Credit, Money, Interest, and Prices[*]

FIRST DRAFT: 2016, CURRENT DRAFT: 2018

Saki Bigio[†] and Yuliy Sannikov[‡]

July 5, 2018

Abstract

A model integrates a modern implementation of monetary policy into an incomplete-markets monetary model. Monetary policy can conduct open-market operations, alter policy rates, and run fiscal transfers. These tools induce different changes on inflation and real spreads and affect the evolution credit, output, and the distribution of wealth. We revisit classic policy and crisis experiments. We describe how different instruments have effects through different transmission channels.

Our framework provides relevant policy insights: (a) monetary policy can move real rates (long and short-run) independently of inflation, (b) running a Taylor rule after a *temporary* credit crunch can lead to *persistent* deflation, (c) coupled with a zero-lower bound on deposit rates, negative rates on reserves increase the loans rates and are recessionary, (d) at a zero-lower bound, fiscal transfers stimulate output when unanticipated, but are recessionary when anticipated.

Keywords: Monetary Economics, Monetary Policy, Credit Channel.

JEL: E31-2, E41-4, E52-2

---

[*]We would like to thank Alex Carrasco for outstanding research assistance. We also thank Pierre-Olivier Weill and Andrew Atkeson and seminar participants at UCLA and the EUI.

[†]Department of Economics, University of California, Los Angeles and NBER, email `sbigio@econ.ucla.edu`

[‡]Stanford Business School and NBER, email `sannikov@gmail.com`

1 Introduction

> Now, toward the end of my career as at the beginning, I see myself as a monetarist. My contributions to monetary theory have been to incorporate the quantity theory into modern modeling. For the empirically well established predictions —long-run links— this job has been accomplished. On the harder questions of monetary economics — the real effects of monetary instability, the roles of inside and outside money, this work contributes examples but little in empirically successful models. It is understandable that in the leading operational macroeconomic models today— the RBC and the New Keynesian models— money as a measurable magnitude plays no role at all, but I hope we can do better than this in the future.
>
> —Robert E. Lucas, 2013
>
> —Final paragraph in the introduction to Collected Papers in Monetary Economics

This paper presents an incomplete-markets model with a role for inside money (deposits) and outside money (currency and reserves). Monetary policy (MP) operates through the provision of bank reserves and by setting of corridor rates, in addition to fiscal transfers.[1] A prevalent view has is that those tools affect real activity and prices through their influence on credit (Bernanke and Blinder, 1992). Although there is empirical support for this view (Kashyap and Stein, 2000; Drechsler et al., 2017), the theoretical foundations of this view, are not well understood. The goal of this paper is to articulate how changes in MP tools lead to predictions about credit, money, interest, and prices. The theory and its predictions provide a unified framework to think of the transmission of MP through the credit channel in conjunction with other transmission channels.[2]

Although the view that central bankers can affect credit volumes and spreads is ubiquitous in the policy narrative, there is a shortage of general-equilibrium models that formalize this point. The reason for this shortage should not be a surprise. Credit creation comes with money creation, and this fact complicates any analysis. A naive, partial-equilibrium, reasoning would suggest that if MP can stimulate bank credit, it should stimulate real activity directly. In general equilibrium, things are more complicated. Banks extend loans by simultaneously creating deposits. However, deposits must be held in equilibrium. Being a claim

[1] A corridor system is a rate on discount-window loans and interest on reserves. The discount rate is the rate at which a Central Bank lends reserves to banks that are below their reserve requirement. The rate on reserves is the rate at which banks are remunerated by holding reserve balances at the Central Bank.

[2] A narrative description of different transmission channels of MP is found in Bernanke and Gertler (1995)'s "Inside the Black box". Kashyap and Stein (2000) presented evidence on the credit channel by exploiting differences in the cross-section of liquidity ratios across banks. Bindseil (2014) describes the modern implementation of MP through banks across countries.

on currency, deposits are a nominal object. Thus, loan creation must impact the price level. This results in the complication that a general equilibrium model with these features must describe the joint determination of credit, money, interests, and prices. If, in addition, that model is to answer the harder questions of monetary economics—in the words of Lucas—monetary instability should beget real outcomes. The model in this paper has all these ingredients.

The model is ultimately motivated by a pragmatic goal. If we conceive the influence on credit markets as the cornerstone to MP, we can also envision that MP can have ambivalent powers. In fact, many view that *that MP can sow the seeds of crises during booms*, while others view it as paramount to *pushing on a string during crises*. Discovering ways in which MP can mitigate the risk of a crunch, and unleash credit when credit is tight, have been at the center of policy debates. These debates go beyond the question of what to do when policy rates are zero. But how can we even frame those debates without a model where credit plays a leading role? The pragmatic goal of this paper is present a first-step model that can enrich policy discussions.

The model. The building block is a heterogeneous-agent continuous-time environment. In essence, this is a Hugget economy (Huggett, 1993) where credit is nominal and intermediated by a banking fringe. Monetary policy operates through its effects on bank lending and borrowing decisions.

The presence of idiosyncratic risk and incomplete markets engenders a demand for some form of money. Just as in a first generation of heterogeneous agent models, namely Lucas (1980) and Bewley (1983), the focus is on how money and MP affect outcomes. However, the focus in Bewley and Lucas was to study the price of outside money when MP was conducted in an old-fashioned way, through fiscal transfers. Issues relating to credit markets and how MP affects it were left out.[3] Credit, of course, has a tradition in heterogeneous agent models that begins with Huggett (1993) and Aiyagari (1994). Unlike in the Hugget or Aiyagari economies, here credit is a nominal claim, is intermediated by banks, and MP affects outcomes through its influence on banks.

In this environment, MP policy is powerful enough to affect credit spreads, real rates (both in the long and short run) and, simultaneously, control the price level. This power stems from special property of outside money: when held as reserves, outside money complements the

[3] In both models, there was a constant supply of outside money. Lucas (1980) studied a stable price equilibrium. Bewley (1983) focused on the case where money earned an interest rate financed with lump-sum taxes, so interest rate had redistributive consequences as it was funded with lump-sum transfers. Ljungqvist and Sargent (2012, Chapter 18) describes shows how policies in Bewley (1983) models are akin to changes in borrowing limits in economies with pure credit.

provision of credit. This complementarity emerges because, as in the real world, banks use reserves to settle deposit transfers. Because transfers are unpredictable, and the interbank market operates with frictions á la Ashcraft and Duffie (2007), some banks face the risk of borrowing reserves at a discount-window rate set by MP. As a result, corridor rates and the quantity of reserves affect the risk of intermediating credit. Any intermediation cost induced by MP is ultimately passed on to consumers as a borrowing-lending spread. This is the credit channel.[4]

The power of MP is limited in two ways. Both are related to different properties of outside money. One limit is imposed by reserve satiation: when MP satiates banks with reserves, outside money and loans become perfect substitutes for banks. Under this satiation limit, open-market operations are neutral on inflation and on real variables. However, changes in policy rates affect inflation. The second limit follows from yet another property: When held as currency, outside money is a perfect substitute with deposits as a store of value for households. This feature induces a zero-lower bound on deposit rates that constraints the effects of different instruments in different ways.

The real effects of MP emerge through a precautionary motive. The mechanism directly links output to credit-market conditions. There is a slight departure to the real side of Huggett (1993) because households have a production decision. Namely, household's operate firms that face a risk-return trade off. As a result of imperfect insurance, as households approach their borrowing limits, they reduce income risk at the expense of their output. Compared with a complete-markets benchmark, excerpting this option is inefficient.[5] Although simplistic, this mechanism captures that financial stress and attitudes towards uncertainty affect output. Since MP indirectly affects the distribution of wealth, it influences the mass of agents that approach borrowing constraints, and hence affects output.

The final section of the paper incorporates real wage rigidity. That extension induces an externality. That externality emerges as agents trade off risk-return with internalizing their effects on other agents' employment. This motivates as to think of how MP can activate real spreads to limit the extent of a crisis.

[4] This way of modeling MP follows from recent work by Afonso and Lagos (2012), Bianchi and Bigio (2017a) and Piazzesi and Schneider (2018).

[5] Although this is a positive paper, MP can be motivated by the desire to provide insurance and to reduce the productive inefficiency. Policy may want to trade-off these goals over time. A Central Bank may want to induce a real borrowing/lending spreads that produces less efficient risk-sharing against the ability to have room to lower credit spreads when credit-market conditions worsen. With other credit market imperfections, this is even more important.

From Instruments to Transmissions Channels. After we present the model and characterize the determination real and nominal variables, we revisit several classical exercises. These exercises study the effects of MP instruments, alone and in combination with a credit crunch. The responses to policy changes depend on the particular monetary doctrine—a combination of policy rules. Each doctrine is associated with a different transmission channels articulated by the literature.

Under the first doctrine MP is designed to eliminate any transmission through the credit channel. This policy is achieved when MP satiates banks with reserves, or eliminates the spread in the corridor rates. Under this doctrine, MP is neutral. Yet, MP has control on nominal rates directly and, thus, indirectly controls the unit of account.[6] Although these other channels are shut down, this doctrine connects the environment to other transmission channels stressed by the literature: interest-rate channel, the inflation-cost channel, and the debt-deflation channel. In each case, the model would need an additional ingredient: nominal rigidities, cash transactions, and long-term debt, respectively. Here, we only underscore that without these frictions, under this doctrine, a Taylor rule will produce a permanent deflation after a credit crunch.

A second doctrine studies the role of lump-sum currency transfers to households. We study this doctrine as a prelude to the credit channel. This is because when the credit channel is activated, MP operations produce fiscal revenues (Hall and Reis, 2015). Hence, the credit channel has a partial effect through fiscal transfers. Because of incomplete markets, fiscal transfers here have non-Ricardian effects because they redistribute wealth. This redistribution channel has output effects: if unexpected, transfers stimulate output as they push borrowers away from their constraints. If the policy is expected, it can be recessionary because it leads more borrowers to hit their borrowing limits.

A third doctrine is to actively manage the credit channel. MP opens a spread in its corridor rates and actively manages open-market operations to indirectly control real spreads. Because MP controls real spreads, it affects the levels of real rates, both in the short and long-run. The control over real does not compromise the ability to control inflation. On the contrary, the model asserts to central bankers what perhaps they already know from practice, that they can control inflation while simultaneously maintaining a grip on credit markets. For empirical work, this observation has important implications because it means we cannot study the

[6] Different from Woodford (1998), this can be achieved without open-market operations, but by issuing reserves. A related result was independently obtained by Hall and Reis (2017).

effects of MP independently from how it affects credit markets in the long-run.

Although the credit channel activates a credit spread, an active management of the credit channel is desirable. The credit channel allows MP to limit the extent of a credit crunch. A policy that keeps an open spread prior to a credit crunch and reduces the the spread during the credit crunch, stabilizes the economy. Thus, the model prescribes a trade-off between a the level and the volatility of output.

During a crunch, MP cannot do more than compressing credit spreads to zero: if in an attempt to reinvigorate lending, if MP floods banks with reserves, the economy reaches its zero-lower bound on deposits rates. At that point, open-market operations seize to function. Furthermore, if MP charges negative interest rates on reserves, banks will increase their lending rate. This shows that at a zero-lower bound on deposits, fiscal transfers are the only expansionary tool available. However, a fiscal transfer can be recessionary if it is anticipated.

The title of our paper is, of course, reminiscent of a sequence of classic titles in the literature. Don Patinkin added Money to the title "Interest and Prices" of a classic piece by Knut Wicksell. Michael Woodford took Money out of Patinkin's title, promoting the view that MP can be studied without reference to money markets.[7] A large body of the literature that analyzes monetary policy follows the no money approach, and we can also do that in this environment. The title of this paper is a counter reaction to that view. Like much of the work we survey below, not only do we think we need to add money back to monetary models, but that we should take the effect on credit markets explicitly.

The organization is the following. A connection with the literature is presented in Section 2. Section 3 lays out the core model. Section 4 describes the determination of credit, interest and prices in the model and how these affect real output. That section also derives implementation conditions for MP. Section 5 studies classic experiments in monetary economics. Section 6 describes the policy effects at the zero-lower bound after a credit crunch. Section 7 presents the effects of the demand externality. Section 8 concludes.

2 Connection with the Literature

Model in Perspective. In perspective, our model differs from the two most common approaches in monetary economics. One approach emphasizes the connection between *money and prices*, and the other emphasize the connection between *interest and prices*. In models

[7] We thank Emmanuel Farhi for pointing this sequence of titles.

with non-interest bearing money plays a transactions role—cash-in-advance or money-search models—there is a tight connection between the quantity of outside money and prices. Under that approach, the real interest rate is pinned down by time preferences. Nominal rates and expected inflation move in tandem. If there are real effects in those models, it is because transactions are carried in non-interest bearing money. In essence, under that workhorse, inflation is a transactions tax. A challenge for that workhorse is that Central Banks do influence real rates. Furthermore, thanks to the financial innovations of the last thirty years, most transactions are carried out through deposits accounts that are interest-rate bearing.

The second common approach is new-Keynesian workhorse. The new-Keynesian approach is all about the connection between *interest and prices*. Under this framework, a central bank controls the real interest rate thanks to two imperfections. One imperfection are sticky prices, but also of fundamental importance is a financial-market imperfection that grants policy control over nominal rates. That imperfection is also a cash-in-advance constraint, so this literature is subject to the same criticism as the first a approach.[8] That environment is understandably appealing to policy makers because it is consistent control over real rates. A problem of the new-Keynesian approach is that it leaves any connection MP and credit markets aside. We argue that our paper provides a bridge between the interest-rate channel and the credit channel.

As we can see, both approaches were developed without an explicit role for credit. After the crisis of 2008, there's been an increased demand for models that can speak about how MP affects credit market. That gap is being filled out quickly and we want to contrast our paper with that battery of recent work.

Models that feature credit must provide a motive for credit. One way is to endow agents with different technologies as in Bernanke and Gertler (1989) and the other is make them subject to idiosyncratic risks. To establish a connection between MP and credit markets, models must have features by which MP impacts credit. A first such model is Bernanke et al. (1999) which incorporated nominal rigidities into the two-sector economy of Bernanke and Gertler (1989). In Bernanke et al. (1999) MP was capable of moving real rates because of nominal rigidities. In that model, and models that follow it, Christiano et al. (2009), credit amplifies the effects of other shocks—through the financial accelerator—or a source of disturbances

[8] This point is not obvious because the new-Keynesian models studies the cash-less limit of a cash-in-advance economy. That is, the limit as the fraction of transactions that require money goes to zero. Under our implementation of MP, the model doesn't require a demand for currency to grant control over nominal rates. Furthermore, the zero-lower bound is modeled explicitly.

when there are exogenous shocks to credit markets.

Guerrieri and Lorenzoni (2012) study the tightening of borrowing limits in a Bewley economy with nominal rigidities.[9] Following that paper, models with nominal rigidities and heterogeneous agents have received substantial attention. Heterogeneity makes the response of aggregate demand to policy changes to depend on the distribution of marginal propensities. Along those lines, Auclert (2016) studies how the number of borrowing constrained agents influences the income sensitivity to changes in policy rates. Werning (2015) shows that the effectiveness also depends on general equilibrium forces. Kaplan et al. (2016) introduce illiquid assets to disconnect interest rate elasticities from the distribution of wealth. Greenwald (2016) and Wong (2016) how interest rate sensitivities to mortgage refinancing. In virtually all of these studies, MP operates through the credit channel, but does not affect credit directly. Instead, the distribution of wealth determines the strength of this channel.

Another class of papers blend models where cash serves a role for transactions with models with an active credit market. Some models emphasize that when inside money is an imperfect substitute to currency, inflation can determine the volume of credit (see for example Berentsen et al., 2007; Williamson, 2012). In a money-search environment Gu et al. (2015) show conditions under which the real value outside and inside money remains constant and is independent of the composition. Lippi et al. (2015) introduce shocks in a two-agent model where outside is held for transactions. MP has effects through the distribution of wealth. Rocheteau et al. (2016) work in a money-search environment with a non-degenerate distribution of money, and study how MP affects activity by changing the relative value of outside money. Gomes et al. (2016) postulate a Fisher equation and study how inflation affects credit markets via the redistribution of wealth when debt is long-term.

A growing body of work is interested in the connection between credit, money, interest and prices. Recent examples are Brunnermeier and Sannikov (2012) . In Brunnermeier and Sannikov (2012), agents face undiversified investment risk. A natural demand for currency emerges without intermediaries. The presence of intermediaries allows some diversification because intermediaries can exchange equity of inside money depending on intermediary net worth. However, reductions in intermediary net worth can reduce the supply of money and thereby increase exposure to idiosyncratic risk. With a decline in the supply of inside money, idiosyncratic risk increases and output falls as this leads to misallocation across sectors and

[9]Following up on that work, McKay et al. (2015) compare the effects of forward-guidance policies in representative agent new-Keynesian models and incomplete markets economies.

less investment. MP in that paper achieves two things: first, it stabilizes asset prices and redistributes wealth towards intermediaries. In that paper, MP is implemented in two ways, either via helicopter drops or through interest payments on outside money holdings. However, that models doesn't feature a channel where spreads are affected directly. Silva (2016) focuses on open-market operations and the effects of expected inflation. In Buera and Nicolinni (2016), the identity between borrowers and lenders is determined by a threshold interest rate. Furthermore, there is an explicit role for outside money as a transactions instruments and MP has real effects by affecting the stock of risk-free bonds which, in turn, affects the threshold identity of borrowers and lenders.

The implementation of MP follows from Bianchi and Bigio (2017a). In Bianchi and Bigio (2017a) financial intermediaries issue deposits to make working capital loans, thus, intermediating between the deposit demand and the demand for loans. Outside money is only held by intermediaries to satisfy payments needs across banks. Monetary policy affects a borrowing and lending spread by increasing the supply of outside money or by affecting borrowing and lending spread in the interbank market. Real effects emerge as raising the cost of debt operates as a labor wedge. The implementation of MP policy in this paper is close to Bianchi and Bigio (2017a). Outside money plays is connected to the provision of credit in the same way in Piazzesi and Schneider (2018), but there, the focus is on how changes in MP affect asset prices. In Ennis (2014) reserves are held to meet regulatory requirements.

A common criticism to both the New-Keynesian approach and the CIA in advance approach is that responses of policies are immediate. The emphasis on the idea that MP works slowly because it has to affect the distribution of wealth is a commonality with models of the liquidity effect like Alvarez et al. (2009).

3 Environment

We begin with a description of the model environment and then proceed to study monetary policy. Time t is continuous and runs to infinity, $t \in [0, \infty)$. The economy is populated by three sets of agents: the public, banks, and a consolidated government which we simply refer to as the Central Bank (CB). There is a single good. The unit of account are units of outside money and the price of goods at instant t is P_t.

The CB is a combination of a central bank and a fiscal authority. The CB chooses policy rates and open-market operations. However, here it is consolidates with a fiscal authority that makes/collects (lump-sum) fiscal transfers to/from households. We present the environment quickly without digressions and leave discussions towards the end of the section.

Notation. Individual state variables are denoted with lower case letters. Aggregate nominal state variables in capital letters. Aggregate real variables are written in calligraphic font.

