A tail of labor supply and a tale of monetary policy*

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Abstract

We study the interaction between monetary policy and labor supply decisions at the household level. We uncover evidence of heterogeneous responses and a strong countercyclicality of hours worked in the left tail of the income distribution, following a monetary policy shock in the U.S. and the U.K. That is, while aggregate hours and labor earnings decline, employed individuals at the bottom of the income distribution increase their hours worked in response to an interest rate hike. Moreover, their response is stronger in magnitude relative to other income groups. We rationalize this using a two-agent New-Keynesian (TANK) model where our empirical findings can be replicated with heterogeneity in the sensitivity of marginal utility of consumption and a stronger income effect for the Hand-to-Mouth households. This setup uncovers a novel channel of transmission of monetary policy via inequality generated by the Hand-to-Mouth substitution of leisure for consumption following a negative income shock. Using a quantitative model with both intensive and extensive margin of labor supply that replicates our evidence, we show that this new channel reduces the amplification of monetary policy via inequality generated by the heterogeneous behavior of unemployment along the income distribution.

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1 Introduction

Do people adjust how much they want to work when the central bank's monetary policy stance shifts? More specifically, does an interest rate hike induce individuals to work more or fewer hours? And does this effect differ across households with different levels of income (or earnings)?

The vast literature on the heterogeneous effects of monetary policy has focused on the inter-temporal channel that affects the consumption and savings plans of households (Bilbiie (2008), Auclert (2019), Cloyne, Ferreira and Surico (2020), Kaplan, Moll and Violante (2018)). However, changes in consumption plans induced by variation in rates also influence the intratemporal allocation between consumption and leisure; i.e., a household's desired supply of labor depends on how each individual can substitute consumption with working time and/or compensate with different sources of income. In standard models, the lower wage rates induced by a contractionary monetary policy have two effects on households' labor supply: a substitution effect that reduces how much households would prefer to work and an income effect that increases it.

The majority of the theoretical macro literature, assumes no or negligible income effects on the labor supply.[1] It is often thought that income effects are small because –being short-lived– monetary policy shocks do not have large effects on lifetime income, which is what matters for an optimizing worker-consumer.[2]

The scope of this paper is to revisit this channel and study the transmission mechanism of monetary policy to the labor supply decisions at the household level.[3] First, we aim to offer novel empirical evidence on the effect of monetary policy on hours worked at a more granular, disaggregated level. To do this, we study the effects of unexpected shifts in the monetary policy stance on the amount of hours worked by households with different income levels using survey data for the U.S. and the U.K. In both countries, we observe an increase in the average hours worked supplied by households with low incomes in response to an interest rate hike, in contrast with conventional macrooeconomic theory. At the same time, aggregate hours and wages across the whole distribution decline. This channel operates through an intensive margin, i.e., individuals who remain in the labor market work more hours. Moreover, their response is more sensitive to interest rate variations compared with other percentiles of income in the population. As the labor supplied by low- and moderate-

[1] E.g. Galí, Smets and Wouters (2012); Dyrda and Pedroni (2022); Wolf (2021); Auclert and Rognlie (2020); amongst others.

[2] However, most of the empirical evidence used to support this assumption focuses on other shocks and not on monetary policy shocks. See the literature review section for more details.

[3] The consequences of monetary policy actions on the labor market dynamics are not only of interest in academic cycles. Policymakers have expressed considerable interest in labor market outcomes across the whole spectrum of the population and in particular in low- and moderate-income communities. E.g. in a Jackson Hole speech on August 27, 2020, J. Powell said in unveiling the new Fed strategy that "our revised statement emphasizes that maximum employment is a broad-based and inclusive goal. This change reflects our appreciation for the benefits of a strong labor market, particularly for many in low- and moderate-income communities."

income households (*the tail of labor supply*) represents both a non-negligible share of the volatility and a relevant proportion of hours worked in the aggregate, this response is also quantitatively relevant from a macro perspective.[4]

The countercyclicality of hours worked in the left tail of the income distribution observed in the data is an equilibrium outcome resulting from the interaction of households' labor supply forces and firms' labor demand factors and consistent with multiple explanations. For example, as the recession induced by a contractionary monetary policy increases the probability of becoming unemployed, households with limited income sources have incentives to work more. Similarly, individuals who are close to their borrowing limits may need to work more hours to meet their debt obligations when interest rates rise. Supply-side explanations suggest that when lacking buffer savings or nonlabor income sources, households with low incomes have less room to maneuver during tough economic times, and by varying their labor supply they can smooth consumption along the business cycle. Alternatively, on the demand side, firms may lay off temporary or part-time workers and adjust the labor's input by utilizing more of their existing labor force inducing selection in the sample. While it is very difficult to isolate the dominant channel responsible for our empirical findings and a combination of these stories is more likely, several pieces of evidence suggest that selection is not the dominant force in this context; i.e. the results carry over when isolating the response of only full time employed workers and hours worked dynamics are found to be different when triggered by different business cycles shocks.

The second contribution of the paper is to study these channels in structural models and their implications for the transmission mechanism (*the tale*) of monetary policy. To gain intuitions, we begin by abstracting from movements in the extensive margin and focus only on the intensive margin. Our starting point is a simple New Keynesian model featuring two agents (TANK), an unconstrained household[5] and a *poor* Hand to Mouth (HtM)[6] household; in these models, the Euler equation does not hold for the HtM household because, for example, they have no access to financial markets or are constrained by a borrowing limit. A strong enough income effect and heterogeneous sensitivity of marginal utility of consumption (via intertemporal elasticity of substitution (IES) heterogeneity) across agents are the key ingredients to replicate our empirical evidence. These assumptions are in line with available estimates of the IES across households in the US and UK (Vissing-Jørgensen 2002, Attanasio, Banks and Tanner 2002) and can be microfounded with *ex-ante* homogeneous Stone-Geary preferences that generate *ex-post* heterogeneous IES increasing in the household's level of consumption in steady state.[7]

[4]We show that these results are robust across different household surveys, and identification strategies, are not primarily driven by fixed characteristics like housing tenure or by composition effects.

[5]By *unconstrained*, we mean a household that has access to complete financial markets and therefore satisfies the Permanent Income Hypothesis.

[6]We follow Kaplan, Violante and Weidner (2014) and denote these *poor* HtM to highlight that our focus is on the proportion of households with very little illiquid wealth and income and not on the *wealthy* HtM.

[7]Another way to generate similar results would be to follow the literature on *consumption commitments* (Chetty

While abstracting from extensive margin considerations, this setup has interesting implications for the effect of income inequality on the transmission of monetary policy. Higher inequality amplifies the transmission of monetary policy with homogeneous IES, as in e.g. Bilbiie (2008). Monetary policy propagation can instead be weaker when the IES of HtM agents is lower than that of unconstrained agents. The value of the elasticity of HtM consumption to aggregate income, the key driver of amplification (Bilbiie (2020), Auclert (2019), Patterson (2021)), is now directly affected by the ratio of the two agents' IES and the proportion of HtM. We show that this elasticity is increasing in the HtMs' IES. Therefore the stronger the income effect on HtMs' labor supply, the stronger the dampening effect on monetary policy would be. This happens because an heterogeneous IES amplifies the heterogeneity in the marginal rate of substitution between consumption and leisure across agents. This allows HtM households more flexibility in response to a decline in their income. Instead of reducing consumption one-to-one with income as in the standard HtM setup, they increase their labor supply, effectively substituting leisure for consumption, which translates into a lower impact of monetary policy on aggregate demand. The *dampening* effect of increasing inequality on the monetary policy transmission, we unveil, is novel in this class of models.

Finally, we want to study how quantitatively relevant this new channel is. To this end, we build a medium-scale general equilibrium model that features various layers of real and nominal frictions and different factors in production. Importantly, we allow for labor market adjustments also along the extensive margin and assume two segmented labor markets where the HtM (non-HtM) households' labor is a substitute (complement) of capital.[8] In downturns, as HtM labor can be easily substituted with capital, firms find it less costly to lay off HtM workers and ask the remaining ones to work longer hours; higher unemployment risks generate a motive for the HtM households to work more. We show that also in this set up, the heterogeneity in the sensitivity of marginal utility of consumption across agents is crucial to capture the different responses of households' labor supply; absent this feature, the hours worked supplied by the HtM workers would not increase after a monetary policy tightening, in contrast with our empirical evidence. We find that relative to the case with homogeneous sensitivity of marginal utility of consumption across households, the transmission of monetary policy is substantially weaker; in particular, the effects on output are 13 percent and 18 percent smaller on impact and cumulated over two years, respectively. The effects of a policy tightening on consumption and hours worked are more striking, where the drop in consumption and hours worked are on impact 30 percent and 34 percent less severe than in the homogeneous case, respectively. In sum, we find that allowing for a strong

and Szeidl (2007)) and have non-homothetic preferences defined over two types of consumption goods: one freely adjustable and one costly to adjust. This would limit the ability of poor households to adjust consumption pushing those with a job to increase their labor supply.

[8] Using data from the Survey of Consumer Finance, in Appendix I we show that poor HtM are mostly unskilled workers.

income effect on the labor supply of HtM households reduces the amplification of monetary policy via the inequality generated by the heterogeneous behavior of unemployment along the income distribution.

The paper is organized as follows: the next subsection discusses the existing literature. Section 2 describes the data and the empirical strategy and presents our empirical evidence. Section 3 presents a structural model that accounts for this evidence and investigates the implication for the transmission of monetary policy. Finally, section 4 provides some concluding remarks.

1.1 Related Literature

This paper relates to the recent literature on monetary policy and household heterogeneity. Empirically, the bulk of this literature has focused on the composition of households' balance sheets and measured the heterogeneity in the marginal propensity to consume (MPC) after a monetary policy shock (Cloyne et al. (2020), Auclert (2019)). Unlike these papers, we focus on the transmission of monetary policy shocks to labor supply decisions at the household level and abstract from the composition of their balance sheet.

Kehoe, Lopez, Pastorino and Salgado (2020) and Amir-Ahmadi, Matthes and Wang (2021) study the responses of hours worked and unemployment, respectively, of different population groups in the US to monetary shocks. In the former, they find that age and college education make household labor less cyclically sensitive. In the latter, they estimate large differences in the response of unemployment across different demographic groups. Unlike these studies, we take a complementary approach and sort the household surveyed population according to earning bins (as opposed to age, education, mortgage, or preassigned groups), from low- to high-income units and focus on the intensive margin of labor supply. Graves, Huckfeldt and Swanson (2023) study the effect of monetary policy on the labor market flows and find that a monetary policy tightening induces an increase in the fraction of labor force non-participants reporting that they want a job and an increase in the number of distinct job search methods by unemployed individuals. Both these margins of adjustments are consistent with an increase in the 'intensive margin' of the labor supply of non-employed individuals. These results are in line with our findings on the increase in the intensive margin of employed workers with low or moderate income.

Several papers have looked at administrative data to study the heterogeneous effects of monetary policy on labor market quantities, in Scandinavian countries (Amberg, Jansson, Klein and Rogantini-Picco (2022), Andersen, Johannesen, Jørgensen and Peydró (2021), Holm, Paul and Tischbirek (2021)). Differently from us, all of them are capturing the combined effect of monetary policy on the extensive and intensive margin of labor supply and focus their analysis on labor income. Hubert and Savignac (2022) do a similar analysis for France and can identify the relative importance of the intensive and extensive margins. They find that most of the variation in labor income in the bottom half of the income distribution is due to the extensive margin. However, they cannot distinguish between hours and wages when looking at the intensive margin. The same is true for Broer, Kramer and Mitman (2022), who find that unemployment risk is heterogeneous in Germany. Job separation is more pro-cyclical following a monetary policy (MP) shock for low-income workers than high-income ones. This implies a larger effect of MP on the labor income of low-income workers and a larger amplification of MP compared to the case without heterogeneous unemployment risk. Our results show that the response of unemployment along the income distribution in the US is consistent with unemployment risk heterogeneity. To quantify the different forces at play, we add such heterogeneity to our quantitative model. However, our contribution here is to identify a different channel of transmission working via the heterogeneous response of the intensive margin of labor supply following a monetary policy shock.

In summary, this literature focuses on labor income and cannot distinguish between hourly wages and the number of hours worked. This implies that their estimates may reflect quantity effects as well as price effects: salary income goes up because workers are employed more hours or because the hourly wage rate increases. To deal with this problem, we use a data set which includes hourly wages and hours worked and allows us to distinguish between price and quantity variations. Moreover, for the quantity variation, we try to get a sense of the intensive vs. extensive margin of variation. Another departure from previous studies is the frequency of the dataset. Except for Broer et al. (2022), all the others use annual data. We use quarterly and monthly series, which allows us to exploit a longer times series dimension to identify the transmission of MP shocks.

As discussed in the introduction, it is very common to assume no or negligible income effects on labor supply in macroeconomics. However, usually most of the evidence used to support this assumption does not focus on monetary policy. Small income effects on labor supply are typically motivated by the evidence on elasticities of idiosyncratic (lottery-induced) variations in income on labor supply. Cesarini, Lindqvist, Notowidigdo and Östling (2017) use Swedish administrative data to estimate wealth effects and labor supply elasticities. Based on these estimations, they conclude that income effects are modest. Golosov, Graber, Mogstad and Novgorodsky (2021) by using US data, question these results and reveal that the earnings responses to lottery winnings are sizable and far from negligible in the U.S.

From a theoretical standpoint, we contribute to the literature that studies micro-level heterogeneity in the New Keynesian model. To date, this literature has focused on how household-level heterogeneity affects the consumption channel of monetary policy while either abstracting from or not focusing on labor supply heterogeneity; see, for example, Bilbiie (2008); Bilbiie (2021); McKay, Nakamura and Steinsson (2016); Kaplan et al. (2018) or Auclert (2019). Athreya, Owens and Schwartzman (2017) show how labor supply decisions are crucial to determine the direction and size of the output effects of fiscal transfers. They highlight the crucial role of the marginal propensities to work across the population in heterogeneous agents' incomplete market models. To the best of our knowledge, we are the first to look at this with regard to monetary policy.

The aggregate demand amplification effect of income inequality has been studied by Bilbiie (2008, 2021), Auclert (2019), Patterson (2021) and Bilbiie, Känzig and Surico (2022) amongst others.[9] Relative to these studies, we show that allowing for hours and IES heterogeneity unveils a novel channel that dampens the impact on aggregate demand. We characterize this channel analytically in combination with the standard amplification one, and then use a richer model to quantify the contribution of different assumptions to the transmission of monetary policy.[10]

Finally, our results on the behavior of the left tail of labor supply are also related to the evidence on households' *consumption commitments* and *inflation inequality*. Regarding the former, Chetty and Szeidl (2007) document regular payments that cannot be easily adjusted and limit households' ability to adjust the consumption margin.[11] Our story here is related to the presence of these commitments being likely to push poor employed households to increase their labor supply following a negative income shock. On the latter, Jaravel (2021) discusses how heterogeneity in consumption baskets generates a dampening effect of monetary policy on aggregate demand.[12] Our dampening channel instead arises from labor supply and income effect heterogeneity.

2 Monetary policy and labor market outcomes along the income distribution

In this section, we describe the data sources and construction and the empirical strategy to identify monetary policy shocks, and we present our empirical evidence about the transmission of these shocks to household level variables. Our main empirical evidence is constructed using the information on US labor earnings and hours worked at the individual level. We also present similar results for the UK.

[9] Bilbiie, Primiceri and Tambalotti (2022) show how cyclical precautionary savings also generate amplification. We abstract from this channel here.

[10] Our results crucially depend on the empirical evidence on IES heterogeneity. Most studies highlight the role of income in driving heterogeneity in IES. This is because poor consumers, whose consumption bundle contains a large share of necessities, tend to substitute less consumption intertemporally than rich households (Andreolli and Surico (2021)). Therefore if subsistence represents an important part of the poor households' consumption, these agents have limited discretion for intertemporal substitution in consumption (Blundell, Browning and Meghir (1994), Attanasio and Browning (1995)). Attanasio and Browning (1995) document that the IES is increasing in the household level of consumption. Attanasio et al. (2002) and Vissing-Jørgensen (2002) show that the IES for stockholders and bondholders in the UK and the US is an order of magnitude higher than that of those who do not hold these assets. Calvet, Campbell, Gomes and Sodini (2021) find a bi-modal distribution of the IES across households in Sweden.

[11] Households allocate a significant part of their expenses to goods and services that are costly to adjust such as mortgage/rental payments, insurance payments, or mobile phone plans.

[12] Households with lower marginal propensities to consume spend more on sectors with higher price rigidity. Moreover, after a monetary policy shock, prices fall more in sectors employing poor workers which increases relative labor demand for the these workers through changes in consumer demand. Therefore monetary policy shocks reduce labor earnings more for the low MPC agents, leading to another dampening channel.

We find evidence that the individuals at the left tail of the income distribution typically increase the weekly amount of hours worked after a monetary policy tightening. Since the unemployment rate increases across all percentiles of the income distribution, the increase in hours worked operates through the intensive margin. Therefore, those individuals who remain in the labor market after a monetary policy tightening tend to supply more hours of work. Moreover, we find that labor market outcomes (unemployment and hours worked) are more sensitive on the left tail of the income distribution.

2.1 Household level data

For the US, our primary source of individual-level data is the Current Population Survey (CPS), sponsored jointly by the U.S. Census Bureau and the U.S. Bureau of Labor Statistics (BLS), which is used to derive our benchmark results. Data from the Consumer Expenditure Survey (CE or CEX) are used as a robustness check.

The CPS survey is conducted at a monthly frequency on a sample of about 60,000 U.S. households; it contains detailed information about the demographic characteristics of households, labor market attitudes, and labor earnings. From the CPS we can use individual-level data on *actual* hours worked and hourly wages, with individuals sorted on labor earnings.[13]

Unless otherwise specified, we apply a filter to the surveys. In particular, we drop respondents that lie in the top and bottom first percentile of the earnings distribution or are aged less than 18 or more than 66. Moreover, at each point in time (e.g. the month for the CPS), we construct different earning percentile groups, e.g. quintiles and deciles. For example, quintile bins refer to five percentile groups of total income or earnings. We refer to less than or equal to the 20^{th} percentile with P_{20}, greater than 20^{th} and less than equal to 40^{th} percentile with P_{20-40} and so on. We also use a model specification where we split the population into two groups, $P_{\leq J}$ and $P_{>J}$ where J ranges from 5 to 95 with increments of 5. For example, when $J = 5$, the two groups consist of respondents who fall below that 5th percentile of hourly earnings or income ($P_{\leq 5}$) and those that lie above the 5th percentile ($P_{>5}$).

For the CPS, individual-level data is obtained for each month from 1994 to 2019. For earnings, we use the outgoing rotation group.[14] Our measure of hourly earnings corresponds to the amount earned per hour in the current job reported by respondents who are paid hourly. For respondents who are paid weekly, we construct earnings by dividing the usual earnings by usual weekly hours in the main job. Earnings are deflated using CPI. We use actual hours worked in the main job as our main measure of weekly hours. We calculate average earnings and average weekly hours for each group using the survey weights. Repeating

[13] Gross income data are not available at the monthly frequency in the CPS.

[14] Households are interviewed in the CPS for four months and then again for four months after an eight-month break. In the fourth and eighth months of interviews, when households are about to rotate out of the interviews, they are asked additional questions about earnings. For more details on the outgoing rotation group, see https://cps.ipums.org/cps/outgoing_rotation_notes.shtml

this across all months in the sample provides a time series of average earnings and average weekly hours by percentile group. We also construct unemployment rates at various percentiles of the earnings distribution. To do that, we use mincer-type regressions to impute hourly earnings for unemployed individuals. In particular, we regress earnings on level of education, a measure of experience (age minus year of schooling minus six), and individual characteristics, including race, sex, and industry of occupation. The fitted values from this regression are used to obtain imputed earnings for unemployed individuals.[15]

The demographic characteristics vary substantially across bins. As we move from the left to the right of the earnings distribution, respondents tend to be older and better educated; they are more likely to work longer hours, own their homes, be white and male, and less likely to be employed in a manual job. In appendix A.1, we provide more details on data constructions and on the demographic characteristics of individuals for different deciles of the earning distribution, e.g. see Figure A.1.

2.2 Empirical Model

To estimate the impact of monetary policy shocks on the labor supply for different slices of the population, we use a factor-augmented VAR (FAVAR) model for the US and the UK, respectively. The model is defined by the VAR:

$$Y_t = c + \sum_{j=1}^{P} \beta_j Y_{t-j} + u_t \tag{1}$$

where $Y_t = \begin{pmatrix} R_t \\ \hat{F}_t \end{pmatrix}$, where R_t denotes a policy interest rate and $\hat{F}_t$ represents factors that summarize information in a panel of macroeconomic and financial series and the survey-based data on income and hours, described above. The factors are estimated using the non-stationary factor model of Barigozzi, Lippi and Luciani (2021). A key advantage of this approach is that it allows us to use the data in levels. This is convenient as we are primarily interested in the impact of policy on the level of wages/labor earnings and hours in the percentile groups. Denote X_t as the $(M \times 1)$ data matrix that contains the panel of macroeconomic and financial series that summarize information about the economy, and also includes the average hours and average income components deflated by the CPI in various earnings/income percentile groups. The observation equation of the FAVAR is defined as:

$$X_t = c + b\tau + \Lambda F_t + \xi_t \tag{2}$$

where c is an intercept, τ denotes a time-trend, F_t are the R non-stationary factors, Λ is a $M \times R$ matrix of factor loadings, and ξ_t are idiosyncratic components that are allowed to

[15] The coefficients of the regression are shocked and used to produce predicted values. These are assigned to unemployed individuals using predicted mean matching. We produce 5 replicates, with the final imputed data taken to be the mean across these replications. We then compute the distribution of earnings (including those unemployed with imputed income) and for each percentile, we compute the fraction of unemployed people. These measures will be useful for isolating the extensive and intensive margins of labor market fluctuations.

be $I(1)$ or $I(0)$. Note that the idiosyncratic components corresponding to the survey-based data can be interpreted as shocks that are specific to those groups and also capture possible measurement errors. In contrast, the shocks to equation (1) represent macroeconomic or common shocks. The response to these common shocks is relevant to our investigation. This ability to estimate the impact of macroeconomic shocks while accounting for idiosyncratic disturbances is a key advantage of the FAVAR over a VAR, where these two sources of fluctuations may be conflated (see De Giorgi and Gambetti (2017)). Moreover, expanding the cross-sectional dimension of the VAR with factors is important also for identification purposes as it reduces the problem of information deficiency (see e.g. Forni and Gambetti (2014)) and shock deformation (see e.g. Canova and Ferroni (2022)).

For the US, the macro and financial data in X_t is obtained from the FRED-MD database. This monthly database contains 137 times series covering real activity, employment, inflation, money, credit, spreads, and asset prices and is trimmed to start in 1994m1, which is the first observation of hours worked constructed in the CPS, while the last observation is 2019m12.

To identify a monetary policy shock, we use an external instrument approach (see e.g. Stock and Watson (2008) and Mertens and Ravn (2013)). The residuals u_t are related to structural shocks ε_t via:

$$u_t = A_0 \varepsilon_t \tag{3}$$

where $cov\,(u_t) = \Sigma = A_0 A_0'$. We denote the shock of interest as ε_{1t} and the remaining disturbances as ε_{-t}. Identification of ε_{1t} is based on the instrument m_t that satisfies the relevance and exogeniety conditions: $cov\,(m_t, \varepsilon_{1t}) = \alpha \neq 0$ and $cov\,(m_t, \varepsilon_{-t}) = 0$. As discussed in appendix C, these conditions can be combined with the covariance restrictions to obtain an estimate of the relevant column of the contemporaneous impact matrix A_0.

For the US, our benchmark instrument used to identify the monetary policy shock is taken from Gertler and Karadi (2015). This instrument is built using intra-day data on three months ahead federal funds futures. Gertler and Karadi (2015) calculate the change in this instrument during a tight window around FOMC meetings. The high frequency of the data makes it likely that changes in the futures over the window reflect unexpected changes in monetary policy. Following Gertler and Karadi (2015), R_t is assumed to be the one-year government bond yield. When using quarterly data and variables from the CEX survey, the quarterly sum of the Gertler and Karadi (2015) shock is used.

The number of factors in each FAVAR model is chosen via the information criteria of Bai and Ng (2002). However, we follow Bernanke, Boivin and Eliasz (2005) and add additional factors if the model selected by the information criterion displays impulse responses of macro variables that are at odds with the theory (e.g. the price puzzle). This procedure suggests the presence of 10 factors. The lag length is set at 12.[16]

The unobserved factors in (2) are estimated using the principal component estimator described in Barigozzi et al. (2021). The parameters of the VAR model in (1) are estimated

[16] Our main results are not sensitive to this choice.

using a Bayesian approach. The Markov chain Monte-Carlo algorithm is described in appendix B. We employ 51,000 iterations, retaining every 10^{th} draw after a burn-in period of 1000.[17]

2.3 Response of aggregate and dis-aggregate variables

The top panel of Figure 1 shows the response of some key aggregate variables to a contractionary monetary policy shock in the US. The results are obtained using the monthly FAVAR which includes data on the distribution of hours from the CPS. The bottom panel reports the response of average hours worked for households below the 20^{th} percentile of the wage (income) distributions obtained using the CPS survey and compares it with the average response of hours in the remaining groups above the 20^{th} percentile and aggregate hours constructed from the CPS survey.[18]

The peak decline in output is about 3 percent and CPI falls by 0.7 percent. Stock market prices react negatively, with a peak response of around 13 percent. Figure D.1 in appendix D displays the response of selected macroeconomic and financial variables from the FAVAR models. The monetary contraction is associated with an increase in the unemployment rate, a fall in housing starts, and the real money supply. There is an increase in short-term interest rates, financial conditions deteriorate, and the exchange rate appreciates. In short, these results accord well with theory.

The bottom panel of Figure 1 shows our main result. There is a persistent decline in aggregate hours after the monetary contraction. For individuals above the 20^{th} percentile of the wage distribution, the impulse response is similar to the aggregate. Hours fall by 0.7 percent at their trough. In contrast, hours display a persistent increase on the left tail of the wage distribution. The peak response of hours occurs at about the 2 year horizon and is estimated to be an increase of 0.3 percent.

So far we have arbitrarily chosen a cutoff percentile at 20 percent. In the next section, we look across the whole earnings distribution to check precisely where this threshold lies.

2.4 Income effect on Labor Supply

In this section, we present the results by running a set of empirical models. We split the population into two groups, with average earnings below or above a certain percentile J, $P_{\leq J}$ and $P_{>J}$, respectively. We repeat the estimation of the FAVAR for $J = 5, 10, ..., 95$. In Figure 2 we show the response of hours worked by individuals below a certain earnings threshold ($P_{\leq J}$) to a monetary policy tightening in the US for different thresholds (J) and different horizons.[19]

[17] The prior distributions for the VAR parameters are standard and described in appendix B.

[18] The responses of hours from the CEX survey are reported in the Appendix and discussed in the robustness section 2.6

[19] Appendix D.4 presents the full three-dimensional IRFs for hours worked, labor income and hourly wages.

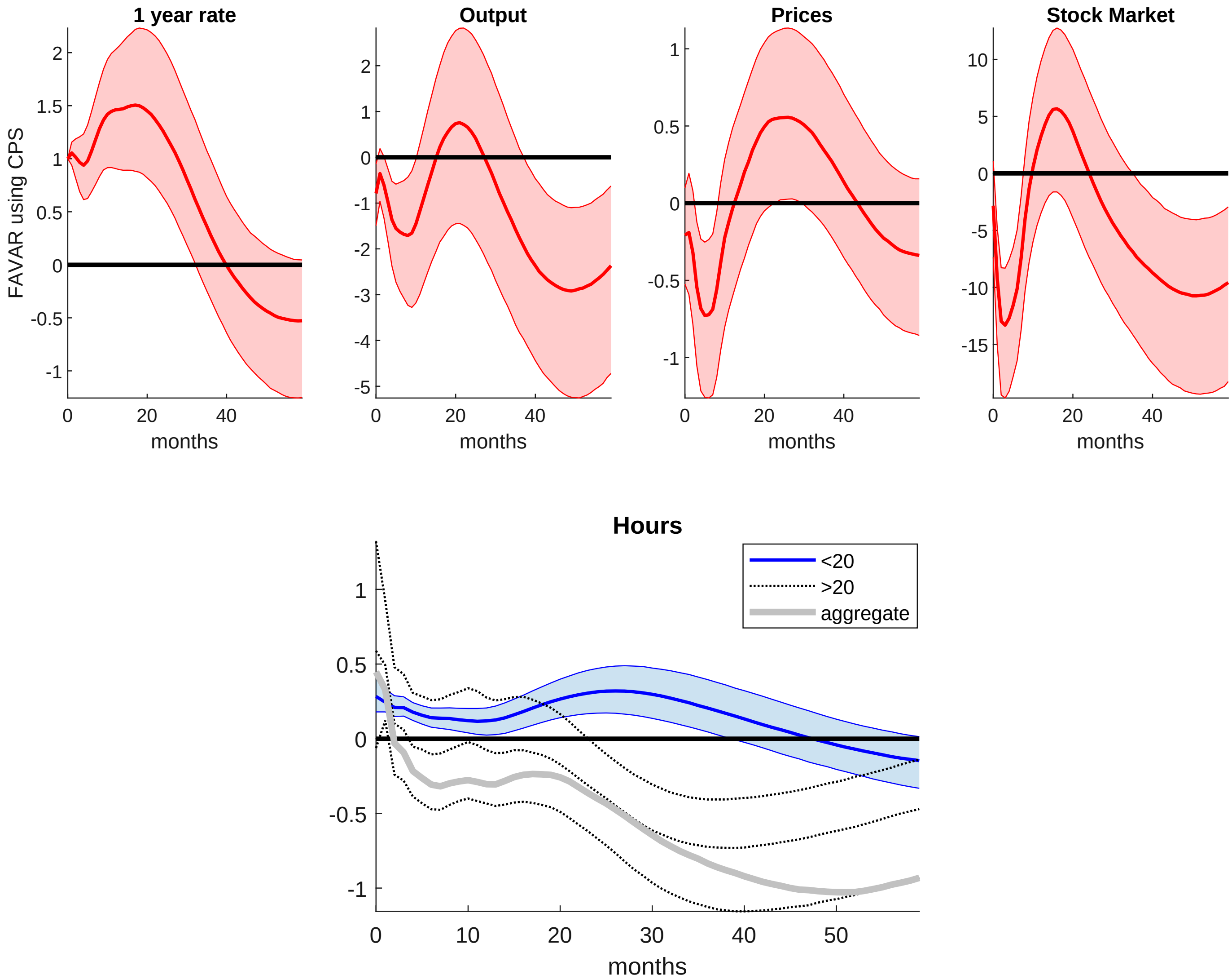

Figure 1: Responses of selected variables to a contractionary monetary policy shock in the US normalized to a 1 percent increase in the one-year interest rate. The response of hours is weighted by the share of total hours in each percentile group. The response above the 20^{th} percentile is the weighted mean of the response of average hours in $20^{th} - 40^{th}$, $40^{th} - 60^{th}$, $60^{th} - 80^{th}$ and above the 80^{th} percentiles. For each variable, we report the median IRF and 68% bands. Except for aggregate hours for which we report only the median IRF.

The estimated response of hours confirms the results presented in the previous section, where we identified a portion of the population that increased their labor supply after a monetary-induced negative demand shock. The estimated model suggests that hours increase for individuals below the 20^{th} percentile after a contractionary shock at the one-year horizon. The response is negative towards the right tail of the earnings distribution, with the decline estimated to be statistically different from zero at the three-year horizon.

The increase in hours worked by households with low earnings could be the result of two mechanisms: more people at the bottom of the earning distribution enter the labor market (extensive margin), or employed people work longer (intensive margin). Since the unemployment rate for low- and moderate-earnings individuals tends to increase after a monetary contraction –see the bottom right panel of Figure 2– we can rule out large movements in the extensive margin. Hence, after a monetary-induced negative demand shock, low-earnings individuals who remain in the labor market tend to work longer hours. Low- and moderate-earnings households tend to explain non-negligible fractions of the variance of total hours worked and this fraction is larger than the analog for consumption. In the CPS survey, the cross-sectional variance of total hours worked explained by the bottom 30 percent of the labor income distribution ($P_{<30}$) is around 22 percent.[20]

One concern with the benchmark results is that the impulse response of hours at the left tail reflects selection. That is, if the increase in the unemployment rate at the left tail of the earnings distribution is concentrated amongst part-time workers, then the increase in average hours could simply reflect the fact that more full-time workers remain in the sample after the shock. There are however several pieces of evidence that suggest that this is not the case. First, in our benchmark CPS sample we are able to construct unemployment rates for full-time and part-time workers, separately. As shown in the appendix, the monetary policy shock has a substantially larger impact on the unemployment rates of full-time workers at lower earnings quantiles than the corresponding rates of part-time workers. Second our benchmark results survive when we restrict the sample to individuals working full-time hours. Finally, as we discuss below, the response of hours at the left tail to general business cycle shock is *negative*. This result goes against the argument that a substitution of full-time for part-time workers is driving our main result.

As discussed in the introduction, the countercyclicality of hours worked in the left tail of the income distribution observed in the data is an equilibrium outcome resulting from the interaction of households' labor supply forces and firms' labor demand factors. Isolating the channel responsible for our empirical finding is difficult with the data we consider and a combination of factors is more likely. One plausible rationale for the countercycliacal behavior of hours worked is a strong income motive pushing households with low incomes to work more hours. In the section where we discuss the theoretical models, we devote particular attention to this channel and study its implications for the transmission of monetary policy.

Finally, it is important to highlight that, even considering just the intensive margin, labor income for individuals with low labor incomes is procyclical (see figure D.6 in the appendix), in line with the evidence offered in Broer et al. (2022) and Hubert and Savignac (2022).

[20]The labor supplied by the left tail of the labor income distribution represents also a relevant proportion of total hours worked in the economy. In the CPS, the average proportion of hours worked by the bottom 15 percent of workers across the labor income distribution is 15 percent. These numbers highlight an important difference between our work and the literature on consumption heterogeneity. The latter has stressed the importance of the *wealthy* HtM households for matching the average marginal propensity to consume, empirically and theoretically (see e.g. Kaplan et al. (2014)). Here, our empirical analysis suggests that the labor supply heterogeneity does not arise for *wealthy* HtM but rather for *poor* HtM households, who appear to be crucial in driving labor supply heterogeneity following a monetary policy shock. Finally, we wish to emphasize that the labor market outcomes for low- and moderate- labor income households are significantly more volatile than for the rest of the population.

This occurs because even if they work more hours, their average hourly earnings decrease relatively more, leading to a decline in their labor earnings. The focus of our analysis is precisely to isolate and study the intensive margin of employed individuals.

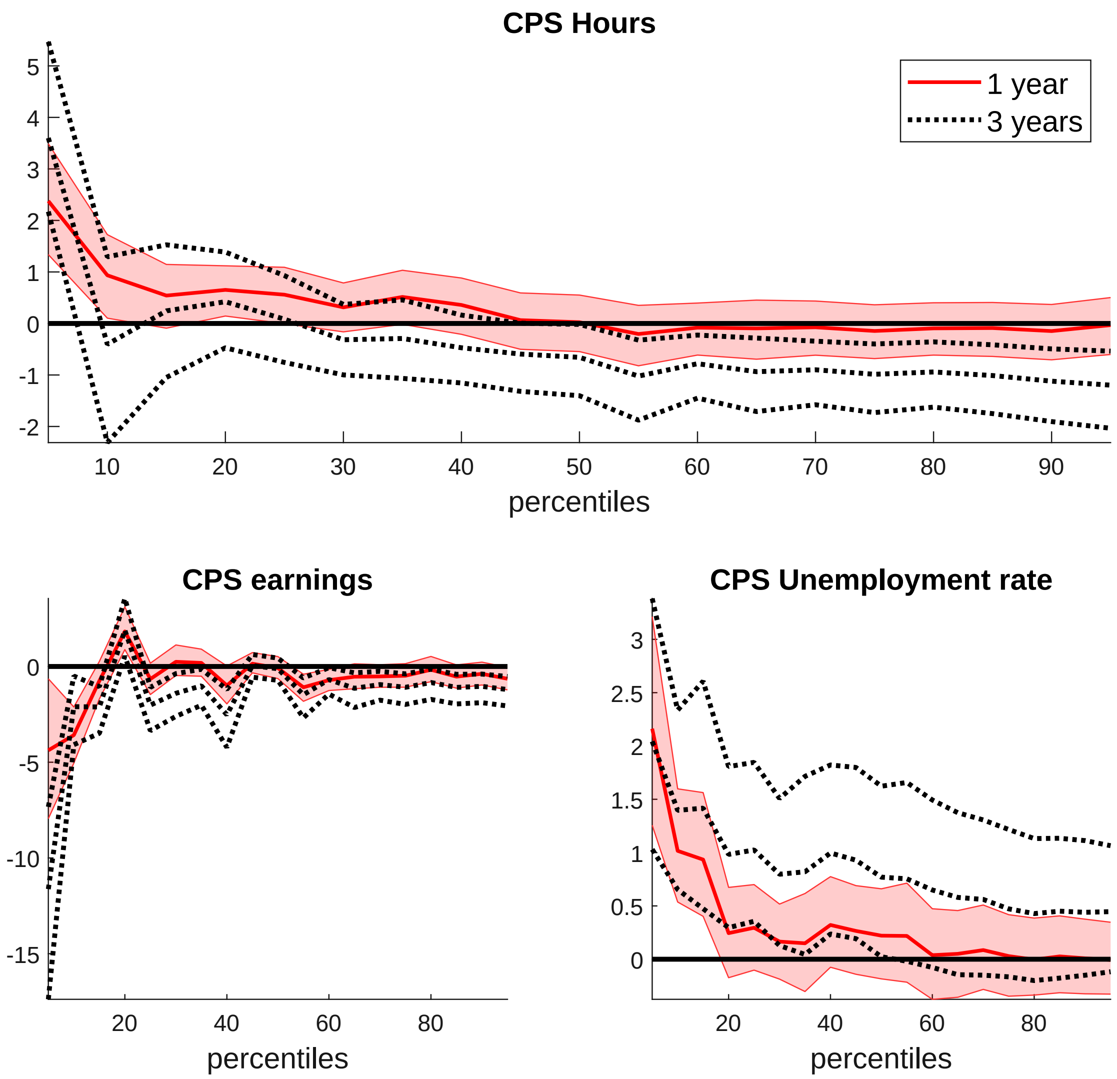

Figure 2: Impulse response of hours, earnings and the unemployment rate at different percentiles of the earnings distribution from the CPS. For each variable, we report the median IRF and 68% bands.

2.5 Results for the UK

The data for the UK are obtained from the Labour Force Survey (LFS), a survey that collects information to construct labor force statistics. Observations that lie in the top and bottom percentiles of the income/earnings distribution are dropped and only individuals older than 18 and younger than 66 are included in the sample. Moreover, we study the same percentile groups as described for the US.

From the LFS we obtain quarterly data for actual hours worked, hourly earnings, and the unemployment rate for the UK for the period 1994 to 2019. As in the case of the US, we use mincer regressions to impute earnings for unemployed individuals. Unemployment rates in earnings quintiles are then constructed by using both employed and unemployed individuals to calculate the earnings distribution. The characteristics of the UK earnings distribution are very similar to that of the US. Low earners in the UK tend to be younger, less educated, and employed in manual labor. They are less likely to be male and to own a home and their average weekly hours are shorter. In appendix A.3, we provide more details on data constructions and on the demographic characteristics for different deciles of the earning distribution, e.g. see Figure A.4. We use 100 macroeconomic and financial variables over the period 1994Q1 to 2019Q4 in the FAVAR model.

