4$, the level of net worth can be rebuilt without deviating from the conventional monetary policy implicit in $\left\{\mathbf{Z}_t^*\right\}$: the paths of inflation, output gap, interest rate and long-term asset prices are consistent with Neutrality.

If, instead, the central bank shortens the duration $\bar{T}$ of zero remittances (i.e. $\bar{T} < 12$), Neutrality breaks down. As the figure shows, the "deferred asset" regime in this case dictates to accelerate the rebuilding of net worth, and thereby requires a change in the conventional monetary policy: the central bank commits to a gradual convergence of the nominal interest rate to its target in order to smooth out the downward response of long-term asset prices and mitigate the financial loss. As a consequence, the shorter the period of zero remittances is, the more inflationary and expansionary on output the change in conventional monetary policy is.

With respect to the benchmark case of Eggertsson and Woodford (2003), particularly interesting is the implied dynamics of the nominal interest rate. Indeed, the existence of political-economy reasons that induce the central bank to limit the duration of financial losses – and the corresponding zero remittances to the treasury – are able to rationalize the gradual exit strategy from a liquidity trap that characterizes the recent behavior of major central banks.

We now consider the second case in which there is full treasury support, but the central bank does not exploit it completely avoiding any loss. Figure 3 illustrates this case under the same scenario as before of an economy in a liquidity trap for some period of time. The figure shows that the central bank, when it holds long-term securities and in order to satisfy the non-negative constraint on its profits, has to engineer a dynamic path for asset prices such that the long-term return does not display the sharp drop when the preference shock brings the natural rate of interest back to steady-state. This requires committing to an interest rate path that remains at the zero-lower bound substantially longer than before, and implies (when the natural rate returns positive) a surge in inflation about five times stronger than in the case $D^C = 0$, and an output boom almost twice as strong. The central bank's remittances to the treasury, as a result, stay at around zero several periods longer than the duration of the shock, following thereafter the path of the nominal interest rate with one period delay. Nominal money supply temporarily increases, when the natural interest rate turns back positive, to accommodate the surge in inflation and the output boom, and central bank's reserves progressively decrease to their steady-state level to ensure that net worth stays constant (not shown in the figure).

In a liquidity trap, therefore, a central bank committed to financial independence signals, when purchasing long-term securities, a change in its conventional monetary policy stance towards temporarily higher inflation, as in the case of Figure 2. There are some important differences between the two cases considered. Here there is a delayed exit strategy from the ZLB, while in the case of Figure 2 the commitment was toward a sooner (though much smoother) liftoff of the nominal rate. This difference depends on the fact that here the central bank self-imposes a zero-lower bound constraint on its profits, but faces no further constraint on the path of its remittances to the treasury, which can be zero for as long as necessary. Instead, in the case of Figure 2, the central bank faces a constraint on the duration of zero remittances and, in the limiting case, it completely avoids them.

5.3 Active fiscal policy

In Section 4.4 we have shown a somewhat surprising result that neutrality can arise even under an active fiscal policy provided the treasury does not change its holdings of long-term assets. The result is of particular interest since the active fiscal policy (34) is often used in the fiscal theory of the price level to uniquely determine the price level. To get a non-neutrality result we should go beyond what is common practice and assume a different fiscal policy. Given the intuition already provided the fiscal rule should not rebate back all gains or losses on central bank's balance sheet to the private sector. A simple active policy that serves this

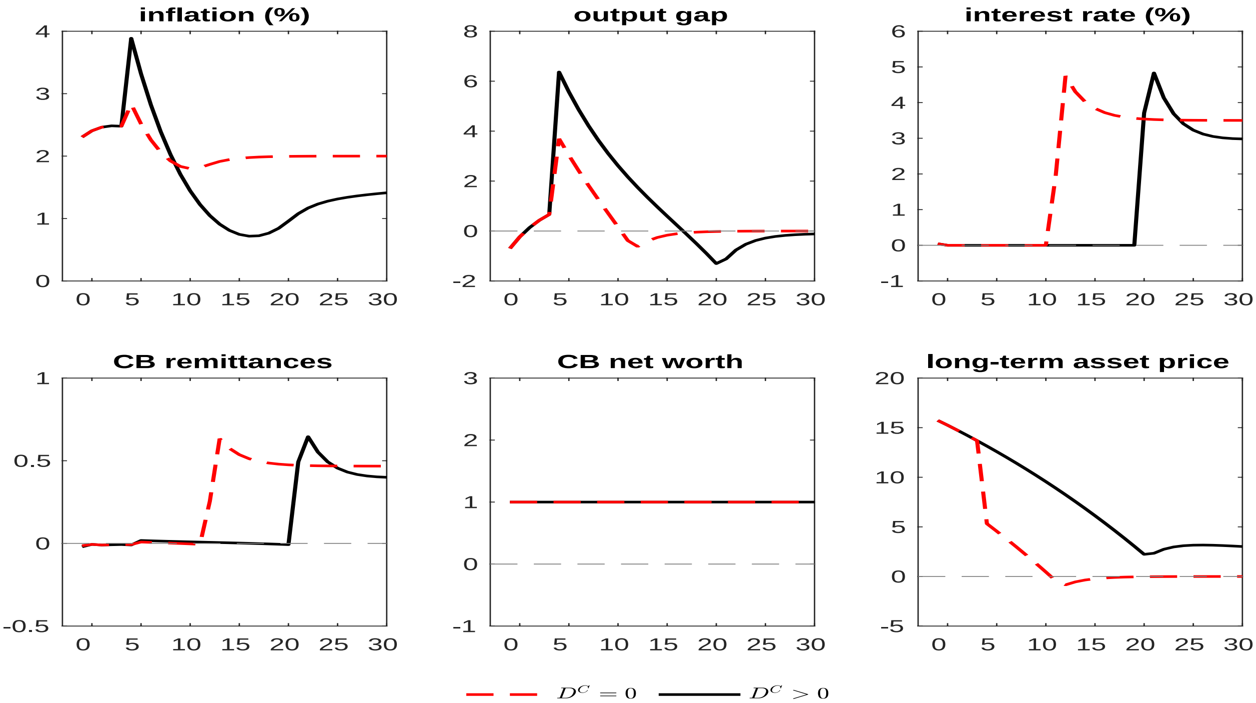

Figure 3: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk: seeking financial independence. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns unexpectedly and permanently to steady state after one year. Red dashed line: central bank holds only short-term assets. Black solid line: central bank holds also long-term assets. Inflation and the nominal interest rate are annual percentage points, the output gap and long-term asset price are percentage deviations from steady state, remittances and net worth are percentages of steady-state central bank's balance sheet. X-axis displays quarters.

purpose is the following

$$\frac{T_t^F}{P_t} = \bar{T}_t^F, \tag{40}$$

where $\bar{T}_t^F$ is a Markov process, with transition density $\pi_F(\bar{T}_{t+1}^F|\bar{T}_t^F)$ and initial distribution f_F. We assume that (π_F, f_F) is such that $\bar{T}_t^F \in [\bar{T}_{\min}^F, \bar{T}_{\max}^F]$.

To get the intuition straight in this section we assume that, for what concerns the remittance policy, there is *full treasury's support*, i.e. $T_t^C = \Psi_t^C$ at each t. As previously shown, this implies a constant central bank's net worth $N_t^C = N_{t_0-1} = \bar{N} > 0$.

Consider an equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ under the fiscal rule (40) and *full treasury's support*, given a *conventional monetary policy* and a *balance-sheet policy*. In this case, we can write the aggregate intertemporal budget constraint (35) as

$$\frac{B_{t-1}^{F*}}{P_t^*} + (1 + r_t^*)\frac{Q_{t-1}^* D_{t-1}^{F*}}{P_t^*} - \frac{\bar{N} + \Psi_t^{C*}}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T}\left[\frac{i_T^*}{1 + i_T^*}\frac{M_T^*}{P_T^*} + \bar{T}_T^F\right], \tag{41}$$

having used the definitions of central bank's net worth and profits. Consider now an alterna-

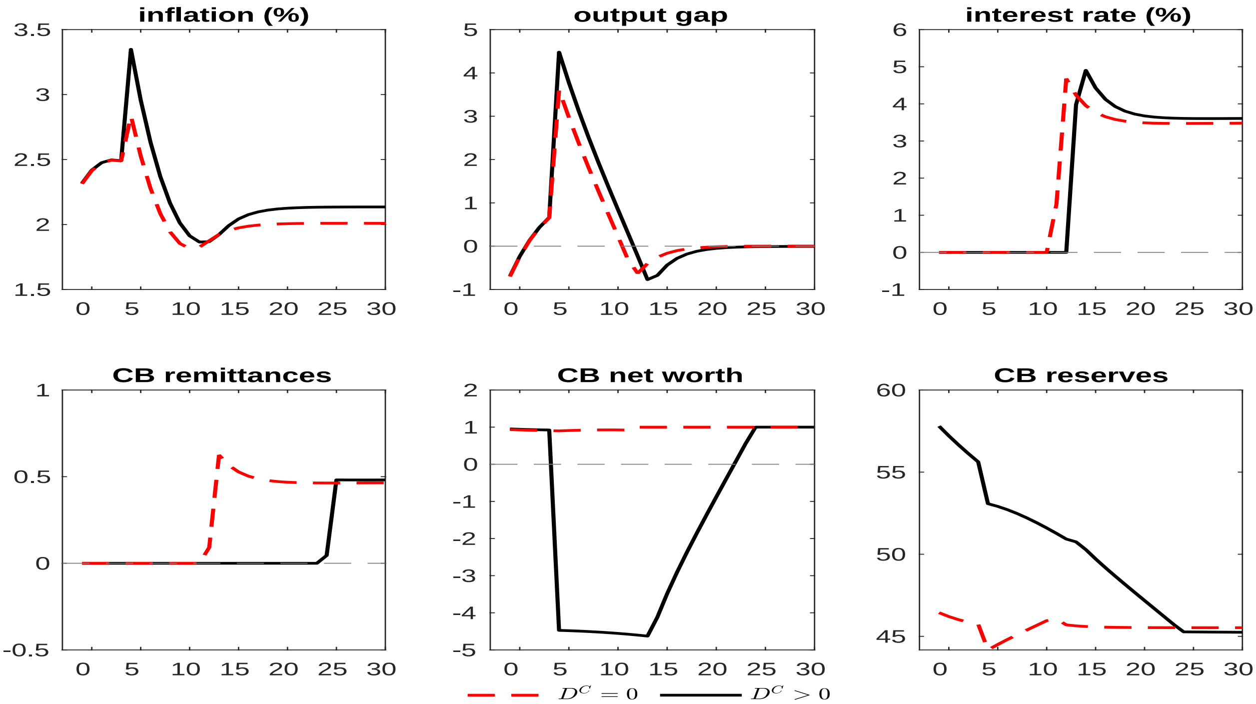

Figure 4: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk: active fiscal policy. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns unexpectedly and permanently to steady state after one year. Red dashed line: central bank holds only short-term assets. Black solid line: central bank holds also long-term assets. Inflation and the nominal interest rate are annual percentage points, the output gap is percentage deviations from steady state, remittances, net worth and reserves are percentages of steady-state central bank's balance sheet. X-axis displays quarters.

tive *balance-sheet policy* that just changes the holdings of long-term securities of the central bank at time $t-1$ to $\tilde{D}^C_{t-1} \neq D^{C*}_{t-1}$ assuming $r^*_t \neq i^*_{t-1}$. It simply follows that $\tilde{\Psi}^C_t \neq \Psi^{C*}_t$ which implies that $\{\mathbf{Z}^*_t\}$ could no longer be consistent with (41) and therefore cannot be part of an equilibrium under the new *balance-sheet policy*. This is again a result of non-neutrality.

If the central bank purchases long-term securities and the treasury does not pass the gains or losses to the private sector, there can be a reallocation of risk in the economy inducing wealth effects on households that move consumption, aggregate demand and then prices.

Figure 4 illustrates the result, assuming a *transfer policy* given by a combination of the *active fiscal policy* (40) and a *"deferred-asset" remittance policy*. When the natural rate turns back positive, a central bank that holds long-term assets suffers losses which are kept in the treasury's balance sheet. The private sector experiences a positive wealth gain that pushes up inflation, both on impact and in the medium run, supported by a longer stay of the nominal interest rate at the zero-lower bound.

6 Conclusions

This work has studied monetary/fiscal policy regimes which may or may not support neutrality results following central bank's generic open-market operations. To preserve tractability, we kept the environment as simple as possible, at the cost of disregarding some important features that we now discuss more extensively.

