Monetary Policy and Heterogeneity: An Analytical Framework[I]

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Abstract

THANK is a tractable heterogeneous-agent New-Keynesian model that captures analytically key micro-heterogeneity channels of quantitative-HANK: cyclical inequality and risk as separate but related channels; idiosyncratic uncertainty and self-insurance, precautionary saving; and realistic intertemporal marginal propensities to consume. I use it for a full-fledged New Keynesian macro analysis: determinacy with interest-rate rules, solving the forward-guidance puzzle, amplification and multipliers, and optimal monetary policy. Amplification requires countercyclical while solving the puzzle requires procyclical inequality: a Catch-22, resolved in theory if the separate "pure" risk channel is procyclical enough. Price-level-targeting ensures determinacy and is puzzle-free, even when both inequality and risk are countercyclical, thus resolving the Catch-22. The same holds for a rule fixing nominal public debt in the model version with liquidity. Optimal policy with heterogeneity features a novel inequality-stabilization motive generating higher inflation volatility—but it is unaffected by risk, insofar as the target equilibrium entails no inequality.

JEL Codes: E21, E31, E40, E44, E50, E52, E58, E60, E62

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1 Introduction

A spectre is haunting Macroeconomics—the spectre of Heterogeneity. Some of the world's leading policymakers have been asking for research on it, and its other name, "Inequality", in connection with stabilization, monetary and fiscal policies. For until recently, research on these two topics has been, with few exceptions, largely disconnected. Yet a burgeoning field emerged as a true synthesis of these two lodes: heterogeneous-agent (HA) and New Keynesian (NK), leading to HANK.[1]

The vast majority of contributions consists of quantitative models, involving heavy machinery for their resolution, the price to pay to achieve the realism conferred by matching the micro data.[2] Yet given that much of the post-crisis bad press of existing DSGE models refers to their being too complex and somewhat black-box, it seems important to build simple tractable representations of these models to gain analytical insights into their underlying mechanisms and make their policy conclusions sharper and easier to communicate. The two, quantitative and analytical approaches are thus strongly *complementary* and reinforce each other.

With this paper, I wish to propose a tractable HANK model, **THANK**, to achieve two purposes.[3] First, argue that it is a good representation, along several key dimensions, for rich-heterogeneity quantitative HANK models. And second, use it for a full-fledged positive and normative NK analysis in closed form: determinacy of interest rate rules, curing the forward guidance puzzle, amplification and fiscal multipliers; and optimal monetary policy. The ethos is thus to maximize micro heterogeneity into a macro model under the constraint of tractability.

THANK is a three-equation model isomorphic to the textbook representative-agent (RANK) model, which it nests; yet it captures several dimensions that the recent quantitative literature finds important for the study of macro fluctuations with heterogeneity. First, it features a key aggregate-demand (AD) amplification: the "New Keynesian cross" present in any HANK model where some households are constrained hand-to-mouth. Heterogeneity shapes aggregate outcomes through *cyclical inequality*: how the distribution of income between constrained and unconstrained changes over the cycle, e.g. who suffers more in recessions. This originates in the TANK model in Bilbiie (2008, 2020) and generalizes to the subsequent rich-heterogeneity literature (Auclert's (2019)). Recently, Patterson (2019) provides empirical evidence for countercyclical inequality.

Second, my analytical model incorporates uninsurable risk, precautionary, self-insurance saving, and the distinction between liquid and illiquid assets, staples of quantitative HANK models, e.g. Kaplan et al (2018). The paper proposes a decomposition of cyclical income risk into one part due to cyclical inequality and one related to skewness: cyclical variations in the likelihood of ending up in the bad state. I argue below that this distinction is key for some of the theoretical properties of the model. Third, the version with liquidity allows an analytical solution for the key "intertemporal MPC" statistics that Auclert, Rognlie, and Straub (2018) introduced in a quantitative HANK for their distinct, "intertemporal" Keynesian cross. Finally, it delivers key stylized statistical properties

[1] The abbreviation is due to Kaplan, Moll, and Violante (2018); the opening sentence is a paraphrase of Marx and Engels.

[2] Overwhelming evidence was long available for the failure of an *aggregate* Euler equation, for a high fraction of households having zero net worth and a high marginal propensity to consume MPC, "hand-to-mouth". Important work clarified the link with liquidity constraints: some "wealthy" households behave as hand-to-mouth if wealth is illiquid (Kaplan and Violante (2014)), perhaps because it consists of a mortgaged house (Cloyne, Ferreira, and Surico (2015)).

[3] The T in THANK stands for "tractable" and for "two" (states/types, and assets), symbolizing the model's bridging HANK and (two-agent) TANK, the *analytical* version in Bilbiie (2008) centered on the asset market participation distinction. Galí, Lopez-Salido and Valles (2007) embedded a different distinction in a *quantitative* model, between holding or not *physical capital*.

of idiosyncratic income emphasized by a large empirical literature: autocorrelation, cyclical variance and (negative) skewness, and leptokurtosis (e.g. Guvenen et al (2014)).

To the best of my knowledge and to this date, THANK is the only tractable framework, of the several reviewed below, that simultaneously captures *all* of these important features of the rich *micro*-heterogeneity models. This, in my view, makes it particularly suitable for the full (short-term) *macro* analysis pursued in this paper.

This paper's contribution is manifold, as afforded by the tractable model. The determinacy analysis provides a first modified Taylor principle in this environment, with the differential effect of pro- and countercyclical inequality on the threshold. Understanding determinacy properties is crucial for being able to even solve quantitative models, and for policy discussions: what rules or institutions can anchor expectations when distributional mechanisms are at play. Another key contribution is the decomposition of "income risk" into income inequality and "pure" (skewness-related) risk, with different implications for transmission. The paper identifies a Catch-22 for HANK models, that amplification and multipliers in HANK models require countercyclical inequality, but this also aggravates the forward guidance puzzle (the resolution of which would require procyclical inequality). This can be resolved if the "pure risk" channel is procyclical, but is further aggravated if it is countercyclical. This paper is also the first to show that a Wicksellian rule delivers determinacy in a HANK economy regardless of how countercyclical inequality and risk are; as a corollary, it allows preserving amplification while ruling out the FG puzzle, thus sidestepping the Catch-22. These are virtues shared with a fiscal rule setting nominal debt originally proposed by Hagedorn, that I prove analytically in the version with liquidity. A final novel contribution is the optimal monetary policy analysis, emphasizing the different roles of inequality and risk.

Adding a standard Phillips curve, I study monetary policy both as interest rate rules and as aggregate welfare-maximizing optimal policy. Under a further inconsequential simplification, the model reduces to *one* first-order difference equation whose root governs aggregate dynamics and depends chiefly on the *cyclicality of inequality*, that is on the constrained agents' income elasticity to aggregate income χ. AD-amplification occurs with countercyclical inequality, $\chi > 1$: a demand increase leads to a disproportionate increase in constrained agents' income and a further demand expansion, the intertemporal version of which delivers *compounding* in the aggregate Euler equation. Conversely, procyclical inequality $\chi < 1$ leads to AD-dampening and Euler-equation *discounting*.

The *determinacy properties of Taylor rules* reflect this intuition. When inequality is countercyclical, the central bank needs to be possibly much more aggressive than the "Taylor principle" (increasing nominal interest more than inflation) to rule out indeterminacy. Whereas in the discounting, procyclical-inequality case, the Taylor principle is sufficient but not necessary: for a large region there is determinacy even under a peg, undoing the Sargent-Wallace result.

The Catch-22 is that the condition for *amplification* relative to RANK and multipliers—that much of the literature uses HANK models for—is countercyclical inequality $\chi > 1$. Yet ruling out the *forward guidance puzzle*—that the later an interest rate cut takes place the larger its effect today (Del Negro, Giannoni, and Patterson, 2012)—requires the *opposite:* procyclical inequality $\chi < 1$ (evidently, $\chi > 1$ implies an *aggravation* of the puzzle). A possible way out is for the *distinct, non-inequality-related* "pure" risk to be *procyclical enough* to compensate. My model includes a novel formalization of such a channel (emphasized previously by others as reviewed below) that can also give rise to Euler discounting, through a different mechanism: if uninsurable risk increases in expansions, precautionary saving leads agents to cut back demand. A nagging policy implication is that empirically, both channels seem to be countercyclical, implying that the puzzle is (double-)aggravated and determinacy requirements with a Taylor rule become very stringent.

While indeterminacy and puzzles are pervasive with heterogeneity under countercyclical inequality and risk, I show that the *Wicksellian* price-level targeting rule, introduced in RANK by Woodford (2003) and Giannoni (2014), ensures determinacy and rules out the FG puzzle in THANK no matter how strong the Euler-equation compounding, resolving the Catch-22. Similar virtues are shared by the debt quantity rule proposed by Hagedorn (2020) and shown by Hagedorn et al (2018) to sidestep the Catch-22. I prove analytically both this and Auclert et al's numerical determinacy criterion based on intertemporal MPCs in my model's version with liquidity.

Optimal monetary policy in quantitative HANK is subject to phenomenal technical challenges, many of which are resolved in a recent contribution by Bhandari, Evans, Golosov, and Sargent (2021); I calculate optimal policy *analytically* in THANK, approximating aggregate welfare to second-order to derive a quadratic objective function for the central bank. This encompasses a novel inequality motive relative to RANK, implying *optimally* tolerating more inflation volatility when more households are constrained. While inequality is of the essence for optimal policy, risk is not—insofar as the policymaker shares society's first-best (perfect-insurance) objective. Risk does matter for implementation: with countercyclical inequality and idiosyncratic risk, the interest-rate rule that implements optimal discretionary policy may entail cutting real rates, when in RANK it would imply increasing them. Furthermore, optimal policy under commitment ensures determinacy regardless of heterogeneity and inequality-cyclicality and, while affected by similar inequality considerations, amounts to a form of price-level targeting.

Related Literature—*Quantitative* HANK models with rich heterogeneity and feedback effects from equilibrium distributions to aggregates are increasingly used to address a wide spectrum of issues in macroeconomic policy.[4] This paper develops an *analytical* representation of the richer-heterogeneity models in order to gain insights into their mechanisms and thus belongs to an emerging literature reviewed below and in more detail in Appendix A, including my own previous work.

Most other analytical studies focus on the role of cyclical income risk without disentangling the role played by cyclical inequality. The clearest example is Acharya and Dogra (2020), which isolates the cyclical-risk channel by using CARA preferences to simplify heterogeneity and shows that intertemporal amplification *may* occur *purely* as a result of income volatility going up in recessions. With this different mechanism, it also studies determinacy and the forward guidance puzzle making explicit reference to the analysis in this paper's previous version.[5] Ravn and Sterk (2020) study a complementary analytical HANK with *endogenous* (through search and matching) unemployment risk and analyze determinacy and shock transmission, while Challe (2020) analyzes optimal monetary policy in that model. Werning (2015) studies the possibility of AD amplification/dampening of monetary policy in a different, general model of cyclical risk and market incompleteness, without discussing the distinction between inequality and risk and without analyzing any of the topics of

[4]The effects of transfers (Oh and Reis, 2012); liquidity traps (Guerrieri and Lorenzoni, 2017); job-uncertainty-driven recessions (Ravn and Sterk, 2017; den Haan, Rendahl, and Riegler, 2018); monetary transmission (Gornemann, Kuester, and Nakajima, 2016; Auclert, 2018; Debortoli and Gali, 2018; Auclert and Rognlie, 2017); portfolio composition (Bayer et al, 2019 and Luetticke, 2021); fiscal policy (Ferrière and Navarro, 2018, Hagedorn, Manovskii, and Mitman, 2018; Auclert, Rognlie, and Straub, 2018; McKay and Reis, 2016; Cantore and Freund, 2019); the FG puzzle (McKay et al, 2016).

[5]Also subsequently to this paper, Auclert et al (2018) provided *numerical* determinacy results emphasizing the cyclicality of risk in quantitative HANK; Acharya and Dogra (2020) stemmed from a discussion of it meant to provide analytical insights.

this paper (determinacy with various rules, Catch-22, optimal policy).[6] Broer, Hansen, Krusell, and Oberg (2020), in another analytical HANK, show that wage rigidity can cure some of the uncomfortable implications brought about by the dynamics and distribution of profits, some of which occur in TANK in Bilbiie (2008). The companion paper Bilbiie (2020) abstracts from cyclical risk and liquidity and analyzes different issues, focusing on the important role of TANK's cyclical-inequality channel in HANK transmission in and of itself. Both that paper and Debortoli and Galí (2018) use the TANK version in Bilbiie (2008) to approximate some aggregate implications of *some* HANK models.

Fiscal multipliers under heterogeneity have been analyzed in several quantitative HANK models cited above and in TANK for spending (Galí et al (2007)), transfers (e.g. Bilbiie, Monacelli and Perotti (2013)) or both, in liquidity traps (Eggertsson and Krugman (2012)). Hagedorn (2020) has shown that the demand for nominal bonds inherent in incomplete-market economies coupled with a supply rule leads to price-level determinacy and rules out the FG puzzle;[7] I provide an analytical application of this in my model. Hagedorn et al (2018) explored the implications of such a rule for fiscal multipliers, and thus for sidestepping what this paper calls the Catch-22.

Finally, this paper is related to studies of optimal policy: in RANK (i.a. Woodford (2003)) and with heterogeneity in TANKs (Bilbiie (2008); Nistico (2016); Curdia and Woodford (2016)). Recent different analytical HANKs providing complementary insights include Challe (2020) and Bilbiie and Ragot (2020). Important optimal-policy studies in rich-heterogeneity quantitative HANK imply deviations from price stability: Bhandari, Evans, Golosov, and Sargent (2021) emphasize inequality motives, and Nuño and Thomas (2021) redistribution with nominal assets.

2 THANK: An Analytical HANK Model

This section outlines THANK, an analytical HANK model that captures several key channels of complex HANK models: cyclical inequality, self-insurance in face of idiosyncratic uncertainty, and a distinction between liquid and illiquid assets. While related to several studies reviewed in the Introduction, the exact model is to the best of my knowledge novel to this paper and its companion Bilbiie (2020), which uses a special cases of it (with *this* paper as its reference for the full model), focusing on AD amplification of monetary and fiscal policies through a "New Keynesian Cross" and on using it as a *one-channel* approximation to richer HANK models.

A unit mass of households $j \in [0,1]$ discount the future at rate β, derive utility from consumption C_t^j and dis-utility from labor supply N_t^j, and have access to two assets: a government-issued riskless bond (with nominal return $i_t > 0$), and shares in monopolistically competitive firms. Households participate infrequently in financial markets and freely adjust their portfolio and receive dividends from firms when they do. When they do not, they receive only the payoff from previously accumulated bonds. Denote these two states, for ease and anticipating what equilibrium we will focus on, S for participating (from "savers") and H from "hand-to-mouth" for non-participants.

The exogenous change of state follows a Markov chain: the probability to *stay* type S and H is respectively s and h, with transition probabilities $1-s$ and $1-h$; later on, I assume that s is a

[6] Holm (2021) shows that the effectiveness of monetary policy is reduced with (yet another model of) procyclical risk; see also Bernstein (2021) and Caramp and Silva (2021) for other recent analytical frameworks.

[7] Other modifications of the NK model that solve its puzzles include changing the information structure (Garcia-Schmidt and Woodford (2019), Gabaix (2019), Angeletos and Lian (2017), Farhi and Werning (2019), Woodford (2018)), pegging interest on reserves (Diba and Loisel (2017)), wealth in the utility function (Michaillat and Saez (2017), Hagedorn (2018)).

function of aggregate activity. I focus on stationary equilibria whereby by standard results the mass of H is the unconditional probability λ (with $1-\lambda$ the mass of S):[8]

$$\lambda = \frac{1-s}{2-s-h}, \tag{1}$$

that is the ergodic distribution (in the idiosyncratic dimension) around which we approximate the model with respect to aggregate shocks. At one extreme stands TANK: permanent idiosyncratic shocks ($s = h = 1$) and λ fixed at its initial free-parameter value. Other useful special cases include $s = h = 0$ with agents *oscillating* between states every other period and $\lambda = \frac{1}{2}$; and iid idiosyncratic shocks $s = 1-h = 1-\lambda$ (being S or H tomorrow is independent on today's state).

Key assumptions on the asset market structure simplify the equilibrium and afford an analytical solution; the precise combination of assumptions used is novel to this paper, although subsets have been used by existing literature. Notably, the setup builds on the seminal contributions of Lucas (1990) and Shi (1997) in monetary theory.[9] First, households belong to a family whose utilitarian (equally-weighted) intertemporal welfare its head maximizes facing limits to risk sharing. Households can be thought of as being in two states or "islands", with all participants on the same island S and all non-participants on island H. The family head can transfer *all* resources across households *within* the island, but can only transfer *some* resources *between* islands.

In face of idiosyncratic risk there is thus full insurance within type, after idiosyncratic uncertainty is revealed, but limited insurance across types. At the beginning of the period, the family head pools resources within the island. The aggregate shocks are revealed and the family head determines the consumption/saving choice for each household in each island. Then households learn their next-period participation status and have to move to the corresponding island accordingly, taking *only bonds* with them. There are no transfers to households *after* the idiosyncratic shock is revealed, and this is taken as a constraint for the consumption/saving choice. Different assets thus have different liquidity: only one of the two assets (bonds) can be used to self-insure before idiosyncratic uncertainty is revealed, i.e. is liquid. While stocks are illiquid—they cannot be used to self-insure.

The *flows across islands* are as follows. The total measure of households leaving island H each period (who participate next period) is $(1-h)\lambda$; the rest λh stay. Likewise, a measure $(1-s)(1-\lambda)$ leaves island S for H at the end of each period. Total welfare maximization implies that the family head pools resources at the beginning of the period in a given island and implements symmetric consumption/saving choices for all households in that island. Denote by B^j_{t+1} per-capita, real *beginning-of-period-*$t+1$ bonds on island $j = S, H$: after the consumption-saving choice, and also after changing state and pooling. The *end-of-period-t* (*after* the consumption/saving choice but *before* agents move across islands) per capita real values are Z^j_{t+1}. We have the following relations (stocks stay on the S island, so we ignore them):

$$\begin{aligned}(1-\lambda) B^S_{t+1} &= s(1-\lambda) Z^S_{t+1} + (1-h)\lambda Z^H_{t+1} \\ \lambda B^H_{t+1} &= (1-s)(1-\lambda) Z^S_{t+1} + h\lambda Z^H_{t+1}.\end{aligned}$$

[8]The stationary distribution (1) is found by solving: $\begin{pmatrix} \lambda & 1-\lambda \end{pmatrix} \begin{pmatrix} h & 1-h \\ 1-s & s \end{pmatrix} = \begin{pmatrix} \lambda & 1-\lambda \end{pmatrix}$.

[9]Closer to this literature, this way of reducing heterogeneity and eliminating the wealth distribution as a state variable also extends Challe and Ragot (2011), Challe et al (2017), Heathcote and Perri (2018) and Bilbiie and Ragot (2020). NK models with two switching types were studied by Curdia and Woodford (2016) and Nistico (2016) but with different insurance structures.

or rescaling by the relative population masses and using (1):

$$B^S_{t+1} = sZ^S_{t+1} + (1-s)\, Z^H_{t+1} \qquad (2)$$
$$B^H_{t+1} = (1-h)\, Z^S_{t+1} + hZ^H_{t+1}.$$

The *program of the family head* is (with π_t denoting the net inflation rate):

$$W\left(B^S_t, B^H_t, \omega_t\right) = \max_{\{C^S_t, Z^S_{t+1} Z^H_{t+1}, C^H_t, \omega_{t+1}\}} \left[(1-\lambda)\, U\left(C^S_t\right) + \lambda U\left(C^H_t\right)\right] + \beta E_t W\left(B^S_{t+1}, B^H_{t+1}, \omega_{t+1}\right)$$

subject to the laws of motion for bond flows (2) and to standard budget constraints for the respective households, where post-tax incomes are Y^j_t, ω_t is the per-capita fraction of the portfolio of shares (with price v_t) held only by S who receive dividends D_t; all households receive the real return on their respective bond holdings and face positive constraints on new bond holdings (4).

$$C^S_t + Z^S_{t+1} + v_t \omega_{t+1} = Y^S_t + \frac{1+i_{t-1}}{1+\pi_t} B^S_t + \omega_t \left(v_t + D_t\right),$$
$$C^H_t + Z^H_{t+1} = Y^H_t + \frac{1+i_{t-1}}{1+\pi_t} B^H_t \qquad (3)$$
$$Z^S_{t+1}, Z^H_{t+1} \geq 0 \qquad (4)$$

The Kuhn-Tucker conditions with complementary slackness are:

$$U'\left(C^S_t\right) \geq \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[sU'\left(C^S_{t+1}\right) + (1-s)\, U'\left(C^H_{t+1}\right)\right]\right\} \qquad (5)$$
$$\text{and } 0 = Z^S_{t+1} \left[U'\left(C^S_t\right) - \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[sU'\left(C^S_{t+1}\right) + (1-s)\, U'\left(C^H_{t+1}\right)\right]\right\}\right];$$
$$U'\left(C^H_t\right) \geq \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[(1-h)\, U'\left(C^S_{t+1}\right) + hU'\left(C^H_{t+1}\right)\right]\right\} \qquad (6)$$
$$\text{and } 0 = Z^H_{t+1} \left[U'\left(C^H_t\right) - \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[(1-h)\, U'\left(C^S_{t+1}\right) + hU'\left(C^H_{t+1}\right)\right]\right\}\right];$$
$$U'\left(C^S_t\right) \geq \beta E_t \left\{ \frac{v_{t+1} + D_{t+1}}{v_t} U'\left(C^S_{t+1}\right)\right\} \text{ and } \omega_{t+1} = \omega_t = (1-\lambda)^{-1}.$$

The key is the Euler equation (5), governing the bond-holding decision of S self-insuring against the risk of becoming H and taking into account that bonds can be used when moving to the H island. Equation (6) determines the bond choice of agents in the H island; both bond Euler conditions are written as complementary slackness conditions. With this market structure, the Euler equations (5) and (6) are of the same form as in fully-fledged incomplete-markets model of the Bewely-Huggett-Aiyagari type. In particular, the probability $1-s$ measures the uninsurable risk to switch to a bad state next period, risk for which only bonds can be used to self-insure—thus generating a demand for bonds for "precautionary" purposes. The last Euler equation corresponds to *illiquid* shares: there is no self-insurance motive, for they cannot be carried to the H state, so it is the same as with a representative agent, merely determining the price v_t residually.[10]

To deliver the simple equilibrium representation, I focus on equilibria where the constraint of H agents always binds and their Euler "equation" (6) is in fact a strict inequality, *whatever the reason*: for instance, because the shock is a "liquidity" or impatience shock making them want to consume more today (decreasing β in (6) to a low enough value β^H), or because their average income in that state is lower enough than in the S state, e.g. if average profits are high enough; or simply because a technological constraint prevents them from accessing any asset markets.

