Policy Persistence and Economic Reform

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Abstract

This paper examines the impact of adjustment costs on the implementation of market-oriented reform programs. I consider the problem of a (benevolent) policy-maker who can introduce efficiency-enhancing changes throughout time to increase the rate of economic growth by inducing, via monetary compensations, reassignments of a scarce resource from low to high productivity agents. Without adjustment costs, aggregate productivity gains are sufficiently large, despite the asymmetry of information between agents and government, to permit the implementation of pure market mechanisms with unanimous support from the population. In the presence of adjustment costs, the government is required to increase compensation to all productivity agents to induce them to adopt any policy change. Since adjustment costs interact with incentives, introducing pure market mechanism creates too onerous financial burdens in the economy. I show that, under some mild assumptions, the policy-maker prefers to introduce a *transition* reform program that exhibits some policy persistence, instead of a full market reform. In a transition policy, persistence is concentrated in intermediate productivity agents for whom the interaction of adjustment costs and individual incentives is most perverse. I characterize the optimal dynamic policy changes and show that persistence is "a feature, not a bug" of the optimal economic reform program.

Keywords Market-Oriented Reforms; Policy Persistence; Adjustment Costs; Productivity Gains.

The weight of the past has sometimes been more present than the present itself. And the repetition of the past has sometimes seemed to be the only foreseeable future.

Enrique Krauze, *Mexico: Biography of Power*

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1 Introduction

This paper examines the impact of adjustment costs on the implementation of market-oriented reform programs. I consider the problem of a (benevolent) policy-maker who can introduce efficiency-enhancing policy changes throughout time. The purpose of the reform program is to increase the rate of economic activity in a given sector by inducing, via monetary compensations that incentivize individual decisions, reassignments of a scarce resource from low to high productivity agents. I characterize the optimal dynamic reform program and show that, while in the long run policies converge to a market mechanism, persistence in the initial phase of the reform is 'a feature, not a bug' of the optimal program.

Previous research on the persistence of economic policies focuses on political economy issues (Alesina and Drazen [2]) or uncertainty about the distributional gains and loses of a reform (Fernandez and Rodrik [9]).[1] Yet even in cases where policies have lost their original rationale and their ineffectiveness has been exposed —so, even in cases where there are clear net societal gains from moving away from the current situation— moving away from an inefficient policy is not simple. Coate and Morris [5] argue that agents respond to the introduction of an economic policy by investing to benefit from it, thereby increasing their future willingness to pay once the policy is introduced. Their model predicts that, as a consequence, in equilibrium a policy is either implemented in every period, or not at all. But policies do change, and different countries at different times experience a drive towards the introduction of market-oriented reforms to modernize their economies. In this context, the pertinent questions are then how does economic reform occurs, and whether or not it contains elements of policy persistence.

To address this issue, I propose a simple dynamic framework to analyze economic reform programs in a sectorial model of an economy. The policy-maker has control over the allocation of a key factor of production, which to fix ideas is termed capital. There is a continuum of capacity constrained agents with different productivities who, upon renting up to one unit of capital, transform it using a linear production function whose returns depend on their individual productivity coefficients. For simplicity, and to focus on the impact of efficiency-enhancing policy changes on the rate of economic growth, I assume a fixed savings rate in this economy. The starting point of my model is the existence of an inefficient policy that assigns the same amount of capital to all agents, irrespective of their productivity coefficients. I do not discuss the reasons behind the presence of this inefficient initial policy, instead putting attention on whether it will (partly) persist over time. Potential explanations include weak property rights, government neglect of incentive provision problems, corruption and related political economy issues, and sectorial disruption created by technological innovation or shocks in the flow of international trade. The savings and depreciation rates are assumed to be such that, without any policy change, there is no capital accumulation and hence no economic growth. In other words, without economic reforms the economy is stagnant.

The policy-maker tries to increase average productivity by shifting the allocation of resources from low to high productivity agents. Because productivity coefficients (sector specific abilities) are privately known by these agents, the policy-maker aims at introducing market-oriented policy

[1] For a broader perspective on institutional persistence, see Acemoglu and Robinson [1].

changes that use monetary transfers (prices, cash subsidies) to guide individual decisions. To study the optimal design of a reform program, i.e., a sequence of policy changes introduced and sustained in time, I use tools and concepts borrowed from the mechanism design literature. To take into account the fact that reforms are long term processes that, in reality, do not occur in an institutional vacuum, I assume that policy changes are proposed and require approval at each period of time. If a proposed policy is vetoed, the previous period policy remains in place.

Neither the fact that the previous policy becomes the current status quo, nor the need to incentivize agents, prevents the policy-maker to introducing a market mechanism in the first period of the reform, and expand market reach thereafter. Thus, without additional frictions, persistence does not occur in equilibrium. The main novelty in my model of economic reform is the explicit incorporation of adjustment costs under the simple premise that meaningful changes in the way resources are allocated or utilized carry temporary frictions. To operationalize this novel element, I assume that individual adjustment costs are proportional to the difference between the current capital allocation of an agent and the allocation received in the immediately preceding period. In equilibrium, adjustment costs are triggered only when an agent changes the amount of capital utilization in the production process. In particular, a displaced agent adjusts only during the period exit from the sector occurs. But the agent also incurs in temporary frictions if, and when, reincorporation occurs. The presence of adjustment costs implies a bias against changes in the allocation of resources. In particular, under adjustment costs, an output efficient reform program implemented via a competitive market mechanism is no longer optimal. To avoid obtaining persistence in a mechanical way, by hard-wiring it into the model, I assume that adjustment costs are linear in the productivity coefficients. As a result, any policy persistence present in the optimal reform program responds to the interaction between adjustment costs and individual incentive provision.

While adjustment frictions have been a feature of search and macroeconomic models,[2] these are absent in the mechanism design literature. In the context of economic reform, temporary adjustment costs have several interpretations. They can be interpreted as frictions in labor market mobility, a feature whose importance, both theoretical and empirical, has been highlighted by the recent literature on labor market adjustments, e.g., Autor et al. [3] and Babcock et al. [4]. Switching jobs (or industries) requires reorientation of skills to new circumstances, a practice regularly imposed in the form of retraining programs which depend on (declared) past experiences and occupations.[3] Adjustment costs can be seen as a reduced-form way to capture individual investments in the status quo policy, the value of which is destroyed when there is a policy change; e.g., Coate and Morris [5]. Adjustment costs can be associated with external costs of learning to navigate the complexities of the new situation, for instance by dealing with different legal or regulatory environments, or other sources of friction.[4] Finally, adjustment costs in labor

[2] See Khan and Thomas [12] for an overview.

[3] An example of this is the requirement to be enrolled in a training program to be eligible for the Trade Adjustment Assistance Program (TAA) in the US — see Autor et al. [3].

[4] A concrete example is the bidding design recently implemented by Feeding America to allocate resources to food banks. A sealed-bid auction running twice a day was adopted instead of a continuous auction, to avoid small banks incurring extra costs of dedicating staff to the bidding process — see Prendergast [14].

markets may incorporate some psychological component, responding to a desire for maintaining a previously acquired occupation.[5] To keep the framework as general as possible, my model is on purpose agnostic about which is of them more relevant.

Not surprisingly, the presence of adjustment costs renders impossible for the policy-maker to adopt a full market reform program without running a costly budgetary deficit. To be precise, a switch from the initial inefficient policy, which provided all agents equal access to the factor of production resource, to a market mechanism that sets a market clearing price and compensates displaced agents, cannot be accomplished without running a period by period deficit. Even as the market mechanism generates maximal growth, and maximal capital accumulation over time, the cost of the budget deficits imply that the output efficient reform program is not the optimal (welfare-maximizing) program.

More interestingly, even when it is welfare enhancing to introduce a market-oriented reform, and this can be accomplished with full support of the population by running a budgetary deficit, the presence of adjustment costs alters the pace of the reform. A transition program considers a dual system during the initial phase of the market-oriented reform. Under the dual system, agents with low productivity coefficients are provided cash subsidies to exit the sector, and high productivity agents rent a unit of capital in a regulated market. However, agents with intermediate productivity coefficients are allowed to stay in the initial policy. These intermediate agents neither receive a cash subsidy nor pay a price for the input, but obtain for free a rationed quantity equal to the quantity assigned to them under the inefficient policy. The advantage of using a transition reform program does not hinge on mechanically reducing adjustments for a segment of the population of agents, although this occurs in equilibrium. Introducing a transition reform program is advantageous because it provides more flexibility in the way monetary compensations are designed. To understand this, consider the difference between the introduction of an output efficient policy and a transition policy in the first period. Under the output efficient policy, incentive considerations imply that the price for renting a unit of capital and the cash subsidy are determined to make a single agent indifferent between staying and exiting the sector. Under a transition policy, because a segment of agents are kept at their reservation utility levels (which is determined by the initial policy), the cash subsidy and the market price are determined to make two distinct agents indifferent. As a result, the cash bonus is lower and the market price higher than in the output efficient policy.

The disadvantage of introducing a transition reform program is that, during the first periods, it does not fully exploit efficiency gains. This affects both the growth rate in these periods and, through the accumulation of capital stock, future growth as well. So, even when a transition program yields to the implementation of the market mechanism is implemented in later periods of the reform, growth rates before reaching the steady-state are lower, and the economy may take longer to reach its steady-state. Despite these shortcomings, I show that the optimal economic

[5] The literature on reference dependence in labor markets (e.g., Eliaz and Spiegler [8], Dellavigna et al. [6]) emphasizes a wage-based reference point. However, there is evidence of displaced workers reluctant to change occupations, looking instead for jobs like their previous one even when prospects are better in different sectors. Katz [11] mentions that this form of long-term 'retrospective wait unemployment' specially affects high tenured workers displaced from declining sectors in the economy.

reform exhibits elements of persistence. Moreover, my model can predict precisely the way the initial inefficient institution winds down and the characteristics of market expansion. Markets consolidate over time by gradually incorporating the most productive agents among the segment of the population in the persistent policy. The analytical framework is sufficiently rich to provide predictions of price dynamics. In a typical period of the persistent phase of the optimal reform program, there is a regulated price for a unit of the capital, which goes down over time. In addition, the policy-maker provides a discount for agents who move from the inefficient policy to the market mechanism. As less productive agents make this transition over time, the discount also diminishes; it disappears once capital accumulation reaches a point where it can sustain a full market mechanism.

The main message of my work is not that adjustment costs inhibit the introduction of market mechanisms as a way of solving allocation problems. Rather, it is that in situations where changes in the economic conditions entail temporal frictions, taking them into account alters the way markets are optimally introduced. In particular, a big push towards markets may be welfare dominate by a more gradual approach that exhibits, for at least a segment of agents, for some time, policy persistence.

The rest of the paper is organized as follows. Section 2 contains the model of economic reform. After stating the initial conditions of the dynamic sectorial model of the economy, I explain how policy changes are affected by adjustment costs. This section also describes requirements imposed on the economic reform programs the policy-maker can introduced. In particular, any reform program must be resource feasible, sequentially incentive compatible, and sequential individually rational. Sequential incentive compatibility is complicated by the fact a policy introduced and approved in the previous period acts as the status quo policy in the current one. Nonetheless, a relatively simple characterization of sequentially incentive compatible policies is obtained.

In Section 3 I discuss the output efficient and the optimal policy introduced by a short-sighted policy-maker. I also discuss how persistence, as a feature of the optimal short term policy, is induced by the interaction between adjustment costs and incentive provision. The optimal policy exhibits persistence but can be implemented simply by a cash bonus to agents who decide to exit the sector, and a market clearing price charged to agents who decide to rent one unit of capital. The problem of a far-sighted policy-maker is studied in Section 4. There, the optimal reform program is obtained and characterized, and the dynamics of implementation via prices and cash subsidies is fully discussed. Section 5 presents a few concluding remarks. All the proofs are presented in Appendix A.

2 Model

I construct a multi-period, sectorial model of an economy where the policy-maker has the ability to introduce efficiency enhancing reforms after the initial period. Throughout, individual payoffs are additively separable across time, with no discounting.

This economy is populated by a mass 1 of small agents. At the beginning of period $t = 0, 1, \dots, T$, the policy-maker controls $0 < K_t < 1$ units of a key factor of production. The exact nature of the input is not essential for my purposes; what is important is that the policy-maker has monopoly power over its allocation. To fix ideas, I treat K_t as the amount of (average) *aggregate capital stock* available at time period t.

Each agent constitutes a single production unit with capacity constraints. Upon renting a fraction $k_t \in [0, 1]$ of capital in period t, an agent with *productivity coefficient* θ produces $\theta\, k_t$ units of output for consumption and investment. I normalize the price of the output to 1. To allow for heterogeneity in the production process, θ is a random variable distributed on the interval $\Theta = [\theta_L, \theta_H]$ according to the c.d.f. $F(\cdot)$, with strictly positive, continuous density function $f(\cdot)$. Individual realizations of the productivity coefficient, which remained unchanged throughout time, are privately observed by agents before any interaction with the policy-maker takes place.[6] I let $\theta_L = 0$ and $\theta_H = 1$ but keep the notation of the lower and upper bounds of Θ to avoid confusion. This normalization is not crucial, but simplifies some of the proofs. To avoid unnecessary complications in the implementation of economic reforms under asymmetric information, the following assumption is made throughout the paper.

Assumption 1. The distribution function $F(\cdot)$ has decreasing reverse hazard rate $f(\theta)/F(\theta)$ and increasing hazard rate $f(\theta)/(1 - F(\theta))$; i.e.,

$$\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \;\geq\; 0 \;\geq\; \frac{d}{d\theta}\left(\frac{1 - F(\theta)}{f(\theta)}\right), \qquad \text{for all } \theta \in (\theta_L, \theta_H).$$

Intuitively, the above condition requires that the relative weight of productivity coefficients above θ, in comparison with those below θ, decreases as θ increases.[7] It is convenient to denote the conditional expectations of productivity as follows:

$$A(\theta) \;\equiv\; \mathbb{E}\big[\tilde{\theta} \mid \tilde{\theta} > \theta\big], \qquad \text{for all } \theta \in (\theta_L, \theta_H).$$

Observe that $A(\cdot)$ is an increasing function of productivity coefficients. With this notation, average productivity in the economy is $A(\theta_L)$.

2.1 Initial Conditions

Aggregate capital stock at the beginning of the initial period is $K_0 = K \in (0, 1)$. Here the parameter K provides an inverse measure of capital scarcity in this economy. I take as given that the initial conditions involve inefficiencies in resource allocation, leaving aside the question of exactly how these inefficiencies emerged. A particularly simple way to capture inefficiencies is to consider an initial policy that lends agents the same amount of capital irrespective of their individual productivities, so that each θ-agent rents $k_0(\theta) = K$ units in the initial period. Individual production is equal to $\theta\, K$, and aggregate output is then

$$Y_0 \;=\; \int_{\theta_L}^{\theta_H} \theta\, k_0(\theta)\, dF(\theta) \;=\; A(\theta_L)\, K.$$

[6] It is entirely plausible that abilities or production capacities evolve over extended periods of time. This assumption, albeit a limitation of the model, is imposed to emphasize the dynamic impact of adjustment costs in policy design.

[7] This condition goes back to Lewis and Sappington [13]. It is satisfied by a number commonly used distributions, including the uniform, the exponential and the normal distribution.

The policy-maker does not need to rely on any monetary transfers to implement $k_0(\cdot)$. Indeed, K can be interpreted as the amount of capital that each agent in the economy is entitled to use, for free, in period 0. However, as a way to allocate resources, the initial policy is inefficient.

