

# Nonparametric Estimates of Demand in the California Health Insurance Exchange \*

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## Abstract

We estimate the demand for health insurance in the California Affordable Care Act marketplace (Covered California) without using parametric assumptions on the distribution of the unobserved components of utility. To do this, we develop a computational method for constructing sharp identified sets in a nonparametric discrete choice model. The method allows for endogeneity in prices (premiums) and for the use of instrumental variables to address this endogeneity. We use the method to estimate bounds on the effect of changes in premium subsidies on coverage choices, consumer surplus, and government spending. We find that a \$10 decrease in monthly premium subsidies would cause between a 2.3% and 11.4% decline in the proportion of low-income adults with coverage. The corresponding reduction in total annual consumer surplus would be between \$43 and \$66 million, while the savings in yearly subsidy outlays would be between \$293 and \$944 million. These nonparametric estimates reflect substantially greater price sensitivity than in comparable logit or probit models.

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# 1 Introduction

Under the Patient Protection and Affordable Care Act of 2010 (“ACA”), the United States federal government spends over \$40 billion per year on subsidizing health insurance premiums for low-income households. The design of the ACA and the regulation of non-group health insurance remain objects of intense debate among policy makers. Addressing several key design issues, such as the structure of premium subsidies, requires estimating demand under counterfactual scenarios.

Recent research on the demand for health insurance has filled this need using discrete choice models in the style of McFadden (1974). For example, Chan and Gruber (2010) and Ericson and Starc (2015) used conditional logit models to estimate demand in Massachusetts’ Commonwealth Care program, Saltzman (2017) used a nested logit to estimate demand in the California and Washington ACA exchanges, and Tebaldi (2017) estimated demand in the California ACA exchange with a variety of logit, nested logit, and mixed (random coefficient) logit models.

These various flavors of logit models differ in the way they deal with the independence of irrelevant alternatives property (see e.g. Goldberg, 1995; Berry, Levinsohn, and Pakes, 1995; McFadden and Train, 2000), and in how they deal with potential endogeneity of prices (e.g. Berry, 1994; Hausman, 1996; Berry et al., 1995; Berry, Levinsohn, and Pakes, 2004). However, they are all fully parametric, with the logistic distribution playing a central role in the parameterization. This raises the concerning possibility that the counterfactual demand predictions generated from these models might be significantly driven by functional form.

In this paper, we use a nonparametric model to estimate the effects of changing premium subsidies on choice behavior, consumer surplus, and government spending in the California ACA exchange (Covered California). The model is a distribution-free counterpart of a standard discrete choice model in which consumers’ indirect utility for insurance options depends on the price (premium) and on their unobserved valuation for the option. In contrast to standard parametric discrete choice models, we do not assume that these valuations follow a specific distribution such as normal (probit) or type I extreme value (logit). The main restriction of the model is that indirect utility is additively separable in premiums and latent valuations. The model allows for premiums to be endogenous (correlated with latent valuations), and allows a researcher to use instrumental variables to address this endogeneity.<sup>1</sup>

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<sup>1</sup> While we develop our methodology with a focus on our health insurance application, we believe that it should also be of wider interest for demand analysis in other markets with product differentiation, as well as for other discrete choice settings more generally.

Point identification arguments in similar nonparametric discrete choice models with exogenous prices are often premised on the assumption of large variation in prices (e.g. Thompson, 1989; Matzkin, 1993). When prices are endogenous, point identification requires large variation in instruments for prices, as well as additional completeness conditions (Chiappori and Komunjer, 2009; Berry and Haile, 2014). In the Covered California data, we only observe limited variation in premiums, so these conditions will not be satisfied. This leads us to consider a partial identification framework.

The primary challenge with allowing for partial identification is finding a way to characterize and compute sharp bounds for target parameters of interest. We develop a characterization based on the insight that in a discrete choice model, many different realizations of latent valuations would lead to identical choice behavior under all relevant observed and counterfactual prices. Using this idea, we partition the space of unobserved valuations according to choice behavior by constructing a collection of sets that we call the Minimal Relevant Partition (MRP). We prove that sharp bounds for typical target parameters of interest can be characterized by considering only the way the distribution of valuations places mass over the MRP. We then use this result to develop estimators of these bounds, which we implement using linear programming.

We combine our empirical methodology with rich administrative data to estimate demand counterfactuals for the California ACA exchange. The focus of our analysis is the choice of metal tier for low-income households who are not covered under employer-sponsored insurance or public programs. Our main counterfactual of interest is how changes in premium subsidies would affect the proportion of this population that chooses to purchase health insurance, as well as their chosen coverage tiers, and their realized consumer surplus. To identify these quantities, we use the additively separable structure of utility in the nonparametric model together with institutionally-induced variation in premiums across consumers of different ages and incomes. We exploit this variation by restricting the degree to which preferences (latent valuations) can differ across consumers of similar age and income who live in the same geographic region.

Since the nonparametric model is partially identified, this strategy yields bounds, rather than point estimates. However, the bounds are quite informative. Using our preferred specification, we estimate that a \$10 decrease in monthly premium subsidies would cause between a 2.3% and 11.4% decline in the proportion of low-income adults with coverage. The average consumer surplus impact of this subsidy decrease would be between \$1.86 and \$2.80 per person, per month, or between \$43 and \$66 million annually when aggregated. Savings on subsidy outlays would be \$293 to \$944 million. When we analyze heterogeneity by income, we find that poorer consumers value health insurance more, and so incur the bulk of the surplus loss from decreasing subsidies.

Overall, our estimates reinforce and amplify the finding that the demand for health insurance in this segment of the population is highly price elastic (e.g. Abraham, Drake, Sacks, and Simon, 2017; Finkelstein, Hendren, and Shepard, 2017).

We show that comparable estimates using logit models tend to yield price responses close to our lower bounds, and so may substantially overstate the value that consumers place on health insurance. This possibility of understated price sensitivity is more severe when considering larger price changes that involve more distant extrapolations. It also remains when considering similar models, such as mixed logit, that allow for valuations to be correlated across options. Our findings provide an example in which the shape of the logistic (or similarly-shaped Gaussian) distribution can have an important impact on empirical conclusions.<sup>2</sup> The nonparametric model we use presents a remedy for this problem, and in this case provides empirical conclusions that differ significantly along a policy-relevant dimension.

The remainder of the paper is organized as follows. In Section 2, we begin with a discussion of the key institutional aspects of Covered California. In Section 3, we develop our nonparametric discrete choice methodology for estimating the demand for health insurance.<sup>3</sup> In Section 4, we discuss our empirical implementation of the method using the Covered California administrative data and our main empirical results. In Section 5 we contrast these results with estimates from standard parametric models. Section 6 contains some brief concluding remarks.

## 2 Covered California

Covered California is one of the largest state health insurance exchanges regulated by the ACA, accounting for more than 10% of national enrollment. As in other states, it primarily serves low-income households with income between 100-400% of the federal poverty level (FPL) by providing these households an option to purchase subsidized insurance if they are not covered by an employer or a public program, such as Medi-Cal (Medicaid) or Medicare. In 2014, over 94% of purchasing households were beneficiaries of premium subsidies. Our analysis will focus exclusively on this large and important subpopulation.

The basic structure of Covered California is determined by federal regulation, and so is common to other ACA marketplaces. The regulation splits states into geographic

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<sup>2</sup> Other examples include Ho and Pakes (2014) and Compiani (2018), who also found that logit models underestimate price elasticities relative to less parametric alternatives, albeit using different methods in different empirical settings.

<sup>3</sup> Appendix A contains a methodological literature review that compares and contrasts our approach with the related semi- and nonparametric literature on discrete choice models.

rating regions comprised of groups of contiguous counties or zip codes. In California, there are 19 such rating regions. Insurers are allowed to vary premiums across (but not within) rating regions, and consumers face the premiums set for their resident region. Each year in the spring, insurers announce their intention to enter a region in the subsequent calendar year and undergo a certification process by the state. Consumers are then able to purchase insurance for the subsequent year during an open enrollment period at the end of the year.

However, Covered California also differs from other ACA marketplaces in several important aspects. One important difference is that an insurer who intends to participate in a rating region is required to offer a menu of four plans classified into metal tiers of increasing actuarial value: Bronze, Silver, Gold and Platinum.<sup>4</sup> Unlike other marketplaces, the insurer must provide the entire menu of four plans in any region where it enters. Moreover, the actuarial features of the plans are standardized to have the characteristics shown in Table 1 (among others). Insurers who enter a rating region must therefore offer each of the plans listed in Table 1 with the features shown there. We will focus our analysis on the choice of tier, i.e. on how much coverage to purchase, rather than modeling which insurer to purchase it from.

Insurers are also regulated in the way in which they can set premiums. Each insurer chooses a base premium for each metal tier in each rating region. This base premium is then transformed through federal regulation into premiums that vary by the consumer's age.<sup>5</sup> The insurer is not permitted to adjust premiums based on any other characteristic of the consumer.<sup>6</sup> Households with annual income below 400% FPL pay lower premiums than received by the insurer, with the difference being made up by premium subsidies. These subsidies are set according to federal regulations based on the household's income (measured in FPL) and number of household members.<sup>7</sup> Besides subsidies, there is also an income tax penalty for remaining uninsured which is assessed at filing.

In addition to premium subsidies, the ACA also contained a provision under which households with income lower than 250% of the FPL receive cost-sharing reduction (CSR) subsidies. In Covered California, CSRs are implemented by changing the actu-

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<sup>4</sup> There is a fifth coverage level called minimum (or catastrophic) coverage, but this is not available to subsidized buyers, so we omit it from our analysis.

<sup>5</sup> This transformation involves multiplying base premiums by an adjustment factor that starts at 1 for individuals at age 21 and increases smoothly to 3 at age 64. These factors are set by the Center for Medicaid & Medicare Services. Individuals 65 and older are covered by Medicare. See Orsini and Tebaldi (2017) for further discussion.

<sup>6</sup> Some states also allow for adjustments based on tobacco use. California is not one of these states.

<sup>7</sup> See Tebaldi (2017) for details on how these subsidies are set.

**Table 1:** *Standardized Financial Characteristics in Covered California*

	Annual deductible	Annual max out-of-pocket	Primary visit	E.R. visit	Specialist visit	Preferred drugs	Actuarial value
Bronze	\$5,000	\$6,250	\$60	\$300	\$70	\$50	60%
Silver (>250% FPL)	\$2,250	\$6,250	\$45	\$250	\$65	\$50	70%
Silver (200-250% FPL)	\$1,850	\$5,200	\$40	\$250	\$50	\$35	74%
Gold	\$0	\$6,250	\$30	\$250	\$50	\$50	79%
Silver (150-200% FPL)	\$550	\$2,250	\$15	\$75	\$20	\$15	88%
Platinum	\$0	\$4,000	\$20	\$150	\$40	\$15	90%
Silver (100-150% FPL)	\$0	\$2,250	\$3	\$25	\$5	\$5	95%

Source: <http://www.coveredca.com/PDFs/2015-Health-Benefits-Table.pdf>

arial terms of the Silver plan for these households. As a result, the terms of a Silver plan vary across households with different FPL levels, with discrete changes at 150%, 200%, and 250% of the FPL; see Table 1. The CSRs make the Silver plan very attractive for low-income households relative to the more expensive Gold and Platinum plans.

### 3 Empirical Framework and Methodology

#### 3.1 Model

We consider a model in which a population of consumers indexed by  $i$  each choose a single health insurance plan  $Y_i$  from a set  $\mathcal{J} \equiv \{0, 1, \dots, J\}$  of  $J+1$  choices. Each plan  $j$  has a premium,  $P_{ij}$ , which is indexed by the consumer,  $i$ , since different consumers face different post-subsidy premiums depending on their sociodemographic characteristics.<sup>8</sup> Choice  $j = 0$  represents the outside option of not choosing any of the insurance plans, and has premium normalized to 0, so that  $P_{i0} = 0$ . When we take the model to the Covered California data in Section 4, we will have five choices ( $J = 4$ ) with options 1, 2, 3, and 4 representing Bronze, Silver, Gold, and Platinum plans, respectively.

Consumer  $i$  has a vector  $V_i \equiv (V_{i0}, V_{i1}, \dots, V_{iJ})$  of valuations for each plan, with the standard normalization that  $V_{i0} = 0$ . The valuations are known to the consumer, but latent from the perspective of the researcher. We assume that consumer  $i$ 's indirect utility from choosing plan  $j$  is given by  $V_{ij} - P_{ij}$ , so that their plan choice is given by

$$Y_i = \arg \max_{j \in \mathcal{J}} V_{ij} - P_{ij}. \quad (1)$$

<sup>8</sup> The premium  $P_{ij}$  can be viewed as net of the tax penalty for remaining uninsured.

Our use of (1) will be nonparametric in the sense that we will not assume that the distribution of  $V_i$  follows a specific functional form such as i.i.d. type I extreme value (logit) or multivariate normal (probit).

Models like (1) in which valuations and premiums are additively separable in indirect utility have been widely used in the recent literature on insurance demand, see e.g. Einav, Finkelstein, and Cullen (2010a), Einav, Finkelstein, and Levin (2010b), and Bundorf, Levin, and Mahoney (2012). In Appendix B, we derive (1) from an insurance choice model similar to the ones in Handel (2013) and Handel, Hendel, and Whinston (2015), in which consumers have quasilinear utility and constant absolute risk aversion preferences. In this model, differences in  $V_i$  across consumers arise from heterogeneity in their unobserved preferences, risk factors (and/or perception), and risk aversion.

The additive separability (quasilinearity) of premiums in (1) imposes restrictions on substitution patterns. In particular, (1) implies that if all premiums were to increase by the same amount, then a consumer who chose to purchase plan  $j \geq 1$  before the premium increase will either continue to choose plan  $j$  after the premium increase, or will switch to the outside option ( $j = 0$ ), but they will not switch to a different plan  $k \geq 1$ ,  $k \neq j$ . This limits the role of income effects to the extensive margin of purchasing any insurance plan versus taking the outside option.

However, it is important to note that (1) is a model of a given consumer  $i$ . When we take (1) to the data, we will be combining observations on many consumers, and so in practice we can allow for income effects by allowing for dependence between a consumer's income and their valuations. We make this formal by treating a consumer's observed characteristics (including their income) as part of a vector,  $X_i$ , and then considering restrictions on the dependence between  $V_i$ ,  $P_i$ , and the various components of  $X_i$ . We discuss these types of assumptions more in Section 3.3.1 and our specific implementation of them in Section 4.2. In general,  $X_i$  can also contain characteristics that vary over choice options, but this will not play a part in our application, due to the regulated homogeneity of tier characteristics in Covered California.

A common parametric specification for discrete choice demand models replaces (1) by

$$Y_i = \arg \max_{j \in \mathcal{J}} X'_{ij} \beta_i - \alpha_i P_{ij} + \xi_j + \epsilon_{ij}, \quad (2)$$

where  $X_{ij}$  and  $P_{ij}$  are as in our notation (indexed as specific to choice  $j$ ),  $\xi_j$  are the unobservable components of choice  $j$  that do not vary across consumers, and  $\epsilon_{ij}$  are idiosyncratic logit (type I extreme value) unobservables that are independent across

choices.<sup>9</sup> In the influential model of Berry et al. (1995), the  $\beta_i$  and  $\alpha_i$  terms are unobservable random coefficients that vary across consumers, and are typically parameterized to be normally distributed.<sup>10</sup> Our motivation in considering (1) is to use the utility maximization structure of a discrete choice model for inference on policy counterfactuals, while avoiding having to make these types of non-economic parametric assumptions.

The specification of indirect utility we use in (1) can be viewed as nesting (2) after dividing through by  $\alpha_i$  and taking  $V_{ij} \equiv \alpha_i^{-1}(X'_{ij}\beta_i + \xi_j + \epsilon_{ij})$ .<sup>11</sup> This observation highlights some considerations for our analysis. First, we want to be careful about assuming that  $V_i$  and  $X_i$  are independent, since the observable characteristics of consumers or insurance options might naturally be related to valuations.<sup>12</sup> Whereas (2) tightly parameterizes the way in which observable characteristics affect valuations, we prefer to leave this relationship nonparametric in our application, in part because we have a large administrative data set. Second, we typically do not want to assume that  $V_i$  and  $P_i$  are independent, since  $V_i$  also depends on unobservable characteristics of plans and consumers (the  $\xi_j$  and  $\epsilon_{ij}$  terms in (2)), and firms likely set premiums with knowledge of these characteristics (Berry, 1994; Hausman, 1996). Third, we want to allow for  $V_{ij}$  and  $V_{ik}$  to be arbitrarily dependent for  $j \neq k$ , in order to avoid imposing the unattractive substitution patterns associated with the logit model (Hausman and Wise, 1978; Goldberg, 1995; Berry et al., 1995; McFadden and Train, 2000).

### 3.2 Target Parameters

The primitive object of model (1) is the distribution of valuations,  $V_i$ , conditional on premiums,  $P_i$ , and other covariates,  $X_i$ . We will assume throughout the paper that this distribution is continuous so that ties between choices in (1) occur with zero probability. In addition to ensuring no ties, this also means we can associate the conditional distribution of valuations with a conditional density function  $f(\cdot|p, x)$  for each realization  $P_i = p$  and  $X_i = x$ .<sup>13</sup>

The function  $f$  will be a key object in the following. However, to compute common

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<sup>9</sup> For example, see equation (6) of Nevo (2011), or equation (1) of Berry and Haile (2015).

<sup>10</sup> However, see Fox, Kim, Ryan, and Bajari (2012) who provide conditions under which this distribution of random coefficients is nonparametrically identified, and Fox, Kim, Ryan, and Bajari (2011) who develop a nonparametric estimator based on discretizing this distribution. Their results maintain the type I extreme value assumption on  $\epsilon_{ij}$ , and require additional structure to allow for endogeneity in  $P_{ij}$ .

<sup>11</sup> This requires the mild assumption that  $\alpha_i > 0$  with probability 1.

<sup>12</sup> For example, this is implied by the motivating model discussed in Appendix B.

<sup>13</sup> More formally, this requires the assumption that the distribution of  $V_i$ , conditional on  $(P_i, X_i) = (p, x)$  is absolutely continuously distributed with respect to Lebesgue measure on  $\mathbb{R}^J$  for every  $(p, x)$  in the support of  $(P_i, X_i)$ .



counterfactual quantities of interest we do not need to consider  $f$  in its entirety. Instead, these quantities can be written as integrals (or sums of integrals) of  $f$ . For example, a natural counterfactual is the proportion of consumers who would choose plan  $j$  (the demand for  $j$ ) if premiums were changed from an observed baseline,  $p$ , to a new vector,  $p^*$ . This proportion can be written in terms of  $f$  as

$$\int \underbrace{\mathbb{1}[v_j - p_j^* \geq v_k - p_k^* \text{ for all } k]}_{\text{choose } j \text{ if premiums were } p^*} f(v|p, x) dv, \quad (3)$$

where we have conditioned on  $X_i = x$  as well. Another natural counterfactual quantity is the associated impact on average consumer surplus from such a premium change. This can be written as

$$\int \underbrace{\left\{ \max_{j \in \mathcal{J}} v_j - p_j^* \right\}}_{\text{consumer surplus under } p^*} f(v|p, x) dv - \int \underbrace{\left\{ \max_{j \in \mathcal{J}} v_j - p_j \right\}}_{\text{consumer surplus under } p} f(v|p, x) dv. \quad (4)$$

Conceptually, we view both (3) and (4) as scalar-valued functions of  $f$ . The functions vary in their form, and will further vary when we consider different counterfactual premiums  $p^*$ , choice probabilities for products for plans other than  $j$  in (3), and different values of (or averages over) the covariates,  $x$ . In Section 4, we will also consider a third class of quantities that measure changes in subsidy outlays.

