# A Theory of Liquidity in Private Equity*

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**Abstract**

We develop a model of private equity in which many empirical patterns in fundraising, fund structure, and returns arise endogenously. Our model rests solely on two critical features of the private equity market: moral hazard and illiquidity. General partners (GPs) possess superior investment skills and raise funds from Limited Partners (LPs) to invest in illiquid investments. The optimal fund structure provides incentives for GPs to maximize investment payoffs by giving them a profit share in the fund, and compensates LPs for liquidity risk by offering them a return premium over liquid investments. Fund size increases with the amount of wealth the GP can co-invest in the fund. When liquidity risk decreases, LPs require a lower return premium for investing in private equity. GPs then react by increasing fund size but keep their profit share constant, leading to a negative relation between LP returns and aggregate fundraising. GPs may inefficiently accelerate investment to ensure that LPs honor their funding commitment. LPs with a lower cost of illiquidity have access to better-performing funds and realize higher net returns. In the secondary market, LP partnership claims trade at a discount to fundamentals when aggregate liquidity is scarce. The secondary market can contribute to the growth of the primary market by enabling LPs with higher illiquidity costs to invest. This effect can be reversed when LPs with lower illiquidity cost choose to focus on the secondary market.

**Keywords**: Private Equity, Liquidity Premium, Secondary Markets.

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# 1 Introduction

Private Equity (PE) firms are stewards of other people's capital. In practice, they typically operate through funds organized as limited partnerships, in which the General Partners (GPs) – employees of the private equity firm itself – raise capital from outside investors, known as Limited Partners (LPs). These investment partnerships are typically structured as closed-end funds with a limited life, in which LPs commit, but do not initially provide, capital. Instead, their capital is called over time by the GP as investment opportunities are identified. Once committed to a private equity fund, the LPs' capital becomes illiquid for two distinct reasons. First, partnership agreements typically prescribe severe penalties on LPs who default on pledged commitments. Second, private equity investments take time to mature and LPs typically wait for several years to realize a positive return on their capital, inducing a cash-flow pattern known as the J-curve.

Since its humble origins in the 1960s and 1970s, the private equity market has grown significantly, both in absolute terms and relative to public markets (Döskeland and Strömberg 2018). Besides growing, aggregate commitments to private equity funds are nevertheless highly pro-cyclical. Robinson and Sensoy (2016) show that net cash flows to LPs in private equity funds are pro-cyclical as well, suggesting that liquidity demands on LPs become more severe during economic downturns. In recent decades, a secondary market for partnership claims has emerged, which allows LPs to exit their PE investments early. Liquidity in the secondary market is still limited, however, and claims often trade at a substantial discount to net asset value, particularly in market downturns (Nadauld et al. 2018).

A large literature has emerged to study private equity and its impact on the economy. Starting with Jensen (1989) and Sahlman (1990), the economic structure of private equity partnerships has been interpreted as the solution to agency problems arising from delegated asset management. Empirically, limited partnership agreements are strikingly similar across funds and over time: the most common structure involves funds with a 10-year life, during which GPs are compensated through a fixed management fee (typically 1.5-2% of LPs' committed capital) and a 20% share of net returns (or *carried interest*; see Robinson and Sensoy 2013). A large number of empirical studies have documented that the efficiency and productivity of companies improve under private equity ownership, suggesting that GPs indeed possess real investment skills (Kaplan and Strömberg 2009; Da Rin et al. 2012).

A distinct literature has demonstrated that private equity funds have experienced higher returns than similarly-timed investments in public equity on a net-of-fee basis.[1] In addition, returns to private equity funds have been shown to be negatively related to aggregate PE fundraising, implying that this excess return to private equity is time-varying (Kaplan and Strömberg 2009). Some observers have interpreted this pattern as a "money chasing deals" phenomenon, whereby high past returns induce LPs to increase their commitments, which in turn leads to an overheated PE market and lower subsequent returns.[2]

Several studies have also examined returns and fundraising at the level of individual private equity funds. Kaplan and Schoar (2005) documented that some private equity firms consistently earned higher net returns than others across the funds they raise over time.[3] They also show that higher past fund returns leads GPs to increase the size of their next fund. Several studies find evidence of return persistence at the LP rather than the GP level (Lerner et al. 2007, Cagnavaro et al. 2018, Dyck and Pomorski 2016), where the PE portfolios of certain institutional investors consistently seem to outperform those of others.[4]

These empirical regularities raise a number of questions. How do the excess returns to private equity investments relate to liquidity risk? Why are PE-fund terms so unresponsive to performance and fundraising conditions, whereas fund sizes vary greatly? Is the tendency of LPs to increase private equity commitments after strong returns consistent with rational behavior, or is it simply a sign of overheated markets and "too much money chasing too few deals"? Why do some GPs seem to consistently deliver higher net returns to their LPs across the funds they raise? If some GPs are simply better than others, why do they leave "money

[1] See Harris et al. (2014a) and Robinson and Sensoy (2016) for recent evidence.

[2] Kaplan and Stein (1993) and Gompers and Lerner (2000) provide evidence that PE boom periods are associated with higher deal valuations, suggesting that the negative relation between fundraising and returns is driven by increasing competition for a limited number of investment opportunities. Brown et al. (2018) illustrates the difficulty that LPs face avoiding this cyclicality.

[3] Harris et al. (2014a) find that the performance persistence in VC funds is equally strong throughout their sample period, while the persistence for buyout funds has weakened considerably for post-2000 vintages. Korteweg and Sørensen (2017) confirm that PE funds exhibit return persistence, but argue that persistence is not investable (partly because performance of the previous fund is not known at the time the next fund is raised, as in Phalippou 2010).

[4] Lerner et al. (2007) compare private equity portfolio returns for different types of LPs and find significant performance differences, with endowments exhibiting the highest returns. They argue that differences are both due to fund selection ability and access to oversubscribed funds. Using more recent data, Sensoy et al. (2014) no longer find any consistent differences in performance across LP types, although subsequent study including some of the same authors (Cagnavaro et al. 2018) do find evidence of performance persistence for individual LPs (rather than LP types). Focusing on public pension funds, Dyck and Pomorski (2016) document that LPs with larger PE portfolios significantly outperforms the ones with smaller portfolios, and attribute this to economies of scale in LP investment activities.

on the table" for LPs, rather than capture these rents in the form of higher fees (along the lines of Berk and Green 2004)? And how are some LPs able to consistently achieve higher returns than others in their private equity portfolio? Is it due to differences in access to superior funds, or due to fund-picking ability? And how will the growth in the secondary market for LP commitments affect fundraising and returns in the primary market?

To address these questions, we construct a theoretical model of delegated portfolio management that builds on two key features. First, GPs must be given incentives to properly manage investments for moral hazard reasons, as in Holmström and Tirole (1997). Second, LPs are exposed to an aggregate liquidity shock – like the consumers in Diamond and Dybvig (1983) – which limit their willingness to commit long-term capital. Using this simple framework, our model is able to rationalize many of the empirical findings above, without resorting to irrational agents, asymmetric information, or heterogenous investment skill.

We explain the private equity fund structure as a contractual solution to the agency conflict between LPs and GPs. It is optimal for LPs to commit capital for a series of investments rather than on a deal-by-deal investment basis: this makes it easier to incentivize GPs to add value to their investments. The compensation to GPs is a function of the overall performance of the fund and resembles the carried interest given to fund managers.[5]

In our model, investment is scalable and GPs face a trade-off between fund size and carried interest share. When PE investment becomes more attractive to LPs, either because liquidity risk decreases or because underlying PE investment opportunities improve, we show that GPs will respond by increasing fund size rather than their fees, consistent with the observed stickiness in carried interest.[6] The amount of external LP capital that GPs can raise is constrained by the co-investment GPs are able to make in their own funds. This suggests a simple explanation for why LPs increase their new fund commitments in response to strong previous performance. Following a successful fund, GP will have accumulated more wealth thanks to the fees they received. They can then provide a larger co-investment and raise more external capital in their next fund. This dynamic carries through to the macro level: following a successful vintage, GPs collectively accumulate more wealth and aggregate

[5]As we explain later, this result is a variant of the benefits of cross-pledging with delegated monitoring in Diamond (1984)

[6]Our model also suggests an explanation for the fact that there is more dispersion in carried interest among VC funds compared to buyout funds, since the VC investment technology is likely to be less scalable (Metrick and Yasuda 2010, Robinson and Sensoy 2013).

PE fundraising should increase. While this would be true even if private equity investments were fully liquid, liquidity risk plays a central role for generating patterns in fundraising and expected returns consistent with empirical evidence.

In particular, liquidity risk affects GPs profit through two distinct channels. First, when LPs face a lower likelihood of a liquidity shock, and/or such liquidity shocks are less costly, the premium they require for long-term investments goes down. This decreases the cost of capital for GPs, who are then able to raise larger funds. As a result, the empirical finding that PE fundraising as well as average fund sizes are negatively related to subsequent returns arises naturally in our model due to liquidity risk. Second, to avoid the risk of default by LPs who may experience liquidity shocks, we show that GPs choose to call more capital in the early life of the fund. Large first investments act as collateral, ensuring that LPs stand by their capital commitments for subsequent investments. However, this behavior reduces the incentive benefits from diversification across investments and constrains GPs to raise smaller funds at the expense of total profit.[7]

We analyze the effect of investor heterogeneity with two types of LPs, "good" and "bad'," with good LPs facing smaller liquidity shocks. Good LPs require a lower premium to invest in PE and their commitment problem is less severe. Raising capital from good LPs therefore allows GPs to run larger and more efficient funds. This creates a distinction between premium capital supplied by the good LPs and the total supply of capital available in the market. When premium capital is abundant, only good LPs invest in private equity. As the demand for LP capital grows, e.g. because underlying investment opportunities improve, premium capital eventually becomes scarce. GPs then choose to raise capital also from bad LPs, who are more exposed to liquidity shocks.

A key finding is that such investor heterogeneity can generate return persistence at both the GP and the LP level. The ability of good LPs to withstand liquidity shocks allows GPs to run more profitable funds by avoiding inefficient acceleration in drawdowns. Some GPs will therefore choose to cater only to good LPs by offering a higher expected return in their fund, while restricting access to bad LPs (i.e. these higher-returning PE funds will become "oversubscribed"). In equilibrium GPs will be indifferent between offering a high expected return to good LPs and a lower expected return to bad LPs. We therefore provide

[7] Ljungqvist et al. (2017) show that GPs sometimes accelerate their draw-downs of LP commitments as a function of both GP characteristics and market conditions.

an explanation for why some GPs systematically generate higher returns to their LPs, and why some LPs consistently are able to invest with these GPs, solely as a function of their different tolerance to liquidity shocks.[8]

In the final step of our analysis, we introduce a secondary market for LP investments. When a liquidity shock hits, LPs who are severely affected by this shock can gain by selling their partnership claims to investors with higher tolerance for illiquidity. This exit option is beneficial to LPs, who can fully or partially realize the value of their investment without holding their claim until maturity. We show that this *liquidity effect* in the secondary market lowers the return required by LPs to commit capital to PE funds and increases the size of the primary market.

Buyers in the secondary market are the LPs who are more resilient to liquidity shocks. They can spend any resource net of their own private equity commitments to buy secondary claims. When their aggregate resources are small, secondary claims trade at a discount as a result of *cash-in-the market* pricing. Discounts compensate buyers to provide liquidity in the secondary market. In period 0, these LPs then have more incentives to hoard cash to benefit from these discounts. This increases the return GPs have to offer in the primary market to these investors. We call this effect the *opportunity cost effect* of a secondary market. Those investors who made the primary market when there was no secondary market may now commit most of their resources to the secondary market.

We show indeed that the secondary market does not only increase the size of the primary market, it leads to a change in the investor base of private equity funds. LPs who are less resilient to liquidity shocks now invest in the primary market while investors who are more resilient focus on the secondary market. The first reason for this change is the *opportunity cost effect*: the latter investors find it more profitable to pick up discounts in the secondary market than to invest in the primary market. The second reason follows from the *liquidity effect*: bad LPs are now willing to invest at a lower return since they can exit when a liquidity shock hits. In addition, the secondary market reduces the default risk of these investors who can exit through a sale rather than through a default.

As a result, we show that segmentation between different funds in the primary market

[8]In an extension, we allow GPs to also have differential skills, and derive conditions under which good LPs match with good GPs. We find that positive assortative matching is sometimes, but not always the equilibrium outcome.

may disappear. GPs then only raise one type of funds instead of two types of funds with different returns in the absence of a secondary market. Hence, our model suggests that the recent growth of the secondary market may explain the decline in GP performance persistence in last private equity vintages documented by Harris et al. (2014a). In our model, there is less of a benefit for GPs to cater to good LPs. In addition, good LPs may require unsustainably high returns in the primary market given their outside option to go for cheap deals in the secondary market. While expected return differences across GPs should decrease, our model implies that the performance persistence among LPs should remain even when segmentation disappears in the primary market. By focusing on the secondary market, good LPs earn higher monetary returns when claims trade at a discount. In addition, good LPs who would invest in the same fund than bad LPs would still realize higher monetary returns because they would not sell their claim at a discount when a liquidity shock hits.[9]

Although our model does not exhibit investor irrationality, asymmetric information, or learning about GP skill, we do not dispute that such features may still be important in practice. Rather, we aim to provide a benchmark with rational and fully informed market participants, against which to assess documented empirical findings in the PE market. In this sense, we try to provide a private equity counterpart to the Berk and Green (2004) model of mutual funds that invest in liquid assets. The stylized structure of our model should also make it applicable to delegated portfolio management in illiquid asset classes more broadly, such as infrastructure, private credit, and real estate funds.

The remainder of the paper is structured as follows. Section 2 connects our paper to existing theoretical work on private equity. We lay out the basic model in Section 3. Section 4 analyzes the optimal fund structure, while Section 5 adds heterogeneity in Limited Partner types. We introduce the secondary market in section 6. Section 7 concludes.

# 2 Related Theoretical Literature

Our model builds on the Holmström and Tirole (1997) model of financing under agency frictions adding liquidity risk for investors. Holmström and Tirole (1997) provides a tractable framework for modeling the optimal contracting problem between LPs and GPs, which

[9]This last statement considers an out of equilibrium choice by good LPs. This is because good LPs either do not invest in the primary market or invest in funds targeted to them that deliver higher returns.

allows us to endogenize the fund structure, the fund size as a function of GP net worth and the compensation scheme. We model liquidity risk in the spirit of Diamond and Dybvig (1983) but with risk-neutral investors to obtain a simple representation of the premium for investing in illiquid assets. The endogenous fund structure and the liquidity risk generate the commitment problem for investors that is key to our analysis.

We are not the first ones to derive the economic structure of private equity funds as an optimal incentive contract between LPs and GPs. Our explanation for why GPs invest through funds, bundling several individual investments together, is very similar to Axelson et al. (2009): investing through funds rather than deal-by-deal creates some "inside equity" which makes it less costly to incentivize the GP. In their model, the quality of an investment is GP private information. Cross-pledging of cash flows prevents the GP from picking bad investments, as long as there is a possibility of finding other good investments. They also show that there is a role for third-party debt financing alongside PE fund capital, as a way of mitigating over-investment in bad projects. Their model takes investment size as given and does not consider determinants of equilibrium fundraising. In contrast to Axelson et al. (2009), we consider a moral-hazard problem for GPs which enables us to endogenize fund size and aggregate fundraising. More importantly, we study the consequences of illiquidity on expected returns and the role of the secondary market.