The stock of aggregate capital in subsequent periods is determined by average savings and capital depreciation. To isolate the source of growth to productivity gains driven by improvements in the utilization of the scarce resources, I consider a fixed saving rate $s \in (0, 1)$ and a fixed capital depreciation rate $\rho \in (0, 1)$. This also allows me to measure of social welfare in terms of output, not consumption. Individual payoffs in the initial period is thus $u_0(\theta) = \theta K$, for all $\theta \in \Theta$. Aggregate capital in period $t = 1, \ldots, T$ follows the accumulation rule

$$K_t = (1 - \rho)K_{t-1} + sY_{t-1}. \tag{1}$$

In this way, savings take the form of undepreciated capital, which is part of individual output of each production unit. The policy-maker collects individual savings in period $t - 1$ to obtain capital stock for period t. It is convenient to assume that the savings rate is not too large in comparison with the depreciation rate; in particular, $sA(\theta_L) = \rho$. With this specification, Equation 1 now yields $K_1 = K$. Readily, one sees that without changes to the initial allocation of resources there is no capital accumulation, and therefore no economic growth.[8] I record this fact in the next result, the proof of which is immediate (thus omitted).

Proposition 1—Stationary Output: *Without changes to the initial allocation policy, aggregate capital in period $t = 1, \ldots, T$ is constant at $K_t = K$, and aggregate output remains stationary at $Y_t = A(\theta_L) K$.*

2.2 Policy Changes under Adjustment Costs

The only source of capital accumulation in this economy is aggregate productivity gains. These can be realized via market-oriented reforms, employing prices and subsidies to guide individual decisions so that more capital ends up with more productive agents. Because individual productivity coefficients are private information, prices need to take into account incentives. I borrow concepts and techniques from the mechanism design literature to explore optimal policy change.

At the beginning of period $t = 1, \ldots, T$, the policy-maker proposes the adoption of a *(market-oriented) allocation policy* $\Gamma_t = \{k_t(\cdot), \tau_t(\cdot)\}$. Here $k_t \colon \Theta \to [0, 1]$ is an allocation function and $\tau_t \colon \Theta \to \mathbb{R}$ is a transfer function. Together they specify the fraction $0 \leq k_t(\theta) \leq 1$ of capital rented to an agent who reports θ in period t and the (net) monetary transfer $\tau_t(\theta)$ received in exchange.[9] To better approximate the introduction of market-oriented reforms, I restrict attention to allocation functions that can be implemented via a system of non-personalized, posted prices. Thus, while the formal treatment is via direct mechanisms, in practice the policy-maker pursues the implementation of economic reforms using a (finite) collection of prices and cash subsidies. Because of this emphasis, I assume that the policy-maker cannot verify individual abilities at the

[8] When $sA(\theta_L) > \rho$ productivity gains in the economy generate *growth acceleration*, not just growth, but the main insights of the model are maintained. Endogenous consumption–savings decisions can be accommodated with additional complications.

[9] Recall the economy is populated by small agents who are subject to capacity constraints.

end of each time period, and therefore cannot condition the allocation at time t on information, truthful or not, collected in previous periods.

The main novelty in my model is the explicit incorporation of frictions in policy design under the premise that meaningful changes in the economic environment involve *costly temporary disruptions*. I operationalize this idea in the following way. Let $k_t(\cdot)$ be the allocation function proposed by the policy-maker in period $t = 1, \ldots, T$, and $k_{t-1}(\cdot)$ the allocation function actually implemented in the previous period. Upon adoption of $k_t(\cdot)$, the payoff of the θ-productivity agent in period t depends on individual output and transfers in an additively separable way. In addition, the payoff includes an *adjustment cost* proportional to the difference between the current and the past allocations.

Adjustment costs admit several interpretations. They can be seen as frictions in labor market mobility (Autor et al. [3] and Babcock et al. [4]). Switching jobs (or industries) requires reorientation of skills to new circumstances, a practice regularly imposed in the form of training programs depended on (declared) past occupations. Thus, even when an agent reports a productivity level other than her own, adjustment costs are incurred if there is a discrepancy between current assignment and perceived past assignment.[10] Adjustment costs can also be associated with firms' external costs of learning to navigate the complexities of the new situation, for instance different legal or regulatory environments. Finally, adjustment costs can be interpreted as a reduced-form approach to capturing individual investments in the status quo policy (Coate and Morris [5]).

To exclude the possibility of obtaining policy persistence as a feature of optimal economic reform in a mechanical way, e.g., by introducing enough concavity in the agents' payoff function, I impose linearity in adjustment costs. Any persistence in the optimal policy will be a consequence of the *interaction* between adjustment frictions and individual incentives. Period t adjustment cost for the θ-productivity agent, when reporting θ', is thus given by

$$\eta\,\theta\ |k_t(\theta') - k_{t-1}(\theta')|\,.$$

Here $0 \leq \eta < 1$ captures the relative weight of the adjustment drag in the overall individual payoff function. The value of this parameter is influenced by institutional, regulatory, technological and even psychological factors. To give an example, a lengthy mandatory training program translates into a larger η, and so on. Note that adjustment costs are increasing on the true productivity level θ. When adjustment costs are instead decreasing on individual productivity, they reinforce the efficiency drive to provide high ability agents with more resources. The more interesting case, and thus the one I focus on, is the opposite. This reflects situations where productivity is sector specific or associated to occupational experience: a higher productivity coefficient thus entails larger switching costs in the event of sectorial displacement.

The total payoff for the θ-productivity agent in period $t = 1, \ldots, T$, when reporting θ', is equal to individual production net of monetary transfers and of the adjustment costs:

$$\theta\, k_t(\theta') + \tau_t(\theta') - \eta\,\theta\,|k_t(\theta') - k_{t-1}(\theta')|.$$

[10] In equilibrium, an agent always reports truthfully and thus adjustment costs will be proportional to the difference between her actual previous and current allocations.

I will work with this utility specification for the remainder of the paper. To simplify notation, let

$$\chi(k_t, k_{t-1}, \theta) \equiv k_t(\theta) - \eta\,|k_t(\theta) - k_{t-1}(\theta)| \tag{2}$$

denote the *adjusted allocation function* for period t. Per period payoffs for the θ-agent can be concisely expressed as adjusted individual production net of transfers:

$$\theta\chi(k_t, k_{t-1}, \theta') + \tau_t(\theta').$$

Remark 1. When frictions are primary psychological, loss aversion may be present in adjustment costs. It is straightforward to incorporate loss aversion into the model. Note that in such case, reference points are backward, not forward, looking. This interpretation seems also pertinent when obtaining zero quantities of the input implies moving out of a given sector or industry and into a new job, which arguably carries more frictions than just increasing production from the past benchmark. The qualitative nature of my results do not depend on loss aversion.

Remark 2. This formulation of adjustment costs implies that the policy-maker can condition period t transfers on the declared allocation for periods t and $t-1$ alone. Thus, it puts a restriction on the space of policies the policy-maker is allowed to choose from. On the other hand, conditioning transfers only on current and immediate past experience highlights the temporary nature of adjustment costs.

2.3 Sequentially Incentive Feasible Reforms

A *(market-oriented) reform program* is a sequence of allocation policies $\Gamma = \{\Gamma_1, \dots, \Gamma_T\}$. To be successfully implemented and sustained through time, a reform program must satisfy certain requirements.

Periodic resource feasibility Because maintaining the initial policy configuration yields capital stock K in every period(Proposition 1), a basic requirement of the reform package is that capital accumulation be non-negative. I also assume that capital availability acts as a hard constraint at every time period. Given $K_t \geq K$ at time t, the allocation function $k_t(\cdot)$ is said to be resource feasible if

$$\int_{\theta_L}^{\theta_H} k_t(\theta)\,dF(\theta) \leq K_t. \tag{3}$$

The reform program Γ is called *periodically resource feasible* if the allocation policy $\Gamma_t = \{k_t(\cdot), \tau_t(\cdot)\}$ specifies a resource feasible allocation function, for every period $t = 1, \dots, T$.

There are many different ways to exhaust aggregate resources. A salient example is the *output efficient allocation function* $k_t^e(\cdot)$, which rents $k_t^e(\theta) = 0$ units to agents with productivity coefficients below a threshold type $\vartheta(K_t) \in \Theta$, and $k_t^e(\theta) = 1$ to agents with productivity levels above $\vartheta(K_t)$. This threshold, which makes Equation 3 bind, is given by

$$\vartheta(K_t) \equiv F^{-1}(1 - K_t). \tag{4}$$

When no confusion arises, I write $\theta_t^e = \vartheta(K_t)$ to denote the threshold of an output efficient allocation function given aggregate capital K_t.

An alternative way to exhaust resources K_t is by employing a *allocation function* $k^c_t(\cdot)$ *with control* θ^c_t defined as follows. It rents $k^c_t(\theta) = 0$ units to agents with productivity levels below $\theta^c_t \in \Theta$, it provides $k^c_t(\theta) = K$ units to agents with productivity coefficients between control θ^c_t and threshold $\zeta(\theta^c_t, K_t) \in \Theta$, and $k^c_t(\theta) = 1$ to agents with productivities above $\zeta(\theta^c_t, K_t)$. This threshold type, which makes the resource constraint in Equation 3 bind, is implicitly defined by

$$\zeta(\theta^c_t, K_t) \;\equiv\; F^{-1}\left(\frac{1 - K_t - K\,F\big(\theta^c_t\big)}{1 - K}\right). \tag{5}$$

I shall write $\theta^d_t = \zeta(\theta^c_t, K_t)$ when there is no risk of confusion. Note that for all $0 < K \leq K_t < 1$,

$$\theta^c_t \;\leq\; \theta^e_t \;\leq\; \theta^d_t,$$

with the second inequality in the above expression binding if, and only if, the control $\theta^c_t = \vartheta(K_t)$. In other words, the output efficient allocation function is a special case of an allocation function $k^c_t(\cdot)$, where the control type θ^c_t is chosen to be equal to the output efficient threshold type θ^e_t.

Sequential incentive compatibility The next requirement imposed on a reform program is that it provides adequate signals to guide individual behavior across time. Policy $\Gamma_t = \{k_t(\cdot), \tau_t(\cdot)\}$ is said to be *incentive compatible given* $k_{t-1}(\cdot)$ if every θ-productivity agent is better off truthfully reporting in period t; i.e., for all $\theta, \theta' \in \Theta$,

$$u_t(\theta) \;\equiv\; \theta\chi(k_t, k_{t-1}, \theta) \;+\; \tau_t(\theta) \;\geq\; \theta\chi(k_t, k_{t-1}, \theta') \;+\; \tau_t(\theta'). \tag{6}$$

The reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ is called *sequentially incentive compatible* if $\Gamma_t = \{k_t(\cdot), \tau_t(\cdot)\}$ is incentive compatible given $k_{t-1}(\cdot)$, for each $t = 1, \ldots, T$. Any sequentially incentive compatible reform program will induce truthful behavior on the equilibrium path. Since off-the-equilibrium individual behavior is inconsequential for aggregate output (except for veto deviations, which I discuss below), the policy-maker offers policy Γ_t after any type profile declaration in the previous period. This is consistent with the assumption that ex-post verification of productivity coefficients, say by direct inspection of individual production, is not feasible.[11]

I use the following characterization of sequentially incentive compatible reform programs.

Proposition 2—Sequential Incentive Compatibility: *The reform program* $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ *is sequentially incentive compatible if, and only if, for all periods* $t = 1, \ldots, T$*, the adjusted allocation function*

$$\chi(k_t, k_{t-1}, \theta) \;=\; k_t(\theta) \;-\; \eta\,|k_t(\theta) - k_{t-1}(\theta)|$$

is non-decreasing in θ *and the indirect utility function* $u_t(\cdot)$ *is expressible as*

$$u_t(\theta) \;=\; u_t(\theta_L) \;+\; \int_{\theta_L}^{\theta} \chi(k_t, k_{t-1}, x)\,dx, \qquad \textit{for all } \theta \in \Theta. \tag{7}$$

Proof. See the appendix. □

[11] For this it matters that the policy-maker plays a game with a continuum of small agents. Individual deviations do not change the aggregate outcome of any stage game.

The proof of Proposition 2 follows standard arguments from the mechanism design literature. Note however that monotonicity is imposed on the adjusted allocation functions. Requiring each allocation function $k_t(\cdot)$ to be weakly increasing is not sufficient to guarantee sequential incentive compatibility of the reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$. The appendix contains a simple counter example to illustrate this point.

Sequential individual rationality Finally, to better approximate the actual conduct of economic policy, in my model reforms are rolled out sequentially. Thus, I assume that the introduction of policy changes requires unanimous support in each time period.[12] Denote by $\hat{u}_t(\theta)$ the reservation utility for the θ-productivity agent in period t, to be defined shortly. A reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ must be *sequentially individually rational*; i.e., for each $t = 1, \ldots, T$,

$$u_t(\theta) \;\geq\; \hat{u}_t(\theta), \qquad \text{for all } \theta \in \Theta. \tag{8}$$

I assume that blocking a proposed policy in period t entails reverting to its immediate predecessor, which acts as the status quo. In other words, the reservation utility of a θ-productivity agent for period t is the payoff associated with maintaining $k_{t-1}(\cdot)$, which can can be attained by vetoing policy Γ_t. The reservation utility for the first period is simply $\hat{u}_1(\theta) = u_0(\theta) = \theta\, K$, since there is no adjustment costs associated with keeping the initial inefficient policy. I write $\hat{\Gamma}_1 = \{k_0(\cdot)\}$ to denote the policy implemented by the policy-maker in period 1 if the proposal Γ_1 is not unanimously approved.

For $t \geq 2$, specifying a period t reservation utility equal to $\theta k_{t-1}(\theta) + \tau_{t-1}(\theta)$ is not possible, as there is no guarantee that $\Gamma_{t-1} = \{k_{t-1}(\cdot), \tau_{t-1}(\cdot)\}$ is incentive compatible given $k_{t-1}(\cdot)$. Indeed, policy Γ_{t-1} was designed to be incentive compatible given k_{t-2} (see Equation 6). To deal with this issue, I define $\hat{\tau}_{t-1}(\cdot)$ as the minimal transfer function satisfying two conditions: it implement $k_{t-1}(\cdot)$ when $k_{t-1}(\cdot)$ acts itself as status quo allocation function and thus no adjustment frictions arise; and it generates a payoff weakly above $u_0(\theta)$, for all $\theta \in \Theta$. Policy $\hat{\Gamma}_t = \{k_{t-1}(\cdot), \hat{\tau}_{t-1}(\cdot)\}$ determines reservation utility $\hat{u}_t(\cdot)$ in period $t \geq 2$. Sequential individual rationality of the reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ is now expressed as

$$u_1(\theta) \;\geq\; \hat{u}_1(\theta) \;=\; \theta\, K \quad \text{and} \quad u_t(\theta) \;\geq\; \hat{u}_t(\theta) \;=\; \theta k_{t-1}(\theta) \;+\; \hat{\tau}_{t-1}(\theta), \quad \text{for all } t \geq 2.$$

To close the model, I need to specify what policy will be proposed at $t + 1$ in the contingency that Γ_t is rejected in period t and $\hat{\Gamma}_t$ is implemented instead. It seems natural to consider that the policy-maker will try to take the reform program off the ground again in the subsequent period. Thus, I assume that upon rejection in period t, policy Γ_t is proposed in period $t + 1$. This specification guarantees that every agent weakly prefers to approve policy changes in every period. In addition, it ensures the continuity of the reservation utility. Thus, I focus on equilibria in the sequential game defined by a reform program Γ that induce full participation in every period and truth-telling equilibrium strategies.

[12]Requiring majority instead of unanimity complicates the analysis. While vital in our understanding of economic reform, political economy issues are orthogonal to the main topic this paper addresses. On the other hand, one can treat Θ as the measure of stakeholders indispensable to introduce any policy change.

Sequential incentive feasibility A reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ is called *sequentially incentive feasible* if it is sequentially incentive compatible, sequentially individually rational, and periodically resource feasible.