I consider two equilibria, according to whether liquidity is supplied or not: as a benchmark, the **zero-liquidity limit** reminiscent of Krusell, Mukoyama and Smith (2011)[11]; and then in section 4, an equilibrium with government-provided liquidity. In the former case, we assume that even though S's demand for bonds is well-defined (their constraint is not binding), the supply is zero so there are no bonds held in equilibrium. Under these assumptions the only equilibrium condition from this part of the model is the Euler equation for bonds of S (5) holding with equality.[12] The H's constraint binding and zero-liquidity implies that they are hand-to-mouth $C_t^H = Y_t^H$. Because transition probabilities are independent of history and there is full insurance within type, all agents who are H in a given period have the same income and consumption.

The rest of the model is exactly like the TANK version in Bilbiie (2008, 2020), nested with $s = 1$. The λ households who are "hand-to-mouth" H do make an optimal labor supply decision determining their income. The income of H is $Y_t^H = W_t N_t^H + \mathcal{T}_t^H$, where W the real wage, N^H hours and $\mathcal{T}_t^H$ fiscal transfers to be spelled out. The remaining $1 - \lambda$ agents also work, and receive the *profits* from the illiquid shares net of taxes and transfer. The choice of hours worked delivers the standard intratemporal optimality condition for each j: $U_C^j \left(C_t^j\right) = W_t U_N^j \left(N_t^j\right)$. Defining $\sigma^{-1} \equiv U_{CC}^j C^j / U_C^j$ as risk aversion and $\varphi \equiv U_{NN}^j N^j / U_N^j$ as the inverse labor supply elasticity, and small letters log-deviations from steady-state (to be discussed below), we have the labor supply for each j: $\varphi n_t^j = w_t - \sigma^{-1} c_t^j$. Assuming for tractability that elasticities are identical across agents, the same holds on aggregate $\varphi n_t = w_t - \sigma^{-1} c_t$.

The **supply, firms' side** is standard and outlined for completion in Appendix A.2. A notable feature is that I assume *as a benchmark* the standard NK optimal sales subsidy inducing marginal cost pricing. This policy is *redistributive*: since steady-state profits are zero $D = 0$, it taxes the firms' shareholders and results in the "full-insurance" steady-state used here as a benchmark $C^H = C^S = C$. Loglinearizing around it, with $d_t \equiv \ln (D_t / Y)$, profits vary inversely with the real wage: $d_t = -w_t$, an extreme form of the general property of NK models. This series of assumptions (optimal subsidy, steady-state consumption insurance, zero steady-state profits) is not necessary for the results and can be easily relaxed, but makes the algebra more transparent.

Firms' optimal pricing under Rotemberg costs implies the loglinearized Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa c_t + u_t, \tag{7}$$

[10]As households pool resources when participating (which would be optimal with $t = 0$ symmetric agents and $t = 0$ trading), they perceive a return conditional on participating next period. This exactly compensates for the probability of not participating next period, generating the same Euler equation as with a representative agent.

[11]Other zero-liquidity HANK include i.a. Ravn and Sterk (2020), Werning (2015), and Broer et al (2020).

[12]The Euler equation prices these possibly non-traded bonds, just like in RANK or TANK—but now the pricing takes into account the possible transition to the constrained H state.

where u_t are cost-push shocks that I abstract from until studying optimal policy in Section 5. To obtain maximum tractability and closed forms, I first focus on the simplest special case:

$$\pi_t = \kappa c_t, \tag{8}$$

nested in (7) above with myopic firms ($\beta = 0$), used previously in a different context in Bilbiie (2019). Appendix A.2 microfounds this assuming that firms pay a Rotemberg cost relative to yesterday's *market average* price index, rather than to their own individual price (the latter leads to (7)). That is, firms ignore the impact of today's price choice on tomorrow's profits. While over-simplified, this nevertheless captures the key supply-side NK trade-off between inflation and real activity and allows us to isolate and focus on the essence of this paper: AD. The results reassuringly generalize to the standard Phillips curve (7), as I show in Appendix C.

The government conducts fiscal and monetary policy. The former consists of a simple *endogenous redistribution* scheme: taxing profits at rate τ^D and rebating the proceedings lump-sum to H who thus receive $\frac{\tau^D}{\lambda} D_t$ per capita; this is key here for the transmission of *monetary policy*, understood as changes in the nominal interest rate i_t. In the version with liquidity, the government also supplies liquid nominal bonds and levies lump-sum uniform taxes/transfers on households.

Market clearing implies for equilibrium in the goods and labor market respectively $C_t \equiv \lambda C_t^H + (1-\lambda) C_t^S = \left(1 - \frac{\psi}{2}\pi_t^2\right) Y_t$ and $\lambda N_t^H + (1-\lambda) N_t^S = N_t$. With uniform steady-state hours $N^j = N$ by normalization and the fiscal policy assumed above inducing $C^j = C$, loglinearization around a zero-inflation steady state delivers $y_t = c_t = \lambda c_t^H + (1-\lambda) c_t^S$ and $n_t = \lambda n_t^H + (1-\lambda) n_t^S$.

2.1 Cyclical Income Risk and Inequality in THANK

A keystone to this paper's analysis is to define and distinguish income inequality and risk, and their cyclicality. I define **income inequality** as the ratio of income in the two states $\Gamma_t \equiv Y_t^S / Y_t^H$; this is proportional to the *unconditional* variance of log income:

$$var\left(\ln Y_t^j\right) = \lambda (1-\lambda) (\ln \Gamma_t)^2;$$

in Appendix B.1, I show that this is also proportional to other standard inequality measures like the Gini coefficient and generalized entropy. Importantly, as we will see in the model's equilibrium inequality is *cyclical*: it depends on aggregate output $\Gamma(Y_t)$.

In the data and in quantitative HANK models alike, **income risk** is generally cyclical. Other analytical HANK frameworks model *cyclical* idiosyncratic risk as either unrelated (Acharya and Dogra (2020)) or differently related (Challe et al (2017); Holm (2021); Ravn and Sterk (2020); Werning (2015)) to liquidity constraints and hand-to-mouth behavior. To capture a component of *cyclical risk* that is distinct from *cyclical inequality* and thus further differentiate from the cited papers, I assume that the probability of becoming constrained depends on tomorrow's aggregate demand $1 - s(Y_{t+1})$.[13] If the first derivative of $1 - s(.)$ is positive $-s'(Y_{t+1}) > 0$, the probability is higher in expansions so, insofar as being constrained leads on average to lower income, this makes income

[13] In a model with endogenous unemployment risk like Ravn and Sterk or Challe et al, this happens in equilibrium through search and matching. This is also related to Werning's Section 3.4, where nevertheless it is unconditional probabilities (and population shares) that are cyclical. Here, to capture purely idiosyncratic (as opposed to "aggregate") variation, λ is invariant.

risk *procyclical* (go up in expansions). Conversely, $-s'(Y_{t+1}) < 0$ makes risk *countercyclical*.

A precise definition of "income risk" is notoriously controversial. The literature often employs the **conditional variance** of idiosyncratic (log) income, found to be countercyclical in the data by Storesletten, Telmer, and Yaron (2004). This is easily calculated in my two-state model as:

$$var\left(\ln Y^S_{t+1} | \ln Y^S_t\right) = s\left(Y_{t+1}\right)\left(1 - s\left(Y_{t+1}\right)\right)\left(\ln \Gamma_{t+1}\right)^2. \tag{9}$$

Of particular interest for the dynamic properties of the model is its *cyclicality*. Taking a first-order Taylor expansion around an arbitrary steady state Γ, this is made of two components:

$$var\left(\ln Y^S_{t+1} | \ln Y^S_t\right) - s\left(1-s\right)\left(\ln \Gamma\right)^2 \simeq 2 \ln \Gamma \left[\underbrace{\frac{\Gamma_Y}{\Gamma} s\left(1-s\right)}_{\text{inequality}} \underbrace{- s_Y \left(s - \frac{1}{2}\right) \ln \Gamma}_{\text{pure risk}}\right] y_{t+1} \tag{10}$$

The first component is due to cyclical *inequality*: when $\Gamma_Y < 0$, inequality is countercyclical and so is risk because income at the bottom overreacts, increasing variance in *both* expansions and recessions. This "intensive" margin of risk operates even when the second channel is absent, i.e. with constant s or symmetric distribution $s = \frac{1}{2}$. The second component is intuitively and formally related to the cyclicality of conditional *skewness* that Guvenen, Ozkan, and Song (2014) argued forcefully in favor of using as a measure of cyclical income risk, see also Mankiw (1986).[14] I derive this formally in Appendix B, but the intuition is simple: skewness is negative whenever $s > \frac{1}{2}$ (there is left-tail risk). When $1 - s(Y)$ is decreasing with aggregate activity $-s_Y < 0$, it becomes more likely to draw from the left tail in recessions; hence, (skewness) risk is countercyclical: upward income movements become less likely and downward movements more likely in a recession. This "extensive" margin of risk operates even with acyclical inequality $\Gamma_Y = 0$.

This simple setup allows for an arbitrary, general relationship between risk and inequality: it depends on both their levels and cyclicalities, and their interactions, as parameterized by Γ, s, Γ_Y and s_Y. It is useful for the further analysis to consider the following polar cases:

a. **no inequality in levels**, $\Gamma = 1$: risk is acyclical to first order only. As we will see, risk has no impact on the first-order equilibrium in this benchmark. (naturally, risk is still cyclical to higher orders and away from this steady state).

b. **no risk in levels**, $s = 1$ or $s = 0$: the former is TANK (no transition between states), the latter has agents oscillating between states every other period and $\frac{1}{2}$ mass in each state.

c. **acyclical inequality**, $\Gamma_Y = 0$; risk is cyclical only through the second channel $s_Y \neq 0$, and only if there is level inequality $\Gamma > 1$ and skewness $s \neq \frac{1}{2}$.

d. **acyclical (pure) risk**, $s_Y = 0$; risk is cyclical only through the inequality channel $\Gamma_Y \neq 0$, *and* (to first order) only if there is level inequality $\Gamma > 1$.

Thus, the model nests several scenarios that are useful to disentangle the importance of the corresponding economic mechanisms, as we will see below. While the precise decomposition is of course model-specific, the general idea and mechanisms and their equilibrium implications derived below transcend the simple model used here.

[14] Appendix B derives higher moments: formally, conditional skewness $\left(1-2s\right) / \sqrt{s\left(1-s\right)} < 0$ when $s > \frac{1}{2}$.

2.2 Cyclical Inequality and Aggregate Demand in THANK

We derive an *aggregate* Euler-IS equation by taking an approximation (in the aggregate-shocks dimension) around the ergodic idiosyncratic distribution with relative shares given by (1). To isolate the role of cyclical inequality, we first approximate around a *symmetric* steady-state with $\Gamma = 1$ and $C^H = C^S$. Start from the individual self-insurance Euler equation (5):

$$c_t^S = sE_t c_{t+1}^S + (1-s) E_t c_{t+1}^H - \sigma (i_t - E_t \pi_{t+1}). \tag{11}$$

To express this in terms of aggregates, we need individual c_t^j as a function of aggregate c_t and y_t. Once idiosyncratic uncertainty is revealed and asset markets clear, this part—one of many possible examples of how the income distribution depends on aggregate income—is exactly as in the TANK model in Bilbiie (2008, 2020), for simplicity. I summarize its main implications here and refer to the reader to the Appendix for a complete derivation and to those papers for a thorough discussion.

In equilibrium, individual consumption/income is related to aggregate income by:

$$\begin{aligned} c_t^H &= y_t^H = \chi y_t, \chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right) \lessgtr 1; \\ c_t^S &= \frac{1-\lambda\chi}{1-\lambda} y_t. \end{aligned} \tag{12}$$

The composite parameter χ is the model's keystone: a sufficient statistic that in this specific model depends on fiscal redistribution $1 - \tau^D/\lambda$ and labor market characteristics φ. It is important to stress that this is but one possible simple theory of the income distribution.[15] Equilibrium **cyclical income inequality** γ_t, the log deviation of Γ_t, reveals that Γ_Y/Γ in (10) is proportional to $1-\chi$:

$$\gamma_t \equiv y_t^S - y_t^H = (1-\chi) \frac{y_t}{1-\lambda}. \tag{13}$$

Inequality is *procyclical* ($\partial\gamma/\partial y > 0$) iff $\chi < 1$ and *countercyclical* ($\partial\gamma/\partial y < 0$) iff $\chi > 1$.

Distributional considerations make χ different from 1. In RANK, such considerations are absent since one agent works and receives all the profits. When aggregate income goes up, labor demand, real wages and marginal cost increase. This decreases profits, but because the *same* agent incurs both the labor gain and the profit loss, the redistribution of income across factors is neutral.

Take now the case with heterogeneity and $\chi > 1$, i.e. fiscal redistribution of profits that is not skewed towards H, $\tau^D < \lambda$ *and* upward-sloping labor supply $\varphi > 0$.[16] If demand goes up, the wage goes up, H's income increases and so does their demand. Thus aggregate demand increases by *more* than initially, shifting labor demand and increasing the wage even further, and so on. In the new equilibrium, the extra demand is produced by S, whose decision to work more is optimal given the income loss from falling profits of which they get a disproportionate share even post-redistribution.

With $\chi < 1$ (when H receive a disproportionate share of the profits $\tau^D > \lambda$) the AD expansion is instead *smaller* than the initial impulse, as H recognize that this will lead to a fall in their income;

[15] Several subsequent models deliver different distributional implications, e.g. using *sticky wages*. See Colciago (2011), Ascari et al (2017) and Walsh (2018) in TANK; Broer et al (2020), Hagedorn et al (2018), and Auclert et al (2018) in HANK.

[16] The benchmark used by Campbell and Mankiw's (1989) seminal paper is $\chi = 1$, which occurs when profits are uniformly redistributed $\tau^D = \lambda$ or labor is infinitely elastic $\varphi = 0$; income inequality is then *acyclical*. See also Bilbiie (2008, footnote 14).

while S, given the positive income effect from profits, optimally work less. As the income of H now moves less than proportionally with aggregate income, inequality is *procyclical*.

Replacing the individual (12) in the self-insurance equation (11), we obtain the *aggregate Euler-IS*:

$$c_t = \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi} \left(i_t - E_t \pi_{t+1} \right), \text{ where } \delta \equiv 1 + (\chi - 1) \frac{1-s}{1-\lambda\chi}. \tag{14}$$

The contemporaneous AD elasticity to interest rates is the TANK one, $\sigma \frac{1-\lambda}{1-\lambda\chi}$, reflecting the described New Keynesian Cross logic. Although the direct effect of interest rates is scaled down by $1-\lambda$, the indirect effect is increasing with λ at rate χ; with $\chi > 1$ the latter dominates, delivering amplification relative to RANK (dampening, for $\chi < 1$). $\lambda\chi$ is thus akin to an *aggregate-MPC* slope of a planned-expenditure line: it yields amplification when $\chi > 1$, for then the slope increase dominates the decrease in the shift of this line due to adding λ agents who are directly insensitive to policy.[17]

The key novelty of THANK's aggregate Euler-IS equation relative to TANK is that it is characterized by **compounding** $\delta > 1$ iff inequality is countercyclical $\chi > 1$ and **discounting** $\delta < 1$ if procyclical $\chi < 1$ (see Proposition 3 in the companion paper Bilbiie (2020)). In RANK and TANK, good future income news imply a one-to-one demand increase today as households (who can) substitute consumption towards the present and, with no assets, income adjusts. Discounting occurs when procyclical inequality meets idiosyncratic uncertainty: When good news about future *aggregate* income arrive, households recognize that in some states they will be constrained and not benefit fully from it. They self-insure, increasing their consumption less than if there were no uncertainty: the saving demand increase cannot be accommodated (there is no asset), so income falls accordingly.

Countercyclical inequality leads to compounding instead. The Keynesian-cross amplification extends *intertemporally*: good aggregate income news boost today's demand because they imply less need for self-insurance. Since future income in states where the constraint binds over-reacts to good aggregate news, households demand *less* saving. With zero savings in equilibrium, households consume more than one-to-one and income increases more than without risk.

The foregoing focuses on cyclical inequality and embeds a notion of idiosyncratic risk that is intimately related to whether liquidity constraints bind or not but is by construction *acyclical* as can be seen by recalling the discussion after (10): locally around $\Gamma = 1$ variance is *acyclical* because it is proportional to $\ln \Gamma$. Idiosyncratic risk itself may still be cyclical, but this has locally no first-order effect on the variance, on precautionary saving, and thus on Euler discounting-compounding.

The other useful special case illustrating the independent role of cyclical inequality is the limit $s = 0$, with no risk at all as the conditional variance is nil.[18] Even in that extreme case, my model implies Euler discounting-compounding with, replacing $s = 0$ and $\lambda = \frac{1}{2}$ in (14):

$$\delta|_{s=0} = \frac{\chi}{2-\chi} \lesseqgtr 1 \text{ iff } \chi \lesseqgtr 1. \tag{15}$$

There is again discounting with pro- and compounding with counter-cyclical inequality. These two observations illustrate clearly that cyclical risk is *not necessary* for Euler-equation discounting/compounding: cyclical inequality is *sufficient*, combined with a self-insurance motive.

[17] As shown in Bilbiie (2020) the aggregate MPC out of *aggregate* income is a convex combination of the two out-of-own-income MPCs weighed by the elasticities to the aggregate, $mpc = (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda} + \lambda \times 1 \times \chi = 1 - \beta(1-\lambda\chi)$.

[18] This limit case is akin to Woodford (1990), abstracting from the endogenous income distribution that is of the essence here.

2.3 Cyclical Inequality and Risk

I now turn cyclical risk back on, both by allowing the probability s to depend on the cycle *and* approximating around a steady state with inequality $\Gamma > 1$. Risk is cyclical through both channels, see (10), and matters *to first order*—the loglinearized aggregate Euler-IS becomes (see Appendix B):

$$c_t = \Big[\underbrace{\tilde{\delta}}_{\text{cycl.-ineq HANK}} + \underbrace{\eta}_{\text{cycl.-risk HANK}} \Big] E_t c_{t+1} - \underbrace{\sigma \frac{1-\lambda}{1-\lambda\chi} \left(i_t - E_t \pi_{t+1} \right)}_{\text{cyclical-inequality TANK}} \tag{16}$$

$$\begin{aligned} \text{with } \tilde{\delta} &\equiv \delta + \left(\Gamma^{1/\sigma} - 1 \right) (\delta - 1) \tilde{s} = 1 + \frac{(\chi - 1)(1 - \tilde{s})}{1 - \lambda\chi} \\ \text{and } \eta &\equiv \frac{s_Y Y}{1-s} \left(1 - \Gamma^{-1/\sigma} \right) (1 - \tilde{s}) \sigma \frac{1-\lambda}{1-\lambda\chi}, \end{aligned}$$

where $1 - \tilde{s} = \frac{(1-s)\Gamma^{1/\sigma}}{s + (1-s)\Gamma^{1/\sigma}} > 1 - s$ is the inequality-weighted transition probability measure of risk.

There are thus two effects of income risk cyclicality. The first comes from income inequality, which around a steady-state with $\Gamma > 1$ and $s > 0$ *also* makes risk cyclical, as discussed above and captured by $\left(\Gamma^{1/\sigma} - 1 \right) (\delta - 1) \tilde{s}$. When $\delta > 1$ there is an additional source of compounding that increases with the *level* of inequality. The second effect encapsulates "pure" (independent on cyclical inequality) cyclical risk through its key determinant: the elasticity $-s_Y Y / (1 - s)$, the second term in (10) above, which determines the novel parameter η. Dampening/amplification of future shocks *only* occurs through η even with acyclical inequality $\delta = 1$. Procyclical risk $\eta < 0$ implies Euler *discounting*: good news generate an expansion today to start with, which increases the probability of moving to the bad state and triggers precautionary saving, containing the expansion. Conversely, countercyclical risk generates *compounding*: an aggregate expansion reduces the probability of moving to the bad state and mitigates the need for insurance, amplifying the initial expansion.[19]

This formalization of cyclical risk has similar reduced-form implications for the link between current and *future* consumption to the cyclical-inequality channel, but the underlying economic mechanism is different. Furthermore, while η is observationally equivalent to Acharya and Dogra's (2020) different formalization with CARA preferences leading to a P(seudo-)RANK, the underlying implications for risk are also different. In my model, η captures the cyclicality of *skewness*, a key element of the reviewed evidence; whereas Acharya and Dogra's PRANK relies on symmetry (normal shocks), abstracting from skewness to focus on variance. Ravn and Sterk (2020)'s model delivers something akin to η based on search and matching, but abstracts from cyclical inequality. While Werning's (2015) general non-linear model contains both channels but without the distinction and decomposition and without identifying their differential effects on transmission.

The pure-risk channel captured by η operates *only if* there is long-run *level* inequality $\Gamma > 1$, the literal *risk* of moving to a *lower income level*. Whereas the cyclical-inequality channel relies only on the cyclicality of income when constrained χ. Both channels capture precautionary saving: the former, through the effect of uncertainty and the third derivative of the utility function (η is proportional to prudence σ); the latter, through the effect of constraints, a separate source of concavity in the consumption function, a manifestation of the general results in Carroll and Kimball (1996).

[19] The Appendix studies an alternative where s depends on current Y_t, delivering contemporaneous amplification.

Comparing (16) with its RANK counterpart reveals directly three separate *endogenous wedges* corresponding to each channel; in a recent contribution, Berger et al (2021) provide a valuable empirical "accounting" of related wedges, including a decomposition between the ability to smooth shocks versus the volatility of labor income. Such exercises may help disentangle the signs of $\delta - 1$ and η which, as we will see next, are crucial for the model's properties.

3 THANK Analytics: Determinacy, Puzzles, and Amplification

This section exploits tractability to conduct a pencil-and-paper, full analysis of some main NK topics: determinacy, solving the FG puzzle, and conditions for amplification-multipliers.

3.1 HANK, Taylor, and Sargent-Wallace

I now solve the model under further assumptions delivering a *one-equation representation* that may be useful in different contexts where reducing dimensionality is necessary for closed-form solutions. In particular, first, the nominal rate i_t follows a Taylor rule (we study other policies momentarily):

$$i_t = \phi \pi_t. \tag{17}$$

The model is completed by adding the simple aggregate-supply, Phillips-curve specification (8); all the results carry through with the more familiar forward-looking (7) as I show in Appendix C.

With this simple RANK-isomorphic HANK we first revisit a classic determinacy result and derive a *HANK* Taylor principle. Replacing (8) and (17) in (14), THANK collapses to *one* equation:

$$c_t = \nu E_t c_{t+1}, \text{ where } \nu \equiv \frac{\delta + \kappa\sigma \frac{1-\lambda}{1-\lambda\chi}}{1 + \phi\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}} \tag{18}$$

captures the effect of *good news* on AD, and the elasticity to interest rate shocks.

There are three channels shaping this key summary statistic. First, the "pure AD" effect through δ coming from cyclical inequality, operating even with fixed prices or fixed real rate $i_t = E_t \pi_{t+1}$. Second, a supply feedback *cum* intertemporal substitution: the inflationary effect (κ) of good income news triggers *ceteris paribus* a fall in the real rate and intertemporal substitution towards today, the magnitude of which depends on static amplification/dampening $\frac{1-\lambda}{1-\lambda\chi}$. Finally, all this demand amplification generates *inflation* and real rate movements. When policy is active $\phi > 1$, a higher real rate and a contractionary effect today ensue, the strength of which depend on cyclical-inequality. These considerations drive Proposition 1 (the case with NKPC (7) is in Appendix C.1).

