ENDOGENOUS LIQUIDITY AND CAPITAL REALLOCATION*

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Abstract

We study economies where firms acquire capital in primary markets, then, after information on idiosyncratic productivity arrives, retrade it in secondary markets. Our secondary markets incorporate bilateral trade with search, bargaining and liquidity frictions. We distinguish between full or partial sales (one firm gets all or some of the other's capital). Both exhibit interesting long- and short-run patterns in data that the model can match. Depending on monetary and credit conditions, more partial sales occur when liquidity is tight. Quantitatively, we find significant steady-state and business-cycle implications. We also investigate the impact of search, taxation, and persistence in firm-specific shocks.

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Used equipment and structures sometimes trade unbundled in that firm 1 buys a machine or building from firm 2, but firm 2 continues to exist. At other times, firm 1 buys firm 2 and thereby gets to own all of firm 2's capital. Jovanovic and Rousseau (2002).

1 Introduction

This paper studies economies where firms first acquire capital in centralized primary markets, as in standard growth theory, then, after new information on idiosyncratic productivity shocks arrives, retrade it in secondary markets. In the interest of realism and generality our decentralized markets incorporate bilateral trade with search, matching, bargaining and liquidity frictions, although as special cases each of these can be suppressed. A novel feature of the analysis is that we distinguish carefully between full sales, sometimes called acquisitions or buyouts, where in a bilateral deal the buyer gets all of the seller's capital, and partial sales, where the buyer gets only some. Under constant returns to scale, it is socially efficient for the firm with higher productivity to get all the capital, but that does not always happen in equilibrium. Our goal is to study how this depends on frictions and policies, and to analyze the macro economic implications.

We use this model to assess the long-run effect of government policy, as well as the business cycle implication. In particular, we show that financial shocks can generate the cyclicality of full sales and partial sales as in the data. Financial shocks are considered as perturbations that originate from the financial sector and that affect firm debt. Financial shocks' impact on employment and new investment during the business cycle is well known (e.g., Jermann and Quadrini (2012) and Del Negro et.al. (2017)). Less known is the impact on capital reallocation especially via decentralized markets.

The analysis begins by documenting that both full and partial sales exhibit interesting empirical patterns. At the business cycle frequency, the ratio of full sales to total capital expenditure (new investment plus reallocation) is procyclical while the ratio of partial sales to total capital expenditure is countercyclical. In the longer run, the ratio of full sales to total capital expenditure has increased while the ratio of partial sales to total capital expenditure has decreased. Given that 42% of full sales are facilitated by cash or cash-equivalent

payments in the data (Thomson Reuters M&A Database, 1971-2018), we examine the relationship between reallocation and the cost of liquidity, which as discussed below is measured by inflation. In the longer run, full sales decrease while partial sales increase with inflation, but at the business cycle frequency the pattern is reversed.

We then develop a formal model of reallocation, where the mix between full and partial sales is determined endogenously. Theory predicts that higher inflation raises the cost of liquidity, decreasing full sales and increasing partial sales, and since full sales are larger, total reallocation decreases. This is consistent with the long-run facts. Then, to get full sales to increase while partial sales decrease with inflation at the business cycle frequency, we introduce shocks to credit conditions. Easier credit increases total reallocation, increases full sales, decreases partial sales, and reduces firms' demand for money, which leads to a short-term jump in inflation. Thus, with credit shocks at business cycle frequencies, total reallocation is procyclical and moves with inflation while partial sales are countercyclical.

While the model is designed to be consistent with these observations, it also provides a framework to study various other issues. For one, it provides insights into how frictions related to search, matching and bargaining affect trade in secondary capital markets, and how they affect the economy more generally. For another, it provides insights into how fiscal policy affects primary and secondary capital markets, and allows us to solve for the optimal capital tax/subsidy, analytically as well as numerically. Additionally, while our base model assumes i.i.d. firm-specific productivity shocks, we also solve it with persistent firm-specific shocks to see how that affects firms' capital and liquidity positions. One finding is that the responses of aggregate variables to aggregate shocks do not depend much on whether the idiosyncratic shocks are i.i.d. or persistent.

As motivation for studying capital reallocation, in general, note that efficiency requires not only getting the right amount of investment over time, but getting capital into the hands of those best able to use it at any point in time. While macro economists traditionally concentrated on the former, the latter is now receiving much attention. Idiosyncratic shocks make capital flow from lower- to higher-productivity firms, in theory, and in data (Maksimovic and Phillips 2001; Andrade et al. 2001; Schoar 2002). Importantly, the ease with

2

which capital can be retraded on secondary markets affects investment in primary markets, as is the case houses, cars and many other assets (Harrison and Kreps 1978; Lagos and Zhang 2020). But the channel is subtle: a well-functioning secondary market encourages primary investment since, if firms have more capital than they need, it is relatively easy to sell in the secondary market; it also discourages primary investment because, if firms want more capital than they have, it is relatively easy to buy in the secondary market. We show precisely how the net results depend on various factors.

As additional motivation, research documents that reallocation is sizable, with purchases of used capital between 25% and 33% of total investment (Eisfeldt and Rampini 2006; Cao and Shi 2016; Dong et al. 2016; Cui 2017; Eisfeldt and Shi 2018). Indeed, these may be underestimates, since they ignore small firms and those that are not publicly traded, neglect mergers, and include purchases but not rentals. Studies also document several stylized facts: reallocation is procyclical while capital mismatch is countercyclical (Eisfeldt and Rampini 2006; Cao and Shi 2016); productivity dispersion is countercyclical (Kehrig 2015); the price of used capital is procyclical (Lanteri 2016); and the ratio of spending on used capital to total investment is procyclical (Cui 2017). As will be shown, our model is consistent with each and every one of these observations.

As for frictional reallocation, many authors argue that secondary capital markets are far from the perfectly competitive ideal (Gavazza 2010, 2011*a,b*; Kurman 2014; Ottonello 2015; Kurmann and Rabinovitz 2018; Horner 2018). Imperfections include financial constraints, difficulties in finding the right counterparty, holdup problems and asymmetric information. We somewhat downplay adverse selection (see, e.g., Li and Whited 2015 or Drenik et al. 2019) to concentrate on other issues. Our secondary market features bilateral trade and bargaining, as in search theory.[1] In addition, it features the use of assets to facilitate payments, as in monetary economics, which implies that whether buyers

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[1]In comparing models of capital with and without search, Ottonello (2015) finds the former fit the facts better and entail more interesting propagation. Horner (2018) shows vacancy rates for industrial, retail and office space resemble unemployment data, suggesting that search is as relevant for that kind of capital as it is for labor, and finds disperse rents on similar structures, inconsistent with frictionless markets. For aircraft, Pulvino (1998), Gilligan (2004) and Gavazza 2011*a,b*) find used sales are thrice new sales, while Gavazza (2011*a*) shows prices vary inversely with search time, and market thickness affects trading frequency, average utilization, utilization dispersion, average price and price dispersion. This work also emphasizes *specificity* – capital is often customized to a firm. This all suggests search and matching are relevant.

get some or all of sellers' capital depends on liquidity. While explicit modeling of this is missing from most previous studies of capital reallocation, Buera et al. (2011), Moll (2014) and others argue that financial frictions are empirically important, even if self financing mitigates the problem to some extent, and we model that explicitly.

To say more about our approach to liquidity, we use the term money but do not narrowly mean currency; it can include any asset that is widely accepted as a payment instrument, or can be converted into something that is accepted, with little cost or delay. Actual economies have a spectrum of assets with varying degrees of acceptability and rates of return, implying there must be a tradeoff between these attributes. Research on the micro foundations of monetary economics tries to analyze this tradeoff explicitly, e.g., Kiyotaki and Wright (1989) formalize it in a relatively clean environment, although one that is far too stylized for this paper, which is an exercise in macro economics.

The essence of macro is aggregation. For instance, in standard models there are just two uses of output: consumption and investment. Similarly, there are typically two uses of time: work and leisure.[2] Similarly, our benchmark model has two assets: capital; and money that is used to buy capital in the secondary market, although firms can use some credit. In reality, cash may be the most liquid asset, but there are substitutes, including at least bank deposits. To be consistent with this, we incorporate banking as in Berentsen et al. (2007), and in calibration money is defined as currency plus checkable deposits.

The key idea is that inflation represents the cost of liquidity because it lowers the return on the most liquid asset, and, in equilibrium, that lowers the return on other liquid assets, thus reducing overall liquidity and hindering reallocation. As Wallace (1980) put it: "Of course, in general [inflation] is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on [inflation]... The models under consideration here imply that the higher the [inflation rate] the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income." While our baseline model has only money and capital, Appendix C adds a liquid real assets to show explicitly how this works.

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[2] There are exceptions, like macro models with home production that have three uses for output and three uses for time, but that does not diminish the point.

The rest of he paper is organized as follows. Section 2 discusses in more detail the macro and micro evidence. Sections 3 and 4 present the model and analytic results. Sections 5 and 6 contain the quantitative exercises. Section 7 concludes. The Appendix includes details about the data and proofs.[3]

2 Evidence

We use US data from 1971 to 2018 (details are in Appendix A). Capital reallocation is from COMPUSTAT, where there is information on full and partial sales, plus total capital expenditures, for each firm. This is summed to obtain an annual aggregate series on full sales, partial sales, and capital expenditures. Reallocation is defined as full plus partial sales. We focus on the reallocation-to-expenditures ratio, called the R share, and the partial-sales-to-reallocation ratio, called the P share. The former captures the importance of reallocation in total investment, and the latter the importance of partial sales in reallocation. In the early part of the sample, the R share varies a lot, but after 1984 it stabilizes, fluctuating around 30%. Similarly, early in the sample the P share was around 70%, but it stabilizes after 1984, fluctuating around 31%.[4]

As discussed above, we entertain the possibility that liquidity plays a role and use inflation as a measure of its cost. In robustness checks we also used the nominal T-bill and AAA corporate bond rates, and the results were similar, which is not surprising since inflation and nominal yields move together. Figure 1 (all Figures are at the end) shows the R and P shares vs inflation, with different panels using the raw data, plus the trend and cyclical components after applying a band-pass filter.

In the longer run, when inflation is high, firms spend less on used capital relative to total investment – in fact spend less on both – while within reallocation there are more partial sales. Given full sales are twice partial sales, when inflation rises the fall in reallo-

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[3]On reallocation, in addition to the above-mentioned, relevant papers include Ramey and Shapiro (1998, 2001), Hsieh and Klenow (2009), Asker et al. (2014), Midrigan and Xu (2014), Cooper and Schott (2016), Ai et al. (2016), David et al. (2016), Lanteri and Gavazza (2019), David and Venkateswaran (2019), and Wright et al. (2018,2020). Papers with money and capital include Shi (1999*a,b*), Shi and Wang (2006) and Aruoba et al. (2011), not to mention classics like Sidrauski (1967) or Tobin (1965).

[4]Because the R and P shares look relatively stationary after 1984, for the calibration below we start the sample in 1984, despite starting here in 1971. The conclusions do not depend on this.

cation is mainly driven by full sales. Of course, other secular changes could have affected reallocation, and the fall in inflation may or may not have resulted from monetary policy. In any case in the longer run lower inflation is associated with more full sales and fewer partial sales. However, at business cycle frequencies, the relationships are reversed. A plausible explanation involves credit conditions: easier credit leads to more full and fewer partial sales, plus it reduces money demand, which puts upward pressure on the price level and thus raises inflation in the short run.

In light of this, we construct a proxy for credit conditions using aggregate firm debt in 2012 dollars (see Appendix A). Figure 2 shows the cyclical components of debt, investment, the R share, the P share, and output. It is evident that debt is procyclical, as is the R share, while the P share is countercyclical. Therefore, when credit conditions ease, firm debt goes up, part of which is used to fund reallocation. This explain why full sales go up and partial sales down while total reallocation goes up.

Notice reallocation needs to be more volatile than investment to get a procyclical R share. As Table 1 shows, the primary capital market (investment) and secondary market (reallocation) positively comove. Also, inflation is procyclical. These business cycle statistics are consistent with the above discussion and suggest money demand and credit conditions are important over the cycle.[5]

Table 1: Business Cycle Statisticss

	Debt	R share	P share	Investment	Inflation	Output
Debt	1	0.52(0.57)	-0.45(-0.57)	0.47(0.77)	0.18(0.46)	0.59(0.79)
R share	-	1	-0.77(-0.89)	0.62(0.73)	0.24(0.48)	0.64(0.68)
P share	-	-	1	-0.50(-0.69)	-0.14(-0.31)	-0.53(-0.68)
Investment	-	-	-	1	0.43(0.69)	0.96(0.97)
Inflation	-	-	-	-	1	0.37(0.59)
Output	-	-	-	-	-	1
Relative SD.	1.19	5.88	8.68	3.74	1.93	1

Note: The numbers in brackets are associated with NBER recession dates only.

Next, we use the disaggregated data in COMPUSTAT to present two pieces of micro evidence. First, we show money holdings have a positive effect on full buyouts and a

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[5]Notice that all the relationships are stronger if we look at recession dates only, suggesting that money demand plays a bigger role during downturns.

negative or insignificant effect on partial buyouts. (In terms of labels, we usually use full and partial sales, but when the focus is on the firm acquiring capital it seems better to say full and partial buyouts, so we do so for this part of the discussion.) Second, we examine how inflation impacts firms' money holdings.

First, we regress full buyouts on liquidity (holdings of cash or cash equivalent, the latter consisting of assets that are readily converted to cash). Two outcomes are considered. The first is a binary variable that equals 1 if a firm engages in a full buyout this year, and 0 otherwise, which captures the extensive margin. We use a linear probability model, but a logistic regression gives similar results. The second approach is to examine full buyout expenditure. For this we take logs, and focus on firms engaging in a full buyout in a given year, which captures the intensive margin. Importantly, for our purposes it does not matter whether money holdings cause full buyouts or the anticipation of full buyouts leads to more money holdings – either one means that cash facilitates reallocation.

For both outcomes, we control for variables that may affect full buyouts and money holdings, such as earnings before interest and taxes (EBIT), total assets, and the leverage ratio calculated as short-term liabilities over shareholder equity (SEQ). All the dependent variables are lagged by one period to avoid simultaneity problems. We also include year or year-industry fixed effects (FE) as controls, defined using the first two digits of SIC codes. All variables except the leverage ratio are in logs, and normalized by total assets of the firm (results are similar if we deflate by the CPI).

In Table 2, the first three columns give results on the probability of a full buyout. The first column includes only firm FE, the second includes firm and year FE and the third includes firm and year-industry FE. In all cases, money holdings have a significantly positive effect on the probability of a full buyout, with a 1% increase in cash raising the probability by around 0.00019 in levels. As the average probability of a full sale is around 0.21, this means a 1% increase in cash increases the full sale probability by close to 0.1%. EBIT has significant positive effects on full buyouts. Total assets have a positive effect and leverage ratios a negative effect. The last three columns of Table 2 report results on full buyout spending, with a 1% increase in money held leading to a 0.2% increase in spending, which is sizeable. Again EBIT has a positive effect on full buyout spending. We

also ran dynamic panel regressions to account for the possibility that full buyouts may be persistent; the results are similar, and the coefficients on lagged buyouts are small and insignificant. All of this indicates that liquidity, as measured by holdings of cash or cash equivalent, encourages full buyouts.

Table 2: Full Sales and Money Holdings

	Prob	Prob	Prob	Spending	Spending	Spending
Money Holdings	0.018***	0.019***	0.018***	0.190***	0.199***	0.200***
	(0.001)	(0.001)	(0.001)	(0.012)	(0.011)	(0.012)
EBIT	0.024***	0.027***	0.028***	0.267***	0.274***	0.264***
	(0.002)	(0.002)	(0.002)	(0.018)	(0.018)	(0.019)
Asset	0.082***	0.063***	0.066***	-0.317***	-0.384***	-0.383***
	(0.003)	(0.003)	(0.004)	(0.017)	(0.021)	(0.022)
Leverage	-0.001**	-0.001**	-0.001*	-0.005	-0.005	-0.005*
	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.003)
Firm FE	Y	Y	Y	Y	Y	Y
Year FE		Y			Y	
Year-Industry FE			Y			Y
# observations	126354	126354	126354	37175	37175	37175

Note: Robust standard errors are in brackets and are clustered at firm level.
$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Now consider partial buyouts. One downside of the COMPUSTAT data is that we cannot identify the buyer in each PPE (property, plant and equipment) purchase and hence cannot assess how buyers' cash matters. However, we can aggregate the data to the industry level (defined by the first two digits of SIC) and investigate how cash held in an industry affects partial sales. If the sales of PPE occur mostly within industry, i.e. firms mostly buy used PPE from other firms in the same industry, then evidence from an industry-level analysis should be similar to a firm-level analysis.

Table 3 shows the results. Purchases of PPE, full buyouts, money held, sales, assets are in logs and normalized by total assets. All the dependent variables are values at the end of the last period, and all regressions include industry FE. The first column reports the effect of cash on purchases of PPEs without year FEs. The coefficient on cash held is negative and significant at the 5% level. The second column adds the year FE, and the

Table 3: Money Holdings on Partial and Full Sales Normalized by Assets

	PPE	PPE	Full Sales	Full Sales
Money Holdings	-0.255**	-0.051	0.425***	0.271***
	(0.129)	(0.103)	(0.080)	(0.076)
Sales	-0.170	-0.245**	0.224***	0.375***
	(0.121)	(0.094)	(0.097)	(0.085)
Log Asset	-0.505***	-0.094	0.057	-0.224***
	(0.084)	(0.091)	(0.075)	(0.070)
Leverage	0.234*	0.134	-0.108	0.167
	(0.133)	(0.112)	(0.220)	(0.168)
Year FE		Y		Y
# observations	2662	2662	2555	2555

Note: Robust standard errors are in brackets and are clustered at industry level.
$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

coefficient is still negative but insignificant. We also include the results on full buyouts. Again cash has a significantly positive effect. The coefficient after introducing year FE is comparable to the firm-level regression. Therefore, it is plausible that capital reallocation happens mostly within an industry. Putting these pieces together, the conclusion is that money held has a significantly positive effect on full buyouts, both statistically and economically, and an insignificant effect on partial buyouts.

