Buying First or Selling First in Housing Markets*

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Abstract

Housing transactions by moving owner-occupiers take two steps, purchase of a new property and sale of the old housing unit. This paper shows how the transaction sequence decision of moving homeowners depends on, and in turn, affects housing market conditions in an equilibrium search model of the housing market. We show that moving homeowners prefer to buy first whenever there are *more* buyers than sellers in the market. This behavior leads to multiple steady state equilibria and to self-fulfilling fluctuations in prices and time-on-market. Equilibrium switches create large fluctuations in the housing market, which are broadly consistent with stylized facts on the housing cycle.

Keywords: housing market, search frictions, order of transactions, strategic complementarity, self-fulfilling fluctuations

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1 Introduction

A large number of households move within the same local housing market every year. Many of these moves are by owner-occupiers who buy a new property and sell their old housing unit. However, it takes time to transact in the housing market, so a homeowner that moves may end up either owning two units or being forced to rent for some period, depending on the sequence of transactions. Either of these two alternatives may be costly.[1] There is anecdotal evidence that the incentives to "buy first" (buy the new property before selling the old property) or "sell first" (sell the old property before buying the new one) may depend on the state of the housing market.[2] If the transaction sequence decisions of moving owner-occupiers in turn affect housing market conditions, there could be powerful equilibrium feedbacks with important consequences for housing market dynamics.

In this paper we study a tractable equilibrium model of the housing market, which explicitly features trading delays and a transaction sequence decision for moving owner-occupiers. We show that the transaction sequence choices of moving homeowners can have powerful effects on the housing market and can lead to large fluctuations in the stock of houses for sale, time-on-market, trading volume, and also prices.

In the model, agents continuously enter and exit a local housing market. They have a preference for owning housing over renting, and consequently, search for a housing unit to buy. The market is characterized by a frictional trading process in the form of search-and-matching frictions. This leads to a positive expected time-on-market for buyers and sellers, which is affected by the tightness in the market - the ratio of buyers to sellers. Once an agent becomes an owner-occupier, he may be hit by an idiosyncratic preference shock over his life cycle. In that case he becomes "mismatched" with his current house and wants to move internally in the same housing market. To do that, the mismatched owner has to choose optimally the order of transactions - whether to buy a new housing unit first and then sell his old unit (buy first), or sell his old unit first and then buy (sell first). Given trading delays, this may lead to the agent becoming a double owner (owning two housing units) or a forced renter (owning no housing) for some time, which is costly. The expected time of remaining in such a state depends on the time-on-market for sellers and buyers, respectively.

Whenever the costs of a double owner or a forced renter are high relative to the costs of mismatch, the mismatched owner prefers buying first over selling first whenever there are *more* buyers than sellers in the market, or more generally, when the buyer-seller ratio is high. What is the reason for such a behavior? First, whenever it is more costly to be a double owner or a forced renter compared to being mismatched, a moving owner wants to minimize the delay between the two transactions.

[1] The following quote from Realtor.com, an online real estate broker, highlights this issue: "If you sell first, you may find yourself under a tight deadline to find another house, or be forced in temporary quarters. If you buy first, you may be saddled with two mortgage payments for at least a couple months." (Dawson (2013))

[2] A common realtor advice to homeowners that have to move is to "buy first" in a "hot" market, when there are more buyers than sellers and prices are high or expected to increase, and "sell first" in a "cold" market, when there are more sellers than buyers and house prices are depressed or expected to fall. Anundsen and Røed Larsen (2014) provides evidence on the response of the intentions of owner-occupiers to buy first or sell first to the state of the housing market using survey data for Norway. In Section 2 we provide direct evidence for this link using data for the Copenhagen housing market.

Second, when there are more buyers than sellers, the expected time-on-market is low for a seller and high for a buyer. Consequently, if a mismatched owner buys first he expects to spend a longer time as a buyer, and hence to remain mismatched longer. However, once he buys, he expects to stay with two houses for a short time while searching for a buyer for his old property. Conversely, choosing to sell first in that case implies a short time-to-sell and a short time of remaining mismatched but a longer time of searching to buy a new unit afterwards. Since the flow costs in between the two transactions are higher than during mismatch, buying first clearly dominates selling first in that case.

However, the order of transactions by moving owner-occupiers affects the buyer-seller ratio. Specifically, when mismatched owner-occupiers buy first, they crowd the buyer side of the market, and so the market ends with more buyers than sellers in steady state. Nevertheless, this high buyer-seller ratio is consistent with the incentives of mismatched owners to buy first. Conversely, when all mistmatched owners sell first, there are more sellers than buyers in steady state. However, a low buyer-seller ratio is consistent with the incentives of mismatched owners to sell first.

Thus, the interaction between the behavior of mismatched owners and the buyer-seller composition of the housing market creates a strategic complementarity in their transaction sequence decisions, which in turn may lead to multiple steady state equilibria. In one steady state equilibrium (a "Sell first" equilibrium), mismatched owners prefer to sell first, the market tightness is low and the expected time-on-market for sellers is high. In the other steady state equilibrium (a "Buy first" equilibrium), mismatched owners prefer to buy first, the market tightness is high and the expected time-on-market for sellers is low.[3]

Switches between the "Buy first" and the "Sell first" equilibria lead to fluctuations in the housing market. Specifically, moving from the "Buy first" to the "Sell first" equilibrium is associated with an increase in the stock of houses for sale, an increase in time-on-market for sellers, and a drop in transactions. This behavior is broadly consistent with evidence on the housing cycle. Also, as we show in a simple numerical example, the fluctuations generated by the model can be substantial.[4]

When house prices respond to changes in the buyer-seller ratio, there can exist equilibria with self-fulfilling fluctuations in prices and market tightness. Since mismatched owners are more likely to buy first (sell first) when they expect price appreciation (depreciation), they end up exerting a destabilizing force on the housing market. For example, if agents expect prices to depreciate, they are more likely to sell first. However, this decreases the buyer-seller ratio, which in turn drags down house prices, and thus, confirms the agents' expectations.

Finally, we show that our main result on the strategic complementarity in mismatched owners' actions and the resulting equilibrium multiplicity is robust across alternative modeling environments beyond our benchmark model. First, while in our benchmark model prices are fixed across steady

[3]Note that we derive this multiplicity under the assumption of a constant returns to scale matching function. Therefore, the strategic complementarity does not arise from increasing returns to scale in matching as in Diamond (1982).

[4]While in our benchmark model house prices are hold fixed across equilibria, the version of our model with prices determined by Nash bargaining also shows that there can be substantial price fluctuations arising from the equilibrium switches.

states (but lie within the bargaining sets of agents), we show that there can be multiple equilibria even in an environment where prices are endogenously determined by Nash bargaining, and hence, both differ across trading pairs and respond to changes in the buyer-seller ratio. The main forces that drive equilibrium multiplicity, the interplay between mismatched owners' incentives to buy first or sell first and the stock-flow conditions that determine the equilibrium market tightness are still present in that environment. Second, we show that there can be equilibrium multiplicity even in an environment with competitive search where agents can trade off prices and time-on-market (queue length).

Related Literature. The paper is related to the growing literature on search-and-matching models of the housing market and fluctuations in housing market liquidity, initiated by the seminal work of Wheaton (1990).This foundational paper is the first to consider a frictional model of the housing market to explain the existence of a "natural" vacancy rate in housing markets and the negative comovement between deviations from this natural rate and house prices. In that model, mismatched homeowners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of becoming a forced renter with no housing is prohibitively large, so that mismatched owners always buy first. As we show in our paper, allowing mismatched owners to endogenously choose whether to buy first or sell first has important consequences for the housing market.

The paper is particularly related to the literature on search frictions and propagation and amplification of shocks in the housing market (Krainer (2001), Novy-Marx (2009), Caplin and Leahy (2011), Diaz and Jerez (2013), Head, Lloyd-Ellis, and Sun (2014), Ngai and Tenreyro (2014), Guren and McQuade (2013), Anenberg and Bayer (2013), and Ngai and Sheedy (2015)). This literature shows how search frictions naturally propagate aggregate shocks due to the slow adjustment in the stock of buyers and sellers. Additionally, they can amplify price responses to aggregate shocks, which in Walrasian models would be fully absorbed by quantity responses.[5]

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must buy first, as well as a model where they must sell first. They show that each model explains some aspects of the data on housing market dynamics pointing to the importance of a model that contains both choices. Other models of the housing market assume that the sequence of transactions is irrelevant, which implicitly assumes that the intermediate step of a transaction sequence for a moving owner is costless regardless of whether he buys first or sells first (Ngai and Tenreyro (2014), Head, Lloyd-Ellis, and Sun (2014), Guren and McQuade (2013), Ngai and Sheedy (2015)).

Ngai and Sheedy (2015) model an endogenous moving decision based on idiosyncratic match quality as an amplification mechanism of sales volume. The paper shows how the endogenous participation decisions of mismatched owners in the housing market can explain why time-on-market for sellers can decrease while the stock of houses for sale increases at the same time, as was the case during the housing boom of the late 90s and early 2000s. In our model we assume that

[5] The paper is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magne and Rady (2006), Glaeser, Gyourko, Morales, and Nathanson (2014)).

mismatched owners always participate and instead focus on their transaction sequence decisions. The implications we draw from our analysis are therefore complementary to the insights in their paper.

Anenberg and Bayer (2013) is a recent contribution that is closest to our paper, particularly in terms of motivation. The paper studies a rich quantitative model of the housing market with two segments, in which some agents are sellers in the first segment, and simultaneously choose whether to also be buyers in the second segment. Shocks to the flow of new buyers in the first segment are transmitted and amplified onto the second segment through the decisions of these agents to participate as buyers in that second segment. Therefore, unlike our paper, there is no strategic complementarity in the decisions of mismatched owners. As discussed above, the feedback between market tightness and the decisions of mismatched owners that creates this strategic complementarity is the key driver of multiplicity, self-fulfilling fluctuations, and housing market volatility in our model. Also, in contrast to our model, "buying first" in that paper is a stochastic outcome rather than an endogenous choice.

Section 6 that extends our results to environments where prices are endogenously determined, relates the paper to the literature with Nash bargaining and competitive search in the housing market. In particular, Albrecht, Anderson, Smith, and Vroman (2007) study a search model of the housing market where buyers and sellers may become "desperate" if they search unsuccessful for too long. Because prices in their model are determined by Nash bargaining, the presence of agents with heterogeneous flow values while searching results in compositional effects that also arise in the extension of our model that features Nash bargaining. However, in their model equal numbers of buyers and sellers enter the market at an exogenous rate, so that there is no transaction sequence decision.[6] Finally, our extension in Section **??** to a model of the housing market with competitive search relates the paper to recent models of competitive search in housing and asset markets (Diaz and Jerez (2013), Albrecht, Gautier, and Vroman (2010), Lester, Visschers, and Wolthoff (2013), and Lester, Rocheteau, and Weill (forthcoming)).

The rest of the paper is organized as follows. In the next section we present some motivating facts using individual level data from Denmark. Section 3 sets up the basic model of the housing market. Section 4 characterizes the decisions of mismatched owners and discusses the equilibrium multiplicity and the implications of equilibrium switches. Section 5 shows how the incentives of mismatched owners to buy first or sell first depend on price expectations and shows that there can exist equilibria with self-fulfilling fluctuations in house prices and tightness. Section 6 shows that there can be equilibrium multiplicity in an environment where prices are determined by Nash bargaining and also in an environment with competitive search. Section 7 includes additional extensions, including allowing mismatched owners to simultaneously participate as buyers and sellers. Section 8 provides a discussion on the institutional details of transacting for several countries and concludes.

[6] Maury and Tripier (2014) study a modification of the Wheaton (1990) model, in which mismatched owners can buy and sell simultaneously, which they use to study price dispersion in the housing market. However, they do not consider the feedback from buying and selling decisions on the stock-flow process and on market tightness. This feedback is key for the mechanisms we explore in our paper.

2 Motivating Facts

We start by providing some motivating facts about the transaction sequence decisions of owner-occupiers for Copenhagen, Denmark. We focus on the Copenhagen urban area for the period 1992-2010. We use the Danish ownership register, which records the property ownership of individuals and legal entities as of January 1st of a given year. We combine that with a record of property sales for each year. The unique owner and property identifiers give us a matched property-owner data set, which we use to keep track of the transactions of individuals over time. We focus on individual owners who are recorded as the primary owner of a property.

We use the ownership records of individual owners over time to identify owner-occupiers who buy and sell in the Copenhagen housing market.[7] We then use the property sales record to determine the agreement dates (the dates the sale agreement is signed) and closing dates (the dates the property formally changes ownership) for the two transactions. Based on those, we construct a variable that measures the time difference between the sale of the old property and the purchase of the new property. Owner-occupiers, for which this difference is negative are classified as "selling first", while those with a positive difference are classified as "buying first".

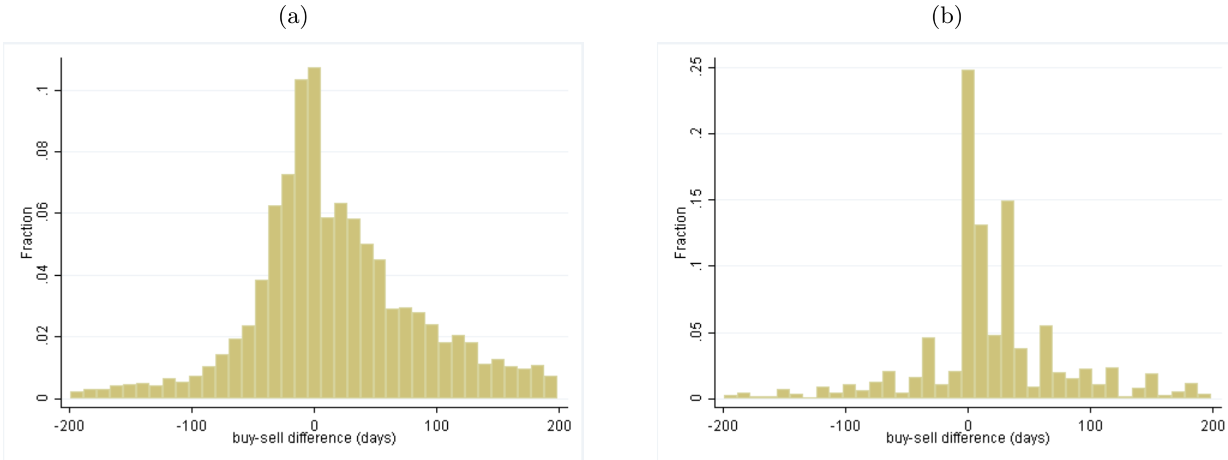
Figure 1 shows the distribution of the time difference between the agreement dates (Panel 1a) and closing dates (Panel 1b) for owner-occupiers who both buy and sell in Copenhagen during our sample period. There is substantial dispersion in the time difference between agreement dates, which suggests that a large fraction of these homeowners cannot synchronize the two transactions on the same date. Specifically, there is substantial mass even in the tails of the distribution. Examining the difference in closing dates shows a similar picture. Even though the distribution is more compressed in that case since homeowners try to a greater extent to synchronize the closing dates, so that they occur on the same day or in a close interval, a large fraction of homeowners face a time difference of a month or more in between closing the two transactions.[8] Overall, these distributions suggest that for homeowners that buy and sell in the same housing market the time difference between transactions can be substantial, confirming the anecdotal evidence cited in the introduction.

Another important observation is that the two distributions are right skewed, so moving home-owners tend to buy first during our sample period. This is confirmed when we examine the time series behavior of the fraction of homeowners that are identified as buying first in a given year from 1993-2008, as Figure 2 shows. Similar to Figure 1, the left-hand panel (Panel 2a) is based on agreement dates, while the right-hand panel (Panel 2b) is based on closing dates. Both panels also contain a price index for single family homes for the Copenhagen housing market. As the figure shows, the fraction of owners that buy first is not constant over time but exhibits wide variations going from a low of around 0.3 in 1994 to a high of 0.8 in 2006 and then back to a low of around 0.4 in 2008. This fraction tracks closely the house price index increasing over most of the sample

[7] The Appendix contains detailed information on the data used and on the procedure for identifying owner-occupiers that buy and sell. Given the way we identify these owner-occupiers, we have a consistent count for the number of owners who buy first or sell first in a given year for the years 1993 to 2008.

[8] It is interesting to note that for the difference between closing dates there are mass points around 30 day multiples. The reason for this is that in Denmark closing dates tend to fall on the first day of a given month.

Figure 1: Distribution of the time difference between "sell" and "buy" agreement dates (a) and closing dates (b) for homeowners who both buy and sell in Copenhagen (1993-2008).



period and peaking in the same year. It is then followed by a substantial drop as house prices start to decline after 2006. Therefore, Figure 2 suggests that the decisions to buy first may be related to the state of the housing market.

A closer examination of the period 2004-2008 strengthens this conjecture. Specifically, Figure 3 illustrates the fluctuations in key housing market variables like the for-sale stock, seller time-on-market, transaction volume and prices for Copenhagen in the period 2004-2008. It also includes our constructed fraction of buy first owners for Copenhagen in the period 2004-2008. During the first half of this period seller time-on-market (TOM), and the for-sale stock are low, while transaction volume and the fraction of buy first owners are high. There is a switch in all of these series around the 3rd quarter of 2006 and a quick reversal during which seller time-on-market and the for-sale stock increase rapidly, while the fraction of moving owners that buy first drops. Transaction volume is also lower during the second half of this period. Prices increase during the first half of the period and then decline.

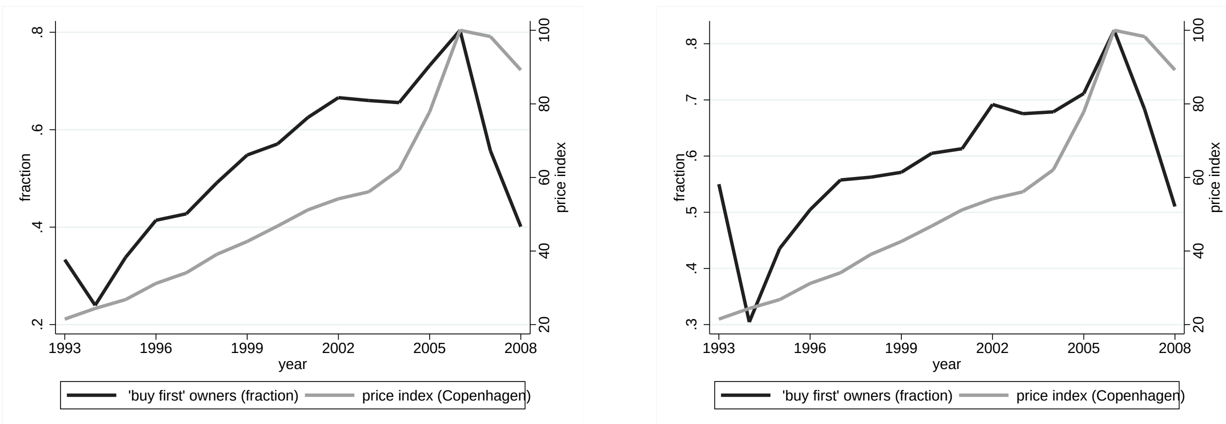
We take these three exhibits as indication that there is a non-trivial transaction sequence choice for owner-occupiers that move in the same housing market, that the time difference between the two transactions can be substantial, and that the decision to buy first or sell first is related to the state of housing markets. These facts motivate our theoretical study below.

3 A Model of the Housing Market

In this section we set up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis.

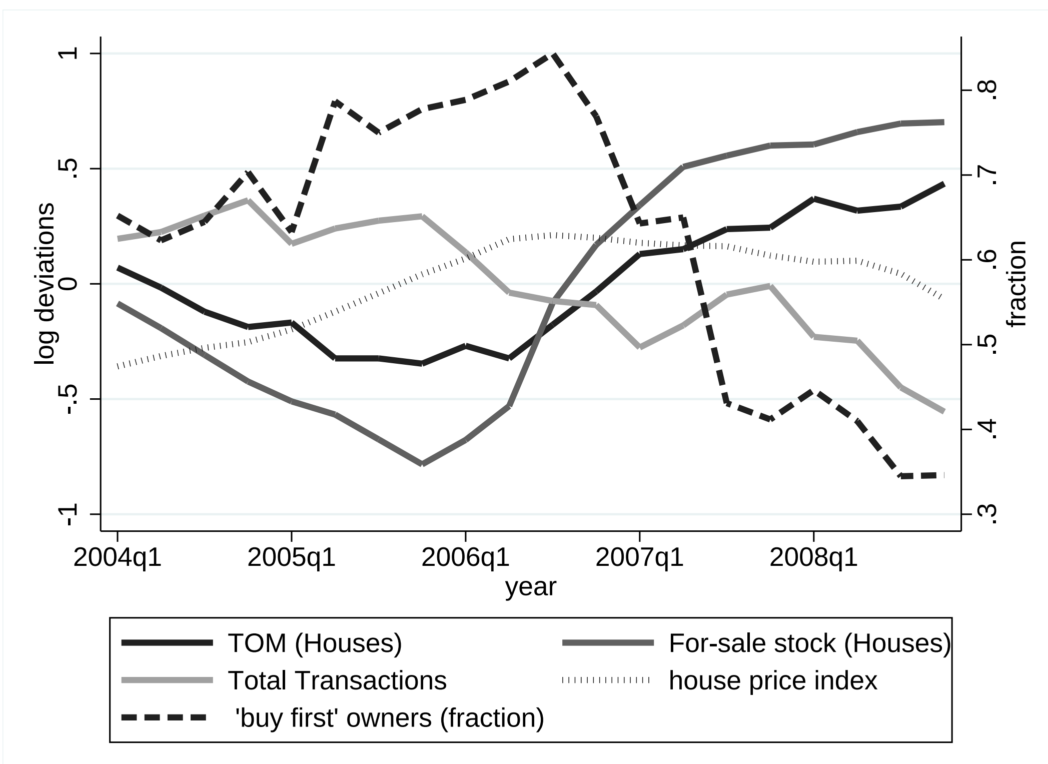
Figure 2: Fraction of owners who “buy first” and housing market conditions in Copenhagen (1993-2008). Panel (a) is based on agreement dates, and panel (b) is based on closing dates.