For the UK, the 1-year rate is used as the policy rate and the shock series proposed by Gerko and Rey (2017) is used for identification. Gerko and Rey (2017) use three-month sterling futures to build their proxy. Their shock is constructed by calculating the change in these future rates around the release of the minutes of the Monetary Policy Committee (MPC) meetings.[21] Using the procedure described above, the number of factors is set to 9. The number of lags is set to 4.

Figure 3 plots the responses of hours and the unemployment rate for $P_{\leq J}$ for $J = 5, 10, ..., 95$ after a monetary policy tightening.[22] The shock triggers an increase of the one-year rate by 1 percent and is associated with a decline in GDP, aggregate hours (constructed using LFS data), and RPI, while the unemployment rate rises (see Figure E.10 in appendix E.8). Financial conditions deteriorate after the shock and the nominal exchange rate appreciates. Monetary policy tightening is also associated with an increase in hours at the 1 year horizon that is statistically different from zero for individuals with earnings less than the 10^{th} percentile. In contrast, the response is zero towards the right tail of the distribution with hours declining at longer horizons. As in the case of the US, the increase in hours coincides with a rise in the unemployment rate, suggesting that the impact on hours is driven by existing workers changing their labour supply.

2.6 Robustness and Limitations

Before looking at our empirical evidence through the lens of a theoretical model, we wish to briefly discuss several additional robustness exercises to complement our empirical evidence. To save space, figures and details are presented in appendix E and we discuss here only the findings.

First, we consider household level hours and earnings taken from the CEX database. Hours from this survey are defined as *usual* hours worked. This is in contrast to *actual*

[21] Over this sample, large announced changes in monetary policy are concentrated around the great financial crisis. As discussed in Gerko and Rey (2017) this makes it likely that changes around MPC meetings are conflated by other shocks.

[22] For the unemployment rate, the largest percentile we consider is $J = 50$. This is because towards the right tail of the earnings distribution, the number of unemployed individuals indicated in the LFS is close to zero and the unemployment rate is imprecisely estimated.

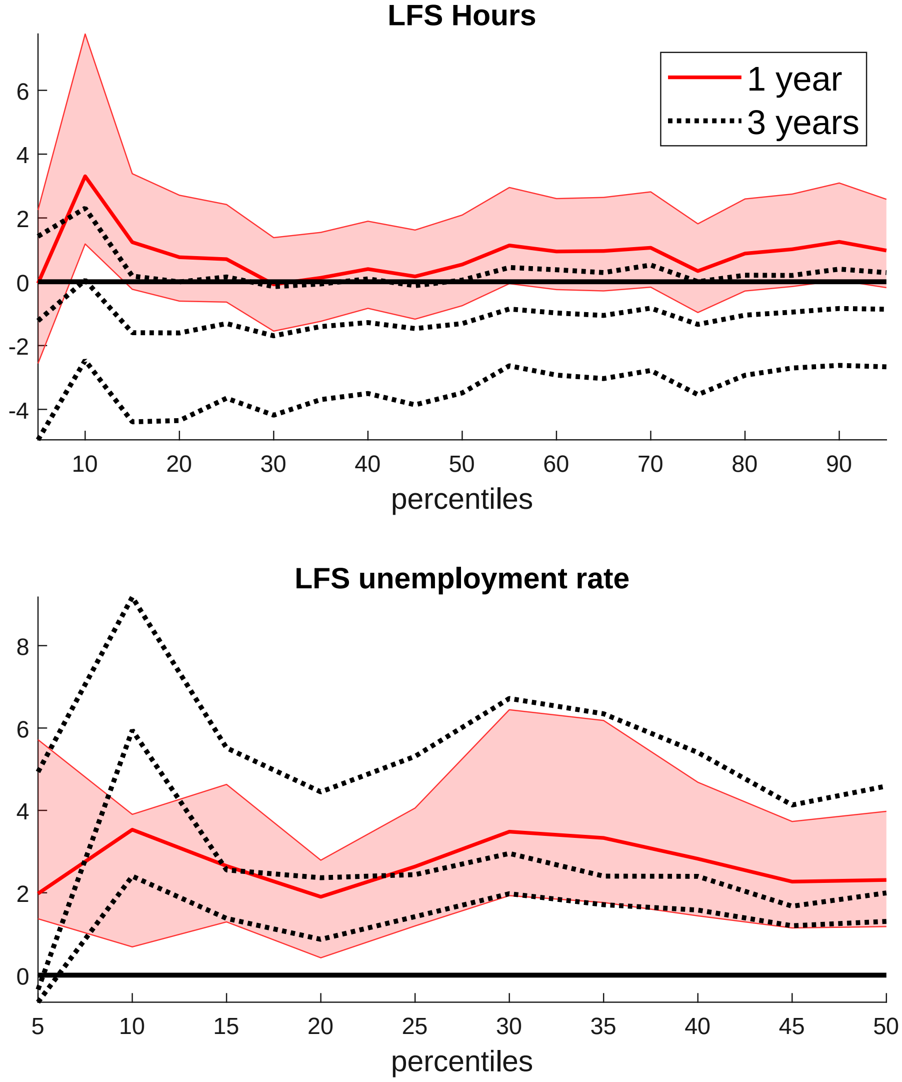

Figure 3: Impulse response of hours, and the unemployment rate at different percentiles of the income distribution for the UK. For each variable, we report the median IRF and 68% bands.

hours obtained from our benchmark, the CPS survey. We include these in a FAVAR model that uses quarterly data from FRED-QD.[23] The monetary policy shock is identified using the Gertler and Karadi (2015) instrument. Figure D.2 presents the response of hours and income to a contractionary monetary policy shock. The response is presented for households below income percentiles ranging from 5 to 95. The estimated response of hours from this CEX-based model is more volatile but suggests a conclusion that is similar to the benchmark case. Hours increase for households below the 30^{th} percentile. In cumulative terms, the impact of the contractionary shock is negative towards the right tail of the income distribution.[24]

[23]The quarterly data runs from 1984Q1 to 2018Q4 and contains 238 series covering real activity, employment, inflation, money, credit, spreads, and asset prices.

[24]In the CEX we observe a non-monotonic pattern. For the group below the 5 percent ($P_{\leq J}$), we see a decline in hours. However, this effect is short-lived.

Next we consider alternative identification approaches. In particular, for the US we use two alternative schemes for identifying the monetary policy shock. First, we utilize the instrument proposed by Miranda-Agrippino and Ricco (2021), who refine the measure used in Gertler and Karadi (2015) by purging the component that represents a signal regarding the central bank's information about the state of the economy. Second, we identify the monetary policy shock via sign restrictions, where a contractionary shock increases the policy rate and reduces GDP, and price indices. The response of hours supports the benchmark results. In particular, the response of hours is positive towards the left tail of the earnings and income distribution.

We have also looked at supply shocks and we find that hours at the bottom of the income distribution exhibit a large procyclical response to supply shocks (see appendix F).[25] These results show that our novel evidence on the response of hours of people remaining in employment at the bottom of the income distribution is specific to a particular demand shock.

Furthermore, we conduct a series of robustness analyses by looking at different household characteristics. We check if the results on hours are driven by mortgage debt and housing tenure. As it can be seen in appendix E.4, hours rise on impact for mortgagors. However, the increase is short-lived and becomes negative after a few quarters, suggesting that having a mortgage is not the primary driver.

To check that what we uncover is mainly a labor supply story and not primarily driven by labor demand, we look at the response of hours by industry of occupation using the CPS. We find substantial heterogeneity across industries but no particular pattern that might point out a labor demand story driving our results.[26]

In the benchmark analysis, hours in the CPS survey pertain to hours worked in the respondent's main job. In appendix E.2, we show that our key results are unaffected if we define hours as those worked in the main and additional jobs. Our results on the supply of hours worked could also be affected by a composition effect of people switching jobs and/or going in and out of the labor force between survey data collection dates. The CPS data is restricted to employed individuals. However, in the CEX, as the reference period covers a year, it could include spells of unemployment, job changes, or other changes in job status. As we show in appendix A.4, the data on hours aggregated from the CEX track quite well data from national accounts. Therefore this composition effect, if present, should not be large enough to affect our results significantly. As a double-check, we also use the occupation information to restrict the sample. We remove observations where the household reference person does not respond to the occupation question or the response does not fit in the five

[25] We have also looked at a general demand shock identified via sign restrictions and found results that look qualitatively similar for the response of hours worked. Results are available upon request.

[26] We find that hours worked increase substantially in industries producing durable goods, utilities, finance, professional services, and the public sector.

options given in the survey.[27] The hours' response of the left tail in the US does not appear to be driven by the *unemployed*.

Finally, we tested for asymmetries in the responses of hours worked to an interest rate cut or hike in the US. We could not find evidence for the latter.[28]

3 Monetary policy and labor supply in TANK

In this section, we turn to a theoretical model to rationalize the empirical results and quantify their implications for the 'tale' of the monetary policy transmission mechanism. We first do so in the most simple and stylized setup, focusing only on the intensive margin of labor supply, where we can obtain closed-form expressions. This helps to gain intuition towards the novel channel of transmission between inequality and monetary policy we uncover. That is, HtM agents that keep their job after an MP hike, give away leisure time to avoid having to drop their consumption one to one with their decline in income. We then construct a quantitative model with both intensive and extensive margins of labor supply that can match our empirical evidence on both margins as well as on wage heterogeneity. It can also be used to quantify the relevance of the new channel of transmission we uncover.

As discussed, our empirical evidence about the response of the left tail of hours worked is consistent with multiple theories. What we observe is an equilibrium outcome resulting from the interactions of households' labor supply forces and firms' labor demand factors. Unfortunately, data limitations prevent us from discriminating empirically between each of those factors. However, it is most likely that our empirical evidence is driven by a combination of those factors. Our approach here is to keep the analysis as simple as possible in order to obtain analytical insights. We do it by abstracting from the extensive margin and focusing on a strong income effect for low-income households. Strong income effects on labor supply can be generated via a low IES (or a high risk aversion (RA)) and/or with the presence of borrowing constraints. Importantly, to match our evidence, it is not enough to have a low IES (or a high RA) for everyone in the economy. We need heterogeneity across the population. This essentially boils down to assuming that poor households have a higher sensitivity of marginal utility of consumption (MUC) than the rest of the population via heterogeneity in the curvature of the utility function with respect to consumption.

We build on the TANK model of Bilbiie (2008)[29] and allow for the MUC to be hetero-

[27] These are: Managerial & professional specialty; Technical, sales, and administrative support; Services; Farming, forestry, and fishing; Precision production, craft, and repair, mechanics, mining; Precision production, craft, and repair, mechanics, mining; and Machine operator, fabricators, transport, and laborers.

[28] In addition, we carried out several robustness for the UK. We obtain results that are similar to the benchmark case, if sign restrictions are used to identify the monetary policy shock. In addition, expanding the definition of hours worked to include all jobs does not have a major impact on the benchmark conclusions. These results are available on request.

[29] Given the focus on aggregate demand dynamics, some of the TANK literature assumes an aggregate wage schedule by imposing an equal labor supply response between agents (see Bilbiie, Känzig and Surico (2022) and Debortoli and Galí (2017)). The seminal work by Bilbiie (2008), as well as his subsequent papers (Bilbiie 2020, 2021), present a model with heterogeneous labor supplies. Bilbiie (2008) shows that the response of the labor

geneous across agent types. Specifically, everything else being equal, HtM households have a larger MUC. Assuming a CRRA separable utility between consumption and leisure, this implies a smaller IES (or higher RA[30]) for HtM compared to Savers (in line with evidence in Attanasio et al. (2002), Vissing-Jørgensen (2002) and Calvet et al. (2021)).

This simple setup has interesting implications for the effectiveness of monetary policy transmission. With homogeneous IES, as long as the Investment-Saving curve is negatively sloped,[31] the presence of HtM agents generates *amplification* of monetary policy. Amplification arises because the elasticity of HtM consumption to aggregate income (χ) is larger than 1. That is, consumption of the HtM moves more than 1 to 1 with aggregate income (*countercycical inequality*).

However, this amplification mechanism implicitly assumes that, following a negative income shock, HtM households have no other option than to cut their consumption to make up for their income loss. We highlight instead an additional channel that allows HtM who remain employed to use their labor supply as an extra margin of adjustment. MUC/IES heterogeneity amplifies the differences in marginal rates of substitution between consumption and leisure across agents. We show that this novel dimension of heterogeneity generates a *dampening* channel that contributes to reducing the value of χ.

As a result, the aggregate demand amplification of monetary policy due to household inequality is reduced compared with what is usually assumed in the literature. At the same time, we show that this novel dampening effect only arises in the slope of the aggregate demand. On the contrary, this simple set up with labor supply and sensitivity of marginal utility of consumption heterogeneity but homogeneous wages, does not imply any changes on the slope of the New Keynesian Phillips curve.[32]

3.1 A Simple TANK model with heterogeneous IES

In this section, we outline the TANK model a la Bilbiie (2008) with IES heterogeneity. The aim is to isolate the effect of the sensitivity of marginal utility of consumption heterogeneity in a simple and transpatent manner. While this heterogeneity is completely ad-hoc here, we micro-found it in section 3.3. In particular, the ex-post IES heterogeneity can be generated by ex-ante identical Stone-Geary type preferences where the IES is increasing in the steady state level of consumption of different households.

supply of HtM depends crucially on the IES parameter which captures the relative strength of the income and substitution effect. All these papers however, do not focus on the response of hours worked to monetary policy shocks.

[30] Given this preferences specification, all the discussion referring to the IES also applies to the inverse of the relative risk aversion. See Shaw (1996) and Chiappori and Paiella (2011) for evidence of decreasing risk aversion along the wealth distribution.

[31] Following Bilbiie (2008), we denote this region as the Standard Aggregate Demand Logic (SADL) region. This is the region of the parameter space where the proportion of HtM is not high enough to make the elasticity of aggregate consumption to the interest rate positive.

[32] To generate an effect on the slope of the Phillips curve one needs also heterogeneity in wages/marginal productivity of labor of the two agents, as our simulations in section 3.3 show.

The economy consists of three sectors: households, firms, and a central bank. The household sector is populated by two different types: *savers* (S) and *hand-to-mouth* (H). A share λ of households are HtM who work and consume all of their income. The remaining $1-\lambda$ are savers who hold bonds and shares in monopolistic firms and get firm profits. Savers price all assets and get all returns, thus there is limited asset market participation. The saver's problem is the same as the one of the standard permanent income hypothesis agent. We assume that both agents enjoy consumption and leisure and have the same $CRRA$ utility function. The firm sector is standard; intermediate goods producers face nominal rigidities in the form of quadratic adjustment costs à la Rotemberg when adjusting their prices. To simplify the analysis and without loss of generality we assume that: i) both agents' labor supply is perfectly substitutable in production and, as a result, they both earn the same wage; ii) there is a production subsidy that induces marginal cost pricing, and therefore no consumption inequality, in steady state (following Bilbiie (2020)); and iii) the central bank chooses the nominal interest rate by responding to inflation expectations. We will relax all of these assumptions in section 3.3.

The key equations of the log-linearized model are reported in Table 1, where all variables with '^' are expressed in log deviation from steady state, and $\hat{x}_{t+1|t}$ represents the time t conditional expectation of variable $\hat{x}$ at time $t+1$. Real quantities are in terms of the consumption good and - unless otherwise stated - denoted by lowercase letters, while nominal variables are denoted by capital letters.[33]

We assume that the typical market clearing and resource constraint conditions hold, i.e. production utilizes labor input from S and H and equals aggregate demand (equations 1 and 2). Aggregate hours and real consumption are defined as $\hat{H}_t$ and $\hat{c}_t$ respectively. All variables with subscripts S or H correspond to savers and HtM. The third and fourth equations are the Euler and the labor supply equation of the agent S respectively; β is the time discount factor, σ_j (with $j = H, S$) controls the intertemporal elasticity of substitution (and risk aversion) and is heterogenous, while φ is the inverse of the Frisch elasticity of the labor supply.[34] $\hat{R}_t$ is the nominal interest rate and $\hat{\Pi}_{t+1|t}$ the expected inflation rate. The following two equations describe the behavior of the HtM household, which is bounded to consume only its labor income proceedings (equation 6) and optimally substitute between consumption and leisure/working time (equation 5). Here $\hat{w}_t$ stands for real wages. We have the standard New Keynesian Phillips curve (equation 7) where κ is the slope of the NKPC linking the real with the nominal side of the economy. To allow for an analytical solution, we assume that monetary policy shocks are the only source of fluctuation in this economy (ϵ_t^m) and that the central bank responds one-to-one to inflation expectations as in Bilbiie

[33] Appendix G presents details about the model derivations from first principles, steady state, and log-linearization. Here we note that, as a result of our simplification assumptions, output and aggregate hours in the economy are equal to aggregate consumption, and their steady state is normalized to 1, without loss of generality.

[34] It is easy to show that in this model heterogeneity in the marginal utility of leisure/Frisch elasticity can affect the magnitude but not the sign of the response of hours to monetary policy shocks.

Log-linearized Conditions		
1:	Aggregate Hours	$(1-\lambda)\hat{H}_t^S + \lambda \hat{H}_t^H = \hat{H}_t$
2:	Aggregate Consumption	$(1-\lambda)\hat{c}_t^S + \lambda \hat{c}_t^H = \hat{c}_t$
3:	Euler Savers	$\hat{c}_t^S = \hat{c}_{t+1\|t}^S - \sigma_S \left(\hat{R}_t - \hat{\Pi}_{t+1\|t}\right)$
4:	Labor Supply Savers	$\varphi \hat{H}_t^S = \hat{w}_t - \frac{1}{\sigma_S}\hat{c}_t^S$
5:	Labor Supply HtM	$\varphi \hat{H}_t^H = \hat{w}_t - \frac{1}{\sigma_H}\hat{c}_t^H$
6:	Budget constraint HtM	$\hat{c}_t^H = \hat{H}_t^H + \hat{w}_t$
7:	Phillips Curve	$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1\|t} + \kappa \hat{w}_t$
8:	Taylor Rule	$\hat{R}_t = \hat{\Pi}_{t+1\|t} + \epsilon_t^m$

Table 1: Log linearized conditions of the simple TANK model with hours and IES heterogeneity.

(2008) and McKay et al. (2016), therefore effectively choosing the real rate.

In summary, the only substantial departure from the standard TANK model in Bilbiie (2008) is that we allow for the IES to be different across agents.[35]

3.1.1 Matching the empirical evidence

Savers reduce their labor supply decision after a monetary policy tightening.[36] On the contrary, it is immediately clear that labor supply decisions of the HtM households are countercyclical to aggregate quantities as long as the income effect dominates. This can be derived by solving the HtM budget constraint and intratemporal optimality condition in terms of the aggregate real wage,

$$\hat{H}_t^H = \frac{\sigma_H - 1}{\sigma_H \varphi + 1}\hat{w}_t; \quad \hat{c}_t^H = \frac{\sigma_H(\varphi+1)}{\sigma_H \varphi + 1}\hat{w}_t. \tag{4}$$

Note that heterogeneity in the Frisch elasticity would not be able to generate a countercyclical behavior of hours worked. In what follows, we restrict to the *standard aggregate demand logic* (SADL) region of the model (Bilbiie (2008)) where $\lambda < \frac{1}{1+\varphi}$.[37] A monetary policy

[35] We also abstract from dividend's redistribution, productivity shock and assume a production subsidy that induces marginal cost pricing instead of fixed costs. See appendix G.

[36] To single out the optimal behavior of savers, assume that the share of HtM is negligible, i.e. $\lambda \to 0$. In this limiting case, we have that $\hat{H}_t^S = \hat{H}_t = \hat{c}_t = \hat{c}_t^S$. By combining the Taylor rule with the Euler equation and solving the latter forward we get that $\hat{c}_t^S = -\sigma_S \epsilon_t^m$. Therefore, the consumption and labor supply decisions of Savers are procyclical after a monetary policy shock.

[37] This is the region of the parameter space where the proportion of HtM is not high enough to make the slope of the aggregate demand function positive.

induced demand shock that pushes real wages down produces an increase in the HtM hours worked when $\sigma_H < 1$. More formally, we can state the following proposition.

Proposition 1 *Under SADL ($\lambda < \frac{1}{1+\varphi}$) and with a sufficiently low IES of the HtM households (e.g. $\sigma_H < 1$), a rate hike induces a decline in total hours worked and an increase in the HtM labor supply.*

The proof is in appendix G.4. In response to a rate hike, unconstrained agents reduce their consumption because of both the intertemporal substitution channel and the higher return from saving relative to the return from working. This effect is strong enough to generate a fall in demand that induces a decline in aggregate wages. For the HtM household, the only source of income is labor. With a low IES ($\sigma_H < 1$), the negative income effect dominates and labor supply must increase to counteract the negative demand shock. This generates a countercyclical behavior of the HtM labor supply.[38] An alternative set up presented in appendix H, with borrowing constrained borrowers as opposed to HtM households, generates the same results. An interest rate hike increases debt repayments for borrowers which pushes up their income effect on labor supply.[39] It is well known that a large fraction of low and moderate income families in the US tend to finance durable consumption with debt and might use payday loans (Stegman (2007), Melzer (2011)).

The heterogeneity of the MUC/IES is important as it allows us to capture the different sensitivities of the households' labor supply decision to aggregate conditions. As the IES of HtM household becomes smaller, HtM labor supply becomes more sensitive to changes in the real wage schedule (see equation (4)); for low enough values of σ_H, the income effect eventually gets so strong that it generates an absolute increase in hours worked larger than the one of the unconstrained agents. Since aggregate hours worked decline in the SADL equilibrium, the fraction of HtM households should not to be too large. These arguments are depicted in Figure 4 where we display the absolute ratio between the HtM and savers households' hours worked after a monetary policy shock for different values of the intertemporal elasticity of substitution of the HtM (σ_H) and their relative share in the economy (λ).[40] We calibrate the IES of the saver $\sigma_S = 1$ as well as the Frisch elasticity of labor supply ($1/\varphi = 1$), so that the SADL region extends for $\lambda \in [0, 0.5)$.[41] Cool (warm) colors indicate that the hours worked of HtM households are smaller (larger) than the (absolute value of) hours worked of saver households. As we move to the northwest corner (low λ and low σ_H), we generate a response of HtM households that is at least as sensitive as the ones of savers and moves

[38]Note that when income and substitution effects are of the same opposite magnitudes (i.e. $\sigma_H = 1$ - log utility) the HtM are insensitive to changes in the monetary policy stance as originally showed by Bilbiie (2008).

[39]We show that in a model with borrowers and savers similar to Bilbiie, Monacelli and Perotti (2013) a rate hike induces an increase in the borrowers' labor supply when $\sigma < \frac{1+\bar{D}\kappa}{\gamma}$ where $\bar{D}$ is the debt limit and $\gamma = 1+\bar{D}(\beta-1) < 1$.

[40]The hours worked responses of the two agents to a monetary policy shock are derived in the proof of proposition 1 (appendix G.4).

[41]The rest of the parameters values are $\beta = 0.99$ and $\kappa = 0.1406$. The value of k is obtained by assuming an elasticity of substitution between goods variety equal to 6 and an average price stickiness of 3.5 quarters.

closer to our empirical evidence. With homogeneous IES this is not possible and we discuss why in the next subsection.

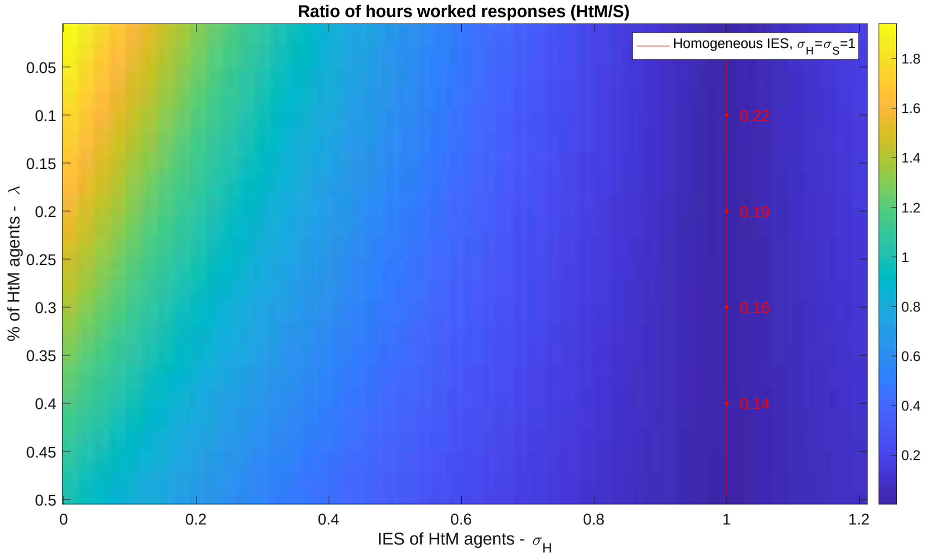

Figure 4: Relative (absolute) magnitude of the response of HtM households and savers' hours worked to a monetary policy shock for different values of σ_H and λ. Values larger than one indicate larger volatility of HtM labor supply relative to savers. The red vertical line indicates the relative ratio when the IES equals 1 for both agents.

3.2 Monetary policy and aggregate demand amplification

What are the implications for monetary policy effectiveness when the proportion of HtM agents changes and IES are different across households? Following Bilbiie (2020) we define $\chi = \frac{\hat{c}_t^H}{\hat{c}_t}$ as the elasticity of HtM consumption to aggregate income.[42] Bilbiie (2008) showed that limited asset market participation amplifies the effect of monetary policy in the SADL region with homogeneous preferences if $\chi > 1$. Here we have:

$$\chi = \frac{\frac{\sigma_H}{\sigma_S}(\varphi+1)(\sigma_S\varphi+1)}{\lambda\left(\frac{\sigma_H}{\sigma_S}-1\right)(\varphi+1)+\sigma_H\varphi+1}. \tag{5}$$

With homogeneous IES ($\sigma_S = \sigma_H$), the latter collapses to $\chi = 1 + \varphi > 1$.[43] In general, however, the elasticity of HtM consumption to aggregate income depends on the relative size of the IES's. Importantly, χ is not independent of the proportion of HtM agents λ. It appears to be increasing (decreasing) in λ if $\frac{\sigma_H}{\sigma_S} < 1$ (> 1). This elasticity is a crucial

[42]Recall that in this set up $\hat{c}_t$ is equal to aggregate income.

[43]See Bilbiie (2020) where he denotes $\chi = 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right)$. Here we abstract from profits redistribution which implies $\tau^D = 0$.

parameter as it affects the slope of the aggregate Euler equation in the model:[44]

$$\hat{c}_t = \hat{c}_{t+1|t} - \sigma_S \frac{1-\lambda}{1-\chi\lambda}(\hat{R}_t - \hat{\Pi}_{t+1|t}). \tag{6}$$

First, we review what this implies in the homogeneous IES case where χ does not depend on λ. An increase in inequality (higher λ) increases the elasticity of consumption to the real interest rate. When the interest rate increases, savers substitute consumption intertemporally, which generates a fall in demand. As a result, real wages fall. Since poor HtM income is made up only of wages, they have to consume less, pushing demand further down. Note that, in this case, the sign of the slope of the Euler equation is independent of σ_H. From (4) when $\sigma_H = \sigma_S = 1$, the consumption of HtM is one to one procyclical with wages $\hat{c}_t = \hat{w}_t$. Otherwise, when $\sigma_S = \sigma_H < 1$, this relationship is not one-to-one ($\hat{c}_t^H = \frac{1+\varphi}{1/\sigma_H+\varphi}\hat{w}_t$) but it remains procyclical and still amplifies the effect of monetary policy shocks. This can be seen by looking at the slope of (6) or by substituting in the Taylor rule and solving it forward $\hat{c}_t = -\sigma_S \frac{1-\lambda}{1-(\varphi+1)\lambda}\ \epsilon_t^m$. Under SADL an increase in λ magnifies the effect of ϵ_t^m on aggregate consumption independently of what happens to the labor supply of HtM. How is this possible even when the labor supply of HtM is countercyclical? This is due to a general equilibrium effect that makes unconstrained agents' labor supply more procyclical. The lower the number of non-HtM agents in the economy (higher λ), the larger the slice of firm profits that each of them receives. As profits are countercyclical, this induces them to work even less after a monetary policy tightening. This is the reason why we need IES heterogeneity to generate HtM labor supply at least as sensitive as the one of savers (cf. Figure 4).

The case of heterogeneous IES generates quite different implications. By plugging (5) into (6) we obtain:

$$\hat{c}_t = \hat{c}_{t+1|t} - \sigma_S \underbrace{\frac{1-\lambda}{1-(1+\varphi)\lambda}}_{(+)\text{ when }\lambda\uparrow} \times \underbrace{\frac{\lambda\left(\frac{\sigma_H}{\sigma_S}-1\right)(\varphi+1)+\sigma_H\varphi+1}{\sigma_H\varphi+1}}_{(-)\text{ when }\lambda\uparrow\text{ if }\frac{\sigma_H}{\sigma_S}<1} \times (\hat{R}_t - \hat{\Pi}_{t+1|t}). \tag{7}$$

The slope of Equation (7) has the same amplification channel as in (6) plus a new one that reduces amplification if $\frac{\sigma_H}{\sigma_S} < 1$.[45] The dampening effect arises as a consequence of χ decreasing in λ if $\frac{\sigma_H}{\sigma_S} < 1$. The source of this dampening can be traced to the additional heterogeneity in the marginal rate of substitution (MRS) between hours and consumption that the heterogenous labor supply implies. Intuitively, heterogeneity in the labor supply gives HtM household an extra tool to use in response to a decline in their income. These hosehold can increase their labor supply, substituting leisure for consumption. Technically,

[44] Following Bilbiie (2020), this can be derived using the equation relating the consumption of savers with aggregate consumption, $\hat{c}_t^S = \frac{1-\chi\lambda}{(1-\lambda)}c_t$, and their Euler equations $\hat{c}_t^S = \hat{c}_{t+1|t}^S - \sigma\left(\hat{R}_t - \hat{\Pi}_{t+1|t}\right)$.

[45] Note that the SADL region stays the same as in the homogeneous IES case. The slope still changes sign with $\lambda > \frac{1}{1+\varphi}$.

when households have the same sensitivity of marginal utility of consumption, the intratemporal optimality condition between hours worked and consumption (for a given wage schedule) is equivalent at the *aggregate* and at the household level, i.e. $\varphi \hat{H}_t + \frac{\hat{c}_t}{\sigma} = \hat{w}_t = \varphi \hat{H}_t^j + \frac{\hat{c}_t^j}{\sigma}$ for $j = H, S$. Thus, in response to a change in the real wage schedule, the MRS adjusts in the same proportion for the aggregate as well as for the two agent' types. With different MUC/IES, this is no longer the case. When we combine[46] the two intratemporal optimality conditions we obtain:

$$\varphi \hat{H}_t + \frac{\hat{c}_t}{\sigma_S} + \lambda \left(1 - \frac{\sigma_H}{\sigma_S}\right) \frac{\hat{c}_t^H}{\sigma_H} = \hat{w}_t.$$

When the real wage drops, HtM households reduce their consumption as they do not have other sources of income. Thus, for the same real wage decline, aggregate quantities (i.e. consumption and hours worked) decline less relative to the case where the IES of households are the same. Moreover, the smaller σ_H, the stronger this channel is. The following proposition formalizes this and shows under which parameterizations of λ and σ_H this novel channel is not strong enough to dominate and overturn monetary policy *amplification* and *countercyclical consumption inequality.*

Proposition 2 *Under SADL* ($\lambda < \frac{1}{1+\varphi}$) *and when* $\sigma_H < \sigma_S$, *it is the case that if* $\sigma_H < \sigma_H^\star$ *an increase in* λ *increases (reduces) the aggregate impact of monetary policy shocks if* $\lambda > \lambda^\star$ ($\lambda < \lambda^\star$). *Where*

$$\lambda^\star = \frac{1}{1+\varphi} - \frac{\sqrt{\sigma_H\, \varphi\, (\sigma_S - \sigma_H)\, (\sigma_S\, \varphi + 1)}}{(\sigma_S - \sigma_H)(1+\varphi)}$$
$$\sigma_H^\star = \frac{\sigma_S}{\sigma_S\, \varphi^2 + \varphi + 1}.$$

The proof is in appendix G.5. Note that $\lambda^\star$ and $\sigma_H^\star$ are decreasing in φ. Proposition 2 can be used to check when the introduction of IES heterogeneity is still empirically plausible – namely the parameterization under which generating countercyclical HtM labor supply still implies countercyclical consumption inequality as in Coibion, Gorodnichenko, Kueng and Silvia (2017) and Mumtaz and Theophilopoulou (2017). For example, if we calibrate $\varphi = 1$ and $\sigma_S = 1$ as above, we get $\sigma_H^\star = 0.333$. In such a case, when $\sigma_H = \sigma_H^\star$, then $\lambda^\star = 0$ and an increase in inequality always generates an amplification in aggregate demand. If we consider a lower value of the HtM IES, say $\sigma_H = 0.1$, we obtain that $\lambda^\star = 0.26$. When inequality increases (up to a maximum share of 25 percent of HtM households), the response of aggregate demand to a monetary policy shock decreases. Next, we show this graphically. Figure 5 plots the impact response of aggregate consumption for different values of λ in the SADL region and selected calibrations of σ_H keeping $\varphi = \sigma_S = 1$.[47] If the HtM household's IES is sufficiently low, we have no amplification.

[46]Taking the weighted average of $\varphi \hat{H}_t^j + \frac{\hat{c}_t^j}{\sigma_j} = \hat{w}_t$ using λ and $1 - \lambda$ as weights.

[47]The rest of the calibration is the same as in footnote 41.

It is important to stress that the dampening effect of increasing inequality on the transmission of monetary policy shock is only a theoretical statement and should not be treated as a quantitative or empirical assessment. The scope of the next section is to build a more realistic model and attempt to quantify the role of this mechanism.

In appendix G.6, we show that IES heterogeneity does not affect the slope of the Phillips curve in this setup where labor services and wages are homogeneous across agents – assumptions that we will relax in the next section.

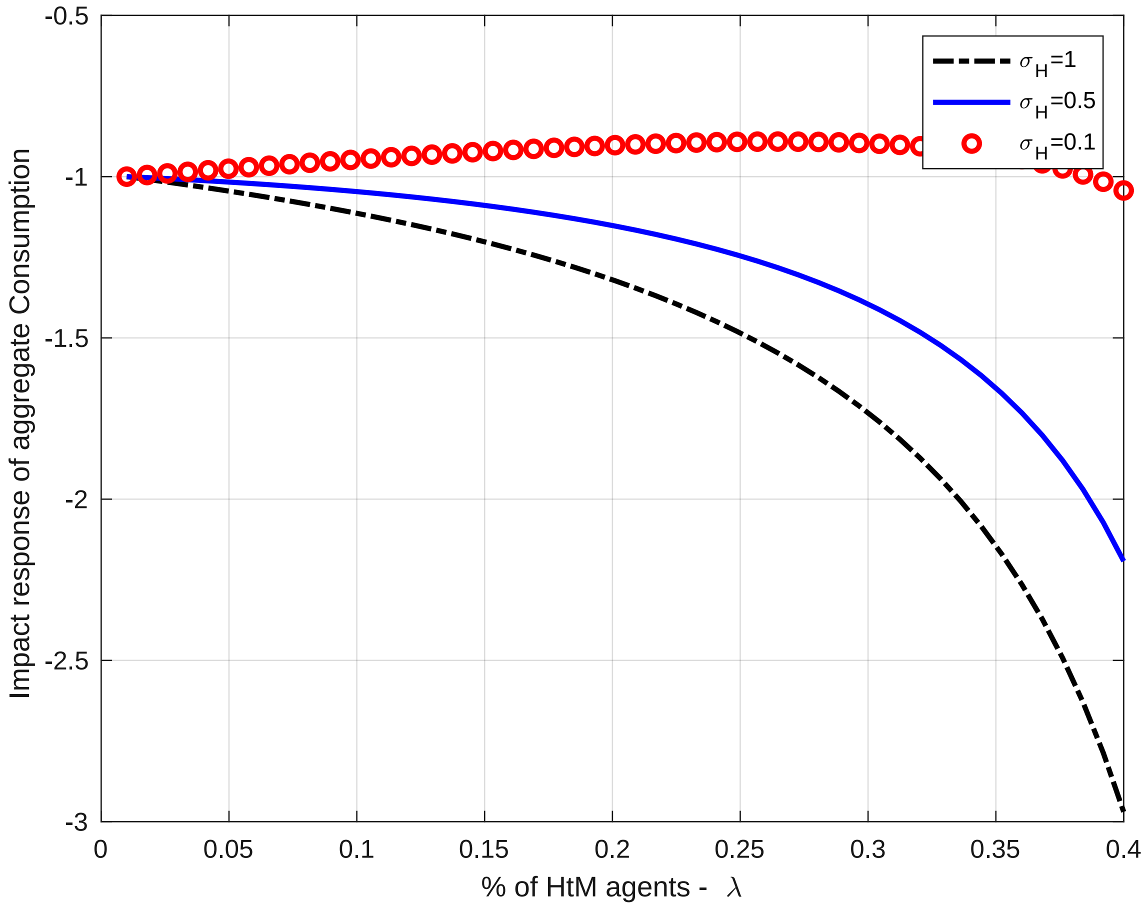

Figure 5: Impact response of Hours/Consumption to a 1 percent tightening with $\sigma_S = \varphi = 1$ for selected values of σ_H.

3.3 Heterogeneity, frictions, and monetary policy amplification

In this section, we study how quantitatively relevant is the new channel of transmission of monetary policy via HtM substitution of leisure for consumption presented in the stylized setup in the previous section. As discussed, our empirical evidence shows the behavior of the equilibrium hours worked of the left tail of the income distribution. This behavior could be the result of different channels such as income heterogeneity, as well as the result of higher unemployment risk in the left tail of the income distribution or the result of firms laying off some workers and increasing the utilization of the remaining workforce on the labor demand side. Therefore, in this section, we build a medium-scale TANK model where all these channels are at play. Moreover, we look at the implications of adding different layers of heterogeneity, frictions, and factors in production. For example, Bilbiie, Känzig and Surico (2022) showed that the complementarity between capital and income inequality in heterogeneous agent models substantially amplifies the effect of monetary policy. So far, we have abstracted from the extensive margin of labor supply, which is the main driver of monetary amplification in the data (see Patterson (2021)).

The model used in this section builds on the one discussed in section 3.1 by adding capital accumulation (with investment adjustment costs), and search and matching frictions with segmented labor markets, driving heterogeneity in wages and unemployment rates between the two types of agents. We also assume that the two types of labor have different degrees of complementarity/substitutability in production (Krusell, Ohanian, Ríos-Rull and Violante (2000)). Moreover, we now propose a microfoundation of the income effect heterogeneity on labor supply presented in section 3.1. We do so by assuming Stone-Geary preferences that generate IES increasing in the level of consumption in steady state. With this model, we are now able to target not only the heterogeneous response of labor supply we observe in the data, but also the heterogeneity in wages and unemployment. While appendix J describes the model in detail, here we focus on the microfoundation of heterogeneous IES.

