Intermediaries in a Decentralized Market with Information Frictions: Facilitators or Bottlenecks?*

Klaus-Peter Hellwig†

New York University

December 3, 2013

[download the latest version]

Abstract

This paper provides a theory of intermediation in a market with asymmetric information between buyers and sellers. Both, buyers and intermediaries have access to a costly screening technology. Unlike existing theories, I find that a technological advantage is not a *necessary* condition for middlemen to be active. Intermediaries can have the same search and screening technology as buyers and still earn positive rents. A *sufficient* condition for intermediation to take place is that a critical mass of intermediaries choose to screen the sellers they meet. This screening and cherry-picking has two general equilibrium effects: 1) It deteriorates the quality distribution among sellers. 2) It improves the value of search to buyers. I show that both externalities make the lemons problem in meetings between buyers and sellers worse, so that bilateral trade is crowded out. If the externalities are strong enough, intermediaries are the only channel of trade in the market. The welfare implications of my model are that more intermediation can reduce the volume of trade in the market even if intermediaries *do* have a technological advantage.

JEL classification: D82; D83; G10

Keywords: Intermediation; Search; Adverse Selection; Information Choice

*I am grateful to Jess Benhabib for his encouragement and advice. I also thank Anmol Bhandari, Jarda Borovicka, Katka Borovickova, Douglas Gale, Martin Hellwig, Boyan Jovanovic, Ricardo Lagos, Tobias Salz, Tom Sargent, Edouard Schaal, and Shengxing Zhang as well as seminar participants at NYU and the Max Planck Institute for all the helpful comments.

†kphellwig@nyu.edu

1 Introduction

It is well known that in a wide array of situations the Walrasian paradigm of homogeneous goods markets that clear at every instant is only of limited use. In many markets it takes time for buyers and sellers to find each other, and the quality of the goods or assets offered can be heterogeneous and difficult to observe. In real life, as in the existing theoretical literature on intermediation, there are middlemen that help other market participants mitigate these difficulties. In real estate markets, brokers who help buyers find a new home are ubiquitous. Workers who want to enhance their job prospects can sign up with an employment agency. And in the market for corporate control, investment banks earn fees for screening takeover targets and finding potential buyers.

The fact that buyers and sellers sometimes forego the opportunity to engage in bilateral trade and instead rely on the services of an intermediary suggests that intermediation makes these market participants better off. Indeed, in the literature, a necessary condition for middlemen to be active is the availability of some superior search (Rubinstein and Wolinsky (1987)) or screening (Biglaiser (1993), Li (1998)) technology that allows them to speed up trade or serve as guarantors of quality to overcome a "Lemons" problem.[1] Contrary to these views, the present paper highlights situations in which intermediaries attract business and earn rents even though they have no such technological advantage. Put differently, I show that a technological advantage is not a necessary condition for intermediaries to be active. A sufficient condition for intermediation to take place is that a critical mass of intermediaries coordinate on screening the sellers they meet. The central welfare result of the paper is that, through the general equilibrium effects of intermediation, middlemen can lower the volume of trade and the overall level of welfare even when they *do* have a better technology at hand.

To illustrate these results, I model a decentralized market in which heterogeneous sellers can either search for buyers by themselves or instead hire an intermediary who then helps them market their good.[2] In addition to the search friction, I assume that, while sellers can observe the quality of their own good, buyers and intermediaries know only the quality distribution across sellers. To overcome this information asymmetry, intermediaries and buyers can choose to inspect the good at a cost before bargaining over the terms of trade. As in Rubinstein and Wolinsky (1987), intermediaries must provide a higher speed of trade in order to attract sellers. But where their work assumed exogenous trading speeds, the ex ante heterogeneity in my model, together with the information asymmetry, leads to speeds of trade that are endogenous. While the rate at which they meet a buyer can be the same for sellers and intermediaries, the probability that a

[1] In a similar vein, Wong and Wright (2011) suggest that some middlemen exist due to superior bargaining skills. For alternative theories of intermediation, see, e.g., Townsend (1978), Diamond (1984), Boyd and Prescott (1986), or Afonso and Lagos (2012).

[2] Throughout the paper I assume that intermediaries and sellers engage in consignment relationships. The question when intermediaries are agents of buyers and when they act on behalf of sellers in a setting with quality uncertainty is addressed elsewhere (e.g., Durbin (1999)).

meeting results in trade depends on whether the seller has hired an intermediary or not. In particular, more screening by intermediaries leads to faster intermediated trade. But at the same time, through its general equilibrium effects, intermediation reduces the speed of direct trade between buyers and sellers.

As in other search models with *ex ante heterogeneity* (e.g., Shimer and Smith (2001)), the value of search to an agent depends on the mass of potential trading partners and their quality distribution. The *information asymmetry* implies that the probability of trade in a given meeting depends not only on the trading partner's types and their outside options, but also on the entire distribution of types that they were drawn from. In the presence of intermediaries, these mechanisms have powerful additional implications, because intermediaries divide the market into two pools: sellers who are matched with intermediaries and sellers who search on their own. Buyers randomly meet both, matched and unmatched sellers, and they know which pool a seller was drawn from. In equilibrium, they use this extra piece of information when making their screening decisions. The lower the quality in the pool of unmatched sellers, the lower the expected gains from bilateral trade with these sellers. And the higher the quality in the pool of matched sellers, the higher the buyers' opportunity cost when dealing with unmatched sellers. Intermediaries influence the quality composition of both pools since, by applying more scrutiny, they direct more high quality goods from the pool of unmatched sellers into the pool of matched sellers. Hence, the more intermediaries decide to screen, the harder it becomes for sellers to trade with buyers directly. If strong enough, this general equilibrium effect can lead to a lemons problem in the pool of sellers – a complete breakdown of bilateral trade between buyers and sellers, so that all trade is carried out through intermediaries.