3.1 The Public

The public is a measure-one continuum of households that run firms, supply labor and face a consumption-savings decision. Their preferences are described by:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right]$$

where the instantaneous utility is $U\left(c_t\right) \equiv \left(c_t^{1-\gamma}-1\right) / \left(1-\gamma\right)$. The γ is the coefficient of risk-aversion.[10]

Each household owns a firm and operates a production technology.[11] Household's are heterogeneous because firm income is stochastic—and there are no insurance markets. The individual state of a household is the stock of real financial claims, s_t, although all assets are nominal claims on outside money. When credit is available, s_t can be negative. At any point in time, there's a distribution $f_t\left(s\right)$ of real financial wealth. Positive wealth is held in deposits, a_t^h, or currency, m_t^h, if the position is positive. If wealth is negative, it is held as loans, l_t^l. The nominal rate on deposits is i_t^a and the nominal rate on loans is i^l. The balance sheet of the household is presented in Appendix A.

Technology. Households can operate their firms with one of two intensities, $u \in \{L, H\}$. The intensity is chosen at every instant. We refer to $u = L$ as the low utilization and $u = H$ as the high utilization intensity. The choice of u determines the production rate, $y\left(u\right)$. The technology with high utilization produces output at a higher rate $y\left(H\right) > y\left(L\right)$. The only reason why a household chooses low utilization is to reduce risk. If $u = H$, the household faces idiosyncratic risk.[12] This idiosyncratic risk is equivalent to $\sigma\left(u\right) dZ_t$ where dZ_t is white noise associated with Brownian motion. Each household faces it's own idiosyncratic risk Z_t,

[10] The model can be easily extended to incorporate a bliss point $\bar{c}$ in consumption. That parameter can be used to generate non-linear expenditure rules as functions of wealth and this is useful to deliver more inequality.

[11] In the extension of section 3, we also imbed a labor supply decision.

[12] This shock can be interpreted as risky output or as demand risk. Demand risk can be introduced easily by assuming that products are heterogeneous and aggregated via an Armington aggregator.

but it controls risk via u. We assume that $\sigma(H) > \sigma(L) = 0$.[13]

Borrowing and Debt Limits. An important feature of the model is that the households cannot borrow without limit. In particular, credit to households is limited by two numbers. The first is a *debt limit* $\bar{s} \leq 0$ which is always the same. The second is a potentially time-varying *borrowing limit* $\tilde{s}_t$. The debt limit states that $s_t \geq \bar{s}$. The borrowing limit is introduced to model a credit crunch in a simple way. In $s \in [\bar{s}, \tilde{s}_t]$, household can increase their debt if $s_t \geq \bar{s}$ but not beyond the accumulation of accrued interests. This means that in $s \in [\bar{s}, \tilde{s}_t]$, banks do not extend the principal of the loan, but they do allow households to role over their debt. Technically, this constraint reads as $ds_t \geq r_t s_t dt$. If we combine the constraint that $s_t \in [\bar{s}, \tilde{s}_t]$ with the household's budget constraint, we obtain:

$$c_t dt \geq dw_t \text{ in } s \in [\bar{s}, \tilde{s}_t].$$

Unless $u_t = L$, the household faces the random shock w_t. Hence, the constraint forces $u_t = L$. Thus, the borrowing limit is equivalent to forcing:

$$u_t = L \text{ and } ds_t \geq r_t s_t dt \text{ in } s \in [\bar{s}, \tilde{s}_t].$$

When we model a credit crunch we study the effects after changes in $\tilde{s}_t$. We discuss why we adopt this constraints before we end this section.

Household's Problem. Real household income is the sum of firm profits, labor income, and net transfers. Households earn $h(u_t, t) = y(u_t) + T_t$. Real lump-sum transfers are denoted by T_t. The stochastic component of real household income is $\sigma(u)dZ_t$. Thus, the stochastic process from real income is $dw_t = h(u,t)\,dt + \sigma(u)dZ_t$. The law of motion of the household's real wealth is:

$$ds_t = \left(r_t(s_t) \left(s_t - \frac{m_t^h}{P_t} \right) - c_t \right) dt + dw_t \text{ where } r_t = \begin{cases} r_t^a \text{ if } s_t > 0 \\ r_t^l \text{ if } s_t \leq 0 \end{cases}.$$

The real rates $\{r_t^a, r_t^l\}$ are defined as $r_t^a \equiv i^a - \dot{P}_t/P_t$ and $r_t^l \equiv i^l - \tau^l - \dot{P}_t/P_t$. Naturally, $\dot{P}_t/P_t$ is the inflation rate. The corresponding Hamilton-Jacobi-Bellman (HJB) equation of the

[13]It is worth saying that this idiosyncratic risk is born by the household and cannot be diversified due to incomplete markets. This induces a Pareto inefficiency when household's chose $u = L$. This follows because Brownian innovations have mean zero. Hence, if agents could diversify this risk, they would want to, and this would create an extra benefit of $y(H) - y(L)$.

household problem is:

Problem 1 *[Household's Problem] The household's value and policy functions are the solutions to:*

$$\rho V(s,t) = \max_{\{c,u,m\}} \; U(c) + V_s' \left(r_t \left(s - \frac{m}{P_t} \right) - \frac{\dot{P}_t}{P_t} m - c + h(u,t) \right) + \frac{1}{2} V_s'' \sigma^2(u) + V_t$$

subject to: $u = L$ *and* $c \in [0, h(u,t)]$ *in* $s \in [\bar{s}, \tilde{s}_t]$.

3.2 Banks

Banks are intermediaries between households with positive (lenders) and negative (borrowers) wealth. There is free entry among banks and banks operate without equity.[14] Banks are price takers. At every t, banks issue nominal deposits a_t^b, nominal loans l_t^b, and maintain reserves balances m_t^b. Let A_t^b, L_t^b, and M_t^b denote the aggregate holdings of deposits, loans and reserves by banks. An individual bank's (nominal) balance sheet is found in Appendix A.

Reserve Positions. The CB sets a reserve requirement coefficient ϱ which is potentially zero. The coefficient indicates he fraction of bank's deposits that must be maintained as reserve balances. A bank violates its reserve requirements, if its reserves are less than the fraction ϱ of its deposits. This balance of reserves is not entirely under the control of a bank. Similar to Bianchi and Bigio (2017a); Piazzesi and Schneider (2018), banks are subject to random payments shocks which does not allow a perfect control of the bank's reserve balances.

We consider that a payment shock occurs within any time interval. In particular, think of a time interval length—to be taken to zero— of size Δ. Between t and $t+\Delta$, a bank receives or loses deposits to other banks. Net deposit flows are settled with reserves. Payment shocks take one of two values $\omega \in \{-\delta, \delta\}$. Each value occurs with equal probability. If $\omega = \delta$, a bank simultaneously receives δa_t deposits and δa_t reserves from other banks. If $\omega = -\delta$, the bank transfers δa_t deposits and δa_t reserves to other banks. Thus, the balance of reserves at $t + \varepsilon$ for the bank that receives the ω shock is:

$$b_{t+\Delta} = m_t^b + \omega a_t - \varrho \left(a_t + \omega a_t \right).$$

Naturally, banks with reserve deficits ($b_{t+\Delta} < 0$) can borrow from banks in surplus ($b_{t+\Delta} > 0$). However, the interbank loans operates in an over-the-counter market that doesn't clear

[14] Adding a bank equity would require and additional state variable. Restrictions such as capital requirements or limited participation would produce bank profits.

perfectly. For that reason, banks cannot borrow their entire deficits from other banks and must resort to the CB's discount window. We study the problem of the bank at the size of time intervals vanishes $\Delta \to 0$. This limit yields some convenient expressions discussed below an maitains the conformity with the rest of the model.[15]

The average benefit (cost) of having and excess (deficit) or reserve balances is summarized by:

$$\chi(b) = \begin{cases} \chi^{-} b & \text{if } b \leq 0 \\ \chi^{+} b & \text{if } b > 0 \end{cases}. \tag{1}$$

The coefficients of this kinked function are $\{\chi^{-}, \chi^{+}\}$. These are endogenous objects whose formulas are presented below. For now, we note that the function follows from an interbank market. Bank profits between t and $t + \Delta$ are:

$$\pi_t^b = \Delta \left(i_t^l l_t^b + i_t^m - i_t^a a_t^b + \mathbb{E}\left[\chi_t\left(b_{t+\varepsilon}\right)|\theta_t\right] \right).$$

Since profits are proportional to Δ, bank policy functions are independent of the interval length—this allows us to take limits of bank policies as $\Delta \to 0$. When we characterize the optimal portfolio choice of a bank, we see how CB policies affect the real spread via its influence on $\dot{\chi}_t$.

The problem of an individual bank is:

Problem 2 *[Bank's Problem] The bank's policy functions are the solutions to the following profit-maximization problem:*

$$\pi_t^b = \max_{\{a,m,l\} \geq \mathbb{R}_+^3} i_t^l l + i_t^m m - i_t^a a + \mathbb{E}\left[\chi_t\left(b\left(a,m\right)\right)|\theta_t\right]$$

subject to $l + m = a$ and

$$b\left(a,m\right) = \begin{cases} m - \varrho a + \left(1-\varrho\right)\delta a \text{ with probability 1/2} \\ m - \varrho a - \left(1-\varrho\right)\delta a \text{ with probability 1/2} \end{cases}.$$

[15] Note that the $b_{t+\varepsilon}$, is a random that we are treating as a stochastic process. If we were to track b_t as a function of time, this stochastic process would not be well defined. This is beause this process would jump discretely, in every instant. However, treating $b_{t+\varepsilon}$ as the single realization of a random variable is a well defined object.

3.3 The Government

The CB's balance sheet is given by:

Assets	Liabilities
L_t^f	M_t
	E_t

.

In this balance sheet, E_t is the net-asset position of the CB. The net-asset position is the monetary base minus L_t^f. The term L_t^f are loans held by the CB. In real terms, the CB's net asset position is $\mathcal{E}_t = E_t/P_t$. This object is critical to describe the Government's intertemporal budget constraint.

Corridor Rates. As a policy instrument, the CB sets a lending rate i_t^{dw} for discount window loans, and the rate on reserves i_t^m. We assume that $i_t^m \leq i_t^{dw}$. The pair $\{i_t^m, i_t^{dw}\}$ are called the corridor rates. The distance $\iota_t = i_t^{dw} - i_t^m$ is the policy corridor.

Open-Market Operations. An open-market operation (OMO) is a simultaneous increase in M_t and L_t^f. There is no distinction between private and public loans. In particular, the Government is allowed to issue debt, which occurs when L_t^f. Whenever $L_t^f < 0$, an increase in L_t^f is interpreted as conventional open-market operation. Instead, when $L_t^f > 0$, an increase L_t^f is an unconventional open-market operation. The accounting entries after OMO's are presented in Appendix A.

Fiscal Transfers. The model admits two forms of nominal fiscal transfers to the public, $P_t T_t$. Fiscal transfers can be financed with the CB's profits or can be entirely unbacked. Unbacked transfers, also called inorganic emissions, occur when the CB issues outside money. Unbacked transfers are: $dM_t - dL_t^f$.

Central Bank Profits and Balance Sheet Evolution. The income flow for the CB is given by:

$$\pi_t^f = i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + \iota_t B_t^-. \tag{2}$$

The first two terms are the interest-rate income and expenses. The CB earns (pays) $i_t^l L_t^f$ on its holdings (issuances) of loans. The CB also pays an interest on reserves i_t^m on the money supply held by reserves—currency does not earn interests on reserves. The third term, $\iota_t B_t^-$ is the income earned at the discount window loans. The fourth term are the fiscal transfers.

The net-asset position evolves according to

$$dE_t = \underbrace{\pi_t^f dt - P_t T_t}_{\text{CB profits - transfers}} = \underbrace{dM_t - dL_t^f}_{\text{inorganic emissions}} .$$

3.4 Markets

Outside Money Market. The CB supplies outside money, M_t. Outside money is held as currency by the public or as reserves by banks. The stock of currency held by the public is:

$$M0_t \equiv \int_0^\infty m_t^h(s) f_t(s) ds.$$

Equilibrium in the outside-money market is:

$$M0_t + M_t^b = M_t. \tag{3}$$

Credit Markets. The credit market has two sides, a deposit and a loans market. In the deposit market, household's save in deposits issued by banks. In the loans market, bank have a claim against households. A distinction between the loans and deposit markets there is an interest rate spread. In particular, i_t^b is instantaneous nominal interest rate charged to borrowers i_t^d is the nominal interest on deposit accounts at banks. All assets/liabilities are claims on currency.

The deposit market satisfies:

$$A_t^b = \int_0^\infty \underbrace{a_t^h(s)}_{P_t s - m_t^h(s)} f_t(s) ds. \tag{4}$$

The left is the supply of deposits and the right is the deposit demand.

The loans market satisfies:

$$L_t^b + L_t^f = \int_{-\infty}^0 l_t^h(s) f_t(s) ds. \tag{5}$$

These quantities define the narrow money aggregates: M_t is the monetary base, $M0_t$ is the currency outstanding and the higher aggregate is $M1_t \equiv A_t^b + M0_t$.

Interbank Market. By t, a bank has a balance b_t of reserves at the Fed. A fraction of those balances, the amount f, are lent (or borrowed) at the interbank market. If the bank is in deficit, $b_t - f_t$ is borrowed from the Fed's discount window i_t^{dw}. The corresponding amounts traded in the interbank market and borrowed from the discount window depend on two probability coefficients $\{\psi_t^+, \psi_t^-\}$. Hence, we have:

$$f = \begin{cases} -\psi^- b & \text{if } b \leq 0 \\ \psi^+ b & \text{if } b > 0 \end{cases} \quad \text{and} \quad b - f = \begin{cases} -(1-\psi^-)b & \text{if } b \leq 0 \\ 0 & \text{if } b > 0 \end{cases}.$$

We define the aggregate deficit and surplus of the system as:

$$B_t^- = -\int b_t \mathbb{I}_{[b>0]} G_t(b) \quad \text{and} \quad B_t^+ = \int b_t \mathbb{I}_{[b>0]} G_t(b).$$

Clearing in the interbank market requires that:

$$\psi_t^- B_t^- = \psi_t^+ B_t^+. \tag{6}$$

We think of the interbank market as an over-the-counter (OTC) market (as in Ashcraft and Duffie, 2007; Afonso and Lagos, 2012). Thus, there are many interbank interest rates. The average interbank rate is an endogenous rate equal to $\bar{\imath}_t^f$.

Given these trading probabilities, the policy rates and the average rate $\bar{\imath}_t^f$, the average rate earned on positive (negative) yields the average coefficients in the cost of funding for banks (1):

$$\chi_t^- = \psi_t^- \bar{\imath}_t^f + (1-\psi_t^-)\iota_t, \text{ and } \chi_t^+ = \psi_t^+ \bar{\imath}_t^f.$$

We adopt the formulation in Bianchi and Bigio (2017b) which presents a an explicit solution for $\left\{\psi_t^+, \psi_t^-, \bar{\imath}_t^f\right\}$. The microfoundation in Bianchi and Bigio (2017b) follows Afonso and Lagos (2012) under special assumptions that yield the analytic expressions we employ below. The idea in the Afonso and Lagos (2012) model is that banks in surplus and deficit trade in sequential trading rounds. During each round, a number of matches between deficit and surplus banks are formed. Upon a match, banks bargain over the rate on interbank loan. The outside option depends on the matching probabilities of the following rounds and the outside options of subsequent rounds. Matching probabilities evolve depending on the evolution of matches. The number of matches depend on the volume of deficit and surplus balances that haven't matched at previous rounds.

Let $\theta_t = B_t^- / B_t^+$ denote the market tightness at time t. The formulas in Bianchi and Bigio (2017b), depend θ_t and two parameters $\{\eta, \lambda\}$. The parameter η is ther bargaining power of lenders and λ captures the efficiency of the OTC market. Given an interbank-market tightness ofθ, we obtain $\bar{\theta}$, a the post-trade tightness. These ratios are related via:

$$\bar{\theta}(\theta) \equiv \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ \left(1 + \left(\theta^{-1} - 1\right)\exp(\lambda)\right)^{-1} & \text{if } \theta < 1 \end{cases}. \tag{7}$$

With this function we obtain the average cost function in (1):

$$\chi^+(\theta, \iota) = \iota \left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta} \left(\frac{\theta^{\eta}\bar{\theta}(\theta)^{1-\eta} - \theta_t}{\bar{\theta}(\theta) - 1}\right) \text{ and } \chi^- = \iota \left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta} \left(\frac{\theta^{\eta}\bar{\theta}(\theta)^{1-\eta} - 1}{\bar{\theta}(\theta) - 1}\right). \tag{8}$$

Naturally, $\chi_t^+ = \chi^+(\theta_t, \iota_t)$, and $\chi_t^- = \chi^-(\theta_t, \iota_t)$. The corresponding formulas for $\{\psi_t^+, \psi_t^-, \bar{\iota}^f\}$ are presented in Appendix B. This object is critical for the FED's ability to affect the real interest rate spread. Figure 11 in Appendix D presents a depiction of the formula (8).

Goods Market. Clearing in the goods market requires:

$$\int_{-\infty}^{\infty} y(u(s,t)) f_t(s)\, ds \equiv Y_t = C_t \equiv \int_{-\infty}^{\infty} c(s,t) f_t(s)\, ds. \tag{9}$$

Evolution of Real Wealth. Let $c(s,t), u(s,t)$ and $m^h(s,t)$ be the solutions to the household's problem. The drift of the household's nominal wealth is

$$\mu(s,t) \equiv r_t(s_t)\left(s_t - \dot{m}_t^h / P_t\right) - c_t + h(u,t).$$

The volatility of household wealth is $\sigma_s^2(s,t) \equiv \sigma^2(u(s,t))$. The the path of the real wealth distribution, $f_t(s)$, is the solution to the following Kolmogorov-Forward equation:

$$\frac{\partial}{\partial t} f_t(s) = -\frac{\partial}{\partial s}\left[\mu(s,t) f_t(s)\right] + \frac{1}{2}\frac{\partial^2}{\partial s^2}\left[\sigma_s^2(s,t) f_t(s)\right]. \tag{10}$$

3.5 Equilibrium Definition

A price path-system is the vector functions $\left\{P(t), i^l(t), i^a(t)\right\} : [0,\infty) \to \mathbb{R}^3_+$. A policy path is the set of functions $\left\{L^f_t, M_t, E_t, i^{dw}_t, i^m_t, T_t, \tau_t\right\} : [0,\infty) \to \mathbb{R}^7_+$. Next, we define an equilibrium path.

Definition 1 *[Perfect Foresight Equilibrium.] Given initial condition for the distribution of household wealth $f_0(s)$, for E_0 and P_0, and a policy path, an equilibrium is (a) a price system, (b) a path of real wealth distribution $f_t(s)$, (c) aggregate bank holdings $\{L^b_t, M^b_t, A^b_t\}_{t\geq 0}$, and (d) household's policy rule and value function, $\{c(s,t), u(s,t), m^h(s,t), V(s,t)\}_{t\geq 0}$:*

1. *the solution to the bank's problem is $\{A^b_t, M^b_t, L^b_t\}_{t\geq 0}$,*
2. *the household's policy rule and value functions solve the household's problem,*
3. *the government's policy path satisfies the governments budget constraint (2)*
4. *the law of motion for $f_t(s)$ is consistent with (10)*
5. *all the asset markets and the goods market clear.*

We characterize some features of the equilibrium dynamics of the model in the next section. A steady state occurs when $\frac{\partial}{\partial t} f_t(s) = 0$ and $\{r^a_t, r^l_t\}$ are constant. An asymptotically stable path is an equilibrium path where $\{r^a_t, f_t(s), r^l_t\}$ asymptotically approaches a steady-state.