Several papers provide models of the excess return of private equity over public equity and its implications for portfolio choice. Sørensen et al. (2014) and Giommetti and Sørensen (2019) investigate the illiquidity cost of private equity to investors in dynamic portfolio-choice models. In their paper, the cost of private equity is that it exposes a risk-averse LP to additional uninsurable risk. Phalippou and Westerfield (2014) also solve a dynamic optimal portfolio allocation problem for a risk-averse investor. Similar to Sørensen et al. (2014), the cost of illiquid assets arise from suboptimal diversification, but they add the feature that fund capital calls are stochastic. They also consider the possibility of LP defaults and secondary market sales, although they take the discount in the secondary market to be an exogenous parameter. In contrast to these papers, we model LPs as risk-neutral investors, who suffer from liquidity shocks along the lines of Diamond and Dybvig (1983). While our model is static, we provide an equilibrium model of delegated portfolio management, where we endogenize PE fund and compensation structures as well as equilibrium returns in the primary and secondary markets.

Similar to us, Haddad et al. (2017) explain variation in buyout activity as a result of time-varying risk premia in an agency framework. As in the papers mentioned above, the excess return on private equity compensates risk-averse investors for holding an undiversified portfolio, which in turn is necessary to provide incentives for adding value to the investment.[10] Haddad et al. (2017) argue that this compensation increases as the overall market risk premium increases, leading to pro-cyclical fundraising activity. Their model does not distinguish between LPs and GPs, but focuses on the relationship between PE investors and their portfolio companies. In contrast, we model the liquidity premium as a compensation to LPs. We also analyze the frictions between GPs and LPs and provide an explanation for return persistence.

Hochberg et al. (2014) provide a theory to explain the documented return persistence for private equity funds. In their model, LPs learn the skill of the GPs in which they invest over time, leading to informational holdup when GPs raise their next fund. This informational holdup reduces the ability of good GPs to increase fees in their next fund, and leads to performance persistence across funds.[11] In contrast, our model can rationalize GP performance persistence without asymmetric information or differences in skill, as a rent provided to the most liquid LPs for providing capital. Our model incorporates a secondary market and this feature can explain why LP return persistence remains, despite the fact that GP return persistence seems to have weakened over time.

Similar to us, Lerner and Schoar (2004) argue that GPs have preference for investors with low costs of illiquidity. In their model, however, investors are uncertain about GPs' skill, and sales of LP claims in the secondary market are interpreted as negative signals of GP ability. While they do not derive an optimal fund structure, they argue that GPs endogenously limit trading of Limited Partnership claims to screen for "good" LPs. Our model allows for different funds to be raised in equilibrium offering different returns to their investors. We allow for an active secondary market and derive endogenous discounts to NAV that are not due to information asymmetries.

Finally, few papers have also modeled the secondary market for private equity fund shares.

[10] Ewens et al. (2013) use a similar mechanism to rationalize the high observed required rates of return that GPs use for evaluating PE investments.

[11] Relatedly, Glode and Green (2011) model the persistence in the returns to hedge fund strategies as a result of learning spillovers.

In Bollen and Sensoy (2016), a risk-averse LP allocates funds between public equity, private equity, and risk-free bonds, and has to sell PE assets at a discount if hit by an exogenous liquidity shock. They do not aim to determine primary and secondary market returns in equilibrium, but instead calibrate their model to data to determine whether observed returns and discounts can be rationalized. In our model, secondary market discounts are instead endogenously determined as a result of "cash-in-the-market pricing" when liquidity is scarce. In our model both the supply of liquidity and the long-term asset supply (private equity fund claims) is endogenous. Finally, the economic mechanism leading to market segmentation in our paper, where more liquid investors focus on the secondary market in the hope of capturing "fire-sale discounts" is reminiscent of Diamond and Rajan (2011).

# 3 Model

The model has three periods, denoted $t = 0, 1, 2$. The economy is populated with investors called LPs (Limited Partners) who have large financial resources and managers called GPs (General Partners) who have long-term investment opportunities but limited financial resources. GPs seek financing from LPs to leverage their investment skills. However, LPs face liquidity shocks that reduce their willingness to commit capital for long-term investments.

**LPs**

There is a mass $M$ of risk-neutral LPs who consume in period 1 and 2. Each LP is endowed initially with 1 unit of cash storable at a rate equal to 0. In period 1, an aggregate liquidity shock hits the economy with probability $\lambda \in (0, 1)$. When the shock hits, LPs strictly prefer to consume early in period 1. Their preferences are given by

$$u(c_1, c_2) = \begin{cases} c_1 + c_2, & \text{with prob.} \quad 1 - \lambda \\ c_1 + \delta c_2 & \text{with prob.} \quad \lambda \end{cases} \tag{1}$$

where $c_t$ denotes consumption in period $t$ and $\delta < 1$ is the LPs discount factor when a liquidity shock hits. These preferences imply that LPs require an extra net return $r(\delta, \lambda)$ to invest in long-term assets. This break-even rate for long-term investments is given by:

$$r(\lambda, \delta) := \frac{1}{1 - \lambda(1-\delta)} - 1 \tag{2}$$

This break-even rate increases with the probability of a liquidity shock $\lambda$ and with the severity of this shock $1-\delta$.

The liquidity shock is meant to capture an event that decreases investors' appetite for long-term assets such as private equity. In practice, this could be a financial crisis associated with a flight to liquidity or a regulatory change in the treatment of the asset class.[12]. In the first part of the paper, all LPs are ex-ante identical and have the same value of $\delta$. We allow for investors heterogeneity in Section 5.

**GPs and Investments**

There is a unit mass of risk-neutral GPs who do not discount future cash flows. GPs have an initial endowment of $A$ units of cash in period 0. They have investment opportunities in period 0 and in period 1. Both these investments mature in period 2. Each investment returns $R$ per unit invested in case of success and 0 in case of failure. Period 0 and period 1 investments have independent returns.

Following Holmström and Tirole (1997) we assume that GPs must exert unobservable effort for an investment to be profitable. An investment succeeds with probability $p$ if the GP exerts effort. If the GP shirks, the probability of success of the investment is $q$ and the GP enjoys a private benefit $B$ per unit of funds invested. The following assumption ensures that an investment has positive NPV only when the GP exerts effort.

[12]For instance, under Solvency II regulation in Europe, unlisted private equity will sit under the "other equities" umbrella, which is currently allocated a shock buffer of 49%, meaning that for every 100 invested the insurer would have to hold up to 49 of capital

**Assumption 1** (Shirking destroys value)

$$pR \geq 1 \geq qR + B$$

The leftmost term in the inequality is the expected payoff when the GP exerts effort. The rightmost term is the monetary payoff plus the non-monetary payoff when the GP shirks. The moral hazard problem vis à vis external investors will imply that GPs must be incentivized with a claim to the investment cash flows. As it is standard, we assume that this claim needs to be large enough so as to constrain financing for GPs.

**Assumption 2** (Limited Pledgeability)

$$p\left(R - \frac{pB}{p^2 - q^2}\right) < 1$$

We will show that the left hand side is the maximum payoff the GPs can promise per unit of investment in a fund.[13] Assumption 2 means that this pledgeable income does not cover the investment cost. Hence, GPs cannot rely only on external finance and must co-invest. Finally, we impose that the resources of LPs are large compared to that of GPs.

**Assumption 3** (Abundant capital)

$$M \geq \frac{A}{1 - p\left(R - \frac{pB}{p^2-q^2}\right)} - A$$

The left hand side is the total resources in the hands of LPs. As will become clear later, the right hand side is the maximum amount of external financing that GPs can credibly raise. An interpretation of this condition is that the human capital of GPs needed to manage the investments is scarce compared to the financial capital available to LPs.

**Partnership Contracts**

In period 0, GPs compete for LPs' capital by offering investment partnership contracts. We assume that the contract terms cannot be made contingent on the realization of the

[13] The usual version of this condition is $p\left(R - \frac{B}{p-q}\right) < 1$. Our condition is more restrictive since, as we will show, pledgeable income is higher when GPs can finance two investments jointly in a fund.

liquidity shock.[14] A contract thus specifies the total fund size $I$ (including the co-investment $A$ by the GP), the share $x \in [0,1]$ of the fund resources $I$ called by the GP for the period 0 investment, and the compensation schedule of the GP

$$\Big\{w(y) \mid y \in \{0, R(1-x), Rx, R\}\Big\}, \qquad 0 \le w(y) \le y \tag{3}$$

The compensation schedule specifies the fee for the GP per unit of investment for each of the four possible cash flows of the fund. For instance, cash flow $Rx$ corresponds to a success of the first investment (share $x$) and a failure of the second investment (share $1-x$). When the unit cash flow is $y$, the total compensation of the GP is then equal to the fee multiplied by the fund size, that is $Iw(y)$. In period 2, the fund cash flows are realized and distributed according to the compensation schedule.

The key friction in our model is the commitment problem of investors. LPs only provide a fraction $x$ of their total capital commitment in period 0. In period 1, LPs hit by a liquidity shock may thus renege on the commitment to provide the remaining fraction $1-x$ of capital. Upon default, LPs lose any claim to the partnership cash flows.[15] To formalize the trade-off faced by LPs in period 1 when a liquidity shock hits, we introduce the net expected return $r_{PE}$ per unit of capital committed. As we will see, there is a direct mapping between the partnership contract features $(I, x, w(.))$ and $r_{PE}$. The no-default constraint writes:

$$\delta(1 + r_{PE}) \ge 1 - x \tag{4}$$

The right hand side of (4) is the benefit from defaulting since the LP then avoids the second capital call $1-x$. The left hand side of (4) is the cost of defaulting equal to the expected value of a unit claim $1 + r_{PE}$ discounted at rate $1/\delta - 1$. LPs make the second capital call if the cost of defaulting exceeds the benefit. Figure 1 summarizes the timeline of the model.

**Discussion of the partnership contract**

A key feature of our model is that sequential investment generates a commitment prob-

[14] While this aggregate shock is observable, contracts contingent on the realization of this shock may not be enforceable by a court. We can replace this assumption by the milder requirement that only the investment size cannot be contingent on the realization of the shock.

[15] Quoting from Ippolito et al. (2017), "*default penalties [for LPs] are often written as long lists of punishments, ranging from relatively mild to very severe, implying the loss of some or all of the profits and the forfeiture of the defaulter's entire stake in the fund*". See also Litvak (2004).

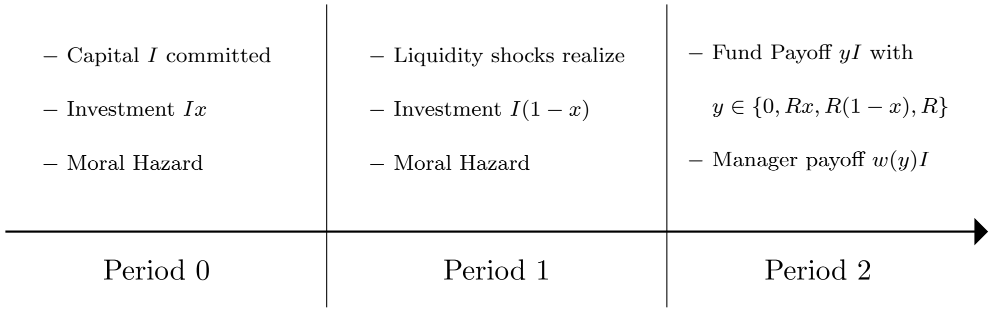


Figure 1: Timeline

lem because LPs face liquidity shocks. The no-default constraint (4) implies that GPs may need to increase the share of capital $x$ called and invested in period 0. We will show that this generate an investment distortion and decreases the profit of GP when liquidity shocks are severe (low $\delta$). In the following paragraphs, we discuss alternative ways to mitigate the commitment problem to explain why they cannot be implemented or why they would also generate costs for the GPs. Appendix B provides details and proofs of the claims below.

First, it may seem that the GP can avoid the commitment problem of LPs by calling extra capital in period 0. He would then hold as cash any capital called in excess of the amount needed for the first contractual investment. However, we show in Appendix B.1 that this arrangement is not incentive compatible. The GP would then deviate by investing all the capital called in period 0 and shirk on the investment. Hence, the original moral hazard problem in delegated investment also implies that GPs cannot credibly solve the commitment problem of their LPs by holding cash.[16]

Second, GPs may consider raising capital in period 1 from new investors to finance partially or fully the second investment. However, we show in Appendix B.2 that the expected cost of capital is higher in period 1 than in period 0. Hence, when allowed to raise capital in both periods, GPs may find it optimal to do so only in period 0 despite the commitment problem of initial investors. Hence, GPs would bear an alternative cost by raising capital in period 1 to mitigate the commitment problem of LPs. To simplify the analysis, we rule out period 1 financing entirely in the main text.[17]

[16] GPs would be unable to engage in this deviation if the cash was held in an escrow account. However, if LPs can earn a higher return on their alternative investment, GPs would have to compensate LPs for the opportunity cost of holding capital as cash. A straightforward extension of our model where LPs' outside option dominates cash would allow us to flesh out this insight. To conclude, the commitment problem of LPs generates a cost in any case.

[17] While our model abstracts from search frictions, we also observe that raising new capital would divert

Third, our assumption that the contract terms cannot be contingent on the realization of the aggregate shock implies that GPs call for capital in period 1 even when the liquidity shock hits. GPs could still introduce state contingency *de facto* by designing a contract such that LPs would default after a liquidity shock. In this event, GPs would then become the sole claimants of the smaller residual fund. The implicit assumption underlying the no-default condition (12) is that GPs find it too costly to downsize the fund. Our assumption can be micro-founded with up-front costs (salaries, searching for deals) that the GPs would pay before the actual investment takes place.

# 4 Fund Design

Our analysis in this section delivers three main results that capture institutional features of private equity. First, GPs face a trade-off between fee and size. In our model with scalable investments, it is optimal to maximize fund size while keeping fees just high enough to incentivize the GP. Second, GPs can raise larger funds and increase profit when LPs are less sensitive to liquidity risk. Finally, our model rationalizes the fund structure whereby LPs commit capital for a series of investments rather than on an investment by investment basis.

**Fee Size Trade-off**

Let us consider a partnership between a GP and LPs. Take as given the share of capital $x \in [0, 1]$ allocated to the first investment, chosen such that the no-default constraint (4) holds. Under Assumption 1, the GP should exert effort on both investments. This requirement defines a set of incentive compensation schedule and we denote by $\underline{W}_x$ the minimum expected fee GPs can credibly charge. Since the GP exerts effort in each period, both investments have an expected return of $pR$ per unit invested. The total expected return of the fund is thus independent of $x$ and it is equal to $pRI$ where $I$ is the size of the fund. Hence, given an expected fee $W \geq \underline{W}_x$ resulting from an incentive-compatible compensation schedule, the total expected payoff net of fees to LPs when investing in a fund of size $I$ is given by

$$(pR - W)I$$

the GP's attention from ongoing investments. In practice, partnership contracts that require GPs to raise new capital might not be feasible.

We can now relate the fund size $I$ and the expected fee $W$ with the buy and hold return $r_{PE}$ required in equilibrium by LPs. GPs act competitively and take this return as given. The contribution of LPs to a fund is equal to $I - A$ since $A$ is the GP's co-investment. Since their total payoff is equal to $(pR - W)I$, LPs earn return $r_{PE}$ on their investment if

$$\frac{(pR - W)I}{I - A} = 1 + r_{PE} \tag{5}$$

The GP must ensure that the fund delivers to investors at least the rate of return required by the market. Obviously, the GP has no incentive to deliver a higher return than $r_{PE}$ so equation (5) holds as an equality.