Proposition 1 ***The HANK Taylor Principle****: The HANK model under a Taylor rule (18) has a determinate, locally unique rational expectations equilibrium if and only if (as long as $\lambda < \chi^{-1}$):*

$$\nu < 1 \Leftrightarrow \phi > \phi^* \equiv 1 + \frac{\delta - 1}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}.$$

The ***Taylor principle*** *$\phi > 1$ is sufficient for determinacy if and only if there is Euler-IS discounting: $\delta \leq 1$.*

The proposition follows by recalling that the determinacy requirement is that the root ν be inside the unit circle; in the discounting case $\delta < 1$, the threshold ϕ is evidently *weaker* than the Taylor principle, while in the compounding case it is *stronger*. With countercyclical inequality and risk $\delta > 1$, a future *aggregate* sunspot increase in income generates a disproportionate increase in income in the bad state, and thus incentives to dis-save and a further demand boost today–making the sunspot self-fulfilling even with a fixed real rate $\phi = 1$. The central bank needs to do more to counteract this and prevent the sunspot from becoming self-fulfilling. (The opposite holds in the *discounting* case and the Taylor principle is sufficient for determinacy.) The Taylor threshold $\phi > 1$ reappears for either of $\chi = 1$ (acyclical inequality), $s \to 1$ (no risk), or $\kappa \to \infty$ (flexible prices). The determinacy region for ϕ squeezes very rapidly with countercyclical inequality because of a complementarity between idiosyncratic and aggregate risk apparent from $\phi^* = 1 + \frac{(\chi-1)(1-\bar{s})}{\kappa\sigma(1-\lambda)}$. The threshold depends on price stickiness because policy responds to inflation, but the relevant amplification goes through real demand's equilibrium response, and price stickiness modulates the relationship between the two. If instead policy responds to real activity $i_t = \pi_t + \phi_c c_t$, the determinacy threshold is $\phi_c > \frac{(\chi-1)(1-s)}{(1-\lambda)\sigma}$ and no longer depends on price stickiness because policy then acts directly on demand.

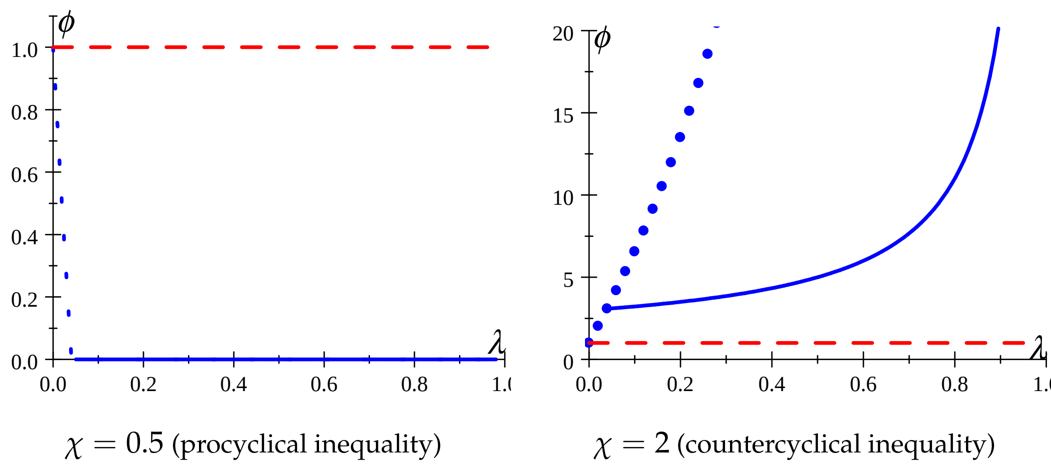

Fig. 1: Taylor threshold ϕ^* with $1 - s = 0$ (dash, TANK); 0.04 (solid); λ (dots). Note: determinacy above the curve.

Figure 1 plots the threshold ϕ^* as a function of λ (for $\lambda < \chi^{-1}$) for different $1 - s$, with procyclical inequality in the left panel and countercyclical in the right. The parametrization assumes $\kappa = 0.02$, $\sigma = 1$, and $\varphi = 1$. In the countercyclical-inequality case, the threshold increases with λ and does so at a faster rate with higher risk $1 - s$. The required response can be large: for the calibration used in Bilbiie (2020) to match Kaplan et al's quantitative HANK aggregate outcomes ($\chi = 1.48$, $\lambda = 0.37$, $1 - s = 0.04$) it is $\phi^* = 2.5$ and can be as high as 5 for other calibrations therein. With procyclical inequality, the Taylor principle is *sufficient* but *not necessary* for determinacy. For a large subset of the region, there is in fact determinacy even under a peg $\phi = 0$, undoing the classic Sargent and Wallace (1975) result, namely if and only if $\phi^* < 0$ or:

$$\nu_0 \equiv \delta + \kappa\sigma\frac{1-\lambda}{1-\lambda\chi} < 1. \tag{19}$$

With enough discounting, the sunspot is ruled out by the economy's endogenous forces, unlike in

RANK where $\nu_0 = 1 + \kappa\sigma \geq 1$; as we shall see, (19) *also* rules out the forward guidance puzzle.

Finally, since adding cyclical risk is isomorphic to a change in δ, the threshold becomes:

$$\phi^* \equiv 1 + \frac{\tilde{\delta} - 1 + \eta}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}, \tag{20}$$

with the different intuition discussed above for AD amplification/dampening through η.[20]

3.2 A Catch-22 for HANK: No Puzzle, No Amplification?

We are now in a position to state the Catch-22: the closed-form conditions for *amplification* in THANK are the opposite of those needed to solve the forward guidance puzzle. To state this formally, we introduce two policy shocks: discretionary exogenous changes in interest rates i_t^* in the Taylor rule $i_t = \phi\pi_t + i_t^*$; and public spending: the government buys an amount of goods G_t with zero steady-state value ($G = 0$) and taxes all agents uniformly to finance it.[21] Straightforward derivation delivers the aggregate Euler-IS, starting with *cyclical inequality* only (extending (14)):

$$c_t = \delta E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t\pi_{t+1}\right) + \zeta\left[\frac{\lambda\left(\chi-1\right)}{1-\lambda\chi}\left(g_t - E_t g_{t+1}\right) + \left(\delta-1\right)E_t g_{t+1}\right], \tag{21}$$

where $\zeta \equiv \varphi\sigma/\left(1+\varphi\sigma\right)$. Together with the static PC $\pi_t = \kappa c_t + \kappa\zeta g_t$ and AR(1) spending $E_t g_{t+1} = \mu g_t$, this delivers Proposition 2 (see Appendix C.2 for the general case with NKPC (7)).

Proposition 2 ***A Catch-22 for HANK:*** *In THANK with cyclical inequality, there is amplification of monetary policy relative to RANK and the fiscal multiplier on consumption is positive if and only if:*

$$\chi > 1,$$

whereas the forward-guidance puzzle is ruled out ($\frac{\partial^2 c_t}{\partial\left(-i_{t+T}^*\right)\partial T} < 0$*) only if*

$$\chi < 1.$$

The first part pertains to amplification with respect to RANK, the focus of the majority of quantitative HANK studies. Kaplan et al (2018) show that HANK yields *monetary* policy amplification through "indirect", general-equilibrium forces; similar insights apply to Auclert (2019), Gornemann et al (2015), and Debortoli and Galí (2018). Such amplification occurs only with countercyclical inequality (Bilbiie (2020) calibrates TANK and the acyclical-risk zero-liquidity THANK to match the

[20]Ravn and Sterk (2020) show that SaM delivers $\eta > 0$ endogenously, making the Taylor principle insufficient. In works *subsequent* to this paper's determinacy Proposition 1: Acharya and Dogra (2020) derived a Taylor principle and Auclert et al (2018) provided numerical simulations in a quantitative HANK on the role of *cyclical risk* for determinacy.

[21]The redistribution of the taxation financing spending is essential for the multiplier, see Bilbiie (2020) in TANK: I sidestep it assuming uniform taxation. See Bilbiie, Monacelli, and Perotti (2013) in TANK, and Oh and Reis (2012), Ferrière and Navarro (2018), Hagedorn et al (2018) and Auclert et al (2018) in quantitative HANK for multipliers with progressivity.

aggregate predictions of these quantitative models). The *fiscal multiplier* in THANK is:

$$\frac{\partial c_t}{\partial g_t} = \frac{1}{1-\nu\mu}\frac{\zeta}{1+\phi\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}\left[\underbrace{(\chi-1)\frac{\lambda(1-\mu)+(1-s)\mu}{1-\lambda\chi}}_{\text{TANK + HANK AD}}-\underbrace{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}(\phi-\mu)}_{\text{RANK AS}}\right]. \tag{22}$$

With fixed prices $\kappa = 0$ and proportional incomes $\chi = 1$, one recovers the benchmark zero-multiplier derived in RANK by Bilbiie (2011) and Woodford (2011). Positive multipliers occur with *countercyclical inequality* $\chi > 1$: spending has a demand effect that increases labor demand, wages, the income of H, and so on: the "new Keynesian cross" channel.[22] If the stimulus is persistent ($\mu > 0$), there is an extra kick through self-insurance: as agents expect higher demand and income, with $\chi > 1$ they expect even higher income in the H state and thus less need to self-insure.[23]

The second part of Proposition 2 pertains to solving the forward guidance puzzle. The condition is $\nu_0 < 1$, i.e. determinacy under a peg, found by iterating forward (21) with $\phi = 0$ to obtain:

$$c_t = \nu_0 E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}i_t^* = \nu_0^{\bar{T}} E_t c_{t+\bar{T}} - \sigma\frac{1-\lambda}{1-\lambda\chi}E_t\textstyle\sum_{j=0}^{\bar{T}-1}\nu_0^j i_{t+j}^*.$$

The time-t response to time-$t+T$ interest rate cut is, for any $T \in (t, \bar{T})$: $\frac{\partial c_t}{\partial(-i_{t+T}^*)} = \sigma\frac{1-\lambda}{1-\lambda\chi}\nu_0^T$, which is decreasing in T iff $\nu_0 < 1$ (the derivative is $\sigma\frac{1-\lambda}{1-\lambda\chi}\nu_0^T \ln\nu_0$). Since with $\nu_0 < 1$ the term $\nu_0^{\bar{T}} E_t c_{t+\bar{T}}$ vanishes in the limit as $\bar{T} \to \infty$, we can solve the equation forward finding a *unique* solution, i.e. the earlier determinacy under a peg ($\phi^* < 0$) result. The condition $\nu_0 < 1$ captures a powerful intuition when rewritten as:

$$1-\delta > \kappa\sigma\frac{1-\lambda}{1-\lambda\chi}.$$

To rule out the puzzle, the HANK-discounting on the left side needs to dominate the right-side AS-compounding of news that is the source of trouble in RANK. This entails *jointly* idiosyncratic uncertainty $1-s > 0$ and procyclical enough inequality $\chi < 1-\sigma\kappa\frac{1-\lambda}{1-s} < 1$. One implication is an interpretation of McKay et al (2016), where *procyclical inequality* holds because profits are redistributed disproportionately to low-productivity households, "as if" $\tau^D > \lambda$ in my model (see also Hagedorn et al (2019) for a quantitative illustration).

Proposition 2 purposefully abstracts from cyclical risk. Turning it back on *can in theory* resolve the Catch-22, providing amplification without the puzzle, as emphasized next.

Proposition 3 *THANK with cyclical inequality and risk resolves the Catch-22 if and only if, with countercyclical inequality ($\chi > 1$), pure risk is procyclical enough:*

$$\eta < 1-\tilde{\delta} \leq 0. \tag{23}$$

The condition requires that risk procyclicality through the separate $s(Y)$ channel dominate the countercyclicality through $\chi > 1$, meaning high enough steady-state inequality *in levels* Γ and pru-

[22] The channel is at work in Gali et al's (2007) earliest quantitative model on this (but convoluted with several other channels), as well as in Bilbiie and Straub (2004), Bilbiie, Meier and Mueller (2008), and Eggertsson and Krugman (2012).

[23] The last term extends the usual RANK channel: spending is inflationary, which when $\phi > 1$ increases *real* rates generating intertemporal substitution towards the future.

dence σ, i.e. a strong enough precautionary motive due to uncertainty:

$$-s_Y Y\left(1-\Gamma^{-1/\sigma}\right)\sigma > (1-s)\,\frac{\chi-1}{1-\lambda} > 0.$$

This reflects the dependence of the risk-cyclicality channel on the *level* of inequality; recall that with no inequality in levels $\Gamma = 1$, risk cyclicality is irrelevant for Euler discounting/compounding, while without risk in levels $(1-s=0)$ the cyclicality of inequality is irrelevant. When the two channels coexist and go in opposite directions with the right relative strengths, the Catch-22 can be resolved.

However, when the $\eta < 1-\tilde{\delta}$ condition is not met, cyclical risk *aggravates* the Catch-22. In particular, when risk and inequality are *both countercyclical* THANK delivers amplification and aggravates the puzzles further, while the determinacy conditions with a Taylor rule become even more stringent, see (20). Existing empirical evidence suggests that this is the more plausible scenario. Insofar as it is a counterpart to η, Guvenen et al's (2014) measure of skewness risk cyclicality (with acyclical variance) can be viewed as implying $\eta > 0$; while Storesletten et al's (2004) countercyclical-variance estimates suggest that the overall sign of the elasticity in the variance approximation (10) is negative, and thus the aggregate Euler (16) is compounded $\tilde{\delta}+\eta > 1$. Recent evidence on inequality controlling for MPCs suggests that it is also countercyclical (Patterson (2019)). A fruitful complementary approach is accounting of Euler wedges directly, see Berger et al (2021).

Two caveats to interpreting this evidence through the model's lens are necessary. First, all the existing evidence pertains to *earnings*, whereas the model-relevant object is total income inclusive of both financial income and transfers. And second, to my knowledge none of the available evidence isolated and disentangled the two channels in a model-equivalent way, i.e. provide identified parameter estimates for Γ, χ, s, and η. In that regard, the paper provides theoretical restrictions to inform that measurement, as well as reasons for why that is important for macro transmission.

Both the Catch-22 and the FG puzzle, however, are model properties contingent upon the Taylor rule and intimately related to determinacy: I next study two different policy rules that alleviate these issues, ensuring determinacy and sidestepping the Catch-22.

3.3 Virtues of a Wicksellian, price-level targeting rule in THANK

Indeterminacy under Taylor rules is pervasive in HANK economies with countercyclical inequality and risk. What *can* the central bank do to anchor expectations, when for a standard calibration it would need to change nominal rates by 5 percent if inflation changed by one percent? One solution is to adopt the *"Wicksellian" policy rule* of price level targeting which, as shown by Woodford (2003) and Giannoni (2014), yields determinacy in RANK:

$$i_t = \phi_p p_t \text{ with } \phi_p > 0, \tag{24}$$

This rule is especially powerful in HANK, as emphasized in the following Proposition.

Proposition 4 ***Wicksellian rule in HANK***: *In the THANK model, the Wicksellian rule (24) leads to local determinacy even when $\delta > 1$. Thus, the model delivers "amplification" without also aggravating the FG puzzle even when both inequality and risk are countercyclical.*

The simple proof is outlined in Appendix C.3 (and extended to NKPC in Appendix C.4); the intuition is that, no matter how strong the AD-amplification, this rule anchors long-run expectations. Agents recognize that bygones are not bygones: adjustment will eventually take place, as inflation will *a fortiori* imply future deflation. The same intuition applies for ruling out the FG puzzle while delivering amplification, resolving the Catch-22. The essence is that under the Wicksellian rule THANK reduces, instead of one difference equation (18), to a *second-order* equation; replacing (8) rewritten with the price level $p_t - p_{t-1} = \kappa c_t$ and (24) in (16) delivers:

$$E_t p_{t+1} - \left[1 + \nu_0^{-1}\left(1 + \sigma \frac{1-\lambda}{1-\lambda\chi}\phi_p \kappa\right)\right] p_t + \nu_0^{-1} p_{t-1} = \sigma \frac{1-\lambda}{1-\lambda\chi}\kappa \nu_0^{-1} i_t^*. \tag{25}$$

It is easy to show that (25) has a unique solution iff $\phi_p > 0$, and that the effect of an interest cut $\partial c_t / \partial \left(-i_{t+T}^*\right)$ decreases with the time T, i.e. the FG puzzle disappears. Intuitively, the puzzle's source is indeterminacy under a peg and a Wicksellian rule provides determinacy under a "quasi-peg": *some*, no matter how small response to the price level anchors long-run expectations. This is particularly important in HANK, for even when heterogeneity *aggravates* the puzzle, the rule restores standard logic and resolves the "Catch-22".[24]

This paper assumes throughout a passive-Ricardian fiscal policy. A different route to determinacy and solving the puzzle is to resort to an active, non-Ricardian fiscal rule that does *not* ensure debt repayment for any price level (Leeper (1991); Woodford (1996); Cochrane (2017)). In incomplete-market economies, yet a different fiscal policy can deliver determinacy, as argued by Hagedorn (2020); to study it in my model, we need to turn to the equilibrium with liquidity.

4 THANK with Liquidity

In the version with liquidity in the form of government bonds used for precautionary saving, I derive the savings demand and study the intertemporal propagation and determinacy implications.

4.1 Savings-Liquidity Demand

Denote by B_{t+1}^N the total *nominal* quantity of government bonds outstanding at the end of each period. In nominal terms, $B_{t+1}^N = (1 + i_{t-1}) B_t^N - P_t T_t$, and in real terms:

$$B_{t+1} = R_t B_t - T_t \tag{26}$$

where $R_t = \frac{1+i_{t-1}}{1+\pi_t}$ is the gross interest rate. The bond market clears $B_{t+1} = \lambda Z_{t+1}^H + (1-\lambda) Z_{t+1}^S$. Recall now that $Z_{t+1}^H = 0$, so that $B_{t+1} = (1-\lambda) Z_{t+1}^S$ and using the flow definitions:

$$B_{t+1}^H = (1-h) Z_{t+1}^S = \frac{1-h}{1-\lambda} B_{t+1} = \frac{1-s}{\lambda} B_{t+1},$$

[24] A hitherto unnoticed to my knowledge corrollary is that in RANK too, the puzzle disappears under a Wicksellian rule.

and for S similarly $B^S_{t+1} = sZ^S_{t+1} = \frac{s}{1-\lambda} B_{t+1}$. The respective budget constraints imply:

$$\begin{aligned} C_t^H &= \hat{Y}_t^H + \frac{1-s}{\lambda} R_t B_t \\ C_t^S + \frac{1}{1-\lambda} B_{t+1} &= \hat{Y}_t^S + \frac{s}{1-\lambda} R_t B_t, \end{aligned} \tag{27}$$

where $\hat{Y}_t^j$ is j's disposable (net of taxes) income. Savers hold all bonds for next period; because bonds are liquid a fraction $\frac{1-s}{\lambda}$ of the payoff, including interest, accrues to next period's hand-to-mouth.

In Appendix D, I derive the steady-state demand for liquidity-bonds, or savings function:

$$B = \frac{1}{1 - R\left(1 - \frac{1-s}{\lambda}\right)} \left[\frac{1}{1 + \left(\frac{1}{\beta R} - 1\right) \frac{1-\lambda}{1-s}} - \frac{1}{1 + (1-\lambda)(\Gamma - 1)} \right], \tag{28}$$

under log utility $\sigma = 1$, normalizing $Y = 1$; (28) is an increasing convex function of R under standard restrictions. The Appendix explores this analytically in detail, but is is worth noticing that as $\beta R \to 1$ debt becomes proportional to $(1-\lambda)(\Gamma - 1)$ and thus tends to zero with no steady-state income inequality $\Gamma = 1$. The condition for a positive liquidity demand $B > 0$ is:

$$\frac{1-s}{\lambda} > 1 - \beta; \tag{29}$$

this is strictly true when $\beta R \to 1$ and translates into more general restrictions on $1-s,\ \lambda,\ \beta$, see appendix D. Essentially, (29) requires *some* idiosyncratic risk and liquidity; it is violated e.g. in TANK. One intuitive interpretation is that positive long-run liquidity, self-insurance savings requires that the present discounted value of the risk of becoming constrained (the infinite discounted sum of $1-s$ at rate β) be larger than the *unconditional*, long-run probability of being constrained.

4.2 Liquidity, Inequality, and Intertemporal MPCs in THANK

This version embeds a distinct amplification channel *orthogonal* to the NK Cross, the "intertemporal Keynesian cross" of Auclert et al (2018) (see also Hagedorn et al (2018)), and allows a novel analytical solution for their key summary statistics, the intertemporal MPCs (iMPCs). Loglinearizing (27) around a zero-liquidity steady state with $R = \beta^{-1}$ delivers (see Appendix D):

$$\begin{aligned} c_t^H &= \hat{y}_t^H + \beta^{-1} \frac{1-s}{\lambda} b_t, \\ c_t^S + \frac{1}{1-\lambda} b_{t+1} &= \hat{y}_t^S + \beta^{-1} \frac{s}{1-\lambda} b_t, \end{aligned} \tag{30}$$

where b_t is in shares of steady-state Y. Aggregating (30), we have:

$$c_t = \hat{y}_t + \beta^{-1} b_t - b_{t+1}. \tag{31}$$

The iMPCs are the partial derivatives of aggregate consumption c_t with respect to *aggregate* disposable income $\hat{y}_{t+k}$ at different horizons k, keeping fixed everything else (i.e. taxes and public debt). To find them, we solve for equilibrium liquidity b_t replacing (30) in the self-insurance equation (11). The general case is analyzed in Appendix D, but the intuition is clearest in the oscillating case $s = 0$, with agents saving when they expect lower income and vice versa:

$$b_{t+1} = \frac{\hat{y}_t^S - E_t \hat{y}_{t+1}^H}{2\left(1+\beta^{-1}\right)} = \frac{(2-\chi)\,\hat{y}_t - \chi E_t \hat{y}_{t+1}}{2\left(1+\beta^{-1}\right)}. \tag{32}$$

The consumption function follows by substituting this into (31), delivering Proposition 5.

$$c_t = \frac{2-\chi+\beta\chi}{2(1+\beta)}\hat{y}_t + \frac{2-\chi}{2(1+\beta)}\hat{y}_{t-1} + \frac{\beta\chi}{2(1+\beta)}\hat{y}_{t+1}; \tag{33}$$

Proposition 5 *The iMPCs for THANK with $s = 0$ in response to a time-T disposable income shock are:*

$$\frac{dc_T}{d\hat{y}_T} = \frac{2-\chi+\beta\chi}{2(1+\beta)};\ \frac{dc_{T+1}}{d\hat{y}_T} = \frac{2-\chi}{2(1+\beta)};\ \frac{dc_{T-1}}{d\hat{y}_T} = \frac{\beta\chi}{2(1+\beta)};\ \frac{dc_t}{d\hat{y}_T} = 0 \textit{ for any } t < T-1 \textit{ or } t > T+1.$$

This illustrates the key points transparently. With acyclical inequality $\chi = 1$ ($\hat{y}_t^j = \hat{y}_t$, Auclert et al's case) a current income shock induces agents to self-insure, saving in liquidity to maintain higher future consumption. While a *future* shock makes them consume in anticipation, depleting liquid savings. The second point concerns adding cyclical inequality: higher income cyclicality when constrained χ makes agents consume more out of news and less out of past and current aggregate income. When self-insuring, agents take into account how the aggregate shock affects income in each state and change their asset demand and equilibrium liquidity consequently. Even this simplest $s = 0$ case can then match the two key iMPCs matched by Auclert et al, the contemporaneous $dc_0/d\hat{y}_0 = 0.55$ and one-year-after $dc_1/d\hat{y}_0 = 0.15$ with $\beta = 0.95$ annually and $\chi = 1.47$.