3.6 Discussion of Environment Features

Discussion of Financial Technology.

Discussion of Financial Constraints. The financial constraints here have a technical and an economic motivation. The technical reason is that we want to introduce a shock that captures the idea of an unexpected tightening of credit conditions. However, there is no way a household can repay a stock of debt instantaneously if the debt limit is suddenly tightened: income only flows continuously so no household can replay an fraction of the stock of debt instantaneously. This inconsistency does not apply when $\tilde{s}_t$ jumps unexpectedly.

These financial constraints also have an economic motivation: A bank that lends more principal increases the bank's liabilities. A bank that rolls-over debt earns interest income. During financial crises, a bank may allow agents to roll-over their debts, but may not want to extend more principal loans because this expands the banks leverage. Another motivation is that forcing a debt repayment immediately can lead to default. Defaults are costly for banks also because they lead to underwrings that eat their capital buffers. Hence, it may be in the interest of banks and borrowers to roll over debt.[16] Hence, $\{\bar{s}, \tilde{s}_t\}$ allow for this distinction.

Discussion of Financial Arrangements. The institutional arrangements of the model capture some important features of the real world. Here, banks issue deposits for two transactions. One is to purchase currency from households —and automatically transfer currency into reserves at the Central Bank. The other transaction is a swap of liabilities with the public: banks issues deposits in exchange loans. This corresponds to the process of inside money creation which we see in practice. The money multiplier is simply A_t^b/M_t^b. The environment is also explicit about a zero-lower bound on nominal deposit rates.[17]

Discussion of Time-Zero Price as a Parameter. Our definition of equilibrium is non-standard because the time-zero price is a parameter. This is not a usual practice in monetary models. The reason why we chose to anchor the time-zero price is because we don't want to take the initial period literally. Instead, we assume that the economy was running in the past, and the CB had already made a commitment to some price. This approach imposes an additional solvency condition.[18]

4 Analysis

4.1 Analysis: From Nominal to Real Variables

Characterization. To characterize equilibria, we specify implementation conditions. Next, we derive some results that enable us to obtain a set of implementability conditions, such that the CB can target real objects and inflation. Define the *liquidity ratio* by $\Lambda_t \equiv M_t^b/A_t$. The

[16]This phenomenon is called evergreening. We do not model this explicitly, but we are guided by this economic interpretation. Our constraint is consistent with the interpretation. Caballero et al. (2008) discuss evergreening feature in a model of zombie lending.

[17] It is not the usual argument of rulling out the arbitrage where individuals can borrow at the bond rate and deposit in currency. Instead, here, by convention deposits are a claim on currency so they are exchanged at par. If the deposit rate is positive, it will not be in their interest to hold currency. If banks offer a negative deposit rate, households would convert all deposits into currency. At zero interest rates on deposits, banks are indifferent between exchanging deposits for reserves on the margin. Below, describe the economy is affected by this constrain and describe how if the CB charges negative rates on reserves, this induces an increase in spreads.

[18]It is the opposite approach to the fiscal theory of the price level where the time-zero price adjusts to deliver solvency.

coefficients $\{\chi_t^+,\chi_t^-\}$ can be written in terms of Λ_t and ι_t only:

$$\theta_t = \theta\left(\Lambda_t\right) \equiv \frac{\sum_{z\in\{-1,1\}} -\frac{1}{2}\min\left\{\varrho-\Lambda_t-\delta z,0\right\}}{\sum_{z\in\{-1,1\}}\frac{1}{2}\max\left\{\Lambda_t-\varrho+\delta z,0\right\}}.$$

We can also write $\bar{\theta}_t = \bar{\theta}\left(\theta_t\left(\Lambda_t\right)\right)$ so $\{\chi_t^+,\chi_t^-\}$ depends only on Λ_t. The following Proposition follows immediately.

Proposition 1 *[χ rates] In equilibrium χ_t is a function of the policy corridor ι_t and the liquidity ratio Λ_t.*

Figure 12 in Appendix D depicts $\bar{\theta}_t = \bar{\theta}\left(\theta_t\left(\Lambda_t\right)\right)$ so $\{\chi_t^+,\chi_t^-\}$ as functions of Λ_t. An immediate consequence is that nominal loan and deposit rates are only functions of $\{i_t^m,\Lambda_t,\iota_t\}$. In particular we have that:

Proposition 2 *[Nominal Rates] Given $\{i_t^m,\Lambda_t,\iota_t\}$ the equilibrium rates $\{i_t^l,i_t^a\}$ are given by:*

$$i_t^l = i_t^m + \frac{1}{2}\left[\chi^+\left(\Lambda_t,\iota_t\right)+\chi^-\left(\Lambda_t,\iota_t\right)\right] \tag{11}$$

$$i_t^a = i_t^m + \frac{1}{2}\left(1-\varrho\right)\left[\left(1+\delta\right)\chi^+\left(\Lambda_t,\iota_t\right)+\left(1-\delta\right)\chi^-\left(\Lambda_t,\iota_t\right)\right]. \tag{12}$$

In addition, when (11) and (12) banks earn zero profits. Furthermore, the real spread is given by:

$$\Delta r_t \equiv r_t^l - r_t^a = \varrho\frac{\chi^+\left(\Lambda_t,\iota_t\right)+\chi^-\left(\Lambda_t,\iota_t\right)}{2} + \delta\left(1-\varrho\right)\frac{\chi^-\left(\Lambda_t,\iota_t\right)-\chi^+\left(\Lambda_t,\iota_t\right)}{2}. \tag{13}$$

Proposition 2 establishes that both the nominal borrowing and lending rates equal the nominal rate plus a constant. Thus, in either case, the rate on reserves acts like a base rate. The other constant depends on the liquidity ratio and the policy corridor, ι. It is easy to verify that because when $\chi^- > \chi^+$, there is a positive spread $i_t^l > i_t^a$. Any spread in nominal rates is a spread in real rates. This is why we can express the spread in real rates (13) only as a function of the liquidity ratio and the policy corridor. This real spread is an important variable for the evolution of output and the distribution of wealth. The credit channel is how MP affects outcomes through its control over that real spread.

So far we have expressed the real spread as a function of the liquidity ratio and the policy corridor. Since banks earn zero profits, any revenue from the spread must be earned by the CB. Because the real spread only depends $\{\Lambda_t,\iota_t\}$, then we have the following immediate corollary.

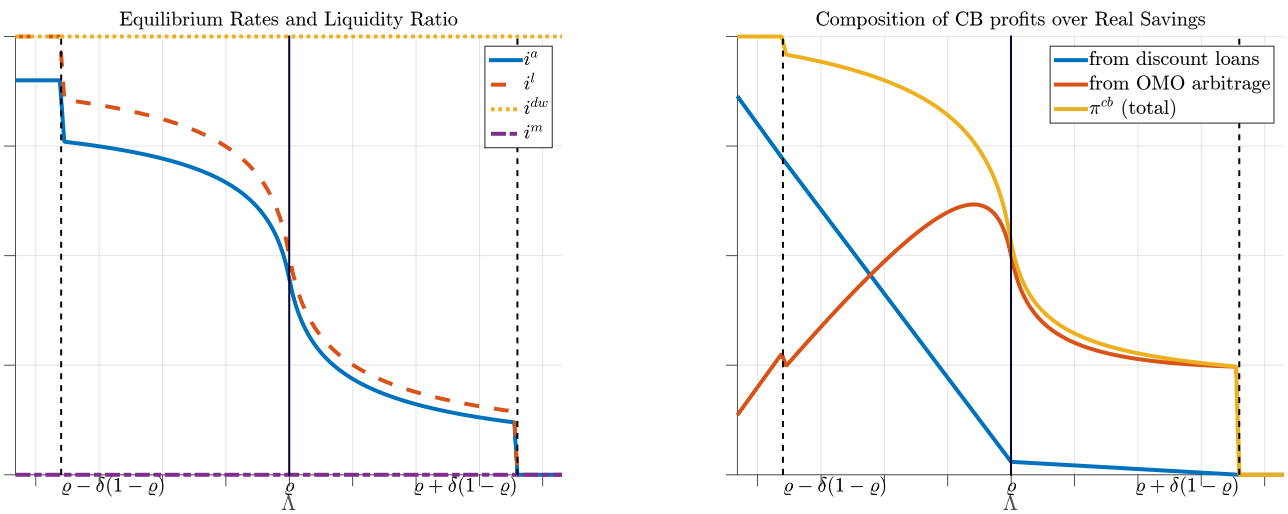

Figure 1: Market Rates and CB profits as Functions of Induced Liquidity

Corollary 1 *[Central Bank Profits] The CB's profit from open-market operations—per deposit—is only a function of ι_t and Λ_t.*

Figure 1 displays the formulas in Propositions 2 and behind Corollary 1. The figure displays two panels. The left panel plots $\{i_t^l, i_t^a\}$ as functions of (11) and (12)—for fixed $\{\iota, i^m\}$. Both rates lie in between i^m and i^{dw}.[19] Both rates feature a spread unless Λ is above a threshold $\varrho + (1-\delta)\,\varrho$. This threshold corresponds to reserve satiation. We can also see that the formulas feature spreads that are decreasing in the liquidity ratio. The right panel shows the components of the CB's profits normalized by the stock of deposits. The profits are decomposed into its components.[20]

A Single Real Asset Market. The next result shows that all market-clearing conditions can be summarized by market clearing of real wealth.

Proposition 3 *[Real Wealth Clearing] Let nominal rates be given by (11) and (12), and let the liquidity*

[19]Credit risk or illiquidity is enough to produce rates above those bands.

[20] The first source are revenues obtained from discount window loans. As CB provides more liquidity, its discount-window profit per unit deposit declines. The second source of revenues is the arbitrage from open-market operations. This source produces the typical Laffer curve that emerges in monetary models. For a fixed amount of deposits, the higher the liquidity ratio, the CB exploits an arbitrage between the loans and the rate on reserves. The larger the open-market operations, the greater the arbitrage. However, as the CB provides more liquidity, the spread $i_t^l - i_t^m$ drops, which explains decreasing profit part of the plot. This is different from the typical Laffer curve that follows from seigniorage. With a decreasing demand for real balances in inflation, monetary models feature a Laffer curve because more inflation provides more marginal revenues. However, an opposite force emerges from reducing the value of real balances.

ratio be given by Λ_t. Then, market-clearing in real terms

$$-\int_{-\infty}^{0} s f_t(s)\, ds = \int_{0}^{\infty} s f_t(s)\, ds + \mathcal{E}_t \tag{14}$$

implies market clearing in all asset markets. Furthermore, when (14) and the Kolmogorov-Forward equation (10) holds, then, the goods market clearing condition (9) holds.

The proposition shows how all nominal asset markets are summarized by one real market, either the real rate on deposits r_t^a or the real rate on loans r_t^l. What this means is that once the real spread is determined by MP, clearing requires just to solve for that real interest rate. Hidden in this proposition is a a continuous-time version of Walras's law, which guarantees that if (14) holds, the goods market also clears. Next, we express the law of motion of $\mathcal{E}_t$ in real terms:

Proposition 4 *[Real Budget Constraint] In equilibrium, $\mathcal{E}_t$ follows from:*

$$d\mathcal{E}_t = \left(\left(r_t^a + \Delta r_t\right) \mathcal{E}_t + \Delta r_t \int_0^{\infty} s f_t(s)\, ds - T_t \right) dt, \tag{15}$$

with $\mathcal{E}_0$ given.

Notice that the CB earns all the profits from intermediation. The reason is that if the CB is able to affect real or nominal spreads in the economy, some agent in the economy must earn the spread between borrowing and lending rates. Here, banks are in perfect competition. Hence, the spread must be earned by the CG.

Solvency Constraint. The CG is not allowed to choose any policy. Here, the CB must satisfy a long-run solvency constraint. At the limit $lim_{t\to\infty}\mathcal{E}_t \geq \underline{\mathcal{E}}$, for some minimum $\underline{\mathcal{E}}$ that guarantees that the government can raise enough revenues and satisfy $d\underline{\mathcal{E}} = 0$. This condition is equivalent to assuming that the the CB's liabilities are not worth zero in equilibrium, that is, that the equilibrium is indeed monetary. Although we don't solve for $\underline{\mathcal{E}}$, in all of the equilibria we study, the policy path converges to some stable government asset position: $lim_{t\to\infty} d\mathcal{E}_t = 0$.

4.2 Analysis: Implementation

So far, we expressed a interest rate spread as a function of $\{\iota, \Lambda\}$. Given a distribution of real wealth and a real spread, the real rate will be given by (14) will. Next, we explain how the CB can indeed control the control real rates by either carrying out OMO or.

Implementation of a Real Rates Target. From equation (13), the real spread Δr_t depends on $\{\Lambda_t, \iota_t\}$. The policy corridor ι is chosen by the CB. In turn, the liquidity ratio Λ_t can be expressed in real terms:

$$\Lambda_t = \frac{M_t}{A_t} = \frac{\left(E_t + L_t^f\right)/P_t}{A_t/P_t} = \frac{\mathcal{E}_t + \mathcal{L}_t^f}{\int_0^\infty s f_t\left(s\right) ds} \equiv \Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right).$$

In this expression, $\mathcal{L}_t^f$ is the real value of CB assets. Since the evolution of the CB's real assets $\mathcal{E}_t$—equation (15)—and the evolution of $f_t\left(s\right)$—equation (10)—do not depend on $\mathcal{L}_t^f$, the evolution of $\{f_t, \mathcal{E}_t\}$ depends on $\mathcal{L}_t^f$ only through the effects on Δr_t. This means that the CB can execute OMO to set $\mathcal{L}_t^f$ to control Λ_t directly. The effect of this operation produces simultaneous change in Δr_t. This spread affects the evolution of $\{f_t, \mathcal{E}_t\}$ slowly. Hence, the real rate r_t^a will move slowly.

Implementation of an Inflation Target. From the definition of real rates, we have that:

$$\frac{\dot{P}_t}{P_t} = = i_t^a - r_t^a = i_t^l - \Delta r_t - r_t^a = i_t^m + \frac{1}{2}\left[\chi^+\left(\Lambda_t, \iota_t\right) + \chi^-\left(\Lambda_t, \iota_t\right)\right] - \Delta r_t - r_t^a. \tag{16}$$

The last term in the expression shows that inflation is linear in i_t^m given $\left\{\iota_t, \mathcal{L}_t^f\right\}$. Since Δr_t is independent of i_t^m, the CB can target a real spread—via a choice of $\{\Lambda_t, \iota_t\}$— and move inflation with i_t^m. This is independent of the real rate:

Policy Lesson (i). The CB can simultaneously target a real spread and inflation.

Summary Result. The CB's policy must be consistent with private agent decisions, the law of motion of real wealth, and the law of motion of $\mathcal{E}_t$. Formally, the implementability conditions are given by:

Proposition 5 *[Implementation Conditions] The equilibrium path is characterized by a path for real variables $\{r_t^a, \Delta r_t, \mathcal{E}_t, f_t, T_t\}$. To implement a desired equilibrium path, the CB chooses $\left\{\iota_t, \mathcal{L}_t^f, T_t, \tau_t\right\}$. The real variables satisfy asset clearing condition (14), the Kolmogorov-Forward equation (10), and (15) and*

$$\begin{aligned}\Delta r_t &= \varrho\frac{\chi^+\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right) + \chi^-\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right)}{2} \\ &\quad + \delta\left(1-\varrho\right)\frac{\chi^-\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right) - \chi^+\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right)}{2}.\end{aligned}$$

To determine inflation the CB chooses i_t^m

$$\dot{P}_t/P_t \;=\; i_t^m + \frac{1}{2}\left[\chi^+\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right) + \chi^-\left(\Lambda\left(\mathcal{E}_t, f_t, \mathcal{L}_t^f\right), \iota_t\right)\right] - \Delta r_t - r_t^a.$$

Proposition 5 states that the CB has to satisfy a certain constraints. . If the CB holds i_t^m constant, and carries open-market operations or moves the policy corridor, it will simultaneously affect affect inflation and output. In the following section, we explain how different policy tools relate to different transmission channels. Next, we explain how the CB faces two additional implementation constraints. Appendix C presents an algorithm that builds on this proposition that we use to solve the model.

4.3 Implementation Restriction 1: Reserve Satiation

Consider a policy where the CB satiates banks with reserves. Satiation occurs when every bank has enough reserves to meet the reserve requirement. This occurs when the liquidity ratio satisfies $\Lambda_t \geq \bar{\Lambda} = \varrho + (1+\delta)$. Under satiation, we have the following results.

Proposition 6 *Let there be some t such that the economy is satiated with reserves. Let $i_t^m \geq 0$, then, $i_t^m = i_t^d = i_t^l = \bar{i}^f$. Furthermore, $\Delta r_t = 0$ and $\dot{P}_t/P_t = i_t^m - r_t^a$.*

4.4 Implementation Restriction 2: Deposit Zero-Lower Bound

FIX DEFINITION OF Λ. In this paper, households can convert deposits into currency. We assume that banks can't. We assume that banks can't hold currency wither by regulation, taxation or physical costs. Hence, $i_t^m < 0$ is possible. Thus, our model allows $i_t^m < 0$ but $i_t^d = 0$. Under this assumption, a zero-lower bound is characterized by:

Proposition 7 *Fix a $\{i_t^m, \Lambda_t\}$ at any time instant. If $\Lambda_t \geq \Lambda_t^{zlb}$ where Λ_t^{zlb} solves*

$$0 = i_t^m + \frac{1}{2}(1-\varrho)\left[(1-\delta)\chi^+\left(\Lambda_t^{zlb}, \iota_t\right) + (1+\delta)\chi^-\left(\Lambda_t^{zlb}, \iota_t\right)\right], \tag{17}$$

then, households hold currency given by $M0_t = M_t - \Lambda_t^{zlb} P_t \int_0^\infty s f_t(s)\, ds > 0$. The real spread then is given by

$$\Delta r_t = \varrho\frac{\chi^+\left(\Lambda_t^{zlb}, \iota_t\right) + \chi^-\left(\Lambda_t^{zlb}, \iota_t\right)}{2} + \delta(1-\varrho)\frac{\chi^-\left(\Lambda_t^{zlb}, \iota_t\right) - \chi^+\left(\Lambda_t^{zlb}, \iota_t\right)}{2}.$$

Furthermore, $\partial \Delta r_t / \partial i_t^m < 0$.