In our model, long-term assets have only a pecuniary return and therefore the focus of our analysis has been restricted on the equilibrium consequences of central bank's losses due to long-term assets' purchases. The literature has instead stressed the importance of considering also non-pecuniary benefits of debt securities, to characterize departures from irrelevance of central bank's asset purchase programs. There are mainly two classes of such models. The first includes limits to arbitrage in the private financial intermediation of certain securities which translates into credit or term premia (see among others Curdia and Woodford, 2011). These excess returns can be relaxed by central bank intervention in these markets through its ability to finance the purchases more easily than the private sector by expanding reserves. In the second class, long-term *assets* have also a non-pecuniary value like, for example, that of relaxing collateral constraints (see among others Araújo et al., 2015, and Reis, 2016). Along this direction central bank's purchases can produce effects on the economy.

Another strand of literature has emphasized the benefits of balance-sheet policies on the ground that the *liabilities* of the central bank can have an advantage in relaxing some liquidity (or collateral) constraints that bind the action of private agents.[45] But in this case the benefits obtained by increasing the central bank's liabilities are the same regardless of whether the central bank purchases short or long-term securities.

Central banks around the world have very different accounting practices, capital requirements and transfer policies. A comprehensive analysis of all the various possibilities is out of the scope of this paper, though some alternative assumptions not made here could affect the results. One that deserves particular attention is the way purchases of long-term securities are accounted in the balance sheet and therefore in the profit-loss statement. We have evaluated them at the market value but some central banks do it at the historical value, like the Federal Reserve.[46]

A more relevant extension for emerging-market economies could be the modelling of reserves in foreign currency. In this case, capital losses can be consequence of exchange rate movements and can affect the conduct of monetary policy also for what concerns its effects on

[45] See Benigno and Nisticò (2017) and Reis (2016).

[46] In general the ECB uses a mark-to-market procedure with the possibility of inputting precautionary reserves in the case of gains. However, in the recent purchases of covered bonds and sovereign debt, it moved to an accounting system at historical costs.

the exchange rate (see Jeanne and Svensson, 2007). In this respect, Adler et al. (2012) have shown that for emerging market economies deviations from standard interest-rate policies can be explained by concerns about the weakness of the central bank's balance sheet.

We have discussed our theoretical results in a cash-in-advance model *à la* Lucas and Stokey (1987) where the asset market opens before the goods market. The results of this paper are robust to other ways of modeling the liquidity friction like money in the utility function or through a cash-in-advance constraint in which the goods market opens before the asset markets.[47] The analysis can be extended also to cashless-limiting economies or to overlapping-generation monetary models.

In our model, the velocity of money is constant and unitary. Qualitative results can be robust to environments in which the velocity is endogenous. An interesting extension is to relate it to the balance-sheet position of the central bank. The possibility that a currency can be substituted with other means of payments, when the balance sheet deteriorates, can impair the long-run profitability of the central bank and leave the currency unbacked if there is no fiscal support.[48]

Finally, our analysis has emphasized the importance of the interaction between monetary and fiscal policy. In practice, it is hard to exactly tell when a regime of full treasury's support is in place or even when fiscal policy is passive. It should be interesting to consider the implications of uncertainty on the monetary and fiscal regimes, like in Leeper (2013), or the political dimension of the strategic interaction between treasury and central bank.[49]

[47] Details of the latter model are available upon request.

[48] Quinn and Roberds (2014) discuss the disappearance of the "bank florin" as an international reserve currency in the late 1700s as a consequence of the central bank's income losses on non-performing loans. In Del Negro and Sims (2015), a specific transaction cost of holding money balances delivers a money demand function elastic with respect to the nominal interest rate. In their analysis real money balances can become zero for an interest rate above a certain finite threshold.

[49] See for an interesting avenue of research Gonzalez-Eiras and Niepelt (2015).

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A Appendix

A.1 Proofs

We collect in this appendix some derivations and proofs.

A.1.1 Equation (26)

The fiscal rule

$$\frac{T_t^F}{P_t} = \bar{T}^F - \gamma_f \frac{T_t^C}{P_t} + \phi_f \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} \right] \tag{A.1}$$

is in the class of passive fiscal policies if and only if $\gamma_f = 1$ *and* $0 < \phi_f < 2$.

Proof. We find the conditions under which the rule (A.1) satisfies the requirements of passive fiscal policy given by Definition 4. First use the flow budget constraint (21) and write it in real terms:

$$Q_t \frac{D_t^F}{P_t} + \frac{1}{1+i_t} \frac{B_t^F}{P_t} = \frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} - \frac{T_t^F}{P_t} - \frac{T_t^C}{P_t}. \tag{A.2}$$

Using (A.1) into (A.2), we obtain

$$Q_t \frac{D_t^F}{P_t} + \frac{1}{1+i_t} \frac{B_t^F}{P_t} = (1-\phi_f) \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} \right] - \bar{T}^F - (1-\gamma_f) \frac{T_t^C}{P_t}. \tag{A.3}$$

Therefore we can write

$$\begin{aligned} E_{T-1} \left\{ R_{T-1,T} \left(Q_T \frac{D_T^F}{P_T} + \frac{1}{1+i_T} \frac{B_T^F}{P_T} \right) \right\} = \\ (1-\phi_f) E_{T-1} \left\{ R_{T-1,T} \frac{(1+r_T)Q_{T-1}D_{T-1}^F + B_{T-1}^F}{P_T} \right\} + \\ - E_{T-1} \left\{ R_{T-1,T} \bar{T}^F \right\} - (1-\gamma_f) E_{T-1} \left\{ R_{T-1,T} \frac{T_T^C}{P_T} \right\}. \end{aligned} \tag{A.4}$$

Note first that the equilibrium conditions (8) and (11) imply

$$E_{T-1} \left\{ R_{T-1,T} \frac{(1+r_T)}{P_T} \right\} = \frac{1}{P_{T-1}}$$

and

$$E_{T-1} \left\{ R_{T-1,T} \frac{1}{P_T} \right\} = \frac{1}{(1+i_{T-1})P_{T-1}},$$

since $R_{T-1,T}/\Pi_T = R^n_{T-1,T}$. We can therefore write (A.4) as

$$E_{T-1}\left\{R_{T-1,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\}=$$
$$(1-\phi_f)\left(Q_{T-1}\frac{D^F_{T-1}}{P_{T-1}}+\frac{1}{1+i_{T-1}}\frac{B^F_{T-1}}{P_{T-1}}\right)$$
$$-E_{T-1}\left\{R_{T-1,T}\bar{T}^F\right\}-(1-\gamma_f)E_{T-1}\left\{R_{T-1,T}\frac{T^C_T}{P_T}\right\}. \quad \text{(A.5)}$$

Pre-multiplying equation (A.5) by $R_{T-2,T-1}$ and taking the expectation at time $T-2$, we can write (A.5) as

$$E_{T-2}\left\{R_{T-2,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\}=$$
$$(1-\phi_f)E_{T-2}\left\{R_{T-2,T-1}\left(Q_{T-1}\frac{D^F_{T-1}}{P_{T-1}}+\frac{1}{1+i_{T-1}}\frac{B^F_{T-1}}{P_{T-1}}\right)\right\}$$
$$-E_{T-2}\left\{R_{T-2,T}\right\}\bar{T}^F-(1-\gamma_f)E_{T-2}\left\{R_{T-2,T}\frac{T^C_T}{P_T}\right\}$$

in which we can substitute on the right hand side equation (A.5) lagged one period to get

$$E_{T-2}\left\{R_{T-2,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\}=(1-\phi_f)^2\left(Q_{T-2}\frac{D^F_{T-2}}{P_{T-2}}+\frac{1}{1+i_{T-2}}\frac{B^F_{T-2}}{P_{T-2}}\right)$$
$$-(1-\phi_f)(1-\gamma_f)E_{T-2}\left\{R_{T-2,T-1}\frac{T^C_{T-1}}{P_{T-1}}\right\}-(1-\gamma_f)E_{T-2}\left\{R_{T-2,T}\frac{T^C_T}{P_T}\right\}$$
$$-(1-\phi_f)E_{T-2}\left\{R_{T-2,T-1}\right\}\bar{T}^F-E_{T-2}\left\{R_{T-2,T}\right\}\bar{T}^F.$$

After reiterating the substitution back to time t, we get

$$E_t\left\{R_{t,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\}=(1-\phi_f)^{T-t}\left(Q_t\frac{D^F_t}{P_t}+\frac{1}{1+i_t}\frac{B^F_t}{P_t}\right)$$
$$-\bar{T}^F E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}-(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T^C_j}{P_j}\right\}. \quad \text{(A.6)}$$

We now study under which conditions (A.6) converges to zero in the limit $T\to\infty$. Consider the first-term on the right-hand side. This clearly converges to zero if and only if $0<\phi_f<2$. Now consider the term

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}, \quad \text{(A.7)}$$

which can be written as

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}=(1-\phi_f)^{T-t}E_t\left\{\sum_{j=t+1}^{T}\left(\frac{\beta}{1-\phi_f}\right)^{j-t}\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\right\}$$

and which converges to zero as $T\to\infty$, provided $|\beta/(1-\phi_f)|\leq 1$, if and only if $0<\phi_f<2$, considering that the stochastic processes ξ_t, Y_t are bounded.

Equivalently, the term (A.7) can be written as

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}=\beta^{T-t}E_t\left\{\sum_{j=t+1}^{T}\left(\frac{\beta}{1-\phi_f}\right)^{j-T}\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\right\}$$

which converges to zero as $T\to\infty$, provided $|\beta/(1-\phi_f)|\geq 1$ since $0<\beta<1$, considering that the stochastic processes ξ_t, Y_t are bounded. Therefore $0<\phi_f<2$ is also necessary and sufficient for the second-term on the right-hand side of (A.6) to converge to zero.

Now consider the last term on the right-hand side of (A.6) which can be written as

$$(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T_j^C}{P_j}\right\}=(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}\beta^j\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{T_j^C}{P_j}\right\}$$

Considering bounded processes for T_t^C, the above term should also converge to zero as $T\to\infty$ for any stochastic processes $\{P_t,i_t,Q_t,M_t\}$ satisfying the conditions of part i) of Definition 2 consistently each with a specified conventional monetary policy.

The sufficiency of $\gamma_f=1$ to grant this convergence is self evident.

To prove also necessity, note that if $i_j=0$ for each $j\geq t$, equation (16) implies $E_t\beta^j\xi_j Y_j^{-\rho}\xi_t^{-1}Y_t^{\rho}P_j^{-1}=P_t^{-1}$. Let us assume that T_t^C has a deterministic path, then if $i_j=0$ for each $j\geq t$ the above expression can be written as

$$(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T_j^C}{P_j}\right\}=(1-\gamma_f)\frac{1}{P_t}\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}T_j^C\right\}.$$

Note that in this case, even if the deterministic process for $\{T_t^C\}$ implies that T_T^C converges to zero as $T\to\infty$, the sum would in general converge (if at all) to a finite number, not necessarily zero. This proves necessity of $\gamma_f=1$.

Therefore $\gamma_f=1$ and $0<\phi_f<2$ are necessary and sufficient conditions for (A.1) to be in the class of passive fiscal policies. ■

A.1.2 Equation (29)

The remittance policy

$$\frac{T_t^C}{P_t} = \bar{T}^C + \gamma_c \frac{\Psi_t^C}{P_t} + \phi_c \frac{N_{t-1}^C}{P_t} \tag{A.8}$$

is in the class of passive remittance policies if and only if $0 < \gamma_c < 2$ *and* $0 < \phi_c < 2$.