Technically, screening by intermediaries can shut down trade between buyers and sellers for two separate reasons. First, the cherry-picking by intermediaries has an information externality. If some intermediaries choose to market only high quality goods, then there are sellers in the market who have been screened and rejected by an intermediary, so that a buyer who meets an unmatched seller should ask: "If your product is that great, why haven't you found an intermediary already?" In other words, not having an intermediary becomes a signal of low quality, so that buyers don't find it worthwhile to deal with sellers directly. I refer to this externality as the *selection effect*.[3]

Second, intermediaries' screening decisions affect the buyers' value of search. The higher the quality of goods sold through intermediaries, the more valuable is the option of waiting for an intermediary.[4] When meeting an unmatched seller, a buyer then refuses to trade simply because the opportunity cost is too high. This mechanism is referred to as the *opportunity cost effect*.

The finding that intermediaries can crowd out bilateral trade formalizes the popular notion that, in many markets with quality uncertainty, a limited number of firms play the role of gatekeepers. For instance,

[3]Similar selection effects have been extensively studied in the labor economics literature: a worker's duration of unemployment carries some information about the outcomes of previous job interviews (see, e.g., Lockwood (1991) or for recent empirical evidence, Kroft et al. (2013)).

[4]I assume that buyers are capacity constrained.

before the emergence of platforms like YouTube or iTunes, the only way for pop musicians to reach the mass market was through a record company. A large share of start-up financing for new technologies is provided through venture capitalists. And before the internet age, the only way to publish a political opinion to a broad audience was through a traditional news outlet. The presence of such gatekeepers has always drawn criticism for limiting the diversity of products. And indeed, with the arrival of new technologies and alternative ways for selling music records, publishing news or books, consumers now have easy access to products that perhaps would not have passed a gatekeeper. On the other hand, in all these examples, gatekeepers apply some scrutiny and ensure that the products that reach the market meet certain quality standards, which might explain why they were not entirely driven out of the market by improvements in the technology (the internet) of bilateral trade. Within the context of my model, I study the impact of intermediation on trade volumes and aggregate transaction costs by comparing these outcomes across equilibria. That is, whenever there exists an equilibrium with intermediation, there exists another equilibrium with only direct bilateral trade. In numerical examples I find that, whether the flow welfare is higher in an equilibrium with intermediaries than without depends on the relative importance of the selection effect and the opportunity cost effect.

The paper adds to a growing body of research on the role of middlemen in search environments. In particular, following the work of Duffie et al. (2005) and Lagos and Rocheteau (2009), search theoretical models are now widely used in the analysis of over-the-counter markets for financial assets. These papers closely follow Rubinstein and Wolinsky (1987) in the assumption that intermediaries have access to a superior trading technology, which leads to straightforward implications regarding the relationship between intermediation and liquidity. The present paper highlights cases in which, by crowding out bilateral trade, intermediaries reduce the liquidity in the market – despite such technological advantages – because, under quality uncertainty, the speeds of trade are endogenous. It thereby further contributes to the literature on dynamic lemons markets. Due to the opacity of many financial assets, the dynamic interaction between an endogenous quality distribution and trade volumes has gained some attention in the context of the last financial crisis, as this channel can generate endogenous market freezes and provides a rationale for policy interventions to restore liquidity (see, e.g., Guerrieri et al. (2010), Chiu and Koeppl (2011), Camargo and Lester (2011), and Guerrieri and Shimer (2012)). The other ingredient of my theory, costly screening in a market with adverse selection, has been studied in environments with an exogenous quality distribution (see, e.g., Dang (2008), Dang et al. (2010), and Farhi and Tirole (2012)). My model merges elements from these related strands of the literature by allowing for both, information structure and quality distribution, to be endogenous. In this respect my paper is closely related to Li (1998) and Hellwig and Zhang (2013). The former also studies the role of intermediaries in a decentralized market with information frictions. The crucial difference between Li's work and my theory is that in my model each act of screening is a costly choice by both, buyers and intermediaries, so that the decision to screen is a function of the quality distribution and the opportunity cost. In Li (1998), by contrast, the costly choice is whether to become an intermediary with a perfect and costless screening technology. Hellwig and Zhang (2013) study a dynamic lemons market in which screening is also a costly choice. The equilibria without intermediation of the present paper are very similar to their model. The distinctive feature of my model is that, once intermediaries are active, they divide the market into two pools. Buyers can then condition their decisions on the identity of the pool. The strategic complementarities that thus arise from statistical discrimination have been highlighted in a different context by Coate and Loury (1993).

The paper is organized as follows. In the next section I introduce the model and characterize equilibria without intermediation. Section 3 discusses conditions for the existence of an intermediation equilibrium. In Section 4, I examine the general equilibrium effects of the model and the effect of intermediaries on bilateral trade. Finally, in Section 5, I give two numerical examples in which I isolate the selection effect and the opportunity cost effect.