GPs face a trade-off between fees $W$ and fund size $I$. Suppose indeed that equation (5) holds and that a GP considers increasing the fund size $I$. Then, he must reduce the expected fee $W$ charged to LPs. Increasing the fund size decreases the co-investment share $A/I$ of the GP. To maintain the same expected return to investors, the GP must then reduce the fee per unit of investment. To derive the solution to this trade-off, let us write the total profit earned by a GP as:

$$\Pi_{GP} = \max_{W \in [\underline{W}_x, pR]} WI \qquad \text{subject to} \quad (5) \tag{6}$$

Observe from equation (5) that choosing $W = pR$ means that the GP only invests his own resources $A$. In our model, investment is scalable and this trade-off has a simple solution. GPs charge the minimum expected fee to maximize fund size.

**Lemma 1** (Fee Size Trade-off)
*GPs raise capital from LPs if and only if LPs' break-even rate is low, that is if*

$$r(\lambda, \delta) < pR - 1 \tag{7}$$

*If condition (7) holds, the equilibrium return on a private equity commitment is*

$$r^*_{PE} = r(\lambda, \delta) \tag{8}$$

*GPs then charge the minimum feasible expected fee $W^* = \underline{W}_x$.*

Since capital supply is abundant by Assumption 3, the equilibrium return on a private equity commitment $r^*_{PE}$ is equal to the investors' break-even rate for long-term investments

$r(\lambda,\delta)$. Hence LPs are indifferent between storing cash and committing capital to private equity funds in equilibrium. Given the rate $r(\lambda,\delta)$ required by LPs, condition (7) for GPs to raise capital has an intuitive interpretation. When it does not hold, LPs would not invest even if they could manage investments themselves. In this case, the cost of capital faced by GPs is too high and they only invest their own resources.

To understand why GPs choose fund size $I$ over fees $W$, it is useful to rewrite the total profit of GPs as:

$$\Pi_{GP} = pRI - (1 + r(\lambda,\delta))(I - A)$$

This payoff is equal to to the total fund cash flows minus the cost of external financing equal to $\left[1 + r(\lambda,\delta)\right](I - A)$. When condition (7) holds, it is optimal to maximize the fund size $I$ and GPs reduce the expected fee they charge to its minimum feasible value $\underline{W}_x$.

To further characterize the equilibrium, we must derive the incentive compatible compensation schedule that minimizes the expected fee for a given value of $x$. The second step is to find the investment split $(x, 1-x)$ that minimizes this fee $\underline{W}_x$ under the no-default constraint (4). Note that the fund composition $x$ does not affect the total expected payoff $pR$ since the two investments are ex-ante identical.

**Minimum Fee**

We first show that there is an intermediate range of values of $x$ that allows the GP to minimize the expected fee they charge. This benefit from diversifying the fund capital justifies why GPs and LPs contract for a series of investments rather than on a deal-by-deal basis. Second, we show that the commitment problem faced by LPs may limit these benefits.

We first derive the minimum expected fee $\underline{W}_x$ charged by GPs, focusing on the case $x \in [1/2, 1]$ without loss of generality. Under risk-neutrality, it is a well known result that the GP should be paid only after the outcome most informative about effort exertion. Since a success of two independent investments is always (weakly) more informative about effort exertion that a success of a single investment, we have

$$w(0) = w(Rx) = w(R(1-x)) = 0 \tag{9}$$

that is the GP should be paid only if both investments succeed.[18] The incentive constraint for the GP is then given by the following inequality:

$$p^2 w(R) \geq \max\left\{q^2 w(R) + B, pqw(R) + Bx, pqw(R) + B(1-x)\right\} \tag{10}$$

where the payoffs on the right hand side correspond respectively to the case where the GP never exerts effort, exerts effort only on the second investment and exerts effort only on the first investment. In each case, the GP receives private benefits proportional to the fraction of the investment for which he shirks. Note that the probability of a joint success of the investments is reduced to $pq$ (resp. $q^2$) when shirking on one (resp. two) investments. Lemma 1 showed that the GP should seek to minimize the fee charged to LPs. Hence, constraint (10) should bind. Since we focus on the case $x \geq 1/2$, the compensation of the GP after a joint success is given by

$$\underline{w}(R) = \begin{cases} \frac{B}{p^2-q^2} & \text{if} \quad x \in \left[\frac{1}{2}, \frac{p}{p+q}\right] \\ \frac{B}{p(p-q)}x & \text{if} \quad x \in \left[\frac{p}{p+q}, 1\right] \end{cases} \tag{11}$$

and the minimum expected fee is simply given by $\underline{W}_x = p^2 \underline{w}(R)$. This expected fee is minimal when $x \in \left[\frac{1}{2}, \frac{p}{p+q}\right]$ since then, the diversification benefits are maximal.[19] Figure 2 illustrates this result by plotting $\underline{W}_x$ as a function of $x$ where the region $[0, 1/2]$ is obtained by symmetry.

To understand the role of the fund structure in lowering the minimum expected fee $\underline{W}_x$, let us consider a contract where the GP compensation is independent across investments. This is equivalent to a scheme where GPs finance investments separately. The compensation

[18] We verify our claim about informativeness in the proof of Proposition 1.

[19] The reader may observe that when $R$ is small, it can be that $R - w^*(R) \leq Rx$. In this case, the LPs' claim would not be monotonic in the fund cash flows. This monotonicity constraint is often imposed on grounds of moral hazard to avoid misreporting of the cash flows by the manager (see for instance Innes 1990). In Online Appendix C, we derive the optimal fund design under the monotonicity constraint on the LPs' claim. Essentially, this constraint makes it harder to give steep incentives to the GP. However, the results from Proposition 1 still hold: diversification across investments is optimal and these benefits are lower when raising funds from investors with a low value of $\delta$.

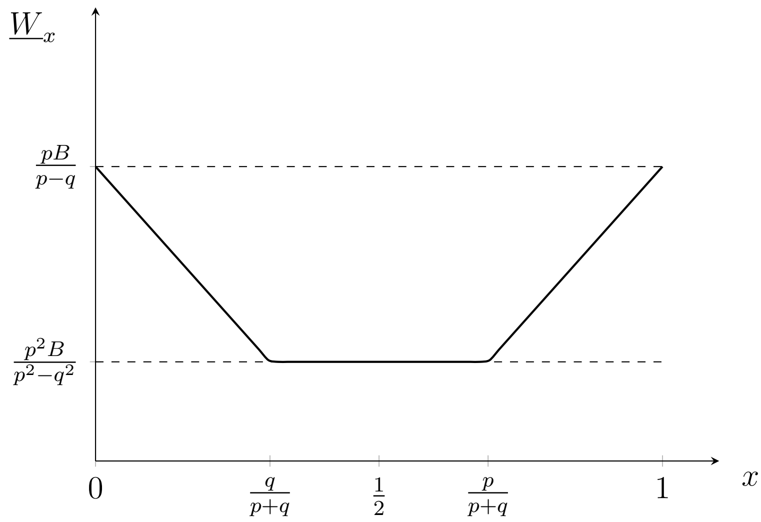


Figure 2: Minimum expected fund fee ($x$: *share of the first investment*)

schedule would then be given by

$$\tilde{w}(0) = 0, \quad \tilde{w}(R(1-x)) = \underline{w}(1-x), \quad \tilde{w}(Rx) = \underline{w}x, \quad \tilde{w}(R) = \underline{w}$$

where $\underline{w} := \frac{B}{p-q}$ is the minimum fee the GP must charge to exert effort. To see this, observe that with such a compensation scheme, the incentive constraint for each investment can be considered in isolation. Per unit of investment, it writes $p\underline{w} \geq q\underline{w} + B$, hence the result above. The expected fee of the GP would then be equal to $\tilde{W} = \frac{pB}{p-q}$ which is strictly higher than the minimum expected fee $\underline{W}_x$ unless $x \in \{0, 1\}$, as shown by Figure 2. The fund structure provides better incentives to the GPs by tying the compensation for one investment to the payoff of the other investment, a result sometimes referred to as *cross-pledging*.

**Investment distortion**

Our analysis thus shows that GPs would choose $x$ within the range $\left[\frac{q}{p+q}, \frac{p}{p+q}\right]$ in the absence of further constraints. However, GPs must also ensure that LPs do not default on their second capital call. Using the result in Lemma 1 that $r_{PE} = r(\lambda, \delta)$, we can rewrite the no-default constraint (4) as

$$x \geq \hat{x}(\lambda, \delta) := 1 - \delta(1 + r(\lambda, \delta)) = 1 - \frac{\delta}{1 - \lambda(1-\delta)} \tag{12}$$

It is intuitive that the no-default constraint ultimately imposes a lower bound $\hat{x}(\lambda, \delta)$ on the share of the fund capital allocated to the first investment. Since LPs lose the proceeds from the first investment when they default on the second capital call, it effectively acts as collateral. The threshold $\hat{x}(\lambda, \delta)$ increases with the severity of the liquidity shock $1-\delta$. Then, the cost from defaulting decreases since LPs attach little weight to the foregone profits on the first investment. Expression (12) shows that $\hat{x}(\lambda, \delta)$ decreases with $\lambda$. The commitment problem becomes less severe if the probability of the aggregate liquidity shock increases. This result arises because the return $r(\lambda, \delta)$ paid to LPs must increase with $\lambda$. This implies that the cost of defaulting goes up together with the value of the claim.

Given the result of Lemma 1, the optimal fund structure is easy to derive. Since GPs seek to minimize the expected fee $\underline{W}_x$, they choose a value of $x$ in the region $[\frac{q}{p+q}, \frac{p}{p+q}]$ that satisfies inequality (12), if any. Otherwise, since $\underline{W}_x$ is increasing over $[\frac{p}{p+q}, 1]$, the optimal choice of $x$ is pinned down by the no-default constraint (12) satisfied as an equality. The following Proposition formalizes these observations.

**Proposition 1** (Diversification Benefits)
*The fee charged by GPs is equal to $W^* = p^2 \underline{w}(R)$ where $\underline{w}(R)$ is the compensation paid only in case of joint success, given by equation (12). The first capital call is given by:*

$$x^* = \begin{cases} x \in [\max\{\underline{x}, \hat{x}(\lambda, \delta)\}, \bar{x}] & \textit{if} \quad \delta \geq \hat{\delta}(\lambda) \\ \hat{x}(\lambda, \delta) & \textit{if} \quad \delta < \hat{\delta}(\lambda) \end{cases} \tag{13}$$

*where $\underline{x} = \frac{q}{p+q}$, $\bar{x} = 1 - \bar{x}$, $\hat{x}(\lambda, \delta)$ is given by equation (12) and*

$$\hat{\delta}(\lambda) := 1 - \frac{p}{p + (1-\lambda)q} \tag{14}$$

Proposition 1 shows that the capital of a private equity fund should optimally be deployed over two investments rather than one. We showed indeed that the expected fee charged by GPs can be minimized when choosing an intermediate value $x \in [\underline{x}, \bar{x}]$ for the share allocated to the first investment.[20] Observe that these diversification benefits are not driven by

[20]The fact that the exact value of $x$ is not pinned down although it can be part of the contract is a realistic feature. The contract only predicts that GPs should not over-invest in a given period. In practice,

risk-sharing motives or complementarity between investments. In our model, diversification is instead a way to discipline GPs at a lower cost. With two independent investments, LPs receives two independent signals about effort instead of one. Tying the compensation of the GP to the joint outcome of these investments then reduces the fee GPs must charge to exert effort per unit of total investment, as we explained above.[21] This benefits GPs who can increase fund size and total profit, as shown by Lemma 1.

The key result from Proposition 1 is that the limited commitment of LPs may induce GPs to inefficiently distort the fund structure. GPs need to call and invest enough capital in period 0 to avoid default on period 1 capital calls by their LPs. When liquidity shocks are really severe, that is when $\delta \leq \hat{\delta}(\lambda)$, the minimum share to be called $\hat{x}(\lambda, \delta)$ lies outside the optimal region $[\underline{x}, \bar{x}]$. GPs are forced to call "too much" capital early and do not reap the full incentive benefits of diversification. This distortion increases the expected fee and it reduces fund size and total profit for GPs thus decreases.[22]

We thus showed that GPs incur a cost when raising capital from LPs who face severe liquidity shocks, beyond the cost of capital captured by the break-even rate $r(\lambda, \delta)$. The expression for the threshold $\hat{\delta}(\lambda)$ in (14) shows that this cost of commitment decreases with $p/q$, or with $\lambda$. When $p/q$ is large, diversification benefits arise for a larger range of values of $x$, as shown by equation (11). This allows GPs to increase the first capital call $x$ in order to avoid default by LPs while preserving the diversification benefits. When $\lambda$ increases, the cost of capital $r(\lambda, \delta)$ and thus the return to LPs goes up. Since the foregone profits would be higher, it is more costly for LPs to walk away.

### Comparative Statics

Using Proposition 1, we can derive the equilibrium fund size and the GP's profit. Dis-

partnership agreements typically specify investment concentration limits. See for example Schell et al. (2019).

[21] This insight about the benefits of diversification in the context of delegated monitoring is originally due to Diamond (1984). Axelson et al. (2009) rely on a similar argument to derive the compensation structure of GPs in a model with asymmetry of information.

[22] In a May 15, 2009 article for *PE Hub, Buyout Insiders*, Erin Griffith observes:

> *How are general partners avoiding that potentially messy situation* [default by investors]*? Some funds have drawn up to 20% of their capital upon closing, panelists at the Masterclass said. Its a way for GPs to make sure their investors have skin in the game from the start.*

tinguishing between the two cases in Proposition 1, we have

$$W^* = \begin{cases} \frac{p^2 B}{p^2 - q^2} & \text{if} \quad \delta \geq \hat{\delta}(\lambda) \\ \left[1 - \delta(1 + r(\lambda, \delta))\right] \frac{pB}{p-q} & \text{if} \quad \delta < \hat{\delta}(\lambda) \end{cases} \tag{15}$$

$$I^* := \frac{A}{1 - \frac{1}{1+r(\lambda,\delta)} \left[pR - W^*\right]} \tag{16}$$

$$\Pi_{GP}^* := W^* I^* \tag{17}$$

The fund size $I^*$ is linear in the GP's own contribution to the fund $A$. The ratio between the two variables is sometimes called the equity multiplier. This equity multiplier is large when capital is cheap, that is $r(\lambda, \delta)$ is low or when the pledgeable income per unit of investment, equal to $pR - W^*$ is high. Both the probability of a liquidity shock $\lambda$ and the severity of this shock $1 - \delta$ increases the cost of capital for GPs. When $\delta < \hat{\delta}(\lambda)$, a more severe liquidity shock further reduces the pledgeable income since the investment schedule is suboptimal. The following results formalize these observations.

**Corollary 1** (Comparative Statics)
*Fund size $I^*$ and GPs' profit $\Pi_{GP}^*$ are decreasing in LPs' liquidity risk $\lambda$ and in the severity of the liquidity shock $1 - \delta$. They are increasing with the investments' payoff $R$ and with the probability of success $q$ when $p - q$ is kept constant.*

The effect of an increase in the probability of the liquidity shock $\lambda$ is not obvious. A higher value of $\lambda$ increases the cost of capital $r(\lambda, \delta)$ but this increase in $r(\lambda, \delta)$ indirectly relaxes the no-default constraint (12). However, the first effect dominates and size and profit decrease with $\lambda$. This finding confirms the implicit result in Proposition 1 that GPs would rather distort investment than increase returns to alleviate the commitment problem.

The effect of an increase in $R$ is intuitive. When investments are more profitable, the total payoff to investors net of fees per unit of investment goes up. GPs can then scale up their funds to bring back investors return at their break-even rate. When varying the probability of success $q$ under shirking, we fix the difference $p - q$. This allows us to isolate the contribution from a better economic environment (higher $q$) from that of having more efficient GPs (higher $p$). When $p$ increases but $q$ does not, investment becomes more profitable but incentives are also cheaper to provide. Both these effects contribute to an increase in size and profit.