(a) (b)



Notes: The series on the fraction of “buy first” owners is from own calculations based on registry data from Statistics Denmark. See the Appendix for a description on how we identify an owner that buys and sells in Copenhagen (a buyer-and seller) as a “buy first” (“sell first”) owner. We compute annual counts of the number of “buy first” and “sell first” owners by looking at the year of the first transaction for each of these owners. The fraction of “buy first” owners is then the proportion of buyer-and-seller owners that buy first. For panel (a) the identification of an owner as buy first/sell first is based on the difference in the two agreement dates. For the second, it is based on the difference in the two closing dates. The price index is a repeat sales price index for single family houses for Copenhagen (Region Hovedstaden) constructed by Statistics Denmark.

Figure 3: Housing market dynamics, Copenhagen 2004-2008



Notes: Data on seller time-on-market (TOM) and the for-sale stock is from the Danish Mortgage Banks’ Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx). These series are shown in log deviations from their sample mean. The total transaction volume is from Statistics Denmark. It is in log deviations from the sample mean after controlling for seasonal effects by quarter-in-year dummies. The fraction of buy first owners is from own calculations based on registry data from Statistics Denmark. See the Appendix for a description on how we identify an owner that buys and sells in Copenhagen (a buyer-and-seller) as a “buy first” (“sell first”) owner. We compute quarterly counts of the number of “buy first” and “sell first” owners by looking at the quarter of the first transaction for each of these owners. The fraction of “buy first” owners is then the proportion of buyer-and-seller owners that buy first. The identification of an owner as buy first/sell first is based on the difference in the two agreement dates

3.1 Preferences

Time is continuous. The housing market consists of a unit measure of durable housing units that do not depreciate, and a unit measure of households, which we refer to as agents. The agents are risk neutral and have access to a perfect credit market with interest rate $r > 0$. When an agent buys a house and becomes a homeowner, he receives a flow utility of $u > 0$. We say that the homeowner is matched. With a Poisson rate γ the matched homeowner is hit by a taste shock, and becomes "mismatched" with his current housing unit. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$. A mismatched owner has to move to another housing unit in order to become matched again. Taste shocks of this form are standard in search models of the housing market (Wheaton, 1990), and create potential gains from trading.[9]

A mismatched owner can choose to "sell first" (and become a "mismatched seller"), selling the housing unit he owns first and then buying a new one. Alternatively, he can choose to "buy first" (becoming a "mismatched buyer"), buying a new housing unit first and then selling his old one. Finally, the mismatched owner may choose to not enter the housing market and stay mismatched.[10]

We also assume that a mismatched owner cannot synchronize the two transactions (the selling and buying). For example, he cannot exchange houses with another mismatched owner, as there is no double coincidence of housing wants among owners.[11] Instead, a mismatched owner has to conduct the two transactions in a sequence. A mismatched buyer ends up holding two housing units simultaneously for some period. In this case we say that he becomes a "double owner". Similarly, a mismatched seller ends up owning no housing. In that case the agent becomes a "forced renter".

We assume that a double owner receives a flow utility of $0 \leq u_2 < u$, while a forced renter receives a flow utility of $0 \leq u_0 < u$. Both of these include some implicit costs, such as maintenance costs in the former case, or restrictions on the use of the rental property imposed by a landlord in the latter case.[12]

Agents are born and die at the same rate g. New entrants start out their life without owning housing, and receive a flow utility $u_n < u$. Also, we assume that $u_n \geq u_0$, so forced renters obtain a lower utility flow when not owning a house compared to new entrants. After a death/exit shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If he

[9] Rather than introducing segmentation in the housing stock, we treat all housing units as homogeneous, so that mismatched owners participate in one integrated market with other agents. Although in reality agents move across housing market segments (whether spatial or size-based) in response to a taste shock of the type we have in mind, modeling explicitly several types of housing would substantially reduce the tractability of the model. Furthermore, defining empirically distinct market segments is not straightforward as in reality households often search in several segments simultaneously (Piazzesi, Schneider, and Stroebel, 2015).

[10] In Section 7, we allow a mismatched owner to search as a buyer and seller simultaneously, subject to a fixed time endowment. Note that searching simultaneously as both a buyer and seller does not mean that the agent can synchronize the two transactions, only that he chooses to receive offers both from potential buyers and sellers.

[11] This is similar to the lack of double coincidence of wants used in money-search models (Kiyotaki and Wright (1993)). In reality, some moving homeowners may be able to synchronize the two transactions as is also evident from Figure 1. Allowing some mismatched owners to synchronize their buy and sell transactions would reduce the tractability of the model without changing its qualitative predictions.

[12] For tractability we also assume that a double owner does not experience mismatching shocks. This ensures that the maximum holdings of housing by an agent will not exceed two units in equilibrium.

owns housing, his housing units are taken over by a real-estate firm, which immediately places them for sale on the market.[13] Real-estate firms are owned by all the agents in the economy with new entrants receiving the ownership shares of exiting agents.

Given the exit shock, agents effectively discount future flow payoffs at a rate $\rho \equiv r + g$. For notational convenience, we will directly use ρ later on. Also, we assume that agents are free to exit the economy in every instant and obtain their reservation utility of 0.

Finally, we assume that there exists a frictionless rental market with a rental price of R.[14] Agents without a house rent a dwelling. A landlord can simultaneously rent out a unit and have it up for sale. Hence double owners rent out one of their units, as do real-estate firms. This, together with free exit from the economy, implies that the equilibrium rental price can take multiple values. Specifically, we will consider rental prices that lie in the set $[0, u_n]$.[15]

3.2 Trading Frictions and Aggregate Variables

The housing market is subject to trading frictions. These frictions are captured by a standard constant returns to scale matching function $m(B(t), S(t))$, mapping a stock $B(t)$ of searching buyers and a stock $S(t)$ of searching sellers to a flow m of new matches. We assume that there is random matching, so different types of agents meet with probabilities that are proportional to their mass in the population of sellers or buyers. Directed search is discussed in Section **??**. We define the market tightness in the housing market as the buyer-seller ratio, $\theta(t) \equiv \frac{B(t)}{S(t)}$. Additionally, $\mu(\theta(t)) \equiv m\left(\frac{B(t)}{S(t)}, 1\right) = \frac{m(B(t),S(t))}{S(t)}$ is defined as the Poisson rate with which a seller successfully transacts with a buyer. Similarly, $q(\theta(t)) \equiv \frac{m(B(t),S(t))}{B(t)} = \frac{\mu(\theta(t))}{\theta(t)}$ is the rate with which a buyer meets a seller and transacts.

Beside the market tightness $\theta(t)$, which will be relevant for agents' equilibrium payoffs, we keep track of the following aggregate stock variables:

- $B_n(t)$ - new entrants;
- $O(t)$ - matched owners;
- $B_1(t)$ - mismatched buyers;
- $S_1(t)$ - mismatched sellers;
- $S_2(t)$ - double owners;
- $B_0(t)$ - forced renters;

[13]For simplicity, we assume that agents are not compensated for their housing upon exiting the economy. We extend our results in Section 7.2 to a case where exiting agents are compensated for their housing by the real-estate firms.

[14]We can alternatively assume that there are search frictions in the rental market as well, with the rental price R exogenously fixed within traders' bargaining sets. Since the measures of non-owners and vacant houses are always equal (see below), it follows that the market tightness in that market would be constant and equal to one.

[15]The equilibrium rental price R may be higher than u_n because of the additional value from homeownership that a new entrant anticipates.

- $A(t)$ - housing units that are sold by real-estate firms.

Therefore, the total measure of buyers is $B(t) = B_n(t) + B_0(t) + B_1(t)$ and the total measure of sellers is $S(t) = S_1(t) + S_2(t) + A(t)$. Since the total population is constant and equal to 1 in every instant, it follows that

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1. \tag{1}$$

Also, since the housing stock does not shrink or expand over time, the following housing ownership condition holds in every instant,

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{2}$$

Summing up, the life-cycle of an agent in the model proceeds as follows. An agent begins his life as a new entrant. With rate $q(\theta)$, he becomes a matched owner. Once matched, he becomes mismatched with rate γ. A mismatched owner chooses to either buy first (mismatched buyer) or sell first (mismatched seller). A mismatched buyer becomes a double owner with rate $q(\theta)$, who in turn sells and reverts to being a matched owner with rate $\mu(\theta)$. A mismatched seller becomes a forced renter with rate $\mu(\theta)$ and after that moves to being a matched owner with rate $q(\theta)$. In every stage of life an agent may exit the economy with rate g.

3.3 House price determination

We begin our analysis by assuming that the house price p is fixed and does not vary with the market tightness θ. However, in the equilibria we consider, the price p lies in the bargaining set of all actively trading pairs.[16] We progressively relax this assumption by assuming that p varies with θ in a reduced form-way in Section 5.2 and by assuming that prices are determined by symmetric Nash bargaining in Section 6.1 or in a competitive search equilibrium in Section **??**. The main insights of our analysis hold in those environments as well, although at a significant reduction in tractability.

Given that the price is assumed to lie in the bargaining sets of all trading pairs, it can also be considered as the market clearing price in a competitive market with frictional entry of traders. In particular, as in Duffie, Garleanu, and Pedersen (2005) or Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B, S)$ with buyers and sellers of different types entering according to their fractions in the population of buyers and sellers, respectively. Once in the market, there is anonymity, and all agents are price takers. The transaction price leaves trading counterparties (weakly) better off from transacting at that price. However, given the limited heterogeneity of agents, there are generically

[16] This is similar to the literature on rigid wages in search-and-matching models (Hall (2005), Gertler and Trigari (2009)). Also, under certain conditions, a unique fixed price that does not vary with tightness or across trading pairs can be microfounded as resulting from bargaining between heterogeneous buyers and sellers, in which the buyer has full bargaining power but does not know the type of the seller. As shown in the Appendix, take-it-or-leave-it offers from buyers under private information about the seller's type can generate a fixed price that is equal to the present discounted value of rental income.

many market clearing prices that leave agents (weakly) better off from transacting, which generates some indeterminacy in the price level. We resolve this indeterminacy by selecting some price p from the set of market clearing prices and examining equilibria in that case.

4 Steady State Equilibria

We start by characterizing steady state equilibria of this economy.[17] We first discuss the value functions of different types of agents in a candidate steady state equilibrium.

4.1 Value functions

We have the following set of value functions for different agents in this economy:

- V^{B1} - value function of a mismatched buyer;
- V^{S1} - value function of a mismatched seller;
- V^{B0}- value function of a forced renter;
- V^{S2} - value function of a double owner;
- V^{Bn} - value function of a new entrant;
- V - value function of matched owner;
- V^{A} - value function of a real-estate firm that holds one housing unit.

Given these notations, we have a standard set of Bellman equations for the agents' value functions in a steady state equilibrium.

First of all, for a mismatched buyer we have

$$\rho V^{B1} = u - \chi + q(\theta) \max\left\{-p + V^{S2} - V^{B1}, 0\right\}, \tag{3}$$

where $u - \chi$ is the flow utility from being mismatched. Upon matching with a seller, a mismatched buyer purchases a housing unit at price p, in which case he becomes a double owner, incurring a utility change of $V^{S2} - V^{B1}$.

A double owner has a flow utility of $u_2 + R$ while searching for a counterparty. Upon finding a buyer, he sells his second unit and becomes a matched owner. Therefore, his value function satisfies the equation[18]

$$\rho V^{S2} = u_2 + R + \mu(\theta)\left(p + V - V^{S2}\right). \tag{4}$$

[17]Informally, in a steady state equilibrium, agents (most importantly mismatched owners) make choices that maximize their discounted payoffs given the market tightness θ, and aggregate variables and agent values are constant over time. Finally, the house price, p, is such that it is privately optimal for agents to transact. A formal definition of a steady state equilibrium of this economy and some parametric restrictions can be found in the Appendix.

[18]We present the value functions of double owners and forced renters assuming that they always trade at the price p, since that will always be the case in the steady state equilibria we consider. For example, for the case of a double owner we have $V + p \geq \frac{u_2+R}{\rho}$. The Appendix provides a set of sufficient conditions for this to hold.

The value function of a mismatched seller is analogous to that of a mismatched buyer apart from the fact that a mismatched seller enters on the seller side of the market first and upon transacting becomes a forced renter. Therefore,

$$\rho V^{S1} = u - \chi + \mu(\theta) \max\left\{p + V^{B0} - V^{S1}, 0\right\}. \tag{5}$$

Finally, for a forced renter we have

$$\rho V^{B0} = u_0 - R + q(\theta)\left(-p + V - V^{B0}\right). \tag{6}$$

The remaining value functions are straightforward and are given in the Appendix. Importantly, given the assumption that a real-estate firm can rent out a housing unit without costs, in any steady state equilibrium,

$$\rho p \geq R, \tag{7}$$

as otherwise real estate firms do not find it optimal to sell housing.

4.2 Optimal choice of mismatched owners

In a steady state equilibrium, the optimal decision of mismatched owners depends on the simple comparison

$$V^{B1} \gtreqless V^{S1}. \tag{8}$$

We can substitute for V^{B0} and V^{S2} from equations (6) and (4) into the value functions for a mismatched buyer and seller to obtain

$$V^{B1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho + q(\theta)} + \frac{q(\theta)(u_2 - (\rho p - R))}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V\right\}, \tag{9}$$

and

$$V^{S1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho + \mu(\theta)} + \frac{\mu(\theta)(u_0 + (\rho p - R))}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V\right\}. \tag{10}$$

We define the effective utility flow for a forced renter as $\tilde{u}_0 \equiv u_0 + \triangle$, and for a double owner as $\tilde{u}_2 \equiv u_2 - \triangle$, where $\triangle \equiv \rho p - R$. Note that in the special case in which $R = \rho p$, $\tilde{u}_0 = u_0$ and $\tilde{u}_2 = u_2$. Hence in this case, the housing price does not influence the flow value, including incomes/expenses from renting, for double owners or forced renters. In our benchmark case we assume that $\tilde{u}_0 = \tilde{u}_2 = c$.

Our analysis focuses on the empirically relevant and realistic case, in which being mismatch gives a higher flow value than being a double owner of a forced renter:

Assumption A1: $u - \chi \geq \max\{\tilde{u}_0, \tilde{u}_2\}$**.**

Anecdotal evidence points to the mismatch state as not particularly costly for the majority of homeowners. As Ngai and Sheedy (2015) argue, mismatch may be so small for many homeowners that they prefer not to move and save on the transaction costs. On the other hand, a comparison between the difference in agreement and closing dates from Figure 1 shows that moving homeowners tend to minimize the delay between the closing of the two transactions with many transactions either occurring simultaneously or within a short period. This suggests that delays between transactions are particularly costly for moving homeowners.

Note that, in the special case with $R = \rho p$, assumption $A1$ can be written as $u - \chi > \max\{u_0, u_2\}$. If, in addition, $u_0 = u_2 = c$, the assumption boils down to $u - \chi \geq c$.

Define $D(\theta) \equiv V^{B1} - V^{S1}$, the difference in value between buying first and selling first (assuming that it is optimal for both a mismatched buyer and mismatched seller to transact). We have

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 \right]. \tag{11}$$

In the benchmark case, where $\tilde{u}_0 = \tilde{u}_2 = c$ equation (11) simplifies to

$$D(\theta) = \frac{(\mu(\theta) - q(\theta))(u - \chi - c)}{(\rho + q(\theta))(\rho + \mu(\theta))}. \tag{12}$$

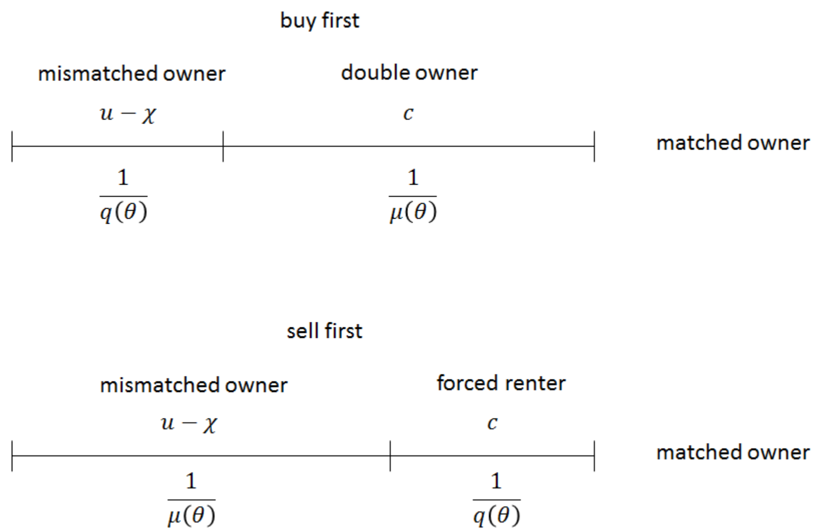
Hence, in the benchmark case, buying first is preferred whenever $\mu(\theta) > q(\theta)$. The expected time on the market for a buyer and a seller are $\frac{1}{q(\theta)}$ and $\frac{1}{\mu(\theta)}$, respectively. Hence buying first is preferred if and only if the time-on-market is higher for a buyer than for a seller. The reason is that a mismatched owner has to undergo two transactions on both sides of the market before he becomes a matched owner. Given this, a mismatched owner wants to minimize the expected time in the situation that is relatively more costly. Since it is more costly to be a double owner or a forced renter than to be mismatched, a mismatched owner would care more about the expected time on the market for the second transaction and would want to minimize the delay between the two transactions.

As an example, suppose that $\theta < 1$ and consider the schematic representation of a mismatched owner's expected payoffs in Figure 4. If the agent buys first (top part of Figure 4), he has a short expected time-on-market as a buyer. However, he anticipates a long expected time-on-market in the next stage when he is a double owner and has to dispose of his old housing unit. In contrast, selling first (bottom part of Figure 4) implies a long expected time-on-market until the agent sells his property but a short time-on-market when the agent is a forced renter and has to buy a new property. Since $u - \chi > c$, then it is more costly to remain in the second stage for a long time (as a double owner or forced renter) rather than to be mismatched and searching. Therefore, selling first is strictly preferred to buying first in that case.

We now formally characterize the optimal action of a mismatched owner given a steady state market tightness θ. We adopt the notation $\theta = \infty$ for the case where the buyer-seller ratio is unbounded.

We define

Figure 4: Buying first versus selling first when $\theta < 1$.



$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \tag{13}$$

Note that if $\tilde{u}_2 = \tilde{u}_0$, then $\tilde{\theta} = 1$, while if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$, and vice versa if $\tilde{u}_2 < \tilde{u}_0$.[19]

In the more general case, in which $\tilde{u}_0$ may differ from $\tilde{u}_2$, the following lemma fully characterizes the incentives of mismatched owners to buy first or sell first given a steady state market tightness θ:

Lemma 1. *Let $\tilde{\theta}$ be as defined in (13). Then for $\theta \in (0, \infty)$, $V^{B1} > V^{S1} \iff \theta > \tilde{\theta}$ and $V^{B1} = V^{S1} \iff \theta = \tilde{\theta}$. For $\theta = 0$ and $\theta = \infty$, $V^{B1} = V^{S1} = \frac{u-\chi}{\rho}$.*

Proof. See Appendix D. □

Lemma 1 shows that, in general, as θ increases, the incentives to buy first are strengthened. For high values of θ, buying first dominates selling first. For low values of θ, selling first dominates buying first.

4.3 Steady state flows and stocks

We turn next to a description of the steady state equilibrium stocks and flows of this model. The full set of equations for these flows are included in the Appendix. Here we just make some important observations on the stock-flow process in the model. First, combining the population and housing ownership conditions (1) and (2) we get that

$$B_n(t) + B_0(t) = A(t) + S_2(t). \tag{14}$$

[19] In what follows we will additionally assume that at $\theta = \tilde{\theta}$, both $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so that a mismatched owner is strictly better off from transacting at $\theta = \tilde{\theta}$. This removes uninteresting steady state equilibria in which mismatched owners never transact. Assumption A2 in the Appendix gives a sufficient condition for this.

Since there are equally many agents and houses, the stocks of agents without a house (forced renters and new entrants) must be equal to the stock of double owners and real-estate firms. This identity implies that in a candidate steady state equilibrium where all mismatched owners buy first (so that there are no forced renters), the market tightness, denoted by $\overline{\theta}$ satisfies

$$\overline{\theta} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n} > 1.$$

Similarly, if $\underline{\theta}$ denotes the market tightness in a candidate steady state where all mismatched owners sell first (so there are no double owners), we have that

$$\underline{\theta} = \frac{B_n + B_0}{A + S_1} = \frac{A}{A + S_1} < 1.$$

Therefore, $\underline{\theta} < 1 < \overline{\theta}$. This points to possible wide variations in market tightness from changes in the behavior of mismatched owners. In Lemma 2 we show that $\overline{\theta}$ solves

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right) = \frac{1}{g} + \frac{1}{\gamma}, \tag{15}$$

and $\underline{\theta}$ solves

$$\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma}. \tag{16}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched owners buy first and sell first, respectively.