2 The Model

The economy is populated by three groups of agents that I call buyers, sellers, and intermediaries. Time is continuous with an infinite horizon, and during each short time interval dt a mass $e_b \times dt$ of buyers, $e_s \times dt$ of sellers, and $e_i \times dt$ of intermediaries enter the world. At birth, each seller is endowed with a single good for which he has no use. Buyers and intermediaries enter the world without any goods. There is heterogeneity in the quality of goods, and buyers enjoy utility $H > 0$ if they obtain a high quality good and utility $L \leq 0$ if the good is of low quality. The share of entering sellers endowed with a high quality good is exogenous and denoted by $\rho_e \in (0, 1)$. When entering the world at time t, buyers join a pool of buyers of size $\mu_b(t)$, and sellers join a pool of mass $\mu_s(t)$. The quality composition within the pool of sellers is endogenous and measured by $\rho_s(t)$, the fraction of high quality goods. Agents exit the world randomly with Poisson rate δ, and they discount the future at rate r.

I assume that every buyer is endowed with a large amount of a numeraire good that is valued equally by all agents. Hence, when buyers and sellers of high quality goods meet, there are gains from trade to be realized if both parties agree on the terms. After such a trade is carried out, both the buyer and the seller leave the world. Buyers and sellers can either trade bilaterally or through an intermediary. Intermediaries and their role are introduced in more detail below, after the analysis of bilateral trade between buyers and sellers. In the following, I first describe the bilateral meetings between buyers and sellers and derive the bargaining outcome. Second, I discuss the role of intermediaries. Then, I present the dynamics of the model more formally. That is, I introduce the laws of motion that describe how the distribution of agents across types depends on the history of past bargaining outcomes and how the agents' valuations depend on their expectations about future bargaining outcomes. Finally, I introduce the equilibrium concept, requiring that the dynamic paths of bargaining outcomes, valuations, and distributions are consistent.

2.1 Buyers and Sellers

There are two frictions that delay or prevent transactions between buyers and sellers. First, due to a search friction, trading opportunities arrive only at random points in time. I let λ denote the matching efficiency. From the perspective of a buyer (seller), meetings with sellers (buyers) follow a Poisson process with rate $\lambda \times \mu_s$ ($\lambda \times \mu_b$, respectively). Second, when a buyer finds a seller, he cannot costlessly observe the true quality of the good. He only has a belief about $\rho_s \in [0,1]$, the probability that a good is of high quality, so that, absent any further information, the bargaining takes place under asymmetric information. It is well known since Akerlof (1970) that, if ρ_s too small, there can be no trade. To overcome this problem, buyers can pay a cost $c_b > 0$ to screen the good and learn its true quality before the bargaining game starts. Combining the information acquisition problem from Hellwig and Zhang (2013) with a simple bargaining mechanism borrowed from Wong and Wright (2011), I assume the following timing protocol for meetings between a buyer and a seller:

1. The buyer decides whether to screen the good at a cost c_b. This choice is observed by the seller.
2. The buyer offers a price for the good.
3. The seller accepts or rejects the offer. If he accepts, the match is consumed and both agents leave the world.
4. If the seller rejected the buyer's initial offer, then a second, final, offer is made. This offer is made by the buyer with probability q and by the seller with probability $1-q$. If the offer is accepted, the match is consumed and both agents leave the world. Otherwise, the meeting ends and both agents continue searching.

Solving the game backwards, I first consider the case of an informed buyer. Let W be the value of search for a buyer and S_H (S_L) the values of a seller with a high (low) quality good. An informed buyer is willing to trade at a positive price only if the good is of high quality. In that case, the seller will ask for the buyer's reservation price which is $H-W$. If the buyer has the bargaining power, he just offers S_H to the seller. The initial offer made by the buyer (in stage 2 of the game) is chosen to make the seller just indifferent between accepting and rejecting. This price is $qS_H + (1-q)(H-W)$ and, since sellers have no reason to reject it, stage 4 of the meeting is never reached.

If the buyer remains uninformed, his reservation price is $\rho_s H + (1-\rho_s)L - W$, his expected utility gain from owning the asset. The high quality seller's reservation price is again S_H. Note that the seller's reservation price is below that of the buyer if and only if $\rho_s H + (1-\rho_s)L - W > S_H$, and hence uninformed buyers are willing to trade if and only if

$$\rho_s \geq \frac{S_H + W - L}{H - L} \equiv \rho^*_{b,s}. \tag{1}$$

If ρ_s is sufficiently high, the initial offer made by the uninformed buyer is $qS_H + (1-q)(\rho H + (1-\rho)L - W)$.

Going back to the first stage of the stage game, buyers compare the value of being informed with the value of remaining uninformed to make their screening decision. The value of being informed is $\rho_s q(H - S_H - W) + W$, their outside option plus the expected surplus from trade in high quality assets, scaled by the bargaining power. The value of remaining uninformed is simply W if ρ_s is small and $q\left[\rho H + (1-\rho)L - W - S_H\right] + W$ if ρ_s is high enough to encourage uninformed buyers to trade. The comparison between the value of being informed and the value of remaining uninformed yields that buyers prefer to acquire information whenever either

$$c_b \leq q(1-\rho_s)\left[W + S_H - L\right] \text{ and } \rho_s \geq \rho^*_{b,s} \tag{2}$$

or

$$c_b \leq \rho_s q(H - S_H - W), \text{ and } \rho_s < \rho^*_{b,s}. \tag{3}$$

The conditions for acquiring information are different for pools with different quality levels because the value of information depends on what uninformed buyers would do. If the average quality in the pool of sellers is high, then uninformed buyers would be willing to trade. Hence, the use of information is that it prevents buyers from acquiring a low quality good (Type 2 error). The probability of such an error is increasing in the share of low quality goods. On the other hand, if the quality in the pool is low and uninformed buyers refrain from trading, the use of information is to avoid rejecting high quality goods (Type 1 error).

