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The Optimal COVID-19 Quarantine and Testing Policies

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Abstract

Many countries are taking measures stopping productive activities to slow down the spread of COVID-19. At times these measures have been criticized as being excessive and too costly. In this paper we make an attempt to understand the optimal response to an infectious disease. We find that the observed policies are very close to a simple welfare maximization problem of a planner who tries to stop the diffusion of the disease. These extreme measures seem optimal in spite of the high output cost that it may have in the short run, and for various curvatures of the welfare function. The desire for cost smoothing reduces the intensity of the optimal lockdown while extending it for longer, but it still amounts to reducing economic activity by a half. We then study the possibility of either complementing or substituting the lockdown policy with random testing. We find that testing generates sizeable welfare gains and can almost eliminate the need for indiscriminate quarantines.

1 Introduction

The arrival of COVID-19 at the beginning of 2020 took most of the world by surprise. It was quickly understood that even though related to SARS, its fatality rate was significantly lower. This drove many governments to deem it as a mild illness, resulting in very few initial measures to stop it.[1] However, soon after the outbreak it became clear that COVID-19 was substantially more contagious than SARS. This worried local and national officials because if the virus could spread freely the hospitals would not be able to treat the large inflow of potential patients: the health systems were facing a capacity constraint.

This per se would not be a daunting feature if it weren’t because the fatality rate among untreated elderly (those above 60 years of age) was alarmingly high, with some studies estimating above 5% for patients between 60 and 70, and around 15%-20% for individuals above 70.[2] The combination of rapid diffusion and the need of intensive care units (ICU) to prevent a high mortality rate resulted in many administrations taking aggressive measures to either stop the infection or, at the very least, slow down the diffusion, which is known as “flattening the curve.”

The approach to deal with the treatment capacity problem has been heterogenous across countries. China took initial drastic measures stopping all economic activity in the most affected areas, while the UK and South Korea have implemented policies to slow down the diffusion without greatly affecting economic activities.[3] As long as the number of affected individuals do not reach the treatment capacity constraint, the disease should be manageable. Other countries have taken intermediate approaches, but the common language appears to be to “flatten the curve”, without necessarily eliminating the threat, mitigation rather than suppression.

Since any intervention that affects the economic activity is costly, and exponentially so as the intervention deepens, these different approaches raise many questions about the right policy: should countries follow the Chinese approach taking drastic measures until the virus is extinct? Or is it better to do enough intervention to keep

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[1]One example of this is the fact that the Wuhan doctor who discovered the virus was initially disciplined for “spreading rumors” that could create paranoia. On the other hemisphere, President Trump in public appearances argued that it was no more than a seasonal flu.

[2]All available data shows that the fatality rate among individual under 40, without pre-existing conditions, is no different than a seasonal flu.

[3]The strategy of trying to eliminate the virus is also termed Suppression as in [Ferguson et al. (2020)], while flattening the curve is termed Mitigation.
the affected population under the capacity constraint? If so, how much is enough? Wouldn’t a combination of the two be better? What should the role of testing be in this context?

In this paper we aim to provide a preliminary answer to the aforementioned questions and try to rationalize the diverse observed policies. To this end we build on Atkeson (2020) and Eichenbaum, Rebelo, and Trabandtz (2020) who incorporate a SIR epidemiology model into well known economic setups. In this environment there is an outbreak of an infectious disease which spreads out continuously over time. Some affected individuals are initially asymptomatic and engage in economic activities (meeting) with healthy, but susceptible, subjects who then contract the illness and pass it to others. Once the subject is symptomatic and recognized as infected, it is contained and cannot transmit the illness. However, in this period she may need medical care. If she is not able to receive medical care, she dies with a higher probability than with proper care. We assume that the country has a capacity constraint on how many people can be treated at a given time. Once the capacity is exceeded, the average fatality rate in the economy starts to sharply rise.

We stress in this paper the two fundamental issues that we think should drive the optimal intervention. The first one, as we mentioned above, is the capacity of the health system to deal with a large inflow of patients. By many accounts COVID-19 does not seem to be an extremely deadly illness when the carriers are properly treated. Hence, the need to incorporate a hospital capacity constraint is a first order issue. Second, but no less important, in the standard SIR model, the main (implicit) friction creating the need for indiscriminate quarantines, is the inability of the policy maker to distinguish the asymptomatic infected (exposed in the terminology of Atkeson (2020)) from the susceptible but still unaffected. If it could, the decision maker would quarantine only the affected, letting the unaffected population to continue with their normal activities. Even though this appears as a natural information friction, the technology available to test individuals, identify them and be able to impose personalized quarantines rather than indiscriminate ones, exists and could be a welfare improving substitute of what is nowadays termed lockdowns. Of course, testing the whole population at once would completely eliminate the problem, but it could be prohibitively expensive. But this is a cost-benefit analysis that should be properly addressed in the current state of affairs. We analyze these two issues in sequential order. First, we take as given the information friction, then we analyze policies that relax it.
When the policy maker cannot separate an exposed, but asymptomatic, from a susceptible individual, it directly stops some or all of the economic activity to avoid the spread of the illness. By doing so, it prevents the realization of meetings that reproduce the virus. How much and for how long should production be restricted? In the current jargon, how strict and how long should the quarantine be?

We calibrate the model to the (very) preliminary data arising from the outbreak in Italy and we find that the observed set of policies are in line with our estimations. A complete lockdown for two weeks appears optimal when the planner is not concerned about consumption smoothing and the measure is implemented at the very beginning of the outbreak. When the measures are implemented 2 months after the initial outbreak, the optimal policy without concern for smoothing calls again for a complete shut-down of all economic activities, but for a month.

If instead the planner has concave preferences that favor cost smoothing, the intensity of the optimal policy is reduced by a half, i.e., half of the economic activity should stop, and the length is doubled. Thus, if the measures are implemented early on, the quarantine should be for a month, while if the implementation is delayed, the quarantine should last for two months.

Placing these results in the Italian context, we estimate that without any intervention there could be as many as 780,000 fatalities. With a very early intervention that number reduces to about 120–230 fatalities, while if the intervention is starts 60 days later, the number of fatalities is between 7,200 and 9,000.

Yet, these policies are drastic with large costs in terms of output, which falls by more than 50% at the peak of the intervention. This brings about the possibility of complementing the quarantines with massive testing to simultaneously decrease the speed COVID-19’s reproduction and being able to put to work a larger share of the population. To do so we take seriously that the main problem is an information friction. We consider the possibility that the government can initiate intense screening trying to identify the exposed individuals. Once identified as positive, the subject is required to endure a (personal) quarantine. This is done by randomly selecting individuals for which there is no information yet: those who have never tested positive before.\footnote{If a subject has been tested before but the results were always negative, it is still susceptible to the illness, and therefore is in the same situation as another who has never been tested. While those who have been affected and tested positive and recover, are from then on immune, so that there is no need to test them again.}
tifying a positive case have three beneficial effects. 1) Once it is known that someone is affected, it is possible to better treat him/her should the individual become ill. 2) It is possible to quarantine the individual, even in a stricter way than the rest of the population to slow down the reproduction of the virus. Last but not least, 3) Once the individual is able to eliminate the virus from its biological system, it is immune and can be allowed to work without any restriction, helping to moderate the extent of the recession. This last contribution, is many times overlooked and it could be of considerable relevance, see for instance Dewatripont et al. (2020), especially when many subjects could remain asymptomatic during the whole duration of the infection. Without the testing, we would quarantine many individuals that are immune and could be working.