We assume that both agents ($j = H, S$) have the same separable Stone-Geary preferences between real consumption c_t^j and hours worked H_t^j:

$$\mathcal{U}(c_t^j, H_t^j) = \frac{(c_t^j - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^j \frac{(H_t^j)^{1+\varphi}}{1+\varphi}, \tag{8}$$

where $\bar{c}$ is subsistence level of consumption, σ^{-1} is the curvature of the utility in consumption, ν the weight of hours in utility, and φ^{-1} is the Frisch elasticity of labor supply. For a given steady state level of consumption $c^j \geq \bar{c}$, the IES is increasing in c^j and given by:

$$-\frac{\mathcal{U}'_{c^j}}{c^j \mathcal{U}''_{c^j}} = \sigma\left(1 - \frac{\bar{c}}{c^j}\right). \tag{9}$$

From (9) we derive the IES of HtM and of Savers respectively:

$$\sigma_H = \sigma\left(1 - \frac{\bar{c}}{c^H}\right) \tag{10}$$

$$\sigma_S = \sigma\left(1 - \frac{\bar{c}}{c^S}\right). \tag{11}$$

It follows that, when $c^S > c^H$ in steady state, then $\sigma_H < \sigma_S$. Consumption inequality in steady-state depends on profits and the different sources of income for the two agents; namely capital and labor. The consumption of savers is larger in steady state because they have positive income from profits and capital, but also because they work longer hours.[48]

[48] Note that here we remove the steady state subsidies that induced 0 profits in steady-state so far to make the algebra simpler. Results are robust to keeping the subsidies in place or introducing fixed costs in production that induce 0 profits in steady state.

We then add capital accumulation and investment adjustment costs by assuming that only savers own and rent capital to the firms (Bilbiie, Känzig and Surico (2022)). Labor market frictions are introduced using the Search and Matching (SAM) model pioneered by Diamond (1982) and Mortensen and Pissarides (1994). In doing so we assume that the two agents supply different types of labor and that there are segmented labor markets, one for the savers and one for the HtM (as in Dolado, Motyovszki and Pappa (2021)). Hence, we can calibrate the labor market frictions for the two agents and therefore generate heterogeneous responses of wages and unemployment rates as per our empirical evidence. In Appendix I we complement the analysis of Kaplan et al. (2014) by using the Survey of Consumer Finance and look at the proportion of *poor* HtM agents that do not have a college education. While we observe an increase over time in the education of people who hold little or no wealth in the US, in 2019 the proportion of them with a college degree was lower than 20%. Given this evidence that most of the *poor* HtM are also unskilled workers, we follow the capital-skill complementary tradition (Krusell et al. (2000)) and assume that firms combine the different types of labor from the two agents with a nested CES production function, where savers's labor supply is complementary to capital while HtM labor supply is a substitute. Importantly, as discussed in appendix J, we follow Thomas (2008), Trigari (2009) and Cantore, Levine and Melina (2014) in modeling hours per worker as being determined by firms and workers in an efficient way (i.e., to maximize the joint surplus of their employment relationship). As a result, hours worked in equilibrium are pinned down by an expression that is the same as in the competitive labor market, where the marginal product of labor equals the marginal rate of substitution between consumption and leisure:

$$\varphi \hat{H}_t^j = \hat{mc}_t - \frac{1}{\sigma_j}\hat{c}_t^j, \tag{12}$$

with $j = H, S$, σ_j as in (10)-(11) and mc the real marginal cost in production. The only difference from the labor supply expressions in Table 1 is that, with labor market frictions, hours are independent of the hourly wage, which is determined in the Nash bargaining process. This shows how the heterogeneity in the income effect on labor supply is still the main driver of the countercyclical behavior of labor supply of the HtM household even in this richer setup.

3.3.1 Calibration

We calibrate a period to be a quarter ($\beta = 0.99$), capital to depreciate at rate $\delta = 0.018$ (NIPA 1954-2019), labor share to be $s^L = 0.68$ (BLS 1954-2019), and the investment adjustment costs to be $\iota = 1.26$, which are in line with the estimates of Bayer, Born and Luetticke (2022).[49] The Taylor rule coefficients on inflation ($\psi_\pi = 2.5$), output ($\psi_y = 0.2$),

[49] We calibrated ι to the value estimated by Bayer et al. (2022) in RANK. This is because in their estimated HANK economy they also have portfolio adjustment costs at the household level which are shown to reduce the estimated value of the investment adjustment costs.

and interest rate smoothing ($\rho^R = 0.79$) are set as estimated by Bayer et al. (2022), as well as the Rotemberg price adjustment costs.[50] Instead, th elasiticity of substitution between goods' variety, $\eta = 11$, follows from Born and Pfeifer (2014).

We calibrate the steady state of hours using individual actual hours of work from the CPS so that Savers work 32 percent and HtM 27 percent of their available time.[51] We also calibrate the unemployment rates as $U^S = 4.4$ percent and $U^H = 9$ percent and the wage ratio to $\tilde{w} = w^S/w^H = 2.11$ from the CPS. The two IES are set to $\sigma_S = 0.8$ and $\sigma_H = 0.1$, in line with the estimates in Vissing-Jørgensen (2002). We calibrate $\zeta^j = 0.33$ for $j = S, H$, which implies a Frisch elasticity of labor supply $\frac{1}{\varphi} = 0.5$ for both agents (Chetty, Guren, Manoli and Weber (2011)), and $\lambda = 0.3$, to be consistent with the average proportion of hours worked by the bottom 30 percent of workers across the labor income distribution in the CPS. For the two elasticities of substitution in production, we set $\sigma_y = 1.67$ and $\sigma_x = 0.67$ as in Krusell et al. (2000).[52] The matching elasticity $\omega^j = 0.5$ and bargaining power $\eta^j = 0.5$ are symmetric across the types of households and follow standard calibrations. We follow Dolado et al. (2021) and calibrate SAM frictions asymmetrically between the two segmented labor markets using the evidence in Wolcott (2021). As a result, the reparation rate $1 - \rho^j$ and labor market tightness θ^j are higher for HtM ($1 - \rho^H = 5.62\% > 1 - \rho^S = 2.45\%$ and $\theta^H = 2.13 > \theta^S = 1.39$).

3.3.2 Results

Figures 6-7 plot selected variables under alternative calibrations. In Figure 6, we show simulations of aggregate variables in the RANK economy ($\lambda = 0$), the TANK economy with homogenous income effects on labor supply ($\sigma_S = \sigma_H = 0.8$), and the TANK economy with heterogenous income effects ($\sigma_S = 0.8 > \sigma_H = 0.1$). Instead, Figure 7 shows the details of agent-specific variables in the two TANK economies.

As usual, household heterogeneity amplifies aggregate demand and inflation. However, with heterogenous IES, we observe less amplification in aggregate demand than in the homogeneous IES case, in line with the analysis presented in section 3.2. This muted amplification of aggregate demand is also evident in the impact and cumulative multipliers of output, consumption, and hours in the model with heterogenous IES relative to the homogenous IES case (Table 3). As Figure 7 shows, this is driven by the substitution of leisure for consumption of the HtM househods in the heterogeneous IES case. As aggregate demand falls, HtM workers who remain employed increase their hours of work and therefore can take a smaller hit on their consumption.

[50] They are calibrated to match their estimated slope of the Phillips curve and equal to 0.15.

[51] We use P_{30} as the cut off between HtM and savers along the labor income distribution consistent with the empirical evidence. The average weekly hours per worker in the CPS are 32.5 for $P_{<30}$ and 38.4 for $P_{>30}$. More details about CPS data are presented in appendix A.1.

[52] Bilbiie, Primiceri and Tambalotti (2022) recently estimated a TANK model with a similar production function and found a much larger degree of substitutability between labor services of the HtM and capital. Our results are not very sensitive to the value of these parameters.

As discussed before, this is also in line with another labor supply channel where higher unemployment risk pushes HtM households to increase their labor supply and/or a channel of labor demand where firms are asking the HtM workers who are not made redundant to increase their hours of work. Figure 7 shows that these channels are also at work here. Unemployment of HtM workers is already higher than that of savers in steady state, plus it increases substantially more after a monetary policy hike. As a result, the model with heterogenous IES generates amplification in unemployment, as shown in Table 3.

The strength of these two other channels, coming from unemployment risk and labor demand, depends on the replacement ratio and the unemployment rate in steady state of the HtM households. Therefore in appendix J.9 we examine whether any of them can also generate a countercyclical response of the hours of HtM agents. Our results show that this is not the case and that a low value of σ_H (strong income effect) is still needed to get a substantial increase in the hours of HtM workers even if we increase their replacement ratio (higher risk of becoming unemployed) or decrease their steady-state unemployment rate (more room for firms to fire HtM workers and ask the remaining workers to work longer hours). Hence, in line with equation (12), our simulations show that the unequal incidence of unemployment alone is not able to generate an increase in HtM labor supply in line with our empirical evidence. The income effect captured by σ_H is still the key to matching the empirical evidence.

It is important to stress that the proposed TANK-SAM model here can match not just the heterogeneous response of hours reported in our empirical analysis, but also the larger impact of MP shocks on unemployment and wages of the HtM households. Moreover, as a side product of the behavior of the labor supply and consumption of the HtM households, our simulations also produce a muted response of consumption inequality compared to the standard case.[53] Consumption inequality is still countercyclical, in line with Coibion et al. (2017) and Mumtaz and Theophilopoulou (2017), but its response is four times smaller than in the case with homogeneous IES. Instead, a consequence of matching the more sensitive response of wages of the HtM workers to the MP shock implies that IES heterogeneity amplifies the response of inflation.[54] Finally, in appendix J.10 we show the robustness of these results to the introduction of real wage rigidities.

[53]Consumption inequality is defined as the ratio of the consumption of the two agents.

[54]This is different from the analysis in section G.6 where we show that with homogenous wages there is no impact of heterogeneous IES on inflation.

Table 2: Calibration

Parameter	Value	Description
$\bar{\Pi}$	1	Steady State Inflation Convention
β	0.99	Discount Factor
σ_H	[0.1, 0.8]	IES, HtM
σ_S	0.8	IES, HtM
δ	0.018	Capital depreciation
s^L	0.68	Labor share
ι	1.26	Investment adjustment costs
η	11	Elasticity of substitution between goods
ϕ^r	0.79	Interest rate smoothing
ϕ^π	2.5	Taylor rule coeff of inflation
ϕ^y	0.2	Taylor rule coeff of output
σ_y	1.67	Elasticity of subs. K/H^S with H^H
σ_x	0.67	Elasticity of subs. K with H^S
λ	0.3	Share of HtM Agents
ϕ^p	66.66	Rotemberg prices
$\tilde{w}$	2.11	Steady State Wage ratio
ω^j	0.5	elasticity of matches wrt unemployment
η	$= \mu$	workers bargaining power
ζ^j	0.33	steady-state relative value of non-work to work activity
$\bar{M}^j$	0.5	matching efficency
$\bar{H}^H$	0.27	Steady State Hours, HtM
$\bar{H}^S$	0.32	Steady State Hours, Savers
$\bar{U}^H$	0.09	Steady State Unemployment, HtM
$\bar{U}^S$	0.044	Steady State Unemployment, Savers
$\bar{\theta}^S$	1.39	labor market tightness, Savers
$\bar{\theta}^H$	2.13	labor market tightness, HtM
$1-\rho_H$	0.0562	job separation rate, HtM
$1-\rho_S$	0.0245	job separation rate, Savers

Table 3: Multipliers relative to the homogenous IES case.

Output	Consumption	Inflation	Hours	Unemployment
Impact multipliers				
0.87	0.69	1.04	0.66	1.12
Cumulative multipliers				
0.82	0.73	1.00	0.55	1.06

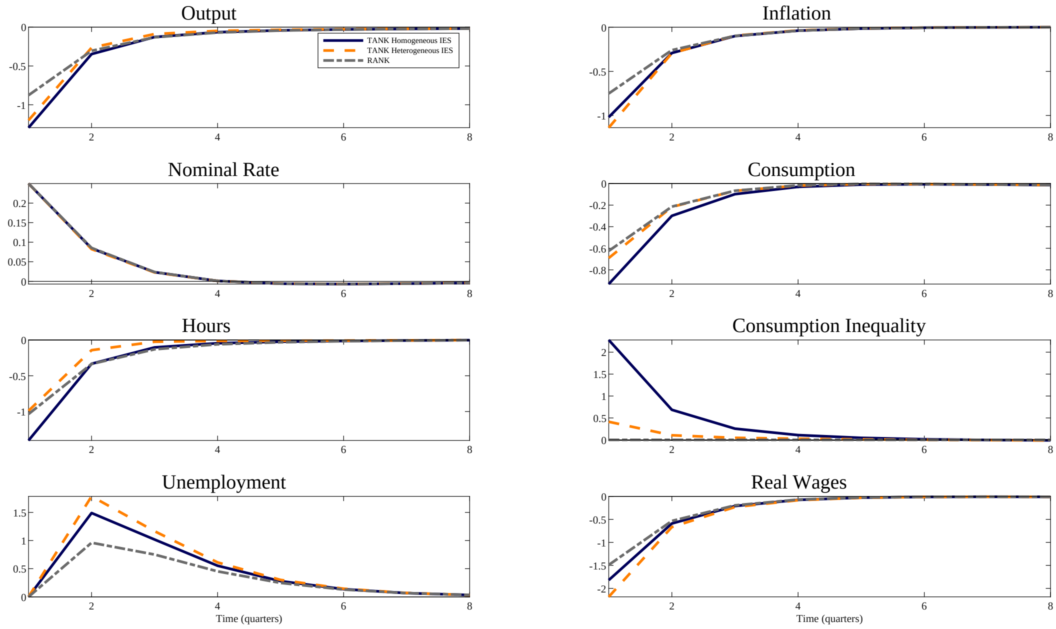

Figure 6: Impulse responses of selected variables in the TANK-SAM model under different calibrations: RANK (grey dashed-dotted line), TANK with homogeneous IES (solid blue line), and TANK with heterogeneous IES (dashed orange line).

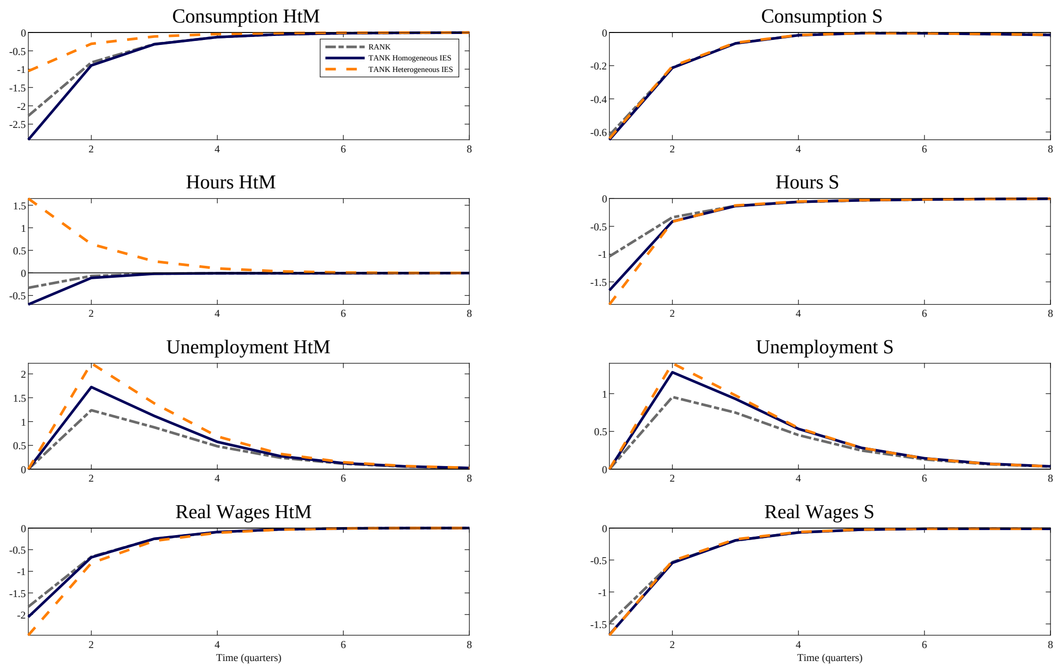

Figure 7: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), and TANK with heterogeneous IES (dashed orange line).

4 Conclusion

In this paper, we study the interaction between monetary policy and labor supply decisions at the household level. Our first contribution is to establish some new empirical facts. Using survey data and a state of the art FAVAR we find that in the US and the UK the response of hours worked to monetary policy shocks across the income distribution is heterogeneous. While aggregate hours decline, the hours worked by households with low incomes, conditional on keeping the job, actually increases. We also uncover that the hours' response at the left tail of the income distribution not only exhibits a different sign than the rest of the economy, but it is also much larger in magnitude. We report a series of robustness checks showing that these results are not primarily driven by the extensive margin, household fixed characteristics, or demographics. Given that the labor supplied by households with low and moderate incomes represents both a non-negligible share of the volatility and a relevant proportion of hours worked in the aggregate, these results appear to be quantitatively relevant from a macro perspective. The countercyclicality of hours worked in the left tail of the income distribution is consistent with multiple explanations, from households' labor supply forces to firms' labor demand factors. Empirically isolating the relative strength of each of these channels is too ambitious with the data we considered and we wish to leave this interesting topic for future research projects.

The second contribution of the paper is to study the implications of the behavior of the left *tail* of labor supply following a monetary policy shock for the *tale* of the monetary policy transmission mechanism. We use a two-agents New-Keynesian model where we model the left tail of the income distribution as *poor* Hand-to-Mouth (HtM). We show that our empirical findings can be replicated with heterogeneity in the sensitivity of marginal utility of consumption (via IES) and a stronger income effect for the HtM households. This setup generates interesting implications for the effectiveness of monetary policy transmission. In particular, we uncover a novel channel of transmission of monetary policy via inequality on aggregate demand. After an interest rake hike, HtM agents give away leisure time to avoid having to reduce their consumption one-to-one with their decline in income. Equipped with a quantitative model with both intensive and extensive margin of labor supply that replicates our empirical evidence, we show that this new channel reduces the amplification of monetary policy via inequality generated by the heterogeneous behavior of unemployment along the income distribution. Another source of amplification of monetary policy via inequality that has been identified in the literature is the cyclical behaviour of precautionary savings due to the presence of idiosyncratic risk (Bilbiie, Primiceri and Tambalotti (2022)). Our analysis abstracts from this, and we leave the interaction of idiosyncratic risk and labor supply heterogeneity for future research. Finally, our theoretical analysis suggests that heterogeneous labor supply can alter the transmission of a particular type of demand shock such as monetary policy, for which we have good measured proxies. In future work, it would be interesting to study whether labor supply heterogeneity and the unequal intensity of income effects could significantly alter the propagation of other demand or financial shocks.

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Online Appendix

A Household level data

A.1 US: CPS Survey

For the US, individual level data is obtained from the current population survey (CPS) for each month from 1994 to 2019. For earnings we use the outgoing rotation group.[55] Our measure of hourly earnings is constructed by using the variable HOURWAGE, the amount earned per hour in current job reported by respondents who are paid hourly. For respondents that are paid weekly, we construct earnings by dividing the variable EARNWEEK (usual weekly earnings) by UHRSWORK1 (usual weekly hours in main job). We use actual hours worked in the main job (AHRSWORK1) as our main measure of weekly hours. We drop respondents that lie in the top and bottom percentile of the earnings distribution or are aged less than 18 or more than 66.

For each month, we construct the earnings percentile groups $P_{\leq J}$ and $P_{>J}$ where Jranges from 5 to 95 with increments of 5. For example when $J = 5$, the two groups consist of respondents the fall below that 5th percentile of hourly earnings ($P_{\leq 5}$) and those that lie above the 5th percentile ($P_{>5}$). We calculate average earnings and average weekly hours for each group using the survey weights. Repeating this across all months in the sample provides a time series of average earnings and average weekly hours by percentile group. Figure A.1 provides information regarding the characterstics of the earnings distribution. Respondents in the right tail of the distribution tend to be older, better educated, are likely to work longer hours, own their home, be white and Male and less likely to be employed in a manual job.

A.2 US: CEX Survey

In the CEX survey, data at the Consumer Unit (CU) level is constructed using the Consumer Expenditure Survey (CEX). The component of income that is the focus of our study is labor income and hours worked. Labor income for CU $i$and time t ($L_{i,t}$) are the amount of wage and salary income before taxes received over the past 12 months. The remaining components of income are grouped into: (1) income from farm and non-farm business, (2) financial income and (3) income from social security.[56] Total hours are defined as the sum of hours worked

[55] Households are interviewed in the CPS for four months and then again for four months after a eight month break. In the fourth and eighth month of interviews, when households are about to rotate out of the interviews, they are asked additional questions regarding earnings. For more details on the outgoing rotation group, see https://cps.ipums.org/cps/outgoing_rotation_notes.shtml

[56] Finanical income = Interest on saving or bonds + Amount of regular income from dividends royalties, estates, or trusts + Amount of income from pensions or annuities from private companies + Amount of net income or loss from roomers or boarders +Amount of net income or loss from other rental units. Social Security income = Amount of Social Security and Railroad Retirement income + Unemployment Compensations + Workers Compensation and veterans' payments + public assistance or welfare including job grants plus food stamps.

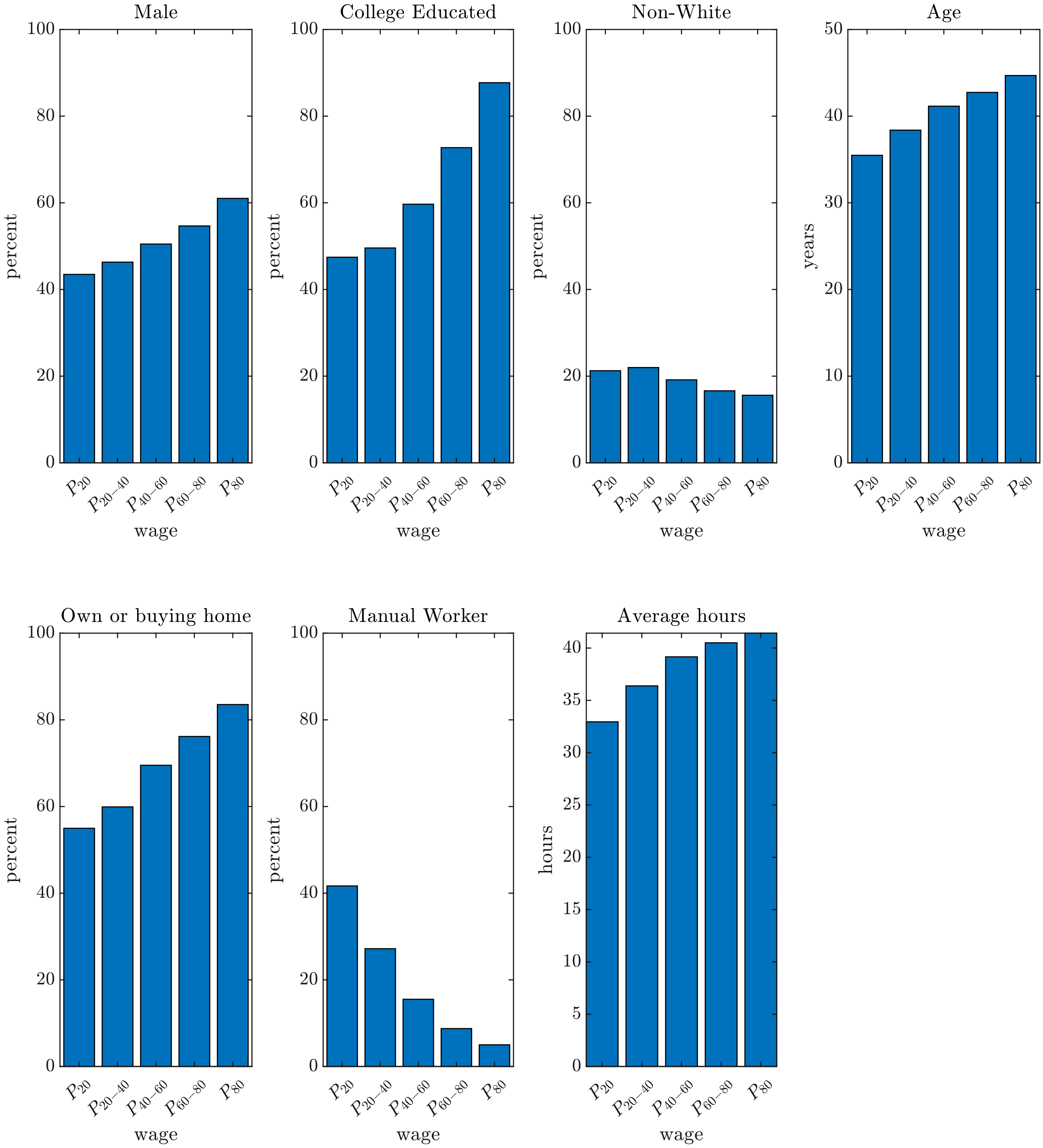

Figure A.1: The earnings distribution for the US in 2010.

by each individual in the CU over the past year. In each year of the sample, we retain CUs from the second or the fifth interview when income data is updated. CUs are then assigned to each quarter of the year by their date of interview (see Cloyne and Surico (2016)), and then sorted into bins by gross income as discussed previously. Average wages, the average of the remaining components of income, and average hours are computed in each of these groups using survey weights. These calculations are repeated for each year in the sample from 1984 to 2018, delivering a quarterly time series for each group[57].

As described below, we impute missing values in the CEX income data in the pre-2004 period. This ensures that the data is consistent with the post-2004 data where imputation is carried out by the Bureau of Labor Statistics (BLS) prior to data release. In each year of the sample, we retain CUs from the second or the fifth interview when income data is updated. We drop observations that lie in the top or bottom 1 percent of total income. We retain CUs where the age of reference person is greater or equal to 18 and less than equal to 66.

CUs are then assigned to each quarter of the year by their date of interview (see Cloyne and Surico (2016)). They are then sorted into bins by gross income. The bins refer to five percentile groups of total income: (1) less than or equal to the 20^{th} percentile (P_{20}), (2) greater than 20^{th} and less than equal to 40^{th} percentile (P_{20-40}), (3) greater than 40^{th} and less than equal to 60^{th} percentile (P_{40-60}) (4) greater than $60^{th}$and less than equal to 80^{th} percentile (P_{60-80}) and (5) greater than 80^{th} percentile (P_{80}). Average wages, the average of the remaining components of income and average hours are computed in each of these groups using survey weights. These calculations are repeated for each year in the sample (1984-2018) delivering a quarterly time-series for each group.

In terms of CEX variables wages are defined as the variable FSALARYX. Salary income from other sources is defined as the sum of income or loss from farm (FFRMINCX) and non-farm business (FNONFRMX) received by CU members over the past year. Financial income is the sum of interest on saving accounts/bonds (INTEARNX), income from dividends royalties, estates, or trusts (FININCX), net income or loss from roomers or boarders (INCLOSSA) and net income or loss from other rental units (INCLOSSB). Income from social security is defined as the sum of social security and railroad retirement income (FRRETIRX), supplemental security income (FSSIX), unemployment compensation (UNEMPLX), workers' compensation and veterans' payments (COMPENSX), public assistance or welfare (WELFAREX) and food stamps (JFDSTMPA).

Total hours for each member of the household are defined as weekly hours worked (INC_HRSQ) times the number of weeks worked over the past year (INCWEEKQ). These are then summed across CU members.

Figure A.2 displays demographic and financial characteristics of the CUs for each income percentile in the US. See Figure A.3 for the evolution over time of the different income components. The CUs in the 1^{st} quintile of income are mostly females, less than 20 percent are college graduates and are non-white survey participants. Most of these households are renters and only 10 percent own a house through a mortgage. A 20 percent is out-right homeowners, usually to inherited properties. In terms of financial assets, the 20^{th} percentile has on average less than 2 percent of its total income in the form of liquid assets (e.g.

[57]The demographic characteristics of CU along the income distribution constructed from the CEX survey are similar to those obtained with the CPS survey. As we move from the left to the right, respondents are more likely to be male, white, college graduates, homeowners with a mortgage, and work longer hours.

checking and savings accounts, bond, stocks and other securities). While labor income is the most important source of total income, it is still the lowest percentage compared to other percentiles (only 50 percent). Social security transfers are a very important source as they comprise around 40 percent of total income. On the other tail of the distribution, the 5^{th}

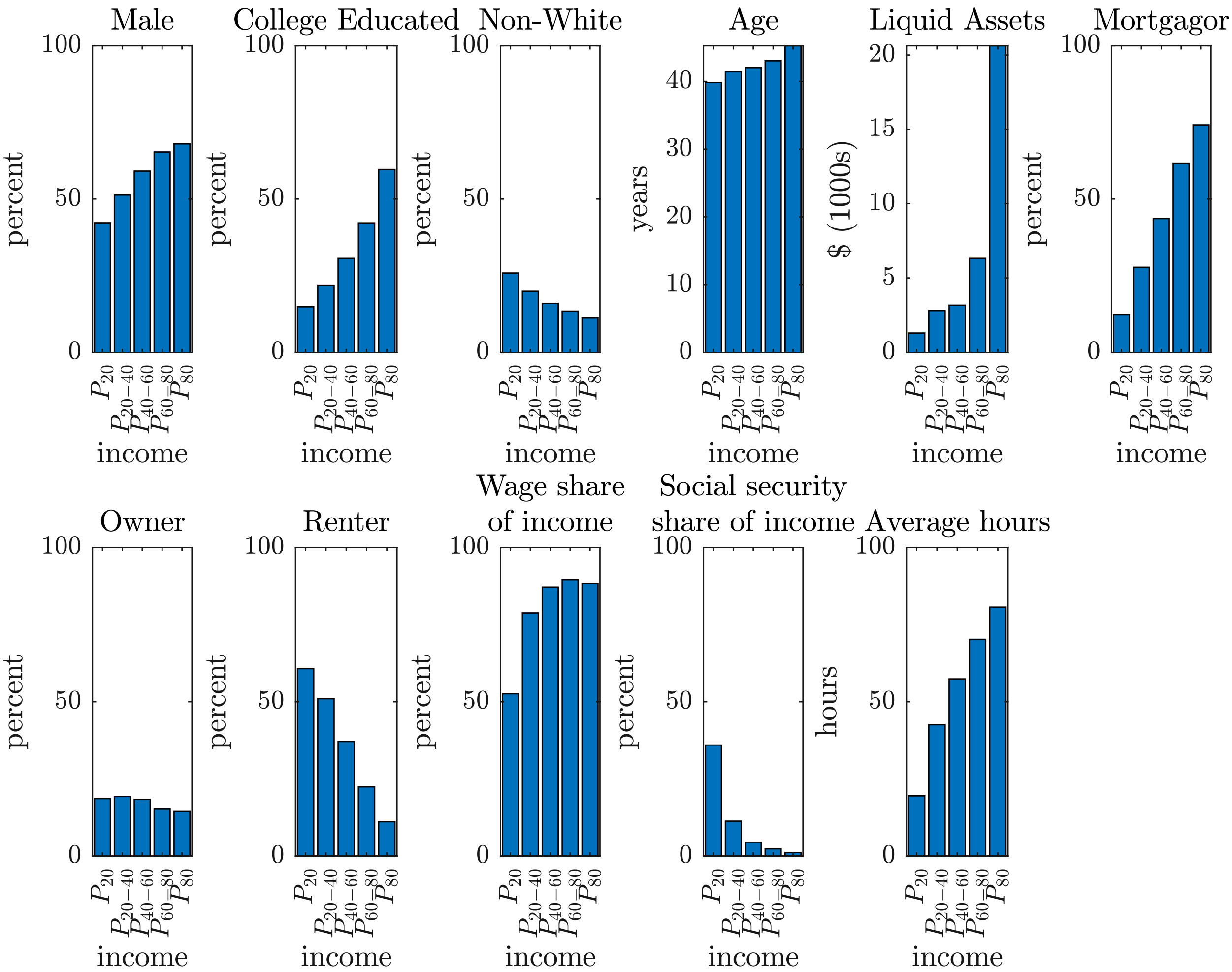

Figure A.2: Components of income in the US by percentile group.

quintile consists mostly of white males, relatively older to the other bins, who are college graduates. Around 70 percent in this bin owns a property through mortgage while less than 10 percent rents one. Their liquid assets comprise 20 percent of their total income which is the highest to all other bins. This percentage may appear relatively low for this income group but as noted by Coibion et al. (2017) for the US, the contribution of financial income is fairly small throughout the sample because the CEX does not include reliable measures of household wealth. Therefore more than 80 percent of total income in this quintile comes from wages while this group scores the highest amount of hours worked per week by the reference person.

Figure A.3 displays the different components of income as a proportion of the total and their evolution over time. They show that their proportion has been relatively constant over time.

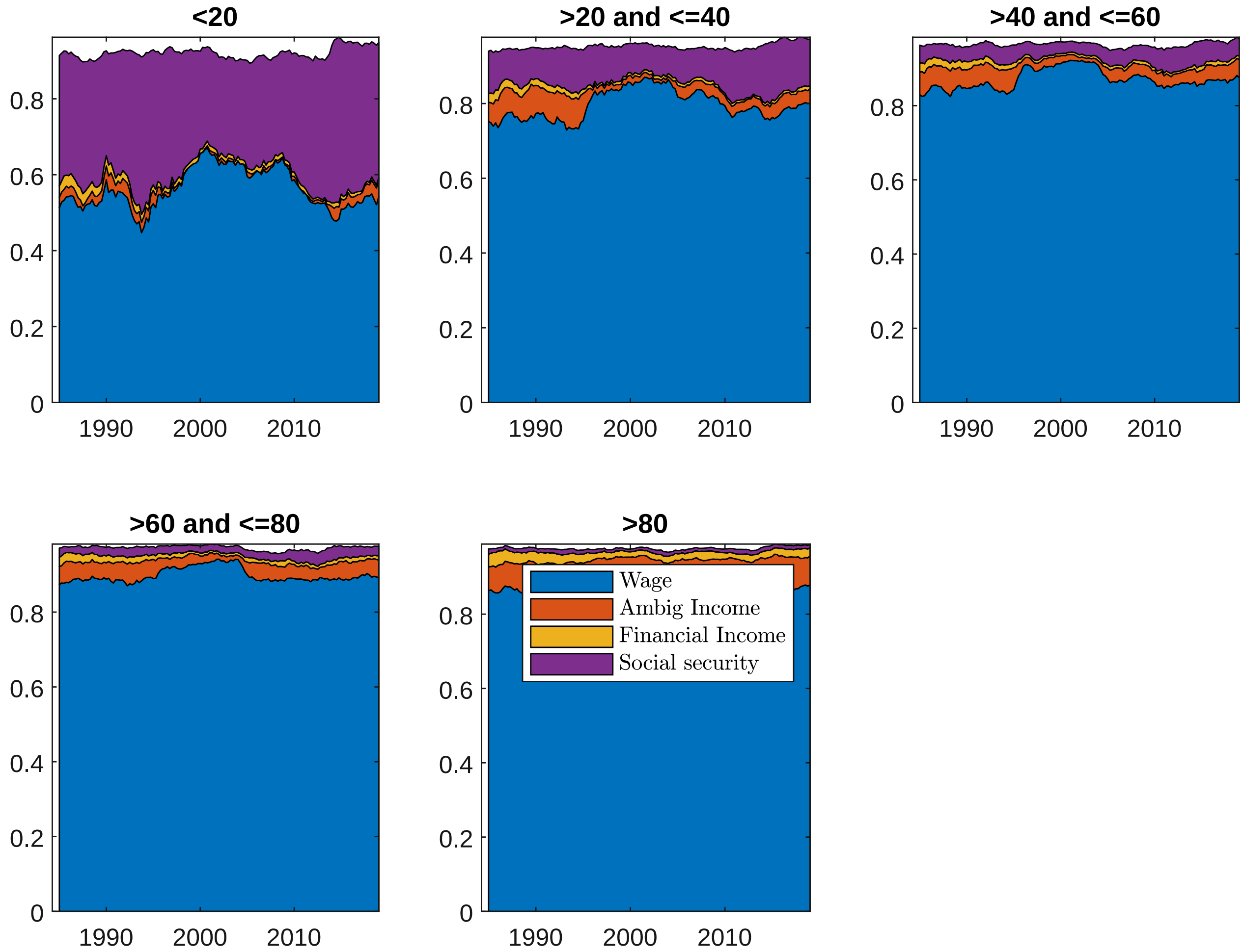

Figure A.3: Components of income in the US by percentile group. The data is smoothed using a 4 quarter moving average.

Imputation of income data After 2004, income data in the CEX is imputed. In order to ensure that pre-2004 data is consistent with this approach, we follow papers such as Coibion et al. (2017) and impute components of total income where there are missing values due to an invalid non-response (see also the description here). We first estimate a logistic regression where the dependent variable is coded as 1 if the CU receives income from a particular source and zero otherwise. The independent variables are the same as those used for the imputation procedure described below. The logistic regression provides a probability that the CU received this income component. If this probability exceeds a random draw from the uniform distribution for a valid non-response/zero response, this observation is changed to an invalid response and is imputed in the next step. The approach to imputation is based on Coibion et al. (2017) who closely replicate the procedure used by CEX. Imputed data is obtained as fitted values from a regression (using sampling weights) of the income component of valid reporters on age, the square of age, month of interview, sex, race, education, number of weeks worked, family size, occupation, region, marital status, total consumption expenditure, number of persions below 18, number of persons below 18, number of persons above 64 and the number of earners. Note that the median is subtracted from the dependent variable before the procedure and added back after the fitted values are obtained. The coefficients are shocked and used to produce predicted values. These are assigned to missing values using predicted mean matching. We produce 5 replicates, with the final imputed data taken to be the mean across these replications. Any income variables that do not allow negative numbers are bottom coded at 0.

This procedure is applied to the data pooled over samples of 5 years. In the regressions described above, we use a quadratic time-trend to account for growth over time. The regressions use survey weights.

A.3 UK: LFS Survey

We obtain quarterly data for the UK for the period 1994 to 2019 from the Labour Force Survey (LFS). Our measure of hourly earnings is the variable HOURPAY. We use actual hours worked for each individual (TTACHR). We apply the filters described above to UK data – observations that lie in the top and bottom percentiles of the wage distribution are dropped and individuals older than 18 and younger than 66 are included in the sample. As in the case of the US, we construct a pseudo panel using the earnings percentile groups $P_{\leq J}$ and $P_{>J}$.

Figure A.4 shows that the characteristics of the UK earnings distribution are very similar to that of the US. Low earners in the UK tend to be younger, less educated, employed in manual labour. They are less likely to be male and to own a home and their average weekly hours are shorter.