Proof. Recall the law of motion of net worth (13) and substitute in the remittances policy. It implies

$$\frac{N_T^C}{P_T} = (1-\phi_c)\frac{N_{T-1}^C}{P_T} + (1-\gamma_c)\frac{\Psi_T^C}{P_T} - \bar{T}^C.$$

Using the definition of profits (15) we can obtain

$$\begin{aligned}\frac{N_T^C}{P_T} &= (1-\phi_c+(1-\gamma_c)i_{T-1})\frac{N_{T-1}^C}{P_T} + (1-\gamma_c)i_{T-1}\frac{M_{T-1}}{P_T} + \\ &+(1-\gamma_c)(r_T - i_{T-1})\frac{Q_{T-1}D_{T-1}^C}{P_T} - \bar{T}^C.\end{aligned}$$

By substituting on the left hand side of the above equation the law of motion for N_{T-1}^C we get

$$\begin{aligned}\frac{N_T^C}{P_T} &= \prod_{j=T-2}^{T-1}(1-\phi_c+(1-\gamma_c)i_j)\frac{N_{T-2}^C}{P_T} + (1-\gamma_c)i_{T-1}\frac{M_{T-1}}{P_T} + \\ &(1-\phi_c+(1-\gamma_c)i_{T-1})(1-\gamma_c)i_{T-2}\frac{M_{T-2}}{P_T} + (1-\gamma_c)(r_T - i_{T-1})\frac{Q_{T-1}D_{T-1}^C}{P_T} \\ &(1-\phi_c+(1-\gamma_c)i_{T-1})(1-\gamma_c)(r_{T-1} - i_{T-2})\frac{Q_{T-2}D_{T-2}^C}{P_T} - \bar{T}^C \\ &-(1-\phi_c+(1-\gamma_c)i_{T-1})\frac{P_{T-1}}{P_T}\bar{T}^C.\end{aligned}$$

Repeating the substitution for N_{T-2}^C and then recursively back to time t we get

$$\begin{aligned}\frac{N_T^C}{P_T} &= \left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T} + (1-\gamma_c)\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T} \\ &+(1-\gamma_c)\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T} - \bar{T}^C\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\end{aligned}$$

in which we have defined

$$\Psi_t \equiv (1-\phi_c+(1-\gamma_c)i_t).$$

We can now pre-multiply the above equation by $R_{t,T}$ and take the expectation at time t to get

$$
\begin{aligned}
E_t\left\{R_{t,T}\frac{N_T^C}{P_T}\right\} &= E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}+(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T}\right\} \\
&+(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\} \\
&-\bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}. \qquad \text{(A.9)}
\end{aligned}
$$

Now consider the first term on the right-hand side, we need to prove that

$$
\lim_{T\to\infty} E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}=0 \qquad \text{(A.10)}
$$

for any vector of stochastic processes $\{\mathbf{Z}_t\}$ satisfying the conditions of part i) of Definition 2 consistently each with a specified conventional monetary policy. We show that a necessary and sufficient condition is that there exists a $\varepsilon>0$ and a corresponding time T_1 such that $|\Psi_t|\le(1-\varepsilon)(1+i_t)$ for each $t\ge T_1$. To see that it is a sufficient condition note that in this case

$$
\begin{aligned}
&E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\} \\
&\qquad\le E_t\left\{R_{t,T}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\left(\prod_{j=T_1}^{T-1}(1+i_j)\right)(1-\varepsilon)^{T-T_1}\frac{N_t^C}{P_T}\right\} \\
&\qquad\qquad= (1-\varepsilon)^{T-T_1}E_t\left\{R_{t,T_1}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\frac{N_t^C}{P_{T_1}}\right\}
\end{aligned}
$$

and

$$
E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}\ge-(1-\varepsilon)^{T-T_1}E_t\left\{R_{t,T_1}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\frac{N_t^C}{P_{T_1}}\right\}.
$$

Therefore the first term on the right-hand side converges to zero as $T\to\infty$ since the upper and lower bounds converge to zero. To prove the necessity of the condition, consider now the allocation $P_T=R_{t,T}P_t$, $i_T=0$ for each $t\ge T$ with M_t that satisfies $M_t\ge P_tY_t$ and $Q_t=E_t\left\{\sum_{j=0}^{\infty}\delta^j(1-\varkappa_{t+j})\right\}$. This allocation satisfies the conditions of part i) of Definition 2. Therefore if (A.10) holds, evaluated at the above allocation, it should be also the case that

$$\lim_{T\to\infty} E_t \left\{ \prod_{j=t}^{T-1} \Psi_j \right\} = 0$$

which requires that $|\Psi_t| < 1$ infinitely many times. Therefore the condition "there exists a $\varepsilon > 0$ and a corresponding time T_1 such that $|\Psi_t| \leq (1-\varepsilon)(1+i_t)$ for each $t \geq T_1$" is a necessary condition since it implies $|\Psi_t| < 1$ infinitely many times. Note indeed that in the allocation used $i_t = 0$ for each $t \geq T$.

We now evaluate under which restriction on the parameters ϕ_c and γ_c the inequality is satisfied

$$|\Psi_t| = |1 - \phi_c + (1-\gamma_c)i_t| \leq (1-\varepsilon)(1+i_t)$$

for any sequence of stochastic processes $\{\mathbf{Z}_t^*\}$ satisfying the conditions of part $i)$ of Definition 2 consistently each with a specified conventional monetary policy. Note that when $i_t = 0$, the above condition is satisfied when $\varepsilon \leq \phi_c \leq 2 - \varepsilon$ while when i_t is unboundedly large, the above condition is satisfied when $\varepsilon \leq \gamma_c \leq 2 - \varepsilon$. These are both necessary conditions. Note that given positive ϕ_c and γ_c, ε can be chosen positive, small enough and such that $\varepsilon < \min(\phi_c, \gamma_c)$. Therefore the necessary conditions are that $0 < \phi_c < 2$ and $0 < \gamma_c < 2$. These are also sufficient conditions given an $\varepsilon < \min(\phi_c, \gamma_c)$. Note also that under the necessary and sufficient conditions $0 < \phi_c < 2$ and $0 < \gamma_c < 2$, the condition "there exists a $\varepsilon > 0$ and a corresponding time T_1 such that $|\Psi_t| \leq (1-\varepsilon)(1+i_t)$ for each $t \geq T_1$" is equivalent to "$|\Psi_t| \leq (1-\varepsilon)(1+i_t)$ at all times and contingencies."

Consider now the second term on the right-hand side of (A.9). We now show that $|\Psi_t| \leq (1-\varepsilon)(1+i_t)$ at all times is a sufficient condition for its convergence to zero. Using $|\Psi_j| \leq (1-\varepsilon)(1+i_j)$ for each j given a positive and small ε, we can write

$$(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T}\right\}$$
$$\leq (1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\}$$

and

$$(1-\gamma_c)E_t\left\{\sum_{j=t}^{T-1}R_{t,j}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_j}\right\}$$
$$\geq -(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\}$$

Consider moreover that

$$
\begin{aligned}
E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\} &= E_t\left\{\sum_{j=t}^{T-1}R_{t,j}(1-\varepsilon)^{T-1-j}\frac{i_j}{1+i_j}\frac{M_j}{P_j}\right\}\\
&= E_t\left\{\sum_{j=t}^{T-1}\beta^j(1-\varepsilon)^{T-1-j}\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{i_j}{1+i_j}Y_j\right\}
\end{aligned}
$$

where in the first line we have used repeatedly (16) while in the second line we have substituted in the expression for $R_{t,j}$ and used (19).

We can now write

$$
E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)\kappa^{T-1-j}i_j\frac{M_j}{P_T}\right\} = (1-\varepsilon)^{T-1}E_t\left\{\sum_{j=t}^{T-1}\left(\frac{\beta}{1-\varepsilon}\right)^j\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{i_j}{1+i_j}Y_j\right\}
$$

whenever $|\beta/(1-\varepsilon)|\leq 1$ and

$$
\begin{aligned}
&E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)\kappa^{T-1-j}i_j\frac{M_j}{P_T}\right\}\\
&\qquad = \beta^{T-1}E_t\left\{\sum_{j=1}^{T-t}\left(\frac{\beta}{1-\varepsilon}\right)^{-j}\frac{\xi_{T-j}Y_{T-j}^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{i_{T-j}}{1+i_{T-j}}Y_{T-j}\right\}
\end{aligned}
$$

whenever $|\beta/(1-\varepsilon)|\geq 1$ which both converge to zero as $T\to\infty$ given that $\varepsilon>0$ and $0<\beta<1$ and all stochastic processes within the curly bracket are bounded. Therefore the existence of a positive and small ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the second term on the right-hand side of (A.9) to converge to zero.

Consider now the third term on the right-hand side of (A.9) under the condition that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times for a positive ε

$$
\begin{aligned}
&(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}\\
&\qquad\leq(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\},
\end{aligned}
$$

$$
(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}
$$
$$
\geq -(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}.
$$

Now the terms on the right-hand side of the above inequalities are zero after noting that (16) and (17) imply

$$
E_t\left\{R_{t,t+1}\frac{(r_{t+1}-i_t)}{\Pi_{t+1}}\right\}=0.
$$

Therefore the existence of a positive ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the third term on the right-hand side of (A.9) to converge to zero.

Finaly, consider the fourth term on the right-hand side of (A.9) under the condition that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times for a positive ε

$$
\bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}\leq \bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\},
$$

$$
\bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}\geq -\bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\}.
$$

Note that

$$
\begin{aligned}
E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\} &= E_t\left\{\sum_{j=t+1}^{T}R_{t,j}(1-\varepsilon)^{T-1-j}\right\}\\
&= E_t\left\{\sum_{j=t+1}^{T}\beta^j(1-\varepsilon)^{T-1-j}\frac{\xi_jY_j^{-\rho}}{\xi_tY_t^{-\rho}}\right\}
\end{aligned}
$$

which converges to zero following previous reasonings. Therefore the existence of a positive ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the fourth term on the right-hand side of (A.9) to converge to zero.

Therefore $0<\phi_c<2$ and $0<\gamma_c<2$ are necessary and sufficient conditions for (A.9) to converge to zero. ■

A.1.3 Proof of Proposition 1

Under a combined regime of passive fiscal policy and passive policy of central-bank remittances the Neutrality Property holds.

Proof. Consider the set $\mathcal{P}$ of rational expectations equilibria characterized by a *transfer* *policy* $\mathcal{T}(\cdot)$ set consistently with the respectively-defined passive policies. Consider a rational expectations equilibrium $\left\{\mathbf{Z}_t^*,\ \mathbf{K}_t^*\right\} \in \mathcal{P}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{P}$. Fix the *conventional monetary policy* and the *transfer policy* and consider an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. The existence of an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ will be clear from the proof that follows. Given this monetary/fiscal policy regime, the vector of stochastic processes $\left\{\mathbf{Z}_t^*\right\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy.* Moreover the stochastic process $\left\{M_t^*\right\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$. Under the passive *transfer policy* $\mathcal{T}(\cdot)$ is such that (25) and (28) hold looking forward from each contingency $t \geq t_0$, together with the sequence of equilibrium conditions (21), (22) given the vector of stochastic processes $\left\{\mathbf{Z}_t^*\right\}$ and finite $D_{t-1}^G, B_{t-1}^G, X_{t-1}, B_{t-1}^C, D_{t-1}^C$. Consider passive *transfer policies* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ and $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$. Evaluate them in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ for initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_\mathbf{t}, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Consider the flow budget constraint (22) in the same contingency at time t_0 and use these results to evaluate it and get $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$

$$Q_{t_0}^* \tilde{D}_{t_0}^C + \frac{\tilde{B}_{t_0}^C}{1+i_{t_0}^*} - \tilde{M}_{t_0} - \frac{X_{t_0}}{1+i_t^*} = (1-\varkappa_t)(1+\delta Q_{t_0}^*) D_{t_0-1}^C + B_{t_0-1}^C - X_{t_0-1} - M_{t_0-1} - \tilde{T}_{t_0}^C$$

where in particular $\tilde{M}_{t_0} = M_{t_0}^*$ if $i_{t_0}^* > 0$ and $\tilde{M}_{t_0}$ can take any other value $\tilde{M}_{t_0} \geq P_{t_0}^* Y_{t_0}$ if $i_{t_0}^* = 0$.[50]

Consider now the flow budget constraint (21) and evaluate it

$$Q_{t_0}^* \tilde{D}_{t_0}^F + \frac{\tilde{B}_{t_0}^F}{1+i_{t_0}^*} = (1-\varkappa_t)(1+\delta Q_{t_0}^*) D_{t_0-1}^F + B_{t_0-1}^F - \tilde{T}_{t_0}^F - \tilde{T}_{t_0}^C.$$

The above equation can then be used to determine $\tilde{B}_{t_0}^F$.[51] Repeating the above steps se-

[50]Note that the requirement of the rational expectations equilibrium that $\tilde{X}_{t_0} \geq 0$ is equivalent to $Q_{t_0}^*(\tilde{D}_{t_0}^C - D_{t_0}^{C*}) + (\tilde{B}_{t_0}^C - B_{t_0}^{C*})/(1+i_{t_0}^*) - (\tilde{M}_{t_0} - M_{t_0}^*) + X_{t_0}^*/(1+i_{t_0}^*) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

[51]Note that the requirement of the rational expectations equilibrium that $\tilde{B}_{t_0}^F \geq 0$ is equivalent to $Q_{t_0}^*(\tilde{D}_{t_0}^F - D_{t_0}^{F*}) - B_{t_0}^{F*}/(1+i_{t_0}^*) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

quentially and taking care at each step of the constraints $\tilde{X}_t \geq 0$ and $\tilde{B}_t^F \geq 0$, it is possible to build stochastic processes $\{\tilde{X}_t, \tilde{B}_t, \tilde{B}_t^C, \tilde{B}_t^F, \tilde{D}_t, \tilde{D}_t^C, \tilde{D}_t^F, \tilde{T}_t^F, \tilde{T}_t^C\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) and (28), given $\{\mathbf{Z}_t^*\}$ and initial conditions $\mathbf{w}_{t_0-1}$.