We lack reliable data on the cost of a test. Thus, we assume that the marginal cost of the first unit is 2 days of daily output per worker and grows quadratically. The speed at which the marginal cost grows is chosen in such a way that it would be economically infeasible to test the entire population at once. We find that testing is intensively used as a substitute of indiscriminate quarantines and generates substantial welfare gains. With the cost function that we assume, the output gains are so large that with the optimal policies the lockdowns are almost not used. When the utility is linear this happens because the duration of the lockdown is reduced to a minimum, while with logarithmic utility the duration of the lockdown is unchanged, but the intensity is reduced to a minimum. Random testing policies seem to be very important and should be studied in more detail.

1.1 Literature review

The literature on epidemiology control dates back to the model proposed by Kermack and McKendrick (1927), also known as the SIR epidemiology model. However, to the best of our knowledge there has not been many applications to economics. With the recent outbreak of COVID-19, economic researchers have started to incorporate SIR models in economic environments to assess the potential economic implications of COVID-19.

Atkeson (2020) computes the projected paths of the disease and evaluates its economic impact. We build on his work deepening the information structure. In this work, it is implicitly assumed that only the symptomatically infected individuals are
contagious. We instead assume that, as it happens in many countries, the symptomatically infected are isolated and therefore do not contribute to the speed of contagion. It is what we call exposed but asymptomatic individuals, potentially unidentified without testing, who actually fuel the spreading of the disease. This extension allows us 1) to better fit the dynamics of the disease and 2) to have a well-defined information friction calling for the need for testing. In addition, we optimize over the set of policies rather than focusing on some, yet interesting, paths.

Eichenbaum, Rebelo, and Trabandtz (2020) construct what they call a SIR-macro model with endogenous consumption and labor supply. The competitive equilibrium in their model is suboptimal due to fact that agents do not fully internalize the externality of their economic interactions. They consider the optimal consumption tax policy that can correct the externality. We differ in many dimensions. First, as Atkeson (2020) they consider only the actively infected as potential carriers. Second, they use a meeting technology that does not allow for congestion. Thus, the dynamics of how infection spreads doesn’t feature our dilutive effect that kicks in later as immunity expands. Third, we consider policies that directly control economic activities, rather than altering marginal decisions.

As us, Alvarez, Argente, and Lippi (2020) also study the optimal lockdown policy. To this end they use a meeting technology similar to Eichenbaum, Rebelo, and Trabandtz (2020). They assume linear preferences, which weights output and the cost of disease. They also explicitly consider the possibility that in some countries the lockdown could be less effective or harder to implement. We differ from them in some dimensions. Our structure and meeting technology allow us to focus on the fundamental information friction in distinguishing types, which necessitates quarantines to stops the contagion or testing to overcome it. We consider also concave preferences, which generate a need to smooth the costs of the intervention, and show that is very relevant reducing the intensity and increasing the length. Similarly, to incorporate the hospital capacity problem, they assume an exogenous fatality rate linearly increasing in the number of infected. Instead, we model hospital capacity explicitly and calibrate it to the Italian situation. Finally, they solve the optimal control problem, without restricting the policy space, while we look for the optimal lockdown intensity in a restricted policy space.

Dewatripont et al. (2020) propose that testing, either prioritized or random, is essential to restart the economy. They argue that mass testing is technological feasible and
a mere logistic issue of scaling up. We assess a random testing policy and find that some degree of testing joint with quarantines are welfare improving.

2 A SIR model of disease contagion

Time is continuous and runs indefinitely, \( t \in [0, \infty) \). At time \( t \) the economy is inhabited by a population \( n_t \) with an initial mass of one: \( n_0 = 1 \). Since the spread of the illness is so fast that it can be measured by the day, we use the convention that one unit of time is one day. At any given time, each individual can be one of four types: susceptible, exposed, infected, and recovered. We denote by \( s \) the number of subjects still unaffected but susceptible to the virus, \( e \) the number of exposed but not symptomatic, \( i \) the number of affected individuals, and \( r \) the number of immune recovered agents. Clearly, it must be the case that

\[
    n_t = s_t + e_t + i_t + r_t.
\]

At \( t = 0 \), the economy is hit by a disease due to a deadly virus. If the exposed population is above a critical mass, \( e > \underline{e} \geq 0 \), then it starts to spread. Otherwise, it’s self-contained and all patients gradually recover. The virus spread through meetings between exposed and unaffected individuals. Those who are initially exposed after an incubation period become symptomatically infected. Those who already have it and are recovered become immune permanently. The number of meetings in the economy depends on the level of economic activity \( 1 - q_t \), and it is governed by the meeting function \( f(s_t, n_t) \). Not all of these meetings generate and infection, only \( \lambda f(s_t, n_t) \) of the meetings generate new affected (exposed) individuals. Once exposed, an individual becomes symptomatically infected with intensity \( \gamma \) per unit of time.

Thus, the law of motion of the exposed type satisfies:

\[
    de_t = \begin{cases} 
    [\lambda f(s_t, n_t) (1 - q_t) - \gamma] e_t dt, & \text{if } e_t > \underline{e} \\
    -\gamma e_t dt, & \text{if } e_t \leq \underline{e}.
    \end{cases}
\]  

\[ (1) \]

\[ ^5 \text{The asymptomatic status prior to becoming symptomatically infected clarifies the effects in production, and it is instrumental in Section 4.2 when we analyze the information friction. Otherwise, we could merge them into a single type as in Alvarez, Argente, and Lippi (2020).} \]
We assume that the symptom appearance intensity $\gamma$ is independent of time and the state of the economy. It just reflects the individual’s strength to fight the virus inside their biological system. The same is true for the intensity of contagion $\lambda$, which is a scale parameter capturing the level of interactions among agents in their daily economic activity. The speed at which the illness spreads is clearly state dependent, increasing in the number of exposed $e_t$ and the share of the population which are still susceptible. The function $f(s_t, n_t)$ could incorporate potential “congestion” effects. For instance, one may think that when most of the population are already affected, most meetings would be between individuals who are either immune or already infected and thus would not generate new infections. We later propose a functional form for $f(.)$ but we experiment with different alternatives.