To handle this generality in the following, we consider all such quantities to be examples of *target parameters*,  $\theta : \mathcal{F} \rightarrow \mathbb{R}^{d_\theta}$ , where  $\mathcal{F}$  is the collection of all conditional density functions on  $\mathbb{R}^J$ . The target parameter is just a function of the conditional density of valuations,  $f$ . In the examples just given, it is a scalar function, so that  $d_\theta = 1$ . However, we may also want to consider cases with  $d_\theta > 1$ , e.g. when thinking about the joint identified set for two related target parameters of interest, such as consumer surplus and government expenditure. Our goal is to infer the values of  $\theta(f)$  that are consistent with both the observed data and our prior assumptions.

### 3.3 Assumptions

We augment (1) with two types of prior assumptions. The first assumption is that one or more components of  $X_i$  are suitable instruments. The second assumption exploits the vertical structure of the metal tiers in the ACA.

### 3.3.1 Instrumental Variables

To describe the first type of assumption, let  $W_i$  and  $Z_i$  be two subvectors (or more general functions) of the observable characteristics,  $P_i$  and  $X_i$ . The  $Z_i$  variables will be assumed to be instruments and satisfy an exogeneity condition discussed ahead. This exogeneity condition will be conditional on  $W_i$ , so this subvector can be viewed as containing controls. Note that either or both of these subvectors could be chosen to be empty.

Stating the instrumental variable assumption requires considering the density of valuations conditional on  $W_i$  and  $Z_i$ . We can construct this object by averaging over  $f$  as follows:

$$f_{V|WZ}(v|w, z) \equiv \mathbb{E} \left[ f(v|P_i, X_i) \middle| W_i = w, Z_i = z \right]. \quad (5)$$

Our assumption that  $Z_i$  is an instrument, conditional on  $W_i$ , can then be stated as the following:

$$f_{V|WZ}(v|w, z) = f_{V|WZ}(v|w, z') \quad \text{for all } z, z', w, \text{ and } v. \quad (6)$$

In words, (6) just says that the distribution of valuations is invariant to shifts in  $Z_i$ , conditional on  $W_i$ . That is,  $Z_i$  is exogenous. A special case of (6) would be to take  $Z_i = P_i$  and  $W_i = X_i$ , which amounts to assuming that premiums are exogenous conditional on other observables,  $X_i$ .

In order for (6) to be a useful assumption, shifts in the instrument  $Z_i$  (still conditioning on  $W_i$ ) *should* have an effect on premiums. This follows the usual intuition: If  $Z_i$  is exogenous, then changes in observed choice shares as  $Z_i$  varies reflect changes in premiums, rather than changes in the unobservable valuations. The more that premiums vary with  $Z_i$ , the more information we will have to pin down different parts of the density of valuations,  $f$ , and hence the target parameter,  $\theta$ .

However, it is important to stress that we do not *require* an instrument to have a large amount of variation, or even to be continuous. These types of assumptions are commonly made to justify point identification of nonparametric discrete choice models, but in our data we can plainly see that they would not be satisfied, even if we were to assume (which we will not) that premiums themselves are exogenous.<sup>14</sup> This reality

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<sup>14</sup> These types of “large support” assumptions, and the closely related concept of identification-at-infinity, have had a prominent role in the literature on nonparametric identification. Early examples of their use include Manski (1985), Thompson (1989), Heckman and Honoré (1990), and Lewbel (2000). More recent applications of this argument to discrete choice include Heckman and Navarro (2007) and Fox and Gandhi (2016).

leads us to consider the partial identification framework discussed in the next section.

### 3.3.2 Vertical Structure

The second assumption we use exploits the vertical structure of the ACA, i.e. the fact that the Bronze plan is actuarially more generous than the Platinum plan. For example, the Bronze plan has a higher deductible and higher out-of-pocket maximum than the Platinum plan (see Table 1). Our assumption is that, for equal premiums, a consumer would always prefer a plan that is more generous to one that is less generous. With  $j = 4$  as the Platinum plan, and  $j = 1$  is the Bronze plan, this means we assume that  $f$  places zero mass on regions where  $v_1 > v_4$  or, equivalently, concentrates all of its mass on regions with  $v_4 \geq v_1$ .<sup>15</sup>

Implementing this assumption in the context of the ACA is complicated by the existence of CSR subsidies. As discussed in Section 2, CSRs are used in Covered California to change the terms of the Silver plan depending on a consumer's income. Lower-income consumers face more generous Silver plans, and this generosity gets gradually phased out at higher incomes. The effect of this is that, depending on a consumer's income, they might prefer a Silver plan (at equal premiums) to a Gold or even a Platinum plan.

With this flexibility in mind, we formalize the verticality assumption as follows. For each realization of  $W_i$  defined as in the previous section, we choose a set  $\bar{\mathcal{V}}(w)$  and then assume that  $f$  is such that

$$\int_{\bar{\mathcal{V}}(w)} f_{V|WZ}(v|w, z) dv = 1 \quad \text{for all } w, z. \quad (7)$$

This assumption captures the idea that the distribution of valuations is concentrated on a given region, e.g. by taking  $\bar{\mathcal{V}}(w) = \{v : v_4 \geq v_1\}$  in the example above. Allowing  $\bar{\mathcal{V}}(w)$  to change with covariates  $w$  will be used in our application to allow the vertical ordering to change with income, so as to account for CSRs. Note that if one does not want to impose a verticality assumption, then one can simply take  $\bar{\mathcal{V}}(w) = \mathbb{R}^J$ , in which case (7) will be satisfied for any conditional density  $f$ .

### 3.4 The Identified Set

We now develop our method for determining the set of possible values that the target parameter  $\theta(f)$  could take over valuation densities  $f$  that both satisfy the assumptions

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<sup>15</sup> Note that we will not assume that consumers prefer any of the plans (inside options) to the outside option.

in the preceding section, and are consistent with the observed data. To do this, we assume that researcher has at their disposal a collection of conditional choice shares, denoted as

$$s(j|p, x) \equiv \mathbb{P}[Y_i = j | P_i = p, X_i = x]. \quad (8)$$

In our setting, these shares will be estimated from a combination of administrative data on enrollment and survey data that we use to estimate the market size. Here, our analysis of identification is premised on the thought experiment of perfect knowledge of these choice shares.<sup>16</sup>

Each density of valuations implies a set of choice shares analogous to (8). In particular, a consumer would choose option  $j$  when faced with a premium  $p$  if and only if they have valuations in the set

$$\mathcal{V}_j(p) \equiv \{(v_1, \dots, v_J) \in \mathbb{R}^J : v_j - p_j \geq v_k - p_k \text{ for all } k\}. \quad (9)$$

The choice shares for good  $j$  implied by the density  $f$  are just determined by the mass that  $f$  places on this set. We denote these implied choice shares by

$$s_f(j|p, x) \equiv \int_{\mathcal{V}_j(p)} f(v|p, x) dv. \quad (10)$$

We say that a valuation density  $f$  is observationally equivalent if its predicted choice shares match the observed choice shares, i.e. if

$$s_f(j|p, x) = s(j|p, x) \quad \text{for all } j, p \text{ and } x. \quad (11)$$

The identified set of valuation densities is the set of all  $f$  that are both observationally equivalent and satisfy the assumptions laid out in the previous section. We call this set  $\mathcal{F}^*$ :

$$\mathcal{F}^* \equiv \{f \in \mathcal{F} : f \text{ satisfies (6), (7), and (11)}\}. \quad (12)$$

However, our real interest centers on the target parameter,  $\theta$ , examples of which include counterfactual choice shares (3) and changes in consumer surplus (4). The identified set for  $\theta$  is just the image of the identified set for  $\mathcal{F}^*$  under  $\theta$ . That is,

$$\Theta^* \equiv \{\theta(f) : f \in \mathcal{F}^*\}.$$

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<sup>16</sup> More formally, it is premised on perfect knowledge of the joint distribution of  $(Y_i, P_i, X_i)$ .

The set  $\Theta^*$  consists of all values of the target parameter that are consistent with both the data and the instrumental variable and verticality assumptions (6) and (7). It is our central object of interest.

The difficulty lies in characterizing  $\Theta^*$ . In the following, we develop an argument that enables us to compute  $\Theta^*$  exactly. The idea is to partition  $\mathbb{R}^J$  into the smallest collection of sets within which choice behavior would remain constant under all premiums that were observed in the data, as well as all premiums that are required to compute the target parameter. We call this set the *minimal relevant partition* (MRP) of valuations. We then reduce the problem of characterizing  $\Theta^*$  from one of searching over densities  $f$  to one of searching over mass functions defined on the sets that constitute the MRP. For cases in which the target parameter is scalar-valued ( $d_\theta = 1$ ), this latter problem can often be solved with two linear programs.

### 3.5 The Minimal Relevant Partition of Valuations

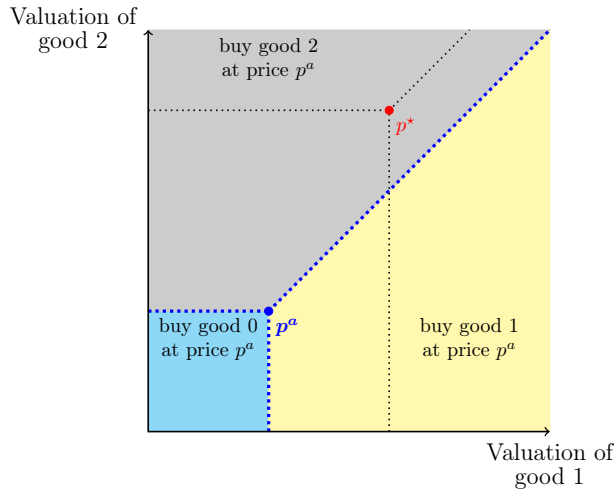
We illustrate the definition and construction of the MRP using a simple example with  $J = 2$ , so that a consumer’s valuations (and the premiums of the plans in their choice set) can be represented as points in the plane. A general (and formal) definition of the MRP is given in Section 3.7.

Suppose that the data consists of a single observed premium vector,  $p^a \equiv (p_1^a, p_2^a)$ , and that we are concerned with behavior under a counterfactual premium vector,  $p^*$ , which we do not observe in the data. The idea behind the MRP in this example is illustrated in Figure 1. Panel (a) shows that considering behavior under premium  $p^a$  divides  $\mathbb{R}^2$  into three sets depending on whether a consumer would choose options 0, 1, or 2 when faced with  $p^a$ . Panel (b) shows the analogous situation under premium  $p^*$ . Intersecting these two three-set collections creates the collection of six sets shown in panel (c). This collection of six sets is the MRP in this example.

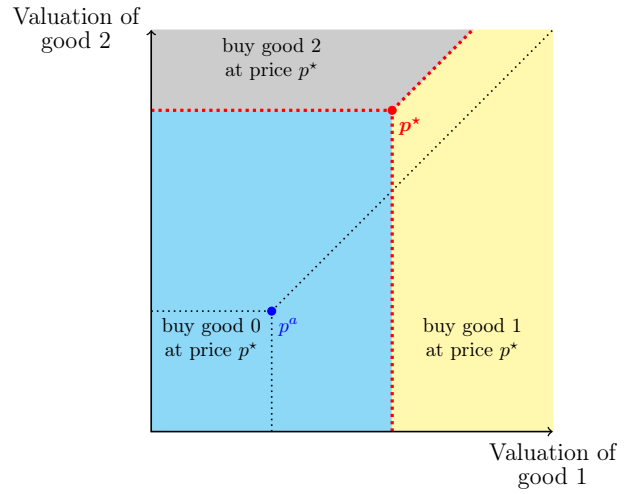
The MRP is “minimally relevant” in the sense that any two consumers who have valuations in the same set would exhibit the same choice behavior under both premiums  $p^a$  and  $p^*$ . Conversely, any two consumers with valuations in different sets would exhibit different choice behavior under at least one of these premiums. For example, consumers with valuations in the set marked  $\mathcal{V}_2$  in Figure 1c make the same choices as those with valuations in  $\mathcal{V}_4$  under  $p^a$ , but make different choices under  $p^*$ , where the first group chooses the outside option, and the second group chooses plan 1. Similarly, consumers with valuations in  $\mathcal{V}_2$  and  $\mathcal{V}_6$  both choose the outside option at  $p^*$ , but at  $p^a$  the first group chooses plan 2 and the second group chooses plan 1.

In Figure 1d, we show how the MRP would change if we were to observe a second

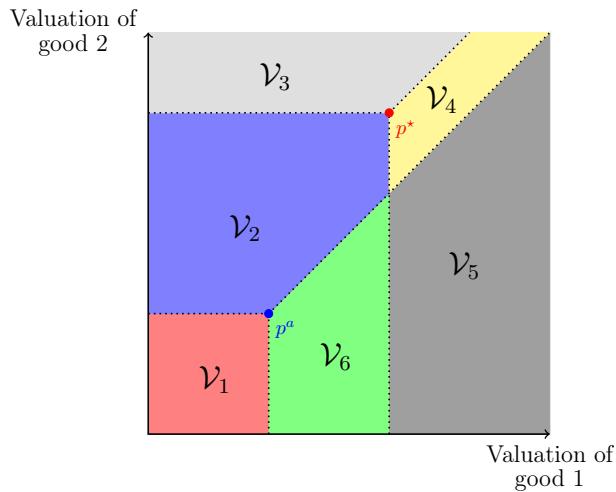
**Figure 1: Partitioning the Space of Valuations**



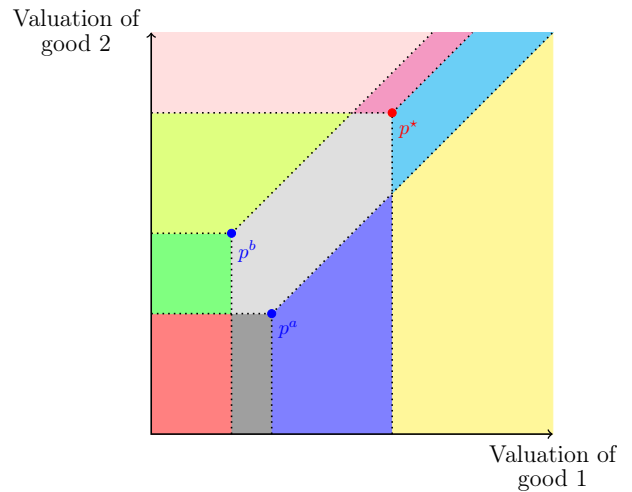
**(a)** Choices if prices were  $p^a$ .



**(b)** Choices if prices were  $p^*$ .



**(c)** The minimal relevant partition (MRP) constructed from  $p^a, p^*$ .



**(d)** The minimal relevant partition (MRP) constructed from  $p^a, p^b, \text{ and } p^*$ .

premium,  $p^b$ . The MRP now consists of ten sets, but the idea is the same: Consumers with valuations within a given set have the same choice behavior under all premiums  $p^a, p^b$ , and  $p^*$ , while consumers with valuations in different sets would make different choices for at least one of these premiums.

The way the MRP is constructed ensures that predicted choice shares for any valuation density can be computed by summing the mass that the density places on sets included in the MRP. For example, in Figure 1c, we can see that the share of consumers who would choose good 1 if premiums were  $p^a$  can be written as

$$s_f(1|p^a, x) = \int_{\mathcal{V}_5} f(v|p^a, x) dv + \int_{\mathcal{V}_6} f(v|p^a, x) dv,$$

while the share of consumers who would choose good 2 is given by

$$s_f(2|p^a, x) = \int_{\mathcal{V}_2 \cup \mathcal{V}_3 \cup \mathcal{V}_4} f(v|p^a, x) dv.$$

This allows us to simplify the determination of whether a given  $f$  is observationally equivalent by considering only the total mass that  $f$  places in sets in the MRP, without having to be concerned with how this mass is distributed within these sets.

Since we constructed the MRP using  $p^*$  too, the same is also true when considering target parameters  $\theta$  that measure choice behavior at  $p^*$ . For example, suppose that our target parameter is the choice share of plan 2 if premiums were changed from  $p^a$  to  $p^*$ . This is a particular case of (3), and can be written in terms of the MRP as

$$\theta(f) = \int_{\mathcal{V}_3} f(v|p^a, x) dv. \tag{13}$$

As another example, we could write the associated *change* in this choice share as

$$\theta(f) = \int_{\mathcal{V}_3} f(v|p^a, x) dv - \int_{\mathcal{V}_2 \cup \mathcal{V}_3 \cup \mathcal{V}_4} f(v|p^a, x) dv = - \int_{\mathcal{V}_2 \cup \mathcal{V}_4} f(v|p^a, x) dv.$$

In both of these quantities, we have kept the density conditional on the observed premium,  $p^a$ , which corresponds to the typical counterfactual of changing prices while keeping the unobservable factors fixed.

### 3.6 Computing Bounds on the Target Parameter

Now suppose that we observe the following choice shares at  $p^a$ :

$$s(0|p^a) = .20, \quad s(1|p^a) = .14, \quad \text{and} \quad s(2|p^a) = .66,$$

where we are assuming that there are no covariates (no  $X_i$ ) for simplicity. Also for simplicity, we will start by assuming that premiums are exogenous, i.e. we limit our attention to  $f$  for which  $f(v|p^a) = f(v|p^*) = f(v)$ .<sup>17</sup> In this case, the observational equivalence condition (11) can be written as

$$\begin{aligned} & \int_{\mathcal{V}_1} f(v) dv = s(0|p^a) = .20, \\ \text{and } & \int_{\mathcal{V}_5} f(v) dv + \int_{\mathcal{V}_6} f(v) dv = s(1|p^a) = .14, \\ \text{and } & \int_{\mathcal{V}_2} f(v) dv + \int_{\mathcal{V}_3} f(v) dv + \int_{\mathcal{V}_4} f(v) dv = s(2|p^a) = .66. \end{aligned} \quad (14)$$

As shown in (13), if our target parameter is the choice share of plan 2 at  $p^*$ , this can be written as

$$\theta(f) = \int_{\mathcal{V}_3} f(v) dv. \quad (15)$$

The key observation is that, even though all of these quantities depend on a density  $f$ , they can be computed with knowledge of just six non-negative numbers:

$$\left\{ \phi_l \equiv \int_{\mathcal{V}_l} f(v) dv \right\}_{l=1}^6.$$

This suggests that we can focus only on the total mass placed on the sets in the MRP without losing any information. To find the largest value that  $\theta(f)$  can take while still respecting (14), we just rephrase everything in terms of  $\{\phi_l\}_{l=1}^6$  and then maximize (15) subject to (14):

$$t^* \equiv \max_{\phi \in \mathbb{R}^6} \phi_3 \quad (16)$$

$$\text{subject to: } \phi_1 = .20$$

$$\phi_5 + \phi_6 = .14$$

$$\phi_2 + \phi_3 + \phi_4 = .66$$

$$\phi_l \geq 0 \quad \text{for } l = 1, \dots, 6.$$

This is a linear program. In this simple example, one can see by inspection that the solution of the program is to take  $\phi_3 = .66$ , so that  $t^* = .66$ . To find the smallest value of  $\theta(f)$  we solve the analogous minimization problem, the optimal value of which we

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<sup>17</sup> In terms of (6), this would be like taking  $W_i$  to be a null (empty) vector, and taking  $Z_i = P_i$ .



call  $t_*$ . In this example,  $t_* = 0$ .