Note that the sum of (25) and (28) given (23) and (24) implies (20). It follows that there is a vector of stochastic processes $\{\tilde{\mathbf{K}}_t\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given $\{\mathbf{Z}_t^*\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Therefore $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ is a rational expectations equilibrium which also belongs to $\mathcal{P}$ with $\{M_t^*\}$ that can take any value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. Moreover, in these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if the latter is specified in terms of $\mathcal{M}(\cdot)$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{P}$ The Neutrality Property holds. ■

A.1.4 Full Treasury Support

A regime of full treasury's support, $T_t^C = \Psi_t^C$ at each date t (and in each contingency at t), is not in the class of passive remittance policies.

Proof. Consider the allocation $P_t = \beta^{t-t_0} P$, $i_t = 0$, $Q_t = 1/(1-\delta)$, and $M_t \geq \beta^t PY$. It is easy to verify that this allocation satisfies the equilibrium conditions (16) to (19) as required by Definition 5 when $\xi_t = \xi$, $Y_t = Y$ and $\varkappa_t = 0$, given some non-negative P and considering the *conventional monetary policy* $i_t = 0$. Moreover, a regime of *full treasury's support* implies that net worth is constant at $N_t^C = N_{t_0-1}^C = \bar{N} > 0$ as shown by the law of motion (13). However, (28) is not satisfied because $\lim_{T\longrightarrow\infty} E_t\left[R_{t,T} N_T^C / P_T\right] = \bar{N}/P_t$, which is not necessarily zero, unless $\bar{N} = 0$.[52] Therefore a regime of full treasury's support is not in the class of passive remittance policies. ■

A.1.5 Proof of Proposition 2

Under a passive fiscal policy and full treasury's support the Neutrality Property holds.

Proof. Consider the set $\mathcal{F}$ of rational expectations equilibria with a *transfer policy* $\mathcal{T}(\cdot)$ set consistently with a *passive fiscal policy* and *full treasury's support.* Consider a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{F}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{F}$.

[52] Recall that $R_{t,T} \equiv \beta^{T-t} \xi_T Y_T^{-\rho} / \xi_t Y_t^{-\rho}$ which is equal to β^{T-t} in this case.

Note that for an equilibrium to be in this set it is necessarily the case that

$$\lim_{T\longrightarrow\infty} E_t\left[\beta^{T-t}\frac{\xi_T Y_T^{-\rho}P_t^*}{\xi_t Y_t^{-\rho}P_T^*}\right]=0. \tag{A.11}$$

Indeed, given a passive fiscal policy (28) should necessarily hold while full treasury's support implies $N_t = N_{t_0-1} = \bar{N} > 0$. Therefore (A.11) necessarily holds. Fix the *conventional monetary policy* and the *transfer policy* and consider an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. The existence of an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ will be clear from the proof that follows. Given this monetary/fiscal policy regime, the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy.* Moreover the stochastic process $\{M_t^*\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^*=0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$.

Consider passive *transfer policies* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$. Evaluate it in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ given initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Full treasury's support implies that at each point in time and contingency $T_t^C = \Psi_t^C$ and $N_t^C = N_{t_0-1}^C = \bar{N} > 0$. Starting from a generic contingency at time t_0, it is possible to determine $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ as

$$\tilde{T}_{t_0}^C = i_{t_0-1}(N_{t_0-1}^C + M_{t_0-1}) + (r_{t_0}^* - i_{t_0-1})Q_{t_0-1}D_{t_0-1}^C.$$

Moreover $N_{t_0} = \bar{N}$ implies that we can determine $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ from

$$Q_{t_0}^*\tilde{D}_{t_0}^C + \frac{\tilde{B}_{t_0}^C}{1+i_{t_0}^*} - \tilde{M}_{t_0} - \frac{\tilde{X}_{t_0}}{1+i_{t_0}^*} = \bar{N},$$

where in particular $\tilde{M}_{t_0} = M_{t_0}^*$ if $i_{t_0}^* > 0$ or $\tilde{M}_{t_0}$ can take any other value $\tilde{M}_{t_0} \geq P_{t_0}^* Y_{t_0}$ if $i_{t_0}^* = 0$.[53] Consider now the flow budget constraint (21) and evaluate it

$$Q_{t_0}^*\tilde{D}_{t_0}^F + \frac{\tilde{B}_{t_0}^F}{1+i_{t_0}^*} = (1-\varkappa_t)(1+\delta Q_{t_0}^*)D_{t_0-1}^F + B_{t_0-1}^F - \tilde{T}_{t_0}^F - \tilde{T}_{t_0}^C.$$

[53]Note that the requirement of the rational expectations equilibrium that $\tilde{X}_{t_0} \geq 0$ is equivalent to $Q_{t_0}^*\tilde{D}_{t_0}^C + \tilde{B}_{t_0}^C/(1+i_{t_0}^*) - \tilde{M}_{t_0} - \bar{N} \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

The above equation can then be used to determine $\tilde{B}^F_{t_0}$.[54] Repeating the above steps sequentially and taking care at each step of the constraints $\tilde{X}_t \geq 0$ and $\tilde{B}^F_t \geq 0$, it is possible to build stochastic processes $\{\tilde{X}_t, \tilde{B}_t, \tilde{B}^C_t, \tilde{B}^F_t, \tilde{D}_t, \tilde{D}^C_t, \tilde{D}^F_t, \tilde{T}^F_t, \tilde{T}^C_t\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) and (28), given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Note that (28) is satisfied because it is the case that $\tilde{N}^C_t = \bar{N}$ at each t and contingency while (A.11) holds given $\{\mathbf{Z}^*_t\}$. Finally, note that the sum of (25) and (28) given (23) and (24) implies (20). It follows that there is a vector of stochastic processes $\{\tilde{\mathbf{K}}_t\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Therefore $\{\mathbf{Z}^*_t, \tilde{\mathbf{K}}_t\}$ is a rational expectations equilibrium which also belongs to $\mathcal{F}$ with $\{M^*_t\}$ that can take any value $\tilde{M}_t \geq P^*_t Y_t$ in each contingency in which $i^*_t = 0$. Moreover, in these contingencies (and only in these contingencies) $\tilde{M}_t \geq P^*_t Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{F}$. The Neutrality Property holds. ■

A.1.6 Proof of Proposition 5

Under a passive fiscal policy and a deferred-asset policy of central bank's remittances the Neutrality Property holds if and only if

$$\frac{\tilde{N}^C_t}{P^*_t} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i^*_T}{1+i^*_T} \frac{M^*_T}{P^*_T} \right) \tag{A.12}$$

in equilibrium at each time t (and in each contingency at t).

Proof. Consider the set $\mathcal{G}$ of rational expectations equilibria with a *transfer policy* $\mathcal{T}(\cdot)$ set consistently with a passive fiscal policy and a deferred-asset policy of central bank's remittances.

We first prove the necessary condition. Assume that the Neutrality Property holds. Consider a rational expectations equilibrium $\{\mathbf{Z}^*_t, \mathbf{K}^*_t\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$. Consider the equilibrium $\{\mathbf{Z}^*_t, \tilde{\mathbf{K}}_t\}$ for which the Neutrality Property holds given an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ with respect to $\{\mathbf{Z}^*_t, \mathbf{K}^*_t\}$. Assume that

[54] Note that the requirement of the rational expectations equilibrium that $\tilde{B}^F_{t_0} \geq 0$ is equivalent to $Q^*_{t_0}(\tilde{D}^F_{t_0} - D^{F*}_{t_0}) - B^{F*}_{t_0}/(1+i^*_{t_0}) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

there is a contingency in which

$$\frac{\tilde{N}_t^C}{P_t^*} \leq -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{A.13}$$

at a generic time t. First note that given a passive fiscal policy, (28) holds looking forward from each t and in each contingency at the equilibrium stochastic processes $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$. Therefore the intertemporal budget constraint should hold in equilibrium

$$\frac{\tilde{N}_t^C}{P_t^*} + E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) = E_t \sum_{T=t+1}^{\infty} R_{t,T} \left(\frac{\tilde{T}_T^C}{P_T^*} \right) \tag{A.14}$$

in each contingency and in particular in the same contingency at time t in which (A.13) holds. In particular, we choose the contingency at time t in a way that the history up to time t shows at least one contingency at a generic time j, with $t_0 \leq j < t$, in which $r_j^* - i_{j-1}^* < 0$. Comparison of (A.13) and (A.14) taking into account the non-negativeness of $\tilde{T}_t^C$ under the deferred-asset regime shows that (A.13) can only hold with equality in equilibrium. Note moreover that the law of motion of net worth implies that

$$\tilde{N}_t^C = \tilde{N}_{t-1}^C + i_{t-1}^*(\tilde{N}_{t-1}^C + M_{t-1}^*) + (r_t^* - i_{t-1}^*) Q_{t-1}^* D_{t-1}^{C*} - \tilde{T}_t^C,$$

which can be solved backward given initial conditions. Use now the fact that Neutrality Property holds. Start from the equilibrium $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ for a balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$, given $\mathcal{T}(\cdot)$ and $\mathcal{I}(\cdot)$ (or $\mathcal{M}(\cdot)$). Consider another balance-sheet policy $\hat{\mathcal{B}}(\cdot)$ which just changes the long-term asset holdings to $\left\{\hat{D}_i^C\right\}_{i=t_0}^{t-1}$ with $\hat{D}_i^C \geq 0$, in a way that the implied $\hat{N}_t^C$ given the same history $\mathbf{s}^t$ and same realized values of the stochastic processes $\mathbf{Z}^*$ up to time t is such that $\hat{N}_t^C < \tilde{N}_t^C$.[55] However, under this alternative balance-sheet policy $\hat{\mathcal{B}}(\cdot)$, $\hat{N}_t^C$ cannot satisfy (A.14), and therefore $\{\mathbf{Z}_t^*, \hat{\mathbf{K}}_t\}$ is not an equilibrium contradicting the fact that the Neutrality Property holds for appropriately-bounded *balance-sheet policies*. We have a contradiction and therefore (A.12) should necessarily hold.

To prove the sufficient condition, consider a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy*

[55]Since at time j $r_j^* - i_{j-1}^* < 0$ choose $\hat{D}_{j-1}^C$ so that central-bank profits at time j are lower than what implied by $\tilde{D}_{j-1}^C$ and moreover negative while $\hat{D}_i^C = \tilde{D}_i^C$ for all other i with $t_0 \leq i \leq t-1$. For profits to be negative at time j, $\hat{D}_{j-1}^C$ should be chosen sufficiently high. Under these assumptions it is also the case that $\hat{T}_j^C \leq \tilde{T}^C$. Moreover note that under this construction $\hat{X}_i \geq \tilde{X}_i$ for $t_0 \leq i \leq t-1$.

$\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$. Assume that

$$\frac{N_t^{*C}}{P_t^*} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{A.15}$$

holds in the equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ at each point in time and contingency. Therefore N_t^{C*}/P_t^* is bounded below since the right-hand side of (A.15) is bounded given (5). Moreover under a *deferred-asset regime* N_t^{C*} is bounded above by $\bar{N}$. Finally, note that under a passive fiscal policy

$$\lim_{T \longrightarrow \infty} E_t \left[R_{t,T} \frac{P_t^*}{P_T^*} N_T^{C*} \right] = 0 \tag{A.16}$$

holds in equilibrium.

Consider now an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. We need to prove that under the condition (A.12), there is a rational expectation equilibrium associated with the same *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$, *transfer policy* $\mathcal{T}(\cdot)$, and alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. Given that the *conventional monetary policy* has not changed, the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy*. Moreover the stochastic process $\{M_t^*\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specifed as $\mathcal{M}(\cdot)$.