**Empirical Relevance**

Our model can rationalize several empirical relationships between PE fund compensation, fundraising, and returns that have been documented in the literature. Fees in private equity vary remarkably little across funds and over time, especially when it comes to the carried interest ($w(R)$ in our model), where 94% of the PE funds in Robinson and Sensoy (2013) have a carried interest of exactly 20%. Instead, average funds size increases in periods of high aggregate fundraising. Our model indeed suggests that GPs should increase fund size rather than fees. Kaplan and Schoar (2005) also find that PE firms raise the size of their funds when previous fund performance has been relatively strong. In our model, successful GPs will have earned higher carried interest and will therefore have more wealth $A$ to invest in their next fund. This, in turn, increases the amount of capital $I - A$ they can raise from LPs.[23] Finally, Kaplan and Strömberg (2009) show that funds raised during strong fundraising periods have lower returns. In our model, GPs indeed raise more capital when the compensation for illiquidity required by investors $r(\lambda, \delta)$ and thus returns are low.

A straightforward extension of our model could also help explain some systematic differences between buyout and VC funds that have been documented in the literature. Our baseline result that GPs prefer to increase size rather than fees relies on the technological assumption that investment is perfectly scalable, that is $R$ or $p$ do not depend on $I$. This assumption is more plausible for buyout funds, where a manager who raises a larger fund can simply acquire larger portfolio companies using a similar investment approach. In contrast, a VC manager investing in early-stage start-ups cannot as easily scale up the amount invested in any given company, since start-ups are almost by definition bounded in size.[24] With limited investment scalability, our model would predict that successful GPs should respond by increasing fees $w(R)$ when they cannot increase $I$. Consistent with this prediction, Robinson and Sensoy (2013) shows that the variation in carried interest is much lower in buyout funds,

[23]Stretching the theory, one could imagine that strong performance in the past fund would lead LPs to increase their expectation of $R$ and/or $p$, which would also lead to an increase in $I$.

[24]This reading is supported by Metrick and Yasuda (2010) who find that buyout managers build on their prior experience by increasing the size of their funds faster than VC managers do, and conclude that the buyout business is more scalable than the VC business. The limited scalability of VC is also supported in Kaplan and Schoar (2005), who find that the sensitivity of fund size to past performance is significantly stronger in buyout compared to VC.

where only 1% of funds have carried interest above 20%, compared to VC funds where this number is 10%.

We conclude this section by highlighting the dual role of liquidity risk which is specific to our model. Investors that are less sensitive to liquidity risk have a lower break-even rate $r(\lambda, \delta)$ for long-term investments. This decreases the cost of capital for GPs who raise larger funds and earn more profit. However the break-even rate does not fully capture the cost of illiquidity for GPs. Among two group of investors with the same break-even rate $r(\lambda, \delta)$, GPs prefer LPs for which $\delta > \hat{\delta}(\lambda)$, that is LPs who are better able to comply with capital calls. The default risk on capital calls for "bad" LPs translates into an investment distortion for GPs who must ultimately raise smaller, less profitable funds. We show in the next section that this second feature explains why "good" LPs can earn higher returns in equilibrium.

# 5 Heterogeneous LPs

In our analysis so far, GPs faced an homogeneous population of LPs with the same preferences for liquidity. In this section, we consider two classes $i \in \{L, H\}$ of LPs who differ according to the severity of the aggregate liquidity shock $1 - \delta_i$.

**Assumption 4** (Heterogeneous LPs)

$$\delta_L < \hat{\delta}(\lambda) < \delta_H \tag{18}$$

According to our previous analysis, Assumption 4 implies that $H$-LPs are better investors than $L$-LPs for private equity. They require a lower rate of return, that is $r(\lambda, \delta_H) < r(\lambda, \delta_L)$ and they do not risk defaulting on capital calls. This assumption is meant to capture the significant heterogeneity in maturity profiles, investment horizons or exposure to regulatory shocks among institutional investors in private equity. The total resources $M$ of investors are divided between $H$-LPs with a share $\mu_H$ and $L$-LPs with a share $1 - \mu_H$.

Our objective it to analyze the impact of a change in $\mu_H$ on the private equity market equilibrium. The variable $\mu_H$ captures the average appetite for illiquidity in the population of investors. For simplicity, we assume that the same contract must be offered to all investors in a given fund. However, GPs can still select the type of LPs they allow in their fund.[25]

[25] If types were not observable, $L$-LPs would pretend to be $H$-LPs and default on their capital calls when

Our assumption will imply that a given GP raises capital from one type of LP. We thus call $i$-fund a fund with $i$-LPs as investors.[26]

In this setting, two pairs of variables describe an equilibrium. First, we define $\alpha_i \in [0,1]$ as the fraction of GPs who raise a $i$-fund. Second, we let $r_{PE,i}$ be the return on a dollar invested by a LP in a $i$-fund. The analysis in Section 4 showed that the expected fee $W_i^*$ in a $i$-fund is given by

$$W_H^* := \frac{p^2 B}{p^2 - q^2}, \qquad W_L^* := \frac{pB}{p-q}\hat{x}(\lambda, \delta_L) > W_H^* \tag{19}$$

Given an expected return $r_{PE,i}$, the supply of capital by $i$-LPs is given by

$$S_i(r_{PE,i}) := \begin{cases} 0 & \text{if} \quad r_{PE,i} < r(\lambda, \delta_i) \\ S \in [0, \mu_i M] & \text{if} \quad r_{PE,i} = r(\lambda, \delta_i) \\ \mu_i M & \text{if} \quad r_{PE,i} > r(\lambda, \delta_i) \end{cases} \tag{20}$$

As we saw, risk-neutral LPs supply all their capital when the return on investing exceeds their break-even rate. Using equation (5), the demand from capital by GPs managing a fund of type $i$ can be expressed as a function of the return required by $i$-LPs:

$$I_i(r_{PE,i}) - A := \frac{A}{\frac{1+r_{PE,i}}{pR - W_i^*} - 1} \tag{21}$$

The profit of a GP when managing such a fund is $\Pi_i(r_{PE,i}) = W_i^* I_i(r_{PE,i})$. An equilibrium is defined as follows:

**Definition 1** (Equilibrium with heterogeneous LPs)
*An equilibrium is given by returns $(r^*_{PE,L}, r^*_{PE,H})$ and a fund composition $(\alpha^*_L, \alpha^*_H)$ such that:*

1. *(Optimal fund choice) $\alpha_i^* > 0$ iff $\Pi_i(r^*_{PE,i}) = \arg\max \left\{ \Pi_L(r^*_{PE,L}), \Pi_H(r^*_{PE,H}) \right\}$.*

2. *(Market Clearing) For $i \in \{L, H\}$, $S_i(r^*_{PE,i}) = \alpha_i^*(I_i(r^*_{PE,i}) - A)$*

the liquidity shock hits. However, there is substantial evidence that GPs care about the liquidity profile of the investors allowed in the fund. See for instance Lerner and Schoar (2004).

[26]If we allow GPs to raise funds with different contracts tailored to each type of LP, funds with mixed investor composition can arise in equilibrium. Intuitively, if capital supplied by $H$-LPs, GPs would try to raise the minimum amount from $H$-LPs that avoids the investment distortion, rather than raising capital only from $H$-LPs. However, our key results survive: $H$-LPs earn higher returns and fund segmentation emerge when $\mu_H$ is low. The formal results are available upon request.

The first equilibrium requirement is that GPs only offer a given type of funds if it delivers the highest profit. It implies that $L$-funds and $H$-funds coexist in equilibrium only if they deliver the same profit. This observation will be key to interpret our main Proposition.

**Proposition 2** (Heterogeneous LPs)

*Under Assumption 4, there exists $(\underline{\mu}_H, \bar{\mu}_H)$ where $0 < \underline{\mu}_H < \bar{\mu}_H < 1$ such that*

1. *When $\mu_H \geq \bar{\mu}_H$, $\alpha_L^* = 0$ ($L$-LPs do not invest) and $r^*_{PE,H} = r(\lambda, \delta_H)$ and .*

2. *When $\mu_H \in [\underline{\mu}_H, \bar{\mu}_H]$, $\alpha_L^* = 0$ and*

$$r^*_{PE,H} = \frac{\bar{\mu}_H(A + \mu_H M)}{\mu_H(A + \bar{\mu}_H M)} [1 + r(\lambda, \delta_H)] - 1 > r(\lambda.\delta_H)$$

3. *When $\mu_H \leq \underline{\mu}_H$, $H$-LPs earn a strictly higher return on their PE investment,*

$$r^*_{PE,H} > r^*_{PE,L} = r(\lambda, \delta_L), \qquad \text{where} \quad W^*_H I_H(r^*_{PE,H}) = W^*_L I_L(r(\lambda, \delta_L))$$

   *The fraction $\alpha_H^* < 1$ of $H$-funds solves $\alpha_H^*(I_H(r^*_{PE,H}) - A) = \mu_H M$.*

The expression for the thresholds $\underline{\mu}_H$ and $\bar{\mu}_H$ can be found in the proof. Proposition 2 contains two important findings. GPs only raise $L$-funds if capital from $H$-LPs is very scarce, that is $\alpha_L^* = 0$ when $\mu_H > \underline{\mu}_H$. When $\mu_H < \underline{\mu}_H$, there are both $H$-funds and $L$-funds but $L$-LPs earn a strictly lower return than $H$-LPs. Hence, there is a pecking order for LPs capital and GPs are willing to pay a premium to $H$-LPs.

The first finding that $L$-LPs do not invest in private equity when $\mu_H$ is high is intuitive. Indeed, $H$-LPs have a lower break-even rate $r(\lambda, \delta_H)$ and they collectively have enough resources to meet the demand for capital $I(r(\lambda, \delta_H)) - A$ from GPs when $\mu_H$ is high. When $\mu_H$ is intermediate, the resources of $H$- LPs become scarce and the market can only clear if the equilibrium rate $r^*_{PE,H}$ increases above the break-even rate of $H$-LPs. As long as $\mu_H > \underline{\mu}_H$, $L$-LPs cannot compete away these rents because the return still falls short of their own break-even rate $r(\lambda, \delta_L)$. When $\mu_H$ decreases further below $\underline{\mu}_H$, the cost of capital for $H$-funds becomes so high that some GPs raise funds $L$-funds. At this point, the competition from $L$-LPs implies that $H$-LPs do not receive higher returns as $\mu_H$ goes down.

Our key finding is that GPs are willing to offer an extra return $r^*_{PE,H} - r^*_{PE,L} > 0$ to

$H$-LPs when both types of funds exist in equilibrium, that is when $\mu_H < \underline{\mu}_H$. GPs must charge a higher expected fee $W_L^* > W_H^*$ in a $L$-fund compared to a $H$-fund. Since total profit is decreasing in the expected fee $W_*$, $H$-funds would be more profitable if $r^*_{PE,H}$ were equal to $r^*_{PE,L}$. Hence, by the optimal fund choice condition of Definition 1, an equilibrium with both types of fund can only exist if the cost of capital is higher for a $H$-fund, that is $r^*_{PE,H} > r^*_{PE,L}$. This premium reflects the higher willingness of GPs to pay for capital supplied by $H$-LPs. In $H$-funds, GPs can optimally diversify their investments by limiting the amount of capital called in period 0. This allows them to raise larger and more profitable funds. GPs who compete for the capital provided by $H$-LPs must pay a premium when this special capital is scarce. Figure 3 illustrates our findings about PE returns with heterogeneous investors.

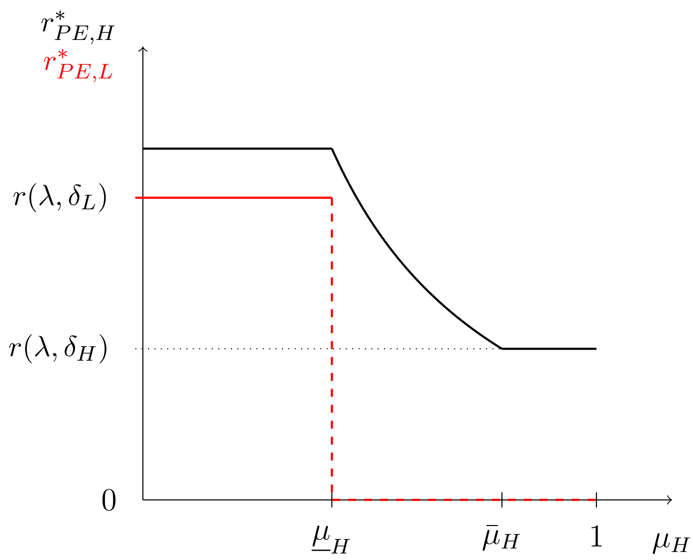


Figure 3: Returns from private equity investment

**Return Persistence for LPs**

Our results resonate with the recent evidence about performance persistence for LPs. Cagnavaro et al. (2018) and Dyck and Pomorski (2016) show that some LPs or types of LPs earn consistently higher returns on their private equity portfolio. Our theory is that GPs "cherry-pick" their LPs based on their liquidity profile. The best LPs earn a higher return compensating their ability to withstand liquidity shocks. GPs value this commitment ability of LPs which allows them to run more profitable funds. Our argument contrasts with the view that some LPs may be better at selecting their GPs. In particular, only our explanation based on LP screening can explain why some GPs ration access to their funds.

**Return Persistence for GPs**

Should return only persist at the investor level or also at the GP level as Kaplan and Schoar (2005) suggests? A simple extension of the model accommodates heterogeneity in GPs' skills as in Berk and Green (2004). Suppose that some GPs have special skills and their investment pays off $R_g$ in case of success while for other GPs, this payoff is only $R_b < R_g$. Our previous analysis suggests that when $\mu_H$ is low, GPs compete for scarce premium capital supplied by $H$-LPs. GPs win the competition if they are willing to pay a higher rate labeled $\bar{r}_{PE,H}(R)$ for premium capital. This rate is pinned down by the indifference condition of a GP between a fund with $H$-LPs and a fund with $L$-LPs:

$$\bar{r}_{PE,H}(R) = \frac{W_L^*\left(pR - W_H^*\right)}{\frac{W_H^*}{1+r(\lambda,\delta_L)}\left(pR - W_L^*\right) + W_L^* - W_H^*} - 1 \tag{22}$$

This expression obtains by solving for $r_{PE,H}$ using the indifference condition in Case 3 of Proposition 2 and $r^*_{PE,L} = r(\lambda, \delta_L)$. Since $\bar{r}_{PE,H}(R)$ is generically strictly monotonic in $R$ as we show below, it follows that one type of GP is willing to pay a higher return to raise a $H$-fund. Hence in heterogeneity in skills are persistent, return persistence also arises at the fund level. Perhaps surprisingly, we also show that $H$-LPs do not always match with the best GPs.

**Corollary 2**

*The return $\bar{r}_{PE,H}(R)$ is increasing in $R$ if and only if $W_H^* < 1+r(\lambda, \delta_L)$. When this condition holds, $H$-funds are raised by the GPs with higher absolute performance $R_g > R_b$.*

Corollary 2 shows that good GPs are not always willing to pay more for premium capital than bad GPs. The intuition is that good GPs also make more profit than bad GPs when raising funds from $L$-LPs. Hence, they may prefer raising cheap $L$-fund when their marginal benefit from investment in these funds $pR-(1+r(\lambda, \delta_L))$ is high. When instead the pledgeable income $pR - W_H^*$ in a $H$-fund is high, good GPs will raise capital from $H$-LPs. Hence, if the condition in Corollary 2 holds, the matching result between good LPs and good GPs naturally follows. When $\mu_H$ is low, $H$-LPs only invest in funds run by GPs with higher willingness to pay for premium capital.