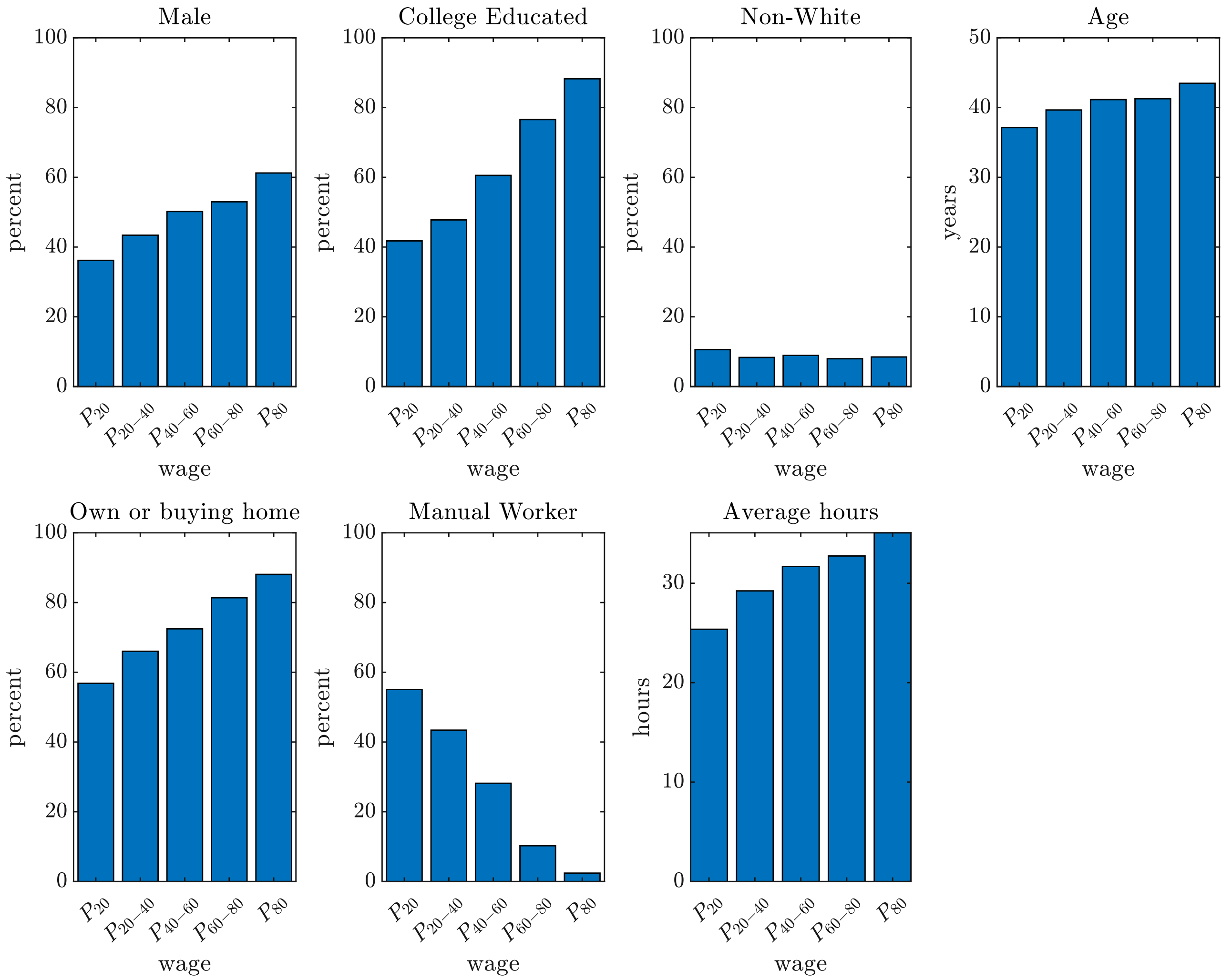

Figure A.4: The earnings distribution for the UK in 2010.

A.4 Comparison with aggregate data

The top panel of Figure A.5 compares aggregate actual hours from the CPS to a measure of monthly hours from the Bureau of Labour statistics (average weekly hours of production and non-supervisory employees). Note that the data is standardised.[58] The CPS data captures the main movements in the aggregate data fairly well. The bottom panel of the figure shows a comparison of the aggregate usual hours constructed from the CEX and total hours in the non-farm business sector from the Bureau of Labour statistics. Once more, the survey-based aggregate appears to be a good approximation of the official aggregate data.

Figure A.6 shows this comparison for the UK. The figure shows that aggregate hours constructed using our LFS sample mimics closely the aggregate one (code YBUY) published by the Office of National Statistics (ONS), with a correlation of 0.9.

[58] As our sample is affected by the filters we apply we do not expect to match the level of the aggregate data. The comparison in this section focusses on the trends and cylical movements in the data.

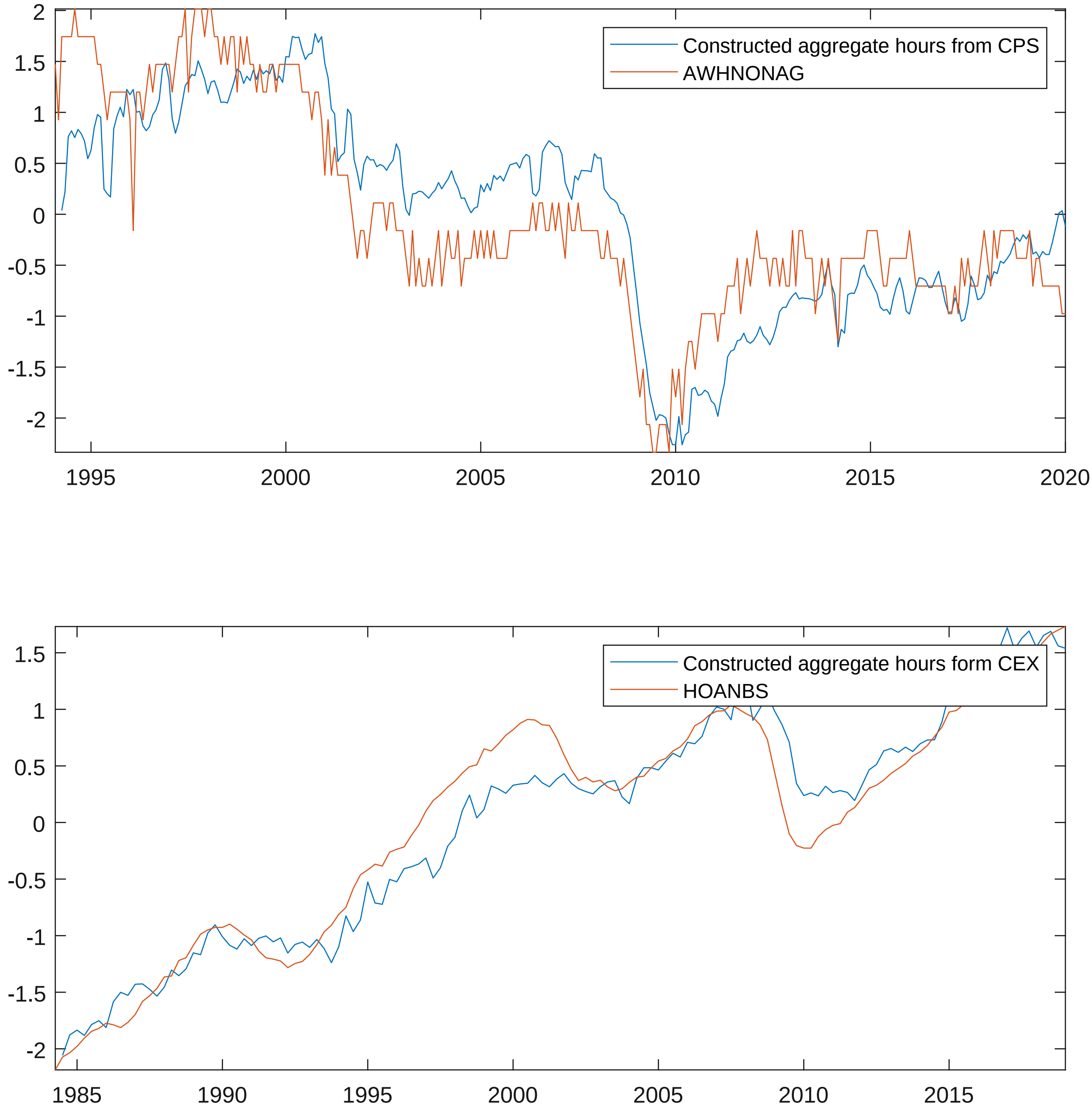

Figure A.5: Comparison of survey based total hours (blue) with aggregate (orange), US.

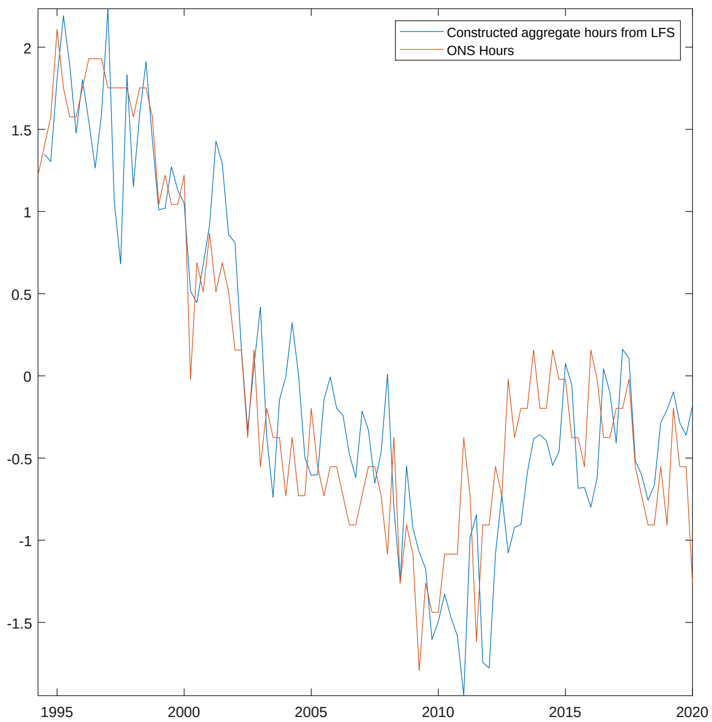

Figure A.6: Comparison of survey based total hours (blue) with aggregate (orange), UK.

B Priors and estimation of the FAVAR

The FAVAR model is defined by the following equations:

$$X_{it} = c_i + b_i\tau + \Lambda_i F_t + \xi_{it} \tag{B.1}$$

$$Y_t = c + \sum_{j=1}^{P} \beta_j Y_{t-j} + u_t \tag{B.2}$$

$$cov\left(u_t\right) = \Sigma = A_0 A_0' \tag{B.3}$$

Where $Y_t = \underbrace{\begin{pmatrix} R_t \\ F_t \end{pmatrix}}_{N\times 1}$, R_t denotes the 1 year interest rate, $i = 1, 2, .., M$ denotes the cross-sectional dimension of the panel data-set X_{it} while $t = 1, 2, .., T$ is the dimension. As described in Barigozzi et al. (2021), the factors can be consistently estimated using a principal components (PC) estimator. In particular, the factor loadings are estimated via PC analysis of the first differenced data ΔX_{it}. With these in hand, the factors are estimated as $\hat{F}_t = \frac{1}{M}\left(\hat{\Lambda}'\tilde{X}_t\right)$. Here,$\Lambda$ is the matrix of factor loadings, $\tilde{X}_t$ is given by $(x_{1t}, x_{2t}, ..., x_{Mt})$ where $x_{it} = X_{it} - \hat{c}_i - \hat{b}_i\tau$ Note that Barigozzi et al. (2021) describe a procedure to check if the ith series contains a linear trend and that $\hat{b}_i$ is different from zero.

Given the estimated factors, the VAR in equations B.2 is estimated using a Bayesian methods.

B.1 Priors

Denote the var coefficients as $B = vec\left([\beta_1, \beta_2, .., \beta_P, c]\right)$. We follow Banbura, Giannone and Reichlin (2007) and use a Natural Conjugate prior implemented via dummy observations. The priors are implemented by the dummy observations y_D and x_D that are defined as:

$$y_D = \begin{bmatrix} \frac{diag(\gamma_1 s_1 ... \gamma_n s_n)}{\kappa} \\ 0_{N\times(P-1)\times N} \\ diag\left(s_1 ... s_n\right) \\ .... \\ 0_{EX\times N} \end{bmatrix}, \quad x_D = \begin{bmatrix} \frac{J_P \otimes diag(s_1 ... s_n)}{\kappa} & 0_{NP\times EX} \\ \multicolumn{2}{c}{..............} \\ \multicolumn{2}{c}{0_{N\times(NP)+EX}} \\ \multicolumn{2}{c}{..............} \\ 0_{EX\times NP} & I_{EX}\times 1/c \end{bmatrix} \tag{B.4}$$

where $J_P = diag(1, 2, ..., P)$, γ_1 to γ_n denote the prior mean for the parameters on the first lag obtained by estimating individual AR(1) regressions, s_1 to s_n is an estimate of the variance of the endogenous variables obtained individual AR(1) regressions, κ measures the tightness of the prior on the autoregressive VAR coefficients, and c is the tightness of the prior on the remaining regressors. We set $\kappa = 0.2$ and $c = 1000$. We also implement priors on the sum of coefficients (see Banbura et al. (2007)). The dummy observations for this prior are defined as:

$$\tilde{y}_D = \frac{diag\left(\gamma_1\mu_1 ... \gamma_n\mu_n\right)}{\tau}, \tilde{x}_D = \begin{pmatrix} \left(1_{1\times P}\right) \otimes \frac{diag(\gamma_1\mu_1 ... \gamma_n\mu_n)}{\tau} & 0_{N\times EX} \end{pmatrix} \tag{B.5}$$

where μ_i is the sample average of the ith variable. As in Banbura et al. (2007) we set $\tau = 10\kappa$. The total number of dummy observations is T_D.

B.2 MCMC algorithm

Banbura et al. (2007) show that posterior distribution can be written as:

$$g(\Sigma|Y) \sim iW\left(\bar{\Sigma}, T_D + 2 + T - K\right) \tag{B.6}$$

$$g(B|\Sigma, Y) \sim N\left(\bar{B}, \Sigma \otimes \left(X_*' X_*\right)^{-1}\right) \tag{B.7}$$

where iW denotes the inverse Wishart distribution, K denotes the number of regressors in each equation of the VAR model. Note that $Y_* = \begin{pmatrix} Y \\ y_D \\ \tilde{y}_D \end{pmatrix}$ and $X_* = \begin{pmatrix} X \\ x_D \\ \tilde{x}_D \end{pmatrix}$, X collects the regressors, and

$$\begin{aligned} \tilde{B} &= \left(X_*' X_*\right)^{-1}\left(X_*' Y_*\right) \\ \bar{B} &= vec\left(\tilde{B}\right) \\ \bar{\Sigma} &= \left(Y_* - X_* \tilde{B}\right)'\left(Y_* - X_* \tilde{B}\right) \end{aligned}$$

Posterior draws can be easily generated by drawing Σ from the marginal distribution in B.6 and then b from the conditional distribution in equation B.7. We set the number of draws to 10,000 with a burn-in of 1,000.

C IV Identification

For a given draw of B, Σ and u_t, we obtain the first column of $A_0$by using the procedure proposed by Mertens and Ravn (2013). We assume that the instrument is relevant and exogenous:

$$\begin{aligned} cov\left(m_t, \varepsilon_{1t}\right) &= \alpha \\ cov\left(m_t, \varepsilon_t^{-}\right) &= 0 \end{aligned}$$

where ε_{1t} denotes the structural shock of interest that is ordered first for convenience, while ε_t^{-}represent all remaining shocks and $\varepsilon_t = \begin{pmatrix} \varepsilon_{1t} & \varepsilon_t^{-} \end{pmatrix}$. Re-writing the relevance and exogeneity conditions in vector form:

$$E(m_t \varepsilon_t') = [\alpha \quad 0] \tag{C.8}$$

$$E(m_t \varepsilon_t' A_0') = [\alpha \quad 0] A_0' \tag{C.9}$$

$$E(m_t u_t') = \alpha a_0 \tag{C.10}$$

where a_0 is a $(1 \times R)$ vector corresponding to the first row of A_0' (hence first column of A_0). An estimate of $E(m_t u_t') = \begin{pmatrix} E(m_t u_{1t}') \\ E(m_t u_{2t}') \\ . \\ . \\ E(m_t u_{Nt}') \end{pmatrix}'$ can be easily obtained by using a linear regression. However, α on the RHS of equation C.10 is unknown. This parameter can be eliminated by normalising the left and the right hand side by dividing by the first element of $E(m_t u_t')$ and a_0, respectively. Therefore the impulse vector to a unit shock is given by

$$\tilde{a}_0 = \begin{pmatrix} 1 \\ \frac{E(m_t u'_{2t})}{E(m_t u'_{1t})} \\ . \\ . \\ \frac{E(m_t u'_{Nt})}{E(m_t u'_{1t})} \end{pmatrix}' .$$

D FAVAR IRF

D.1 IRF of selected US variables from the FAVAR

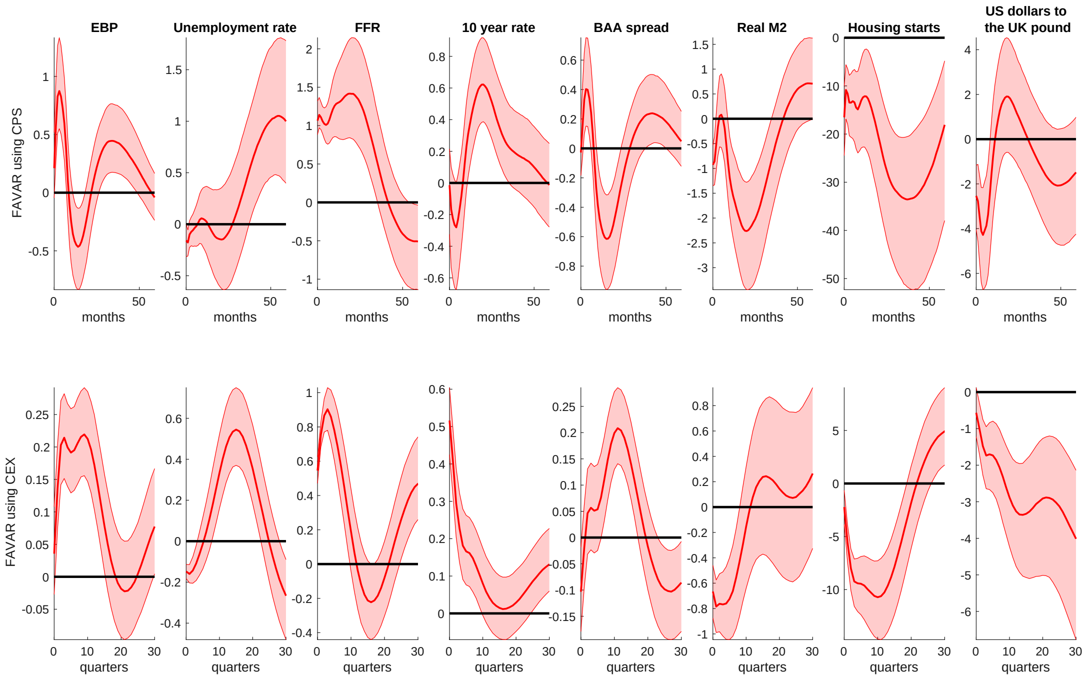

Figure D.1: Impulse responses of selected macroeconomic variables for the US. For each variable, we report the median IRF and 68% bands.

D.2 Response of hours and earnings from the FAVAR that uses CEX data

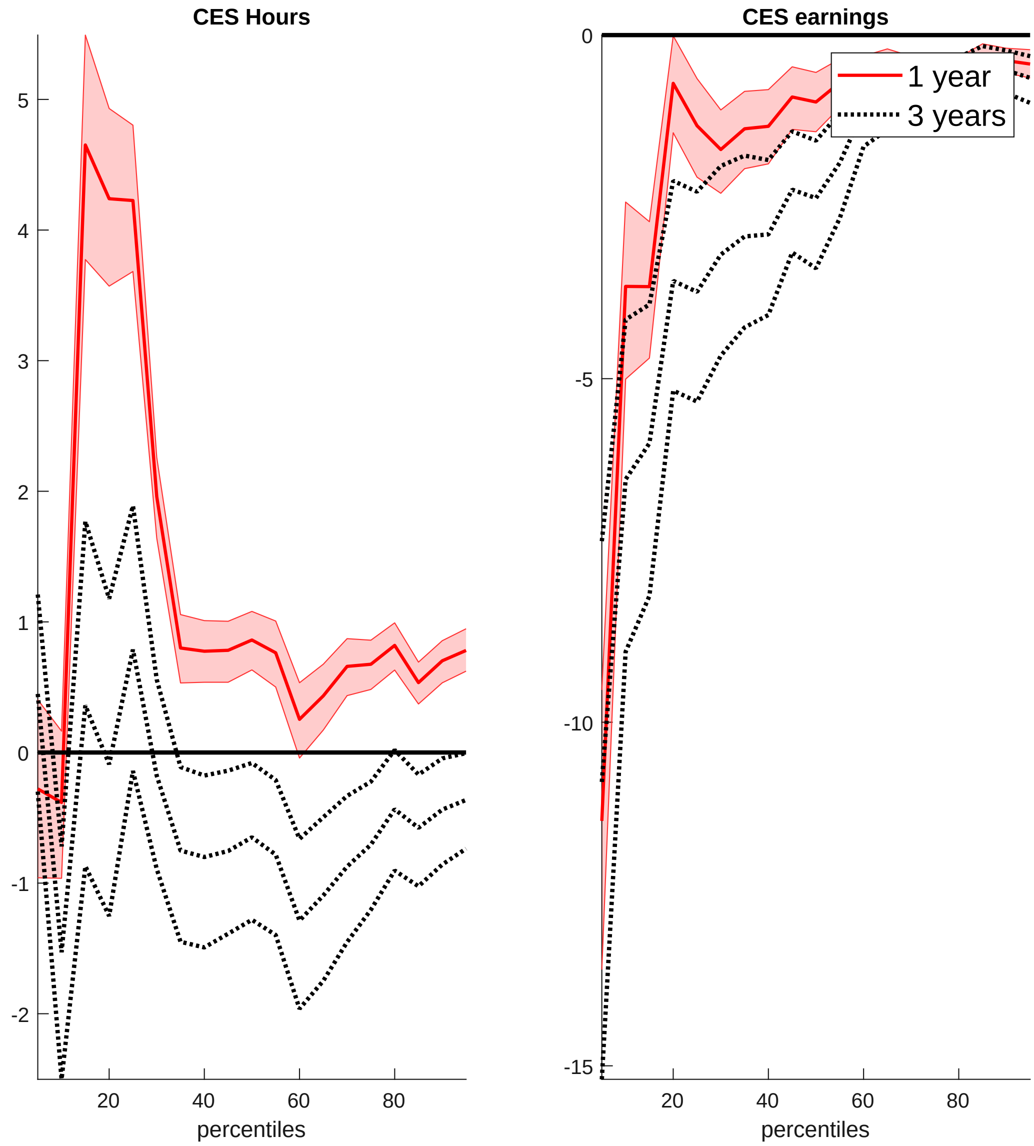

Figure D.2: Impulse responses of selected macroeconomic variables for the US. For each variable, we report the median IRF and 68% bands.

D.3 IRF of hours worked at different percentile groups

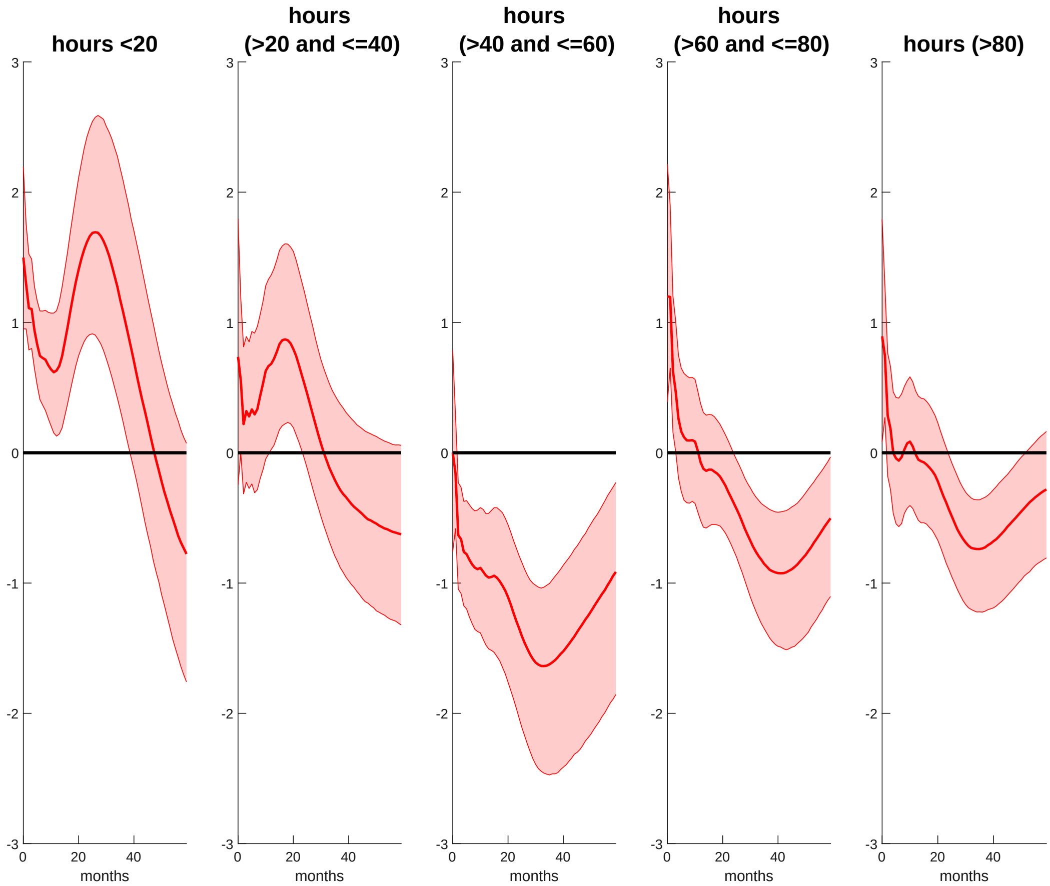

Figure D.3: Impulse responses of hours worked in each percentile group for the US. The data for Hours are coming from the CPS. For each variable, we report the median IRF and 68% bands.

D.4 3D responses

D.4.1 US CPS

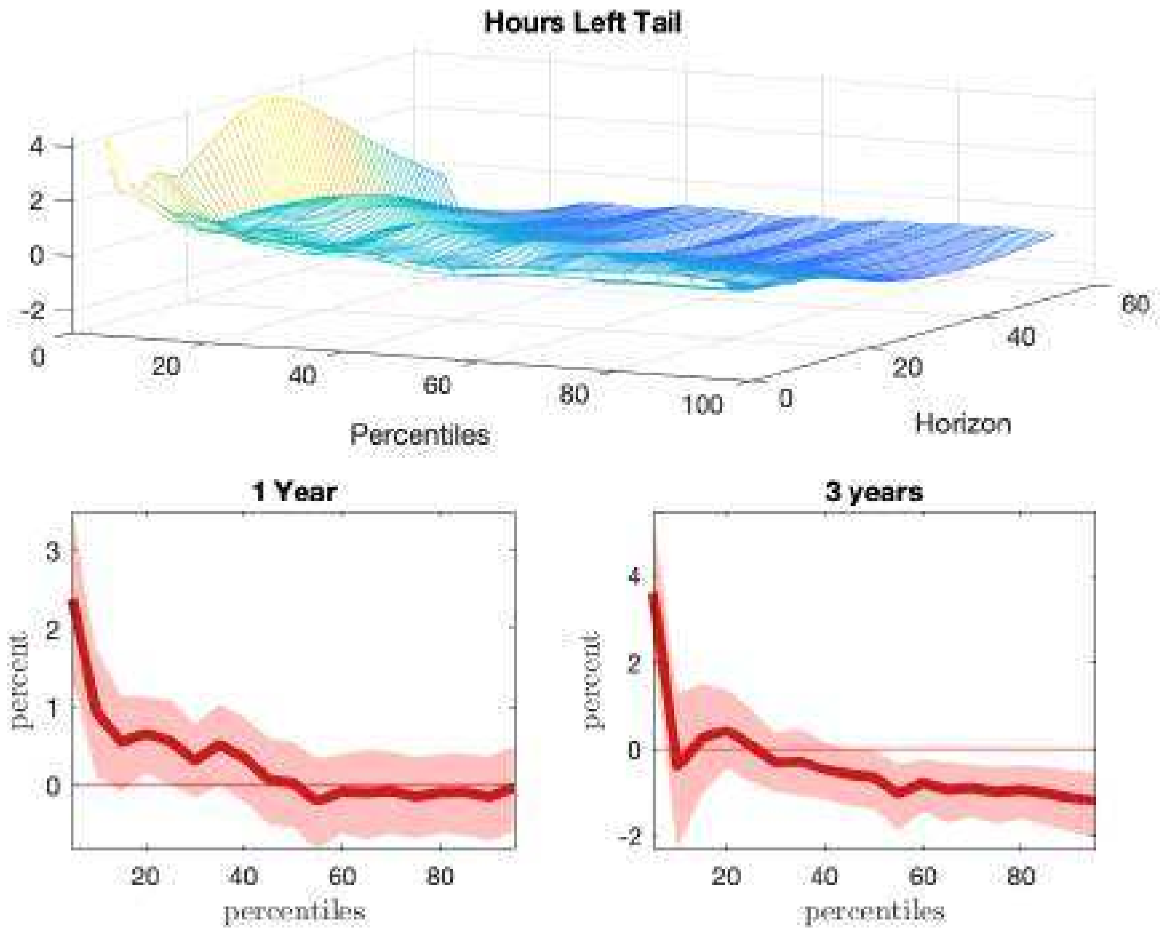

Figure D.4: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CPS). The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

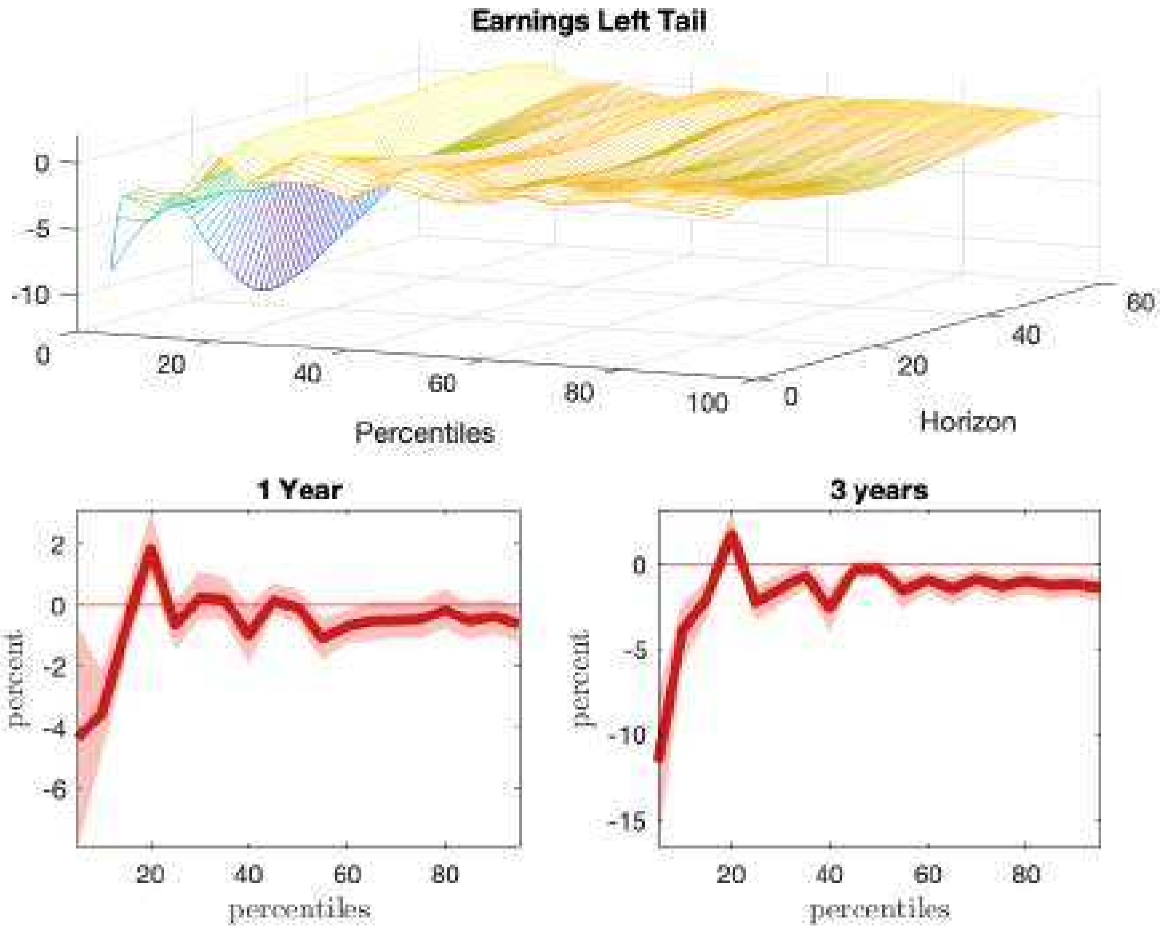

Figure D.5: 3D Impulse response of hourly earnings of the left tail of the income distribution in the US (CPS). The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

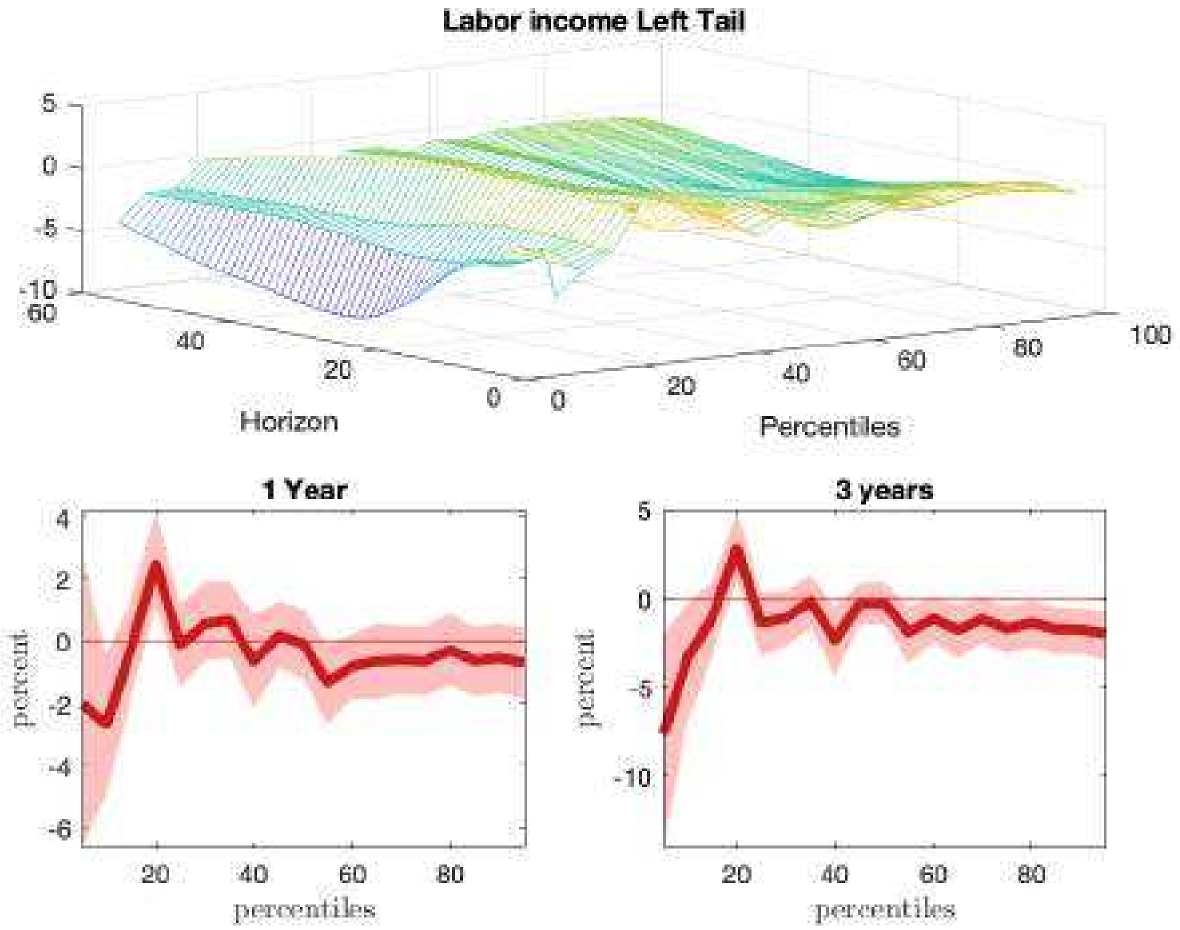

Figure D.6: 3D Impulse response of labor income of the left tail of the income distribution in the US (CPS). The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

D.4.2 US CEX

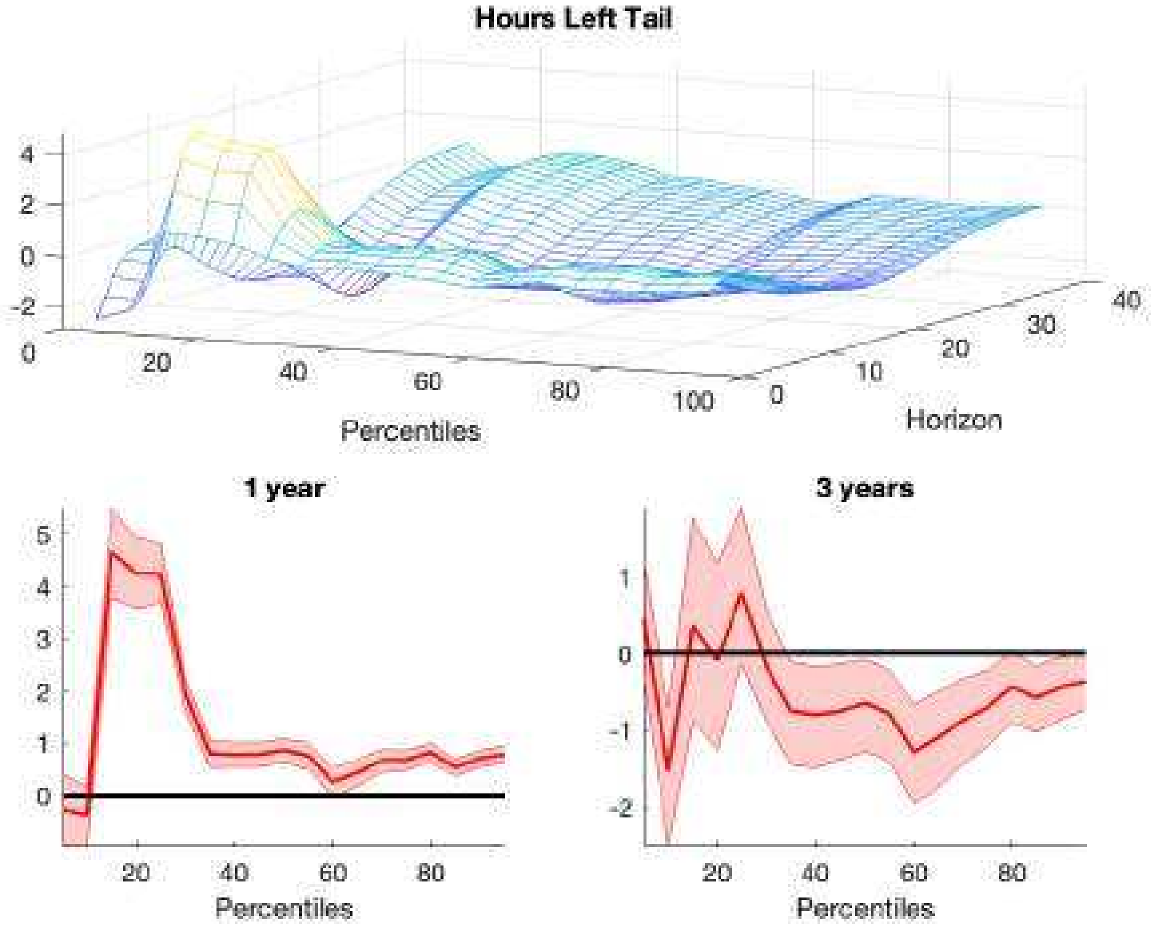

Figure D.7: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CEX). The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

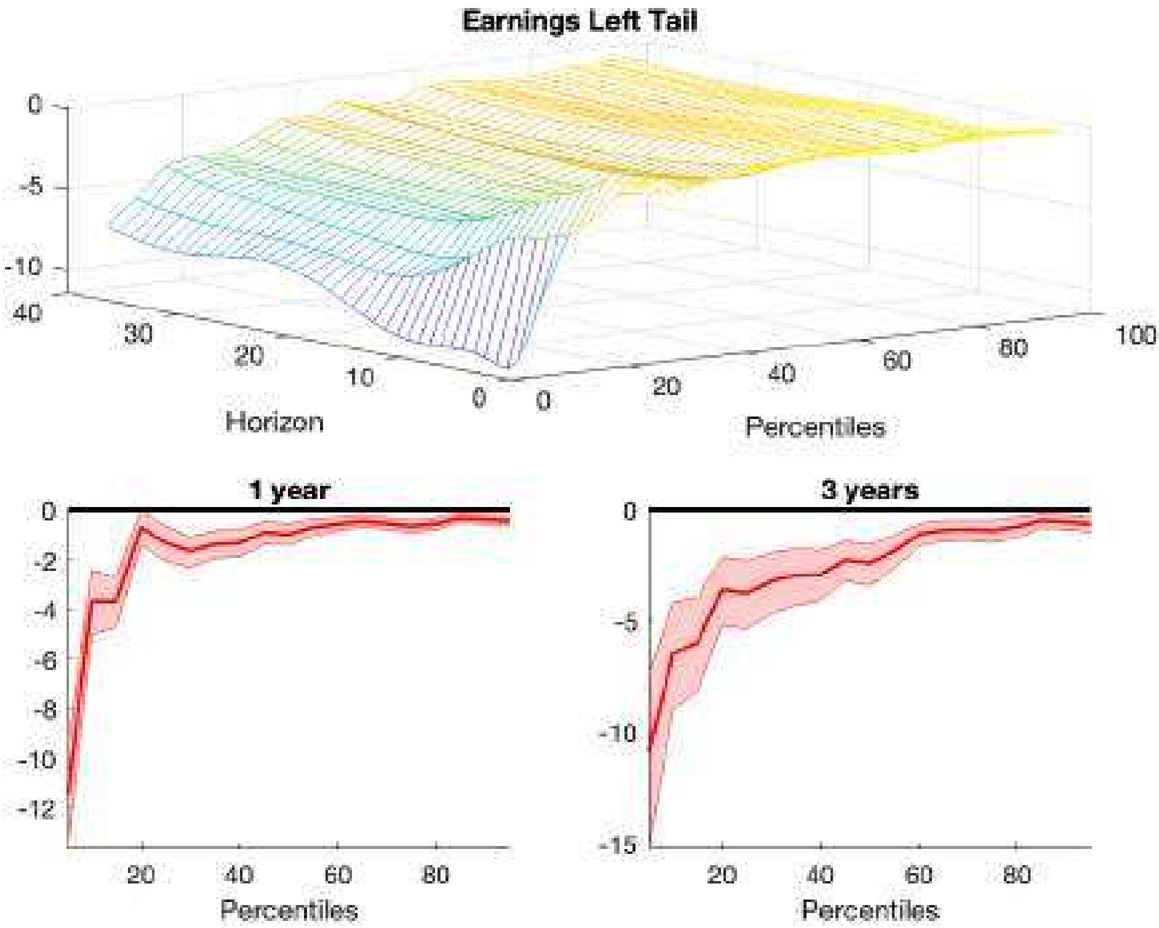

Figure D.8: 3D Impulse response of hourly wages of the left tail of the income distribution in the US (CEX). These are constructed by subtracting the IRFs of hours from the ones of labor income. The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

E Robustness

E.1 Identification

We employ the instrument of Miranda-Agrippino and Ricco (2021). Miranda-Agrippino and Ricco (2021) argue that high frequency instruments such as those used in Gertler and Karadi (2015) contain information about the policy shock and a signal regarding central bank's information about the state of the economy. Miranda-Agrippino and Ricco (2021) construct their proxy as the high frequency changes in federal funds futures that are orthogonal to Greenbook forecasts and data revisions.

