The Rise of Closet Indexing[*]

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Abstract

We develop a model to analyze how increased industry competition affects funds' incentives to closet index (in which funds charge active management fees while closely mimicking a benchmark). Our analysis shows that lower fees and higher competition for abnormal returns increase closet indexing and decrease information production. Closet index funds strategically add variance to their returns, complicating identification of closet indexing. If given the ability to signal via a performance-based fee, high-skill funds forgo the option and only middle-skill funds adopt the signaling mechanism.

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1 Introduction

The mutual fund industry has transformed in recent decades: (i) the menu of products has expanded, (ii) information is disseminated more efficiently, and (iii) fees for both passively and actively managed funds have decreased sharply. These stylized facts are consistent with increased competition and corresponding benefits to investors. On the other hand, closet indexing, an agency conflict in which investors pay for the service of active management and do not receive it, has increased during this same period (Cremers and Petajisto (2009)). We develop a model to explore the relations among these trends, showing that increased competition is not necessarily a panacea for investors.

In our model, actively managed funds supply portfolio management services and excess returns over two periods. The cost of providing portfolio management is normalized to zero, while excess returns depend on each fund manager's unobserved effort and type. In the first period, all funds appear identical to investors and earn identical revenues (management fee times assets under management). Realized returns provide a noisy signal of fund managers' effort and type. In the second period, investors revise their beliefs and reallocate their investments, rewarding funds with superior first-period performance. The setup lends itself to a typical adverse selection problem in which more skilled managers attempt to signal their abilities and separate (earn high returns) by expending effort. Naturally, the effort that managers are willing to expend is proportional to the benefit of separation. The key insight of our model is that the separation benefit decreases as measures of competition for providing the two services increase. Competition in providing portfolio management manifests through lower fees, which decreases the the value of attracting new assets via superior performance. Competition in providing excess returns implies fewer profitable trades, muting fund in-flows following superior performance. Closet indexing is a natural result of increased competition.

Extending the base model provides novel insights regarding the consequences of closet indexing. First, we find that the prevalence of closet indexing is positively related to fund flow sensitivities, because greater adverse selection implies stronger reactions to abnormal returns. Second, we show that closet index funds may engage in signal-jamming to improve their ability to pool with truly active funds. Although signal-jamming does not affect expected performance, it adds variance. This implies that closet indexing may be more widespread than indicated by measures that rely on comparing portfolio weights to benchmark weights. Third, we consider the addition of a performance-based fee as a means to mitigate closet indexing. We show that a performance-based

fee (i.e., a fulcrum fee) has an ambiguous effect on closet indexing; in some settings it can actually increase the incentives to closet index. Finally, we allow funds to endogenously choose between a fee schedule that contains a performance-based component and one that does not. We show that highly-skilled active managers forgo the performance-based fee. Conversely, marginally-skilled active managers choose performance-based compensation.

The base model considers a two-period game of delegated asset management between investors and fund managers. Fund managers are endowed with unobservable skill and their objectives are to maximize fee revenue over the two periods. Managers may choose to expend costly resources into their strategies (becoming a truly active manager) or invest passively in the fund's benchmark (becoming a closet indexer). Managers' choices are unobservable, however, their distributions of trading profits first order stochastically dominate if they choose the former over the latter. The degree to which active managers' distributions dominate are positively related to managers' skills. This implies that the expected trading profits for a highly-skilled active manager are greater than those of a marginally-skilled active manager, all else equal.

Realized trading profits are passed through to investors in exchange for a management fee. We decompose the management fee into two components: a component attributed to an investor's opportunity cost of managing her own assets (referred to as the passive fee) and a component to compensate fund managers for engaging in active management (referred to as the active fee). In the first period, investors allocate capital to funds based on their initial beliefs regarding both fund managers' skills and their choices to be active. The equilibrium level of assets under management (AUM) equates the marginal benefit (expected abnormal return) to the cost (the management fee). Subsequently, funds realize their first period performances and pass them through to investors. Investors observe the performances and update their beliefs regarding both the managers' skills and their choices to be truly active or not.

In the second period, investors reallocate investments based on first-period performance, leading to increased AUM for successful performance and decreased AUM for unlucky active funds and closet indexers. The increase in second-period AUM (and the accompanying increase in fee revenue) is the benefit which entices funds to expend costly resources on active management. The model's equilibrium is characterized by a threshold rule based on managers' unobservable skill. A fund manger with skill above the threshold has a relatively high probability of success in producing first-period abnormal returns, and is more likely to earn the higher second-period revenues associated with success. The marginal fund manager exactly equates the expected benefit of greater second-period revenues and the cost of being truly active. The higher the threshold, the lower the

proportion of truly active funds and the higher the proportion of closet indexers.

Our analysis shows that this threshold is higher (more closet indexing) when measures of competition are also high. While investors benefit directly from the lower portfolio management costs, they also incur two indirect costs: (i) investors are paying for active management and not receiving it, and (ii) less active management implies less research, which may lead to less efficient prices.[1]

We supplement the base model by considering the effect of closet indexing on fund flow sensitivities. First, fund flow sensitives exhibit positive correlation with the threshold; as closet indexing increases, fund in-flows and out-flows are more pronounced. Second, and consistent with empirical evidence, the model predicts that the magnitudes of fund in-flows and out-flows are asymmetric; as closet indexing increases, investors reward positive performance more strongly than they punish negative performance.

The benchmark analysis also provides several testable empirical predictions. First, we predict that closet indexing is greater and fund flow sensitivities are more pronounced within highly-competitive or niche strategies as compared to uncrowded or broad market strategies. This prediction relates to strategy "capacity" (i.e., the ability to scale up a strategy), that is likely relatively low for highly-competitive or niche strategies. Second, closet indexing and fund flow sensitivities are more severe as investors become better informed and more sophisticated. This is directly related to the relationship between closet indexing and the passive fee as we imagine that increasing investor sophistication leads to lower passive fees.

Next, we consider an extension in which closet indexers can add strategic variance to their distributions of returns. In practice, this could be achieved by taking uninformed bets via under and over-weighting benchmark components. The analysis demonstrates that a closet index fund manager always engages in this behavior. Furthermore, the magnitude of signal-jamming is positively related to the threshold level, i.e., greater closet indexing suggests greater strategic variance. The finding implies that closet indexing levels may be much greater than previously measured because conventional measures do not account for strategic variance.

We explore whether or not performance-based fees mitigate closet indexing. With a performance-based fee, a fund captures a fraction of its performance directly (as opposed to passing it through to investors and being compensated via fee revenue). The addition of a performance-based fee has an ambiguous effect on the level of closet indexing. On one hand, the marginally-skilled fund manager more directly internalizes his choice. On the other hand, by retaining a portion of trading profits, the fund decreases its AUM and its ability to extract portfolio management rents from investors.

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[1]Bond, Edmans, and Goldstein (2012) survey the literature considering the real effects of financial markets.

The two tensions push the marginally-skilled manager in opposite directions and neither strictly dominates.

We then allow funds to endogenously choose whether or not to charge a performance-based fee. Given the ability to signal via a performance-based fee, highly-skilled managers forgo the option and only marginally-skilled managers adopt the signaling mechanism. On the surface, the finding appears to be in conflict with standard results from agency theory. However, considering that managed funds are pass-through entities, the finding is intuitive. Namely, if a fund retains a portion of its performance it mechanically lowers the incentives of investors to invest in the fund. This decreases the asset management rents that the fund is able to extract from investors (via the passive fee component), which is costly. As such, when the passive fee component is non-zero, performance-based fees are relatively more costly for highly-skilled managers (who will likely separate via performance) than they are for marginally-skilled managers.[2]

Our analysis concludes with a discussion of extensions and insights. First, we discuss a model setup including index funds (i.e., explicitly passive funds). We conjecture that competition among index funds and the advent of new products like exchange traded funds (ETFs) has a spill-over effect in the active fund space. Namely, as the prices of passive alternatives decline, the fraction of closet indexing increases. Second, we consider what we term as "sticky investors" — investors that remain with a fund even if realized performance is poor. The fraction of sticky investors needs to be sufficiently large to challenge the base model's results.

2   Related Literature

Our paper is related to the delegated asset management literature. First, to our knowledge, our work is the first attempt to provide theoretical justification for the time series trends regarding closet indexing, as documented by Cremers and Petajisto (2009) and Cremers et al. (2014). Furthermore, our analysis provides new predictions that should be tested.

Our work also builds on and adds to the broader theoretical literature. As in Berk and Green (2004), Pástor and Stambaugh (2012), and Brown and Wu (2014), we assume investors compete away abnormal returns, removing persistence in funds' abnormal returns, and that fund flows reflect unobserved managerial skill. Consequently, investors in our model chase performance, however, past performance is uninformative in predicting future performance.[3] We depart from the existing

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[2]The result holds as long as both fee schedules are supported in equilibrium. It is possible that only one fee schedule survives in equilibrium, i.e., all types choose to include a performance-based fee or all types choose a flat management fee.

[3]For empirical work regarding the so-called "fund flow anomaly", see Ippolito (1992), Chevalier and Ellison (1997),

literature in two distinct and novel ways. First, we endogenize the information production of fund managers. Second, we decompose management fees into two parts: one part attributed to the opportunity cost of managing one's own portfolio and one part attributed to active management.[4] These two features allow us to reconcile two concurrent trends: many metrics of competition have increased with the prevalence of closet indexing.[5,6] Huang et. al. (2007) characterize fund flow sensitivities in a framework in which investors have heterogenous participation costs. In our setup, however, fund flow sensitivities are related to the equilibrium level of closet indexing and not investor characteristics.

Our base analysis takes fees as given, acknowledging that funds generally cluster on similar fee schedules. While our work does not make normative statements or policy recommendations, it follows the literature of optimal contracting in delegated portfolio management, e.g., Bhattacharya and Pfleiderer (1985), Heinkel and Stoughton (1994), and Huddart (1999), and the literature examining strategic fee choices and structures, e.g., Huberman and Kandel (1993), Nanda et. al. (2000), Das and Sundaram (2002), and Kacperczyk et. al. (2005). In an extension we consider a fund's endogenous choice to charge a performance fee.[7]

3 Base Model

Consider a two period model in which there exist a set of managed funds and an infinite number of investors. Each fund is run by a risk neutral manager whose objective is to maximize profit. A fund's profit is determined through a price channel (management fee) and a quantity channel (assets under management). Investors are competitive and allocate their capital to equate the expected

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Sirri and Tufano (1998).

[4]The fee decomposition necessarily implies that the fee revenue collected by a fund is a measure of both managerial skill and investors' opportunity costs of self-management. This is consistent with Berk and van Binsbergen (2013).

[5]French (2008) documents that the annual costs of open-end funds shrunk from 2.19% of AUM in 1980 to 1.00% in 2006. More recently, the Investment Company Institute (2014) reports that expense ratios of actively managed equity funds has fallen from 106 bps to 89 bps from 2000 to 2013. Actively managed bond fund expense ratios have fallen from 78 bps to 65 bps over the same window. The trend is persistent among index funds as well. From 2000 to 2014 expense ratios have fallen from 27 bps to 12 bps for index equity funds and from 21 bps to 11 bps for index bond funds. Front-end load fees have also decreased. Wahal and Wang (2011) provide empirical evidence that, since 1998, the decline in management fees is attributed to an increase in competition in the managed fund space.

[6]The Investment Company Institute (2014) reports that the number of managed funds has grown more than eightfold between 1982 and 2013.

[7]The 1970 Amendment to the Investment Advisors Act of 1940 introduced a performance-based-fee option for mutual funds. The performance-based fee, however, is only admissible if it a "fulcrum" fee, i.e., the performance component must be symmetric around a predetermined benchmark. The 1970 Amendment implicitly prohibits the standard asymmetric performance-based fees used in other managed products, e.g., the "2-and-20" schedule used by many hedge funds.

benefit (i.e., excess return) and cost (management fee).[8] The discount rate between periods is normalized to zero.

Each fund's manager has a private skill type θ that is drawn from the unit continuum $[0,1]$. The managerial skill types are individually and identically distributed according to a continuous and strictly positive probability density function $g(\tilde{\theta})$,

$$\theta \sim g(\tilde{\theta}), \tag{1}$$

which has the cumulative density function $G\tilde{\theta})$. A manager's skill corresponds to his fund's ability to earn excess returns; a high draw of θ implies a greater ability to earn trading profits than a low draw. This is expounded on shortly. Skill types are privately observed and cannot be credibly disclosed.

A fund's ability to earn excess returns is also influenced by whether or not the fund engages in costly research. Funds that choose to engage in research incur an incremental cost,

$$c > 0, \tag{2}$$

which is not observable. The funds that incur c are defined *active* managed funds (and the subscript A is used to characterize those funds hereafter). The funds that do not incur the cost are defined *closet index* funds (and the subscript C is used to characterize those funds).

The cost, c, represents the resources spent on identifying profitable opportunities. A fund that engages in costly research achieves a distribution of excess returns (relative to its benchmark) that first order stochastically dominates the excess return distribution without research. This feature is captured simply by allowing funds that pay c the chance to earn trading profits,

$$\gamma > 0, \tag{3}$$

in each period. The parameter proxies for the trading profits that are obtainable via active management. Later, in Section 3.3, γ is normally distributed. In the base model, all trading profits flow through to investors. In Section 4, however, funds are permitted to retain a portion of trading profits through a performance component in the fee schedule.

An active fund's ability to earn γ is influenced by its manager's skill and the elasticity of trading profits relative to assets under management (AUM). The elasticity parameter is denoted

$$\eta \geq 0. \tag{4}$$

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[8]The assumption of perfectly elastic investor capital is commonplace in the delegated portfolio management literature, e.g. Berk and Green (2004), and Pástor and Stambaugh (2012). See also Brown and Wu (2014).