Lemma 2. *Let $\overline{\theta}$ and $\underline{\theta}$ denote the steady-state market tightness when all mismatched owners buy first and sell first, respectively. Then $\overline{\theta}$ and $\underline{\theta}$ are unique. Furthermore, $\overline{\theta} > 1$, $\underline{\theta} < 1$, and $\overline{\theta}$ is increasing in γ and $\underline{\theta}$ is decreasing in γ.*

Proof. See Appendix D. □

It is illustrative to consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. From equations (15) and (16), we have that

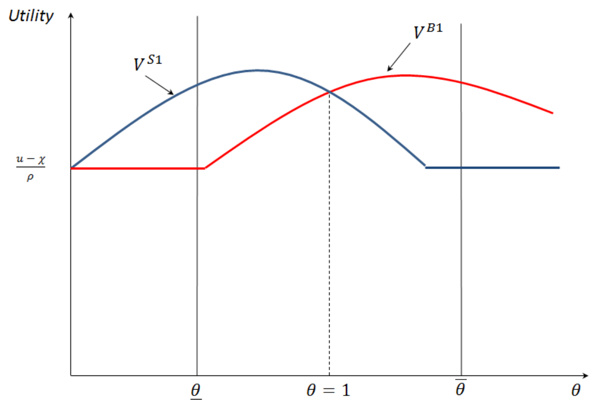
$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \overline{\theta} = 1 + \kappa, \tag{17}$$

and

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \underline{\theta} = \frac{1}{1 + \kappa}. \tag{18}$$

Thus, the more important mismatched owners are in housing transactions (the higher is $\kappa = \gamma/g$), the larger the variation in market tightnesses from changes in mismatched owners' actions.

Figure 5: Equilibrium multiplicity with $\tilde{u}_0 = \tilde{u}_2 = c$.



4.4 Equilibrium characterization

We now combine the observations on the optimal choice of mismatched owners and the steady state stocks from the previous two sections to characterize equilibria of our model. We first characterize steady state equilibria in the benchmark case where there is symmetry in the flow payoffs of a double owner and a forced renter, so $\tilde{u}_0 = \tilde{u}_2 = c$.

Proposition 3. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exist three steady state equilibria:*

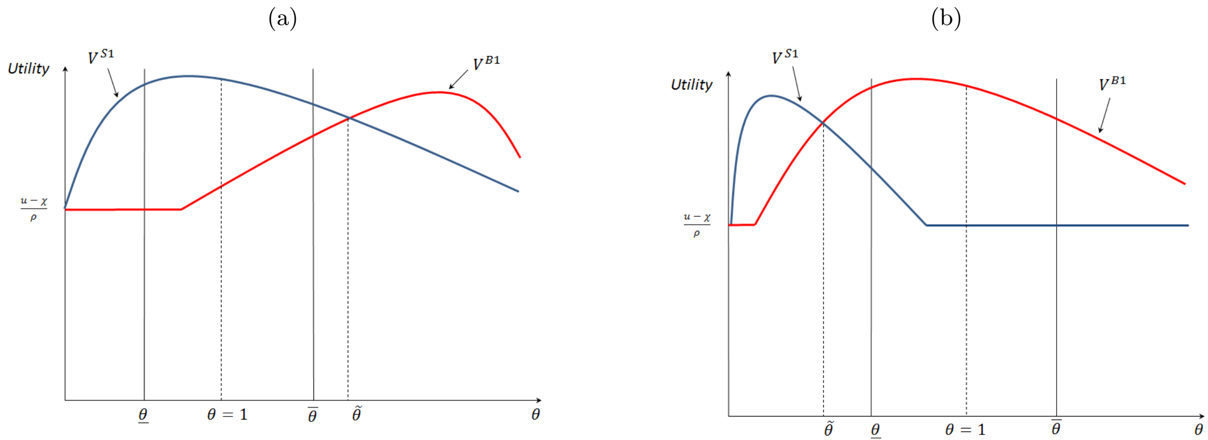
1. *A sell first equilibrium with $\theta = \underline{\theta}$, in which mismatched owners sell first.*
2. *A buy first equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first.*
3. *An equilibrium with $\theta = 1$, in which the mismatched owners are indifferent between buying first and selling first, and half of them buy first.*

Proof. See Appendix D. □

Proposition 3 shows that in the benchmark case there exist multiple steady state equilibria. Intuitively, the equilibrium multiplicity arises because the feedback from the transaction sequence decisions of mismatched owners to the steady state equilibrium market tightness creates a form of strategic complementarity in their actions. When mismatched owners are buying first, the steady state buyer-seller ratio $\theta > 1$, so that it is individually rational for any mismatched owner to buy first. Conversely, when mismatched owners are selling first, the steady state buyer-seller ratio $\theta < 1$, and it is individually rational to sell first. Figure 5 illustrates this equilibrium multiplicity and the equilibrium value functions of mismatched owners.[20]

[20]For illustrative purposes, in the figures below we assume that V^{B1} and V^{S1} as defined in (9) and (10) are single peaked, i.e. there is a $\hat{\theta}^{B1}$, s.t. V^{B1} is increasing for $\theta < \hat{\theta}^{B1}$ and decreasing for $\theta > \hat{\theta}^{B1}$ and similarly for V^{S1}. Also, from the figure one can conclude that the steady state equilibrium with $\theta = 1$ is generically unstable in the following sense: if slightly more mismatched owners start to buy first than the equilibrium prescribes, all the mismatched owners will have an incentive to buy first and vice versa.

Figure 6: Examples with unique equilibria in the case of $\tilde{\theta} > \overline{\theta}$ (a) and $\tilde{\theta} < \underline{\theta}$ (b).



Apart from this symmetric payoff case, there can be multiple equilibria more generally when $\tilde{u}_0 \neq \tilde{u}_2$. However, if the payoff asymmetry is sufficiently strong, there will be a unique equilibrium. In particular, if $\tilde{u}_0$ is sufficiently low relative to $\tilde{u}_2$, there is a unique equilibrium in which mismatched owners buy first and vice versa when $\tilde{u}_2$ is sufficiently low relative to $\tilde{u}_0$. Whether there is equilibrium uniqueness or multiplicity depends on a comparison of $\underline{\theta}$ and $\overline{\theta}$ against the value of $\tilde{\theta}$ at which a mismatched owner is indifferent between buying first and selling first as defined in condition (13).

Proposition 4. *Consider the above economy. Let $\tilde{\theta}$ be defined by condition (13), and $\overline{\theta}$ and $\underline{\theta}$ be defined by (15) and (16) with $\overline{\theta}, \underline{\theta} \in (0, \infty)$.*

1. *If $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, then there exists a steady state equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first. There also exists a steady state equilibrium with $\theta = \underline{\theta}$ in which mismatched owners sell first.*

2. *If $\tilde{\theta} < \underline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners buy first.*

3. *If $\tilde{\theta} > \overline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners sell first.*

Proof. See Appendix D. □

Therefore, depending on the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$, there can exist multiple equilibria or a unique equilibrium.[21] Figure 6 shows examples in which only one equilibrium may exists. In Figure 6a, $\tilde{\theta} > \overline{\theta}$, so that only a "Sell first" equilibrium exists with $\theta = \underline{\theta}$. Figure 6b shows the opposite case when $\tilde{\theta} < \underline{\theta}$, so that only a "Buy first" equilibrium exists with $\theta = \overline{\theta}$.

[21] Additionally, whenever there are multiple equilibria there can also be a third equilibrium with market tightness $\theta = \tilde{\theta}$ where mismatched owners mix between buying first and selling first.

Since $\tilde{\theta}$ depends on flow payoffs of mismatched owners, shocks to these payoffs can lead to equilibrium switches.[22] Apart from payoff shocks, equilibrium switches may also occur because of changes in agents' believes. Next, we discuss the implications of such equilibrium switches for transaction volume, time-on-market, and the stock of houses for sale.

4.5 Equilibrium switches

We now discuss equilibrium switching. To simplify the analysis we consider the limit economy introduced in Section 4.3, where $g \to 0$ and $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, $\overline{\theta} = 1 + \kappa$, and $\underline{\theta} = \frac{1}{1+\kappa} = \frac{1}{\overline{\theta}}$. Suppose that the economy starts in a "Buy first" equilibrium with market tightness $\theta = \overline{\theta}$. In that case

$$\overline{\theta} = \frac{\overline{B}}{\overline{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{19}$$

where $\overline{B}$ and $\overline{S}$ denote the stocks of buyers and sellers in the "Buy first" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first. In that case, the new market tightness becomes

$$\theta' = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \frac{1}{\overline{\theta}},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Therefore, θ' is the reciprocal of the market tightness before the switch. In the limit economy, that reciprocal value is exactly $\underline{\theta}$. Therefore, there is an immediate switch from the "Buy first" to the "Sell first" buyer-seller ratio and there is no dynamic adjustment in θ.

What are the implications of this switch? First of all, clearly average time-on-market for sellers, $\frac{1}{\mu(\theta)}$ increases. Second, consider the ratio of the stock of sellers before and after the switch. That ratio is exactly

$$\frac{\overline{S}}{S'} = \frac{B_n}{B_n + B_1} = \frac{1}{\overline{\theta}} < 1. \tag{20}$$

Therefore, there is an increase in the stock of houses for sale since some of the previous buyers become sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, suppose that we have a Cobb-Douglas matching function, so $m(B, S) = \mu_0 B^{\alpha} S^{1-\alpha}$, for $0 < \alpha < 1$, and consider the ratio of transaction volumes before and after the switch. This ratio is given by

$$\frac{m\left(\overline{B}, \overline{S}\right)}{m\left(B', S'\right)} = \frac{\mu\left(\overline{\theta}\right)\overline{S}}{q\left(\underline{\theta}\right)B'} = \frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)}. \tag{21}$$

[22] As an example of such a payoff shock, suppose that the payoff of a double owner, u_2, includes costs associated with obtaining a mortgage that allows him to finance the downpayment on his new property prior to the sale of his old property. When financial markets function normally, these costs are relatively low. Suppose that in that case $\tilde{u}_2 > \tilde{u}_0$ and $\tilde{\theta} < \underline{\theta}$. Therefore, the "Sell first" equilibrium does not exist. Conversely, suppose that there is a shock to financial markets so that obtaining a bridging mortgage becomes very costly, and thus $\tilde{u}_2 < \tilde{u}_0$ and $\tilde{\theta} > \overline{\theta}$. As a result, after the shock, buying first is no longer optimal and the "Buy first" equilibrium no longer exists.

The latter ratio is

$$\frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)}=\frac{\mu_0\overline{\theta}^{\alpha}}{\mu_0\underline{\theta}^{\alpha-1}}=\left(1+\kappa\right)^{2\alpha-1}.$$

Hence, transaction volume falls after the equilibrium switch if $\alpha>\frac{1}{2}$ and increases if $\alpha<\frac{1}{2}$. The reason is that for $\alpha>\frac{1}{2}$ buyers are more important than sellers in generating transactions. When mismatched owners switch from buying first to selling first this leads to a reduction in the number of buyers and an increase in the number of sellers, and hence, to a fall in the transaction rate. Genesove and Han (2012) estimate a value of $\alpha=0.84$. At that value, transaction volume would drop after the switch from a "Buy first" equilibrium to a "Sell first" equilibrium.

Consequently, a switch from a "Buy first" equilibrium to a "Sell first" equilibrium implies a behavior for key housing market variables like the for-sale stock, average time-to-sell, and transaction volume that is broadly consistent with evidence on housing cycles (Diaz and Jerez (2013), Guren (2014)). This behavior is also consistent with the evidence on the housing cycle in Copenhagen as shown in Figure 3.

4.6 Quantitative relevance

In this section we provide a numerical example to assess the quantitative relevance of our mechanism. We use data from the Copenhagen housing market to determine plausible values for the rate of mismatch, γ, and the entry and exit rate, g.

We use two quantities to determine these parameters. The first is the fraction of owners that we identify as both buying and selling, and who are recorded as owning two properties. Those correspond to the stock of double owners in our model. The fraction of such owners is fairly low at around 0.04% during the 90s but quickly increases and reaches a high of 0.3% in 2006.[23] During the period 2005-2006, which we take as the period where the market is in the "Buy first" steady state with mismatched owners buying first, the average fraction of such owners is 0.22%. The second quantity is the fraction of owners that exit the market within a year. During the period 2005-2006 the average fraction of owners that exit within a year is 4.7%.

We assume that the matching function is Cobb-Douglas $\mu\left(\theta\right)=\mu_0\theta^{\alpha}$ with $\alpha=0.84$, following Genesove and Han (2012). We also set the matching efficiency parameter μ_0 to match an average time-on-market for a seller of around 3 month, which corresponds to the time-on-market for single-family homes in Copenhagen during 2005-2006. As already mentioned, we assume that the housing market is in a "Buy first" equilibrium during 2005-2006.

Matching the fractions of double owners and owners that exit the market, we find a value of $\gamma=0.01$ and a value of $g=0.05$. This implies an average duration for homeowners of around 17 years and an annual turnover rate of 6%.

Given these parameters, the implied market tightness in a "Buy first" equilibrium is $\overline{\theta}=1.196$. The market tightness in a "Sell first" equilibrium is $\underline{\theta}=0.836$. Note that these values are almost

[23] Housing ownership is observed at year-end. It may well be that the number of double owners is lower than average at this date, if they try to avoid to sit with two housing units over Christmas. If this is the case, our quantitative exercise underestimates γ and underestimates the difference between the regimes.

reciprocal since the estimated values of γ and g are small. This suggests that the focus on the limit economy in Section 4.5 is reasonable.

Therefore, going back to our equilibrium switching example in the previous section, switching from a "Buy first" equilibrium to the "Sell first" equilibrium decreases the market tightness by around 30%. This is associated with a 20% increase in the stock of houses for sale, and a 36% increase in the average time-to-sell. Looking at the ratio of transactions before and after the switch, we find that transactions fall by around 14%.[24]

5 House Price Fluctuations

Up to now we considered a constant house price p that agents are willing to transact at. In this section, we first examine the implications of expected changes in the house price for the behavior of mismatched owners. We then construct dynamic equilibria with self-fulfilling fluctuations in prices and tightness.

5.1 Exogenous house price movements

We first show that expected future changes in the house price affect the incentives of mismatched owners to buy first or sell first. We consider the benchmark case with $u_0 = u_2$ and with $p = \frac{R}{\rho}$, so the effective flow payoffs of a forced renter and double owner are equal, $\tilde{u}_0 = \tilde{u}_2 = c$. Consider a simple exogenous process for the house price p. With rate λ the house price p changes to a permanent new level p_N.[25]

We compare the utility from buying first relative to selling first for a mismatched owner before the price change. If the price change occurs between the two transactions, the mismatched owner will make a capital gain of $p^N - p$ if he buys first and a capital loss of the same amount if he sells first. If the shock happens before the first transaction or after the second transaction, it will not influence the decision to buy first or sell first.[26]

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions $D(\theta) = V^{B1} - V^{S1}$ takes the form

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)} 2\lambda (p_N - p). \tag{22}$$

An expected price decrease, leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease is sufficiently large, so that even at $\theta = \overline{\theta}$, $D\left(\overline{\theta}\right) < 0$, then selling

[24] In Section 6.1 we also discuss the quantitative implications for house prices in the model with Nash bargaining.

[25] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$.

[26] We assume that θ remains constant over time, so the only change occurs in the house price p. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of the house price p.

first will dominate buying first for any value of θ that is consistent with equilibrium. Similarly, a sufficiently large expected price increase, will imply that $D(\underline{\theta}) > 0$, so buying first will dominate selling first for any value of θ that is consistent with equilibrium. We summarize these observations in the following

Proposition 5. *Consider the modified economy with an exogenous house price change. Then for every $\lambda > 0$ and $\theta \in [\underline{\theta}, \overline{\theta}]$, a mismatched owner prefers to sell first for sufficiently low values of p_N. Analogously, a mismatched owner prefers to buy first for sufficiently high values of p^N.*

Proof. See Appendix D. □

5.2 Self-fulfilling house price fluctuations

In the steady-state analysis, we showed that the model may exhibit multiple equilibria, with different market tightnesses. If a high market tightness is associated with a high price, this may lead to a destabilizing effect on housing prices.

To see this, suppose $X(t)$ follows a two-state Markov chain $X(t) \in \{0, 1\}$. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable. The price in state 1 is assumed to be given by a smooth function $p_1 = f(\theta_1)$. The price in state 0 is assumed to be implicitly given by a smooth function $p_0 = f(\theta_0, \lambda(p_1 - p_0))$, increasing in both arguments, and with $f(\theta, 0) \equiv f(\theta)$. We take these relationships as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of housing prices and market liquidity conditions with the transaction decisions of mismatched owners. Again we look at a limit economy as γ and g go to zero, keeping $\kappa = \gamma/g$ fixed. We also assume that $R_i = \rho p_i$, $i = 1, 2$.

We consider an equilibrium in which the economy starts out in a "Buy first" regime ($X(t) = 0$) in which 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \overline{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Sell first" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Sell first" regime forever. Since $\bar{\theta} > \underline{\theta}$, it follows that $p_0 > p_1$.[27] It is trivial to show that as $\lambda \to 0$, the pay-offs when buying first and selling first converge to the pay-offs without regime switching. Hence, in the limit, buying first in state zero is an equilibrium strategy if $\bar{\theta} > \tilde{\theta}$, while selling first is an equilibrium strategy in state 1 if $\underline{\theta} < \tilde{\theta}$, where $\tilde{\theta}$ is defined by proposition 4. Hence multiple equilibria exists whenever

$$1 + \kappa > \frac{u - \chi - u_2}{u - \chi - u_0} > \frac{1}{1 + \kappa} \tag{23}$$

This equation is clearly satisfied when $u_2 = u_0$

Proposition 6. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, and with the sunspot process described above. Suppose further that equation (23) is satisfied. Then there is a $\overline{\lambda}$, such*

[27] Suppose $p_0 \leq p_1$. Then $p_0 = f(\bar{\theta}, \lambda(p_1 - p_0)) \geq f(\bar{\theta})$. But then $p_0 \geq f(\bar{\theta}) > f(\underline{\theta}) = p_1$, a contradiction.

that for $\lambda < \overline{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 = \overline{\theta}$ and mismatched owners buy first. In the second regime, $\theta_1 = \underline{\theta}$, $p_1 < p_0$, and mismatched owners sell first. The economy starts in regime 0 and transitions to regime 1 with rate λ.

Proof. See Appendix D.

□

6 Robustness

In this section we show that there can be multiple equilibria even in an environment where prices are determined by Nash bargaining, and in an environment with competitive search where agents can trade off prices and time-on-market. With endogenous price formation, the model becomes algebra-intensive, as we have to keep track of which agent types trade with one another, as well as the stocks and flows of the different types. For that reason, the details of the analysis are deferred to the appendix. For the same reason we consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is constant in the limit, and where $\overline{\theta} = 1 + \kappa$ and $\underline{\theta} = \frac{1}{1+\kappa}$.

6.1 Endogenous Prices Determined by Nash Bargaining

In this section we assume that prices are determined by Nash bargaining, so that buyers and sellers split the surplus of a match equally between them as in Pissarides (2000). Therefore, there is no longer a single transaction price p, but prices depend on the types of the trading counterparties. Note that the real-estate firms are different from the other agents in the economy, as they receive no utility from owning a house, and the gain from transacting is the price, which is a transfer and hence does not influence the match surplus. As a result, equilibrium allocation becomes asymmetric, and tilt towards the "Buy first" equilibrium even with $u_2 = u_0$. It turns out that symmetry, in the sense that mismatched owners are indifferent between buying first and selling first if $\theta = 1$, is re-established if $u - u_2 = u_n - u_0$. In what follows we therefore assume that this is the case.

First, consider a "Sell first" equilibrium candidate. The sellers are the mismatched owners and the real-estate firms, while the buyers are the forced renters and new entrants. Forced renters are the most eager buyers (in the sense that they obtain the lowest utility flow while searching), and will therefore trade with both seller types. Otherwise it is not obvious that a meeting ends in a transaction, as this will depend on parameter values. Here we assume that they do, and give the relevant parameter restrictions in the appendix.

Since the mismatched sellers and real-estate firms have the same transaction rate, the fraction of mismatched sellers to firms in the limit economy is equal to the fraction of the inflows $\gamma/g = \kappa$. Hence the fractions of mismatched sellers and firms to the total number of sellers are $1 - \underline{\theta}$ and $\underline{\theta}$, respectively. Furthermore, in the limit, as no one dies in the selling queue, the fraction of forced renters to new entrants is also $\gamma/g = \kappa$. Thus, the fractions of new entrants and forced renters to the total number of buyers are also $\underline{\theta}$ and $1 - \underline{\theta}$, respectively. Similarly, in a "Buy first" equilibrium the fraction of sellers that are real-estate firms is $A/S = 1/\overline{\theta}$ and so is the fraction of new entrants relative to buyers, $\frac{B_n}{B}$.