Next consider how liquidity costs impact money holdings, shown in Table 4. In all regressions we control for firm FE. The first column is a static panel regression. It suggests that a 1% increase in the level of inflation reduces money held by more than 3.3%. However, money holdings today may also depend on previous money holdings; to allow for that we estimate a dynamic panel model using the Arellano-Bond (1990) estimator, shown in the second column. The coefficient on past cash is about 0.24 and highly significant, while the effect of inflation is somewhat smaller but still significant both statistically and economically. In our sample, inflation decreases and money holdings increase over time, and negative coefficients on inflation may result from the trends. To partly address this, we use a Band-Pass filter to remove the trend component of inflation. The last two columns in Table 4 repeat the regressions using the cyclical component. Coefficients on

Table 4: Money Holdings and Liquidity Cost

Money Holding				
Inflation	-3.348***	-1.939***		
	(0.122)	(0.163)		
Inflation (Cycle)			-4.020***	-2.667***
			(0.268)	(0.196)
Sales	0.195***	0.083***	0.185***	0.083***
	(0.005)	(0.005)	(0.005)	(0.005)
Log Asset	-0.122***	-0.096***	-0.114***	-0.081***
	(0.002)	(0.016)	(0.002)	(0.016)
Leverage	-0.000***	-0.000	-0.000***	-0.000
	(0.000)	(0.001)	(0.000)	(0.001)
Capital Exp.	-0.138***	-0.153***	-0.145***	-0.155***
	(0.004)	(0.006)	(0.004)	(0.006)
Lag Liquidity		0.238***		0.236***
		(0.012)		(0.013)
# observations	155133	99977	150468	96509

Robust standard errors are in brackets and are clustered at firm level.
* p<0.10, ** p<0.05, *** p <0.01

inflation remain negative and highly significant; in fact the magnitude is larger.

While more could always be done, the micro and macro results presented above suffice for our purposes, i.e., to motivate the study of both types of capital reallocation, taking into account credit conditions, liquidity and its cost measured by inflation.

3 Model

A $[0, 1]$ continuum of ex ante homogeneous agents live forever in discrete time. As shown in Figure 3, at each date t they interact in two distinct markets that convene sequentially: a frictional decentralized market, or DM; and a frictionless centralized market, or CM. This alternating-market structure, adapted from Lagos and Wright (2005), is ideal for our purposes because in a stylized way the CM and DM correspond to primary and secondary capital markets. Moreover, it features an asynchronization of expenditures and receipts – reallocation occurs in the DM while profit accrues in the CM – crucial to any analysis of

money or credit. It is also tractable and flexible.[6]

In the CM agents consume a numeraire good c, supply labor hours h, produce, settle debt d and adjust their portfolios of capital k and money m. In the (preceding) DM, they meet bilaterally and potentially retrade k, where gains from trade arise due to idiosyncratic productivity shocks. Agents discount between the CM and (the next) DM using $\beta \in (0,1)$, but not between the DM and CM. Given time endowment 1, their utility over consumption and leisure is $U(c, 1-h) = u(c) + \xi(1-h)$, where $\xi > 0$ is a parameter, and $u'(c) > 0 > u''(c)$. As is well known, quasi-linear utility greatly enhances tractability in environments like this, as described in Lemma 1 and 2 below. However, following Wong (2016), one can show that the results actually hold for any monotone, concave $U(c, 1-h)$ such that $U_{11}U_{22} = U_{12}^2$.

Alternatively, as in Rocheteau et al. (2008), one can show that the results hold for any monotone and concave $U(c, 1-h)$ if labor is indivisible, $h \in \{0, 1\}$, and agents trade using employment lotteries as in Rogerson (1988). This is relevant when we compare our results on business cycles to those from other models, including the one in Hansen (1985). We consider Hansen (1985) the textbook model in RBC (real business cycle) theory, because it jettisons many of the bells and whistles in Kydland and Prescott (1982) without a big impact on results, then improves on its performance by adding indivisible labor. A special case of our environment, with no idiosyncratic productivity shocks, is exactly Hansen's (1985) indivisible-labor model.[7]

We interpret agents as owning their own firms (they can own shares in many firms, but given Lemma 1 there is no demand for that kind of diversification). These firms have production functions satisfying CRS (constant returns to scale),

$$f(k, h) = (A\varepsilon k)^{1-\eta} h^\eta,$$

where k is capital, h is labor, ε is an idiosyncratic productivity factor and A is aggregate

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[6]By flexible we mean the environment accommodates many different specifications for search, price determination, etc. See Lagos et al. (2017) for a survey of papers using it in a variety of applications.

[7]One potential virtue of assuming indivisible labor and lotteries is that the model generates unemployment, not just leisure, but we do not dwell on that except for a brief digression in fn. 19. Note also that we consider Hansen (1985) the textbook model, as opposed to the monetary version in Cooley and Hansen (1989), because in that version money is used by household to buy consumption while here it is used by firms for capital reallocation. Therefore, if our idiosyncratic shocks are shut down, as a special case, the formulation reduces to Hansen and not Cooley-Hansen.

productivity. As in standard growth theory, investment in capital at t becomes productive at $t+1$, and we assume it is productive whether or not it gets traded in the DM at $t+1$. Aggregate uncertainty comes from A, but it is assumed constant until Section 5. The firm-specific ε is drawn from a time-invariant distribution $F(\varepsilon)$. While the cross section of ε is constant, for a firm it can be persistent: using subscript $+$ to indicate next period, ε_+ is drawn from conditional distribution $Q(\cdot|\varepsilon)$, realized at the start of the DM after the CM closes. These shocks generate gains from trade in the secondary capital market because ε is realized after firms choose k in the primary market, capturing the idea that it can be desirable to adjust k when new information arrives.

In the CM, a firm with (k,ε) chooses labor demand $\tilde{h}$ to maximize profit,

$$\Pi(k,\varepsilon) = \max_{\tilde{h}}\{(A\varepsilon k)^{1-\eta}\tilde{h}^\eta - w\tilde{h}\}.$$

Note Π also depends on w, but that remains implicit. to ease notation, The solution is

$$\tilde{h}(k,\varepsilon) = \left(\frac{\eta}{w}\right)^{\frac{1}{1-\eta}}A\varepsilon k, \tag{1}$$

and therefore $\Pi(k,\varepsilon) = B(w)\varepsilon k$, where

$$B(w) \equiv \left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}}(1-\eta)A. \tag{2}$$

Since profit is linear in k, socially efficient reallocation entails full sales, which is why we assume CRS (with DRS, full sales are possible but partial sales that equalize the marginal product of k across traders are more likely).

Denote the CM and DM value functions by W and V, respectively. Then

$$W(\Omega,\varepsilon) = \max_{c,h,\hat{k},\hat{z}}\left\{u(c) - \xi h + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_+(\hat{k},\hat{z})\right\}, \tag{3}$$
$$\text{st } c = \Omega + (1-\tau_h)wh - \hat{z}\phi/\phi_+ - \hat{k},$$

where Ω is wealth, k and z are capital and real money balances at the start of the CM while $\hat{k}$ and $\hat{z}$ are capital and real money balances at the end, τ_h is a labor income tax, and $\mathbb{E}_{\hat{\varepsilon}|\varepsilon}$ denotes the expectation wrt $\hat{\varepsilon}$ conditional on ε. Note that $z = \phi m$ is defined as money m times its price ϕ in terms of c, and the price of real balances next period $\hat{z}$ in terms of current c is the gross inflation rate $1 + \pi = \phi/\phi_+$. Wealth is given by

$$\Omega = (1-\tau_k)B(w)\varepsilon k + (1-\delta)k + z - d - T,$$

12

where δ is the depreciation rate, τ_k is a capital income tax, T is a lump-sum tax that could be negative, and d is debt brought in from the previous DM.

Assume an interior solution for $h \in [0,1]$ and strict concavity of V_+, which can be verified later. Then use the budget constraint to eliminate h and rewrite (3) as

$$
\begin{aligned}
W\left(\Omega, \varepsilon\right) \ = \ & \frac{\xi\Omega}{(1-\tau_h)w} + \max_c \left\{ u(c) - \frac{\xi c}{(1-\tau_h)w} \right\} \\
& + \max_{\hat{k},\hat{z}} \left\{ -\frac{\xi\left(\hat{z}\phi/\phi_+ + \hat{k}\right)}{(1-\tau_h)w} + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_+\left(\hat{k},\hat{m},\hat{\varepsilon}\right) \right\}.
\end{aligned}
\tag{4}
$$

Conveniently, the choice of $(\hat{k}, \hat{z})$ is isolated in the last term, which immediately yields these results:

Lemma 1 $W(\Omega, \varepsilon)$ *is linear in Ω with slope $\xi/[(1-\tau_h)w]$.*

Lemma 2 $(\hat{k}, \hat{z})$ *solves*

$$
-\frac{\xi}{(1-\tau_h)w} + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}\frac{\partial V_+(\hat{k},\hat{z},\hat{\varepsilon})}{\partial\hat{k}} \ \leq \ 0, \ = 0 \ \textit{if } \hat{k} > 0;
\tag{5}
$$

$$
-\frac{\xi}{(1-\tau_h)w}\frac{\phi}{\phi_+} + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}\frac{\partial V_+(\hat{k},\hat{z},\hat{\varepsilon})}{\partial\hat{z}} \ \leq \ 0, \ = 0 \ \textit{if } \hat{z} > 0.
\tag{6}
$$

This means that $(\hat{k}, \hat{z})$ is the same for all agents with the same ε, although in general agents with different ε choose different $(\hat{k}, \hat{z})$.

To complete the CM problem, let $\hat{z}(\varepsilon)$ and $\hat{k}(\varepsilon)$ solve (5)-(6), which depend on ε because $\hat{\varepsilon}$ does, and note from (4) that c solves $u'(c) = \xi/[(1-\tau_h)w]$. Then the budget constraint gives labor supply,

$$
h\left(\Omega, \varepsilon\right) = \frac{c + \hat{k}\left(\varepsilon\right) + \hat{z}\left(\varepsilon\right)\phi/\phi_+ - \Omega}{(1-\tau_h)w},
\tag{7}
$$

where dependence on w is implicit. Letting Γ be the distribution of (k, z, ε) at the start of a period, we have the law of motion

$$
\Gamma_+\left(k, z, \varepsilon\right) = \int_{\hat{k}(x)\leq k, \hat{z}(x)\leq z} Q\left(\varepsilon|x\right)dF\left(x\right).
\tag{8}
$$

13

Without aggregate shocks, while agents move around in the distribution the cross section is constant; however, that is not true with aggregate shocks. In any case, while there can be a distribution across agents leaving the CM, tractability is preserved since $\hat{k}$ and $\hat{z}$ depend only on ε (and not on past DM trades, as in models like Molico 2006 or Molico and Zhang 2006).

Now consider the DM, where with probability α each firm (owner) is randomly matched to a potential trading partner.[8] In a pairwise meeting the state variables of the pair are $\mathbf{s} = (k, z, \varepsilon)$ and $\tilde{\mathbf{s}} = (\tilde{k}, \tilde{z}, \tilde{\varepsilon})$. When $\varepsilon > \tilde{\varepsilon}$ there are gains from trade where the ε firm, called *the buyer*, gets some quantity $q(\mathbf{s}, \tilde{\mathbf{s}})$ of capital from the $\tilde{\varepsilon}$ firm, called *the seller*. Let $p(\mathbf{s}, \tilde{\mathbf{s}})$ be the value of any cash payment by the buyer, and $d(\mathbf{s}, \tilde{\mathbf{s}})$ the value of any debt, which means a promise of payment deferred to the next CM, as discussed in more detail below. Then

$$V(k, z, \varepsilon) = W(\Omega, \varepsilon) + \alpha \int_{\varepsilon > \tilde{\varepsilon}} S^b(\mathbf{s}, \tilde{\mathbf{s}}) d\Gamma(\tilde{\mathbf{s}}) + \alpha \int_{\varepsilon < \tilde{\varepsilon}} S^s(\tilde{\mathbf{s}}, \mathbf{s}) d\Gamma(\tilde{\mathbf{s}}), \qquad (9)$$

where $S^b(\cdot)$ and $S^s(\cdot)$ are buyer and seller surpluses, which by Lemma 1 are

$$S^b(\mathbf{s}, \tilde{\mathbf{s}}) = \frac{\xi \left\{ \left[(1 - \tau_k) \varepsilon B(w) + 1 - \delta \right] q(\mathbf{s}, \tilde{\mathbf{s}}) - p(\mathbf{s}, \tilde{\mathbf{s}}) - d(\mathbf{s}, \tilde{\mathbf{s}}) \right\}}{w(1 - \tau_h)}; \qquad (10)$$

$$S^s(\tilde{\mathbf{s}}, \mathbf{s}) = \frac{\xi \left\{ p(\tilde{\mathbf{s}}, \mathbf{s}) + d(\tilde{\mathbf{s}}, \mathbf{s}) - \left[(1 - \tau_k) \tilde{\varepsilon} B(w) + 1 - \delta \right] q(\tilde{\mathbf{s}}, \mathbf{s}) \right\}}{w(1 - \tau_h)}. \qquad (11)$$

We distinguish between two types of reallocation: a full sale, $q(\mathbf{s}, \tilde{\mathbf{s}}) = \tilde{k}$, which might be described as one firm taking over the other; and a partial sale, $q(\mathbf{s}, \tilde{\mathbf{s}}) \in (0, \tilde{k})$, which leaves both operating in the next CM. While full sales are socially efficient, they may not happen if buyers are limited in their ability to pay. Up-front cash payments are constrained by feasibility, $p \leq z$, while deferred payments are constrained by $d \leq D$, where in general the debt limit is

$$D = \chi_0 + \chi_\Pi \Pi + \chi_k (1 - \delta) k + \chi_q (1 - \delta) q.$$

---

[8]It may be interesting to consider competitive search, with directed rather than random matching and price posting rather than bargaining, but it would be complicated by having to solve for a cutoff, say ε^*, where firms above (below) ε^* act as buyers (sellers). Having said that, random matching is arguably nice if α is interpreted as the probability of meeting an *appropriate* counterparty – e.g., you meet someone for sure, but only with probability α do they have the kind of k you want or want the kind you have, capturing *specificity* notion mentioned in fn. 1. This is invoked in other applications of search, like labor, where you might think you can always find a firm but they may not be hiring in your area. Our model also has a matching friction in that a seller (buyer) of k would like to meet someone with the highest (lowest) ε, but we assume they meet at random (see Section 6 for a discussion of alternatives)..

The first term represents unsecured debt, which can be taken as a parameter or can be endogenized as in Kehoe and Levine (1993). The second represents debt secured by future profit, as in Holmstrom and Tirole (1998). The third and fourth represent debt secured by a existing and newly-acquired capital, as in Kiyoaki and Moore (1997).

As is standard, we call χ_Π, χ_k and χ_q pledgeability parameters and impose $\chi_j \in [0,1]$. The usual story for $\chi_j < 1$ is that only some of your assets can be pledged as collateral, since if you renege on a payment only a fraction χ_j can be seized while you abscond with the rest. An alternative approach based on private information is provided by Li et al. (2011): holding more assets than can be used to secure debt signals high quality. In any case, we set $\chi_\Pi = 0$, but the other χ's will be discussed, in various contexts, below.[9]

Next, we determine the DM terms of trade, $q(\mathbf{s},\tilde{\mathbf{s}})$, $p(\mathbf{s},\tilde{\mathbf{s}})$ and $d(\mathbf{s},\tilde{\mathbf{s}})$. Here we use Kalai's (1977) bargaining solution, with θ denoting buyers' share, which is found by solving

$$\max_{d,p,q} S^b(\mathbf{s},\tilde{\mathbf{s}}) \text{ st } (1-\theta)S^b(\mathbf{s},\tilde{\mathbf{s}}) = \theta S^s(\tilde{\mathbf{s}},\mathbf{s}), \tag{12}$$

with our additional constraints $q \leq \tilde{k}$, $p \leq z$ and $d \leq (1-\delta)(\chi_q q + \chi_k k)$.[10]

Proposition 1 *In any DM meeting $(\mathbf{s},\tilde{\mathbf{s}})$ with $\varepsilon > \tilde{\varepsilon}$, there is threshold*

$$\bar{\varepsilon}_b = \bar{\varepsilon}(\mathbf{s},\tilde{\mathbf{s}}) \equiv \frac{[z + \chi_k(1-\delta)k]/\tilde{k} - (1-\delta)(1-\chi_q)}{(1-\tau_k)(1-\theta)B(w)} - \frac{\theta\tilde{\varepsilon}}{1-\theta}, \tag{13}$$

with the following properties: (i) If $\varepsilon > \bar{\varepsilon}_b$ there is a partial sale $q = Q < \tilde{k}$, where

$$Q = Q(\mathbf{s},\tilde{\mathbf{s}}) \equiv \frac{z + (1-\delta)\chi_k k}{(1-\tau_k)B(w)[(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + (1-\delta)(1-\chi_q)}, \tag{14}$$

and payments satisfy the constraints at equality, $p = z$ and $d = (1-\delta)(\chi_q Q + \chi_k k)$. (ii) If $\varepsilon < \bar{\varepsilon}_b$ there is a full sale $q = \tilde{k}$, and the total payment is

$$p + d = \{(1-\tau_k)B(w)[(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + 1 - \delta\}\tilde{k},$$

but the mix between p and d is irrelevant as long as $p \leq z$ and $d \leq (1-\delta)(\chi_q \tilde{k} + \chi_k k)$.