We want to emphasize that the presence of the minimum critical mass $\varepsilon$ could be very important for the prescribed policy interventions. When $\varepsilon = 0$ the virus never dies, it could be forced to affect a negligible number of people, but it would be always around to re-surface and spread again. Instead, when $\varepsilon > 0$ it could be possible to take drastic measures to force the affected population below the critical mass, so that the virus disappears and the infection is definitively defeated. Instead, when $\varepsilon = 0$, since the virus would eventually spread anyway, a policy maker could choose to simply regulate the speed at which the number of exposed and infected subjects arrive. This would be important when we bound the capacity of the health system to treat the illness.

Exposed subjects become symptomatically infected at rate $\gamma$. Once they are infected they would require medical assistance and potential hospitalization. When treated individuals recover at rate $\eta$ and die at the rate $\Delta_t$ per unit of time. The law of motion of (symptomatic) infected individuals satisfies:

$$\text{di}_t = [\gamma e_t - (\eta + \Delta_t)i_t]dt.$$  

As with $\gamma$, here again $\eta$ is independent of the economy’s state. The process by which the body is able to eliminate the virus from the system is not affected by the health

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6 Another way to think about $\varepsilon$ is as a way to prevent the modeling strategy from forcing policy prescriptions. For instance, because growth is proportional, one we can divide a positive number indefinitely by other positive number and it would always be strictly positive. In the context of our model we could end up with less than a person infected, which is not physically possible, but it would imply that the infection would reappear in the future. $\varepsilon > 0$ makes sure that, whenever fewer than a minimum amount of person are infected, the disease would disappear.
system, it only depends on the strength of the subject’s immune system, conditional on surviving. But notice that \( \Delta_t \) does depend on the state of economy. One may think that the way in which the illness affect a particular individual depends only on her/his biological characteristics and therefore should be independent of how other individuals are affected. However, here we assume that the death rate depends on the capacity of the health system to treat patients.

Hospitals can optimally treat only \( h \) patients at a time. Once that capacity is exceeded the treatment received by each patient is diluted resulting in a suboptimal treatment. Those who are optimally treated die with intensity \( \theta \), while does who are treated in an overcrowded system die with intensity \( \delta > \theta \). As a result, the average daily death intensity in the economy satisfies:

\[
\Delta_t = \theta \min \left\{ 1, \frac{h}{i_t} \right\} + \delta \max \left\{ 1 - \frac{h}{i_t}, 0 \right\}.
\]

Given the previous assumptions, the number of recovered patients and total population evolve according to:

\[
\begin{align*}
\frac{dr_t}{dt} &= \eta i_t dt, \\
\frac{dn_t}{dt} &= -\Delta_t i_t dt.
\end{align*}
\]

From the previous structure it is straightforward, see Appendix [A], to compute the average death rate from the illness and the duration of sickness. This of course would depend on whether the patients are treated or not. When all sick individuals are treated, a patient recovers in \( \frac{\eta}{(\eta+\theta)^2} \) days, and on average a fraction \( \frac{\theta}{(\eta+\theta)} \) of the patients die. When left untreated, the recovery happens in \( \frac{\eta}{(\eta+\delta)^2} \) days, and the average death rate is \( \frac{\delta}{(\eta+\delta)} \). These are moments that it is possible to match with the already available data. Note that due to selection, patients would appear to recover faster in countries with higher fatality rates.

For simplicity we assume that the production technology is linear. Each meeting produces one unit of output per individual involved in the meeting. Infected hospitalised individuals are unable to produce. In normal times the total production would be \( y_t = n_t \) per day. However, during the spreading of the virus, only the unaffected, fully recovered and those still undetected but yet exposed can produce. Hence, if the
government allows for undistorted economic activity, i.e. $q_t = 0$, the total production would be $y_t = s_t + r_t + e_t$. To prevent the spread of the virus the government could ban certain activities. It does so by forcing quarantines among the population. Since the government is unable to distinguish $s_t$ from $e_t$, it cannot condition the quarantine on each individual status, it simply ban a fraction $q_t \in [0, 1]$ of all economic activity.

As result, the total production after a policy intervention is $y_t = (1 - q_t)(s_t + e_t) + r_t$.

We assume a closed economy. The only produced good is non-storable, and there is no possibility of borrowing or saving in financial assets. This implies that consumption is equal to production in every period: $c_t = y_t$. All individuals, and therefore also society as whole, discount the future at rate $\rho > 0$. The government chooses a path $\{q_t\}_{t=0}^{\infty}$ to maximize society’s welfare:

$$\max \int_0^\infty e^{-\rho t} u ((1 - q_t)(s_t + e_t) + r_t) \, dt; \quad (P1)$$

subject to equations (1), (3), and (4).

Notice that this setup allows for a variety of possibilities. A solution could be a forced quarantine for every individual for a limited period. For instance, we can think about Wuhan’s suppression policy intervention as setting $q_t = 1$ for all $t \leq \bar{\tau}$ and $q_t = 0$ for all $t > \bar{\tau}$, for some $\bar{\tau} > 0$. In this case if in some point $e_t < e$ the virus dies and never recovers. This problem would reduce to choosing the optimal length $\bar{\tau}$ of a complete lock down. Alternatively, one can think about policies that are more mitigated, mitigation policies, with $q_t < 1$, but that last for a longer interval. Here the strategy would be to try to maintain $i_t$ below $h$ at all $t$ until most of the population becomes immune.

The choice of the welfare function is by no means trivial. In Problem (P1) we have purposely excluded the welfare lost due to fatalities. We have done so because it is, at the very least, highly controversial how to compare losses due to foregone consumption with welfare losses due to fatalities. What is the value of a life? If one believes that a human life is more important than everything else, then the correct welfare function should only minimize the number of fatalities. In this case, as long as $\theta > 0$

\[7\] Notice that we are allowing the recovered subjects to return to work. This is clearly optimal and the recover status is fully observable. In spite of this, most, if not all, countries include inefficiently the recovered in the mandatory quarantines.
the solution to (P1) is almost trivial, setting \( q_t = 1 \) for as long as is needed to locate \( e_t \) below \( \varepsilon \). If \( \theta = 0 \) and \( \delta > 0 \), then only policy paths that maintain \( i_t \leq h \) would be part of the solution. Here the output cost becomes the relevant factor pinning down the optimal path.

However, (P1) is also problematic because current and past choices reveal that societies are willing to trade off human life for economic activity. For instance, the U.S. Center for Disease Control and Prevention (CDC) estimates that between 12,000 and 61,000 people die annually due to influenza. Yet, governments are not willing to stop the economic activity to prevent it. Similarly, in 2018 around 36,000 people died in car accidents in the U.S. But there has never been a discussion about banning circulation in motor vehicles. One can interpret these choices as balancing individual and collective responsibility. As long as the fatalities are not too large, society prefers to delegate the choice of the “acceptable risk” to the individual, while if the fatality rate is too high there maybe some frictions that prevent individuals from properly asses the risk. Then, it becomes a collective responsibility and the government must intervene. For this reason we also consider and alternative problem where the policy maker trades off economic activity and lives:

\[
\max_{\{q_t: t \geq 0\}} \int_0^\infty e^{-\rho t} \left[ u((1 - q_t)(s_t + e_t) + r_t) - v(\Delta_t i_t) \right] dt; \quad (P2)
\]

subject to equations (1), (3), and (4).