This simple extension thus explains why the same funds may consistently deliver higher returns to their investors. However, our result suggests that positive assortative matching between good GPs and good LPs needs not be the equilibrium outcome. We also stress that return persistence at the fund level would disappear in our model if the only source of heterogeneity is GP skills. Better GPs would simply raise larger funds. Hence, according to our model, differences in LPs liquidity profile is a fundamental source of return persistence while heterogeneity in GPs skills is not. In the remainder of the text, we return to our benchmark where all GPs have the small investment skills.

We provide additional comparative statics for the premium earned by $H$-LPs. Corollary 2 already showed that investment profitability has an ambiguous effect on this premium. We show below that the premium increases in the probability of a liquidity shock but may decrease in the severity of the shock.

**Corollary 3** (Return Premium)
*When premium capital is scarce, that is $\mu_H < \underline{\mu}_H$, the premium $\frac{r^*_{PE,H} - r^*_{PE,L}}{1+r^*_{PE,L}}$:*
*– decreases in the probability of the liquidity shock $\lambda$.*
*– may increase or decrease in the severity of the shock $1 - \delta_L$ for $L$-LPs.*

When the probability of a liquidity shock or the severity of the shock increases, the break-even rate $r(\lambda, \delta_L)$ of $L$-LPs goes up. However, since there is not a full pass-trough to the rate $r^*_{PE,H}$ earned by $H$-LPs, the premium tends to go down. The overall effect of the severity of the shock $1 - \delta_L$ is ambiguous because funds raised from $L$-LPs become less efficient when the commitment problem of investors worsens. This increases GP's willingness to pay for premium capital. The overall effect of $1 - \delta_L$ on the return premium is thus ambiguous.

We conclude this section noting that the commitment friction studied in Section 4 is key to explain return persistence in our model. Suppose that we modify condition 4 assuming now that $\hat{\delta}(\lambda) < \delta_L < \delta_H$ so that the commitment problem of $L$-LPs is also moot. Obviously, $L$-LPs still have a higher break-even rate for investment than $H$-LPs. Our previous analysis thus implies that for high $\mu_H$, $H$-LPs are the only investors in private equity. However, because the commitment problem of $L$-LPs has no bite, GPs have the same willingness to pay for capital from $H$-LPs or $L$-LPs. This implies that $H$-funds and $L$-funds would deliver the same return to their investors in equilibrium when $\mu_H$ is low.

# 6 Secondary Market

We now consider a secondary market for LPs claims. The secondary market opens in period 1 after liquidity shocks are realized. The market in period 0 is now called the primary market. When a liquidity shock hits, the secondary market allows $L$-LPs who invested in the primary market to sell their claim to $H$-LPs. These gains from trade arise because $H$-LPs value period 2 cash flows more under Assumption 4. A claim entitles his new owner to the cash flow rights attached to \$1 of capital committed. This normalization implies that the initial LP makes the second capital call before selling the claim.[27]

The objective of this section is to understand the pricing of claims in the secondary market and the effect of the secondary market on the primary market for private equity. We call $P_L$ the secondary market price of a claim in a $L$-fund, that is a fund initially raised with $L$-LPs. Observe that the secondary market for claims in $H$-funds is inactive. When a $H$-LP is hit by a liquidity shock, no investor has a strictly higher valuation for his claim. Hence there is no gain from trade. We now explicit the role of the secondary market on investors' choice in period 1 and period 0.

**Secondary market in period** 1

As we observed, only $L$-fund claims trade in the secondary market. Given their primary market commitment $S_L(r_{PE,L}, P_L)$, the supply of claims from $L$-LPs is given by:

$$S^{sec}(r_{PE}, P_L) = \begin{cases} S \in [0, S_L(r_{PE,L}, P_L)] & \text{if} \quad P_L = \delta_L(1 + r_{PE,L}) \\ S_L(r_{PE,L}, P_L) & \text{if} \quad P_L > \delta_L(1 + r_{PE,L}) \end{cases} \tag{23}$$

By linearity of preferences, $L$-LPs sell their entire participation if the price exceeds their reservation value $\delta_L(1 + r_{PE,L})$. On the demand side, $H$-LPs can buy claims using any resources net of their own capital commitments $S_H(r_{PE,H}, r_{PE,L}, P_L)$ in the primary market. The demand for claims is thus given by:

$$D^{sec}(r_{PE,H}, r_{PE,L}, P_L) = \begin{cases} \frac{\mu_H M - S_H(r_{PE,H}, r_{PE,L}, P_L)}{P_L} & \text{if} \quad P_L < \delta_H(1 + r_{PE,L}) \\ D \in \left[0, \frac{\mu_H M - S_H(r_{PE,H}, r_{PE,L}, P_L)}{P_L}\right] & \text{if} \quad P_L = \delta_H(1 + r_{PE,L}) \end{cases} \tag{24}$$

[27] We could alternative assume that the new LP makes the capital call. The price of the claim would decrease by $1 - x$, reflecting the liability acquired by the buyer. The economics are unaffected.

With linear preferences, $H$-LPs spend all their available resources to buy claims when the price $P_L$ is lower than their reservation value $\delta_H(1 + r_{PE,L})$.

**Period 0 Fundraising by GPs**

In period 0, GPs raise funds taking the cost of capital $r_{PE,i}$ for $i$-funds as given. In a $i$ fund, GPs must also chose the share $x_i$ of capital called in period 0. As we showed in Section 4, the choice of $x$ is constrained by the commitment problem of LPs. The presence of a secondary market changes the no-default constraint of $L$-LPs who can now sell their claim. Condition (4) then writes:

$$\max\{\delta_L(1 + r_{PE,L}), P_L\} \geq 1 - x \tag{4b}$$

Equation (4b) shows that when $P_L$ is high enough, GPs need not worry about default risk from $L$-LPs. While $L$-LPs might have defaulted otherwise, the possibility to sell the claim at a high price increases their willingness to make the second capital call. The choice of $x_i$ by GPs is thus given by

$$x^*_H = \bar{x} \tag{25}$$

$$x^*_L = \max\left\{\bar{x}, 1 - \max\{\delta_L(1 + r_{PE,L}), P_L\}\right\} \tag{26}$$

While Proposition 1 showed that there is an optimal range of values of $x$, equations (25) and (26) are without loss of generality since $\bar{x}$ is the highest value in this range. Note that we showed that the commitment problem of $H$-LPs is moot under Assumption 4.

We now write the capital demand from GPs for each type of fund, using equation (20):

$$I_i(r_{PE,i}, x_i) - A = \frac{A}{\frac{1+r_{PE,i}}{pR - \underline{W}_{x_i}} - 1} \tag{27}$$

where $\underline{W}_x$ is the expected fee charged by the GPs, given by equation (11), when he calls a fraction $x$ of the capital in period 0. The GPs profit with a $i$-fund is thus given by $\Pi_i(r_{PE,i}, x_i) = \underline{W}_{x_i} I_i(r_{PE,i}, x_i)$. The secondary market price has a direct effect on the capital demand for $L$-funds since $x_L$ depends on $P_L$ by equation (26). We will show that in equilibrium the secondary market price also affects the capital demand indirectly through the return $r_{PE,i}$ required by $i$-LPs.

**LPs portfolio choice in period** $0$

We now consider the portfolio choice of LPs in period 0. We saw that in the absence of a secondary market, a $i$-LP commits all his resources to private equity if the return offered $r_{PE,i}$ exceeds his break-even rate $r(\lambda, \delta_i)$. With a secondary market, this comparison is not relevant anymore. To describe this new trade-off, we define $u_{PE,i}$ and $u_{c,i}$ as the net return in utils from one unit of capital invested in a PE fund and stored in cash respectively. For $L$-LPs, we have:

$$1 + u_{PE,L} = \lambda \max\{P_L, \delta_L(1 + r_{PE,L})\} + (1 - \lambda)(1 + r_{PE,L}) \tag{28}$$

$$1 + u_{c,L} = 1 \tag{29}$$

With a secondary market, $L$-LPs will sell their claim when the price $P_L$ exceeds their reservation value $\delta_L(1 + r_{PE,L})$ for the claim. In particular, if $P_L$ strictly exceeds this value, the return $r_{PE,L}$ that makes $L$-LPs indifferent is strictly lower than $r(\lambda, \delta_L)$. Since $L$-LPs can increase their return by selling their claim in the secondary market when a liquidity shock hits, they accept a rate lower than their break-even rate $r(\lambda, \delta_L)$ to commit capital in the primary market. We call this effect the *liquidity* effect of a secondary market.

The secondary market alters the $H$-LPs portfolio choice in a different way. These investors do not use the secondary market as a source of liquidity for their primary market investment but rather as buyers of claims sold by $L$-LPs. Their returns on a PE fund investment and cash are given respectively by:

$$1 + u_{PE,H} = \lambda\delta_H(1 + r_{PE,H}) + (1 - \lambda)(1 + r_{PE,H}) \tag{30}$$

$$1 + u_{c,H} = \lambda \max\left\{\frac{\delta_H(1 + r_{PE,L})}{P_L}, 1\right\} + 1 - \lambda \tag{31}$$

The difference between equations (30) and (28) arises because $H$-LPs hold on to their primary market investment even when a liquidity shock hits. Hence, if the outside option of $H$-LPs would deliver a zero net return, their minimum required return would be given again by their break-even rate $r(\lambda, \delta_H)$. This can be seen by setting $u_{PE,H}$ to 0 in equation (30). However, the utility weighted return on cash $u_{c,H}$ is now endogenous since $H$-LPs can buy claims in the secondary market. In particular, this return strictly exceeds 1 when $P_L < \delta_H(1 + r_{PE_L})$. At such a price, secondary claims trade cheap from the point of view of $H$-LPs. Hence, unlike for $L$-LPs, the minimum return required by $H$-LPs now exceeds their break-even rate $r(\lambda, \delta_H)$. We call this effect the *opportunity cost effect* of a secondary market.

The utility weighted returns allow us to write the portfolio choice of LPs in the primary market in a simple way. LPs invest in PE funds if the return on their investment exceeds the return on cash. The capital supply in the primary market is then given by:

$$S_i = \begin{cases} 0 & \text{if} \quad u_{PE,i} < u_{c,i} \\ S \in [0, \mu_i M] & \text{if} \quad u_{PE,i} = u_{c,i} \\ \mu_i M & \text{if} \quad u_{PE,i} > u_{c,i} \end{cases} \tag{32}$$

Equation (32) is similar to its counterpart (20) but the minimum returns required by LPs now depend on the endogenous pricing of claims in the secondary market. An equilibrium with a secondary market is defined as follows:

**Definition 2** (Equilibrium with a secondary market)
*An equilibrium is given by returns $(r^*_{PE,L}, r^*_{PE,H})$, a fund composition $(\alpha^*_L, \alpha^*_H)$ and a secondary market price $P^*_L$ such that the following conditions are satisfied:*

1. *(Optimal fund choice) $\alpha^*_i > 0$ iff $\Pi_i(r^*_{PE,i}, x^*_i) = \arg\max \left\{ \Pi_L(r^*_{PE,L}, x^*_L), \Pi_H(r^*_{PE,H}, x^*_H) \right\}$.*

2. *(Primary Market Clearing) For $i \in \{L, H\}$, $S_i = \alpha^*_i (I_i(r^*_{PE,i}, x^*_i) - A)$*

3. *(Secondary Market Clearing) $D^{sec}(r^*_{PE,L}, P^*_L) = S^{sec}(r^*_{PE,L}, P^*_L)$ if $\alpha_L > 0$*
   *and $P^*_L = \delta_H(1 + r^*_{PE,L})$ otherwise.*

*where $x^*_H$ and $x^*_L$ are given by equations (25) and (26).*

The definition of an equilibrium builds on Definition 1 adding the secondary market clearing condition. Note that if $\alpha^*_L = 0$, no $L$-fund is raised in equilibrium and there is no secondary market trading. GPs who contemplate raising $L$-funds must still form expectations about $P^*_L$. In this case, the equilibrium price is set to the highest valuation for the claim among all investors in the economy. This selection device avoids coordination problems whereby GPs do not raise $L$-funds because they expect low secondary market prices.

It will be useful to introduce the familiar concept of discount to Net Asset Value in the secondary market. In our model, the endogenous equilibrium discount $D^*$ is given by:

$$D^* = 1 - \frac{P_L^*}{1 + r^*_{PE,L}} \in [1 - \delta_H, 1 - \delta_L] \tag{33}$$

The bounds obtain because, using equations (23) and (24), the secondary market price can only clear if $P_L^*$ lies between $\delta_L(1 + r^*_{PE,L})$ and $\delta_H(1 + r^*_{PE,L})$. Note that in our model the lowest possible discount is strictly positive. When a liquidity shock hits, even the investors with the highest valuation for the claim discount period 2 cash flows. Hence, the existence of a discount follows mechanically from the assumption that $\delta_H < 1$. More interestingly, we will show that when liquidity is scarce in the secondary market, the discount increases above its baseline value.[28]

Before stating the main proposition, we show that in the presence of a secondary market, $L$-LPs require a lower return on their private equity investment than $H$-LPs. For a given discount $D$, we define this minimum return $\underline{r}_{PE,i}(D)$ as the value of $r_{PE,i}$ such that the return on cash $u_{c,i}$ is equal to the return on private equity $u_{PE,i}$ using equations (28)-(31).

**Lemma 2** (Cost of Capital)
*Let $D \in [1 - \delta_H, 1 - \delta_L]$ be the discount to NAV in the secondary market. The difference $\underline{r}_{PE,H}(D) - \underline{r}_{PE,L}(D)$ in the minimum rate of return required by $H$-LPs compared to $L$-LPs is equal to 0 when $D = 1 - \delta_H$ and it is increasing in $D$.*

Remember that without a secondary market, the opposite result holds since $H$-LPs break-even rate $r(\lambda, \delta_H)$ lies below the break-even rate $r(\lambda, \delta_L)$ of $L$-LPs. To understand why this inequality is now reversed, observe that the secondary market lowers the minimum rate required by $L$-LPs through the *liquidity effect*. Simultaneously, the minimum rate required

[28] In practice, the expected return is not known and must be estimated by GPs. Empirical studies like Albuquerque et al. (2018) show that PE fund claims sometimes trade at a premium over NAV (a negative discount). Our measure does not allow for negative discounts because we use a slightly different definition. The usual concept of discount to NAV only applies to the drawn portion of the commitment while we normalize by the expected values of all the commitments. In our framework, the standard definition would read

$$\hat{D} = 1 - \frac{P}{x(1 + r_{PE})}$$

Proposition 3 then shows that premia to NAV arise in equilibrium under this definition. Essentially, the standard concept of discount to NAV prices the remaining capital calls at cost while we use the expected *fair* value. Our measure can also be related to that used in Nadauld et al. (2018). In our model, $P$ is the "return to a seller" while $(1 + r_{PE})/P$ is the "the return to a buyer".

by $H$-LPs increases because of the *opportunity cost* effect. Lemma 2 thus shows that the sum of these two effects reverse the ranking between the minimum required rates of return.