---

[9]Note is that even at $\chi_q = 1$ credit secured only using $(1-\delta)q$ as collateral is not enough to generate DM trade: sellers outside option is at least $(1-\delta)q$, as they can get that before even producing. Hence buyers must have additional means of payment – cash or another line of credit – to trade.

[10]The Kalai solution emerges as the outcome of his axioms, but similar to generalized Nash, it can be characterized by a simple maximization problem. Here Kalai and generalized Nash are the same if liquidity constraints are slack, but not if they bind. Since Aruoba et al. (2007), Kalai bargaining is known to have several advantages in models of money or credit, and hence has become very popular in the literature.

Proposition 1 is proved in Appendix B and illustrated in Figure 4. The results accord well with intuition: if a buyer is flush with liquidity there is a full sale and we say *the seller stocks out*; but if liquidity is tight there is a partial sale and we say *the buyer cashes out*. Inefficient partial sales occur when the unconstrained price (i.e., the one that prevails with perfect credit) is high, which, as Figure 4 shows, means $\varepsilon - \tilde{\varepsilon}$ is big. Hence, meetings with the most social value may yield only partial sales as the buyer cashes out.

The last step before defining equilibrium is to discuss market clearing in the CM, where m, c and h are traded. By Walras' law one can be ignored, and we choose h; indeed, h does not appear in the definition below, but can always be recovered using (7). For m, let the aggregate supply M grow at rate μ, with changes engineered in the CM as follows: add the seigniorage to revenue from τ_h and τ_k, subtract government spending G, and set the lump sum T to balance the public budget each period. Then money and goods market clearing are given by

$$\phi M_+ = \int \hat{z}(\varepsilon) \, dF(\varepsilon) \text{ and } c + K_+ + G = Y + (1-\delta) K, \tag{15}$$

where $Y = B(w) \bar{K}/(1-\eta)$ is total output, $K_+ = \int \hat{k}(\varepsilon) \, dF(\varepsilon)$ is gross investment, and $\bar{K}$ is effective capital weighted by productivity after DM trade,

$$\begin{aligned} \bar{K} &= \alpha \iint_{\varepsilon > \tilde{\varepsilon}} \varepsilon \left[k + q(\mathbf{s}, \tilde{\mathbf{s}}) \right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) \\ &\quad + \alpha \iint_{\varepsilon < \tilde{\varepsilon}} \varepsilon \left[k - q(\tilde{\mathbf{s}}, \mathbf{s}) \right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) + (1-\alpha) \int \varepsilon k \, d\Gamma(\mathbf{s}). \end{aligned} \tag{16}$$

This leads to the following definition.[11]

Definition 1 *Given initial conditions for (z, k) and paths for (M, G, τ_h, τ_k) equilibrium is a list of nonnegative, bounded paths for $(c, \hat{z}, \hat{k}, q, p, d, \phi, w, \Gamma)$, where $\hat{z} = \hat{z}(\varepsilon)$ and $\hat{k} = \hat{k}(\varepsilon)$ for each agent while $q = q(\varepsilon, \tilde{\varepsilon})$, $p = p(\varepsilon, \tilde{\varepsilon})$ and $d = d(\varepsilon, \tilde{\varepsilon})$ for each pair, satisfying at every date: (i) in the CM $(c, \hat{z}, \hat{k})$ solves (3); (ii) in the DM (q, p, d) are given by Proposition 1; (iii) markets clear as defined by (15); and (iv) Γ evolves according to (8). It is a monetary equilibrium if $\hat{z} > 0$.*

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[11]When we say paths are bounded in this defintion it means the usual transversality conditions in these kinds of models, $\beta^t u'(x_t) \hat{k}_t \to 0$ and $\beta^t u'(x_t) \hat{z}_t \to 0$ (e.g., see Rocheteau and Wright 2013):

To discuss steady state, let G, τ_h, τ_k and μ be constant. If $\mu \neq 0$ then ϕ generally changes over time, but $\phi M = z$ does not if $\phi/\phi_+ = 1 + \mu$. This leads to the following definition:

Definition 2 *Steady state is a time-invariant list* $(c, z, k, q, p, d, w, \Gamma)$ *satisfying Definition 1 except for the initial conditions. It is a monetary steady state if* $z > 0$.

Note that while monetary policy is specified above in terms of μ, we can instead target inflation π or an illiquid nominal rate ι, and in steady state these are equivalent: pegging μ is the same as pegging $\pi = \mu$ or $\iota = (1 + \pi)/\beta - 1$. To be precise, define illiquid interest rates as follows: $1 + r$ is the amount of c agents require in the next CM to give up 1 unit in this CM; and $1 + \iota$ is similar except m replaces c. If there is another asset, as in Appendix C, it is equivalent to target its nominal rate; sometimes for some assets it is possible to target real rates, but not always (e.g., in steady state the illiquid real rate $1 + r = 1/\beta$ is independent of policy). In any case, we impose $\iota > 0$, but consider the limit as $\iota \to 0$, which is what Monetarists call the Friedman rule and Keynesians call the zero lower bound.

Continuing towards a characterization of the equilibrium, the next step is to derive the marginal value of capital entering the DM,

$$
\frac{\partial V}{\partial k} = \frac{\xi}{(1 - \tau_h)w} \left\{ (1 - \tau_k)B(w) \left[\varepsilon + \alpha(1 - \theta) \int\limits_{\mathcal{S}_s(\mathbf{s})} (\tilde{\varepsilon} - \varepsilon)\, d\Gamma(\tilde{\mathbf{s}}) \right. \right. \tag{17}
$$
$$
\left. \left. + \alpha\theta(1 - \delta)\chi_k \int\limits_{\mathcal{S}_b(\mathbf{s})} \frac{\varepsilon - \tilde{\varepsilon}}{\Delta(\varepsilon, \tilde{\varepsilon})}\, d\Gamma(\tilde{\mathbf{s}}) \right] + 1 - \delta \right\},
$$

where to save space $\Delta(\varepsilon, \tilde{\varepsilon})$ denotes the denominator in (14), while

$$
\mathcal{S}_s(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} > \varepsilon, \tilde{\varepsilon} < \bar{\varepsilon}(\tilde{\mathbf{s}}, \mathbf{s})\} \text{ and } \mathcal{S}_b(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} < \varepsilon, \varepsilon > \bar{\varepsilon}(\mathbf{s}, \tilde{\mathbf{s}})\} \tag{18}
$$

are the set of meetings where you sell and are constrained in capital, and the set of meetings where you buy and are constrained in cash, respectively. In words, a marginal unit of k has four potential benefits: (i) You can get its contribution to CM production, the first term in square brackets, which is ε because $(1 - \tau_k)B(w)$ outside the brackets

converts εk into disposable income. (ii) You can get its value from a DM sale, the second term in brackets, since you sell all of your k when you meet someone with $\tilde{s} \in \mathcal{S}_s(\mathbf{s})$ and enjoy a share $1 - \theta$ of the surplus. (iii) You can get its DM collateral value, captured by the third term in brackets, since you hit your liquidity constraint when you buy from someone with $\tilde{s} \in \mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of the surplus. (iv) You can get the CM resale value of $1 - \delta$ in the last term.

Similarly, derive

$$\frac{\partial V}{\partial z} = \frac{\xi}{(1 - \tau_h)w} \left[1 + (1 - \tau_k)B(w)\alpha\theta \int_{\mathcal{S}_b(\mathbf{s})} \frac{\varepsilon - \tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})} d\Gamma(\tilde{\mathbf{s}}) \right]. \tag{19}$$

In words, a marginal unit of z has two potential benefits: (i) You can get its CM purchasing power. (ii) You can get its DM purchasing power, since you hit your liquidity constraint as a buyer when you meet someone with $\tilde{s} \in \mathcal{S}_b(\mathbf{s})$ and you enjoy a share θ of the surplus.

Now combine (17)-(19) with the FOCs in Lemma 2 at equality to get

$$\frac{1}{w} = \frac{\beta(1 - \tau_k)B(w_+)}{w_+}\mathbb{E}_{\varepsilon_+|\varepsilon} \left[\varepsilon_+ + \alpha(1 - \theta)I_s + \alpha\theta(1 - \delta)\chi_k I_b \right] + \frac{\beta(1 - \delta)}{w_+} \tag{20}$$

$$\frac{Z}{w} = \frac{\beta Z_+}{w_+(1 + \mu)}\mathbb{E}_{\varepsilon_+|\varepsilon} \left[1 + (1 - \tau_k)B(w_+)\alpha\theta I_b \right], \tag{21}$$

where Z denotes aggregate real balances, and we define

$$I_s \equiv \int_{\mathcal{S}_s(\mathbf{s}_+)} (\tilde{\varepsilon}_+ - \varepsilon_+) d\Gamma_+(\tilde{\mathbf{s}}_+) \text{ and } I_b \equiv \int_{\mathcal{S}_b(\mathbf{s}_+)} \frac{\varepsilon_+ - \tilde{\varepsilon}_+}{\Delta(\varepsilon_+,\tilde{\varepsilon}_+)} d\Gamma(\tilde{\mathbf{s}}_+).$$

These Euler equations, (20)-(21), play a big role in what follows.

Before presenting more substantive results, we need to say how banks enter the picture. So far this was not mentioned, as it seemed best to first present the framework without banking, but we want it for the quantitative work below. To begin, suppose after the CM closes and before the DM opens some information is revealed that affects agents' desired liquidity. Then, as in Berentsen et al. (2007), banks can help reallocate $\hat{z}$. We want the revealed information to be minimal, because different signals generally entail different demands for $\hat{z}$, making it harder to solve the model. Therefore it is assumed that the only revelation is whether an agent will have a meeting in the upcoming DM. Hence

there are just two types: those that will not have a DM meeting, who hold excess cash; and those that will have a meeting, who could use more.

This liquidity mismatch introduces a role for banks similar to the one in the large literature based on Diamond and Dybvig (1982).[12] However, different from a typical Diamond-Dybvig bank, here they take deposits and make loans in money, not goods. Also different from many banking models, here the infinite horizon is crucial both for having money valued in the first place and for incentives based on reputation. In particular, it is hard for agents to trade liquidity among themselves using promises of repayment in the next CM, for the same reason it is hard for them to trade capital using such promises: lack of commitment and lack of concern for reputation. However, it is reasonable to say that bankers have more concern for reputation, as they are not anonymous, so their promises to honor deposits are relatively credible, and they can have comparative advantage in collecting debts. Then in the model, as in reality, bankers can serve as intermediaries facilitating the exchange of liquidity across agents.[13]

For our purposes, the important implication is that banks prop up money demand. The nice insight in Berentsen et al. (2007) is that banking makes it less costly to hold money, because if you find yourself with an abundance you can put/keep it in the bank, at interest financed by loans to those who want more. Thus banking reduces the cost of getting stuck with idle cash balances. Moreover, the only change in the equilibrium conditions is that α no longer appears in (21). To see the equations explicitly one can look at He et al. (2015) for a related application. Here we think the intuition should suffice: by lending $\hat{z}$ when you will have no meeting in the upcoming DM, you get the same marginal benefit as those that will have a meeting. Thus α drops out of the Euler equation for $\hat{z}$, and that means monetary equilibrium exists for a bigger range of parameters. While we could just set $\alpha = 1$ and similarly prop up money demand, that has other implications (e.g., for DM

---

[12]One way to envision this is to have agents making trips to the bank between the CM and DM. However, we do not want them to trade capital at the bank, as that is supposed to happen in the DM. To that end we invoke "sequential service" as discussed in the context of Diamond-Dybvig by Wallace (1988). Then agents with k never meet each other, so they cannot trade with each other, but bankers can still issue (lend) them liquid claims to each others' deposits.

[13]We think this captures in spirit the role of banking in, e.g., both Gertler and Kiyotaki (2015) and Gu et al. (2013), even if the former are more concerned with macro applications and the latter with micro foundations. While it would be interesting to flesh out in more detail how banking works, and perhaps derive additional implications, that would take us too far afield for this project.

trading volume) and hence that is not equivalent. In fact, in the calibration below we find α far less than 1 is necessary to match the facts.

4   More Analytic Results

To develop a few more results and insights, suppose here that ε is i.i.d. This simplifies the analysis because then $\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_{+}(\hat{k},\hat{z})$ is independent of ε, so everyone has the same $(\hat{k},\hat{z}) = (K_{+}, Z_{+})$ leaving the CM. To see what this implies, first, define

$$L \equiv \frac{(Z + \chi_0)/K - (1 - \delta)\left(1 - \chi_q - \chi_k\right)}{(1 - \tau_k)B\left(w\right)}, \tag{22}$$

a normalized notion of liquidity determining when the constraint binds. Also, we now include unsecured credit, χ_0. Using L and abusing notation slightly, write

$$\mathcal{S}_b\left(L\right) \equiv \left\{(\varepsilon, \tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\ \varepsilon > \frac{L - \theta\tilde{\varepsilon}}{1 - \theta}\right\} \text{ and } \mathcal{S}_s\left(L\right) \equiv \left\{(\varepsilon, \tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\ \tilde{\varepsilon} < \frac{L - \theta\varepsilon}{1 - \theta}\right\}$$

for the sets of meetings where partial and full sales occur, as defined in (18), except here they are functions of L. Also use L to write the effective capital stock defined in (16) as $\bar{K} = J\left(L, w\right)K$, with

$$J\left(L, w\right) \equiv \mathbb{E}\varepsilon + \alpha I_s\left(L\right) + \alpha\left[(1 - \tau_k)B\left(w\right)L + (1 - \delta)\left(1 - \chi_q\right)\right]I_b\left(L\right),$$

where, with another slight abuse of notation,

$$I_b\left(L\right) = \iint\limits_{\mathcal{S}_b(L)} \frac{\varepsilon - \tilde{\varepsilon}}{\Delta\left(\varepsilon, \tilde{\varepsilon}\right)}dF(\varepsilon)dF(\tilde{\varepsilon}) \text{ and } I_s\left(L\right) = \iint\limits_{\mathcal{S}_s(L)} \left(\tilde{\varepsilon} - \varepsilon\right)dF(\varepsilon)dF(\tilde{\varepsilon}).$$

Given the above notation, in steady state the Euler equations become

$$r + \delta = \left[\mathbb{E}\varepsilon + \alpha\left(1 - \theta\right)I_s\left(L\right) + (1 - \delta)\chi_k\iota\right]B\left(w\right)\left(1 - \tau_k\right) \tag{23}$$

$$\iota = \alpha\theta \iint\limits_{\mathcal{S}_b(L)} \frac{\left(\varepsilon - \tilde{\varepsilon}\right)dF(\tilde{\varepsilon})dF\left(\varepsilon\right)}{(1 - \theta)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)B(w)}}, \tag{24}$$

where r and ι are the illiquid real and nominal rates defined earlier. Also, goods market clearing becomes

$$u'^{-1}\left[\frac{\xi}{(1 - \tau_h)w}\right] + G = \left[\frac{B\left(w\right)J\left(L, w\right)}{1 - \eta} - \delta\right]K. \tag{25}$$

Notice (23)-(25) constitute three equations in (K, Z, w), while if ε were not i.i.d., K and Z would be functions of ε, not just numbers.

For what it's worth, (23) and (24) can legitimately be called the IS and LM curves, setting the demand for Investment to the supply of Savings, and the demand for Liquidity to the (real) supply of Money. Both depend on credit conditions as captured by the χ's, fiscal policy as captured by G and the τ's, and monetary policy as captured by ι, π or μ. While the framework is evidently different from the IS-LM model taught in some undergrad macro classes, it can be used the same way to make predictions by shifting curves. The only complication is that wages here are endogenous, but in principle one can solve (25) for w and insert it into (23)-(24) to get two equations in (K, Z).[14]

In practice, it is better to regard (23)-(24) as two equations in (L, B), independent of other variables, then, given (L, B), we get Z/K and w from (12) and (22), and K from (25). In Figure 5 an intersection of (23)-(24) in (L, B) space is a monetary steady state.

Proposition 2 *If θ is not too small, while χ_0 and χ_k are not too big, there exists a unique monetary steady state if and only if $\iota < \bar{\iota}$, where $\bar{\iota} > 0$.*

It is no surprise how the results in Proposition 2 depend on parameters: as is known from related work, under Kalai bargaining existence of monetary equilibrium requires θ above and ι below thresholds. Naturally it also it requires χ_0 and χ_k below thresholds because too-easy credit makes money irrelevant. We need no restrictions on χ_q, however, other than the maintained $\chi_q \leq 1$.

Table 5 shows the effects of policy and credit conditions on (K, Z, w), output Y and a new variable Φ defined as the probability of a full sale conditional on a meeting. Here $\pm$ means the result can go either way, and the nonmonotonicity of different variables as functions of ι will be confirmed below in the numerical work. A perhaps surprising result is the ambiguous impact of ι on Z, although $\partial Z / \partial \iota < 0$ holds below at the calibrated parameters. Notice $\partial Z / \partial \chi_0 < 0$ is unambiguous: related to the discussion on credit conditions and money demand, higher χ_0 reduces the need for cash, putting upward

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[14]One trick that reduces the algebra is to assume $f(k, h) = h + \tilde{f}(k)$ to pin down $w = 1$; however, that it is obviously an extreme specification. Another is to set $\theta = 1$ so that (23) pins down $B(w)$, thus determining w endogenously but independently of other variables; however, that is also extreme, in the sense that varying θ has interesting positive and normative implications that we do not want to miss.