Here the function \( v(x) \) would be key in determining the number of acceptable deaths. Admittedly, it is hard to parameterize it, but we use data for alternative activities that generate fatalities to discipline its implications. In the sense that, if for instance, an activity is allowed when the fatalities are caused by influenza, it should also be allowed when caused by COVID-19.

2.1 Functional forms

We consider alternative functional forms. We start assuming that welfare is linear in consumption, so that the planner is only concern about productive efficiency. In this case:

\[
u(c) = c.\]

Of course, variations of consumption across time could also be important for the planner. Thus, we also present welfare results in which the elasticity of intertemporal substitution is not zero. In this case we use two alternatives:

\[ u(c) = \log(c) \quad \text{and} \quad u(c) = c - \frac{b}{2}c^2. \]

All the functional forms are mathematically tractable and meaningful in one dimension or another, allowing for a wide range of interpretations.

The choice of \( v(x) \) is less straightforward. One may think that a quadratic loss function \( v(x) = \frac{d}{2}x^2 \) would be appropriate, because the cost grows exponentially with the number of fatalities. However, it also has the potentially unappealing feature that the planner would be willing to accept many fatalities if they are sufficiently spread out over time, while it wouldn’t accept it if all fatalities happen in a concentrated interval of time. For instance, 1 dead today and 1 tomorrow is much better that 2 dead either today or tomorrow. An alternative is to use a linear function \( v(x) = dx \), and choose \( d \) in such a way that there is an “optimal” upper bound for the number of fatalities. Alternatively, we could consider the cumulative number of fatalities and define

\[
\int_{0}^{\infty} e^{-\rho t} v(\Delta t_i) dt = \frac{d}{2} \left[ \int_{0}^{\infty} \Delta t_i dt \right]^2.
\]

Regarding the meeting function we do most of our calculations using a standard proportion function:

\[ f(s_t, n_t) = \frac{s_t}{n_t}. \]

We also present robustness using a Cobb-Douglas meeting function which allows for more flexibility at targeting congestion on diffusion technologies.

## 3 Quantitative implications

### 3.1 Parametrization

There are several key parameters in the model. First, the parameter \( \gamma \) determines how long an individual can be contagious without potentially showing any clear
symptoms. This parameter is related to the incubation period, which is 6.5 days of generation time according to Ferguson et al. (2020), so that \( 1/\gamma = 6 \).

Regarding the fatality rate there is much controversy about its “true” value, especially when all the available data is too raw to provide a concrete answer. Most studies tend to state that on average 1% of the infected die. However, this value could significantly change with the demographic structure of the population. In particular, the fatality rate appears to sharply increase with age and the lack of proper treatment. Since we are focusing on the Italian case, both factors are first order issues for our estimations. The study by Ferguson et al. (2020) estimates that the average fatality rate in Wuhan is around 0.99%. The same paper states that around 4.4% of the infected subjects require hospitalization. They also estimate that 30% of the hospitalised cases require critical care; and even when the patient receives proper critical care she dies with 0.5 probability. If we assumed that without critical care the subject dies with certainty, that implies that the fatality rate for the untreated is twice the analogous for the treated. This indicates that \( \delta \approx 2 \times \theta \).

Another calculation to determine the difference between \( \theta \) and \( \delta \) is to compare the fatality rates in a country that didn’t hit the health capacity with another that did. A candidate for the first is South Korea, while Italy is a clear candidate for a country in which the health system was overwhelmed. In Appendix B we present the information for the fatality rates by age for both countries. To obtain the average fatality rate, we multiply each age-specific rate by the relative weight of that age group in the population. We obtain that the average rate for South Korea is 1.22%, while for Italy is significantly larger at 4.09%. But, how much of this total difference is due to the different age distributions and how much due to the difference in the health systems? We made an intermediate calculation where we recompute the average death rate for South Korea, but using the population weights of Italy, which delivers 1.92%. We interpret the difference 1.92% − 1.22% = 0.7% as the pure age composition effect. This number is by itself substantial and informative about the significant risk that COVID-19 represents for an “old” country as Italy. Still, the observed fatality rate in Italy is, so far, more than 4% and, if our adjustment is correct, the additional two percentage points are not explained by the age composition. If we attribute the difference to the lack of proper medical attention, we obtain again that \( \delta \approx 2 \times \theta \). Since these two independent sources deliver consistent estimations, we calibrate our model with \( \delta = 2 \times \theta \).
### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contagion rate</td>
<td>$\lambda$</td>
<td>21%</td>
<td>27 days from 100 to 50,000 cases</td>
</tr>
<tr>
<td>Exposed to infected rate</td>
<td>$\gamma$</td>
<td>1/6</td>
<td>6 days incubation period</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>$\eta$</td>
<td>10.8%</td>
<td>9 days to recovery</td>
</tr>
<tr>
<td>Daily death rate if treated</td>
<td>$\theta$</td>
<td>0.16%</td>
<td>1.5% fatality rate if treated</td>
</tr>
<tr>
<td>Daily death rate if untreated</td>
<td>$\delta$</td>
<td>0.32%</td>
<td>3% fatality rate if untreated</td>
</tr>
<tr>
<td>Hospital capacity</td>
<td>$h$</td>
<td>0.00674</td>
<td>5,343 ICUs for 60 million population</td>
</tr>
<tr>
<td>Initial exposed</td>
<td>$e_0$</td>
<td>$10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Critical mass</td>
<td>$\varepsilon$</td>
<td>$10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Daily discount rate</td>
<td>$\rho$</td>
<td>0.05/365</td>
<td>interest rate</td>
</tr>
</tbody>
</table>

In the previous section we show that $\eta$ and $\theta$ jointly determine the average time to recovery and the average death rate when the subject is treated. The relationship is:

$$\frac{\eta}{(\eta + \theta)^2} = \text{Days}; \quad \frac{\theta}{(\eta + \theta)} = d.$$  

The solution to this system is:

$$\eta = \frac{(1 - d)^2}{Days}; \quad \theta = \frac{d}{1 - d}\eta,$$

where $\text{Days}$ is the average number of days until recovery and $d$ is the average death rate. The previous discussion hinted that the average death rate with treatment is very close to 1%, and that for Italy can be as high as 1.92% due to demographics. Yet, since it is still too early to know with reasonable certainty and given that the fatality rates tend to decrease over time, we calibrate the model to deliver an average fatality rate with proper treatment of 1.5%. Assuming that the time to recovery is 9 days, we obtain $\eta = (1 - 0.015)^2/9 = 0.1078$ and $\theta = 0.015/(1 - 0.015) \times 0.1078 = 0.16\%$. Given the previous discussion we set $\delta = 2 \times 0.16\% = 0.32\%$.