We assume that the buyers and the sellers bargain over the match surplus, and split it evenly. It follows that the value function of a mismatch seller is (with $\rho = r$ in the limit)

$$rV^{S1} = u - \chi + \frac{1}{2}\mu(\underline{\theta})[\underline{\theta}\Sigma_{S1Bn} + (1-\underline{\theta})\Sigma_{S1B0}]$$

where $\Sigma_{S1Bn} = V + V^{B0} - V^{Bn} - V^{S1}$ is the match surplus when a mismatched seller meets a new entrant, and $\Sigma_{S1B0} = V - V^{S1}$ is the match surplus when a mismatched seller meets a forced renter. In the Appendix we show that V^{S1} can be written as

$$V^{S1} = \frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V - \underline{\theta}\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{\rho+\frac{1}{2}\mu\left(\underline{\theta}\right)}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\underline{\theta}\right)}.$$

We then consider a mismatched owners who deviates (permanently) and buys first. In this case, it is not obvious whether a mismatched seller and a mismatched buyer will trade. The algebra simplifies when they do not trade, and we therefore assume that this is the case and give conditions on parameters in the appendix. It follows that the gain from deviating, $D(\underline{\theta}) = V^{B1} - V^{S1}$, can be written as

$$D\left(\underline{\theta}\right) = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)} - \frac{u-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\right). \tag{24}$$

Given our assumptions on utility flows, $D(1) = 0$. A decrease in $\underline{\theta}$ (equivalently, an increase in κ) leads to a decrease in $D\left(\underline{\theta}\right)$ since the expression in parenthesis, decreases. This decrease comes from two effects. First, $\mu\left(\underline{\theta}\right)$ decreases and $q\left(\underline{\theta}\right)$ increases, so the second term in the parenthesis becomes more negative (given that $u_2 < u - \chi < u$) and the first term decreases (since $u_n > u_0$). This effect is tightly linked to the main mechanism in our basic set-up discussed in Section 4.2. Specifically, as before, a decrease in $\underline{\theta}$ decreases the value of buying first given a higher expected time-on-market for double owners, while it increases the value of selling first, given a shorter expected time-on-market for forced renters. It follows that the mismatched owners strictly prefer to sell first in the "Sell first" equilibrium candidate.

Additionally, the fraction of new entrants, $\underline{\theta}$, decreases. This additionally strengthens the incentives to sell first, and arises via a compositional effect on the buyer side of the market. In particular, as $\underline{\theta}$ falls there are relatively fewer new entrants and relatively more forced renters among the pool of buyers. Since forced renters have a lower outside option compared to new entrants (given the lower utility flow), there is a higher surplus from transacting with a forced renter, which provides an additional incentive for mismatched owners to sell first.

Consider then a "Buy first" equilibrium candidate. Again we make parameter assumptions so that all agents trade in the candidate equilibria, and that a deviating mismatched seller does not trade with a mismatched buyer. In this equilibrium candidate, $D(\overline{\theta}) = V^{B1} - V^{S1}$ is given by

$$D(\bar{\theta}) = \frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\left(\frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - \frac{1}{\overline{\theta}}\frac{u - u_2}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)}\right).$$

Given our assumptions on the utility flows, $D(\overline{\theta})= 0$ for $\bar{\theta} = 1$. Again an increase in $\bar{\theta}$ increases D, and hence makes it more attractive to buy first. It follows that the mismatched agents strictly prefer to buy first in the "Buy first" equilibrium candidate.

As already mentioned, we have to make restrictions on the parameter set, so that our assumptions regarding the matching sets are satisfied. However, we conjecture that if these restrictions are not satisfied, and other matching sets emerge, these may also have multiple equilibria with the same structure.[28]

Proposition 7. *Consider the limit economy with prices determined by symmetric Nash bargaining. Suppose that conditions B1-B4 given in the Appendix hold. Then there exists a steady state equilibrium, in which all mismatched owners buy first and the equilibrium market tightness converges to* $\overline{\theta} = 1 + \kappa$. *Also, there exists a steady state equilibrium, in which all mismatched owners sell first and the equilibrium market tightness converges to* $\underline{\theta} = \frac{1}{1+\kappa}$.

Proof. See Appendix E. □

How much can prices fluctuate across the two equilibria in the model with Nash bargaining? To assess this, we use the values of γ and g, the matching function, and the implied market tightnesses in the two equilibria from the numerical example in Section 4.6. Depending on preference parameters (χ, u_0, and u_2), the average transaction prices can decrease by up to 10% across the two equilibria. This constitutes around one half of the observed decline in house prices in Copenhagen in the period 2007-2012. Therefore, our mechanism can also lead to quantitatively significant fluctuations in house prices.

6.2 Competitive Search

In competitive search equilibrium, the sellers post prices, and buyers direct their search towards the sellers they find most attractive, taking into account that a high price means a long waiting time in order to transact. Hence, the market splits up in submarkets, and the different agents choose which submarket to enter. As shown in Garibaldi, Moen and Sommervoll (2014), the most patient buyers (who are most willing to trade off a short waiting time for a high price) will search for the most impatient sellers (who are most willing to trade of a low price for a short waiting time). Analogously, the least patient buyers search for the most patient sellers.

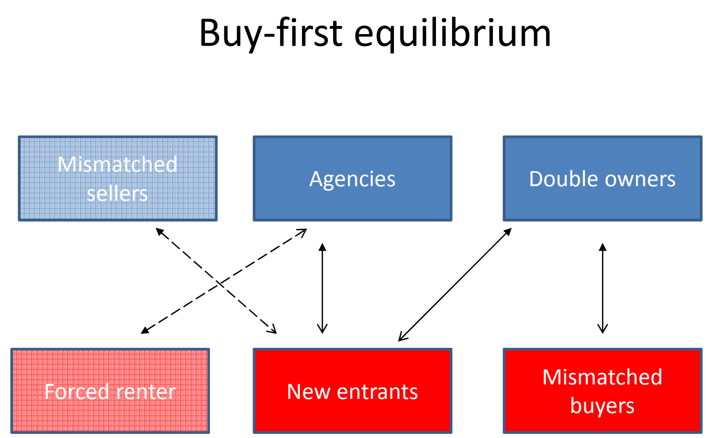
Depending on parameter values, different submarket constellations may emerge. In order to control which submarkets are active in equilibrium, we assume that the cost of being mismatched, χ, is low, and that the flow utilities of being a forced renter (u_0) and a double owner (u_2) are also

[28]Simulations confirm that multiple equilibria exist also for parameter values that do not satisfy conditions B1-B4.

low. In the market constellations we study, a submarket with new entrants and real estate firms always operates. We refer to this as market 1. Furthermore, it can be shown that as χ goes to zero, the tightness in this market goes to 1. We study the limit equilibrium with small flows.

Buy-first equilibrium

In the buy-first equilibrium, the buyers are mismatched owners and new entrants, while the sellers are real estate firms and double owners. The most patient buyer is the mismatched owner, while the most impatient sellers are the double owners. Hence these agents transact. The least patient buyers are the new entrants, while the most patient sellers are the real estate firms. Hence submarkets for real estate firms and new entrants will always exist (market 1), as will a submarket for mismatched buyers and double owners (market 3). One can easily show that the simplest equilibrium candidate, with these two markets as the only active markets, cannot exist.[29] In addition, a market for new entrants and double owners will also open (market 2). The market constellation is illustrated in the figure below, where blue indicate sellers and red buyers. We have also included a deviating agent (weaker color)



As $\chi \to 0$, $S_2/B_1 \to 0$ and hence $\theta_3 = \infty$. Let V^m denote the value of being mismatched. Note that $\lim_{\chi \to 0} V^m = V$. We can then easily calculate the asset values of new entrants and real estate firms, V^{Bn} and V^A. Given the value of new entrants, we can calculate the asset value of double

[29] In such an equilibrium, stock-flow conditions imply that the tightness in market 1, $\theta_1 = 1$, and hence, that $A = B_n$,while we know that $B_n = A + S_2$ by definition

owners, which is the maximum they can obtain given that their trading partners obtain V^{Bn}.[30] Finally, note that the match surplus Σ^{S2B1} between a mismatched buyer and a double owner is given by

$$\Sigma^{S2B1} = V + V^{S2} - V^m - V^{S2} = V - V^m > 0$$

Hence, the economy gets going.

A mismatched owner that deviates and sells first will be the most patient than both the real estate firms and the double owners, and will therefore transact with the most impatient buyer among the non-deviating buyers, the new entrants. It will then become a new buyer type - a forced renter - that is even more impatient than the new entrant. He will therefore transact with the most patient sellers, which are the real estate firms. We assume that a new submarket opens up for the deviating agent. In particular, the value he receives from being a forced renter, V^{B0}, maximizes his gain from search given the value of the real estate firm. Below we show that V^{B0} is strictly lower than V^{Bn}, also in the limit as $\chi \to 0$. (Below we show that it falls without bounds when u_0 does).

The match surplus between the deviating mismatched owner and the new entrant, Σ^{S1B_n}, can be written as

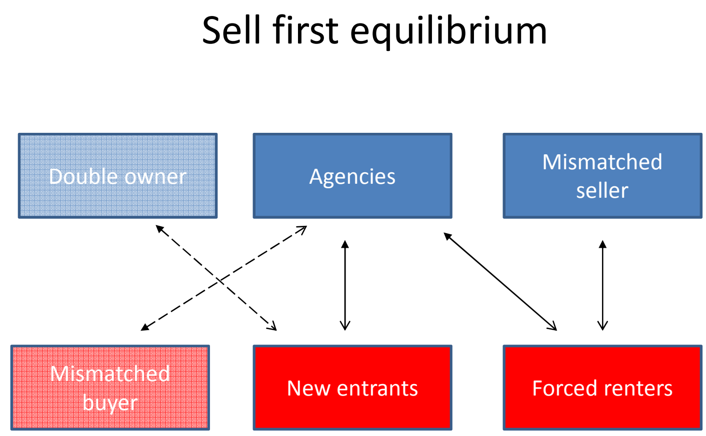
$$\Sigma^{S1Bn} = V + V^{B0} - V^{Bn} - V^m$$

Since $V^{Bn} > V^{B0}$, the match surplus is certainly negative for small values of χ and/or u_0. Hence the mismatched owner cannot gain by deviating, and the buy-first equilibrium exists.

Sell-first equilibrium

In the sell-first equilibrium, the sellers are mismatched homeowners and real estate firms, while the buyers are new entrants and forced renters. The most patient buyers are the new entrants, and the most patient sellers the mismatched owners. The active submarkets will be with new entrants and real estate firms (market 1), mismatched buyers and forced renters (market 3), and forced renters and real estate firms (market 2). Again the markets (together with a deviating agent) are illustrated in the figure below:

[30]Since S_2 is small relative to B_n and A, the behavior in market 2 (the $S_2 - B_n$ market) does not influence V^{Bn}or V^A.



The asset values in market 1 are as in the buy-first equilibrium. The asset values for forced renters are the same as for the deviating forced renter in the buy-first equilibrium. Furthermore, the match surplus between a forced renter and a mismatched buyer, Σ^{S1B0}, is given by

$$\Sigma^{S1B0} = V + V^{B0} - V^m - V^{B0} = V - V^m > 0$$

Now, consider a mismatched agent that deviates and buys first. He will be the more patient than both new entrants and forced renters, and will transact with the real estate firm, the most impatient seller. He will then become a double owner, and hence more impatient than the real estate firm, and therefore transact with the new entrants. A new submarket will open up, and he will receive the same value V^{S2} as the double owner in the buy-first equilibrium. Let Σ^{AB1} denote the match surplus between the mismatched buyer and the real estate firm. It follows that

$$\Sigma^{AB1} = V^{S2} - V^m - V^A$$

For low values of u_2, V^{S2} is low (this is shown below), hence the surplus is negative. It follows that the deviation is unprofitable, so that also the sell-first equilibrium exists. Hence, the model still exhibits multiple equilibria.

Asset values and proofs

Asset values, buy-first . In the limit,

$$rV^{Bn} = u_n + \frac{\alpha q_1}{r+q_1}(u-u_n) - R \tag{25}$$

which is strictly between 0 and V, as long as R is not too large. Similarly, V^A is given by

$$rV^A = R + \frac{(1-\alpha)q_1}{r+q_1}(u-u_n) \tag{26}$$

which is also strictly between 0 and V if R is not too large. Note that $V^A+V^{Bn}= u_n+\frac{q_1}{r+q_1}(u-u_n)$, which is strictly between 0 and V.

It follows that V^{S2} is uniquely determined as

$$rV^{S2} = \max_{p,\theta}\left\{u_2 + R + \mu(\theta)(V+p-V^{S2})\right\}$$

subject to $u_n - R + q(\theta)(V-p-V^{Bn}) = rV^{Bn}$, where V^{Bn} is defined in (25) above. Note that $V^{S2} < 2V$. Furthermore, V^{S2} goes to negative infinity when u_2 does.[31]

Match surpluses, buy-first The match surplus from different matches reads

$$\begin{aligned}
\Sigma^{ABn} &= V - V^A - V^{Bn} > 0\\
\Sigma^{S2B1} &= V - V^m > 0\\
\Sigma^{AB1} &= V^{S2} - V^A - V^m\\
\Sigma^{S2Bn} &= 2V - V^{Bn} - V^{S2}
\end{aligned}$$

If u_2 is sufficienctly low (and hence V^{S2} is low), it follows that $\Sigma^{AB1}< 0$ and $\Sigma^{S2Bn}> 0$.

Deviations, buy-first Consider a deviation. A mismatched owner sells first. Then he becomes a forced renter. As a forced renter, the deviator, after successfully selling his dwelling, will buy from the real estate firm, the most patient of the sellers. A new submarket opens up, and real estate firms flow into this submarket up to the point where they are indifferent between selling to the deviator and to a new agent. Hence, in effect the price and tightness in the new submarket is set, so as to maximize the value of the deviator given the indifference condition of the real estate firm. It follows that

$$rV^{B0} = \max_{p,\theta}\left\{u_0 - R + q(\theta)(V-p-V^{B0})\right\}$$

subject to $R+\mu(\theta)(p-V^A)= rV^A$, where V^A is determined in (26) above. It follows that V^{B0} goes to $-\infty$ when u_0 falls without bounds. Note also that as $V^{Bn} - V^{B0} > 0$. Let q^tdenote the trading

[31]Suppose not. Then $\mu(\theta)$ must go to infinity, and hence θ must go to infinity. But then q goes to zero, and for the new entrants to get their outside option, p must go to $-\infty$, in which case V^{S2} goes to negative infinity. A contradiction.

rate for the forced renter. Note that q^t is finite, following an argument similar to that in footnote 31 above. It follows that $V^{Bn} - V^{B0} > \frac{u_n - u_0}{r + q^t}$.

Suppose the deviator sells to a new entrant. Then the match surplus reads

$$\Sigma^{S1Bn} = V^{B0} + V - V^m - V^{Bn} < 0$$

since, as we have seen, $V^{Bn} - V^{B0} > \frac{u_n - u_0}{r + q^t}$. Suppose instead that he sells to the mismatched agent. Then

$$\Sigma^{S1B1} = V^{S2} + V^{B0} - 2V^m < 0$$

for sufficiently low values of u_0 and u_2.

sell-first Let us then consider a sell-first equilibrium. In this equilibruim, mismatched sellers sell their house to a forced renter. The match surplus from this trade is clearly positive, as

$$\Sigma^{S1B0} = V - V^m > 0$$

A mismatched owner will not sell to a new entrant, as

$$\Sigma^{S1Bn} = V - V^m + V^{B0} - V^{Bn} < 0$$

since, as we have seen, $V^{Bn} - V^{B0} > \frac{u_n - u_0}{r + q} > \chi$ for small values of χ. Furthermore,

$$\Sigma^{AB0} = V - V^{B0} - V^A > V - V^{Bn} - V^A > 0$$

(see derivations above).

Deviation: Now consider a deviator that buys first. Potential sellers are real estate firms and mismatched homeowners. Since the real estate firms are the most impatient among the sellers, they will trade with the deviator. Note that V^{S2} is as in the buy-first equilibrium. Then

$$\Sigma^{AB1} = V^{S2} - V^A - V^m < 0$$

for sufficiently low values of u_2. Finally, and for completeness, note that the match surplus between the deviating forced renter buying first and selling first,

$$\Sigma^{S1B1} = V^{B0} + V^{S2} - 2V^m < 0$$

is also negative.

Proposition 8. *Consider the limit economy with small flows (γ and g goes to infinity, while $\kappa = \gamma/g$ is constant) with competitive search. Suppose also that χ, u_0 and u_2 are all small. Then the economy exhibits multiple equilibria. In one equilibrium, all mismatched owners buy first. In another equilibrium, all mismatched owners sell first.*

We have no reason to believe that the conditions on χ is necceesary to obtain multiple equilibria.

However, without this assumption the model becomes less tractable, as it is not clear from the outset what market constelations that will be realized.

7 Additional Extensions

7.1 Simultaneous Entry as Buyer and Seller

Our benchmark model assumes that a mismatched owner has to choose to enter the housing market either as a buyer or as a seller. In this section, we examine the optimal behavior of a mismatched owner that can choose to be both a buyer and a seller at the same time. We show that a mismatched owner strictly prefers to either only enter as a buyer or as a seller for any $\theta \neq \tilde{\theta}$, where $\tilde{\theta}$ is defined as in equation (13). Intuitively, since the decision to enter as both a buyer and a seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only, whenever entering as a buyer only is dominated by entering as a seller only, then entering as both a buyer and a seller is also dominated by entering as a seller only, and vice versa.

We assume that a mismatched owner can allocate a fixed amount of time (normalized to 1 unit) to search in the housing market as a buyer or a seller. A mismatched owner that chooses to enter as a buyer or seller only allocates all of his time to one activity. Otherwise, a mismatched owner that enters as both a buyer and a seller can allocate a fraction $\phi \in (0, 1)$ of his time to searching as buyer, and searches the remaining $1 - \phi$ of his time as seller. For a given market tightness θ, the value function V^{SB} for a mismatched owner that enters as both buyer and seller satisfies the following equation in a steady state equilibrium:

$$\rho V^{SB} = u - \chi + (1 - \phi)\mu(\theta) \max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta) \max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

We then show the following

Proposition 9. *For $\theta \in \left(0, \tilde{\theta}\right)$, $V^{S1} > V^{SB}$, for any $\phi \in (0, 1)$. Also, for $\theta \in \left(\tilde{\theta}, \infty\right)$, $V^{B1} > V^{SB}$, for any $\phi \in (0, 1)$.*

Proof. See Appendix. □

Finally, note that under payoff symmetry (i.e. $\tilde{u}_0 = \tilde{u}_2 = c$) the possibility to enter as both buyer and seller while allocating each an equal amount of time can result in an equilibrium with a market tightness of $\theta = 1$. Specifically, at $\theta = 1$, $\mu(\theta) = q(\theta) = \mu(1)$. At these flow rates it can easily be seen that if $\tilde{u}_0 = \tilde{u}_2 = c$, then $V^{B1} = V^{S1} = V^{SB}$ for any ϕ. Finally, a tightness of $\theta = 1$ can result from mismatched owners entering as buyers and sellers simultaneously and allocating each an equal amount of time (so $\phi = 0.5$).

This is analogous to the equilibrium described in Proposition 3, with the only difference that now agents follow symmetric strategies compared to asymmetric strategies with one half of mismatched owners buying first and the other half selling first.

7.2 Homeowners compensated for their housing unit upon exit

In this section we show that our main results continue to hold under the alternative assumption that homeowners are compensated for the value of their housing units when they exit the economy.

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A(\theta)$, homeowners receive $V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0(\theta, g) \equiv u_0 + \triangle - gV^A(\theta)$ and $\tilde{u}_2(\theta, g) = u_2 - \triangle + gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D(\theta) \equiv V^{B1} - V^{S1}$, can be written as

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}\left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g)\right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[32] Note that in the limit as $g \to 0$, assumption A1 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 4.

8 Institutional Details and Concluding Comments

In this section we compare the process of housing sales in several countries, and then provide brief concluding comments.

8.1 Institutional Details

Actual housing markets in different countries differ in their institutional characteristics. Naturally, our model of the housing market abstracts from many of these peculiarities. As a result, the fit between the model and the way that houses are bought and sold may vary across countries. Nonetheless, we think our model captures essential elements of housing transactions for many countries, including Denmark, Norway, the Netherlands, and the United States. In these countries, the institutional set-up for the process of housing transactions is such that homeowners are concerned about the order of buying and selling, at least to some extent. In principle, the same issue occurs

[32] Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.

in the United Kingdom, but there the phenomenon of housing chains (see Rosenthal (1997)) may provide a way to accommodate the risks associated with moving in the owner-occupied housing market. Because of the widespread usage of housing chains in the UK, our model may be less suited for capturing the way houses are bought and sold in that country. Instead it describes more closely the housing markets of countries where housing chains are rare or non-existent.[33]

Additionally, in England and Wales buyers and sellers are not legally bound to an agreed transaction until late in the process, so that both sides easily renege on offers (Rosenthal, 1997). As a result, if a household is not able to complete a second transaction as fast as desired, it may just withdraw from a first transaction in order to avoid the costly period in between. As shown in the Appendix in Table 1 for buyers and Table 2 for sellers, commitment to an agreed transaction is significantly larger outside the UK. The tables show whether the law requires a grace period, what the penalty is for reneging during and after this grace period, which conditions that allow to dissolve a contract are usually included in the contract, and for what period parties can still refer to these conditions. In Denmark (where our transactions data are from) for instance, only buyers enjoy a grace period of 6 days, in which they can cancel the transaction at a cost of one percent of the transaction price. Afterwards, buyers are liable for the full amount, while sellers can be taken to court if they do not transfer the house. Sometimes purchase offers allow for contingencies such as the ability to secure financing or the approval of one's own lawyer, but referral to these conditions requires proof and is restricted in time. The picture that emerges from the tables is that in Denmark, Norway, the Netherlands, and the United States it may be very costly to renege on a transaction once a purchase offer has been made or a conditional contract has been signed.