In the next section, we formally prove that  $\Theta^* = [t_*, t^*]$ . This result shows that the procedure of reducing  $f$  to a collection of six numbers  $\{\phi_l\}_{l=1}^6$  is a sharp characterization of  $\Theta^*$  in the sense that it entails no loss of information. The intuition behind the sharpness is as follows. First, for any value in  $t \in \Theta^*$ , there must exist (by definition) an  $f \in \mathcal{F}^*$  such that  $\theta(f) = t$ . This  $f$  generates a collection of numbers  $\{\phi_l = \int_{\mathcal{V}_l} f(v) dv\}_{l=1}^6$ , which must satisfy the constraints in (16), since every  $f \in \mathcal{F}^*$  satisfies (14). Conversely, given any value of  $t \in [t_*, t^*]$ , there exists a set of numbers  $\{\phi_l\}_{l=1}^6$  satisfying the constraints in (16), and such that  $\phi_3 = t$ .<sup>18</sup> From this set of numbers  $\{\phi_l\}_{l=1}^6$ , we can construct a density  $f$  that satisfies (14) by distributing mass in the amount of  $\phi_l$  arbitrarily within each  $\mathcal{V}_l$ . Evidently, this density will also satisfy  $\theta(f) = \phi_3 = t$ . Thus,  $\Theta^* = [t_*, t^*]$ .

Now suppose that we have a second observed premium,  $p^b$ , so that the MRP is as shown in Figure 1d. In this case, the MRP contains 10 sets, so the linear program analogous to (16) will have 10 variables of optimization. In addition to the observational equivalence constraints for  $p^a$  in (16), these variables will also need to satisfy three more observational equivalence constraints corresponding to the observed shares for  $p^b$ , which we will suppose here are given by

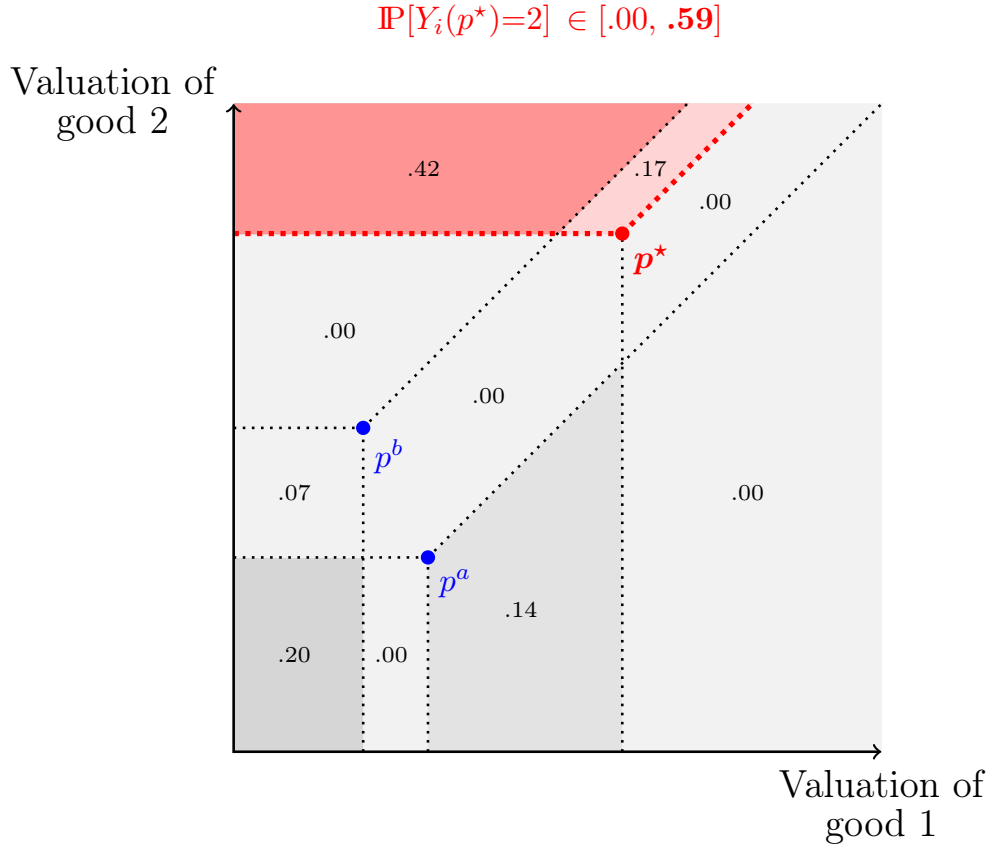
$$s(0|p^b) = .27, \quad s(1|p^b) = .31, \quad \text{and} \quad s(2|p^b) = .42.$$

Reasoning through the solution to the resulting program is more complicated. Since the observed shares for  $p^a$  still need to be matched, it is still the case that a total mass of .66 must be placed over consumers who would choose plan 2 under  $p^a$ . Some of these consumers might choose the outside option under  $p^b$ . In fact, as shown in Figure 2, this must be the case for a proportion of at least  $s(0|p^b) - s(0|p^a) = .07$  of consumers. Given this new requirement, the maximum amount of mass remaining to distribute over consumers who would choose plan 2 under  $p^*$  has decreased from .66 to  $.66 - .07 = .59$ . This is the new upper bound,  $t^*$ . The fact that it is smaller than the previous upper bound reflects the additional information contained in  $p^b$ . The lower bound,  $t_*$ , is still zero, because it is still possible to match the observed choice shares for  $p^a$  and  $p^b$  by concentrating all mass to the south of  $p^*$ .

When we take this procedure to the data, the linear programs will have thousands of variables and constraints, which makes this sort of case-by-case reasoning impossible.

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<sup>18</sup> This follows because the constraint set in (16) is closed and connected and the objective function is continuous.



**Figure 2:** The numbers in each set show a solution to the linear program when the target parameter is the proportion of consumers who choose plan 2 at  $p^*$  and the objective is to find the upper bound (maximize) this proportion. Matching the share of consumers who choose the outside option at the new observed premium,  $p^b$ , means there is now .07 less mass to devote to this objective.

Instead, we will use state-of-the-art solvers to obtain  $t^*$  and  $t_*$ .<sup>19</sup> In practice, we will also not assume that premium are exogenous. This makes a graphical interpretation unweildy, since a separate diagram like Figure 2 would be needed for each value of the conditioning variables. The mass placed over sets within each diagram gets linked together by imposing constraints on these masses that are analogous to the instrumental variable assumption (6). Part of the formal analysis in the next section involves showing that such a procedure retains sharpness.

<sup>19</sup> In particular, we use Gurobi (Gurobi Optimization, 2015). We formulate and presolve the problems using AMPL (Fourer, Gay, and Kernighan, 2002).

### 3.7 Formalization

In this section, we formalize the discussion in the previous three sections in the following ways. First, we provide a precise definition of the MRP. Second, we generalize the transformation from densities  $f$  to mass functions over the sets in the MRP, which, as in the previous section, we refer to as  $\phi$ . Third, we show how to compute bounds for any target parameter under the instrumental variable and verticality assumptions. Fourth, we provide the general statement and proof of the result that these bounds are sharp. Lastly, we consider the conditions under which these bounds can be computed by solving linear programs. Throughout the analysis, we model  $(P_i, X_i)$  as discretely distributed with finite support, although this is not essential to the methodology.

Beginning with the MRP, we let  $\mathcal{P}$  denote a finite set of premiums that is chosen by the researcher and always contains at least the marginal support of premiums,  $\text{supp}(P_i)$ . The premiums in  $\mathcal{P}$  are used to construct the MRP, so a given MRP depends on  $\mathcal{P}$ . For example, in Figure 1c we had  $\mathcal{P} = \{p^a, p^*\}$ , while in Figure 1d,  $\mathcal{P} = \{p^a, p^b, p^*\}$ .

The choice of which additional points to include in  $\mathcal{P}$  is determined by the parameter of interest,  $\theta$ . In Figure 1, we were focusing on demand at a new premium,  $p^*$ , so  $\mathcal{P}$  had to include  $p^*$ . This restriction will be formalized below as the statement that  $\theta(f)$  can be evaluated for any  $f$  by only considering the total mass that  $f$  places on sets in the MRP. Additional points can always be added to  $\mathcal{P}$  in an effort to make this restriction hold.

We use the set  $\mathcal{P}$  to formally define the MRP as follows.

**Definition 1.** Let  $Y(v, p) \equiv \arg \max_{j \in \mathcal{J}} v_j - p_j$  for any  $(v_1, \dots, v_J), (p_1, \dots, p_J) \in \mathbb{R}^J$ , where  $v \equiv (v_0, v_1, \dots, v_J)$  and  $p \equiv (p_0, p_1, \dots, p_J)$  with  $v_0 = p_0 = 0$ . The minimal relevant partition of valuations (MRP) is a collection  $\mathbb{V}$  of sets  $\mathcal{V} \subseteq \mathbb{R}^J$  for which the following property holds for almost every  $v, v' \in \mathbb{R}^J$  (with respect to Lebesgue measure):

$$v, v' \in \mathcal{V} \text{ for some } \mathcal{V} \in \mathbb{V} \quad \Leftrightarrow \quad Y(v, p) = Y(v', p) \text{ for all } p \in \mathcal{P}. \quad (17)$$

Definition 1 creates a collection of sets that is “minimally relevant” in the sense that any two consumers who have valuations in a set in the collection would exhibit the same choice behavior for every premium vector in  $\mathcal{P}$ . Conversely, any two consumers with valuations in different sets would exhibit different choice behavior for at least one premium in  $\mathcal{P}$ . Constructing the MRP is intuitive, but somewhat involved both notationally and algorithmically. Since the details of constructing the MRP are not necessary for understanding the methodology, we relegate our discussion of this to

Appendix C.<sup>20</sup>

The utility of the MRP as a concept is that it allows us to express the choice probabilities associated with any density of valuations,  $f$ , in terms of the mass that  $f$  places on sets in  $\mathbb{V}$ . In particular, for every  $p \in \mathcal{P}$  and  $j \in \mathcal{J}$ , let  $\mathbb{V}_j(p) \subseteq \mathbb{V}$  denote the sets in the MRP for which a consumer with valuations in these sets would choose  $j$  when facing premiums  $p$ .<sup>21</sup> Then the probability that a consumer chooses  $j$  under premiums  $p$  is the probability that  $V_i$  lies in the union of  $\mathcal{V} \in \mathbb{V}_j(p)$ . Since sets in  $\mathbb{V}$  are disjoint, the observational equivalence condition (10) can be written as the sum of the masses that a given  $f$  places on sets in  $\mathbb{V}_j(p)$ , i.e.

$$s_f(j|p, x) = \sum_{\mathcal{V} \in \mathbb{V}_j(p)} \int_{\mathcal{V}} f(v|p, x) dv. \quad (18)$$

Having defined the MRP, we now define mass functions over the MRP. To do this, let  $\phi(\cdot|\cdot, \cdot)$  denote a function with domain  $\mathbb{V} \times \text{supp}(P_i, X_i)$ . Such a function  $\phi$  can be viewed as an element of  $\mathbb{R}^{d_\phi}$ , where  $d_\phi$  is the product of the cardinalities of these sets. Let  $\mathbb{R}_+^{d_\phi}$  denote the subset of  $\mathbb{R}^{d_\phi}$  whose elements are all non-negative and define

$$\Phi \equiv \left\{ \phi \in \mathbb{R}_+^{d_\phi} : \sum_{\mathcal{V} \in \mathbb{V}} \phi(\mathcal{V}|p, x) = 1 \quad \text{for all } (p, x) \in \text{supp}(P_i, X_i) \right\}. \quad (19)$$

The set  $\Phi$  contains all functions that could represent a conditional probability mass function with domain given by the finite collection of sets,  $\mathbb{V}$ .

Each conditional valuation density  $f$  generates a mass function  $\phi_f \in \Phi$  defined by

$$\phi_f(\mathcal{V}|p, x) \equiv \int_{\mathcal{V}} f(v|p, x) dv. \quad (20)$$

We assume that the value of the target parameter for any  $f$  is fully determined by  $\phi_f$ . Formally, the assumption is that there exists a known function  $\bar{\theta}$  with domain  $\Phi$  such that  $\theta(f) = \bar{\theta}(\phi_f)$  for every  $f \in \mathcal{F}$ . Since  $\Phi$  depends on the MRP, and the MRP depends on  $\mathcal{P}$ , satisfying this requirement is a matter of choosing  $\mathcal{P}$  to be sufficiently

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<sup>20</sup> We should, however, note two small misnomers in our terminology that become evident in the construction, or perhaps by inspecting Figure 1. First, the MRP may not be a strict partition, because adjacent sets in  $\mathbb{V}$  could overlap on their boundary. Since we are focusing on continuously distributed valuations, this distinction does not have any practical or empirical relevance, and does not violate Definition 1. Second, and for the same reason, although we have described the MRP as “the” MRP, it may not be unique, since one could consider a boundary region to be in either of the sets to which it is a boundary without violating (17) on a set of positive measure. Again, this is not important for our analysis given our focus on continuously distributed valuations.

<sup>21</sup> Using the notation of Definition 1,  $\mathbb{V}_j(p) \equiv \{\mathcal{V} \in \mathbb{V} : Y(v, p) = j \text{ for almost every } v \in \mathcal{V}\}$ .

rich to evaluate the target parameter of interest,  $\theta$ .

We have now phrased both the target parameter and observational equivalence condition in terms of  $\phi$ . The last step is to translate the instrumental variable and verticality assumptions into statements about  $\phi$ . For the instrumental variable assumption, we first define for any  $\phi \in \Phi$  a function  $\phi_{\mathbf{V}|WZ}$  in analogy to (5) as

$$\phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z) \equiv \mathbb{E} \left[ \phi(\mathcal{V}|P_i, X_i) \middle| W_i = w, Z_i = z \right], \quad (21)$$

where  $W_i$  and  $Z_i$  are as in the statement of that condition. Then, a condition appropriately analogous to (6) is

$$\phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z) = \phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z') \quad \text{for all } z, z', w, \text{ and } \mathcal{V}. \quad (22)$$

Similarly, for the verticality assumption, we define in analogy to (7),

$$\sum_{\mathcal{V} \in \bar{\mathbf{V}}(w)} \phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z) = 1 \quad \text{for all } w, z, \quad (23)$$

where  $\bar{\mathbf{V}}(w)$  is the subset of  $\mathbf{V}$  that intersects  $\bar{\mathcal{V}}(w)$ , i.e.  $\bar{\mathbf{V}}(w) \equiv \{\mathcal{V} \in \mathbf{V} : \lambda(\mathcal{V} \cap \bar{\mathcal{V}}(w)) > 0\}$ , with  $\lambda$  denoting Lebesgue measure on  $\mathbb{R}^J$ .

The next proposition shows that  $\Theta^*$  can be characterized exactly by solving systems of equations in  $\phi$ .

**Proposition 1.** *Let  $t \in \mathbb{R}^{d_\theta}$ . Then  $t \in \Theta^*$  if and only if there exists a  $\phi \in \Phi$  such that*

$$\bar{\theta}(\phi) = t, \quad (24)$$

$$\sum_{\mathcal{V} \in \mathbf{V}_j(p)} \phi(\mathcal{V}|p, x) = s(j|p, x) \quad \text{for all } j \in \mathcal{J} \text{ and } (p, x), \quad (25)$$

$$\phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z) = \phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z') \quad \text{for all } z, z', w, \text{ and } \mathcal{V}, \quad (26)$$

$$\text{and} \quad \sum_{\mathcal{V} \in \bar{\mathbf{V}}(w)} \phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z) = 1 \quad \text{for all } w, z. \quad (27)$$

Observe that each of (25)–(27) are linear in  $\phi$ .<sup>22</sup> If  $\bar{\theta}$  is also linear in  $\phi$ , then Proposition 1 shows that  $\Theta^*$  can be exactly characterized by solving linear systems of equations. This linearity is satisfied for common target parameters, such as choice shares and consumer surplus.<sup>23</sup> One byproduct of this linearity is that  $\Theta^*$  will be connected, and so when  $d_\theta = 1$  it can also be characterized by solving two linear programs. We record

<sup>22</sup> This requires noting from (21) that  $\phi_{\mathbf{V}|WZ}(\mathcal{V}|w, z)$  is itself a linear function of  $\phi$ .

<sup>23</sup> The former is clear from e.g. (15), but the latter is not obvious; see Appendix E.

this point in the following proposition.

**Proposition 2.** *If  $\bar{\theta}$  is continuous on  $\Phi$ , then  $\Theta^*$  is a compact, connected set. In particular, if  $d_\theta = 1$ , then  $\Theta^* = [t_*, t^*]$ , where*

$$t_* \equiv \min_{\phi \in \Phi} \bar{\theta}(\phi) \quad \text{subject to (25)–(27),} \quad (28)$$

and with  $t^*$  defined as the solution to the analogous maximization problem.

In practice, one often finds that the feasible set in (28) is empty, so that  $\Theta^*$  is also empty.<sup>24</sup> This is an indication of either sampling error in the observed shares,  $s(j|p, x)$ , or model misspecification, or both. Instead of reporting empty identified sets, we modify Proposition 2 to construct a set estimator of  $\Theta^*$ . The estimator uses a procedure analogous to the method of moments for point identified models. First, we minimize a criterion function that measures the extent to which the observational equivalence equality in (25) is violated. Second, we find the set of values  $t$  that  $\bar{\theta}(\phi)$  can take while coming close to the optimal value of the criterion.<sup>25</sup> By choosing an absolute deviations criterion, this procedure amounts to solving three linear programs in cases where  $\bar{\theta}$  is linear. We provide more detail on the estimation procedure in Appendix F.

## 4 Demand in Covered California

### 4.1 Data

Our primary data source is administrative data containing the universe of households who purchased a plan through Covered California in 2014. The data contains unique person and household identifiers for each individual in each household, as well as their age, income (as a percentage of the FPL), gender, zipcode of residence, and choice of plan. We focus on the subpopulation of subsidy-eligible households (100-400% FPL) in which the uninsured members consist of either one or two adults aged 27 and older.<sup>26</sup> In addition, we drop the relatively small number of purchasing households with income under 140% of FPL, since these households are likely eligible for other public health programs. These restrictions reduce our analysis sample to 630,924 of the 877,365 households who purchased coverage.

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<sup>24</sup> We follow the usual convention here of letting  $t_* = +\infty$  and  $t^* = -\infty$  if the feasible set is empty, in which case  $\Theta^* = \emptyset$ .

<sup>25</sup> This second step would not be needed if the model were point identified.

<sup>26</sup> That is, the household either is childless, or the children are insured through a public program such as Medi-Cal.

We characterize each household by a vector  $X_i$  of observables consisting of age, income, and rating region. Household age is defined as the age of a single household member, or the average age of a couple, which we discretize into 19 two-year bins running from age 27 and 64. We discretize household income into 20 bins of either 10% or 20% of the FPL.<sup>27</sup> When crossed with the 19 rating regions in Covered California, this yields 7,220 unique age  $\times$  income  $\times$  region bins of the observable characteristics,  $X_i$ . Since the number of households per bin varies greatly by region, we will report parameters that average over  $X_i$ , and therefore put greater weight on larger geographic markets.

We reconstruct the post-subsidy premiums faced by each household by using their demographic information together with knowledge of insurers’ base prices and the ACA-mandated age-rating and income subsidies. As described in Section 2, pre-subsidy premiums for a given metal tier, rating region, and insurer only vary by age, while the post-subsidy premiums also vary by income. As a consequence, the post-subsidy premiums faced by consumers ( $P_{ij}$ ) are a deterministic function of household characteristics,  $X_i$ . In order to infer the impact of premiums on plan choice (i.e. to infer demand), we will therefore need to consider variation across  $X_i$ . The assumptions we use to do this are described in the next section.