We also experiment with a death rate of 1.92% for treated patients. This implies $\theta = 0.21\%$. For consistency in this case we set $\eta = 0.1069$ and $\delta = 0.42\%$. As we show in the next section, this calibration delivers a better fit to the data.

To pin down the dynamics of the outbreak, we calibrated the initial exposed mass such that the number of days that takes to go from 100 cases to 50,000 cases is 27
days, as it happened in Italy. We achieve this number by setting $\lambda = 1/4.7$.

Regarding the hospital capacity, in our model we do not distinguish between patients who require critical care v.s. those who don’t. For this reason we need to scale the observed capacity to the number of infected. At the time of the outbreak in Italy there were 5,343 beds for intensive care and a population of 60,000,000 people. Since only $0.3 \times 4.4\% = 1.32\%$ of the infected need critical care, the country can treat no more than $5,343/0.0132$ infected individuals at a time. Thus, the country is prepared to treat only $100 \times (5,343/0.0132)/60,000,000 = 0.67\%$ of the population. For this reason, in the baseline scenario we assume that $h$ is constant and equal to 0.00674. Of course, governments are taking measures to increase this capacity. We will analyze this issue later. For instance the increased capacity in Italy after the outbreak would implied that the new $h$ is $0.0106 \approx 1.1\%$. We present all the parameter values in Table 1.

### 3.2 Simulated paths without intervention

With these parameter values we can estimate what would be the evolution of the illness, and its economic impact, if it were to spread in an unrestricted fashion, i.e., with $q_t = 0$ for all $t$. In Figures 1 and 2 we present simulated paths with two different values for $\gamma$, our baseline calibration scenario with $\gamma = 1/6$ and a faster growing scenario with $\gamma = 1/6.5$. In both figures the patterns are very similar, only the relative quantitative relevance of the virus changes. Thus, to understand the patterns we can focus on Figure 1. The economy starts with an initial mass of $0.001\%$ of exposed individuals and 0 infected. Initially the exposed move around and engaged in economic activities without necessarily knowing that they are carriers. Soon after, some “confirmed” infected start to arise, but still those numbers are very small, and definitively smaller than the number of exposed individuals. In this period, the growth rate of the infection is large, around 100\% per day, but the quantities do not seem alarming due to the still small number of affected individuals. After 15 days, the number of infected are about the same as the number of exposed. At this point, if a policy maker

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*Source: FT analysis of John Hopkins University, CSSE; Worldometers. Data updated March 22.*

*Source: President Conte’s national speech on March 24th, 2020. He also mentioned that due to the outbreak the numbers of beds increased to 8,370.*

*Note that this is a very conservative estimation, the age distribution in Italy also increases the number patients in need of critical care. However, we maintain the estimations from China to account for emergency increases in the number of beds.*
Figure 1: Potential path: calibrated incubation

![Graphs showing the potential path of the disease including infected, total cases, population, death rate, output, and quarantine over time.](image)

takes a picture of the situation, it can only see the type $i$ individuals, but the number of carriers is $2 \times i$.

Nevertheless, the number of infected cases are still small, although piling up over time. The initial slope is steep, with the growth in the number of total cases in an apparent explosive path. Matters get even worse after around 110 days when the hospital capacity is reached. Then the fatality rate that was low at the beginning starts to rise due to the affected individuals that are either untreated or badly treated. After 200 days the number of infected is at its maximum, with around 3% of the population symptomatically infected and around 1.7% have been affected but do not show symptoms yet.
At this point, the growth rate of the infected starts to decrease. The main reason for this is that the number of susceptible people reached a point, around 70% of the population with our calibration, such that the reproduction rate of the virus \( \lambda(s/n) \) is smaller than the death rate of the virus, given by \( \eta + \Delta \). After that, the virus starts to die by itself. Eventually, it disappears with 30% of the population who have developed and immunity. The cost in lives is large, without intervention 1% of the population dies, with the analogous effect on total production and consumption. In Figure\[2\] we present the same simulations but using \( \gamma = 1/6.5 \). As it is clear from there, the patterns are very similar, the only difference is that the fatality rate is higher, so that the population decreases by 1.3% and the required immune population to stop the virus is 40% rather than 30% as in Figure\[1\].

There are two important takeaways from these simulations. First, the virus needs unaffected individuals to reproduce. As the infection spreads, the number of susceptible unaffected individuals decreases. More and more meetings start to happen between exposed and already immune individuals. It is true that still some new subjects become infected, but every period less individuals are becoming infected than the people who are either recovering or dying. For our calibration, 30% of the population immune is enough to eradicate the viruses. Of course, that depends on the meeting function. With other meeting functions that could be different. This is important because it determines the feasibility of a mitigation (flattening the curve) policy. To take that approach, one must have a very good knowledge about the features of the matching function. Second, notice that there is a rebound in production after the illness reaches the peak. This happens because the recovered patients are allowed to return to their jobs. The contribution is not trivial and it must be properly considered when designing contention policies. For instance, the current quarantines do not distinguish between subjects that are already immune from those with uncertain status. If the recovered status is known to the planner, they must be allowed to work.
4 Optimal intervention

4.1 Simple quarantines

In most countries affected by COVID-19 the approach has been to impose quarantines with fixed intensity for a determined period of time. We can think about these type of policies as restricting the set of policies \( q_t \) to be a two-parameter step function such
that, for some $\tilde{q} \in [0, 1]$ and some $\tau \geq 0$, the government intervention satisfies:

$$q_t = \begin{cases} 
\tilde{q}, & \text{if } t \leq \tau \\
0, & \text{if } t > \tau.
\end{cases}$$

(5)

For instance, a complete shutdown of all economic activities for two weeks would be represented by $\tilde{q} = 1$ and $\tau = 15$. There is no reason, a priori, to believe that these type of fixed intensity policies are close to the optimum. Even if this step-function feature were close to the optimum, it is not known what would be the optimal intensity and its duration, and how the duration is related to the “feasible” intensity. In Appendix C we characterize the optimal path for $q_t$ in an unrestricted functional space. Nevertheless, in this section we compute the optimal “quarantine” duration and intensity to provide some intuition about the forces at play and to compare it with policies that have actually been implemented.