The optimal policy decision is summarized graphically in Figure 1. The vertical line represents the cutoff $\rho^*_{b,s}$ that marks the optimal decision of an uninformed buyer. If ρ_s is above that level, uninformed buyers are willing to trade. The cutoff rule (2) for information choice, given that ρ_s high, is illustrated by the downward sloping line, while the decision rule (3) for low values of ρ_s is given by the upward sloping line.

Taken together, the three cutoffs lead to the optimal decision rule described in the right panel of Figure 1. If c_b is small, enough then agents choose to become informed. But the value of information depends on ρ. If ρ_s is either very small or very close to one, then there is almost no uncertainty, so that additional information is worthless. Uninformed buyers agree to trade as long as ρ_s is high enough.

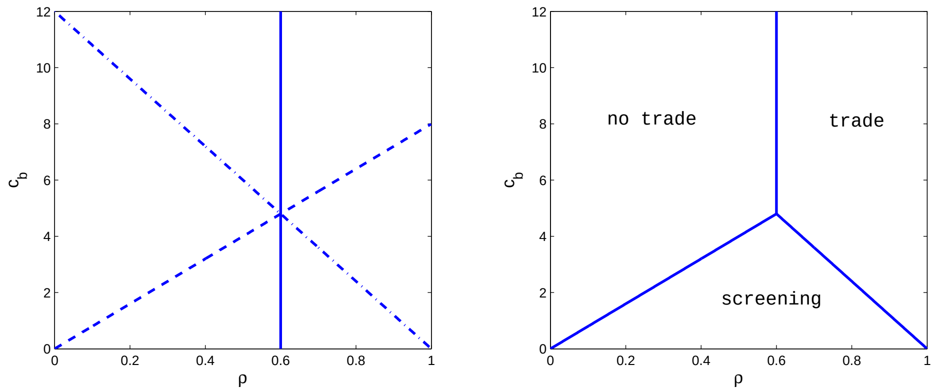


Figure 1: Buyer's cutoff values (left panel) and optimal policy (right panel) when meeting a seller. In the "screening" region, only high quality goods are traded, whereas in the "trade" region, all goods are traded without any screening.

2.2 Intermediaries

Intermediaries are agents who have no good and have no use for goods either. Hence, intermediaries distinguish themselves from buyers mainly by their preferences. Intermediaries can earn rents by helping sellers find a buyer in exchange for a share of the price. Upon entering the world, intermediaries join a pool of size $\mu_{i,0}(t)$ of unmatched intermediaries. From this pool they can either exit by exogenous random death or they can meet a seller and write an intermediation contract. If such a contract is agreed upon, the intermediary and the seller join a pool $\mu_{i,1}(t)$ of matched intermediaries. The values of matches between intermediaries and sellers are J_H and J_L, depending on the seller's type. The value of being an unmatched intermediary is J. I now describe the meetings between buyers and matched intermediaries and meetings between unmatched intermediaries and sellers.

Matched Intermediaries and Buyers Meetings between buyers and matched intermediaries occur at random points in time, again with Poisson arrival rate $\lambda \times \mu_b(t)$ (or, from the buyers perspective, with $\lambda \times \mu_{i,1}(t)$, respectively). They unfold in the same way as meetings between buyers and sellers, and q again denotes the buyer's bargaining power. Note that these assumptions imply that the speed at which agents meet buyers is the same for intermediaries and sellers, and that intermediaries have the same bargaining power when meeting a seller as buyers. In analogy to the outcome of bilateral meetings, as summarized by (1),(2), and (3), uninformed buyers and matched intermediaries trade whenever

$$\rho_i \geq \frac{J_H + W - L}{H - L} \equiv \rho^*_{b,i}. \tag{4}$$

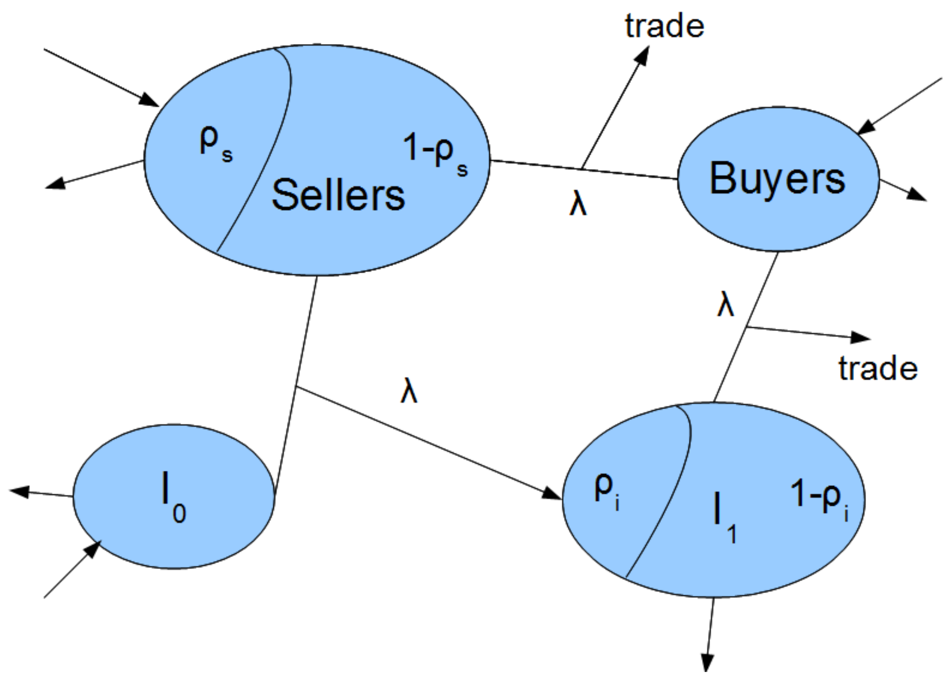


Figure 2: The model dynamics for sellers, buyers, matched (I_1) and unmatched (I_0) intermediaries. Buyers can trade with sellers and matched intermediaries. The two pools may have different quality compositions ($\rho_s(t)$ and $\rho_i(t)$). Sellers can either search for a seller on their own or sign up with an unmatched intermediary and join the pool of matched intermediaries. Entry is exogenous, whereas exit occurs both exogenously and endogenously through trade.