The parameter η captures the notion that active funds experience diminishing-returns-to-fund-size.[9,10] A parameter of $\eta_j = 0$ ($= \infty$), represents perfectly inelastic (elastic) trading profits with respect to AUM. The parameter is related to the scalability of a fund's strategy and to the level of competition within the fund's strategy space. Broad market categories likely have lower values of η, or more strategy capacity, than specific sectors. Similarly, strategies facing more competition likely have larger values of η than those in less competitive spaces.[11] While fund capacity is included for consistency with the existing literature, the model's main results do not require that $\eta > 0$.

Each fund in the active space A earns $\gamma > 0$ in period $t \in \{1, 2\}$ with a probability that is determined by its manger's skill, the fund's capacity, and the fund's assets under management at t. Namely, the probability of success (S) is given by,

$$\Pr(S|\theta, K_t, \eta, A) \equiv \frac{\theta}{1 + \eta K_t}, \tag{5}$$

where K_t is the fund's AUM at t. The expression in (5) requires that the fund is active, i.e., the fund has paid c (for simplicity, the probability that a closet index fund earns γ is equal to zero). The probability in (5) is increasing in θ, and decreasing in both K_t and η_j. That is, a high-type fund expects more trading profits ex ante than a low-type fund. Furthermore, the probability of earning trading profits is diminishing in the product of the fund's AUM and the parameter η. If the parameter is near zero, the fund's AUM has a small effect on the probability of success. If, however, the parameter is large, the fund's AUM has a more adverse effect on the probability of success.

If a fund successfully earns γ, investors observe it directly through the fund's performance because all trading profits are captured by the fund's investors. Conversely, if a fund does not earn γ, investors cannot distinguish whether the fund was active and failed or if it never undertook costly research in the first place. Investors' inability to observe if a fund paid c lends plausible deniability to the funds that do not succeed. This feature of the model captures the agency conflict coined closet indexing.

Each investor is indifferent between managing a dollar of her own assets or paying,

$$F_P > 0, \tag{6}$$

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[9]See Berk and Green (2004) and Pástor and Stambaugh (2012)

[10]The inclusion of the parameter is supported by empirical evidence. See Chen et. al. (2004), Fung et. al (2008), and Pollet and Wilson (2008).

[11]Hoberg et al. (2014) show that funds only experience persistent performance when facing few rivals and Pástor et al. (2014) show decreasing- returns-to-scale at the industry level.
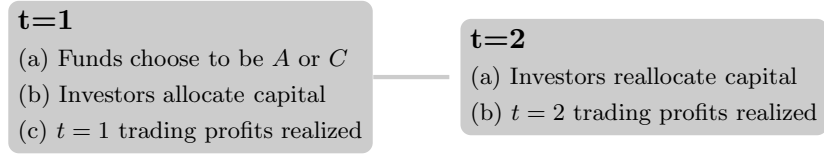
Figure 1: Managed Fund Game Timing

to have that dollar managed for her. As such, F_P is interpreted as a proxy that captures both investors' financial literacy and the direct costs of portfolio management. Each fund charges an incremental fee above F_P,

$$F_E > 0, \tag{7}$$

so that the total management fee charged by the fund is $F_E + F_P$. We refer to F_E as the active fee and F_P as the passive fee. The management fee is sticky and does not change between periods 1 and 2.

Investors are competitive and collectively allocate their capital in each of the two periods so that the expected net benefit of investment in each fund is equal to investing on their own. Investors are rational and have accurate beliefs about the funds. As such, at $t = 1$, investors allocate their capital according to their initial beliefs. At the conclusion of that period, trading profits are realized (or not) and passed through to investors. Investors subsequently update their beliefs about each fund and, at the beginning of $t = 2$, reallocate their capital based on their refined beliefs. The model's timing is summarized in Figure 1.

The model's setup requires that investors allocate their capital at $t = 1$ to a fund so that they are indifferent between investing in that fund or their outside option of self-management. The indifference condition yields,

$$-F_P = -(F_E + F_P) + \Pr(S_1|K_1, \eta)\frac{\gamma}{K_1}, \tag{8}$$

where $\Pr(S_1|K_t, \eta)$ is the probability of success that investors assess to the fund based on their beliefs. Similarly, investors reallocate their capital at $t = 2$ based on their refined beliefs,

$$-F_P = -(F_E + F_P) + \Pr(S_2|K_2, \eta, \mathbb{1}_{S_1})\frac{\gamma}{K_2}, \tag{9}$$

where $\mathbb{1}_{S_1}$ is an indicator function that equals one if the fund succeeded in earning trading profits at $t = 1$ and equals zero otherwise. For clarity, define the following shorthand notation,

$$\Pr(S_1^\star) \equiv E[\theta|A], \tag{10}$$

$$\Pr(S_2^\star|\mathbb{1}_{S_1}) \equiv E[\theta|\mathbb{1}_{S_1}, A]. \tag{11}$$

The preceding notation implies,

$$\Pr(S|K_1, \eta, A) = \frac{\Pr(S_1^\star)}{1 + \eta K_1}, \tag{12}$$

$$\Pr(S|K_2, \eta, A) = \frac{\Pr(S_2^\star | \mathbb{1}_{S_1})}{1 + \eta K_2}. \tag{13}$$

The following lemma utilizes the expressions in (8) and (9) to define the capital allocation rule for the fund in each period.

Lemma 1. *A fund's capital allocations in periods* 1 *and* 2 *are given by,*

$$K_1 = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4\eta^2 F_E}}, \tag{14}$$

$$K_2(\mathbb{1}_{S_1}) = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{4\eta^2 F_E}}. \tag{15}$$

The comparative statics of the capital allocation in Lemma 1 are straightforward: investors allocate more capital to a fund when possible trading profits γ are large, when the incremental fee F_E is low, and when the parameter η is small. The lemma also dictates that the capital allocations are increasing in the investors' beliefs about the manager's skill θ.

Each fund's profit is determined by the total management fee $F_E + F_P$ and the amount of capital it attracts.[12] Denote a fund's profit as Π_j,

$$\Pi_j(a_j, F_E) = \begin{cases} (F_E + F_P)(K_1 + K_2(0)) & \text{if } a_j = C \\ (F_E + F_P)(K_1 + K_2(\mathbb{1}_{S_1})) - c & \text{if } a_j = A, \end{cases} \tag{16}$$

where a_j represents a fund's choice to be A or C.

The competition among investors implies that each fund captures all expected trading profits (from an investor perspective). Each fund captures the expected trading profits via fund flows (changes in quantity) because the active fee (price) is fixed — this is a point highlighted by both Berk and Green (2004) and Pástor and Stambaugh (2012). Each fund's profit also includes the rents associated with delegated portfolio management via the passive fee. Consequently, there are two profit components: expected trading profits (from an investor perspective) and the cumulative value of delegated portfolio management. "From an investor perspective" is emphasized because investor beliefs about expected trading profits and a fund's beliefs are almost certainly different. The presence of adverse selection, which is discussed in detail shortly, implies that high (low) type funds believe that trading profits will be larger (smaller) than investors do. In Section 4, this

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[12]For tractability, fees are earned on beginning-of-period assets under management.

mismatch of beliefs is addressed by allowing funds to charge a performance-based fee and retain a portion of trading profits.

A fund's choice to be an active fund or closet index fund is endogenous. This is demonstrated by first defining $\Delta(\theta)$ as a benefit-cost function for a fund with type θ,

$$\Delta(\theta) \equiv \Pi_j(A, F_E|\theta) - \Pi_j(C, F_E|\theta), \tag{17}$$

$$= \frac{\theta(F_E + F_P)}{1 + \eta K_1}(K_2(1) - K_2(0)) - c. \tag{18}$$

The function $\Delta(\theta)$ compares the fund's expected profits if it chooses to be an active fund relative to the fund's expected profits if it chooses to be a closet indexer. It is clear from (18) that the benefit of being active is increasing with the separation that occurs at $t = 2$, which is the quantity

$$K_2(1) - K_2(0). \tag{19}$$

Highlighting the benefit of separation is useful in understanding the model's tensions. The function in (18) is weakly increasing in θ and $\Delta(0) < 0$. If $\Delta(1) > 0$, there exists some fund type $\theta \in (0, 1)$ for which the fund is indifferent between being an active fund or a closet index fund. The following lemma formalizes the intuition,

Lemma 2. *There exists a unique equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^* \in (0, 1]. \tag{20}$$

A fund with type $\theta < \theta^*$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^*$ chooses to pay c and is an active fund (A).

According to Lemma 2, investors face two forms of adverse selection when they choose a fund manager. The first form of adverse selection, *active skill*, is exogenous in the model. Conditional on choosing a fund that engages in active research, an investor cannot distinguish if the fund is marginal (i.e., a type near θ^*) or superior (i.e., a type near 1). The second form of adverse selection in the model, *active illusion*, arises endogenously. Investors cannot distinguish between the funds that are engaging in costly research and the ones that masquerade as if they are. Indeed, an investor choosing a fund will fall prey to the *active illusion* with probability $G(\theta^*)$. For the remainder of the analysis, the threshold itself is considered, as it is a sufficient statistic for the fraction of funds that are closet indexers. The following proposition characterizes the threshold.

Proposition 1. *The threshold θ^* is*

(i) increasing in c,

(ii) decreasing in γ,

(iii) increasing in η,

(iv) and decreasing in F_P.

The comparative static of θ^* with respect to F_E is equivocal.

The first two comparative statics in Proposition 1 are natural: one would expect the fraction of closet index funds to increase with the cost of being truly active and decrease with potential trading profits. The comparative statics with respect to η and F_P are less obvious.

Considering η, the fraction of closet index funds increases as trading profits become more sensitive to AUM (due to increased competition or reduced strategy capacity). The result is best understood by considering a fund-flow hedging effect associated with η, and its corresponding effect on the separation that occurs at $t = 2$. If a fund succeeds at $t = 1$, investors update their beliefs upward and more capital flows into the fund. This, however, has an adverse effect on expected trading profits because an increase in AUM decreases the probability of success due to η. Consequently, flows to the fund are dampened. Similarly, if a fund fails at $t = 1$, investors update their beliefs downward and capital flows from the fund. This has a positive effect on expected trading profits because a decrease in AUM increases the probability of success via η. Again, flows from the fund are dampened. Consequently, η hedges fund flow volatility and necessarily dampens the spread $K_2(1) - K_2(0)$. Larger values of η result in more hedging, decreasing separation at $t = 2$ and the benefit of being truly active (as is clear in (18)).

Now considering F_P, an increase in F_P leads to a smaller fraction of closet-indexing funds. Recall, F_P proxies as investors' cost of self-managing \$1. The cost of self-management is attributed to the direct costs of portfolio management and the costs associated with financial literacy. Funds are able to extract rents from investors by providing a delegated portfolio service, and the aggregate size of those rents is determined by a fund's AUM. Higher passive fees increase the benefit of being truly active and more funds want to succeed at $t = 1$ to increase AUM at $t = 2$ (as is clear in (18)).

Finally, the comparative static with respect to F_E is generally consistent with the comparative static with respect to F_P. However, there is a technical consideration when F_E gets sufficiently large. As F_E approaches infinity, a fund's AUM approaches zero. The fund still captures its expected trading profits, but the rents associated with delegated portfolio management diminish. This effect causes the sign on the comparative static to switch. The point is merely academic, and

we conjecture that the comparative static with respect to F_E mirrors the comparative static with respect to F_P under reasonable parameters.

3.1 Fund Flow Sensitivity

The model of Berk and Green (2004) demonstrates that the evolution of investor beliefs is captured through fund flows. In their model, no agency conflicts exist: there is no moral hazard and there is not private information. Our framework is consistent with their setup, with the addition of a layer of adverse selection coined *active illusion*. In this subsection, fund flow sensitivities are characterized in the presence of *active illusion*.

Define a fund's relative net fund flow as,

$$\Delta K(\mathbb{1}_{S_1}) \equiv \frac{K_2(\mathbb{1}_{S_1}) - K_1}{K_1}. \tag{21}$$

The expression in (21) immediately extends itself to the following lemma.

Lemma 3. *If $\theta^* < 1$, a fund that succeeds (fails) at $t = 1$ experiences a positive (negative) net fund flow at $t = 2$.*

Lemma 3 is natural — as long as there is a fraction of funds that are truly active, fund flows are positively related to realized performance. The sensitivity of those fund flows are now characterized.

Corollary 1.1. *Fund flow sensitivities are*

(i) increasing in c,

(ii) decreasing in γ,

(iii) increasing in η,

(iv) and decreasing in F_P.

The comparative static with respect to F_E is equivocal.

A quick comparison of Proposition 1 to Corollary 1.1 provides straightforward intuition: fund flow sensitives increase when the fraction of closet index funds increases. The linkage between the results is attributed to the adverse selection of *active illusion*: a greater fraction of closet index funds implies that investor beliefs are more sensitive to abnormal returns. Because fund flows are the channel that communicates investor expectations, this implies that greater adverse selection implies greater fund flow volatility.

One can nest a reduced-form version of the Berk and Green (2004) model in our framework by setting $c = 0$ and reinterpreting a fund's inability to credibly communicate its type. In their setting, it is dominant for all funds to be active because it is costless to do so. Certainly, there is less adverse selection in a setting without closet index funds. Consequently, fund flow sensitivities in the Berk and Green (2004) model are dampened as compared to the fund flow sensitivities in our model.

Although our results suggest that fund flow sensitivities increase with the fraction of funds that are closet indexers, we have not yet examined the relationship between fund in-flows and out-flows.