2.4 Optimality and Policy Persistence

Because in this economy agents do not make consumption–savings decisions, adjusted output (output net of adjustment costs incurred) reflects individual welfare. The utilitarian social welfare in period $t = 1, \ldots, T$ associated to a reform program Γ is equal to aggregate adjusted output net of any borrowing costs necessary to finance the intervention:

$$SW_t \;=\; \int_{\theta_L}^{\theta_H} \Big\{ \theta\chi(k_t, k_{t-1}, \theta) \;-\; \beta\,\tau_t(\theta) \Big\}\, dF(\theta).$$

Here $0 < \beta < 1$ captures the distortionary effects of raising funds via borrowing or negotiating assistance programs with an international donor.[13]

Relying on Proposition 2, for the remainder of the paper I express sequentially incentive feasible programs in terms of the allocation functions and the (per-period) indirect utility functions: $\Gamma_t = \{k_t(\cdot), u_t(\cdot)\}$, for all $t = 1, \ldots, T$. Replacing the incentive compatible transfers $\tau_t(\theta) = u_t(\theta) - \theta\chi(k_t, k_{t-1}, \theta)$ in the expression for social welfare in period t yields

$$SW_t \;=\; \int_{\theta_L}^{\theta_H} \Big\{ (1+\beta)\,\theta\chi(k_t, k_{t-1}, \theta) \;-\; \beta\,u_t(\theta) \Big\}\, dF(\theta). \tag{9}$$

A reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ is said to be *optimal* if it is sequentially incentive feasible and maximizes inter-temporal social welfare

$$SW \;=\; \sum_{t=1}^{T} SW_t.$$

$\Gamma_t = \{k_t(\cdot), u_t(\cdot)\}$ exhibits *policy persistence* if there is a subset $\Theta' \subset \Theta$ of positive measure for which one has $k_t(\theta) = k_0(\theta) = K$, for all $\theta \in \Theta'$. Any policy that includes an allocation function $k_t^c(\cdot)$ with control $\theta_t^c \neq \vartheta_t(K_t)$ will exhibit policy persistence (see Equation 5).

Remark 3. The choice of technology highlights the effects of adjustment costs on policy persistence. Indeed, under a linear production function and without any sort of frictions, the optimal allocation of resources (under asymmetric information) involves corner solutions. Since adjustment costs are also linear, the presence of persistence in the optimal reform responds to the interaction between adjustment costs and incentives.

3 Short-Run Policy Changes

The dynamic aspects of the economic reform model complicate the analysis. To gain intuition, in this section I focus on changes devised by a short-sighted policy-maker. I investigate the long-run optimal reform program in Section 4.

[13]This borrowing cost is standard in the economic reform literature; e.g., Dewatripont and Roland [7].

3.1 Virtual Productivity and Welfare

Let $\Gamma_1 = \{k_1(\cdot), u_1(\cdot)\}$ be incentive feasible given $k_0(\cdot)$. By Proposition 2, I restrict attention to first-period allocation rules for which the adjusted allocation function $\chi(k_1, k_0, \theta)$ is non-decreasing in the productivity coefficient θ. On the other hand, the participation constraint is endogenous. In particular, when persistence is a feature of the optimal policy change, it binds for a positive measure of types. To handle this, I employ the techniques introduced by Jullien [10] in the mechanism design literature.

Using Equation 7 and integration by parts, I write the policy-maker's objective function as

$$\begin{aligned} SW_1 &= \int_{\theta_L}^{\theta_H} \Big\{(1+\beta)\,\theta\,\chi(k_1,k_0,\theta) - \beta\, u_1(\theta)\Big\}\, dF(\theta) + \int_{\theta_L}^{\theta_H} \beta\, u_1(\theta)\, d\lambda_1(\theta) \\ &= \int_{\theta_L}^{\theta_H} \Big\{(1+\beta)\,\theta\,\chi(k_1,k_0,\theta) + \beta\,\frac{F(\theta)-\lambda_1(\theta)}{f(\theta)}\,\chi(k_1,k_0,\theta)\Big\}\, dF(\theta). \end{aligned}$$

The Lagrange multipliers $\{\lambda_1(\theta)\colon \theta \in \Theta\}$ capture the first period participation constraints. One can show that $\lambda_1(\theta) \geq 0$ for all θ, $\lambda_1(\theta_H) = 1$, and $\lambda_1(\cdot)$ is non-decreasing —that is, $\lambda_1(\cdot)$ is a distribution function, constant whenever the participation constraint is slack, continuous on any interval where it binds.[14] The expression in the brackets of the second line of the above equation has the usual interpretation of virtual welfare, now under endogenous participation *and* adjustment costs. With a linear production process, virtual welfare for the θ-productivity agent is the product of *virtual productivity* $\nu(\theta, \lambda_1(\theta))$ and the adjusted allocation:

$$\left((1+\beta)\,\theta + \beta\,\frac{F(\theta)-\lambda_1(\theta)}{f(\theta)}\right)\chi(k_1,k_0,\theta) =: \nu\,(\theta,\lambda_1(\theta))\,\chi(k_1,k_0,\theta). \tag{10}$$

Virtual productivity captures both incentive and participation constraints, the last one via the Lagrange multipliers. Equation 10 makes explicit the fact that any feature of optimal policy design, including persistence, responds to the interaction between incentive constraints, participation constraints, and adjustment costs. I record some useful properties of the virtual productivity function.

Lemma 1—Properties of Virtual Productivity: *The function* $\nu(\cdot,\cdot)$ *defined on* $\Theta \times [0,1]$ *by*

$$\nu\,(\theta,\lambda) = (1+\beta)\,\theta + \beta\,\frac{F(\theta)-\lambda}{f(\theta)}$$

satisfies the following:

(a) $\nu(\cdot,\lambda)$ *is continuous and strictly increasing in* θ*, for all* $\lambda \in [0,1]$.
(b) $\nu(\theta,\cdot)$ *is strictly decreasing in* λ*, for all* $\theta \in \Theta$.
(c) For each $\lambda \in [0,1]$*, there exists a unique type* $\theta(\lambda) \in \Theta$ *such that* $\nu(\theta(\lambda),\lambda) = 0$.
(d) Moreover, $\theta(\cdot)$ *is increasing in* λ *with* $\theta_L = \theta(0) < \theta(1) < \theta_H$*, where type* $\theta(1)$ *is implicitly defined by*

$$\theta(1) = \frac{\beta}{1+\beta}\,\frac{1-F(\theta(1))}{f(\theta(1))}.$$

[14] See Jullien [10], and Seierstad and Sydsaeter [15] for further details.

Proof. See the appendix. □

To avoid optimal displacement of low productivity agents purely due to incentive issues, henceforth attention is restricted to the case of acute capital scarcity as a starting point in the economy.

Assumption 2. Let $0 < \beta < 1$, $0 < K < 1$ and $F(\cdot)$ be such that $\theta(1) < \vartheta(K)$.[15]

3.2 Optimal Policy Changes

Given aggregate capital stock $K_1 = K$ $(= K_0)$, the output efficient allocation function $k_1^e(\cdot)$ with threshold type $\theta_1^e = \vartheta(K_1)$ generates maximal growth:

$$Y_1^e \;\equiv\; \int_{\theta_L}^{\theta_H} \theta\, k_1^e(\theta)\, dF(\theta) \;=\; A\left(\theta_1^e\right) K_1 \;>\; A(\theta_L)\, K_0 \;=\; Y_0.$$

Without frictions in the economy, the output efficient intervention would be implemented by establishing a competitive market where highly productive agents rent a unit of the capital input at a market clearing price determined by resource availability. Less able agents find themselves priced out of the market but receive a cash subsidy as compensation (so that the individual rationality constraints are satisfied).

This logic applies, with some modifications, to a setting with adjustment costs. Here the corresponding adjusted allocation function takes values

$$\chi(k_1^e, k_0, \theta) \;=\; -\,\eta\, K \quad \text{for } \theta < \theta_1^e \quad \text{and} \quad \chi(k_1^e, k_0, \theta) \;=\; 1 - \eta + \eta\, K \quad \text{for } \theta \geq \theta_1^e. \tag{11}$$

It follows by Proposition 2 that $k_1^e(\cdot)$ is incentive compatible given $k_0(\cdot)$. To implement $k_1^e(\cdot)$ with minimal financing costs the policy-maker chooses indirect utilities $u_1^e(\cdot)$ so that the participation constraint binds only at threshold type θ_1^e, and is slack everywhere else. The *output efficient policy* $\Gamma_1^e = \{k_1^e(\cdot), u_1^e(\cdot)\}$ is therefore incentive feasible (it satisfies resource feasibility by construction). I show in the appendix that the indirect utility $u_1^e(\cdot)$ associated with the implementation of policy Γ_1^e is strictly decreasing for all $\theta < \theta_1^e$ and strictly increasing for all $\theta > \theta_1^e$ —this is illustrated in Figure 1. In other words, intermediate productivity agents are mostly affected by the adjustment costs triggered by this policy intervention.

In particular, since the participation constraint of agents with productivity coefficients below θ_1^e is slack, their Lagrangian multipliers are equal to a constant; call it λ_1^e. From Lemma 1, the virtual productivity of types between $\theta(1)$ and θ_1^e is positive, which means these agents exhibit a negative virtual welfare:

$$\nu(\theta, \lambda_1^e)\, \chi(k_1^e, k_0, \theta) \;=\; -\,\nu(\theta, \lambda_1^e)\, \eta\, K \;<\; 0, \qquad \text{for all } \theta(1) \leq \theta \leq \theta_1^e.$$

The monetary compensations to all low productivity agents must be equal to the compensation received by the threshold agent θ_1^e. This compensation is constructed to make him indifferent between fully exiting the sector and renting a unit of the capital input at the market clearing price. But either option involves a large adjustment cost for agent θ_1^e, and thus is expensive to finance.

[15] Assumption 2 does not impose stringent conditions. For instance, when $F(\cdot)$ is the uniform distribution on $[0, 1]$ and $K = 1/2$, it holds for any $0 < \beta < 1$.

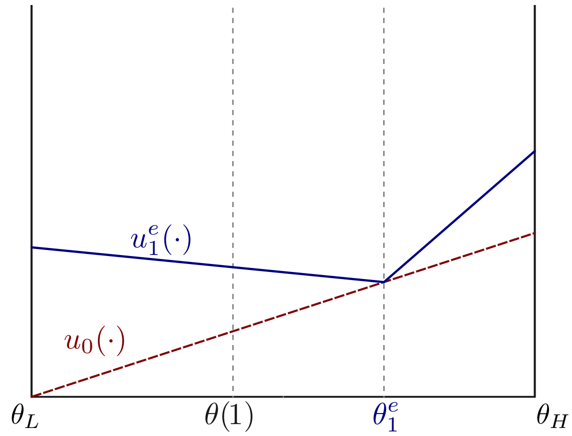

Figure 1. Indirect utilities $u_0(\cdot)$ and $u_1^e(\cdot)$.

The immediate implication is that the output efficient policy Γ_1^e requires substantial external financing.

Aiming at lowering the budget deficit, the policy-maker can reassign resources to agents with productivity coefficients lower than, but sufficiently close to θ_1^e to reduce their virtual welfare loses. Such change also lowers participation costs. Because capital is scarce, it also requires reducing the allocation of types immediately to the right of θ_1^e. In addition, any such rearrangement has to be accomplished in a way that preserves incentive compatibility given $k_0(\cdot)$. There are many ways to accomplish such objective, some involving complex allocation rules. A particularly simple one is by means of an allocation function $k_1^c(\cdot)$ with control θ_1^c and threshold $\theta_1^d = \zeta(\theta_1^c, K_1)$ that specifies

$$k_1^c(\theta) = 0 \quad \text{for } \theta < \theta_1^c, \qquad k_1^c(\theta) = 1 \quad \text{for } \theta > \theta_1^d,$$
$$\text{and} \quad k_1^c(\theta) = K \quad \text{for } \theta_1^c \le \theta \le \theta_1^d.$$

Using Equation 2, the adjusted allocation function corresponding to $k_1^c(\cdot)$ takes values

$$\chi(k_1^c, k_0, \theta) = -\eta K \quad \text{for } \theta < \theta_1^c, \qquad \chi(k_1^c, k_0, \theta) = 1 - \eta + \eta K \quad \text{for } \theta > \theta_1^d,$$
$$\text{and} \quad \chi(k_1^d, k_0, \theta) = K \quad \text{for } \theta_1^c \le \theta \le \theta_1^d. \tag{12}$$

Since $\chi(k_1^c, k_0, \cdot)$ is non-decreasing in θ, it follows that $k_1^c(\cdot)$ is incentive compatible given $k_0(\cdot)$. Resource feasibility is satisfied by construction; what remains to show is individual rationality. The reservation utility and Equation 7 for period 1 pin down the indirect utility function $u_1^c(\cdot)$ associated with $k_1^c(\cdot)$:

$$u_1^c(\theta) = \begin{cases} \theta_1^c(1+\eta)K - \theta\eta K & \text{for } \theta_L \le \theta < \theta_1^c, \\ \theta K & \text{for } \theta_1^c \le \theta \le \theta_1^d, \\ \theta(1-\eta+\eta K) - \theta_1^d(1-\eta)(1-K) & \text{for } \theta_1^d < \theta \le \theta_H. \end{cases} \tag{13}$$

Readily, agents with productivity coefficients between θ_1^c and θ_1^d are at their reservation utility, which is equal to their payoff from the initial period policy; everyone else is strictly better off. A

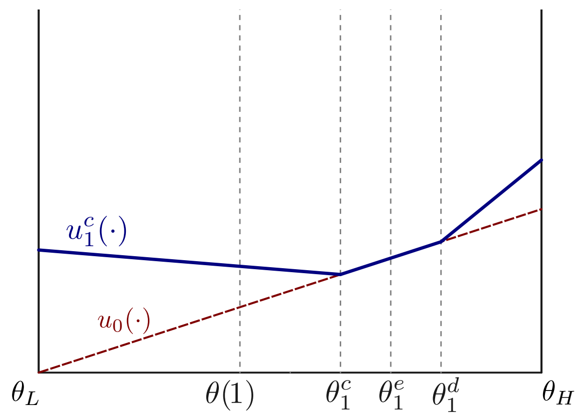

Figure 2. Indirect utilities $u_0(\cdot)$ and $u_1^c(\cdot)$.

transition policy $\Gamma_1^c = \{k_1^c(\cdot), u_1^c(\cdot)\}$ is thus incentive feasible. It exhibits persistence for intermediate productivity agents whenever $\theta_1^c \neq \vartheta(K_1)$, and thus $\theta_1^c < \theta_1^e < \theta_1^d$ —see Figure 2.

The implementation of a transition policy is not more complex than the implementation of Γ_1^e. The difference is that, while the latter uses a market clearing price mechanism, the former involves a combination of market segmentation and regulated prices. The upper segment of the market phases no rationing but rents the capital good at a price above the competitive price. The intermediate segment of the market pays a zero price, but is effectively rationed to rent only K units. The lower range is effectively priced out of the market.

Proposition 3—Implementation (short-run): *The output efficient policy Γ_1^e can be implemented via a cash bonus b_1^e paid to agents who voluntarily decide to exit the sector; and a market clearing price p_1^e charged to agents who rent one unit of capital.*

A transition policy Γ_1^c with control $\theta_1^c < \vartheta(K_1) = \theta_1^e$ and threshold type $\theta_1^d = \zeta(\theta_1^c, K_1)$ can be implemented via a cash bonus b_1^c paid to agents who leave the sector, and a regulated price p_1^c charged to agents who rent one unit of capital. All other agents rent K units of capital for free.

These prices/subsidies satisfy:

$$b_1^c \ = \ \theta_1^c(1+\eta)K \ < \ \theta_1^e(1+\eta)K \ = \ b_1^e,$$
$$p_1^c \ = \ \theta_1^d(1-\eta)(1-K) \ > \ \theta_1^e(1-\eta)(1-K) \ = \ p_1^e.$$

Proof. Immediate from the indirect utility function $u_1^c(\cdot)$ in Equation 13 and the indirect utility function $u_1^e(\cdot)$ in Equation 19 (in the appendix). □

In the short-run, the policy-maker will favor introducing a regulated market associated with a transition policy, over a competitive market associated with the output efficient policy.