Our analysis is focused on a household’s choice of coverage level (metal tier). The implicit assumption here is that a household’s health insurance choice problem is separable in coverage level and insurer. We view this as a reasonable assumption for Covered California because the regulations ensure that the metal tiers offered—as well as the characteristics of the tiers—do not vary by insurer. We define premiums for each tier in each market by taking the median post-subsidy premium across insurers.

As in most other discrete choice demand settings, we do not directly observe individuals who chose the outside option, i.e. to not purchase a plan through Covered California. This means that we first need to transform data on quantities chosen for the inside choices into choice shares by estimating the size of the market. To do this, we use the 2013 American Community Survey public use file (via IPUMS, Ruggles et al., 2015) to estimate uninsurance rates conditional on  $X_i$ . Our estimation procedure for this part closely follows those used by Finkelstein et al. (2017) and Tebaldi (2017). For more detail, see Appendix G.

Table 2 provides some summary statistics for the data we use in estimation. The participation rate in Covered California is on average 32%, and varies widely across

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<sup>27</sup> These bins are  $[140, 150), [150, 160), \dots, [270, 280), [280, 300), [300, 320), \dots, [380, 400]$ . The 20% bins are used for households with income higher than 280% of FPL, who account for around 15% of purchasing households.

**Table 2: Summary Statistics**

	Markets	Mean	St. Dev.	P-10	Median	P-90
Number of buyers	7,220	270.4	302.2	37	172	628
Age	7,220	42.9	11.4	29	43	59
Income (FPL%)	7,220	238.4	69.5	150	230	340
Takeup rate	7,220	0.318	0.258	0.037	0.260	0.705
Average premium paid (\$/month)	7,220	155.4	75.3	65	143	262

	Bronze		Silver		Gold		Platinum	
	Premium(\$/m)	Share	Premium(\$/m)	Share	Premium(\$/m)	Share	Premium(\$/m)	Share
By age:								
27-34	104	0.064	158	0.134	212	0.013	254	0.014
35-49	92	0.072	158	0.201	227	0.016	281	0.016
50-64	75	0.093	182	0.291	299	0.021	391	0.017
By income (FPL%):								
140-150	4	0.015	56	0.418	133	0.006	194	0.008
150-200	21	0.060	87	0.367	166	0.010	228	0.011
200-250	67	0.102	145	0.199	225	0.019	288	0.017
250-400	160	0.082	243	0.083	323	0.021	387	0.019

markets, with a 10th percentile of 4% and a 90th percentile of over 70%. Older and poorer buyers are more likely to purchase coverage. The impact of the CSRs is evident in the table: Buyers with income below 200% face premiums of less than \$100 per month to purchase a Silver plan with actuarial value of 88% or more. Likely as a consequence, that plan is chosen by roughly 40% of such consumers—many fewer than the 10% of consumers with income over 250% FPL who face a more expensive and less generous Silver plan.

## 4.2 Identifying Assumptions

In this section, we describe our specific implementations of assumptions (6) and (7).

An insurer’s primary decision in Covered California is the base price for a rating region and a coverage level. This decision likely depends on differences in demand and cost specific to each rating region, for example due to the underlying socioeconomic or health characteristics of the residents in a region, or due to differences in hospitals of medical providers. These factors are unobserved in our data, so we will not assume that variation in premiums across regions is exogenous. That is, we will not impose any restriction on how preferences (the density of valuations  $f$ ) varies across regions.



Instead, we will assume—in a limited way—that preferences are invariant to changes in age and income. Since premiums vary with age due to the age-rating, and with income, due to the premium subsidies, this will provide variation in premiums that we can use to help identify demand counterfactuals. The way in which premiums evolve with age and income is prescribed by the regulations in Covered California, so the behavior of insurers is not likely to be an important threat to this strategy. Rather, the main concern we have is that valuations also change with age or income due to changes in latent risk factors or preferences. For this reason, we will look only at very local variation in age and income.

We formulate this approach using the notation of Section 3 by letting  $W_i$  denote a coarser aggregate of a group of  $X_i$  realizations. For each region, we group  $X_i$  into age bins given by  $\{27\text{--}32, 33\text{--}38, 39\text{--}44, 45\text{--}50, 51\text{--}56, 57\text{--}64\}$  and income bins given in percentage of FPL by  $\{140\text{--}170, 170\text{--}200, 200\text{--}220, 220\text{--}250, 250\text{--}280, 280\text{--}340, 340\text{--}400\}$ . A value of  $W_i$  is then taken to be a region indicator crossed between all possibilities of these coarser age-income bins. Conditioning on a value of  $W_i$ , we observe multiple premiums corresponding to variation in age and income within the  $W_i$  bin. Our assumption is that the distribution of latent valuations does not change as  $X_i$  varies within this coarser bin.

For example, one value of  $W_i = w$  corresponds to the North Coast rating region, ages between 39–44 and incomes between 170–200% of the FPL. Within this bin, we have 9 values of  $X_i$ , comprised of the finer age bins  $\{39\text{--}40, 41\text{--}42, 43\text{--}44\}$  crossed with the finer income bins  $\{170\text{--}180, 180\text{--}190, 190\text{--}200\}$ . For each of these 9 values we observe a different premium vector. Since the variation we want to use is now in  $X_i$ , conditioning on a value of  $W_i$ , the notation we developed in Section 3 corresponds to taking  $Z_i = X_i$ . The assumption we use is now precisely (6) in that discussion, repeated here for emphasis:

$$\underbrace{f_{V|WZ}(v|w, z) = f_{V|WZ}(v|w, z')}_{\text{within a coarse bin } (W_i = w), \text{ valuations are invariant to age and income } (z \neq z')} \quad \text{for all } z, z', w, \text{ and } v. \quad (29)$$

The income aspect of assumption (29) gives empirical content to the separability between income and valuations in (1). The implied behavioral restriction is that a change in income that leaves a household inside a given  $W_i$  bin would not affect their choice of metal tier, although it could lead a household to change to or from the outside option. That is, the assumption is that there are no income effects with respect to the choice of metal tier within a coarse income bin. This type of assumption must hold as the width of the  $W_i$  bins gets smaller. It is also commonly assumed in parametric

implementations such as (2), which impose exogeneity of all non-price explanatory variables.

The other aspect of (29) is the invariance to age. We are more concerned about this assumption, since health risks certainly increase in age, and likely at an increasing rate. We begin with (29) primarily for ease of interpretability. In Section 4.4, we relax assumption (6)/(29) to a strictly weaker “imperfect instrument” assumption that allows for some variation with age. We view our estimates there as constituting our most credible and interesting results.

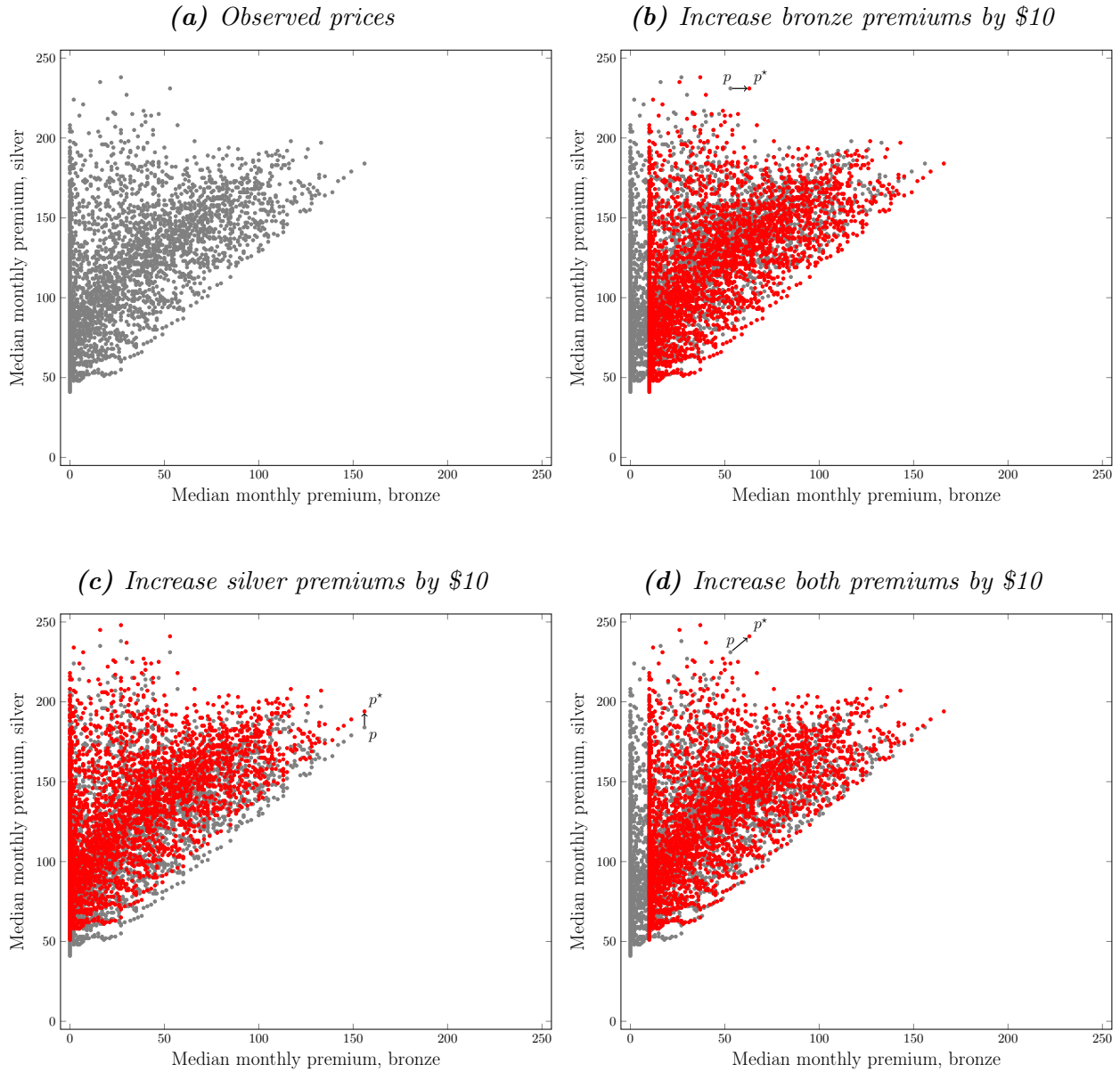
The other assumption we utilize is the verticality assumption (7), adjusted to account for CSRs as shown in Table 1. To account for the CSRs, we have chosen the coarse bins ( $W_i$ ) so as not to cross the CSR thresholds of 200 and 250% of the FPL. For consumers with income above 200% FPL, we assume that for equal prices everyone would prefer Platinum over Gold, Gold over Silver, and Silver over Bronze. Below 200% FPL range, we assume that Silver is preferred to Gold, and Gold is preferred to Bronze, however we do not assume a relationship between Silver and Platinum, since the actuarial values are close, and the terms of the two plans are significantly different. In no case do we assume that any of the plans are preferred to the outside option.

### 4.3 Results

The focus of our analysis is on measuring the effect of an equal change in post-subsidy premiums for all consumers on aggregate choice shares, consumer surplus, and government subsidy expenditure. Let  $\pi(X_i) = P_i$  denote the observed premium vector, which we recall is a deterministic function of  $X_i$  in Covered California. We consider new counterfactual premium vectors of the form  $\pi(X_i) + \delta$ , which reflect an equal change in premiums for all consumers. That is, the counterfactuals we consider can be represented as the impact of shifting every households’ price from the observed price,  $P_i \equiv \pi(X_i)$  to a counterfactual price,  $P_i^* \equiv \pi(X_i) + \delta$  for various choices of  $\delta$ . For each value of  $W_i$ , we construct the MRP using the set formed from all  $P_i$  and  $P_i^*$ .

Figure 3 illustrates Bronze and Silver observed and counterfactual prices for buyers with income lower than 250% of the FPL. We use this case for illustration since it can be plotted on the plane, and because most buyers (over 93%) in this income range choose Bronze and Silver, presumably due to the CSR subsidies. In Figure 3b, the counterfactual corresponds to increasing the price of the Bronze plan by \$10 for all consumers, while Figure 3c illustrates the analogous change in the Silver plan. In Figure 3d, both the Bronze and the Silver plan are increased by \$10, which from the consumer’s perspective would be equivalent to a \$10 reduction in premium subsidies if

**Figure 3: Observed and Counterfactual Prices**



**Note:** The figure shows observed and counterfactual prices of Bronze and Silver plans for households with income between 140-250% of the FPL. Panel (a) plots the prices observed in the data in grey, where each observation is a unique region-age-income combination. Panel (b) overlays in red the counterfactual prices representing an increase in \$10 per person, per month for Bronze premiums. Panel (c) is like Panel (b), but the price increases are for Silver premiums. Panel (d) is like Panels (b) and (c) with price increases of \$10 for both Silver and Bronze premiums.

Bronze and Silver were the only two choices.<sup>28</sup>

The first set of target parameters we consider is the change in choice shares for each good. For market  $x$  and good  $j$ , this can be written as

$$\Delta\text{Share}_j(f|x) \equiv \int_{\mathcal{V}_j(\pi(x)+\delta)} f(v|x) dv - \int_{\mathcal{V}_j(\pi(x))} f(v|x) dv, \quad (30)$$

where  $\mathcal{V}_j(p)$  was defined in (9). Note that in (30), and in the following, the densities  $f$  are only conditional on  $X_i$ , since premiums are a deterministic function of  $X_i$ . We also omit the dependence on the price change,  $\delta$ , since this will be clear from the way we present our results. In order to aggregate (30) into a single measure, we average it over markets  $x$  and report

$$\Delta\text{Share}_j(f) \equiv \sum_x \Delta\text{Share}_j(f|x) \mathbb{P}[X_i = x]. \quad (31)$$

We will average other parameters in an analogous way. In the notation of Section 3,  $\Delta\text{Share}_j$  is an example of a target parameter,  $\theta$ .

Table 3 reports estimated bounds for  $\Delta\text{Share}_j$  across the four metal tier choices together with bounds on overall participation, i.e.  $1 - \Delta\text{Share}_0$ . The rows of Table 3 reflect different types of premium increases,  $\delta$ . The nominal premium increase is taken to be \$10 per person, per month, which represents a moderate to large price increase for many consumers (see Table 2). Our estimated bounds are quite informative. For example, for the full sample in panel (a), we estimate that a simultaneous \$10 increase in all premiums reduces the proportion of households that purchase coverage by between 2.5 and 10.3%. Panel (b) shows that these estimates are larger in magnitude for low-income households, at between 3.4 and 14.7%, and panel (c) shows that they are smaller in magnitude for higher-income households, who we estimate would reduce participation in Covered California by between 1.3 and 4.4%. Comparing panels (b) and (c) more generally, we find a pattern of higher price sensitivity for low-income households.

The other columns of Table 3 measure substitution patterns within and between coverage tiers. For example, panel (a) shows that an increase in Bronze premiums by \$10 per person, per month would lead to a decrease of between 0.9 and 4.7% in the share of consumers choosing the Bronze plan, and an increase in the share choosing Silver of between 0.1 and 4.0%. The increase in the share choosing Gold or Platinum

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<sup>28</sup> In this example with  $J = 2$ ,  $j = 1$  denoting Bronze, and  $j = 2$  denoting Silver, Figures 3b, 3c, and 3d would correspond to taking  $\delta = (10, 0)$ ,  $\delta = (0, 10)$  and  $\delta = (10, 10)$ , respectively, where the price of the outside option is always fixed at 0.

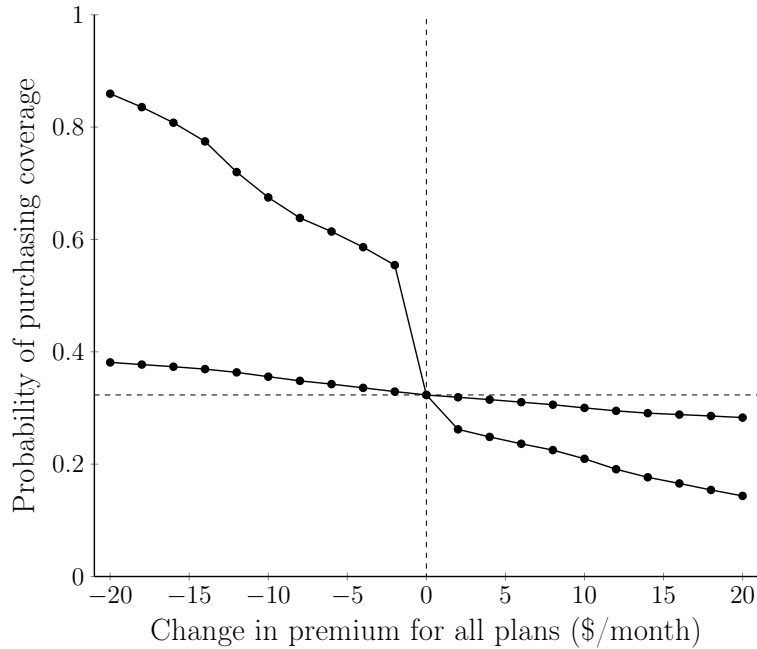
*Table 3: Substitution Patterns*

\$10/month premium increase for	Change in probability of choosing									
	Bronze		Silver		Gold		Platinum		Any Plan	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Panel (a): 140 - 400% FPL										
Bronze	-0.047	-0.009	+0.001	+0.040	+0.000	+0.017	+0.000	+0.022	-0.026	-0.002
Silver	+0.001	+0.104	-0.146	-0.021	+0.000	+0.042	+0.000	+0.092	-0.070	-0.004
Gold	+0.000	+0.004	+0.000	+0.008	-0.010	-0.001	+0.000	+0.008	-0.004	-0.000
Platinum	+0.000	+0.004	+0.000	+0.008	+0.000	+0.007	-0.011	-0.001	-0.005	-0.000
All plans	-0.027	-0.005	-0.073	-0.016	-0.005	-0.000	-0.005	-0.000	-0.103	-0.025
Panel (b): 140 - 250% FPL										
Bronze	-0.054	-0.011	+0.002	+0.047	+0.000	+0.016	+0.000	+0.026	-0.030	-0.002
Silver	+0.002	+0.157	-0.217	-0.031	+0.000	+0.047	+0.000	+0.143	-0.108	-0.007
Gold	+0.000	+0.003	+0.000	+0.006	-0.007	-0.001	+0.000	+0.006	-0.003	-0.000
Platinum	+0.000	+0.005	+0.000	+0.009	+0.000	+0.006	-0.012	-0.001	-0.005	-0.000
All plans	-0.032	-0.006	-0.112	-0.024	-0.004	-0.000	-0.006	-0.000	-0.147	-0.034
Panel (c): 250 - 400% FPL										
Bronze	-0.038	-0.007	+0.001	+0.031	+0.000	+0.019	+0.000	+0.017	-0.020	-0.002
Silver	+0.000	+0.034	-0.053	-0.007	+0.000	+0.036	+0.000	+0.025	-0.021	-0.001
Gold	+0.000	+0.005	+0.000	+0.010	-0.013	-0.001	+0.000	+0.010	-0.005	-0.000
Platinum	+0.000	+0.003	+0.000	+0.006	+0.000	+0.008	-0.009	-0.001	-0.004	-0.000
All plans	-0.020	-0.005	-0.022	-0.005	-0.006	-0.001	-0.004	-0.000	-0.044	-0.013

is significantly smaller, reflecting the closer substitutability of the Bronze and Silver plans. The extensive margin change of participation for a Bronze premium increase is between 0.2 and 2.6%, which is naturally both smaller and tighter than the change when all premiums are increased together. In contrast, increasing Platinum premiums by the same amount would lead to a much smaller decline in the proportion of buyers not purchasing coverage. Overall, Table 3 indicates substitution patterns inconsistent with the independence of irrelevant alternatives property of the logit model.