And a buyer screens whenever

$$c_b \leq q(1-\rho_i)\,(W + J_H - L) \text{ and } \rho_i \geq \rho^*_{b,i} \tag{5}$$

or

$$c_b \leq \rho_i q(H - J_H - W), \text{ and } \rho_i < \rho^*_{b,i}. \tag{6}$$

Unmatched Intermediaries and Sellers When an unmatched intermediary meets a seller, the two can decide whether to sign an intermediation contract. For simplicity, I assume that the terms of the contract are exogenous. The contract specifies that

1. The intermediary searches on behalf of the seller, and the seller cannot search on his own.
2. If the seller's good is sold, the intermediary obtains a share α of the price. While α is a model parameter, it could as well be determined as an equilibrium outcome of a bargaining process similar to the one outlined above.
3. The contract ends either if the seller exits or if the good is sold, but not before.[5]

Before the intermediary decides whether to accept the contract, he can inspect the good. By paying a fixed cost c_i, he can learn the true quality. Therefore, the outcome of a meeting between sellers and intermediaries consists of three decisions. The intermediary chooses whether to participate and whether to screen. And the seller decides whether to participate or not. The seller participates only if

$$(1-\alpha)J_H(t) \geq S_H(t). \tag{7}$$

That is, his share of the match value has to exceed his value of search. An uninformed intermediary's value from participating is $\alpha\rho_s J_H(t) + \alpha(1-\rho_s)J_L(t)$, which has to be greater than his value of continued search $J(t)$. Anticipating the equilibrium behavior, I claim that uninformed intermediaries never refuse to sign an intermediation contract. The reason why uninformed intermediaries never refuse to trade is that they have nothing to lose. If they are matched with a high quality seller, they can get a share of the upside. And if they are matched with a lemon seller, they get at least zero.

Claim. $\alpha\rho_s J_H(t) + \alpha(1-\rho_s)J_L(t) > J(t)$, $\forall t \geq 0$.

Under the claim, the only decision left to the intermediary is whether he wants to screen to avoid being matched with a lemon. This decision is analogous to the buyer's information choice in (2) and (5).

[5]i.e., I assume that the intermediary is committed to the contract even if he later learns unfavorable news about the seller's quality.

Intermediaries prefer to screen whenever their cost of information is below the expected loss from being matched with a low quality seller, or

$$c_i \leq (1 - \rho_s(t)) \left(J(t) - \alpha J_L(t)\right). \tag{8}$$

2.3 Dynamics

To characterize how the distribution of types is determined and to study how they are affected by the outcomes of bilateral meetings, it is useful to introduce some notation that summarizes the agents' strategies. Let $\phi_{b,s}(t) \in [0,1]$ denote the probability that a buyer screens a seller in a meeting at time t. And let $\psi_{b,s}(t) \in [0,1]$ be the probability that, conditional on being uninformed, a buyer is willing to trade (i.e., $\psi_{b,s}(t) = 1$ if $\rho_s(t) > \rho^*_{b,s}(t)$ and only if $\rho_s(t) \geq \rho^*_{b,s}(t)$). We allow all agents to play mixed strategies and assume that they play symmetric strategies. Then the share of meetings between buyers and sellers at time t that result in trade is

$$\rho_s(t)\phi_{b,s}(t) + (1 - \phi_{b,s}(t))\psi_{b,s}(t).$$

I use a similar notation to describe meeting between buyers and intermediaries ($\phi_{b,i}(t)$, $\psi_{b,i}(t)$). In meetings between intermediaries and sellers, intermediaries screen with probability $\phi_{i,s}(t)$, and sellers accept the intermediation contract with probability $\xi_{i,s}(t)$.

Distributions Given the exposition above, these masses of each type of agents evolve according to the following laws of motion:

$$\dot{\mu}_b(t) = \underbrace{e_b}_{\text{entry}} - \underbrace{\delta\mu_b(t)}_{\text{exit}} \quad \underbrace{-\lambda\mu_b(t)\mu_s(t)\left(\phi_{b,s}(t)\rho_s(t) + (1-\phi_{b,s}(t))\psi_{b,s}(t)\right)}_{\text{bilateral trade}} \tag{9}$$

$$\underbrace{-\lambda\mu_b(t)\mu_{i,1}(t)\left(\phi_{b,i}(t)\rho_i(t) + (1-\phi_{b,i}(t))\psi_{b,i}(t)\right)}_{\text{trade with matched intermediaries}} \tag{10}$$

$$\dot{\mu}_s(t) = e_s - \delta\mu_s(t) \quad \underbrace{-\lambda\mu_b(t)\mu_s(t)\left(\phi_{b,s}(t)\rho_s(t) + (1-\phi_{b,s}(t))\psi_{b,s}(t)\right)}_{\text{bilateral trade}} \tag{11}$$