Proposition 4—Existence (short-run): *There exists a transition policy Γ_1^c that exhibits persistence and strictly welfare dominates the output efficient policy Γ_1^e in period* 1. *It also strictly welfare dominates implementing the initial policy Γ_0.*

Proof. See the appendix. □

The proof of Proposition 4 is somewhat involved, but the intuition is neat. The policy-maker considers the trade-off between lowering adjustment costs and hurting output efficiency. For a control type θ_1^c sufficiently close, but not equal to θ_1^e, this trade-off pays off. Aggregate output loses are small compared to the welfare gains from lowering the budget deficit, which come from two sources. The exit bonus paid to all types with productivities below θ_1^c is reduced. At the same time, the price charged to all types with productivity coefficients above θ_1^d is increased. This extra flexibility introduced by a transition policy is key to the result.

Transition policies may be only one way to achieve welfare improvements over the initial policy. For instance, in addition to excluding the lowest segment of the population and fully integrating into a market the highest productivity agents, the policy-maker could assign an amount of capital $0 < K' < K$ to low productivity agents, K intermediate productivity agents, and $K < K'' < 1$ to high productivity agents, in a why that respects resource feasibility. It turns out that this additional complexity in policy intervention does not translate into welfare gains. The main result of this section is the following proposition.

Proposition 5—Optimal Policy Change (short-run): *The optimal policy for period* 1 *is a transition policy* $\Gamma_1^* = \{k_1^*(\cdot), u_1^*(\cdot)\}$ *that exhibits persistence; i.e., it is characterized by a control type* θ_1^* *such that*

$$\theta_L \; < \; \theta_1^* \; < \; \vartheta(K_1) \; < \; \zeta(\theta_1^*, K_1) \; < \; \theta_H.$$

Proof. See the appendix. □

I stress that transition policies outperform the competitive market mechanism not because they reduce adjustment costs to a segment of the population, although this does occur. A transition policy outperforms the competitive market intervention because it lowers the information rents that intermediate productivity agents can extract, by keeping them at their reservation utility. In doing so, it also reduces the information rents that all low productivity and high productivity agents are able to extract in equilibrium. Thus, persistence is a feature of the optimal short-run policy. In the long run, however, allocative efficiency issues become important, as lower aggregate output in period 1 translates into lower aggregate resources for the future.

4 Long-Run Economic Reforms

In the long run the design of a market-oriented reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ is complicated by inter-temporal dependencies between policies: $\Gamma_{t-1} = \{k_{t-1}(\cdot), u_{t-1}(\cdot)\}$ affects adjustment costs and reservation utility of period t via the allocation function $k_{t-1}(\cdot)$, which acts now as a status quo allocation in the determination of adjustment costs. An additional consideration is that capital endowment in period t is endogenously determined in previous stages of the reform. Thus, there is tension between static and dynamic welfare maximization. From a static perspective, policy persistence reduces adjustment costs. But persistence hinders capital accumulation and hence long term growth.

4.1 The Output Efficient Reform Program

As benchmark, consider the performance of the output efficient reform program, denoted by $\Gamma^e = \{\Gamma_1^e, \ldots, \Gamma_T^e\}$, which introduces output efficient policies in every period and never exhibits persistence.

Let $K(1)$ denote the amount of capital that would be required to implement the output efficient allocation function with threshold type is $\theta(1)$; i.e., from Equation 4

$$K(1) \;=\; 1 \;-\; F(\theta(1)).$$

A consequence of Lemma 1 is that $\theta(1)$ is the lowest productivity coefficient that can support an output efficient allocation function. Therefore $K(1)$ is the maximal amount of capital stock supported in this economy. While additional capital would allow the market to expand beyond the lower bound $\theta(1)$, doing so while preserving incentive feasibility generates a negative virtual surplus for agents with productivity coefficients below $\theta(1)$, destroying welfare gains.

With this in mind, I can decompose the reform program Γ^e in two phases. In the *expansionary phase*, a market for the capital good is created and then expanded throughout time. The economy grows at its maximal rate, accumulating capital in each period. Capital accumulation (eventually) reaches the maximal level $K(1)$ in some period, say $T^e \leq T$. Afterwards, the reform program enters its *steady-state phase*, where there is no additional capital accumulation and aggregate output becomes stationary.

To define Γ^e formally, and somewhat abusing notation, let now $K_1^e = K$ denote the stock of capital at the beginning of the reform. In the first period, Γ_1^e prescribes the output efficient allocation function $k_1^e(\cdot)$ given K_1^e. Aggregate capital at the beginning of period $t \geq 2$ generated by implementing Γ_{t-1}^e, which I denote by K_t^e, is

$$K_t^e \;=\; \min\left\{ \left[1 \;-\; \rho \;+\; s\,A\left(\vartheta(K_{t-1}^e)\right)\right] K_{t-1}^e,\; K(1)\right\}.$$

Capital is thus accumulated according to the transition rule in Equation 1 as long as capital stock is below $K(1)$, and remains constant thereafter. The output efficient allocation function $k_t^e(\cdot)$ specifies

$$k_t^e(\theta) \;=\; \begin{cases} 0, & \text{for } \theta < \theta_t^e = \max\{\vartheta(K_t^e), \theta(1)\}, \\ 1, & \text{for } \theta \geq \theta_t^e = \max\{\vartheta(K_t^e), \theta(1)\}. \end{cases}$$

Aggregate output from the reform program Γ^e in period $t \geq 1$ is thus

$$Y_t^e \;=\; A\left(\max\left\{\vartheta(K_t^e), \theta(1)\right\}\right) K_t^e.$$

The output efficient program Γ^e implements a market mechanism in every period of the reform. During the expansionary phase, it is incentive feasible to accumulate capital in period $1 \leq t < T^e$. As a consequence, the market expands in period $t+1$ and the threshold type $\theta_{t+1}^e = \vartheta(K_{t+1}^e)$, which acts as a lower bound on market coverage, decreases: $\theta_{t+1}^e < \theta_t^e$. Expansion stops at the end period T^e, a period at which the availability of capital according to the accumulation rule would have exceeded $K(1)$. Thus, $\theta_{T^e}^e = \theta(1)$ is the lower bound for the market in the last period of the

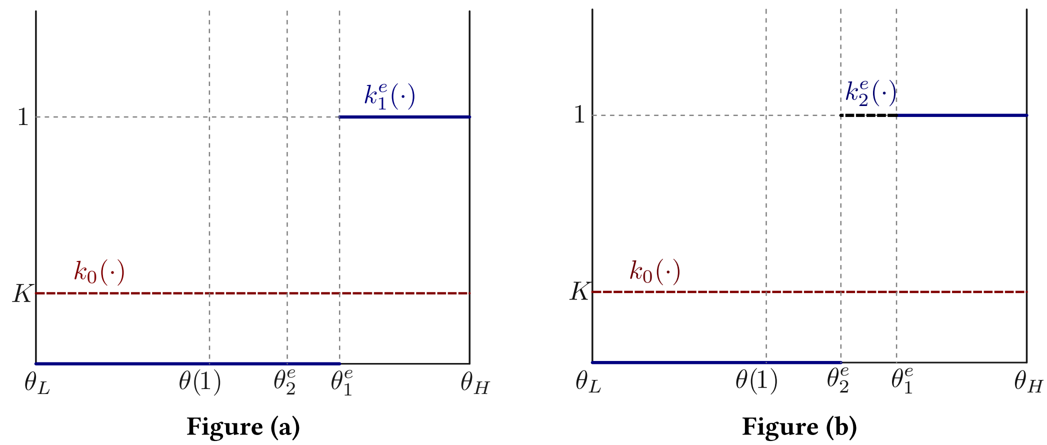

Figure 3. Output efficient allocation functions $k_1^e(\cdot)$, left panel, and $k_2^e(\cdot)$, right panel.

expansionary phase. Now the reform enters the steady-state phase, where capital accumulation stops and market size becomes constant, fixed by threshold type $\theta(1)$.[16]

During the expansionary phase the allocation function in period $t+1$ differs from the allocation function in the previous period only for the fraction of the population incorporated into the market in the current period — agents with productivity coefficients between θ_{t+1}^e and θ_t^e experience temporary adjustment costs. But once incorporated into the market, a θ-productivity agent does not incur in adjustment costs any longer. One can show that the adjusted allocation function $\chi(k_t^e, k_{t-1}^e, \theta)$, is non-decreasing for all $t = 1, \ldots, T$. The per-period indirect utility $u_t^e(\cdot)$ associated with each output efficient policy Γ_t^e is pinned down by the endogenous participation constraint and Equation 7. As it turns out, the policy-maker can set indirect utilities so that, in every period of the expansionary phase, only the participation constraint of the threshold type θ_t^e binds —in the steady-state phase, all agents are at their reservation utility. Because of this, no agent has an incentive ever to veto the adoption of the output efficient reform program. Thus, we obtain the following result.

Proposition 6—Output Efficient Program: *The reform program* $\Gamma^e = \{\Gamma_1^e, \ldots, \Gamma_T^e\}$ *is sequentially incentive feasible and never exhibits policy persistence. Moreover, among all sequentially incentive feasible reforms,* Γ^e *generates maximal growth and therefore reaches the steady-state in the shortest time frame.*

Proof. See the appendix. □

An interesting observation is that the implementation of the output efficient allocation function in periods $2 \leq t \leq T^e$ requires, in addition to a cash bonus and a market price, a discount to be offered to intermediate productivity agents who reenter the sector. To understand this, it suffices

[16] Implicitly, I am considering that savings rate in this case adjust to compensate capital depreciation without accumulating additional capital.

to examine the implementation of policy Γ_2^e, where $K_1^e < K_2^e < K(1)$. Agents with productivity coefficients below threshold $\theta_2^e = \vartheta(K_2^e) < \vartheta(K_1^e) = \theta_1^e$ remain outside the sector and are content receiving a cash bonus b_2^e. This bonus is lower than the first period bonus b_1^e, since the temporary disruption caused by the reform was already accounted for in period 1. High productivity agents with coefficients above θ_1^e, i.e., those who were already renting a unit of the capital input in the previous period, continue doing so in the current period at the full price p_2^e. For agents with productivity levels between thresholds θ_2^e and θ_1^e, renting a unit of the capital good triggers adjustment costs, since these agents were effectively operating outside the sector in the previous period. To reincorporate them, the policy-maker offers a reduced price r_2^e for a unit the capital good to compensate the reallocation costs of these intermediate agents, which must be designed to prevent higher productivity agents to misreport.

Proposition 7—Implementation: *The output efficient reform program* Γ^e *is implemented in the first period by an cash bonus* b_1^e *and a market clearing price* p_1^e *per unit of capital such that*

$$\begin{aligned} b_1^e &= \theta_1^e(1+\eta)K, \\ p_1^e &= \theta_1^e(1-\eta)(1-K). \end{aligned}$$

In subsequent periods of the expansionary phase, i.e., $2 \le t \le T^e$, *the output efficient reform* Γ^e *is implemented by a cash bonus* b_t^e, *a full price* p_t^e, *and a reduced price* r_t^e *offered to those agents who are reincorporated into the sector in period* t *and experience adjustment costs:*

$$\begin{aligned} b_t^e &= \theta_1^e K, \\ p_t^e &= \theta_t^e(1-\eta) + \theta_{t-1}^e\eta - \theta_1^e K, \\ r_t^e &= \theta_t^e(1-\eta) - \theta_1^e K. \end{aligned}$$

In the steady-state phase, i.e., $T^e < t \le T$, *the reform* Γ^e *is implemented by a cash bonus* $b_t^e = \theta_1^e K$ *and a market clearing price* $p_t^e = \theta(1) - \theta_1^e K$.

Proof. See the appendix. □

The presence of two distinct prices for the capital good remains in force in the expansion phase of the market reform, until the economy reaches its steady-state. It will disappear thereafter, leaving policy implementation via a constant cash bonus and a unique market clearing price. So even a radical introduction of the market mechanism to replace inefficient assignment systems requires some governmental intervention, not only in the way of subsidies but as price discounts to a segment of the population of agents. This intervention winds down gradually, as the market expands, becoming moot in the steady state phase.

Corollary 1—Price Dynamics: *The cash bonus goes down in period* 2 *and remains constant for all subsequent periods:*

$$b_1^e > b_t^e, \quad \text{for all } t = 2, \ldots, T.$$

The full price p_t^e *diminishes over time in all periods after period* 2 *until reaching the steady-state, and remains constant thereafter:*

$$p_t^e > p_{t+1}^e, \quad \text{for all } 2 \le t \le T^e, \qquad p_t^e = \theta(1) - \theta_1^e K, \quad \text{for all } t \ge T^e.$$

The reduced price r_t^e decreases over time for all periods after period 2 until the steady-state is reached, and becomes ineffective thereafter:

$$r_t^e \; > \; r_{t+1}^e, \quad \textit{for all } 2 \leq t \leq T^e.$$

Proof. See the appendix. □

Some facts of price dynamics are worth highlighting. First, the long-run cash bonus to agents who exit the sector, $b_t^e = \theta_1^e K$, depends on the availability of initial capital K and the distribution of productivity coefficients — recall θ_1^e is determined via Equation 4, setting $K_1 = K$. Because K and θ_1^e move in opposite directions, it is likely that intermediate levels of capital secure a higher level of cash bonus. This, in turn, would generate a low steady-state rental price of capital.

Second, the sign of the difference between full prices in periods 1 and 2, namely

$$p_2^e \; - \; p_1^e \; = \; (\theta_2^e - \theta_1^e)(1 - \eta) \; + \; \theta_1^e \, \eta \, (1 - K),$$

is ambiguous because it responds to two opposite forces. On the one hand, aggregate capital is more abundant in the second period, which puts a downward pressure on its price. This is captured by the negative sign of the first term of the right-hand side in the expression above. On the other, in equilibrium agents who pay the full market price in period 2 do not suffer any adjustment, and this absence of frictions implies the policy-maker can charge these highly productive agents more for the resource to recover the implicit discount given to them in period 1. This corresponds to the second, positive, term in the above expression.

Third, note that from period 2 onwards the movement of prices responds to two forces pushing in the same direction. As before, more availability of capital puts pressure towards a decrease in price. In addition, the discount offered to lower productivity agents who are re-entering the sector diminishes over time, because less productive agents are incorporated back as time passes by. This implies that the full market price itself will decrease. Put it differently, before the steady state is reached, the reduced price r_t^e paid by agents who are re-incorporated into the sector in period t depends on capital availability, which determines the minimal adjusted productivity gains — the term $\theta_t^e(1 - \eta)$ in the expression for r_t^e. The full price p_t^e takes into account, in addition, a premium charged to agents who already were on the market because, in equilibrium, they don't experience adjustment costs in period t. This premium is limited by the adjusted productivity of the least productive agent among them —the term $\theta_{t-1}^e \eta$ in the expression for p_t^e. Over time, these expressions decrease and the sequence of full prices converges to the steady-state price $\theta(1) - \theta_1^e K$, while the reduce price disappears after period T^e.

4.2 The Optimal Reform Program

Now I turn back to the problem of a long-sighted policy-maker in charge of designing a sequentially incentive feasible reform program $\Gamma = \{\Gamma_1, \ldots, \Gamma_T\}$ to maximize inter-temporal social welfare $SW = \sum_t SW_t$. As before, the design of reform programs is complicated by the fact that the current period policy affects the participation and the incentive constraints in the subsequent periods, in addition to the availability of the aggregate resources.