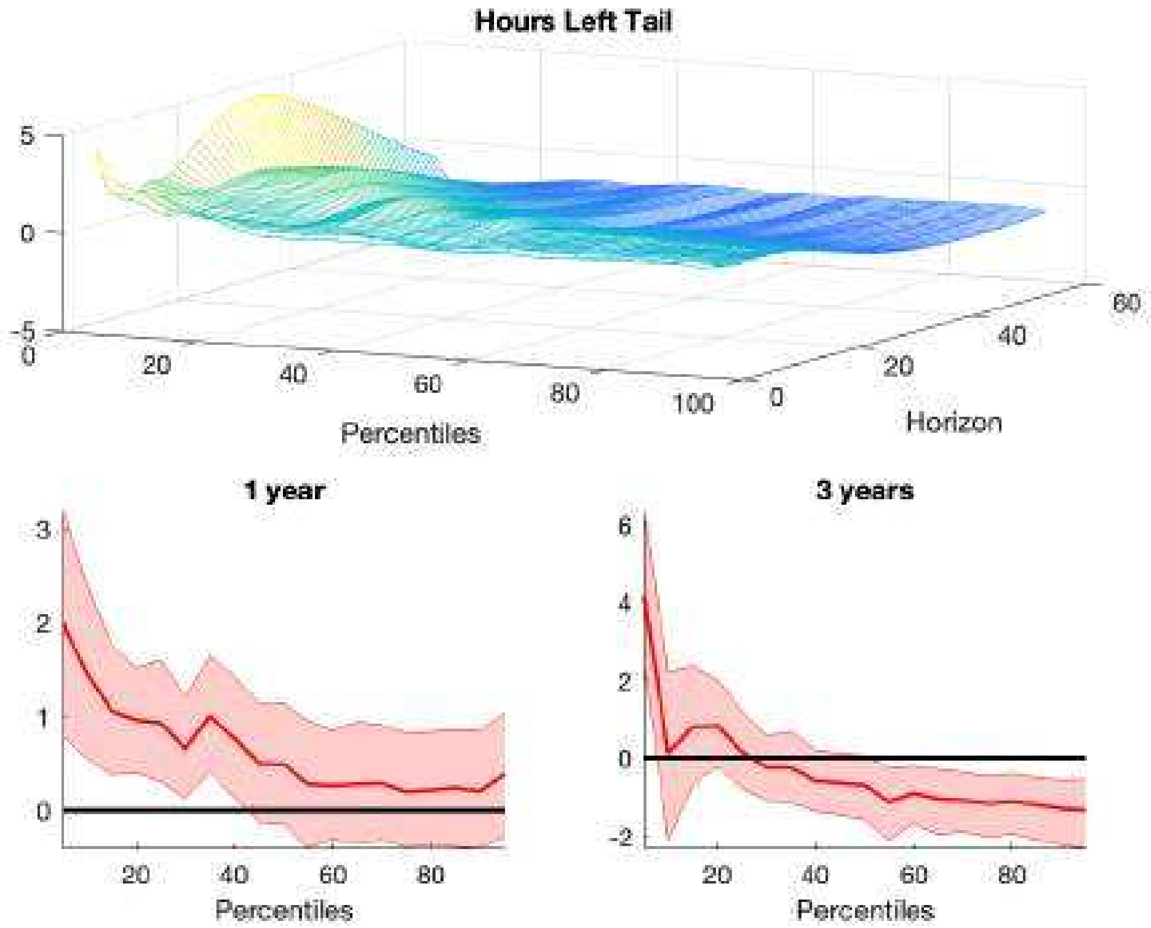

Figure E.1: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CPS). Identification using the instrument of Miranda-Agrippino and Ricco (2021). The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

Figure E.1 shows the response of the distribution of hours to a contractionary policy shock obtained from this model. As in the benchmark case, hours increase towards the left tail of the earnings distribution.

As an alternative identification scheme, we employ sign restrictions. We assume that a contractionary monetary policy shock increases the one year rate and the excess bond

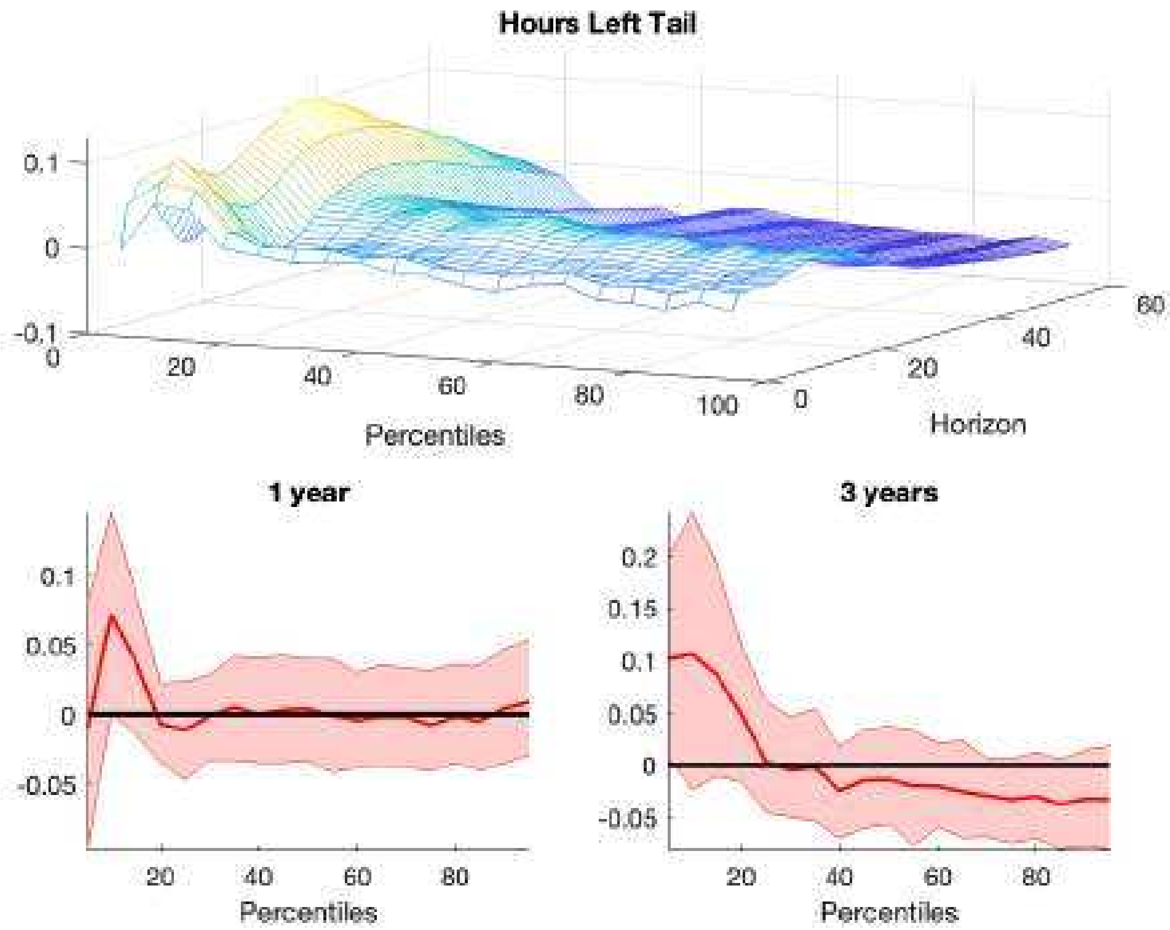

Figure E.2: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CPS). Identification using sign restrictions. The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

premium and leads to a decrease in CPI and industrial production. The restrictions are imposed on the first six periods after the shock. Figure E.2 shows the response of hours to one standard deviation shock. While the response are less precisely estimated, the point to the conclusions obtained from the benchmark model. A contractionary monetary policy shock leads to an increase in hours at the left tail of the distribution.

Figure E.3 presents the response of CEX-based hours using these two identification results. The response of hours is similar to the benchmark case.

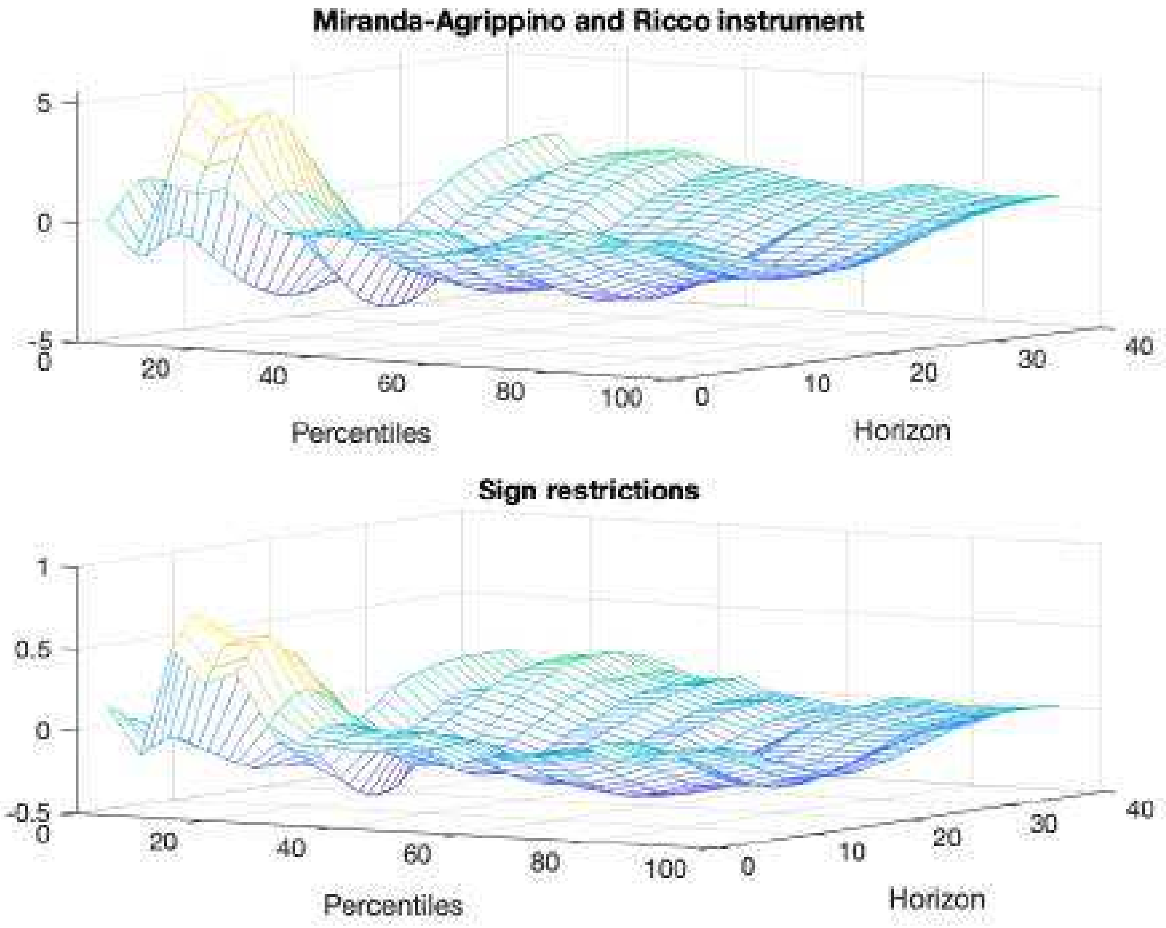

Figure E.3: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CEX). Identification using the Miranda-Agrippino and Ricco (2021) instrument and sign restrictions. Both panels report only the median IRFs.

E.2 Hours from all jobs from CPS

The benchmark result are based on CPS data on hours worked in the main job of the respondents. In this section, we use hours of respondents worked in all jobs. Figure E.4 presents the impulse response of this measure of hours to a contractionary policy shock. As in the benchmark case, hours rise at the left tail of the earnings distribution.

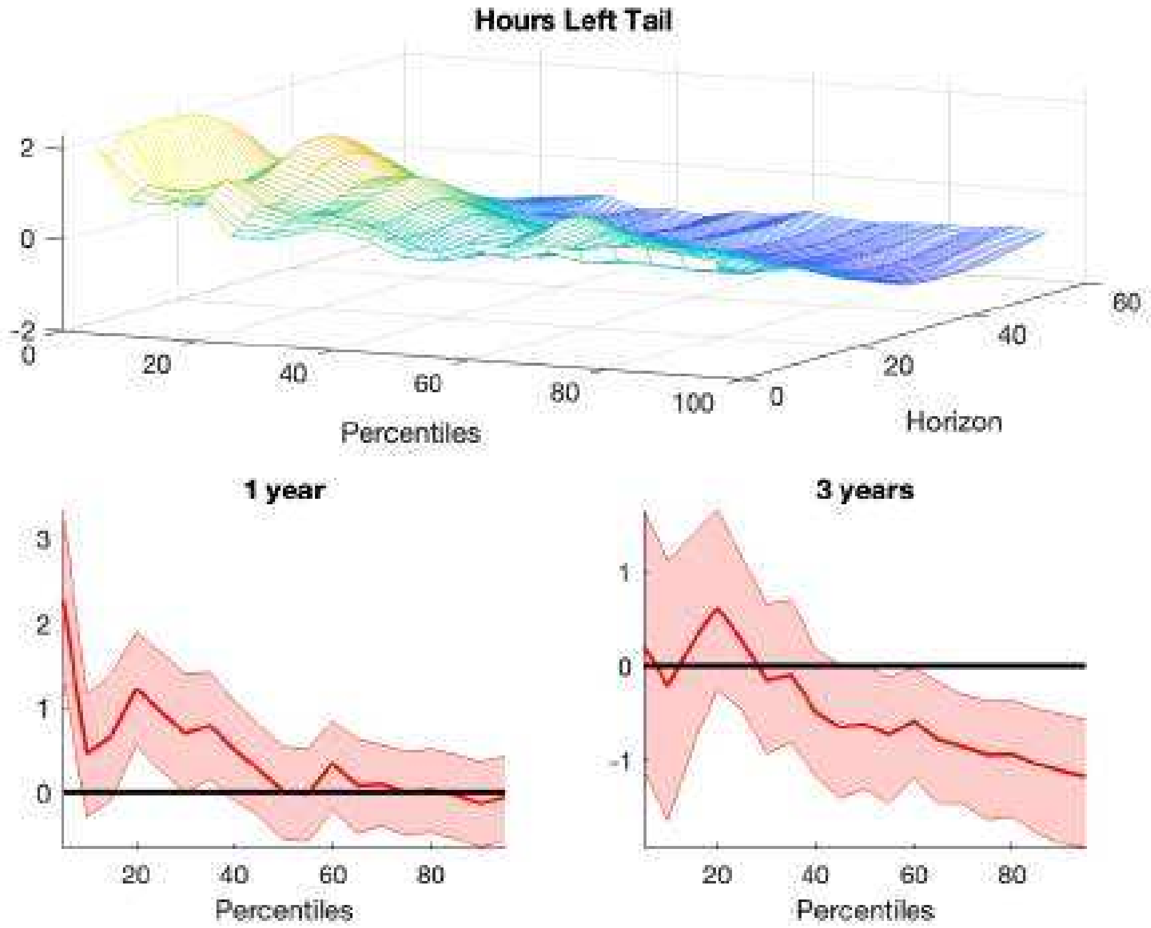

Figure E.4: 3D Impulse response of hours worked of the left tail of the income distribution in the US (CPS). Using hours worked in all jobs. The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

E.3 Hours by industry

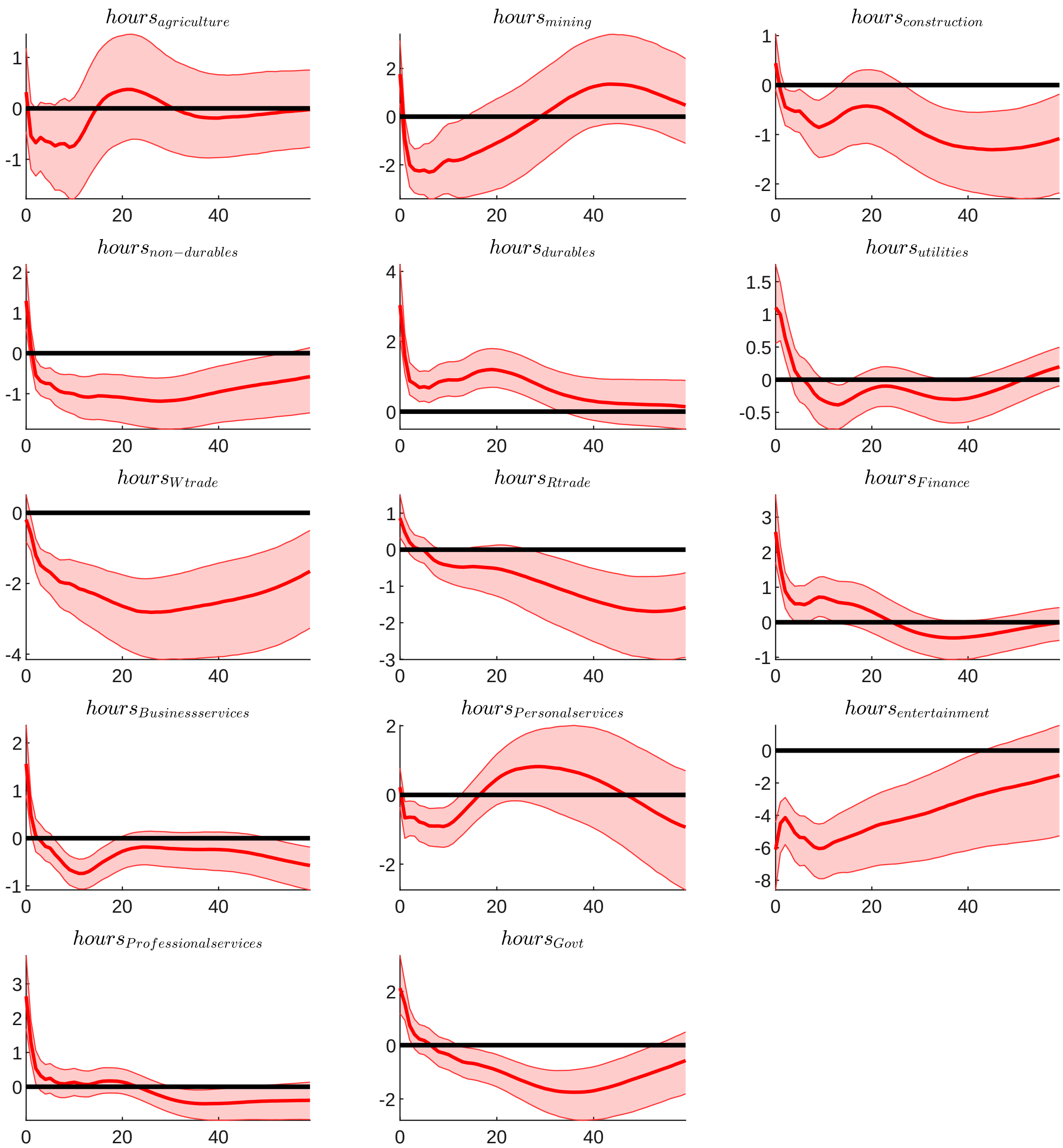

Figure E.5: Impulse responses of hours by industry of employment in the US. For each variable, we report the median IRF and 68% bands.

E.4 Housing Tenure

We consider the response of CEX based hours by housing tenure. Note that the CPS does not contain consistent information on housing status over our sample of interest. Households are grouped according to tenure status. Renters are generally households with lower income, while mortgagors earn typically higher income. Proportion of outright owners is fairly constant across income groups.

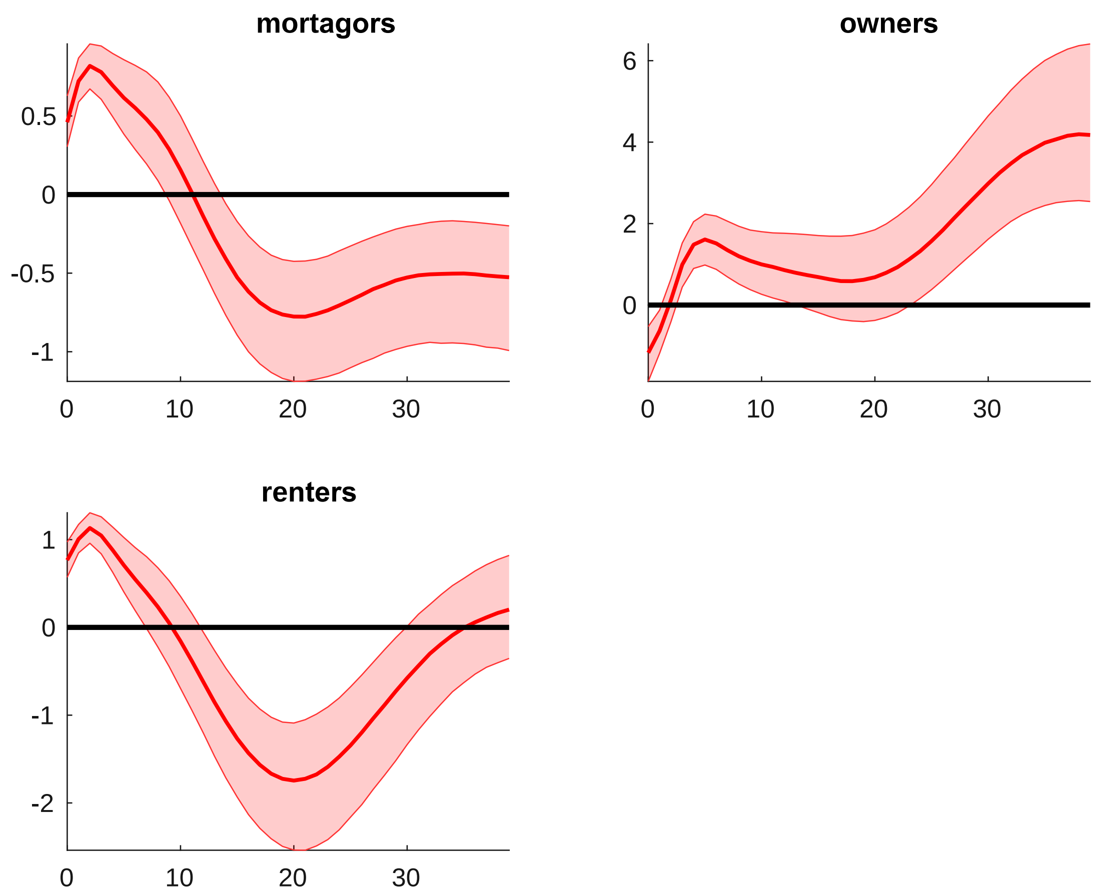

Figure E.6: Impulse responses of hours by housing tenure in the US. For each variable, we report the median IRF and 68% bands.

E.5 Sample restricted to Employed

Observations restricted to include only the ones where the reference person reports being in one of the occupations asked by the survey.

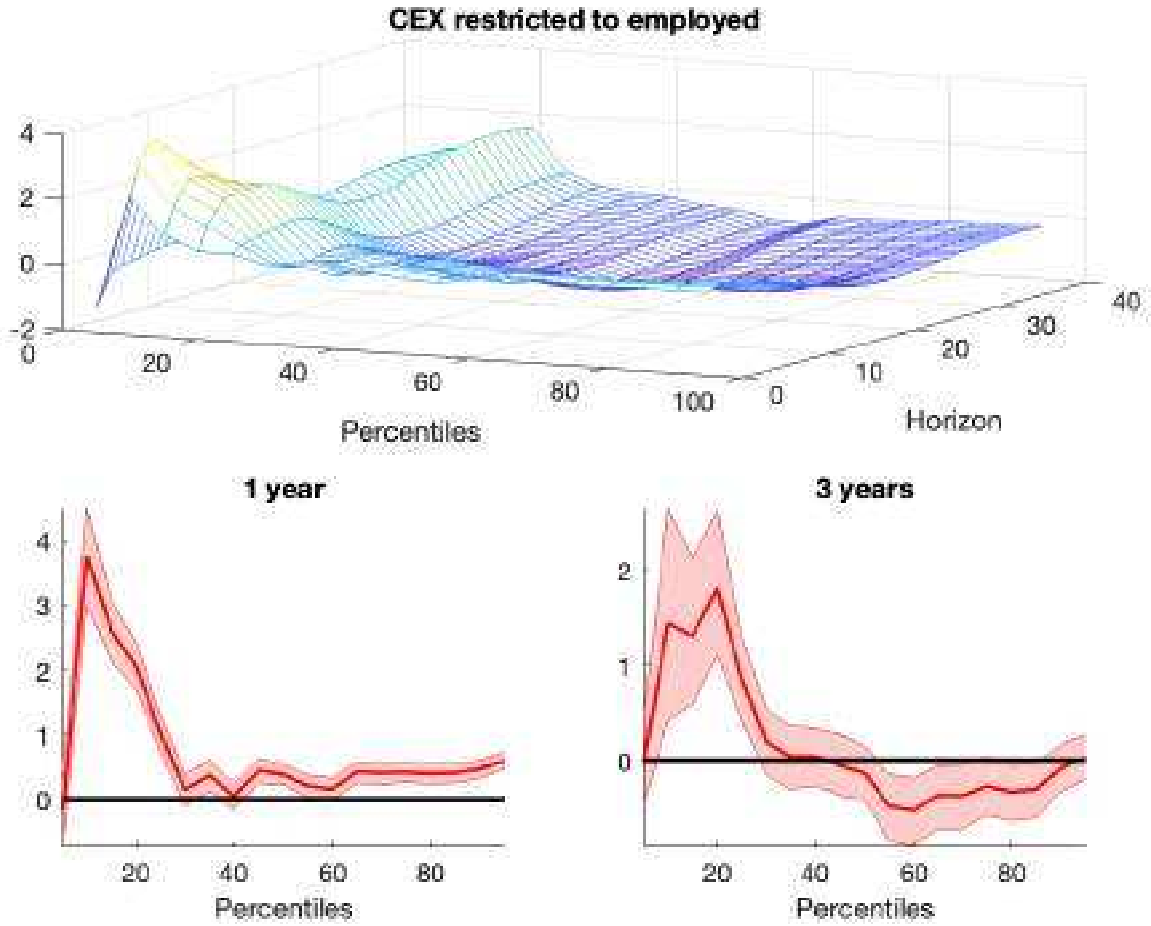

Figure E.7: 3D Impulse response of hours worked of the left tail of the income distribution in the US(CEX). Sample restricted to employed. The top panel reports only the median IRFs while the bottom two report the median IRFs and 68% bands.

E.6 Response of unemployment rates of full-time and part-time workers

We use CPS data to construct unemployment rates for full-time and part-time workers. First we use the CPS variable WKSTAT to select individuals that are either working full-time or part-time hours. We also include unemployed workers who indicate that they are looking for full-time or part-time jobs (i.e. WKSTAT equals either 50 or 60). As explained in the main text, we use mincer type regressions to impute earnings for unemployed individuals in each category and compute the corresponding unemployment rates in each earnings group.

Figure E.8 shows the response of these unemployment rates to a monetary contraction. Unemployment increases for both part-time and full-time workers, especially at the left tail

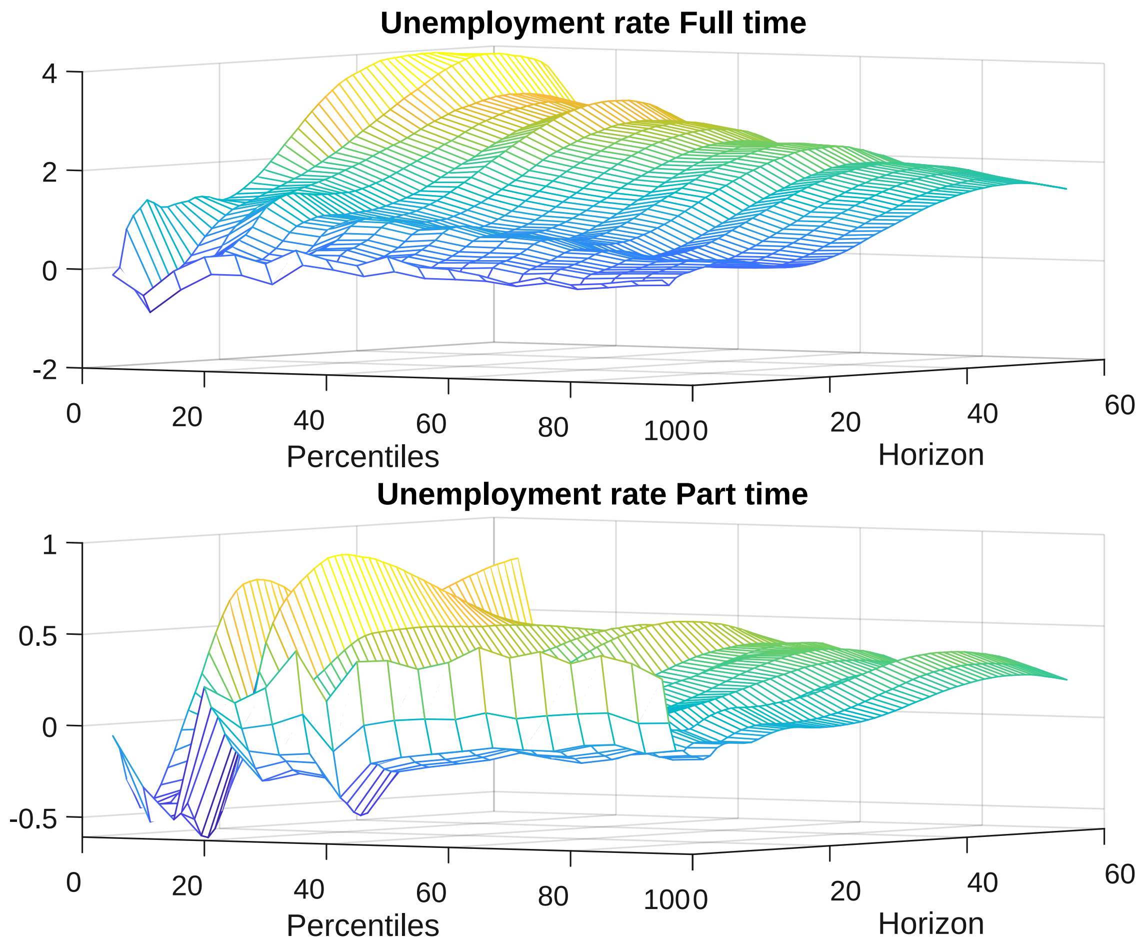

Figure E.8: 3D Impulse response of unemployment rates of full-time and part-time workers across earnings percentiles

of the earnings distribution. However, this increase is much larger for full-time workers - unemployment rates increase about 4 percent at long horizons for full-time workers while the rise for part-time workers is less than one percent.

E.7 Response of hours for full-time workers

We restrict the sample to full-time workers in the CPS using two methods. First we use the variable WKSTAT and select individuals that report working full-time hours (we retain individuals for which this variable equals 11, 13, 14 or 15). Second, we drop individuals who work less than 35 hours per-week. As shown in figure E.9, hours increase at the left tail of the earnings distribution after a monetary contraction. This increase is largest at the 2 to 3 year horizon.

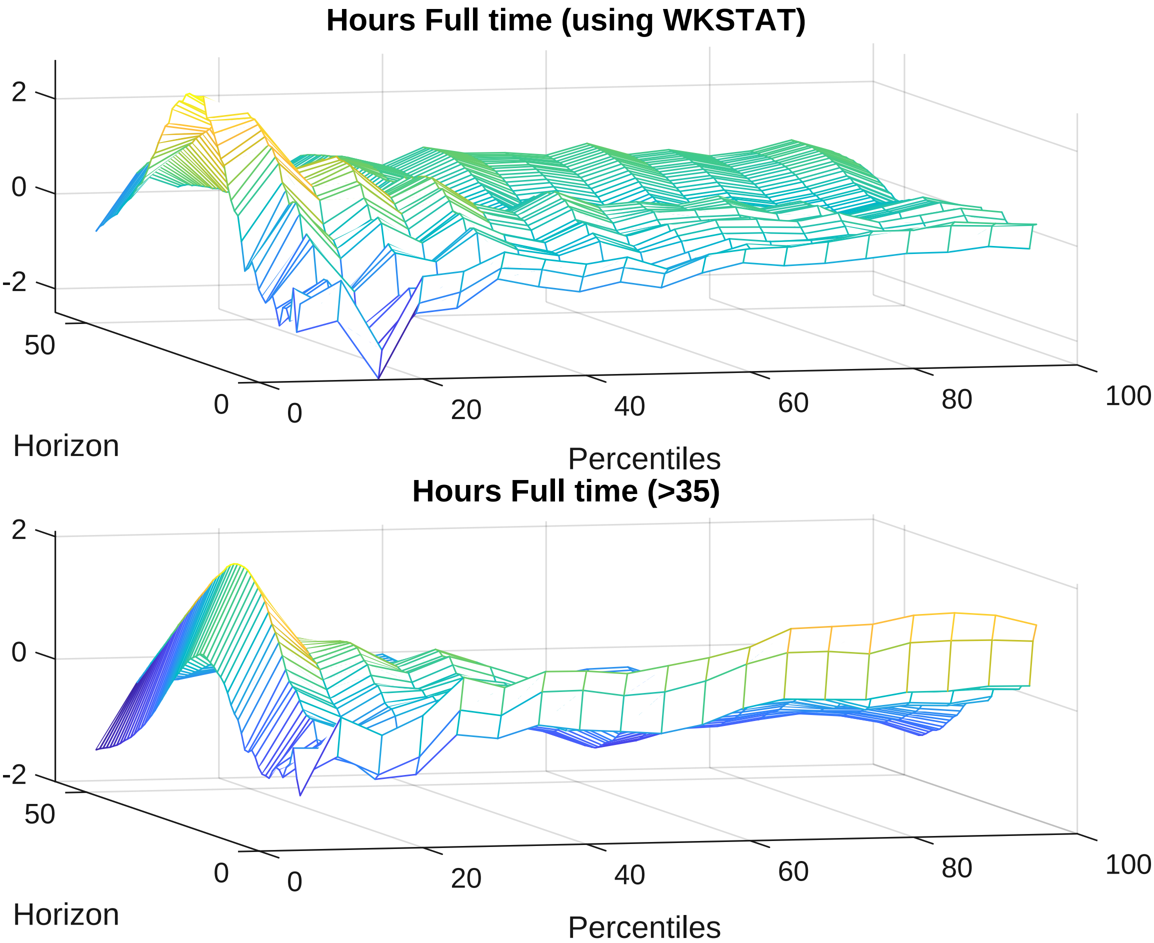

Figure E.9: 3D Impulse response of hours of full-time workers across earnings percentiles

E.8 Additional results for the UK

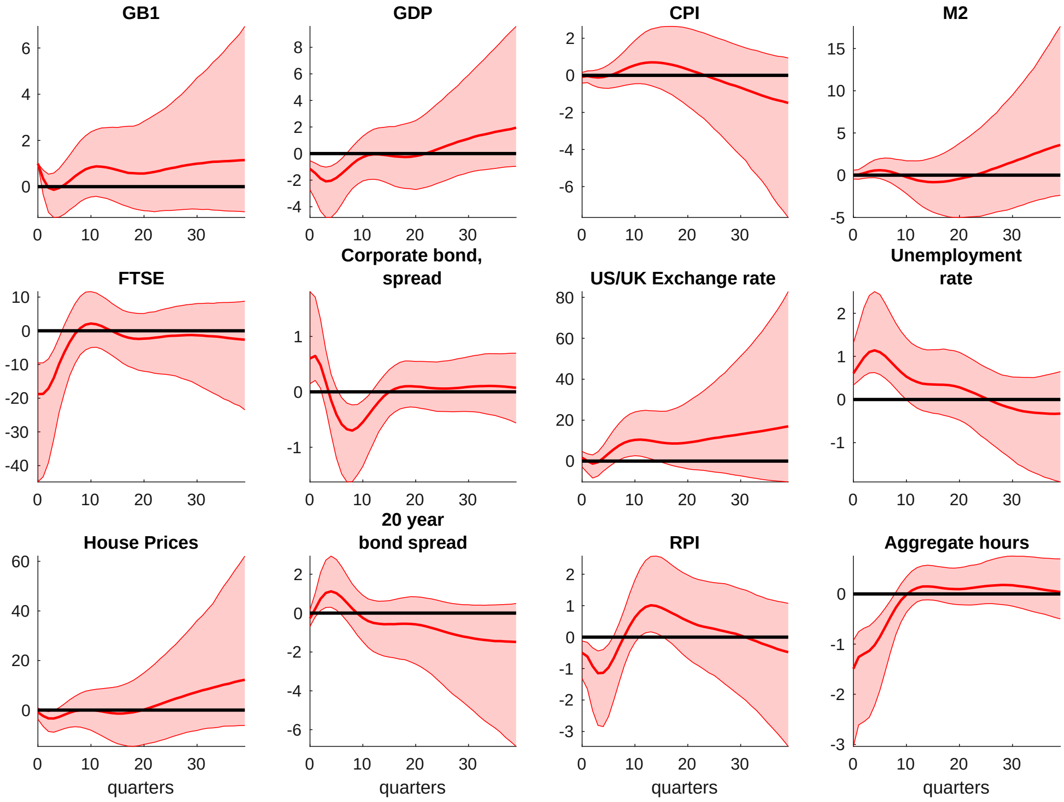

Figure E.10: Response of selected variables to a monetary policy shock in the UK. For each variable, we report the median IRF and 68% bands.