Lemma 2 implies that the cost of capital in a $H$-fund necessarily exceeds that of a $L$-fund. This implies that the only rationale left for GPs to raise $H$-funds is to avoid the potential investment distortion when raising a $L$-fund. However, as we discussed, GPs may not even face this distortion if the secondary market is liquid enough. Furthermore, even if the claims trade at a large discount, raising a $H$-fund may still not be attractive. Indeed, Lemma 2 shows that the extra return GPs must pay to $H$-LPs increases in this discount $D$. In fact, we will show that under the following assumption, GPs never raise $H$-funds when the discount to NAV is higher than its benchmark value $1-\delta_H$

**Assumption 5**

$$\left[1+r(\lambda,\delta_H)\right]\left[1-\lambda+\frac{\lambda\delta_H(p+(1-\lambda)q)}{(1-\lambda)q}\right]\geq pR$$

Assumption 5 implies that the return $H$-LPs can make in the secondary market if claims trade at a discount is so high that GPs may never attract $H$-LPs in the primary market.[29] We may now state the main result of this section.

**Proposition 3** (Secondary Market Equilibrium)
*There are thresholds* $(\mu_{H,1},\mu_{H,2},\mu_{H,3})$ *such that*

*1) For* $\mu_H\leq\mu_{H,1}$, $r^*_{PE,L}=r(\lambda,\delta_L)$, $D^*=1-\delta_L$, $x^*_L=\hat{x}(\lambda,\delta_L)$ *and* $\alpha^*_H=0$

*2) For* $\mu_H\in[\mu_{H,1},\mu_{H,2}]$, $x^*_L=1-P^*_L>\bar{x}$, $\alpha^*_H=0$ *and* $(r^*_{PE,L},P^*_L)$ *solve*

$$r^*_{PE}=\underline{r}_{PE,L} \tag{34}$$

$$\left[I\left(r^*_{PE},x^*_L\right)-A\right]P^*_L=\mu_H M \tag{35}$$

*3) For* $\mu_H\in[\mu_{H,2},\mu_{H,3}]$, $x^*_L=\bar{x}$, $\alpha^*_H=0$ *and* $(r^*_{PE},P^*_L)$ *solve* (34)-(35).

*4) For* $\mu_H\geq\mu_{H,3}$, $r^*_{PE,L}=r^*_{PE,H}=r(\lambda,\delta_H)$, $D^*=1-\delta_H$, $x^*_L=\bar{x}$.

[29] Our objective is to highlight that the *opportunity cost effect* can be so strong that GPs may stop raising $H$-funds when secondary claims trade at a discount. We show however in Appendix A.8 that this result is not generic. When $\lambda$ is low enough, $H$-LPs also invest in the primary market in funds that offer higher returns than $L$-funds. Hence, the result in Proposition 2 that different funds with different returns coexist in equilibrium is weaker but it may survive.

The equilibrium allocation is uniquely pinned down up to the primary market allocation when $\mu_H \geq \mu_{H,3}$. Then, $H$-LPs and $L$-LPs are identical investors from the point of view of GPs. They require the same return $r(\lambda, \delta_H)$ on their capital and since $L$-LPs sell their claim at high price in the secondary market, GPs need not distort the investment schedule in a $L$-fund. The proof shows that $\mu_{H,3} < \bar{\mu}$ where $\bar{\mu}$ was the threshold below which the cost of capital exceeded $r(\lambda, \delta_H)$ without a secondary market. Hence, GPs can raise capital at the minimum rate of $r(\lambda, \delta_H)$ for lower values of $\mu_H$. The key intuition is that $H$-LPs need less resources overall to sustain prices in the secondary market than to make the primary market. The scarce resources of the $H$-LPs are redeployed towards their more efficient use in the secondary market. In general, we can show that the introduction of a secondary market reduces the cost of capital and increases profit for GPs for all values of $\mu_H$.

When $\mu_H \leq \mu_{H,3}$ however, $H$-LPs capital becomes too scarce to sustain fair secondary market prices. Then, secondary claims trade at a discount $D^* > 1-\delta_H$. From Lemma 2, this implies that $H$-LPs require a strictly higher minimum rate of return than $L$-LPs. Hence, GPs only raise $L$-funds while $H$-LPs allocate their resources entirely to the secondary market. As we mentioned, secondary claims trade at a discount when $\mu_H \in [\mu_{H,2}, \mu_{H,3}]$. But secondary market liquidity is still high enough to sustain the price $P_L^*$ above $1 - \bar{x}$. This allows GPs to choose an optimal investment schedule in $L$-funds. The higher cost of capital for $H$-funds and the absence of investment distortion in $L$-funds explain why $H$-funds are dominated for GPs. When $\mu_H \leq \mu_{H,2}$ however, liquidity is so scarce that the secondary market price falls below $1 - \bar{x}$. Then $L$-LPs would again default rather than sell their claim if GPs were to call $1 - \bar{x}$ in period 1. As a result, GPs increases the share of capital $x_L^*$ called in period 0 so that the modified no-default constraint (4b) holds.

As $\mu_H$ decreases below $\mu_{H,2}$, the secondary market price $P_L^*$ falls while the cost of capital $r^*_{PE,L}$ increases, forcing GPs to call an increasing share of the fund capital in period 0. The reduction in secondary market liquidity as $\mu_H$ goes down thereby reduces the profit of GPs through two channels. Investors require a higher return $r^*_{PE,L}$ and GPs need to distort more their investment schedule. When $\mu_H$ reaches $\mu_{H,1}$, the secondary market price attains its minimum level equal to the reservation value of $L$-LPs. Then, in the region $[0, \mu_{H,1}]$, $L$-LPs are indifferent between selling and holding their claim. This implies that the cost of capital for $L$-funds is equal to the break-even rate $r(\lambda, \delta_L)$ of $L$-LPs. Then, from the point of view of $L$-LPs, selling in the secondary market is the same as holding the claim. On the buy-side, $H$-LPs benefit from large discounts. The *opportunity cost effect* thus implies that they would require high returns to invest in the primary market. Under Assumption 5, this required return is so high that GPs may not break even with $H$-funds compared to $L$-funds although they can implement an optimal investment schedule $x_H^* = \bar{x}$ with the former. This explains why there is no $H$-fund in equilibrium also in the region $\mu_H \leq \mu_{H,2}$.

**Secondary market and competition**

The secondary market generates additional competition on the investor side. In particular, the provision of liquidity by $H$-LPs in the secondary market undermines the investment gains they make in the absence of a secondary market. As we observed before, the threshold $\mu_{H,3}$ in Proposition 3 lies below the threshold $\bar{\mu}_H$ in Proposition 2 without a secondary market. In each case, the thresholds are the lower bound of the region where $H$-LPs earn a strictly positive return. Hence, when $\mu \in [\mu_{H,3}, \bar{\mu}_H]$, $H$-LPs lose from the introduction of a secondary market. This result arises because $H$-LPs enact the competition from $L$-LPs in the primary market for private equity by providing liquidity in the secondary market.

**Return Persistence with a secondary market**

Under Assumption 5, we showed that essentially one type of funds is offered in equilibrium. This contrast with our result in Proposition 2 where in the absence of a secondary market, two types of funds with different returns necessarily coexist for $\mu_H$ low enough. Our model thus suggests that the presence of a secondary market tends to reduce segmentation in the primary market (see also footnote 29). Hence, part of the recent decrease in fund-level persistence documented by Harris et al. (2014b) may be attributed to the growth of the secondary market for LP partnership claims. Consistently with this prediction, these authors show that persistence is more robust for VC funds where the secondary market is less mature.

However, our model does not imply that return persistence should also disappear at the LP level when premium capital from $H$-LPs is scarce. Accounting for secondary market investments, $H$-LPs still realize higher returns on their private equity portfolio than $L$-LPs.

The average monetary return on a dollar committed to a $L$-fund for a $L$-LP is given by

$$1 + r^{avg}_{PE,L} = (1 - \lambda)(1 + r_{PE,L}) + \lambda P_L = 1 \tag{36}$$

since the investor sells his claim in the secondary market when a liquidity shock hits. The second equality follows from the participation constraint of $L$-LPs. The average monetary return on a dollar committed to the secondary market by a $H$-LP is equal to

$$1 + r^{avg}_{c,H} = \lambda + (1 - \lambda)\frac{1 + r_{PE,L}}{P_L} > 1 \tag{37}$$

The inequality follows from the observation that $P_L \leq \delta_H(1 + r_{PE,L})$ in equilibrium. Hence, $H$-LPs still earn a higher monetary return than $L$-LPs because they focus on the secondary market.

Interestingly, $H$-LPs would also perform better even if they could only invest in the primary market. Their average monetary return from investing in the $L$-funds offered in equilibrium is given by

$$1 + r^{avg}_{PE,H} = 1 + r_{PE,L} > 1 \tag{38}$$

The difference between equations (36) and (38) follows from the fact that unlike a $L$-LP, a $H$-LP would not sell his claim when hit by a liquidity shock. Since secondary claims trade at a discount, the observed return is higher for a $H$-LP. Note finally that in the main text we focused for simplicity on a case without primary market segmentation in equilibrium. As we show in Appendix A.8, under alternative parameter configurations, segmentation is still an equilibrium outcome. Then, the source of return differences between LPs would be the same than in Section 5 since $H$-funds would offer higher returns than $L$-funds.

# 7 Conclusion

This paper provides a model of delegated investment in private equity funds where investors are subject to liquidity risk. We derive the optimal partnership between GPs and LPs with a fund structure and a compensation contract that resemble actual partnership agreements. Because investors face liquidity risk, there is a pecking order for LPs' capital. GPs prefer to raise capital from LPs who are less sensitive to liquidity risk. These good LPs supply capital at a lower cost and are more likely to stand by their capital commitment. When this high quality capital is scarce, GPs pay a premium to good LPs. This finding rationalizes persistent differences in returns between PE investors. Our model thereby suggests that GPs cherry-pick their investors for their ability to provide long-term capital. We also study the introduction of a secondary market for LPs claims. Good LPs migrate from the primary market to the secondary market. Discounts to NAV endogenously arise when secondary market liquidity is scarce. Finally, our model suggests that fund-level persistence may disappear with a secondary market while LP-level persistence remains.

Our analysis rests solely on two factors: the agency problem between fund managers and investors and the investors' exposure to liquidity shocks that makes them averse to providing long-term capital. The fact that our model does not exhibit investor irrationality, or asymmetric information and/or learning about GP skill differences, does not mean we necessarily believe such features are not important in practice. Instead, we provide a benchmark against which to be able to judge whether the observed patterns are consistent with agents being informed and rational. In this sense, we offer a counterpart to the Berk and Green (2004) model of mutual fund for investments in liquid assets. We believe the stylized structure of our model makes it applicable to delegated portfolio management in other illiquid asset classes, such as infrastructure, private credit, or real estate funds.

Finally, although our model is static, the insights can be useful to understand some private equity market dynamics. It is a well documented fact that private equity fundraising and dealmaking are highly procyclical. While demand effects may be partially driving these fluctuations, our model suggests two potential supply-side effects. In downturns, investors might require a higher illiquidity premium and GPs' wealth is low. Both these effects contribute to a decrease in fund size and a lower level of fundraising.

# Appendix

## A Proofs

### A.1 Proof of Proposition 1

Building on our analysis in the main text, we are left to show two results. The first is that equation (9) holds, that is the GP is only compensated after a joint success. The second result is that it is suboptimal for the GP to increase the promised return over $r(\lambda, \delta)$ in order to relax the no-default constraint (4).

**Proof that (9) holds**

As stated in the text, since the GP is risk-neutral, he should only be compensated after the outcome most informative about effort exertion. For each relevant unit payoff $y$ in (3), we can then define an informativeness ratio

$$\mathcal{I}(y) = \frac{\mathbf{Pr}[\tilde{y} = y|\text{effort}]}{\mathbf{Pr}[\tilde{y} = y|\text{shirk}]} \tag{A.1}$$

The higher $\mathcal{I}(y)$, the better signal of effort is payoff $y$. Two cases need to be considered: either the GP shirks on both investments or he only shirks on the first investment. By symmetry, the argument is similar if he only shirks on the second investment. If the GP shirks on both investments, the probability of a joint success if $q^2$. We thus have

$$\mathcal{I}(R) = \frac{p^2}{q^2} > \frac{p(1-p)}{q(1-q)} = \mathcal{I}(Rx) = \mathcal{I}(R(1-x))$$

When the GP shirks on both investments, a single success obtains with probability $q(1-q)$. The strict inequality follows from $p > q$. Suppose now that the GP only shirks on the first investment. This means that the probability of a joint success when shirking is $pq$. The probability of a success of the first investment only is $q(1-p)$ and the probability of a success of the second investment only is $p(1-q)$. We now have

$$\mathcal{I}(R) = \frac{p^2}{pq} = \frac{p(1-p)}{q(1-p)} = \mathcal{I}(Rx) > \frac{p(1-p)}{q(1-q)} = \mathcal{I}(R(1-x))$$

This shows that is always weakly optimal to compensate the GP only in the state $y = R$ and that equation (9) holds.

**Proof that $r^*_{PE} = r(\lambda, \delta)$**

To verify our claim in the main text, we have to prove that when $\delta < \hat{\delta}(\lambda)$, a GP never finds it optimal to decrease $x$ from $\hat{x}(\lambda, \delta)$ by offering a return strictly above $r(\lambda, \delta)$. Under the binding no-default constraint (4), the expected return to the LPs would be given by $\hat{r}_{PE}(x) = (1-x)/\delta$ where $x \in [\bar{x}, \hat{x}(\lambda, \delta)]$ is chosen by the GP. The expected compensation of the GP is given by $\underline{W}_x = \frac{pBx}{p-q}$. The profit of the GP as a function of $x$ is this alternative fund is given by

$$\hat{\Pi}(x) = \frac{A\underline{W}_x}{1 - \frac{\delta}{1-x}(pR - \underline{W}_x)}$$

We are left to show that $\hat{\Pi}(\hat{x}(\delta)) > \hat{\Pi}(\bar{x})$. Since the numerator is increasing in $x$, it is enough to show that the denominator is decreasing in $x$. We thus have

$$\begin{aligned} 0 \leq \frac{\partial \hat{\Pi}}{\partial x} \quad &\Leftarrow \quad 0 \leq -\frac{\delta}{1-x}\frac{pB}{p-q} + \frac{\delta}{(1-x)^2}(pR - \frac{pBx}{p-q}) \\ &\Leftarrow \quad 0 \leq \frac{\delta}{(1-x)^2}\left[pR - \frac{pB}{p-q}\right] \end{aligned}$$

The last inequality follows from Assumption 1. This proves that $\frac{\partial \hat{\Pi}}{\partial x} \geq 0$ and that GPs do not increase the promised return to relax the no-default constraint (4).

## A.2 Proof of Corollary 1

– *Effect of* $1 - \delta$

This result follows directly from the discussion in the text.

– *Effect of* $\lambda$

When $\delta \geq \hat{\delta}(\lambda)$, an increase in $\lambda$ increases the required rate of return $r(\lambda, \delta)$. This lowers fund size and profit. We now turn to the case $\delta \leq \hat{\delta}(\lambda)$. In order to use our previous results, let us then write $\Pi^*_{GP}$ and $I^*$ as a function of $x$. We have

$$I^* = \frac{A}{1 - \frac{\delta}{1-x}\left(pR - \frac{pBx}{p-q}\right)}, \qquad \Pi^*_{GP} = I^* \frac{pBx}{p-q}$$

We showed in the Proof of Proposition 1 that $I^*$ and $\Pi^*_{GP}$ are increasing in $x$. When $\delta \leq \hat{\delta}(\lambda)$, by equation (14), we have that $x = \hat{x}(\lambda, \delta)$ is a decreasing function of $\lambda$. This proves that profit and fund size are decreasing in $\lambda$.

– *Effect of R*

This result follows immediately from the inspection of equation (16) and (17).