I argue that it is without loss of generality to focus solely on transition policies, drastically simplifying the policy-maker's problem. This is because any incentive feasible policy Γ_t influences future periods via two channels. The first one is associated to allocative efficiency: a lower (higher) output in period t translates to lower (higher) capital stock available at the beginning of period $t+1$. The second channels is associated with the outside option in period $t+1$, which is determined by allocation function $k_t(\cdot)$. The next result states that for any incentive feasible policy Γ_t it is possible to find a transition policy Γ_t^c that achieves the same aggregate output, without increasing the next period outside option, nor decreasing short term welfare. The linearity of the adjustment costs plays a role here. There is no additional gains or loses in shifting any adjustment between periods t and $t+1$.[17] In reading the statement of the next proposition, recall that an output efficient policy is a special case of a dual transition policy.

Proposition 8—Domination: *For* $t = 1, \dots, T$, *let* $\Gamma_t = \{k_t(\cdot), u_t(\cdot)\}$ *be an incentive feasible policy given* $k_{t-1}(\cdot)$. *There exists a transition policy* $\Gamma_t^c = \{k_t^c(\cdot), u_t^c(\cdot)\}$ *that replicates aggregate output generated by* $k_t(\cdot)$, *weakly lowers the value of the reservation utility in period* $t+1$, *and weakly dominates* Γ_t *in terms of short-term welfare.*

Proof. See the appendix. □

An important implication of Proposition 8 is that the optimal reform program consists of a sequence of transition policies. Indeed, given any sequentially incentive feasible reform program Γ, the policy-maker can replace policy Γ_T with a transition policy Γ_T^c. Welfare in the last period of the reform will not be negatively affected by this change, and may be possibly increased. In period $T-1$, the policy-maker can replace Γ_{T-1} with a transition policy Γ_{T-1}^c. By Proposition 8, aggregate output in $T-1$ will not be negatively affected, hence aggregate capital in period T does not diminish. Moreover, the last period reservation utility resulting from this policy change is weakly lower than in the original program, so that no additional financial resources are necessary to sustain the reform because no extra compensation is needed for agents to unanimously accept the new policy change.[18]

A transition reform program $\Gamma^c = \{\Gamma_1^c, \dots, \Gamma_T^c\}$ may involve three distinct phases.

Persistence phase In the first part of the reform, a regulated market is introduced to partially replace $k_0(\cdot)$, but a fraction of the population keeps their original claims on capital. Aggregate productivity gains generate growth and capital accumulation. Additional capital is used to incorporate less productive agents into the regulated market, leaving a smaller fraction of agents operating in the initial policy. Thus, this phase exhibits policy persistence that gradually diminishes over time. At some period, say $1 \le T^c \le T$, the market is fully consolidated and, thereafter, additional capital is employed to increase market reach leading to the expansionary phase.

Expansionary phase In the second phase of the reform a market mechanism utilizes additional capital to reincorporate low productivity agents who had left the sector in the past. Policy persistence is no longer present, but the policy-maker still intervenes in the market to offer a temporary discount to low productivity agents when they reenter the sector.

[17] My model does not consider an inter-temporal discount factor. Under linearity, this remains true if the discount factor is sufficiently large.

[18] Note that I leave open the possibility that Γ_t in the original program is an output efficient policy, in which case $\Gamma_t^c = \Gamma_t$.

Steady-state phase Market expansion lasts until the economy reaches the steady-state phase, at time $T^c \leq T^s \leq T$, when no additional capital is accumulated and aggregate output becomes stationary. Because of the output inefficiencies associated with the persistence phase of a reform program, reaching the steady-state may take a long time —compared to the time to reach the steady-state under the output efficient program Γ^e.

The transition reform program $\Gamma^c = \{\Gamma_1^c, \ldots, \Gamma_T^c\}$ is formally defined as follows. To keep notation consistent, let $K_1^c = K\ (= K_0)$ denote the stock of capital at the beginning of Γ^c. The first period policy $\Gamma_1^c = \{k_1^c(\cdot), u_1^c(\cdot)\}$ implements an allocation function $k_1^c(\cdot)$ with control θ_1^c and threshold type $\theta_1^d = \zeta(\theta_1^c, K_1^c)$. Observe the policy-maker never chooses a control below type $\theta(1)$. This policy generates aggregate output given by

$$Y_1^c \;=\; \int_{\theta_1^c}^{\theta_1^d} \theta\, K\, dF(\theta) \;+\; \int_{\theta_1^d}^{\theta_H} \theta\, dF(\theta) \;=\; K \int_{\theta_1^c}^{\theta_1^d} \theta\, dF(\theta) \;+\; A\big(\theta_1^d\big)\Big(1 - F(\theta_1^d)\Big)\,.$$

Using Equation 5 and the fact that $K = K_1^c$, the above expression can be written as

$$\begin{aligned} Y_1^c \;&=\; \left[A\big(\zeta(\theta_1^c, K_1^c)\big) \;-\; \int_{\theta_1^c}^{\zeta(\theta_1^c, K_1^c)} \big\{A\big(\zeta(\theta_1^c, K_1^c)\big) \;-\; \theta\big\}\, dF(\theta)\right] K_1^c \\ &\equiv\; \mathscr{A}(\theta_1^c, K_1^c)\, K_1^c. \end{aligned} \tag{14}$$

Aggregate output Y_1^c is determined by the average productivity of the segment of the population that obtains one unit of the resource, discounted by the efficiency loses coming from policy persistence —i.e., allocation of resources to agents between control θ_1^c and threshold $\theta_1^d = \zeta(\theta_1^c, K_1^c)$. Observe the adjusted aggregate productivity $\mathscr{A}(\theta_1^c, K_1^c)$ in Equation 14 depends on the control θ_1^c and the initial level of capital stock. It is clear that, since $k_1^c(\cdot)$ assigns resources more efficiently than $k_0(\cdot)$ and weakly less efficiently than $k_1^e(\cdot)$, aggregate outputs satisfy $Y_0 < Y_1^c \leq Y_1^e$. Equality between Y_1^c and Y_1^e holds if and only if the control type θ_1^c is equal to $\vartheta(K_1^c)$, in which case efficiency loses in the allocation of resources disappear.

Aggregate capital at the beginning of period 2, which I denote by K_2^c, is the minimum of the capital stock resulting from the accumulation process (see Equation 1) and $K(1)$. To ease notation, I assume without loss of generality it is the former, so that

$$K_1^c \;<\; K_2^c \;=\; \big[1 \;-\; \rho \;+\; s\,\mathscr{A}(\theta_1^c, K_1^c)\big]\, K_1^c \;<\; K(1).$$

It follows that there is additional capital to allocate in the second period. I have argued in Proposition 8 that the policy-maker optimally chooses a transition policy in period 2. It can rent the extra units of capital to agents with productivity coefficients left of θ_1^d, who under $k_1^c(\cdot)$ rented only $K < 1$ units, effectively consolidating the market mechanism. But additional capital could be also used to bring back to the sector agents who left in period 1; i.e., agents for whom $k_1^c(\theta) = 0$. The linearity of the payoff function simplify the analysis. To understand intuitively which is the most welfare improving way to allocate the additional capital in period 2 suffices to focus on the

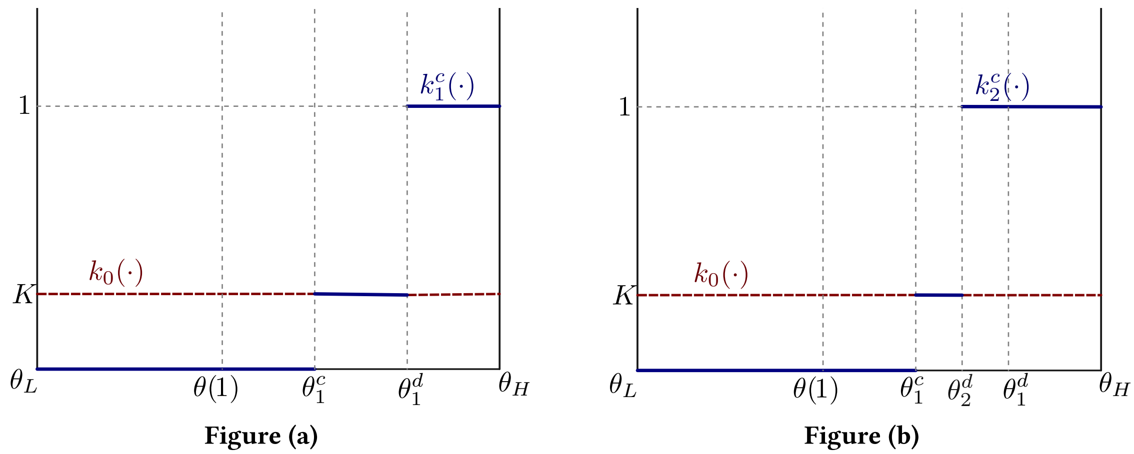

Figure 4. Allocation functions $k_1^c(\cdot)$, left panel, and $k_2^c(\cdot)$, right panel.

adjusted allocation function $\chi(k_2^c, k_1^c, \cdot)$, which incorporates adjustment costs in the social welfare function for the second period.[19]

For agents who received zero capital in period 1, an increase in the allocation determined by $k_2^c(\theta)$ has a constant *marginal* effect of $1-\eta$. For types between θ_1^c and θ_1^d, the *marginal* effect of an increase in the generalized allocation determined by $k_2^c(\theta)$ is $1+\eta$ for $k_2^c(\theta) < K$, and $1-\eta$ for $k_2^c(\theta) \geq K$. Since for all λ, the function $v(\theta,\lambda)$ is increasing function in θ (see Lemma 1), and $\chi(k_2^c, k_1^c, \theta)$ is non-decreasing in θ, it follows that the welfare maximizing allocation of the extra capital available in period 2 entails market consolidation: the allocation function $k_2^c(\cdot)$ is characterized by a control $\theta_2^c = \theta_1^c$ and a threshold type $\theta_2^d = \zeta(\theta_1^c, K_2^c) < \theta_1^d$. This is illustrated in Figure 4a, for the first period allocation function is represented, and Figure 4b, showing the second period allocation function under Γ^c.

The persistence phase of the reform program Γ^c continues in periods $2 \leq t < T^c$ as long as capital stock K_t^c is below $K(1)$ *and* $\theta_1^c < \vartheta(K_t^c)$; i.e., as long as capital is not sufficiently large to support a full market. The policy-maker optimally introduces a transition policy $\Gamma_t^c = \{k_t^c(\cdot), u_t^c(\cdot)\}$ with control $\theta_t^c = \theta_1^c$ and threshold type $\theta_t^d = \zeta(\theta_1^c, K_t^c) < \zeta(\theta_1^c, K_{t-1}^c) = \theta_{t-1}^d$, assigning

$$k_t^c(\theta) \; = \; 0 \quad \text{for } \theta < \theta_t^c = \theta_1^c, \qquad\qquad k_t^c(\theta) \; = \; 1 \quad \text{for } \theta > \theta_t^d,$$
$$\text{and} \quad k_t^c(\theta) \; = \; K \quad \text{for } \theta_t^c \leq \theta \leq \theta_t^d.$$

In words, any additional units of the aggregate resources are exhausted in the transition to a market mechanism. Since in period $2 \leq t < T^c$ the allocation function $k_{t-1}^c(\cdot)$ acts as the status quo triggering adjustment costs, I still need to verify that Γ_t^c is incentive compatible given $k_{t-1}^c(\cdot)$. The generalized allocation function $\chi(k_t^c, k_{t-1}^c, \cdot)$ can be shown to be non-decreasing. By Proposition 2, Γ_t^c is incentive compatible given $k_{t-1}^c(\cdot)$.

[19]One still has to make sure the participation constraint multipliers $\lambda(\theta)$ for that period are chosen in an appropriate way, but is of secondary importance.

To simplify notation, I shall assume that at time T^c capital stock is such that a full market can be established with threshold type $\theta^c_{T^c} = \vartheta(K^c_{T^c}) = \theta^c_1$. Afterwards, the market expands without persistence: $\vartheta(K^c_{T^c+1}) < \theta^c_1$. The second phase of the economic reform thus begins at $T^c + 1$. For all $T^c + 1 \leq t < T^s$ in the expansionary phase, the policy-maker introduces a policy $\Gamma^c_t = \{k^c_t(\cdot), u^c_t(\cdot)\}$ with control type $\theta^c_t = \vartheta(K^c_t)$, assigning

$$k^c_t(\theta) \; = \; 0 \quad \text{for all } \theta < \theta^c_t \quad \text{and} \quad k^c_t(\theta) \; = \; 1 \quad \text{for all } \theta \geq \theta^c_t.$$

Observe that aggregate output in periods $t \geq T^c$ reverts to $Y^c_t = A(\theta^c_t)\, K^c_t$. The more efficient allocation of resources translates, in the expansionary phase of the reform, to an acceleration of the growth rate (per unit of capital). The second phase lasts until capital stock reaches $K(1)$, say at time T^s, at which the reform enters its final stationary phase. For all $T^s \leq t \leq T$, there is no additional capital accumulation and thus aggregate output is stationary at the steady-state level, thus $\theta^c_t = \theta(1)$ for all $t \geq T^s$.

Despite its apparent complexity, the optimal reform program is determined solely via the choice of the first period control θ^c_1. Indeed, θ^c_1 and $K^c_1 = K$ jointly determine threshold θ^d_1, output Y^c_1 and thus capital stock K^c_2. In subsequent periods of the persistence phase, the control is $\theta^c_t = \theta^c_1$, and threshold θ^d_t is jointly determined by θ^c_1 and K^c_t. In the expansionary phase, control θ^c_t is solely determined by capital availability K^c_t. Finally, the control for the steady-state phase is fixed at $\theta^c_t = \theta(1)$. Thus, one can fully characterize the optimal economic reform program via the determination of the first period control θ^c_1.

The main result of this paper is the following proposition.

Proposition 9—Optimal Reform Program (long-run): *The optimal reform program $\Gamma^* = \{\Gamma^*_1, \ldots, \Gamma^*_T\}$ consists of a sequence of incentive feasible transition policies $\Gamma^*_t = \{k^*_t(\cdot), u^*_t(\cdot)\}$ introduced in each period $t = 1, \ldots, T$. This program is characterized by a (unique) control θ^*_1 such that $\theta(1) < \theta^*_1 < \vartheta(K)$, and (potentially) three different phases.*

In the persistence phase, the optimal reform program exhibits capital accumulation and aggregate output growth below the levels associated with the output efficient reform program; i.e.,

$$Y^*_t \; = \; \mathscr{A}\left(\theta^*_1, K^*_t\right) K^*_t \qquad \text{and} \qquad K^*_{t+1} \; = \; \left[1 \; - \; \rho \; + \; s\, \mathscr{A}\left(\theta^*_1, K^*_t\right)\right] K^*_t,$$

for all $1 \leq t < T^c$. Policy persistence affects a segment $[\theta^*_1, \zeta(\theta^*_1, K^*_t)]$ of the population, but diminishes gradually over time: $\zeta(\theta^*_1, K^*_{t+1}) < \zeta(\theta^*_1, K^*_t)$.

Persistence disappears in period T^c and the expansionary phase begins right after. Now the optimal reform program exhibits maximal capital accumulation and maximal growth; i.e.,

$$Y^*_t \; = \; A\left(\vartheta(K^*_t)\right) K^*_t \qquad \text{and} \qquad K^*_{t+1} \; = \; \left[1 \; - \; \rho \; + \; s\, A\left(\vartheta(K^*_t)\right)\right] K^*_t,$$

for all $T^c \leq t < T^s$. The expansionary phase lasts until capital stock in the economy reaches the maximum limit $K(1)$ in period T^s, at which point in time capital accumulation stops and the economy enters the steady-state phase, where output is

$$Y^*_t \; = \; A\left(\theta(1)\right) K(1).$$

Proof. All the statements of the proposition, with one exception, follow from Proposition 8 and the arguments given above. It remains to show that the optimal reform program Γ^* is incentive feasible. I provide the proof of this last part in the appendix. □

Several comments are in order. First, the above proposition makes clear that some kind of persistence will be an element of the optimal reform policy, the degree of which can be measured in terms of the mass of agents who remain in the inefficient policy during the first period; i.e., the interval $[\theta_1^*, \zeta(\theta_1^*, K)] \subset \Theta$. This affects the length of time that the optimal reform will take to attain the expansionary phase and finally to reach the steady-state. How severe is persistence in the optimal program depends on the details of the model. A lengthier persistence phase is related to higher financing costs. Note that the relative duration of the persistence phase is inversely related to the degree of scarcity of the capital good. This is because the policy-maker may be obliged to favor a more efficient first-period policy to take advantage of the larger gains under a lower initial capital stock.