8.2 Concluding Comments

The transaction sequence decision of moving owner-occupiers depends on housing market conditions, such as the expected time-on-market for buyers and sellers and expectations about future house price appreciation. However, these decisions in turn exert important effects on the buyer-seller ratio of the housing market. This creates a coordination problem for moving owner occupiers, resulting in multiple equilibria. Equilibrium switches are associated with large fluctuations in the stock of units for sale, average time-on-market, transactions, and also prices.

The tractable equilibrium model that we study in this paper to show these effects is deliberately simplified, and so lacks heterogeneity in many important dimensions. In particular, there is no heterogeneity in the costs of being a double owner versus a forced renter, which are likely to vary substantially across households and also to vary over time in response to aggregate shocks. In addition, we assumed constancy of the rate of mismatch and entry into and exit from the market. Nevertheless, endogenous fluctuations in γ and g are likely to additionally amplify and propagate aggregate shocks. Enriching the model along these dimensions will allow for a detailed quantitative model of the housing market, which can be taken to the data. We view this as an important step

[33] There is anecdotal evidence that innovations in mortgage financing in recent years may have decreased the importance of housing chains in the UK market as well.

for future research.

References

Albrecht, J., A. Anderson, E. Smith, and S. Vroman (2007): "Opportunistic Matching in the Housing Market," *International Economic Review*, 48(2), 641–664.

Albrecht, J., P. A. Gautier, and S. Vroman (2010): "Directed Search in the Housing Market," CEPR Discussion Paper No. 7639.

Anenberg, E., and P. Bayer (2013): "Endogenous Sources of Volatility in Housing Markets: The Joint Buyer-Seller Problem," NBER Working Paper No. 18980.

Anundsen, A. K., and E. Røed Larsen (2014): "Strategic sequencing behavior among owner-occupiers: The role played by sell-first movers in a housing recovery," *Journal of European Real Estate Research*, 7(3), 295–306.

Caplin, A., and J. Leahy (2011): "Trading Frictions and House Price Dynamics," *Journal of Money, Credit, and Banking*, 43, 283–303.

Dawson, M. (2013): "Buy or Sell First?," available from: http://www.realtor.com/home-finance/real-estate/general/buy-or-sell-first.aspx.

Diamond, P. (1982): "Aggregate Demand Management in a Search Equilibrium," *Journal of Political Economy*, 90, 881–894.

Diaz, A., and B. Jerez (2013): "House Prices, Sales, and Time on the Market: A Search Theoretic Framework," *International Economic Review*, 54:3, 837–872.

Duffie, D., N. Garleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73:6, 1815–1847.

Genesove, D., and L. Han (2012): "Search and Matching in the Housing Market," *Journal of Urban Economics*, 72, 31–45.

Gertler, M., and A. Trigari (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117:1, 38–86.

Glaeser, E. L., J. Gyourko, E. Morales, and C. G. Nathanson (2014): "Housing Dynamics: An Urban Approach," *Journal of Urban Economics*, 81, 45–56.

Guren, A. (2014): "The Causes and Consequences of House Price Momentum," mimeo.

Guren, A., and T. McQuade (2013): "How Do Foreclosures Exacerbate Housing Downturns?," mimeo.

Hall, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95:1, 50–65.

Head, A., H. Lloyd-Ellis, and H. Sun (2014): "Search, Liquidity and the Dynamics of House Prices and Construction," *American Economic Review*, 104:4, 1172–1210.

Hosios, A. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57:2, 279–98.

Kiyotaki, N., and R. Wright (1993): "A Search-Theoretic Approach to Monetary Economics," *American Economic Review*, 83:1, 63–77.

Krainer, J. (2001): "A Theory of Liquidity in Residential Real Estate Markets," *Journal of Urban Economics*, 49, 32–53.

Lester, B., G. Rocheteau, and P.-O. Weill (forthcoming): "Competing for Order Flow in OTC Markets," *Journal of Money, Credit, and Banking.*

Lester, B., L. Visschers, and R. Wolthoff (2013): "Competing with Asking Prices," mimeo.

Maury, T.-P., and F. Tripier (2014): "Search strategies on the housing market and their implications on price dispersion," *Journal of Housing Economics*, 26, 55–80.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105:2, 385–411.

Ngai, L. R., and K. D. Sheedy (2015): "Moving House," CEPR Discussion Paper No. 10346.

Ngai, L. R., and S. Tenreyro (2014): "Hot and Cold Seasons in the Housing Market," *American Econimc Review*, 104(12), 3991–4026.

Novy-Marx, R. (2009): "Hot and Cold Markets," *Real Estate Economics*, 37:1, 1–22.

Ortalo-Magne, F., and S. Rady (2006): "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studies*, 73, 459–485.

Piazzesi, M., M. Schneider, and J. Stroebel (2015): "Segmented Housing Search," NBER Working Paper No. 20823.

Pissarides, C. (2000): *Equilibrium Unemployment Theory.* MIT Press.

Rocheteau, G., and R. Wright (2005): "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73:1, 175–202.

Rosenthal, L. (1997): "Chain-formation in the Owner-Occupied Housing Market," *The Economic Journal*, 107, 475–488.

Stein, J. (1995): "Prices and Trading Volume in the Housing Market: a Model with Down-Payment Effects," *Quarterly Journal of Economics*, 110, 379–406.

Wheaton, W. (1990): "Vacancy, Search, and Prices in a Housing Market Matching Model," *Journal of Political Economy*, 98:6, 1270–1292.

Appendix

A. Data Description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other datasets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, sale (agreement), and takeover (closing) dates. Furthermore, they contain the property identifiers used in the EJER dataset, which allows for linking of the two datasets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[34]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a non-missing agreement date. We drop properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3312520 property-year observations. These comprise 199812 unique properties and 345943 unique individual owners over our sample period.

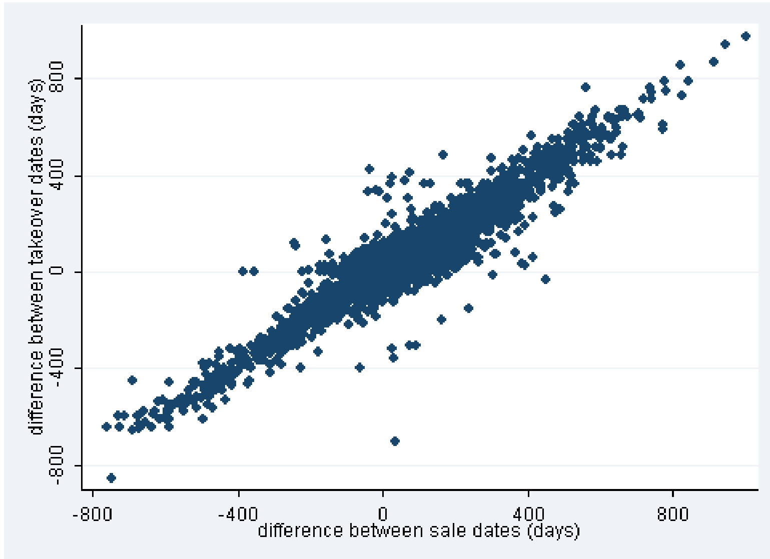
To identify an individual owner as a buyer-and-seller we rely on the information from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consecutive years. An old property is defined as a property which an individual is registered as owning over at least 2 consecutive years.[35] Also, we do not count individuals that are recorded as holding two properties for two or more consecutive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our sample. In particular, in a given year in our sample from 1991-2009 there are on average only around

[34] Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

[35] We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale. Adding back those agents has a very small effect on the pattern we uncover.

Figure 7: Difference in agreement dates vs. difference in closing dates, Copenhagen (1993-2008)



0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the agreement data for sale of the old property and the agreement date for the purchase of the new property. Similarly, we compute the time period (in days) between the closing date that of the buyer-and-seller's old property by the new owner and the closing date for his new property. We then denote a buyer-and-seller for which the time period between agreement dates is negative (sale date is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive (sale date is after purchase date) as "buying first". We also do the same classification but based on closing dates rather than agreement dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who "buy first" vs. "sell first" in a given year for the years 1993 to 2008.

In principle, and as Figures 1 and 2 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the agreement dates and closing dates are highly correlated with a correlation coefficient of 0.9313. Figure 7 visualizes this strong correlation by plotting a scatter plot of the two time differences.

B. Institutional details

Table 1: Institutional details for buyers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract (requires proof)	Period to refer to dissolving conditions
Denmark	6 days	1% of price	Liable for full amount	May include as conditions: • Ability to secure financing • Lawyer reservation	As specified in the purchase offer
Norway	None	N/A	Liable for full amount	May include as condition: • Ability to secure financing	As specified in the purchase offer
Netherlands	3 days	None	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	Standard contract: • Ability to secure financing • Applicability for national mortgage insurance • Structural inspection	As specified in the contract: usually not for more than a few weeks after signing it
United States	None (3 days in NJ only)	N/A	Losing the Earnest Money Deposit, ranging from \$500 to 10% of the price	Standard purchase offer: • Ability to secure financing • Appraisal • Structural inspection	Usually not for more than a few weeks after signing the offer, e.g. 17 days in CA
England & Wales	Pull out until exchange of (unconditional) contracts	Occasionally one loses holding deposit, ranging from £500 to £1000	10% price	None	N/A

Table 2: Institutional details for sellers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract
Denmark	None	N/A	Court	None
Norway	None	N/A	Court	None
Netherlands	None	N/A	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	None
United States	None (3 days in NJ only)	N/A	Court	None
England & Wales	Pull out until exchange of contracts	None	Court	None

The information in Tables 1 and 2 is based on:

- http://boligejer.dk/koebsaftale/ for Denmark
- http://www.eiendomsrettsadvokaten.no/advokathjelp/kjop-og-salg-av-eiendom/bolig-eiendom-kjop-salg-eierskifte-forsikring-avhending-avhendingslov-opplysning-undersokelse-tinglysning-budgivning-skjote-kontraktsinngaelse/ for Norway
- http://www.eigenhuis.nl/juridisch/ for the Netherlands
- http://www.realtor.com and https://www.doorsteps.com/ for the United States
- http://hoa.org.uk/advice/guides-for-homeowners for England and Wales

C. Equilibrium concept and parameter restrictions for the basic model

First of all, the steady state value functions for a new entrant, a matched owner, and a real estate firm satisfy the following equations:

$$\rho V^{Bn} = u_n - R + q(\theta)\left(-p + V - V^{Bn}\right), \tag{27}$$

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right), \tag{28}$$

and

$$\rho V^{A} = R + \mu(\theta)\left(p - V^{A}\right). \tag{29}$$

Importantly, in every steady state equilibrium, V satisfies $V \geq \tilde{V}$, where $\tilde{V} = \frac{u}{\rho+\gamma} + \frac{\gamma}{\rho+\gamma}V^m$, with $V^m = \frac{u-\chi}{\rho}$. Hence, $\tilde{V}$ is the value of a matched owner who never transacts. Therefore, $V \geq \tilde{V} = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$ in any steady state equilibrium.

Parameter restrictions

Sufficient conditions for new entrants, forced renters and double owners to prefer transacting and becoming matched owners are given by

$$\frac{u_n - R}{\rho} \leq \tilde{V} - p, \tag{30}$$

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p, \tag{31}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p. \tag{32}$$

Since $u_n \geq u_0$, we can disregard (31), as it is implied by (30). Conditions (30) and (32) imply restrictions for the values of the house price, p, that are sufficient for these agents to be willing to transact at p, namely $p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]$.

From (29) a real estate firm is willing to transact iff $p \geq \frac{R}{\rho}$. Therefore, equilibrium is defined for a house price p, that satisfies

$$p \in \left[\max\left\{\frac{u_2}{\rho} - \tilde{V}, 0\right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]. \tag{33}$$

For $u - \chi \geq \max\{u_0, u_2\}$, which is the condition we will use to characterize equilibria, it follows that $\frac{u_2}{\rho} - \tilde{V} < 0$ and so the set for prices is given by

$$p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]. \tag{34}$$

Finally, a sufficient condition for $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$ at $\theta = \tilde{\theta}$, with $\tilde{\theta}$ as defined in (13) is

Assumption A2: $\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu(\tilde{\theta})} + \frac{\mu(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\tilde{u}_0 + \frac{\mu(\tilde{\theta})q(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\left(\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}\right)$.
Note that $\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho} \leq V$, $\forall\theta$, so the right hand side of this expression is lower than the value of V^{S1} at $\theta = \tilde{\theta}$.

Steady state flow conditions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 3.2 must satisfy. We have that in a steady state equilibrium, given a market tightness θ, the steady state values of B_n, B_0, B_1, S_1, S_2, O, and A must satisfy the following system of flow conditions:

$$g = (q(\theta) + g) B_n, \tag{35}$$

$$\mu(\theta) S_1 = (q(\theta) + g) B_0, \tag{36}$$

$$\mu(\theta) S_2 + q(\theta)(B_n + B_0) = (\gamma + g) O, \tag{37}$$

$$\gamma x_b O = (q(\theta) + g) B_1, \tag{38}$$

$$\gamma x_s O = (\mu(\theta) + g) S_1, \tag{39}$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2, \tag{40}$$

$$g(O + B_1 + S_1 + 2S_2) = \mu(\theta) A, \tag{41}$$

$$x_b + x_s = 1, \tag{42}$$

where x_b, and x_s are the equilibrium fractions of mismatched buyers and sellers, respectively. Apart from these conditions, the aggregate variables must satisfy the population constancy and housing ownership conditions (1) and (2). Finally, the equilibrium market tightness θ, satisfies

$$\theta = \frac{B}{S} = \frac{B_n + B_0 + B_1}{S_1 + S_2 + A}. \tag{43}$$

Equilibrium definition

We define a steady state equilibrium for this economy in the following way:

Definition 10. A steady state equilibrium consists of a house price p, equilibrium rental rate R, value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^A, market tightness θ, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, and aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A such that:

1. The house price $p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]$;

2. The equilibrium rental rate $R \in [0, u_0]$;

3. The value functions satisfy equations (3)-(6) and (27)-(29) given θ, p, and R;

4. Mismatched owners choose $x \in \{b, s\}$, to maximize $\overline{V} = \max\left\{V^{B1}, V^{S1}\right\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

5. The market tightness θ solves (43) given B_n, B_0, B_1, S_1, S_2, O, and A;

6. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (35)-(41) given θ and mismatched owners' optimal decisions reflected in x_b and x_s.

D. Proofs

Proof of Lemma 1

Proof. First of all, note that the function $D(\theta)$, defined in (11) crosses zero only at $\theta = \tilde{\theta}$. To see this, notice that

$$\lim_{\theta \to 0} D(\theta) = \frac{\tilde{u}_2 - (u - \chi)}{\rho} < 0,$$

and

$$\lim_{\theta \to \infty} D(\theta) = \frac{u - \chi - \tilde{u}_0}{\rho} > 0.$$

Away from these two limiting values, $D(\theta) > 0$, whenever

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 > 0,$$

which is equivalent to $\tilde{\theta} < \theta$. Therefore, $D(\theta) > 0$ iff $\theta \in \left(\tilde{\theta}, \infty\right)$ and $D(\theta) < 0$ iff $\theta \in \left(0, \tilde{\theta}\right)$. Therefore, $D(\theta) = 0$, iff

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 = 0,$$

or $\theta = \tilde{\theta}$. Note that $D(\theta)$ fully summarizes the incentives of a mismatched owner to buy first/sell first apart from at $\theta = 0$ and $\theta = \infty$. To see this, let

$$\begin{aligned}\tilde{V}^{B1} &= \frac{u - \chi}{\rho + q(\theta)} + \frac{q(\theta)\,\tilde{u}_2}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))}V - \frac{u - \chi}{\rho} \\ &= \frac{q(\theta)}{\rho + q(\theta)}\left(\frac{\tilde{u}_2}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)}V - \frac{u - \chi}{\rho}\right),\end{aligned}$$

and

$$\begin{aligned}\tilde{V}^{S1} &= \frac{u-\chi}{\rho+\mu(\theta)} + \frac{\mu(\theta)\tilde{u}_0}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V - \frac{u-\chi}{\rho} \\ &= \frac{\mu(\theta)}{\rho+\mu(\theta)}\left(\frac{\tilde{u}_0}{\rho+q(\theta)} + \frac{q(\theta)}{\rho+q(\theta)}V - \frac{u-\chi}{\rho}\right).\end{aligned}$$

The functions $\tilde{V}^{B1}$ and $\tilde{V}^{S1}$ give the difference between the value of transacting and never transacting for a buyer first and seller first, respectively.

By Assumption A2, at $\tilde{\theta}$, $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so at $\theta = \tilde{\theta}$, $\tilde{V}^{B1} > 0$, and $\frac{\tilde{u}_2}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)}V - \frac{u-\chi}{\rho} > 0$. Furthermore, this latter inequality holds for any $\theta > \tilde{\theta}$, and so $\tilde{V}^{B1} > 0$ for any $\theta > \tilde{\theta}$. Therefore, for any $\theta > \tilde{\theta}$, a mismatched owner who buys first is better off transacting than not transacting. Similarly, for $\theta < \tilde{\theta}$ the mismatched owner who sells first is better off transacting than not transacting.

Therefore, for $\theta \in (0,\infty)$, if $D(\theta) > 0$, a mismatched owners is better off buying first (and transacting) compared to selling first (and transacting or not transacting) and similarly, if $D(\theta) < 0$, a mismatched owner is better off selling first (and transacting) compared to buying first (and transacting or not transacting). At $D(\theta) = 0$, he is indifferent between buying first (and transacting) and selling first (and transacting).

Finally, clearly if $\theta \to \infty$, $\tilde{V}^{B1} \to 0$, so $V^{B1} \to \frac{u-\chi}{\rho} = V^{S1}$. Similarly, if $\theta \to 0$, $\tilde{V}^{S1} \to 0$, and so $V^{S1} \to \frac{u-\chi}{\rho} = V^{B1}$. □

Proof of Lemma 2

Proof. For the case where mismatched owners buy first ($x_s = 0$), the stock-flow conditions are

$$g = (q(\theta) + q) B_n,$$

$$\gamma O = (q(\theta) + g) B_1,$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2,$$

$$g = (\mu(\theta) + g) A,$$

$$B_n + B_1 + S_2 + O = 1,$$

and

$$B_n = A + S_2.$$

It follows that $B_n = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g} B_n$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta - 1) B_n$ and so $O = \frac{1}{\gamma}(q(\theta) + g)(\theta - 1) B_n$. Substituting into the population constancy condition, we have that

$$\theta B_n + B_n - \frac{q(\theta) + g}{\mu(\theta) + g} B_n + \frac{1}{\gamma}(q(\theta) + g)(\theta - 1) B_n = 1,$$

which, after substituting for B_n and re-arranging we can write as

$$\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta+\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)=\frac{1}{g}+\frac{1}{\gamma}.$$

This is exactly equation (15). At $\theta=1$, the left-hand side equals

$$\frac{1}{q(1)+g}+\frac{1}{\gamma}<\frac{1}{g}+\frac{1}{\gamma}.$$

Furthermore, note that $\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (15) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta=1$. Therefore, it has a unique solution for $\theta>1$. We call this solution $\overline{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\overline{\theta}$ is increasing in γ.