Lemma 4. *The magnitude of fund in-flows and fund out-flows is asymmetric. In general, fund out-flows are more pronounced if θ^* is small and fund in-flows are more pronounced if θ^* is large.*

Chevalier and Ellison (1997) shows that flows are stronger for a fund that beats its benchmark as opposed to a fund that fails to beat it. Our result is consistent with their findings and suggests the increased closet indexing increases the convexity of this relation.

3.2 Empirical Predictions

Proposition 1 and Corollary 1.1 provide several testable hypotheses. We begin with cross sectional predictions regarding fund flow sensitivities and proportions of closet indexing across different fund categories. Earlier in Section 3, it was conjectured that broad market funds have greater strategy capacity than specialized funds. The comparative statics in Proposition 1 and Corollary 1.1 with respect to η lend themselves to the following prediction.

Empirical Prediction 1. *Relative to generalized funds or funds in low-competition strategies, specialized funds or funds in high-competition strategies have*

- *higher fund-flow sensitivity to returns, and*

- *higher proportions of closet indexing.*

The preceding empirical prediction is novel. Existing models that consider diminishing-returns-to-fund-size, e.g., Berk and Green (2004) and Pástor and Stambaugh (2012), do not differentiate the strength of the tension within different fund categories.

Also in Section 3, we discussed the benefit to truly active funds when F_P rises. We conjecture that the advent of online trading platforms for retail investors, and increased financial literacy via the availability of information implies a *decrease* in F_P over the past two decades. The comparative

statics in Proposition 1 and Corollary 1.1 with respect to F_P yield a time series prediction that should be tested over an event window beginning with the technology and Internet boom of the 1990s.

Empirical Prediction 2. *As F_P (e.g. financial literacy costs or investors' opportunity cost of self-management) decreases*

- *fund-flow sensitivity to returns increases, and*

- *the proportion of closet indexing increases.*

There is already suggestive evidence that the latter part of the prediction holds, e.g., Cremers and Petajisto (2009) and Cremers et. al. (2014) show that their measure of closet indexing has increased over time, but more empirical work is needed.

3.3 Normally Distributed Trading Profits

The base model treats trading profits as a binary outcome. The simple setup yields several intuitive insights regarding closet indexing. Allowing trading profits to be normally distributed provides richer results regarding the behavior of closet indexers. However, the additional richness comes at a cost; closed form solutions and analytic tractability are lost. Consequently, we use numerical methods to solve the model incorporating normally distributed profits.

The main assumptions from the base model are maintained: each fund manager's skill is individually and identically distributed according to the distribution $g(\tilde{\theta})$ (uniform in the numerical methods), funds that engage in active management pay a fixed cost $c > 0$, and fees contain both a passive management component F_P and an active management component F_E. In this extension, however, an active fund's trading profits $\tilde{\gamma}_{j,t}$ are distributed normally,

$$\tilde{\gamma}_{j,t} \sim N\left(\frac{\gamma\theta}{1 + \eta K_t}, \sigma_A^2\right). \tag{22}$$

A closet index fund's trading profits, $\epsilon_{j,t}$, are zero in expectation, but realized outcomes are noisy,

$$\epsilon_{j,t} \sim N(0, \sigma_C^2), \tag{23}$$

where σ_C^2 is endogenously chosen by the closet funds. A closet index fund's variance choice is achieved by making uninformed bets via over- and under-weighting many of the benchmark's components.

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The setup implies that the distribution of trading profits for a fund that pays c first order stochastically dominates the distribution if the fund did not. The setup also better captures the underlying portfolio problem: a highly-skilled manager (θ near 1) will identify many profitable trades and expects to earn abnormal returns. A marginally-skilled manager (θ near θ^*) will be able to identify some profitable trades but will also make some mistakes. Nonetheless, the marginally-skilled manager expects more profitable trades than unprofitable ones. This feature is captured by parameterizing the mean of an active manager's distribution by his type. Furthermore, for consistency with the base model, the mean is scaled by $\frac{1}{1+\eta K_t}$ to capture the idea that funds have limited capacity.

The setup allows a skilled manager to under-perform; even a highly-skilled manager has positive probability on losing money. Conversely, the setup allows a closet index fund to get lucky and achieve positive abnormal returns. Despite these possibilities, numerical analysis of the model yields similar insights as in Proposition 1. The following remark formalizes this point,

Remark 1. *In a setup with normally distributed trading profits, there exists a unique equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^* \in (0, 1]. \tag{24}$$

A fund with type $\theta < \theta^*$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^*$ chooses to pay c and is an active fund (A). Furthermore, the threshold's comparative statics match those in Proposition 1 and the threshold is increasing in σ_A^2.

Remark 1, which is supported by numerical analysis, reveals that θ^* increases with σ_A^2. The finding is intuitive; the incentive to closet index increases as realized performance becomes a nosier signal of both managerial skill and active management.

Remark 2. *A closet index fund chooses a finite and strictly positive variance σ_C^2. The level of σ_C^2 is positively correlated with θ^*.*

The preceding remark implies that a closet index fund necessarily deviates from its benchmark with uninformed bets. The extent of this "signal-jamming" behavior is positively related to the level of closet indexing. As more funds closet index, the expected performance gap between active funds and closet index funds increases. This leads closet index funds to increase variance, directing more probabilistic weight to the expected performance outcomes of truly active funds.[13] The

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[13]Heinkel and Stoughton (1994) consider a similar problem.

finding suggests that measures of closet indexing should control for strategic, uninformed deviations. Furthermore, the finding suggests that existing closet indexing measures that compare portfolio weightings to benchmark weightings may perceive increased closet indexing as increased active management.

4 Adverse Selection and Performance-Based Fees

In the preceding section, we emphasized that a fund captures all expected trading profits from an investor perspective. The fund's expectations, however, likely differ from investors'. In this section, we introduce a performance component to the fee schedule, allowing a fund to retain a portion of its trading profits. In addition to charging a management fee of $F_E + F_P$, a fund retains a fraction of trading profits

$$\lambda > 0. \tag{25}$$

Similar to the expressions in (8) and (9) from Section 3, investors allocate their capital according to,

$$-F_P = -(F_E + F_P) + \Pr(S_1 | K_1^\lambda, \eta, \lambda) \frac{(1 - \lambda)\gamma}{K_1}, \tag{26}$$

$$-F_P = -(F_E + F_P) + \Pr(S_1 | K_1^\lambda, \eta, \lambda, \mathbb{1}_{S_1}) \frac{(1 - \lambda)\gamma}{K_2}. \tag{27}$$

The preceding functions differ from (8) and (9) only with regards to the trading profits that are potentially available. The following lemma provides closed-form solutions for the capital allocations.

Lemma 5. *A fund's capital allocations in periods* 1 *and* 2 *are given by*

$$K_1 = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4(1 - \lambda)\gamma\eta\Pr(S_1^\star)}{4\eta^2 F_E}}, \tag{28}$$

$$K_2(\mathbb{1}_{S_1}) = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4(1 - \lambda)\gamma\eta\Pr(S_2^\star | \mathbb{1}_{S_1})}{4\eta^2 F_E}}. \tag{29}$$

The comparative statics of K_1 and $K_2(\mathbb{1}_{S_1})$ with respect to λ are natural: a fund's AUM is diminishing as the fund retains a larger portion of trading profits.

Funds endogenously separate as active funds or as closet indexers depending on those actions' payoffs. Similar to the base model, there exists a unique threshold for which a fund with skill level above the threshold chooses to be an active fund and a fund with a skill level below it chooses to be a closet indexer.

Lemma 6. *In a setup in which funds charge both a management fee and a performance-based fee, there exists a unique equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^\lambda \in (0, 1]. \tag{30}$$

A fund with type $\theta < \theta^\lambda$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^\lambda$ chooses to pay c and is an active fund (A).

So far, the results of a setup with a performance-based fee are redolent of a setup with only a fixed management fee. However, as the proof of Lemma 7 demonstrates, the performance-based fee's level has vacillating implications on the fraction of closet index funds.

Lemma 7. *The comparative static of θ^λ with respect to λ is equivocal.*

An increase in λ introduces three competing effects. The first and second effects relate to assets under management. Large values of λ mechanically decrease AUM because less expected trading profits are available to investors. Less AUM is particularly costly to the fund when investors are willing to pay amply for portfolio management, that is, when F_P is large. This first effect decreases the incentive to pay c. However, less AUM correlates to greater probability of earning excess returns, so the second effect increases the incentive to pay c. Third, a large value of λ implies that the fund greater internalizes its actions. The direction of this tension is not obvious. In the limiting case, when $\lambda = 1$, the fund fully internalizes its actions and does not suffer from any form of asymmetric information. As such, if the fund would choose to pay c in a world of perfect transparency, i.e., one in which both the manager's skill and paying c are observable, an increase in λ moves them closer to that decision. The fund type that is just indifferent in paying c in a perfect transparency setup is the relevant benchmark, which is referred to as the fully transparent benchmark (θ^{FT}). If θ^{FT} is increasing in λ, θ^λ should mirror it and converge in the limit. If, however, θ^{FT} is decreasing in λ, then θ^λ should follow suit.

Lemma 7 states that none of the three effects dominate. This is depicted the in two graphs of Figure 2. Figure 2(a) illustrates the case when the portfolio management benefit is highly valued by investors, that is, the passive fee is relatively large. As λ increases, a fund passes less trading profits to investors which mechanically lowers AUM. Consequently a fund extracts lower rents associated with F_P. This makes succeeding in the first period less attractive. Consequently, θ^λ, which is represented by the solid line, increases, implying a greater fraction of closet index funds.

Figure 2(b) illustrates the case in which the the portfolio management benefit is not as highly valued, i.e., the passive fee is relatively small. In that case, an increase in λ increases the probability
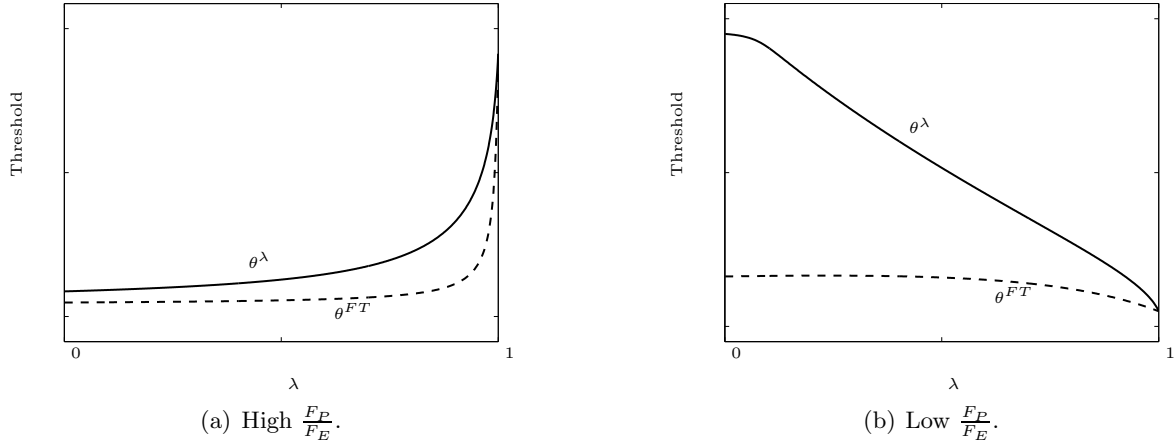
(a) High $\frac{F_P}{F_E}$.

(b) Low $\frac{F_P}{F_E}$.

Figure 2: Comparative static of θ^λ with respect to λ. The two figures depict the non-monotonic relationship between λ and θ^λ. In Figure 2(a) the passive fee is relatively large and in Figure 2(b) the passive fee is relatively small. In both figures, θ^λ approaches the fully transparent benchmark θ^{FT} as λ goes to one.

that a fund will succeed in both periods. The fund captures a $(1 - \lambda)$ share of expected trading profits from an investor perspective (via fee revenue) and a λ share of expected trading profits from the fund's perspective. All else equal, both shares of trading profits are higher when λ is larger, which incentivizes a marginal fund to pay c. This decreases θ^λ, which is represented by the solid line, implying that the fraction of closet index funds is lower.

In both settings depicted in Figure 2, an increase in λ induces more alignment between the fund and its actions. In the two figures, the fully transparent benchmark is depicted as the dashed line. In Figure 2(a), θ^{FT} increases with λ because the rents associated with the passive fee decrease. This coincides with an increase in θ^λ, and the two converge at $\lambda = 1$. In Figure 2(b), θ^{FT} decreases with λ due to increase in probability of success. Again, the decrease in fully transparent benchmark coincides with a decrease in θ^λ and the two converge at $\lambda = 1$.

4.1 Strategic Fee Choice

In this section, funds choose from two fee schedules: one in which the fund charges a management fee only and one in which the fund retains a portion of the profits, i.e., a performance-based fee, in addition to the management fee.[14] Again, the management fee charged under both fee

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[14]While we do not formally analyze funds' ex-ante (prior to knowing their own types) optimal fee schedule, our setup results in optimal fees approaching one of two extremes. When capturing the rents associated with providing the portfolio management service are relatively more valuable than trading profits, funds will optimally set the excess fee near zero, leading to the largest possible base of AUM. Alternatively, if trading profits are relatively more valuable

schedules is $F_E + F_P$. The setup allows for endogenous separation on a flat management fee and a management fee plus a fulcrum fee.[15] Here, active funds are characterized in two ways: active without a performance component (these funds are referred to as A_χ hereafter) and active with a performance-based fee (these funds are referred to as A_λ). The action set for a fund now includes three elements,

$$\text{a} \in \{C, A_\lambda, A_\chi\}. \tag{31}$$

The funds that choose a $= C$ necessarily mix between the two possible fee schedules. Define the mixing probability that closet funds use to mix between fee schedules as,

$$\rho \in [0, 1], \tag{32}$$

where closet funds choose the flat management fee schedule with probability ρ. In this setup, we concern ourselves only with the set of parameters that lends itself to an equilibrium in which both fee schedules co-exist. Indeed, there are some parameter sets for which only one of the two fee schedules survives in equilibrium (which were discussed in Sections 3 and 4).