Table 5: Comparative Statics

	ι	χ_0	χ_k	χ_q	τ_k	τ_h
K	$\pm$	0	$+$	$-^*$	$-$	$-$
Z	$\pm$	$-$	$\pm$	$\pm$	$\pm$	$-$
w	$+^*$	0	$+$	$+$	$-$	0
Y	$\pm$	0	$+$	$-^*$	$-$	$-$
Φ	$-$	0	$+$	$+$	$+$	0

Note: All assume χ_q is big; $*$ also assumes θ is big.

pressure on the price level and hence measured inflation, even if it is pinned down in the long run by money growth, $\phi/\phi_+ = 1 + \mu$ (by definition of stationary equilibrium). Also related to that discussion, the effects on Φ are unambiguous: in the long run the fraction of full sales goes down with ι and up with χ_k or χ_q.

These effects can be illustrated by shifting the IS and LM curves in Figure 5, and we make use of that in proving the results in Appendix B. Consider for the sake of illustration the case $\chi_k = 0$, which is easy because it implies changes in ι shift LM but not IS. Intuitively, higher ι increases the cost of liquidity, so agents only want to hold the same Z if the benefit to reallocation is higher, which means that B must rise. As shown, LM shifts to the northwest. That leads to lower L, which implies fewer full sales, more partial sales, and less total reallocation; it also leads to higher B, which implies more profit per unit of capital. By continuity this is still valid if $\chi_k > 0$ is not too big. However, if χ_k is too big, while L is still decreasing in ι, B can be increasing or decreasing. Having $\partial k/\partial \iota > 0$ is similar to the Mundell-Tobin effect in the sense that higher ι makes agents want to shift out of Z and into K, but the reason here is that Z and K are substitutes as payment instruments when $\chi_k > 0$.

In terms of efficiency, the Appendix solves the planner problem and compares it to equilibrium with no fiscal distortions, $\tau_k = \tau_h = 0$. For q to be efficient in equilibrium we need full sales in all meetings, and if this is to happen for an arbitrary distribution of ε shocks we need $\iota \to 0$. At the same time, for K to be efficient we need sellers in the DM to reap the full benefit of their investments, which requires $\theta = 0$, but $\theta = 0$ implies monetary equilibrium cannot exist and more generally θ low implies there is underinvestment in Z. This is a two-sided holdup problem: given $\iota > 0$, a big θ is necessary to encourage money

demand; a small θ is necessary to encourage capital demand; and no $\theta \in [0, 1]$ delivers both. However, for any θ steady state is efficient if there is no monetary or labor wedge, $\iota \to 0$ and $\tau_h = 0$, and we implement a corrective subsidy on capital formation:

Proposition 3 *Efficiency is not possible at $\iota > 0$. When $\iota \to 0$ monetary steady state is efficient if $\tau_h = 0$ and $\tau_k = \tau_k^*$, where $\tau_k^* \leq 0$ with strict inequality unless $\theta = 0$, is given by*

$$\tau_k^* = 1 - \frac{\int_0^\infty \hat{\varepsilon} \, dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon} < \tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon})}{\int_0^\infty \hat{\varepsilon} \, dF(\hat{\varepsilon}) + \alpha (1 - \theta) \int_{\hat{\varepsilon} < \tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon})}. \tag{26}$$

Before moving to the quantitative work, let us consider a pure-credit setup, where pure means: (i) $\phi M = 0$ (no money); and (ii) $\chi_0 > 0$, $\chi_j = 0$ for $j \neq 0$ (no collateral). Pure credit does not mean perfect credit unless χ_0 is big, of course, since DM trade is still constrained by $d \leq \chi_0$. The capital equation (23) and market clearing (25) are similar to those seen above, but now they immediately yield two equations in (K, w). Based on this the Appendix proves the following:[15]

Proposition 4 *With pure credit, steady state exists and is unique if α is not too big. It is efficient if χ_0 is sufficiently large, $\tau_h = 0$ and $\tau_k = \tau_k^*$ as given in (26).*

5   Quantitative Results

As many of our parameters are standard in the RBC literature, we follow that approach where possible (e.g., see Gomme and Rupert 2007). In particular, while in general all the parameters are set simultaneously to hit all the targets, it is useful to discuss how each one is set to match one observation when that observation has the most obvious or important impact. Another preliminary remark is that ε is i.i.d. for now, but that is relaxed in Section 6. Also, the sample period is 1984 to 2018 for reasons given in fn. 4.

5.1   Calibration

The calibrated parameters are in Table 6. Average annual (CPI) inflation π over the period is 2.68%. For the average illiquid nominal rate, the AAA corporate bond nominal yield ι

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[15]A reason to present this result is that most papers on capital reallocation do not have money. While we consider that a deficiency, a nonmonetary version of our framework facilities comparison with that work.

over the period is 6.72%. We considered alternatives, including the T-bill yield, but settled on this for three reasons. (i) We are, after all, studying corporations. (ii) Yields on corporate bonds are somewhat high, presumably in part because they are somewhat risky, but our agents are effectively risk neutral on that dimension by Lemma 1, so the measure corresponds well to our definition of ι (the money agents require in the next CM to give up a dollar in this CM). (iii) It is generally accepted in finance that corporate bonds are less liquid than T-bills, even if this is sometimes described as less "convenient" (e.g., Krishnamurthy and Vissing-Jorgensen 2012). Given the AAA corporate bond yield and inflation, we get the real illiquid rate $1 + r = (1 + i)/(1 + \pi) = 1.0393$ and $\beta = 1/(1 + r) = 0.962$.

We made an effort to be careful with β, as it seems central to our approach, but results are not too sensitive over a reasonable range. For the other preference parameters, we use $u(c) = \log(c)$ and set the coefficient on leisure to $\xi = 2.38$ so hours worked as a fraction of discretionary time are 33%, as reliably reported in time-use surveys for the representative US household, but the results are not very sensitive to this. For technology, we normalize $A = 1$ and set labor's share to $\eta = 0.60$, which is within the range people use, although perhaps in the lower part of the range (a classic discussion by Christiano 1988 suggests that at the time one could reasonably claim labor's share is anywhere between 0.57 and 0.75). That is partially offset by having depreciation somewhat high, $\delta = 0.125$, to yield a reasonable $K/Y = 1.8$; but we also report results for $\delta = 0.1$ (see below). Then, G is 20% of steady state Y, while $\tau_k = 0.25$ and $\tau_h = 0.22$, reasonable approximations to US fiscal policy (Gomme and Rupert 2007).

Other parameters are less standard but can still be set with discipline. The idiosyncratic productivity shock distribution is assumed to be log-normal, $\log \varepsilon \sim N(\log \bar{\varepsilon}, \sigma^2)$, with $\bar{\varepsilon}$ set to normalize $\mathbb{E}\varepsilon = 1$, and $\sigma = 1.15$, in line with previous studies.[16] The DM arrival rate is set to $\alpha = 0.118$ to match an R share of 30%, as reported in Section 2, and it is clear that this is a good parameter for hitting that target. For credit frictions, we set $\chi_q = 0.836$ to match a P share of 31% and $\chi_k = 0.0652$ to match firm money holdings over output of 5%, and it is clear that these are good parameters for hitting the fraction of partial sales in

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[16]A usual procedure is to fit an AR(1) process for idiosyncratic productivity shocks, with an estimated persistence parameter 0.7 and a standard deviation of 0.25 in COMPUSTAT data. See, e.g., Imrohoroglu and Tuzel (2014).

Table 6: Calibrated Parameters and Targets

	Value	Explanation/Target		Value	Explanation/Target
ι	0.067	nominal AAA bond yield	α	0.118	R share 30% (35%)
β	0.962	real AAA bond yield	χ_q	0.836 (0.860)	P share 31%
ξ	2.38	labor hours h	χ_k	0.065 (0.057)	cash/output 5%
η	0.60	labor's share	τ_k	0.25	capital tax rate
δ	0.125 (0.10)	depreciation rate	τ_h	0.22	labor tax rate
$\bar{\varepsilon}$	-0.66	normalization	G	0.11	gov't 20% of Y
σ	1.15	COMPUSTAT	θ	0.50	symmetry

Note: numbers in parentheses are for the calibration that implies R share is 35%.

reallocation and firm cash holdings.[17]

The remaining parameter, bargaining power, is set to $\theta = 1/2$, as a benchmark, for several reasons. First, that is symmetric bargaining, which is natural with ex ante identical agents. Second, it seems conservative, as we could only do better by choosing θ to match something. Third, when we tried freeing up θ it came out as very close $1/2$. Finally, plenty of time is spent below discussing how the results vary with θ, and so it is not crucial what we take as a benchmark.

As mentioned, we also try $\delta = 0.1$. Recalibrating the other parameters, they do not change much: as Table 6 shows by the numbers in parentheses, the only ones that move up to three digits are χ_q and χ_k. Using $\delta = 0.1$ raises K/Y to 2.08, somewhat better than $\delta = 0.125$, but it turns out that it cannot deliver an R share of 30%, so we end up settling for 35%. The tradeoff is between having δ a bit big and hitting the R share, or having δ and K/Y slightly better but overshooting the R share. An argument for the former is that because we made a big deal of the R share earlier we really should get it right here. A counter argument uses the conventional wisdom that the R share is underestimated (recall the data neglect small firms, those that are not publicly traded, mergers, and rentals). While 30% may well be an underestimate, it is hard to say if 35% is too low, too high or just right. Hence, we consider both, using as a benchmark the calibration with an R share of 30%, and indicating where results change when we used 35%.

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[17]Firm cash is from FRED, Nonfinancial Corporate Business; Checkable Deposits and Currency, and dividing by nominal GNP makes the series stationary, with an average of 5%. For the record, we did not target money used in reallocation, which as mentioned above is 42% by value (Thomson Reuters M&A Database, 1971-2018), yet the model turns out to match that extremely well – a nice consistency check.

In summary, in matching the model and data, we cannot get everything perfect, but it is very close. Moreover, the bar was set high – e.g., most studies of reallocation do not try to match money demand and vice versa. It is also worth emphasizing that the targets are all first moments, except the standard deviation of ε, and they are for the most part fairly well measured.

5.2   Long-Run Results

Consider the impact of inflation on steady state. While Table 5 has analytic results on that, some can go either way, and moreover we want the magnitude, not just the sign. See Figure 6, where the x-axis is labeled ι since in steady state $1 + \iota = (1 + \pi)/\beta$ means that either π or ι can measure inflation. This picture is drawn with $\delta = 0.125$, but it is very similar with $\delta = 0.1$. Results are given for three bargaining powers: the benchmark $\theta = 1/2$, $2/3$ and $3/4$. We could set lower $\theta < 1/2$, too, and for a few issues mentioned below that is interesting, but recall low θ implies monetary equilibria exist only for low ι, if they exist at all. To be more precise, the vertical lines show the $\bar{\iota}$ from Proposition 2 at which monetary equilibria vanish.[18]

The top row in Figure 6 shows standard macro variables. They are normalized to 100 at $\alpha = 0$, which can be interpreted as their values in the textbook model (adding ε shocks has no impact on these variables when $\alpha = 0$). The first thing to note is the nonmonotonicity in K vs ι. The economic logic is straightforward: on the one hand, higher ι taxes selling k in the DM, and that decreases incentives to invest in the CM; on the other hand, higher ι taxes buying k in the DM, and that increases incentives to invest in the CM; so the net effect depends on ι as well as θ, with the latter especially important because it determines whether buyers or sellers get the lion's share of the bilateral surplus. The impact of ι on K leads to a similar impact on Y and notice two things: the ι where effects turn from negative to positive is empirically relevant; and the effects are not small.[19]

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[18]In case it is hard to read, $\theta = 0.5 \Rightarrow \bar{\iota} = 12\%$; $\theta = 2/3 \Rightarrow \bar{\iota} = 18\%$; and $\theta = 3/4 \Rightarrow \bar{\iota} = 22\%$. If so desired, we could further raise $\bar{\iota}$ by, e.g., adding a demand for money by households buying goods, or adding a motive for firms to hold cash other than reallocation.

[19]While Y and K are nonmonotone, H is increasing in ι for these parameters. As mentioned in fn. 7, in indivisible-labor models people typically call $1 - H$ unemployment. On that interpretation, we get a stable, long-run Phillips curve, exploitable by policy: unemployment falls with inflation, although not a lot. This contrasts with a related paper, Berentsen et al. (2011), where unemployment increases with ι. While the

The middle row in Figure 6 is especially important for us, as it focuses on reallocation variables. As in the data, in the long run inflation decreases the R share and increases the P share. Also, the probability of full sales in meetings goes down with ι, consistent with $\partial\Phi/\partial\iota < 0$ in Table 5, and with economic intuition: higher ι makes it more likely that buyers cash out before sellers stock out. Higher ι also raises the real DM price $(p+d)/q$, but not much. Note this graph shows the average price, because the model generates price dispersion across meetings, depending on the ε of both buyer and seller. This is consistent with empirical work showing that the law of one price is not upheld in used-capital markets (recall fn. 1), but as interesting as that may be, detailed analysis of the equilibrium price distribution is left to future work.

The last row illustrates subsidiary results. First, average productivity is decreasing because ι hinders reallocation, so more capital ends up at less productive firms. Also shown is welfare measured as usual as the percent of c an agent would give up to reduce ι to 0. Here $\iota = 0$ is optimal, but that is not always true – e.g., given other parameters, $\iota > 0$ is optimal at $\theta = 1$.[20] Also shown is cash over Y, which resembles well money demand curve by firms in the data, and while the mean was targeted, the elasticity was not. Also shown is credit over Y, credit being the non-cash part of DM payment. There is an arbitrariness to this, because in meetings where liquidity constraints are slack agents are indifferent between cash or credit, and we adopt the convention that they use cash first. A different convention may shift the curve up, but that does not alter the point that as ι rises agents substitute out of cash and into credit.

The calibrated model thus generates several insights about the long run. One is that investment and output are decreasing in ι when it is low, but increasing when it is moderately higher, while employment is increasing for all ι at calibrated parameters; hence inflation can stimulate investment, output and employment, but that does not mean it is good for welfare. Another lesson is that we can get the R and P shares to depend on ι in

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models have many differences, the key for this is that that model has no capital.

[20]A common summary statistic is the welfare cost of 10% inflation, which for our calibration is a rather big 2.92%. The number does not depend on δ but, as is well understood (e.g., Craig and Rocheteau 2008) it is senstive to θ, e.g. $\theta = 2/3$ implies 1.40% and $\theta = 1/3$ implies 3.61%. For comparison, consider models with consumers using cash to buy goods, rather than firms using it to buy capital. Roughly speaking, reduced-form models, that seem to only use Walrasian pricing, find 0.5% (e.g., Lucas 2000), while search-and-bargaining models, with θ calibrated, can get around 5.0% (e.g., Lagos and Wright 2005).

ways consistent with the long-run evidence, which was a prime goal. Given reallocation depends on ι, so does measured productivity. Another lesson concerns the impact of θ on investment, reallocation, money demand, etc., which one would miss if never considering anything but Walrasian pricing. Other insights, related to search efficiency, fiscal policy and productivity persistence, are discussed in Section 6.

5.3 Short-Run Results

The next step is to investigate how the model accounts for business cycle observations. Suppose there are shocks to aggregate productivity A, and potentially also to credit conditions as captured by χ_q, motivated by the earlier discussion and empirical findings. In general the specification is

$$
\begin{aligned}
\ln A_t &= \rho_A \ln A_{t-1} + \varsigma_{A,t} \\
\ln \chi_{q,t} - \ln \chi_q &= \rho_\chi \left(\ln \chi_{q,t-1} - \ln \chi_q \right) + \varsigma_{\chi,t},
\end{aligned}
$$

where $\varsigma_{A,t} \sim N(0, \sigma_A^2)$ and $\varsigma_{\chi,t} \sim N(0, \sigma_\chi^2)$ are i.i.d.

We set $\rho_A = \rho_\chi = 0.825$, as is standard in yearly models, roughly corresponding to 0.95 in quarterly models. Then we set $\sigma_A = 4.33\%$ and $\sigma_\chi = 20.8\%$ to match the volatility of output and the R share (filtered), and ask how well the model captures the volatility and correlation with output for other variables. Table 7 provides answers for the calibration with $\delta = 0.125$ and an R share of 30%, while Table 8 does so with $\delta = 0.1$ and an R share of 35%. The first column of numbers has standard deviations from the data; the second has standard deviations from the model with both shocks; and the third standard deviations from the model with only A shocks. The remaining are columns similarly have correlations of the variables with output.[21]

The first observation is that Tables 7 and 8 are very similar. So the business cycle results do not depend much on whether the R share is 30% or 35%, and we can focus on Table 7. The next observation is that for the standard macro variables Y, C, I and H, the model does a good job by the standards of the literature accounting for volatility and correlation with output. Indeed, with respect to these variables, it generates results

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[21]Statistics from the model and data are computed the same way, by first taking logs and filtering out lower frequencies, as is a common (if not the only) way to proceed.

very similar to the textbook model. Moreover, this is true with only A shocks, or with both shocks: although the numbers statistics are not the same in the two cases, it is hard to say which better resembles the data. This is not a big surprise or success, but a minimal requirement: the new features introduced here do not impair performance on capturing the basic business cycle facts.