Our model is explicit about the zero-lower bound. Proposition 7 describes how if the CB attempts to lower the deposit rate below zero, by setting $i_t^m \leq 0$, the public will react by drawing currency. This happens if the liquidity ratio is too high, i.e., $\Lambda_t \geq \Lambda_t^{zlb}$. This result is different from the ZLB that emerges in models with cash-in advance constraints. There, a ZLB emerges if the CB floods the public with savings instruments so that the asset clears at negative rates. This opens the door to an arbitrage. An interesting feature of the model is that at the zero-lower bound, lowering rates on reserves increases the real interest spread. The reason is that negative rates on reserves act like a tax on deposit holdings. Thus, banks require a higher lending rate to compensate them. Ours is one of the first models that are explicit about currency holding at the zero-lower bound and the first to articulate this force.[21] Appendix C presents how the solution algorithm has to be adapted to admit currency holdings at the zero-lower bound. We conclude with:

Policy Lesson (ii). The CB can simultaneously target a real spread and inflation.

4.5 Discussion of Results

Alternative Implementations.

An alternative to the control of the real spread directly through OMO is to target an interbank-market rate $\bar{i}^f$. Observe that $\bar{i}^f = i_t^m + \chi^+(\Lambda, \iota) / \psi^+(\theta(\Lambda))$. For a given ι, we can obtain a value of Λ consistent with a target $\bar{i}^f$. Hence, there is a map from a policy target $\bar{i}^f$ to a real spread Δr_t. In practice, central banks are explicit about a nominal interbank target. The interpretation of our model is that this affects the real spread too.

Implementing an Inflation Target with a Policy Corridor. In practice (Bindseil, 2014, see for example), CB's adopt either a floor systems or corridor systems. These are different way to implement a monetary target. Corridor systems usually fix a policy corridor ι. When we explore policy experiments we work with a fix ι. In that case, the CB can affect real spreads moving Λ_t via (13) and inflation moving i_t^m given (16). The US currently runs a corridor system.

A floor system imposes the restriction that $i_t^m = 0$ and the CB changes $i^{dw} = \iota$. That was the operating framework in the US prior to the Great Recession. Under a floor system, the CB

[21] In Rognlie (2016), negative rates are possible because there are costs of holding physical currency. He also models currency holdings. If banks could transform reserves into cash at zero cost, then $i_t^m \geq 0$. However, if banks have a physical holding cost as in Rognlie (2016), the increase in lending rates would follow.

	Instrument				Channel		
Doctrine	i_t^m	T_t	ι_t	L_t^f	Fisherian	Non-Ricardian	Credit
Interest Target (sec 5.1)	✓				✓		
Fiscal Transfers (sec 5.2)		✓				✓	
Credit Target (sec 5.3)		✓	✓	✓		✓	✓

Table 1: Instruments and Transmission Channels

can simultaneously control inflation and a real spread—it can use equations (13) and (16) and moves$\{\Lambda_t, \iota_t\}$ to obtain a given target.[22] In summary, we have an important policy lesson:

5 From Instruments to Transmission Channels

A monetary doctrine is a set of MP rules designed to achieve a result. In this section we study three doctrines. First, we describe an doctrine where MP eliminates any spread. MP can target inflation directly through its control over the nominal interest rate. This doctrine relates to three transmission mechanisms articulated by the literature. The second doctrine are unbacked fiscal transfers. This related the model to standard exercises about money injections. That section is important also to understand the credit channel. When the credit-channel is activated, the CB generates revenues that must be distributed at some point. The final section studies MP when it targets the level of credit. Table 1 presents a summary of the instruments used under each doctrine and the effects that these generate.

5.1 Interest Targets and the Fisherian Channels

In this section, we present policy doctrine where MP neutralizes the effect on real spreads and its fiscal consequences. This doctrine is a useful benchmark because it will show how the CB can control inflation through its choice of i^m, but non the less induces neutrality. We call this benchmark a Wicksellian doctrine following Woodford (1998, 2001) belief that MP should be conducted without reference to outside and setting $\iota = 0$. We have the following:

Proposition 8 *Consider a policy path such that $E_t = T_t = 0$, all t. Let the CB either:*

(a) *satiate banks with reserves, $\Lambda_t > \bar{\Lambda} = \varrho + (1+\delta)$, or*

(b) *eliminate the corridor spread, $\iota_t = 0$,*

[22] If a system sets a fixes the distance between $i^{dw} - \bar{i}^f$ and works with floor system, it looses the ability to control inflation and spreads simulataneously. That was the policy regime in the US prior to the Great Recession.

at all t, then an equilibrium features:

1. ***(No Spread)*** $\Delta r_t = 0$

2. ***(Neutrality)*** *The evolution of* $\{r^a_t, f_t(s)\}$ *is unaffected by policy.*

3. ***(Fisherian Transmission)*** *Inflation is controlled by* i^m_t *through the Fisher equation (16).*

The outcomes Proposition 8 holds under possible policy regimes. In each regime, the CB eliminates all the frictions from the interbank market, either by directing any distortion through an intermediation tax, or by satiating banks with reserves, or by eliminating the policy corridor. There are three relevant outcomes. First, the CB bank eliminates the credit spread. Second, because the CB eliminates credit spreads, the real interest in the economy coincides with the non-monetary version of the model. The third outcome is that, as in Woodford (1998), the CB can control inflation directly by setting i^m_t and the appropriate scale of open market operations.[23] In essence, under this doctrine, CB simply controls the unit of account. We discuss how this result relates to other transmission mechanisms and study a credit crunch under this Wicksellian doctrine.

The Fisherian Channels. Let's discuss where classic transmission mechanisms fit in the model. In New-Keynesian models, the transmission mechanism is the real interest-rate channel. Because of nominal rigidities, the CB can generate changes in relatives prices that change real output. The equilibrium real rate follows from the household's Euler equations. We could add nominal rigidities to our environment, without problem. This featured would lead to a version of the Kaplan et al. (2016) model. Proposition 8 is however, a microfoundation for the cash-less limit of new-Keynesian models that relates to the implementation of monetary policy through banks. Whether it is desirable to eliminate all the interbank market frictions is controversial.

In new-Monetarist models, e.g. Lagos and Wright (2005), real rates are fixed discount factions. Inflation follows directly from a choice of nominal rates. Those models feature a transmission mechanism that absent here: in those models inflation acts like tax on transactions because since transactions must be carried out in currency. If deposits could be used for transactions, there would be no costs from inflation.

In our model, debt is paid instantaneously. With long-term debt, changes in policy rates i^m_t would not be neutral, as long as these are unexpected (see for example Gomes et al., 2016).

[23] In the Proposition, we set $E_t = 0$ for the sake of exposition, but the result holds with little loss in generality. Under condition (c), if the CB eliminates the corridor system, the CB can control inflation even if it issues zero reserve balances.

The reason is that any surprise in the price level alters the real distribution of debt. Our model can be adapted along those lines to allow for that channel.

5.1.1 Application: Credit Crunch

We now study a tightening of the borrowing limit. Because this happens under the Wicksellian doctrine, the effect captures the purely real effects. This also will help us understand the results under other doctrines. We introduce a temporal in $\tilde{s}_t$ that occurs at $t = 15$, but is anticipated at time 0 (credit crunch). The wealth distribution is initiated at steady state. We present the effects of anticipated shocks because as the dynamics after the shock date are similar to the dynamics that follow if the shock is unanticipated. Thus, anticipated shocks allow us to illustrate the main forces in anticipation of the shock and after the arrival of an unanticipated shock.

Figure 2 shows the dynamics of the real-side of the economy during the experiment. The effects can be divided into ex-ante and ex-post effects. When the credit crunch is expected, borrowers want to avoid a wealth position that falls above the borrowing limit. If they do, borrowers will have to adopt the safe, but unproductive technology. In preparation to that event, borrowers increase their desire to save away from the constraint. Naturally, if borrowers increase their savings, savers must reduce their savings along the transition. This leads to the compression in the distribution of wealth that appears in Panel (a). Panel (b) shows how both real deposits and loans fall along the transition. In equilibrium, real interest rates must fall, to discourage savers from savings. The threat of falling in the borrowing-constrained region induces borrowers to save, despite the low interest rate regime. Rates fall gradually reaching a trough when the shock arrives. Panel (c) shows the path of real rates. In the ex-ante phase, output actually expands (Panel (d)). The reason is the desire to avoid the borrowing constrained region during the crunch, leads to a lower mass of households at the constrained region, prior to the crunch. This allows those households to produce more efficiently soon after they abandon the region.

Upon the credit crunch, a mass of households is found in the borrowing constrained region, $s_t \in [\bar{s}, \tilde{s}_t]$. This can be seen through the mass concentration at the borrowing constrained region. This forces those households to switch to the safe technology. The consequence an immediate output collapse. Output continuously falls during crunch, as more and more households are dragged into the borrowing constrained region. The expectation of a recovery produces an increasing path of real interest rates—consumption smoothing—but as the crunch vanishes, interest rates jump back to accommodate the increased demand for credit. The evolution of the credit volume is interesting: at an initial phase, credit continues to decrease during the crunch. However, as the recovery is expect, credit begins to expand as borrowers wish to to smooth consumption—borrowers are more sensitive to interest rates than savers.

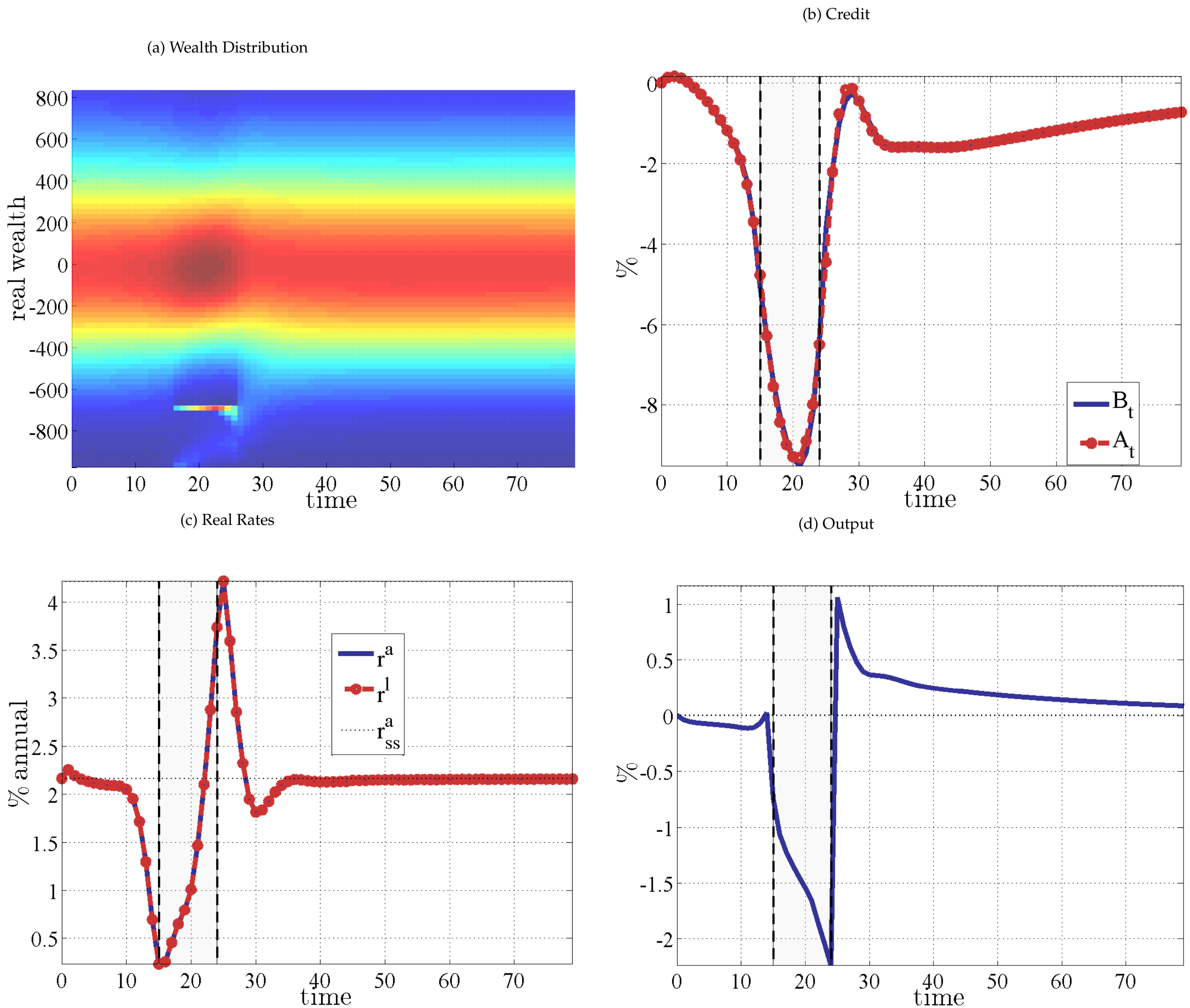

Figure 2: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

5.1.2 Taylor Rule and *Persistent* Deflation after *a Temporary* Credit Crunch

Next, we an apparently paradoxical in light of the new-Keynesian model. Our model predicts that running a Taylor rule after a *temporary* credit-crunch leads to *permanent* deflation.[24] For this section, we assume that the CB runs the following rule:

$$i_t^m = 1.5\frac{\dot{P}_t}{P_t} + 0.5\left(Y_t - Y_{ss}\right).$$

We also adopt a policy where the CB maintains a constant money supply M_t and an open corridor as in case (a) of Proposition 8. Figure 3, Panel (a) presents a decomposition of Fisher's equation. The real side of the economy is still that displayed in Figure 2, so it features a full recovery. Although the money supply is kept fixed, and the economy featured zero steady-state inflation prior to the shock, the period after the crunch, the economy enters a path of steady-state deflation.

The components of the Fisher equation explain this effect: as we learned Figure 2, the credit crunch is associated with a period of low real interest rates that eventually revert. During the beginning of the crunch, the Taylor rule is consistent with inflation. However, as real rates revert to above steady state values, the economy begins a deflationary path. The reason is that the Taylor rule, sets low nominal rates when lagged inflation is low. Since the reversal of real rates is rapid, it enters a trap where nominal rates are kept below real rates.

An interesting aspect of the model is what happens with the monetary quantities. As noted, during the credit crunch, nominal deposits shrink. Once there is deflation a certain number of periods, the price level will accumulate a decline for certain periods. Then, when real credit is normalized and the real value of deposits increases, nominal deposits should be lower than their pre-crisis level. Therefore, the liquidity ratio remains permanently low, leading to permanently low level of nominal rates. Thus, the policy leads to a new stead-state level of interest rates. This result, although in the context of a model that features monetary neutrality, is already important for its policy lessons. It stresses the importance of the implementation of monetary policy.[25]

[24] This is a theoretical possibility that has been discussed informally by John Cochrane and Stephen Williamson which is labeled a Neo-Fisherian effect. ***Add blog not.***

[25] Although this is a force not present in the model so far, if the economy were to feature debt-deflation, the CB's policy may actually worsen outcomes. Policy makers following the Taylor rule, and the advice of the New-Keynesian model may be permanently trapped in a low inflation environment. This is a potential policy trapped discussed in Williamson (2012), that our model is capable of rationalizing.

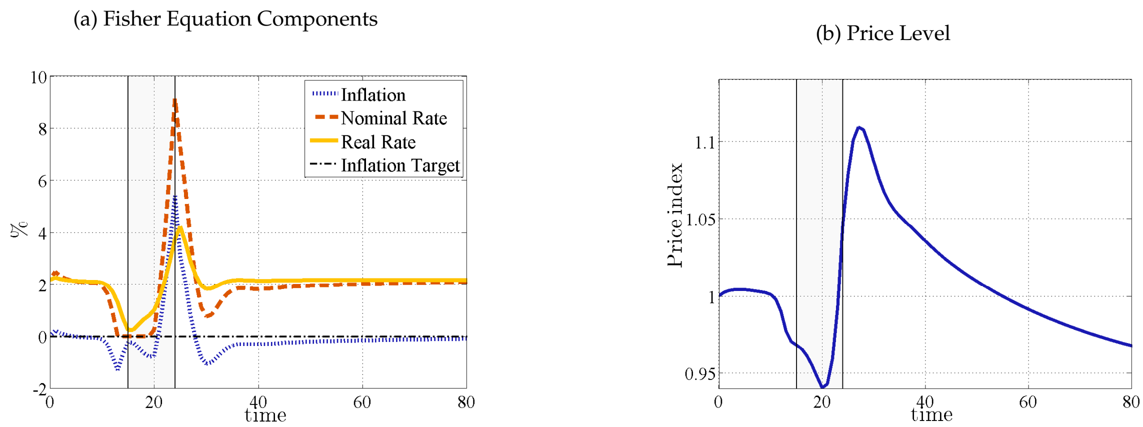

Figure 3: Effects of a Taylor Rule after a Credit Crunch.

Policy Lesson (iii). A Taylor rule can lead to persistent deflation after a credit crunch.

5.2 Fiscal Transfers and the Non-Ricardian Channel

We now study the effects of fiscal transfers. There are two reasons to study this policy. First, although we are ultimately interested in the credit channel, the credit channel necessarily produce a fiscal transfer because an increase in credit spreads produces fiscal revenues. Hence, analyzing the effects of fiscal transefers is an essential part of the credit channel. Second, when the zero-lower bound is reached, fiscal transefers are the last remaining tool available in the CB's arsenal. A final advantage is that fiscal transfers allow a connection with monetary-fiscal theories (Ljungqvist and Sargent, 2012, , chapter 26).

For now, we maintain the assumption that the CB provides a loan subsidy that neutralizes the effects on spreads, but drop the assumption that $\{E_t, T_t\} = 0$.

5.2.1 Expected vs. Unexpected Transfers

In this experiment we maintain a zero policy corridor spread, keep the rate on reserves constant and study fiscal transfers. The entire policy experiment begins at period 15 the CB begins a transfer program that lasts 10 periods. Each period, the FA transfers xxx% of GDP in currency, as shown in Figure . The policy is announced at time 0. Prior to the operation, $\{E_t, T_t\} = 0$. After the transfer, by period 25, the policy is slowly reverted. The policy variables are shown in Figure 4. Figure 5 plots the macroeconomic responses.[26]

[26] As explained in section xxx, the operation is carried out with the FA increasing it's debt, the CB increasing reserves and holding public debt.

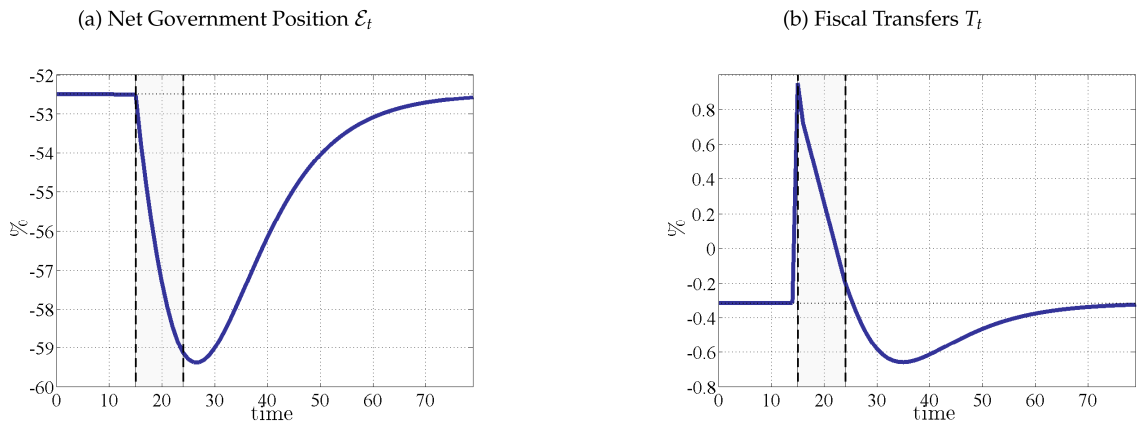

Figure 4: Fiscal Transfer Policy.