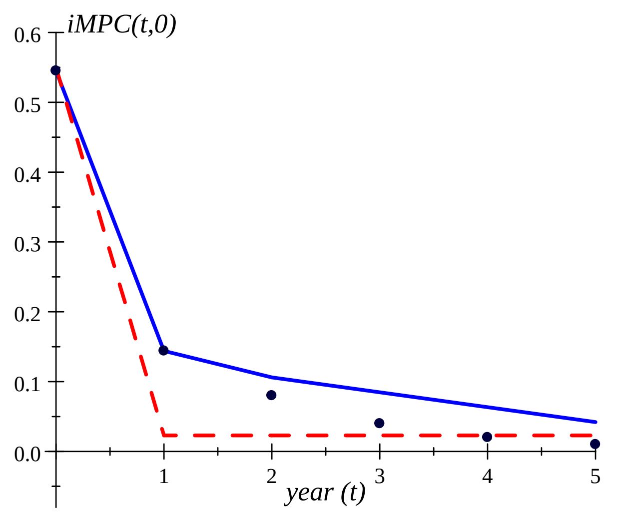

Figure 2: iMPCs in THANK (blue solid); TANK (red dash); Data (dots)

The expressions for THANK with $s > 0$ are still analytical and convey the same intuition, but are more tedious (see Proposition 9, Appendix D). Figure 2 plots the iMPCs for THANK, along with TANK and the data from Fagereng et al. In THANK, I match the two target MPCs with $\lambda = 0.33$,

$s = 0.82$ (0.96 quarterly) and $\chi = 1.4$.[25] The intertemporal path is remarkably in line with both the data and Auclert et al's quantitative HANK: the effect dies off a few years after, whereas TANK misses this intertemporal amplification altogether, the iMPCs being:

$$\frac{dc_T}{d\hat{y}_T} = \lambda\chi + (1-\lambda\chi)(1-\beta)\beta^T; \frac{dc_t}{d\hat{y}_T} = (1-\lambda\chi)(1-\beta)\beta^T \; \forall t \neq T.$$

4.3 Determinacy with Liquidity

I now provide two determinacy results that are analytical counterparts of results in the quantitative literature: a criterion based on iMPCs as in Auclert et al; and a debt quantity rule à la Hagedorn.

My HANK Taylor principle in Proposition 1 is intimately related to subsequent results developed for quantitative HANK by Auclert et al (2019) showing that the Taylor principle is sufficient when the sum of iMPCs is larger than 1. The intuition is that if the total MPC out of an income shock far into the future is larger than 1, the model is "explosive" (stable forward) and thus determinate. The connection can be clearly seen by summing up the iMPCs in Proposition 5 to obtain:

$$\mu_{impc} = 1 + (1-\chi)\frac{1-\beta}{1+\beta} > 1 \text{ iff } \chi < 1. \tag{34}$$

There is determinacy (the Taylor principle is sufficient) with procyclical inequality and indeterminacy otherwise. This holds in the general model, as I prove analytically in Appendix D.

In an incomplete-markets economy, a further route to determinacy was discovered by Hagedorn (2020). If the government chooses the quantity of nominal debt, the price level can be determined without an interest-rate rule. Consider a rule that chooses nominal debt and sets nominal taxes so as to balance the budget intertemporally for any price level, thus making policy passive-Ricardian and ruling out fiscal theory. In this arrangement, the central bank sets freely the nominal interest rate that clears the liquid-bond market, with no need to respond to any endogenous variable.

Given the steady-state demand for bonds (28), determinacy of the *steady-state* price level is immediate: the proof is exactly as in Hagedorn (2020).[26] More interesting is that local determinacy under a debt quantity rule holds in my model too, as shown in the next Proposition.

Proposition 6 ***A nominal debt rule****: The THANK model with a well-defined demand for liquid bonds ((29) holds) leads to local determinacy even when $\delta > 1$ under the nominal-debt quantity rule:*

$$B^N_{t+1} = B^N \textit{ fixed } \rightarrow b^N_{t+1} = 0. \tag{35}$$

Intuitively, condition (29) requires that H agents receive a fraction of savings that is larger than the interest income, $\beta^{-1}\frac{1-s}{\lambda} > \beta^{-1} - 1 = r$; more generally, it requires that in steady state H agents receive positive net income from liquidity, see Appendix D.

The proposition generalizes to $b^N_{t+1} = \phi_b p_t$ with $\phi_b < 1$ so that real debt $b_{t+1} = (\phi_b - 1) p_t$ falls when the price level increases. Furthermore, it can be easily shown using the same proof structure

[25] This is coincidentally close to the calibration in Bilbiie (2020) matching *general-equilibrium* statistics with the zero-liquidity model. Figure D1 in the Appendix provides a comparison of different calibrations, and iMPCs in response to future shocks. See Cantore and Freund (2021) for a subsequent, simpler analytical iMPC calculation with portfolio adjustment costs.

[26] In a nutshell, monetary policy chooses steady-state i, which given π determines R and thus the steady-state real B. The fiscal authority's choice of nominal B^N (and its growth rate) then immediately determines P (and steady-state π).

as for the Wicksellian rule that it holds with forward-looking Phillips curve and that it rules out the FG puzzle even in the "amplification" region, thus sidestepping the Catch-22. See Hagedorn (2020) for further discussion and a more general version of those arguments, and Hagedorn et al (2018) for an analysis of the implications for fiscal multipliers and the earliest illustration of how a quantitative HANK model with this policy rule sidesteps the Catch-22.

5 Optimal Policy in THANK

THANK is also useful for studying optimal monetary policy analytically, in the version **without liquidity**. This provides a benchmark that helps elucidate some key mechanisms operating in the rich-heterogeneity quantitative-HANK studies featuring several additional relevant channels, such as Bhandari et al (2021). I build on Woodford's (2003, Ch. 6) analysis in RANK. In Appendix E, I spell out the full Ramsey problem and derive a linear-quadratic problem equivalent to it under certain conditions, taking a second-order approximation to aggregate welfare around a flexible-price equilibrium that is *efficient*. The target equilibrium of the central bank is the socially-desirable, *perfect-insurance* equilibrium induced by a fiscal policy generating zero profits to first order under flex prices, following the TANK analysis in Bilbiie (2008, Proposition 5). This delivers Proposition 7.

Proposition 7 *Solving the welfare maximization problem is equivalent to solving:*

$$\min_{\{c_t,\pi_t\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \underbrace{\pi_t^2 + \alpha_y y_t^2}_{RANK} + \underbrace{\alpha_\gamma \gamma_t^2}_{inequality\text{-}THANK} \right\}, \tag{36}$$

s.t. (7),(13), and (14),

where the optimal weights on output and inequality stabilization are, respectively:

$$\alpha_y \equiv \frac{\sigma^{-1} + \varphi}{\psi};\ \alpha_\gamma \equiv \lambda \left(1 - \lambda\right) \sigma^{-1} \varphi^{-1} \alpha_y.$$

Several results are worth emphasizing. While the weight on output (gap) stabilization α_y is the same as in RANK, there is an additional term pertaining to *income inequality*.[27] This evidently affects the central bank's stabilization tradeoff, introducing a redistribution motive. However, idiosyncratic risk and its cyclicality are *irrelevant* for optimal policy, insofar as the target flexible-price equilibrium is the first-best with perfect insurance, without inequality; thus, the aggregate implications of the distributional channel for optimal policy in THANK happen to be the same as in TANK.

This is different from Challe (2020), which abstracts from inequality altogether but where an isomorphism occurs between *RANK* and a *different* analytical HANK with cyclical risk through search and matching. The common point is that my framework too features irrelevance of income risk, but in THANK relative to TANK. Both the optimal allocation *and* the interest rate policy instrument are radically different, and depend crucially on the cyclicality of inequality here. Furthermore, as we shall see, in my framework cyclical risk does not matter for implementation either.

[27] Other studies found additional stabilization motives using different TANK extensions, e.g. Nistico (2016) and Curdia and Woodford (2016) for a financial-stability motive, and Bilbiie and Ragot (2016) for a liquidity-insurance motive with an imperfect-insurance target equilibrium giving rise to a *linear* term in the approximation.

An important observation concerns the interest rate, which is both *residual* to the policy problem and unaffected by *risk cyclicality*. Recalling that we approximate around the efficient equilibrium $\Gamma = 1$, the IS curve (14) is *not* a constraint: as in RANK, it just determines i_t once we found the optimal allocation (y_t, π_t). And since the IS curve approximated around $\Gamma = 1$ is also independent of *cyclical* risk, so will the interest rate that implements optimal policy.[28]

Consider for simplicity *only* shocks that drive *no wedge* between *inequality* and *aggregate output* gap, which stay proportional: (13) holds; the analysis of shocks that *do* drive a wedge is relevant for capturing further mechanisms in richer HANK, but beyond the scope of this paper and pursued in follow-up work. We can simplify the problem by replacing (13), obtaining the per-period loss:

$$\pi_t^2 + \alpha y_t^2, \text{ with } \alpha \equiv \alpha_y \left(1 + \frac{\lambda}{1-\lambda}\sigma^{-1}\varphi^{-1}(\chi - 1)^2\right) \tag{37}$$

The inequality motive thus amounts, in my benchmark THANK relative to RANK, to a higher weight on output stabilization that increases with λ. Importantly, this holds regardless of whether inequality is counter- or pro-cyclical, as long as it is cyclical: the extra stabilization motive is proportional to $(\chi - 1)^2$. The simple intuition is based, as in TANK, on the key role of *profits* which are eroded by inflation volatility. With higher λ, less agents receive profits; the weight on inflation falls, and vanishes in the $\lambda \to 1$ limit, where there is no rationale for stabilizing profit income.

We can now study optimal policy in THANK starting with **discretion**, or Markov-perfect equilibrium. The ability to study this more realistic, time-consistent policy is an appealing feature of the tractable framework, since computing Markov-perfect optimal policies in quantitative models is cumbersome. This is obtained by solving (36) assuming that the central bank lacks commitment and treats expectations parametrically, without internalizing its actions' effect on them; this amounts to re-optimizing every period subject to (7) with fixed expectations at the decision time t. The problem being mathematically identical to RANK, we go directly to the solution:

$$\pi_t = -\frac{\alpha}{\kappa} y_t. \tag{38}$$

This *targeting rule under discretion* requires engineering an aggregate demand decrease for a given inflation increase. Assuming AR(1) cost-push shocks $E_t u_{t+1} = \mu u_t$, the equilibrium is:

$$\pi_t^d = \frac{\alpha}{\kappa^2 + \alpha(1 - \beta\mu)} u_t; \;\; y_t^d = -\frac{\kappa}{\kappa^2 + \alpha(1 - \beta\mu)} u_t. \tag{39}$$

Optimal policy under discretion implies that both output and inflation deviate from target: a trade-off between inflation and output stabilization. Since α is increasing in λ, it follows directly that optimal policy in THANK requires *greater inflation volatility and lower output volatility* than in RANK.

One instrument rule implementing this equilibrium is found by using the aggregate IS (14):

$$i_t = \phi_d^* E_t \pi_{t+1}, \text{ with } \phi_d^* \equiv 1 + \left(\mu^{-1} - \delta\right) \frac{\kappa(1 - \lambda\chi)}{\alpha\sigma(1 - \lambda)}.$$

[28] This is no longer the case—and risk then matters—in the model with liquidity, where the interest rate has direct distributional consequences and thus novel interactions with fiscal policy that are left to future work.

Unlike in RANK, the instrument rule implementing optimal policy may be *passive* $\phi_d^* < 1$ with enough compounding $\delta > \mu^{-1}$, i.e. with countercyclical enough inequality: optimal policy requires a real rate *cut* in THANK when in RANK it would require an increase. Whereas with procyclical inequality $\delta < 1$, the required instrument rule is even more active than in RANK. However, it is independent of the cyclicality of *risk* in this benchmark.

Optimal (timeless-) commitment policy, the time-inconsistent Ramsey equilibrium, requires committing to the different targeting rule, by similar arguments as in RANK (Woodford, 2003, Ch. 7):

$$\pi_t = -\frac{\alpha}{\kappa}\left(y_t - y_{t-1}\right). \tag{40}$$

It is straightforward to show that commitment to (40) delivers determinacy regardless of heterogeneity. The difference from RANK is still captured by the inequality motive shaping α, but optimal commitment policy still amounts to price-level targeting, like in RANK.

6 Conclusions

THANK, a tractable HANK model with two types and two assets, captures analytically several key channels of quantitative HANK models. I use it for a full analysis of the main themes of the NK literature of the past decades: determinacy properties of interest rate rules, amplification, multipliers, resolving the forward guidance puzzle, and optimal monetary policy.

The key channel is *cyclical inequality*: whether the income of constrained hand-to-mouth agents comoves more or less with aggregate income. This channel is already the main focus of TANK in Bilbiie (2008), but interacts with idiosyncratic uncertainty and self-insurance in THANK, as in quantitative HANK models. Procyclical inequality delivers discounting in the aggregate Euler equation, which makes the Taylor principle not necessary for determinacy and can cure the forward guidance puzzle. Conversely, countercyclical inequality generates Euler-equation compounding, making the Taylor principle insufficient for determinacy and aggravating the puzzle. This is a Catch-22, for countercyclicality is precisely the condition for amplification or multipliers in HANK models, which is what many studies focus on, exploiting a New Keynesian cross inherent therein.

The paper proposes a decomposition of cyclical variations in income risk into one component related to cyclical income inequality, and one due to cyclical skewness—variations in the probability of the bad, constrained state. The Catch-22 can be resolved in theory if the latter channel is procyclical enough, as it delivers Euler-discounting without mitigating amplification. As discussed in text, available evidence does not speak directly to this decomposition—the theory developed here can in fact be viewed as potentially informing such measurement. However, existing empirical studies reviewed in text seems to support the view that both (inequality and pure-risk) channels are countercyclical, case in which determinacy conditions become very stringent and the puzzle is aggravated. Nevertheless, I show that a Wicksellian rule of price-level targeting resolves this tension by making THANK determinate and puzzle-free, even with countercyclical inequality *and* risk. This virtue is shared by a rule setting nominal debt proposed by Hagedorn (2020), as I show analytically in my model's version with liquidity.

Optimal monetary policy, solved for analytically in THANK, requires a separate inequality objective, in addition to stabilizing inflation and real activity around an efficient perfect-insurance equilibrium. Regardless of risk, optimal policy implies tolerating more inflation volatility as a result of distributional concerns when inequality is cyclical. While timeless-optimal commitment policy ultimately still amounts to price-level targeting, even though along the adjustment path there it still entails tolerating more inflation.

It is conceivable that for the analysis of many important macroeconomic questions the tractable HANK framework proposed here, THANK, is *sufficient* and one does not always *need* a full-heterogeneity model; the latter is certainly needed for many important questions, e.g. for identifying the most relevant micro heterogeneity dimensions. To date and to the best of my knowledge, THANK is the only tractable framework, among the several reviewed, to capture all of these channels found to be key in rich-heterogeneity models: cyclical income inequality, precautionary self-insurance saving, intertemporal marginal propensities to consume, and features of idiosyncratic income uncertainty and risk (cyclical variance and skewness, and kurtosis).

As models of the economy as a whole become larger and more complex, with many sectors, frictions, and sources of heterogeneity, the quest for tractable representations seems important for entropic reasons. It is my hope that this framework is thus useful for policymakers and central banks, for communicating to the larger public, for students and colleague economists from other fields seeking to enter the fascinating realm of macro stabilization policy in a world where heterogeneity and inequality are of the essence.

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Appendix to **Monetary Policy and Heterogeneity: An Analytical Framework**

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A Model and Literature Details

This Appendix presents in detail the model and reviews the connection to the literature.

A.1 Relation to Literature: Details

Relation to other analytical HANK Others studies also provide different analytical frameworks, both because they isolate *different* HANK mechanisms and focus on different questions. The clearest separation in terms of channels is illustrated by the subsequent paper by Acharya and Dogra (2020) reviewed in text, that is explicitly set to isolate cyclical risk using CARA preferences. That paper shows that indeed intertemporal amplification *may* occur *purely* as a result of uninsurable income volatility going up in recessions, even when inequality is acyclical. (the paper also studies determinacy and puzzles referring to this paper's results from the previous version.) In a previous contribution, Werning (2015) similarly emphasizes the possibility of AD amplification/dampening of monetary policy relative to RANK in a more general model of income risk and market incompleteness where inequality and risk coexist. My paper's subject is very different, a full analysis of NK topics. So is the mechanism, although some of its equilibrium implications pertaining to intertemporal amplification or dampening have a similar flavor. But the key here is *cyclical inequality:* the *distribution* of income (between labor and "capital" understood as monopoly profits) and how it depends on aggregate income, as summarized through χ, the chief feature of my earlier TANK model Bilbiie (2008). Whereas the discussion in Werning emphasizes the *cyclicality of income risk*: as uninsurable income risk goes up in a recession, agents increase their precautionary savings and decrease their consumption, amplifying the initial recession which further increases idiosyncratic risk, and so on—a mechanism previously emphasized through endogenous unemployment risk by Ravn and Sterk (2020) and Challe et al (2020). My model's mechanism is instead an *intertemporal* extension of the cornerstone amplification (dampening) mechanism in TANK, when any agent can become constrained in any future period and self-insures (imperfectly) using liquid assets against the (acyclical) risk of doing so. This puts the *cyclicality of income of constrained*, and thus of inequality, at the core of transmission; whereas Werning emphasizes the cyclicality of income *risk*, although the two *are* convoluted in the different, more general framework therein.

To incorporate this distinction, I embed a separate cyclical-risk channel in THANK, assuming that the probability of becoming constrained is a function of aggregate output. With this different formalization, the two different channels of cyclical inequality and risk jointly determine AD amplification. Not only *are* the two channels naturally separate: my analysis implies that they *better be distinct*, for in order to resolve the Catch-22 they *need* to go in opposite directions. Which channel prevails empirically is a very interesting and hitherto unexplored topic that I pursue currently.

Additionally, my analysis is conducted in a loglinearized NK model that nests not only the three-equation textbook RANK but also: TANK, a HANK with cyclical inequality and acyclical risk, and a HANK with cyclical risk and acyclical inequality. Since it is so simple and transparent and close to standard NK craft, it may be of independent interest to some researchers.

My results imply an analytical reinterpretation of McKay et al's (2016, 2017) incomplete-markets based resolution of the FG puzzle. My framework underscores the *procyclicality of inequality* as sufficient for delivering Euler-equation discounting in the presence of (albeit *acyclical*) idiosyncratic risk. Procyclicality of inequality occurs in my model through labor market features and fiscal redistribution making the income of constrained agents vary less than one-to-one with the cycle $\chi < 1$. If inequality is instead *countercyclical*, the Euler equation is *compounded* in my model, implying an aggravation of the FG puzzle. Furthermore, my paper addresses a wide range of NK topics as mentioned above.

Broer, Hansen, Krusell, and Oberg (2020) study a simplified HANK whose equilibrium has a two-agent representation, underscoring the implausibility of some of the model's implications for monetary transmission through income effects of profit variations on labor supply—and showing that a sticky-wage version features a more realistic transmission mechanism; Walsh (2017) provides another analytical model with heterogeneity emphasizing the role of sticky wages (see Colciago (2011), Ascari, Colciago, and Rossi (2017), and Furlanetto (2011) for earlier sticky-wage TANK).

Auclert, Rognlie, and Straub (2018) also use a "Keynesian cross" version to capture a distinct, complementary HANK channel. In particular, they abstract from the cyclical-inequality channel emphasized here to focus on the role of *liquidity* in the form of public debt; they unveil key summary statistics pertaining to the marginal propensities to consume out of past and future income (labelled iMPCs) and how they shape the responses of the economy to past and future income shocks. Their quantitative HANK model with liquid and illiquid assets can in fact be viewed as the closest generalization of my THANK model; or alternatively, it is among the wide spectrum of quantitative HANK models the one to which my THANK model is the closest reduced representation. I use THANK to calculate analytically Auclert et al's iMPCs and provide insights into the important propagation mechanism they emphasize. Indeed, self-insurance to idiosyncratic risk is necessary and sufficient in the presence of liquidity to generate the tent-shaped path of iMPCs in THANK; whereas cyclical inequality is not of the essence to generate persistent iMPCs, but is important to fit the magnitudes under realistic calibrations.

Ravn and Sterk (2020) also study an analytical HANK but with search and matching (SaM), that is different from and complementary to my model and focusing on a different (sub)set of the issues studied here; Challe (2020) studies optimal monetary policy therein. Their models include *endogenous* unemployment risk through labor SaM, risk against which workers self-insure. The simplifying assumptions used to maintain tractability, in particular pertaining to the asset market, are orthogonal to mine.[29] Their framework delivers an interesting feedback loop from precautionary saving to aggregate demand (see also Challe et al (2017)). My benchmark model does much the opposite: in the zero-liquidity case, it gains tractability assuming *exogenous* transitions and a different asset market structure, but emphasizes the NK-cross feedback loop through the *endogenous* constrained income that is absent in Ravn and Sterk and Challe. While my extension to cyclical risk can be viewed as a reduced-form formalization of their channel. This paper addresses additional topics: forward guidance and the FG puzzle, (restoring determinacy under a peg), the Catch-22 and a way out of it, the virtues of a Wicksellian rule of price-level targeting, the version with liquidity, and optimal monetary policy.

Relation to Bilbiie (2020) and (2008) The THANK model proposed here is an extension of the TANK model in Bilbiie (2008), which analyzed monetary policy introducing the distinction between the two types based on *asset markets participation*:[30] H have no assets, while S own all the assets, i.e. price bonds and shares in firms through their Euler equation. That paper analyzed AD amplification of monetary policy and emphasized the key role of *profits* and their distribution, as well as of fiscal redistribution, for this in an analytical 3-equation TANK model isomorphic to RANK. In recent work, Bilbiie (2020) and Debortoli and Galí (2018) both used this TANK model to argue that it can approximate reasonably well some aggregate implications of *some* HANK models: several models from the HANK literature cited above, for the former; and the authors' own, for the latter. This suggests that the cyclical-inequality channel plays an important role in HANK transmission in and of itself.