Second, Proposition 9 obtains a sharp characterization of the optimal reform program via the determination of a single parameter value. This implies a strong path dependency in the development of the market oriented reform. Moreover, after the occurrence of any shock that takes the reform away from the path prescribed by θ_1^*, the policy-maker can re-optimize by introducing a transition policy given the last period allocation function. This new path will depend on a (new) control in a way described in Proposition 9. This sharp characterization is an artifact of the linear production technology assumption, and one would expect messier characterizations with different technologies. However, the main message of the proposition, namely that the optimal reform involves policy persistence that gradually disappears over time, remains.

Third, the implementation of the optimal reform program entails setting different prices for the same amount of rented capital. In a typical period of the persistence phase, one has a regulated price paid by all high productivity types who were incorporated into the market in the previous period, and thus do not experience any adjustment cost in the current period. But one also has a discounted price paid by intermediate productivity agents who were kept at the original inefficient policy (those for which persistence is exhibited), as they experience a temporal adjustment cost associated with their increased allocation from K to one unit of capital. Finally, agents kept at the inefficient policy receive K units of capital for free.

Proposition 10—Implementation (long-run): *The optimal reform program Γ^* is implemented in the first period by an cash bonus b_1^* and a regulated price p_1^* per unit of capital such that*

$$\begin{aligned} b_1^* &= \theta_1^*(1+\eta)K, \\ p_1^* &= \zeta(\theta_1^*, K_1^*)(1-\eta)(1-K). \end{aligned}$$

In periods $2 \le t \le T^c$, Γ^* *is implemented by a cash bonus* b_t^**, a regulated price* p_t^**, and a reduced price* r_t^* *offered to those agents who are incorporated to the market mechanism in period* t*:*

$$\begin{aligned} b_t^* &= \theta_1^* K, \\ p_t^* &= \left[\zeta(\theta_1^*, K_t^*)(1-\eta) + \zeta(\theta_1^*, K_{t-1}^*)\,\eta\right](1-K), \\ r_t^* &= \zeta(\theta_1^*, K_t^*)(1-\eta)(1-K). \end{aligned}$$

In periods $T^c < t \le T^s$, Γ^* *is implemented by a cash bonus* b_t^**, a full price* p_t^**, and a reduced price* r_t^* *offered to agents who reenter the sector in period* t*:*

$$\begin{aligned} b_t^* &= \theta_1^* K, \\ p_t^* &= \vartheta(K_t^*)(1-\eta) + \vartheta(K_{t-1}^*)\,\eta - \theta_1^* K, \\ r_t^* &= \vartheta(K_t^*)(1-\eta) - \theta_1^* K. \end{aligned}$$

In the steady-state phase, $T^s < t \le T$*, the reform is implemented by a cash bonus* $b_t^* = \theta_1^* K$ *and a full price* $p_t^* = \theta(1) - \theta_1^* K$*.*

Proof. See the appendix. □

Observe that, as in the case of the output efficient reform program Γ^e, the steady-state price $p_t^* = \theta(1) - \theta_1^* K$ and cash subsidy $b_t^* = \theta_1^* K$ in the optimal reform program Γ^* depend only on the initial capital stock K in the economy and the distribution of productivity coefficients. However, as $\theta_1^* \le \theta_1^e$, one has that the long-run price of capital is above its output efficient level, and the long-run subsidy below its out efficient level. We can obtain additional price dynamics for the optimal reform program, which I summarize below.

Corollary 2—Price Dynamics: *Assume* $0 < \eta < 1/2$*. In the optimal reform program* Γ^**, the cash bonus goes down in period* 2 *and remains constant for all subsequent periods:*

$$b_1^* > b_t^*, \quad \text{for all } t = 2, \ldots, T.$$

The market price p_t^* *decreases in time for all periods* after *period* 2 *until reaching the steady-state in period* T^s*, and remains constant thereafter:*

$$p_t^* > p_{t+1}^*, \quad \text{for all } 2 \le t \le T^s, \qquad p_t^* = \theta(1) - \theta_1^* K, \quad \text{for all } t > T^s.$$

The reduced price r_t^* *decreases in time for all periods* after *period* 2 *until reaching the steady-state at time* T^s*, becoming ineffective thereafter:*

$$r_t^* > r_{t+1}^*, \quad \text{for all } 2 \le t \le T^s.$$

Proof. See the appendix. □

Note that to obtain clear price dynamics after period 2, I have assumed that the utility weight of the adjustment cost is less than one-half. When the weight of the adjustment frictions is sufficiently high, the policy-maker does not need to lower the price of capital in response to more availability from one period to another. Agents who were already renting one unit of capital will accept a slightly higher price and still prefer this to paying the reduced price, because this last option comes with adjustment costs —in the form of mandatory training, for example. As in the case of the output efficient reform, the movement of prices between the first and the second period depends on the parameters of the model, because it responds to two opposite forces. On the one hand, more capital stock puts downward pressure on its price. On the other, agent who rented one unit of capital in period 1 do not suffer adjustment costs in period 2, and thus the rental price could increase.

Finally, note that the effective discount enjoyed by those who enter the market and buy rent one unit of the capital good, in any given period, goes down in time. In a sense, the governmental support for these agents, via access to reduced prices, diminishes as less and less productive agents are incorporated into the market mechanism.

5 Concluding Remarks

I've presented a dynamic framework of economic reform that incorporates adjustment costs to study the persistence of economic policies. The main insight of my model is that, under some conditions, persistence is a feature of the optimal reform program, which exhibits a gradual move away from the inefficient policy and towards market mechanisms. My model emphasizes the role of the government, via regulation of markets and cash subsidies, in implementing efficiency enhancing reforms.

While the model is sufficiently rich to allow for the study of implementation and price dynamics, the exact configuration of the optimal reform, in terms of timing and duration of the persistence and the expansionary phase, depend on the particular details of the model. Also, I have simply assumed away additional economic opportunities outside the main sector of the economy. This is clearly a gross simplification, which misses any dynamics of what happens in different parts of the economy. Similarly, political economy issues have been assumed away. I leave these issues for future study.

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A Appendix

The appendix contain the proofs of the results omitted in the main text. The last section contains an example of a policy that is not incentive compatible, despite considering monotone increasing allocation functions.

A.1 Omitted Proofs

Proof of Proposition 2. The first part is standard, so I focus in showing that $\chi(k_t, k_{t-1}, \cdot)$ non-decreasing in θ and the integral representation of the indirect utility ensure sequential incentive compatibility of the reform program. Using the integral representation, we define incentive transfers $\tau_t(\cdot)$ in period t as

$$\tau_t(\theta) \;=\; u_t(\theta_L) \;+\; \int_{\theta_L}^{\theta} \chi(k_t, k_{t-1}, x)\, dx \;-\; \theta\chi(k_t, k_{t-1}, \theta).$$

With this transfer function, write payoffs of the θ-agent, when reporting θ', as

$$u_t(\theta, \theta' \mid k_t, k_{t-1}, \tau_t) \;=\; (\theta - \theta')\chi(k_t, k_{t-1}, \theta') \;+\; \int_{\theta_L}^{\theta'} \chi(k_t, k_{t-1}, x)\, dx \;+\; u_t(\theta_L).$$

From the monotonicity of the adjusted allocation function follows that

$$u_t(\theta, \theta \,|\, k_t, k_{t-1}, \tau_t) \;-\; u_t(\theta, \theta' \,|\, k_t, k_{t-1}, \tau_t) \;=\; \int_{\theta'}^{\theta} \chi(k_t, k_{t-1}, x)\, dx \;-\; (\theta - \theta')\chi(k_t, k_{t-1}, \theta')$$
$$\geq\; 0.$$

This holds for all θ, θ' in Θ, as desired. □

Proof of Lemma 1. (a) It is clear that $v(\cdot, \lambda)$ is continuous on Θ. Also note that

$$\frac{\partial v(\theta, \lambda)}{\partial \theta} \;=\; 1 + \beta \;+\; \beta \frac{d}{d\theta}\left(\frac{F(\theta) - \lambda}{f(\theta)}\right) \;>\; 0,$$

since by Assumption 1 the ratio $(F(\theta) - \lambda)/f(\theta)$ is weakly increasing in θ, for all $0 \leq \lambda \leq 1$ and $0 < \beta < 1$.

(b) Immediate from the definition of $v(\theta, \lambda)$.

(c)–(d) Note that $v(\theta_L, 0) = 0$, thus $\theta(0) = \theta_L$. Now fix $0 < \lambda \leq 1$, and notice that $v(\theta_L, \lambda) < 0$ and $v(\theta_H, \lambda) > 0$. Since $v(\cdot, \lambda)$ is continuous and strictly increasing, there exists a unique $\theta(\lambda)$ such that $v(\theta(\lambda), \lambda) = 0$, as desired. The fact that $\theta(\lambda)$ is strictly increasing in λ follows immediately from part (b) of the proposition. □

Let $\Gamma_1 = \{k_1(\cdot), u_1(\cdot)\}$ be an incentive feasible policy given $k_0(\cdot)$. First period welfare is

$$SW_1 \;=\; \int_{\theta_L}^{\theta_H} v\left(\theta, \lambda_1(\theta)\right) \chi(k_1, k_0, \theta)\, dF(\theta).$$

The virtual productivity is increasing in θ, for all λ_1. Note that to maximize social welfare the policy-maker reassigns resources from low productivity agents to high productivity agents. To do so, it needs to take into account the value of the adjusted allocation function and the resource feasibility constraint. In addition, any incentive compatible policy change must have a non-decreasing adjusted allocation function. In the proof of some of the results below, I invoke the following arguments.

Lemma 2—Auxiliary: *Let $\Gamma_1 = \{k_1(\cdot), u_1(\cdot)\}$ be an incentive feasible policy given $k_0(\cdot)$ such that the allocation function $k_1(\cdot)$ assigns*

(a) $0 \leq K_1^c < K$ units of capital to all types in the subinterval $\Theta_1^c = [\theta_L, \theta_1^c]$;

(b) $K < K_1^d \leq 1$ units to all types in $\Theta_1^d = [\theta_1^d, \theta_H]$, for $\theta_1^c \leq \theta_1^d$;

(c) K units of capital to all types in $\Theta \setminus (\Theta_1^c \cup \Theta_1^d)$.

Moreover, let the resource feasibility constraint bind under $k_1(\cdot)$. Then there are types $x \in \Theta_1^c$ and $y \in \Theta_1^d$ and a positive constant M such that change in social welfare from using $k_1(\cdot)$ instead of $k_0(\cdot)$ can be approximated by

$$M\left[(1-\eta)v\,(y, 1) \;-\; (1+\eta)v\,(x, 0)\right]. \tag{15}$$

Proof. Let $\Gamma_1 = \{k_1(\cdot), u_1(\cdot)\}$ be an incentive feasible policy given $k_0(\cdot)$ satisfying the conditions in the lemma. Since the threshold types θ_1^c and θ_1^d are determined in a way that makes the resource constraint in Eq. (3) bind, one has

$$(K - K_1^c)\left[F(\theta_1^c) - F(\theta_L)\right] \;=\; (K_1^d - K)\left[F(\theta_H) - F(\theta_1^d)\right] \;\equiv\; M > 0.$$

Note also that if $K_1^c = 0$ and $K_1^d = 1$, then $\theta_1^c = \theta_1^d = \theta_1^e$ necessarily, in which case $k_1(\cdot)$ coincides with $k_1^e(\cdot)$ and the unique threshold type $\theta_1^e = \vartheta(K)$ is given by Eq. (5).

For all types $\theta \in \Theta_1^c$, I can compute the change in the adjusted allocation function $\chi(\cdot)$ from using $k_1(\cdot)$ instead of $k_0(\cdot)$. This is

$$K_1^c \;-\; \eta(K - K_1^c) \;-\; K \;=\; -(1+\eta)(K - K_1^c).$$

This yields to a change in social welfare approximated by:

$$\begin{aligned} \triangle SW_1(K_1^c, \theta_1^c) \;&=\; -(1+\eta)(K - K_1^c)\int_{\theta_L}^{\theta_1^c} \nu\left(\theta, \lambda_1(\theta)\right) f(\theta)\, d\theta \\ &=\; -(1+\eta)\, \nu\,(x, 0)\,(K - K_1^c)\left[F(\theta_1^c) - F(\theta_L)\right], \quad \text{for some } x \in \Theta_1^c. \end{aligned} \tag{16}$$

The last expression follows from the Mean Value Theorem for Riemann integrals and the fact that, since the participation constraint is slack in Θ_1^c (except for, maybe, θ_1^c), the Lagrangian multiplier is $\lambda_1(\theta) = 0$ on Θ_1^c and therefore $\nu(\cdot, 0)$ is continuous in Θ_1^c.

Similarly, the change in the adjusted allocation from using $k_1(\cdot)$ instead of $k_0(\cdot)$, for all types $\theta \in \Theta_1^d$, is given by

$$K_1^d \;-\; \eta(K_1^d - K) \;-\; K \;=\; (1-\eta)(K_1^d - K).$$

This yields to the following social welfare change for this subinterval:

$$\begin{aligned} \triangle SW_1(K_1^d, \theta_1^d) \;&=\; (1-\eta)(K_1^d - K)\int_{\theta_1^d}^{\theta_H} \nu\left(\theta, \lambda_1(\theta)\right) f(\theta)\, d\theta \\ &=\; (1-\eta)\, \nu\,(y, 1)\,(K_1^d - K)\left[F(\theta_H) - F(\theta_1^d)\right], \quad \text{for some } y \in \Theta_1^d. \end{aligned} \tag{17}$$

Note that in this case the participation constraint of all types above θ_1^d is slack, and therefore the Lagrangian multiplies is $\lambda_1(\theta) = 1$ on Θ_1^d. Thus, social welfare change from using $k_1(\cdot)$ instead of $k_0(\cdot)$ evaluated on the intervals Θ_1^c and Θ_1^d amounts to

$$\triangle SW_1(K_1^c, \theta_1^c) \;+\; \triangle SW_1(K_1^d, \theta_1^d) \;=\; M\left[(1-\eta)\nu\,(y, 1) \;-\; (1+\eta)\nu\,(x, 0)\right],$$

as desired. □

Proof of Proposition 4. Consider two exhaustive cases. In the first one, the repetition of Γ_0 welfare dominates Γ_1^e, and thus I show that there exists a transition policy that performs strictly better than Γ_0. In the second one, Γ_1^e welfare dominates Γ_0. In this case I show the existence of a transition mechanism that performs strictly better than Γ_1^e.

Case 1. When $K_1^c = 0$ and $K_1^d = 1$, the allocation function $k_1(\cdot)$ constructed in Lemma 2 becomes a control allocation function. I rename it by $k_1^c(\cdot)$, with control type θ_1^c and threshold type $\theta_1^d = \zeta(\theta_1^c, K)$; see Eq. (4). The policy-maker can choose a small θ_1^c to ensure $\theta_1^d > \theta_1^e = \vartheta(K) > \theta(1)$. When θ_1^c is sufficiently close to θ_L, the continuity of $v(\theta, 0)$ implies that type $x \in [\theta_L, \theta_1^c]$ in Eq. (16) has a virtual productivity sufficiently close to zero, since $v(\theta_L, 0) = 0$. On the other hand, the virtual productivity for type $y \in [\theta_1^d, \theta_H]$ in Eq. (17) is strictly positive. Since $0 \leq \eta < 1$, it follows from Eq. (15) that $\triangle SW_1(0, \theta_1^c) + \triangle SW_1(1, \theta_1^d) > 0$.