In the expectation of the shock, we observe an increase in the common real interest (Panel (c)), a growth in credit (Panel (b)) and a decline in GDP (Panel (d)). The reason for this pattern is because fiscal transfers reduce the precautionary behavior by borrowers. The reason is that in a world without transfers, reaching a borrowing limit is not only a static state. Leaving the borrowing limit is very difficult do to the increase in marginal utility of consumption. If borrowers expect a transfer, they are aware that if they are unlucky to follow into a borrowing limit, the fiscal transfer will push them away from that constraint. As a result there is a relative increase in the demand for debt. This results in an increase in real rates. Although the policy induces more credit, it is recessionary in the ex-ante phase. The reason is that because the reduction in precautionary behavior leads more agents to the borrowing limit (see the wealth distribution in Panel (a)). In that state, they switch technology and output falls.

Once the policy is implemented, we observe a decline in real rates, a decrease in private borrowing, and an expansion of production. The fiscal transfer pushes a mass of borrowers away from the constrained region. Real rates suddenly increase despite a reduction in the volume of loans. The reason is the expansion of output. Because the precautionary is eased by the policy which pushes borrowers away from their constraints, output expands. The transfer increases the volume of deposits: borrowers use the transfer to clear their debts. Savers exchange their transfers for deposits. As a result, banks increase their reserve balances.

By period 25, the FA gradually reverses the transfers and smoothly reduces the stock of Government Debt back to steady state. Real rates then return to steady state, Output and credit variables mirror the behavior of the expansion phase. The policy is clearly non-Ricardian because the market incompleteness.

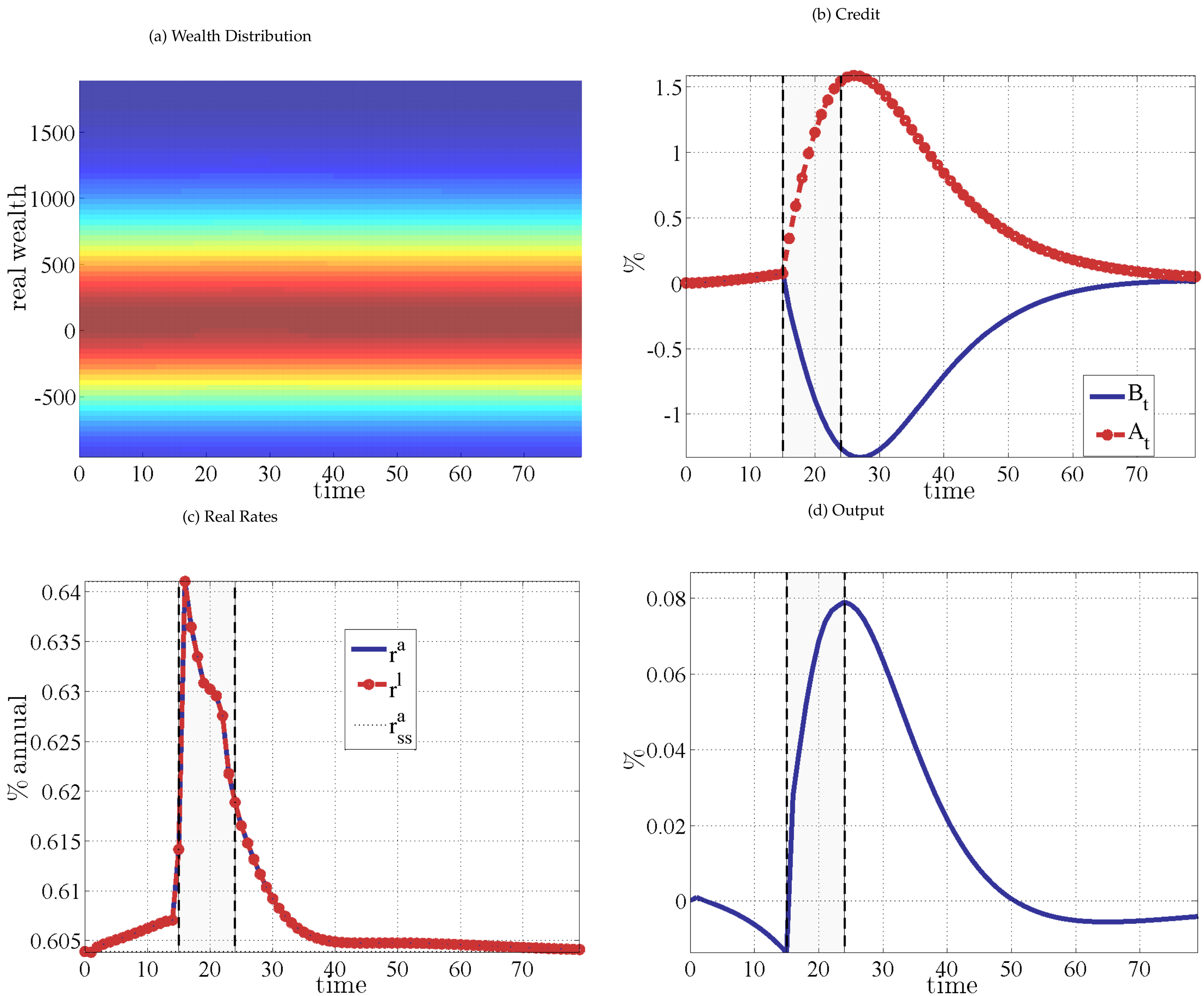

Figure 5: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

Connection to Monetary-Fiscal Theories. Before we proceed to the credit channel, we want to establish contact with classic monetary issues. The model inherits classical properties of a textbook analysis of the effects of deficits financed with issuances of outside money in Bewley economies (Bewley (1983); Ljungqvist and Sargent (2012, , Chapter 18.11)). For example, assume a policy that from a certain period t, consistently increases fiscal transfers from then on. For simplicity, assume that the initial inflation is zero. In the long-run, inflation will increase at a constant rate. If the CB keeps nominal i_t^m constant, the real rate will drop precisely to the point where it adjusts to the increase in inflation. As a result, the distribution of real wealth changes.[27]

There is sense in which the quantity theory holds. As long as the path for the real net asset position of the CG, $\mathcal{E}_t$, is given a new equilibrium can be found by scaling every nominal variable: prices and the nominal quantity of money by any multiple. This observation, however, does not mean that that an increase in fiscal transfers is neutral. This policy can lead to an entirely new steady state, with a new distribution of real wealth.

5.3 Credit Spread Target and the Credit Channel

In the previous two sections we discussed two policies. We are now ready to present the effects of monetary policy through the credit channel.

5.3.1 Activating the Credit-Channel

We now consider a policy where the CB targets a reduction in real rates. As in all examples, the policy is announced at time 0, but is effective for 10 periods beginning with period 15. As explained earlier, the policy has both the effect of a reduction in credit spreads and alters the profits of the CB. Because the CB earns profits, the FA distributes them as lump-sum taxes. In the previous examples, we showed how these effects where small. The policy targets a positive spread in steady state of 250bps. By period fifteen, open-market operations are carried out and these reduce the credit spread to 100bps. As before, we proceed with a description of ex-ante and ex-post effects.

Ex-ante effects. The macroeconomic policy effects are shown in Figure 6. The effects of an expected reduction in credit spreads stimulate borrowers. The reason is that, as with expected transfers, the precautionary motive of borrowers is eased. This produces an increase in the

[27] For this example to work, we need to abandon the assumption that p_0 is given. Instead, we need the initial price level to adjust.

demand for loans relative to the supply of deposits. However, since savers are more elastic to real rates, the real rate increases in response to the expected decline in spreads. We can see that credit expands faster than savings, and this has to do with a reduction in the CG net asset position. Because agents expect an the easing of credit spreads, they reach their borrowing limits faster, something that produces a decline output.

This policy is achieved thanks to a large scale open-market operation (panel (a) in Figure 7). Notice that since the policy assumes fixed nominal interest-rate on reserves, the policy is deflationary given the increase in real rates—the increase in the liquidity ratio reduces both nominal rates. This leads to a deflationary episode. Note that the credit channel operates throughout the supply-side as it leads to a direct effect over bank's balances. An ex-ante deflation is observed in panel (b) of Figure 7.

Ex-post phase. Immediately after the policy is implemented, the reduction in the real spread pushes a mass of borrowers away from the constrained region and starts a phase of output recovery. The expansion in credit continues during the period of low spreads, but slowly begins to revert. This follows because the period of low spreads is also known to end. This aspect is also shown in the declining path of real rates.

The easing of credit spreads allows a large mass of agents to leave the borrowing-constrained region. This produces an expansion that eventually overcomes the prior reduction in the level of output. As the policy is reverted, real rates are normalized and the distribution of real wealth stabilizes. The economy converges to a one where the price level is lower than before, although reserves in real terms are back to steady state.

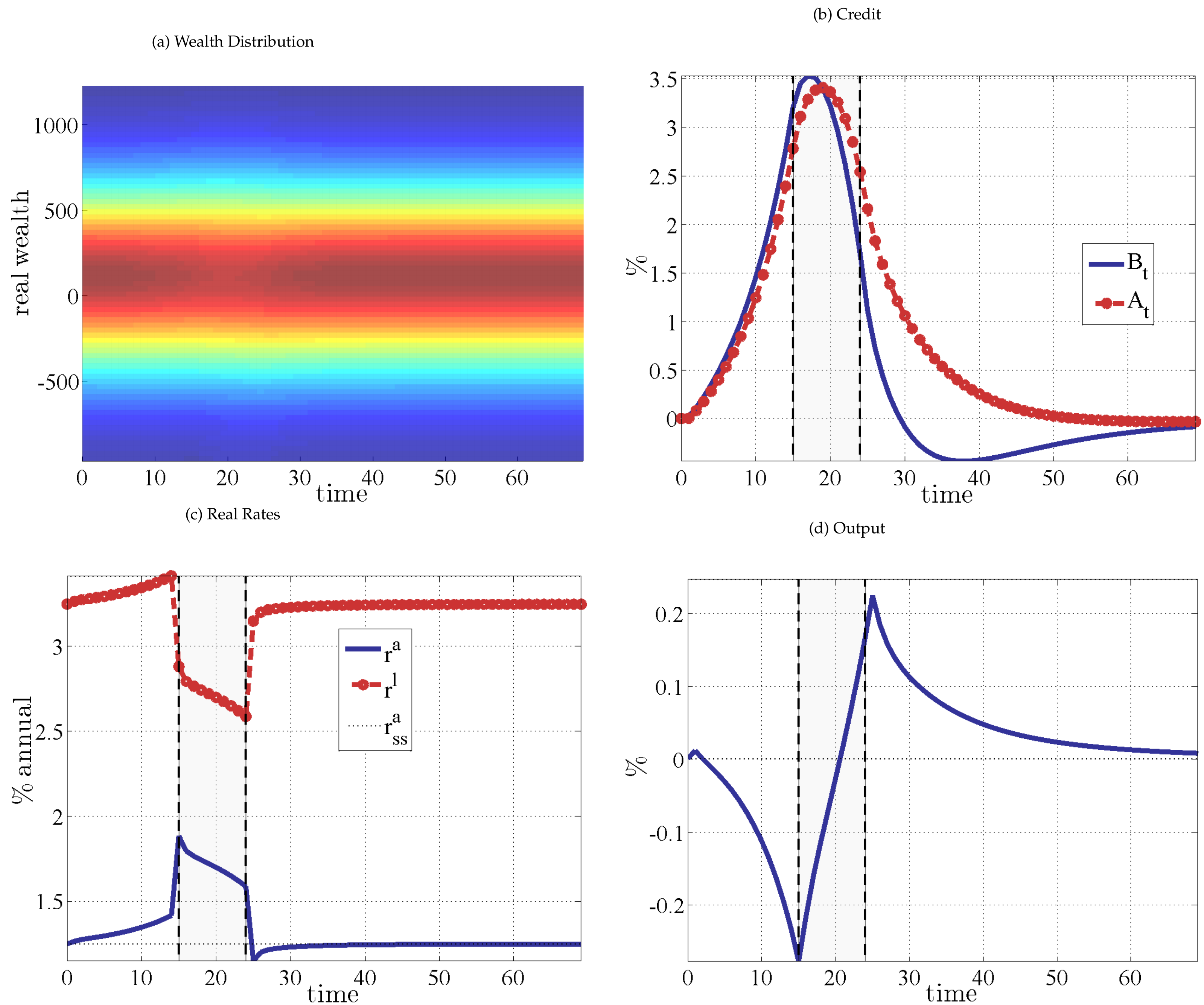

Figure 6: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

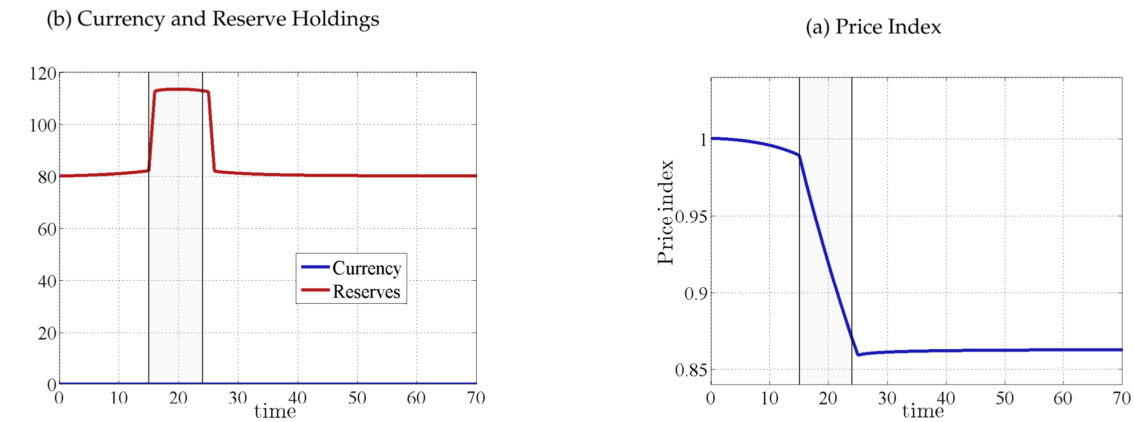

Figure 7: Components of (Outside) Money Supply and Inflation.

5.3.2 Discussion: Does it matter how we implement the credit channel?

We discussed how an implementation of the credit channel with open-market operations. In the model, the same effects can be obtained by moving ι. In practice, these instruments can have different fiscal consequences.

6 A Credit Crunch and the Zero-Lower Bound

6.1 Credit Crunch and Deposit Zero-Lower Bound

We now study the effects of a credit crunch in conjunction with an aggressive open-market operation. The policy is aggressive enough to lead the economy to a zero-lower bound on deposit rates. The macroeconomic patterns are described in Figure 8. By time 15, we observe a a credit crunch. Prior to the crunch, the patter follows a similar path to the one that occurs in absence of a policy reaction: output expands as the volume of credit drops, something that shows in a partial decline in real rates. When the shock is realized, monetary policy reacts with an aggressive program of open-market operations. This expansion is observed in Figure xxx. The result is a reduction in credit spreads during the crunch. The presence of the zero-lower bound is reflected in the increase in cash holdings by households. The spread is still active because the policy is conducted with a reduction in the interest in reserves. An even stronger expansion leads only to an increase in currency.

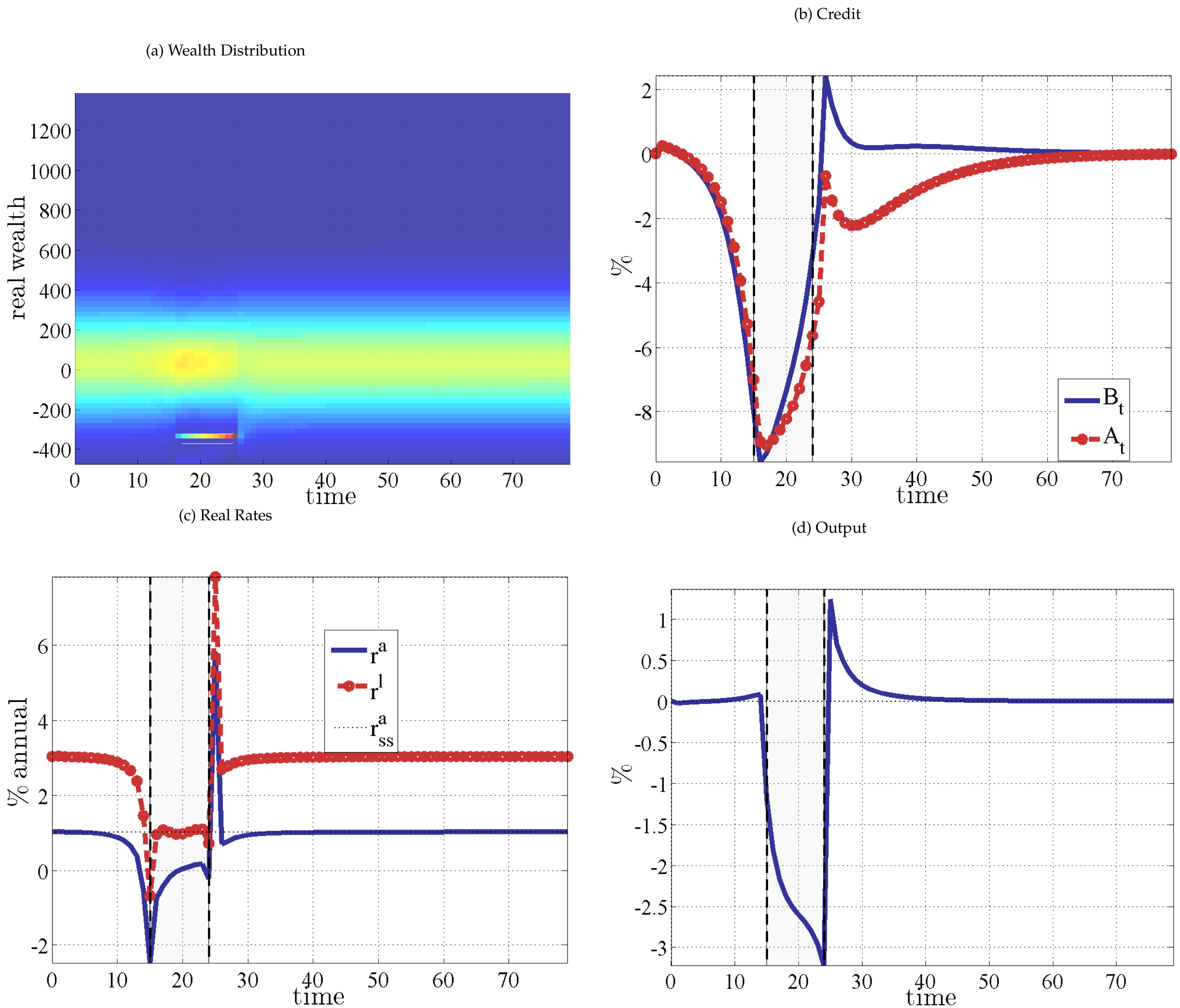

Figure 8: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

Discussion: Negative Rate on Reserves. In this model, negative rate on reserves, are detrimental. A negative rate on reserves will be the prescription in a model with nominal rigidities as means to containing deflation. In this model, it is a policy mistake because it leads to an aggravation of credit spreads. Cite in proposition and Brunnermeier Coby. To see this, Figure 9, performs the same experiment as before, but in conjunction with the large scale operation, the policy is conducted together with reduction in the rate on reserves. As we can observe, the policy leads to an aggravation of credit spreads, as anticipated by Proposition xxx.