$$\underbrace{-\lambda\mu_s(t)\mu_{i,0}(t)\xi_{i,s}(t)\left(\phi_{i,s}(t)\rho_s(t) + (1-\phi_{i,s}(t))\right)}_{\text{new intermediation contracts}} \tag{12}$$

$$\dot{\mu}_{i,0}(t) = e_i - \delta\mu_{i,0}(t) \quad \underbrace{-\lambda\mu_s(t)\mu_{i,0}(t)\xi_{i,s}(t)\left(\phi_{i,s}(t)\rho_s(t) + (1-\phi_{i,s}(t))\right)}_{\text{new intermediation contracts}} \tag{13}$$

$$\dot{\mu}_{i,1}(t) = -\delta\mu_{i,1}(t) \quad \underbrace{+\lambda\mu_s(t)\mu_{i,0}(t)\xi_{i,s}(t)\left(\phi_{i,s}(t)\rho_s(t) + (1-\phi_{i,s}(t))\right)}_{\text{new intermediation contracts}} \tag{14}$$

$$\underbrace{-\lambda\mu_b(t)\mu_{i,1}(t)\left(\phi_{b,i}(t)\rho_i(t) + (1-\phi_{b,i}(t))\psi_{b,i}(t)\right)}_{\text{bilateral trade}} \tag{15}$$

Among sellers, the share of goods that are of high quality evolves according to

$$\dot{\rho}_s(t) = \frac{e_s}{\mu_s(t)}(\rho_e - \rho_s(t)) - \underbrace{\lambda\mu_b(t)\left(1-\rho_s(t)\right)\phi_{b,s}(t)\rho_s(t)}_{\text{selection effect of bilateral trade}}$$
$$- \underbrace{\lambda\mu_{i,0}(t)\left(1-\rho_s(t)\right)\xi_{i,s}(t)\left[1+\rho_s(t)\phi_{i,s}(t)\right]}_{\text{selection effect of intermediation}} \tag{16}$$

The second and third terms on the right-hand side of (16) capture the externality of an agents' screening decision on the quality distribution in the future. If some buyers or intermediaries increase their screening activity, the average quality in the pool deteriorates because sellers of high-quality goods are matched faster. On the other hand, if no screening takes place in the economy, the first term implies that ρ_s converges to ρ_e over time. I call the effect of screening on the quality distribution a *selection effect.* Due to these selection effects, the quality composition at time t depends on the entire history of trades.

Among those sellers who are matched with intermediaries, the quality composition ρ_i evolves according to

$$\dot{\rho}_i(t) = \xi_{i,s}(t)\lambda \frac{\mu_{i,0}(t)\mu_s(t)}{\mu_{i,1}(t)}(\rho_s(t) - \rho_i(t)) + \underbrace{\lambda \frac{\mu_{i,0}(t)\mu_s(t)}{\mu_{i,1}(t)}\xi_{i,s}(t)\phi_{i,s}(t)(t)\rho_i(t)(1 - \rho_s(t))}_{\text{screening by intermediaries}} - \underbrace{\lambda\mu_b(t)\phi_{b,i}(t)\,(1 - \rho_i(t))\,\rho_i(t)}_{\text{selection effect}} \quad (17)$$

Screening by buyers has a similar effect on ρ_i as it has on ρ_s – a faster outflow of high quality goods through trade. Screening by intermediaries, by contrast, controls the inflow into the pool of matched intermediaries. By screening more, intermediaries raise ρ_i over time. When there is no intermediation, that is, whenever $\mu_{i,1}(t) = 0$, I assume that $\rho_i(t) = 0$.

Value Functions Omitting he time argument, the value of search to a buyer is given by the Hamilton-Jacobi-Bellman equation

$$\begin{aligned}(r+\delta)W =& \underbrace{\lambda\mu_s q\left[\rho_s\phi_{b,s}\left(H - W - S_H\right) + (1-\phi_{b,s})\psi_{b,s}\left(\rho_s H + (1-\rho_s)L - S_H - W\right)\right]}_{\text{trade with sellers}} \\ &+ \underbrace{\lambda\mu_{i,1} q\left[\rho_i\phi_{b,i}(H - W - J_H) + (1-\phi_{b,i})\psi_{b,i}\left(\rho_i H + (1-\rho_i)L - J_H - W\right)\right]}_{\text{trade with matched intermediaries}} \\ &- \lambda\left(\mu_s\phi_{b,s} + \mu_{i,1}\phi_{b,i}\right)c_b + \dot{W}.\end{aligned} \quad (18)$$

Searching is valuable to buyers if they can expect gains from trade with either matched or unmatched sellers. The valuations of sellers and the match values between sellers and intermediaries are as follows:

$$\begin{aligned}(r+\delta)S_H =& \lambda\mu_b(1-q)\left(\phi_{b,s}(H - W - S_H) + (1-\phi_{b,s})\psi_{b,s}\left(\rho_s H + (1-\rho_s)L - W - S_H\right)\right) \quad (19)\\ &+ \lambda\mu_{i,0}\xi_{i,s}\left((1-\alpha)J_H - S_H\right) + \dot{S}_H\end{aligned}$$

$$(r+\delta)S_L = \lambda\mu_b(1-\phi_{b,s})\psi_{b,s}\left[(1-q)\left(\rho_s H + (1-\rho_s)L - W - S_H\right) + S_H - S_L\right] + \dot{S}_L \quad (20)$$

$$(r+\delta)J_H = \lambda\mu_b(1-q)\left(\phi_{b,i}(H - W - J_H) + (1-\phi_{b,i})\psi_{b,i}\left(\rho_i H + (1-\rho_i)L - W - J_H\right)\right) + \dot{J}_H \quad (21)$$

$$(r+\delta)J_L = \lambda\mu_b(1-\phi_{b,i})\psi_{b,i}\left[(1-q)\left(\rho_i H + (1-\rho_i)L - W - J_H\right) + J_H - J_L\right] + \dot{J}_L \quad (22)$$

And finally, the value of search to unmatched intermediaries is

$$(r+\delta)J = \lambda\mu_s\xi_{i,s}\left[\alpha\left(\rho_s J_H + (1-\rho_s)(1-\phi_{i,s})J_L\right) - \left(1 - (1-\rho_s)(1-\phi_{i,s})\right)J\right] - \lambda\mu_s\phi_{i,s}c_i + \dot{J} \quad (23)$$

.