To help put our estimates in context, consider the lower-income buyers in panel (b). The sociodemographic characteristics of these buyers are similar to buyers in the more widely studied Commonwealth Care health exchange implemented in Massachusetts before the ACA (e.g. Chan and Gruber, 2010; Ericson and Starc, 2015; Finkelstein

**Figure 4:** Extensive Margin Demand Changes for Different Counterfactuals



et al., 2017). Finkelstein et al. (2017) estimate demand in this market by exploiting two discontinuities in subsidies. They estimate that a \$40 increase in monthly premiums leads to a 24% reduction in the probability of enrollment for buyers 150% of the FPL, a 20% reduction for buyers at 200% FPL, and a 14% reduction for 250% FPL. Assuming these effects are linear in premium, and dividing by a factor of four suggests that a 10\$ premium increase would lead to a reduction in enrollment of between 3.5% and 6%. This estimate is consistent with our 3.4 to 14.7% bounds on the same quantity. Our direct estimates of the effect of a \$40 increase in all premiums on participation are between 8% and 38% for buyers with income below 200% of the FPL, between 6% and 28% for buyers with income between 200–250% of the FPL, and between 6% and 16% for buyers with income between 250–300% of the FPL. Despite important differences in institutions and econometric methodology, our estimates are remarkably aligned with the point estimate of demand responses in the Massachusetts exchange.

One consequence of adopting a partial identification framework is that the amount of information that the data and assumptions yield about a specific counterfactual quantity is reflected in the width of the bounds. The bounds for more ambitious (more distant) counterfactuals will be wider than for more modest counterfactuals that are closer to what was observed in the data. This situation is evident in Figure

4, which plots the average extensive margin (enrollment) response as a function of a given increase or decrease in all premiums. Our bounds are relatively tight for small changes in premiums, and then widen as the premiums get farther from what was observed in the data. We consider this an attractive feature of our approach, since it reflects the increasing difficulty of drawing inference about objects that involve larger departures from the observed data, and so captures an important dimension of model uncertainty. In contrast, a fully parametric model point identifies any counterfactual quantity regardless of how distant the extrapolation involved.<sup>29</sup>

The second set of parameters we consider measure the effects of changing premium subsidies on consumer surplus and government spending. From the household's perspective, a decrease in premium subsidies is the same as an increase in premiums faced.<sup>30</sup> Such a subsidy change generates an average change in consumer surplus for a household in market  $x$  of

$$\Delta\text{CS}(f|x) \equiv \int \max_{j \in \mathcal{J}} \{v_j - \pi_j(x) - \delta_j\} dv - \int \max_{j \in \mathcal{J}} \{v_j - \pi_j(x)\} dv,$$

which we aggregate by averaging over markets into

$$\Delta\text{CS}(f) \equiv \sum_x \Delta\text{CS}(f|x) \mathbb{P}[X_i = x].$$

We will be interested in contrasting the change in consumer surplus to the change in government spending on premium subsidies. For market  $x$ , this is given by

$$\begin{aligned} \Delta\text{GS}(f|x) \equiv & \sum_{j>0} (\text{sub}_j(x) - \delta_j) \times \left[ \int_{\mathcal{V}_j(\pi(x)+\delta)} f(v|x) dv \right] \\ & - \sum_{j>0} \text{sub}_j(x) \times \left( \int_{\mathcal{V}_j(\pi(x))} f(v|x) dv \right), \end{aligned}$$

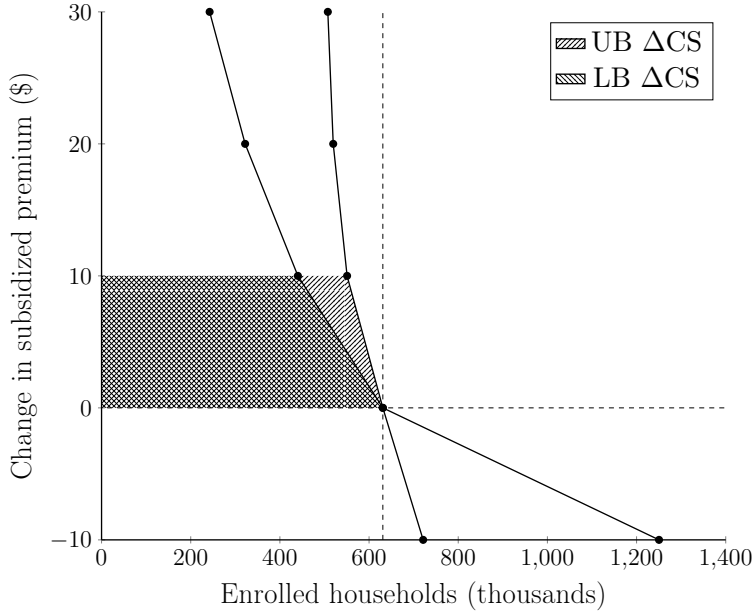
where  $\text{sub}_j(x)$  denotes the baseline premium subsidy for purchasing plan  $j$  in market

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<sup>29</sup> Note that confidence intervals on point estimates from a parametric model will tend to widen as one extrapolates further. However, for the parametric models we consider in Section 5, the width of these confidence intervals is basically zero even for distant extrapolations.

<sup>30</sup> Our analysis here requires maintaining a partial equilibrium framework in which there are no other supply side responses in base prices due to an adjustment in subsidy schemes. Integrating our approach with a model of insurance supply is beyond the scope of the current paper, but an interesting avenue for future research.

**Figure 5:** Change in Consumer Surplus Resulting from a Change in Premiums



$x$ . We denote aggregated government spending as

$$\Delta\text{GS}(f) \equiv \sum_x \text{GS}(f|x) \mathbb{P}[X_i = x].$$

Both  $\Delta\text{CS}$  and  $\Delta\text{GS}$  are examples of target parameters  $\theta$ .<sup>31</sup>

Figure 5 depicts our bounds on  $\Delta\text{CS}$  for a \$10 decrease in subsidies as the shaded areas between the two demand curves. The lower bound on the change in consumer surplus is the area to the left of the less-steep demand curve, while the upper bound also includes the entire area to the right of the steeper demand curve. Intuitively, the lower bound is attained at the upper bound (smallest magnitude) of price elasticity for the extensive margin, while the upper bound of the change in consumer surplus is attained at the lower bound (largest magnitude) this price elasticity. Note that while the bounds on  $\Delta\text{CS}$  shown here are sharp and unique, the demand curves we have plotted are not, since there are many ways to draw a demand curve up to a \$10 premium increase that can yield the same area to the left, while still respecting the data and assumptions.

Table 4 tabulates the estimated bounds on  $\Delta\text{CS}$  for the same \$10 decrease in

<sup>31</sup> In Appendix E, we show how to construct sharp bounds on  $\Delta\text{CS}$  by deriving corresponding  $\bar{\theta}$  functions that are linear in  $\phi$ .



**Table 4:** *The Impacts of Reducing Monthly Subsidies by \$10*

	140 - 400% FPL		140 - 250% FPL		250 - 400% FPL		140 - 400% FPL	
	Change in consumer surplus		Change in consumer surplus		Change in consumer surplus		Associated change in subsidy outlays	
	LB	UB	LB	UB	LB	UB	LB	UB
Average (\$/person-month)	-2.62	-1.80	-3.58	-2.45	-1.35	-0.94	-36.20	-13.03
Aggregate (\$ million/year)	-61.49	-42.26	-47.84	-32.74	-13.64	-9.52	-848.05	-305.16

premium subsidies. The first column shows estimated bounds using the entire sample, while the second and third columns split the estimates into lower and higher income samples. In the fourth column of Table 4, we report bounds on the corresponding reduction in government spending that results from the lower subsidies. We estimate these by fixing average consumer surplus at its lower or upper bound, then solving for the bounds on government spending that could be realized for this consumer surplus change.<sup>32</sup>

Our bounds imply that a \$10 decrease in monthly subsidies would lead to a reduction in average monthly consumer surplus of between \$1.80 and \$2.62 per person. The impacts for the lower-income sample are estimated to be approximately twice as large, which is a consequence of the higher price elasticity for this group found in Table 1. Both estimates are dwarfed by the corresponding change in government expenditure on premium subsidies, which we estimate to be between \$13.03 and \$36.20 per consumer, per month. The large magnitude of the expenditure savings is due to the large number of marginal buyers who exit the market due to the post-subsidy premium increase. When these buyers exit, they relinquish their entire premium subsidy, which in most cases is significantly more than \$10.

The bottom row of Table 4 shows the aggregate yearly impact of a \$10 reduction in subsidies in Covered California. The total consumer surplus impact would be between \$42 and \$61 million, with the majority of the losses concentrated among households with income below 250% of the FPL. At the same time, government subsidy outlays would decline by between \$305 and \$848 million per year. Overall, our findings suggest that consumers value health insurance significantly less than it would cost in premium subsidies to induce them to purchase a plan. This finding is consistent with a growing number of empirical analyses, see e.g. Finkelstein et al. (2017). In interpreting this finding, we caution that our estimates do not account for the existence of potentially large

<sup>32</sup> We do this because a given consumer surplus change could be attained in a variety of different ways, each of which might be associated with different changes in government spending.

externalities such as the cost of uncompensated care, debt delinquency, or bankruptcy (Finkelstein et al., 2012; Mahoney, 2015; Garthwaite, Gross, and Notowidigdo, 2018).

#### 4.4 Allowing Valuations to Change Within Coarse Age Bins

The primary assumption that drives our results is (29). As we noted, the part of this assumption that imposes independence between valuations and age within coarse age bins is probably questionable, since valuations likely change with risk factors, and risk factors change with age.<sup>33</sup> In this section, we consider a strictly weaker version of (29) that allows for some deviations away from perfect invariance. This can be viewed as a sensitivity analysis, and is similar in spirit to proposals by Conley, Hansen, and Rossi (2010), Nevo and Rosen (2012), and Manski and Pepper (2017).

The way in which we do this is to relax (29) into two inequalities controlled by a slackness parameter. The relaxed assumption is that

$$(1 - \kappa(z, z'))f_{V|WZ}(v|w, z') \leq f_{V|WZ}(v|w, z) \leq (1 + \kappa(z, z'))f_{V|WZ}(v|w, z')$$

for all  $z, z', w$ , and  $v$ ,

(32)

where  $\kappa(z, z') \geq 0$  is the slackness parameter. We specify  $\kappa$  in the following way:

$$\kappa(z, z') = \begin{cases} \kappa, & \text{if } z \text{ and } z' \text{ differ only in age, and only by a single bin} \\ 0, & \text{if } z \text{ and } z' \text{ differ only in income} \\ +\infty, & \text{otherwise.} \end{cases}$$

In words, the assumption is that within any coarse bin (i.e., conditional on  $W_i = w$ ), the pointwise difference in conditional valuation densities corresponding to any two adjacent two-year age bins (with identical income) can be no greater than  $\kappa\%$ . The constant  $\kappa$  is a value that we choose and will vary. Taking  $\kappa = 0$  reduces (32) back to our previous assumption of (29). Alternatively, taking  $\kappa = +\infty$  completely relaxes the age restriction, so that the only variation we are using is with respect to income.

Table 5 reports bounds on some of our main target parameters under (29) for different values of  $\kappa$ .<sup>34</sup> The row with  $\kappa = 0$  are the same as the estimates reported in the previous section,  $\kappa = +\infty$  corresponds to estimates that only use variation in income. Our preferred specification sets  $\kappa = 0.4$ , which can be interpreted as allowing

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<sup>33</sup> Indeed, the importance of age heterogeneity in health insurance demand is the emphasis of existing work, see e.g. Ericson and Starc (2015), Geruso (2017), and Tebaldi (2017).

<sup>34</sup> Note that it is straightforward to modify the sharp characterization in Proposition 1 to allow for an assumption like (29) instead of (6). The difference in implementation just amounts to replacing (26) with an inequality analogous to (29).

**Table 5:** Allowing for Valuations to Vary Within Coarse Age Bins

Allowed variation in preferences with age	Change in probability of not enrolling if all premiums increase by \$10/month		Change in consumer surplus (\$/person-month) if subsidies decrease by \$10/month		Change in government spending (\$/person-month) if subsidies decrease by \$10/month	
	LB	UB	LB	UB	LB	UB
	$\kappa=0$	+0.025	+0.103	-2.62	-1.80	-36.20
$\kappa=0.2$	+0.024	+0.108	-2.73	-1.85	-38.31	-12.75
$\kappa=0.4$	+0.023	+0.114	-2.80	-1.86	-40.29	-12.52
$\kappa=0.8$	+0.020	+0.125	-2.89	-1.80	-44.15	-11.77
$\kappa=1$	+0.018	+0.131	-2.94	-1.74	-46.33	-11.15
$\kappa=3$	+0.015	+0.143	-3.01	-1.63	-50.21	-10.22
$\kappa=+\infty$	+0.013	+0.154	-3.07	-1.47	-55.12	-8.98

for a change of up to 40% in valuations between adjacent two-year age bins, conditional on income. This seems fairly conservative to us. The figures we reported in the abstract and introduction are for  $\kappa = .4$ , but we include a variety of choices of  $\kappa$  so that the reader can make their own judgment.<sup>35</sup> Overall, our findings remain similar in that we find evidence of high price elasticity. As before, this leads to an estimated effect of decreasing subsidies on consumer surplus that is small, at between \$1.86 and \$2.80 per person, per month, while the impact on government spending of between \$12.52 and \$40.29 is considerably larger.

## 5 Comparison to Parametric Models

Our motivation in this paper has been to provide estimates of key policy parameters using a model that does not use parametric distributional assumptions. In this section, we compare our nonparametric bounds to estimates from some fully parametric logit and probit models which do use such assumptions. These models all follow a

<sup>35</sup> Note that the bounds generally widen with  $\kappa$ , since larger values correspond to weaker assumptions. However, this is not always the case, due to the fact that we are estimating these bounds using the procedure in Appendix F. Essentially, that procedure works by restricting attention to densities that come close to fitting the observed choice shares the best. This fit mechanically improves as  $\kappa$  increases, because more densities are considered. As a result, densities that seemed to fit well for smaller values of  $\kappa$  might no longer be deemed to fit well when  $\kappa$  increases, since the best fit has improved. This creates a countervailing effect to changing  $\kappa$ , which can lead to non-monotonicity in the estimated bounds even though monotonicity must hold for the population bounds.

specification similar to (2), which we write here as

$$Y_i = \arg \max_{j \in \mathcal{J}} \mathbb{1}[j \geq 1] (\gamma_i + \beta_i AV_{ij}) - \alpha_i P_{ij} + \epsilon_{ij}, \quad (33)$$

where  $\gamma_i$  is an individual-specific intercept, and  $AV_{ij}$  is the actuarial value of tier  $j$  for individual  $i$  (see Table 1). The presence of the indicator sets the contribution of these terms to 0 for the outside option ( $j = 0$ ). The logit class of models restrict  $\epsilon_{ij}$  to follow a type I extreme value distribution, independently across  $j$ , while probit models restrict  $\epsilon_{ij}$  to follow a standard normal distribution.

The first model we estimate is a logit in which the price parameter,  $\alpha_i$ , is constant, but both  $\gamma_i$  and  $\beta_i$  vary with observables in a rich way.<sup>36</sup> The second model is a probit with the same specification.<sup>37</sup> We then consider three mixed logit models. In all of these models,  $\gamma_i$  and  $\beta_i$  vary with observables as in the baseline model, and now the premium coefficient  $\alpha_i$  varies with the region. The three models differ in whether  $\gamma_i$ ,  $\alpha_i$ , or both have an additional unobservable component that is normally distributed with unknown variance. In the latter case, we also assume that the two unobservable components are uncorrelated.

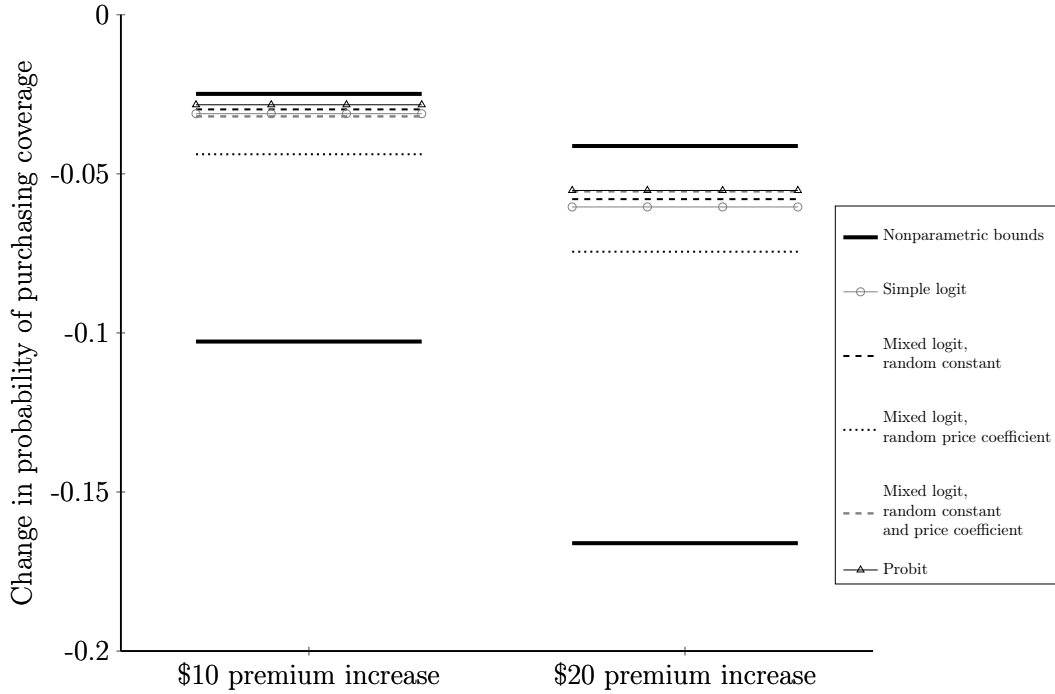
Figure 6 illustrates how our nonparametric bounds on the extensive margin responses compare to the estimates one obtains from these five parametric models. The estimates shown are for the counterfactuals of a \$10 and \$20 increase in all premiums (or decrease in subsidies). All of the point estimates are within the nonparametric bounds, but clustered near the upper bound, where price sensitivity is smallest. The implication is that different distributional assumptions on  $\epsilon_{ij}$  other than logit and probit could yield estimates near the lower bound, while still preserving the same degree of fit to the observed choice shares. As we showed in Table 4, these estimates would have substantially different policy implications in terms of consumer surplus and government spending. Thus, the assumption of a type I extreme value (or similarly-shaped normal) distribution appears here to have a significant impact on the empirical conclusions that would be drawn.

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<sup>36</sup> The specification allows  $\beta_i$  to vary freely by region with a different value in each of the following four age bins: {27–34, 35–44, 45–54, 54–64}. It allows  $\gamma_i$  to also vary freely by region, and within each region restricts  $\gamma_i = \gamma_i^{\text{Inc}} + \gamma_i^{\text{Age}}$ , where  $\gamma_i^{\text{Inc}}$  varies in three FPL income bins {140–200, 200–250, 250–400}, and  $\gamma_i^{\text{Age}}$  varies in the same four age bins as  $\beta_i$ .