– *Effect of q when $p = q + \alpha$ with $\alpha$ constant*

Expression (16) shows that the fund size $I^*$ depends on $q$ only through the pledgeable income

$$pR - W^* = (q+\alpha)R - \begin{cases} \frac{(q+\alpha)^2}{\alpha(2q+\alpha)}B & \text{if} \quad \delta \geq \hat{\delta}(\lambda) \\ \frac{q+\alpha}{\alpha}B\hat{x}(\lambda,\delta) & \text{if} \quad \delta < \hat{\delta}(\lambda) \end{cases}$$

The result when $\delta < \hat{\delta}(\lambda)$ follows directly from the fact that $R \geq B/\alpha$ by Assumption 1. In the case where $\delta \geq \hat{\delta}(\lambda)$, a similar conclusion arises since $\frac{\partial(q+\alpha)}{\partial q} > 0$ and $\frac{\partial\left(\frac{q+\alpha}{2q+\alpha}\right)}{\partial q} < 0$ The profit $\Pi^*_{GP}$ is equal to the expected fee $W^*$ multiplied by the fund size $I^*$. The expression above shows that the expected fee is increasing in $q$. Since we showed that the fund size is also increasing in $q$, this proves that the profit of the GP is increasing in $q$.

## A.3 Proof of Proposition 2

We first prove a preliminary result before analyzing the three cases of Proposition 2.

**Proof that if $\alpha^*_L > 0$, then $r^*_{PE,L} = r(\lambda, \delta_L)$**

That $r^*_{PE,L} \geq r(\lambda, \delta_L)$ is obvious since $r(\lambda, \delta_L)$ is the minimum rate of return required by $L$-LPs. To prove the reverse inequality, we proceed by contradiction. If $r^*_{PE,L} > r(\lambda, \delta_L)$, $L$-LPs invest all their resources in PE funds, that is $S^L = \mu_L M$. Let us now prove that $H$-LPs would also invest all their resources in PE funds to arrive at a contradiction. Observe first that it must be that $\alpha^*_H > 0$ when $\mu_H > 0$. If $\alpha^*_H = 0$, from equation (20) and by market clearing it must be that $r^*_{PE,H} \leq r(\lambda, \delta_L)$. But GPs would then strictly prefer to raise funds from $H$-LPs since

$$\Pi^H(r^*_{PE,H}) \geq \Pi^H(r(\lambda, \delta_H)) > \Pi^L(r(\lambda, \delta_L)) > \Pi^L(r^*_{PE,L})$$

The first and the last inequality hold because $\Pi^i$ is decreasing in $r_{PE,i}$. The middle inequality was proved in Section 1. Hence, this cannot be an equilibrium and it must be that $\alpha^*_H > 0$ when $\alpha^*_L > 0$. Finally, by optimality of GPs' decision, the return offered to $H$-LPs, denoted $r^*_{PE,H}$ would then satisfy

$$\Pi^H(r^*_{PE,H}) \geq \Pi^L(r(\lambda, \delta_L)) \tag{A.2}$$

so that in particular $r^*_{PE,H} > r(\lambda, \delta_H)$. But equation (20) then implies that $S^H = \mu_H M$. Hence, the total supply of capital by LPs is $M$ while by assumption 3, the demand from GPs is strictly below $M$. This cannot be an equilibrium since markets would not clear. Our analysis also establishes that $r^*_{PE,H} \in [r(\lambda, \delta_H), \bar{r}_{PE,H}]$ where $\bar{r}_{PE,H}$ is the value of $r_{PE,H}$ that satisfies (A.2) as an equality. In addition, we must have $\alpha^*_L = 0$ when $r^*_{PE,H} < \bar{r}_{PE,H}$. We now examine the three possible cases.

**Three different cases**

*Case i)* Suppose first that $r^*_{PE,H} = r(\lambda, \delta_H)$. This implies that $\alpha^*_L = 0$. With the supply of capital by $H$-LPs given by the second case in equation (20), market clearing requires that $I_H(r(\lambda, \delta_H)) \leq \mu_H M + A$. This holds if $\mu_H \geq \bar{\mu}_H$ where

$$\bar{\mu}_H = \frac{I_H(r(\lambda, \delta_H)) - A}{M} \tag{A.3}$$

This proves the first case of Proposition 2.

*Case ii)* Suppose now that $r^*_{PE,H} \in (r(\lambda, \delta_H), \bar{r}_{PE,H})$. Once again, we have $\alpha^*_L = 0$. From (20), the supply of capital from $H$-LPs is given by $\mu_H M$ so market clearing requires that

$$\mu_H M + A = I_H(r^*_{PE,H}) \tag{A.4}$$

which implicitly defines $r^*_{PE,H}$ as as strictly decreasing function of $\mu_H$. Comparing (A.3) and (A.4), the inequality $r^*_{PE,H} > r(\lambda, \delta_H)$ implies that this outcome can only be an equilibrium if $\mu_H \leq \bar{\mu}_H$. Similarly, the inequality $r^*_{PE,H} < \bar{r}_{PE,H}$ imposes a lower bound $\underline{\mu}_H$ on $\mu_H$. Since $\bar{r}_{PE,H}$ is implicitly defined by the condition $\Pi^H(\bar{r}_{PE,H}) = \Pi^L(r(\lambda, \delta_L))$, using that $\Pi^H(r_{PE,H}) = W^*_H I_H(r_{PE,H})$ together with equation (A.4), this lower bound is given by

$$\underline{\mu}_H := \frac{\frac{\Pi^L(r(\lambda, \delta_L))}{W^*_H} - A}{M}$$

Re-arranging equation (A.4) and using the definition of $\bar{\mu}_H$ in (A.3),

$$\begin{aligned} 1 + r^*_{PE,H} = (pR - W^*_H)\frac{A + \mu_H M}{A} &= (1 + r(\lambda, \delta_H))\frac{I_H(r(\lambda, \delta_H)) - A}{I_H(r(\lambda, \delta_H))}\frac{A + \mu_H M}{A} \\ &= \frac{\bar{\mu}_H(A + \mu_H M)}{\mu_H(A + \bar{\mu}_H M)}\big(1 + r(\lambda, \delta_H)\big) \end{aligned}$$

which proves the expression in Case 2 of Proposition 2.

*Case iii)* Finally, consider the case where $r^*_{PE,H} = \bar{r}_{PE,H}$. In this case, the capital supply from $L$-LPs is indeterminate since $r^*_{PE,L} = r(\lambda, \delta_L)$. The supply of capital from $H$-LPs is $S_H = \mu_H M$

since $r^*_{PE,H} > r(\lambda, \delta_H)$. Hence, market clearing for funds with $H$ type investors requires that

$$\alpha^*_H (I_H(\bar{r}_{PE,H}) - A) = \mu_H M$$

which pins down $\alpha^*_H$. This equation implies that $\alpha^*_H$ is an increasing function of $\mu_H$ over $[0, \underline{\mu}_H]$ with $\alpha^*_H(\underline{\mu}_H) = 1$.

Hence, we showed that the allocation in Proposition 2 is an equilibrium. Our analysis of the three cases also shows that this is the unique equilibrium.

## A.4 Proof of Corollary 2

Using equation (22), we obtain

$$\frac{\partial \bar{r}_{PE,H}(R)}{\partial R} \propto W^*_L - W^*_H - \frac{W^*_L W^*_H}{1 + r(\lambda, \delta_L)} + \frac{W^*_H}{1 + r(\lambda, \delta_L)} W^*_H = (W^*_L - W^*_H) \left[ 1 - \frac{W^*_H}{1 + r(\lambda, \delta_L)} \right]$$

which proves our first result.

We now prove the matching result when $W^*_H < 1 + r(\lambda, \delta_L)$. We need to show that the equilibrium return for $H$-LPs is strictly above the threshold $\bar{r}_{PE,H}(R_b)$ when $\mu_H$ is low. Suppose by contradiction that $r^*_{PE,H} \leq \bar{r}_{PE,H}(R_b)$. Under the condition above, good GPs strictly prefer to raise funds from $H$-LPs since $\bar{r}_{PE,H}(R_g) > \bar{r}_{PE,H}(R_b)$. Their demand for capital is thus strictly bounded below by 0. But as $\mu_H \to 0$, the supply of capital from $H$-LPs converges to 0. This cannot be an equilibrium. Hence, when $\mu_H$ is too low, it must be that $r^*_{PE,H} > \bar{r}_{PE,H}(R_b)$ to clear the market. When this is the case, $H$-LPs only supply capital to good GPs.

## A.5 Proof of Corollary 3

– *Effect of* $\lambda$

Observe that $\lambda$ affects the premium through $r(\lambda, \delta_L)$ and $W^*_L$. Given that an increase in $\lambda$ increases $r(\lambda, \delta_L)$ and decreases $W^*_L$, it follows that the premium is decreasing in $\lambda$ since the premium is negatively affected by $r(\lambda, \delta_L)$ and positively affected by $W^*_L$.

– *Effect of* $\delta_L$

We showed that $W^*_L$ and $r(\lambda, \delta_L)$ are decreasing in $\delta_L$. By the argument above, it follows that the effect of $\delta_L$ on the premium is ambiguous.

– *Effect of $R$*

Let us call $rp$ the return premium. We have that

$$\begin{aligned}\frac{\partial rp}{\partial R} &\propto (W_L^* - W_H^*)(1 + r(\lambda, \delta_L)) + W_H^*(pR - W_L^*) - W_H^*(pR - W_H^*)) \\ &= (W_L^* - W_H^*)(1 + r(\lambda, \delta_L) - W_H^*)\end{aligned}$$

The result follows since $W_L^* > W_H^*$.

## A.6 Proof of Lemma 2

Rewriting equations (28) and (31) using the definition of the discount in equation (33), we obtain

$$\begin{aligned}1 + u_{PE,L} &= (1 + r_{PE,L})\left[1 - \lambda + \lambda(1 - D)\right] \\ 1 + u_{c,H} &= 1 - \lambda + \lambda\delta_H \frac{1}{1 - D}\end{aligned}$$

By definition, $\underline{r}_{PE,L}(D)$ and $\underline{r}_{PE,H}(D)$ are respectively the values of $r_{PE,L}$ and $r_{PE,H}$ such that $u_{PE,L} = u_{c,L}$ and $u_{PE,H} = u_{c,H}$. We thus have

$$\underline{r}_{PE,L}(D) = \frac{\lambda D}{1 - \lambda D} \tag{A.5}$$

$$\underline{r}_{PE,H}(D) = \frac{\lambda\delta_H D}{(1 - D)\left[1 - \lambda(1 - \delta_H)\right]} \tag{A.6}$$

Subtracting these two equations, we obtain

$$\begin{aligned}\underline{r}_{PE,H}(D) - \underline{r}_{PE,L}(D) &= \frac{\lambda D}{1 - \lambda(1 - \delta_H)}\left[\frac{\delta_H}{1 - D} - \frac{1 - \lambda(1 - \delta_H)}{1 - \lambda D}\right] \\ &= \frac{(1 - \lambda)\lambda D}{1 - \lambda(1 - \delta_H)} \frac{D - (1 - \delta_H)}{(1 - D)(1 - \lambda D)}\end{aligned}$$

Since the numerator is increasing in $D$ and the denominator is decreasing, this proves that $\underline{r}_{PE,H}(D) - \underline{r}_{PE,L}(D)$ is increasing in $D$. The expression above also shows that the difference is equal to 0 when $D = 1 - \delta_H$.

## A.7 Proof of Proposition 3

The proof is in two steps. We first characterize the equilibrium under the conjecture that $H$-LPs do not participate in the primary market when $D > 1 - \delta_H$. Then, we verify that it is optimal for GPs not to raise $H$-funds.

**Equilibrium characterization**

Following the proof of Proposition 2., we prove the result by construction. For each possible value of the discount $D_*$, we characterize the equilibrium and the range of values for $\mu_H$ where this equilibrium exists.

Case 1. $D^* = 1 - \delta_H$.

From Lemma 2, we obtain that $\underline{r}_{PE,L} = \underline{r}_{PE,H} = r(\lambda, \delta_H)$.

Since

$$P_L^* = \delta_H(1 + r_{PE,L}) > 1 - \bar{x}$$

by Assumption (4), the capital call in a $L$-fund is given by $x_L = \bar{x}$. Hence, by optimality of the fund choice, it must be that $r^*_{PE,L} = r^*_{PE,H}$. By the clearing condition in the primary market, we further obtain that $r^*_{PE,L} = r(\lambda, \delta_H)$ since otherwise the supply of funds from LPs would exceed the demand by GPs. This allocation can be an equilibrium if and only if the supply of claims in the secondary market exceeds the supply at price $P_L^* = \delta_H(1 + r(\lambda, \delta_H))$, that is

$$\big[I(r(\lambda, \delta_H), \bar{x}) - A - S_H\big]\delta_H\big[1 + r(\lambda, \delta_H)\big] \leq \mu_H M - S_H \tag{A.7}$$

Note that since $\delta_H\big[1 + r(\lambda, \delta_H)\big] < 1$, this inequality is easier to satisfy when $S_H = 0$. Hence, this allocation is an equilibrium for $\mu_H \geq \mu_{H,3}$ where $\mu_{H,3}$ is the minimum value of $\mu_H$ such that equation (A.7) holds when setting $S_H = 0$.

Case 2: $D^* = 1 - \delta_L$.

This implies that $\underline{r}_{PE,L} = r(\lambda, \delta_L)$. Since GPs only raise $L$-funds, the clearing condition in the primary market implies that $r^*_{PE,L} = r(\lambda, \delta_L)$. Combining this result with $D = 1 - \delta_L$, we obtain $x^* = \hat{x}(\lambda, \delta_L)$. By equation (23), the supply of claims in the secondary market is indeterminate. This outcome is an equilibrium if the maximum supply of claims in the secondary market exceeds the demand at price $P_L^* = \delta_L(1 + r(\lambda, \delta_L))$. Using equation (24), the condition writes

$$\big[I(r(\lambda, \delta_L), \hat{x}(\lambda, \delta_L)) - A\big]\delta_L(1 + r(\lambda, \delta_L)) \geq \mu_H M \tag{A.8}$$

which we rewrite as $\mu_H \leq \mu_{H,1}$. Since $I(r, x)$ is decreasing in $r$ and $x$ for $x \geq \bar{x}$ and $\delta(1 + r(\lambda, \delta))$ is increasing in $\delta$, the comparison between equations (A.7) and (A.8) shows that $\mu_{H,1} < \mu_{H,3}$.

Case 3: $D^* \in (1 - \delta_H, 1 - \delta_L)$.

Since $P_L^* > \delta_L(1 + r^*_{PE,L})$, $L$-LPs strictly prefer to sell their claims by equation (23). Since

$P^*_L < \delta_H(1 + r^*_{PE,L})$, the demand for claims from $H$-LPs is given by $\mu_H M$. Hence, the market clearing condition on the secondary market writes

$$\left[I(r^*_{PE,L}, x^*_L) - A\right] P^*_L = \mu_H M \tag{A.9}$$

The binding participation constraint of $L$-LPs implies that

$$r^*_{PE,L} = \underline{r}_{PE,L} = \frac{1 - \lambda P^*_L}{1 - \lambda} \tag{A.10}$$

By definition, $I(r, x)$ is strictly decreasing in $r$ and weakly decreasing in $x$ (strictly over the range $[\bar{x}, \hat{x}(\lambda, \delta_L)]$). Since $r^*_{PE,L}$ and $x^*_L$ are themselves strictly and weakly decreasing in $P^*$ respectively. the left hand side of (A.9) is strictly increasing in $P^*_L$ and thus decreasing in $D^*$. Hence, condition $D^* \in (1 - \delta_H, 1 - \delta_L)$ defines a range of values of $\mu_H$ where equation (A.9) may hold. Comparing equations (A.9) to equations (A.7) and (A.8), the upper bound and lower bound of this region are given respectively by $\mu_{H,3}$ and $\mu_{H,1}$. Over this region, $P^*$ and $r^*_{PE}$ are strictly monotone in $\mu_H$.