F Supply shocks

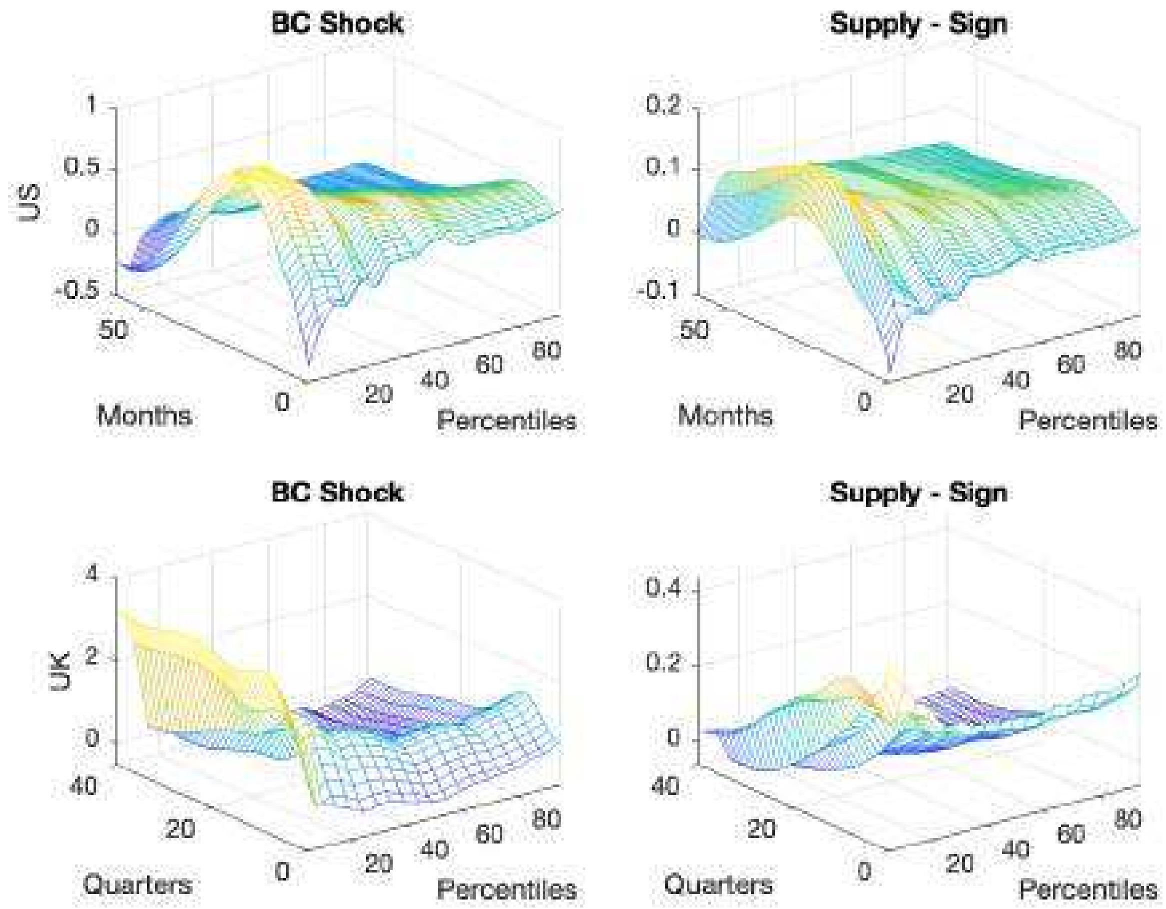

Figure F.11: Response of Hours worked at different percentiles of the income distribution in the US (CPS) and the UK to supply shocks. We estimated supply shocks following two identification methods: a) defining the shock as the business cycle one in Angeletos et al. (2020) (BC shock); b) using sign restrictions. We report the median IRFs.

G Simple TANK with *poor* Hand to Mouth

Throughout, time is discrete and denoted t = 0, 1, 2,... . Real quantities are in terms of the consumption good and - unless otherwise stated - denoted by lower case letters while nominal variables are denoted by capital letters. Steady-state variables are without time subscript and log-linear variables in deviation from their steady state will be denoted by a $\hat{}$.

This model follows the TANK model in Bilbiie (2008, 2020) allowing for heterogeneity in the intertemporal elasticities (IES) of substitution across agents. As in Bilbiie (2020), there is a production subsidy that induces marginal cost pricing which implies that the steady state of marginal costs is 1 and simplifies substantially the steady state and the log-linearized conditions. Differently from Bilbiie (2008, 2020), and for simplicity, we abstract from fiscal redistribution of profit income.

The economy consists of three sectors: households, firms, and a central bank. The household sector is populated by two different types: **savers** S and **hand-to-mouth** H. The firm sector is the standard one in New Keynesian models. Nominal rigidities are introduced by assuming that intermediate goods producers face quadratic adjustment costs a la Rotemberg (1982) in adjusting their prices. The central bank follows a Taylor type rule to choose the real interest rate. This assumption is made to obtain simpler analytical expressions.

G.1 Model

There is a continuum of households $[0, 1]$. There are two types of households: A share λ of households are HtM (indexed by H) who work and consume all of their labor income, having no access to bonds nor to firm ownership and dividends. The remaining $1 - \lambda$ are savers (indexed by S) who work, hold bonds and shares in monopolistic firms and get firm profits. Savers are standard PIH households.

Savers Savers maximize their lifetime utility subject to their budget constraint, taking prices and wages as given:

$$\max_{c_t^S, b_t^S, H_t^S} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^t \left(\frac{(c_{t+s}^S)^{1-\frac{1}{\sigma_S}}}{1-\frac{1}{\sigma_S}} - \nu^S \frac{(H_{t+s}^S)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^S + b_t^S = \frac{d_t}{1-\lambda} + H_t^S w_t + \frac{R_{t-1}}{\Pi_t} b_{t-1}^S,$$

where c_t^S is consumption, b_t^S bonds, H_t^S hours, Π_t is inflation, w_t are real wages, R_t is the gross nominal interest rate on assets and d_t are firm profits. σ_S is the inter-temporal elasticity of substitution, $\frac{1}{\varphi}$ is the Frish elasticity of labor supply and ν^S indicates how leisure is valued relative to consumption.

The solution to the savers problem yields the standard Euler and labor supply equations:

$$1 = \beta E_t \left[\left(\frac{{c^S}_{t+1}}{{c^S}_t} \right)^{-\frac{1}{\sigma_S}} \frac{R_t}{\Pi_{t+1}} \right] \tag{G.11}$$

$$w_t = \nu^S \left(H_t^S\right)^{\varphi} (c_t^S)^{\frac{1}{\sigma}}. \tag{G.12}$$

HtM HtM households have no access to bonds nor firms' shares and therefore rely solely on labor earnings. Using the same CRRA form of utility as for the savers, and allowing for IES heterogeneity, their static problem reads:

$$\max_{c_t^H, H_t^H} \left(\frac{(c_t^H)^{1-\frac{1}{\sigma_H}}}{1-\frac{1}{\sigma_H}} - \nu^H \frac{(H_t^H)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^H = H_t^H w_t$$

where parameters and variables with H superscript have the same interpretations as the ones for savers.

Since preferences are monotonic, HtM will just consume all of their labor income:

$$c_t^H = H_t^H w_t \tag{G.13}$$

$$w_t = \nu^H \left(H_t^H\right)^{\varphi} (c_t^H)^{\frac{1}{\sigma_H}}. \tag{G.14}$$

Firms The firm sector is split into two. A representative competitive final goods firm aggregates intermediate goods according to a CES technology and a continuum of intermediate goods producers that produce different varieties using labor as an input. To the extent to which the intermediate goods are imperfect substitutes, with ϵ denoting the elasticity of substitution, there is a downward-sloping demand for each intermediate variety, giving the intermediate producers some pricing power. However, importantly, intermediate goods producers are also subject to some costs (ϕ^p) in adjusting prices (Rotemberg (1982)). This generates sticky prices. The production function is CRS and linear in labor. This supply-side setup is the standard one in NK models and follows directly from Bilbiie (2020).[59] We assume that the government implements the standard NK optimal subsidy inducing marginal cost pricing: $\tau^S = (\epsilon - 1)^{-1}$. The optimal firm behavior after having imposed the symmetric equilibrium is characterized by

$$y_t = H_t$$

$$w_t = mc_t \frac{y_t}{H_t}$$

$$\left(1+\tau^S\right)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p (\Pi_t - 1) + \beta^S E_t \left[\left(\frac{c_{t+1}^S}{c_t^S} \right)^{-\frac{1}{\sigma}} \Pi_{t+1} \phi_p (\Pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \right] = 0.$$

[59] And we refer to it for details.

Central Bank To allow for an analytical solution we assume that the central bank responds one-to-one to inflation expectations as in Bilbiie (2008) and McKay et al. (2016):

$$\frac{R_t}{R} = \left(\frac{\Pi_{t+1}}{\Pi}\right) e^{\epsilon_t^m}.$$

Aggregation and market clearing Aggregate bonds are in 0 net supply and aggregate consumption and hours are:

$$0 = (1-\lambda)b_t^S \tag{G.15}$$

$$c_t = \lambda c_t^H + (1-\lambda)c_t^S \tag{G.16}$$

$$H_t = \lambda H_t^H + (1-\lambda)H_t^S. \tag{G.17}$$

Using firm's profits definition and the two agent's budget constraints we obtain the resource constraint:

$$c_t = y_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 y_t.$$

	Equilibrium Conditions	
1:	Labor Supply S	$w_t = \nu^S \left(H_t^S\right)^{\varphi} (c_t^S)^{\frac{1}{\sigma_S}}$
2:	Labor Supply H	$w_t = \nu^H \left(H_t^H\right)^{\varphi} (c_t^S)^{\frac{1}{\sigma_H}}$
3:	Euler S	$1 = \beta E_t \left[\left(\frac{c^S{}_{t+1}}{c^S{}_t}\right)^{-\frac{1}{\sigma_S}} \frac{R_t}{\Pi_{t+1}}\right]$
4:	Budget constraint H	$c_t^H = H_t^H w_t$
5:	Marginal prod. of labor	$w_t = mc_t \frac{y_t}{H_t}$
6:	Phillips Curve	$\left(1+\tau^S\right)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p \left(\Pi_t - 1\right) + \beta E_t \left[\left(\frac{c_{t+1}^S}{c_t^S}\right)^{-\sigma_S} \Pi_{t+1}\phi_p \left(\Pi_{t+1} - 1\right) \frac{y_{t+1}}{y_t}\right] = 0$
7:	Production Function	$y_t = H_t$
8:	Profits	$d_t = \left(1 - mc_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2\right) y_t$
9:	Aggregate C	$c_t = \lambda c_t^H + (1-\lambda)c_t^S$
10:	Aggregate H	$H_t = \lambda H_t^H + (1-\lambda)H_t^S$
11:	Resource constraint	$c_t = y_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 y_t$
12:	Taylor Rule	$\frac{R_t}{R} = \left(\frac{\Pi_{t+1}}{\Pi}\right) e^{\epsilon_t^m}$

Table G.1: Non linear TANK.

Note: S budget constraint is not included because it is implied by Walras law.

G.2 Steady State

We assume that inflation is zero in steady state, $\Pi = 1$. From the Euler equation of savers we have $R = \frac{1}{\beta}$. From the optimal pricing equation, we have $mc = \frac{\left(1+\tau^S\right)(\epsilon-1)}{\epsilon} = \mathcal{M}^{-1}$. From the production function, and the resource constraint, we have that $c = y = H$. Imposing the optimal subsidy $\tau^s = \frac{1}{\epsilon-1}$ we have $\mathcal{M}^{-1} = mc = w = 1$ which implies, combining both agents budget constraints, $c = c^S = c^H$. Finally we can normalize total hours $H = 1$ which implies also $y = c = 1$.

G.3 Log-linear Model

We log-linearize the model around the steady state. We assume that inflation is zero in steady state. Variables with a ˆ denote log-deviations from steady state. We log-linearize all variables ($\hat{x}_t = \frac{x_t - x}{x}$) except total profits, which we linearize and denote as a share of total output, i.e. $\tilde{d}_t = \frac{d_t - d}{y}$.

The model in Table G.1 can be simplified further removing output and total hours ($y_t = c_t = H_t$) and marginal costs and profits ($w_t = mc_t = -d_t$).

The log-linearization of the equations is straightforward and standard. We denote the slope of the Phillips curve is by $\kappa = \frac{\epsilon}{\phi^p}$. The full list of the log-linear equilibrium conditions is in Table 1 of the main text.

G.4 Derivation of Proposition 1

Solving the Euler equation forward we have

$$\hat{c}_t^S = -\sigma_S \epsilon_t^m$$

Combining the HtM BC and labor supply decision we have

$$\hat{H}_t^H = \frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H$$

Taking the difference between the S and H labor supplies we have

$$\begin{aligned}
\varphi \hat{H}_t^S - \varphi \hat{H}_t^H &= -\frac{1}{\sigma_S} \hat{c}_t^S + \frac{1}{\sigma_H} \hat{c}_t^H \\
\varphi \hat{H}_t^S - \varphi \frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H &= \epsilon_t^m + \frac{1}{\sigma_H} \hat{c}_t^H \\
\varphi \hat{H}_t^S &= \epsilon_t^m + \frac{1}{\sigma_H} \hat{c}_t^H + \varphi \frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H \\
\hat{H}_t^S &= \frac{1}{\varphi} \epsilon_t^m + \frac{1 + \varphi \sigma_H}{\sigma_H(\varphi + 1)\varphi} \hat{c}_t^H
\end{aligned}$$

Combining aggregate C and H, we have

$$\lambda \hat{H}_t^H - \lambda \hat{c}_t^H = (1 - \lambda) \hat{c}_t^S - (1 - \lambda) \hat{H}_t^S$$

Combining the last equations we get

$$
\begin{aligned}
\lambda\frac{\sigma_H - 1}{\sigma_H(\varphi+1)}\hat{c}_t^H - \lambda\hat{c}_t^H &= -(1-\lambda)\sigma_S\epsilon_t^m - (1-\lambda)\hat{H}_t^S \\
\lambda\frac{\sigma_H - 1 - \sigma_H(\varphi+1)}{\sigma_H(\varphi+1)}\hat{c}_t^H &= -(1-\lambda)\sigma_S\epsilon_t^m - (1-\lambda)\hat{H}_t^S \\
\lambda\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m + (1-\lambda)\left[\frac{1}{\varphi}\epsilon_t^m + \frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H\right] \\
\lambda\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\hat{c}_t^H - (1-\lambda)\frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m + \frac{1-\lambda}{\varphi}\epsilon_t^m \\
\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\left[\lambda - \frac{1-\lambda}{\varphi}\right]\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m + \frac{1-\lambda}{\varphi}\epsilon_t^m \\
\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\left[\frac{\lambda\varphi - 1 + \lambda}{\varphi}\right]\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\varphi}\epsilon_t^m \\
\frac{\lambda\varphi - 1 + \lambda}{\sigma_H(\varphi+1)}\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\epsilon_t^m \\
\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\,\frac{\sigma_H(\varphi+1)}{\lambda(\varphi+1)-1}\epsilon_t^m
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\hat{H}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\,\frac{\sigma_H - 1}{\lambda(\varphi+1)-1}\epsilon_t^m \\
\hat{H}_t^S &= \frac{1}{\varphi}\epsilon_t^m + \frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H \\
&= \frac{1}{\varphi}\epsilon_t^m\left[1 + \frac{(1+\varphi\sigma_S)(1-\lambda)}{\lambda(\varphi+1)-1}\right] \\
&= \frac{\lambda + (1-\lambda)\sigma_S}{\lambda(\varphi+1)-1}\epsilon_t^m
\end{aligned}
$$

Aggregate hours become

$$
\begin{aligned}
\hat{H}_t &= \lambda\hat{H}_t^H + (1-\lambda)\hat{H}_t^S \\
&= \frac{\sigma_S(1-\lambda)}{\lambda(\varphi+1)-1}\,\frac{\lambda(1+\varphi)(\sigma_H/\sigma_S - 1) + (1+\varphi\sigma_H)}{\sigma_H\varphi+1}\epsilon_t^m \\
&= \frac{\sigma_S(1-\lambda)}{\lambda(\varphi+1)-1}\left[1 - \frac{\lambda(1+\varphi)(1-\sigma_H/\sigma_S)}{\sigma_H\varphi+1}\right]\epsilon_t^m
\end{aligned}
$$

The numerator in the first term is always positive; the denominator is negative when $\lambda < \frac{1}{1+\varphi}$. When $\lambda < \frac{1}{1+\varphi}$, the terms in square brackets is always positive since $\sigma_H > 0 > \frac{\lambda(\varphi+1)-1}{\sigma_S\varphi+(1+\varphi)\lambda}\ \sigma_S$. This makes the all term multiplying the monetary positive shock negative. Hence aggregate hours decline while hours worked by HtM household increase.

G.5 Derivation of Proposition 2

Substituting the Taylor rule in (7) and solving it forward we get:

$$\hat{c}_t = \sigma_S \frac{1-\lambda}{1-(1+\varphi)\lambda} \frac{\lambda\left(\frac{\sigma_H}{\sigma_S}-1\right)(\varphi+1)+\sigma_H\varphi+1}{\sigma_H\varphi+1}\epsilon_t^m. \tag{G.18}$$

$\frac{\partial \hat{c}_t}{\partial \lambda}$ is a second order equation in λ:

$$\frac{\partial \hat{c}_t}{\partial \lambda} = \frac{1}{\sigma_S\,(\sigma_H\,\varphi+1)\,(\lambda+\lambda\,\varphi-1)^2}(\sigma_H - \sigma_S - 2\,\lambda\,\sigma_H + 2\,\lambda\,\sigma_S + \sigma_H\,\varphi + \lambda^2\,\sigma_H - ...$$
$$-\lambda^2\,\sigma_S + 2\,\lambda^2\,\sigma_H\,\varphi - 2\,\lambda^2\,\sigma_S\,\varphi + \sigma_H\,\sigma_S\,\varphi^2 + \lambda^2\,\sigma_H\,\varphi^2 - \lambda^2\,\sigma_S\,\varphi^2 - 2\,\lambda\,\sigma_H\,\varphi + 2\,\lambda\,\sigma_S\,\varphi). \tag{G.19}$$

Solving $\frac{\partial \hat{c}_t}{\partial \lambda} = 0$ for λ we obtain:

$$\lambda^1 = \frac{\sigma_H - \sigma_S + \sqrt{-\sigma_H\,\varphi\,(\sigma_H-\sigma_S)\,(\sigma_S\,\varphi+1)}}{\sigma_H - \sigma_S + \sigma_H\,\varphi - \sigma_S\,\varphi}$$
$$\lambda^2 = -\frac{\sigma_S - \sigma_H + \sqrt{-\sigma_H\,\varphi\,(\sigma_H-\sigma_S)\,(\sigma_S\,\varphi+1)}}{\sigma_H - \sigma_S + \sigma_H\,\varphi - \sigma_S\,\varphi}$$

which can be rewritten as

$$\lambda^1 = \frac{1}{1+\varphi} - \frac{\sqrt{\sigma_H\,\varphi\,(\sigma_S-\sigma_H)\,(\sigma_S\,\varphi+1)}}{(\sigma_S-\sigma_H)(1+\varphi)}$$
$$\lambda^2 = \frac{1}{1+\varphi} + \frac{\sqrt{\sigma_H\,\varphi\,(\sigma_S-\sigma_H)\,(\sigma_S\,\varphi+1)}}{(\sigma_S-\sigma_H)(1+\varphi)}$$

Since λ_2 is outside the SADL region, $\lambda^\star = \lambda^1$ as in proposition 2. Finally, $\sigma^\star$ can be found by solving $\lambda^2 = 0$ for σ_H.

G.6 Labor supply income effect heterogeneity and the slope of the Phillips curve

Here we explore the implications of heterogeneity in the income effect of labor supply in the simple TANK model on the slope of the New Keynesian Phillips curve (NKPC).

Using the aggregate labor supply as in Galí (2015) it is easy to show that for the case $\sigma_H = \sigma_S$ the NKPC is given by:

$$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1|t} + \kappa \left(\frac{1}{\sigma_S} + \varphi \right) \hat{c}_t.$$

Substituting $\hat{c}_t = -\sigma_S \frac{1-\lambda}{1-(\varphi+1)\lambda}\ \epsilon_t^m$ we have

$$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1|t} - \kappa \frac{(1-\lambda)(\sigma_S \varphi + 1)}{1-(\varphi+1)\lambda}\ \epsilon_t^m. \tag{G.20}$$

So in the standard TANK with homogenous MUC the slope of the NKPC is increasing in λ.

With $\sigma_H \neq \sigma_S$ instead we have:

$$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1|t} + \underbrace{\frac{\kappa \left(\frac{1}{\sigma_S} + \varphi \right)}{1 - \lambda \left(1 - \frac{\sigma_H}{\sigma_S} \right) \frac{(1+\varphi)}{\sigma_H \varphi + 1}}}_{\Uparrow \text{ when } \lambda \uparrow \& \frac{\sigma_H}{\sigma_S} < 1} \hat{c}_t$$

Solving forward the aggregate Euler equation:

$$\hat{c}_t = -\frac{(1-\lambda)\sigma_S}{1-(1+\varphi)\lambda} \frac{\lambda \left(\frac{\sigma_H}{\sigma_S} - 1 \right) (\varphi + 1) + \sigma_H \varphi + 1}{\sigma_H \varphi + 1} \epsilon_t^m.$$

Substituting this back in the NK Phillips curve we get the exact same expression as in (G.20). Hence the slope of the Phillips curve is still increasing in λ but is independent of σ_H. This is due to the assumption of homogeneous wages which we relax in the next section.

H TANK model with Borrowers and Savers

An alternative way to characterize the household heterogeneity is to assume that some agents are net *borrower* (indexed with B) and some other net *saver*, as in Bilbiie et al. (2013).[60] The key features of this class of models are that borrowers are more impatient than savers, have no access to government bonds, and can borrow up to a limit. Importantly, to keep the algebra simple, here we abstract from IES heterogeneity and assume that both agents have the same IES, σ. However, to generate a dampening effect like the one in the model with HtM we would still need IES heterogeneity here.

The key equations of the log-linearized model are reported in table H.2. Equilibrium

Log-linearized Conditions		
1:	Labor Supply S	$\varphi \hat{H}_t^S = \hat{w}_t - \sigma^{-1}\hat{c}_t^S$
2:	Euler S	$\hat{c}_t^S = \hat{c}_{t+1\|t}^S - \sigma\left(\hat{R}_t - \hat{\Pi}_{t+1\|t}\right)$
3:	Labor Supply B	$\varphi \hat{H}_t^B = \hat{w}_t - \sigma^{-1}\hat{c}_t^B$
4:	Budget constraint B	$\hat{c}_t^B\gamma + \bar{D}(\hat{R}_{t-1} - \hat{\Pi}_t) = \left(\hat{w}_t + \hat{H}_t^B\right)$
5:	Phillips Curve	$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \kappa\hat{w}_t$
6:	Aggregate C	$\hat{c}_t = \lambda\gamma\hat{c}_t^B + (1-\lambda\gamma)\hat{c}_t^S$
7:	Aggregate B	$\hat{c}_t = \hat{H}_t = \lambda\hat{H}_t^B + (1-\lambda)\hat{H}_t^S$
8:	Taylor Rule	$\hat{R}_t = \hat{\Pi}_{t+1\|t} + \epsilon_t^m$

Table H.2: Log-linearized Conditions of Savers/Borrowers model

conditions in table H.2 and Table 1 are similar (leaving aside IES heterogeneity) with the only difference being the borrowers budget constraint and aggregate consumption. The assumption that borrowers discount more future consumption, $\beta^B < \beta^S = \beta$, implies that they become net borrower in equilibrium with the borrowing limit ($\bar{D}$) always binding.[61] γ is a steady state parameter which captures the consumption inequality between borrowers and savers, i.e. $\gamma = c^B/c = 1 + \bar{D}(\beta - 1) < 1$. Notice that when $\bar{D} = 0 \rightarrow \gamma = 1$ and the model is identical to the one with HtM consumers, and homogenous IES. In this model, as well, a fraction of agents are not on the Euler equation and cannot optimize intertemporally. The key difference with the previous set up is that a change in the nominal rate will have an impact not only on the time t consumption and labor supply decision but also on the $t+1$ decisions because the debt repayments at $t+1$ depend on the time t interest rates.

Under mild conditions, borrowers have an incentive to increase their labor supply after an interest rate hike; this is formalized in the following proposition.

[60] See their paper for details on the model derivation. Relative to them we simplify it further by abstracting from government debt, expenditure, and redistribution concerns. As in the previous model, we follow Bilbiie (2020) and assume that there is a production subsidy that induces marginal cost pricing which implies that the steady state of marginal costs is 1 which simplifies substantially the steady state and the log-linearized conditions.

[61] Note that in equation (4) in table H.2 $\bar{D}$ is effectively divided by the steady state of total income/consumption. But this is =1 one in this simple set up.

Proposition 3 *Under SADL ($\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)}$) and $\sigma < \frac{1+\bar{D}\kappa}{\gamma}$, a rate hike at time t induces an increases in the borrowers labor supply both at time t and $t+1$.*

The proof is in appendix H.2. At the core of this result, we have that consumption and the labor supply of the borrowers move in opposite directions. This can be appreciated when combining the time $t+1$ optimal response of borrowers in terms of consumption and labor supply after a monetary policy shock which are[62]

$$\hat{H}^B_{t+1} = \frac{\bar{D}(\varphi\lambda\gamma\sigma - 1 + \lambda)}{(\varphi\lambda - 1 + \lambda)(1 + \gamma\varphi\sigma)}\epsilon^m_t$$
$$\hat{c}^B_{t+1} = \frac{\varphi\sigma\bar{D}}{(\varphi\lambda - 1 + \lambda)(1 + \gamma\varphi\sigma)}\epsilon^m_t$$

Combining the latter two equations we have that

$$\hat{c}^B_{t+1} = \frac{\varphi\sigma}{\varphi\lambda\gamma\sigma - 1 + \lambda}\hat{H}^B_{t+1}$$

The numerator is positive. Notice that if the conditions of Proposition 3 hold, we have $\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)} < \frac{1}{1+\varphi\sigma\gamma}$, which implies that $(1 - \lambda - \varphi\sigma\lambda\gamma) > 0$; hence the denominator is negative. As mentioned this model nests the HtM model when $\bar{D} = 0$. This holds true also for the sufficient conditions of Proposition 3; notice that when $\bar{D} = 0 \rightarrow \gamma = 1 \rightarrow \sigma < 1$ and $\lambda < \frac{1}{1+\varphi}$.

H.1 Hours ratio in Borrowers/Savers model

Here we check how the Borrowers/Savers model performs in capturing the different sensitivity of the household's labor supply decision to a monetary policy shock. As the tightness of the borrowing constraint increases (larger $\bar{D}$), the borrower's labor supply becomes more sensitive to changes in the nominal rate; for large enough values of $\bar{D}$ and low enough value of σ, the income effect eventually gets so strong that generates an absolute increase in hours worked larger than one of the unconstrained agents. Since aggregate hours worked decline in the SADL equilibrium, the fraction of HtM households ought not to be too large. These arguments are depicted in Figures H.1-H.2 where we display the absolute ratio between the borrower's and savers' hours worked after a monetary policy shock for different values of $\bar{D}$ and λ. Figures H.1 is obtained by calibrating $\sigma = 1$ and $\varphi = 1$. Cool (warm) colors indicate a smaller (larger) value of the hour's ratio. As we move to the northeast corner (larger $\bar{D}$ and low λ, we generate a relatively more sensitive response of borrower households. However, the hour's ratio remains well below unity and far from our empirical evidence. This is because with $\sigma = 1$ the income effect on labor supply is still not enough to generate a larger sensitivity of the borrower's labor supply. Figure H.2 shows that to move closer to our empirical evidence we need to reduce the economy-wide IES to 0.5.

62 The time t optimal responses are analogous but more involved. To ease the notation we only discuss the time $t+1$ decisions.

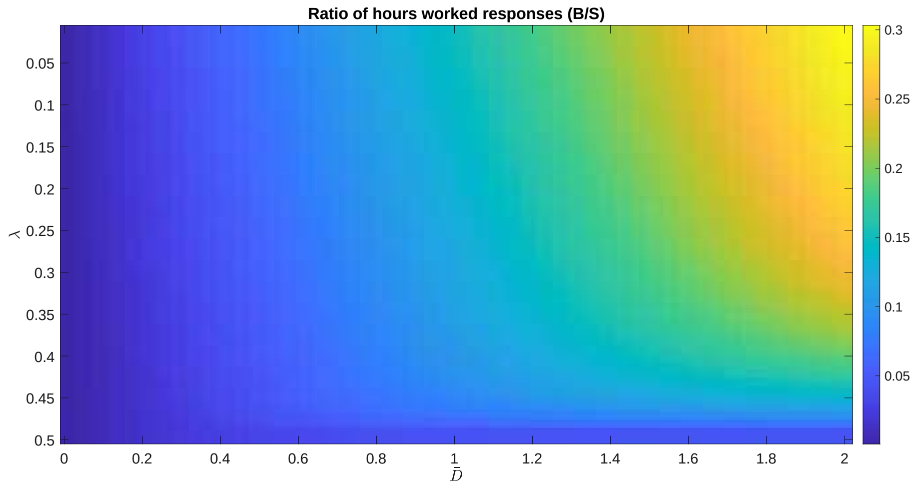

Figure H.1: Relative (absolute) magnitude of the response of borrowers and savers hours worked to a monetary policy shock for different values of $\bar{D}$ and λ and with $\sigma = 1$. Values larger than one indicate larger volatility of borrowers labor supply relative to Savers.

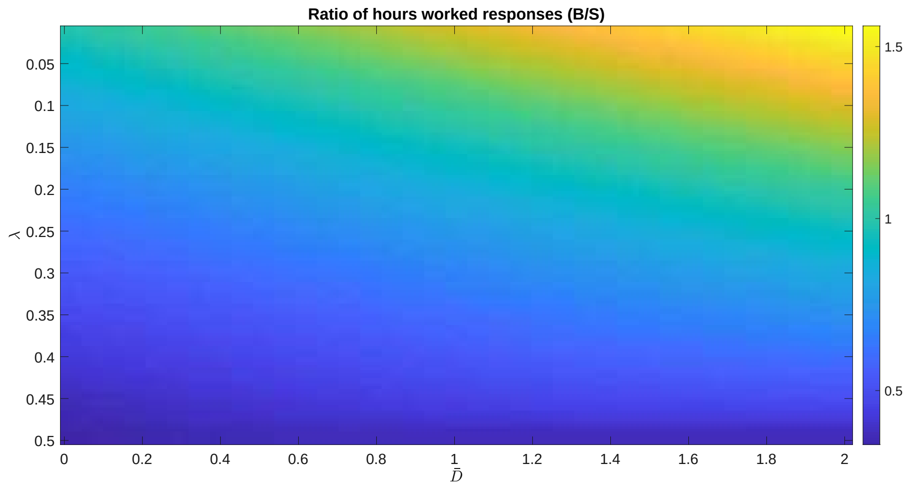

Figure H.2: Relative (absolute) magnitude of the response of borrowers and savers hours worked to a monetary policy shock for different values of $\bar{D}$ and λ and with $\sigma = 0.5$. Values larger than one indicate larger volatility of borrowers labor supply relative to Savers.

H.2 Derivation of Proposition 3

The derivation of the proposition 3 is more involved since R_t enters in the budget constrain of the Borrowers at $t+1$. This requires a backward induction solution approach, i.e. we first solve $t+1$, then given $t+1$ we solve for t. For convenience we report below the time $t+1$ equilibrium conditions.