The possibility that active funds employ mixing strategies over the two fee schedules has not been ruled out. The following lemmas, however, demonstrates that a fund that pays c chooses a fee schedule based on its type and chooses that fee schedule with probability 1.

Lemma 8. *There exists a unique threshold,*

$$\overline{\theta} \in (0, 1), \tag{33}$$

for which any fund with type $\theta \geq \overline{\theta}$ chooses a fee schedule without a performance component (A_χ).

Lemma 9. *There exists a unique threshold,*

$$\underline{\theta} \in (0, \overline{\theta}), \tag{34}$$

for which any fund with type $\theta < \underline{\theta}$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \in [\underline{\theta}, \overline{\theta})$ chooses to pay c and chooses a fee schedule with a performance-based fee (A_λ).

The preceding set of lemmas imply that active funds separate on fee schedules according to managerial skill. This separation is explored in detail shortly, but first the setup's equilibrium is defined.

than those rents, funds prefer an excess fee as high as possible, which minimizes AUM and increases the probability of achieving trading profits (i.e., excess returns). Without additional tensions, an internal optimal fee does not result.

[15] According to a Wall Street Journal article regarding mutual funds in the United States (Gay (2011)), 4.1% of equity mutual funds charge performance fees, which represents approximately 9.5% of assets in managed equity funds. In bond funds, only 1.1% charge a performance fee, which represents only 0.4% of assets in managed bond funds.

Proposition 2. *An equilibrium in which both fee schedules co-exist consists of,*

(i) high-type funds ($\theta \in [\overline{\theta}, 1]$) select the fee schedule consisting of only a management fee (A_χ),

(ii) medium-type funds ($\theta \in [\underline{\theta}, \overline{\theta})$) select the fee schedule consisting of a management fee and a performance-based fee (A_λ),

(iii) low-type funds ($\theta \in [0, \underline{\theta})$) are closet index funds (C) and mix between the two fee schedules with probability ρ^*.

The relevant thresholds, $\overline{\theta}$ and $\underline{\theta}$, are determined by the indifference conditions implied by Lemmas 8 and 9. The equilibrium mixing probability $\rho^* \in (0, 1)$ equates the payoffs of closet indexing under both fee schedules. If $\rho^* = 1$, closet indexing on the fee schedule without a performance component strictly dominates.

Proposition 2 analytically shows that a marginally-skilled manager chooses to charge performance-based fees (as in mutual funds with fulcrum fees or other performance-based compensation) and a highly-skilled manager charges only a flat fee. The intuition for this separation relies on the high-type fund's ability to separate via performance.[16] While a high-type fund is relatively likely to succeed in the first period, a medium-type fund is likely to be pooled with the closet index funds, and as a result, significantly punished. Therefore, the medium-type fund is willing to use a costly method to separate from the closet indexers. Specifically, it sacrifices the rents associated with providing the portfolio management service, via F_P, that could be earned by passing profits through to investors, and in exchange it keeps a portion of its profits and pools with a smaller population of closet indexers. Unfortunately, tractability prevents analytic comparative statics, and we use numerical methods to provide insights into how the model's exogenous parameters affect the proportion of closet indexing.[17] Our numerical analysis confirms that the comparative statics in this setting are consistent with those in Proposition 1.

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[16]Gervais and Strobl (2012) provide a result that, on the surface, appears to be in conflict with ours; they show that managers with an average skill level self-select into transparent funds with flat fees, e.g., a mutual fund with disclosure requirements, and highly-skilled managers self-select into opaque funds with performance-based fees, e.g., a hedge fund. Although the end results differ, both results hinge on the same tension: higher-skilled managers rely on their quality to separate from low-skill managers, while average-skill managers utilize a costly mechanism to signal their skill. The costly mechanism in their model is disclosure (revealing fund strategy), while the mechanism in our model is retaining a portion of trading profits, which lowers the passive fee rents that can be extracted from investors.

[17]See Appendix B for details on our numerical methods.

5 Concluding Discussion

Our analysis provides insights into how measures of industry competition (e.g. fee levels, sensitivity of trading opportunities to AUM, and cost of information) affect a fund's decision to closet index. The model's implications fit the established empirical regularities, provide additional testable predictions, and suggest refinements to existing measures of closet indexing. Furthermore, the model suggests that active managers with performance-based compensation are less skilled than their flat management fee counterparts. We conclude by providing a discussion of potential extensions for the model. Although the model does not explicitly capture the subsequent analysis, it does provide useful insights that can be used for future research.

5.1 On the Role of Passive Managed Funds

Investors in the model are uniform and face the same outside incremental cost of self-management, F_P. In reality, investors are not homogeneous in this regard. Some investors are sophisticated and have a lower cost of self-management than those that are financially illiterate.[18] Furthermore, investors differ in regards to their individual wealth which further confounds the issue, i.e., the incremental cost of managing one additional dollar for a large sum of money is less than for a small sum. Consequently, it would be more appropriate to model F_P as the *marginal investor's* cost of self-management for the last dollar in his portfolio. This cost is pinned down by considering a distribution of investor self-management costs. Denote the distribution function as,

$$H(w_t), \tag{35}$$

where w_t is the sum of wealth delegated by all investors in period t. It is important to note that $H(w_t)$ is not a probability distribution and that $H(w_t)$ is weakly decreasing in w_t: if w_t is small, only individuals with high self-management costs delegate their portfolio. According to the distribution, the marginal investor's cost of self-management is equal to $H(w_t)$. Determining the level of w_t requires an aggregation of all funds' AUM in each period. In the model, the focus is solely on the active fund space; the role of index funds, i.e., passive funds, is not considered. Passive funds, although absent from the analysis, play an important role for investors as they solely provide the benefit of delegated portfolio management. Suppose there are J funds in the active space and

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[18]The NASD Literacy Survey (2003) demonstrates that retail investors have limited sophistication regarding financial products. Lusardi and Mitchell (2007) show that financial literacy is in short-supply among retiring Baby Boomers. See also Alexander, Jones, and Nigro (1998) and Agnew and Szykman (2005).

L in the passive space. This implies that w_t is given by,

$$w_t = \sum_{j=1}^{J} K_{j,t} + \sum_{l=1}^{L} K_{l,t}. \tag{36}$$

Recall that, in the base model, fund allocations were pinned down by setting F_P equal to the expected payoff of active management. In the alternative setup described here, the F_P on the left-hand side of (8) and (9) is no longer an exogenous variable: investors will allocate their capital to managed funds until the marginal investor is indifferent between self-managing his last dollar or delegating it to a fund. This naturally implies that the capital flows to each fund are influenced by the aggregate number of dollars under management by all other funds. Explicit modeling of passive managed funds is a non-trivial task: one must consider how those funds compete for investors, the role of entry costs, and how investors locate. However, even without an explicit equilibrium model of passive and active funds, several insights are obvious and consistent with Empirical Prediction 2.

First, if there is a shock to investor wealth and more dollars accrue to individuals with high costs of self-management, all funds will experience an increase in AUM, ceteris paribus. Consistent with the intuition from Proposition 1, this should be accompanied by a decrease in the fraction of funds that choose to be closet indexers. As such, we would predict that the information production among active funds is higher when there is a shock to domestic wealth, e.g., periods of rampant home price appreciation.

Second, if competition among passive funds increases or the entry costs for passive funds decreases, the cost of self-management for the marginal investor will decrease. Consistent with the intuition from Proposition 1, this should be accompanied with an increase in the fraction of funds that choose to be closet indexers. We predict that the advent of index products like exchange traded funds (ETFs) and similar constructs has lead to more closet indexing among active funds.

5.2 Sticky Investors and Marketing Expenses

The base model is generous in its treatment of investor behavior; investors are fully rational and allocate their capital efficiently in each period. However, the empirical evidence suggests that investors are not entirely rational nor entirely attentive.[19,20] Consequently, the model could be

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[19]Barber, Odean, and Zheng (2005) show that investors gravitate towards mutual funds with low up-front fees, even if operating expenses are considerably higher. There is also considerable evidence that investors are not fully attentive. See Klibanoff, Lamont, and Wizman (1998), Huberman and Regev (2001), Hou and Moskowitz (2005), DellaVigna and Pollet (2007), Hong, Torous, and Valkanov (2007), and Hirshleifer, Lim, and Teoh (2009).

[20]Many empirical studies demonstrate that actively managed fund performance, net of fees, is negative, e.g., French (2008). As a caveat, Edelen (1999) shows that the negative return performance of mutual funds is largely attributed to

adapted to capture imperfect investors by making a fraction of AUM "sticky." Suppose that a fraction ϕ of investors allocate their capital to a fund and then subsequently ignore the fund's performance. It is not obvious whether the existence of sticky investors influences the results in Sections 3 and 4.[21]

Consider a fund that starts with AUM of K_1. If the fund succeeds and earns trading profits at $t = 1$ the fund should have $K_2(1)$ in AUM in the subsequent period. As $K_2(1) > K_1$, the existence of sticky investors should not lead to over- or under-investment in the fund. Our earlier results are consistent with this event.

Conversely, if the fund fails at $t = 1$ the AUM should drop to $K_2(0)$ in the following period. If $\phi K_1 \leq K_2(0)$, the non-sticky investors will exit until the fund is not over- or under-invested in. Again, our results are not compromised. If, however, $\phi K_1 > K_2(0)$, all non-sticky investors will exit the fund, but there will still be too many dollars under management. Consequently, the existence of sticky investors challenges the model's results only if ϕ is sufficiently large, i.e.,

$$\phi > \frac{K_2(0)}{K_1}, \tag{37}$$

Such a case creates an incentive for funds to shirk on producing research, so we predict that the fraction of closet funds is increasing with the population of sticky investors.[22] Furthermore, if funds can expend resources on capturing sticky investors, and the expense is not perfectly observable, funds will undertake it. This is because the presence of sticky investors provides a hedge against fund outflows. We expect funds to expend resources on capturing sticky investors until the marginal cost equates the marginal benefit of the hedge. Consequently, assuming all funds face the same cost function, low-type funds should, in equilibrium, spend more on marketing than high-type funds. Furthermore, closet funds should spend more on marketing than any active fund, as the hedging benefit is largest for them.

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the costs of liquidity motivated trading and Glode (2010) shows that negative unconditional performance is consistent with managers expending more effort (and providing better returns) in bad states of the world.

[21] Del Guercio and Reuter (2014) consider a clientele effect in the managed fund industry. The authors show that different market segments compete for different types of investors. The first segment markets funds directly to retail investors and face the strongest incentives to invest in active management. The second segment markets its products to brokers, who subsequently sell them to clients. The latter, the authors conjecture, face the weakest incentives to invest in active management. The authors show that including the clientele effect has important implications: directly marketed funds outperform index funds, while funds sold through brokers do not.

[22] Consistent with our intuition, Gil-Bazo and Ruiz-Verdu (2009) find a negative relationship between before-fee performance and fees. The results seem to imply that funds with sticky investors charge higher management fees.

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Appendix A

Proof of Lemma 1:

The expressions in (14) and (15) are rewritten as,

$$K_1 = \Pr(S_1|K_1, \eta)\frac{\gamma}{F_E}, \tag{A1}$$

$$K_2 = \Pr(S_2|K_2, \eta, \mathbb{1}_{S_1})\frac{\gamma}{F_E}. \tag{A2}$$

The expressions are expanded to incorporate (5),

$$K_1 = \frac{\Pr(S_1^\star)}{1 + \eta K_{1,t}}\frac{\gamma}{F_E}, \tag{A3}$$

$$K_2 = \frac{\Pr(S_2^\star|\mathbb{1}_{S_1})}{1 + \eta K_{2,t}}\frac{\gamma}{F_E}. \tag{A4}$$

The non-negative solutions to the preceding expressions are given by,

$$K_1 = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4\eta^2 F_E}}, \tag{A5}$$

$$K_2(\mathbb{1}_{S_1}) = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|\mathbb{1}_{S_1})}{4\eta^2 F_E}}. \tag{A6}$$

∎

Proof of Lemma 2:

Without loss of generality, consider that all funds above $\theta^* \in [0, 1]$ choose to pay c. Define,

$$\Pr(S_1^\star) \equiv \Pr(S_1|K_1, \eta)(1 + \eta K_1), \tag{A7}$$

$$\Pr(S_2^\star|\mathbb{1}_{S_1}) \equiv \Pr(S_2|K_2, \eta, \mathbb{1}_{S_1})(1 + \eta K_2). \tag{A8}$$

The explicit forms of $\Pr(S_1^\star)$, $\Pr(S_2^\star|1)$, and $\Pr(S_2^\star|0)$ are now derived, starting with $\Pr(S_1^\star)$,

$$\Pr(S_1^\star) = \Pr(S_1^\star|A)\Pr(A) + \Pr(S_1^\star|C)\Pr(C) \tag{A9}$$

$$= \Pr(S_1^\star|A)\Pr(A) \tag{A10}$$

$$= \frac{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 g(\theta)\,\mathrm{d}\theta}\left(\int_{\theta^*}^1 g(\theta)\,\mathrm{d}\theta\right) \tag{A11}$$

$$= \int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta. \tag{A12}$$

Now, consider $\Pr(S_2^\star|1)$,

$$\Pr(S_2^\star|1) = \Pr(S_2^\star|A,1)\Pr(A|1) + \Pr(S_2^\star|C,1)\Pr(C|1) \tag{A13}$$

$$= \Pr(S_2^\star|A,1)\Pr(A|1) \tag{A14}$$

$$= \frac{\Pr(S_2^\star \cap 1|A)}{\Pr(1|A)}\Pr(A|1) \tag{A15}$$

$$= \frac{\int_{\theta^*}^1 \theta^2 g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}. \tag{A16}$$