Table 7: Business Cycle Statistics with $\delta = 0.125$ and Steady State R Share 30%

	SD			Corr with output		
	Data	A and χ_q	A only	Data	A and χ_q	A only
Output	1.90	1.90	1.90	1.00	1.00	1.00
Consumption	0.63	0.50	0.60	0.91	0.87	0.97
Investment	3.74	3.44	3.04	0.86	0.98	0.99
Employment	0.74	0.61	0.45	0.90	0.91	0.94
TFP	0.62	0.74	0.80	0.79	0.98	0.99
R share	5.88	5.88	2.32	0.64	0.31	-0.97
P share	8.68	10.14	0.48	-0.53	-0.56	0.13
Inflation	1.93	3.83	0.07	0.35	0.44	-0.98

Note: SD is standard deviation relative to output's, except for output itself.

Table 8: Business Cycle Statistics with $\delta = 0.10$ and Steady State R Share 35%

	SD			Corr with output		
	Data	A and χ_q	A only	Data	A and χ_q	A only
Output	1.90	1.90	1.90	1.00	1.00	1.00
Consumption	0.63	0.41	0.53	0.91	0.83	0.97
Investment	3.74	3.90	3.36	0.86	0.98	0.99
Employment	0.74	0.70	0.50	0.90	0.94	0.96
TFP	0.62	0.67	0.76	0.79	0.98	0.99
R share	5.88	5.88	2.37	0.64	0.46	-0.98
P share	8.68	12.95	0.42	-0.53	-0.67	0.10
Inflation	1.93	4.33	0.06	0.35	0.63	-0.99

Note: SD is standard deviation relative to output's, except for output itself.

Now consider reallocation dynamics. On that dimension, the model with only A shocks is way off – really a long way off. For the R share, the standard deviation is too small, and

the correlation with output is -0.97 instead of $+0.64$. For the P share, the standard deviation is far too small, only 0.48, compared to 8.68 in the data, and the correlation with output again takes the wrong sign, this time $+0.13$ instead of -0.53. The inescapable conclusion is that with only aggregate productivity shocks the model cannot capture reallocation dynamics at all well.

What about the model with both χ_q and A shocks? From the Table, it obviously does better, and by the standards of the literature it accounts for the data quite well. Of course, it matches the volatility of the R share, because σ_χ is calibrated to that; but we did not target the correlation between the R share and output, which is much better than before, with a magnitude in the ballpark and the sign no longer wrong. For the P share, the standard deviation and correlation with output in the model look quite good compared to the data, certainly far better than when there are only A shocks.

While the above results are the ones we want to stress, there are subsidiary findings. First, notice that with only A shocks inflation is counterfactually countercyclical and its volatility is too small, while with both shocks, for the correlation with output the sign is right and the for volatility the magnitude is not too bad. We do not play this up, however, because here we are less interested in inflation dynamics, especially given our simple specification for monetary policy. Second, the average DM price is procyclical (not in the Table, but the correlation with output is 0.67). Third, productivity dispersion measured by the coefficient of variation is countercyclical (the correlation with output is -0.57). Putting these results together, the model is consistent with each and every one of the stylized facts mentioned in the Introduction and stressed in the capital reallocation literature.[22]

The conclusion is this: with shocks to aggregate productivity and credit conditions, the framework does fairly well at accounting for reallocation dynamics, as well still matching standard macro facts and some other observations.

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[22]The result on the DM price is driven mainly by A shocks, while the result on productivity dispersion is driven by χ_q shocks. We also mention in passing an advantage of having two shocks stressed by Christiano and Eichenbaum (1992): it breaks the tight relationship between A and H seen in the model but not in the data. We cannot do justice to that issue here, but can report the correlation between A and H is 0.61 with both shocks, which is a step in the right direction compared to the model with only A shocks.

6   Extensions and Other Applications

We now investigate briefly the impact of search frictions, fiscal policy and persistence in the firm-specific shocks.

6.1   Effects of Search in Different Specifications

One might like to know how equilibrium depends on search efficiency measured by α. Additionally, given that a version without ε shocks is the textbook model, with no need for a DM or money, we can start from that and move to our main model gradually, by first adding ε shocks, then adding other components one at a time. In the interest of space, let us concentrate on three versions: (i) we start with search frictions in the DM, but easy credit so that liquidity constraints never bind and hence money is not valued; (ii) then we tighten credit constraints, so they bind at least sometimes, but do not yet introduce money; and (iii) then we add money to get the main model.[23]

The first finding is that in response to aggregate shocks all versions generate statistics for standard macro variables that are about the same, i.e., close to the textbook model and the data. We do not show analogs to Table 7 for each specification, because the results are similar and not too surprising, although it is still good to verify that adding different features does not reduce performance in terms of standard business cycle facts. However, in terms of reallocation, the specifications are quite different. An obvious difference is that with perfect credit there are no partial sales, making it a nonstarter for explaining the two types of reallocation. Still, we can examine case (i) on other dimensions.

The outcome is depicted in Figure 7 as α varies for different values of θ. Here we deviate from Figure 6 and show one θ above and one below the benchmark $1/2$, because we do not need high θ when we are not trying to sustain monetary equilibrium, and because lower θ is interesting in this exercise. As shown in the top row, for low θ both output and investment increase with α, while for high θ both decrease with α, and for intermediate

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[23]We also studied versions where agents in the DM trade with probability α, but for all those that trade, the highest ε firm gets the capital of all the others, which can be interpreted as having search but not matching problems. Do that with $\alpha = 1$ eliminates search and matching entirely, which makes the DM completely frictionless if one also assumes perfect credit and Walrasian pricing. In the interest of space those results are omitted, but they can be made available on request.

θ output increases while investment decreases. Related to the intuition offered for the effects of ι in the monetary economy, higher α means better DM trading opportunities, and since θ determines how buyers and sellers split the surplus it determines incentives to invest less or more in the CM. At the same time higher α always increases reallocation, which can raise Y even if it lowers K, as happens at $\theta = 1/2$.

Now consider case (ii), tighter credit but still no money. As Figure 8 shows, for these parameters credit limits are tight enough that we get only partial sales. The results also differ from the previous case in the ordering of the curves by θ: before all curves shift down as θ rises; now they all shift up as θ rises. Still, case (i) and (ii) have some features in common: productivity and welfare always go up with α; all variables move monotonically with α; and the ordering of the curves by θ does not change with α.

Now consider case (iii), the monetary economy. Looking at Figure 9 compared to 7 and 8, there is a lot more going on – e.g., monetary equilibria only exist for α above a threshold, and output can be nonmonotone, although welfare is still monotone increasing, in α. Also, there are now partial and full sales, and should be expected, for the following reasons. For monetary equilibria to exist credit has to be tight enough that without money there are not full sales in all meetings. Then money can be valued, but there must still be some partial sales because $\iota > 0$ implies buyers must be constrained with positive probability (basically, by the envelope theorem). At the same time, if ι is not too big there will also be some full sales. Hence, monetary equilibria tend to have both.

6.2 Fiscal Policy

Now consider taxation. While there are various experiments one could run, Figure 10 plots the usual slate of variables in steady state against τ_k, ranging from the calibrated $\tau_k = 0.25$ down to large negative values. Capital taxation has big effects on the standard macro variables, as is true in most models (e.g., McGrattan et al. 1997). Interestingly, τ_k hardly matters for reallocation, in the sense that the P and R shares are flat;thus, the absolute amount of reallocation changes a lot, but the R share is approximately constant as investment moves with τ_k. We do see a noticeable impact on money demand, presumably because reallocating capital is less lucrative, and hence liquidity is less important, when

the income is more heavily taxed.

In Figure 10 any impact of changing τ_k on public revenue is made up by adjusting the lump sum tax T. That is perhaps not very realistic, but still an interesting exercise. To pursue it further, recall from the theoretical analysis that (26) gives the τ_k^* that supports the first best in steady state, for any θ, when other fiscal and monetary distortions are eliminated by setting $\tau_h = 0$ and letting $\iota \to 0$. Putting numbers on it, for $\theta = (0.25, .0.5, .0.75, 1)$ the optimal subsidy (i.e., $-\tau_k^*$) in percentage terms is $(1.64, 3.33, 5.08, 6.88)$. Higher θ implies a bigger subsidy because there is a bigger holdup problem when selling capital in the DM, yet even at $\theta = 1$ it is less than 10%.

In addition to using the lump sum T to sterilize changes in τ_k, the above experiment concerns fully optimal policy, with $\tau_h = 0$ and $\iota \to 0$. Consider instead the subsidy that maximizes steady-state welfare at the calibrated values of τ_h and ι. Ignoring $\theta = 0$ (there is no monetary equilibrium at $\theta = 0$ when $\iota > 0$), for $\theta = (0.25, 0.50, 0.75)$ the optimal subsidy is $-\tau_k^* = (44.5, 48.5, 50.0)$.[24] One thing we learn from these exercises is that θ matters. Pursuing the implications further would be interesting, perhaps by solving a Ramsey tax problem. Aruoba and Chugh (2010) show that solutions to these problems change qualitatively and quantitatively when liquidity is modeled with better micro foundations, and it may be worth checking how that plays out with capital reallocation.

6.3 Persistent Shocks

The firm-specific shocks used above were i.i.d. Now, suppose ε can be decomposed into a persistent component a and a transient component ϵ:

$$\log \varepsilon = \log a + \log \epsilon.$$

Assume $a \in \{1-x, 1+x\}$, with $x \in [0, 1)$, so a is a two-state Markov process. Also assume $\log \epsilon$ is independent of $\log a$ and i.i.d. normal. The generalization to an N-state process is straightforward, in principle, but computationally more intense (and it is already intense because of numerical integration). Firms' (k, z) choices in the CM now depend on their

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[24]These numbers are much larger than the previous experiment in part because they involve steady state comparisions, while the ones above take into account dynamics.

persistent component a. Hence, we lose the degeneracy of the (k, z) distribution at the close of the CM but maintain history independence (i.e., Lemma 2 holds for each a).

We are interested in the impact of x, measuring the gap between high and low a. To this end, we keep all other parameters the same and use a switching probability $1 - p$ to get a stationary distribution where half of the firms have $a = 1 - x$ and half have $a = 1 + x$. In particular, we set $p = 0.75$ and vary x without changing average productivity, but increasing the difference between high and low a firms. Results are shown in Figure 11, where the horizontal axis is the productivity gap, $2x$. Notice that some of these charts differ from earlier experiments: since there is firm heterogeneity in the CM, we show capital, cash, and cash/output for both high and low productivity firms.

Table 9: Business Cycle Statistics with Persistent Idiosyncratic Shocks

	SD			Corr with output		
	Data	A and χ_q	A only	Data	A and χ_q	A only
Output	1.90	1.90	1.90	1.00	1.00	1.00
Consumption	0.63	0.49	0.60	0.91	0.93	0.97
Investment	3.74	3.53	3.05	0.86	0.99	0.99
Employment	0.74	0.58	0.45	0.90	0.95	0.95
TFP	0.62	0.72	0.80	0.79	0.99	0.99
R share	5.88	5.88	0.68	0.64	0.71	-0.97
P share	8.68	3.68	0.36	-0.53	-0.69	-0.04
Inflation	1.93	2.66	0.10	0.35	0.62	-0.78

Note: SD is standard deviation relative to output's, except for output itself.

Consider first the impact on aggregate variables. At the benchmark $\theta = 0.5$, as x increases, output, investment and employment all increase, capturing the option value of reallocation: because firms with low productivity can sell their k to high productivity firms, albeit in a frictional market, more dispersion is good. This is similar to the classic result in rudimentary job search theory that more more wage dispersion is good for those seeking employment. The effects are sizable: Y increases about 0.4% if we change x from 0 to 0.025, which means average productivity of the high a firms is 5% higher than low a firms. If θ is higher, macro aggregates can be nonmonotone, as higher θ reduces sellers' DM surplus and hence reduces this option value effect.

As x increases, capital goes up for high a firms and down for low a firms, naturally. As for money holdings, higher x can increase or decrease it for both high and low a firms, depending on θ. There are counteracting effects. On the one hand, low a firms hold less capital, so buyers are less likely to be constrained, which reduces money demand. On the other hand, high a firms hold more capital, making buyers more likely to be constrained, which increases money demand. If θ is low (high) the former (latter) effect dominates. In aggregate, total money demand may increase or decrease with x.

The above results concern levels. Table 9 shows how the introduction of a gap affects second moments over the cycle, where we recalibrate parameters to hit the steady-state targets and the volatilities of output and reallocation. Comparing this to Section 5.3, for the standard macro variables the statistics do not depend a lot on x. Moreover, the conclusions about reallocation continue to hold: the version with only A shocks fails to account for reallocation dynamics, while the version that also has χ_q shocks does well. Hence the main findings are robust to adding persistence in firm-specific productivity.

This is reminiscent of Rios-Rull (1996). He asks how results in models like Kydland and Prescott (1982) change when one incorporates realistic life cycles, rather than representative agents. His answer is that the responses of aggregate variables to aggregate shocks do not change much, which is convenient, because representative-agent economies are much easier to solve. But note that life-cycle economies generate more than aggregate statistics – e.g., cross sections due to age heterogeneity – so there are more predictions one could take to the data (Gomme et al. 2004).

Analogously, our model with persistent shocks generates aggregate statistics that are similar to the i.i.d. case, which is convenient, because the latter is much easier to solve. But it also generates more statistics – e.g., cross sections of money and capital. In principle, one could try to match the distribution of (k, z) across firms and study how it varies over the cycle. We looked at that, but in-depth analysis is relegated to future work mainly because it seems be important to go beyond a two-state Markov process for the persistent component. The two-state version generates a two-point distribution for (k, z) at the end of the CM, and while that spreads out endogenously after DM trade, the result tends to be bimodal. An N-state version seems promising, if computationally intense.

7 Conclusion

This paper developed a model consistent with empirical relationships related to different types of capital reallocation and inflation. Theory predicts higher inflation lowers liquidity, which decreases (increases) full (partial) sales. This captures long-run patterns in the data. With credit shocks, easier credit reduces the demand for money, increasing inflation in the short run while increasing (decreasing) full (partial) sales. This captures business-cycle patterns in the data. The framework also provides insights into how bargaining, search efficiency and fiscal policy affect reallocation and other variables. Additionally, it allows us to study how persistence in idiosyncratic shocks affects capital and liquidity positions.

The model yields analytic results on existence, uniqueness and comparative statics, and is not hard to calibrate. This suggests there may be other applications for the framework in future work. A potentially useful extension is to allow persistent firm-specific shocks to follow a more general process to study the distribution of money and capital across firms. Another is to model heterogeneity in types or vintages of capital, where partial sales may arise not only due to liquidity, but because a buyer wants some but not all types of a seller's capital. Another possibility is to consider firms selling off capital to raise cash, which does not happen here, but could be incorporated. Abstracting from such interesting ideas, we have left much scope for additional research.

Appendix A: More on Data

Financial data are from the Flow of Funds Accounts (Z1 Report of the Federal Reserve Board). We use the Coded Table released on December 8, 2016; new editions may use different coding, so one should take that into account. Total business (corporate and non-corporate) debt in nominal terms is the sum of Debt Securities (Table F.102, item 30) and Loans (Table F.102, item 34), with the GDP implicit price deflator (Table 1.1.9 in NIPA) used to put this in 2009 dollars. Aggregate consumption and investment in 2009 dollars are from Table 1.1.3 in NIPA, excluding residential investment, consumer durables, government expenditures and net exports. The AAA corporate (nominal) bond yield and (CPI) inflation are from FRED at the St. Louis Fed.

For capital reallocation, COMPUSTAT (North America) contains useful information on ownership changes of productive assets starting in 1971. We measure capital reallocation by sales of property, plant and equipment (SPPE, data item 107 with combined data code entries excluded), plus full buyouts (AQC, data item 129 with combined data code entries excluded) from 1971 to 2018. We also use capital spending (CAPX, data item 128). Since capital spending in COMPUSTAT excludes full sales, the level of capital expenditures of each firm is calculated as the sum of AQC and CAPX. Finally, we mention for the empirical work on micro data in Section 2, industries are excluded with standard industry classification (SIC) codes below 1000 (agriculture, forestry and fishing), above 9000 (public and non-classified), and between 6000 and 6500 (financial).

Appendix B: Proofs

Proof of Proposition 1. Clearly, either the constraints on p and d both bind or they are both slack. Suppose they bind, and consider solving (12) ignoring the constraint on q. The Kalai condition $(1 - \theta) S^b (\mathbf{s}, \tilde{\mathbf{s}}) = \theta S^s (\tilde{\mathbf{s}}, \mathbf{s})$ yields $q = Q$. If $Q < \tilde{k}$ then the true (i.e., constrained) solution is $q = Q$ and the constraints on p and d at equality, which is case (i). If $Q > \tilde{k}$ then the true solution is $q = \tilde{k}$ and the Kalai condition gives the total payment, which is case (ii). Finally, the threshold in (13) comes from rearranging $Q < \tilde{k}$. ∎

Proof of Proposition 2. Consider the IS curve. If $L \leq \underline{L} \equiv (1 - \theta) \varepsilon_L$, the integral in (23)

is 0, and
$$B = \overline{B} \equiv \frac{r + \delta}{(1 - \tau_k)\mathbb{E}\varepsilon + (1 - \delta)\,\chi_k \iota}.$$

The IS curve is decreasing and $B \to \widetilde{B}$ as $L \to \infty$, where

$$\widetilde{B} \equiv \frac{r + \delta}{(1 - \tau_k)\left[\mathbb{E}\varepsilon + \alpha\,(1 - \theta) \iint\limits_{\varepsilon < \tilde{\varepsilon}} (\tilde{\varepsilon} - \varepsilon)\,dF(\tilde{\varepsilon})dF\,(\varepsilon)\right]}.$$

Intuitively, if L is larger the liquidity constraint is looser and opportunities for resale are better, so firms invest in more k even if the benefit from production B is low.