Log-linearized Conditions		
1:	Labor Supply S	$\varphi \hat{H}^S_{t+1} = \hat{w}_{t+1} - \sigma^{-1}\hat{c}^S_{t+1}$
2:	Euler S	$\hat{c}^S_{t+1} = \hat{c}^S_{t+2\|t+1} - \sigma\left(\hat{R}_{t+1} - \hat{\Pi}_{t+2\|t+1}\right)$
3:	Labor Supply B	$\varphi \hat{H}^B_{t+1} = \hat{w}_{t+1} - \sigma^{-1}\hat{c}^B_{t+1}$
4:	Budget constraint B	$\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{R}_t - \hat{\Pi}_{t+1}) = \left(\hat{w}_{t+1} + \hat{H}^B_{t+1}\right)$
5:	Phillips Curve	$\hat{\Pi}_{t+1} = \beta\hat{\Pi}_{t+2\|t+1} + \kappa\hat{w}_{t+1}$
6:	Aggregate C	$\hat{c}_{t+1} = \lambda\gamma\hat{c}^B_{t+1} + (1-\lambda\gamma)\hat{c}^S_{t+1}$
7:	Aggregate B	$\hat{c}_{t+1} = \lambda\hat{H}^B_{t+1} + (1-\lambda)\hat{H}^S_{t+1}$
8:	Taylor Rule	$\hat{R}_{t+1} = \hat{\Pi}_{t+2\|t+1} + \epsilon^m_{t+1}$

Table H.3: Log-linearized Conditions of Savers/Borrowers model

Recall that $\epsilon^m_{t+j} = 0$ for $j > 0$ and $\epsilon^m_t \neq 0$. This implies that from $t+2$ onward the economy is back to steady states and all quantities are zero. This means also that $\hat{R}_{t+j} = \hat{\Pi}_{t+j+1|t+j}$ for $j > 0$, which implies that

$$\hat{c}^S_{t+1} = 0$$

The saver labor supply becomes

$$\varphi \hat{H}^S_{t+1} = \hat{w}_{t+1}$$

Using the borrowers BC we have

$$\begin{aligned}
\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{R}_t - \hat{\Pi}_{t+1}) &= \left(\hat{w}_{t+1} + \hat{H}^B_{t+1}\right) \\
\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{\Pi}_{t+1|t} + \epsilon^m_t - \hat{\Pi}_{t+1}) &= (1+\varphi)\hat{H}^B_{t+1} + 1/\sigma\hat{c}^B_{t+1} \\
1/\sigma\hat{c}^B_{t+1} &= \frac{1+\varphi}{\gamma\sigma - 1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma-1}\epsilon^m_t
\end{aligned}$$

notice that in absence of shocks in $t+1$ $\hat{\Pi}_{t+1} = \hat{\Pi}_{t+1|t}$. Combining the to labor supply conditions we have

$$\begin{aligned}
\varphi\hat{H}^S_{t+1} + \sigma^{-1}\hat{c}^S_{t+1} &= \varphi\hat{H}^B_{t+1} + \sigma^{-1}\hat{c}^B_{t+1} \\
\varphi\hat{H}^S_{t+1} &= \varphi\hat{H}^B_{t+1} + \frac{1+\varphi}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma-1}\epsilon^m_t \\
\varphi\hat{H}^S_{t+1} &= \frac{1+\gamma\varphi\sigma}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma-1}\epsilon^m_t
\end{aligned}$$

Combining the aggregate conditions we have

$$
\begin{aligned}
\lambda\gamma\hat{c}^B_{t+1} + (1-\lambda\gamma)\hat{c}^S_{t+1} =& \lambda\hat{H}^B_{t+1} + (1-\lambda)\hat{H}^S_{t+1} \\
\lambda\gamma\sigma\left[\frac{1+\varphi}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma-1}\epsilon^m_t\right] =& \lambda\hat{H}^B_{t+1} + (1-\lambda)\left[\frac{1+\gamma\varphi\sigma}{\varphi(\gamma\sigma-1)}\hat{H}^B_{t+1} - \frac{\bar{D}}{\varphi(\gamma\sigma-1)}\epsilon^m_t\right] \\
\hat{H}^B_{t+1}\left[\frac{(1+\varphi)\lambda\gamma\sigma}{\gamma\sigma-1} - \lambda - (1-\lambda)\frac{1+\gamma\varphi\sigma}{\varphi(\gamma\sigma-1)}\right] =& \left[\frac{\bar{D}\lambda\gamma\sigma}{\gamma\sigma-1} - (1-\lambda)\frac{\bar{D}}{\varphi(\gamma\sigma-1)}\right]\epsilon^m_t \\
\hat{H}^B_{t+1}\left[\frac{(1+\varphi)\varphi\lambda\gamma\sigma - \lambda\varphi(\gamma\sigma-1) - (1-\lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)}\right] =& \frac{\varphi\lambda\gamma\sigma-(1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t \\
\hat{H}^B_{t+1}\left[\frac{\varphi\lambda(\gamma\sigma+\varphi\gamma\sigma-\gamma\sigma+1) - (1-\lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)}\right] =& \frac{\varphi\lambda\gamma\sigma-(1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t \\
\hat{H}^B_{t+1}\frac{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)} =& \frac{\varphi\lambda\gamma\sigma-(1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t
\end{aligned}
$$

which yield to

$$
\hat{H}^B_{t+1} = \frac{\varphi\lambda\gamma\sigma-1+\lambda}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t \tag{H.21}
$$

This implies that borrower consumption at time $t+1$ is

$$
\begin{aligned}
\hat{c}^B_{t+1} =& \frac{\sigma(1+\varphi)}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\sigma\bar{D}}{\gamma\sigma-1}\epsilon^m_t \\
=& \frac{\sigma(1+\varphi)}{\gamma\sigma-1}\frac{\varphi\lambda\gamma\sigma-1+\lambda}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t - \frac{\sigma\bar{D}}{\gamma\sigma-1}\epsilon^m_t \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{(1+\varphi)(-1+\lambda(1+\varphi\gamma\sigma)) - (\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{-(1+\varphi)+(1+\varphi)\lambda(1+\varphi\gamma\sigma) - \varphi\lambda(1+\gamma\varphi\sigma) + (1-\lambda)(1+\gamma\varphi\sigma)}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{-1-\varphi+1+\gamma\varphi\sigma}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\varphi\sigma\bar{D}}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}
\end{aligned}
$$

and wages

$$
\begin{aligned}
\hat{w}_{t+1} =& \varphi\hat{H}^B_{t+1} + 1/\sigma\hat{c}^B_{t+1} \\
=& \varphi\frac{\varphi\lambda\gamma\sigma-1+\lambda}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t + \epsilon^m_t\frac{\varphi\bar{D}}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)} \\
=& \epsilon^m_t\bar{D}\varphi\frac{\varphi\lambda\gamma\sigma-1+\lambda+1}{(\varphi\lambda-1+\lambda)(1+\gamma\varphi\sigma)} \\
=& \epsilon^m_t\frac{\bar{D}\varphi\lambda}{\varphi\lambda-1+\lambda}
\end{aligned}
$$

and inflation

$$
\hat{\Pi}_{t+1} = \beta\hat{\Pi}_{t+2|t+1} + \kappa\hat{w}_{t+1} = \epsilon^m_t\frac{\bar{D}\varphi\lambda\kappa}{\varphi\lambda-1+\lambda}
$$

Now, we are in a position to solve for time t. Solving the Euler equation forward we have

$$
\hat{c}^S_t = -\sigma\epsilon^m_t
$$

From the NKP we have an expression for today inflation

$$\hat{\Pi}_t = \beta\hat{\Pi}_{t+1|t} + \kappa\hat{w}_t = \epsilon_t^m \frac{\bar{D}\varphi\lambda\kappa\beta}{\varphi\lambda - 1 + \lambda} + \kappa\hat{w}_t$$

Using the borrowers BC we have

$$\gamma\hat{c}_t^B + \bar{D}(\hat{R}_{t-1} - \hat{\Pi}_t) = \hat{w}_t + \hat{H}_t^B$$

$$\gamma\hat{c}_t^B - \epsilon_t^m \bar{D}\frac{\bar{D}\varphi\lambda\kappa\beta}{\varphi\lambda - 1 + \lambda} - \bar{D}\kappa\varphi\hat{H}_t^B - \bar{D}\kappa/\sigma\hat{c}_t^B = \varphi\hat{H}_t^B + 1/\sigma\hat{c}_t^B + \hat{H}_t^B$$

which yields to

$$\hat{c}_t^B = \frac{\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\gamma\sigma - 1 - \bar{D}\kappa}\hat{H}_t^B + \frac{\bar{D}^2\sigma\varphi\lambda\kappa\beta}{e_1 e_0}\epsilon_t^m$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. Combining the labor supply decision we have

$$\begin{aligned}
\varphi\hat{H}_t^S + \sigma^{-1}\hat{c}_t^S &= \varphi\hat{H}_t^B + \sigma^{-1}\hat{c}_t^B \\
\varphi\hat{H}_t^S - \epsilon_t^m &= \varphi\hat{H}_t^B + \frac{1 + \varphi + \varphi\bar{D}\kappa}{\gamma\sigma - 1 - \bar{D}\kappa}\hat{H}_t^B + \frac{\bar{D}^2\varphi\lambda\kappa\beta}{(\gamma\sigma - 1 - \bar{D}\kappa)(\varphi\lambda - 1 + \lambda)}\epsilon_t^m
\end{aligned}$$

which yields to

$$\hat{H}_t^S = \frac{1 + \varphi\gamma\sigma}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)}\hat{H}_t^B + \frac{\bar{D}^2\varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1}\epsilon_t^m$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. Combining the aggregate conditions we have

$$\lambda\gamma\hat{c}_{t+1}^B + (1 - \lambda\gamma)\hat{c}_{t+1}^S = \lambda\hat{H}_{t+1}^B + (1 - \lambda)\hat{H}_{t+1}^S$$

$$\begin{aligned}
&\lambda\gamma\left[\frac{\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\gamma\sigma - 1 - \bar{D}\kappa}\hat{H}_t^B + \frac{\bar{D}^2\sigma\varphi\lambda\kappa\beta}{e_1 e_0}\epsilon_t^m\right] + (1 - \lambda\gamma)[-\sigma\epsilon_t^m] \\
&\quad = \lambda\hat{H}_{t+1}^B + (1 - \lambda)\left[\frac{1 + \varphi\gamma\sigma}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)}\hat{H}_t^B + \frac{\bar{D}^2\varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1}\epsilon_t^m\right]
\end{aligned}$$

$$\begin{aligned}
&\hat{H}_t^B\left[\frac{\lambda\gamma\varphi\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} - \frac{\lambda\varphi(\gamma\sigma - 1 - \bar{D}\kappa)}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} - \frac{(1 - \lambda)(1 + \varphi\gamma\sigma)}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)}\right] \\
&\quad = \epsilon_t^m\left[\sigma(1 - \lambda\gamma) - \gamma\lambda\frac{\bar{D}^2\sigma\varphi\lambda\kappa\beta}{e_1 e_0} + (1 - \lambda)\frac{\bar{D}^2\varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1}\right]
\end{aligned}$$

Focusing on terms inside the left hand side square bracket we have

$$\begin{aligned}
&\varphi^{-1}e_1^{-1}\left[\varphi\lambda(\sigma\gamma + \sigma\gamma\varphi(1 + \bar{D}\kappa)) - \lambda\varphi(\gamma\sigma - 1 - \bar{D}\kappa) - (1 - \lambda)(1 + \varphi\gamma\sigma)\right] \\
&= \varphi^{-1}e_1^{-1}\left[\varphi\lambda(1 + \sigma\gamma\varphi)(1 + \bar{D}\kappa) - (1 - \lambda)(1 + \varphi\gamma\sigma)\right] \\
&= \varphi^{-1}e_1^{-1}(1 + \sigma\gamma\varphi)(\varphi\lambda(1 + \bar{D}\kappa) - 1 + \lambda)
\end{aligned}$$

Focusing on terms inside the right hand side square bracket we have

$$\begin{aligned}
&(\varphi e_0 e_1)^{-1}\left[\varphi\sigma(1 - \lambda\gamma)e_0 e_1 - \varphi\gamma\lambda(\bar{D}^2\sigma\varphi\lambda\kappa\beta) + (1 - \lambda)(\bar{D}^2\varphi\lambda\kappa\beta + e_0 e_1)\right] \\
&= (\varphi e_0 e_1)^{-1}\left[\varphi\sigma(1 - \lambda\gamma)e_0 e_1 - \varphi\gamma\lambda(\bar{D}^2\sigma\varphi\lambda\kappa\beta) + (1 - \lambda)\bar{D}^2\varphi\lambda\kappa\beta + (1 - \lambda)e_0 e_1\right] \\
&= (\varphi e_0 e_1)^{-1}\left[e_0 e_1(\varphi\sigma(1 - \lambda\gamma) + 1 - \lambda) - \varphi\gamma\lambda\sigma\bar{D}^2\varphi\lambda\kappa\beta + (1 - \lambda)\bar{D}^2\varphi\lambda\kappa\beta\right] \\
&= (\varphi e_0 e_1)^{-1}\left[\varphi\sigma e_0 e_1 + e_0 e_1(1 - \lambda - \varphi\sigma\lambda\gamma) + \bar{D}^2\varphi\lambda\kappa\beta(1 - \lambda - \varphi\sigma\lambda\gamma)\right] \\
&= (\varphi e_0 e_1)^{-1}\left[\varphi\sigma e_0 e_1 + (e_0 e_1 + \bar{D}^2\varphi\lambda\kappa\beta)(1 - \lambda - \varphi\sigma\lambda\gamma)\right]
\end{aligned}$$

Rearranging terms we have

$$\hat{H}_t^B = \frac{\varphi\sigma e_0 e_1 + (e_0 e_1 + \bar{D}^2\varphi\lambda\kappa\beta)(1 - \lambda - \varphi\sigma\lambda\gamma)}{(1 + \sigma\gamma\varphi)(\varphi\lambda(1 + \bar{D}\kappa) - 1 + \lambda)(\varphi\lambda - 1 + \lambda)}\ \epsilon_t^m \tag{H.22}$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. While still not very tractable we can derive a set of sufficient conditions such that the latter expression becomes positive. These conditions are

1. $\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)}$
2. $\sigma < \frac{1+\bar{D}\kappa}{\gamma}$

Condition 1 implies that $e_1 = [\varphi\lambda(1+\bar{D}\kappa)-1+\lambda] < 0$; this implies also that $(\varphi\lambda-1+\lambda) < 0$. Thus, if condition 1 holds, the denominator is positive. Condition 2 implies that $e_0 = (\sigma\gamma - 1 - \bar{D}\kappa) < 0$; this implies also that $e_0 e_1 > 0$. Moreover, if condition 2 hold, it is the case that

$$\lambda < \frac{1}{1 + \varphi(1 + \bar{D}\kappa)} < \frac{1}{1 + \varphi\sigma\gamma}$$

The latter implies that $(1 - \lambda - \varphi\sigma\lambda\gamma) > 0$. Therefore also the numerator is positive. These conditions imply also that the coefficient in (H.21) is positive.

I *Poor* Hand-to-mouth and college education

Here we use data from the Survey of Consumer Finances (SCF). The sample is 1989-2019. The estimation of Hand-to-mouth agents follows Kaplan et al. (2014) where one can distinguish between *poor* and *wealthy* HtM agents by using information on liquid and illiquid wealth. We use data on the education of the reference person to categorize agents among skilled (college-educated) and unskilled (no college education).

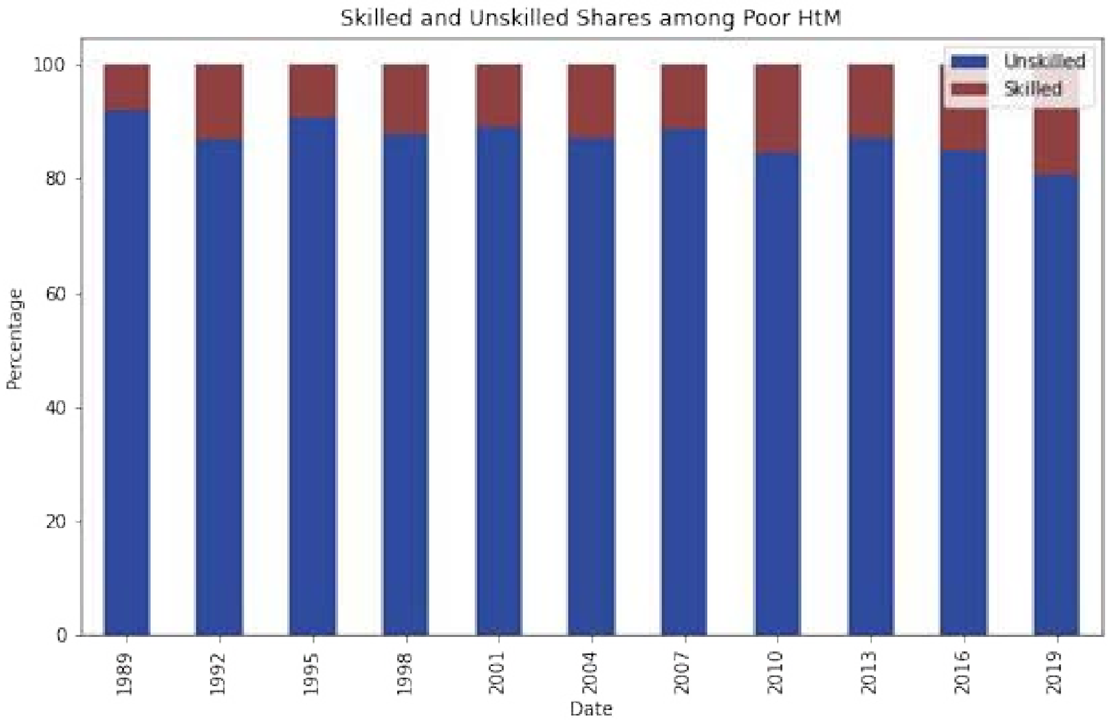

Figure I.3: Share of college-educated (skilled) vs non-college educated (unskilled) amongst the *poor* Hand-to-mouth. Source: Survey of Consumer Finance.

J Medium scale TANK+SAM

In this section we describe the medium scale TANK model with Search and Matching used in section 3.3. We build on the TANK model of Bilbiie (2008), the sticky price version of the model in Monacelli, Perotti and Trigari (2010) adding intensive margin of labor supply as in Thomas (2008), Trigari (2009) and Cantore, Levine and Melina (2014) assuming a separable utility function between consumption and leisure[63] and the presence of capital accumulation. We abstract from the presence of unemployment benefits for simplicity by assuming that income is pooled between the employed and unemployed in the same household.

There is a continuum of households $[0, 1]$. There are two types of households: A share λ of households are HtM (indexed by H) who work and consume all of their labor income, having no access to physical capital, bonds nor to firm ownership and dividends. The remaining $1-\lambda$ are savers (indexed by S) who work, hold physical capital, bonds and shares in monopolistic firms and get firm profits. Savers are standard PIH households.

J.1 Labor Market

Unemployment is introduced using the Search and Matching set up pioneered by Diamond, Mortensen and Pissarides (DMP) (Diamond (1982), Mortensen and Pissarides (1994)). Importantly we allow both for intensive and extensive margin of labor supply.

We assume that the two agents supply different types of labor supply and that there are segmented labor markets, one for the savers and one for the HtM. Hence, for each type of agent $j = S, H$, there is a matching function defining the number of new employed workers M_t^j as a function of total unemployed U_t^j and total vacancies V_t^j of each type:

$$M_t^j = \bar{M}^j (U_t^j)^{\omega^j} (V_t^j)^{1-\omega^j}, \tag{J.23}$$

where

$$U_t^j = 1 - N_{t-1}^j, \tag{J.24}$$

and N_t^j is the employment rate. $\bar{M}^j$ represents the efficiency in the matching process and $\omega^j \in (0, 1)$ is the elasticity of the number of matches to unemployment. These assumptions imply that the probability of filling a vacancy is:

[63] Monacelli et al. (2010) use non separable preferences because without intensive margin the marginal dis-utility of not working is not time varying.

$$q_t^j = \frac{M_t^j}{V_t^j}$$
$$q_t^j = \frac{\bar{M}^j (U_t^j)^{\omega^j} (V_t^j)^{1-\omega^j}}{V_t^j}$$
$$q_t^j = \bar{M}^j \left(\frac{U_t^j}{V_t^j}\right)^{\omega^j}$$
$$q_t^j = \bar{M}^j \left(\theta_t^j\right)^{-\omega^j}, \tag{J.25}$$

where $\theta^j = \frac{V_t^j}{U_t^j}$ denotes labor market tightness. Similarly, the probability to find a job for agent j is given by:

$$p_t^j = \frac{M_t^j}{U_t^j}$$
$$p_t^j = \bar{M}^j \left(\theta_t^j\right)^{1-\omega^j}. \tag{J.26}$$

The timing in the labor market is the following. In period t:

- Firm i starts the period with $N_{t-1}^S(i)$ and $N_{t-1}^H(i)$ workers employed.
- Firm i post $V_t^S(i)$ and $V_t^H(i)$ vacancies, and $U_t^S(i)$ and $U_t^H(i)$ unemployed search for a job.
- At firm i, $M_t^S(i)$ and $M_t^H(i)$ vacancies are filled.
- A fraction $1 - \rho^S$ of savers and a fraction $1 - \rho^H$ HtM lose their job. They can search for a new job in $t + 1$.

The law of motions of employment of the two types of agents in firm i is:

$$N_t^j(i) = \rho^j N_{t-1}^j(i) + V_t^j(i) q_t^j(i). \tag{J.27}$$

Importantly, opening a new vacancy is costly for the firms. And we denote the real cost real cost of opening a new vacancy κ_V^j.

J.2 Households

Each household of type j includes members that are employed and unemployed. We assume that they can perfectly insure themselves against idiosyncratic shocks, i.e. household income is pooled between the employed and unemployed in the same household. Then, the household's instantaneous utility function is given by:

$$U(c_t^j, N_t^j, H_t^j) = N_t^j \mathcal{U}(c_t^j, H_t^j) + (1 - N_t^j)\mathcal{U}(c_t^j, H_t^j), \tag{J.28}$$

where c_t^j is real consumption and H_t^j hours worked. Households have the same separable Stone-Geary preferences between consumption and hours, $\mathcal{U}(c_t^j, H_t^j) = \left(\frac{(c_t^j - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^j \frac{(H_t^j)^{1+\varphi}}{1+\varphi}\right)$, where $\bar{c}$ represents the subsistence level of consumption, σ is the curvature with respect to consumption and φ the inverse of the Frish elasticity of labor supply. Combining this expression with (J.28) we have:

$$U(c_t^j, N_t^j, H_t^j) = \left(\frac{(c_t^j - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^j N_t^j \frac{(H_t^j)^{1+\varphi}}{1+\varphi} \right). \tag{J.29}$$

Importantly, households do not choose directly employment and hours of work. These are determined through wage bargaining, as described below.

Savers

Savers maximize their lifetime utility subject to their budget constraint, taking prices and wages as given:

$$\max_{c_t^S, b_t^S, k_t^S, i_t^S} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^t \left(\frac{(c_{t+k}^S - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^S N_{t+k}^S \frac{(H_{t+k}^S)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^S + b_t^S + i^S = \frac{d_t}{1-\lambda} + H_t^S N_t^S w_t^S + \frac{R_{t-1}}{\Pi_t} b_{t-1}^S + r_t^K k_{t-1}^S,$$

$$k_t^S = \left(1 - \frac{\iota}{2} \log\left(\frac{i_t^S}{i_{t-1}^S} \right)^2 \right) i_t^S + (1-\delta)\, k_{t-1}^S.$$

b_t^S are bonds, Π_t is inflation, w_t^S are real wages of savers, R_t is the gross nominal interest rate on assets, r_t^K the rental rate of capital (k_t^S), i_t^S is investment and d_t are firm profits. ν^S indicates how leisure is valued relative to consumption, δ is capital depreciation and ι adjustment costs to investment.

Let's define the marginal utility of consumption $\Lambda_{c,t}^S$, marginal dis-utility of employment $\Lambda_{N,t}^S$, marginal dis-utility of an extra hour of work $\Lambda_{NH,t}^S$:

$$\Lambda_{c,t}^S = (c_t^S - \bar{c})^{-\frac{1}{\sigma}} \tag{J.30}$$

$$\Lambda_{N,t}^S = -\nu^S \frac{(H_t^S)^{1+\varphi}}{1+\varphi} \tag{J.31}$$

$$\Lambda_{NH,t}^S = -\nu^S (H_t^S)^{\varphi}. \tag{J.32}$$

The solution to the savers problem yields the standard Euler equations for bonds, capital and investment:

$$1 = \beta E_t \left[\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \frac{R_t}{\Pi_{t+1}} \right], \tag{J.33}$$

$$\Lambda_{c,t}^S = \Lambda_{c,t+1}^S \frac{\beta\, \mathbb{E}_t \left(r_{t+1}^K + (1-\delta)\, q_{t+1} \right)}{q_t}, \tag{J.34}$$

$$q_t \left(1 - \frac{\iota}{2} \log\left(\frac{i_t^S}{i_{t-1}^S} \right)^2 - \frac{i_t^S}{i_{t-1}^S} \frac{\iota \log\left(\frac{i_t^S}{i_{t-1}^S} \right)}{\frac{i_t^S}{i_{t-1}^S}} \right) +$$
$$+\mathbb{E}_t \left(q_{t+1} \frac{\beta\, \Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \frac{\iota \log\left(\frac{i_{t+1}^S}{i_t^S} \right)}{\frac{i_{t+1}^S}{i_t^S}} \left(\frac{i_{t+1}^S}{i_t^S} \right)^2 \right) = 1. \tag{J.35}$$

Finally the surplus of the savers household in the bargaining process, $S_t^{H,S}$, can be computed as the value of having an additional household member employed. By using the envelope condition for employment, we obtain

$$S_t^{H,S} = w_t^S H_t^S - \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta E_t \left[\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} S_{t+1}^{H,S} (\rho^S - p_{t+1}^S) \right]. \tag{J.36}$$

$\frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S}$ is the current *marginal value of non-work activity*. In this set up, where we abstract from unemployment benefits and home production is equivalent to the marginal value of leisure.

HtM

HtM households have no access to bonds nor firms' shares and therefore rely solely on labor earnings. Using the same form of utility as for the savers, their static problem reads:

$$\max_{c_t^H, H_t^H} \left(\frac{(c_t^H - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^H N_t^H \frac{(H_t^H)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^H = H_t^H N_t^H w_t^H,$$

where parameters and variables with H superscript have the same interpretations as the ones for savers.

Since preferences are monotonic, HtM will just consume all of their labor income:

$$c_t^H = H_t^H N_t^H w_t^H. \tag{J.37}$$

Following exactly what we did for savers we have:

$$\begin{aligned}
\Lambda_{c,t}^H &= (c_t^H - \bar{c})^{-\frac{1}{\sigma}} && \text{(J.38)}\\
\Lambda_{N,t}^H &= -\nu^H \tfrac{(H_t^H)^{1+\varphi}}{1+\varphi} && \text{(J.39)}\\
\Lambda_{NH,t}^H &= -\nu^H (H_t^H)^{\varphi}. && \text{(J.40)}
\end{aligned}$$

Finally the surplus of the HtM household in the bargaining process, $S_t^{H,H}$, can be computed as the value of having an additional household member employed. By using the envelope condition for employment, we obtain

$$S_t^{H,H} = w_t^H H_t^H - \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta E_t \left[\frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} S_{t+1}^{H,H} (\rho^H - p_{t+1}^H) \right]. \tag{J.41}$$

J.3 Firms

The firm sector is split into two. A representative competitive final goods firm aggregates intermediate goods according to a CES technology and a continuum of intermediate goods producers that produce different varieties using both types of labor and capital as an input. We assume that output is produced combining different types of labor and capital via a constant elasticity of substitution (CES) production function as in Krusell et al. (2000) where capital and savers labor are complementary between each other while they substitute HtM labor. Formally we use a two nested CES system normalized as in Cantore, León-Ledesma, McAdam and Willman (2014) which is equivalent to the set up of Krusell et al. (2000). Hence we have the system:

$$\frac{y_t^w(i)}{y^w(i)} = \left(\alpha \left(\frac{x_t(i)}{x(i)} \right)^{\frac{\sigma_y - 1}{\sigma_y}} + (1-\alpha) \left(\frac{\lambda N_t^H(i)\, H_t^H(i)}{\lambda N^H(i)\, H^H(i)} \right)^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{\sigma_y}{\sigma_y - 1}}, \tag{J.42}$$

$$\frac{x_t(i)}{x(i)} = \left(\gamma \left(\frac{k_{t-1}(i)}{k(i)} \right)^{\frac{\sigma_x - 1}{\sigma_x}} + (1-\gamma) \left(\frac{(1-\lambda) N_t^S(i)\, H_t^S)(i)}{(1-\lambda) N^S(i)\, H^S(i)} \right)^{\frac{\sigma_x - 1}{\sigma_x}} \right)^{\frac{\sigma_x}{\sigma_x - 1}}, \tag{J.43}$$

$$y_t(i) = y_t^w(i) - F. \tag{J.44}$$

y_t is output, y_t^w is the two-level nested CES production function and F is a fixed cost to ensure 0 profits in steady state. The production function combines HtM type labor and x_t which is a CES function of capital and savers labor where α and γ are shares and σ_y and σ_x elasticities of substitution. Recall that employment at firm level i evolves according to $N_t^j(i) = \rho^j N_{t-1}^j(i) + V_t^j(i) q_t^j(i)$. And firm i pays κ_V^j for each new vacancy posted. To the extent to which the intermediate goods are imperfect substitutes, with ϵ denoting the elasticity of substitution, there is a downward-sloping demand for each intermediate variety, giving the intermediate producers some pricing power. However, importantly, intermediate goods producers are also subject to some costs (ϕ^p) in adjusting prices (Rotemberg (1982)). This generates sticky prices.

The profit maximization problem of the generic firm i is the following:

$$\max_{\{p_t(i), k_t(i)(1-\lambda)N_t^S(i), \lambda N_t^H(i), V_t^S(i), V_t^H(i), y_t(i)\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_{c,t}^S}{\Lambda_{c,0}^S} \left[\frac{p_t(i)}{p_t} y_t(i) - w_t^S N_t^S(i) H_t^S(i) + \right.\right.$$
$$\left.\left. - w_t^H N_t^H(i) H_t^H(i) - r_t^K k_{t-1}(i) - \kappa_V^S V_t^S(i)) - \kappa_V^H V_t^H(i) - \frac{\phi_p}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t \right] \right\}$$

$$\text{s.t.} \begin{cases} y_t(i) = y_t^w(i) - F \\ y_t(i) = y_t(i) \left(\frac{p_t(i)}{p_t} \right)^{-\varepsilon} \\ N_t^S(i) = \rho^S N_{t-1}^S(i) + V_t^S(i) q_t^S(i) \\ N_t^H(i) = \rho^H N_{t-1}^H(i) + V_t^H(i) q_t^H(i). \end{cases}$$

Eliminate one constraint and write the lagrangian as follows:

$$\mathcal{L}^i = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda^S_{c,t}}{\Lambda^S_{c,0}} \left[\frac{p_t(i)}{p_t} y_t \left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon} - w^S_t H^S_t(i) N^S_t(i) - w^H_t H^H_t(i) N^H_t(i) - r^K_t k_{t-1}(i) - \kappa^S_V V^S_t(i) + \right.\right.$$
$$\left. - \kappa^H_V V^H_t(i) - \frac{\phi_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 y_t \right] + mc_t(i)\left[y^w_t(i) - F - y_t(i)\left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon}\right] +$$
$$\left. - \mu^S_t(i)\left[N^S_t(i) - \rho^S N^S_{t-1}(i) - q^S_t(i) V^S_t(i)\right] - \mu^H_t(i)\left[N^H_t(i) - \rho^H N^H_{t-1}(i) - q^H_t(i) V^H_t(i)\right] \right\},$$

where $mc_t(i)$ is the lagrange multiplier on the production function and it represents the real marginal cost of producing an additional unit of output; $\mu^j_t(i)$ are the lagrangian multiplier on the employment constraints, which capture the value of an additional employee for each type of labor. The first order condition with respect to capital is:

$$r^K_t = mc_t(i) mpk_t(i), \tag{J.45}$$

where the marginal product of capital is $mpk_t(i) = \frac{\partial y_t(i)}{\partial k_t(i)}$.

The first order condition with respect to employment is:

$$w^j_t H^j_t = mc_t(i) mpn^j_t(i) - \mu^j_t(i) + \beta \rho^j \mathbb{E}_t \left[\mu^j_{t+1}(i) \frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \right]$$
$$\mu^j_t(i) = mc_t(i) mpn^j_t(i) - w^j_t H^j_t + \beta \rho^j \mathbb{E}_t \left[\mu^j_{t+1}(i) \frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \right], \tag{J.46}$$

Where the marginal product of labor for savers is $mpl^S_t(i) = \frac{\partial y_t(i)}{\partial (1-\lambda) N^S_t(i) H^S_t(i)}$ while for HtM is $mpl^H_t(i) = \frac{\partial y_t(i)}{\partial \lambda N^H_t(i) H^H_t(i)}$. At the margin the product of an employee j $mpn^j_t(i) = mpl^j_t(i) H^j_t(i)$.

The first order condition with respect to V^j_t is:

$$\kappa^j_V = q^j_t(i) \mu^j_t. \tag{J.47}$$

The first order condition with respect to $p_t(i)$, imposing the symmetric equilibrium, is:

$$(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p (\Pi_t - 1) + \beta^S E_t \left[\frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \Pi_{t+1} \phi_p (\Pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \right] = 0. \tag{J.48}$$

Combining (J.46) and (J.47):

$$\frac{\kappa^j_V}{q^j_t} = mc_t mpn^j_t - w^j_t H^j_t + \beta \rho^j \kappa^j_V \mathbb{E}_t \left(\frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t} q^j_{t+1}} \right). \tag{J.49}$$

Clearly, in the absence of vacancy costs $\kappa^j_V = 0$ we have the standard competitive labor market condition $mc_t mpn^j_t = w^j_t H^j_t \rightarrow mc_t mpl^j_t = w^j_t$.

J.4 Wage setting and Hours worked bargaining

Bargaining takes place along two dimensions for each type of worker: the real wage and hours worked. We assume Nash bargaining.

Firm i and the worker j choose the real wage w_t^j to maximize the Nash product of each other surplus, weighted by each other bargaining power. The surplus of the firm from employment at the margin is given by $\mu_t^j(i)$. In analogy with the notation for household's surplus let's define $S_t^{F,j} = \mu_t^j(i)$.

The wage is set through a Nash bargaining process, where

$$w_t^j = argmax(S_t^{H,j})^{\eta^j}(S_t^{F,j})^{1-\eta^j}.$$

We can write the problem in logs, as follows:

$$\max_{w_t^j} \eta^j \log\left\{w_t^j H_t^j - \frac{\Lambda_{N,t}^j}{\Lambda_{c,t}^j} + \beta\mathbb{E}_t\left[\frac{\Lambda_{c,t+1}^j}{\Lambda_{c,t}^j} S_{t+1}^{H,j}\left(\rho^j - p_{t+1}^j\right)\right]\right\}$$
$$+(1-\eta^j)\log\left[mc_t mpn_t^j - w_t^j H_t^j + \beta\rho^j\kappa_V^j\mathbb{E}_t\left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S q_{t+1}^j}\right)\right].$$

FOC:

$$\begin{aligned}
\frac{\eta^j}{S_t^{H,j}} &= \tfrac{1-\eta^j}{S_t^{F,j}}\\
S_t^{F,j} &= \tfrac{1-\eta^j}{\eta^j} S_t^{H,j}\\
S_t^{F,j} + S_t^{H,j} &= \tfrac{1-\eta^j}{\eta^j} S_t^{H,j} + S_t^{H,j}\\
S_t &= \tfrac{1}{\eta^j} S_t^{H,j}\\
S_t^{H,j} &= \eta^j S_t,
\end{aligned}$$

where $S_t^j = S^{F,j} + S^{H,j}$ is the total surplus from a job of type j. Workers get a fraction η^j of the surplus, firms get a fraction $1-\eta^j$.

Using the previous expression and equations (J.36) and (J.46) the equilibrium wage for savers is given by:

$$S_t^{F,S} = \frac{1-\eta^S}{\eta^S} S_t^{H,S}$$

$$mc_t mpn_t^S - w_t^S H_t^S + \beta\rho^S \mathbb{E}_t \left[S_{t+1}^{F,S} \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \right] = ...$$

$$... = \frac{1-\eta^S}{\eta^S} \left\{ w_t^S H_t^S - \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta E_t \left[\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} S_{t+1}^{H,S} (\rho^S - p_{t+1}^S) \right] \right\}$$

$$\frac{1-\eta^S}{\eta^S} w_t^S H_t^S + w_t^S H_t^S = ...$$

$$... = mc_t mpn_t^S + \beta\rho^S \mathbb{E}_t \left[S_{t+1}^{F,S} \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \right] + \frac{1-\eta^S}{\eta^S} \left\{ \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} - \beta E_t \left[\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} S_{t+1}^{H,S} (\rho^S - p_{t+1}^S) \right] \right\}$$

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta E_t \left\{ \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \left[\eta^S \rho^S S_{t+1}^{F,S} - (1-\eta^S) S_{t+1}^{H,S} (\rho^S - p_{t+1}^S) \right] \right\}$$

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta E_t \left\{ \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \left[\eta^S \rho^S S_{t+1}^{F,S} - \eta^S S_{t+1}^{F,S} (\rho^S - p_{t+1}^S) \right] \right\}$$

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta\eta^S E_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} S_{t+1}^{F,S} p_{t+1}^S \right)$$

Using (J.25) and (J.47):

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta\eta^S \kappa_V^S E_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \frac{p_{t+1}^S}{q_{t+1}^S} \right)$$

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta\eta^S \kappa_V^S E_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \frac{V_{t+1}^S}{U_{t+1}^S} \right)$$

$$w_t^S H_t^S = \eta^S mc_t mpn_t^S + (1-\eta^S) \frac{\Lambda_{N,t}^S}{\Lambda_{c,t}^S} + \beta\eta^S \kappa_V^S E_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \theta_{t+1}^S \right). \tag{J.50}$$

This expression implies that the wage paid to the employee is a weighted average of the marginal product of the employee plus the savings from job continuation, net of the cost of posting vacancies, and the opportunity cost of working, which is increasing with the disutility of working activities.

Following the same process for HtM by using $S_t^{H,H} = \eta^H S_t$ with equations (J.53) and (J.46) the equilibrium wage for HtM is given by:[64]

[64] Note that the discount factor in equation (J.46) is still the one of savers as they are the only one owning the firm.

$$S_t^{F,H} = \frac{1-\eta^H}{\eta^H} S_t^{H,H}$$

$$mc_t mpn_t^H - w_t^H H_t^H + \beta\rho^H \mathbb{E}_t \left[S_{t+1}^{F,H} \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \right] = \ldots$$

$$\ldots = \frac{1-\eta^H}{\eta^H} \left\{ w_t^H H_t^H - \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta E_t \left[\frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} S_{t+1}^{H,H} (\rho^H - p_{t+1}^H) \right] \right\}$$

$$\frac{1-\eta^H}{\eta^H} w_t^H H_t^H + w_t^H H_t^H = \ldots$$

$$\ldots = mc_t mpn_t^H + \beta\rho^H \mathbb{E}_t \left[S_{t+1}^{F,H} \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \right] + \frac{1-\eta^H}{\eta^H} \left\{ \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} - \beta E_t \left[\frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} S_{t+1}^{H,H} (\rho^H - p_{t+1}^H) \right] \right\}$$

$$w_t^H H_t^H = \eta^H mc_t mpn_t^H + (1-\eta^H) \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta E_t \left\{ \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \eta^H \rho^H S_{t+1}^{F,H} - \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} (1-\eta^H) S_{t+1}^{H,H} (\rho^H - p_{t+1}^H) \right\}$$

$$w_t^H H_t^H = \eta^H mc_t mpn_t^H + (1-\eta^H) \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta E_t \left\{ \frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} \eta^H \rho^H S_{t+1}^{F,H} - \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \eta^H S_{t+1}^{F,H} (\rho^H - p_{t+1}^H) \right\}$$

$$w_t^H H_t^H = \eta^H mc_t mpn_t^H + (1-\eta^H) \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta\eta^H E_t \left\{ S_{t+1}^{F,H} \left(\left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} - \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \right) \rho^H + \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} p_{t+1}^H \right) \right\}$$

Use (J.25) and (J.47):

$$w_t^H H_t^H = \eta^H mc_t mpn_t^H + (1-\eta^H) \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta\eta^H \kappa_V^H E_t \left\{ \left(\left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} - \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \right) \frac{\rho^H}{q_{t+1}^H} + \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \frac{p_{t+1}^H}{q_{t+1}^H} \right) \right\}$$

$$w_t^H H_t^H = \eta^H mc_t mpn_t^H + (1-\eta^H) \frac{\Lambda_{N,t}^H}{\Lambda_{c,t}^H} + \beta\eta^H \kappa_V^H E_t \left\{ \left(\left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S} - \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \right) \frac{\rho^H}{q_{t+1}^H} + \frac{\Lambda_{c,t+1}^H}{\Lambda_{c,t}^H} \theta_{t+1}^H \right) \right\} \quad \text{(J.51)}$$

We now follow Thomas (2008), Trigari (2009) and Cantore, Levine and Melina (2014) in modelling hours per worker as being determined by firms and workers in a privately efficient way (ie., in order to maximize the joint surplus of their employment relationship). The joint surplus for an employee j is given by S_t^j:

$$S_t^j = S^{F,j} + S^{H,j} \quad \text{(J.52)}$$

$$S_t^j = mc_t mpn_t^j - w_t^j H_t^j + \beta\rho^j \kappa_V^j \mathbb{E}_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S q_{t+1}^j} \right) + w_t^j H_t^j - \frac{\Lambda_{N,t}^j}{\Lambda_{C,t}^j} + \beta E_t \left[\frac{\Lambda_{c,t+1}^j}{\Lambda_{c,t}^j} S_{t+1}^{H,j} (\rho^j - p_{t+1}^j) \right] \quad \text{(J.53)}$$

By maximizing S_t with respect to H^j we obtain the hours-determination condition:

$$mc_t mpl_t^j = -\frac{\Lambda_{NH,t}^j}{\Lambda_{c,t}^j}, \quad \text{(J.54)}$$

using again $mpn_t^j = mpl_t^j H_t^j$.

According to this expression the marginal revenue product of labor is equal to the marginal rate of substitution between consumption and leisure. Therefore hours are independent of the hourly wage. Quoting from Trigari (2009): *Hence, the first order condition* *of hours worked is exactly the same as in the competitive labor market. This happens because the correct measure of the cost of hours to the firm is the marginal rate of substitution, rather than the wage. In other words, the wage is not allocational for hours.*

As a result the two equilibrium conditions determining hours worked for the two agents are:

$$mc_t = \nu^S (H_t^S)^\varphi (c_t^S - \bar{c})^{\frac{1}{\sigma}} \tag{J.55}$$

$$mc_t = \nu^H (H_t^H)^\varphi (c_t^H - \bar{c})^{\frac{1}{\sigma}}. \tag{J.56}$$

J.5 Monetary policy and Aggregation

Central Bank

We assume that the central bank follows a standard Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left(\frac{\Pi_t}{\Pi}\right)^{(1-\rho^R)\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{(1-\rho^R)\phi_y} \epsilon_t^m.$$

ρ^R sets the strength of inertia in nominal interest rate, ϕ_π its sensitivity to inflation, ϕ_y its sensitivity to output and ϵ_t^m is the monetary policy shock.