Now, consider $\Pr(S_2^\star|0)$,

$$\Pr(S_2^\star|0) = \Pr(S_2^\star|A,0)\Pr(A|0) + \Pr(S_2^\star|C,1)\Pr(C|0) \tag{A17}$$

$$= \Pr(S_2^\star|A,0)\Pr(A|0) \tag{A18}$$

$$= \Pr(S_2^\star|A,0)\frac{\Pr(A \cap 0)}{\Pr(0)} \tag{A19}$$

$$= \frac{\Pr(S_2^\star \cap 0|A)}{\Pr(0|A)}\frac{\Pr(0|A)\Pr(A)}{\Pr(0|A)\Pr(A) + \Pr(0|C)\Pr(C)} \tag{A20}$$

$$= \left(\frac{\frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{1-G(\theta^*)}}{\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{1-G(\theta^*)}}\right)\left(\frac{(1-G(\theta^*))\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{(1-G(\theta^*))}}{(1-G(\theta^*))\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{(1-G(\theta^*))} + G(\theta^*)}\right) \tag{A21}$$

$$= \left(\frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}\right)\left(\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)}\right) \tag{A22}$$

$$= \frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)} \tag{A23}$$

Note that $\Pr(S_1^\star)$ is decreasing in θ^*, $\Pr(S_2^\star|1)$ is increasing in θ^*, and $\Pr(S_2^\star|0)$ is decreasing in θ^*,

$$\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} = -\theta^* g(\theta^*) \tag{A24}$$

$$\leq 0. \tag{A25}$$

$$\frac{\partial \Pr(S_2^\star|1)}{\partial \theta^*} = \frac{\theta^* g(\theta^*)}{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}\left(\Pr(S_2^\star|1) - \theta^*\right), \tag{A26}$$

which is positive because $\int_{\theta^*}^1 (\theta^2 - \theta^*\theta)g(\theta)\,d\theta \geq 0$,

$$\geq 0. \tag{A27}$$

$$\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*} = \frac{-\theta^*(1-\theta^*)g(\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,d\theta + G(\theta^*)\right) - \theta^* g(\theta^*)\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,d\theta}{\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,d\theta + G(\theta^*)\right)^2} \tag{A28}$$

$$= -g(\theta^*)\theta^* \frac{(1-\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,d\theta + G(\theta^*)\right) + \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,d\theta}{\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,d\theta + G(\theta^*)\right)^2} \tag{A29}$$

$$\leq 0. \tag{A30}$$

The profit functions in (16) are rewritten using (14) and (15),

$$\Pi_j(a_j, F_E) = \begin{cases} (F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4\eta^2 F_E}} + \sqrt{\frac{F_E + 4\gamma\eta E[\theta|0]}{4\eta^2 F_E}} - \frac{1}{\eta}\right) & \text{if a} = C, \\ (F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4\eta^2 F_E}} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|\mathbb{1}_{S_1})}{4\eta^2 F_E}} - \frac{1}{\eta}\right) - c & \text{if a} = A. \end{cases} \tag{A31}$$

The profit functions in A31 yield an explicit form of $\Delta(\theta)$. The function $\Delta(\theta)$ defined in (18), is explicitly given by,

$$\Delta(\theta) = \frac{\theta(F_E + F_P)}{1 + \eta K_1}\left(\sqrt{\frac{F_E + 4\gamma\eta E[\theta|1]}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta E[\theta|0]}{4\eta^2 F_E}}\right) - c. \tag{A32}$$

If an internal value of θ satisfies $\Delta(\theta) = 0$, the following equality implicitly defines θ^*,

$$\theta^* = \frac{c\left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\right)}{(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0)}{4\eta^2 F_E}}\right)}. \tag{A33}$$

∎

Proof of Proposition 1:

Before proving the proposition, we derive the following expression for any two values of $x \geq 0$

and $y \geq 0$. The expression is helpful in signing the comparative statics we study hereafter,

$$\frac{x}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\eta^2 F_E}}} - \frac{y}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\eta^2 F_E}}} \gtreqqless 0 \tag{A34}$$

$$\frac{x^2}{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\eta^2 F_E}} \gtreqqless \frac{y^2}{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\eta^2 F_E}} \tag{A35}$$

$$\frac{x^2}{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)} \gtreqqless \frac{y^2}{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)} \tag{A36}$$

$$x^2\left(F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)\right) \gtreqqless y^2\left(F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)\right) \tag{A37}$$

$$\frac{F_E}{4\gamma\eta} \gtreqqless \frac{y^2\Pr(S_2^\star|1,\hat\theta) - x^2\Pr(S_2^\star|0,\hat\theta)}{(x^2 - y^2)}. \tag{A38}$$

Note if $x = a + b\Pr(S_2^\star|1,\hat\theta)$ and $y = a + b\Pr(S_2^\star|0,\hat\theta)$ for two constants a and b that the expression equals,

$$\frac{F_E}{4\gamma\eta} \gtreqqless \frac{a^2 - b^2\Pr(S_2^\star|1,\hat\theta)\Pr(S_2^\star|0,\hat\theta)}{b^2(\Pr(S_2^\star|1,\hat\theta) + \Pr(S_2^\star|0,\hat\theta)) + 2ab}. \tag{A39}$$

Therefore,

$$\frac{a + b\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\eta^2 F_E}}} - \frac{a + b\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\eta^2 F_E}}} \to \begin{cases} \geq 0 & \text{if } \frac{F_E}{4\gamma\eta} \geq \frac{a^2 - b^2\Pr(S_2^\star|1,\hat\theta)\Pr(S_2^\star|0,\hat\theta)}{b^2(\Pr(S_2^\star|1,\hat\theta) + \Pr(S_2^\star|0,\hat\theta)) + 2ab}, \\ < & \text{otherwise.} \end{cases} \tag{A40}$$

Now we begin the proof of the proposition. Define

$$\Psi(\hat\theta) \equiv \hat\theta - \frac{c\left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\right)}{(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)}. \tag{A41}$$

Note that $\Psi(\theta^*) = 0$. The function $\Psi(\hat\theta)$ can be utilized with the implicit function theorem to characterize θ^*,

$$\frac{\partial\theta^*}{\partial\omega} = -\left.\frac{\partial\Psi/\partial\omega}{\partial\Psi/\partial\hat\theta}\right|_{\hat\theta=\theta^*}, \tag{A42}$$

for $\omega \in \{c, F_P, \gamma, \eta, F_E\}$. First we sign the partial derivatives, beginning with $\partial \Psi / \partial \hat{\theta}$,

$$\frac{\partial \Psi}{\partial \hat{\theta}} = 1 + \frac{c\gamma \left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}} \right) \left(\frac{\partial \Pr(S_2^\star|1,\hat{\theta})/\partial\hat{\theta}}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{\eta^2 F_E}}} - \frac{\partial \Pr(S_2^\star|0,\hat{\theta})/\partial\hat{\theta}}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{\eta^2 F_E}}} \right)}{\eta F_E (F_E + F_P) \left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}} \right)^2}$$

$$- \frac{c\gamma\eta \frac{\partial \Pr(S_1^\star|\hat{\theta})}{\partial\hat{\theta}}}{2F_E(F_E + F_P)\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}} \left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}} \right)}. \tag{A43}$$

Note that $\partial \Pr(S_2^\star|1,\hat{\theta})/\partial\hat{\theta} \geq 0$, $\partial \Pr(S_2^\star|0,\hat{\theta})/\partial\hat{\theta} \leq 0$, and $\partial \Pr(S_1^\star|\hat{\theta})/\partial\hat{\theta} \leq 0$,

$$\geq 0. \tag{A44}$$

Now, we consider $\partial \Psi / \partial c$,

$$\frac{\partial \Psi}{\partial c} = - \frac{\left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}} \right)}{(F_E + F_P) \left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}} \right)} \tag{A45}$$

$$\leq 0. \tag{A46}$$

Now, we consider $\partial \Psi / \partial F_P$,

$$\frac{\partial \Psi}{\partial F_P} = \frac{c \left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}} \right)}{(F_E + F_P)^2 \left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}} \right)} \tag{A47}$$

$$\geq 0. \tag{A48}$$

Now, we consider $\partial\Psi/\partial\gamma$,

$$
\begin{aligned}
\frac{\partial\Psi}{\partial\gamma} &= \frac{c\left(\frac{1}{2} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\right)\left(\frac{\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}}} - \frac{\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}}\right)}{2\eta F_E(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)^2} \\[2ex]
&\quad - \frac{c\eta\Pr(S_1^\star|\hat\theta)}{2F_E(F_E + F_P)\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)}. \quad\text{(A49)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c\left(\frac{\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}}} - \frac{\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}}\right)}{4\eta F_E(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)^2} \\[2ex]
&\quad + \frac{c\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\left(\frac{\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}}} - \frac{\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}}\right)}{2\eta F_E(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)^2} \\[2ex]
&\quad - \frac{c\eta\Pr(S_1^\star|\hat\theta)}{2F_E(F_E + F_P)\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)}. \quad\text{(A50)}
\end{aligned}
$$

It is simple to show that the first term of preceding expression is positive using (A40) where $a = 0$ and $b = 1$. The combination of the second and third terms, however is not as obvious. We now show that the combination of the second and third terms is positive,

$$0 \leq \frac{c\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}}\left(\frac{\Pr(S_2^\star|1,\hat{\theta})}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}}} - \frac{\Pr(S_2^\star|0,\hat{\theta})}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}}}\right)}{2\eta F_E(F_E + F_P)\left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}}\right)^2}$$

$$- \frac{c\eta \Pr(S_1^\star|\hat{\theta})}{2F_E(F_E + F_P)\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}}\right)}. \tag{A51}$$

$$= \frac{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}\left(\frac{\Pr(S_2^\star|1,\hat{\theta})}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4F_E}}} - \frac{\Pr(S_2^\star|0,\hat{\theta})}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4F_E}}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}}\right)^2}$$

$$- \frac{\Pr(S_1^\star|\hat{\theta})\left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat{\theta})}{4\eta^2 F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat{\theta})}{4\eta^2 F_E}}\right)^2}, \tag{A52}$$

which simplifies to,

$$
= \left(\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{4F_E} \left(\frac{\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}{4F_E}}} - \frac{\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}{4F_E}}} \right) \right)
$$
$$
- \left(\Pr(S_1^\star|\hat\theta) \left(\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}{4F_E}} - \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}{4F_E}} \right) \right), \tag{A53}
$$

$$
= \left((F_E + 4\gamma\eta \Pr(S_1^\star)) \left(\frac{\Pr(S_2^\star|1,\hat\theta)\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)} - \Pr(S_2^\star|0,\hat\theta)\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}}{\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}} \right) \right)
$$
$$
- \left(\Pr(S_1^\star|\hat\theta) \left(\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)} - \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)} \right) \right), \tag{A54}
$$

$$
= (F_E + 4\gamma\eta \Pr(S_1^\star)) \left(\Pr(S_2^\star|1,\hat\theta)\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)} - \Pr(S_2^\star|0,\hat\theta)\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)} \right)
$$
$$
- \Pr(S_1^\star|\hat\theta) \left(F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta) \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}
$$
$$
+ \Pr(S_1^\star|\hat\theta) \left(F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta) \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}, \tag{A55}
$$

$$
= \left(\Pr(S_2^\star|1,\hat\theta) (F_E + 4\gamma\eta \Pr(S_1^\star)) - \Pr(S_1^\star|\hat\theta) \left(F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta) \right) \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}
$$
$$
- \left(\Pr(S_2^\star|0,\hat\theta) (F_E + 4\gamma\eta \Pr(S_1^\star)) - \Pr(S_1^\star|\hat\theta) \left(F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta) \right) \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}, \tag{A56}
$$

$$
= \left(\Pr(S_2^\star|1,\hat\theta)F_E - \Pr(S_1^\star|\hat\theta)F_E \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}
$$
$$
- \left(\Pr(S_2^\star|0,\hat\theta)F_E - \Pr(S_1^\star|\hat\theta)F_E \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}, \tag{A57}
$$

$$
= \left(\Pr(S_2^\star|1,\hat\theta)F_E - \Pr(S_1^\star|\hat\theta)F_E \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0,\hat\theta)}
$$
$$
+ \left(\Pr(S_1^\star|\hat\theta)F_E - \Pr(S_2^\star|0,\hat\theta)F_E \right) \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|1,\hat\theta)}, \tag{A58}
$$
$$
\geq 0. \tag{A59}
$$

Therefore, the comparative static is,

$$
\frac{\partial \Psi}{\partial \gamma} \geq 0 \tag{A60}
$$

Now, we consider $\partial\Psi/\partial\eta$,

$$\frac{\partial\Psi}{\partial\eta} = -\frac{8c\left(\frac{F_E+2\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\eta^2 F_E}}} - \frac{F_E+2\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\eta^2 F_E}}}\right)}{\eta^3 F_E(F_E+F_P)\left(\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)^2}$$
$$-\frac{c\gamma\Pr(S_1^\star|\hat\theta)}{2F_E(F_E+F_P)\sqrt{\frac{F_E+4\gamma\eta\Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)}. \tag{A61}$$