It remains to show that a policy Γ_1^c that contains $k_1^c(\cdot)$ can be constructed to satisfy the incentive and participation constraints (the resource constraint binds by construction). Its adjusted allocation function $\chi(k_1^c, k_0, \theta)$ is given in Eq. (12). Clearly, it is non-decreasing on Θ. I can now use Eq. (7) for $t = 1$ and the fact that the participation constraint is binding for all θ between θ_1^c and θ_1^d to construct the indirect utility function $u_1^c(\cdot)$ associated with Γ_1^c — see Eq. (13) in the main text. By Proposition 2, Γ_1^c is incentive compatible given $k_0(\cdot)$. By inspection of the indirect utility, Γ_1^c is individually rational given $k_0(\cdot)$. This shows that there exists a dual allocation policy Γ_1^c that welfare dominates the status quo policy Γ_0 for the first period.

Case 2. Consider the output efficient policy Γ_1^e implemented during the first period. The adjusted allocation associated with $k_1^e(\cdot)$ is given in Eq. (11). Notice that the participation constraint binds only for the threshold agent θ_1^e. It follows that the Lagrangian multiplier function takes value of $\lambda^e(\theta) = \lambda^e < 1$ for all $\theta < \theta_1^e$ and $\lambda^e(\theta) = 1$ for all $\theta \geq \theta_1^e$. Thus, short-run social welfare associated with Γ_1^e can be written as

$$\begin{aligned} SW_1^e &= \int_{\theta_L}^{\theta_1^e} v\left(\theta, \lambda^e\right)(-\eta K)\, dF(\theta) + \int_{\theta_1^e}^{\theta_H} v(\theta, 1)(1 - \eta(1-K))\, dF(\theta) \\ &= -\eta K\, v(x^e, \lambda^e) F(\theta_1^e) + (1 - \eta + \eta K)\, v(y^e, 1)\left(1 - F(\theta_1^e)\right), \end{aligned}$$

where by the Mean Value Theorem $x^e \in [\theta_L, \theta_1^e]$ and $y^e \in [\theta_1^e, \theta_H]$. I can choose $0 < \lambda^e < 1$ so that x^e attains a negative virtual productivity: $v(x^e, \lambda^e) < 0$.

Consider now a transition policy Γ_1^c with threshold types θ_1^c and θ_1^d, where these are to be determined in a manner that makes the resource constraint bind. It is clear that the participation constraint is satisfied with equality only for types in the interval $[\theta_1^c, \theta_1^d]$. Thus, select the Lagrangian multipliers to be $\lambda^c(\theta) = \lambda^e < 1$ for all $\theta < \theta_1^c$, $\lambda^c(\theta) = 1$ for all $\theta \geq \theta_1^d$, and an increasing continuous function everywhere else. Using the adjusted allocation function $\chi(k_1^c, k_0, \theta)$ in Eq. (12), I can write social surplus associated with Γ_1^c as

$$\begin{aligned} SW_1^c &= \int_{\theta_L}^{\theta_1^c} v\left(\theta, \lambda^e\right)(-\eta K)\, dF(\theta) + \int_{\theta_1^c}^{\theta_1^d} v\big(\theta, \lambda^c(\theta)\big) K\, dF(\theta) + \int_{\theta_1^d}^{\theta_H} v(\theta, 1)(1 - \eta + \eta K)\, dF(\theta) \\ &= -\eta K\, v(x^c, \lambda^e) F(\theta_1^c) + \int_{\theta_1^c}^{\theta_1^d} v\big(\theta, \lambda^c(\theta)\big) K\, dF(\theta) + (1 - \eta + \eta K)\, v(y^c, 1)\big(1 - F(\theta_1^d)\big), \end{aligned}$$

where $x^c \in [\theta_L, \theta_1^c]$ and $y^c \in [\theta_1^d, \theta_H]$.

The difference between social welfare created by these two policies is then expressed as

$$SW_1^c - SW_1^e = \eta K\left[v(x^e,\lambda^e)F(\theta_1^e) - v(x^c,\lambda^e)F(\theta_1^c)\right] + \int_{\theta_1^c}^{\theta_1^d} v\big(\theta,\lambda^c(\theta)\big)K\,dF(\theta) + (1-\eta+\eta K)\left[v(y^c,1)\big(1-F(\theta_1^d)\big) - v(y^e,1)\big(1-F(\theta_1^e)\big)\right]. \tag{18}$$

Notice that $v(\cdot,\lambda)$ is strictly increasing in θ, for all $\lambda\in[0,1]$. Thus, we have $x^c<x^e$ and $y^c>y^e$, hence $v(x^e,\lambda^e)>v(x^c,\lambda^e)$ and $v(y^c,1)>v(y^e,1)$. It follows that the first and second terms in Eq. (18) are strictly positive. Moreover, when choosing θ_1^d sufficiently close to θ_1^e, the last term is non-negative. Thus, it follows that there exists a transition policy Γ_1^c that welfare dominates the output efficient policy Γ_1^e in the first period of the reform. □

Proof of Proposition 5. Consider any incentive feasible policy $\Gamma_1=\{k_1,u_1\}$ implemented in period 1. If $\Gamma_1=\Gamma_1^e$ or $\Gamma_1=\Gamma_0$, from Proposition 4 it follows that we can find a transition policy Γ_1^c that welfare dominates Γ_1. Thus, suppose Γ_1 is not a transition policy. Then, without loss of generality, let $\Theta_1^c=[\theta_L,\theta_1^c)$ and $\Theta_1^d=(\theta_1^d,\theta_H]$ be such that $0<k_1(\theta)=K_1^c<K$ for all θ in Θ_1^c and $K<k_1(\theta)=K_1^d<1$ for all θ in Θ_1^d. From Lemma 2 it follows that by the linearity of adjustment costs, the change in welfare between implementing $k_1(\cdot)$ instead of $k_0(\cdot)$ can be approximated by $\triangle SW_1(K_1^c,\theta_1^c)$ for Θ_1^c and $\triangle SW_1(K_1^d,\theta_1^d)$ for Θ_1^d. These terms are given in Eq. (16) and Eq. (17), respectively. Notice from these expressions that welfare changes are increasing in K_1^d and decreasing in K_1^c. Therefore, the optimal policy will set $K_1^c=0$ and $K_1^d=1$. But this implies policy Γ_1 is either an output efficient policy or a transition policy that exhibits persistence. From Proposition 4, the optimal policy is indeed the latter. □

Proof of Proposition 6. Given $\Gamma^e=\{\Gamma_1^e,\ldots,\Gamma_T^e\}$, the first period generalized allocation $\chi(k_1^e,k_0,\theta)$ is non-decreasing, see Eq. (11). Using the integral representation

$$u_1^e(\theta) = u_1^e(\theta_L) + \int_{\theta_L}^{\theta}\chi(k_1^e,k_0,x)\,dx,$$

and the participation constraint $u_1^e(\theta)\ge\theta K_1^e$, one readily obtains $u_1^e(\theta_L)=\theta_1^e(1+\eta)K_1^e$. This fully specifies the indirect utility function generated by the introduction of the output efficient allocation rule $k_1^e(\cdot)$, which is given by

$$u_1^e(\theta) = \begin{cases} -(\eta K_1^e)\theta + \theta_1^e(1+\eta)K_1^e & \text{for } \theta<\theta_1^e, \\ (1-\eta+\eta K_1^e)\theta - \theta_1^e(1-\eta)(1-K_1^e) & \text{for } \theta\ge\theta_1^e. \end{cases} \tag{19}$$

It follows that every type is in favor of policy change in period 1. Thus, Γ_1^e is incentive feasible given $k_0(\cdot)$, as the resource constraint is satisfied by construction. From this expression, one immediately concludes that Γ_1^e can be implemented via a cash bonus $b_1^e=\theta_1^e(1+\eta)K_1^e$ and a posted price for additional resources $p_1^e=\theta_1^e(1-\eta)(1-K_1^e)$.

Capital accumulates in period $t \geq 2$ and the accumulation process stops once the threshold $K(1)$ has reached. Without loss of generality, let T^e be such that

$$K_t^e = \big[1 - \rho + s\,A\big(\vartheta(K_{t-1}^e)\big)\big]K_{t-1}^e, \quad \text{for all } 2 \leq t < T^e,$$

and further

$$K_t^e = K(1), \quad \text{for all remaining periods } t \geq T^e.$$

This determines threshold types $\theta_t^e = \vartheta(K_t^e) > \theta(1)$ for all $t < T^e$ and $\theta_t^e = \theta(1)$ for all $t \geq T^e$. Note I am leaving open the possibility of $T^e > T$, in which case the game stops at T and the steady-state is not reached under the reform program Γ^e.

Since the default option for period $t \geq 2$ is $k_{t-1}^e(\cdot)$, the generalized allocation function is

$$\chi(k_t^e, k_{t-1}^e, \theta) = \begin{cases} 0 & \text{for } \theta < \theta_t^e, \\ 1-\eta & \text{for } \theta_t^e \leq \theta \leq \theta_{t-1}^e, \\ 1 & \text{for } \theta > \theta_{t-1}^e. \end{cases}$$

From Proposition 2, it follows that Γ_t^e is incentive compatible given $k_{t-1}^e(\cdot)$. Therefore, the reform program Γ^e is sequentially incentive compatible. Using the integral representation in Eq. (7) to compute the indirect utility function associated with Γ_t^e obtains

$$u_t^e(\theta) = \begin{cases} u_t^e(\theta_L) & \text{for } \theta < \theta_t^e, \\ u_t^e(\theta_L) + (1-\eta)\theta - (1-\eta)\theta_t^e & \text{for } \theta_t^e \leq \theta \leq \theta_{t-1}^e, \\ u_t^e(\theta_L) - \eta\theta_{t-1}^e - (1-\eta)\theta_t^e + \theta & \text{for } \theta > \theta_{t-1}^e. \end{cases}$$

To pin down $u_t^e(\theta_L)$, I use the period t reservation utility, namely $\hat{u}_t(\theta)$, as the value of the outside option for each type. Recall that in this case, $\hat{u}_t(\theta) = \theta k_{t-1}(\theta) + \hat{\tau}_{t-1}(\theta)$, where this last is the minimal transfer that implements $k_{t-1}(\cdot)$ without adjustment costs present, and in addition leaves every agent with $\hat{u}_t(\theta) \geq u_0(\theta)$. Simple calculations now show that $\hat{\tau}_{t-1}(\theta) = \theta_1^e K_1^e$ for all $\theta < \theta_1^e$, and therefore $u_t^e(\theta_L) = \theta_1^e K_1^e$. This obtains:

$$u_t^e(\theta) = \begin{cases} \theta_1^e K_1^e & \text{for } \theta < \theta_t^e, \\ (1-\eta)\theta + \theta_1^e K_1^e - (1-\eta)\theta_t^e & \text{for } \theta_t^e \leq \theta \leq \theta_{t-1}^e, \\ \theta + \theta_1^e K_1^e - (1-\eta)\theta_t^e - \eta\theta_{t-1}^e & \text{for } \theta > \theta_{t-1}^e. \end{cases} \tag{20}$$

From inspection, it follows that the participation constraint is satisfied for all $\theta \in \Theta$, all $t \geq 2$. It follows that the reform program Γ^e is sequentially individually rational. Since resources feasibility is satisfied by construction, the output efficient reform program Γ^e is sequentially incentive feasible, as desired.

Finally, since at each point in time the reform program Γ^e specifies utilization of the output efficient allocation rule, given availability of aggregate capital and the incentive feasibility constraints, it is clear that it generates maximal growth among all sequentially incentive feasible reform programs. □

Proof of Proposition 7 and Corollary 1. Immediately from the proof of Proposition 6 presented above —in particular, see the expressions for the indirect utility $u_t^e(\cdot)$ associated to Γ^e. □

Proof of Proposition 8. Let $\Gamma_t = \{k_t(\cdot), u_t(\cdot)\}$ be an incentive feasible policy given $k_{t-1}(\cdot)$. Let $0 < K_t < K(1)$ be the stock of capital for period t, this is without loss of generality. If Γ_t is a dual transition policy (including the outcome efficient policy given K_t), then there is nothing to show. So, assume that policy Γ_t is not a transition policy.

In such case, let $\Theta_t^\ell = [\theta_t^{1\ell}, \theta_t^{2\ell}]$ be an interval of Θ such that each $\theta \in \Theta_t^\ell$ receives $0 < K_t^\ell < K$ under $k_t(\cdot)$ and every type left of $\theta_t^{1\ell}$ receives 0 units of capital. Similarly, let $\Theta_t^h = [\theta_t^{1h}, \theta_t^{2h}]$ be such that each $\theta \in \Theta_t^h$ is allocated $K < K_t^h < 1$, and every type right of θ_t^{h2} receives 1 unit of capital. Notice that at least one of these two subsets has positive measure. It is clearly without loss of generality (given Lemma 2) that I assume both subsets to be of positive measure. Moreover, I assume that under $k_{t-1}(\cdot)$ each $\theta \in \Theta_t^\ell$ is assigned K_{t-1}^ℓ and each $\theta' \in \Theta_t^h$ is assigned K_{t-1}^h. Given that I focus on allocation functions that can be expressed as step functions, this is without loss of generality. Indeed, I can take $\hat{\Theta}_t^\ell$ to be a subset of Θ_t^ℓ for which this holds true, and similarly for $\hat{\Theta}_t^h$ and Θ_t^h, and re-write the arguments in the proof using these alternatives subsets. Obviously, K_{t-1}^ℓ, K_t^ℓ, K_{t-1}^h and K_t^h are such that the adjusted allocation $\chi(k_t, k_{t-1}, \cdot)$ is weakly increasing in θ. Suppose for a moment that that $K_{t-1}^\ell = 0 < K_t^\ell$ and $K_{t-1}^h < K_t^h$. I will show later how to accommodate the proof to handle the other cases.

Now, the idea of the proof is to reassign resources from Θ_t^ℓ to Θ_t^h in a way that respects the resource feasibility constraint. I then argue that this improves social welfare in period t and, because allocative efficiency is improved, increases aggregate output for this period. Moreover, this can be done without increasing the reservation utility of period $t+1$.

I start with the observation that subintervals Θ_t^ℓ and Θ_t^h can be chosen so that one replaces $k_t(\theta) = K_t^\ell$ with 0 for all $\theta \in \Theta_t^\ell$ and $k_t(\theta) = K_t^h$ with 1 for all $\theta' \in \Theta_t^h$. Abusing notation, I let these subintervals be equal to Θ_t^ℓ and Θ_t^h. Thus, by Eq. (3), it must be that

$$K_t^\ell\big[F(\theta_t^{2\ell}) - F(\theta_t^{1\ell})\big] = (1-K_t^h)\big[F(\theta_t^{2h}) - F(\theta_t^{1h})\big] \equiv \triangle K_t.$$

Construct an alternative allocation function $k_t'(\cdot)$ that coincides with $k_t(\cdot)$ everywhere except on Θ_t^ℓ, where it assigns 0 to every type, and on Θ_t^h, where it assigns 1 to every type. This new allocation function is resource feasible and incentive compatible given $k_{t-1}(\cdot)$. Using arguments similar to those exposed in Lemma 2, I compute the change in welfare from replacing $k_t(\cdot)$ with $k_t'(\cdot)$. To do this, notice that the change in the adjusted allocation for all types in Θ_t^ℓ is equal to

$$\triangle\chi(\Theta_t^\ell) = -K_t^\ell(1-\eta). \tag{21}$$

Similarly, the change in the adjusted allocation from implementing $k_t'(\cdot)$ instead of $k_t(\cdot)$ is

$$\begin{aligned}\triangle\chi(\Theta_t^h) &= 1 - \eta(1-K_{t-1}^h) - K_t^h + \eta(K_t^h - K_{t-1}^h)\\ &= (1-\eta)(1-K_t^h).\end{aligned} \tag{22}$$

Using Eq. (21), Eq. (22), and appealing to the Mean Value Theorem for Riemann integrals again, I can approximate the change in social welfare by the expression

$$\begin{aligned}\triangle SW_t &= -K_t^\ell\big[F(\theta_t^{2\ell}) - F(\theta_t^{1\ell})\big](1-\eta)\,\nu(x,\lambda_t^\ell) + (1-K_t^h)\big[F(\theta_t^{2h}) - F(\theta_t^{1h})\big](1-\eta)\,\nu(y,\lambda_t^h)\\ &= \triangle K_t(1-\eta)\big[\nu(y,\lambda_t^h) - \nu(x,\lambda_t^\ell)\big],\end{aligned}$$

for some $x \in \Theta_t^\ell$ and $y \in \Theta_t^h$. Since $y > x$ and for both subintervals the participation constraint is slack, one can choose the multipliers λ_t^ℓ and λ_t^h sufficiently close to each other so that $v(y, \lambda_t^h) > v(x, \lambda_t^\ell)$, as desired.