<sup>37</sup> We have had difficulty estimating a similar probit with correlated  $\epsilon_{ij}$  because the likelihood is very flat, suggesting a potential failure of point identification.

**Figure 6:** Comparison Between Nonparametric Bounds and Parametric Estimates



## 6 Conclusion

We estimated the demand for health insurance in California’s ACA marketplace using a new nonparametric methodology. While we designed our methodology with health insurance in mind, it should be applicable to other discrete choice problems as well. The central idea of the method is to divide realizations of a consumer’s valuations into sets for which behavior remains constant. We showed how to define the collection of such sets, which we referred to as the minimal relevant partition (MRP) of valuations. Using the MRP, we developed a computationally reliable linear programming procedure for consistently estimating sharp identified sets for target parameters of interest.

Our estimates of demand using this methodology point to the possibility of substantially greater price sensitivity than would be recognized using comparable parametric models. This is consistent with the commonly-heard folklore that logits are “flat” models. We showed that this finding has potentially important policy implications, since it implies that the impact of decreasing subsidies on consumer surplus could be much smaller—and the impact of government expenditure much larger—than would be recognized using standard parametric methods. More broadly, our results provide a clear example in which functional form assumptions are far from innocuous, and

actually play a leading role in driving empirical conclusions.

## A Methodology Literature Review

In this section, we discuss the relationship of our methodology to the existing literature. We focus our attention first on semi- and non-parametric approaches to unordered discrete choice analysis. This literature can be traced back to Manski (1975). The focus of Manski’s work, as well as most of the subsequent literature, has been on relaxing parameterizations on the distribution of unobservables, while the observable component of utility is usually assumed to be linear-in-parameters.<sup>38</sup> The motivation of our approach is also to avoid the need to parameterize distributions of latent variables, however we have chosen to keep the entire analysis nonparametric.<sup>39</sup>

Our approach has three key properties that, when taken together, make it distinct in the literature on semi- and nonparametric discrete choice. First, much of the literature has focused on identification of the observable components of indirect utility, while treating the distribution of unobservables as an infinite-dimensional nuisance parameter. For example, in (2), this would correspond to identifying  $\alpha_i$  and  $\beta_i$  when these random coefficients are restricted to be constant. Examples of work with this focus include Manski (1975), Matzkin (1993), Lewbel (2000), Fox (2007), Pakes (2010), Ho and Pakes (2014), Pakes, Porter, Ho, and Ishii (2006, 2015), Pakes and Porter (2016), and Shi, Shum, and Song (2016). Identification of the relative importance of observable factors for explaining choices is insufficient for our purposes, because the policy counterfactuals we are interested in, such as choice probabilities and consumer surplus, also depend on the distribution of unobservables. Treating this distribution as a nuisance parameter would not allow us to make sharp statements about quantities relevant to these counterfactuals.

Second, we allow for prices (premiums in our context) to be endogenous in the sense of being correlated with the unobservable determinants of utility. This differentiates our paper from work that focuses on identification of counterfactuals, but which assumes exogenous explanatory variables. Examples of such work includes Thompson (1989), Manski (2007, 2014), Briesch, Chintagunta, and Matzkin (2010), Chiong, Hsieh, and Shum (2017), and Allen and Rehbeck (2017). The importance of allowing for endogenous explanatory variables in discrete choice demand analysis was emphasized by

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<sup>38</sup> Matzkin (1991) considered the opposite case in which the distribution of the unobservable component is parameterized, but the observable component is treated nonparametrically. See also Briesch, Chintagunta, and Matzkin (2002).

<sup>39</sup> Extending our methodology to a semiparametric model is an interesting avenue for future work, but not well-suited to our application since there is no variation in choice (plan) characteristics in Covered California. Conceptually though, one could use our strategy with a semiparametric model by fixing the parametric component and then repeatedly applying our characterization argument, similar to the strategy in Torgovitsky (2018).

Berry (1994) and Hausman, Leonard, and Zona (1994), and motivated the influential work of Berry et al. (1995, 2004). In our application, it is essential that we can make statements about demand counterfactuals while still recognizing that premiums could be dependent with unobservable valuations.

This leads us to the third way that our approach differs from existing literature, which is that we do not place strong demands on the available exogenous variation in the data. In particular, we do not require the existence of a certain number of instruments, or that such instruments satisfy strong support or rank conditions. For example, Lewbel (2000) and Fox and Gandhi (2016) require exogenous “special regressors” with large support, which are not available in our data. Alternatively, Chiappori and Komunjer (2009) and Berry and Haile (2014) provide identification results that require a sufficient number of continuous instruments that satisfy certain “completeness” conditions, which can be viewed as high-level analogs to traditional rank conditions.<sup>40</sup> Besides the difficulty of finding a sufficient number of continuous instruments, one might also be concerned with the interpretability and/or testability of the completeness condition (Canay, Santos, and Shaikh, 2013). Not maintaining these types of support and completeness conditions leads naturally to a partial identification framework.

Other authors have also considered taking a partial identification approach to unordered discrete choice models. Pakes (2010), Ho and Pakes (2014), Pakes et al. (2006, 2015), Pakes and Porter (2016) developed moment inequality approaches that can be used to bound coefficients on observables in specifications like (2) without parametric assumptions on the unobservables. As noted, this is insufficient for our purposes, since we are concerned with demand counterfactuals. Manski (2007), Chiong et al. (2017) and Allen and Rehbeck (2017) bound counterfactuals, but assume that all explanatory variables are exogenous. In parametric contexts, Nevo and Rosen (2012) have considered partial identification arising from allowing instruments to be partially endogenous, and Gandhi, Lu, and Shi (2017) treated the problem of non-purchases in scanner data as one of partial identification.

On a more specific technical level, our work is related to a literature on computational approaches to characterizing identified sets in the presence of partial identification. In particular, the linear programming structure we exploit has been noted by many other authors, see e.g. Balke and Pearl (1994, 1997) and Hansen, Heaton, and Luttmer (1995) for early examples. Previous work that has implemented linear programming to characterize sharp identified sets includes Honoré and Tamer (2006), Honoré and Lleras-Muney (2006), Manski (2007, 2014), Laffers (2013), Freyberger and

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<sup>40</sup> See also Compiani (2018), who has shown how to construct and implement estimators based on the results of Berry and Haile (2014).



Horowitz (2015), Demuynck (2015), Kline and Tartari (2016), Torgovitsky (2016, 2018), Kamat (2017), and Mogstad, Santos, and Torgovitsky (2018). Of this work, ours is closest to Manski (2007), who also considered discrete choice problems. Methodologically, our work differs from Manski’s because we maintain and exploit more structure on preferences (via (1)), and in addition we do not assume that explanatory variables (or choice sets in Manski’s framework) are exogenous.

## B A Model of Insurance Choice

In this section, we provide a model of choice under uncertainty for a risk averse consumer which leads (1). The model is quite similar to those discussed in Handel (2013, pp. 2660–2662) and Handel et al. (2015, pp. 1280–281). Throughout, we suppress observable factors other than price (components of  $X_i$ ) that could affect a consumer’s decision. All quantities can be viewed as conditional on these observed factors, which is consistent with the nonparametric implementation we use in the main text.

Suppose that each consumer  $i$  chooses a plan  $j$  to maximize their expected utility taken over uncertain medical expenditures, so that

$$Y_i = \arg \max_{j \in \mathcal{J}} \int U_{ij}(\text{ex}) dF_{ij}(\text{ex}), \quad (34)$$

where  $U_{ij}(\text{ex})$  is consumer  $i$ ’s ex-post utility from choosing plan  $j$  given realized expenditures of  $\text{ex}$ , and  $F_{ij}$  is the distribution of these expenditures, which varies both by consumer  $i$  (due to risk factors) and by plan  $j$  (due to coverage levels). Assume that  $U_{ij}$  takes the constant absolute risk aversion (CARA) form

$$U_{ij}(\text{ex}) = -\frac{1}{A_i} e^{-A_i C_{ij}(\text{ex})}, \quad (35)$$

where  $A_i$  is consumer  $i$ ’s risk aversion, and  $C_{ij}(\text{ex})$  is their ex-post consumption when choosing plan  $j$  and realizing expenditures  $\text{ex}$ . We assume that ex-post consumption takes the additively separable form

$$C_{ij}(\text{ex}) = \text{Inc}_i - P_{ij} - \text{ex} + \tilde{V}_{ij}, \quad (36)$$

where  $\text{Inc}_i$  is consumer  $i$ ’s income,  $P_{ij}$  is the price they paid for plan  $j$ , and  $\tilde{V}_{ij}$  is an idiosyncratic preference parameter.

Substituting (36) into (35) and then into (34), we obtain

$$Y_i = \arg \max_{j \in \mathcal{J}} -\frac{1}{A_i} \left[ e^{A_i(P_{ij} - \text{Inc}_i - \tilde{V}_{ij})} \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right]$$

Transforming the objective using  $u \mapsto -\log(-u)$ , which is strictly increasing for  $u < 0$ , we obtain an equivalent problem

$$\begin{aligned} Y_i &= \arg \max_{j \in \mathcal{J}} -\log \left( \frac{1}{A_i} \left[ e^{A_i(P_{ij} - \text{Inc}_i - \tilde{V}_{ij})} \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right] \right) \\ &= \arg \max_{j \in \mathcal{J}} -\log \left( \frac{1}{A_i} \right) + A_i \left( \text{Inc}_i - P_{ij} + \tilde{V}_{ij} \right) + \log \left( \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right). \end{aligned}$$

Eliminating additive terms that don't depend on plan choice yields

$$Y_i = \arg \max_{j \in \mathcal{J}} -A_i P_{ij} + A_i \tilde{V}_{ij} + \log \left( \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right).$$

Suppose that  $A_i > 0$ , so that all consumers are risk averse.<sup>41</sup> Then we can express the consumer's choice as

$$Y_i = \arg \max_{j \in \mathcal{J}} \left[ \tilde{V}_{ij} + \frac{1}{A_i} \log \left( \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right) \right] - P_{ij},$$

which takes the form of (1) with

$$V_{ij} \equiv \left[ \tilde{V}_{ij} + \frac{1}{A_i} \log \left( \int e^{A_i \text{ex}} dF_{ij}(\text{ex}) \right) \right].$$

Examining the components of  $V_{ij}$  reveals the factors that contribute to heterogeneity in valuations in this model. Heterogeneity across  $i$  can come from variation in risk aversion ( $A_i$ ), from differences in risk factors or beliefs ( $F_{ij}$ ), and from idiosyncratic differences in the valuation of health insurance ( $\tilde{V}_{ij}$ ). Differences in valuations across  $j$  arise from the interaction between risk factors and the distribution of corresponding expenditures ( $F_{ij}$ ), as well as from idiosyncratic differences in valuations across plans ( $\tilde{V}_{ij}$ ). The main restrictions in this model are the assumption of CARA preferences in (35) and the quasilinearity of ex-post consumption in (36). However, as noted in the main text, it is important to realize that for these restrictions to have empirical content, they must be combined with an assumption about the dependence between income (here called  $\text{Inc}_i$ ) and the preference parameters,  $A_i$  and  $\tilde{V}_{ij}$ .

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<sup>41</sup> Showing that (1) would arise from risk neutral consumers is immediate.

## C Construction of the Minimal Relevant Partition

We first observe that any price (premium) vector  $p \in \mathbb{R}^J$  divides  $\mathbb{R}^J$  into the sets  $\{\mathcal{V}_j(p)\}_{j=0}^J$ , as shown in Figures 1a and 1b. Intuitively, we view such a division as a partition, although formally this is not correct, since these sets can overlap on the hyperplanes like  $v_j - p_j = v_k - p_k$  where ties occurs. These regions of overlap have Lebesgue measure zero in  $\mathbb{R}^J$ , so this caveat is unimportant given our focus on continuously distributed valuations. To avoid confusion, we refer to a collection of sets that would be a partition if not for regions of Lebesgue measure zero as an almost sure (a.s.) partition.

**Definition 2.** Let  $\{\mathcal{A}_m\}_{m=1}^M$  be a collection of Lebesgue measurable subsets of  $\mathbb{R}^J$ . Then  $\{\mathcal{A}_m\}_{m=1}^M$  is an almost sure (a.s.) partition of  $\mathbb{R}^J$  if

- a)  $\bigcup_{m=1}^M \mathcal{A}_m = \mathbb{R}^J$ ; and
- b)  $\lambda(\mathcal{A}_m \cap \mathcal{A}_{m'}) = 0$  for any  $m \neq m'$ , where  $\lambda$  denotes Lebesgue measure on  $\mathbb{R}^J$ .

Next, we enumerate the price vectors in  $\mathcal{P}$  as  $\mathcal{P} = \{p_1, \dots, p_L\}$  for some integer  $L$ . Let  $\mathcal{Y} \equiv \mathcal{J}^L$  denote the collection of all  $L$ -tuples from the set of choices  $\mathcal{J} \equiv \{0, 1, \dots, J\}$ . Then, since  $\{\mathcal{V}_j(p_l)\}_{j=0}^J$  is an a.s. partition of  $\mathbb{R}^J$  for every  $p_l$ , it follows that

$$\{\tilde{\mathcal{V}}_y : y \in \mathcal{Y}\} \quad \text{where} \quad \tilde{\mathcal{V}}_y \equiv \bigcap_{l=1}^L \mathcal{V}_{y_l}(p_l) \quad (37)$$

also constitutes an a.s. partition of  $\mathbb{R}^J$ .<sup>42</sup> Intuitively, each vector  $y \equiv (y_1, \dots, y_L)$  is a profile of  $L$  choices under the price vectors  $(p_1, \dots, p_L)$  that comprise  $\mathcal{P}$ . Each set  $\tilde{\mathcal{V}}_y$  in the a.s. partition (37) corresponds to the subset of valuations in  $\mathbb{R}^J$  for which a consumer would make choices  $y$  when faced with prices  $\mathcal{P}$ .

The collection  $\mathbb{V} \equiv \{\tilde{\mathcal{V}}_y : y \in \mathcal{Y}\}$  is the MRP, since it satisfies Definition 1 by construction. To see this, note that if  $v, v' \in \tilde{\mathcal{V}}_y$  for some  $y$ , then by (37),  $v, v' \in \mathcal{V}_{y_l}(p_l)$  for all  $l = 1, \dots, L$ , at least up to collections of  $v, v'$  that have Lebesgue measure zero. Recalling (9), this implies (using the notation of Definition 1) that  $Y(v, p) = Y(v', p)$  for all  $p \in \mathcal{P}$ . Conversely, if  $Y(v, p) = Y(v', p)$  for all  $p \in \mathcal{P}$ , then taking

$$y \equiv (Y(v, p_1), \dots, Y(v, p_L)) = (Y(v', p_1), \dots, Y(v', p_L)), \quad (38)$$

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<sup>42</sup> Note that these sets are Lebesgue measurable, since  $\mathcal{V}_j(p)$  is a finite intersection of half-spaces and  $\tilde{\mathcal{V}}_y$  is a finite intersection of sets like  $\mathcal{V}_j(p)$ .

yields an  $L$ -tuple  $y \in \mathcal{Y}$  such that  $v, v' \in \mathcal{V}_{y_l}(p_l)$ , again barring ambiguities that occur with Lebesgue measure zero.

From a practical perspective, this is an inadequate representation of the MRP, because if choices are determined by the quasilinear model (1), then many of the sets  $\tilde{\mathcal{V}}_y$  must have Lebesgue measure zero. This makes indexing the partition by  $y \in \mathcal{Y}$  excessive; for computation we would prefer an indexing scheme that only includes sets that are not already known to have measure zero. For this purpose, we use an algorithm that starts with the set of prices  $\mathcal{P}$  and returns the collection of choice sequences  $\bar{\mathcal{Y}}$  that are not required to have Lebesgue measure zero under (1). We use this set  $\bar{\mathcal{Y}}$  in our computational implementations. Note that since  $\tilde{\mathcal{V}}_y$  has Lebesgue measure zero for any  $y \in \mathcal{Y} \setminus \bar{\mathcal{Y}}$ , the collection  $\mathbb{V} \equiv \{\tilde{\mathcal{V}}_y : y \in \bar{\mathcal{Y}}\}$  still constitutes an a.s. partition of  $\mathbb{R}^J$  and still satisfies the key property (17) of the MRP in Definition 1.

The algorithm works as follows.<sup>43</sup> We begin by partitioning  $\mathcal{P}$  into  $M$  sets (or blocks) of prices  $\{\mathcal{P}_m\}_{m=1}^M$  that each contain (give or take)  $\mu$  prices. For each  $m$ , we then construct the set of all choice sequences  $\bar{\mathcal{Y}}_m \subseteq \mathcal{J}^{|\mathcal{P}_m|}$  that are compatible with the quasilinear choice model in the sense that  $y^m \in \bar{\mathcal{Y}}_m$  if and only if the set

$$\{v \in \mathbb{R}^J : v_{y_l^m} - p_{y_l^m} \geq v_j - p_j \text{ for all } j \in \mathcal{J} \text{ and } p \in \mathcal{P}_m\} \quad (39)$$

is empty. In practice, we do this by sequentially checking the feasibility of a linear program with (39) as the constraint set. The sense in which we do this sequentially is that instead of checking (39) for all  $y^m \in \mathcal{J}^{|\mathcal{P}_m|}$ —which could be a large set even for moderate  $\mu$ —we first check whether it is nonempty when the constraint is imposed for only 2 prices in  $\mathcal{P}_m$ , then 3 prices, etc. Finding that (39) is empty when restricting attention to one of these shorter choice sequences implies that it must also be infeasible for all other sequences that share the short component. This observation helps speed up the algorithm substantially.

Once we have found  $\bar{\mathcal{Y}}_m$  for all  $m$ , we combine blocks of prices into pairs, then repeat the process with these larger, paired blocks. For example, if we let  $\mathcal{P}_{12} \equiv \mathcal{P}_1 \cup \mathcal{P}_2$ —i.e. we pair the first two blocks of prices—then we know that the set of  $y^{12} \in \mathcal{J}^{|\mathcal{P}_1|+|\mathcal{P}_2|}$  that satisfy (39) must be a subset of  $\{(y_1, y_2) : y_1 \in \bar{\mathcal{Y}}_1, y_2 \in \bar{\mathcal{Y}}_2\}$ . We sequentially check the non-emptiness of (39) for all  $y^{12}$  in this set, eventually obtaining a set  $\bar{\mathcal{Y}}_{12}$ . Once we have done this for all pairs of price blocks, we then combine pairs of pairs of blocks (e.g.  $\mathcal{P}_{12} \cup \mathcal{P}_{34}$ ) and repeat the process. Continuing in this way, we eventually

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<sup>43</sup> We expect that this algorithm leaves room for significant computational improvements, but we leave more sophisticated developments for future work. In practice, we also use some additional heuristics based on sorting the price vectors. These have useful but second-order speed improvements that are specific to our application, so for brevity we do not describe them here.

end up with the original set of price vectors,  $\mathcal{P}$ , as well as the set of all surviving choice sequences,  $\overline{\mathcal{Y}} \subseteq \mathcal{Y}$ .

The key input to this algorithm is the number of prices in the initial price blocks, which we have denoted by  $\mu$ . The optimal value of  $\mu$  should be something larger than 2, but smaller than  $L$ . With small  $\mu$ , the sequential checking of (39) yields less payoff, since each detection of infeasibility eliminates fewer partial choice sequences. On the other hand, large  $\mu$  makes the strategy of combining pairs of smaller blocks of prices into larger blocks less fruitful. For our application, we use  $\mu = 8$ – $10$ , which seems to be fairly efficient, although it is likely specific to our setting.