It is simple to show that the first term of preceding expression is negative using (A40) where $a = F_E$ and $b = 2\gamma\eta$ and the second term is strictly negative,

$$\leq 0. \tag{A62}$$

The sign of $\partial\Psi/\partial F_E$, however, is equivocal,

$$\frac{\partial\Psi}{\partial F_E} = \frac{c\left(\frac{F_E^2+2\eta\gamma(F_E-F_P)\Pr(S_2^\star|1,\hat\theta)}{\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{\eta^2 F_E}}} - \frac{F_E^2+2\eta\gamma(F_E-F_P)\Pr(S_2^\star|0,\hat\theta)}{\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{\eta^2 F_E}}}\right)}{2F_E^2\eta^2(F_E+F_P)^2\left(\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)^2}$$
$$+\frac{\frac{c\gamma\Pr(S_1^\star|\hat\theta)}{F_E^2}}{2(F_E+F_P)\sqrt{\frac{F_E+4\gamma\eta\Pr(S_1^\star)}{4F_E}}\left(\sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|1,\hat\theta)}{4\eta^2 F_E}} - \sqrt{\frac{F_E+4\gamma\eta\Pr(S_2^\star|0,\hat\theta)}{4\eta^2 F_E}}\right)}. \tag{A63}$$

The sign on the first term in the preceding expression is determined using (A40) where $a = F_E^2$ and $b = 2\gamma\eta(F_E - F_P)$. A substitution into (A40) yields,

$$\frac{F_E}{4\gamma\eta} \gtreqqless \frac{F_E^4 - 4\gamma^2\eta^2(F_E-F_P)^2\Pr(S_2^\star|1,\hat\theta)\Pr(S_2^\star|0,\hat\theta)}{4\gamma^2\eta^2(F_E-F_P)^2(\Pr(S_2^\star|1,\hat\theta)+\Pr(S_2^\star|0,\hat\theta)) + 4F_E^2\gamma\eta(F_E-F_P)}. \tag{A64}$$

The unknown inequality is reorganized as,

$$0 \gtreqqless F_E^4 - 4\gamma^2\eta^2(F_E-F_P)^2\Pr(S_2^\star|1,\hat\theta)\Pr(S_2^\star|0,\hat\theta)$$
$$-\left(F_E\gamma\eta(F_E-F_P)^2(\Pr(S_2^\star|1,\hat\theta)+\Pr(S_2^\star|0,\hat\theta)) + F_E^3(F_E-F_P)\right), \tag{A65}$$

which implies that the sign on the first term of the comparative static $\partial\Psi/\partial F_E$ is negative as $F_E \to F_P$ and is positive if F_E is sufficiently large. Notice, that the second term is strictly positive, but goes to zero as F_E gets large.

The following summarizes the comparative statics,

$$\frac{\partial \theta^*}{\partial c} = - \left. \frac{\partial \Psi / \partial c}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta} = \theta^*} \tag{A66}$$

$$\geq 0. \tag{A67}$$

$$\frac{\partial \theta^*}{\partial F_P} = - \left. \frac{\partial \Psi / \partial F_P}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta} = \theta^*} \tag{A68}$$

$$\leq 0. \tag{A69}$$

$$\frac{\partial \theta^*}{\partial \gamma} = - \left. \frac{\partial \Psi / \partial \gamma}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta} = \theta^*} \tag{A70}$$

$$\leq 0. \tag{A71}$$

$$\frac{\partial \theta^*}{\partial \eta} = - \left. \frac{\partial \Psi / \partial \eta}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta} = \theta^*} \tag{A72}$$

$$\geq 0. \tag{A73}$$

$$\frac{\partial \theta^*}{\partial F_E} = - \left. \frac{\partial \Psi / \partial F_E}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta} = \theta^*} \tag{A74}$$

$$\gtreqless 0 \text{ the comparative static is equivocal.} \tag{A75}$$

∎

Proof of Lemma 3:

An explicit form for $\Delta K(\mathbb{1}_{S_1})$ is given by,

$$\Delta K(\mathbb{1}_{S_1}) = \frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1. \tag{A76}$$

Now, consider that a fund succeeds. According to (A76), the fund's net fund flow is given by,

$$\Delta K(1) = \frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta E[\theta|1]}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1, \tag{A77}$$

which is strictly positive because $E[\theta|1] > \Pr(S_1^\star)$,

$$> 0. \tag{A78}$$

A similar analysis demonstrates that $\Delta K(0) < 0$.

∎

Proof of Corollary 1.1:

Before computing the comparative statics, the following analysis provides useful insights that are subsequently used in signing the comparative statics. Define $\nu(\mathbb{1}_{S_1}, x, y)$ as,

$$\nu(\mathbb{1}_{S_1}, x, y) \equiv \frac{\left(\frac{\partial \Pr(S_2^\star | \mathbb{1}_{S_1})}{\partial \theta^*} + x\right)\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}} - \frac{\left(\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} + y\right)\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}}$$

(A79)

First, consider $\nu(1, 0, 0)$,

$$\nu(1, 0, 0) \equiv \frac{\frac{\partial \Pr(S_2^\star | 1)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 1)}{F_E}}} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 1)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}}$$

(A80)

$$\geq 0,$$

(A81)

because $\frac{\partial \Pr(S_2^\star | 1)}{\partial \theta^*} \geq 0$ and $\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} \leq 0$, according to (A24) and (A26). Now, consider $\nu(0, 0, 0)$,

$$\nu(0, 0, 0) \equiv \frac{\frac{\partial \Pr(S_2^\star | 0)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}}} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}}$$

(A82)

Note that $\frac{\partial \Pr(S_2^\star | 0)}{\partial \theta^*} \leq 0$ and $\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} \leq 0$. This implies that the preceding expression is negative if,

$$0 \leq \frac{-\frac{\partial \Pr(S_2^\star | 0)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}}} - \frac{-\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}}.$$

(A83)

The preceding expression becomes,

$$-\frac{\partial \Pr(S_2^\star | 0)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}$$

$$\geq -\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}}\right)\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}{F_E}},$$

(A84)

which is rewritten as,

$$\frac{\frac{\partial \Pr(S_2^\star | 0)}{\partial \theta^*}}{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}} \geq \frac{\left(-\sqrt{F_E} + \sqrt{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}\right)\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star | 0)}}{\left(-\sqrt{F_E} + \sqrt{F_E + 4\gamma\eta \Pr(S_1^\star)}\right)\sqrt{F_E + 4\gamma\eta \Pr(S_1^\star)}}.$$

(A85)

The right-hand side of the preceding inequality is at its largest in the limit as $F_E \to \infty$. As such, if the inequality holds in the limit, it must hold for all $F_E \geq 0$. Note that the limit of the right-hand

side is equal to $\Pr(S_2^\star|0)/\Pr(S_1^\star)$,[23]

$$\frac{\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}} \geq \frac{\Pr(S_2^\star|0)}{\Pr(S_1^\star)}. \tag{A86}$$

The expression is expanded using (A12), (A23), (A24), and (A29),

$$\frac{-g(\theta^*)\theta^* \frac{(1-\theta^*)\left(\int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right) + \int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\left(\int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right)^2}}{-\theta^* g(\theta^*)} \geq \frac{\frac{\int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)}}{\int_{\theta*}^1 \theta g(\theta)\,\mathrm{d}\theta}, \tag{A87}$$

which simplifies to,

$$\frac{(1-\theta^*)\left(\int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right) + \int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)} \geq \frac{\int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta*}^1 \theta g(\theta)\,\mathrm{d}\theta}. \tag{A88}$$

The inequality expands to,

$$\int_{\theta*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\int_{\theta*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta + \int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta*}^1 \theta g(\theta)\,\mathrm{d}\theta$$

$$\geq \int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta. \tag{A89}$$

The inequality holds because,

$$\int_{\theta*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta \geq \int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta, \tag{A90}$$

and,

$$G(\theta^*)\int_{\theta*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \geq G(\theta^*)\int_{\theta*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta. \tag{A91}$$

Therefore, $\nu(0,0,0) \leq 0$. Furthermore, note that if $x \geq y$ then $\nu(1,x,y) \geq 0$ and if $x \leq y$ then $\nu(0,x,y) \leq 0$.

Now, consider the comparative statics of a fund's net fund flow with respect to $\{c, F_P, \gamma, \eta, F_E\}$.

---

[23]Evaluating the limit of the function requires an infinite applications of L'Hopital's rule, which converge to $\Pr(S_2^\star|0)/\Pr(S_1^\star)$.

First, consider the comparative static with respect to c,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial c} = \frac{\partial}{\partial c} \left(\frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1 \right) \tag{A92}$$

$$= \frac{\frac{2\gamma\eta}{F_E} \frac{\partial \theta^*}{\partial c}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} \right)^2} \left(\frac{\frac{\partial \Pr(S_2^\star | \mathbb{1}_{S_1})}{\partial \theta^*} \left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E} \right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}} \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right.$$

$$\left. - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} \left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E} \right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}} \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right) \tag{A93}$$

$$= \frac{\frac{2\gamma\eta}{F_E} \frac{\partial \theta^*}{\partial c}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} \right)^2} \nu(\mathbb{1}_{S_1}, 0, 0) \tag{A94}$$

$$= \begin{cases} \geq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \leq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A95}$$

Now, consider the comparative static with respect to F_P,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial F_P} = \frac{\partial}{\partial F_P} \left(\frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1 \right). \tag{A96}$$

The prior analysis with respect to c implies that the preceding expression simplifies to,

$$= \frac{\frac{2\gamma\eta}{F_E} \frac{\partial \theta^*}{\partial F_P}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} \right)^2} \nu(\mathbb{1}_{S_1}, 0, 0) \tag{A97}$$

$$= \begin{cases} \leq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \geq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A98}$$

Now, consider the comparative static with respect to γ,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left(\frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1 \right) \tag{A99}$$

$$= \frac{\frac{2\gamma\eta}{F_E}\frac{\partial\theta^*}{\partial\gamma}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)^2} \left(\frac{\left(\frac{\partial \Pr(S_2^\star | \mathbb{1}_{S_1})}{\partial\theta^*} + \frac{\Pr(S_2^\star | \mathbb{1}_{S_1})}{\gamma}\right)\left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right.$$

$$\left. - \frac{\left(\frac{\partial \Pr(S_1^\star)}{\partial\theta^*} + \frac{\Pr(S_1^\star)}{\gamma}\right)\left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right) \tag{A100}$$

$$= \frac{\frac{2\gamma\eta}{F_E}\frac{\partial\theta^*}{\partial\gamma}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)^2} \nu\left(\mathbb{1}_{S_1}, \frac{\Pr(S_2^\star | \mathbb{1}_{S_1})}{\gamma}, \frac{\Pr(S_1^\star)}{\gamma}\right) \tag{A101}$$

$$= \begin{cases} \leq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \geq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A102}$$

Now, consider the comparative static with respect to η,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial \eta} = \frac{\partial}{\partial \eta} \left(\frac{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}}{-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} - 1 \right) \tag{A103}$$

$$= \frac{\frac{2\gamma\eta}{F_E}\frac{\partial\theta^*}{\partial\eta}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)^2} \left(\frac{\left(\frac{\partial \Pr(S_2^\star | \mathbb{1}_{S_1})}{\partial\theta^*} + \frac{\Pr(S_2^\star | \mathbb{1}_{S_1})}{\eta}\right)\left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right.$$

$$\left. - \frac{\left(\frac{\partial \Pr(S_1^\star)}{\partial\theta^*} + \frac{\Pr(S_1^\star)}{\eta}\right)\left(-\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}} + \frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}\right)}{\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_2^\star | \mathbb{1}_{S_1})}{F_E}}\sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}} \right) \tag{A104}$$

$$= \frac{\frac{2\gamma\eta}{F_E}\frac{\partial\theta^*}{\partial\eta}}{\left(-1 + \sqrt{\frac{F_E + 4\gamma\eta \Pr(S_1^\star)}{F_E}}\right)^2} \nu\left(\mathbb{1}_{S_1}, \frac{\Pr(S_2^\star | \mathbb{1}_{S_1})}{\gamma}, \frac{\Pr(S_1^\star)}{\gamma}\right) \tag{A105}$$

$$= \begin{cases} \geq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \leq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A106}$$

∎

Proof of Lemma 4:

We are concerned with the relative fund flow sensitivity comparison of success to failure,

$$\left| \frac{K_2(1) - K_1}{K_1} \right| - \left| \frac{K_2(0) - K_1}{K_1} \right|. \tag{A107}$$

The preceding expression is rewritten as,

$$\frac{K_2(1) - K_1}{K_1} + \frac{K_2(0) - K_1}{K_1}, \tag{A108}$$

or alternatively as,

$$\frac{K_2(1)}{K_1} + \frac{K_2(0)}{K_1} - 2. \tag{A109}$$

The sign on the preceding expression is determined by,

$$\underbrace{(K_2(1) - K_1)}_{a} - \underbrace{(K_1 - K_2(0))}_{b}, \tag{A110}$$

Clearly, because $\frac{\partial \Pr(S_2^\star|1)}{\partial \theta^*} \geq 0$ and $\frac{\partial \Pr(S_1^\star)}{\partial \theta^*} \leq 0$, the component labeled a is at its smallest when $\theta^* = 0$. Now consider the component of the inequality labeled b. That component is globally decreasing in θ^*, but it remains to be shown that it is at its largest value when $\theta^* = 0$. To see this, consider the change in b with respect to θ^*,

$$\frac{\partial b}{\partial \theta^*} = \frac{\partial K_1}{\partial \theta^*} - \frac{\partial K_2(0)}{\partial \theta^*} \tag{A111}$$

$$= \frac{4\gamma\eta \frac{\partial \Pr(S_1^\star)}{\partial \theta^*}}{\sqrt{(4\eta^2 F_E)(F_E + 4\gamma\eta \Pr(S_1^\star))}} - \frac{4\gamma\eta \frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}{\sqrt{(4\eta^2 F_E)(F_E + 4\gamma\eta \Pr(S_2^\star|0))}}, \tag{A112}$$

which is negative if,

$$0 \geq \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}}{\sqrt{F_E + 4\gamma\eta \Pr(S_1^\star)}} - \frac{\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}{\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0)}}, \tag{A113}$$