Consider passive *transfer policy* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$. Evaluate it in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ considering initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Under a deferred-asset regime $T_{t_0}^C = \Psi_{t_0}^C$ if $\Psi_{t_0}^C \geq 0$ otherwise $T_{t_0}^C = 0$ and therefore it is possible to write $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ where $\tilde{T}^C(\cdot)$ is a non-negative function.

Moreover equation (22) shows that whenever $N_{t-1}^C = \bar{N}$ and $\Psi_t^C \geq 0$, in which case $T_t^C = \Psi_t^C$, reserves X_t are going to be determined by

$$\frac{X_t - B_t^C}{1+i_t} = \frac{(X_{t-1} - B_{t-1}^C)}{1+i_{t-1}} + (Q_t D_t^C - Q_{t-1} D_{t-1}^C) - (M_t - M_{t-1}),$$

while whenever $N_{t-1}^C < \bar{N}$, in which case $T_t^C = 0$, reserves X_t are going to be determined by

$$\frac{X_t - B_t^C}{(1+i_t)} = (X_{t-1} - B_{t-1}^C) + Q_t D_t^C - (1+r_t) Q_{t-1} D_{t-1}^C - (M_t - M_{t-1}).$$

It is then possible to apply one of the above two conditions in the contingency at time t_0, depending on whether $\Psi^C_{t_0} \geq 0$ or $\Psi^C_{t_0} < 0$ and evaluate it using $\tilde{D}^C_{t_0}$, $\tilde{B}^C_{t_0}$ and $Q^*_{t_0}, i^*_{t_0}, r^*_{t_0}$ while $\tilde{M}_{t_0}$ is such that $\tilde{M}_{t_0} = M^*_{t_0}$ if $i^*_{t_0} > 0$ or can take any other value $\tilde{M}_{t_0} \geq P^*_{t_0} Y_{t_0}$ if $i^*_{t_0} = 0$. The above equations (one of the two depending on the case) can be used to obtain $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$.[56]

Consider now the flow budget constraint (21) and evaluate it in the same contingency at time t_0 using previous results to obtain $\tilde{B}^F_{t_0}$. Repeating the above steps sequentially, it is possible to build stochastic processes $\left\{\tilde{X}_t, \tilde{B}_t, \tilde{B}^C_t, \tilde{B}^F_t, \tilde{D}_t, \tilde{D}^C_t, \tilde{D}^G_t, \tilde{T}^F_t, \tilde{T}^C_t\right\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) given initial conditions $\mathbf{w}_{t_0-1}$ and $\left\{\mathbf{Z}^*_t\right\}$. It is also required that the implied $\tilde{N}^C_t$ satisfies

$$\frac{\tilde{N}^C_t}{P^*_t} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i^*_T}{1+i^*_T} \frac{M^*_T}{P^*_T} \right)$$

at each point in time and contingency. Since under the deferred-asset regime $\tilde{N}^C_t \leq \bar{N}$, it follows that $\tilde{N}^C_t$ satisfies exactly the same upper and lower bounds of N^{C*}_t and should therefore also satisfy (A.16). We have therefore proved that if (A.12) holds at each point in time and contingency, there is a vector of stochastic processes $\left\{\tilde{\mathbf{K}}_t\right\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given initial conditions $\mathbf{w}_{t_0-1}$ and $\left\{\mathbf{Z}^*_t\right\}$. Therefore $\left\{\mathbf{Z}^*_t, \tilde{\mathbf{K}}_t\right\}$ is a rational expectations equilibrium which also belongs to $\mathcal{G}$ with $\left\{M^*_t\right\}$ that can take any value $\tilde{M}_t \geq P^*_t Y_t$ in each contingency in which $i^*_t = 0$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{G}$. The Neutrality Property holds. ■

Furthermore, we can show two special cases in which the necessary and sufficient condition for neutrality is precisely that net worth returns back to $\bar{N}$ in a finite period of time. In the first, we consider that the exogenous stochastic processes have an absorbing state after some finite period of time. In the second, we allow for only temporary central bank's purchases of long-term securities

Proposition 6 *Consider either i) the case in which all the exogenous stochastic disturbances have an absorbing state starting from time τ or ii) the case in which $D^C_t = 0$ for each $t \geq \tau$. Under a passive fiscal policy and a deferred-asset policy of central bank's remittances the Neutrality Property holds if and only if $\tilde{N}^C_t = \bar{N}$ in equilibrium for each $t \geq \tau_1$ with $\tau_1 \geq \tau$.*

[56] The requirement that $\tilde{X}_t \geq 0$ for each t and in each contingency imposes appropriate bounds on the *balance-sheet policies* that can be considered.

Proof. Consider first case i). Note that for each $t \geq \tau$ the law of motion of net worth is given by

$$N_t^C = N_{t-1}^C + i_{t-1}(N_{t-1}^C + M_{t-1}) \tag{A.17}$$

since $r_t = i_{t-1}$ for each $t \geq \tau$ as it is implied by conditions (16) and (17) in a deterministic model. Consider case ii), the law of motion of net worth is again given by (A.17) because now $D_t^C = 0$ for each $t \geq \tau$. The following reasonings apply to both cases. We need to show that the condition $\tilde{N}_t^C = \bar{N}$ in equilibrium for each $t \geq \tau_1$ with $\tau_1 \geq \tau$ is equivalent to the necessary and sufficient condition of Proposition 5. Note first that if net worth $\tilde{N}_t^C$ reaches $\bar{N} > 0$ after period τ then $\tilde{N}_t^C$ is going to stay at $\bar{N}$ thereafter, given the deferred-asset regime. Indeed in both cases, profits are never negative and always positive if $i_t > 0$ given that $\Psi_t^C = i_{t-1}(\bar{N} + M_{t-1})$.

Given these observations consider first one of the rational expectations equilibria $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ identified by Proposition 5 for which the Neutrality Property holds and one of the corresponding $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$. Assume by contradiction that $\tilde{N}_t^C < \bar{N}$ for each $t \geq \tau_1$.[57] If $\tilde{N}_t^C < \bar{N}$ for each $t \geq \tau_1$ remittances to the treasury are always zero for each $t \geq \tau_1$ because of the deferred-asset regime. The equilibrium condition (39) implies

$$\frac{\tilde{N}_t^C}{P_t^*} = -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1 + i_T^*} \frac{M_T^*}{P_T^*} \right)$$

at each time for each $t \geq \tau_1$ (and contingency only for case ii). Therefore the necessary condition (A.12) of Proposition 5 is violated at each point in time $t \geq \tau_t$. It should be that $\tilde{N}_t = \bar{N}$ in equilibrium $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ for each $t \geq \tau_1$ with $\tau_1 \geq \tau$.

Consider now a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$ of Proposition 5. Consider now an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ and assume that given this *balance-sheet policy* $\tilde{N}_t^C = \bar{N}$ in equilibrium $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ for each $t \geq \tau_1$ with $\tau_1 \geq \tau$. Note that since fiscal policy is passive a *conventional monetary policy* that sets $i_t = 0$ infinitely many times after period τ is not an equilibrium under cases i) and ii) for the same reasons discussed in Section A.1.4. This implies that when $\tilde{N}_t^C = \bar{N}$ for each $t \geq \tau_1$ profits are strictly positive. Using this result into (39), it implies that the sufficient condition (A.12) is satisfied at each point in time and contingency. ■

[57] As discussed above it is not possible that $\tilde{N}_t$ reaches $\bar{N}$ at some point after period τ and then falls below. The only case that contradicts the Proposition is assuming that $\tilde{N}_t < \bar{N}$ at all times after period τ.

A.2 General Model

In this section, we describe the additional features of the general model used for the numerical simulations.

We assume that preferences are of the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{(L_t(j))^{1+\eta}}{1+\eta} dj \right] \tag{A.18}$$

where C is a consumption bundle of the form

$$C \equiv \left[\int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}};$$

$C(j)$ is the consumption of a generic good j produced in the economy and θ, with $\theta > 1$, is the intratemporal elasticity of substitution between goods; $L(j)$ is hours worked of variety j which is only used by firm j to produce good j while η is the inverse of the Frisch elasticity of labor supply, with $\eta > 0$. Each household supplies all the varieties of labor used in the production. The asset markets now change to

$$M_t + \frac{B_t + X_t}{1+i_t} + Q_t D_t \leq B_{t-1} + X_{t-1} + (1-\varkappa_t)(1+\delta Q_t) D_{t-1} + \int_0^1 W_{t-1}(j) L_{t-1}(j) dj - \tilde{T}_t^F + \Phi_{t-1} + (M_{t-1} - P_{t-1} C_{t-1}). \tag{A.19}$$

In the budget constraint (A.19), $W(j)$ denotes wage specific to labor of quality j. Wage income for each variety of labor j, $W_{t-1}(j) L_{t-1}(j)$, and firms' profits, Φ_{t-1}, of period $t-1$ are deposited in the financial account at the beginning of period t; $\tilde{T}_t^F$ are lump-sum taxes levied by the treasury.

Given that in this general model labor supply is endogenous first-order conditions of the household's problem imply that the marginal rate of substitution between labor and consumption, for each variety j, is given by

$$\frac{(L_t(j))^{\eta}}{C_t^{-\rho}} = \frac{1}{1+i_t} \frac{W_t(j)}{P_t}, \tag{A.20}$$

which is shifted by movements in the nominal interest rate, reflecting the financial friction. Wage income, indeed, can be used to purchase goods only with one-period delay.

We now turn to the supply of goods. We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor $Y(j) = A_t L(j)$, in which A is a stochastic productivity disturbance which is assumed to follow a Markov process, with transition density $\pi_a(A_{t+1}|A_t)$ and initial distribution f_a. We assume that (π_a, f_a) is such that $A \in [A_{\min}, A_{\max}]$. Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta} Y$ where in equilibrium aggregate output is equal to consumption

$$Y_t = C_t. \tag{A.21}$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1-\alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price. The remaining fraction α of firms indexes their previously-adjusted prices to the inflation target $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place until period T with probability α^{T-t}:

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\bar{\Pi}^{T-t} P_t(j) Y_T(j) - (1-\varrho_T) \frac{W_T(j)}{A_T} Y_T(j) \right],$$

where ϱ_t is a subsidy on firms' labor costs. We assume that ϱ_t is a stochastic disturbance which is assumed to follow a Markov process, with transition density $\pi_\varrho(\varrho_{t+1}|\varrho_t)$ and initial distribution f_ϱ. We assume that (π_ϱ, f_ϱ) is such that $\varrho \in [\varrho_{\min}, \varrho_{\max}]$. The optimality condition implies

$$\frac{P_t^*(j)}{P_t} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \mu_T \frac{W_T(j)}{A_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T P_t \bar{\Pi}^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} Y_T \right\}} \tag{A.22}$$

in which we have used the demand function $Y(i) = (P(i)/P)^{-\theta} Y$ and have defined $\mu_t \equiv \theta(1-\varrho_t)/(\theta-1)$.[58] We can also replace in the previous equation $\lambda_t = C_t^{-\rho} \xi_t / P_t$ and $W_t(j)/P_t$

[58] An interesting result is that the efficient steady state of the model can be implemented by setting the steady-state employment subsidy to $\varrho \equiv 1 - (1-1/\theta)/(1+\bar{\imath})$ where $\bar{\imath}$ is the steady-state level of the nominal interest rate. One needs to use only one instrument of policy to offset both the monopolistic distortion and the financial friction, since both create an inefficient wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor. Moreover, given this result, the steady-state level of the nominal interest rate can be different from zero, while the inflation rate can be set at the target $\bar{\Pi}$. The steady-state version of equation (8) relates the nominal interest rate to the inflation rate $\beta(1+\bar{\imath}) = \bar{\Pi}$. This result crucially depends on the assumption that all consumption requires cash. It would fail in a model with cash and credit goods.

from (A.20) together with the demand function, $Y(i) = (P(i)/P)^{-\theta}Y$, to obtain

$$\left(\frac{P_t^*}{P_t}\right)^{1+\theta\eta} = \frac{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)}(1+i_T)\mu_T\left(\frac{Y_T}{A_T}\right)^{1+\eta}\xi_T\right\}}{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1}Y_T^{1-\rho}\xi_T\right\}}$$

where P_t^* is the common price chosen by the firms that can adjust it at time t.