The main results can be seen in Table 2. Clearly the optimal intervention depends on the time at which it is implemented. It is harder to stop the disease when it is already spread out, say at day 60, than when it is just starting. It is also important to consider whether there is a preference for consumption smoothing. It could potentially be better to smooth out the cost of the intervention rather than concentrating it in a few days or weeks. For this reason, in Table 2 we have included different combination of scenarios. In columns (2) and (3) we present the optimal policy when implemented at day 60, which is around the time in which most countries started to intervene. In column (2) is the optimal policy when the government is only concerned about production efficiency (Linear utility), while in column (3) is the optimal policy when society prefers to smooth out the costs. Columns (4) and (5) present the same results, but considering that the government starts the intervention as soon as they found out about the illness. Finally, column (1) shows some analogous statistics for the scenario without intervention. In what follows we focused on the set of results with the estimated death rate for treated patients of 1.92%, which is in the last three rows. We also present estimations with a conservative fatality rate for treated subjects of 1.5%.

Under both fatality rate scenarios the optimal intervention is the same, shown in the second and third rows.

\footnote{See Alvarez, Argente, and Lippi (2020) for a solution in the unrestricted policy space.}
Table 2: Optimal fixed intensity quarantine

<table>
<thead>
<tr>
<th></th>
<th>No Lockdown (1)</th>
<th>Day 60 Lockdown (2)</th>
<th>Log Utility (3)</th>
<th>Day 0 Lockdown (4)</th>
<th>Log Utility (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarantine initial day</td>
<td>-</td>
<td>60</td>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Quarantine duration</td>
<td>-</td>
<td>27</td>
<td>59</td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>Quarantine intensity</td>
<td>-</td>
<td>0.99</td>
<td>0.59</td>
<td>0.98</td>
<td>0.59</td>
</tr>
<tr>
<td>Total cases</td>
<td>41%</td>
<td>0.6%</td>
<td>0.8%</td>
<td>0.01%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Conservative death rate for treated patients 1.5%

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Death rate (per person)</td>
<td>1%</td>
<td>0.01%</td>
<td>0.012%</td>
<td>0.00015%</td>
</tr>
<tr>
<td>Number of fatalities</td>
<td>600,000</td>
<td>6,000</td>
<td>7,200</td>
<td>90</td>
</tr>
<tr>
<td>Welfare improvement</td>
<td>-</td>
<td>0.7%</td>
<td>0.3%</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

Estimated death rate for treated patients 1.92%

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Death rate (per person)</td>
<td>1.3%</td>
<td>0.012%</td>
<td>0.015%</td>
<td>0.0002%</td>
</tr>
<tr>
<td>Number of fatalities</td>
<td>780,000</td>
<td>7,200</td>
<td>9,000</td>
<td>120</td>
</tr>
<tr>
<td>Welfare improvement</td>
<td>-</td>
<td>0.9%</td>
<td>0.6%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

There are many interesting results worth mentioning. First, if the concern is only about total production and the virus is discovered early on, it is optimal to implement a complete lockdown for roughly two weeks (exactly 12 days as shown in column (4)), which is very close to the initial recommendations by experts in epidemiology and the announced initial time of quarantine by most governments. This policy is extremely effective in reducing the number of total cases from 41% to 0.01% of the population, reducing also the total fatalities from 1% to 0.0002% of the population. However, comparing with column (5), when there is a desire for cost smoothing, the optimal policy reduces the intensity by about a half and it duplicates the length of the quarantine. This “mild” quarantine last for a month rather than two weeks. The smoothed policy has a cost in terms of lives though, the fatality rate is 0.00038% rather than 0.0002%, which implies 90% more fatalities. To put it in a context, with the population of Italy, the number of deaths without intervention would be around 780,000 people, with the optimal linear utility intervention is only 120 individuals and with the optimal smooth intervention is 230 individuals. The last row shows the welfare gains in terms of consumption equivalent, ranging from 1.0% to 1.2%.

The situation is substantially worse when the intervention happens late. If the intervention is implemented in day 60, as shown in columns (2) and (3), the cost of the
illness and the extent of the intervention grow exponentially. Now with linear utility
the optimal intervention is a complete shutdown of all economic activity (including
essentials) for a little less than a month. Still the reduction in fatalities is consider-
able, it falls from 1% to 0.012%, which is a sizeable effect. In numbers of people, it
is a drop from 780,000 to 7,200 fatalities. But the increase in fatalities compared to
the early intervention is strickenly high, from 120 to 7,200 individuals. If instead the
policy maker opts for a smoother intervention, column (3), the lockdown is milder,
with half the intensity and twice the time, 2 months rather than 27 days. This gener-
ates an increase of 20% in the fatality rate, from 0.012% to 0.015% with respect to the
complete lock-down. In terms of lives for the Italian example, this is around 1,800
more fatalities.

It is worth noting that the optimal smooth intervention resembles the preliminary
calculations by Guiso and Terlizzese (2020), who estimate that the Italian lockdown is
affecting around 47% of the economic activity. From this point of view, and given the
expectations about the duration of intervention we can argue that the Italian govern-
ment is implementing a policy that is close to the optimal. Moreover, the estimated
number of fatalities in the no intervention scenario, which ranges from 600,000 to
780,000, is in line with the calculations by the panel of experts in Walker et al. (2020),
who estimate around 645,000 fatalities for Italy without any intervention.

It is interesting that the optimal intervention is not significantly affected by the as-
sumed death rates. Here we present only two scenarios, one very conservative and
the other, estimated with preliminary data, which from many points of view can also
be considered conservative. The stability of the optimal intervention respect to the
death rates also happens when we calibrate the model with much higher, even 3 times
larger, values for it. This happens because, 1) in this baseline calibration we do not
incorporate the direct costs of fatalities. Adding it will certainly increase the extend
and intensity of the interventions. 2) Under the optimal intervention the path of in-
fected subjects is kept always in a downwards trend, at low levels, which represent a
relatively low rate of flow of fatalities at each instant. More intensity or duration, only
diminishes a number that is already low and close to its lower bound. 3) The initial
mass of exposed individual is also important. Since a time 60 the mass is still man-
 ageable and easy to locate below the hospital capacity, the death rate becomes less

As of March 26th, 2020 the number of fatalities respect to confirmed cases in Italy is above 8%.
Twice the number we assumed for the exceeded capacity scenario.
important. If we were to start the intervention at day, say 120, the death probability would play a more important role.

In panel a) of Figure 3 we plot the extent of the intervention in each scenario, which corresponds to $q_t$ in our model. The blue continuous line corresponds to outcomes without intervention, the red dashed line to optimal intervention with linear utility and the light orange dotted line to the optimal intervention with log utility. In the panel b) of Figure 3 we show the implied paths for the total cases. In Figure ?? we illustrate the effects on output and fatalities.
4.2 Combining testing and quarantines

The main assumption generating the necessity of an indiscriminate quarantine policy, is the inability of the policy maker to distinguish the exposed subjects from those that are susceptible, but have not been affected yet. If the government knew at each time who are the virus’ carriers, it could simply quarantine the exposed subjects and allow everyone else to work to avoid the output cost. The technology to do so is certainly available, but it could be prohibitively costly to undertake such an approach over a vast proportion of the population. However, since the immediate output cost of the quarantine appears to be also very large, it is worth evaluating how much the planner would be willing to spend on testing to reduce the cost of the quarantine.