Now consider LM. If $L \leq \underline{L}$, buyers are constrained in all transactions and $B = \underline{B}$ where $\underline{B}$ solves

$$\iota = \iint\limits_{\varepsilon > \tilde{\varepsilon}} \frac{\alpha B \theta (1 - \tau_k)\,(\varepsilon - \tilde{\varepsilon})}{\Delta\,(\varepsilon, \tilde{\varepsilon})} dF(\tilde{\varepsilon})dF\,(\varepsilon).$$

Notice that $\underline{B}$ increases with ι and $B = 0$ at $\iota = 0$. As L increases, buyers become less constrained. To make them willing to hold money it must be that the benefit B from reallocation is higher. Notice $B \to \infty$ as $L \to \tilde{L}$, where $\tilde{L}$ solves

$$\iota = \iint\limits_{\mathcal{S}_1(\tilde{L})} \frac{\alpha \theta\,(\varepsilon - \tilde{\varepsilon})}{(1 - \theta)\,\varepsilon + \theta\tilde{\varepsilon}} dF(\tilde{\varepsilon})dF\,(\varepsilon).$$

This implies that when a monetary steady state exists, it uniquely pins down B and L, and they uniquely determine w and Z/K. It remains to show K is unique. By the definition of $J\,(L, w)$ and (23), $J\,(L, w)\,B\,(w) \geq (r + \delta)/(1 - \tau_k) > r$. Then there is a unique $K > 0$ solving (25), finishing the uniqueness result. Existence is standard in this kind of model (e.g., see Gu and Wright 2016), so the details are omitted. ∎

Proof of Proposition 3. We solve the planner problem given the DM frictions. First note that in the CM labor should be allocated to firms according to

$$h^*(k, \varepsilon) = \left[\frac{\eta u'\,(c)}{\xi}\right]^{\frac{1}{1 - \eta}} A\varepsilon k. \tag{27}$$

Aggregating across firms gives total hours, and $h \leq 1$ is assumed slack. Also, when two firms meet in the DM the higher ε firm should get all the capital. Given these observations, consider a planner choosing a path for k to maximize utility of the representative agent, subject to an initial k_0 and resource feasibility after government takes G_t units of x. Assuming ε is i.i.d., for simplicity, $\hat{k}$ is the same for all agents in the CM.

Then the problem can be written

$$W^*(k_0) = \max_{k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \xi h_t \right] \tag{28}$$

$$\text{st } c_t = y_t + (1-\delta) k_t - G_t - k_{t+1}$$

$$y_t = (1-\alpha) \int_0^{\infty} (A\hat{\varepsilon} k_t)^{1-\eta} h^*(k_t, \hat{\varepsilon})^{\eta} dF(\hat{\varepsilon})$$

$$+ \alpha \int_{\hat{\varepsilon} > \tilde{\varepsilon}} (A\hat{\varepsilon} 2k_t)^{1-\eta} h^*(2k_t, \hat{\varepsilon})^{\eta} dF(\hat{\varepsilon}) dF(\tilde{\varepsilon})$$

where output y_t includes production by the $1-\alpha$ measure of firms that did not have a DM meeting, the α measure that had a meeting and increased k, plus the α measure that had a meeting and decreased k. Routine methods yield the Euler equation

$$r_t + \delta = (1-\alpha) \int_0^{\infty} (1-\eta) (A\hat{\varepsilon})^{1-\eta} \left[\frac{h^*(k_{t+1}, \hat{\varepsilon})}{k_{t+1}} \right]^{\eta} dF(\hat{\varepsilon}) \tag{29}$$

$$+ 2\alpha \int_{\hat{\varepsilon} > \tilde{\varepsilon}} (1-\eta) (A\hat{\varepsilon})^{1-\eta} \left[\frac{h^*(2k_{t+1}, \hat{\varepsilon})}{2k_{t+1}} \right]^{\eta} dF(\tilde{\varepsilon}) dF(\hat{\varepsilon})$$

where r_t satisfies $1 + r_t = u'(c_t) / \beta u'(c_{t+1})$.

Next, use (27) to write

$$r_t + \delta = (1-\eta) A \left[\frac{\eta u'(c_{t+1})}{\xi} \right]^{\frac{\eta}{1-\eta}} \left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon} < \tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) dF(\tilde{\varepsilon}) dF(\hat{\varepsilon}) \right]. \tag{30}$$

Recall that in equilibrium with $\iota \to 0$, transactions are efficient in the DM and the Euler equation for k_t is

$$r_t + \delta = (1-\tau_k) B(w_{t+1}) \left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon} < \tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) dF(\tilde{\varepsilon}) dF(\hat{\varepsilon}) \right],$$

where $1 + r_t = u'(c_t) / \beta u'(c_{t+1})$, $B(w) = (\eta/w)^{\frac{\eta}{1-\eta}} (1-\eta) A$ and $u'(c) = \xi / (1-\tau_h) w$. Therefore, in equilibrium

$$r_t + \delta = (1-\eta) (1-\tau_k) A \left[\frac{\eta (1-\tau_h) u'(c_{t+1})}{\xi} \right]^{\frac{\eta}{1-\eta}}$$

$$\left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon} < \tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) dF(\tilde{\varepsilon}) dF(\hat{\varepsilon}) \right].$$

Comparing this with (30), one can see that $\theta > 0$ implies agents do not fully internalizes the benefits of investment, so there is under accumulation of capital under under at $\tau_k = \tau_h = 0$. But if $\tau_h = 0$ and τ_k is given by (26), the first best is achieved. $\blacksquare$

Proof of Proposition 4. (23) defines a unique k for any $w \in (\underline{w}, \bar{w})$, where

$$B\left(\underline{w}\right) = \frac{r+\delta}{\mathbb{E}\varepsilon(1-\tau_k)}, \ B\left(\bar{w}\right) = \frac{r+\delta}{\left[\mathbb{E}\varepsilon + \alpha\left(1-\theta\right)I_s\left(\infty\right)\right]\left(1-\tau_k\right)}.$$

Suppose ε is bounded away from 0 and ∞. If $w = \underline{w}$, any sufficiently large K solves (23). If $w = \bar{w}$, any sufficiently small K solves (23). Also, (25) implies w is increasing in k. Moreover $w = 0$ if $k = 0$ and k is finite if $w = \bar{w}$. By continuity, there is a steady state. If α is not too big, the curve defined by (23) is decreasing in w, implying uniqueness. ∎

Comparative statics wrt ι: If $\theta = 1$ then

$$\frac{r+\delta}{B\left(w\right)\left(1-\tau_k\right)} = \mathbb{E}\varepsilon + \chi_k\iota; \tag{31}$$

$$\iota = \iint_{\mathcal{S}_1(L)} \frac{\alpha\left(\varepsilon - \tilde{\varepsilon}\right)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon} + (1-\delta)\frac{\left(1-\chi_q\right)}{(1-\tau_k)B}} dF(\tilde{\varepsilon})dF\left(\varepsilon\right); \tag{32}$$

$$u'^{-1}\left[\frac{\xi}{(1-\tau_h)w}\right] + G = \left[\frac{B\left(w\right)J\left(w,L\right)}{1-\eta} - \delta\right]K. \tag{33}$$

Hence $B\left(w\right)$ is uniquely determined by (31), which determines w and c. Then, L is determined by (32). If ι increases, both B and L decrease. Therefore, w increases and Φ increases if θ is not too small. Moreover, if $\chi_k = 0$ then $B\left(w\right)$ is constant and $J\left(w,L\right)$ decreases. To see this, first rewrite

$$J\left(L,w\right) \equiv \int \varepsilon dF(\varepsilon) + \alpha\iint_{\varepsilon > \tilde{\varepsilon}} \left(\varepsilon - \tilde{\varepsilon}\right)\min\left\{1, \frac{Z+\chi_0}{\Delta\left(\varepsilon,\tilde{\varepsilon}\right)K}\right\} dF(\varepsilon)dF(\tilde{\varepsilon}). \tag{34}$$

Recall $\Delta\left(\varepsilon,\tilde{\varepsilon}\right)$ involves only B and hence does not change. Therefore, $J\left(L,w\right)$ decreases if L decreases because

$$L \equiv \frac{\left(Z+\chi_0\right)/K - \left(1-\delta\right)\left(1-\chi_q\right)}{(1-\tau_k)B\left(w\right)}.$$

As a result,

$$K = \frac{u'^{-1}\left[\frac{\xi}{(1-\tau_h)w}\right] + G}{\frac{B(w)J(w,L)}{1-\eta} - \delta}$$

increases, because w does not change with ι and J decreases. Constant w implies constant c. Then, higher K implies a higher Y. Moreover, $\left(Z+\chi_0\right)/K$ decreases with ι because L decreases with ι and B is unchanged. By continuity, K and Y increase, $\left(Z+\chi_0\right)/K$ decreases with ι if θ is close to 1 and χ_k not too big.

Comparative statics wrt χ_0: As χ_0 does not affect (31)-(33), w, K, Y and L stay the same. Therefore $(Z + \chi_0)/K$ is constant. If χ_0 increases, Z decreases.

Comparative statics wrt χ_k: Higher χ_k shifts the IS curve down and does not affect the LM curve. Hence B and L decrease, so w increases. If $\chi_q = 1$ then L stays constant and B decreases. Thus w and K increase. Additionally, Y increases because both c and K increase. By continuity, the same is true if χ_q is not too small.

Comparative statics wrt χ_q: If χ_q increases LM shifts down and IS stays the same. Hence L increases and B decreases, w goes up and Φ increases. If θ is close to 1, the change in B is close to 0. As a result, w and c are almost unchanged. At the same time, $B(w) J(w, L)/(1 - \eta)$ increases because L increases, so K and Y decrease. Because L increases, so does Φ.

Comparative statics wrt τ_k: This shifts up both LM and IS, so $B(w)$ increases, L increases if χ_q close to 1, and w decreases. So Φ increases, and since χ_q is close to 1, $B(w) J(w, L)/(1 - \eta)$ increases, so K, c and Y decrease.

Comparative statics wrt τ_h: This does not change B or L, so w and Φ stay the same, while c decreases. Then K decreases, which implies Y decreases. Also, Z decreases. By continuity, this holds for large $\theta < 1$.

Appendix C: Multiple Liquid Assets

As in Lester et al. (2012), in addition to z there is a long-lived real asset a in fixed supply 1, with CM price ψ and dividend ρ. Both z and a are used for DM payments, but we allow general χ_z and χ_a, and set $\chi_q = 1$ and $\chi_k = 0$ to ease notation. Then

$$
\begin{aligned}
W(\Omega, \varepsilon) &= \max_{c, h, \hat{k}, \hat{z}} \left\{ u(c) - \xi h + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon}) \right\} \\
\text{st } c &= \Omega + (1 - \tau_h) w h - \hat{z}\phi/\phi_+ - \hat{k} - \psi \hat{a},
\end{aligned}
$$

where $\Omega = (1 - \tau_k) B(w) \varepsilon k + (1 - \delta)k + z - d - T + (\psi + \rho) a$. FOCs are

$$
\begin{aligned}
-\frac{\xi}{(1 - \tau_h)w} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{k}} &\leq 0, \ = 0 \text{ if } \hat{k} > 0 \\
-\frac{\xi}{(1 - \tau_h)w} \frac{\phi}{\phi_+} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{z}} &\leq 0, \ = 0 \text{ if } \hat{z} > 0 \\
-\frac{\xi}{(1 - \tau_h)w} \psi + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{z}} &\leq 0, \ = 0 \text{ if } \hat{a} > 0.
\end{aligned}
$$

In the DM there are three types of meetings: with probability α_1 only z is accepted; with probability α_2 only a is accepted, and with probability α_3 both are accepted. Emulating the methods in the text, we arrive at

$$\frac{r+\delta}{B(w)(1-\tau_k)} = \mathbb{E}\varepsilon + (1-\theta)\left[\alpha_1 I_s(L_1) + \alpha_2 I_s(L_2) + \alpha_3 I_s(L_3)\right].$$

$$\iota = \alpha_1 \chi_z \lambda(L_1) + \alpha_3 \chi_z \lambda(L_1 + L_2)$$

$$rZ_a = (1+r)\chi_a \rho + \beta Z_a \chi_a \left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1 + L_2)\right]$$

where $Z_a = (\rho + \psi)\chi_a$,

$$\lambda(L) \equiv \iint\limits_{\mathcal{S}_b(L)} \frac{\alpha\theta(\varepsilon - \tilde{\varepsilon})\,dF(\tilde{\varepsilon})dF(\varepsilon)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon}}, \ \ L_1 \equiv \frac{\chi_z Z}{(1-\tau_k)B(w)K}, \ \ L_2 \equiv \frac{\chi_a Z_a}{(1-\tau_k)B(w)K},$$

Here L_1 and L_2 are liquidity per unit of effective capital. This determines the fraction of unconstrained meetings and the fraction of capital traded in constrained meetings. If $\theta = 1$, B is constant and L_1 and L_2 are proportional to the per capital liquidity.

$$\iota = \chi_z\left[\alpha_1 \lambda(L_1) + \alpha_3 \lambda(L_1 + L_2)\right]$$

$$r = C + \chi_a\left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1 + L_2)\right],$$

where $C = (1+r)\chi_a\rho/(1-\tau_k)B(w)KL_2$. If ρ is close to 0 so are $\partial C/\partial L_2$ and $\partial C/\partial L_2$.

Because $\lambda' < 0$, we have

$$\frac{dL_1}{d\iota} \approx \frac{\partial C}{\partial L_2} + \chi_a\left[\alpha_2 \lambda'(L_2) + \alpha_3 \lambda'(L_1 + L_2)\right] < 0$$

$$\frac{dL_2}{d\iota} \approx \frac{\partial C}{\partial L_1} - \chi_a \alpha_3 \lambda'(L_1 + L_2) > 0.$$

Therefore, if ρ is not too big

$$\frac{d(L_1 + L_2)}{d\iota} \approx \chi_a \alpha_2 \lambda'(L_2) + \frac{dC}{dL_2} - \frac{\partial C}{\partial L_1} < 0.$$

As ι increases, liquidity per productive unit of k falls. By continuity, if $\theta < 1$ is not too small and $\rho > 0$ is not too big the same result holds. Exactly as suggested by Wallace (1980), inflation reduces total liquidity even if it directly taxes only z, and not a, due to general equilibrium effects as agents try to move out of z and into a.

References

[1] H. Ai, K. Li and F. Yang (2016) "Financial Intermediation and Capital Reallocation," *mimeo*.

[2] S. Aruoba and S. Chugh (2010) "Optimal Fiscal and Monetary Policy When Money Is Essential," *JET* 145,1618-47.

[3] S. Aruoba, G. Rocheteau and C. Waller (2007) "Bargaining and the Value of Money," *JME* 54, 2636-55.

[4] S. Aruoba, C. Waller and R. Wright (2011) "Money and Capital," *JME* 58, 98-116.

[5] J. Asker, A. Collard-Wexler and J. D. Loecker (2014) "Dynamic Inputs and Resource (Mis)allocation," *JPE*, 112, 1013-1063.

[6] A. Berentsen, G. Camera and C. Waller (2007) "Money, Credit and Banking," *JET* 135, 171-95.

[7] F. Buera, J. Kaboski and Y. Shin (2011) "Finance and Development: A Tale of Two Sectors," *AER* 101, 1964-2002.

[8] M. Cao and S. Shi (2016) "Endogenously Procyclical Liquidity, Capital Reallocation, and q," *mimeo*.

[9] W. Cui (2017) "Macroeconomic Effects of Delayed Capital Liquidation," *mimeo*.

[10] L. Christiano (1988) "Why Does Inventory Investment Fluctuate So Much? *JME* 21, 247-280.

[11] L. Christiano and M. Eichenbaum (1992) "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *AER* 82, 430-450.

[12] T. Cooley and G. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *AER* 79, 733-48.

[13] R. Cooper and I. Schott (2016) "Capital Reallocation and Aggregate Productivity," *mimeo*.

[14] B. Craig and G. Rocheteau (2008) "Inflation and Welfare: A Search Approach," *JMCB* 40, 89-119.

[15] J. David, H. Hopenhayn and V. Venkateswaran (2016) "Information, Misallocation, and Aggregate Productivity," *QJE* 131, 943-1005.

[16] J. David and V. Venkateswaran (2019) "The Sources of Capital Misallocation," *AER* 109, 2531-67.

[17] M. Del Negro, G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017) "The great escape? a quantitative evaluation of the fed's liquidity facilities, " *AER* 107 (3), 824-857.

[18] F. Dong, P. Wang and Y. Wen (2016) "A Searched-Based Framework of Capital Reallocation," *mimeo*.

[19] A. Eisfeldt and A. Rampini (2006) "Capital Reallocation and Liquidity," *JME* 53, 369-399.

[20] A. Eisfeldt and A. Rampini (2008) "Managerial Incentives, Capital Reallocation, and the Business Cycle," *JFE* 87, 177-199.

[21] A. Eisfeldt and Y. Shi (2018) "Capital Reallocation," *Annual Rev Fin Econ* 10, 361-38.