To finish the equilibrium characterization, let us define $\mu_{H,2}$ as the value of $\mu_H \in [\mu_{H,1}, \mu_{H,3}]$ such that equation (A.9) holds with $x^*_L = \bar{x}$, $P^*_L = 1 - \bar{x}$ and where $r_{PE,L}$ is given by equation (A.10). Since we showed that $P^*_L$ is increasing over $[\mu_{H,1}, \mu_{H,3}]$, this implies that $x^*_L = \bar{x}$ for $\mu_H \in [\mu_{H,2}, \mu_{H,3}]$ using equation (26) for the optimal choice of $x_L$ by GPs..

Note that the parameter values for an equilibrium with two different values of $D^*$ are mutually exclusive. This implies that we characterized the only equilibrium that satisfies our conjecture. We now verify this conjecture that $H$-LPs do not invest in the primary market when $D* > 1 - \delta_H$, that is when $\mu_H < \mu_{H,3}$.

**No fund with $H$-LPs when $\mu_H \leq \mu_{H,3}$**

To prove this result, we show that the minimum return $\underline{r}_{PE,H}(D^*)$ required by $H$-LPs exceeds the maximum return $\bar{r}_{PE,H}$ that GPs are willing to pay. This return is the value of $r_{PE,H}$ such that GPs are indifferent between $H$-funds and $L$-funds. Adapting the equation we already derived in (22), we obtain

$$1 + \bar{r}_{PE,H} = \frac{\underline{W}_{x^*_L}(pR - \underline{W}_{\bar{x}})}{\underline{W}_{\bar{x}}(pR - \underline{W}_{x^*_L}) + (\underline{W}_{x^*_L} - \underline{W}_{\bar{x}})(1 + r^*_{PE,L})}(1 + r^*_{PE,L}) \tag{A.11}$$

When $\mu_H \in [\mu_{H,2}, \mu_{H,3}]$, we showed that $x^*_L = \bar{x}$. Hence, equation (A.11) becomes $\bar{r}_{PE,H} = r^*_{PE,L}$. Since $D^* > 1 - \delta_H$, the minimum rate required by $H$-LPs satisfies $\underline{r}_{PE,H}(D^*) > r^*_{PE,L}$ by

Lemma 2. This proves our claim in this case.

For $\mu_H \leq \mu_{H,2}$, we write $\bar{r}_{PE,H}$ as a function of $x_L^*$ using equation (26) and (A.10) to substitute for $r_{PE,L}^*$ in (A.11). We obtain

$$1 + \bar{r}_{PE,H} = \frac{\underline{W}_{x_L^*}\left(pR - \underline{W}_{\bar{x}}\right)\left[1 - \lambda(1 - x_L^*)\right]}{(1-\lambda)\underline{W}_{\bar{x}}\left(pR - \underline{W}_{x_L^*}\right) + \left(\underline{W}_{x_L^*} - \underline{W}_{\bar{x}}\right)\left[1 - \lambda(1 - x_L^*)\right]} \tag{A.12}$$

Since $\bar{r}_{PE,H}$ is increasing in $\underline{W}_{x_L^*}$ and $r_{PE,L}^*$ which are themselves increasing in $x_L^*$, we obtain that $\bar{r}_{PE,H}$ is increasing in $x_L^*$. We can similarly write $\underline{r}_{PE,H}$ as a function of $x_L^*$ using equation (A.6) and writing $D^*$ as a function of $x_L^*$. We obtain

$$1 + \underline{r}_{PE,H} = 1 + \frac{\lambda \delta_H x_L^*}{(1-\lambda)(1 - x_L^*)\left[1 - \lambda(1 - \delta_H)\right]} \tag{A.13}$$

which is also increasing in $x_L^*$. We showed that $x_L^*$ is strictly decreasing in $\mu_H$ for $\mu_H \in [\mu_{H,1}, \mu_{H,2}]$. For $\mu_H \leq \mu_{H,1}$, $x_L^*$ is constant and equal to $\hat{x}(\lambda, \delta_L)$. Hence, in order to prove that $\underline{r}_{PE,H} > \bar{r}_{PE,H}$ for all values of $\mu_H \in [0, \mu_{H,2}]$, it is enough to show that $\underline{r}_{PE,H}(x = \bar{x}) \geq \bar{r}_{PE,H}(x = \hat{x}(\lambda, \delta_L))$. An upper bound on $\bar{r}_{PE,H}(x = \hat{x}(\lambda, \delta_L))$ is given by $pR - 1$. On the other hand, we have

$$\underline{r}_{PE,H}(x = \bar{x}) = \frac{\lambda \delta_H p}{(1-\lambda) q \left[1 - \lambda(1 - \delta_H)\right]}$$

Hence, under Assumption 5, the result obtains. This concludes the proof.

## A.8 Segmentation with secondary market

We prove the claim in footnote 29 that essentially different $H$-funds and $L$-funds may coexist in the primary market when $\lambda$ is low enough.

**Claim 1.** *There exists $\hat{\lambda} > 0$ and $\epsilon > 0$ such that when $\lambda \leq \hat{\lambda}$ and $\delta_H - \delta_L < \epsilon$, both $L$-funds and $H$-funds are offered in equilibrium with $r_{PE,H}^* > r_{PE,L}^*$ for $\mu_H \leq \mu_{H,1}$.*

*Proof.* We proceed by contradiction and first show that the allocation in Proposition 3 cannot be an equilibrium under the conditions of Claim 1. In particular, the fund optimality condition and the market clearing condition of Definition 2 are inconsistent. Given our analysis in the main text, this is equivalent to showing that the maximum rate GPs are willing to pay for $H$-funds $\bar{r}_{PE,H}$ exceeds the minimum rate $\underline{r}_{PE,H}$ $H$-LPs are willing to accept.

When $\mu_H \leq \mu_{H,1}$, the first capital call in a $L$-fund is $x_L^* = \hat{x}(\lambda, \delta_L)$ in the conjectured equilibrium. By continuity, it is enough to prove the result for $\delta_H = \hat{\delta}(\lambda)$. Using equation (A.13), we obtain

$$1+\underline{r}_{PE,H} = (1+r(\lambda,\hat{\delta}(\lambda))\left[1-\lambda+\lambda\hat{\delta}(\lambda)\frac{1-\lambda(1-\hat{x}(\lambda,\delta_L))}{(1-\lambda)(1-\hat{x}(\lambda,\delta_L))}\right]$$

Taking further the limit when $\delta_L \to \hat{\delta}(\lambda)$, we have

$$\lim_{\delta_L\to\hat{\delta}(\lambda)}(1+\underline{r}_{PE,H}) = 1+r(\lambda,\hat{\delta}(\lambda)) = \lim_{\delta_L\to\hat{\delta}(\lambda)}(1+\bar{r}_{PE,H})$$

where the second equality follows from (A.12). Hence, to show the desired result and since $\hat{x}(\lambda,\delta_L)$ is decreasing in $\delta_L$, it is enough to show that

$$\frac{\partial(1+\underline{r}_{PE,H})}{\partial x}_{x=\hat{x}(\lambda,\hat{\delta}(\lambda))} < \frac{\partial(1+\bar{r}_{PE,H})}{\partial x}_{x=\hat{x}(\lambda,\hat{\delta}(\lambda))} \tag{A.14}$$

We obtain that

$$\frac{\partial(1+\underline{r}_{PE,H})}{\partial x}_{x=\hat{x}(\lambda,\hat{\delta}(\lambda))} = \frac{(1+r(\lambda,\hat{\delta}(\lambda))\lambda\hat{\delta}(\lambda)}{(1-\lambda)\left[1-\hat{x}(\lambda,\hat{\delta}(\lambda))\right]^2} = \frac{\lambda(p+q)}{(1-\lambda)q} \tag{A.15}$$

For the right hand side of (A.14), using equation (A.12), we have

$$\begin{aligned}
\frac{\partial(1+\bar{r}_{PE,H})}{\partial x} &= \frac{pB}{p-q}\frac{\partial\bar{r}_{PE,H}}{\partial\underline{W}_x}+\frac{\partial\bar{r}_{PE,H}}{\partial x}\\
&= \frac{pB}{p-q}\frac{(pR-\underline{W}_{\bar{x}})(1-\lambda(1-x))\underline{W}_{\bar{x}}\left[pR(1-\lambda)-1+\lambda(1-x)\right]}{\left[(1-\lambda)\underline{W}_{\bar{x}}\left(pR-\underline{W}_x\right)+(\underline{W}_x-\underline{W}_{\bar{x}})(1-\lambda(1-x))\right]^2}\\
&\quad+\frac{\lambda(1-\lambda)\underline{W}_x(pR-\underline{W}_{\bar{x}})\underline{W}_{\bar{x}}(pR-\underline{W}_x)}{\left[(1-\lambda)\underline{W}_{\bar{x}}\left(pR-\underline{W}_x\right)+(\underline{W}_x-\underline{W}_{\bar{x}})(1-\lambda(1-x))\right]^2}\\
&= \frac{pB}{p-q}\underline{W}_{\bar{x}}(pR-\underline{W}_{\bar{x}})\frac{(1-\lambda(1-x))\left[pR(1-\lambda)-1+\lambda(1-x)\right]+x\lambda(1-\lambda)(pR-\underline{W}_x)}{\left[(1-\lambda)\underline{W}_{\bar{x}}\left(pR-\underline{W}_x\right)+(\underline{W}_x-\underline{W}_{\bar{x}})(1-\lambda(1-x))\right]^2}
\end{aligned}$$

Setting $x=\hat{x}(\lambda,\hat{\delta}(\lambda))=\bar{x}$ in the expression above, we obtain

$$\frac{\partial(1+\bar{r}_{PE,H})}{\partial x}_{|x=\hat{x}(\lambda,\hat{\delta}(\lambda))} = \frac{\lambda}{1-\lambda}+\frac{(1-\lambda(1-\bar{x})\left[pR-\frac{1-\lambda(1-\bar{x})}{1-\lambda}\right]}{(1-\lambda)\bar{x}\left(pR-\underline{W}_{\bar{x}}\right)} \tag{A.16}$$

Note that the second term of (A.16) does not converge to 0 as $\lambda\to 0$. Hence, comparing (A.16) and (A.15), it follows that the required condition (A.14) holds when $\delta_H$ and $\delta_L$ are close enough to $\hat{\delta}$ and $\lambda$ is small enough. This implies that the allocation of Proposition 3 cannot be an equilibrium

The last step is to show that $H$-funds deliver higher return than $L$-funds in equilibrium. According to Lemma 2, this is true if the equilibrium discount is strictly higher than $1-\delta_H$. Proposition showed that an equilibrium with a discount $D^*=1-\delta_H$ can only exist if $\mu_H\le\mu_{H,3}$ where

$\mu_{H,3} > \mu_{H,1}$. Hence, under the parameter configuration of Claim 1, the discount is strictly higher than $1 - \delta_H$ which proves the claim. $\square$

# B Contract Robustness

## B.1 Calling excess capital

We show that GPs cannot avoid the commitment problem of LPs by calling excess capital in period 0. The key intuition for the result is that GPs would deviate by choosing to invest in period 0 all the capital called. We assume that LPs observe the realized investment before the GP chooses the effort level and that

$$R \geq \frac{B}{(p-q)^2} \tag{B.1}$$

These two assumptions will ensure that LPs cannot commit to punish the GP by seizing all the cash flows. Inequality (B.1) is compatible with assumptions 1 and 2 in the main text. In our analysis, we focus on the case $\delta < \hat{\delta}(\lambda)$ since otherwise, Proposition 1 shows that the commitment problem of LPs is moot.

**Claim 2.** *When $\delta < \hat{\delta}(\lambda)$, GPs cannot increase profit by holding LPs' capital as cash.*

*Proof.* To avoid default by LPs, the GP must call at least a fraction $\hat{x}(\lambda, \delta)$ of the fund capital in period 0 where $\hat{x}(\lambda, \delta)$ is defined in Proposition 1. Without loss of generality, we assume that the GP calls exactly $\hat{x}(\lambda, \delta)$. We denote by $x_{inv} \in \left[\frac{p}{p+q}, \hat{x}(\lambda, \delta)\right]$ the amount that the GP should invest in period 0 according to the partnership contract. The lower bound on $x_{inv}$ is without loss of generality since Proposition 1 shows that the diversification benefits are maximized for this value. Given the fund size $I$, the GP expected compensation if he invests according to the contractual schedule is given by

$$\Pi_{GP} = \frac{pBx_{inv}}{p-q} I$$

The first deviation the GP may contemplate is to invest nothing in period 0. However, LPs can block this deviation by credibly committing to (i) confiscate the first capital call - not yet invested - and (ii) withdraw the second capital call.

Hence, the relevant deviation by the GP is to invest all the capital $\hat{x}(\lambda, \delta)I$ called in period 0. The LPs can punish the GP by withdrawing the second capital call. However, under assumption (B.1), they cannot commit to seize all the cash flows. If they do, the GP will react by exerting no effort and LPs earn $qxI$ in expectation. To ensure that GPs exert effort, LPs can pay $\tilde{w}(R)xI$ if the first project succeeds where $\tilde{w}(R) = B/(p-q)$. In this case, the payoff to the LPs would be

$$p(R - \tilde{w}(R))xI \geq qxI$$

where the inequality follows from assumption (B.1). Hence the profit of the GP following a deviation is given by

$$\tilde{\Pi}_{GP} = p\tilde{w}(R)x(\lambda,\delta)I = p\frac{Bx(\lambda,\delta)}{p-q}I > \Pi_{GP}$$

This proves that the proposed contract is not incentive-compatible. Given that the GP would always deviate by investing all the capital $x(\lambda,\delta)I$ called in period 0, the contract might as well specify that $x_{inv} = x(\lambda,\delta)$, which is the contract considered in the main text. This concludes the proof.

□

## B.2 Raising capital in period 1

### Cost of capital

Let us define $r_0$ as the cost of capital raised in period 0 and $\tilde{r_1}$ as the ex-post cost of capital raised in period 1. We can prove the following result

**Claim 3.** *It is cheaper to raise capital in period* 0*, that is* $r_0 \leq \mathbb{E}[\tilde{r_1}]$.

*Proof.* We show in the main text that

$$1 + r_0 = 1 + r(\lambda,\delta) = \frac{1}{1-\lambda+\lambda\delta} = \frac{1}{\mathbb{E}[\tilde{\delta}]}, \quad \text{where} \quad \tilde{\delta} = \begin{cases} \delta & (\text{prob } \lambda) \\ 1 & (\text{prob } 1-\lambda) \end{cases}$$

In period 1, the cost of capital for GPs is either 1 if the liquidity shock does not hit or $1/\delta$ if it does. The expected cost of capital in period 1 is thus given by

$$\mathbb{E}[1+\tilde{r_1}] = \lambda\frac{1}{\delta} + 1 - \lambda = \mathbb{E}\left[\frac{1}{\tilde{\delta}}\right] > \frac{1}{\mathbb{E}[\tilde{\delta}]} = 1 + r_0$$

where the result follows from Jensen's inequality applied to the convex function $x \mapsto 1/x$. □

Proposition 1 then implies that raising capital in period 1 is strictly sub-optimal when $\delta \geq \hat{\delta}(\lambda)$, that is when the commitment problem of LPs does not bind in the benchmark fund. When $\delta < \hat{\delta}(\lambda)$, it is still optimal to raise capital only in period 0 if the investment distortion is small enough that is if $\delta$ is sufficiently close to $\hat{\delta}(\lambda)$.

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