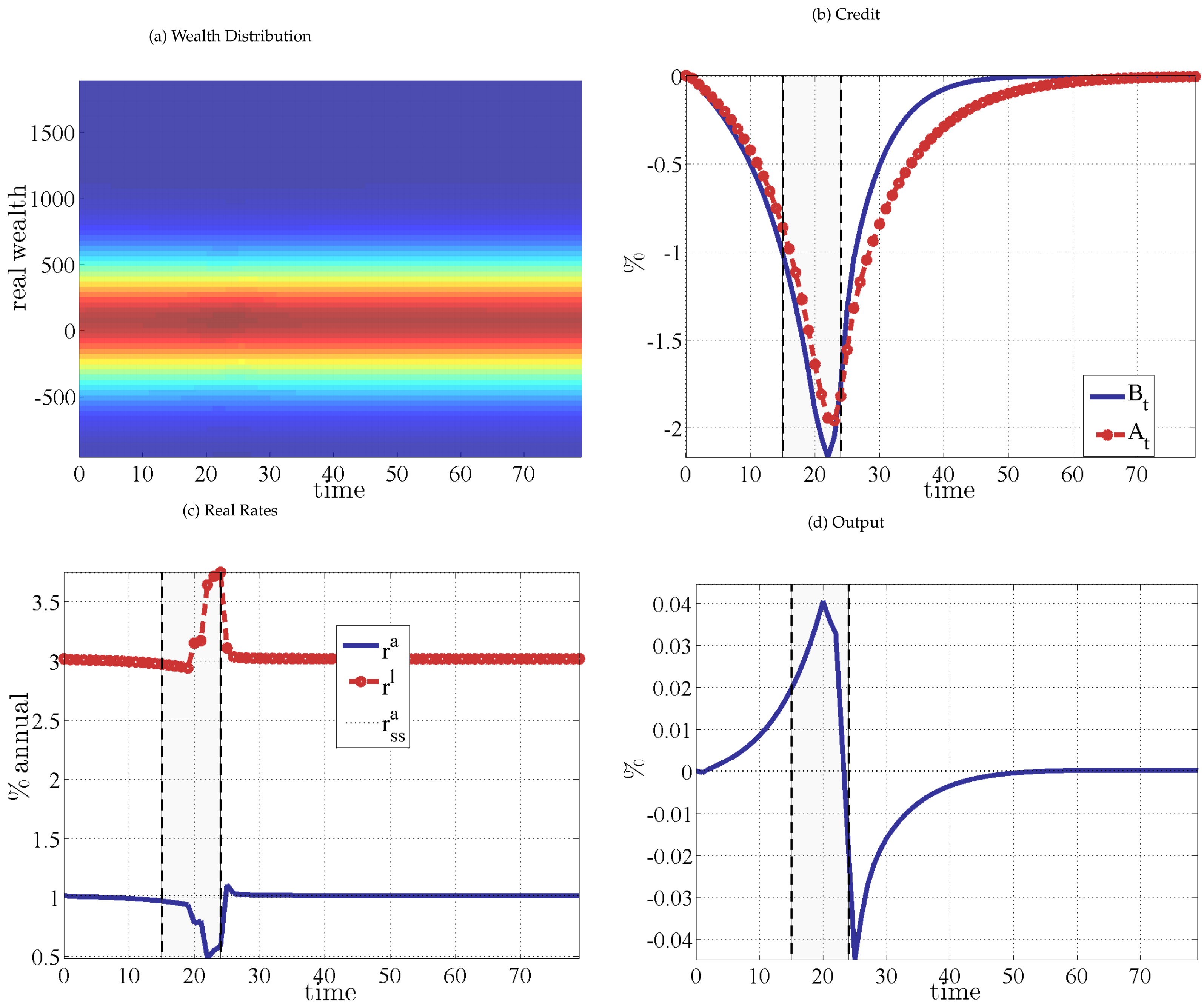

Figure 9: Macroeconomic Aggregates after a Credit Crunch (Wicksellian Doctrine).

Discussion: Fiscal Transfers at the ZLB? So far we described how at the zero-lower bound on deposits, open-market operations only increase the volume of currency in the economy. By contrast, a further reduction in real rates, deepens the effects. MP is left only with fiscal transfers as a tool. This tool can be seen as a helicopter drop, a direct injection of liquidity to households. As we noted, if the policy is anticipated, the policy can have recessionary effects.

7 Aggregate Demand Externality

In this section, we modify our model to allow for a labor-demand externality. This externality will amplify the effects of the credit-crunch and will make a stronger point to lower spreads. We show that in this case, monetary is more powerful than in normal instances.

For that purpose, we now endow households with a continuum mass $\bar{n}$ of labor endowments. Like in Hansen (1985) and Rogerson (1988), labor endowments are indivisible —so a unit of labor is active or inactive. This supply is perfectly inelastic. Also, the labor endowment of one household cannot be employed in the firm owned by that household. The only role for labor is to introduce an aggregate demand externality that is one of two motivations for an active monetary policy.

In this model, the only reason to activate the safe technology is to avoid hitting a borrowing constraint. Since if every entrepreneur chooses the safe technology is equivalent to choosing the total number of workers in the economy.

Each technology requires a specific amount of workers $n(u)$. Naturally, $n(H) > n(L)$ and we also normalize $n(H) = \bar{n}$, so if all entrepreneurs operate with the high intensity technology, all workers are employed.

The Labor Market. The labor market suffers an imperfection because there is a labor hold-up problem as in Caballero and Hammour (1998). Once an entrepreneur hires a worker output becomes specific to the worker. In particular, the workers at a firm can threat the household to divert the fraction $(1 - \eta_l)$ output of the firm. Thus, after being hired, workers are in a position to bargain over the total output produced per unit of time.[28] As a result, ex-post output must split into η_l destines to the entrepreneur and $1 - \eta_l$ to the worker. More

[28]This construction can be approximated by a limit. Suppose that technologies are fixed over specific time intervals $\Delta t, 2\Delta t$, ... For every interval, assume that once the technology is chosen and workers are hired, contracts are negotiated on the spot and according to a bargaining problem. In particular, workers may threaten the entrepreneur not to work in which case they receive no output. Presumably, this hold-up problem leads to an output split according to some Nash-bargaining problem —also a la Rubinstein. In that case, output is divided in η and $(1 - \eta)$ shares to entrepreneurs and workers correspondingly.

importantly, η_l captures the extent of a demand externality. If $\eta_l = 1$, the firm's choice of utilization has only an incidence on its own income, but $\eta_l < 1$, the firm's choice has an incidence on other household's income. Since real wages given u are fixed, workers and firms cannot contract on a technology. Instead, the firm unilaterally chooses a technology and then splits output accordingly.

Since each household has a continuum of workers, labor income risk is perfectly diversified. Considering this diversification workers of each household receive a common labor income flow of:

$$w_t^l = (1 - \eta_l) \int_0^\infty y\left(u\left(s, t\right)\right) f_t\left(s\right) ds.$$

Because the technology choice affects the amount of workers hired, the model can feature unemployment. where $f_t\left(s\right)$ is the distribution of wealth and $u\left(s, t\right)$ technology choice of the employing household with wealth s at time period t. The only change in the household's problem is that now its real income flow is the sum of firm profits, labor income, and net transfers. Thus, per unit of time they earn:

$$h\left(u_t, t\right) = \eta_l y\left(u_t\right) + w_t^l + T_t$$

where $\eta_l y_t\left(u\right)$ is the household's entrepreneurial wealth, w_t^l the wage described earlier and T_t.

Labor market-clearing must be consistent with a level of unemployment:

$$Y\left(t\right) = \int_0^\infty \left[1 - \left(\mathbb{I}\left[u\left(s, t\right) = H\right] - 1\right) \frac{n\left(L\right)}{\bar{n}}\right] f_t\left(s\right) ds.$$

Application: Employing the Credit-Channel. In this section we present the effects of a credit crunch once we activate the demand externality. Figure 10 presents the macroeconomic effects of a credit crunch under three scenarios: the first scenario is the response according to Figure xxx where there's no spread policy. The second scenario activates the externality. The final scenario is the case when the aggregate demand externality is present and the CB activates a credit spread an maintains it open. We can observe two things: First, that the presence of the aggregate demand externality amplifies the extent of the crisis. Second, that by producing an active spread ex-ante, the CB is able to contain the impact of the crisis. The reason is that the CB suppresses credit ex-ante. This means that the economy will feature less borrowing, but when the crunch is realized, less agents hit their borrowing constraints. The policy is so strong, that the impact is mitigated that the path is even smoother than without the externality. This observations lead us to conclude that in the model with a demand externality, MP should trade-off ex-ante inefficiencies against the depth of an ex-post crisis.

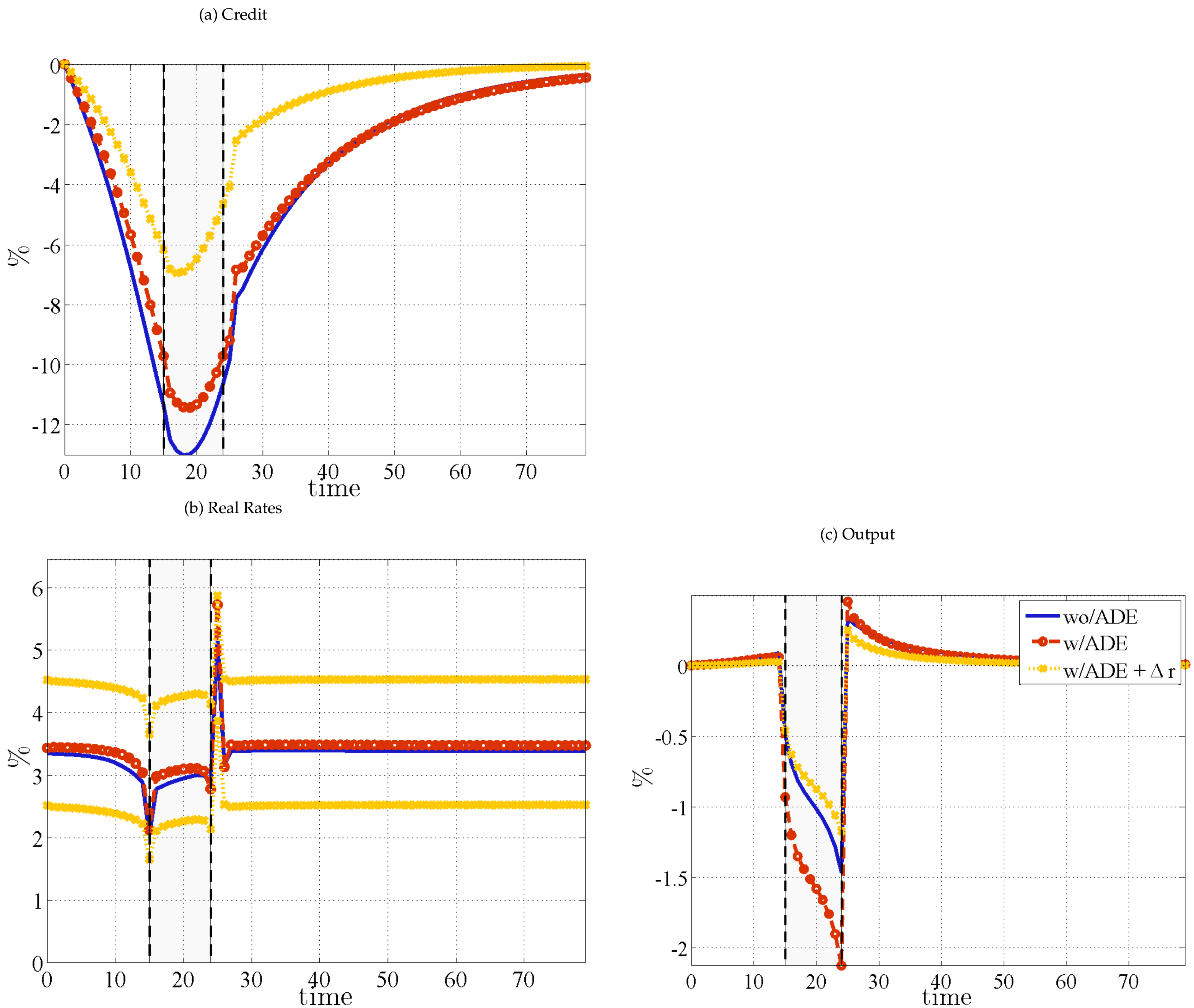

Figure 10: Macroeconomic Aggregates after a Credit Crunch - Comparison.

8 Conclusion

We began this paper with a quote from Robert Lucas. We can conclude by returning to that quote. Lucas's quote is inspiring because it summarizes his contributions to monetary economics, but also shows dissatisfaction with the lack of credit in monetary models. For him, his agenda was successful in rationalizing the long-run connection between money growth and inflation, a correlation that was disputed in the past. Introspectively, Lucas shows genuine scientific dissatisfaction with the workhorse models he help build, emphasizing the lack of a connection of monetary models with credit markets.

Our paper actually builds on one of Lucas earlier models, Lucas (1980). Ours is one of the several recent attempts to integrate credit into monetary theory. Here, outside money (reserves) are an input for inside money creation (deposits and loans). We tried this attempt trying to stay close to modern implementations of monetary policy. We also tried to articulate how different policy tools are tide to different transmission mechanisms stressed by the literature. We drew lessons for policy that we hope can be qualified empirically in the future, they present serious warnings on how monetary policy should be conducted.

References

Afonso, Gara and Ricardo Lagos, "Trade Dynamics in the Market for Federal Funds," 2012. ederal Reserve Bank of Minneapolis Research Department, Working Paper 708 710.

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

Alvarez, Fernando, Andrew Atkeson, and Chris Edmond, "Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand," *The Quarterly Journal of Economics*, 2009, *124* (3), 911–967.

Ashcraft, Adam B. and Darrell Duffie, "Systemic Illiquidity in the Federal Funds Market," *American Economic Review*, 2007, *97* (2), 221–225.

Auclert, Adrien, "Monetary Policy and the Redistribution Channel," January 2016.

Berentsen, Aleksander, Gabriele Camera, and Christopher Waller, "Money, credit and banking," *Journal of Economic Theory*, 2007, *135* (1), 171 – 195.

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 1989, *79* (1), pp. 14–31.

Bernanke, Ben S and Alan S Blinder, "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review*, September 1992, *82* (4), 901–921.

Bernanke, Ben S. and Mark Gertler, "Inside the Black Box: The Credit Channel of Monetary Policy Transmission," *The Journal of Economic Perspectives*, 1995, *9* (4), 27–48.

_ , _ , and Simon Gilchrist, "Chapter 21 The financial accelerator in a quantitative business cycle framework," in John B. Taylor and Michael Woodford, eds., *John B. Taylor and Michael Woodford, eds.*, Vol. 1, Part 3 of *Handbook of Macroeconomics*, Elsevier, 1999, pp. 1341 – 1393.

Bewley, Truman, "A Difficulty with the Optimum Quantity of Money," *Econometrica*, 1983, *51* (5), 1485–1504.

Bianchi, Javier and Saki Bigio, "Banks, Liquidity Management, and Monetary Policy," 2017.

_ and _ , "A Note on OTC Markets," 2017. Mimeo, Federal Reserve Bank of Minneapolis https://www.dropbox.com/s/az3zznfspoqqb2j/Note$_O TC.pdf?dl = 0$.

Bindseil, Ulrich, *Monetary Policy Operations and the Financial System*, first edition ed., Oxford University Press, 2014. ISBN-10: 0198716907.

Brunnermeier, Markus K. and Yuliy Sannikov, "The I-Theory of Money," 2012.

Buera, Francisco J. and Juan Pablo Nicolinni, "Liquidity Traps and Monetary Policy: Managing a Credit Crunch," May 2016.

Caballero, Ricardo J. and Mohamad L. Hammour, "The Macroeconomics of Specificity," *The Journal of Political Economy*, 1998, *106* (4), 724–767.

_ , Takeo Hoshi, and Anil K. Kashyap, "Zombie Lending and Depressed Restructuring in Japan," *The American Economic Review*, 2008, *98* (5), 1943–1977.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno, "Financial Factors in Economic Fluctuations," 2009.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "The Deposits Channel of Monetary Policy*," *The Quarterly Journal of Economics*, 2017, *132* (4), 1819–1876.

Ennis, Huberto, "A Simple General Equilibrium Model of Large Excess Reserves," 2014. Working Paper 14-14 Federal Reserve Bank of Richmond.

Gomes, Joao, Urban J. Jermann, and Lukas Schmid, "Sticky Leverage," February 2016.

Greenwald, Daniel L., "The Mortgage Credit Channel of Macroeconomic Transmission," 2016.

Gu, Chao, Fabrizio Mattesini, and Randall Wright, "Money and Credit Redux," 2015. workingpaper.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings and the Liquidity Trap," 2012.

Hall, Robert E. and Ricardo Reis, "Maintaining Central-Bank Financial Stability under New-Style Central Banking," 2015.

_ and _ , "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves," 2017.

Hansen, Gary D., "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 1985, *16* (3), 309 – 327.

Huggett, Mark, "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 1993, *17* (5), 953 – 969.

Kaplan, Greg, Ben Moll, and Gianluca Violante, "Monetary Policy According to HANK," 2016.

Kashyap, Anil K and Jeremy C. Stein, "What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?," *The American Economic Review*, 2000, *90* (3), pp. 407–428.

Lagos, Ricardo and Randall Wright, "Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 2005, *113* (3), 463–484.

Lippi, Francesco, Stefania Ragni, and Nicholas Trachter, "Optimal monetary policy with heterogeneous money holdings," *Journal of Economic Theory*, 2015, *159, Part A*, 339 – 368.

Ljungqvist, Lars and Thomas J. Sargent, *Recursive Macroeconomic Theory*, 3rd edition ed., Boston: MIT Press, 2012.

Lucas, Robert E., "Equilibrium in a Pure Currency Economy," *Economic Inquiry*, 1980, *18* (2), 203–220.

McKay, Alisdair, Emi Nakamura, and Jon Steinsson, "The Power of Forward Guidance Revisited," December 2015. forthcoming at the American Economic Review.

Piazzesi, Monika and Martin Schneider, "Payments, Credit and Asset Prices," 2018. April 2018.

Rocheteau, Guillaume, Tsz-Nga Wong, and Pierre-Olivier Weill, "Working through the Distribution: Money in the Short and Long Run," May 2016.

Rogerson, Richard, "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, 1988, *21* (1), 3 – 16.