The above inequality implies the following inequality,

$$\frac{\sqrt{F_E + 4\gamma\eta \Pr(S_1^\star)}}{\sqrt{F_E + 4\gamma\eta \Pr(S_2^\star|0)}} \geq \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}}{\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}, \tag{A114}$$

and the left-hand side is necessarily larger than 1. Consequently, the inequality is satisfied if,

$$1 \geq \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}}{\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}, \tag{A115}$$

which holds according to the explicit forms outlined in (A26) and (A29). Now, because the expression in (A110) is at its smallest at $\theta^* = 0$, at its largest at $\theta^* = 1$, and continuous in θ^*, we

can evaluate the function at each extreme and determine whether or not the sign changes. First, consider $\theta^* = 0$ and note that

$$\Pr(S_2^\star|1) + \Pr(S_1^\star|0) - 2\Pr(S_1^\star)|_{\theta^*=0} = 0, \tag{A116}$$

which implies that,

$$\Pr(S_1^\star) = \left.\frac{\Pr(S_2^\star|1) + \Pr(S_1^\star|0)}{2}\right|_{\theta^*=0}. \tag{A117}$$

Notice,

$$\sqrt{m + nx_1} + \sqrt{m + nx_2} - 2\sqrt{m + n\frac{x_1 + x_2}{2}} \le 0, \tag{A118}$$

for all combinations of $m > 0$ and $n > 0$. This implies,

$$(K_2(1) - K_1) - (K_1 - K_2(0))|_{\theta^*=0} < 0. \tag{A119}$$

Now, consider $\theta^* = 1$. Clearly, as $\theta^* \to 1$, $\Pr(S_2^\star|1) \to 1$, $\Pr(S_1^\star|0) \to 0$, $\Pr(S_1^\star) \to 0$. This naturally implies,

$$(K_2(1) - K_1) - (K_1 - K_2(0))|_{\theta^*=1} > 0. \tag{A120}$$

Therefore, because (A110) is strictly negative at $\theta^* = 0$, strictly positive at $\theta^* = 1$, and continuous in θ^*, there exists a value $\theta^S \in (0, 1)$, for which the fund flows for both $S = 0$ and $S = 1$ are symmetric.

∎

Proof of Lemma 5:

The proof follows the proof of Lemma 1.

∎

Proof of Lemma 7:

The profit function for a fund with type θ is given by,

$$\Pi_j(a_j, \theta) = \begin{cases} (F_E + F_P)\,(K_1 + K_2(0)) & \text{if } a_j = C \\ (F_E + F_P)\left(K_1 + \frac{\theta}{1+\eta K_1}K_2(1) + \left(1 - \frac{\theta}{1+\eta K_1}\right)K_2(0)\right) \\ + \frac{\theta\lambda\gamma}{1+\eta K_1} + \frac{\theta^2\lambda\gamma}{(1+\eta K_2(1))(1+\eta K_1)} + \left(1 - \frac{\theta}{(1+\eta K_1)}\right)\frac{\theta\lambda\gamma}{(1+\eta K_2(0))} - c & \text{if } a_j = A. \end{cases} \tag{A121}$$

Using A153, define $\Delta^\lambda(\theta)$ as a benefit-cost function for a fund with type θ,

$$\Delta^\lambda(\theta) \equiv \Pi_j(A, \theta) - \Pi_j(C, \theta) \tag{A122}$$

$$= \frac{\theta(F_E + F_P)}{1 + \eta K_1}(K_2(1) - K_2(0)) + \frac{\theta\lambda\gamma}{1 + \eta K_1}$$

$$+ \frac{\theta^2\lambda\gamma}{(1 + \eta K_2(1))(1 + \eta K_1)} + \left(1 - \frac{\theta}{(1 + \eta K_1)}\right)\frac{\theta\lambda\gamma}{(1 + \eta K_2(0))} - c, \tag{A123}$$

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and note that $\Delta^\lambda(0) < 0$.

If an internal threshold exists, it is implicitly defined by $\Delta^\lambda(\theta^\lambda) = 0$, otherwise $\theta^\lambda = 1$. Consider that θ^λ is an internal value. The expression in (A123) equals zero at,

$$0 = \frac{\theta(F_E + F_P)}{1 + \eta K_1}(K_2(1) - K_2(0)) + \frac{\theta\lambda\gamma}{1 + \eta K_1}$$
$$+ \frac{\theta^2\lambda\gamma}{(1 + \eta K_2(1))(1 + \eta K_1)} + \left(1 - \frac{\theta}{(1 + \eta K_1)}\right)\frac{\theta\lambda\gamma}{(1 + \eta K_2(0))} - c. \tag{A124}$$

Define

$$x = \frac{1}{1 + \eta K_1} \tag{A125}$$

$$y = \frac{1}{1 + \eta K_2(1)} \tag{A126}$$

$$z = \frac{1}{1 + \eta K_2(0)} \tag{A127}$$

And note that $y \le x \le z$. The expression becomes,

$$0 = \theta^\lambda(F_E + F_P)x\,(K_2(1) - K_2(0)) + \theta^\lambda x\left(\lambda\gamma + \lambda\gamma\theta^\lambda y - \lambda\gamma\theta^\lambda z\right) + \theta^\lambda\lambda\gamma z - c. \tag{A128}$$

$$0 = \theta^{\lambda 2}\underbrace{x\lambda\gamma\,(y - z)}_{a} + \theta^\lambda\underbrace{((F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z)}_{b} - \underbrace{c}_{c}. \tag{A129}$$

The quadratic formula yields,

$$\theta^\lambda = -\frac{(F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z}{2x\lambda\gamma\,(y - z)}$$
$$\pm \frac{\sqrt{((F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z)^2 + 4x\lambda\gamma\,(y - z)\,c}}{2x\lambda\gamma\,(y - z)}. \tag{A130}$$

Consider the first term in the quadratic formula above (and notice the negative sign drops when you switch the term of the denominator to $(z - y)$ instead of $(y - z)$),

$$\frac{(F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z}{2x\lambda\gamma\,(z - y)} = \underbrace{\frac{(F_E + F_P)\,(K_2(1) - K_2(0))}{2\lambda\gamma\,(z - y)}}_{\ge 0} + \underbrace{\frac{1}{2\,(z - y)}}_{\ge \frac{1}{2}} + \underbrace{\frac{z}{2x\,(z - y)}}_{\ge \frac{1}{2}}$$
$$\tag{A131}$$
$$\ge 1, \tag{A132}$$

because

$$1 \le \frac{1}{(z - y)}, \tag{A133}$$

$$1 \le \frac{z}{x}. \tag{A134}$$

Therefore, if an internal solution exists, it is unique and it must be of the form,

$$\theta^\lambda = \frac{(F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z}{2x\lambda\gamma\,(z - y)}$$
$$- \frac{\sqrt{((F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\gamma x + \lambda\gamma z)^2 - 4x\lambda\gamma\,(z - y)\,c}}{2x\lambda\gamma\,(z - y)}. \tag{A135}$$

■

Proof of Lemma 7:

To prove the proposition, we consider the two limiting cases, when $\lambda = 0$ and when $\lambda = 1$, and show that the threshold is increasing globally for some parameter sets and decreasing for others. If $\lambda = 0$, the threshold θ^λ is equal to the threshold θ^* introduced in Lemma 2 and is implicitly defined by,

$$0 = \frac{\theta^\lambda_{\lambda=0}(F_E + F_P)}{1 + \eta K_1}\,(K_2(1) - K_2(0)) - c. \tag{A136}$$

If $\lambda = 1$, the threshold θ^λ is implicitly defined by,

$$0 = 2\gamma\theta^\lambda_{\lambda=1} - c. \tag{A137}$$

The two preceding equations imply $\theta^\lambda_{\lambda=1} < \theta^\lambda_{\lambda=0}$ if

$$2\gamma > \frac{(F_E + F_P)}{1 + \eta K_1}\,(K_2(1) - K_2(0)), \tag{A138}$$

and vice versa if the inequality flips. Consider the case when $\eta = 0$ for simplicity. The expression in (A136) simplifies to,

$$0 = \frac{\gamma(F_E + F_P)(\Pr(S_2^\star|1) - \Pr(S_2^\star|0))}{F_E} - c. \tag{A139}$$

The preceding expression implies that $\theta^\lambda_{\lambda=1} < \theta^\lambda_{\lambda=0}$ if

$$2 > \frac{(F_E + F_P)(\Pr(S_2^\star|1) - \Pr(S_2^\star|0))}{F_E}. \tag{A140}$$

Because $\Pr(S_2^\star|1) - \Pr(S_2^\star|0)$ is bound between $(0, 1]$, it is straightforward to see that the inequality does not hold if F_P is sufficiently larger than F_E. A sufficient condition for the inequality to hold is $F_P \le F_E$.

■

Proof of Lemma 8:

Investors observe the fee schedule charged by each fund and allocate their capital based on their beliefs. Similar to the expression from Section 3 and Section 4, investors make their allocations at

$t = 1$ according to,

$$-F_P = -(F_E + F_P) + \Pr(S_1|K_1^{\lambda}, \eta, \lambda, \rho)\frac{(1-\lambda)\gamma}{K_1^{\lambda}}, \tag{A141}$$

$$-F_P = -(F_E + F_P) + \Pr(S_1|K_1^{\chi}, \eta, \rho)\frac{\gamma}{K_1^{\chi}}, \tag{A142}$$

where $\Pr(S_1|K_t^{\lambda}, \eta, \lambda, \rho)$ is the probability of success that investors assess to fund j if it charges a performance-based fee and $\Pr(S_1|K_t^{\chi}, \eta, \rho)$ is the probability of success that investors assess to fund j if it does not charge a performance-based fee. Similarly, investors reallocate their capital at $t = 2$ based on their refined beliefs,

$$-F_P = -(F_E + F_P) + \Pr(S_2|K_2^{\lambda}, \eta, \lambda, \rho, \mathbb{1}_{S_1})\frac{(1-\lambda)\gamma}{K_2^{\lambda}}, \tag{A143}$$

$$-F_P = -(F_E + F_P) + \Pr(S_2|K_2^{\chi}, \eta, \rho, \mathbb{1}_{S_1})\frac{\gamma}{K_2^{\chi}}, \tag{A144}$$

The preceding investor indifference equations imply that the capital allocations are given by,

$$K_1^{\lambda} = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4(1-\lambda)\gamma\eta\Pr(S_1^{\star}|A_{\lambda})}{4\eta^2 F_E}}, \tag{A145}$$

$$K_1^{\chi} = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_1^{\star}|A_{\chi})}{4\eta^2 F_E}}, \tag{A146}$$

$$K_2^{\lambda}(\mathbb{1}_{S_1}) = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4(1-\lambda)\gamma\eta\Pr(S_2^{\star}|A_{\lambda}, \mathbb{1}_{S_1})}{4\eta^2 F_E}}, \tag{A147}$$

$$K_2^{\chi}(\mathbb{1}_{S_1}) = -\frac{1}{2\eta} + \sqrt{\frac{F_E + 4\gamma\eta\Pr(S_2^{\star}|A_{\chi}, \mathbb{1}_{S_1})}{4\eta^2 F_E}}. \tag{A148}$$

where

$$\Pr(S_1^{\star}|A_{\lambda}) = E[\theta|A_{\lambda}], \tag{A149}$$

$$\Pr(S_1^{\star}|A_{\chi}) = E[\theta|A_{\chi}], \tag{A150}$$

$$\Pr(S_2^{\star}|A_{\lambda}, \mathbb{1}_{S_1}) = E[\theta|A_{\lambda}, \mathbb{1}_{S_1}], \tag{A151}$$

$$\Pr(S_2^{\star}|A_{\chi}, \mathbb{1}_{S_1}) = E[\theta|A_{\chi}, \mathbb{1}_{S_1}]. \tag{A152}$$

For a fund with type θ, the possible expected payoffs are,

$$\Pi_j(a_j, \theta) = \begin{cases} (F_E + F_P)\left(\rho\left(K_1^{\lambda} + K_2^{\lambda}(0)\right) + (1-\rho)\left(K_1^{\chi} + K_2^{\chi}(0)\right)\right) & \text{if } a_j = C \\ (F_E + F_P)\left(K_1^{\chi} + \frac{\theta}{1+\eta K_1^{\chi}}K_2^{\chi}(1) + \left(1 - \frac{\theta}{1+\eta K_1^{\chi}}\right)K_2^{\chi}(0)\right) - c & \text{if } a_j = A_{\chi} \\ (F_E + F_P)\left(K_1^{\lambda} + \frac{\theta}{1+\eta K_1^{\lambda}}K_2^{\lambda}(1) + \left(1 - \frac{\theta}{1+\eta K_1^{\lambda}}\right)K_2^{\lambda}(0)\right) \\ + \frac{\theta\lambda\gamma}{1+\eta K_1^{\lambda}} + \frac{\theta^2\lambda\gamma}{(1+\eta K_2^{\lambda}(1))(1+\eta K_1^{\lambda})} + \left(1 - \frac{\theta}{(1+\eta K_1^{\lambda})}\right)\frac{\theta\lambda\gamma}{(1+\eta K_2^{\lambda}(0))} - c & \text{if } a_j = A_{\lambda} \end{cases} \tag{A153}$$

Any fund that chooses to be active chooses its fee schedule according to the following benefit-cost function,

$$\Delta\lambda(\theta) \equiv \Pi_j(A_\lambda, \theta) - \Pi_j(A_\chi, \theta). \tag{A154}$$