2.4 Stationary Equilibrium

At each point in time, the outcomes of the bargaining games depend on the quality shares and on the agents' value functions. On the other hand, the laws of motion for the distribution and for the value functions both depend on the outcomes of the bargaining games. An equilibrium is a path along which the outcomes of the bargaining games, the distributions, and value functions are mutually consistent. Throughout the paper we are are only interested in stationary outcomes.

Definition. A *stationary equilibrium* is a distribution $M = (\mu_b, \mu_s, \mu_{i,0}, \mu_{i1}, \rho_s, \rho_i)$, a collection of stationary values $Q = (W, J, J_H, J_L, S_H, S_L)$, and a stationary strategy profile $\Gamma = (\phi_{b,s}, \psi_{b,s}, \phi_{b,i}, \psi_{b,i}, \phi_{i,s}, \xi_{i,s})$ such that

1. Given W, S_H, and ρ_s, $(\phi_{b,s}, \psi_{b,s})$ is a SPNE of the game played between buyers and sellers.
2. Given W, J_H, and ρ_i, $(\phi_{b,i}, \psi_{b,i})$ is a SPNE of the game played between buyers and intermediaries.
3. Given J, J_H, J_L, S_H and ρ_s, $(\phi_{i,s}, \xi_{i,s})$ is a SPNE of the game played between intermediaries and sellers.
4. M is the stationary distribution associated with the strategy profile Γ.
5. The elements of Q are the stationary values associated with the strategy profile Γ and the distribution M.

Before characterizing the stationary equilibria, I first state the following existence result. The proof involves a simple fixed-point argument and can be found in the Appendix.

Proposition 1. *A stationary equilibrium exists.*

It is easy to see that any equilibrium is uniquely characterized by the vector Γ. Each such vector implies a particular distribution M and a particular set of stationary values Q. We are interested in finding conditions under which a particular strategy profile Γ is consistent with our equilibrium definition. In principle, this can be done by following a simple algorithm. We first guess a strategy profile Γ, then compute the resulting distribution and values and finally we verify that, given these values and distributions, our candidate strategy profile solves all agents' problems.

2.5 Equilibria without Intermediation

Given the assumption on off-equilibrium beliefs regarding ρ_i, an equilibrium where $\phi_{b,i} = \psi_{b,i} = \phi_{i,s} = \psi_{i,s} = 0$ always (trivially) exists. Equilibria without intermediation may differ with respect to the amount of screening and trade, as illustrated in Figure 3. The figure divides the parameter space into regions for which different types of equilibria occur. A more formal characterization of the regions is given in Proposition 3 in the Appendix. For extreme values of ρ_e or c_b, the buyers' equilibrium strategies mirror the optimal policy decision described in Figure 1. For example, when ρ_e is close to 0 or close to 1 – or when c_b is high, there is not enough quality uncertainty to induce agents to screen, so that $\phi_{b,s} = 0$ (Regions A and B). In Region C where c_b is small, all buyers want to screen ($\phi_{b,s} = 1$). For economies in Region D, the unique stationary equilibrium is such that buyers randomize between screening and not trading at all. Those buyers who do screen have a selection effect on the quality distribution ρ_s (see equation (16)) and thereby reduce the value of information (as given by the right-hand side of (3)) for subsequent buyers. In equilibrium, the share of buyers who do screen is such that the equilibrium distribution ρ_s makes all subsequent buyers indifferent between acquiring information and remaining uninformed.

Economies in regions B and C exhibit multiple equilibria. Here, either no buyer acquires information, so that $\rho_s = \rho_e$. Or some buyers acquire information and drive down the average quality to some value $\rho_s < \rho_e$ low enough to make the information acquisition optimal.[6]

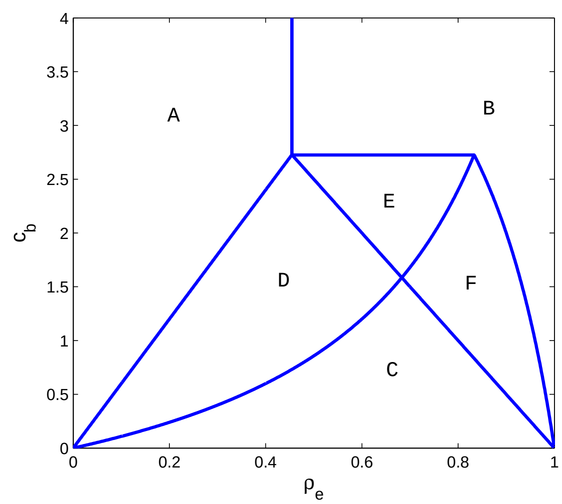


Figure 3: Characterization of equilibria without intermediation for different parameter values ρ_e and c_b

[6]The strategic complementarity in information choice has a flavor of similar results in Farhi and Tirole (2012) and Hellwig and Zhang (2013). However, here the mechanism supporting the multiplicity is entirely different, in that it comes from the selection effect. By contrast, the aforementioned papers keep the quality fixed and obtain the indeterminacy through changes in beliefs about future resale opportunities.