For the case where mismatched owners sell first ($x_s=1$) the stock-flow conditions become

$$g=(q(\theta)+q)B_n,$$

$$\mu(\theta)S_1=(q(\theta)+g)B_0,$$

$$\gamma O=(\mu(\theta)+g)S_1,$$

$$g=(\mu(\theta)+g)A,$$

$$B_n+B_0+S_1+O=1,$$

and

$$B_n+B_0=A.$$

It follows that $A=\frac{g}{\mu(\theta)+g}=B_0+B_n$, $S_1=\frac{1-\theta}{\theta}A$ and $O=\frac{1}{\gamma}(\mu(\theta)+g)\frac{1-\theta}{\theta}A$. Therefore, substituting for these in the population constancy condition, we have that

$$\frac{1}{\theta}A+\frac{1}{\gamma}(\mu(\theta)+g)\frac{1-\theta}{\theta}A=1.$$

Substituting for A, we obtain an equation for θ of the form

$$\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}=\frac{1}{g}+\frac{1}{\gamma},$$

which is equation (16). At $\theta=1$, the left-hand side equals

$$\frac{1}{\mu(1)+g}+\frac{1}{\gamma}<\frac{1}{g}+\frac{1}{\gamma}.$$

Note also that $\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta\to\infty$. Also it asymptotes to ∞ as $\theta \to 0$. Therefore, the equation has a unique solution for $\theta < 1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ. □

Proof of Proposition 3

Proof. With regard to Items 1 and 2, by Lemma 2 $\overline{\theta}$ satisfies the stock-flow conditions when all mismatched owners buy first, and similarly $\underline{\theta}$ satisfies the stock-flow conditions if they sell first. Then by Lemma 1 their actions are optimal given these market tightnesses. With regard to Item 3, by Lemma 1, for $\tilde{u}_0 = \tilde{u}_2 = c$, mismatched owners are indifferent between buying first and selling first at $\theta = 1$. Also, by Assumption A2, they are strictly better off from transacting than not transacting. Finally, to show that the stock-flow conditions are satisfied, suppose that $x_s = x_b = \frac{1}{2}$. We have

$$\gamma \frac{1}{2} O = \left(q\left(\theta\right) + g\right) B_1, \tag{44}$$

and

$$\gamma \frac{1}{2} O = \left(\mu\left(\theta\right) + g\right) S_1. \tag{45}$$

At $\theta = 1$, $\mu\left(\theta\right) = q\left(\theta\right) = \mu\left(1\right)$, so $B_1 = S_1$. Also, $B_n = A = \frac{g}{\mu(1)+g}$ and $B_0 = S_2 = S_1 \frac{\mu(1)}{\mu(1)+g}$. Finally, population constancy implies that

$$2S_1 \frac{\mu\left(1\right)}{\mu\left(1\right) + g} + 2S_1 + 2S_1 \frac{\mu\left(1\right) + g}{\gamma} = \frac{\mu\left(1\right)}{\mu\left(1\right) + g},$$

which is satisfied for some $S_1 \in \left(0, \frac{1}{2}\right)$. □

Proof of Proposition 4

Proof. Clearly, Lemma 2 that determines the values of $\overline{\theta}$ and $\underline{\theta}$ is independent of the agents' payoffs. With regard to Item 1, a direct application of Lemma 1 shows that if $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, then at $\theta = \underline{\theta}$ a mismatched owner is (weakly) better off from selling first and at $\theta = \overline{\theta}$, he is (weakly) better off buying first. Consequently, agents' actions are optimal given θ and the steady state value of θ is consistent with agents' actions. Considering Item 2, by the same logic a steady state equilibrium in which mismatched owners buy first and $\theta = \overline{\theta}$ exists. To see that it is the only symmetric steady state equilibrium, remember from Lemma 1 that mismatched owners only sell first for $\theta < \tilde{\theta}$, which contradicts $\tilde{\theta} < \underline{\theta}$. The same logic applies to Item 3. □

Proof of Proposition 5

Proof. Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases.

$$D(\theta) = \frac{\mu(\theta)\left(1-\frac{1}{\theta}\right)\left(u-\chi-c+\lambda\left(\overline{V}_N - v^{B0}\right)\right)}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)} + \frac{\frac{\lambda\mu(\theta)\left(1-\frac{1}{\theta}\right)q(\theta)}{(r+\mu(\theta))(r+q(\theta))}[\rho V - c] + \mu(\theta)\left(1+\frac{1}{\theta}\right)\lambda(p_N - p)}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{46}$$

Consider the case of $1 < \theta \le \overline{\theta}$, so $\overline{V}_N = V_N^{B1}$. If $\overline{V}_N = V_N^{B1}$, where V_N^{B1} denotes the value of buying first after the price change, this difference simplifies further to

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+q(\theta)}\right)(u-\chi-c) + \left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{47}$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right) = \frac{\lambda(p-p_N)}{(u-\chi-c)}. \tag{48}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \ge 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \le \overline{\theta}$, is that $\theta_{B1}^{PR} > \overline{\theta}$.

Similarly, consider the case of $\underline{\theta} \le \theta < 1$, so $\overline{V}_N = V_N^{S1}$, where V_N^{S1} denotes the value of selling first after the price change. In that case the difference in value functions can be written as

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)(u-\chi-c) + \left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \tag{49}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right) = \frac{\lambda(p-p_N)}{(u-\chi-c)}. \tag{50}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \le 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \le \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

Proof of Proposition 6

Proof. First, we consider the second regime $X(t) = 1$. In that regime the equilibrium market tightness, $\theta_1 = \underline{\theta}$ and agents' payoffs are as in Section 4.1. Therefore, by Lemma 1 and given the assumption for $\tilde{\theta}_1$, mismatched owners prefer to sell first at $\underline{\theta}$, and so $\underline{\theta}$ is consistent with the behavior of mismatched owners.

Second, consider the first regime. The value function of a mismatched owner who buys first in the first regime (and transacts) is given by

$$V_0^{B1} = \frac{u-\chi}{\rho+q\left(\overline{\theta}\right)+\lambda} + \frac{q\left(\overline{\theta}\right)}{\rho+q\left(\overline{\theta}\right)+\lambda}\left(V_0^{S2}-p_0\right) + \frac{\lambda}{\rho+q\left(\overline{\theta}\right)+\lambda}V^{S1},$$

where

$$V_0^{S2} = v^{S2}\left(\overline{\theta},p_0\right) + \frac{\lambda}{\rho+\mu\left(\overline{\theta}\right)+\lambda}\left(v^{S2}\left(\underline{\theta},p_1\right) - v^{S2}\left(\overline{\theta},p_0\right) + p_1 - p_0\right) + p_0,$$

with

$$v^{S2}\left(\theta,p\right) = \frac{u_2+R-\rho p}{\rho+\mu\left(\theta\right)} + \frac{\mu\left(\theta\right)}{\rho+\mu\left(\theta\right)}V,$$

and V^{S1} is given in (10), where the third term arises since in the second regime a mismatched owner sells first. For the value of selling first we have

$$V_0^{S1}\left(\overline{\theta}\right) = \frac{u-\chi}{\rho+\mu\left(\overline{\theta}\right)+\lambda} + \frac{\mu\left(\overline{\theta}\right)}{\rho+\mu\left(\overline{\theta}\right)+\lambda}\left(V_0^{B0}+p_0\right) + \frac{\lambda}{\rho+\mu\left(\overline{\theta}\right)+\lambda}V^{S1},$$

where

$$V_0^{B0} = v^{B0}\left(\overline{\theta},p_0\right) + \frac{\lambda}{\rho+q\left(\overline{\theta}\right)+\lambda}\left(v^{B0}\left(\underline{\theta},p_1\right) - v^{B0}\left(\overline{\theta},p_0\right) + p_0 - p_1\right) - p_0,$$

with

$$v^{B0}\left(\theta,p\right) = \frac{u_0-R+\rho p}{\rho+q\left(\theta\right)} + \frac{q\left(\theta\right)}{\rho+q\left(\theta\right)}V.$$

Consider the difference $D_0\left(\overline{\theta}\right) = V_0^{B1}\left(\overline{\theta}\right) - V_0^{S1}\left(\overline{\theta}\right)$. And note that

$$\lim_{\lambda\to 0} D_0\left(\overline{\theta}\right) = \frac{\mu\left(\theta\right)}{\left(\rho+q\left(\theta\right)\right)\left(\rho+\mu\left(\theta\right)\right)}\left[\left(1-\frac{1}{\theta}\right)\left(u-\chi-\tilde{u}_2\right) - \tilde{u}_0 + \tilde{u}_2\right] > 0,$$

by the assumption on $\tilde{\theta}_0$. Since $V_0^{B1}\left(\overline{\theta}\right)$ and $V_0^{S1}\left(\overline{\theta}\right)$ are continuous in λ, it follows that $D_0\left(\overline{\theta}\right)$ is continuous in λ as well, so that $D_0\left(\overline{\theta}\right) > 0$ will also be the case for λ sufficiently close to 0. Therefore, there exists a $\overline{\lambda}$ such that for $\lambda < \overline{\lambda}$, $V_0^{B1}\left(\overline{\theta}\right) > V_0^{S1}\left(\overline{\theta}\right)$ and mismatched owners prefer to buy first. Finally, $\overline{\theta}$ is consistent with the behavior of mismatched owners. □

Proof of Proposition 9

Proof. To show the first part, suppose the opposite, so $V^{S1} \leq V^{SB}$. Then

$$\mu(\theta)\max\left\{0,p+V^{B0}-V^{S1}\right\} \leq (1-\phi)\mu\left(\theta\right)\max\left\{0,p+V^{B0}-V^{SB}\right\} + \phi q\left(\theta\right)\max\left\{0,-p+V^{S2}-V^{SB}\right\}.$$

Under the assumption that $V^{S1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{B1} < V^{S1}$ for $\theta \in \left(0, \tilde{\theta}\right)$, it must then be the case that

$$\mu(\theta) \max\left\{0, p + V^{B0} - V^{S1}\right\} \leq (1-\phi)\mu\left(\theta\right) \max\left\{0, p + V^{B0} - V^{S1}\right\} + \phi q\left(\theta\right) \max\left\{0, -p + V^{S2} - V^{B1}\right\},$$

which does not hold because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > 0$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Assumption A2, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > q\left(\theta\right)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(0, \tilde{\theta}\right)$, as in Lemma 1.

To show the second part, suppose the opposite, so $V^{B1} \leq V^{SB}$. Then

$$q(\theta) \max\left\{0, p + V^{S2} - V^{B1}\right\} \leq (1-\phi)\mu\left(\theta\right) \max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q\left(\theta\right) \max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

Under the assumption that $V^{B1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{S1} < V^{B1}$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, it must then be the case that

$$q(\theta) \max\left\{0, p + V^{S2} - V^{B1}\right\} \leq (1-\phi)\mu\left(\theta\right) \max\left\{0, p + V^{B0} - V^{S1}\right\} + \phi q\left(\theta\right) \max\left\{0, -p + V^{S2} - V^{B1}\right\},$$

which does not hold because $q\left(\theta\right)\left(-p + V^{S2} - V^{B1}\right) > 0$ for $\theta \in \left(\tilde{\theta}, \infty\right)$ by Assumption A2, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) < q\left(\theta\right)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, as in Lemma 1.[36] □

E. A model with prices determined by Nash bargaining

We will show our main analytical result for the model with Nash bargaining under the following parametric assumptions:

Assumption B1: $u_2 - u_0 = u - u_n$.

Assumption B2: $r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi > 0$.

Assumption B3: $r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi\right]$.

Assumption B4: $\frac{\gamma}{g} \leq \kappa^*$

The first assumption ensures that buying first and selling first are equally attractive at $\theta = 1$. The roles of assumption B2-B4 are to ensure that the trading pattern described in the main text emerges in equilibrium.

Let Σ_{ij} denote the surplus from trade between agents of type i and type j. It follows that

$$rV^{B0} = u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{AB0} + \left(1 - \underline{\theta}\right)\Sigma_{S1B0}\right),$$

[36] Note also that for $\theta = 0$ and $\theta \to \infty$, mismatched owners are indifferent between remaining mismatched and any search strategy, because $V^{B1} = V^{S1} = V^{SB} = \frac{u-\chi}{\rho}$, but that such tightnesses cannot occur in steady state by Lemma 2.

and

$$rV^{Bn}=u_n-R+\frac{1}{2}q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{ANB}+\left(1-\underline{\theta}\right)\Sigma_{S1NB}\right),$$

so

$$V^{Bn}-V^{B0}=\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}. \tag{51}$$

Also,

$$rV^A=R+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}\left(V-V^{NB}-V^A\right)+\left(1-\underline{\theta}\right)\left(V-V^{B0}-V^A\right)\right),$$

or

$$V^A=\frac{R}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V-V^{B0}-\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Analogous with equation 51

$$V^{S2}-V^A=\frac{u_2+\frac{1}{2}\mu\left(\underline{\theta}\right)V}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}.$$

This in turn implies that

$$V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^A=\frac{u-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^A.$$

Turning to the value functions of mismatched owners, an owner that sells first has a value function given by

$$rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V-V^{S1}-\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right),$$

which can be re-written as

$$V^{S1}=\frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V-\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\theta\right)}.$$

For the value function of a deviating mismatched owner who buys first, assuming that trade takes place when he meets a real-estate firm but not when he meets a mismatched seller, writes

$$rV^{B1}=u-\chi+\frac{1}{2}q\left(\underline{\theta}\right)\underline{\theta}\Sigma_{AB1}.$$

Or

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)V^{B1}=u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^A\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^A-V+\underline{\theta}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Substituting for $V^{S2}-V^{A}-V$, we get that

$$V^{B1}-V^{S1}=\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\frac{u_2-u}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Note that at $\underline{\theta}=1$ (i.e. for $\kappa=0$),

$$\frac{u_2-u}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}=0,$$

given Assumption B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ moves towards infinity), we have that $\frac{u_2-u}{r+\frac{1}{2}\mu(\underline{\theta})}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1}<V^{S1}$ for $\underline{\theta}<1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta<1$.

Finally, we verify that our conjectures for the surpluses Σ_{S1Bn}, Σ_{AB1}, and Σ_{S1B1} are correct. We have that in the limit we consider

$$\begin{aligned}
\Sigma_{S1Bn} &= V-V^{Bn}+V^{B0}-V^{S1}=V-V^{S1}-\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\\
&= \frac{\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}-1\right)-r}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\underline{\theta}\right)\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}-1\right)\left(u_n-u_0\right)}{\left(r+\frac{1}{2}q\left(\underline{\theta}\right)\right)\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)}.
\end{aligned}$$

Therefore, at $\underline{\theta}=1$, $\Sigma_{S1Bn}>1$ if

$$r\left(\chi+u_0-u_n\right)+\frac{1}{2}\mu_0\chi>0.$$

Note that given Assumption B1, this is equivalent to

$$r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu_0\chi>0,$$

which holds by Assumption B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1>0$, such that for $\kappa<\kappa_1$, $\Sigma_{S1Bn}>0$. Similarly, in the limit we consider

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2}-V^{B1}-V^{A}=V^{S2}-V^{A}-\frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V^{S2}-V^{A}\right)\\
&= \frac{r\left(V^{S2}-V^{A}\right)-\left(u-\chi\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}=\frac{\frac{r}{r+\frac{1}{2}\mu(\underline{\theta})}u_2+\frac{\frac{1}{2}\mu(\underline{\theta})}{r+\frac{1}{2}\mu(\underline{\theta})}u-\left(u-\chi\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\\
&= \frac{r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu\left(\underline{\theta}\right)\chi}{\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)^2}.
\end{aligned}$$

At $\underline{\theta}=1$, $\Sigma_{AB1}>0$ if $r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu_0\chi>0$, which is our parametric Assumption B2.

Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} \\
&= \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}$$

At $\underline{\theta} = 1$,

$$\begin{aligned}
\frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^{A} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^{A} - R\right) - \left(r + \frac{1}{2}\mu_0\right)(u-\chi)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}$$

Substituting for Σ_{AB1}, we get

$$\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u-\chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^{A} - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$r\left(u_0 + u_2 - 2\left(u-\chi\right)\right) + \mu_0\chi \leq 0,$$

or

$$r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u-\chi\right)\right) + \frac{1}{2}\mu_0\chi\right],$$

which is our parametric assumption B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\left\{\kappa_1, \kappa_2, \kappa_3\right\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

We follow the same steps to show the existence of a "Buy first" equilibrium in which no mismatched owners sell first ($S_1 = 0$) and $\theta = \overline{\theta} > 1$. Again, we assume that $\Sigma_{AB1} > 0$ and show a sufficient condition for that later. The outflow rate of mismatched agents is equal to the outflow rate of new entrants, hence $B_1/B_n = \kappa$. The fraction of buyers that are entrants is therefore $\frac{B_n}{B_n+\kappa B_n} = \frac{1}{1+\kappa} = \frac{1}{\overline{\theta}}$. As no one dies in the queue, it follows that this is also the fraction of real-estate firms to the total number of sellers. Similarly to the sell first case, we have that

$$\left(r + \frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{A} = R + \frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V - V^{NB}\right) + \frac{\overline{\theta} - 1}{\overline{\theta}}\left(V^{S2} - V^{B1}\right)\right),$$

and

$$\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{S2}=u_2+\left(\rho+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^A+\frac{1}{2}\mu\left(\overline{\theta}\right)V.$$

Therefore, as in the "sell first" case,

$$V-V^{S2}=\frac{\rho V-u_2}{\rho+\frac{1}{2}\mu\left(\overline{\theta}\right)}-V^A.$$

Also, as in the previous case,

$$V^{Bn}-V^{B0}=\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Turning to the value functions of a mismatched buyer, we have that

$$rV^{B1}=u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V^{S2}-V^{B1}-V^A\right)+(1-\frac{1}{\overline{\theta}})\left(V-V^{B1}\right)\right),$$

For the value function of a deviating agent who chooses to sell first, we have that

$$\rho V^{S1}=u-\chi+\frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\max\left\{0,\Sigma_{S1Bn}\right\}+\frac{\overline{\theta}-1}{\overline{\theta}}\max\left\{0,\Sigma_{S1B1}\right\}\right).$$

Assume that $\Sigma_{S1Bn}>0$ and $\Sigma_{S1B1}<0$. Then in the limit,

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)V^{S1}=u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)V+\frac{1}{2}q\left(\overline{\theta}\right)\frac{u_0-u_n}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Therefore, the difference between $V^{B1}-V^{S1}$ satisfies

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\right).$$

At $\overline{\theta}=1$, we have that

$$\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}=0,$$

by Assumption B1. As $\overline{\theta}$ increases, we have that $\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu(\overline{\theta})}+\frac{u_n-u_0}{r+\frac{1}{2}q(\overline{\theta})}$ increases, so $V^{B1}>V^{S1}$ for $\overline{\theta}>1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta>1$.

Finally, we verify that our conjectures for the surpluses Σ_{AB1}, Σ_{S1Bn}, and Σ_{S1B1} are correct in the buy first case as well. Very similar to the sell first case, in the limit we consider

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^{A} = V^{S2} - V^{A} - V + V - \frac{u-\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)} - \frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\left[\frac{1}{\overline{\theta}}\left(V^{S2}-V^{A}-V\right)+V\right] \\
&= \frac{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)} + \frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(u_2-(u-\chi)\right)+\frac{1}{2}\mu\left(\overline{\theta}\right)\chi+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\left(u_2-u\right)}{\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\overline{\theta}=1$, Σ_{AB1} in the buy first case is the same as the sell first case. Therefore, there is a $\kappa_4>0$, such that for $\kappa<\kappa_4$ and $\overline{\theta}=1+\kappa$, $\Sigma_{AB1}>0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)} - \frac{r}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\overline{\theta}\right)\chi}{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)^2},
\end{aligned}$$

which at $\overline{\theta}=1$ is again the same as for the sell first case. Therefore, there is a $\kappa_5>0$, such that for $\kappa<\kappa_5$, $\Sigma_{S1Bn}>0$. Finally,

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0}-(u-\chi)+R+\frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)\left(V^{S2}-V^{B1}-V^{A}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)} - V^{A} \\
&= \left(1+\frac{\frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\right)\Sigma_{AB1} + \frac{rV^{B0}-(u-\chi)+R}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.
\end{aligned}$$

At $\overline{\theta}=1$, showing that $\Sigma_{S1B1}<0$ in the buy first case therefore follows the sell first case, so that $\Sigma_{S1B1}<0$ for $\kappa<\kappa_6$, for some $\kappa_6>0$. Taking $\overline{\kappa}=\min\{\kappa_4,\kappa_5,\kappa_6\}$, we have that for $\kappa<\overline{\kappa}$, there is a "Buy first" equilibrium with a market tightness given by $\overline{\theta}=1+\kappa$. Finally, taking $\kappa^*=\min\{\overline{\kappa},\underline{\kappa}\}$, we arrive at the desired result.

F. A model with competitive search

We define a competitive search equilibrium for the economy described in Section ??. Let $(\mathcal{P},\Theta)$ denote the active market segments in the economy, i.e. segments that attract a positive measure of buyers and sellers. The following equations describe the steady state value functions of agents. For new entrants we have:

$$\rho V^{Bn} = u_n - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V - V^{Bn} \right) \right\}. \tag{52}$$

Similarly, for a real estate firm, we have

$$\rho V^{A} = R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p - V^{A} \right) \right\}. \tag{53}$$

For mismatched owners that buy first, we have

$$\rho V^{B1} = u - \chi + \max \left\{ 0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V^{S2} - V^{B1} \right) \right\} \right\}, \tag{54}$$

where the value function takes into account the possibility that a mismatched owner that buys first may be better off not searching. Similarly, if the mismatched owner sells first, we have

$$\rho V^{S1} = u - \chi + \max \left\{ 0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p + V^{B0} - V^{S1} \right) \right\} \right\}. \tag{55}$$

A double owner solves

$$\rho V^{S2} = u_2 + R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta) \left(p + V - V^{S2} \right) \right\}, \tag{56}$$

while a forced renter solves

$$\rho V^{B0} = u_0 - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta) \left(-p + V - V^{B0} \right) \right\}. \tag{57}$$

Finally, for a matched owner we have

$$\rho V = u + \gamma \left(\max \left\{ V^{B1}, V^{S1} \right\} - V \right). \tag{58}$$

Next, we describe the steady state stock-flow conditions. Let

$$\left(p^{Bn}, \theta^{Bn} \right) \in \left(\mathcal{P}^{Bn}, \Theta^{Bn} \right) \equiv \arg\max_{(p,\theta)} \left\{ q(\theta) \left(-p + V - V^{Bn} \right) \right\} \subset (\mathcal{P}, \Theta) \tag{59}$$

denote a market segment that maximizes the value of searching for a new entrant. We define $\left(p^j, \theta^j\right)$ and $\left(\mathcal{P}^j, \Theta^j\right)$ analogously for an agent type $j \in \{A, B1, S1, B0, S2\}$. For agents $j \in \{B1, S1\}$, we adopt the convention that $\Theta^j = \emptyset$ if they choose not to search.