## D Proofs for Propositions 1 and 2

### D.1 Proposition 1

If  $t \in \Theta^*$ , then by definition there exists an  $f \in \mathcal{F}^*$  such that  $\theta(f) = t$ . Let  $\phi_f$  be defined as in (20), which we reproduce here for convenience:

$$\phi_f(\mathcal{V}|p, x) \equiv \int_{\mathcal{V}} f(v|p, x) dv. \quad (20)$$

Note that  $\phi_f \in \Phi$ , because the MRP  $\mathbb{V}$  is (almost surely) a partition of  $\mathbb{R}^J$ , and  $f$  is a conditional probability density function on  $\mathbb{R}^J$ . Due to the assumed properties of  $\bar{\theta}$ , we also know that  $\bar{\theta}(\phi_f) = \theta(f) = t$ . To see that  $\phi_f$  satisfies (25), observe that

$$\sum_{\mathcal{V} \in \mathbb{V}_j(p)} \phi_f(\mathcal{V}|p, x) \equiv \sum_{\mathcal{V} \in \mathbb{V}_j(p)} \int_{\mathcal{V}} f(v|p, x) dv = s_f(j|p, x) = s(j|p, x),$$

where the first equality follows by definition (20), the second follows from (18), and the third follows from the definition of  $\mathcal{F}^*$ . Similarly,  $\phi_f$  satisfies (26) because

$$\begin{aligned} \mathbb{E}[\phi_f(\mathcal{V}|P_i, X_i)|W_i = w, Z_i = z] &= \mathbb{E} \left[ \int_{\mathcal{V}} f(v|P_i, X_i) dv \middle| W_i = w, Z_i = z \right] \\ &= \int_{\mathcal{V}} \mathbb{E} \left[ f(v|P_i, X_i) \middle| W_i = w, Z_i = z \right] dv \\ &= \int_{\mathcal{V}} \mathbb{E} \left[ f(v|P_i, X_i) \middle| W_i = w, Z_i = z' \right] dv \\ &= \mathbb{E}[\phi_f(\mathcal{V}|P_i, X_i)|W_i = w, Z_i = z'] \end{aligned}$$

where the second equality follows by Tonelli's Theorem (e.g. pg. 82 of Shorack, 2000), the third uses (6), which holds (by assumption) for all  $f \in \mathcal{F}^*$ , and the final equality reverses the steps of the first two equalities. That  $\phi_f$  also satisfies (27) follows using a

similar argument and the hypothesis that  $f \in \mathcal{F}^*$  satisfies (7), i.e.

$$\begin{aligned}
\sum_{\mathcal{V} \in \overline{\mathbb{V}}(w)} (\phi_f)_{\mathbb{V}|WZ}(\mathcal{V}|w, z) &= \sum_{\mathcal{V} \in \overline{\mathbb{V}}(w)} \int_{\mathcal{V}} \mathbb{E} \left[ f(v|P_i, X_i) | W_i = w, Z_i = z \right] dv \\
&= \int_{\cup\{\mathcal{V}: \mathcal{V} \in \overline{\mathbb{V}}(w)\}} f_{\mathbb{V}|WZ}(v|w, z) dv \\
&\geq \int_{\overline{\mathbb{V}}(w)} f_{\mathbb{V}|WZ}(v|w, z) dv = 1,
\end{aligned} \tag{40}$$

where the inequality follows because the definition of  $\overline{\mathbb{V}}(w)$ , together with the fact that  $\mathbb{V}$  is an a.s. partition of  $\mathbb{R}^J$ , implies that  $\overline{\mathbb{V}}(w)$  is contained in the union of sets in  $\overline{\mathbb{V}}(w)$ . This inequality implies that  $\phi_f$  satisfies (27), because

$$\begin{aligned}
\sum_{\mathcal{V} \in \overline{\mathbb{V}}(w)} (\phi_f)_{\mathbb{V}|WZ}(\mathcal{V}|w, z) &\leq \sum_{\mathcal{V} \in \mathbb{V}} (\phi_f)_{\mathbb{V}|WZ}(\mathcal{V}|w, z) \\
&= \mathbb{E} \left[ \sum_{\mathcal{V} \in \mathbb{V}} \phi_f(\mathcal{V}|P_i, X_i) | W_i = w, Z_i = z \right] = 1,
\end{aligned}$$

as a result of  $\phi_f$  being an element of  $\Phi$ . We have now established that if  $t \in \Theta^*$ , then there exists a  $\phi \in \Phi$  satisfying (25)–(27) for which  $\bar{\theta}(\phi) = t$ .

Conversely, suppose that such a  $\phi \in \Phi$  exists for some  $t$ . Recall that  $W_i$  was assumed to be a subvector (or more generally, a function) of  $(P_i, X_i)$ , and denote this function by  $\omega$ , so that  $W_i = \omega(P_i, X_i)$ . Then define

$$f_\phi(v|p, x) \equiv \sum_{\mathcal{V} \in \overline{\mathbb{V}}(\omega(p, x))} \frac{\mathbb{1}[v \in \mathcal{V} \cap \overline{\mathbb{V}}(\omega(p, x))]}{\lambda(\mathcal{V} \cap \overline{\mathbb{V}}(\omega(p, x)))} \phi(\mathcal{V}|p, x),$$

noting that the summands are well-defined by the definition of  $\overline{\mathbb{V}}(w)$ . We will show that  $t \in \Theta^*$  by establishing that  $f_\phi \in \mathcal{F}^*$  and  $\theta(f_\phi) = t$ .

First observe that for any  $\mathcal{V} \in \mathbb{V}$ ,

$$\begin{aligned}
\int_{\mathcal{V}} f_\phi(v|p, x) dv &\equiv \sum_{\mathcal{V}' \in \overline{\mathbb{V}}(\omega(p, x))} \int_{\mathcal{V}} \frac{\mathbb{1}[v \in \mathcal{V}' \cap \overline{\mathbb{V}}(\omega(p, x))]}{\lambda(\mathcal{V}' \cap \overline{\mathbb{V}}(\omega(p, x)))} \phi(\mathcal{V}'|p, x) dv \\
&= \mathbb{1}[\mathcal{V} \in \overline{\mathbb{V}}(\omega(p, x))] \phi(\mathcal{V}|p, x),
\end{aligned} \tag{41}$$

since the sets in  $\mathbb{V}$  and hence  $\overline{\mathbb{V}}(\omega(p, x))$  are disjoint (almost surely). Using (41), we

have that

$$\int_{\mathbb{R}^J} f_\phi(v|p, x) dv = \sum_{\mathcal{V} \in \mathbb{V}} \int_{\mathcal{V}} f_\phi(v|p, x) dv = \sum_{\mathcal{V} \in \bar{\mathbb{V}}(\omega(p, x))} \phi(\mathcal{V}|p, x) = 1, \quad (42)$$

where the first equality uses the fact that  $\mathbb{V}$  is a partition. The final equality is implied by the hypothesis that  $\phi$  satisfies (27), since

$$1 = \sum_{\mathcal{V} \in \bar{\mathbb{V}}(w)} \phi_{\mathbb{V}|WZ}(\mathcal{V}|w, z) = \mathbb{E} \left[ \sum_{\mathcal{V} \in \bar{\mathbb{V}}(\omega(P_i, X_i))} \phi(\mathcal{V}|P_i, X_i) \Big| W_i = w, Z_i = z \right],$$

and every  $\phi \in \Phi$  satisfies

$$\sum_{\mathcal{V} \in \bar{\mathbb{V}}(\omega(p, x))} \phi(\mathcal{V}|p, x) \leq \sum_{\mathcal{V} \in \mathbb{V}} \phi(\mathcal{V}|p, x) = 1.$$

Thus, from (42), and since  $f_\phi$  inherits non-negativity from  $\phi \in \Phi$ , we conclude that  $f_\phi$  is a conditional density, i.e.  $f_\phi \in \mathcal{F}$ .

To see that  $f_\phi$  satisfies (6), notice that

$$\begin{aligned} (f_\phi)_{\mathcal{V}|WZ}(v|w, z) &\equiv \mathbb{E} \left[ f_\phi(v|P_i, X_i) \Big| W_i = w, Z_i = z \right] \\ &\equiv \mathbb{E} \left[ \sum_{\mathcal{V} \in \bar{\mathbb{V}}(w)} \frac{\mathbf{1}[v \in \mathcal{V} \cap \bar{\mathcal{V}}(w)]}{\lambda(\mathcal{V} \cap \bar{\mathcal{V}}(w))} \phi(\mathcal{V}|P_i, X_i) \Big| W_i = w, Z_i = z \right] \\ &= \sum_{\mathcal{V} \in \bar{\mathbb{V}}(w)} \frac{\mathbf{1}[v \in \mathcal{V} \cap \bar{\mathcal{V}}(w)]}{\lambda(\mathcal{V} \cap \bar{\mathcal{V}}(w))} \phi_{\mathbb{V}|WZ}(\mathcal{V}|w, z) \\ &= \sum_{\mathcal{V} \in \bar{\mathbb{V}}(w)} \frac{\mathbf{1}[v \in \mathcal{V} \cap \bar{\mathcal{V}}(w)]}{\lambda(\mathcal{V} \cap \bar{\mathcal{V}}(w))} \phi_{\mathbb{V}|WZ}(\mathcal{V}|w, z') = (f_\phi)_{\mathcal{V}|WZ}(v|w, z'), \end{aligned}$$

where the fourth equality uses (26), and the final equality reverses the steps of the first four. The satisfaction of the verticality condition, (7), follows in a similar way from

(27) and Tonelli's Theorem, since

$$\begin{aligned}
\int_{\overline{\mathcal{V}}(w)} (f_\phi)_{\mathcal{V}|WZ}(v|w, z) dv &\equiv \int_{\overline{\mathcal{V}}(w)} \mathbb{E} \left[ f_\phi(v|P_i, X_i) | W_i = w, Z_i = z \right] dv \\
&= \mathbb{E} \left[ \sum_{\mathcal{V} \in \overline{\mathcal{V}}(w)} \phi(\mathcal{V}|P_i, X_i) | W_i = w, Z_i = z \right] \\
&= \sum_{\mathcal{V} \in \overline{\mathcal{V}}(w)} \phi_{\mathcal{V}|WZ}(\mathcal{V}|w, z) = 1.
\end{aligned}$$

That  $f_\phi$  satisfies the observational equivalence condition (11) follows from (18), (25), and (41), i.e.

$$\begin{aligned}
s_{f_\phi}(j|p, x) &\equiv \sum_{\mathcal{V} \in \mathbb{V}_j(p)} \int_{\mathcal{V}} f_\phi(v|p, x) dv \\
&= \sum_{\mathcal{V} \in \mathbb{V}_j(p) \cap \overline{\mathcal{V}}(\omega(p, x))} \phi(\mathcal{V}|p, x) \\
&= \sum_{\mathcal{V} \in \mathbb{V}_j(p)} \phi(\mathcal{V}|p, x) - \sum_{\mathcal{V} \in \mathbb{V}_j(p) \cap \overline{\mathcal{V}}(\omega(p, x))^c} \phi(\mathcal{V}|p, x) = s(j|p, x),
\end{aligned}$$

for all  $j \in \mathcal{J}$  and  $(p, x) \in \text{supp}(P_i, X_i)$ . The last equality here follows using (42) because

$$0 \leq \sum_{\mathcal{V} \in \mathbb{V}_j(p) \cap \overline{\mathcal{V}}(\omega(p, x))^c} \phi(\mathcal{V}|p, x) \leq \sum_{\mathcal{V} \in \overline{\mathcal{V}}(\omega(p, x))^c} \phi(\mathcal{V}|p, x) = 1 - \sum_{\mathcal{V} \in \overline{\mathcal{V}}(\omega(p, x))} \phi(\mathcal{V}|p, x) = 0.$$

Finally, note that in the notation of (20), (41) says

$$\phi_{f_\phi}(\mathcal{V}|p, x) = \mathbb{1}[\mathcal{V} \in \overline{\mathcal{V}}(\omega(p, x))] \phi(\mathcal{V}|p, x).$$

This equality implies that  $\phi_{f_\phi}(\mathcal{V}|p, x) = \phi(\mathcal{V}|p, x)$  for all  $\mathcal{V}$ , since for  $\mathcal{V} \notin \overline{\mathcal{V}}(\omega(p, x))$  we must have  $\phi(\mathcal{V}|p, x) = 0$ , as implied by (42). Thus,  $\theta(f_\phi) = \bar{\theta}(\phi_{f_\phi}) = \bar{\theta}(\phi) = t$ , and therefore  $t \in \Theta^*$ . *Q.E.D.*

## D.2 Proof of Proposition 2

Observe that  $\Phi$  is a compact and connected subset of  $\mathbb{R}^{d_\phi}$ . Since (25)–(27) are linear equalities, the subset of  $\Phi$  that satisfies them is also compact and connected. Thus, if  $\bar{\theta}$  is continuous on this subset and  $d_\theta = 1$ , it follows that its image over it—which Proposition 1 established to be  $\Theta^*$ —is compact and connected as well. If  $d_\theta = 1$ , then



$\Theta^*$  is a compact interval, so by definition its endpoints must be given by  $t_*$  and  $t^*$ .  
*Q.E.D.*

## E Implementing Bounds on Consumer Surplus

In this section, we show that setting the target parameter to be the change in consumer surplus (as defined in (4)) results in a reduced target parameter function ( $\bar{\theta}$ ) that is linear in  $\phi$ . For shorthand, we denote average consumer surplus at price  $p^*$ , conditional on  $(P_i, X_i) = (p, x)$  as

$$\text{CS}_{p^*}(f|p, x) \equiv \int \left\{ \max_{j \in \mathcal{J}} v_j - p_j^* \right\} f(v|p, x) dv.$$

Suppose that  $\mathbb{V}$  is a minimal relevant partition constructed from a set of premiums  $\mathcal{P}$  that contains both  $p$  and  $p^*$ . Then

$$\text{CS}_{p^*}(f|p, x) = \sum_{\mathcal{V} \in \mathbb{V}} \int_{\mathcal{V}} \left\{ \max_{j \in \mathcal{J}} v_j - p_j^* \right\} f(v|p, x) dv, \quad (43)$$

since the MRP is an (almost sure) partition of  $\mathbb{R}^J$ . By definition of the MRP, the optimal choice of plan is constant as a function of  $v$  within any MRP set  $\mathcal{V}$ . That is, using the notation in Definition 1,  $\arg \max_{j \in \mathcal{J}} v_j - p_j \equiv Y(v, p) = Y(v', p) \equiv Y(\mathcal{V}, p)$  for all  $v, v' \in \mathcal{V}$  and any  $p \in \mathcal{P}$ . Consequently, we have from (43) that

$$\text{CS}_{p^*}(f|p, x) = \sum_{\mathcal{V} \in \mathbb{V}} \int_{\mathcal{V}} v_{Y(\mathcal{V}, p^*)} f(v|p, x) dv - p_{Y(\mathcal{V}, p^*)}^*$$

Replacing  $p^*$  by  $p$ , it follows that the change in consumer surplus resulting from a shift in prices from  $p \rightarrow p^*$  can be written as

$$\begin{aligned} \Delta \text{CS}_{p \rightarrow p^*}(f|p, x) &\equiv \text{CS}_{p^*}(f|p, x) - \text{CS}_p(f|p, x) \\ &= \sum_{\mathcal{V} \in \mathbb{V}} \int_{\mathcal{V}} (v_{Y(\mathcal{V}, p^*)} - v_{Y(\mathcal{V}, p)}) f(v|p, x) dv + p_{Y(\mathcal{V}, p)} - p_{Y(\mathcal{V}, p^*)}^*. \end{aligned}$$

Now define the smallest and largest possible change in valuations within any partition set  $\mathcal{V}$  as

$$\begin{aligned} \underline{v}_{p \rightarrow p^*}(\mathcal{V}) &\equiv \min_{v \in \mathcal{V}} v_{Y(\mathcal{V}, p^*)} - v_{Y(\mathcal{V}, p)}, \\ \text{and } \bar{v}_{p \rightarrow p^*}(\mathcal{V}) &\equiv \max_{v \in \mathcal{V}} v_{Y(\mathcal{V}, p^*)} - v_{Y(\mathcal{V}, p)}. \end{aligned}$$

Since we do not restrict the distribution of valuations within each MRP set, the sharp lower bound on a change in consumer surplus is attained when this distribution concentrates all of its mass on  $\underline{v}_{p \rightarrow p^*}(\mathcal{V})$  in every  $\mathcal{V} \in \mathbb{V}$ . That is,

$$\Delta \text{CS}_{p \rightarrow p^*}(f|p, x) \geq \sum_{\mathcal{V} \in \mathbb{V}} \underline{v}_{p \rightarrow p^*}(\mathcal{V}) \phi_f(\mathcal{V}|p, x) + p_{Y(\mathcal{V}, p)} - p_{Y(\mathcal{V}, p^*)}^* \equiv \underline{\Delta \text{CS}}_{p \rightarrow p^*}(f|p, x).$$

Similarly, the sharp upper bound for any  $f$  is given by

$$\Delta \text{CS}_{p \rightarrow p^*}(f|p, x) \leq \sum_{\mathcal{V} \in \mathbb{V}} \bar{v}_{p \rightarrow p^*}(\mathcal{V}) \phi_f(\mathcal{V}|p, x) + p_{Y(\mathcal{V}, p)} - p_{Y(\mathcal{V}, p^*)}^* \equiv \overline{\Delta \text{CS}}_{p \rightarrow p^*}(f|p, x).$$

Therefore, a sharp upper bound on the change in consumer surplus can be found by taking  $\theta(f) \equiv \underline{\Delta \text{CS}}_{p \rightarrow p^*}(f|p, x)$ , setting

$$\bar{\theta}(\phi) \equiv \sum_{\mathcal{V} \in \mathbb{V}} \underline{v}_{p \rightarrow p^*}(\mathcal{V}) \phi(\mathcal{V}|p, x) + p_{Y(\mathcal{V}, p)} - p_{Y(\mathcal{V}, p^*)}^*.$$

and applying Propositions 1 or 2. The key requirement that  $\theta(f) = \bar{\theta}(\phi_f)$  can be seen to be satisfied here by examining the expression for  $\underline{\Delta \text{CS}}_{p \rightarrow p^*}(f|p, x)$  above. The sharp upper bound is found analogously.

## F Estimation

Our analysis in Section 3 concerns the identification problem under which the joint distribution of  $(Y_i, P_i, X_i)$  is treated as known. In practice, features of this distribution, such as the choice shares  $s(j|p, x)$ , need to be estimated from a finite data set, so we want to model them as potentially contaminated with statistical error. In this section, we show how to modify Proposition 2 to account for such error in our primary case of interest with  $\bar{\theta}$  linear. A formal justification for this procedure is developed in Mogstad et al. (2018).

The estimator proceeds in two steps. First, we minimize the discrepancy in the observational equivalence conditions (25) by solving

$$\hat{Q}^* \equiv \min_{\phi \in \Phi} \hat{Q}(\phi) \quad \text{subject to (26)–(27),}$$

$$\text{where } \hat{Q}(\phi) \equiv \sum_{j, p, x} \hat{\mathbb{P}}[P_i = p, X_i = x] \left| \hat{s}(j|p, x) - \sum_{\mathcal{V} \in \mathbb{V}_j(p)} \phi(\mathcal{V}|p, x) \right|, \quad (44)$$

with  $\hat{s}(j|p, x)$  the estimated share of choice  $j$ , conditional on  $(P_i, X_i) = (p, x)$ , and

$\widehat{\mathbb{P}}[P_i = p, X_i = x]$  an estimate of the density of  $(P_i, X_i)$ . The use of absolute deviations in the definition of  $\widehat{Q}$  means that (44) can be reformulated as a linear program by replacing terms in absolute values by the sum of their positive and negative parts.<sup>44</sup> We weight these absolute deviations by the distribution of  $(P_i, X_i)$  so that regions of smaller density do not have an outsized impact on the estimated bounds.