[22] A. Gavazza (2010) "Asset Liquidity and Financial Contracts: Evidence From Aircraft Leases," *JFE* 95, 62-84.

[23] A. Gavazza (2011a) "The Role of Trading Frictions in Real Asset Markets," *AER* 101, 1106-1143.

[24] A. Gavazza (2011b) "Leasing and Secondary Markets: Theory and Evidence From Commercial Aircraft," *JPE* 119, 325-377.

[25] T. Gilligan (2004) "Lemons and Leases in the Used Business Aircraft Market," *JPE* 112, 1157-1186.

[26] M. Gertler and N. Kiyotaki (2015) "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *AER* 105, 2011-43.

[27] P. Gomme and P. Rupert (2007) "Theory, Measurement and Calibration of Macroeconomic Models," *JME* 54, 460-497.

[28] P. Gomme, P. Rupert, R. Rogerson and R. Wright (2004) "The Business Cycle and the Life Cycle," *NBER Macro Annual*.

[29] C. Gu, F. Mattesini, C. Monnet and R. Wright (2013) "Banking: A New Monetarist Approach," *RES* 80, 636-662.

[30] C. Gu and R. Wright (2016) "Monetary Mechanisms," *JET* 163, 644-657.

[31] G. Hansen (1985) "Indivisible Labor and the Business Cycle," *JME* 16, 309-37.

[32] J. Harford (2005) "What Drives Merger Waves?" *JFE* 77, 529-560.

[33] J. Harrison and D. Kreps (1978) "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations." *QJE* 92, 323-36.

[34] C. He, R. Wright and Y. Zhu (2015) "Housing and Liquidity," *RED* 18, 435-55.

[35] B. Holmstrom and J. Tirole (1998) "Private and Public Supply of Liquidity," *JPE* 106, 1-40.

[36] K. Horner (2018) "Capital Unemployment with Search-in-Use," *mimeo*.

[37] C. Hsieh and P. Klenow (2009) "Misallocation and Manufacturing TFP in China and India," *QJE* 124, 1403-1448.

[38] A. Imrohoroglu and S. Tuzel (2014): "Firm-level Productivity, Risk, and Return," *Management Science* 60, 2073-2090

[39] U. Jermann and V. Quadrini. (2012): "Macroeconomic effects of financial shocks, " *AER* 102(1), 238-71.

[40] B. Jovanovic and P. Rousseau (2002) "The Q-Theory of Mergers," *AER* 92, 198-204.

[41] E. Kalai (1977) "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica* 45, 1623-30.

[42] T. Kehoe and D. Levine (1993) "Debt-Constrained Asset Markets," *RES* 60, 865-88.

[43] M. Kehrig (2015) "The Cyclical Nature of the Productivity Distribution," *mimeo*.

[44] N. Kiyotaki and J. Moore (1997) "Credit Cycles," *JPE* 105, 211-48.

[45] N. Kiyotaki and R. Wright (1989) "On Money as a Medium of Exchange," *JPE* 97, 927-54.

[46] A. Krishnamurthy and A Vissing-Jorgensen (2012) "The Aggregate Demand for Treasury Debt," *JPE* 120, 233-67.

[47] A. Kurmann (2014) "Holdups and Overinvestment in Capital Markets," *JET* 151, 88-113.

[48] A. Kurmann and S. Rabinovich (2018) "Dynamic Inefficiency in Decentralized Capital Markets," *JET* 173, 231-256.

[49] R. Lagos, G. Rocheteau and R. Wright (2017) "Liquidity: A new monetarist perspective," *JEL* 55, 371-440.

[50] R. Lagos and R. Wright (2005) "A Unified Framework for Monetary Theory and Policy Analysis," *JPE* 113, 463-84.

[51] R. Lagos and S. Zhang (2020) "Turnover Liquidity and the Transmission of Monetary Policy," *AER* 110, 1635-1672.

[52] A. Lanteri (2016) "The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle," *mimeo*.

[53] A. Lanteri and A. Gavazza (2019) "Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets," *mimeo*.

[54] B. Lester, A. Postlewaite and R. Wright (2012) "Information, Liquidity, Asset Prices, and Monetary Policy," *RES* 79, 1209-38.

[55] S. Li and T. Whited (2015) "Capital Reallocation and Adverse Selection," Simon Business School WP FR 14-16.

[56] Y. Li, G. Rocheteau and P. Weill (2012) "Liquidity and the Threat of Fraudulent Assets," *JPE* 120, 815-46.

[57] R. Lucas (2000) "Inflation and Welfare." *Econometrica* 68, 247-74.

[58] E. McGrattan, R. Rogerson and R. Wright (1997) "An Equilibrium Model of the Business Cycle, with Home Production and Fiscal Policy," *IER* 38, 267-290.

[59] V. Midrigan and D. Xu (2014) "Finance and Misallocation: Evidence from Plant-Level Data," *AER* 104, 422-58.

[60] M. Molico (2006) "The Distribution of Money and Prices in Search Equilibrium," *IER* 47, 701-22.

[61] M. Molico and Y. Zhang (2006) "Monetary Policy and the Distribution of Money and Capital," *Computing in Economics and Finance* 136, 1-31.

[62] B. Moll (2014) "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *AER* 104, 3186-3221.

[63] P. Ottonello (2015) "Capital Unemployment," *mimeo*.

[64] T. Pulvino (1998) "Do Asset Fire Sales Exist? An Empirical Investigation of Commercial Aircraft Transactions," *JF* 53, 939-978.

[65] V. Ramey and M. Shapiro (1998) "Costly Capital Reallocation and the Effects of Government Spending," *Carnegie-Rochester Conference Series* 48, 145-194.

[66] V. Ramey and M. Shapiro (2001) "Displaced Capital: A Study of Aerospace Plant Closings," *JPE* 109, 958-992.

[67] V. Rios-Rull (1996) "Life-Cycle Economies and Aggregate Fluctuations," *RES* 63, 465-490.

[68] G. Rocheteau, P. Rupert, K. Shell and R. Wright (2008) "General Equilibrium with Nonconvexities and Money," *JET* 142, 294-317.

[69] G. Rocheteau and R. Wright (2013) "Liquidity and Asset Market Dynamics," *JME* 60, 275-94.

[70] G. Rocheteau, R. Wright and S. Xiao (2018) "Open Market Operations," *JME*, in press.

[71] R. Rogerson (1988) "Indivisible Labor, Lotteries and Equilibrium." *JME* 21, 3-16.

[72] S. Shi (1999a) "Search, Inflation and Capital Accumulation," *JME* 44, 81-103.

[73] S. Shi (1999b) "Money, Capital, and Redistributive Effects of Monetary Policies," *JEDC* 23, 565-590.

[74] S. Shi and W. Wang (2006) "The Variability of the Velocity of Money in a Search Model," *JME* 53, 537-571.

[75] M. Sidrauski (1967) "Inflation and Economic Growth," *JPE* 75, 796-810.

[76] J. Tobin (1965) "Money and Economic Growth," *Econometrica* 33, 671-684.

[77] N. Wallace (1980) "The Overlapping Generations Model of Fiat Money." In *Models of Monetary Economies*, edited by J. Kareken and N. Wallace. FRB Minneapolis.

[78] N. Wallace (1988) "Another Attempt to Explain an Illiquid Banking System: The Diamond and Dybvig Model With Sequential Service Taken Seriously," FRB Minneapolis *Quarterly Review*.

[79] T. Wong (2016) "A Tractable Monetary Model under General Preferences," *RES* 83, 402-420.

[80] R. Wright, S. Xiao and Y. Zhu (2018) "Frictional Capital Reallocation I: Ex Ante Heterogeneity," *JEDC* 89, 100-116.

[81] R. Wright, S. Xiao and Y. Zhu (2020) "Frictional Capital Reallocation with Ex Post Heterogeneity," *RED* 37, S227-S253.

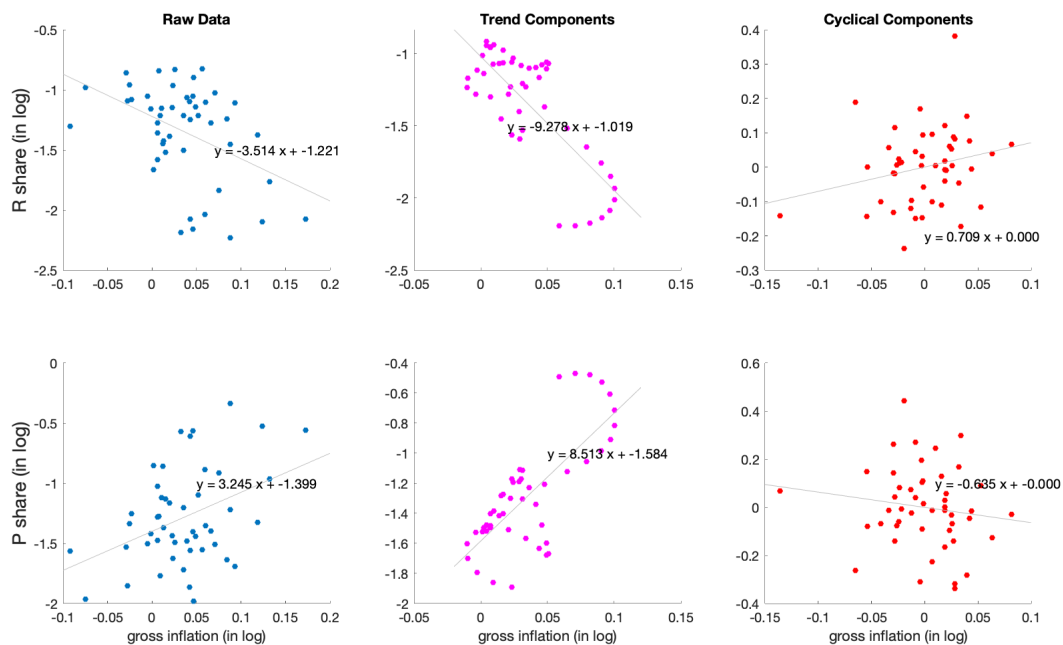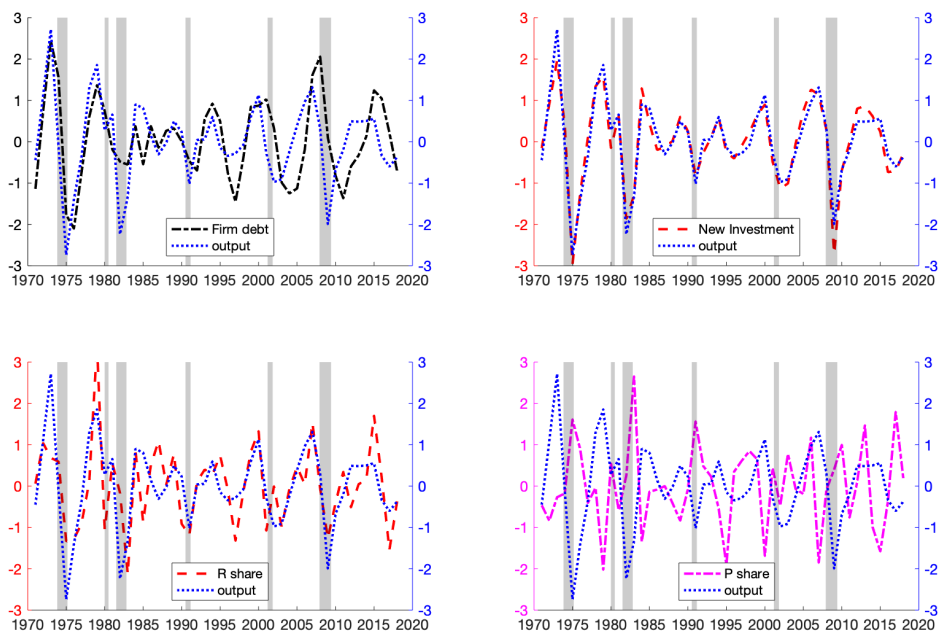Figure 1: Reallocation and the Cost of Liquidity

Figure 2: Debt, Investment and Reallocation

Note: shaded areas denote NBER recession dates.
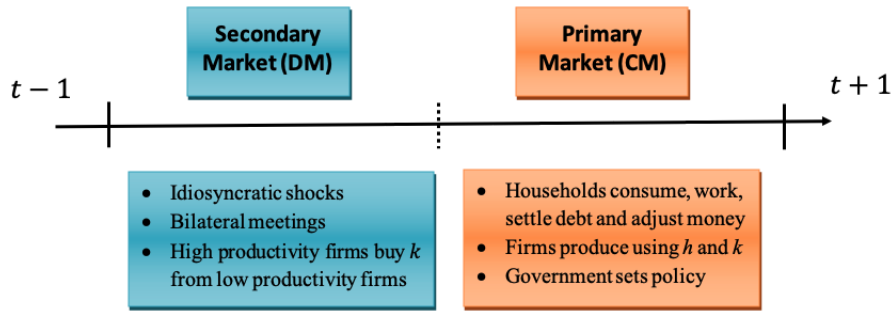
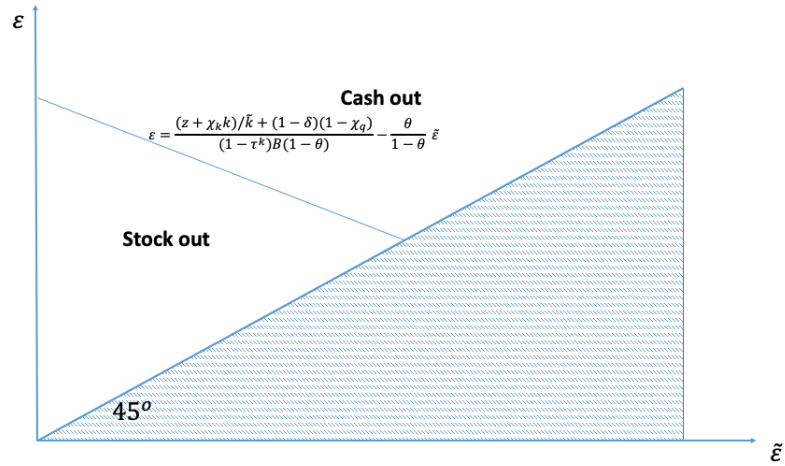## Figure 3: Events in Period t

Figure 4: DM Trade: Stock Out or Cash Out

$$\varepsilon = \frac{(z+\chi_k k)/\bar{k} + (1-\delta)(1-\chi_q)}{(1-\tau^k)B(1-\theta)} - \frac{\theta}{1-\theta}\tilde{\varepsilon}$$