We have the following stock-flow conditions

$$g = \left(\sum_{\theta\in\Theta} x^{Bn}(\theta)\, q(\theta) + g \right) B_n, \tag{60}$$

$$\sum_{\theta\in\Theta} x^{S1}(\theta)\,\mu(\theta)\,S_1 = \left(\sum_{\theta\in\Theta} x^{B0}(\theta)\,q(\theta) + g\right) B_0, \tag{61}$$

$$\gamma x_b O = \left(\sum_{\theta\in\Theta} x^{B1}(\theta)\,q(\theta) + g\right) B_1, \tag{62}$$

$$\gamma x_s O = \left(\sum_{\theta\in\Theta} x^{S1}(\theta)\,\mu(\theta) + g\right) S_1, \tag{63}$$

$$\sum_{\theta\in\Theta} x^{B1}(\theta)\,q(\theta)\,B_1 = \left(\sum_{\theta\in\Theta} x^{S2}(\theta)\,\mu(\theta) + g\right) S_2, \tag{64}$$

$$g\left(O + B_1 + S_1 + 2S_2\right) = \sum_{\theta\in\Theta} x^{A}(\theta)\,\mu(\theta)\,A, \tag{65}$$

$$x_b + x_s = 1, \tag{66}$$

with

$$\sum_{\theta\in\Theta} x^{j}(\theta) = 1\ \forall j \in \{Bn, A, B0, S2\}, \tag{67}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$ and, if a mismatched owner that buys first/sells first chooses to search,

$$\sum_{\theta\in\Theta} x^{j}(\theta) = 1 \text{ for } j \in \{B1, S1\}, \tag{68}$$

with $x^j(\theta) = 0$ if $\theta \notin \Theta^j$. In the above expressions $\mathbf{x}^j(\theta) \geq 0$ is the vector of mixing probabilities over segments in Θ for an agent $j \in \{Bn, A, B1, S1, B0, S2\}$. Market tightnesses in each segment are given by

$$\theta = \frac{x^{Bn}(\theta)\,B_n + x^{B1}(\theta)\,B_1 + x^{B0}(\theta)\,B_0}{x^{A}(\theta)\,A + x^{S2}(\theta)\,S_2 + x^{S1}(\theta)\,S_1}, \tag{69}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$.

Finally, we have the population constancy and housing ownership conditions

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1, \tag{70}$$

and

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{71}$$

Following Moen (1997), we additionally require that the active market segments $(\mathcal{P}, \Theta)$ are such that the equilibrium allocation is a "no-surplus" allocation. Formally, we make the following requirement.

No-surplus allocation Let $\mathcal{B} \subset \{Bn, B1, B0\}$ and $\mathcal{S} \subset \{A, S1, S2\}$ denote the sets of *active* buyers and sellers in a steady state equilibrium, that is agents that have a strictly positive measure in steady state. Given the set of active segments $(\mathcal{P},\Theta)$ and agents' steady state value functions $\left\{V^{Bn},V^{B1},V^{B0},V^{A},V^{S1},V^{S2}\right\}$, there exists no pair $(p,\theta)\notin(\mathcal{P},\Theta)$, such that $V^{i}(p,\theta)>V^{i}$, for some $i\in\{Bn,B1,B0\}$, and $V^{j}(p,\theta)\geq V^{j}$ for some $j\in\mathcal{S}$, or $V^{i}(p,\theta)>V^{i}$, for some $i\in\{A,S1,S2\}$, and $V^{j}(p,\theta)\geq V^{j}$ for some $j\in\mathcal{B}$, where $V^{i}(p,\theta)$ denotes the steady state value function of an agent that trades in segment (p,θ), for $i\in\{Bn,B1,B0,A,S1,S2\}$.

Informally, the no-surplus allocation condition requires that in equilibrium there are no agents that would be strictly better off from deviating and opening a new market segment that would be (weakly) more attractive for some active agents (buyers or sellers) compared to their equilibrium values.

We can now define a symmetric steady state competitive search equilibrium of this economy as follows

Definition 11. A symmetric steady state competitive search equilibrium of this economy consists of a set of active market segments $(\mathcal{P},\Theta)$, steady state value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A, distributions of agent types over active market segments $\left\{\mathbf{x}^{j}\right\}_{j\in\{Bn,A,B1,S1,B0,S2\}}$, and set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, such that

1. The value functions satisfy equations (52) - (58) and the mixing distributions $\left\{\mathbf{x}^{j}\right\}_{j}$ are consistent with the agents' optimization problems.

2. Mismatched owners choose to buy first or sell first, to maximize $\overline{V}=\max\left\{V^{B1},V^{S1}\right\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b=\int_i I\left\{x_i=b\right\}di,$$

   where $i\in[0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

3. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (60)-(65) and (70)-(71) given Θ, $\left\{\mathbf{x}^{j}\right\}_{j}$ and mismatched owners' optimal decisions, reflected in x_b and x_s.

4. Every $\theta\in\Theta$ satisfies equation (69) given B_n, B_0, B_1, S_1, S_2, O, A, and $\left\{\mathbf{x}^{j}\right\}_{j}$;

5. The set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, is consistent with mismatched owners' optimal decisions;

6. $(\mathcal{P},\Theta)$ and agents' steady state value functions satisfy the "No-surplus allocation" condition.

Proof of Proposition ??

Proof. Consider first the "Buy first" equilibrium. The stock-flow conditions for this equilibrium are

$$B_n = \frac{g}{x^{Bn} q\left(\theta_1^{B1}\right) + \left(1 - x^{Bn}\right) q\left(\theta_2^{B1}\right) + g}, \tag{72}$$

$$A = \frac{g}{\mu\left(\theta_1^{B1}\right) + g}, \tag{73}$$

$$B_1 = \frac{\gamma O}{q\left(\theta_3^{B1}\right) + g},$$

$$S_2 = \frac{q\left(\theta_3^{B1}\right) B_1}{x^{S2} \mu\left(\theta_2^{B1}\right) + \left(1 - x^{S2}\right) \mu\left(\theta_3^{B1}\right) + g},$$

$$B_n + B_1 + S_2 + O = 1,$$

and

$$B_n = A + S_2, \tag{74}$$

where x^{Bn} is the probability with which a new entrant visits segment $\left(p_1^{B1}, \theta_1^{B1}\right)$ and x^{S2} is the probability with which a double owner visits segment $\left(p_2^{B1}, \theta_2^{B1}\right)$. The market tightnesses in each active segment satisfy

$$\theta_1^{B1} = \frac{x^{Bn} B_n}{A}, \tag{75}$$

$$\theta_2^{B1} = \frac{\left(1 - x^{Bn}\right) B_n}{x^{S2} S_2}, \tag{76}$$

and

$$\theta_3^{B1} = \frac{B_1}{\left(1 - x^{S2}\right) S_2}. \tag{77}$$

Observe that (72), (73), (74), and (75) imply that $x^{Bn} < 1$, as otherwise, (72), (73) and (75) give

$$\theta_1^{B1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{B1}\right) + g}{q\left(\theta_1^{B1}\right) + g},$$

which has a unique solution at $\theta_1^{B1} = 1$. However, this is inconsistent with (74).

Let Σ_{ij}, for $i \in \{B_n, B_0, B_1\}$ and $j \in \{A, S_1, S_2\}$ denote the match surplus from trading between a buyer i and seller j. The No Surplus Allocation equilibrium condition determines the equilibrium prices in each segment as a function of the steady state values of agents. Define

$$\begin{aligned} \overline{V}^{Bn} &= q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) \\ &= q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right), \end{aligned} \tag{78}$$

$$\overline{V}^{A} = \mu\left(\theta_1^{B1}\right)\left(p_1^{B1} - V^{A}\right), \tag{79}$$

$$\overline{V}^{B1} = q\left(\theta_3^{B1}\right)\left(-p_3^{B1} + V^{S2} - V^{B1}\right), \tag{80}$$

and

$$\begin{aligned}\overline{V}^{S2} &= \mu\left(\theta_2^{B1}\right)\left(p_2^{B1} + V - V^{S2}\right)\\ &= \mu\left(\theta_3^{B1}\right)\left(p_3^{B1} + V - V^{S2}\right),\end{aligned} \tag{81}$$

as the maximized value of searching for each trader. The No-Surplus Allocation condition implies that

$$\left(p_1^{B1}, \theta_1^{B1}\right) = \arg\max_{p,\theta} \mu(\theta)\left(p - V^A\right),$$

$$\text{s.t. } q(\theta)\left(-p + V - V^{Bn}\right) \geq \overline{V}^{Bn}.$$

Solving for p_1^{B1} and θ_1^{B1} gives the well-known Hosios rule (Hosios (1990)),

$$p_1^{B1} - V^A = (1-\alpha)\,\Sigma_{BnA},$$

or equivalently,

$$p_1^{B1} = (1-\alpha)\left(V - V^{Bn}\right) + \alpha V^A.$$

Therefore,

$$\overline{V}^{Bn} = \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2},$$

or

$$q\left(\theta_1^{B1}\right)\Sigma_{BnA} = q\left(\theta_2^{B1}\right)\Sigma_{BnS2}. \tag{82}$$

We have similar surplus sharing rules between the other trading pairs, which determine p_2^{B1} and p_3^{B1}. There is additionally one more indifference condition for a double owner that relates θ_2^{B1} and θ_3^{B1}. Specifically,

$$\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}. \tag{83}$$

The ranking of tightnesses across segments follows from the indifference conditions (82) and (83) and from observing that $\Sigma_{BnA} < \Sigma_{BnS2}$, whenever α is sufficiently close to one, and that $\Sigma_{BnS2} > \Sigma_{B1S2}$, whenever χ is sufficiently small. We show both of these below. Similarly, the ranking of prices across segments comes from the indifference conditions. Specifically, (82) implies that

$$q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) = q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right), \tag{84}$$

so

$$\frac{q\left(\theta_1^{B1}\right)}{q\left(\theta_2^{B1}\right)} = \frac{-p_2^{B1} + V - V^{Bn}}{-p_1^{B1} + V - V^{Bn}}.$$

$\theta_1^{B1} < \theta_2^{B1}$ and $q(.)$ decreasing imply that $p_1^{B1} > p_2^{B1}$. Similarly, (83) imply that $p_2^{B1} > p_3^{B1}$.

Finally, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{85}$$

$$\rho V^{A} = R + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{86}$$

$$\rho V^{B1} = u - \chi + \alpha q\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{87}$$

$$\rho V^{S2} = u_2 + R + (1-\alpha)\,\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{88}$$

and

$$\rho V = u + \gamma\left(V^{B1} - V\right). \tag{89}$$

Therefore, equations (72)-(77), combined with the two indifference conditions (82) and (83), and the value function equations (85)-(89) jointly determine the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "buy first" equilibrium.

Now we show that the surpluses satisfy the ranking used for ranking tightnesses. Using V^{Bn} and V^{A} from (85) and (86) to solve for Σ_{BnA}, we get

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{B1}\right) + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)}.$$

Similarly, solving for V^{B1} from equation (87), we get

$$V^{B1} = \frac{u-\chi}{\rho + \alpha q\left(\theta_3^{B1}\right)} + \frac{\alpha q\left(\theta_3^{B1}\right)}{\rho + \alpha q\left(\theta_3^{B1}\right)}V,$$

so

$$\Sigma_{B1S2} = V - V^{B1} = \frac{\rho V - (u-\chi)}{\rho + \alpha q\left(\theta_3^{B1}\right)}.$$

Finally, solving for Σ_{BnS2}, we get

$$\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)}.$$

Note that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{B1} = \frac{u}{\rho}$, so $\lim_{\chi\to 0}\Sigma_{B1S2} = 0$. This in turn implies that $\lim_{\chi\to 0}\theta_3^{B1} = \infty$ and $\lim_{\chi\to 0} x^{S2} = 1$. To see this, suppose to the contrary that as $\chi \to 0$, θ_3^{B1} remains bounded and x^{S2} is strictly below one. Therefore, $\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2} \to 0$, so indifference condition (83) implies that $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$. This in turn means that $\theta_2^{B1} \to 0$ and $x^{Bn} \to 1$. However, as $\theta_2^{B1} \to 0$, $q\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to \infty$, which is inconsistent with $x^{Bn} \to 1$, since new entrants would be strictly better off participating in the second market segment. Thus, we arrive at a contradiction.

This argument also implies that as $\chi \to 0$, $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2}$ remains bounded away from zero, which means that θ_2^{B1} is bounded away from zero. Next, note that condition (82) implies that θ_2^{B1} must be bounded from above, since otherwise $q\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$, which implies that $q\left(\theta_1^{B1}\right)\Sigma_{BnA} \to 0$

and θ_1^{B1} is unbounded as well. However, note that $\theta_1^{B1} < 1$, where $\theta = 1$ is the tightness in the first segment if $x^{Bn} \to 1$. Therefore, it must be the case that $q\left(\theta_2^{B1}\right)\Sigma_{BnS2}$ is bounded away from zero, and so θ_2^{B1} is finite. Since θ_2^{B1} is bounded away from zero and finite, it follows that Σ_{BnS2} is bounded away from zero for any $\chi > 0$. Also, note that the above observations hold for any value of $\alpha \in [0,1]$.

Since for any χ, Σ_{B1S2} is largest for $\alpha = 0$, there exists a $\overline{\chi}_1 > 0$, such that for any $\alpha \in [0,1]$ and any $\chi < \overline{\chi}_1$, $\Sigma_{BnS2} > \Sigma_{B1S2}$. Finally, note that $\Sigma_{BnS2} > \Sigma_{B1S2}$ implies that

$$V - V^{Bn} > V^{S2} - V^{B1},$$

so a new entrant is more impatient than a mismatched owner that buys first in the sense that the direct utility gain from transacting is higher for a new entrant compared to a mismatched owner that buys first.[37]

Next, note that $\rho V > \rho V^{S2} - R$, for α sufficiently close to 1. To show this, first note that

$$\begin{aligned}
\rho V^{S2} - R &= u_2 + R + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} - R \\
&= u_2 + \frac{(1-\alpha)\,\mu\left(\theta_2^{B1}\right)(2\rho V - u_n - u_2)}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)} \\
&= \frac{\left(\rho + \alpha q\left(\theta_2^{B1}\right)\right)u_2 + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)(2\rho V - u_n)}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)} \to u_2,
\end{aligned}$$

as $\alpha \to 1$. Since $u_2 < u - \chi < \rho V$ for any $\chi > 0$, it follows that there exists an $\overline{\alpha}_1 < 1$, such that for any $\chi > 0$, $\rho V > \rho V^{S2} - R$. This has two implications. First, it implies that $\Sigma_{BnA} < \Sigma_{BnS2}$. To see this, substitute for the respective value functions from the surplus conditions to get

$$V - V^{Bn} - V^A < V - V^{Bn} + V - V^{S2}.$$

Simplifying and multiplying by ρ, we get

$$-\rho V^A < \rho V - \rho V^{S2}.$$

Adding R on both sides, we have

$$-\rho V^A + R < \rho V - \rho V^{S2} + R.$$

As $\alpha \to 1$, the left hand side goes to 0, while the right hand side goes to $\rho V - u_2 > u - \chi - u_2 > 0$. Again this is true for any value of $\chi > 0$, so that also the second surplus ranking required for the ranking of the tightnesses holds. Secondly, it implies that a mismatched owner that buys first never wants to deviate and open a market segment with a real estate firm, because it implies that for any

[37] This also implies that a new entrant has steeper sloped indifference curves in the $\theta - p$ space, so he is willing to trade-off a higher price for the same decrease in market tightness compared to a mismatched owner that buys first.

$\chi > 0$ the surplus from trading in that case is negative, i.e.

$$\Sigma_{B1A} = V^{S2} - V^{B1} - V^{A} < 0.$$

To see this, re-write the inequality as

$$-V^{A} < V^{B1} - V^{S2}.$$

Again multiplying by ρ and adding R on both sides, we have

$$\rho V^{A} + R < \rho V^{B1} - \rho V^{S2} + R.$$

As $\alpha \to 1$, the left hand side goes to 0, while the right hand side goes to $\rho V^{B1} - u_2 \geq u - \chi - u_2 > 0$. Therefore, for any $\chi > 0$, there exists a $\overline{\alpha}_1 < 1$, such that for $\alpha > \overline{\alpha}_1$, $\Sigma_{BnA} < \Sigma_{BnS2}$ and $\Sigma_{B1A} < 0$.

Consider now a mismatched owner that deviates and sells first. Specifically we allow both a mismatched owner that sells first and a forced renter to open new market segments with active agents as counterparties. First, observe that $V^{Bn} > V^{B0}$, that is a new entrant is always better off than a forced renter. This ranking comes from the assumption that $u_0 < u_n$ and from a revealed preference argument. Specifically, suppose to the contrary that $V^{B0} > V^{Bn}$. Suppose that it is optimal for a forced renter to trade with a real estate firm (the argument for the case where the forced renter trades with a double owner is analogous). The No Surplus Allocation condition again implies that the Hosios condition holds, so

$$\rho V^{B0} = u_0 - R + \alpha q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right),$$

where $\tilde{\theta}$ is such that a real estate firm is indifferent between trading in this new segment and trading in the segment with a tightness of θ_1^{B1} and a price of p_1^{B1}. In contrast, we have that

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right).$$

Since $u_0 < u_n$ but $V^{B0} > V^{Bn}$, it follows that $q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$ and so $\tilde{\theta} < \theta_1^{B1}$. But then a new entrant is better off deviating and trading in the segment with tightness $\tilde{\theta}$, since $q\left(\tilde{\theta}\right)\left(V - V^{Bn} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$. Furthermore, $\Sigma_{BnA} > \Sigma_{B0A}$, so a real estate firm is in fact also strictly better off trading with a new entrant in the segment with tightness $\tilde{\theta}$. However, this is not consistent with $\left(p_1^{B1}, \theta_1^{B1}\right)$ not violating the No-Surplus Allocation condition. Therefore, in an equilibrium where $\left(p_1^{B1}, \theta_1^{B1}\right)$ are consistent with the No-Surplus Allocation, we must have $q\left(\tilde{\theta}\right) < q\left(\theta_1^{B1}\right)$. However, this means that $V^{B0} < V^{Bn}$, and we arrive at a contradiction.

Given that $V^{Bn} > V^{B0}$, consider the trading surplus Σ_{BnS1} between a new entrant and a sell first mismatched owner. Note that

$$\Sigma_{BnS1} = V - V^{Bn} + V^{B0} - V^{S1} \leq V - V^{Bn} + V^{B0} - \frac{u-\chi}{\rho},$$

where the second inequality follows from the fact that a mismatched owner that sells first can always choose not to trade and in that case obtains a value of $\frac{u-\chi}{\rho}$. Consider the difference $\rho V^{Bn} - \rho V^{B0}$ and note that

$$\rho V^{Bn} - \rho V^{B0} = u_n - u_0 + \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} - \alpha q\left(\tilde{\theta}\right)\Sigma_{B0A}$$

is bounded away from zero for any values of $\chi > 0$. The reason for this is that $\tilde{\theta}$ is pinned down by an indifference condition for a real estate firm of the form

$$\mu\left(\theta_1^{B1}\right)\Sigma_{BnA} = \mu\left(\tilde{\theta}\right)\Sigma_{B0A}.$$

As argued above, θ_1^{B1} is bounded away from zero for any $\chi > 0$, so $\tilde{\theta}$ is also bounded away from zero. Furthermore, this indifference condition implies that

$$q\left(\tilde{\theta}\right)\Sigma_{B0A} = \frac{\theta_1^{B1}}{\tilde{\theta}} q\left(\theta_1^{B1}\right)\Sigma_{BnA}.$$

Therefore,

$$\rho V^{Bn} - \rho V^{B0} = u_n - u_0 + \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA}\left(1 - \frac{\theta_1^{B1}}{\tilde{\theta}}\right) > \epsilon > 0,$$

for any $\chi > 0$. On the other hand, as $\chi \to 0$, $V - \frac{u-\chi}{\rho} \to 0$. Therefore, there exists a $\overline{\chi}_2 > 0$, such that for $\chi < \overline{\chi}_2$, $\Sigma_{BnS1} < 0$. Note, however, that $\Sigma_{BnS1} > \Sigma_{B1S1}$, for $\chi < \overline{\chi}_1$, since, as shown above, in that case $V - V^{Bn} > V^{S2} - V^{B1}$, meaning that a new entrant is more impatient than a mismatched owner that buys first. Therefore, for $\chi < \min\{\overline{\chi}_1, \overline{\chi}_2\}$, $\Sigma_{B1S1} < \Sigma_{BnS1} < 0$. In that case a mismatched owner that deviates and sells first is better off not trading. However, not trading is dominated buy buying first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from buying first in a "Buy first" equilibrium.