The first extension here pertains to introducing self-insurance to idiosyncratic uncertainty: the risk of becoming constrained in the future despite not being constrained today, a key HANK mechanism

[29] In my model savers hold, price, and receive the payoff (profits) of shares. In Ravn and Sterk, hand-to-mouth workers get the return on shares but do not price them. Their mechanism creates an "unemployment-trap", a breakup of the Taylor principle complementary to the one here, and fixes the puzzling NK effects of supply shocks in a LT, which I abstract from.

[30] Thus abstracting from *physical investment*, the element of distinction in previous two-agent studies: Mankiw (2000) had used a growth model with this distinction, due to pioneerig work by Campbell and Mankiw (1989), to analyze long-run fiscal policy issues. Galí, Lopez-Salido and Valles (2007) embedded this same distinction in a NK model and studied numerically the business-cycle effects of government spending, with a focus on obtaining a positive multiplier on private consumption. They also analyzed numerically determinacy properties of interest rate rules, that Bilbiie (2008) derived analytically.

that is absent in TANK; this gives the model another margin to fit the aggregate findings of quantitative HANK, as shown in Bilbiie (2020).[31] That paper introduces the New Keynesian Cross as a graphical and analytical apparatus for the AD side of HANK models, expressing its key objects—MPC and multipliers—as functions of heterogeneity parameters. It studies the implications for monetary and fiscal multipliers, the link between MPC and multipliers with the "direct-indirect" decomposition of Kaplan et al, and the ability of this simple model to replicate some aggregate equilibrium implications of several quantitative, micro-calibrated HANK models. Finally, Bilbiie and Ragot (2021) builds a different analytical HANK with three assets—one ("money") liquid and traded in equilibrium, two (bonds and stock) illiquid—and studies Ramsey-optimal monetary policy as liquidity provision.

This paper's novel elements include: adding cyclical risk from several sources, related or unrelated to inequality, and pertaining to either variance or skewness; liquidity and a calculation of the iMPCs; an aggregate supply side and closed-form conditions for determinacy with Taylor rules (the HANK Taylor principle), for determinacy under price-level targeting, and for ruling out the forward-guidance puzzle; a formal statement of the "Catch-22" and of the conditions on the cyclicalities of risk and inequality to rule it out; an analysis of optimal monetary policy.

A.2 Aggregate Supply: New Keynesian Phillips Curve

All households consume an aggregate basket of individual goods $k \in [0,1]$, with constant elasticity of substitution $\varepsilon > 1$: $C_t = \left(\int_0^1 C_t(k)^{(\varepsilon-1)/\varepsilon} dk\right)^{\varepsilon/(\varepsilon-1)}$ yielding the standard demand $C_t(k) = (P_t(k)/P_t)^{-\varepsilon} C_t$ and aggregate price index $P_t^{1-\varepsilon} = \int_0^1 P_t(k)^{1-\varepsilon} dk$. Good are produced by monopolistic firms using labor: $Y_t(k) = N_t(k)$, with real marginal cost W_t.

The profit function is: $D_t(k) = \left(1+\tau^S\right)\left[P_t(k)/P_t\right] Y_t(k) - W_t N_t(k) - T_t^F$.

The individual goods producers solve:

$$\max_{P_t(k)} E_0 \sum_{t=0}^{\infty} Q_{0,t}^S \left[\left(1+\tau^S\right) P_t(k) Y_t(k) - W_t N_t(k) - \frac{\psi}{2}\left(\frac{P_t(k)}{P_{t-1}^{**}} - 1\right)^2 P_t Y_t\right],$$

where I consider two possibilities for the reference price level P_{t-1}^{**}, with respect to which it is costly for firms to deviate. In the first scenario, this is the aggregate price index P_{t-1} which small atomistic firms take as given—this delivers the static Phillips curve. In the second, P_{t-1}^{**} is firm k's own individual price as in standard formulations. $Q_{0,t}^S \equiv \beta^t \left(P_0 C_0^S / P_t C_t^S\right)^{\sigma^{-1}}$ is the marginal rate of intertemporal substitution of participants between times 0 and t, and τ^S the sales subsidy. Firms face demand for their products from two sources: consumers and firms themselves (in order to pay for the adjustment cost); the demand function for the output of firms z is $Y_t(k) = (P_t(k)/P_t)^{-\varepsilon} Y_t$. Substituting this into the profit function, the first-order condition is, after simplifying, for each case:

Static PC case $P_{t-1}^{**} = P_{t-1}$

$$0 = Q_{0,t}\left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} Y_t \left[\left(1+\tau^S\right)(1-\varepsilon) + \varepsilon \frac{W_t}{P_t}\left(\frac{P_t(k)}{P_t}\right)^{-1}\right] - Q_{0,t}\psi P_t Y_t \left(\frac{P_t(k)}{P_{t-1}} - 1\right)\frac{1}{P_{t-1}}$$

In a symmetric equilibrium all producers make identical choices (including $P_t(k) = P_t$); defining net inflation $\pi_t \equiv (P_t/P_{t-1}) - 1$, this becomes:

$$\pi_t (1+\pi_t) = \frac{\varepsilon - 1}{\psi}\left[\frac{\varepsilon}{\varepsilon - 1} w_t - \left(1+\tau^S\right)\right],$$

[31] That paper also discusses the differences with earlier work using type-switching to analyze monetary policy, e.g. Nistico (2016) and Curdia and Woodford (2016). I spell out the differentiating assumptions below.

loglinearization of which delivers the static PC in text (8).[32]

Dynamic PC case $P^{**}_{t-1} = P_{t-1}$; the first-order condition is

$$0 = Q_{0,t} \left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} Y_t \left[\left(1+\tau^S\right)(1-\varepsilon) + \varepsilon \frac{W_t}{P_t}\left(\frac{P_t(k)}{P_t}\right)^{-1}\right]$$
$$- Q_{0,t}\psi P_t Y_t \left(\frac{P_t(k)}{P_{t-1}(k)} - 1\right)\frac{1}{P_{t-1}(k)} + E_t\left\{Q_{0,t+1}\left[\psi P_{t+1} Y_{t+1}\left(\frac{P_{t+1}(k)}{P_t(k)} - 1\right)\frac{P_{t+1}(k)}{P_t(k)^2}\right]\right\}$$

In a symmetric equilibrium, using again the definition of net inflation π_t, and noticing that $Q_{0,t+1} = Q_{0,t}\beta\left(C^S_t / C^S_{t+1}\right)^{\sigma^{-1}}(1+\pi_{t+1})^{-1}$, this becomes:

$$\pi_t(1+\pi_t) = \beta E_t[(\frac{C^S_t}{C^S_{t+1}})^{1/\sigma}\frac{Y_t}{Y_{t+1}}\pi_{t+1}(1+\pi_{t+1})] + \frac{\varepsilon}{\psi}\left(w_t - \frac{1}{m}\right),$$

with the post-subsidy markup $m \equiv \frac{1}{1+\tau^S}\frac{\varepsilon}{\varepsilon-1}$. Loglinearizing this delivers the NKPC in text (7); notice that this nests the static PC when the discount factor of firms $\beta = 0$.

I assume as a benchmark that the government implements the standard NK optimal subsidy inducing marginal cost pricing $m = 1$, i.e. $\tau^S = (\varepsilon - 1)^{-1}$. Financing its total cost by taxing firms $T^F_t = \tau^S Y_t$ gives total profits $D_t = Y_t - W_t N_t$. This policy is *redistributive*: since steady-state profits are zero $D = 0$, it taxes the firms' shareholders and results in the "full-insurance" steady-state used here as a benchmark $C^H = C^S = C$. Loglinearizing around this and denoting $d_t \equiv \ln(D_t/Y)$ we have $d_t = -w_t$: profits vary inversely with the real wage an extreme form of the general property of NK models.

B Income Processes, Inequality, and Cyclical Idiosyncratic Risk

B.1 Inequality, Gini Coefficient, and Generalized Entropy

This section discusses the relationship between our measure of inequality Γ_t and the more standard measures: first, Gini coefficient, and then generalized entropy.

The **income Gini** with two levels is given by

$$\Phi_t = \frac{(1-\lambda)Y^S_t}{Y_t} - (1-\lambda) = (1-\lambda)\left(\frac{Y^S_t}{Y_t} - 1\right)$$

and is between 0 and λ (when S get all income). Rewrite it using our measure as

$$\Phi_t = (1-\lambda)\left(\frac{\Gamma_t}{\lambda + (1-\lambda)\Gamma_t} - 1\right) = \lambda\frac{(1-\lambda)(\Gamma_t - 1)}{1 + (1-\lambda)(\Gamma_t - 1)}$$

and conversely $\Gamma_t = 1 + \frac{\Phi_t}{(\lambda - \Phi_t)(1-\lambda)}$. Using the log-deviation of inequality $\gamma_t \equiv \frac{\Gamma_t - \Gamma}{\Gamma} = y^S_t - y^H_t$ we have the log-deviation of the Gini:

$$v_t = (1-\lambda)\frac{Y^S}{Y}\left(y^S_t - y_t\right) = \frac{\lambda(1-\lambda)\Gamma}{\lambda + (1-\lambda)\Gamma}\gamma_t,$$

which around a symmetric SS simplifies to $v_t = \lambda(1-\lambda)\gamma_t$.

A **generalized entropy** measure (with largest sensitivity to small incomes) is:

[32] In a Calvo setup, this amounts to assuming that each period a fraction of firms f keep their price fixed, while the rest can re-optimize freely *but* ignoring that this price affects future demand. This reduces to $\beta_f = 0$ *only* in the firms' problem (not recognizing that today's reset price prevails with some probability in future periods).

$$\Xi_t = -\lambda \ln \frac{Y_t^H}{Y_t} - (1-\lambda) \ln \frac{Y_t^S}{Y_t}$$

Subtracting the steady-state value of this same measure we obtain the deviation (note, in a uniform steady-state this measure is zero so we express this deviation in levels)

$$\begin{aligned}
\xi_t &= \Xi_t - \Xi = -\lambda \ln \frac{Y_t^H Y}{Y^H Y_t} - (1-\lambda) \ln \frac{Y_t^S Y}{Y_t Y^S} \\
&= y_t - \lambda y_t^H - (1-\lambda) y_t^S = \lambda \left(\frac{Y^H}{Y} - 1 \right) y_t^H + (1-\lambda) \left(\frac{Y^S}{Y} - 1 \right) y_t^S \\
&= \lambda (1-\lambda) \frac{Y^S - Y^H}{Y} \gamma_t = \lambda (1-\lambda) \frac{\Gamma - 1}{\lambda + (1-\lambda) \Gamma} \gamma_t = \frac{\Gamma - 1}{\Gamma} v_t.
\end{aligned}$$

B.2 Income processes

Higher moments of the income process are readily calculated in my model, since individual income follows a two-state Markov chain with values Y_t^S and Y_t^H in the respective states. The analytical characterization of this process' key moments is useful both to illustrate a key dimension along which this model is a representation of complex HANK models, and for calibration and quantitative analysis. (Note that this is essentially just an analytical two-state version of the Rouwenhorst method.)

Conditional variance is found using $E_t \left(\ln Y_{t+1}^S | \ln Y_t^S \right) = s \left(Y_{t+1} \right) \ln Y_{t+1}^S + \left(1 - s \left(Y_{t+1} \right) \right) \ln Y_{t+1}^H$ as:

$$var \left(\ln Y_{t+1}^S | \ln Y_t^S \right) = s \left(Y_{t+1} \right) \left(1 - s \left(Y_{t+1} \right) \right) \left(\ln \frac{Y_{t+1}^S}{Y_{t+1}^H} \right)^2 = s \left(Y_{t+1} \right) \left(1 - s \left(Y_{t+1} \right) \right) \left(\ln \Gamma_{t+1} \right)^2 . \tag{B.1}$$

Conditional skewness and **kurtosis** are easily calculated as:

$$\begin{aligned}
skew \left(\ln Y_{t+1}^S | \ln Y_t^S \right) &= \frac{1 - 2s \left(Y_{t+1} \right)}{\sqrt{s \left(Y_{t+1} \right) \left(1 - s \left(Y_{t+1} \right) \right)}}; & \text{(B.2)} \\
kurt \left(\ln Y_{t+1}^S | \ln Y_t^S \right) &= \left[s \left(Y_{t+1} \right) \left(1 - s \left(Y_{t+1} \right) \right) \right]^{-1} - 3.
\end{aligned}$$

The **first-order autocorrelation** of the income process for any of the two states $j = S, H$:

$$corr \left(\ln Y_{t+1}^j, \ln Y_t^j \right) = s + h - 1 = 1 - \frac{1-s}{\lambda}. \tag{B.3}$$

As standard for Bernoulli distributions there is *negative skewness* for $s > .5$ and *leptokurtosis* (positive excess kurtosis $kurt\,(.) - 3$) outside of the $\frac{1}{2} \pm \frac{1}{\sqrt{12}}$ interval, i.e. for s smaller than 0.21 or larger than 0.79. Notice that $s > 0.79$ ensures both negative skewness and leptokurtosis, with $s \geq 1 - h$ ensuring positive autocorrelation.

Of special importance to fit key micro facts on income distribution in the cross-section are the relative skewness and kurtosis of the two types: evidence in e.g. Guvenen et al (2014) suggests that the income of an empirical proxy of S is relatively more negatively skewed and more leptokurtic. It can be easily shown, comparing (B.2) with the equivalent formulae for H that both properties are satisfied in the model if and only if: $s > h$. This simple two-state model features, albeit in a stylized way, some key elements of the literature pertaining to income heterogeneity and uncertainty: conditional idiosyncratic variance that can be cyclical, autocorrelated income processes with left-skewness and leptokurtosis. The combined conditions for matching the key micro facts are $s > 1 - h$, $s > h$ and both s and h larger than .79. We use this when calibrating the model in text.

The cyclicality of skewness is:

$$\frac{d\,(skew)}{dY} = -\frac{s_Y}{2\left[s\,(1-s)\right]^{\frac{3}{2}}} \tag{B.4}$$

and is entirely determined by the cyclicality of the probability to become constrained. When the probability to become constrained $1-s$ is increasing in recessions, $-s_Y < 0$, *risk* (in the Mankiw and Guvenen et al sense) is countercyclical: negative skewness becomes more negative in recessions, making upward income movements less likely and downward income movements more likely therein. Notice that this does not depend on the size of income inequality.

THANK thus captures analytically persistent and conditionally volatile idiosyncratic income, and also, albeit in a stylized way given the coarse two-state implicit discretization, the key feature of concomitant left-skewness and leptokurtosis that a discretization with more states matches very well. Using (B.2), the conditional skewness and excess kurtosis are, for the calibration used in text to match iMPCs, -1.66 and 0.77 respectively; while the quarterly autocorrelation is $(s+h-1)^{1/4} = (1-(1-s)/\lambda)^{1/4} = 0.819$ (corresponding to the quarterly transition probability $1-s = 1-0.82^{1/4}= 1-0.952 \simeq .04$). Given the coarse two-state discretization, it is no surprise that these moments are not perfectly aligned with the micro data.

B.3 Cyclical inequality: derivations

Take first the hand-to-mouth, who consume all *their* income and loglinearize the budget constraint: $c_t^H = y_t^H = w_t + n_t^H + \frac{\tau^D}{\lambda} d_t$. Substituting the wage schedule derived using the economy resource constraint, production function, and aggregate labor supply $w_t = \left(\varphi + \sigma^{-1}\right) c_t$; the profit function $d_t = -w_t$; and their labor supply, we obtain H's consumption function given in text: $c_t^H = y_t^H = \chi y_t$, with

$$\chi \equiv 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right) \lesseqgtr 1.$$

Cyclical distributional effects make χ different from 1. The other agents, S, with income $y_t^S = w_t + n_t^S + \frac{1-\tau^D}{1-\lambda} d_t$, face an additional (relative to RANK) *income effect* of the real wage, which reduces their profits $d_t = -w_t$. Using this and their labor supply, we obtain: $c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t$, so whenever $\chi < 1$ S's income elasticity to aggregate income is *larger* than one, and vice versa.

In RANK, such distributional considerations are absent since one agent works and receives all the profits. When aggregate income goes up, labor demand goes up and the real wage increases. This drives down profits (wage=marginal cost), but because the *same* agent incurs both the labor gain and the "capital" (monopolistic rents) loss, the distribution of income between the two is neutral.

Income distribution matters under heterogeneity; to understand how, start with no fiscal redistribution, $\tau^D = 0$ and $\chi > 1$. If demand goes up and, with upward-sloping labor supply $\varphi > 0$, the wage goes up, H's income increases. Their demand increases proportionally, as they do not get hit by profits falling. Thus aggregate demand increases by *more* than initially, shifting labor demand and increasing the wage even further, and so on. In the new equilibrium, the extra demand is produced by S, whose decision to work more is optimal given the income loss from falling profits. Since the income of H goes up and down more than proportionally with aggregate income, inequality is *countercyclical*: it goes down in expansions and up in recessions.

Redistribution $\tau^D > 0$ dampens this channel, lowering χ. Through the transfer, H start internalizing the negative income effect of profits, and increase demand by less. The benchmark considered by Campbell and Mankiw's (1989) seminal paper is $\chi = 1$, which occurs when the distribution of profits is uniform $\tau^D = \lambda$ (the income effect disappears) or when labor is infinitely elastic $\varphi = 0$ (all households' consumption comoves perfectly with the wage); income inequality is then *acyclical*.

Finally, $\chi < 1$ occurs when H receive a disproportionate share of the profits $\tau^D > \lambda$. The AD expansion is now *smaller* than the initial impulse, as H recognize that this will lead to a fall in their income; while S, given the positive income effect from profits, optimally work less. As the income of H now moves less than proportionally with aggregate income, inequality is *procyclical*.

B.4 Cyclical risk: derivations

The self-insurance equation when the probability depends on aggregate demand (tomorrow) is

$$\left(C_t^S\right)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[s\left(Y_{t+1}\right)\left(C_{t+1}^S\right)^{-\frac{1}{\sigma}} + \left(1 - s\left(Y_{t+1}\right)\right)\left(C_{t+1}^H\right)^{-\frac{1}{\sigma}} \right] \right\}. \tag{B.5}$$

We loglinearize this around a steady-state with inequality; in the context of our model, that requires assuming that steady-state fiscal redistribution is imperfect and that a sales subsidy does not completely undo market power (generating zero profits). In particular, we focus on a steady state with no subsidy, so that the profit share is $D/C = 1/\varepsilon$ and the labor share $WN/C = (\varepsilon - 1)/\varepsilon$. Under the same arbitrary redistribution scheme, the consumption shares of each type are respectively

$$\begin{aligned} \frac{C^H}{C} &= \frac{WN + \frac{\tau^D}{\lambda} D}{C} = 1 - \frac{1}{\varepsilon}\left(1 - \frac{\tau^D}{\lambda}\right) \\ \frac{C^S}{C} &= \frac{WN + \frac{1-\tau^D}{1-\lambda} D}{C} = 1 + \frac{1}{\varepsilon}\frac{\lambda}{1-\lambda}\left(1 - \frac{\tau^D}{\lambda}\right) > \frac{C^H}{C} \text{ iff } \tau^D < \lambda. \end{aligned}$$

Denoting steady-state inequality $\frac{C^S}{C^H} \equiv \Gamma$ we loglinearize around a steady state:

$$1 = \beta\left(1+r\right)\left[s\left(Y\right) + \left(1 - s\left(Y\right)\right)\Gamma^{\frac{1}{\sigma}}\right], \tag{B.6}$$

where I restrict attention to cases with positive real interest-rate r (the topic of "secular stagnation" in this framework is interesting in its own right—it can occur for high enough risk and high enough inequality). Loglinearization delivers, denoting by r_t the ex-ante real interest rate for brevity, and the steady-state value of the probability by $s\left(C\right) = s$ and its elasticity relative to the cycle (consumption) is $-\frac{s'(Y)Y}{1-s(Y)}$:

$$c_t^S = -\sigma\left(i_t - E_t\pi_{t+1}\right) + \frac{s}{s + (1-s)\Gamma^{1/\sigma}} E_t c_{t+1}^S + \frac{(1-s)\Gamma^{1/\sigma}}{s + (1-s)\Gamma^{1/\sigma}} E_t c_{t+1}^H + \left(-\frac{s'\left(Y\right)Y}{1 - s\left(Y\right)}\right)\frac{\sigma\left(1-s\right)\left(1 - \Gamma^{1/\sigma}\right)}{s + (1-s)\Gamma^{1/\sigma}} E_t c_{t+1}$$

which replacing individual consumption levels as function of aggregate becomes

$$c_t^S = -\sigma\frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t\pi_{t+1}\right) + \left(1 + \frac{1-\tilde{s}}{1-\lambda\chi}\left(\chi - 1\right) - \left(-\frac{s'\left(Y\right)Y}{1-s\left(Y\right)}\right)\left(1-\tilde{s}\right)\frac{\sigma\left(1-\lambda\right)}{1-\lambda\chi}\left(1 - \Gamma^{-1/\sigma}\right)\right) E_t c_{t+1}$$

denote by $1 - \tilde{s} = \frac{(1-s)\Gamma^{1/\sigma}}{s + (1-s)\Gamma^{1/\sigma}} > 1 - s$ the inequality-weighted transition probability, the relevant inequality-adjusted measure of risk given steady-state inequality coming from financial income $\Gamma \equiv Y^S/Y^H \geq 1$. There can be discounting as long as risk is procyclical enough $\eta > \frac{\Gamma^{1/\sigma}(\chi-1)}{\sigma(1-\lambda)\left(\Gamma^{1/\sigma}-1\right)}$. But the contemporary AD elasticity to interest rates is unaffected by the cyclicality of risk (this is thus isomorphic to Acharya and Dogra's different formalization of cyclical risk based on CARA utility).

B.5 Current aggregate demand

For the case where the probability depends on *current* (today) aggregate demand $s\left(Y_t\right)$, the aggregate Euler-IS is

$$c_t^S = -\sigma\left(i_t - E_t\pi_{t+1}\right) + \beta\left(1+r\right)sE_tc_{t+1}^S + \beta\left(1+r\right)\left(1-s\right)\Gamma^{\frac{1}{\sigma}}E_tc_{t+1}^H + \sigma\beta\left(1+r\right)\left(-\frac{s'\left(Y\right)Y}{1-s\left(Y\right)}\right)\left(1-s\right)\left(1-\Gamma^{\frac{1}{\sigma}}\right)c_t$$

Replacing $\beta\left(1+r\right)$

$$c_t^S = -\sigma\left(i_t - E_t\pi_{t+1}\right) + \frac{s}{s+\left(1-s\right)\Gamma^{\frac{1}{\sigma}}}E_tc_{t+1}^S + \frac{\left(1-s\right)\Gamma^{\frac{1}{\sigma}}}{s+\left(1-s\right)\Gamma^{\frac{1}{\sigma}}}E_tc_{t+1}^H + \left(-\frac{s'\left(Y\right)Y}{1-s\left(Y\right)}\right)\frac{\sigma\left(1-s\right)\left(1-\Gamma^{\frac{1}{\sigma}}\right)}{s+\left(1-s\right)\Gamma^{\frac{1}{\sigma}}}c_t$$

Replace the consumption functions of H and S we obtain:

$$\begin{aligned} c_t &= \theta\delta E_tc_{t+1} - \theta\sigma\frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t\pi_{t+1}\right) \\ \text{with } \theta &\equiv \left[1+\left(-\frac{s'\left(Y\right)Y}{1-s\left(Y\right)}\right)\left(1-\Gamma^{-1/\sigma}\right)\left(1-\tilde{s}\right)\sigma\frac{1-\lambda}{1-\lambda\chi}\right]^{-1}, \end{aligned} \tag{B.7}$$

where the notation is as previously. Notice that now the two channels (cyclical inequality and cyclical risk via $s'(.)$) are intertwined for both the amplification/dampening of current interest rates and for future consumption. A previous working paper version contained a full analysis of this version of the model and its implications for curing puzzles and the Catch-22.