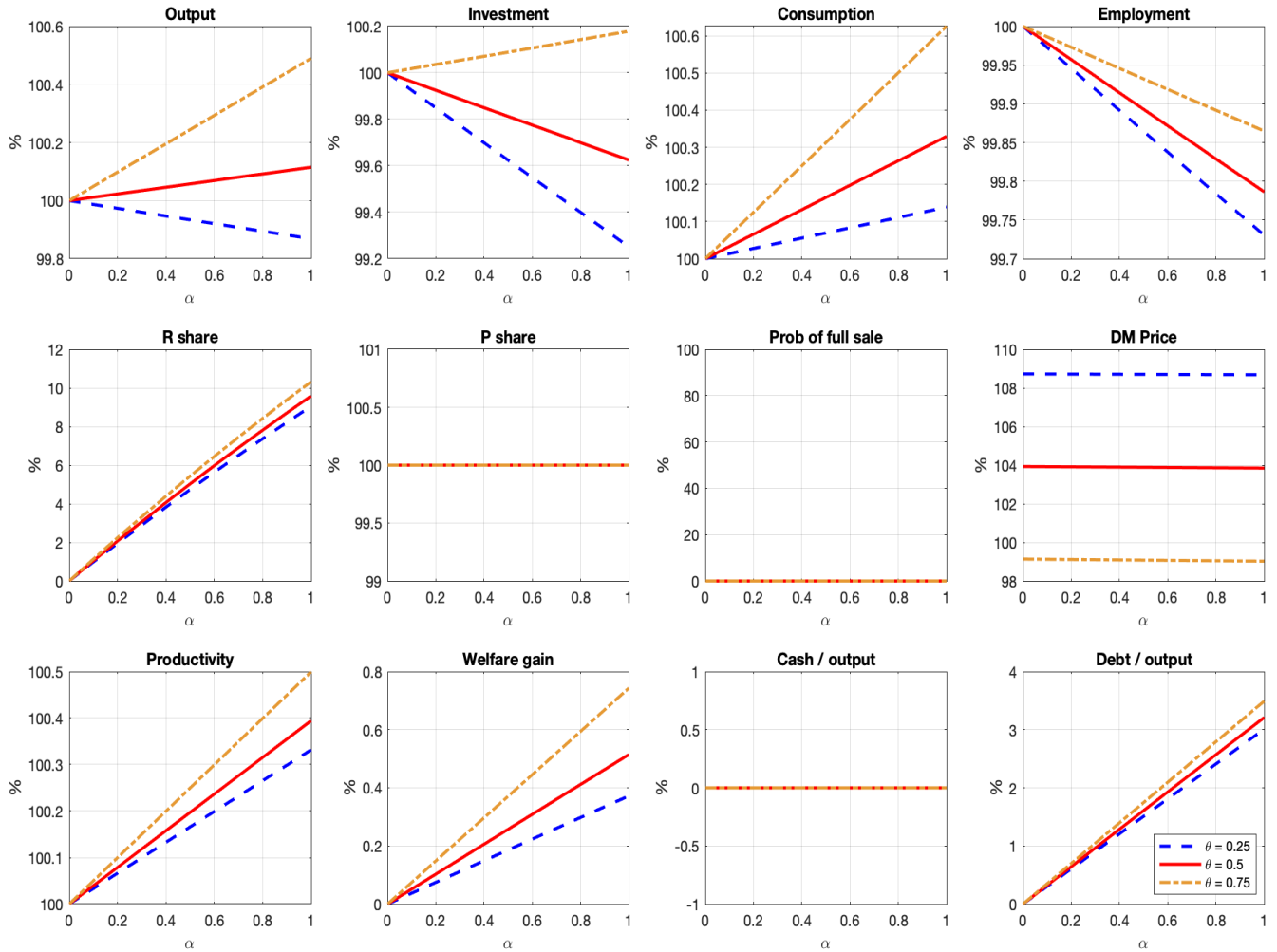## Figure 5: Steady-State Equilibrium
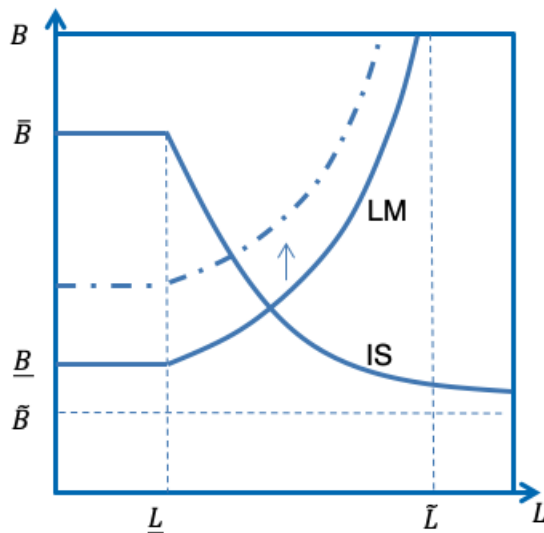
Figure 6: The Long-Run Effects of Inflation
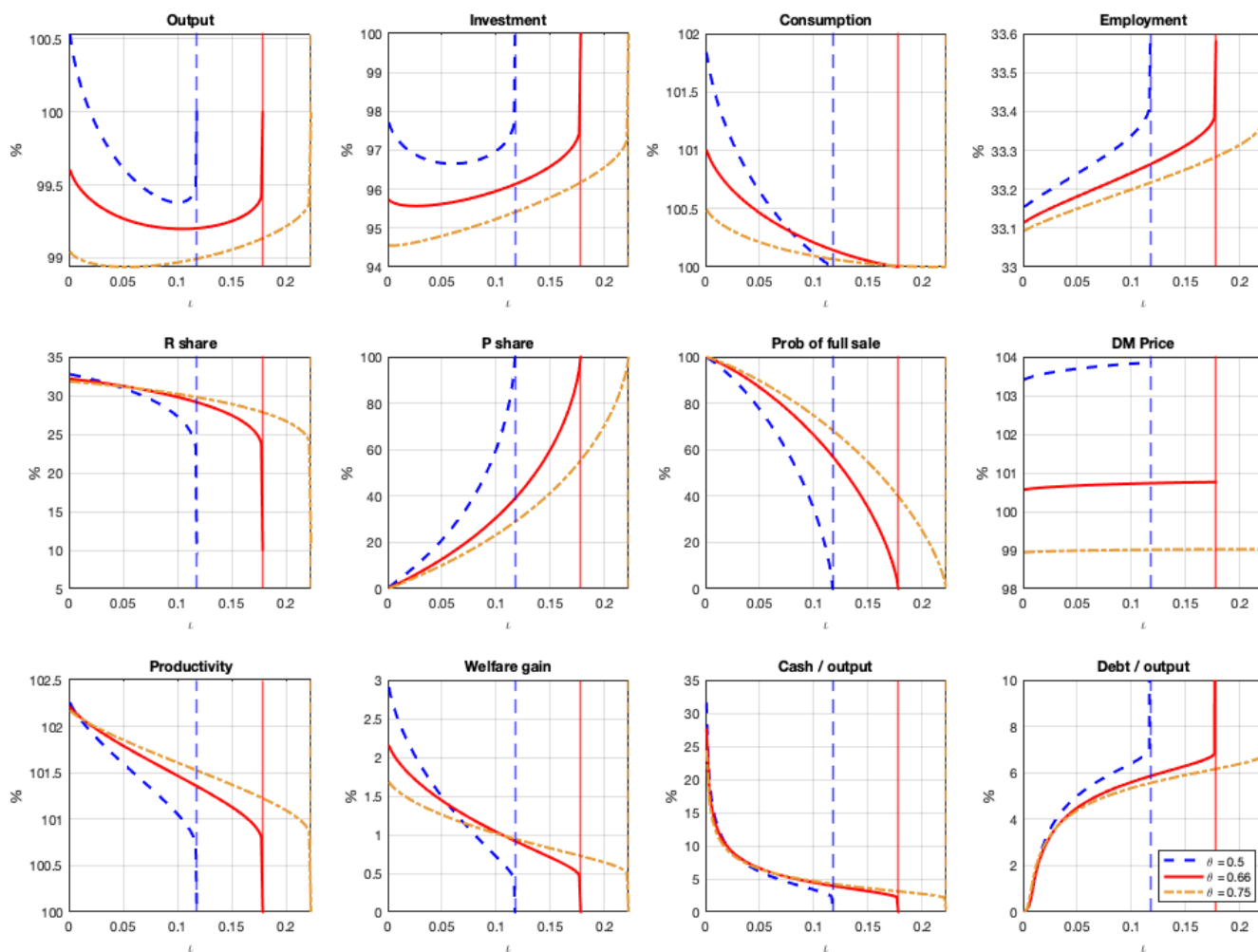
Note: The vertical lines divide non-monetary (left) and monetary (right) regions. For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

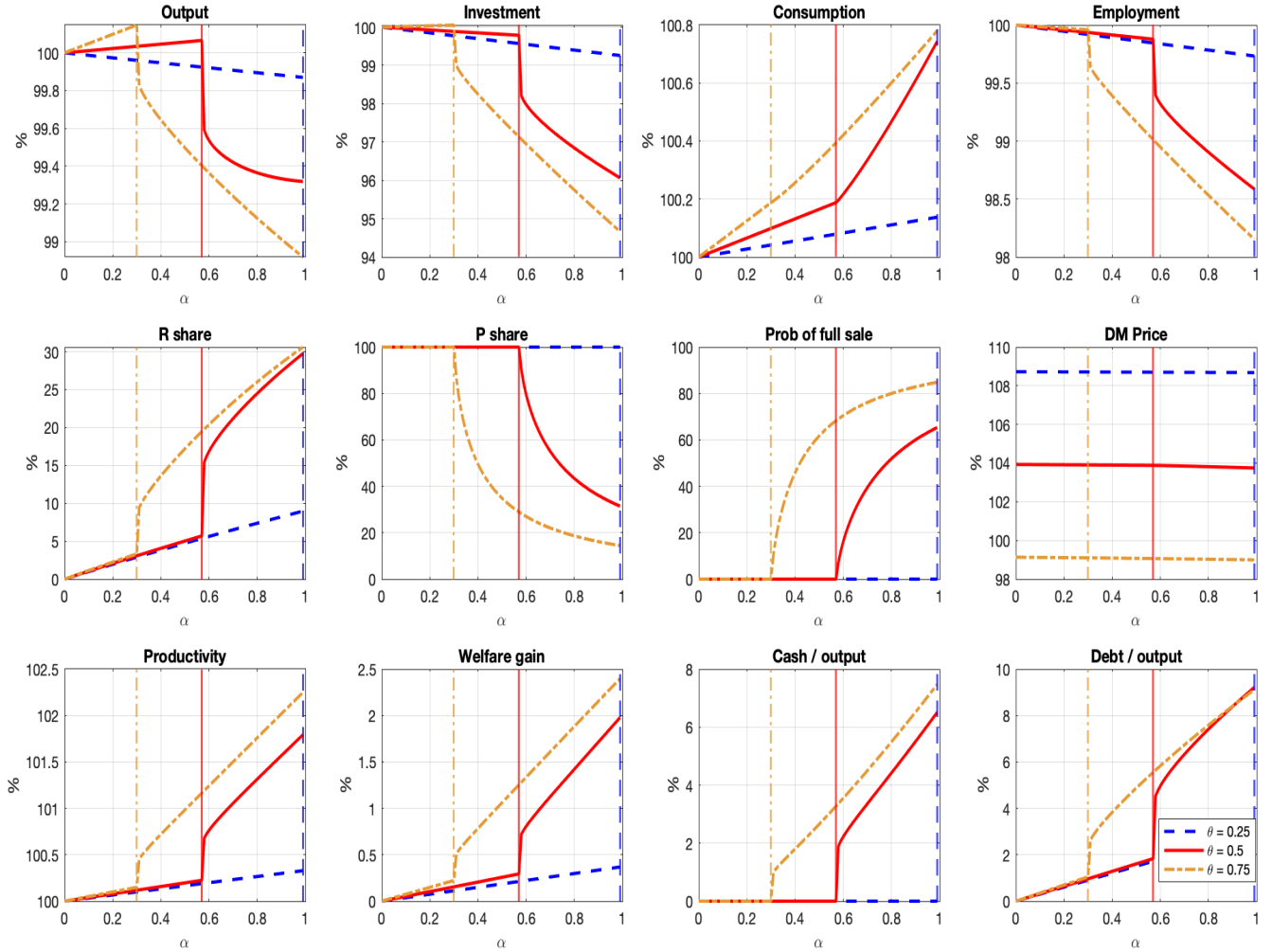# Figure 7: Long-run Effects of Search Frictions with Perfect Credit
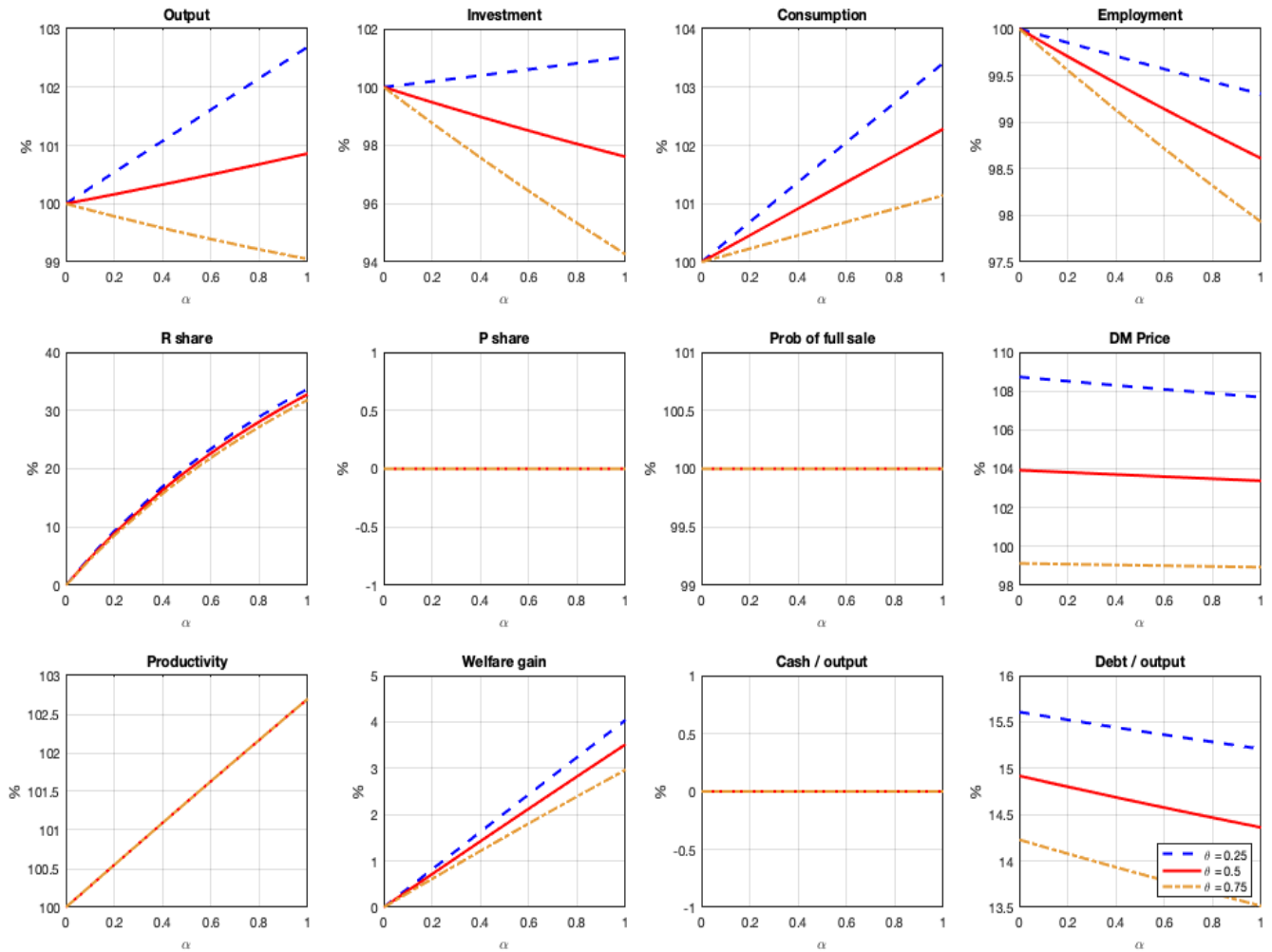
Note: For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 8: Long-run Effects of Search Frictions with Imperfect Credit and No Money

Note: For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 9: Long-run Effects of Search Frictions with Money

Note: The vertical lines divide non-monetary (right) and monetary (left) regions. For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

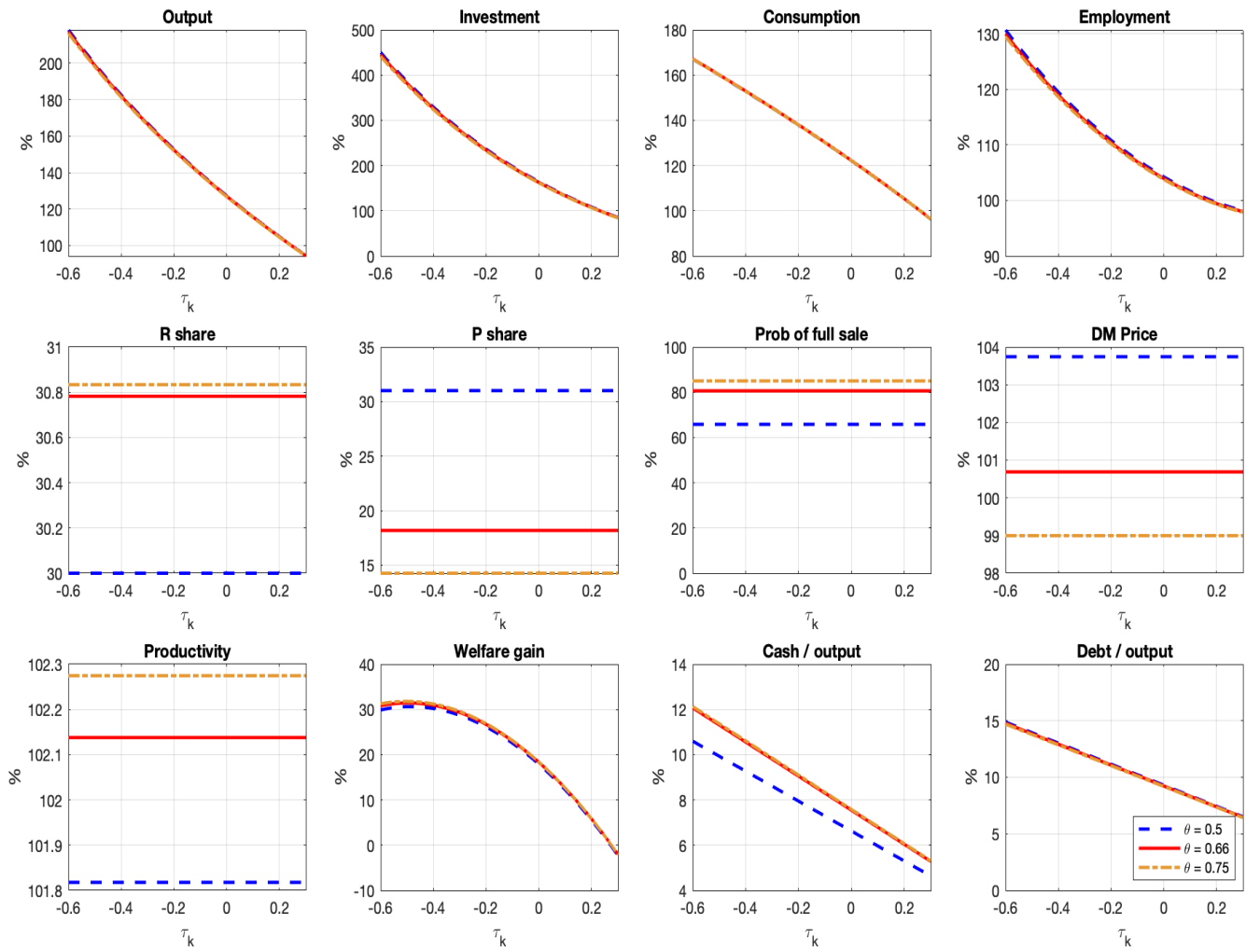# Figure 10: Long Run Effect of Capital Tax With Money

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Figure 11: Effect of Differences in Persistent Component of Productivity