Calvo's model further implies the following law of motion of the general price index

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\bar{\Pi}^{1-\theta}, \tag{A.23}$$

through which we can write the aggregate supply equation as

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)}(1+i_T)\mu_T\left(\frac{Y_T}{A_T}\right)^{1+\eta}\xi_T\right\}}{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1}Y_T^{1-\rho}\xi_T\right\}}. \tag{A.24}$$

The additional difference with respect to the model of Section 2 is now in the flow budget constraint of the government which is given by

$$Q_tD_t^G + \frac{B_t^G}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^G + B_{t-1}^G - T_t^F - T_t^C$$

where

$$T_t^F \equiv \tilde{T}_t^F - \varrho_t\int_0^1 W_t(j)L_t(j).$$

A.2.1 Equilibrium

Here, we describe in a compact way the equations that characterize the equilibrium allocation in the general model:

$$\frac{1}{1+i_t} = E_t\left\{\beta\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\xi_tY_t^{-\rho}}\frac{1}{\Pi_{t+1}}\right\}, \tag{A.25}$$

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{F_t}{K_t}, \tag{A.26}$$

$$F_t = \mu_t(1+i_t)\xi_t\left(\frac{Y_t}{A_t}\right)^{1+\eta} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}F_{t+1}\right\}, \tag{A.27}$$

$$K_t = \xi_t Y_t^{1-\rho} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta-1}\bar{\Pi}^{1-\theta}K_{t+1}\right\}, \tag{A.28}$$

$$\Delta_t = \Delta\left(\frac{\Pi_t}{\bar{\Pi}}, \Delta_{t-1}\right) \equiv \alpha\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta(1+\eta)}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}}, \tag{A.29}$$

$$Q_t = E_t\left\{\beta\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{(1-\varkappa_{t+1})(1+\delta Q_{t+1})}{\Pi_{t+1}}\right\}, \tag{A.30}$$

$$M_t \geq P_t Y_t, \tag{A.31}$$

$$i_t(M_t - P_t Y_t) = 0, \tag{A.32}$$

$$E_t\left\{\sum_{T=t}^{\infty}\beta^{T+1-t}\xi_{T+1}Y_{T+1}^{-\rho}\left[Y_T + \frac{i_T}{1+i_T}\frac{M_T}{P_T}\right]\right\} < \infty, \tag{A.33}$$

$$\lim_{T\longrightarrow\infty} E_t\left[\beta^{T-t}\frac{\xi_T Y_T^{-\rho}}{P_T}\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right] = 0, \tag{A.34}$$

$$Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^F + B_{t-1}^F - T_t^F - T_t^C, \tag{A.35}$$

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C, \tag{A.36}$$

$$B_t^F = B_t + B_t^C, \tag{A.37}$$

$$D_t^F - D_t = D_t^C. \tag{A.38}$$

A rational expectations equilibrium is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$, satisfying each of the conditions in equations (A.25) to (A.38) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of a monetary/fiscal policy regime and given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions given by the vector $\mathbf{w}_{t_0-1}$ which at least includes $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^F, D_{t_0-1}^C, D_{t_0-1}^F$.

A.2.2 Optimal Policy

Optimal policy maximizes the utility of the consumers, the welfare metric can be written as

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{Y_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\frac{\Delta_t}{A_t^{1+\eta}}\right]. \tag{A.39}$$

We consider the following –partial– specification of the monetary/fiscal policy regime: a *transfer policy* $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$ and $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and a *balance-*

sheet policy $B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ which includes all the cases we are going to consider in our numerical exercises. This specification leaves one degree of freedom along which we choose the optimal policy. The optimal policy is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$, satisfying each of the conditions in equations (A.25) to (A.38) at each time $t \geq t_0$ (and in each contingency at t) consistently with $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$, $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and $B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ that maximizes (A.39) given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$.

To compute the optimal policy, we consider the associated Lagrangian problem maximizing (A.39) and attaching Lagrange multipliers $\lambda_{j,t}$ for $j = 1...15$ to the following constraints (which rewrite those above)

$$\xi_t Y_t^{-\rho} = \beta(1+i_t)E_t\left\{\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\Pi_{t+1}}\right\}$$

$$F_t = \mu_t(1+i_t)\xi_t\left(\frac{Y_t}{A_t}\right)^{1+\eta} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}F_{t+1}\right\}$$

$$K_t = \xi_t Y_t^{1-\rho} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta-1}\bar{\Pi}^{1-\theta}K_{t+1}\right\}$$

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} K_t = F_t$$

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta(1+\eta)}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}}$$

$$i_t \geq 0$$

$$Q_t\xi_t Y_t^{-\rho} = E_t\left\{\beta\xi_{t+1}Y_{t+1}^{-\rho}\frac{(1-\varkappa_{t+1})(1+\delta Q_{t+1})}{\Pi_{t+1}}\right\}$$

$$m_t = Y_t$$

$$r_t Q_{t-1} = (1-\varkappa_t)(1+\delta Q_t) - Q_{t-1}$$

$$n_t^C = Q_t d_t^C - m_t - \tilde{x}_t$$

$$\xi_t Y_t^{-\rho}(t_t^C - \psi_t^C + n_t^C) = \xi_t Y_t^{-\rho} n_{t-1}^C \Pi_t^{-1}$$

$$\psi_t^C \Pi_t = i_{t-1}(n_{t-1}^C + m_{t-1}) + (r_t - i_{t-1})Q_{t-1}d_{t-1}^C$$

$$t_t^C = \bar{T}^C + \gamma_c\psi_t^C + \phi_c n_{t-1}^C \Pi_t^{-1}$$

$$Q_t d_t^F + \frac{1}{1+i_t} b_t^F = (1+r_t) Q_{t-1} d_{t-1}^F \Pi_t^{-1} + b_{t-1}^F \Pi_t^{-1} - t_t^F - t_t^C$$

$$t_t^F = \bar{T}^F - \gamma_f t_t^C + \phi_f \left[(1+r_t) Q_{t-1} d_{t-1}^F \Pi_t^{-1} + b_{t-1}^F \Pi_t^{-1} \right]$$

where lower-case variables denote the real counterpart of the upper-case variable, while $\tilde{x}_t \equiv (X_t - B_t^C)/(P_t(1+i_t))$.

The first-order conditions with respect to the vector $(Y_t, i_t, \Pi_t, K_t, F_t, \Delta_t, m_t, Q_t, r_t, t_t^C, n_t, \tilde{x}, \psi_t^C, t_t^F, b_t^F)$ are respectively:

$$\begin{aligned} 0 &= \xi_t Y_t^{-\rho} - \xi_t Y_t^{\eta} \Delta_t A_t^{-(1+\eta)} - \rho \lambda_{1,t} \xi_t Y_t^{-\rho-1} + \lambda_{1,t-1} \rho (1+i_{t-1}) \xi_t Y_t^{-\rho-1} \Pi_t^{-1} \\ & -\lambda_{2,t}(1+\eta)\mu_t(1+i_t)\xi_t \frac{1}{Y_t} \left(\frac{Y_t}{A_t}\right)^{1+\eta} - \lambda_{3,t}(1-\rho)\xi_t Y_t^{-\rho} - \lambda_{7,t}\rho Q_t \xi_t Y_t^{-\rho-1} \\ & +\lambda_{7,t-1}\rho\xi_t Y_t^{-\rho-1} \frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t} - \lambda_{8,t} \end{aligned}$$

$$0 = \lambda_{1,t}\beta E_t \left\{ \xi_{t+1} Y_{t+1}^{-\rho} \Pi_{t+1}^{-1} \right\} + \lambda_{2,t}\mu_t \xi_t \left(\frac{Y_t}{A_t}\right)^{1+\eta} - \lambda_{6,t} + \beta E_t \left\{ \lambda_{12,t+1} \right\} (n_t + m_t - Q_t d_t^C) + \frac{\lambda_{14,t}}{(1+i_t)^2} b_t^F$$

$$\begin{aligned} 0 &= \lambda_{1,t-1}(1+i_{t-1})\xi_t Y_t^{-\rho}\Pi_t^{-2} + \lambda_{4,t} K_t \alpha \frac{1+\theta\eta}{1-\alpha} \left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{\theta(1+\eta)}{1-\theta}} \Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\ & -\lambda_{2,t-1} F_t \alpha\theta(1+\eta)\Pi_t^{\theta(1+\eta)-1}\bar{\Pi}^{-\theta(1+\eta)} - \lambda_{3,t-1} K_t \alpha(\theta-1)\Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\ & -\lambda_{5,t}\Delta_{t-1}\alpha\theta(1+\eta)\Pi_t^{\theta(1+\eta)-1}\bar{\Pi}^{-\theta(1+\eta)} + \lambda_{5,t}\alpha\theta(1+\eta) \left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{1+\theta\eta}{\theta-1}} \Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\ & +\lambda_{7,t-1}\xi_t Y_t^{-\rho} \frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t^2} + \lambda_{11,t}\xi_t \frac{Y_t^{-\rho}}{\Pi_t^2} n_{t-1}^C + \lambda_{12,t}\psi_t^C + \lambda_{13,t}\phi_c n_{t-1}^C \Pi_t^{-2} \\ & +(\lambda_{14,t} + \phi_f\lambda_{15,t})[(1+r_t)Q_{t-1}d_{t-1}^F + b_{t-1}^F]\Pi_t^{-2} \end{aligned}$$

$$0 = \lambda_{4,t} \left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{1+\theta\eta}{1-\theta}} + \lambda_{3,t} - \lambda_{3,t-1}\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}$$

$$0 = -\lambda_{4,t} + \lambda_{2,t} - \lambda_{2,t-1}\alpha\Pi_t^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}$$

$$0 = -\xi_t \frac{Y_t^{1+\eta}}{1+\eta} A_t^{-(1+\eta)} + \lambda_{5,t} - \alpha\beta E_t \left\{ \lambda_{5,t+1} \Pi_{t+1}^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)} \right\}$$

$$0 = \lambda_{8,t} + \lambda_{10,t} - \beta i_t E_t \lambda_{12,t+1}$$

$$\begin{aligned} 0 &= \lambda_{7,t}\xi_t Y_t^{-\rho} - \lambda_{7,t-1}\delta(1-\varkappa_t)\xi_t Y_t^{-\rho}\Pi_t^{-1} + \beta E_t\left\{\lambda_{9,t+1}(1+r_{t+1})\right\} - (1-\varkappa_t)\delta\lambda_{9,t} \\ &\quad -\beta E_t\left\{(\lambda_{14,t+1}+\phi_f\lambda_{15,t+1})(1+r_{t+1})d_t^F\Pi_{t+1}^{-1}\right\} - \lambda_{10,t}d_t^C - \beta d_t^C E_t\left\{\lambda_{12,t+1}(r_{t+1}-i_t)\right\} \\ &\quad +\lambda_{14,t}d_t^F \end{aligned}$$

$$\lambda_{9,t}Q_{t-1} - \lambda_{12,t}Q_{t-1}d_{t-1}^C - (\lambda_{14,t}+\phi_f\lambda_{15,t})Q_{t-1}d_{t-1}^F\Pi_t^{-1} = 0$$

$$0 = \lambda_{11,t}\xi_t Y_t^{-\rho} + \lambda_{13,t} + \lambda_{14,t} + \gamma_f\lambda_{15,t}$$

$$0 = \lambda_{10,t} - \beta E_t\left\{\lambda_{11,t+1}\xi_{t+1}Y_{t+1}^{-\rho}\Pi_{t+1}^{-1}\right\} + \lambda_{11,t}\xi_t Y_t^{-\rho} - \beta i_t E_t\left\{\lambda_{12,t+1}\right\} - \beta\phi_c E_t\left\{\Pi_{t+1}^{-1}\lambda_{13,t+1}\right\}$$

$$\lambda_{10,t} = 0$$

$$\lambda_{12,t}\Pi_t - \lambda_{11,t}\xi_t Y_t^{-\rho} - \gamma_c\lambda_{13,t} = 0$$

$$\lambda_{14,t} + \lambda_{15,t} = 0$$

$$\frac{\lambda_{14,t}}{1+i_t} - \beta E_t\left\{(\lambda_{14,t+1}+\phi_f\lambda_{15,t+1})\Pi_{t+1}^{-1}\right\} = 0.$$

A.2.3 Solution Method, Calibration and Simulated Experiments

We study the optimal policy problem using linear-quadratic methods. We approximate, around a non-stochastic steady state, the objective welfare function to second order, and the models equilibrium conditions to first order. We solve and simulate the model using the piecewise-linear algorithm developed by Guerrieri and Iacoviello (2015): the approximated system of linear equations is treated as a regime-switching model, where the alternative regimes depend on whether specific constraints are binding or not. In particular, in our model there are two distinct constraints that may occasionally bind. The first one is the familiar zero-lower bound on the nominal interest rate, while the second is a non-negativity constraint that may affect central bank's remittances under some specifications of the transfer policies.