C The 3-equation THANK with NKPC

This section derives the same results as in text but with the forward-looking NKPC (7).

C.1 The HANK Taylor Principle: Equilibrium Determinacy with Interest Rate Rules

Determinacy can be studied by standard techniques, extending the result in text (there will now be two eigenvalues). Necessary and sufficient conditions are provided i.a. in Woodford (2003) Proposition C.1. With the Taylor rule (17), the system becomes $\begin{pmatrix} E_t\pi_{t+1} & E_tc_{t+1} \end{pmatrix}' = A\begin{pmatrix} \pi_t & c_t \end{pmatrix}'$ with transition matrix:

$$A = \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \delta^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\left(\phi_\pi - \beta^{-1}\right) & \delta^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\beta^{-1}\kappa\right) \end{bmatrix}$$

with determinant $\det A = \beta^{-1}\delta^{-1}\left(1+\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_\pi\right)$ and trace $\text{tr}A = \beta^{-1}+\delta^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\beta^{-1}\kappa\right)$.

Determinacy can obtain in either of two cases. Case 2. ($\det A - \text{tr}A < -1$ and $\det A + \text{tr}A < -1$) can be ruled based on sign restrictions. Case 1. requires three conditions to be satisfied jointly:

$$\det A > 1;\ \det A - \text{tr}A > -1;\ \det A + \text{tr}A > -1$$

The third condition is always satisfied under the sign restrictions, so the necessary and sufficient conditions are:

$$\phi_\pi > 1 + \frac{\left(\delta-1\right)\left(1-\beta\right)}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}} \tag{C.1}$$

together with $\phi_\pi > \max\left(\frac{\beta\delta-1}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}},\ 1+\frac{(1-\beta)(\delta-1)}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}\right)$. The second term is larger than the first iff $\left(2\beta-1\right)\delta < \kappa\sigma\frac{1-\lambda}{1-\lambda\chi}+\beta$, which holds generically for most plausible parameterizations. Condition (C.1) thus generalizes the HANK Taylor principle to the case of forward-looking Phillips curve.

C.2 Ruling out the FG Puzzle

The analogous of Proposition 2 for the case with NKPC (7) is:

Proposition 8 *The analytical HANK model (with (7)) under a peg is locally determinate and solves the FG puzzle* ($\frac{\partial^2 c_t}{\partial(-i^*_{t+T})\partial T} < 0$) *if and only if:*

$$\delta + \sigma \frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta} < 1,$$

Notice that the condition nests the one of Proposition 2 when $\beta \to 0$. Indeed, it has exactly the same interpretation with $\delta + \sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta}$ being the "long-run" effect of news, and $\frac{\kappa}{1-\beta}$ being the slope of the long-run NKPC.

Point 1. (determinacy under a peg with NKPC) follows directly from (C.1): a peg is sufficient if both $\delta < \beta^{-1}$ and $1 + \frac{(1-\beta)(\delta-1)}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}} < 0$, the latter implying $\delta < 1 - \frac{\kappa}{1-\beta}\sigma\frac{1-\lambda}{1-\lambda\chi} < \beta^{-1}$, which delivers the threshold in the Proposition.

Point 2 requires solving the model; focusing therefore on the case where the condition holds, and the model is determinate under a peg, we rewrite the model in forward (matrix) form as:

$$\begin{pmatrix} \pi_t \\ c_t \end{pmatrix} = A^{-1}\begin{pmatrix} E_t\pi_{t+1} \\ E_t c_{t+1} \end{pmatrix} - \sigma\frac{1-\lambda}{1-\lambda\chi}\begin{pmatrix} \kappa \\ 1 \end{pmatrix} i^*_t \tag{C.2}$$

where

$$A^{-1} = \begin{pmatrix} \beta + \kappa\sigma\frac{1-\lambda}{1-\lambda\chi} & \kappa\delta \\ \sigma\frac{1-\lambda}{1-\lambda\chi} & \delta \end{pmatrix}$$

is the inverse of matrix A defined above under a peg $\phi = 0$. To find the elasticity of $\begin{pmatrix} \pi_t & c_t \end{pmatrix}'$ to an interest rate cut at T, $-i^*_{t+T}$ we iterate forward (C.2) to obtain $\sigma\frac{1-\lambda}{1-\lambda\chi}\left(A^{-1}\right)^T\begin{pmatrix} \kappa \\ 1 \end{pmatrix}$. But notice that we know by point 1 that the eigenvalues of A are both outside the unit circle; it follows by standard linear algebra results that the eigenvalues of A^{-1} are both inside the unit circle and therefore $\left(A^{-1}\right)^T$ is decreasing with T. (the eigenvalues to the power of T appear in the Jordan decomposition used to compute the power of A^{-1}). This proves that the FG puzzle is eliminated.

Point 3 requires computing the equilibrium given an AR1 interest rate with persistence μ as before $E_t i^*_{t+1} = \mu i^*_t$; since we are in the determinate case, the equilibrium is unique and there is no endogenous persistence, so the persistence of endogenous variables is equal to the persistence of the exogenous process. Replacing $E_t c_{t+1} = \mu c_t$ and $E_t\pi_{t+1} = \mu\pi_t$ in (C.2) we therefore have:

$$\begin{pmatrix} \pi_t \\ c_t \end{pmatrix} = -\sigma\frac{1-\lambda}{1-\lambda\chi}\left(I - \mu A^{-1}\right)^{-1}\begin{pmatrix} \kappa \\ 1 \end{pmatrix} i^*_t.$$

Computing the inverse we obtain

$$\left(I - \mu A^{-1}\right)^{-1} = \frac{1}{\det}\begin{bmatrix} 1-\delta\mu & \kappa\delta\mu \\ \sigma\frac{1-\lambda}{1-\lambda\chi}\mu & 1 - \left(\beta + \sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\right)\mu \end{bmatrix},$$

where $\det \equiv \mu^2\beta\delta - \mu\left(\delta + \sigma\frac{1-\lambda}{1-\lambda\chi}\kappa + \beta\right)\mu + 1$. Replacing in the previous equation, differentiating, and simplifying, the effects are:

$$\begin{pmatrix} \frac{\partial\pi_t}{\partial i^*_t} \\ \frac{\partial c_t}{\partial i^*_t} \end{pmatrix} = -\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{1}{\det}\begin{pmatrix} \kappa \\ 1-\mu\beta \end{pmatrix}$$

Therefore, neo-Fisherian effects are ruled out iff det > 0, i.e.:

$$\delta < \frac{1-\beta\mu-\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\mu}{\mu\left(1-\beta\mu\right)}.$$

But this is always satisfied under the condition in the proposition (for determinacy under a peg) $\delta < 1 - \frac{\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa}{1-\beta} \leq \frac{1-\beta\mu-\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\mu}{\mu(1-\beta\mu)}$ where the second inequality can be easily verified, it implies $\left(1-\beta\mu\right)\left(1-\beta\right)+\beta\sigma\kappa\mu \geq 0$.

Figure C1 illustrates the threshold level of endogenous redistribution sufficient to deliver determinacy under a peg and thus rule out the FG puzzle, as a function of λ and for different $1-s$. Close to the TANK limit (small $1-s$), no level of redistribution delivers this (red dash); as idiosyncratic risk $1-s$ increases (blue solid), the region expands and is largest in the iid case (blue dots).

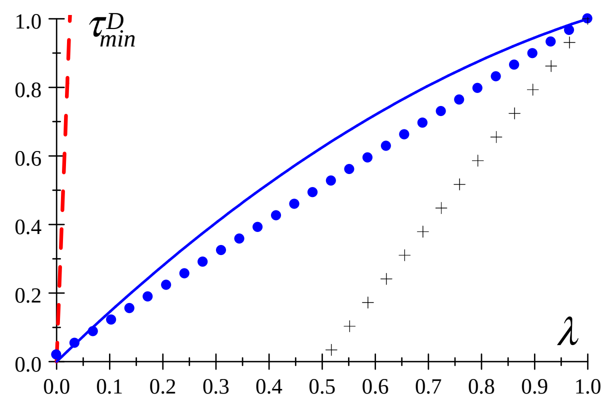

Fig. C1: Redistribution threshold $\tau^D_{\min}$ in TANK $1-s \longrightarrow 0$ (dash); 0.04 (solid); λ (dots).
Note: The crosses represent the threshold above which the IS slope is positive $\lambda\chi < 1$.

C.3 Ruling out puzzles with Wicksellian rule and Contemporaneous PC

Replacing (8) and the policy rule (24) in the aggregate Euler-IS (16) we have

$$c_t = \nu_0 E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}\left(\phi_p p_t + i_t^*\right); \tag{C.3}$$

the static PC rewritten in terms of the price level is:

$$p_t - p_{t-1} = \kappa c_t. \tag{C.4}$$

Combining, we obtain:

$$E_t p_{t+1} - \left[1+\nu_0^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)\right]p_t + \nu_0^{-1}p_{t-1} = \sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}i_t^*. \tag{C.5}$$

Notice that the RANK model is nested here for $\lambda = 0$ (or $\chi = 1$, the Campbell-Mankiw benchmark), which would yield a simplified version of Woodford and Giannoni's analyses.

Recall that we are interested in the case whereby $\nu_0 \geq 1$ (as the paper shows, for $\nu_0 < 1$ there there is determinacy under a peg in HANK and thus no puzzles). The model has a locally unique equilibrium (is determinate) when the above second-order equation has one root inside and one outside the unit circle. The characteristic polynomial is $J\left(x\right) = x^2 - \left[1+\left(\nu_0\right)^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)\right]x + \nu_0^{-1}$ where by standard results, the roots' sum is $1+\nu_0^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)$ and the product is $\nu_0^{-1}<1$. So at least one root is inside the unit circle, and we need to rule out that both are; Since we have $J(1)=-\nu_0^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa$ and $J(-1)=2+2\nu_0^{-1}+\nu_0^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa$, the necessary and sufficient condition for the second root to be outside the unit circle is precisely $\phi_p>0$—coming from $J(1)<0$ and $J(-1)>0$. This completes the proof of Proposition 4.

To find the solution, denote the roots of the polynomial by $x_+>1>x_->0$; the difference equation is solved by standard factorization: The roots of the characteristic polynomial are

$$\begin{aligned} x_\pm &= \frac{1+\nu_0^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)\pm\sqrt{\left[1+\nu_0^{-1}\left(1+\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)\right]^2-4\nu_0^{-1}}}{2} \\ x_+ &> 1>x_->0 \end{aligned}$$

Factorizing the difference equation (25):

$$\left(L^{-1}-x_-\right)\left(L^{-1}-x_+\right)p_{t-1}=\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}i_t^*$$

we obtain:

$$\begin{aligned} p_t &= x_-p_{t-1}-\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}x_+^{-1}\frac{1}{1-(x_+L)^{-1}}i_t^* \\ &= x_-p_{t-1}-\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}x_+^{-1}\sum_{j=0}^{\infty}x_+^{-j}i_{t+j}^* \end{aligned}$$

Let $\Delta_{t+j}\equiv-\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}x_+^{-1}i_{t+j}^*$ denote the rescaled interest rate *cut*:

$$p_t=x_-^{t+1}p_{-1}+\left[\sum_{j=0}^{\infty}(x_+)^{-j}\Delta_{t+j}+x_-\sum_{j=0}^{\infty}(x_+)^{-j}\Delta_{t-1+j}+...+x_-^{t-1}\sum_{j=0}^{\infty}(x_+)^{-j}\Delta_{1+j}+x_-^t\sum_{j=0}^{\infty}(x_+)^{-j}\Delta_j\right]$$

Normalizing initial value to zero (since $x_-<1$ it vanishes when t goes to infinity), the solution is made of a forward and a backward component:

$$p_t=\frac{1-\left(x_-x_+^{-1}\right)^{t+1}}{1-x_-x_+^{-1}}\sum_{j=0}^{\infty}\left(x_+^{-1}\right)^j\Delta_{t+j}+\sum_{k=0}^{t-1}x_-^{1+k}\frac{1-\left(x_-x_+^{-1}\right)^{t-k}}{1-x_-x_+^{-1}}\Delta_{t-1-k}$$

Lagging it once and taking the first difference we obtain the solution for inflation:

$$\begin{aligned} \pi_t &= \frac{1-\left(x_-x_+^{-1}\right)^{t+1}}{1-x_-x_+^{-1}}\sum_{j=0}^{\infty}\left(x_+^{-1}\right)^j\Delta_{t+j}-\frac{1-\left(x_-x_+^{-1}\right)^{t}}{1-x_-x_+^{-1}}\sum_{j=0}^{\infty}\left(x_+^{-1}\right)^j\Delta_{t-1+j} \\ &\quad+\sum_{k=0}^{t-1}x_-^{1+k}\frac{1-\left(x_-x_+^{-1}\right)^{t-k}}{1-x_-x_+^{-1}}\Delta_{t-1-k}-\sum_{k=0}^{t-2}x_-^{1+k}\frac{1-\left(x_-x_+^{-1}\right)^{t-1-k}}{1-x_-x_+^{-1}}\Delta_{t-2-k} \\ &= A(t)\sum_{j=0}^{\infty}\left(x_+^{-1}\right)^j\Delta_{t+j}+\Psi_{t-1}. \end{aligned}$$

where $A(t) \equiv \frac{1-\left(x_+^{-1}\right)+(x_-)^t\left(x_+^{-1}\right)^{t+1}-\left(x_- x_+^{-1}\right)^{t+1}}{1-x_- x_+^{-1}}$ (if we put ourselves at time 0 this simply becomes $A(0) = \sigma\frac{1-\lambda}{1-\lambda\chi}\nu_0^{-1}$), while in Ψ_{t-1} we grouped all terms that consist of lags of Δ_t (Δ_{t-1} and earlier) which are predetermined at time t and will not be used in any of the derivations of interest here—where we consider shocks occurring at t or thereafter. This delivers, for consumption:

$$c_t = -A(t)\, E_t \sum_{j=0}^{\infty} \left(x_+^{-1}\right)^{j+1} i^*_{t+j} + \Psi_{t-1} \tag{C.6}$$

where Ψ_{t-1} is a weighted sum of past realizations of the shock and $A(t) > 0$ is a function only of calendar date; both Ψ_{t-1} and $A(t)$ are irrelevant for our purpose because they are invariant to current and future shocks.

The effect of a one-time interest rate cut at $t+T$ is now:

$$\frac{\partial c_t}{\partial\left(-i^*_{t+T}\right)} = A(t)\left(x_+^{-1}\right)^{T+1}$$

which, since $A(.) > 0$ and $x_+ > 1$, is a decreasing function of T: *the FG puzzle disappears*.[33] Notice that the Wicksellian rule *also* cures the FG puzzle in the (nested) RANK model (this follows immediately by replacing $\lambda = 0$ or $\chi = 1$ above).

C.4 Determinacy with Wicksellian rule and NKPC

Rewrite the system made of (14), (7) and the definition of inflation as (ignoring shocks):

$$\begin{aligned}
c_t &= \delta E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p p_t + \sigma\frac{1-\lambda}{1-\lambda\chi}E_t\pi_{t+1} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa c_t \\
p_t &= \pi_t + p_{t-1}
\end{aligned}$$

Substituting and writing in canonical matrix form $\begin{pmatrix} E_t c_{t+1} & E_t\pi_{t+1} & p_t \end{pmatrix}' = A \begin{pmatrix} c_t & \pi_t & p_{t-1} \end{pmatrix}'$ with transition matrix A given by

$$A = \begin{pmatrix} \delta^{-1}\left(1+\beta^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\right) & \delta^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\left(\phi_p - \beta^{-1}\right) & \delta^{-1}\sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p \\ -\beta^{-1}\kappa & \beta^{-1} & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

We can apply Proposition C.2 in Woodford (2003, Appendix C): determinacy requires two roots outside the unit circle and one inside. The characteristic equation of matrix A is:

$$J(x) = x^3 + A_2 x^2 + A_1 x + A_0 = 0$$

[33] Likewise for neo-Fisherian effects: take an AR(1) process for i^*_t with persistence μ as before; the solution is now both 1. uniquely determined (by virtue of determinacy proved above) and 2. in line with standard logic—an increase in interest rates leads to a fall in consumption and deflation in the short run: $\frac{\partial c_t}{\partial i^*_t} = -A(t)\frac{1}{x_+ - \mu}$,which is negative as $A(.) > 0$ and $x_+ > 1 > \mu$. Notice that in the long-run, i.e. if there is a permanent change in interest rates, the economy moves to a new steady-state and the uncontroversial. long-run Fisher effect applies as usual.

with coefficients:

$$
\begin{aligned}
A_2 &= -\frac{1}{\beta}-\frac{1}{\delta}\left(\frac{\sigma\kappa}{\beta}\frac{1-\lambda}{1-\lambda\chi}+1\right)-1<0\\
A_1 &= \frac{1}{\beta}+\frac{1}{\delta}\left[\frac{\sigma\kappa}{\beta}\frac{1-\lambda}{1-\lambda\chi}\left(1+\phi_p\right)+1+\frac{1}{\beta}\right]>0\\
A_0 &= -\frac{1}{\beta\delta}
\end{aligned}
$$

To check the determinacy conditions, we first calculate:

$$
\begin{aligned}
J(1) &= 1+A_2+A_1+A_0=\frac{1}{\delta}\frac{\sigma\kappa}{\beta}\frac{1-\lambda}{1-\lambda\chi}\phi_p>0\\
J(-1) &= -1+A_2-A_1+A_0\\
&= -2-\frac{2}{\beta}-\frac{1}{\delta}\left[2\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa}{\beta}+\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa}{\beta}\phi_p+2+\frac{2}{\beta}\right]<0
\end{aligned}
$$

Since $J(1)>0$ and $J(-1)<0$ we are either in case Case II or Case III in Woodford Proposition C.2;

Case III in Woodford implies that $\phi_p>0$ is sufficient for determinacy if the additional condition is satisfied:

$$
A_2<-3\rightarrow\delta<\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa+\beta}{2\beta-1}. \tag{C.7}
$$

This is a fortiori satisfied in RANK (and delivers determinacy there), but not here with $\delta>1$. Therefore, we also need to check Case II in Woodford and to that end we need to check the additional requirement (C.15) therein:

$$
A_0^2-A_0A_2+A_1-1>0,
$$

which replacing the expressions for the A_is delivers:

$$
\phi_p>\frac{(1-\beta)(\delta-1)+\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa}{\sigma\frac{1-\lambda}{1-\lambda\chi}\kappa\delta\beta}(1-\delta\beta)
$$

Since the ratio is positive, this requirement is only stronger than the already assumed $\phi_p>0$ when

$$
\delta<\beta^{-1}; \tag{C.8}
$$

It can be easily checked that the δ threshold C.8 is always smaller than the threshold C.7; therefore, whenever $\delta<\beta^{-1}$, Case III applies and $\phi_p>0$ is sufficient for determinacy. While when C.7 fails (for large enough δ), Case II applies and $\phi_p>0$ is still sufficient for determinacy.

D Liquidity in THANK

D.1 Model derivations

The two budget constraints (after asset market clearing) for the case with liquidity (27), evaluated at the steady state deliver, respectively, with Y^j denoting income of agent j:

$$C^H = Y^H + \left[\left(\frac{1-s}{\lambda} - 1\right) R + 1\right] B \tag{D.1}$$

$$C^S = Y^S - \frac{\lambda}{1-\lambda}\left[\left(\frac{1-s}{\lambda} - 1\right) R + 1\right] B, \tag{D.2}$$

where we already imposed the steady-state budget $T = (R-1)B$ and assumed uniform taxation. The steady-state self-insurance Euler equation for bonds is:

$$1 = \beta R \left[s + (1-s)\left(\frac{C^S}{C^H}\right)^{\frac{1}{\sigma}}\right] \tag{D.3}$$

To derive the steady-state savings function (demand for liquidity), replace individual consumptions in the Euler equation

$$1 = \beta R \left[s + (1-s)\left(\frac{Y^S - \frac{\lambda}{1-\lambda}\left[\left(\frac{1-s}{\lambda}-1\right)R+1\right]B}{Y^H + \left[\left(\frac{1-s}{\lambda}-1\right)R+1\right]B}\right)^{\frac{1}{\sigma}}\right],$$

which rewritten and under the particularly tractable case of log utility $\sigma \to 1$ becomes:

$$1 = \beta R \left[1 + \frac{1-s}{1-\lambda}\left(\frac{Y}{Y^H + \left[\left(\frac{1-s}{\lambda}-1\right)R+1\right]B} - 1\right)\right]$$

Using SS inequality $\Gamma = Y^S/Y^H$ to write:

$$\frac{Y^H}{Y} = \frac{1}{1+(1-\lambda)(\Gamma-1)}; \frac{Y^S}{Y} = \frac{\Gamma}{1+(1-\lambda)(\Gamma-1)}$$

we finally obtain (normalizing $Y=1$ wlog) (28) in text:

$$B = \frac{\frac{1}{1+\frac{1-\lambda}{1-s}\left(\frac{1}{\beta R}-1\right)} - \frac{1}{1+(1-\lambda)(\Gamma-1)}}{\left(\frac{1-s}{\lambda}-1\right)R+1}.$$

Notice that SS inequality is determined in the model: using symmetric SS hours and the expression for the real wage $w = m^{-1}$ where m is the steady-state markup *with arbitrary* subsidy $m \equiv \frac{1}{1+\tau^S}\frac{\varepsilon}{\varepsilon-1}$, we have $Y^H = WN^H + \frac{\tau^D}{\lambda}D = \frac{1}{m}N + \frac{\tau^D}{\lambda}\frac{m-1}{m}Y = \frac{1}{m}\left(1+\frac{\tau^D}{\lambda}(m-1)\right)Y$ and for savers $Y^S = WN^S + \frac{1-\tau^D}{1-\lambda}D = \frac{1}{m}\left(1+\frac{1-\tau^D}{1-\lambda}(m-1)\right)Y$. Replacing both of these:

$$\Gamma = \frac{1+\frac{1-\tau^D}{1-\lambda}(m-1)}{1+\frac{\tau^D}{\lambda}(m-1)},$$

so there is SS inequality $\Gamma > 1$ iff $m > 1$ and $\tau^D < \lambda$.