Rognlie, Matthew, "What Lower Bound? Monetary Policy with Negative Interest Rates," 2016.

Silva, Dejanir, "The Risk Channel of Unconventional Monetary Policy," 2016. Job Market Paper.

Werning, Ivan, "Incomplete Markets and Aggregate Demand," October 2015.

Williamson, Stephen, "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," 2012. forthcoming, American Economic Review, 2012.

Wong, Arlene, "Population Aging and the Transmission of Monetary Policy to Consumption," 2016.

Woodford, Michael, "Doing without Money: Controlling Inflation in a Post-Monetary World," *Review of Economic Dynamics*, 1998, *1* (1), 173 – 219.

_ , "Monetary Policy in the Information Economy," Technical Report, Economic Policy for the Information Economy. Kansas City: Federal Reserve Bank of Kansas City 2001.

A Accounting in the Model

Household Balance Sheet. The household's balance sheet in in nominal terms is:

Assets	Liabilities
m_t^h	l_t^h
a_t^h	$P_t s_t$

.

Bank Balance Sheet. The balance sheet of an individual bank is:

Assets	Liabilities
m_t^b	a_t^b
l_t^b	

.

Accounting of OMO. To make this interpretation more clear, consider F_t is an outstanding amount of nominal bonds issued by the FA. Let $F_t^{cb} < F_t$ be the stock of government bonds held at the Central Bank. In that case, the balance sheet of the CG is

Assets	Liabilities
F_t^{cb}	$M_t + F_t$
	E_t

=

Assets	Liabilities
$F_t^{cb} - F_t$	M_t
	E_t

.

Thus, $L_t^f = F_t^{cb} - F_t < 0$ is the stock of government bonds held by banks and E_t is the stock of government liabilities net of CB purchases. A conventional open-market operation is simply an increase in F_t^{cb} funded with an increase in M_t. From the government's income flow, we can see that this operation would yield profits to the CB if there's a spread $i_t^l > i_t^m$.

B Formulas for Interbank Market Trades

The parameter λ captures a the matching efficiency of the interbank market.[29] The corresponding trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1 - e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \psi^{-}(\theta) \equiv \begin{cases} \frac{1-e^{-\lambda}}{\theta} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}.$$

[29] This can be shown very easily using a differential form.

The resulting average interbank market rate is determined by the average of Nash bargains over the positions and is given by:

$$\bar{i}^{f}\left(\theta, i^{m}, \iota\right) \equiv \begin{cases} i^{m} + \iota - \left(\left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta} - 1\right)\left(\frac{\theta}{\theta-1}\right)\left(\frac{\iota}{e^{\lambda}-1}\right) & \text{if } \theta > 1 \\ i^{m} + \iota(1-\eta) & \text{if } \theta = 1 \\ i^{m} + \iota - \left(1 - \left(\frac{\bar{\theta}(\theta)}{\theta}\right)^{\eta}\right)\left(1 + \frac{\theta/\bar{\theta}(\theta)}{1-\theta}\right)\left(\frac{\iota}{e^{\lambda}-1}\right) & \text{if } \theta < 1 \end{cases}$$

where η is a parameter associated with the bargaining power of banks with reserve deficits. It can be verified that

$$\psi_t^{-} B_t^{-} = \psi_t^{+} B_t^{+}, \tag{18}$$

which is a market clearing condition for the interbank market. Thus, the path for $\left\{\psi_t^{+}, \psi_t^{-}, \bar{\imath}^{f}\right\}$ is given by $\psi_t^{+} \equiv \psi^{+}\left(\theta_t\right), \psi_t^{-} \equiv \psi^{-}\left(\theta_t\right)$ and $\bar{\imath}_t^{f} \equiv \bar{\imath}^{f}\left(\theta_t\right)$.

C Solution Algorithm

Propositions 2, 3 and 4 are the objects we need to solve the model. They allow us to solve the model entirely by solving for the equilibrium path of a single price. For example, we can solve the model by solving the path for a real deposit rate r_t^{a}. The spread Δr_t follows immediately from Proposition 2 if we know the path for ι_t and Λ_t set by the CB. The real spread gives us r_t^{l}. To solve the household's problem, we need the path for $\left\{r_t^{a}, r_t^{l}, T_t\right\}$. The path for T_t is must be consistent with (15) and this yields a path for real government liabilities, $\mathcal{E}_t$. Then, $\mathcal{E}_t$ together with the evolution of $f_t\left(s\right)$ obtained from the household's problem, yield two sides of one equation enters 14. The rate equilibrium rate r_t^{a} must be the one that solves 14 implicitly.

Before we study the effects of monetary policy under different policy doctrines, we want to explain the implementability constraints faced by the CB. Then, we briefly discuss the behavior of the model at the zero-lower bound on deposits rates and when the CB satiates the economy with reserves.

Solution at ZLB.

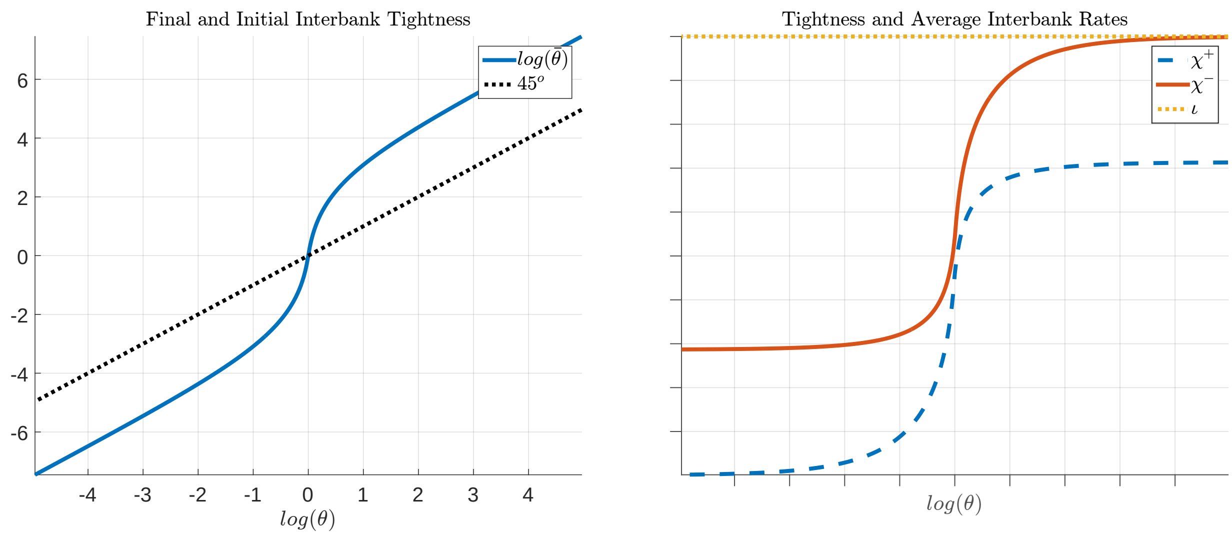

Figure 11: Interbank-Market Conditions and Interbank-Market tightness.

D Additional Plots

D.1 Additional Plots in Model Section

Here we present an example of the formula 8. The left panel presents a mapping from θ to $\bar{\theta}$. The right panel plots $\{\chi_t^+, \chi_t^-\}$ for a given $\{\eta, \lambda, \iota\}$.[30] The formulas for $\{\chi^+, \chi^-\}$ show how the average costs of intermediation depend on the policy corridor ι on the policy spread ι and the amount of outside money M_t. The reason why the CB can affect outcomes is because, in turn, these intermediation costs affect bank decisions.

Figure 12 has two panels. The left panel plots the numerator and denominator in the definition of $\theta(\Lambda_t)$. As Λ_t increases, on aggregate, banks have less reserve deficits and a greater reserve surplus. There are bounds at the left and right of the figure, at the points where all banks are in deficit and at the point where all banks are satiated with reserves. The right panel shows the map from Λ to $log(\theta_t)$. Because θ_t is only a function of the liquidity ratio, (8) we obtain $\chi_t^+ = \chi^+(\theta(\Lambda_t), \iota_t)$ and $\chi_t^- = \chi^-(\theta(\Lambda_t), \iota_t)$.

[30] As the interbank market is tighter average rates for short and long positions increase and approach the width of the corridor window. Instead, as the tightness drops, both rates get closer to zero. We use these formulas later to map the stance of monetary policy to a market tightness, and through the interbank market spread, we obtain formulas for real spreads.

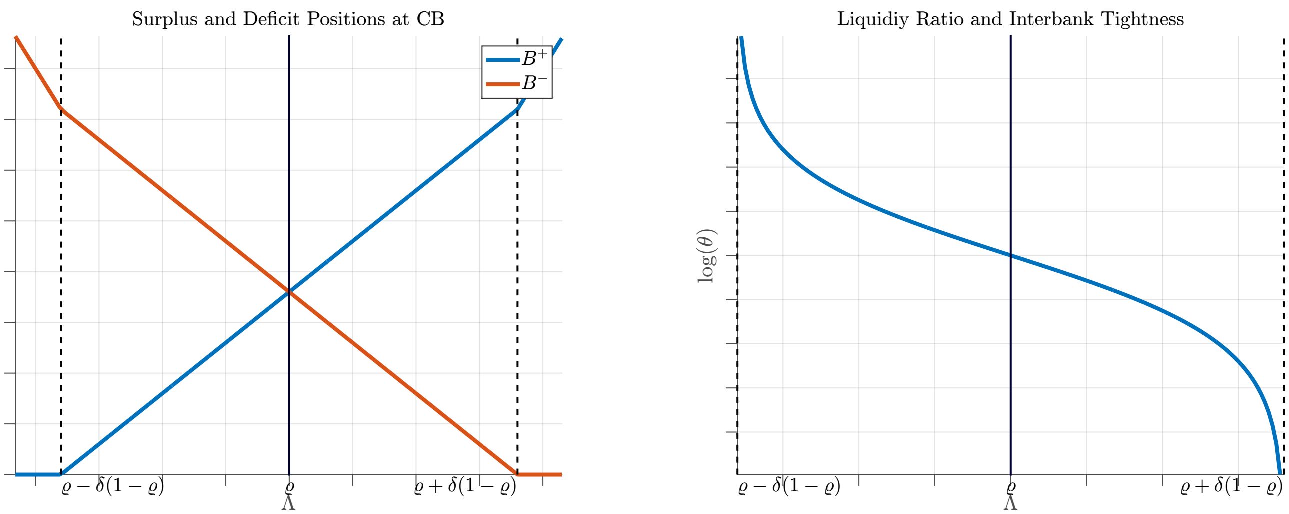

Figure 12: Interbank-Market Conditions and Liquidity Ratio.

D.2 Additional Plots - Wicksellian Policy

D.2.1 Extensions to Section .

Decomposition of Fisher Equation under Monetary Expansion.

Decomposition of Fisher Equation under Fiscal Transfer.

Decomposition of Fisher Equation under Credit Channel and ZLB

Decomposition of Fisher Equation under Credit Channel and ZLB

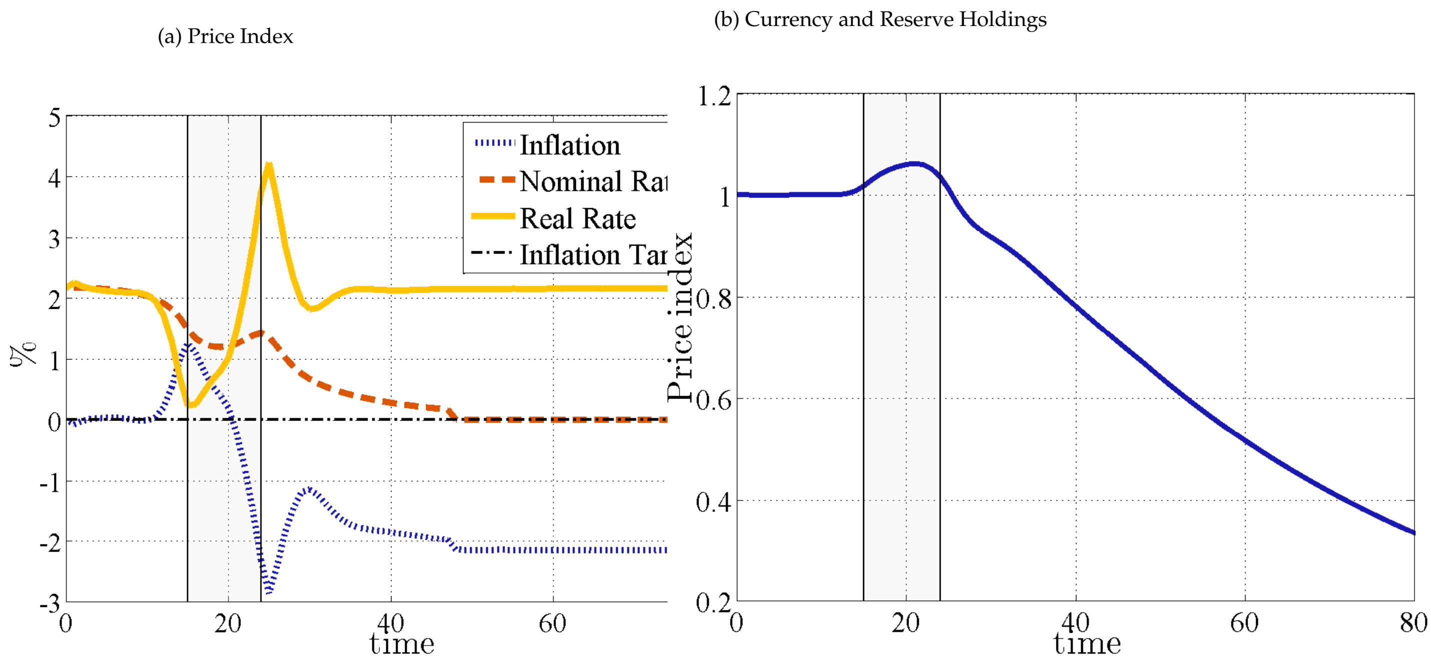

Figure 13: xxx.

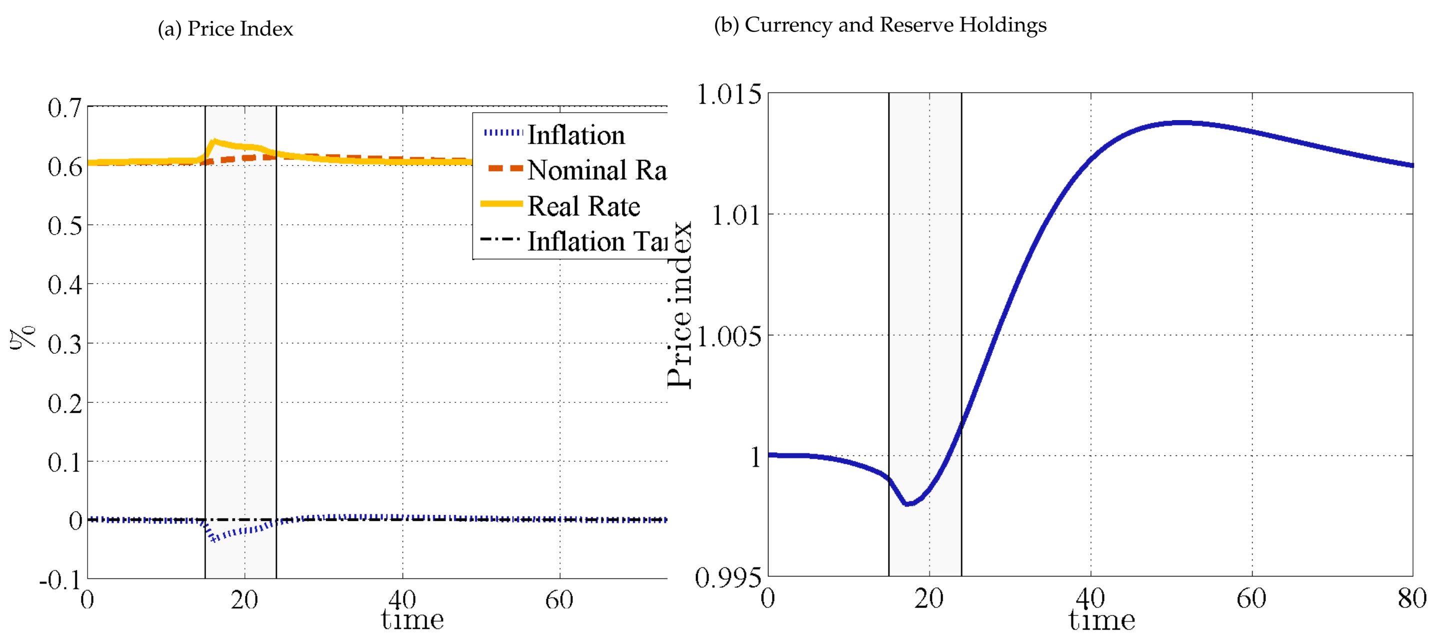

Figure 14: xxx.

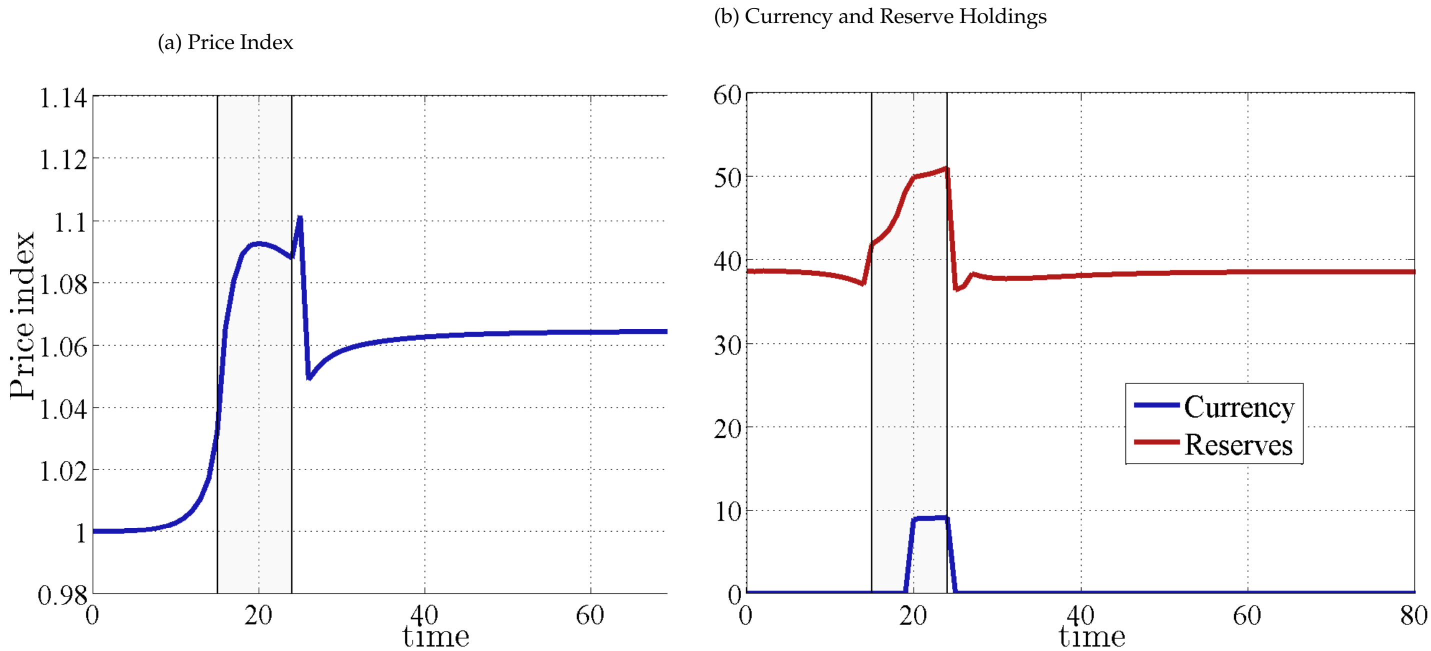

Figure 15: xxx.