The model is calibrated (quarterly) as follows. We set the steady-state inflation rate and nominal interest rate on short-term bonds to 2% and 3.5%, respectively and in annualized terms; accordingly, we set $\beta = (1+\bar{\pi})/(1+\bar{\imath})$. We calibrate the composition of central bank's balance sheet considering as initial steady state the situation in 2009Q3, when the economy had already been in a liquidity trap for about three quarters. Accordingly we set the share of money to total liabilities equal to 53%, the share of net worth to total liabilities to 1%, and the share of long-term asset to total assets to 72%. This calibration implies that the steady-state quarterly remittances to the treasury are equal to about 0.6% of the central bank's assets and that the central bank's position on short-term interest-bearing liabilities (central bank reserves) amounts to 46% of the central bank's balance sheet. The duration of long-term assets is set to ten years (accordingly, $\delta = .9896$). Moreover, we set the ratio of long-term public debt to GDP in the initial steady state equal to $\bar{Q}\bar{D}^G/(4\bar{Y}\bar{P}) = 0.35$, in annual terms, as reported by the US Bureau of Public Debt for 2009Q3. In particular, we consider the stock of publicly-held marketable government debt including securities with maturity above one year. Finally, following Benigno et al. (2016), we set the relative risk-aversion coefficient to $\rho = 1/.66$, the inverse of the Frisch-elasticity of labor supply to $\eta = 1$, the elasticity of substitution across goods to $\theta = 7.88$, the parameter α capturing the degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ($\alpha = 0.75$). As a result, the slope of the Phillips Curve is $\kappa = .024$. To calibrate the initial level of the natural interest rate, we follow Benigno et al. (2016), who show that the extent of households' debt deleveraging observed since 2008 in the U.S. is consistent with a fall of the natural interest rate to about -6% from a steady-state level of 1.5%. See also Gust et al. (2016), who provide consistent empirical evidence.

We evaluate the implications for Neutrality of interest-rate and credit risks.

With respect to interest-rate risk, we run the following experiment. We simulate an economy which at time $t_0 - 1$ is already in a liquidity trap, because of a preference shock (ξ) that hit sometime in the past and turned the natural interest rate negative. At time t_0 the central bank first chooses whether to stick to its past balance-sheet policy ($D^C = 0$) or to engage in large-scale asset purchases ($D^C > 0$) and then commits to a state-contingent path for the endogenous variables from t_0 onward, conditional on the chosen balance-sheet policy. One year later (at time $t_0 + 4$) an unexpected preference shock hits, turning the natural interest rate positive again. At this time, the path of current and future short-term rates changes, producing an unexpected fall in the price of long-term securities and therefore implying income losses for the central bank, in the case it holds long-term assets.

With respect to credit risk, we consider an economy starting at steady state, and a credit event hitting at time t_0, which implies default on a share $\varkappa$ of long-term debt. After period t_0 no other credit event or other shocks are either expected or actually occur. As clear from equation (14), when a credit event occurs, the central bank might experience a loss on its balance sheet if it holds long-term securities. To simulate the optimal response to an increase in the probability of future credit events, we use the result of the above experiment to compute the response of the economy in the contingency in which the credit event occurs. We then use the respective equilibrium decision rules and the probability of a credit event to characterize the one-period-ahead expectations for the relevant variables, and solve a linear-quadratic approximation of the optimal monetary-policy problem at time 0.

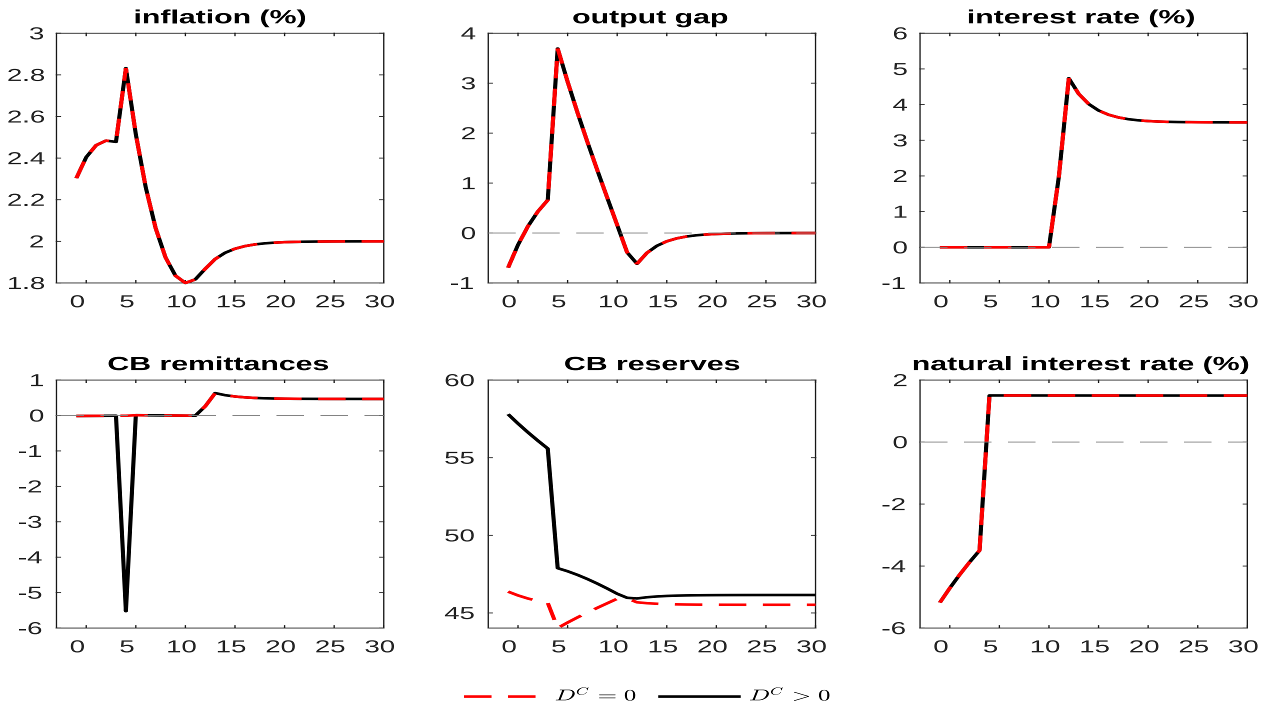

Figure 5: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk under *passive transfer policies*. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

A.2.4 Additional Simulations

Here we discuss some additional simulations not reported in the main text.

Consider a regime with *passive transfer policies*, combining a *passive fiscal policy* and a *passive remittance policy*. The top panels of Figure 5 display the path of inflation, the output gap and the nominal interest rate, and show the familiar result, already discussed in Eggertsson and Woodford (2003), that committing to a higher inflation for the periods after the liftoff of the natural rate of interest allows to limit the deflationary impact of the negative shock, despite the nominal interest rate cannot be cut as much as needed because of the zero floor. This commitment translates into maintaining the policy rate at the zero bound for several periods after the natural rate has turned back positive (in the specific case of Figures 5, for six quarters more).

The bottom panels show instead the evolution of two key variables related to the balance sheet of the central bank – as well as the path of the natural interest rate: the quarterly real remittances to the treasury T_t^C/P_t and the central bank's real reserves X_t/P_t, all expressed as a share of the steady-state balance sheet of the central bank. Consistently with Proposition 1, the central bank's real net worth remains constant at its initial level of 1% (not shown) and the dynamics of profits (and remittances) reflect the specific composition of the central

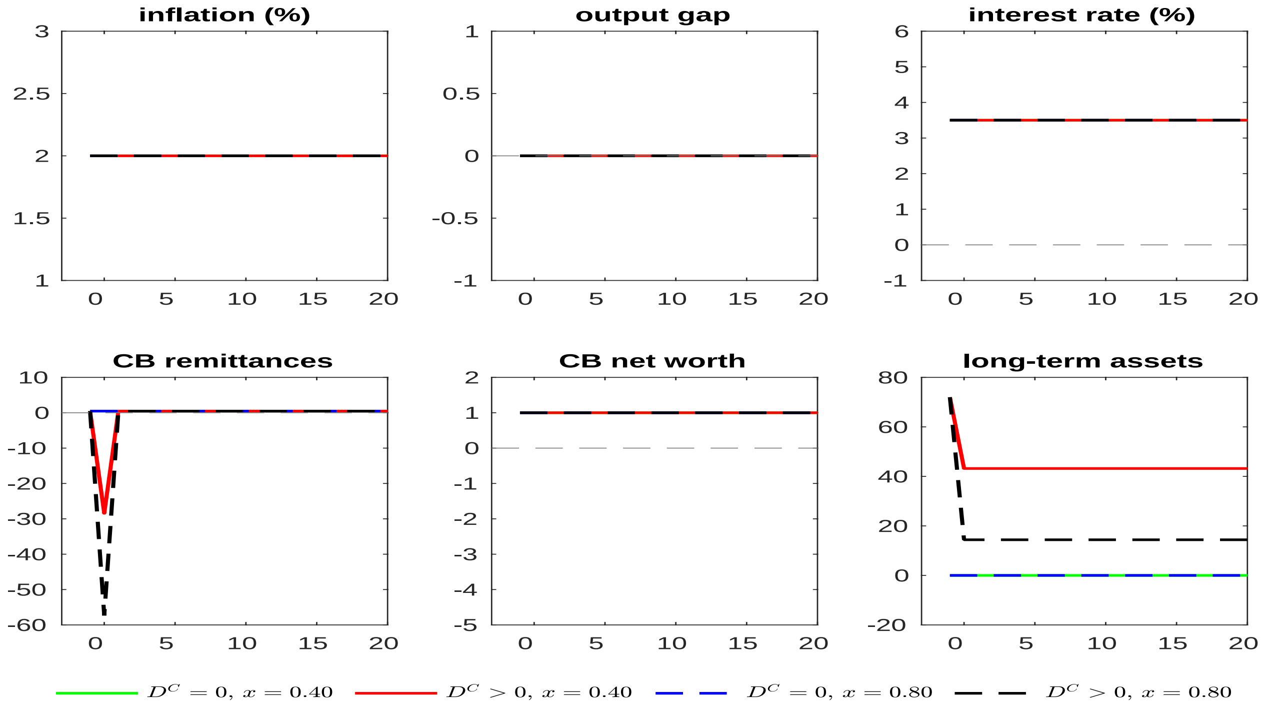

Figure 6: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies and *passive transfer policies*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

bank's balance sheet. When the central bank has only short-term assets, remittances are non-negative while with long-term assets they mainly follow their return. As the natural rate unexpectedly turns positive, the expectation that the nominal interest rate will jump up a few periods later is enough to bring down long-term asset prices and their return, thereby implying negative profits for the central bank. Under *passive remittance policy*, negative profits trigger a transfer of resources from the treasury to the central bank (negative remittances), so that net worth does not move. Central bank's reserves instead fall as a consequence of the lower valuation of the long-term assets.