Constructing a "Sell first" equilibrium follows similar steps. First, the stock-flow conditions in that case become

$$B_n = \frac{g}{q\left(\theta_3^{S1}\right) + g}, \tag{90}$$

$$A = \frac{g}{x^A \mu\left(\theta_2^{S1}\right) + \left(1 - x^A\right)\mu\left(\theta_3^{S1}\right) + g}, \tag{91}$$

$$S_1 = \frac{\gamma O}{\mu\left(\theta_1^{S1}\right) + g},$$

$$B_0 = \frac{\mu\left(\theta_1^{S1}\right) S_1}{x^{B0} q\left(\theta_1^{S1}\right) + \left(1 - x^{B0}\right) q\left(\theta_2^{S1}\right) + g},$$

$$B_n + B_0 + S_1 + O = 1,$$

and

$$B_n + B_0 = A, \tag{92}$$

where x^{B0} is the probability with which a forced renter visits segment $\left(p_1^{S1}, \theta_1^{S1}\right)$ and x^A is the probability with which a real estate firm visits segment $\left(p_2^{S1}, \theta_2^{S1}\right)$. The market tightnesses in each active segment satisfy

$$\theta_1^{S1} = \frac{x^{B0} B_n}{S_1}, \tag{93}$$

$$\theta_2^{S1} = \frac{\left(1 - x^{B0}\right) B_0}{x^A A}, \tag{94}$$

and

$$\theta_3^{S1} = \frac{B_N}{\left(1 - x^A\right) A}. \tag{95}$$

Similarly, to before, observe that (90), (91), (92), and (93) imply that $x^A < 1$, as otherwise, (90), (91) and (93) give

$$\theta_3^{S1} = \frac{B_n}{A} = \frac{\mu\left(\theta_3^{S1}\right) + g}{q\left(\theta_3^{S1}\right) + g},$$

which has a unique solution at $\theta_3^{S1} = 1$. However, this is inconsistent with (92). As before, the No-Surplus Allocation implies that the match surpluses between trading pairs are split according to the Hosios rule. Finally, there are two indifference conditions for real estate firms and forced renters given by

$$\mu\left(\theta_3^{S1}\right) \Sigma_{BnA} = \mu\left(\theta_2^{S1}\right) \Sigma_{B0A}, \tag{96}$$

and

$$q\left(\theta_2^{S1}\right) \Sigma_{B0A} = q\left(\theta_1^{S1}\right) \Sigma_{B0S1}, \tag{97}$$

respectively. The ranking of tightnesses across segments follows from the indifference conditions (96) and (97) and from observing that $\Sigma_{BnA} < \Sigma_{B0A}$, and $\Sigma_{B0A} > \Sigma_{B0S1}$, whenever χ is sufficiently small (see below). The first ranking holds since $V^{Bn} > V^{B0}$, which can be shown by a revealed preference argument. Similarly, the ranking of prices across segments comes from the indifference conditions as in the case of a "Buy first" equilibrium. Finally, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_3^{S1}\right) \Sigma_{BnA}, \tag{98}$$

$$\rho V^A = R + (1 - \alpha) \mu\left(\theta_3^{S1}\right) \Sigma_{BnA}, \tag{99}$$

$$\rho V^{S1} = u - \chi + (1 - \alpha) q\left(\theta_1^{S1}\right) \Sigma_{B0S1}, \tag{100}$$

$$\rho V^{B0} = u_0 - R + \alpha \mu\left(\theta_1^{S1}\right) \Sigma_{B0S1}, \tag{101}$$

and

$$\rho V = u + \gamma\left(V^{S1} - V\right). \tag{102}$$

The stock-flow and market tightness equations combined with the two indifference conditions (96) and (97) and the above value functions fully characterize the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "sell first" equilibrium. The above value functions allow us to solve for the surpluses as follows:

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_3^{S1}\right) + (1-\alpha)\mu\left(\theta_3^{S1}\right)}.$$

$$\Sigma_{B0S1} = V - V^{S1} = \frac{\rho V - (u-\chi)}{\rho + (1-\alpha)\mu\left(\theta_1^{S1}\right)},$$

and

$$\Sigma_{B0A} = \frac{\rho V - u_0}{\rho + \alpha q\left(\theta_2^{S1}\right) + (1-\alpha)\mu\left(\theta_2^{S1}\right)}.$$

Note that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$, so $\lim_{\chi\to 0}\Sigma_{B0S1} = 0$. Then, a set of arguments similar to the case of the "Buy first" equilibrium shows that θ_2^{S1} remains bounded from zero and finite as $\chi \to 0$, so that Σ_{B0A} is bounded away from zero and finite for any $\chi > 0$. Therefore, there exists a $\overline{\chi}_3 > 0$, such that for $\chi < \overline{\chi}_3$, and for any $\alpha \in [0,1]$, $\Sigma_{B0A} > \Sigma_{B0S1}$.

Finally, note that $\Sigma_{B0A} > \Sigma_{B0S1}$ implies that

$$-V^{A} > V^{B0} - V^{S1},$$

so a real estate firm is more impatient than a mismatched owner that sells first in the sense that the direct utility gain from transacting is higher for a real estate firm compared to a mismatched owner that sells first.

Going forward, there exists a $\overline{\chi}_4$ such that for $\chi < \overline{\chi}_4$, $\Sigma_{BnS1} < 0$. Showing this is equivalent to showing the same condition for the case of a "Buy first" equilibrium and a deviating agent that sells first. Therefore, if $\chi < \overline{\chi}_4$, a mismatched owner that sells first never wants to deviate and open a market segment with a new entrant, since the surplus from trading in that case is negative.

Consider now a mismatched owner that deviates and buys first. We allow both a mismatched owner that buys first and a double owner to open new market segments with active agents. First, note that for α sufficiently close to 1 $\rho V - \rho V^{S2} - R > u - \chi - u_2 > 0$, for any $\chi > 0$, and for any market segment that a double owner deviates to and trades in. Specifically, there exists a $\overline{\alpha}_2 < 1$, such that for $\alpha > \overline{\alpha}_2$, $\rho V - \rho V^{S2} - R > u - \chi - u_2 > 0$. Therefore, as in the case of a "buy first" equilibrium for any $\chi > 0$,

$$\Sigma_{B1A} = V^{S2} - V^{B1} - V^{A} < 0.$$

Since $-V^{A} > V^{B0} - V^{S1}$ for $\chi < \overline{\chi}_3$, this also implies that

$$\Sigma_{B1S1} = V^{S2} - V^{B1} + V^{B0} - V^{S1} < \Sigma_{B1A} < 0.$$

It follows that for $\chi < \min\{\overline{\chi}_3, \overline{\chi}_4\}$ and $\alpha > \overline{\alpha}_2$, a mismatched owner that deviates and buys first is better off not trading. However, not trading is dominated by selling first since $V^{S1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from selling first in a "Sell first" equilibrium. Finally, setting $\overline{\chi} = \min\{\overline{\chi}_1, \overline{\chi}_2, \overline{\chi}_3, \overline{\chi}_4\}$ and $\overline{\alpha} = \max\{\overline{\alpha}_1, \overline{\alpha}_2\}$, we arrive at our result. □

G. A fixed price as the outcome of take-it-or-leave-it offers under private information

In this section we show that a fixed price equal to the present discounted value of rental income can be microfounded as the outcome of bargaining under private information about types, with full bargaining power for buyers. Suppose therefore in this section that buyers make take-it-or-leave-it offers, but do not know the type of the seller. However, buyers do know the fractions of the types in the economy. Because of heterogeneity among sellers, their reservation prices vary. Matching is still random, so that buyers cannot direct their search to the seller type with the lowest reservation price but meet a particular seller type with a probability equal to their proportion in the population of sellers. The question is then whether buyers, upon meeting a seller, make an offer that only sellers with a low reservation price would accept (and thus trade only if they have met a seller of this type), or make an offer that all sellers would accept (and therefore trade for sure).

We consider the symmetric case with $\tilde{u}_0 = \tilde{u}_2 = c$ (which for $p = \frac{R}{\rho}$ amounts to $u_0 = u_2 = c$), so that $\tilde{\theta} = 1$. In addition, we maintain Assumptions A1 and A2 and assume that $u_n < u - \chi$, so that both mismatched owners and new entrants are strictly better off to enter the market. As in the model with symmetric Nash bargaining, we focus on steady state equilibria with value functions, market tightness θ, and the stocks of different agent types constant over time. Moreover, although results hold more generally, we again consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. Remember that in this case $\overline{\theta} \to 1 + \kappa$ and $\underline{\theta} \to \frac{1}{1+\kappa}$. We will show that under these conditions both in a "buy first" and in a "sell first" equilibrium no buyer has an incentive to deviate from targeting both types of sellers by demanding a lower price than the unique prevailing price $p = \frac{R}{\rho}$.

Still denoting the value of a matched owner that remains passive upon mismatch by $\tilde{V}$, note first that at $\tilde{\theta} = 1$ Assumption A2 can be simplified to

$$\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu_0} + \frac{\mu_0}{(\rho+\mu_0)^2}c + \frac{{\mu_0}^2}{(\rho+\mu_0)^2}\tilde{V},$$

$$\Leftrightarrow \frac{u-\chi}{\rho} < \frac{c}{\rho+\mu_0} + \frac{\mu_0}{\rho+\mu_0}\tilde{V},$$

$$\Leftrightarrow 0 < \rho\left(c - (u-\chi)\right) + \mu_0\left(\rho\tilde{V} - (u-\chi)\right),$$

which, for future reference, is not greater than $\rho\left(c - (u-\chi)\right) + \mu_0\left(\rho V - (u-\chi)\right)$.

Under the unique price to be proven, the value functions are given by equations (3)-(6) and (27)-(29), given θ and R. We first show that in an equilibrium in which mismatched owners "buy first", buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: double owners and real estate agents. As before, the lowest price that a real estate agent would be willing to accept is $p^A = V^A = \frac{R}{\rho}$. The lowest price that a double owner would be willing to accept is $p^{S2} = V^{S2} - V$. Substituting these prices in the value functions, in an equilibrium with price dispersion $p^{S2} < p^A$, since

$$\rho\left(V^{S2} - V - V^A\right) = u_2 + R + \mu\left(\theta\right)\left(p^{S2} + V - V^{S2}\right) - \rho V - R - \mu\left(\theta\right)\left(p^A - V^A\right),$$

$$\Leftrightarrow \rho\left(V^{S2} - V - V^A\right) = u_2 - \rho V < 0,$$

$$\Leftrightarrow V^{S2} - V < V^A.$$

For that reason, under full information buyers would like to buy from a double owner, but the question is whether under private information they will make an offer that only double owners would accept. Note that for any $p \geq p^{S2}$ double owners are willing to sell, while for $p < p^{S2}$ they are not. As a result, since buying a house is preferred to being passive, among all possible deviations no offer is more profitable than demanding $V^{S2} - V$. The proof can therefore be restricted to this deviating offer. Note also that a deviating mismatched owner that sells first has zero mass, so that its presence doesn't affect the take-it-or-leave-it offers that buyers make.

First considering new entrants, for them to demand p^A it must be the case that

$$V - V^{Bn} - p^A \geq \frac{S_2}{S}\left(V - V^{Bn} - p^{S2}\right).$$

Substituting prices and using that $S = S_2 + A$ yields

$$\frac{A}{S}\left(V - V^{Bn} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{103}$$

From the value functions we have that

$$\rho\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n + R - q(\theta)\left(V - V^{Bn} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow (\rho + q(\theta))\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n,$$

and

$$\rho\left(V - V^{S2} + \frac{R}{\rho}\right) = \rho V - u_2 - R - \mu\left(\theta\right)\left(V - V^{S2} + \frac{R}{\rho}\right) + R,$$

$$\Leftrightarrow (\rho + \mu(\theta))\left(V - V^{S2} + \frac{R}{\rho}\right) = \rho V - u_2. \tag{104}$$

Moreover, in the limit we consider, we know from the section on Nash bargaining that $\frac{A}{S} = \frac{1}{\overline{\theta}}$ and $\frac{S_2}{S} = \frac{\overline{\theta}-1}{\overline{\theta}}$, so that (103) amounts to

$$\frac{1}{\overline{\theta}} \left(\rho + \mu\left(\overline{\theta}\right)\right)\left(\rho V - u_n\right) \geq \frac{\overline{\theta} - 1}{\overline{\theta}} \left(\rho + q\left(\overline{\theta}\right)\right)\left(\rho V - u_2\right).$$

where both sides are positive, but where the right-hand side can be made arbitrarily close to zero by moving closer to $\overline{\theta} = 1$. Therefore, it follows that there is a $\kappa_7 > 0$, such that for $\kappa < \kappa_7$, new entrants in a "buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a selller. Substituting u_0 for u_n, the same condition holds for a deviating mismatched owner that sells first, and then becomes a forced renter. Therefore, there is a $\kappa_8 > 0$, such that for $\kappa < \kappa_8$, forced renters in a "buy first" equilibrium make the same offer.

For mismatched owners that buy first to demand p^A it must be the case that

$$V^{S2} - V^{B1} - p^A \geq \frac{S_2}{S}\left(V^{S2} - V^{B1} - p^{S2}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{105}$$

Rearranging the value functions yields

$$\rho\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 + R + \mu(\theta)\left(V - V^{S2} + \frac{R}{\rho}\right) - R - (u - \chi) - q(\theta)\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right),$$

$$\Leftrightarrow (\rho + q(\theta))\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 - (u - \chi) + \mu(\theta)\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{106}$$

Substituting the steady state fractions and (106) into (105), in the limit we consider we have that

$$\frac{1}{\overline{\theta}}\left(u_2 - (u - \chi) + \mu\left(\overline{\theta}\right)\left(\frac{R}{\rho} + V - V^{S2}\right)\right) \geq \frac{\overline{\theta} - 1}{\overline{\theta}}\left(\rho + q\left(\overline{\theta}\right)\right)\left(V - V^{S2} + \frac{R}{\rho}\right),$$

$$u_2 - (u - \chi) \geq \left[\left(\overline{\theta} - 1\right)\left(\rho + q\left(\overline{\theta}\right)\right) - \mu\left(\overline{\theta}\right)\right]\left(V - V^{S2} + \frac{R}{\rho}\right).$$

Substituting (104) yields

$$\left(\rho + \mu\left(\overline{\theta}\right)\right)\left(u_2 - (u - \chi)\right) \geq \left[\left(\overline{\theta} - 1\right)\left(\rho + q\left(\overline{\theta}\right)\right) - \mu\left(\overline{\theta}\right)\right]\left(\rho V - u_2\right),$$

$$\Leftrightarrow \rho\left(u_2 - (u - \chi)\right) + \mu\left(\overline{\theta}\right)\left(\rho V - (u - \chi)\right) \geq \left(\overline{\theta} - 1\right)\left(\rho + q\left(\overline{\theta}\right)\right)\left(\rho V - u_2\right).$$

The left-hand side is positive for any $\overline{\theta} \geq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_9 > 0$, such that for $\kappa < \kappa_9$, mismatched owners that buy first in a "buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Taking $\overline{\kappa}' = \min\{\kappa_7, \kappa_8, \kappa_9\}$, we have that for $\kappa < \overline{\kappa}'$, all buyers demand $p = \frac{R}{\rho}$ upon meeting a seller in a "buy first" equilibrium with a market tightness given by $\overline{\theta} = 1 + \kappa$.

Secondly, we show that in an equilibrium in which mismatched owners sell first, buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: mismatched owners that sell first, and real estate agents. The lowest price that a real estate agent would be willing to accept is still $p^A = V^A = \frac{R}{\rho}$. The lowest price that a mismatched owner would be willing to accept is $p^{S1} = V^{S1} - V^{B0}$. It must be the case that $V^{B0} - V^{S1} + p^A \geq 0$, because mismatched owners don't remain passive by Assumption A2. It follows that $p^{S1} \leq p^A$, so that with full information buyers would like to buy from a mismatched owner. Again the question is whether under private information buyers will make an offer that only mismatched owners would accept. Similar to the "buy first" equilibrium, the proof can be restricted to the deviation of demanding $V^{S1} - V^{B0}$.

First considering forced renters, for them to demand p^A it must be the case that

$$V - V^{B0} - p^A \geq \frac{S_1}{S}\left(V - V^{B0} - p^{S1}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right). \tag{107}$$

Rearranging the value functions yields

$$\rho\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0 + R - q(\theta)\left(V - V^{B0} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow (\rho + q(\theta))\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0, \tag{108}$$

and

$$\rho\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - R + q(\theta)\left(-\frac{R}{\rho} + V - V^{B0}\right) - (u - \chi) - \mu(\theta)\left(\frac{R}{\rho} + V^{B0} - V^{S1}\right) + R,$$

$$\Leftrightarrow (\rho + \mu(\theta))\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - (u - \chi) + q(\theta)\left(V - V^{B0} - \frac{R}{\rho}\right). \tag{109}$$

Substituting (109), (107) therefore amounts to

$$\frac{A}{S}(\rho + \mu(\theta))\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi) + q(\theta)\left(V - V^{B0} - \frac{R}{\rho}\right)\right),$$

$$\Leftrightarrow \left[\frac{A}{S}(\rho + \mu(\theta)) - \frac{S_1}{S}q(\theta)\right]\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}(u_0 - (u - \chi)).$$

Substituting (108) yields

$$\left[\frac{A}{S}(\rho + \mu(\theta)) - \frac{S_1}{S}q(\theta)\right](\rho V - u_0) \geq \frac{S_1}{S}(\rho + q(\theta))(u_0 - (u - \chi)),$$

$$\Leftrightarrow \frac{A}{S}(\rho + \mu(\theta))(\rho V - u_0) \geq \frac{S_1}{S}\rho(u_0 - (u - \chi)) + \frac{S_1}{S}q(\theta)(\rho V - (u - \chi)).$$

From the section on Nash bargaining we know that $\frac{A}{S} = \underline{\theta}$ and $\frac{S_1}{S} = 1 - \underline{\theta}$. Substituting these steady state fractions, we have that

$$\underline{\theta}\left(\rho + \mu\left(\underline{\theta}\right)\right)\left(\rho V - u_0\right) \geq \left(1 - \underline{\theta}\right)\left[\rho\left(u_0 - (u - \chi)\right) + q\left(\underline{\theta}\right)\left(\rho V - (u - \chi)\right).\right]$$

Again, by moving towards $\underline{\theta} = 1$ the right-hand side can be made arbitrarily close to zero while the left-hand side remains positive. Therefore, there exists a $\kappa_{10} > 0$, such that for $\kappa < \kappa_{10}$, forced renters in a "sell first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Substituting u_n for u_0 the same condition holds for a new entrant, so that there is a $\kappa_{11} > 0$, such that for $\kappa < \kappa_{11}$, new entrants make the same offer.

Finally, for a deviating mismatched owner that buys first to demand p^A it must be the case that

$$V^{S2} - V^{B1} - p^A \geq \frac{S_1}{S}\left(V^{S2} - V^{B1} - p^{S1}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right). \tag{110}$$

Rearranging the value functions yields

$$\rho\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 + R + \mu\left(\theta\right)\left(\frac{R}{\rho} + V - V^{S2}\right) - u - \chi + q\left(\theta\right)\left(-\frac{R}{\rho} + V^{S2} - V^{B1}\right) - R,$$

$$\Leftrightarrow \left(\rho + q(\theta) + \mu\left(\theta\right)\right)\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 - (u - \chi) + \mu\left(\theta\right)\left(V - V^{B1}\right),$$

with

$$\mu\left(\theta\right)\left(V - V^{B1}\right) = \mu\left(\theta\right)\left(V - \frac{u - \chi}{\rho}\right) - \mu\left(\theta\right)q\left(\theta\right)\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right).$$

From (109) we know that

$$\left(\rho + \mu(\theta)\right)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{B0} - \frac{R}{\rho} + V^{S1} - V^{S1}\right),$$

$$\Leftrightarrow \left(\rho + q(\theta) + \mu(\theta)\right)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{S1}\right),$$

with

$$q\left(\theta\right)\left(V - V^{S1}\right) = q\left(\theta\right)\left(V - \frac{u - \chi}{\rho}\right) - \mu\left(\theta\right)q\left(\theta\right)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right).$$

Therefore, (110) simply amounts to

$$\frac{A}{S}\left(u_2 - (u - \chi) + \mu\left(\theta\right)\left(V - \frac{u - \chi}{\rho}\right)\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi) + q\left(\theta\right)\left(V - \frac{u - \chi}{\rho}\right)\right).$$

Substituting the steady state fractions, we have that

$$\underline{\theta}\left(u_2-(u-\chi)+\mu\left(\underline{\theta}\right)\left(V-\frac{u-\chi}{\rho}\right)\right)\geq\left(1-\underline{\theta}\right)\left(u_0-(u-\chi)+q\left(\underline{\theta}\right)\left(V-\frac{u-\chi}{\rho}\right)\right).$$

The left-hand side is positive for any $0<\underline{\theta}\leq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_{12}>0$, such that for $\kappa<\kappa_{12}$, deviating mismatched owners that buy first in a "sell first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a seller. Taking $\underline{\kappa}'=\min\left\{\kappa_{10},\kappa_{11},\kappa_{12}\right\}$, we have that for $\kappa<\underline{\kappa}'$, all buyers demand $p=\frac{R}{\rho}$ upon meeting a seller in a "sell first" equilibrium with a market tightness given by $\underline{\theta}=\frac{1}{1+\kappa}$. Finally, taking $\kappa'=\min\left\{\overline{\kappa}',\underline{\kappa}'\right\}$, we have that both in a "buy first" and in a "sell first" equilibrium, the take-it-or-leave-it offer that buyers make is equal to $p=\frac{R}{\rho}$.