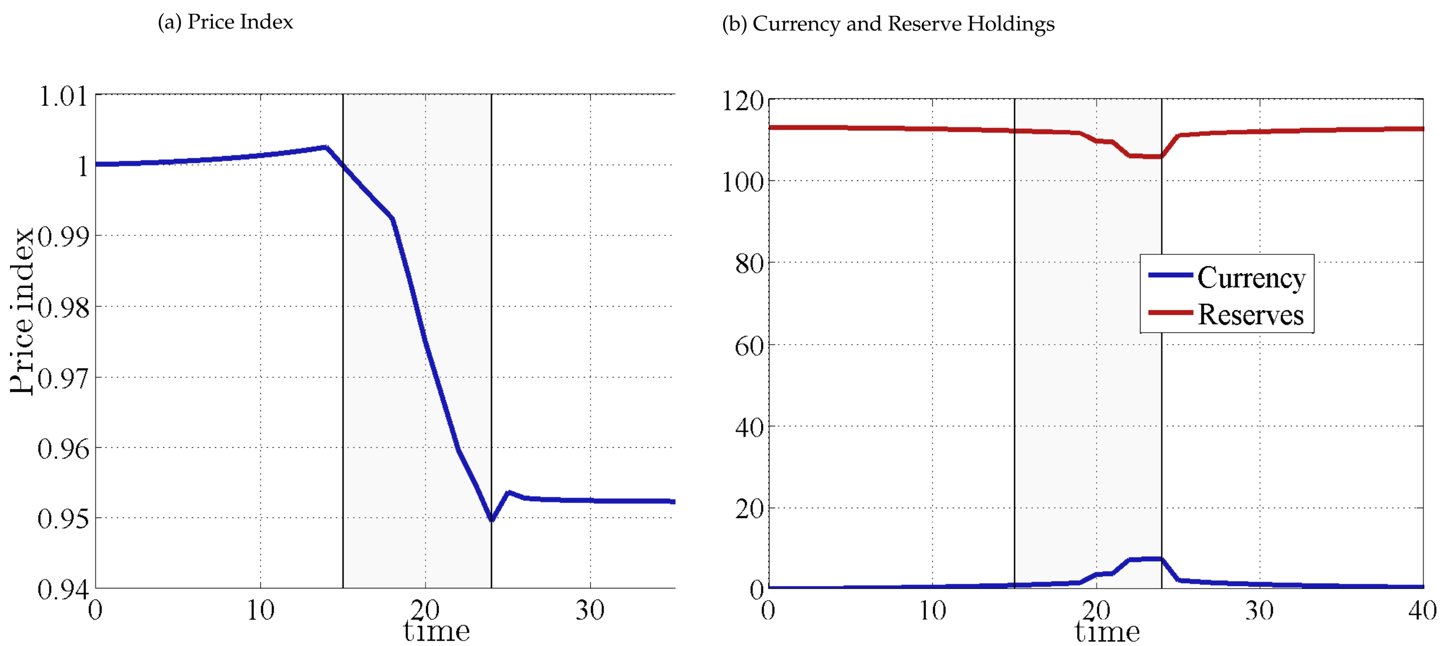

Figure 16: xxx.

E Proofs

E.1 Proposition 1

Proposition 9 *The equilibrium χ_t is a function of the corridor spread $\imath_t$ and the liquidity ratio Λ_t as given by*

$$\chi^+(\Lambda_t, \imath_t) = \imath_t \left(\frac{\bar{\theta}(\Lambda_t)}{\theta(\Lambda_t)} \right)^{\eta} \left(\frac{\theta(\Lambda_t)^{\eta} \bar{\theta}(\Lambda_t)^{1-\eta} - \theta(\Lambda_t)}{\bar{\theta}(\Lambda_t) - 1} \right) \tag{19}$$

$$\chi^-(\Lambda_t, \imath_t) = \imath_t \left(\frac{\bar{\theta}(\Lambda_t)}{\theta(\Lambda_t)} \right)^{\eta} \left(\frac{\theta(\Lambda_t)^{\eta} \bar{\theta}(\Lambda_t)^{1-\eta} - 1}{\bar{\theta}(\Lambda_t) - 1} \right) \tag{20}$$

$$\theta(\Lambda_t) \equiv \frac{-\sum_{z \in \{-1,1\}} \min\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}}{\sum_{z \in \{-1,1\}} \max\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}} \tag{21}$$

where $\bar{\theta}(\Lambda_t) = \bar{\theta}(\theta(\Lambda_t))$.

Proof. Since the balance of reserve for an individual bank is given by $b_t = m_t^b + \omega a_t - \varrho(a_t + \omega a_t) = m_t^b - \varrho a_t + (1-\varrho)\omega a_t$ where $\omega \in \{-\delta, \delta\}$ with equal probability, then the aggregate balance of reserves in bank's sector is

$$B_t = \sum_{z \in \{-1,1\}} \frac{1}{2}(M_t^b - \varrho A_t + (1-\varrho)\delta z A_t)$$
$$B_t^+ = \sum_{z \in \{-1,1\}} \frac{1}{2} \max\{M_t^b - \varrho A_t + (1-\varrho)\delta z A_t, 0\}$$
$$B_t^- = -\sum_{z \in \{-1,1\}} \frac{1}{2} \min\{M_t^b - \varrho A_t + (1-\varrho)\delta z A_t, 0\}$$

Thus, the market tightness of the interbank market can be rewritten as a function of the liquidity ratio

$$\theta_t = \theta(\Lambda_t) \equiv \frac{B_t^-/A_t}{B_t^+/A_t} = \frac{-\sum_{z \in \{-1,1\}} \min\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}}{\sum_{z \in \{-1,1\}} \max\{\Lambda_t - \varrho + (1-\varrho)\delta z, 0\}}$$

Finally, let $\bar{\theta}(\Lambda_t) \equiv \bar{\theta}(\theta(\Lambda_t))$ be the end-of-day tightness as a function of the liquidity ratio, hence we can replace these definitions in the expressions for $\chi_t^+(\theta_t)$, $\chi_t^-(\theta_t)$, and χ_t. □

E.2 Proof of Proposition 2

Proposition 10 *(Nominal Rates). Take Λ_t, i_t^m, and $\imath_t$ along an equilibrium path as given. Then, $\{i_t^l, i_t^a\}$ are given by:*

$$i_t^l = i_t^m + \frac{1}{2}\Big[\chi^+(\Lambda_t, \imath_t) + \chi^-(\Lambda_t, \imath_t)\Big] \tag{22}$$

$$i_t^a = i_t^m + \frac{1}{2}(1-\varrho)\Big[(1+\delta)\chi^+(\Lambda_t, \imath_t) + (1-\delta)\chi^-(\Lambda_t, \imath_t)\Big] \tag{23}$$

Proof. Taking $\{\Lambda_t, i_t^m, \imath_t\}$ as given, an individual bank solves

$$\max_{a,l} \quad i_t^l l + i_t^m(a-l) - i_t^a a + \mathbb{E}\Big\{\chi_t[b(a, a-l)]|\theta_t\Big\}$$

$$\max_{a,l} \quad i_t^l l + i_t^m(a-l) - i_t^a a + \Big[\frac{1}{2}\chi^+(\Lambda_t, \imath_t)(a - l - \varrho a + (1-\varrho)\delta a) + \frac{1}{2}\chi^-(\Lambda_t, \imath_t)(a - l - \varrho a - (1-\varrho)\delta a)\Big]$$

$$\max_{a,l} \quad \Big[i_t^l - i_t^m - \frac{1}{2}\chi^+(\Lambda_t, \imath_t) - \frac{1}{2}\chi^-(\Lambda_t, \imath_t)\Big] l - \Big[i_t^a - i_t^m - \frac{1}{2}(1-\varrho)(1+\delta)\chi^+(\Lambda_t, \imath_t) - \frac{1}{2}(1-\varrho)(1-\delta)\chi^-(\Lambda_t, \imath_t)\Big] a$$

Because of linearity, the first order condition for banks are equivalent to non-arbitrage conditions bank1 and bank2. □

E.3 Proof of Proposition 2

Let the market rates be given by bank1 and bank2. Then, banks earn zero profits.

Proof. From

$$\pi_t^b = \max_{a,l} \quad \Big[i_t^l - i_t^m - \frac{1}{2}\chi^+(\Lambda_t, \imath_t) - \frac{1}{2}\chi^-(\Lambda_t, \imath_t)\Big] l - \Big[i_t^a - i_t^m - \frac{1}{2}(1-\varrho)(1+\delta)\chi^+(\Lambda_t, \imath_t) - \frac{1}{2}(1-\varrho)(1-\delta)\chi^-(\Lambda_t, \imath_t)\Big] a$$

it is not hard to see that $\pi_t^b = 0$ whenever market rates are given by crefbank1 and bank2. □

E.4 Proof of Proposition 2

The CB operational profits from open-market operations per unit of deposits are only functions of Λ_t and $\imath_t$.

Proof. The total profits for the consolidated government

$$\pi_t^f = i_t^l L_t^f - i_t^m M_t^b - \mathbb{E}\left[\chi_t\left(B_t\right)|\theta_t\right] - \tau_t^l L_t - P_t T_t$$

$$\pi_t^f = \underbrace{i_t^l E_t - \tau_t^l L_t - P_t T_t}_{\text{Fiscal Deficit}} + \underbrace{\left(\underbrace{\left(i_t^l M_t - i_t^m M_t^b\right)}_{\substack{\text{OMO}\\\text{Profits}}} - \underbrace{\mathbb{E}\left[\chi_t\left(B_t\right)|\theta_t\right]}_{\substack{\text{Discount}\\\text{Loans}}}\right)}_{\substack{\text{CB Operational}\\\text{Profits (CB Op)}}} \tag{24}$$

Since banks earn zero profits, then $-\mathbb{E}\left[\chi_t\left(B_t\right)|\theta_t\right] = i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t$

$$\begin{aligned} CB\ Op &= i_t^l M_t - i_t^m M_t^b + i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t \\ &= i_t^l (M_t + A_t - M_t^b) - i_t^a A_t \\ &= i_t^l M0_t + (i_t^l - i_t^a) A_t \end{aligned}$$

Whenever $i_t^a > 0$ households do not hold currency, i.e., $M0_t = 0$, hence

$$CB\ Op = \Delta i_t A_t$$

We have already show that Δi_t is only a function of Λ_t and $\imath_t$ (see prop2). □

E.5 Proof of Proposition 2

(Real Wealth Clearing). Let nominal rates be given by bank1 and bank2, and let the liquidity ratio be given by Λ_t. Then, asset market-clearing in real terms

$$-\int_{-\infty}^{0} s f_t(s) ds = \int_{0}^{\infty} s f_t(s) ds + \mathcal{E}_t. \tag{25}$$

Proof. Deposits and loans markets clearing condition are

$$\begin{aligned} A_t &= \int_0^{\infty} a_t^h(s) f_t(s) ds \\ L_t^b + L_t^f &= \int_{-\infty}^{0} l_t^h(s) f_t(s) ds \end{aligned}$$

Since household's deposit and loan policy functions are given by: $a_t^h(s) = P_t s - m_t^h(s)$ and $l_t^h(s) = m_t^h(s) - P_t s$, we can combine these clearing-market conditions with policy functions as

$$A_t - L_t^b = P_t \int_{-\infty}^{\infty} s f_t(s) ds - M0_t + L_t^f$$
$$M_t^b + M0_t = P_t \int_{-\infty}^{\infty} s f_t(s) ds + M_t + E_t$$

Using money market clearing-condition, $M_t^b + M_t^0 = M_t$, dividing by the price index, and rearranging:

$$-\int_{-\infty}^{0} s f_t(s) ds = \int_{0}^{\infty} s f_t(s) ds + \mathcal{E}_t$$

□

E.6 Proof of Proposition 2

Proposition 11 *Assume that the real-asset market clearing conditions holds, asset, that the labor-market clears and the Kolmogorov-Forward equation holds. Then, the goods market clearing is satisfied.*

Proof. Integrate :

$$\frac{\partial \int_{-\infty}^{\infty} s f_t(s) ds}{\partial t} = -r_t^b L_t + r_t^a A_t -$$

□

E.7 Proof of Proposition 2

Proposition 12 *(Real Budget Constraint). A government policy path is feasible if and only if:*

$$d\mathcal{E}_t = \left((r_t^a + \Delta r_t)\mathcal{E}_t + \Delta r_t \int_0^{\infty} s f_t(s) ds - T_t \right) dt \tag{26}$$

Proof. The evolution of government's real equity must satisfy

$$
\begin{aligned}
d\mathcal{E}_t &= \left(\frac{\pi_t^f}{P_t} - \mathcal{E}_t \frac{\dot{P}_t}{P_t}\right) dt \\
&= \left(i_t^l \mathcal{E}_t - \tau_t^l \frac{L_t}{P_t} - T_t + i_t^l \frac{M0_t}{P_t} + \Delta r_t \frac{A_t}{P_t} - \mathcal{E}_t \frac{\dot{P}_t}{P_t}\right) dt \\
&= \left(\left(i_t^l - \frac{\dot{P}_t}{P_t}\right) \mathcal{E}_t - \tau_t^l \left(\frac{A_t}{P_t} + \frac{M0_t}{P_t} + \mathcal{E}_t\right) + i_t^l \frac{M0_t}{P_t} + \Delta r_t \frac{A_t}{P_t} - T_t\right) dt \\
&= \left(\left(i_t^l - \frac{\dot{P}_t}{P_t} - \tau_t^l\right) \mathcal{E}_t + (i_t^l - \tau_t^l) \frac{M0_t}{P_t} + (\Delta r_t - \tau_t^l) \frac{A_t}{P_t} - T_t\right) dt \\
&= \left(\left(r_t^a + \Delta i_t - \tau_t^l\right) \mathcal{E}_t + (i_t^l - \tau_t^l) \frac{M0_t}{P_t} + (\Delta i_t - \tau_t^l) \frac{A_t}{P_t} - T_t\right) dt
\end{aligned}
$$

If $i_t^a > 0$, then $M0_t$. Defining Δr_t as the real effective spread faced by households, i.e., $\Delta r_t = \Delta i_t - \tau_t^l$, then

$$
d\mathcal{E}_t = \left((r_t^a + \Delta r_t)\mathcal{E}_t + \Delta r_t \int_0^\infty s f_t(s) ds - T_t\right) dt
$$

□

E.8 Proof of ...

Proposition 13 *Along an equilibrium path for $\{r_t^a, \mathcal{E}_t, f_t, \Delta r_t, \tau_t^l, T_t\}$ the set of implementable nominal interbank rates and inflation rates is the set of $\{\dot{P}_t / P_t, \bar{i}_t^f\}$ where*

$$
\frac{\dot{P}_t}{P_t} = i_t^m + \frac{1}{2}\left[\chi^+(\Lambda_t, \imath_t) + \chi^-(\Lambda_t, \imath_t)\right] - \Delta r_t - r_t^a \tag{27}
$$

$$
\bar{i}_t^f = \chi^+(\Lambda_t, \imath_t) / \psi^+(\theta(\Lambda_t)) \tag{28}
$$

for any $\{i_t^m, \imath_t, \mathcal{L}_t^f\}$ such that

$$
\begin{aligned}
\Delta r_t &= \Delta i_t - \tau_t^l \\
\Delta i_t &= \varrho \frac{\chi^+(\Lambda_t, \imath_t) + \chi^-(\Lambda_t, \imath_t)}{2} + \delta(1 - \varrho) \frac{\chi^-(\Lambda_t, \imath_t) - \chi^+(\Lambda_t, \imath_t)}{2} \\
\mathcal{L}_t^f &\le \int_{-\infty}^0 s f_t(s) ds, \quad (\imath_t, i_t^m) \in \mathbb{R}_+^2.
\end{aligned}
$$

Proof. Equations (27) and (28) steams form definitions for nominal, real and interbank rate.

The implementation constraint $\mathcal{L}_t^f \leq \int_{-\infty}^{0} s f_t(s) ds$ simply tells that there must be enough private liabilities to set $\mathcal{L}_t^f$. $\square$

E.9 Proof of ...

Proposition 14 *Let there be some t such that the economy is strictly satiated with reserves. Let $i_t^m \geq 0$, then $i_t^m = i_t^d = i_t^l = \bar{i}_t^f$. Furthermore, $\Delta i_t = 0$ and $\dot{P}_t / P_t = i_t^m - r_t^a$.*

Proof. The interbank market is satiated with reserves if $\Lambda_t \geq \bar{\Lambda} = \varrho + (1 - \varrho)\delta$. Then the interbank market tightness is $\theta(\Lambda_t) = 0$ for any $\Lambda_t \geq \bar{\Lambda} = \varrho + (1 - \varrho)\delta$. First, we must take the following limit

$$\lim_{\theta \to 0} \frac{\bar{\theta}(\theta)}{\theta} = \lim_{\theta \to 0} \frac{1}{\theta[1 + (\theta^{-1} - 1)\exp(\lambda)]} = \lim_{\theta \to 0} \frac{1}{\theta + (1 - \theta)\exp(\lambda)} = \exp(-\lambda)$$

Then, given (η, λ), for any $\Lambda_t \geq \bar{\Lambda}$:

$$\chi^+(\Lambda_t, \imath_t) = \lim_{\theta \to 0} \imath_t \theta \left(\frac{\bar{\theta}(\theta)}{\theta} \right)^{\eta} \left(\frac{[\bar{\theta}(\theta)/\theta]^{1-\eta} - 1}{\bar{\theta}(\theta) - 1} \right) = 0$$

$$\chi^-(\Lambda_t, \imath_t) = \lim_{\theta \to 0} \imath_t \left(\frac{\bar{\theta}(\theta)}{\theta} \right)^{\eta} \left(\frac{\theta[\bar{\theta}(\theta)/\theta]^{1-\eta} - 1}{\bar{\theta}(\theta) - 1} \right) = \imath_t \exp(-\eta\lambda)$$

Although $\chi_t^- > 0$, there are not banks with reserves deficit, thus

$$\mathbb{E}\left\{ \chi_t[b(a, a - l)] | \theta_t \right\} = \chi^+(\Lambda_t, \imath_t)\,(a - l - \varrho a) = 0$$

Hence, the bank's problem becomes

$$\pi_t^b = \max_{a,l} (i_t^l - i_t^m) l_t - (i_t^a - i_t^m) a_t$$

and by FOCs we obtain that $i_t^m = i_t^a = i_t^l = \bar{i}_t^f$.

$\square$

E.10 Proof of ...