To deal with this problem we divide the population of exposed individuals in two groups: the unidentified exposed and the exposed population that has been designated as a positive carrier of the virus. We maintain the notation $e_t$ for those subjects that carry the virus, but do not know it. These individuals are indistinguishable from those in the group $s_t$. To separate them, the government can test randomly a subset of individuals in the set $s_t + e_t$. If the test result is positive, it means that the subject carries the virus, it is identified with the new group $e^p_t$, and it is forced into mandatory quarantine, as the group $i_t$, until she fully recovers. This group of individuals maybe asymptomatic and they may remain so until they are fully recovered or develop symptoms. Now the total population is:

$$n_t = s_t + e_t + e^p_t + i_t + r_t.$$  

To understand the relevance of testing it is useful to first present the new law of motion for $e_t$. Suppose the government randomly screens $\alpha_t$ of the individuals in the group $s_t + e_t$, it can identify $\alpha_t e_t$ individuals as positive carriers. Then, the new law of motion for $e_t$ is:

$$de_t = \begin{cases} 
(\lambda s_t \frac{e_t}{n_t} (1 - q_t) - \gamma - \alpha_t) e_t dt, & \text{if } e_t > \varepsilon \\
-(\gamma + \alpha_t) e_t dt, & \text{if } e_t \leq \varepsilon. 
\end{cases} \quad (6)$$

Equation (6) shows the first positive contribution of testing to welfare. Recall that only the group $e_t$ can spread the disease, so the smaller the group, the smaller the contagion rate. Comparing (6) with (1) we can see that testing adds a downwards
drift $\alpha_t$ to the population of unidentified exposed individuals. Before the group was reducing only when the individuals were becoming actively infected at rate $\gamma$, but now some individuals are also exiting the group because some are identified, at rate $\alpha_t$, as positive carriers and, hence, cannot infect anyone else. The exposed subjects identified as positive can eventually become symptomatic and join the group of infected, which happens at the same rate $\gamma$ as the unidentified exposed; but they can also recover, in the sense that they eventually test negative even when they never show symptoms, which happens at rate $\sigma$. If this happens, they join the group of recovered and are therefore allowed to work. Thus, the law of motion for $e_t^p$ and the new law of motion for $r_t$ satisfy:

$$de_t^p = \alpha_t e_t dt - (\gamma + \sigma) e_t^p dt$$

$$dr_t = (\eta i_t + \sigma e_t^p) dt. \quad (7)$$

Comparing equation (8) to (3) we can see the second important contribution of testing. Since the recovered are immune and allowed to work, as they recover they rejoin the labor force at rate $\sigma$, which is useful in reducing the output costs of the quarantine. In short, the group of positively tested individuals generate a bulk that reduces the speed of contagion and increases the available resources to get by the quarantine times. This is especially important when the exposed may never be symptomatic. Without the testing, they would never be sick, and therefore they would always be treated as susceptible population subject to quarantines.

In this new environment the law of motion of infected subjects is slightly modified to:

$$di_t = [\gamma (e_t + e_t^p) - (\eta + \Delta_t) i_t] dt. \quad (9)$$

the only difference with the previous section is the inflow of positive exposed subjects which happens at rate $\gamma$. The population’s law of motion remains exactly the same $dn_t = -\Delta_t i_t dt$, since the infection only affects the population by the death rate; and to die a subject must show symptoms first, which only happens if they previously were part of the $i_t$ group. Finally, the production feasibility set remains the same as before, with the mass $e_t + s_t$ subject to quarantines but the $r_t$ allowed to work. We only subtract the cost of the tests.

Suppose the government test $x_t$ individuals at each instant, then the flow cost is gov-
Table 3: Optimal quarantine and testing policies

<table>
<thead>
<tr>
<th></th>
<th>No Quarantine Only</th>
<th>Quarantine Only</th>
<th>Quarantine &amp; Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lin. Utility (2)</td>
<td>Log Utility (3)</td>
<td>Lin. Utility (4)</td>
</tr>
<tr>
<td></td>
<td>Log Utility (5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarantine initial day</td>
<td>-</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Quarantine duration</td>
<td>-</td>
<td>27</td>
<td>59</td>
</tr>
<tr>
<td>Quarantine intensity</td>
<td>-</td>
<td>0.99</td>
<td>0.59</td>
</tr>
<tr>
<td>Testing frac. of unidentified</td>
<td>-</td>
<td>0.94</td>
<td>0.02</td>
</tr>
<tr>
<td>Total cases</td>
<td>41%</td>
<td>0.6%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Death rate (per person)</td>
<td>1%</td>
<td>0.01%</td>
<td>0.012%</td>
</tr>
<tr>
<td>Number of fatalities</td>
<td>600,000</td>
<td>6,000</td>
<td>7,200</td>
</tr>
<tr>
<td>Welfare improvement</td>
<td>-</td>
<td>0.7%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

The first thing to notice is that testing is used intensively, 84% of the unidentified are tested when the utility is linear and 12% when the utility is logarithmic. These numbers, even with the logarithmic utility, are far larger than the observed testing...
strategies. Also, in both cases there are welfare improvements. With the logarithmic utility the consumption equivalent gain is twice the value of the policy without testing. As expected, since testing is costly, there are important differences depending on the curvature of the utility function. When the utility is linear, so that the concern is more about productive efficiency, testing completely replaces the duration of the quarantine. Rather than doing indiscriminate and inefficient quarantines, it is optimal to stop production for one day, test unidentified subjects and resume production as soon as possible.

When smoothing is a concern, the optimal policy generates the same duration of the quarantine as when testing is not possible, but the intensity is reduced to a minimum. One can think that only activities with high contagion risk are stopped. The low
intensity is replaced with a continuous testing policy that entails to little more than 10% of the unidentified population.\textsuperscript{13}

Panels a) and b) of Figure 4 illustrate how testing can substitute for indiscriminate quarantines. When cost smoothing is not a concern, the intervention spikes in one day through an intense quarantine and large-scale mass testing. With enough exposed individuals being identified through testing, combined with a quarantine that contains the spread, the intervention makes sure that the unidentified exposed group hits the critical mass immediately and thus the contagion stops, as shown in panel c). When smoothing is desired, the intervention is still long lasting. However, testing reduces the intensity to a minimum level.

5 Conclusions

In this paper we have extended the standard epidemiologic SIR model allowing for asymptomatic subjects to be tested and consider the trade-off with output losses. We show that if the government have not means to identify the carriers of the virus, the observed mandatory quarantines around the world seem to be close to what it can be considered optimal.

However, if the government can increase the intensity of testing over subjects, that is a far superior strategy. We acknowledge that ultimately this statement depends on the cost of actually performing those tests. The results of this paper indicate that carefully analyzing and assessing this possibility should be a priority.