Aggregation and market clearing

Aggregate bonds are in 0 net supply and aggregate consumption, hours, employment, unemployment, real wages, investment and capital are:

$$0 = (1-\lambda) b_t^S \tag{J.57}$$

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S \tag{J.58}$$

$$H_t = \frac{\lambda H_t^H N_t^H + (1-\lambda) H_t^S N_t^S}{N_t} \tag{J.59}$$

$$N_t = \lambda N_t^H + (1-\lambda) N_t^S \tag{J.60}$$

$$U_t = 1 - N_{t-1} \tag{J.61}$$

$$w_t = \frac{\lambda w_t^H H_t^H N_t^H + (1-\lambda) w_t^S H_t^S N_t^S}{N_t H_t} \tag{J.62}$$

$$i_t = (1-\lambda) i_t^S \tag{J.63}$$

$$k_t = (1-\lambda) k_t^S \tag{J.64}$$

Using firm's profits definition and the two agent's budget constraints we obtain the resource constraint:

$$c_t + i_t = y_t - \kappa_V^S V_t^S - \kappa_V^H V_t^H - \frac{\phi_p}{2} (\Pi_t - 1)^2 y_t.$$

J.6 Equilibrium

Equilibrium condition symmetric for both agents $j = S, H$:

$$M_t^j = \bar{M}^j (U_t^j)^{\omega^j} (V_t^j)^{1-\omega^j} \tag{J.65}$$

$$U_t^j = 1 - N_{t-1}^j \tag{J.66}$$

$$q_t^j = \frac{M_t^j}{V_t^j} \tag{J.67}$$

$$\theta^j = \frac{V_t^j}{U_t^j} \tag{J.68}$$

$$p_t^j = \frac{M_t^j}{U_t^j} \tag{J.69}$$

$$N_t^j = \rho^j N_{t-1}^j + M_t^j \tag{J.70}$$

$$\frac{\kappa_V^j}{q_t^j} = mc_t mpn_t^j - w_t^j H_t^j + \beta \rho^j \kappa_V^j \mathbb{E}_t \left(\frac{\Lambda_{c,t+1}^S}{\Lambda_{c,t}^S q_{t+1}^j} \right) \tag{J.71}$$

$$S_t^{H,j} = w_t^j H_t^j + mrs_t^j + \beta E_t \left[\frac{\Lambda_{c,t+1}^j}{\Lambda_{c,t}^j} S_{t+1}^{H,j} (\rho^S - p_{t+1}^S) \right] \tag{J.72}$$

$$S_t^j = \frac{\kappa_V^j}{q_t^j} + S^{H,j} \tag{J.73}$$

$$mc_t mpl_t^j = -\frac{\Lambda_{NH,t}^j}{\Lambda_{c,t}^j} \tag{J.74}$$

$$\tag{J.75}$$

Equilibrium conditions specific to Savers:

$$\Lambda^S_{c,t} = (c^S_t - \bar{c})^{-\frac{1}{\sigma}} \tag{J.76}$$

$$\Lambda^S_{N,t} = -\nu^S \frac{(H^S_t)^{1+\varphi}}{1+\varphi} \tag{J.77}$$

$$\Lambda^S_{NH,t} = -\nu^S (H^S_t)^{\varphi} \tag{J.78}$$

$$mpn^S_t = mpl^S_t H^S_t \tag{J.79}$$

$$mpl^S_t = (1-\gamma)\ \alpha \left(\frac{y^w}{x}\right)^{\frac{\sigma_y - 1}{\sigma_y}} \left(\frac{x}{(1-\lambda) N^S H^S}\right)^{\frac{\sigma_x - 1}{\sigma_x}} \left(\frac{y^w_t}{x_t}\right)^{\frac{1}{\sigma_y}} \left(\frac{x_t}{(1-\lambda) N^S_t H^S_t}\right)^{\frac{1}{\sigma_x}} \tag{J.80}$$

$$mrs^S_t = -\frac{\Lambda^S_{N,t}}{\Lambda^S_{c,t}} \tag{J.81}$$

$$w^S_t H^S_t = \eta^S mc_t mpn^S_t - (1-\eta^S) mrs^S_t + \beta \eta^S \kappa^S_V E_t \left(\frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \theta^S_{t+1}\right) \tag{J.82}$$

$$1 = \beta E_t \left[\frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \frac{R_t}{\Pi_{t+1}}\right] \tag{J.83}$$

$$\Lambda^S_{c,t} = \Lambda^S_{c,t+1} \frac{\beta\, \mathbb{E}_t \left(r^K_{t+1} + (1-\delta)\ q_{t+1}\right)}{q_t} \tag{J.84}$$

$$\begin{aligned} q_t & \left(1 - \frac{\iota}{2} \log\left(\frac{i^S_t}{i^S_{t-1}}\right)^2 - \frac{i^S_t}{i^S_{t-1}} \frac{\iota \log\left(\frac{i^S_t}{i^S_{t-1}}\right)}{\frac{i^S_t}{i^S_{t-1}}}\right) + \\ + \mathbb{E}_t & \left(q_{t+1} \frac{\beta\, \Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} \frac{\iota \log\left(\frac{i^S_{t+1}}{i^S_t}\right)}{\frac{i^S_{t+1}}{i^S_t}} \left(\frac{i^S_{t+1}}{i^S_t}\right)^2\right) = 1 \end{aligned} \tag{J.85}$$

$$k^S_t = \left(1 - \frac{\iota}{2} \log\left(\frac{i^S_t}{i^S_{t-1}}\right)^2\right) i^S_t + (1-\delta)\ k^S_{t-1} \tag{J.86}$$

Equilibrium conditions specific to HtM:

$$\Lambda^H_{c,t} = (c^H_t - \bar{c})^{-\frac{1}{\sigma}} \tag{J.87}$$

$$\Lambda^H_{N,t} = -\nu^H \frac{(H^H_t)^{1+\varphi}}{1+\varphi} \tag{J.88}$$

$$\Lambda^H_{NH,t} = -\nu^H (H^H_t)^{\varphi} \tag{J.89}$$

$$mpn^H_t = mpl^H_t H^H_t \tag{J.90}$$

$$mpl^H_t = (1-\alpha) \left(\frac{y^w}{\lambda N^H H^H}\right)^{\frac{\sigma_y - 1}{\sigma_y}} \left(\frac{y^w_t}{\lambda N^H_t H^H_t}\right)^{\frac{1}{\sigma_y}} \tag{J.91}$$

$$mrs^H_t = -\frac{\Lambda^H_{N,t}}{\Lambda^H_{c,t}} \tag{J.92}$$

$$\begin{aligned} w^H_t H^H_t &= \eta^H mc_t mpn^H_t - (1-\eta^H) mrs^H_t + \\ + \beta \eta^H \kappa^H_V E_t & \left\{ \left(\left(\frac{\Lambda^S_{c,t+1}}{\Lambda^S_{c,t}} - \frac{\Lambda^H_{c,t+1}}{\Lambda^H_{c,t}} \right) \frac{\rho^H}{q^H_{t+1}} + \frac{\Lambda^H_{c,t+1}}{\Lambda^H_{c,t}} \theta^H_{t+1} \right) \right\} \end{aligned} \tag{J.93}$$

$$c^H_t = H^H_t N^H_t w^H_t \tag{J.94}$$

Remaining equilibrium conditions

$$c_t = \lambda c_t^H + (1-\lambda)c_t^S \tag{J.95}$$

$$H_t = \frac{\lambda N_t^H H_t^H + (1-\lambda)N_t^S H_t^S}{N_t} \tag{J.96}$$

$$N_t = \lambda N_t^H + (1-\lambda)N_t^S \tag{J.97}$$

$$U_t = 1 - N_{t-1} \tag{J.98}$$

$$w_t = \frac{\lambda w_t^H H_t^H N_t^H + (1-\lambda)w_t^S H_t^S N_t^S}{N_t H_t} \tag{J.99}$$

$$i_t = (1-\lambda)i_t^S \tag{J.100}$$

$$k_t = (1-\lambda)k_t^S \tag{J.101}$$

$$r_t^K = mc_t mpk_t \tag{J.102}$$

$$mpk_t = \gamma\,\alpha \left(\frac{y_t^w}{x_t}\right)^{\frac{1}{\sigma_y}} \left(\frac{y^w}{(x)}\right)^{\frac{\sigma_y-1}{\sigma_y}} \left(\frac{(x)}{k}\right)^{\frac{\sigma_x-1}{\sigma_x}} \left(\frac{x_t}{k_{t-1}}\right)^{\frac{1}{\sigma_x}} \tag{J.103}$$

$$mpx_t = \alpha \left(\frac{y_t^w}{x_t}\right)^{\frac{1}{\sigma_y}} \left(\frac{y^w}{x}\right)^{\frac{\sigma_y-1}{\sigma_y}} \tag{J.104}$$

$$\frac{y_t^w}{y^w} = \left(\alpha \left(\frac{x_t}{x}\right)^{\frac{\sigma_y-1}{\sigma_y}} + (1-\alpha)\left(\frac{\lambda N_t^H\, H_t^H}{\lambda N^H\, H^H}\right)^{\frac{\sigma_y-1}{\sigma_y}}\right)^{\frac{\sigma_y}{\sigma_y-1}} \tag{J.105}$$

$$\frac{x_t}{x} = \left(\gamma \left(\frac{k_{t-1}}{k}\right)^{\frac{\sigma_x-1}{\sigma_x}} + (1-\gamma)\left(\frac{(1-\lambda)N_t^S\, H_t^S)}{(1-\lambda)N^S\, H^S}\right)^{\frac{\sigma_x-1}{\sigma_x}}\right)^{\frac{\sigma_x}{\sigma_x-1}} \tag{J.106}$$

$$y_t = y_t^w - F \tag{J.107}$$

$$c_t + i_t = y_t - \kappa_V^S V_t^S - \kappa_V^H V_t^H - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 y_t \tag{J.108}$$

$$d_t = y_t(1 - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2) - w_t H_t N_t - r_t^K k_{t-1} - \kappa_V^S V_t^S - \kappa_V^H V_t^H \tag{J.109}$$

$$\begin{aligned}(1-\epsilon) + \epsilon mc_t - \Pi_t\phi_p\left(\Pi_t - 1\right) + \\ +\beta^S E_t\left[\left(\frac{\Lambda_{c,t+1}}{\Lambda_c}\right)\Pi_{t+1}\phi_p\left(\Pi_{t+1}-1\right)\frac{y_{t+1}}{y_t}\right] = 0\end{aligned} \tag{J.110}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R}\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}{\epsilon^m}_t \tag{J.111}$$

$$r_t = \frac{R_t}{\Pi_{t+1}}. \tag{J.112}$$

J.7 Steady State

Variables without time index denote the steady-state level. We calibrate ex ante the proportion of HtM (λ), the labor share ($s^L = \frac{wHN}{y}$), the wage gap ($\tilde{w} = \frac{w^S}{w^H}$), the unemployment rate (U^j), labor market (θ^j) tightness, the steady state *relative* value of non-work to work activity ($\zeta^j = -\frac{\frac{\Lambda_N^j}{\Lambda_C^j}}{mc\frac{y}{N^j}}$), hours worked ($H^j$) and implied IES by Stone-Geary preferences for each agents (σ_j). Given the ex ante calibration of $\left\{\lambda, s^L, \tilde{w}, U^j, \theta^j, \zeta^j, H^j, \sigma_S, \sigma_H\right\}$ the following parameters are computed ex post: $\left\{\alpha, \gamma, F, \kappa_V^j, p^j, \nu^j, \varphi, \sigma, \bar{c}\right\}$. Moreover, we also calibrate the steady state of hours worked, H^j and normalize $y^w = x = 1$.[65]

Find N^j:

$$N^j = 1 - U^j.$$

Find $\bar{M}^j$:

$$\bar{M}^j = \frac{N^j {\theta^j}^{\omega^j - 1}(\rho^j - 1)}{N^j - 1}$$

Find p^j:

$$p^j = \bar{M}^j(\theta^j)^{1-\omega^j}$$

Find q^j:

$$q^j = \bar{M}^j(\theta^j)^{-\omega^j}.$$

Aggregate employment and hours are:

$$N = \lambda N^H + (1-\lambda)N^S$$

$$H = \frac{\lambda N^H H^H + (1-\lambda)H^S N^S}{N}.$$

Vacancies:

$$V^j = \theta^j U^j.$$

Total matches:

$$M^j = \bar{M}^j (U^j)^{\omega^j}(V^j)^{1-\omega^j}.$$

Assuming zero inflation in steady state and using as normalization point for the nested CES production function its Cobb-Douglas limit ($\sigma_y = \sigma_x \to 1$):

[65] See Cantore, León-Ledesma, McAdam and Willman (2014).

$$\Pi = 1$$
$$R = \frac{1}{\beta}$$
$$mc = \frac{\epsilon - 1}{\epsilon}$$
$$r^K = \frac{1}{\beta} - 1 + \delta$$
$$k = \left(\frac{r^K}{mc\,\alpha\gamma(\lambda H^H N^H)^{1-\alpha}((1-\lambda)H^S N^S)^{(1-\gamma)\alpha}} \right)^{\frac{1}{\alpha\gamma - 1}}$$
$$x = k^{\gamma}((1-\lambda)H^S N^S)^{(1-\gamma)}$$
$$y^w = x^{\alpha}(\lambda H^H N^H)^{1-\alpha}$$
$$y = y^w - F$$
$$mpl^S = (1-\gamma)\alpha \frac{y^w}{(1-\lambda)N^S H^S}$$
$$mpl^H = (1-\alpha)\frac{y^w}{\lambda H^H N^H}$$
$$mpn^j = mpl^j H^j$$
$$mpk = \alpha\gamma \frac{y^w}{k}$$
$$mpx = \alpha \frac{y^w}{x}$$

Take the wage equation in steady state and use the definition of ζ^j:

$$w^j H^j = \eta^j mc\,mpn^j + (1-\eta^j)\frac{\Lambda_N^j}{\Lambda_c^j} + \beta\eta^j\kappa_V^j\theta^j$$
$$w^j H^j = \eta^j mc\,mpn^j - (1-\eta^j)\zeta^j mc\frac{y}{N^j} + \beta\eta^j\kappa_V^j\theta^j$$
$$w^j = \frac{mc\,mpn^j(\eta^j - (1-\eta^j)\zeta^j) + \beta\eta^j\kappa_V^j\theta^j}{H^j}$$

Use the last equation in (J.49)

$$\frac{\kappa_V^j}{q^j} = mc\,mpn^j - w^j H^j + \beta\rho^j\frac{\kappa_V^j}{q^j}$$
$$\frac{\kappa_V^j}{q^j} = mc\,mpn^j - mc\,mpn^j(\eta^j - (1-\eta^j)\zeta^j) - \beta\eta^j\kappa_V^j\theta^j + \beta\rho^j\frac{\kappa_V^j}{q^j}$$
$$\frac{\kappa_V^j}{q^j} = mc\,mpn^j(1 - \eta^j + (1-\eta^j)\zeta^j) - \beta\eta^j\kappa_V^j\theta^j + \beta\rho^j\frac{\kappa_V^j}{q^j}$$
$$\kappa_V^j\left[\frac{1}{q^j}(1-\beta\rho^j) + \beta\eta^j\theta^j\right] = mc\,mpn^j((1-\eta^j)(1+\zeta^j))$$
$$\kappa_V^j = \frac{mc\,mpn^j((1-\eta^j)(1+\zeta^j))}{\frac{1}{q^j}(1-\beta\rho^j) + \beta\eta^j\theta^j}$$

Aggregate wage:

$$w = \frac{\lambda w^H H^H N^H + (1-\lambda) w^S H^S N^S}{NH}.$$

Aggregate and agents consumption:

$$\begin{aligned} i &= \delta k \\ c &= y - i - \kappa_V^H V^H - \kappa_V^S V^S \\ d &= y - wNH - r^K k - \kappa_V^H V^H - \kappa_V^S V^S \\ c^H &= w^H H^H N^H \\ c^S &= \frac{c - \lambda c^H}{1-\lambda} \end{aligned}$$

The utility function (J.29) implies that, for a given steady state level of consumption $c^j \geq \bar{c}$ the IES is increasing in c^j and given by:

$$-\frac{U'_{c^j}}{c^j U''_{c^j}} = \sigma\left(1 - \frac{\bar{c}}{c^j}\right). \tag{J.113}$$

To simplify the notation, in line with equation (J.113) above we define $\sigma_H = \sigma\left(1 - \frac{\bar{c}}{c^H}\right)$ and $\sigma_S = \sigma\left(1 - \frac{\bar{c}}{c^S}\right)$. This implies $\sigma = \frac{c^H \sigma_H - c^S \sigma_S}{c^H - c^S}$ and $\bar{c} = \left(1 - \frac{\sigma_H}{\sigma}\right) c^H$.

Labor dis-utility and inverse of Frish elasticity:

$$\begin{aligned} \zeta^j &= \frac{\frac{\nu^j (H^j)^{\varphi+1} (c^j - \bar{c})^{\frac{1}{\sigma}}}{\varphi+1}}{mc\, mpn^j} \\ \nu^j &= \frac{\zeta^j mc\, mpn^j (\varphi+1)}{(c^j - \bar{c})^{\frac{1}{\sigma}} (H^j)^{\varphi+1}}, \end{aligned}$$

Using the condition for hours:

$$\begin{aligned} mc\, mpn^j \frac{1}{H^j} &= \nu^j (H_t^j)^{\varphi} (c_t^j - \bar{c})^{\frac{1}{\sigma}} \\ mc\, mpn^j \frac{1}{H^j} &= \frac{\zeta^j mc\, mpn^j (\varphi+1)}{(c^j - \bar{c})^{\frac{1}{\sigma}} (H^j)^{\varphi+1}} (H_t^j)^{\varphi} (c_t^j - \bar{c})^{\frac{1}{\sigma}} \\ \varphi &= -\frac{\zeta^j - 1}{\zeta^j}. \end{aligned}$$

Hence in order to have homogeneous Frish elasticity across agents, as assumed so far, we need to calibrate $\zeta^H = \zeta^S$.

Finally we find α, γ and F numerically by solving for:

$$\begin{aligned} s^L + \alpha\gamma \frac{(y^w\, mc)}{y} &+ \kappa_V^H V^H + \kappa_V^S V^S - 1 \\ F = y^w - w\, H\, N - r^K k &- \kappa_V^S V^S - \kappa_V^H V^H \\ \tilde{w} &= \frac{w^S}{w^H}. \end{aligned}$$

J.8 The behaviour of Hours worked

Hours of the HtM are independent from the wage and are jointly determined by equating the marginal rate of substitution between consumption and leisure and the marginal product of labor. Log-linearizing (J.56):

$$\hat{mc}_t \hat{mpl}_t^H = \varphi \hat{H}_t^H + \frac{1}{\sigma} \hat{c}_t^H \frac{c^H}{c^H - \bar{c}}$$
$$\hat{H}_t^H = \frac{1}{\varphi} \left(\hat{mc}_t \hat{mpl}_t^H - \frac{c^H}{\sigma(c^H - \bar{c})} \hat{c}_t^H \right)$$

Hence, once again it is σ that drives the response of Hours. The marginal product of labor is always procyclical, as well as the consumption of HtM. The lower σ the more countercyclical is H_t^H.

J.9 Robustness: Alternative calibration

Here we show sensitivity of the simulations to alternative calibrations of the job separation rate $(1 - \rho_H)$, unemployment rate $(\bar{U}^H)$ and replacement ratio (ζ^H) of HtM agents. Figures J.4-J.5 show that increasing the separation rate of HtM from 5.6 percent to 7 percent has a positive impact on their labor supply. But only when combined with IES heterogeneity is able to generate a flip in the sign of the IRF.[66] Figures J.6-J.7 show that reducing the steady state unemployment rate of HtM from 9 percent to 7 percent has a positive impact on hours of the HtM. But again only when combined with IES heterogeneity is able to generate a countercylical response. Finally Figures J.8-J.9 show that changing the replacement ratio of HtM, which affects their Frisch elasticity, has little impact on their response of hours worked to a monetary policy shock.

[66] For values of $(1 - \rho_H) > 0.1$ the Blanchard and Kahn conditions are not satisfied.

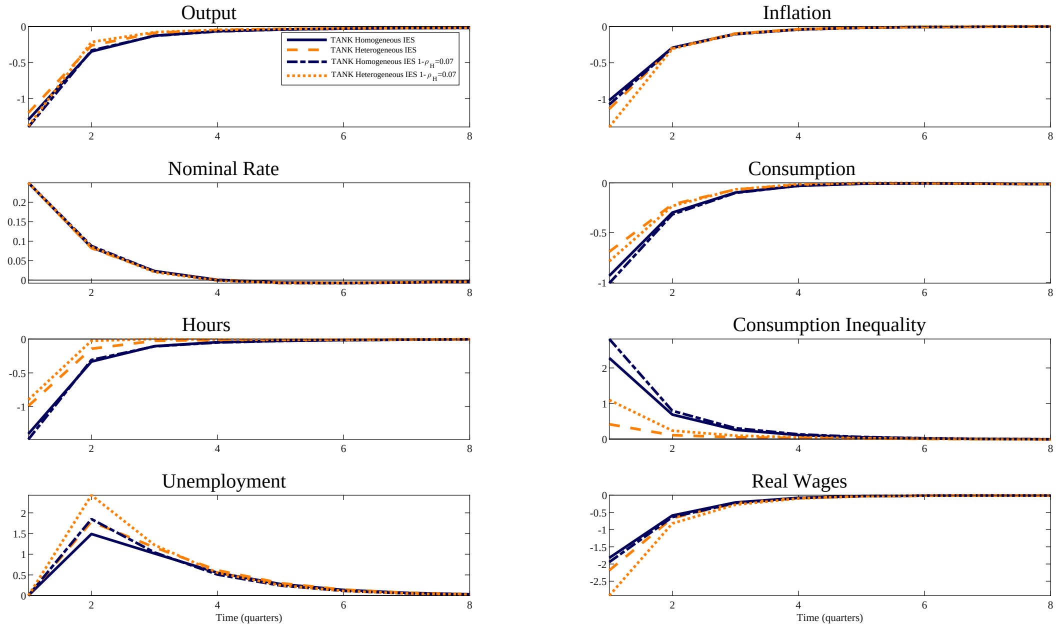

Figure J.4: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $1 - \rho_H = 0.07$ (dashed dotted blue line), and TANK with heterogeneous IES and $1 - \rho_H = 0.07$ (dotted orange line).

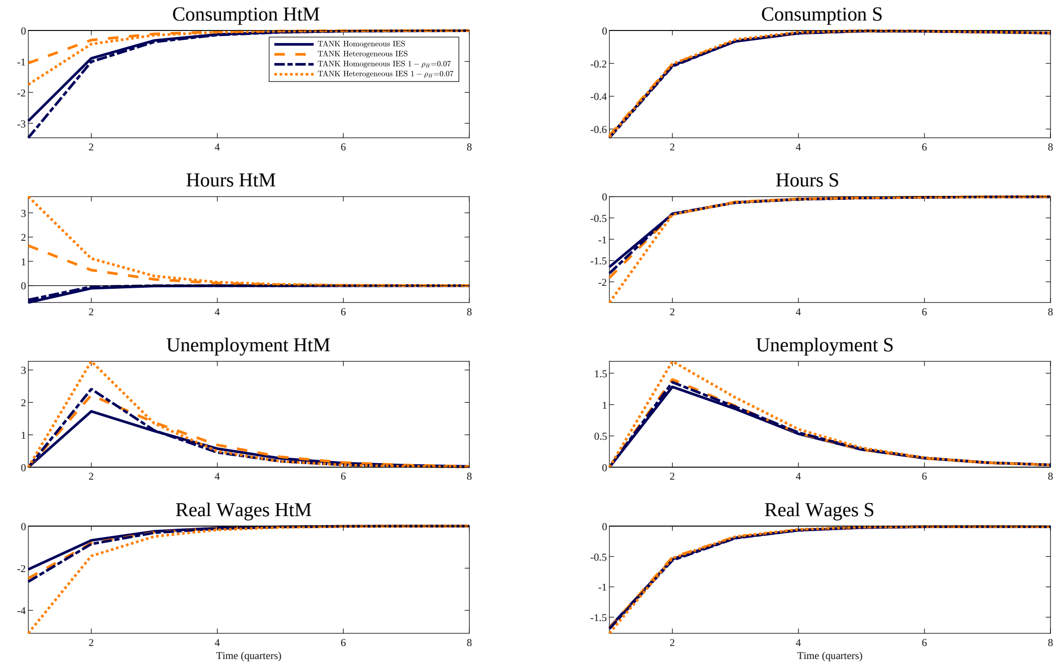

Figure J.5: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $1 - \rho_H = 0.07$ (dashed dotted blue line), and TANK with heterogeneous IES and $1 - \rho_H = 0.07$ (dotted orange line).

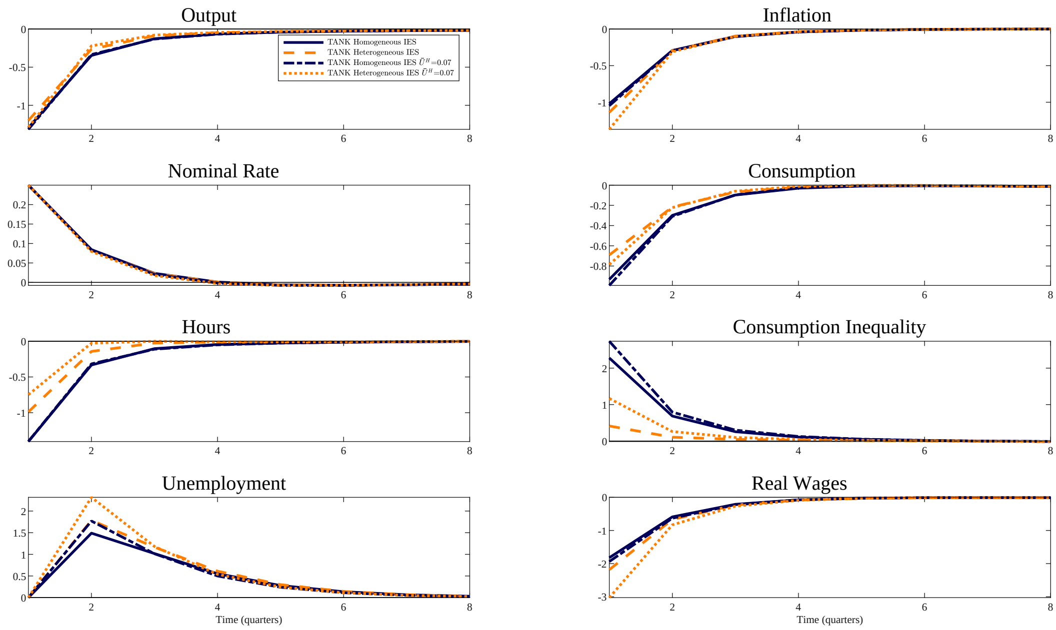

Figure J.6: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $\bar{U}^H = 0.07$ (dashed dotted blue line), and TANK with heterogeneous IES and $\bar{U}^H = 0.07$ (dotted orange line).

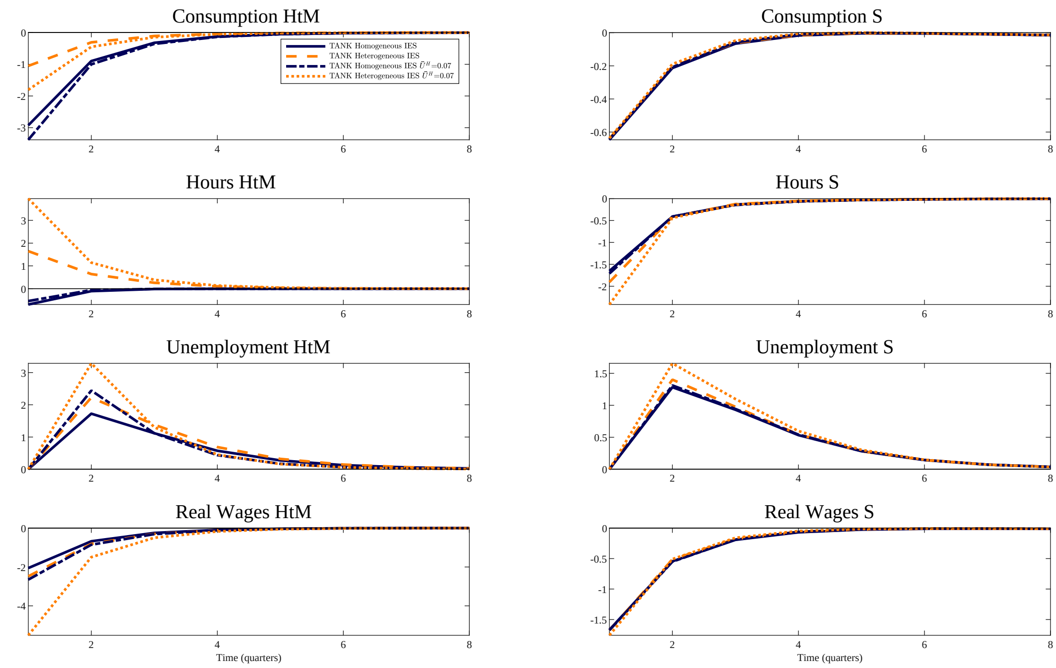

Figure J.7: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $\bar{U}^H = 0.07$ (dashed dotted blue line), and TANK with heterogeneous IES and $\bar{U}^H = 0.07$ (dotted orange line).

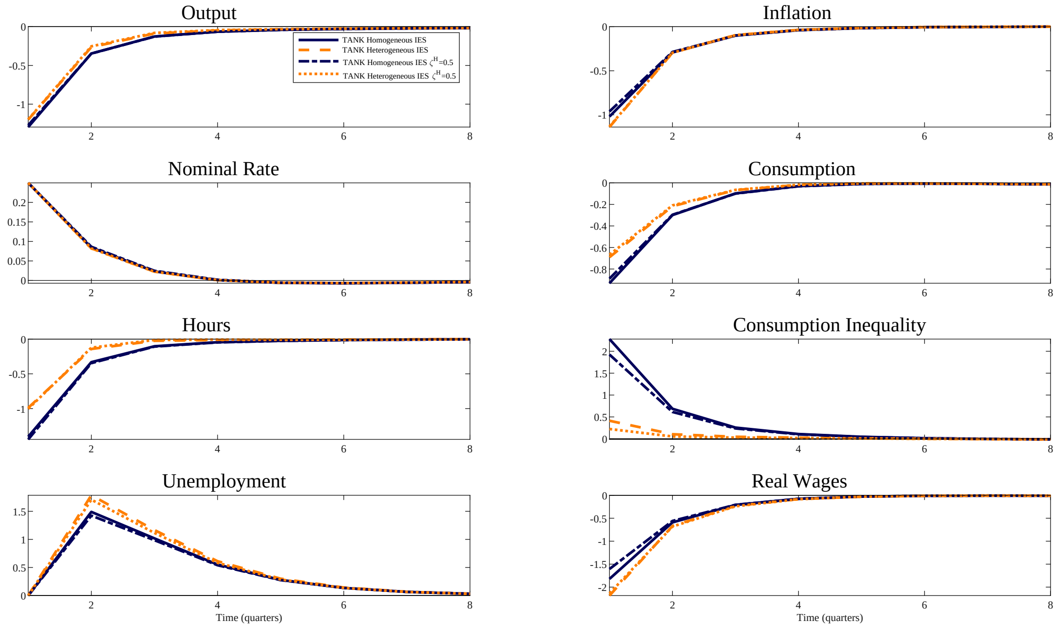

Figure J.8: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $\zeta^H = 0.5$ (dashed dotted blue line), and TANK with heterogeneous IES and $\zeta^H = 0.5$ (dotted orange line).

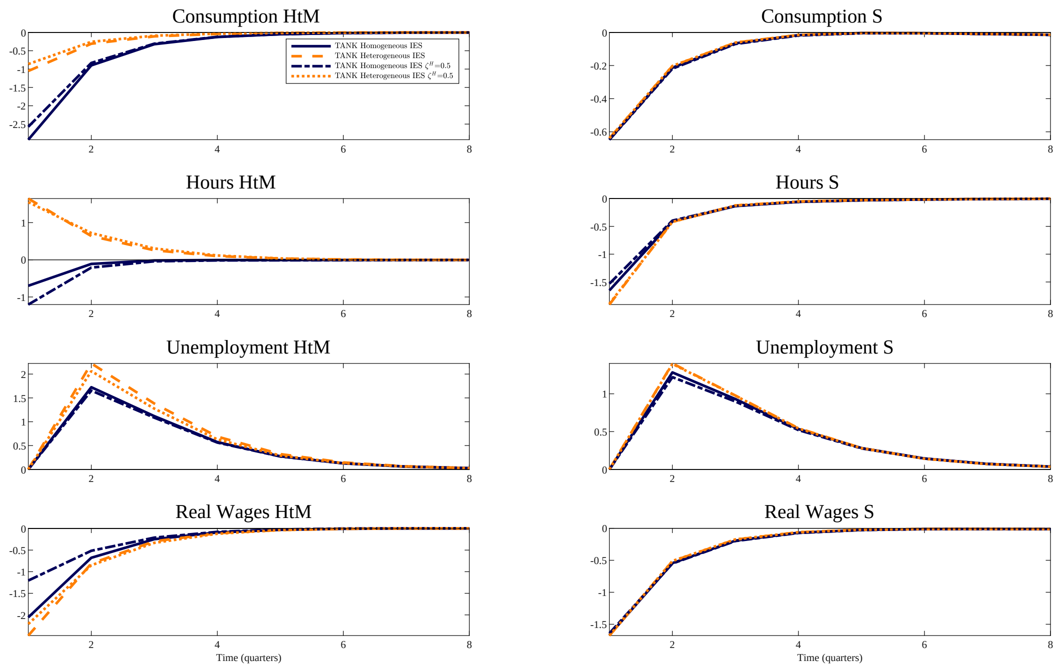

Figure J.9: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), TANK with heterogeneous IES (dashed orange line), TANK with homogeneous IES and $\zeta^H = 0.5$ (dashed dotted blue line), and TANK with heterogeneous IES and $\zeta^H = 0.5$ (dotted orange line).

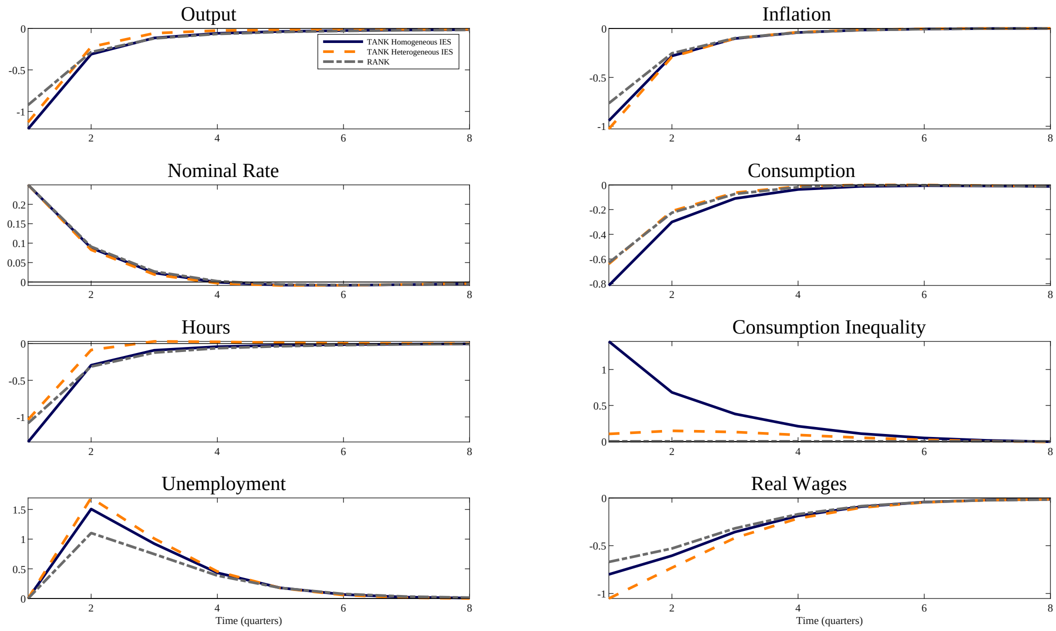

Figure J.10: Impulse responses of selected variables in the TANK-SAM model under different calibrations: RANK (grey dashed-dotted line), TANK with homogeneous IES (solid blue line), and TANK with heterogeneous IES (dashed orange line).

J.10 Robustness: Real wage rigidities

Following Monacelli et al. (2010) we extended the model to incorporate real wage rigidity. We do this through a simple wage adjustment rule. Recall that w_t^j is the wage determined by the Nash bargaining solution as before. Let's call the actual wage, $w_t^{a,j}$, and assume that it id determined following the rule:

$$w_t^{a,j} = (1 - \theta_w^j) w_t^j + \theta_w^j w_{t-1}^{a,j}, \tag{J.114}$$

where θ_w^j is a partial adjustment parameter that reflects the degree of wage rigidity. When $\theta_w^j = 0$ the actual wage corresponds to the Nash bargained wage and we recover the baseline case.

The steady state of the model is unchanged with $w^{a,j} = w^j$.

Figures and Table below replicate the one in the main text with $\theta_w^H = \theta_w^S = 0.5$. Sticky wages reduce the impact of IES heterogeneity on the amplification of aggregate demand and unemployment however qualitatively and quantitatively the main takeovers of our analysis still apply.

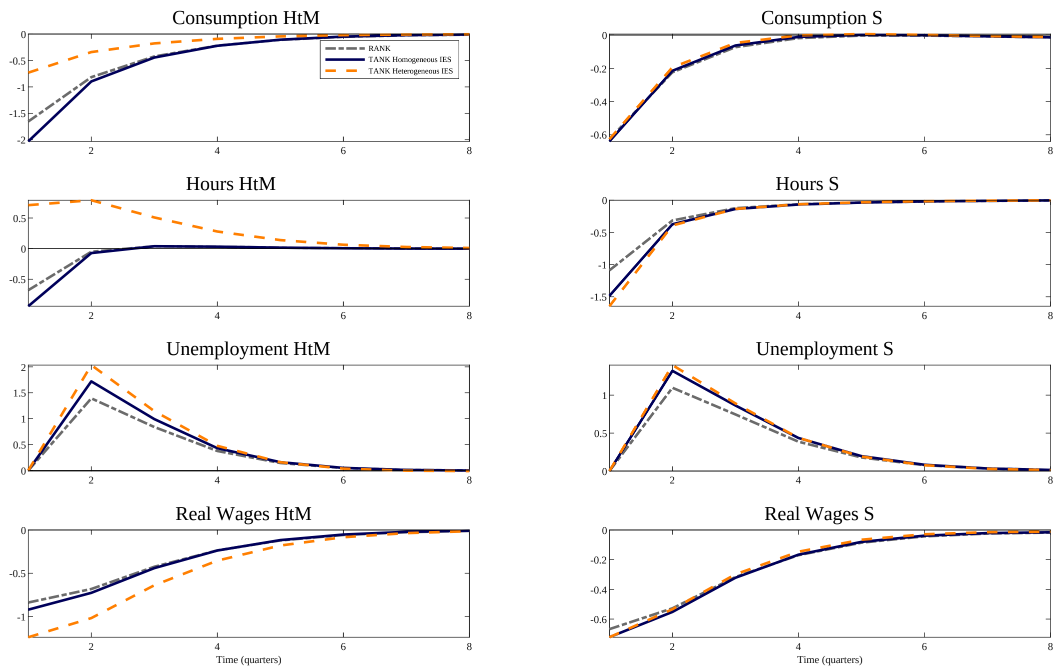

Figure J.11: Impulse responses of selected variables in the TANK-SAM model under different calibrations: TANK with homogeneous IES (solid blue line), and TANK with heterogeneous IES (dashed orange line).

Table J.4: Multipliers relative to the homogenous IES case.

Output	**Consumption**	**Inflation**	**Hours**	**Unemployment**
		Impact multipliers		
0.89	0.75	1.04	0.74	1.07
		Cumulative multipliers		
0.79	0.72	1.01	0.52	1.02