If the preceding expression is positive, the fund chooses the fee schedule with the performance-based fee. If the expression is negative, the fund chooses a fee schedule without a performance component. The explicit form of the preceding benefit-cost function is given by,

$$\Delta\lambda(\theta) = (F_E + F_P)\left(K_1^\lambda - K_1^\chi + \theta\left(\frac{K_2^\lambda(1)}{1+\eta K_1^\lambda} - \frac{K_2^\chi(1)}{1+\eta K_1^\chi}\right) + K_2^\lambda(0) - K_2^\chi(0)\right.$$
$$\left. - \theta\left(\frac{K_2^\lambda(0)}{1+\eta K_1^\lambda} - \frac{K_2^\chi(0)}{1+\eta K_1^\chi}\right)\right)$$
$$+ \frac{\theta\lambda\gamma}{1+\eta K_1^\lambda} + \frac{\theta^2\lambda\gamma}{(1+\eta K_2^\lambda(1))(1+\eta K_1^\lambda)} + \left(1 - \frac{\theta}{(1+\eta K_1^\lambda)}\right)\frac{\theta\lambda\gamma}{(1+\eta K_2^\lambda(0))} \tag{A155}$$

First, we consider an equilibrium in which closet funds mix between the two fee schedules. In equilibrium, a fund that chooses to not pay c and employs a mixing strategy over the fee schedules (i.e., a closet fund) is indifferent between its payoffs under the choice. The indifference condition is used to simplify (A155) as an equilibrium condition,

$$\Delta\lambda(\theta) = \theta\left((F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1+\eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1+\eta K_1^\chi}\right)\right.$$
$$\left. + \frac{\lambda\gamma}{1+\eta K_1^\lambda} + \frac{\theta\lambda\gamma}{(1+\eta K_2^\lambda(1))(1+\eta K_1^\lambda)} + \left(1 - \frac{\theta}{(1+\eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1+\eta K_2^\lambda(0))}\right), \tag{A156}$$

because

$$K_1^\lambda + K_2^\lambda(0) = K_1^\chi + K_2^\chi(0), \tag{A157}$$

due to the presence of closet funds. The expression in (A156) implies that funds that pay c necessarily choose a deterministic strategy. This is because each fund takes $K_{j,t}^\lambda$ and $K_{j,t}^\chi$ as givens. The quantity within the exterior parenthesis is linear in the fund's type θ and the entire quantity is then scaled by the fund's type. As such, for every fund, it is strictly dominant to choose the performance-based fee schedule if $\Delta\lambda(\theta) \geq 0$ and the flat fee schedule otherwise. More importantly, the expression in (A156) implies at most a single crossing (if any). This is because the function is quadratic in θ and one of the roots is $\theta = 0$, meaning that, at most, one internal root exists.

Now we consider the fund that is just indifferent between the two fee schedules. We define this

fund's type as $\theta = \overline{\theta}$, where $\Delta\lambda(\overline{\theta}) = 0$. At $\theta = \overline{\theta}$, the equation in (A156) simplifies to,

$$
0 = \underbrace{(F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1 + \eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1 + \eta K_1^\chi}\right)}_{Cost}
$$
$$
+ \underbrace{\frac{\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{\overline{\theta}\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{\overline{\theta}}{(1 + \eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}}_{Benefit}. \tag{A158}
$$

Notice that the expression termed "*Benefit*" is strictly positive. This implies that "*Cost*" is negative if $\overline{\theta}$ exists. Now, we show that the benefit-cost function in (A155) is initially increasing and then subsequently decreasing. Consider the derivative of (A155) with respect to θ,

$$
\frac{d\Delta\lambda(\theta)}{d\theta} = (F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1 + \eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1 + \eta K_1^\chi}\right)
$$
$$
+ \frac{\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{\theta\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{\theta}{(1 + \eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}
$$
$$
+ \frac{\theta\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} - \frac{\theta}{(1 + \eta K_1^\lambda)}\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}. \tag{A159}
$$

The expression simplifies to,

$$
= (F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1 + \eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1 + \eta K_1^\chi}\right)
$$
$$
+ \frac{\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{2\theta\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{2\theta}{(1 + \eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}. \tag{A160}
$$

A substitution of $\overline{\theta} + x_j$ for θ where $x_j = \theta - \overline{\theta}$ yields,

$$
= (F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1 + \eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1 + \eta K_1^\chi}\right)
$$
$$
+ \frac{\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{2(\overline{\theta} + x_j)\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{2(\overline{\theta} + x_j)}{(1 + \eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}. \tag{A161}
$$

The preceding expression is expanded to,

$$
= (F_E + F_P)\left(\frac{K_2^\lambda(1) - K_2^\lambda(0)}{1 + \eta K_1^\lambda} - \frac{K_2^\chi(1) - K_2^\chi(0)}{1 + \eta K_1^\chi}\right)
$$
$$
+ \frac{\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{\overline{\theta}\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{\overline{\theta}}{(1 + \eta K_1^\lambda)}\right)\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}
$$
$$
+ \frac{(\overline{\theta} + 2x_j)\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} - \frac{(\overline{\theta} + 2x_j)}{(1 + \eta K_1^\lambda)}\frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}, \tag{A162}
$$

and simplifies to,

$$= \frac{(\overline{\theta} + 2x_j)\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} - \frac{(\overline{\theta} + 2x_j)}{(1 + \eta K_1^\lambda)} \frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}. \tag{A163}$$

A reverse substitution of $x_j = \theta - \overline{\theta}$ yields,

$$= \frac{(2\theta - \overline{\theta})\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} - \frac{(2\theta - \overline{\theta})}{(1 + \eta K_1^\lambda)} \frac{\lambda\gamma}{(1 + \eta K_2^\lambda(0))}, \tag{A164}$$

which is positive if,

$$0 \le (2\theta - \overline{\theta})(1 + \eta K_2^\lambda(0)) - (2\theta - \overline{\theta})(1 + \eta K_2^\lambda(1)), \tag{A165}$$

or equivalently if,

$$0 \le -(2\theta - \overline{\theta})\left(K_2^\lambda(1) - K_2^\lambda(0)\right) \tag{A166}$$

Therefore, the benefit function is increasing for funds with a type

$$\theta \le \frac{\overline{\theta}}{2}. \tag{A167}$$

The condition in (A167) suggests that all funds with type $\theta \ge \overline{\theta}$ choose the fee schedule without a performance component and all funds with $\theta < \overline{\theta}$ choose the fee schedule with the performance-based fee.

Now, suppose that one fee schedule is strictly dominant for those funds that choose to not pay c. This implies that

$$K_1^\lambda + K_2^\lambda(0) > K_1^\chi + K_2^\chi(0) \text{ with } \rho = 0, \tag{A168}$$

or,

$$K_1^\lambda + K_2^\lambda(0) < K_1^\chi + K_2^\chi(0) \text{ with } \rho = 1. \tag{A169}$$

First, suppose $K_1^\lambda + K_2^\lambda(0) > K_1^\chi + K_2^\chi(0)$ and $\rho = 0$. For the inequality to be strictly greater than, it must be the case that high-type active funds choose the performance-based option and medium-type active funds choose the option without a performance component. This implies that A155 simplifies to,

$$\Delta\lambda(\theta) = (F_E + F_P)\left(K_1^\lambda + K_2^\lambda(0) - K_1^\chi - \theta\left(\frac{K_2^\chi(1)}{1 + \eta K_1^\chi}\right) - \left(1 - \frac{\theta}{1 + \eta K_1^\chi}\right)K_2^\chi(0)\right) + c. \tag{A170}$$

A fund with type $\overline{\theta}$ is exactly indifferent between the two options,

$$0 = (F_E + F_P)\left(K_1^\lambda + K_2^\lambda(0) - K_1^\chi - \overline{\theta}\left(\frac{K_2^\chi(1)}{1 + \eta K_1^\chi}\right) - \left(1 - \frac{\overline{\theta}}{1 + \eta K_1^\chi}\right)K_2^\chi(0)\right) + c. \tag{A171}$$

A reorganization of the preceding inequality yields,

$$(F_E + F_P)\left(K_1^\lambda + K_2^\lambda(0)\right)$$
$$= (F_E + F_P)\left(K_1^\chi + \overline{\theta}\left(\frac{K_2^\chi(1)}{1 + \eta K_1^\chi}\right) + \left(1 - \frac{\overline{\theta}}{1 + \eta K_1^\chi}\right)K_2^\chi(0)\right) - c, \tag{A172}$$

which implies that the fund type that is indifferent between charging the fee with the performance-based component and charging the fee without a performance component is *also* indifferent between paying c and not. As such, no equilibrium exists in which both fee schedules co-exist and $\rho = 0$.

Now consider that $K_1^\lambda + K_2^\lambda(0) < K_1^\chi + K_2^\chi(0)$ and $\rho = 1$. For the inequality to be strictly less than, it must be the case that high-type active funds choose the option without a performance component and medium-type active funds choose the performance-based option. This implies that A155 simplifies to,

$$\Delta\lambda(\theta) = (F_E + F_P)\left(K_1^\lambda - K_1^\chi + \frac{\theta K_2^\lambda(1)}{1 + \eta K_1^\lambda} + K_2^\lambda(0) - K_2^\chi(0) - \frac{\theta K_2^\lambda(0)}{1 + \eta K_1^\lambda}\right)$$
$$+ \frac{\theta\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{\theta^2\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{\theta}{(1 + \eta K_1^\lambda)}\right)\frac{\theta\lambda\gamma}{(1 + \eta K_2^\lambda(0))}, \tag{A173}$$

and a fund with type $\overline{\theta}$ is exactly indifferent between the two options if,

$$0 = (F_E + F_P)\left(K_1^\lambda - K_1^\chi + \frac{\overline{\theta} K_2^\lambda(1)}{1 + \eta K_1^\lambda} + K_2^\lambda(0) - K_2^\chi(0) - \frac{\overline{\theta} K_2^\lambda(0)}{1 + \eta K_1^\lambda}\right)$$
$$+ \frac{\overline{\theta}\lambda\gamma}{1 + \eta K_1^\lambda} + \frac{\overline{\theta}^2\lambda\gamma}{(1 + \eta K_2^\lambda(1))(1 + \eta K_1^\lambda)} + \left(1 - \frac{\overline{\theta}}{(1 + \eta K_1^\lambda)}\right)\frac{\overline{\theta}\lambda\gamma}{(1 + \eta K_2^\lambda(0))}. \tag{A174}$$

The preceding condition may hold for some parameter sets.

∎

Proof of Lemma 9:

The proof follows the proof of Lemma 7, where $\underline{\theta}$ is implicitly defined by,

$$0 = \Pi_j(A_\lambda, \underline{\theta}) - \Pi_j(A_\lambda, \underline{\theta}|C). \tag{A175}$$

Unlike the proof of Lemma 7, however, only a fraction $1 - \rho^*$ choose to mimic the fee schedule with a performance component. The equilibrium mixing probability is determined by,

$$\Pi_j(A_\lambda, \theta|C, \rho)|_{\rho=\rho^*} = \Pi_j(A_\chi, \theta|C, \rho)|_{\rho=\rho^*}. \tag{A176}$$

■

Proof of Proposition 2:

The proof follows directly from the results of Lemma 8 and Lemma 9.

■

Appendix B

B.1   Numerical Methods for Section 4.1

Solving the model when incorporating a fund's strategic choices to charge an additional performance-based fee appears intractable analytically. Consequently, we utilize numerical methods to derive comparative statics and fund-flow sensitivities. Building from our earlier analysis, we leverage several equilibrium conditions in order to solve the model. First, for a funding having a type equal to $\overline{\theta}$, that fund must be indifferent between charging the flat fee schedule or the flat fee plus the performance-based fee. This is the condition given in (A154). Second, for a fund having a type equal to $\underline{\theta}$, that fund must be indifferent between closet indexing or producing information and charging the flat fee plus the performance-based fee (as we have shown in Lemma 8 that the high-types charge the flat fee schedule). This condition is given in (A175). Third, the payoffs to mimicking the flat fee schedule and the flat fee plus performance-based fee must be equal. This implies that the proportion of funds that choose to mimic each must equate these payoffs as in (A176).

We iteratively solve the model by searching for fixed points over finer and finer grids of values for $\underline{\theta}$ and $\overline{\theta}$. For each given $\underline{\theta}$ and $\overline{\theta}$ pair, we solve for the proportion of funds that mimic each fee schedule (the third condition above).[24] Let the proportion mimicking the flat fee schedule be ρ^*. Given $\underline{\theta}$, $\overline{\theta}$, and ρ^*, we determine, using Equations (A145), (A146), (A147), and (A148), the values of capital for a fund in each period based on its fee schedule and past returns. Using these capital values, $\underline{\theta}$, $\overline{\theta}$ and ρ, we can then calculate the payoff in each of the first and second conditions. Using these payoffs, for each $\underline{\theta}$, we estimate a $\overline{\theta}_1$ and $\overline{\theta}_2$ that would make conditions 1 and 2 hold with equality, respectively. A solution in which $\overline{\theta}_1(\underline{\theta}) = \overline{\theta}_2(\underline{\theta})$ implies that we have an equilibrium. However, this is not likely to be the case in the first iteration, and therefore we interpolate where the functions $\overline{\theta}_1(\underline{\theta})$ and $\overline{\theta}_2(\underline{\theta})$ cross, and the repeat the process using the closest points of the first grid as the bounds in the next iteration. This process continues until the sum of the absolute values of the deviations from zero for conditions one, two and three is less than a threshold tolerance, which is set to 1×10^{-6}. When the threshold tolerance is reached, we consider the resulting numerical equilibrium to be sufficiently close to the true equilibrium.

---

[24]It is possible that the third condition does not hold with equality, but only if all closet indexers mimic one fee schedule. It is also possible that $\overline{\theta} = \underline{\theta}$, in which case all funds subscribe to an identical fee schedule.