\textsuperscript{13}We want to emphasize that the percentage is with respect to the unidentified $s_t + e_t$, not with respect to the entire population. So that the number of tests is continuously decreasing over time.
References


Appendix

A Model calculations

Mapping to data moments: If untreated, the density function for dying after $s$ units of time is

$$f^u(s) = \delta e^{-(\eta+\delta)s}$$

The death rate is

$$\int_0^\infty f^u(s) \, ds = \int_0^\infty \delta e^{-(\eta+\delta)t} \, dt = \frac{\delta}{\eta + \delta}.$$  

The density function for recovering after $s$ units of time is

$$g^u(s) = \eta e^{-(\eta+\delta)s}.$$  

The average recovery duration is

$$\int_0^\infty g^u(s) \, s \, ds = \int_0^\infty \eta e^{-(\eta+\delta)s} \, s \, ds = \frac{\eta}{(\eta + \delta)^2}.$$  

Similarly, if treated, the death rate is $\frac{\theta}{\eta + \theta}$. The average recovery duration is $\frac{\eta}{(\eta + \theta)^2}$.

B Death rate data

Table 4: Fatality rates South Korea and Italy

<table>
<thead>
<tr>
<th>Classification</th>
<th>South Korea</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cases</td>
<td>Fatal cases</td>
</tr>
<tr>
<td></td>
<td>Number (%)</td>
<td>Number (%)</td>
</tr>
<tr>
<td>All</td>
<td>9,137</td>
<td>100</td>
</tr>
<tr>
<td>Above 80</td>
<td>406</td>
<td>4.4</td>
</tr>
<tr>
<td>70–79</td>
<td>611</td>
<td>6.7</td>
</tr>
<tr>
<td>60–69</td>
<td>1154</td>
<td>12.6</td>
</tr>
<tr>
<td>50–59</td>
<td>1724</td>
<td>18.9</td>
</tr>
<tr>
<td>40–49</td>
<td>1246</td>
<td>13.6</td>
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<td>30–39</td>
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<td>10–19</td>
<td>475</td>
<td>5.2</td>
</tr>
<tr>
<td>0–9</td>
<td>105</td>
<td>1.2</td>
</tr>
</tbody>
</table>

C Optimal control problem

Choose the path of quarantine policies:

$$\max_{\{q: t \geq 0\}} \int_0^\infty e^{-\rho t} u(c_t) \, dt$$

28
\begin{align*}
\text{s.t. } c_t &= n_t - i_t - s_t q_t \\
\text{and } &\text{ (1), (3), and (4)}
\end{align*}

Utility options: linear, log, quadratic $c - \frac{b}{2} c^2$.

Hamiltonian:

$$
\max_{(q,t)\geq 0} \int_0^\infty e^{-\rho t} U\left(n_t - i_t - s_t q_t\right) dt
$$

subject to

\begin{align*}
ed_t &= \left[\frac{\lambda s}{n} (1 - q_t) - \gamma\right] e_t dt \\
di_t &= [\gamma e_t - (\eta + \Delta) i_t] dt \\
dr_t &= \eta i_t dt, \\
dn_t &= -\Delta i_t dt.
\end{align*}

$$
\mathcal{H}(q, e, i, r, n) = U\left(n - i - sq\right) - \phi_1 \left[\frac{\lambda s}{n} (1 - q) - \gamma\right] e - \phi_2 \left[\gamma e - (\eta + \Delta) i\right] + \phi_3 \eta i - \phi_4 \Delta i.
$$

Keep notation $c = n - i - uq$.

\begin{align*}
[q] &\quad U'(c) \leq \phi_1 \frac{\lambda e}{n} \\
[e] &\quad U'(c) q + \phi_1 \frac{\lambda e}{n} (1 - q) - \phi_1 \left[\frac{\lambda s}{n} (1 - q) - \gamma\right] - \phi_2 \gamma = \rho \phi_1 - \dot{\phi}_1 \\
i] &\quad -U'(c) (1 - q) + \phi_1 \frac{\lambda e}{n} (1 - q) + \phi_2 \left[\eta + \Delta + \frac{\partial \Delta}{\partial i} i\right] + \phi_3 \eta - \phi_4 \left(\Delta + \frac{\partial \Delta}{\partial i} i\right) = \rho \phi_2 - \dot{\phi}_2 \\
r] &\quad U'(c) q + \phi_1 \frac{\lambda e}{n} (1 - q) = \rho \phi_3 - \dot{\phi}_3 \\
n] &\quad U'(c) (1 - q) + \phi_1 \frac{\lambda e}{n} \frac{n - s}{n} (1 - q) = \rho \phi_4 - \dot{\phi}_4.
\end{align*}

This will imply that with concave utility for example log utility, since in the beginning $n = s$ and $i = 0$, $c = n (1 - q)$, the optimal $q$ is in the interior.

Steady state: $e = 0$, $q = 0$, $\phi_4 = \frac{1}{\rho} U'(c)$, $\phi_3 = 0$,

$$
-U'(c) + \phi_2 (\eta + \Delta) - \frac{1}{\rho} U'(c) \left(\Delta + \frac{\partial \Delta}{\partial i} i\right) = \rho \phi_2
$$

$$
\phi_2 = \frac{1}{\rho} \frac{\Delta + \rho}{\eta + \Delta - \rho} U'(c)
$$

$$
U'(c) - \phi_1 \left[\frac{\lambda s}{n} - \gamma\right] - \phi_2 \gamma = \rho \phi_1
$$

Initial conditions: $e(0) = e_0$, $i(0) = 0$, $r(0) = 0$, $n(0) = 1$. 

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Terminal condition: $e(T) = 0, i(0) = 0, q(T) = 0, \phi_3(T) = 0, \phi_4(T) = \frac{1}{\rho} U'(n(T))$

Variables: $e, i, r, n, q, \phi_1, \phi_2, \phi_3, \phi_4$.

Simplifying, only when the eq [q] is with equality:

\[
\begin{align*}
[q] & \quad U'(c) = \phi_1 \frac{e}{n} \\
[e] & \quad U'(c) - \phi_1 \left[ \frac{\lambda s}{n} (1 - q) - \gamma \right] - \phi_2 \gamma = \rho \phi_1 - \dot{\phi}_1 \\
[i] & \quad \phi_2 \left( \eta + \Delta + \frac{\partial \Delta}{\partial i} i \right) + \phi_3 \eta - \phi_4 \left( \Delta + \frac{\partial \Delta}{\partial i} i \right) = \rho \phi_2 - \dot{\phi}_2 \\
[r] & \quad U'(c) = \rho \phi_3 - \dot{\phi}_3 \\
[n] & \quad U'(c) \left( 1 - q \right) \left( 1 + \frac{n - s}{n} \right) = \rho \phi_4 - \dot{\phi}_4.
\end{align*}
\]