Changing $k_t(\cdot)$ to $k_t'(\cdot)$ in this manner therefore increases social welfare and, because more resources are allocated to more productive agents, also raises aggregate output for period t. Finally, note that the allocation of types in Θ_t^ℓ under $k_t'(\cdot)$ is the same as that for types in $[\theta_L, \theta_t^{1\ell})$. Since this determines the minimal value of the reservation utility $\hat{u}_{t+1}$, and this is constructed in a way that minimizes the need to raise external funds by the policy-maker (see the discussion in Section 2.3), it follows that such a change does not increase the reservation utility for the next period.

To finish the proof, I consider what happens in case $K_{t-1}^\ell \neq 0$ and $K_{t-1}^h > K_t^h$ — still assuming that $k_t(\cdot)$ is incentive feasible given $k_{t-1}(\cdot)$. In the first case, I need to include the term $-2\eta K_{t-1}^\ell$ in the right-hand side of Eq. (21), as this terms responds to adjustment costs now present. But this only affects the change in social welfare in a positive way. In the second case, I need to replace Eq. (22) for

$$\begin{aligned}
\triangle\chi(\Theta_t^h) &= 1 - \eta(1 - K_{t-1}^h) - K_t^h + \eta K_{t-1}^h - \eta K_t^h \\
&> 1 - \eta(1 - K_{t-1}^h) - K_t^h \\
&> (1 - \eta)(1 - K_t^h).
\end{aligned}$$

Therefore, as before, the change in social welfare can only increase.

Finally, note that if the new policy $\Gamma_t' = \{k_t'(\cdot), u_t'(\cdot)\}$ is not a transition policy, we can repeat the above arguments until we arrive at a transition policy $\Gamma_t^c = \{k_t^c(\cdot), u_t^c(\cdot)$ that satisfies all the requirements of the proposition. This completes the proof. □

Proof of Proposition 9. Fix a reform program $\Gamma^c = \{\Gamma_1^c, \ldots, \Gamma_T^c\}$, where in each period the policy-maker introduces a transition policy. I show that this program is incentive feasible and is solely characterized by the choice of the first period control θ_1^c.

In the first period of the reform, policy $\Gamma_1^c = \{k_1^c(\cdot), u_1^c(\cdot)\}$ with control θ_1^c and threshold type $\theta_1^d = \zeta(\theta_1^c, K)$ is incentive feasible give $k_0(\cdot)$ —see Proposition 4. Its indirect utility is given by Eq. (13). Aggregate output $Y_1^c > Y_0$ and capital stock in period 2 is then $K_2^c > K_1^c = K$. Assume that $K_2^c < K(1)$ and $\theta_1^c < \vartheta(K_2^c)$, so that period 2 is of consolidation rather than expansion. The policy-maker implements an allocation function $k_2^c(\cdot)$ with control $\theta_2^c = \theta_1^c$ and threshold $\theta_2^d = \zeta(\theta_1^c, K_2^c) < \theta_1^d$. Every type below θ_1^c receives zero units of capital, and every type above θ_2^d obtains 1 unit. Note however that agents with productivity between θ_2^d and θ_1^d suffer an adjustment cost, as they are recently incorporated into the market mechanism. Agents between θ_1^c and θ_2^d are kept at the inefficient policy.

The generalized allocation function $\chi(k_2^c, k_1^c, \cdot)$ is

$$\chi(k_2^c, k_1^c, \theta) = \begin{cases} 0 & \text{for } \theta < \theta_1^c, \\ K & \text{for } \theta_1^c \leq \theta \leq \theta_2^d, \\ 1-\eta+\eta K & \text{for } \theta_2^d < \theta < \theta_1^d, \\ 1 & \text{for } \theta \geq \theta_1^d. \end{cases}$$

By Proposition 2, Γ_t^c is incentive compatible given $k_{t-1}^c(\cdot)$. To obtain the indirect utility in period 2, I need first to specify the reservation utility $\hat{u}_2(\cdot)$. By assumption, in case of veto the allocation $k_1(\cdot)$ is implemented in period 2 via the minimal transfers that leave agents weakly above $u_0(\cdot)$. Readily, agents that do not receive the input in case of veto can secure at least $\theta_1^c K$, which is the minimal subsidy that leaves control type indifferent between being excluded and receiving K units of capital. Using the expression for the indirect utility in Equation 7, simple calculations obtain

$$u_2^c(\theta) = \begin{cases} \theta_1^c K & \text{for } \theta < \theta_1^c, \\ \theta K & \text{for } \theta_1^c \leq \theta \leq \theta_2^d, \\ \theta(1-\eta+\eta K) - \theta_2^d(1-\eta)(1-K) & \text{for } \theta_2^d < \theta < \theta_1^d, \\ \theta - \theta_2^d(1-\eta)(1-K) - \theta_1^d\eta(1-K) & \text{for } \theta \geq \theta_1^d. \end{cases} \tag{23}$$

Since $u_2^c(\theta) \geq \hat{u}_2^c(\theta)$ for all θ, it follows that the transition policy $\Gamma_2^c = \{k_2^c(\cdot), u_2^c(\cdot)\}$ is incentive feasible given $k_1^c(\cdot)$. Aggregate output Y_2^d is larger than Y_1^d due to the efficiency gains carried on by assigning extra capital to the high productivity types, thus capital increases for next period: $K_3^c > K_2^c$.

It should be clear that the above analysis carries over the entire transition phase; i.e., for all $2 \leq t \leq T^c$, as long as capital does not exceed $K(1)$ and $\theta_1^c < \vartheta(K_{T^c}^c)$. In other words, T^c indicates the last period for which capital gains are insufficient to introduce a full market mechanism. For all these periods, the policy-maker introduces a transition policy $\Gamma_t^c = \{k_t^c(\cdot), u_t^c(\cdot)\}$ with control θ_1^c and threshold type $\theta_t^d = \zeta(\theta_1^c, K_t^c) < \theta_{t-1}^d = \zeta(\theta_1^c, K_{t-1}^c)$. The periodic indirect utility in this phase, with the obvious modifications, is given in Eq. (23).

In period $T^c + 1$, capital stock is sufficiently high to introduce a pure market mechanism. Thus, policy Γ_{T^c+1} specifies an allocation function $k_{T^c+1}(\cdot)$ with control $\theta_{T^c+1}^c = \vartheta(K_{T^c+1}^c) < \theta_1^c$ that assigns zero units of capital for all $\theta < \theta_{T^c+1}^c$, and one unit of capital for all $\theta \geq \theta_{T^c+1}^c$. The generalized allocation function for period $T^c + 1$ is given by

$$\chi(k_{T^c+1}^c, k_{T^c}^c, \theta) = \begin{cases} 0 & \text{for } \theta < \theta_{T^c+1}^c, \\ 1-\eta & \text{for } \theta_{T^c+1}^c \leq \theta < \theta_1^c, \\ 1-\eta(1-K) & \text{for } \theta_1^c \leq \theta < \theta_{T^c}^d, \\ 1 & \text{for } \theta \geq \theta_{T^c}^c. \end{cases}$$

This shows that the allocation function $k_{T^c+1}^c(\cdot)$ is incentive compatible given $k_{T^c}^c(\cdot)$. Taking into account the reservation utility for this period, simple calculations show that the indirect utility

$u^c_{T^c+1}(\cdot)$ is given by

$$u^c_{T^c+1}(\theta) \;=\; \begin{cases} \theta^c_1 K & \text{for } \theta < \theta^c_{T^c+1}, \\ \theta(1-\eta) \,-\, \theta^c_{T^c+1}(1-\eta) \,+\, \theta^c_1 K & \text{for } \theta^c_{T^c+1} \le \theta < \theta^c_1, \\ \theta(1-\eta+\eta K) \,-\, \theta^c_{T^c+1}(1-\eta) \,+\, \theta^c_1 K(1-\eta) & \text{for } \theta^c_1 \le \theta < \theta^d_{T^c}, \\ \theta \,-\, \theta^c_{T^c+1}(1-\eta) \,+\, \theta^c_1 K(1-\eta) \,-\, \theta^d_{T^c}\eta(1-K) & \text{for } \theta \ge \theta^d_{T^c}. \end{cases}$$

Once the reform program enters the market expansion phase, the transition policy Γ^c_t implements an allocation function $k^c_t(\cdot)$ with control $\theta(1) \le \theta^c_t = \vartheta(K^c_t) < \vartheta(K^c_{t-1}) = \theta^c_{t-1}$, for all time periods $T^c + 1 < t \le T^s$. The generalized allocation function in the expansion phase is given by

$$\chi(k^c_t, k^c_{t-1}, \theta) \;=\; \begin{cases} 0 & \text{for } \theta < \theta^c_t, \\ 1-\eta & \text{for } \theta^c_t \le \theta \le \theta^c_{t-1}, \\ 1 & \text{for } \theta > \theta^c_{t-1}. \end{cases}$$

From Proposition 2, it follows that Γ^c_t is incentive compatible given $k^c_{t-1}(\cdot)$. Taking into account the reservation utility obtains, for all $T^c + 1 < t \le T^s$:

$$u^c_t(\theta) \;=\; \begin{cases} \theta^c_1 K & \text{for } \theta < \theta^c_t, \\ (1-\eta)\theta \,+\, \theta^c_1 K \,-\, (1-\eta)\theta^c_t & \text{for } \theta^c_t \le \theta \le \theta^c_{t-1}, \\ \theta \,+\, \theta^c_1 K \,-\, (1-\eta)\theta^c_t \,-\, \eta\theta^c_{t-1} & \text{for } \theta > \theta^c_{t-1}. \end{cases}$$

Since $u^c_t(\theta) \ge \hat{u}^c_t(\theta)$ for all θ and the generalized allocation $\chi(k^c_t, k^c_{t-1}, \theta)$ is still non-decreasing, it follows that the reform program is incentive feasible in the market expansion phase.

Finally, note that the above two equations also describe the steady-state phase, once we let $K^c_t = K(1)$ and $\theta^c_t = \theta(1)$ for all $t > T^s$ — recall T^s is the last period of capital accumulation.

To conclude the proof, notice that in the consolidation phase all controls are $\theta^c_t = \theta^c_1$, and all thresholds are $\theta^d_t = \zeta(\theta^c_1, K^c_t)$. In this phase, aggregate output is given by

$$Y^c_t \;=\; \mathscr{A}\left(\theta^c_1, K^c_t\right) K^c_t,$$

and aggregate capital by

$$K^c_t \;=\; \left[1 \,-\, \rho \,+\, s\,\mathscr{A}\left(\theta^c_1, K^c_{t-1}\right)\right] K^c_{t-1},$$

where to simplify notation I write $K^c_1 = K$. Similarly, in the expansion phase all controls are given by $\theta^c_t = \vartheta(K^c_t)$. Aggregate output and aggregate capital are

$$Y^c_t \;=\; A\left(\vartheta(K^c_t)\right) K^c_t \qquad \text{and} \qquad K^c_t \;=\; \left[1 \,-\, \rho \,+\, s\,A\left(\vartheta(K^c_{t-1})\right)\right] K^c_{t-1}.$$

Finally, the controls in the steady-state are all $\theta^c_t = \theta(1)$, with aggregate output and capital equal to

$$Y^c_t \;=\; A(\theta(1))\,K(1) \qquad \text{and} \qquad K^c_t \;=\; K(1).$$

This shows that all economic variables endogenously determined by the economic reform program, including the rate of capital accumulation and growth, depend on the optimal selection of the first period control θ^c_1, which I denote θ^*_1. □

Proof of Proposition 10 and Corollary 2. Immediately from the proof of Proposition 9 presented above —in particular, see the expressions for the indirect utility $u_t^c(\cdot)$ associated to Γ^c, for all $t = 1, \ldots, T$.

To observe the role of the assumption $0 < \eta < 1/2$ in the proof of the corollary, note that for all $2 \leq t < T^c$, I can write

$$p_2^* - p_1^* = (1-K)\big[(1-\eta)\,\zeta(\theta_1^*, K_2^*) - (1-2\eta)\,\zeta(\theta_1^*, K_1^*)\big], \quad \text{and,}$$
$$p_{t+1}^* - p_t^* = (1-K)\big[\zeta(\theta_1^*, K_{t+1}^*)(1-\eta) - \zeta(\theta_1^*, K_t^*)(1-2\eta)\zeta(\theta_1^*, K_t^*) - \eta\zeta(\theta_1^*, K_{t-1}^*)\big].$$

Since $\zeta(\theta_1^*, K_{t+1}^*) > \zeta(\theta_1^*, K_{t-1}^*)$ and $1 - 2\eta > 0$, this yields to

$$p_{t+1}^* - p_t^* < (1-K)(1-2\eta)\big[\zeta(\theta_1^*, K_{t+1}^*) - \zeta(\theta_1^*, K_t^*)\big] < 0.$$

Similarly, the movement of reduced prices in the persistence phase, that is for $2 \leq t \leq T^c$, is given by

$$r_{t+1}^* - r_t^* = (1-K)(1-\eta)\big[\zeta(\theta_1^*, K_{t+1}^*) - \zeta(\theta_1^*, K_t^*)\big] < 0.$$

The price movements, for the full and reduced prices, of the expansionary phase of the reform Γ^* follow similar patterns, with the obvious modifications, than the price movements for the expansionary phase of the reform Γ^e. □

A.2 Counter-Example

The presence of adjustment costs does impose a dynamic restriction to the class of implementable reform programs. Simple monotonicity of the allocation rule $k_t(\cdot)$ period by period does not suffice, as the following example shows.[20]

Example 1. Let $\Theta = [0, 1]$, and assume types are uniformly distributed on Θ. Let $K = 1/2$, and consider the following allocation functions:

$$k_1(\theta) = \begin{cases} 3/12 & \text{if } \theta < 1/4, \\ 7/12 & \text{if } \theta \geq 1/4, \end{cases}$$

and

$$k_2(\theta) = \begin{cases} 3/8 & \text{if } \theta < 3/4, \\ 7/8 & \text{if } \theta \geq 3/4. \end{cases}$$

Observe that both allocation functions are non-decreasing. However, simple calculations show that the generalized allocation function $\chi(k_2, k_1, \theta)$ for period 2 is

$$\chi(k_2, k_1, \theta) = \begin{cases} 3/8 - \eta(3/24) & \text{if } \theta < 1/4, \\ 3/8 - \eta(5/24) & \text{if } 1/4 \leq \theta < 3/4, \\ 7/8 - \eta(7/24) & \text{if } \theta \geq 3/4, \end{cases}$$

[20] I owe this example to Nick Netzer.

which is not increasing. By Proposition 2, $k_2(\cdot)$ is not incentive compatible given $k_1(\cdot)$. Indeed, one can verify without much effort that there is no transfer function $\tau_2(\cdot)$ that implements $k_2(\cdot)$ given the adjustment costs generated by $k_1(\cdot)$. ⋄