In Figure 6, under the same calibration, we consider a mild and a strong credit event with default rate respectively of 40% and 80% (i.e. $\varkappa = 0.40$ or $\varkappa = 0.80$, displayed by the continuous and dashed lines in Figure 2). The top panels show that the optimal monetary policy requires to completely stabilize inflation, output and interest rate at their targets. Indeed, the shock $\varkappa$ does not appear in either the objective function (A.39) or the constraints that are relevant under Neutrality (A.25)–(A.29). Given the *transfer policy* assumed, the optimal monetary policy is also not affected by the alternative *balance-sheet policy*. The

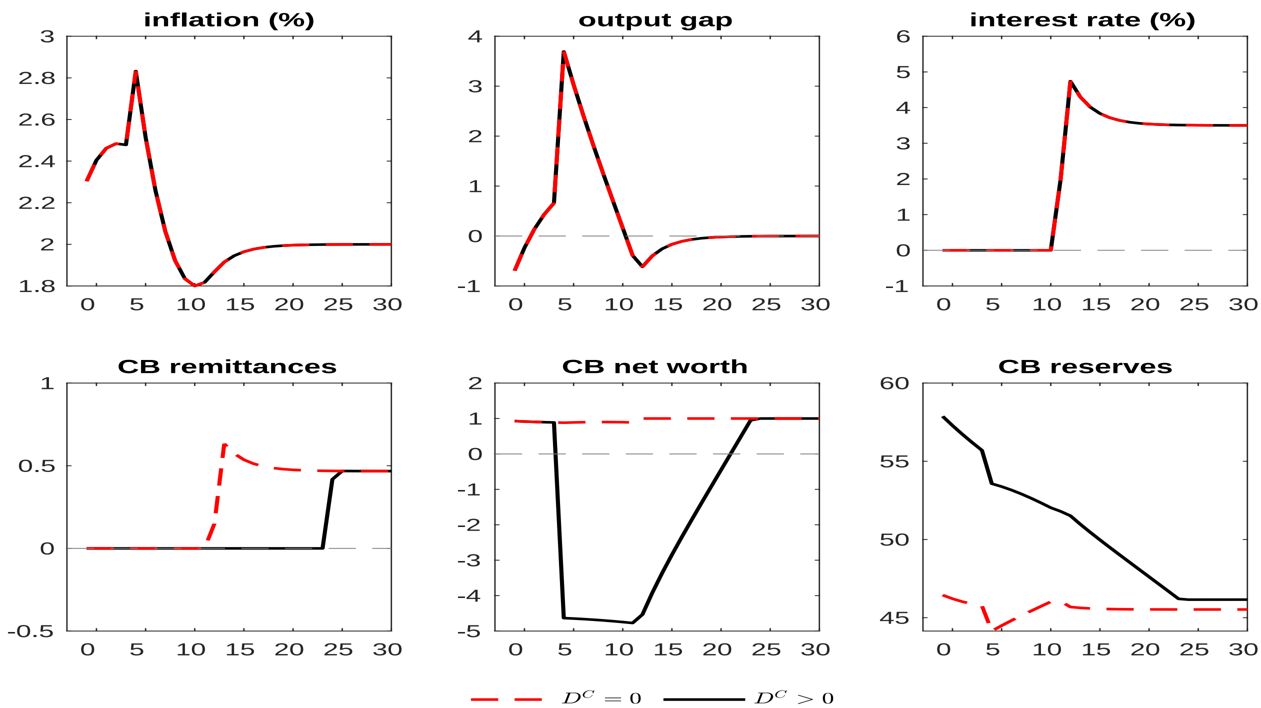

Figure 7: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk. Regime ii): *"lack of treasury's support"*. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

difference is in the remittances to the treasury. In the case of a standard composition of the balance-sheet ($D_t^C = 0$), profits and remittances are always positive while when the central bank holds long-term securities losses are covered by the treasury, given *passive remittance policy*, and the more so the higher the default rate.

We consider now the *"deferred-asset" regime*. Figures 7 through 9 analyze the same scenarios as Figures 5 and 6, respectively, maintaining the assumption of *passive fiscal policy*, the same *balance-sheet policies* but changing the remittance policy to a *"deferred-asset" regime* analogous to the one specified in Definition 6.[59]

With only interest-rate risk, as shown in Figure 7, the responses of inflation, output and interest rate do not change across the two alternative balance-sheet policies. This case is indeed consistent with the necessary and sufficient conditions for neutrality of Proposition 5. Indeed, losses are not large enough to impair the profitability of the central bank and violate condition (38) under the optimal monetary policy. As central bank's profits turn negative, remittances to the treasury fall to zero and stay at this level even when central bank's profits start to be positive as long as real net worth is below its long-run level, thereby allowing the

[59]In particular, since we simulate a linear approximation of the model, we adapt the rules introduced in the previous section to ensure a stationary real net worth (rather than nominal). This adjustment will also apply later when we deal with the case of financial independence.

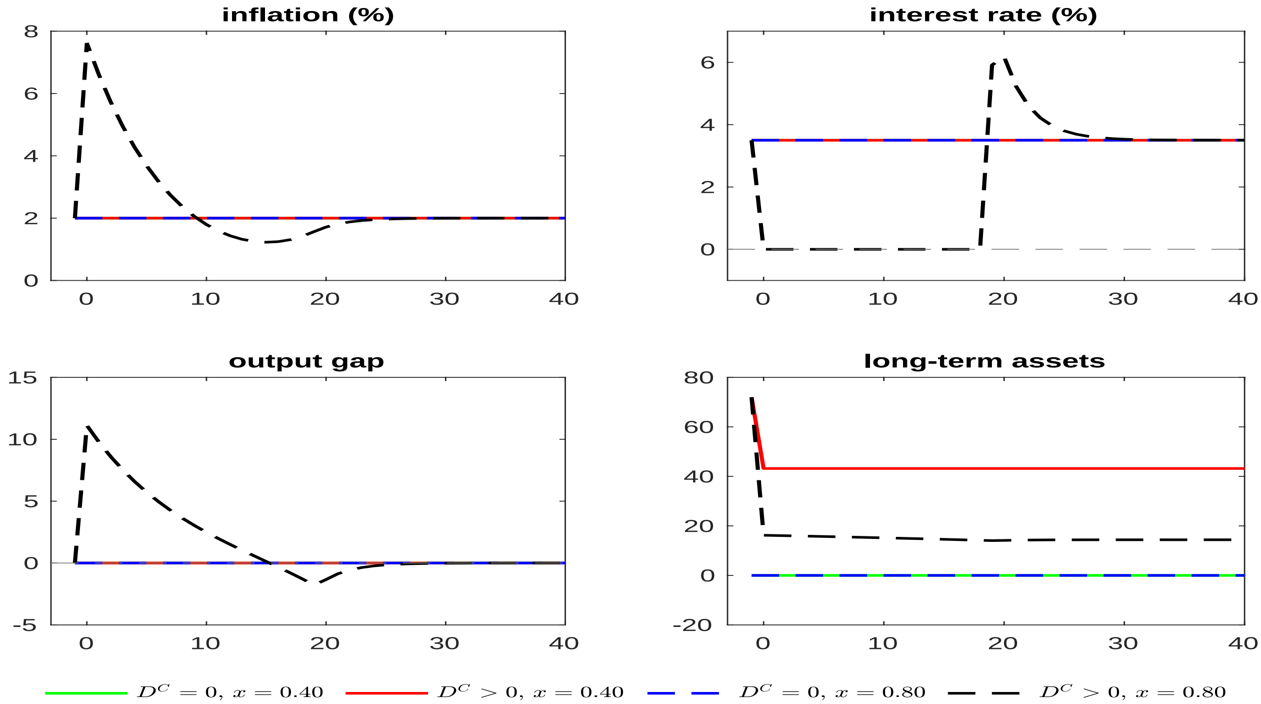

Figure 8: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies. Regime ii): *"lack of treasury's support"*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

latter to converge back to 1% of the balance sheet within a few quarters. After net worth is back at the initial value of 1%, central bank's profits are again rebated to the treasury. The implication is that central bank's reserves are temporarily higher than under passive remittance policy, and are paid back by next-period profits.

Figure 8, in the case of credit risk, shows instead a non-neutrality result when the credit event is significant (i.e. $\varkappa = 0.80$) and the central bank holds long-term assets ($\tilde{D}^C > 0$). Indeed, in this case losses are strong enough to impair the profitability of the central bank: without a change in prices and output with respect to the case $D_t^C = 0$, profits would remain indefinitely negative. The conditions for neutrality of Propositions 5 and 6 are violated. Instead, if the credit event is not too strong (i.e. $\varkappa = 0.40$), neutrality emerges and the central bank is therefore able to return to the steady-state level of net worth in a finite period of time without changing equilibrium prices and output with respect to the case in which $D_t^C = 0$, as shown in the Figure.

Figure 9 further shows the path of remittances, nominal money supply and central bank's net worth under the mild and strong credit events of Figure 8 given the two balance-sheet

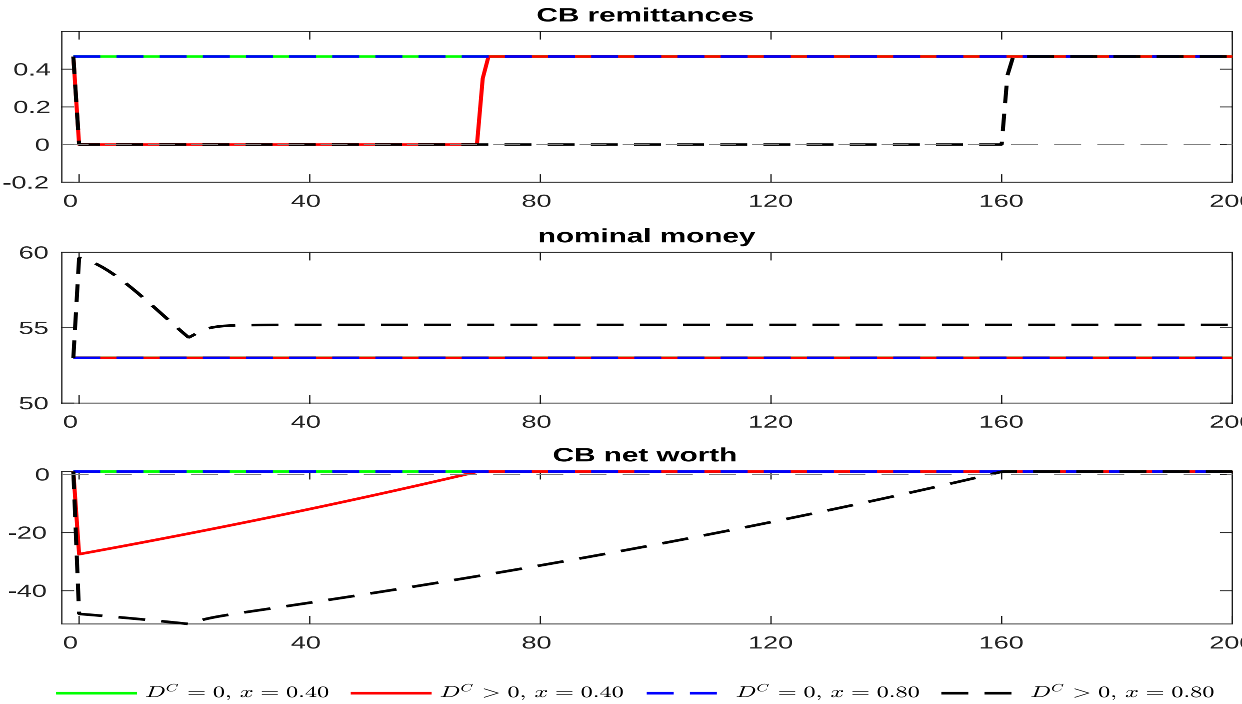

Figure 9: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies. Regime ii): *"lack of treasury's support"*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

policies $D_t^C = 0$ and $\tilde{D}^C > 0$. The solid line, capturing the mild-credit event (when $\tilde{D}^C > 0$), shows that the fall in net worth, as a consequence of the income loss at t_0, is not enough to impair the ability of the central bank to produce positive gains from seigniorage in the future (i.e. $N_t^C + M_t^* > 0$ for each $t \geq \tau$). Such positive profits, therefore, will be possible without the need for the path of nominal money supply to deviate from the equilibrium associated with $D^C = 0$ (second panel of Figure 9). Moreover, these gains will be used to repay the deferred asset over a period in which remittances are zero and net worth can be rebuilt (first and third panels of Figure 9, respectively).

Results substantially change if the credit event is strong. In this case, the nominal stock of non-interest bearing liabilities, $N_t^C + M_t^*$, if evaluated at the inflation rate of the equilibrium with $D_t^C = 0$, would turn negative within the first quarters and violate afterward the solvency condition of the central bank at the initial equilibrium prices. The dashed lines in Figure 9 shows how to optimally deal with a shock of this size. The central bank should commit to substantially raise the stock of nominal money supply in the short-run – to compensate for the fall in nominal net worth – and set it at a permanently higher level in the long-run. Such commitment will ensure that the stock of non-interest bearing liabilities eventually reverts to positive values and produces the profits needed to repay the deferred asset and rebuild net worth (although over an extremely long time). To generate such a path of nominal money supply, the central bank should be accommodative enough to push up prices and inflation. In particular, as the dashed line in Figure 8 shows, inflation and output should go well above their target on impact, which in turn requires the nominal interest rate to fall down to the zero-lower bound. In the specific case displayed in Figures 8 and 9, it takes about 30 quarters for real variables to converge back to the path they would follow under neutrality, and for nominal money supply to stabilize on a new, higher, level.