Start by noticing that to have a self-insurance motive $C^S > C^H$ the standard condition from incomplete-market models applies:

$$R < \frac{1}{\beta}$$

When is SS debt positive? The denominator being positive requires:

$$R < \frac{1}{1-\frac{1-s}{\lambda}}, \tag{D.4}$$

which essentially implies, see (D.1), that H get positive income from liquidity, net of taxes. A positive numerator instead requires:

$$R > \frac{1}{\beta\left[1+(1-s)(\Gamma-1)\right]}. \tag{D.5}$$

For an equilibrium interest rate to exist we thus need

$$\frac{1}{\beta\left[1+(1-s)(\Gamma-1)\right]} < \frac{1}{1-\frac{1-s}{\lambda}} \rightarrow \frac{1-s}{\lambda} > 1-\beta\left[1+(1-s)(\Gamma-1)\right]$$

With large enough Γ, always satisfied as RHS negative, $\beta^{-1}-1<(1-s)(\Gamma-1)$

The combined condition is thus:

$$\frac{1}{\beta\left[1+(1-s)(\Gamma-1)\right]} < R < \min\left(\frac{1}{\beta}, \frac{1}{1-\frac{1-s}{\lambda}}\right),$$

which in the zero-liquidity (no SS inequality) case used in the text is:

$$R = \beta^{-1} < \left(1-\frac{1-s}{\lambda}\right)^{-1}.$$

Several properties are worth noticing. As $\beta R \rightarrow 1$ debt tends to $B \rightarrow \bar{B} = \frac{1}{1+\beta^{-1}\left(\frac{1-s}{\lambda}-1\right)}\frac{(1-\lambda)(\Gamma-1)}{1+(1-\lambda)(\Gamma-1)}$ which is > 0 iff $\frac{1-s}{\lambda} > 1-\beta$. Moreover, $B \rightarrow 0$ when $R \rightarrow \beta^{-1}\left(1+(1-s)(\Gamma-1)\right)^{-1}$. Thus, in the "bondless", zero-liquidity limit we have

$$R \leq \beta^{-1},$$

with equality in the no-SS-inequality case $\Gamma = 1$.

As standard in incomplete-market models, the demand for debt features and asymptote. In particular, $B \rightarrow \infty$ when

$$R \rightarrow \min\left\{\beta^{-1}\frac{1}{1-\frac{1-s}{1-\lambda}}, \left(1-\frac{1-s}{\lambda}\right)^{-1}\right\}.$$

First, at given numerator debt becomes infinite whenever $1+R\left(\frac{1-s}{\lambda}-1\right) \rightarrow 0$ so $R \rightarrow \left(1-\frac{1-s}{\lambda}\right)^{-1} > \beta^{-1}$ under the positive-debt requirement. Second, at given denominator, B can also become infinite when $1+\left(\frac{1}{\beta R}-1\right)\frac{1-\lambda}{1-s} \rightarrow 0$ so $R \rightarrow \beta^{-1}\frac{1}{1-\frac{1-s}{1-\lambda}} > \beta^{-1}$ (assuming $s > \lambda$). The former threshold is smaller than the latter whenever $1-\frac{1-s}{\lambda} < \beta\left(1-\frac{1-s}{1-\lambda}\right)$. Notice, however, that the threshold is always larger than β^{-1}.

Loglinearization. The loglinearized budget constraint of H agents (27) is:

$$\frac{C^H}{Y}c_t^H = \frac{Y^H}{Y}y_t^H - t_t + \frac{1-s}{\lambda}Rb_t + \frac{1-s}{\lambda}RB_Y r_t, \tag{D.6}$$

where $b_t \equiv (B_t - B)/Y$.

D.2 Derivation of analytical iMPCs

For analytical convenience, for this section I derive results loglinearizing around a long-run steady-state with zero public debt (and thus zero liquidity) $B = 0$, implying $R = \beta^{-1}$ and no consumption inequality $C^H = C^S$, so (D.6) and the corresponding equation for S agents become (30), where we imposed asset market clearing.

The equilibrium dynamics of private liquid assets b_t are found by replacing these individual budget constraints (30) into the loglinearized self-insurance Euler equation for bonds (11), with $\hat{y}_t^j \equiv y_t^j - t_t$ denoting disposable income as in text, obtaining:

$$\begin{aligned} E_t b_{t+2} - \Theta b_{t+1} + \beta^{-1} b_t &= \frac{1-\lambda}{s}\left[s E_t \hat{y}_{t+1}^S + (1-s) E_t \hat{y}_{t+1}^H - \hat{y}_t^S\right], \\ \text{where } \Theta &\equiv \frac{1}{s} + \beta^{-1}\left[1 + \frac{1-s}{s}\left(\frac{1-s}{\lambda} - 1\right)\right]. \end{aligned} \tag{D.7}$$

As clear from (D.7), finding the derivatives of b_{t+k} with respect to $\hat{y}_t$ requires a model of how individual disposable incomes are related to aggregate, such as this paper's. Furthermore, since the calculation of iMPCs keeps *fixed by definition* all the other variables (in particular taxes, their distribution, and thus public debt), the partial derivatives of individual disposable incomes with respect to aggregate disposable income are respectively $d\hat{y}_t^H = \chi d\hat{y}_t$ and $d\hat{y}_t^S = \frac{1-\lambda\chi}{1-\lambda} d\hat{y}_t$.[34] Solving the asset dynamics equation taking this into account delivers:

$$db_{t+1} = x_b db_t + \frac{1-\lambda\chi}{s} \sum_{k=0}^{\infty} (\beta x_b)^{k+1} \left(d\hat{y}_{t+k} - \delta d\hat{y}_{t+k+1}\right), \tag{D.8}$$

where the roots of the characteristic polynomial of (D.7) are $x_b = \frac{1}{2}\left(\Theta - \sqrt{\Theta^2 - 4\beta^{-1}}\right)$ and $(\beta x_b)^{-1}$, with $0 < x_b < 1$ as required by stability whenever $\beta > 1 - \frac{1-s}{\lambda}$.

Substituting (D.8) in (31) delivers the aggregate consumption function, the key equation for calculating the analytical iMPCs in Proposition 9:

$$dc_t = d\hat{y}_t + \beta^{-1}(1 - \beta x_b) db_t + \frac{1-\lambda\chi}{s} \sum_{k=0}^{\infty} (\beta x_b)^{k+1} \left(\delta d\hat{y}_{t+k+1} - d\hat{y}_{t+k}\right). \tag{D.9}$$

Proposition 9 *The intertemporal MPCs (iMPCs) for the THANK model, in response to a one-time shock to disposable income at any time T and for any $t \geq 0$: (i) are given by:*

$$\frac{dc_t}{d\hat{y}_T} = \begin{cases} \frac{1-\lambda\chi}{s}\frac{\delta - \beta x_b}{1-\beta x_b^2}(\beta x_b)^{T-t}\left(1 - x_b + x_b(1-\beta x_b)\left(\beta x_b^2\right)^t\right), & \text{if } \ t \leq T-1; \\ 1 - \frac{1-\lambda\chi}{s}\beta x_b - (\delta - \beta x_b) x_b \frac{1-\lambda\chi}{s}(1-\beta x_b)\frac{1-\left(\beta x_b^2\right)^T}{1-\beta x_b^2}, & \text{if } \ t = T; \\ \frac{1-\lambda\chi}{s}\frac{1-\beta x_b}{1-\beta x_b^2} x_b^{t-T}\left(1 - x_b\delta + x_b(\delta - \beta x_b)\left(\beta x_b^2\right)^T\right), & \text{if } \ t \geq T+1. \end{cases}$$

and (ii) are increasing with the cyclicality of inequality χ when $t < T$ and decreasing with χ when $t \geq T > 0$ (keeping fixed the time-0 contemporaneous MPC $dc_0/d\hat{y}_0$).

[34] In particular, any model would deliver a reduced-form $\hat{y}_t^H = \chi\hat{y}_t + \chi_{tax} t_t$, χ_{tax} being an equilibrium elasticity depending on the tax distribution, labor elasticity, etc. But for calculating iMPCs, we look at a partial equilibrium wherein $dt_t/\hat{y}_t = 0$.

It is useful, in order to isolate this *liquidity-amplification channel*, to follow Auclert et al's paper that discovered it and start with the benchmark of acyclical inequality $\chi = 1$. This amounts to replacing individual disposable incomes with aggregate disposable income $\hat{y}_t^j = \hat{y}_t$, obtaining the expressions in Proposition 9 with $\chi = 1$ and $\delta = 1$. The path of the iMPCs is apparent in this special case: faced with a current income shock, agents optimally self-insure, saving in liquid wealth to maintain a higher consumption in the future. While when facing a future income shock agents consume in anticipation, decreasing their stock of liquid savings.

Ceteris paribus, countercyclical inequality $\chi > 1$ leads to a higher contemporaneous MPC but to lower future MPCs (without affecting persistence as described by x_b which is independent of χ). Persistence is instead increasing with the share of hand-to-mouth and decreasing with the level of idiosyncratic risk (it can be directly verified that $\partial x_b / \partial \lambda > 0$ and $\partial x_b / \partial (1-s) < 0$).

Figure D.1 illustrates this by plotting the iMPCs for four models: TANK, and three cases of THANK (encompassing both liquidity and cyclical inequality) for pro- and counter-cyclical inequality, and the benchmark acyclical-inequality akin to Auclert et al's quantitative HANK, respectively. The left panel looks at a date-0 aggregate income shock and calibrates the THANK with acyclical inequality to closely follow Auclert et al, i.e. $\beta = 0.8$ and $\lambda = 0.5$; this requires $s = 0.84$ to match both the contemporaneous and next-year MPCs (0.55 and 0.15 respectively). The discount rate is very large, even for the yearly calibration adopted here; in the models with cyclical inequality (both TANK and THANK) I set $\beta = 0.95$ and match the two target MPCs with $\lambda = 0.33$, $s = 0.82$ and $\chi = 1.4$. This is coincidentally close to the calibration used in Bilbiie (2020) to match other (aggregate, *general-equilibrium*) objects with the same model.

The intertemporal path of the iMPCs is remarkably in line with that documented by Fagereng et al and Auclert et al in the data; in particular, the effect of the income shock dies off a few years after; whereas the model with acyclical inequality implies unrealistically high persistence while TANK implies no persistence at all. The reverse side of it is that, as clear from the right panel that compares iMPCs out of current and future income shocks for THANK with acyclical and countercyclical inequality, the latter implies larger iMPCs out of future income—an illustration of part (ii) of the Proposition; this is due, intuitively, to the same self-insurance forces that generate Euler-compounding in general equilibrium illustrated in the previous section. Direct differentiation of the analytical expressions in Proposition 9 reveals in fact that the iMPCs out of future income (news) are increasing in χ while the iMPCs out of past income are decreasing in χ.

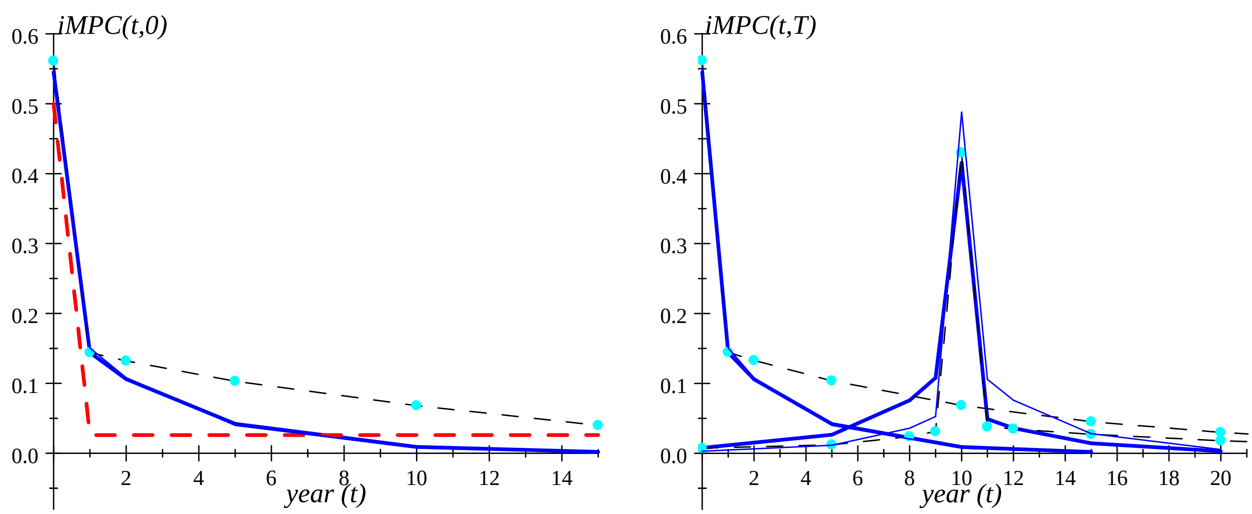

Figure D.1: iMPCs in THANK with $\chi = 1$ (thin black dot-dash); TANK (red dash); THANK with counter- and pro-cyclical inequality (thick and thin blue solid). Left: $T = 0$; right: $T = 0; 10$

An important remark is that *counter*cyclical inequality is, nevertheless, *not necessary* for the THANK model to match the iMPCs. Indeed, the model with *pro*cyclical inequality $\chi < 1$ also does it. To illustrate this, consider the model with $\chi = 0.8$. Clearly, we need to re-calibrate the model for a lower χ implies, by the logic of the cyclical-inequality channel, a lower contemporaneous MPC and a higher MPC out of past income; matching the two MPCs thus requires re-calibrating $\lambda = 0.64$ and $s = 0.74$. The resulting path (the thin solid line in the Figure) illustrates our intuition: the MPC out of past income is virtually identical, which is not surprising since we matched the one-period-ago MPC. But the whole path of the "forward" MPCs is below the countercyclical-inequality case (with the acyclical-inequality case between the two), which is a direct implication of the Euler discounting through δ discussed at length above. Notice, however, that while discounting/compounding in the Euler equation is not *per se* of the essence for matching the iMPCs (although it certainly matters quantitatively), idiosyncratic risk *is*.

D.3 Proof of Proposition 9

The solution of the asset-accumulation equation implies the following recursions for the responses of assets to income shocks:

$$\begin{aligned}
t &\leq T-1: \frac{db_{t+1}}{d\hat{y}_T} = x_b \frac{db_t}{d\hat{y}_T} + \frac{1-\lambda}{s} \left(\beta x_b\right)^{T-t} \left(\beta x_b - \delta\right) \\
t &= T: \frac{db_{t+1}}{d\hat{y}_T} = x_b \frac{db_t}{d\hat{y}_T} + \frac{1-\lambda}{s} \beta x_b \\
t &> T: \frac{db_{t+1}}{d\hat{y}_T} = x_b \frac{db_t}{d\hat{y}_T}
\end{aligned}$$

The solutions of these equations are (setting initial debt equal to steady-state without loss of generality):

$$\begin{aligned}
t &\leq T-1: \frac{db_{t+1}}{d\hat{y}_T} = \left(\beta x_b\right)^{T-t} \frac{1-\lambda\chi}{s} \left(\beta x_b - \delta\right) \frac{1-\left(x_b \beta x_b\right)^{t+1}}{1 - x_b \beta x_b} \\
t &= T: \frac{db_{T+1}}{d\hat{y}_T} = x_b \beta x_b \frac{1-\lambda\chi}{s} \left(\beta x_b - \delta\right) \frac{1-\left(x_b \beta x_b\right)^{T}}{1 - x_b \beta x_b} + \frac{1-\lambda\chi}{s} \beta x_b \\
t &\geq T+1: \frac{db_{t+1}}{d\hat{y}_T} = x_b^{t-T} \frac{db_{T+1}}{d\hat{y}_T}
\end{aligned}$$

Taking derivatives of the consumption function D.9, we have:

$$\begin{aligned}
t &\leq T-1: \frac{dc_t}{d\hat{y}_T} = \beta^{-1} \left(1 - \beta x_b\right) \frac{db_t}{d\hat{y}_T} + \frac{1-\lambda\chi}{s} \left(\beta x_b\right)^{T-t} \left(\delta - \beta x_b\right) \\
t &= T: \frac{dc_t}{d\hat{y}_T} = 1 + \beta^{-1} \left(1 - \beta x_b\right) \frac{db_t}{d\hat{y}_T} - \frac{1-\lambda\chi}{s} \beta x_b \\
t &> T: \frac{dc_t}{d\hat{y}_T} = \beta^{-1} \left(1 - \beta x_b\right) \frac{db_t}{d\hat{y}_T}
\end{aligned}$$

Replacing the solution for assets:

$$
\begin{aligned}
t &\leq T-1: \frac{dc_t}{d\hat{y}_T} = \beta^{-1}(1-\beta x_b)\ (\beta x_b)^{T-t+1} \frac{1-\lambda\chi}{s}(\beta x_b - \delta)\frac{1-(x_b\beta x_b)^t}{1-x_b\beta x_b} + \frac{1-\lambda\chi}{s}(\beta x_b)^{T-t}(\delta - \beta x_b) \\
t &= T: \frac{dc_t}{d\hat{y}_T} = 1+\beta^{-1}(1-\beta x_b)\beta x_b \frac{1-\lambda\chi}{s}(\beta x_b-\delta)\frac{1-(x_b\beta x_b)^T}{1-x_b\beta x_b} - \frac{1-\lambda\chi}{s}\beta x_b \\
t &\geq T+1: \frac{dc_t}{d\hat{y}_T} = \beta^{-1}(1-\beta x_b)\frac{db_t}{d\hat{y}_T} = \beta^{-1}(1-\beta x_b) x_b^{t-T-1}\frac{db_{T+1}}{d\hat{y}_T} \\
&= \beta^{-1}(1-\beta x_b) x_b^{t-T-1}\left(x_b\beta x_b \frac{1-\lambda\chi}{s}(\beta x_b-\delta)\frac{1-(x_b\beta x_b)^T}{1-x_b\beta x_b} + \frac{1-\lambda\chi}{s}\beta x_b\right)
\end{aligned}
$$

Rewriting and simplifying, we obtain the expressions in Proposition 9. Notice that, as argued by Auclert et al, the present discounted sum of the iMPCs needs to be 1 (the increase in income is consumed entirely, sooner or later). To prove that the iMPCs in THANK derived here satisfy this property, replace the respective solution into the sum:

$$
\sum_{t=0}^{T-1} \beta^{t-T}\frac{dc_t}{d\hat{y}_T} + \frac{dc_T}{d\hat{y}_T} + \sum_{t=T+1}^{\infty}\beta^{t-T}\frac{dc_t}{d\hat{y}_T}
$$

obtaining

$$
\begin{aligned}
& \frac{1-\lambda\chi}{s}\frac{\delta-\beta x_b}{1-\beta x_b^2}\beta^T x_b\left[1-\left(\beta x_b^2\right)^T\right] \\
& +1-\frac{1-\lambda\chi}{s}\beta x_b - (\delta-\beta x_b)x_b\frac{1-\lambda\chi}{s}(1-\beta x_b)\frac{1-(x_b\beta x_b)^T}{1-x_b\beta x_b} \\
& +\frac{1-\lambda\chi}{s}\frac{\beta x_b}{1-\beta x_b^2}\left(1-x_b\delta + x_b(\delta-\beta x_b)\left(\beta x_b^2\right)^T\right) \\
=\ & 1
\end{aligned}
$$

The **calibration** in text following Auclert et al concerns two iMPCs, $\frac{dc_0}{d\hat{y}_0} = 1-\frac{1-\lambda\chi}{s}\beta x_b$ and $\frac{dc_1}{d\hat{y}_0} = \frac{1-\lambda\chi}{s}(1-\beta x_b)x_b$.

Part (ii) of the Proposition concerns the dependence on χ (and δ Euler-compounding), keeping fixed the time-0 contemporaneous MPC $\frac{dc_0}{d\hat{y}_0}$; denote this by:

$$
m_{00} \equiv \frac{dc_0}{d\hat{y}_0} = 1-\frac{1-\lambda\chi}{s}\beta x_b
$$

Replacing in the Proposition and rewriting the iMPCs, taking the derivative with respect to the cyclicality of inequality χ we obtain:

$$
\frac{\partial \frac{dc_t}{d\hat{y}_T}}{\partial \chi}|_{\overline{m_{00}}} = \frac{\partial}{\partial\chi}\begin{cases} \frac{1-m_{00}}{\beta x_b}\frac{\delta-\beta x_b}{1-\beta x_b^2}(\beta x_b)^{T-t}\left(1-x_b+x_b(1-\beta x_b)\left(\beta x_b^2\right)^t\right), & \text{if} \quad t\leq T-1; \\ 1-\frac{1-m_{00}}{\beta x_b}\beta x_b - (\delta-\beta x_b)\frac{1-m_{00}}{\beta}(1-\beta x_b)\frac{1-\left(\beta x_b^2\right)^T}{1-\beta x_b^2}, & \text{if} \quad t=T; \\ \frac{1-m_{00}}{\beta x_b}\frac{1-\beta x_b}{1-\beta x_b^2}x_b^{t-T}\left(1-x_b\delta+x_b(\delta-\beta x_b)\left(\beta x_b^2\right)^T\right), & \text{if} \quad t\geq T+1. \end{cases}
$$

It follows directly that "anticipation iMPCs" ($t<T$) are increasing in χ (using $\frac{\partial\delta}{\partial\chi} = (1-s)\frac{1-\lambda}{(1-\lambda\chi)^2} > 0$); iMPCs out of past income ($t>T$) are decreasing in χ (the derivative is proportional to $-x_b\left(1-\left(\beta x_b^2\right)^T\right)\frac{\partial\delta}{\partial\chi}<0$), and decrease the contemporaneous MPC at given T.

D.4 Determinacy and iMPCs

Auclert et al (2019) show that determinacy occurs when the unweighted sum of iMPCs for an income shock occurring at $T \to \infty$ is larger than 1. In my model, this object is calculated using the expressions in Proposition 9.

$$\mu_{impc} = \lim_{T\to\infty} \left(\sum_{t=0}^{T-1} \frac{dc_t}{d\hat{y}_T} + \frac{dc_T}{d\hat{y}_T} + \sum_{t=T+1}^{\infty} \frac{dc_t}{d\hat{y}_T} \right)$$

Replacing the expressions in Proposition 9 and taking the limit for $T \to \infty$ we obtain, for the first term:

$$\frac{1-\lambda\chi}{s} \left(\delta - \beta x_b\right) \beta x_b \frac{1-x_b}{\left(1-x_b\beta x_b\right)\left(1-\beta x_b\right)}$$

for the second term (contemporaneous iMPC):

$$1 + \left(1-\beta x_b\right) x_b \frac{1-\lambda\chi}{s} \left(\beta x_b - \delta\right) \frac{1}{1-x_b\beta x_b} - \frac{1-\lambda\chi}{s} \beta x_b$$

and for the third sum:

$$\left(1-\beta x_b\right) x_b \frac{1-\lambda\chi}{s} \frac{1}{1-x_b} \frac{1-x_b\delta}{1-x_b\beta x_b}.$$

Taking the total sum:

$$\begin{aligned} \mu_{impc} &= 1 + \frac{1-\lambda\chi}{s} x_b \left[\begin{array}{c} \left(\delta - \beta x_b\right) \frac{\beta(1-x_b)}{(1-x_b\beta x_b)(1-\beta x_b)} - \beta \\ -\left(\delta - \beta x_b\right) \frac{1-\beta x_b}{1-x_b\beta x_b} + \left(1-\beta x_b\right) \frac{1}{1-x_b} \frac{1-x_b\delta}{1-x_b\beta x_b} \end{array} \right] \\ &= 1 + \left(1-\delta\right) \frac{1-\lambda\chi}{s} \frac{\left(1-\beta\right) x_b}{\left(1-\beta x_b\right)\left(1-x_b\right)} \end{aligned} \tag{D.10}$$

Thus, the condition for determinacy (and for the Taylor principle to be sufficient) of Auclert et al $\mu_{impc} > 1$ is equivalent to my condition $\delta < 1$.