In the second step, we collect values of  $\bar{\theta}(\phi)$  among  $\phi$  that come close to minimizing (44). That is, we construct the set:

$$\widehat{\Theta}^* \equiv \left\{ \bar{\theta}(\phi) : \phi \in \Phi, \text{ and } \widehat{Q}(\phi) \leq \widehat{Q}^* + \eta_n, \text{ and } \phi \text{ satisfies (26)–(27)} \right\} \quad (45)$$

The qualifier “close” here reflects the tuning parameter  $\eta_n$ , which must converge to zero at an appropriate rate with the sample size,  $n$ . The purpose of this tuning parameter is to smooth out possible discontinuities caused by set convergence. In our empirical estimates, we set  $\eta_n = .1$ , and found very little sensitivity to values of  $\eta_n$  that were bigger or smaller by an order of magnitude. However, there are currently no theoretical results to guide the choice of this parameter.

In our main case of interest when  $\bar{\theta}$  is linear and scalar-valued, we estimate  $\widehat{\Theta}^*$  by solving two linear programs that replace (25) with the condition in (45). That is, we solve

$$\widehat{t}_* \equiv \min_{\phi \in \Phi} \bar{\theta}(\phi) \quad \text{s.t.} \quad \widehat{Q}(\phi) \leq \widehat{Q}^* + \eta_n, \quad (46)$$

and an analogous maximization problem defining  $\widehat{t}^*$ . The set estimator for  $\Theta^*$  is then  $\widehat{\Theta}^* \equiv [\widehat{t}_*, \widehat{t}^*]$ . For this case, Mogstad et al. (2018) show that  $\widehat{t}_*$  and  $\widehat{t}^*$  are consistent for  $t_*$  and  $t^*$  under weak conditions on  $\widehat{s}$ . When  $\bar{\theta}$  is linear, (46) can be reformulated as a linear program, again by appropriately rephrasing the absolute value terms in terms of their positive and negative parts. In this case, the overall procedure of the estimator is to solve three linear programs: One for (44), one for (46), and one for the analogous maximization problem.

## G Estimation of Potential Buyers

In this section, we describe how we use the American Community Survey (ACS) to estimate the number of potential buyers in each market, i.e. in each age  $\times$  income  $\times$  region bin, or each value of  $X_i = x$ . As is often the case in empirical studies of demand (see, e.g. Berry, 1994, pg. 247), in our data we only observe individuals who

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<sup>44</sup> This is a common reformulation argument, see e.g. Bertsimas and Tsitsiklis (1997, pp. 19–20).

buy health insurance in Covered California, but not those who were eligible yet chose the outside option. That is, we do not have data on the quantity who chose choice 0. Instead, we will construct conditional choice probability (market shares) by estimating the number of potential buyers, constructing shares of the inside choices ( $j \geq 1$ ) by dividing quantity by potential buyers, and then taking the difference between the sum of the inside shares and 1 to be the share of the outside choice.

The key step here is estimating the number of potential buyers (market size),  $M_x$ , for each bin  $X_i = x$ . We do this using the California 2013 3-year subsample of the American Community Survey (ACS) public use file, downloaded from IPUMS (Ruggles et al., 2015).<sup>45</sup> We define an individual as a potential buyer, denoted by the indicator  $I_i = 1$ , if they report being either uninsured or privately insured. Individuals with  $I_i = 0$  include those who are covered by employer-sponsored plans, Medi-Cal (Medicaid), Medicare, or other types of public insurance. Then our estimator of  $M_x$  is

$$\widehat{M}_x = \sum_{i=1}^N \text{weight}_i I_i \mathbb{1}[X_i = x], \quad (47)$$

where  $\text{weight}_i$  are the individual sampling weights provided in the ACS, and  $N$  is the total sample size. This sample reflects the selection rules discussed in Section 4.1. To impose the restriction to households with 1 or 2 adults, we combine age with the IPUMS definition of a health insurance unit (HIU), and keep only individuals in HIUs of size 1 or 2.

An adjustment to this procedure is needed to account for the fact that the PUMA (public use micro area) geographic identifier in the ACS can be split across multiple counties, and so in some cases also multiple ACA rating regions. For a PUMA that is split in such a way, we allocate HIUs to each rating region it overlaps using the population of the zipcodes in the PUMA as weights. This is the same adjustment factor used in the PUMA to county crosswalk.<sup>46</sup> Since the definition of a PUMA changed after 2011, we also use this adjustment scheme to convert the 2011 PUMA definitions to 2012–2013 definitions.

A final adjustment is needed for situations in which our estimate  $\widehat{M}_x$  is smaller than the number of enrollees in the Covered California administrative data. Some fix

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<sup>45</sup> The 3 year sample includes information from 2011 to 2013. We use the entire 3 year sample to increase our sample size.

<sup>46</sup> For example, suppose that an HIU is in a PUMA that spans counties A and B, and that this HIU has a total sampling weight of 10, so that it represents 10 observationally equivalent households. If the adjustment factor is 0.3 in county A and 0.7 in county B, we assume there are 3 identical HIUs in county A and 7 in county B.

for such a case is needed in order to keep all shares bounded between 0 and 1. The fix we use is to replace  $\widehat{M}_x$  by the total enrollment observed in the administrative data, so that the estimated share of the outside option is 0. In practice, we find that this only happens for smaller  $x$  bins in sparsely-populated rating regions, and we expect the cause is statistical error in  $\widehat{M}_x$ . While our solution is not ideal, it seems to be the best that we can do given the available data. Since our estimates are weighted by bin size, the adjustment we use turns out to affect our results little when compared to some other (also ad hoc) adjustments we have tried.

## References

- ABRAHAM, J., C. DRAKE, D. W. SACKS, AND K. SIMON (2017): “Demand for health insurance marketplace plans was highly elastic in 2014–2015,” *Economics Letters*, 159, 69–73. 3
- ALLEN, R. AND J. REHBECK (2017): “Identification with Additively Separable Heterogeneity,” *Working paper*. 38, 39
- BALKE, A. AND J. PEARL (1994): “Counterfactual Probabilities: Computational Methods, Bounds, and Applications,” in *Proceedings of the Conference on Uncertainty in Artificial Intelligence (UAI-94)*, ed. by R. Lopez de Mantras and D. Poole, 46–54. 39
- (1997): “Bounds on Treatment Effects From Studies With Imperfect Compliance,” *Journal of the American Statistical Association*, 92, 1171–1176. 39
- BERRY, S. AND P. HAILE (2015): “Identification in Differentiated Products Markets,” *Annu. Rev. Econ.*, –. 7
- BERRY, S., J. LEVINSOHN, AND A. PAKES (1995): “Automobile Prices in Market Equilibrium,” *Econometrica*, 63, 841–890. 1, 7, 39
- (2004): “Differentiated Products Demand Systems from a Combination of Micro and Macro Data: The New Car Market,” *Journal of Political Economy*, 112, 68–105. 1, 39
- BERRY, S. T. (1994): “Estimating Discrete-Choice Models of Product Differentiation,” *The RAND Journal of Economics*, 25, 242–262. 1, 7, 39, 50
- BERRY, S. T. AND P. A. HAILE (2014): “Identification in Differentiated Products Markets Using Market Level Data,” *Econometrica*, 82, 1749–1797. 2, 39
- BERTSIMAS, D. AND J. N. TSITSIKLIS (1997): *Introduction to linear optimization*, vol. 6, Athena Scientific Belmont, MA. 50
- BRIESCH, R. A., P. K. CHINTAGUNTA, AND R. L. MATZKIN (2002): “Semiparametric Estimation of Brand Choice Behavior,” *Journal of the American Statistical Association*, 97, 973–982. 38
- (2010): “Nonparametric Discrete Choice Models With Unobserved Heterogeneity,” *Journal of Business & Economic Statistics*, 28, 291–307. 38
- BUNDORF, M. K., J. LEVIN, AND N. MAHONEY (2012): “Pricing and Welfare in Health Plan Choice,” *American Economic Review*, 102, 3214–3248. 6
- CANAY, I. A., A. SANTOS, AND A. M. SHAIKH (2013): “On the Testability of Identification in Some Nonparametric Models With Endogeneity,” *Econometrica*, 81, 2535–2559. 39
- CHAN, D. AND J. GRUBER (2010): “How Sensitive are Low Income Families to Health Plan Prices?” *American Economic Review*, 100, 292–296. 1, 28
- CHIAPPORI, P.-A. AND I. KOMUNJER (2009): “On the Nonparametric Identification of Multiple Choice Models,” *Working paper*. 2, 39
- CHIONG, K. X., Y.-W. HSIEH, AND M. SHUM (2017): “Counterfactual Estimation in Semiparametric Multinomial Choice Models,” *SSRN Electronic Journal*. 38, 39

- COMPIANI, G. (2018): “Nonparametric Demand Estimation in Differentiated Products Markets,” *Working paper*, 3, 39
- CONLEY, T. G., C. B. HANSEN, AND P. E. ROSSI (2010): “Plausibly Exogenous,” *Review of Economics and Statistics*, 94, 260–272. 33
- DEMUYNCK, T. (2015): “Bounding average treatment effects: A linear programming approach,” *Economics Letters*, 137, 75–77. 40
- EINAV, L., A. FINKELSTEIN, AND M. R. CULLEN (2010a): “Estimating Welfare in Insurance Markets Using Variation in Prices,” *Quarterly Journal of Economics*, 125, 877–921. 6
- EINAV, L., A. FINKELSTEIN, AND J. LEVIN (2010b): “Beyond Testing: Empirical Models of Insurance Markets,” *Annual Review of Economics*, 2, 311–336. 6
- ERICSON, K. M. AND A. STARC (2015): “Pricing Regulation and Imperfect Competition on the Massachusetts Health Insurance Exchange,” *The Review of Economics and Statistics*, forthcoming, 97, 667–682. 1, 28, 33
- FINKELSTEIN, A., N. HENDREN, AND M. SHEPARD (2017): “Subsidizing Health Insurance for Low-Income Adults: Evidence from Massachusetts,” Working Paper 23668, National Bureau of Economic Research. 3, 22, 28, 29, 32
- FINKELSTEIN, A., S. TAUBMAN, B. WRIGHT, M. BERNSTEIN, J. GRUBER, J. P. NEWHOUSE, H. ALLEN, K. BAICKER, AND O. H. S. GROUP (2012): “The Oregon health insurance experiment: evidence from the first year,” *The Quarterly journal of economics*, 127, 1057–1106. 33
- FOURER, R., D. M. GAY, AND B. W. KERNIGHAN (2002): *AMPL: A Modeling Language for Mathematical Programming*, Cengage Learning. 17
- FOX, J. T. (2007): “Semiparametric estimation of multinomial discrete-choice models using a subset of choices,” *The RAND Journal of Economics*, 38, 1002–1019. 38
- FOX, J. T. AND A. GANDHI (2016): “Nonparametric identification and estimation of random coefficients in multinomial choice models,” *The RAND Journal of Economics*, 47, 118–139. 9, 39
- FOX, J. T., K. I. KIM, S. P. RYAN, AND P. BAJARI (2011): “A simple estimator for the distribution of random coefficients,” *Quantitative Economics*, 2, 381–418. 7
- (2012): “The random coefficients logit model is identified,” *Journal of Econometrics*, 166, 204–212. 7
- FREYBERGER, J. AND J. L. HOROWITZ (2015): “Identification and shape restrictions in non-parametric instrumental variables estimation,” *Journal of Econometrics*, 189, 41–53. 39
- GANDHI, A., Z. LU, AND X. SHI (2017): “Estimating Demand for Differentiated Products with Zeroes in Market Share Data,” *Working paper*. 39
- GARTHWAITE, C., T. GROSS, AND M. J. NOTOWIDIGDO (2018): “Hospitals as Insurers of Last Resort,” *American Economic Journal: Applied Economics*, 10, 1–39. 33
- GERUSO, M. (2017): “Demand Heterogeneity in Insurance Markets: Implications for Equity and Efficiency,” *Quantitative Economics*, 8, 929–975. 33

- GOLDBERG, P. K. (1995): “Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry,” *Econometrica*, 63, 891–951. 1, 7
- GUROBI OPTIMIZATION, I. (2015): “Gurobi Optimizer Reference Manual,” . 17
- HANDEL, B., I. HENDEL, AND M. D. WHINSTON (2015): “Equilibria in Health Exchanges: Adverse Selection versus Reclassification Risk,” *Econometrica*, 83, 1261–1313. 6, 40
- HANDEL, B. R. (2013): “Adverse Selection and Inertia in Health Insurance Markets: When Nudging Hurts,” *American Economic Review*, 103, 2643–82. 6, 40
- HANSEN, L. P., J. HEATON, AND E. G. J. LUTTMER (1995): “Econometric Evaluation of Asset Pricing Models,” *The Review of Financial Studies*, 8, 237–274. 39
- HAUSMAN, LEONARD, AND ZONA (1994): “Competitive Analysis with Differentiated Products,” *Annales d’Économie et de Statistique*, 159. 39
- HAUSMAN, J. A. (1996): *Valuation of New Goods under Perfect and Imperfect Competition*, University of Chicago Press, 207–248. 1, 7
- HAUSMAN, J. A. AND D. A. WISE (1978): “A Conditional Probit Model for Qualitative Choice: Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences,” *Econometrica*, 46, 403–426. 7
- HECKMAN, J. J. AND B. E. HONORÉ (1990): “The Empirical Content of the Roy Model,” *Econometrica*, 58, 1121–1149. 9
- HECKMAN, J. J. AND S. NAVARRO (2007): “Dynamic discrete choice and dynamic treatment effects,” *Journal of Econometrics*, 136, 341–396. 9
- HO, K. AND A. PAKES (2014): “Hospital Choices, Hospital Prices, and Financial Incentives to Physicians,” *American Economic Review*, 104, 3841–84. 3, 38, 39
- HONORÉ, B. E. AND A. LLERAS-MUNEY (2006): “Bounds in Competing Risks Models and the War on Cancer,” *Econometrica*, 74, 1675–1698. 39
- HONORÉ, B. E. AND E. TAMER (2006): “Bounds on Parameters in Panel Dynamic Discrete Choice Models,” *Econometrica*, 74, 611–629. 39
- KAMAT, V. (2017): “Identification with Latent Choice Sets: The Case of the Head Start Impact Study,” *Working paper*. 40
- KLINE, P. AND M. TARTARI (2016): “Bounding the Labor Supply Responses to a Randomized Welfare Experiment: A Revealed Preference Approach,” *American Economic Review*, 106, 972–1014. 40
- LAFFÉRS, L. (2013): “A note on bounding average treatment effects,” *Economics Letters*, 120, 424–428. 39
- LEWBEL, A. (2000): “Semiparametric qualitative response model estimation with unknown heteroscedasticity or instrumental variables,” *Journal of Econometrics*, 97, 145–177. 9, 38, 39
- MAHONEY, N. (2015): “Bankruptcy as Implicit Health Insurance,” *American Economic Review*, 105, 710–46. 33



- MANSKI, C. F. (1975): “Maximum score estimation of the stochastic utility model of choice,” *Journal of Econometrics*, 3, 205–228. 38
- (1985): “Semiparametric analysis of discrete response: Asymptotic properties of the maximum score estimator,” *Journal of Econometrics*, 27, 313–333. 9
- (2007): “Partial Identification of Counterfactual Choice Probabilities,” *International Economic Review*, 48, 1393–1410. 38, 39, 40
- (2014): “Identification of income-leisure preferences and evaluation of income tax policy,” *Quantitative Economics*, 5, 145–174. 38, 39
- MANSKI, C. F. AND J. V. PEPPER (2017): “How Do Right-to-Carry Laws Affect Crime Rates? Coping with Ambiguity Using Bounded-Variation Assumptions,” *The Review of Economics and Statistics*. 33
- MATZKIN, R. L. (1991): “Semiparametric Estimation of Monotone and Concave Utility Functions for Polychotomous Choice Models,” *Econometrica*, 59, 1315–1327. 38
- (1993): “Nonparametric identification and estimation of polychotomous choice models,” *Journal of Econometrics*, 58, 137–168. 2, 38
- McFADDEN, D. (1974): “Conditional logit analysis of qualitative choice behavior,” in *Frontiers of Econometrics*, ed. by P. Zarembka, Academic Press: New York, 105–142. 1
- McFADDEN, D. AND K. TRAIN (2000): “Mixed MNL Models for Discrete Response,” *Journal of Applied Econometrics*, 15, 447–470. 1, 7
- MOGSTAD, M., A. SANTOS, AND A. TORGOVITSKY (2018): “Using Instrumental Variables for Inference about Policy Relevant Treatment Parameters,” *Econometrica (forthcoming)*. 40, 49, 50
- NEVO, A. (2011): “Empirical Models of Consumer Behavior,” *Annual Review of Economics*, 3, 51–75. 7
- NEVO, A. AND A. M. ROSEN (2012): “Identification With Imperfect Instruments,” *Review of Economics and Statistics*, 94, 659–671. 33, 39
- ORSINI, J. AND P. TEBALDI (2017): “Regulated Age-based Pricing in Subsidized Health Insurance: Evidence from the Affordable Care Act,” *Available at SSRN 2464725*. 4
- PAKES, A. (2010): “Alternative Models for Moment Inequalities,” *Econometrica*, 78, 1783–1822. 38, 39
- PAKES, A. AND J. PORTER (2016): “Moment Inequalities for Multinomial Choice with Fixed Effects,” Tech. rep. 38, 39
- PAKES, A., J. PORTER, K. HO, AND J. ISHII (2006): “Moment Inequalities and Their Application,” *Working paper*. 38, 39
- (2015): “Moment Inequalities and Their Application,” *Econometrica*, 83, 315–334. 38, 39

- RUGGLES, S., K. GENADEK, R. GOEKEN, J. GROVER, AND M. SOBEK (2015): “Integrated Public Use Microdata Series: Version 6.0 [dataset],” *Minneapolis: University of Minnesota*. 22, 51
- SALTZMAN, E. (2017): “Demand for Health Insurance: Evidence from the California and Washington ACA Marketplaces,” . 1
- SHI, X., M. SHUM, AND W. SONG (2016): “Estimating Semi-Parametric Panel Multinomial Choice Models Using Cyclic Monotonicity,” *SSRN Electronic Journal*. 38
- SHORACK, G. R. (2000): *Probability for Statisticians*, Springer-Verlag. 44
- TEBALDI, P. (2017): “Estimating Equilibrium in Health Insurance Exchanges: Price Competition and Subsidy Design under the ACA,” *Working paper*. 1, 4, 22, 33
- THOMPSON, T. S. (1989): “Identification of Semiparametric Discrete Choice Models,” *Discussion paper 249, Center for Economics Research, Department of Economics, University of Minnesota*. 2, 9, 38
- TORGOVITSKY, A. (2016): “Nonparametric Inference on State Dependence with Applications to Employment Dynamics,” *SSRN Working Paper 2564305*. 40
- (2018): “Partial Identification by Extending Subdistributions,” *Quantitative Economics*, Forthcoming. 38, 40