D.5 Determinacy with a nominal debt quantity rule

In this Appendix, I prove Proposition (6) for the case of static Phillips curve. As we shall see, the algebra and intuition are both very similar to the proof of determinacy under a Wicksellian rule—and so is the generalization to forward-looking Phillips curve, which I ignore for brevity.

The loglinearized government budget constraint is (with B_Y steady-state debt share in output):

$$b_{t+1} + t_t = R\left(b_t + B_Y i_{t-1} - B_Y \pi_t\right). \tag{D.11}$$

Next, notice that one important dimension of Hagedorn's policy is that taxes adjust automatically to ensure that the government budget constraint is indeed a constraint for any price level (thus ruling out fiscal-theory equilibria); that is, t_t is endogenous and determined by (D.11) residually

To derive the aggregate Euler equation for the model with liquidity and with steady-state inequality and non-zero steady-state debt, I adopt the following assumption that simplifies the algebra without losing generality: to obtain that consumption is equalized in steady-state across agents (while income is not, and there is positive debt), assume a pure redistributive transfer taken as given by agents (so it does not preclude bonds demand). Since without the transfer the share of consumption of H in Y

is $C^H/Y = [1+(1-\lambda)(\Gamma-1)]^{-1}$, this transfer—an additive term in (D.1)—is trivially equal to $1 - \left[1+\frac{1-\lambda}{1-s}\left(\frac{1}{\beta R}-1\right)\right]^{-1}$, while for S agents we have a tax equal to the same amount times $\lambda/(1-\lambda)$. In addition, let again χ denote the equilibrium elasticity: $\frac{Y^H}{Y}y_t^H = \chi y_t$.

Under these assumptions, the budget constraint of H (D.6) and S become:

$$\begin{aligned}
c_t^H &= \chi y_t - t_t + \frac{1-s}{\lambda}Rb_t + \frac{1-s}{\lambda}RB_Y(i_{t-1}-\pi_t)\\
c_t^S &= \frac{1}{1-\lambda}c_t - \frac{\lambda}{1-\lambda}\left(\chi y_t - t_t + \frac{1-s}{\lambda}Rb_t + \frac{1-s}{\lambda}RB_Y(i_{t-1}-\pi_t)\right)
\end{aligned}$$

Replacing these in the self-insurance Euler equation we obtain (with the usual δ notation):

$$\begin{aligned}
&c_t = \delta E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}(i_t - E_t\pi_{t+1})\\
&-\frac{\lambda}{1-\lambda\chi}t_t - \frac{1-\lambda-s}{1-\lambda\chi}E_t t_{t+1} + \frac{1-\lambda-s}{1-\lambda\chi}\frac{1-s}{\lambda}Rb_{t+1} + \frac{\lambda}{1-\lambda\chi}\frac{1-s}{\lambda}Rb_t\\
&+\frac{1-\lambda-s}{1-\lambda\chi}\frac{1-s}{\lambda}RB_Y(i_t - E_t\pi_{t+1}) + \frac{\lambda}{1-\lambda\chi}\frac{1-s}{\lambda}RB_Y(i_{t-1}-\pi_t)
\end{aligned}$$

When is the equilibrium determinate under the debt-quantity rule fixing the nominal amount of debt proposed by Hagedorn? Replace the rule $b_{t+1}^N = 0 \to b_{t+1} = b_{t+1}^N - p_t = -p_t$, taxes from (D.11), the assumed exogenous interest rates $i_t = i_t^*$, and the static Phillips curve $c_t = \kappa^{-1}(p_t - p_{t-1})$ to obtain, rearranging (take log utility wlog):

$$\begin{aligned}
&\left[\delta\kappa^{-1} + \frac{1-\lambda}{1-\lambda\chi} - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)^2}{1-\lambda\chi}RB_Y - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)}{1-\lambda\chi}\right]E_t p_{t+1}\\
&-\left\{(1+\delta)\kappa^{-1} + \frac{1-\lambda}{1-\lambda\chi} - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)^2}{1-\lambda\chi}RB_Y + \frac{\lambda\left[1+\left(\frac{1-s}{\lambda}-1\right)^2 R\right]}{1-\lambda\chi} + \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)}{1-\lambda\chi}RB_Y\right\}p_t\\
&+\left[\kappa^{-1} - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)}{1-\lambda\chi}R + \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)}{1-\lambda\chi}RB_Y\right]p_{t-1} =\\
&\left(\frac{1-\lambda}{1-\lambda\chi} - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)^2}{1-\lambda\chi}RB_Y\right)i_t^* - \frac{\lambda\left(\frac{1-s}{\lambda}-1\right)}{1-\lambda\chi}RB_Y i_{t-1}^*
\end{aligned}$$

This is a second-order difference equation with characteristic polynomial

$$J(x) = A_2x^2 - A_1x + A_0,$$

where $A_2,\ A_1,\ A_0$ are the coefficients on $E_tp_{t+1},\ p_t,\ p_{t-1}$. By the same standard results invoked before, the necessary and sufficient condition for determinacy is $J(1)J(-1) < 0$, where

$$\begin{aligned}
J(1) &= -\frac{1-s}{1-\lambda\chi}\left[1+R\left(\frac{1-s}{\lambda}-1\right)\right]\\
J(-1) &= 2\kappa^{-1}(1+\delta) + 2\frac{1-\lambda}{1-\lambda\chi} + 2\frac{\lambda}{1-\lambda\chi}\left(\frac{1-s}{\lambda}-1\right)RB_Y\left(2-\frac{1-s}{\lambda}\right)\\
&\quad + \frac{\lambda}{1-\lambda\chi}\left(2-\frac{1-s}{\lambda}\right)\left[1+\left(1-\frac{1-s}{\lambda}\right)R\right]
\end{aligned}$$

First, we have $J(1) < 0$ if:

$$1 + R\left(\frac{1-s}{\lambda} - 1\right) > 0 \rightarrow R < \frac{1}{1 - \frac{1-s}{\lambda}},$$

which is always satisfied (see the restrictions needed for the steady-state debt demand). Next, note that $J(-1) > 0$ holds in e.g. the most extreme case of pure oscillation $s = 0$, as $2\kappa^{-1}(1+\delta) + 2\frac{1-\lambda}{1-\lambda\chi} > 0$. In the more general case, with $1 - s < \lambda$, the third term in $J(-1)$ can in principle become negative when $B_Y > 0$ but we can in fact prove that the sufficient condition for its being positive (sum of third and fourth term is always positive): $2\left(\frac{1-s}{\lambda} - 1\right)RB_Y + 1 + \left(1 - \frac{1-s}{\lambda}\right)R > 0$ implies (using $\left(1 - \frac{1-s}{\lambda}\right)R < 1$)

$$B_Y < \frac{\frac{1}{\left(1-\frac{1-s}{\lambda}\right)R} + 1}{2} > 1$$

Replacing the equilibrium expression for debt, this implies

$$-\left(\frac{\frac{1-\lambda}{1-s}\left(\frac{1}{\beta R} - 1\right)}{1 + \frac{1-\lambda}{1-s}\left(\frac{1}{\beta R} - 1\right)} + \frac{1}{1 + (1-\lambda)(\Gamma - 1)}\right) < \frac{1 + \left(\left(\frac{1-s}{\lambda} - 1\right)R\right)^2}{2\left(1 - \frac{1-s}{\lambda}\right)R},$$

which is always satisfied since the LHS is negative and the RHS positive. This proves the Proposition in the general case with $B_Y \geq 0$. In the special case, zero-liquidity limit $B_Y = 0$ the determinacy conditions are much simpler with $J(-1) > 0$ following immediately and $J(1) < 0$ requiring:

$$\frac{1-s}{\lambda} > 1 - \beta.$$

The proof that the debt quantity rule eliminates the FG puzzle and expansionary interest rate increases is entirely analogous to the one for a Wicksellian rule outlined in Appendix C.3 above, with the appropriate change of notation (implying different roots x_+ and x_-). Likewise, the extension to forward-looking Phillips curve is also analogous.

Indeed, notice that in the iid idiosyncratic risk case $1 - s = \lambda$, the key difference equation becomes (using the same notation $\nu_0 = \delta + \kappa\frac{1-\lambda}{1-\lambda\chi}$):

$$E_t p_{t+1} - \left[1 + \nu_0^{-1}\left(1 + \lambda\frac{\kappa}{1-\lambda\chi}\right)\right]p_t + \nu_0^{-1}p_{t-1} = \frac{1-\lambda}{1-\lambda\chi}\kappa\nu_0^{-1}i_t^*.$$

This has *exactly* the same form as the equation under Wicksellian rule, but with λ instead of $\sigma(1-\lambda)\phi_p$ inside the bracket of the coefficient on p_t; here, the real demand for liquidity is elastic to the price because of risk, combined with a fixed nominal quantity; under the Wicksellian rule, because of the direct effect $\sigma(1-\lambda)$ of interest rates (and their dependency on p via ϕ_p) on the saving decision of S.

Considering instead the **extension whereby the debt rule responds to the price level** $b_{t+1}^N = \phi_b p_t \rightarrow b_{t+1} = (\phi_b - 1)p_t$ and replacing in the Euler equation above doing the same (tedious) manipulations, we obtain the determinacy-relevant characteristic polynomial objects:

$$J(1) = -(1-\phi_b)\frac{\lambda}{1-\lambda\chi}\frac{1-s}{\lambda}\left[1 + \left(\frac{1-s}{\lambda} - 1\right)R\right]$$

$$
\begin{aligned}
J(-1) = {} & 2\kappa^{-1}(1+\delta) + 2\frac{1-\lambda}{1-\lambda\chi} + 2\frac{\lambda}{1-\lambda\chi}\left(2-\frac{1-s}{\lambda}\right)\left(\frac{1-s}{\lambda}-1\right)RB_Y \\
& + (1-\phi_b)\frac{\lambda}{1-\lambda\chi}\left(2-\frac{1-s}{\lambda}\right)\left[1+\left(1-\frac{1-s}{\lambda}\right)R\right]
\end{aligned}
$$

These satisfy the necessary and sufficient condition for determinacy if $\phi_b < 1$ so that $J(1) < 0$ and $J(-1) > 0$.

E Optimal Policy in THANK

First, we write explicitly the Ramsey problem, and then we derive the second-order approximation around an efficient equilibrium that allows transforming it into a linear-quadratic problem.

E.1 The Ramsey Problem in THANK

The Ramsey problem of maximizing a utilitarian welfare objective is:

$$
\begin{aligned}
& \max_{\{C_t^H, C_t^S, N_t^H, N_t^S, \pi_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \{\lambda U\left(C_t^H, N_t^H\right) + (1-\lambda) U\left(C_t^S, N_t^S\right) \qquad \text{(E.1)} \\
& + \varsigma_{1,t}\left(\frac{U_N\left(N_t^S\right)}{U_C\left(C_t^S\right)} - \frac{U_N\left(N_t^H\right)}{U_C\left(C_t^H\right)}\right) \\
& + \varsigma_{2,t}\left(C_t^H + \frac{U_N(N_t^H)}{U_C(C_t^H)}N_t^H - \frac{\tau^D}{\lambda}(1-\frac{\psi}{2}\pi_t^2 + \frac{U_N(N_t^H)}{U_C(C_t^H)})(\lambda N_t^H + (1-\lambda)N_t^S)\right) \\
& + \varsigma_{3,t}\left(\lambda C_t^H + (1-\lambda)C_t^S - (1-\frac{\psi}{2}\pi_t^2)(\lambda N_t^H + (1-\lambda)N_t^S)\right) \\
& + \varsigma_{4,t} \begin{array}{c} \{\pi_t(1+\pi_t) - \beta E_t[\frac{U_C(C_{t+1}^S)}{U_C(C_t^S)}\frac{\lambda N_{t+1}^H + (1-\lambda)N_{t+1}^S}{\lambda N_t^H + (1-\lambda)N_t^S}\pi_{t+1}(1+\pi_{t+1})] \\ + \frac{\varepsilon-1}{\psi}[\frac{\varepsilon}{\varepsilon-1}\frac{U_N(N_t^H)}{U_C(C_t^H)} + 1 + \tau^S]\} \end{array}
\end{aligned}
$$

where $\varsigma_{j,t}$ the co-state Lagrange multipliers associated to them (with arbitrary initial values).

In the above Ramsey constraints, we already substituted $C_t = \left(1-\frac{\psi}{2}\pi_t^2\right)Y_t = \left(1-\frac{\psi}{2}\pi_t^2\right)N_t...$, $W_t = -U_N\left(N_t^S\right)/U_C\left(C_t^S\right) = -U_N\left(N_t^H\right)/U_C\left(C_t^H\right)$, and eliminated $D_t = \left(1-\frac{\psi}{2}\Pi_t^2 - W_t\right)\left(\lambda N_t^H + (1-\lambda)N_t^S\right)$

Importantly, notice that the self-insurance equation is not a constraint—just as in RANK the Euler-IS curve is not a constraint. In other words, the equation

$$
U_C(C_t^S) = \beta E_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(s(C_{t+1})U_C(C_{t+1}^S) + (1-s(C_{t+1}))U_C(C_{t+1}^H)\right)\right]
$$

determines i_t residually once we found the allocation.[35]

Note that it is trivial to show that the *first-best* equilibrium amounts to perfect insurance. And solving the above Ramsey problem and finding the optimal steady-state inflation can be easily shown to deliver long-run price stability ($\pi = 0$) as the optimal long-run target.

[35]The interest rate is no longer orthogonal in models where it affects the mapping between the income and consumption distribution and aggregate income, for instance because it determines the MPC as in the Acharya and Dogra's PRANK model. See Acharya, Challe, and Dogra (2020) for a recent exploration of the optimal policy implications of this.

E.2 A Second-Order Approximation to Welfare

We approximate the economy around an efficient equilibrium, defined as an equilibrium with both flexible prices and perfect insurance; this is the case in our baseline economy under the assumed steady-state fiscal policy, because the optimal subsidy inducing zero profits in steady state implies that consumption shares are equalized across agents. In particular, since the fiscal authority subsidize sales at the constant rate τ^S and redistribute the proceedings in a lump-sum fashion T^S such that in steady-state there is marginal cost pricing, and profits are zero. The profit function becomes $D_t(k) = \left(1+\tau^S\right) P_t(k)Y_t(k) - W_t N_t(k) - \frac{\psi}{2}\left(\frac{P_t(k)}{P^{**}_{t-1}} - 1\right)^2 P_t Y_t + T^S_t$ where by balanced budget $T^S_t = \tau^S P_t(k) Y_t(k)$. Efficiency requires $\tau^S = (\varepsilon - 1)^{-1}$, such that under flexible prices $P^*_t(k) = W^*_t$ and hence profits are $D^*_t = 0$ (evidently, with sticky prices profits are not zero as the mark-up is not constant). Under this assumption we have that in steady-state:

$$\frac{U_N\left(N^H\right)}{U_C\left(C^H\right)} = \frac{U_N\left(N^S\right)}{U_C\left(C^S\right)} = \frac{W}{P} = 1 = \frac{Y}{N},$$

where $N^j = N = Y$ and $C^j = C = Y$.

Suppose further that the social planner maximizes a convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U^H\left(C^H_t, N^H_t\right) + [1-\lambda]\, U^S\left(C^S_t, N^S_t\right)$. The second-order approximation to type $j's$ utility around the **efficient flex-price equilibrium** delivers:

$$\begin{aligned} \hat{U}_{j,t} &\equiv U_j\left(C_{j,t}, N_{j,t}\right) - U_j\left(C^*_{j,t}, N^*_{j,t}\right) = \\ &= U_C C^j\left[c^j_t + \frac{1-\sigma^{-1}}{2}\left(c^j_t\right)^2\right] - U_N N^j\left[n^j_t + \frac{1+\varphi}{2}\left(n^j_t\right)^2\right] + t.i.p + O\left(\parallel \zeta \parallel^3\right), \end{aligned} \tag{E.2}$$

where we used that flex-price values are equal to steady-state values (because of our assumption of no shocks to the natural rate) $c^{j*}_t \left(\equiv \log \frac{C^{j*}_t}{C}\right) = c^*_t = 0$ and $n^{j*}_t \left(\equiv \log \frac{N^{j*}_t}{N}\right) = n^*_t = 0$.

Approximating the goods market clearing condition to second order delivers:

$$\begin{aligned} \lambda C_{H,t} + (1-\lambda)\, C_{S,t} &\simeq \lambda c_{H,t} + (1-\lambda)\, c_{S,t} + \frac{1}{2}\left(\lambda c^2_{H,t} + (1-\lambda)\, c^2_{S,t}\right) \\ &= \lambda N_{H,t} + (1-\lambda)\, N_{S,t} \simeq \lambda n_{H,t} + (1-\lambda)\, n_{S,t} + \frac{1}{2}\left(\lambda n^2_{H,t} + (1-\lambda)\, n^2_{S,t}\right) \end{aligned}$$

The linearly-aggregated first-order term is thus found from this second-order approximation of the economy resource constraint as:

$$\lambda c_{H,t} + (1-\lambda)\, c_{S,t} - \lambda n_{H,t} - (1-\lambda)\, n_{S,t} + \frac{1}{2}\left(\lambda c^2_{H,t} + (1-\lambda)\, c^2_{S,t} - \left(\lambda n^2_{H,t} + (1-\lambda)\, n^2_{S,t}\right)\right) = 0 \tag{E.3}$$

The economy resource constraint is

$$C_t = \left(1 - \frac{\psi}{2}\pi^2_t\right) Y_t = \left(1 - \frac{\psi}{2}\pi^2_t\right) N_t$$

which approximated to second order is:

$$c_t = n_t - \frac{\psi\pi}{1-\frac{\psi}{2}\pi^2}\pi_t - \frac{1}{2}\frac{\psi}{1-\frac{\psi}{2}\pi^2}\pi^2_t$$

It is straightforward to show that the optimal long-run inflation target in this economy is, just like in RANK, $\pi = 0$. Replacing, we obtain the second-order approximation of the resource constraint around the optimal long-run equilibrium:

$$c_t = n_t - \frac{\psi}{2}\pi_t^2, \tag{E.4}$$

where the second term captures the welfare cost of inflation.

Note that since $U_C C^j$ and $U_N N^j$ are equal across agents we can aggregate the approximations of individual utilities above (E.2), using (E.3) and (E.4) to eliminate linear terms, into:

$$\hat{U}_t = -U_C C\left\{\frac{\sigma^{-1}}{2}\left[\lambda\left(c_t^H\right)^2 + (1-\lambda)\left(c_t^S\right)^2\right] + \frac{\varphi}{2}\left[\lambda\left(n_t^H\right)^2 + (1-\lambda)\left(n_t^S\right)^2\right] + \frac{\psi}{2}\pi_t^2\right\} + t.i.p + O\left(\parallel \zeta \parallel^3\right).$$

Quadratic terms can be expressed as a function of aggregate consumption (output). Notice that in evaluating these quadratic terms we can use first-order approximations of the optimality conditions (higher order terms imply terms of order $O\left(\parallel \zeta \parallel^3\right)$). Recall that up to first order, we have that $c_t^H = \chi y_t$ and $c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t$ and (after straightforward manipulation for hours worked):

$$n_t^H = \left(1 + \varphi^{-1}\sigma^{-1}(1-\chi)\right) y_t;\ n_t^S = \left(1 + \varphi^{-1}\sigma^{-1}\frac{\lambda}{1-\lambda}(\chi - 1)\right) y_t$$

To second order we thus have

$$\begin{aligned}
\left(c_t^H\right)^2 &= \chi^2 y_t^2 + O\left(\parallel \zeta \parallel^3\right);\ \left(n_t^H\right)^2 = \left[1 + \varphi^{-1}\sigma^{-1}(1-\chi)\right]^2 y_t^2 + O\left(\parallel \zeta \parallel^3\right) \\
\left(c_t^S\right)^2 &= \left(\frac{1-\lambda\chi}{1-\lambda}\right)^2 y_t^2 + O\left(\parallel \zeta \parallel^3\right);\ \left(n_t^S\right)^2 = \left[1 + \varphi^{-1}\sigma^{-1}\frac{\lambda}{1-\lambda}(\chi-1)\right]^2 y_t^2 + O\left(\parallel \zeta \parallel^3\right)
\end{aligned}$$

Replacing in the per-period welfare and considering only the "real" part (abstracting from inflation) for notational convenience, and ignoring terms independent of policy and of order larger than 2 we have:

$$\begin{aligned}
&\frac{\sigma^{-1}}{2}\left[\lambda(\chi c_t)^2 + (1-\lambda)\left(\frac{1-\lambda\chi}{1-\lambda}c_t\right)^2\right] \\
&+\frac{\varphi}{2}\left[\lambda\left(\left[1 + (\varphi\sigma)^{-1}(1-\chi)\right]c_t\right)^2 + (1-\lambda)\left(\left[1 - \frac{\lambda}{1-\lambda}(\varphi\sigma)^{-1}(1-\chi)\right]c_t\right)^2\right]
\end{aligned}$$

Collecting terms, multiplying by 2 for simplicity, and rearranging, using $c_t = y_t$:

$$\begin{aligned}
&\sigma^{-1}\left(1 + \frac{\lambda}{1-\lambda}(\chi-1)^2\right) y_t^2 + \varphi\left(1 + \frac{\lambda}{1-\lambda}\left[(\varphi\sigma)^{-1}(\chi-1)\right]^2\right) y_t^2 \\
= &\left(\sigma^{-1} + \varphi + \sigma^{-1}\frac{\lambda}{1-\lambda}(\chi-1)^2\left(1 + (\varphi\sigma)^{-1}\right)\right) y_t^2 \\
= &\left(\sigma^{-1} + \varphi\right) y_t^2 + \sigma^{-1}\frac{\lambda}{1-\lambda}\left(1 + (\varphi\sigma)^{-1}\right)\left[(\chi-1)y_t\right]^2 \\
= &\left(\sigma^{-1} + \varphi\right)\left[y_t^2 + \sigma^{-1}\varphi^{-1}\lambda(1-\lambda)\gamma_t^2\right]
\end{aligned}$$

where the last line used the expression for γ_t (13). Adding back the inflation term, we obtain the loss function in (36) in Proposition 7 in text.