3 Equilibrium Intermediation

I now turn to equilibria with intermediation. That is, I investigate conditions under which intermediaries can earn rents in the model described above. For the remainder of the paper, I restrict the analysis to economies with a relatively low average quality among entering sellers.

Assumption 1. $\rho_e < \frac{-L}{H-L}$

This assumption implies that in bilateral meetings with sellers, buyers always face a lemons problem ($\psi_{b,s} = 0$). Moreover, it ensures that the economy is in either Region A, C, or D from Figure 3, so that the equilibrium without intermediation is unique.

To build intuition for why sellers of high quality goods may be willing to hire an intermediary, it is useful to revisit Rubinstein and Wolinsky's (1987) sufficient condition for intermediation. In their model, sellers favor intermediaries over independent search if intermediaries offer a higher speed of trade. In the present model, the speeds of trade are endogenous, but if $\alpha = 0$, then the sufficient condition remains that

$$\lambda\mu_b\phi_{b,i} > \lambda\mu_b\phi_{b,s}. \tag{24}$$

If intermediaries can convince buyers to screen more (as measured by $\phi_{b,i}$), then they can promise sellers a shorter expected duration of search.[7] Consider the case when intermediaries do enough screening to make $\rho_i > \rho_s$ but not enough to offer a pool without a lemons problem $\rho_i < \rho^*_{b,i}$. Through the buyer's value of information on the right-hand side of (6), $\phi_{b,i}$ in (24) is increasing in ρ_i. Intermediaries can induce buyers to screen more if they coordinate on a high screening effort themselves to increase ρ_i. Obviously, if $\alpha = 0$, intermediaries have no reason to do so since they cannot recover the sunk cost of screening. Hence, an additional condition applies that α is large enough to give the intermediary some rent.

In this section, to find conditions under which screening by intermediaries can be an equilibrium outcome, I restrict the analysis to economies with very few intermediaries. That is, I assume that e_i is small, so that $\mu_{i,1}$ and $\mu_{i,0}$ are close to zero and any general equilibrium effects of the intermediaries' behavior become negligible. In absence of these general equilibrium effects, the right-hand side of (24) is given by the equilibrium without intermediation. In the next section, I then gradually increase the inflow of intermediaries e_i to allow for some general equilibrium effects of intermediation and to show how these effects tend to diminish the right-hand side of (24), making intermediation more easily sustainable.

Before I begin the analysis, it is useful to make a few general observations the help reducing the set of candidate strategy profiles. First, note that an intermediary has an incentive to screen only if he can expect

[7] Note that under *complete information* the model has no equilibrium with intermediation. In that case, $J_H = S_H$, since intermediaries provide no value-added.

that in a future meeting the buyer also screens. If the buyer will doesn't screen, it means that either he refuses to trade altogether, in which case the good is worthless to the intermediary. Or the buyer is willing to trade without checking the quality, in which case the intermediary is not concerned about quality either, since $J_H = J_L$.

Lemma 1. *There is no stationary equilibrium with* $\phi_{b,i} = 0$ *and* $\phi_{i,s} > 0$.

Under Assumption 1, we can make the stronger statement that in an intermediation equilibrium at least some buyers must screen when meeting an intermediary. Otherwise, if $\psi_{b,i} > 0$ and $\phi_{b,i} = 0$, intermediaries would not screen either, so that $\rho_i = \rho_s$. Under Assumption 1, this would imply a lemons problem and render $\psi_{b,i} > 0$ unsustainable.

Lemma 2. *There is no stationary equilibrium with* $\phi_{b,i} = 0$ *and* $\psi_{b,i} > 0$.

Second, whenever some intermediaries screen, other intermediaries are willing buy without screening. In equilibrium, they have to be indifferent. Otherwise, if all intermediaries strictly preferred to screen, only high quality goods would be offered to sellers, so that $\rho_i = 1$. In that case, sellers had no reason to screen and, by Lemma 1, no intermediary would want to screen.

Lemma 3. *There is no stationary equilibrium with* $\phi_{i,s} = 1$.

More generally, one can show that in any equilibrium, uninformed intermediaries are willing to trade without screening, which confirms our claim from the previous section.

Lemma 4. *In any stationary equilibrium,* $\rho_s \alpha J_H + (1 - \rho_s)\alpha J_L - J \geq 0$.

Finally, note that in an equilibrium with intermediation, at least some intermediaries need to screen. Otherwise, $\rho_i \leq \rho_s$, so that the speed of trade through intermediaries cannot be higher than the speed of bilateral trade. Taken together with the previous results, this means that in an intermediation equilibrium intermediaries are always indifferent between screening and remaining uninformed.

Proposition 2. *In any equilibrium with intermediation,* $\phi_{i,s} \in (0, 1)$ *and* $\phi_{b,i} > 0$.

In the following, we are interested in conditions under which intermediation can be sustained as an equilibrium outcome under the assumption that e_i is small. That is, we are looking for conditions under which unmatched sellers are willing to participate in the intermediation contract. When there are few potential intermediaries entering the economy, the mass of both matched and unmatched intermediaries is very small. By continuity, the equilibrium outcome of a meeting between a buyer and a seller and their distributions and value functions are barely affected by the intermediaries' behavior. Therefore, if we let $\hat{x}$ be the equilibrium value of a variable x in a stationary equilibrium *without* intermediation, then the following applies.

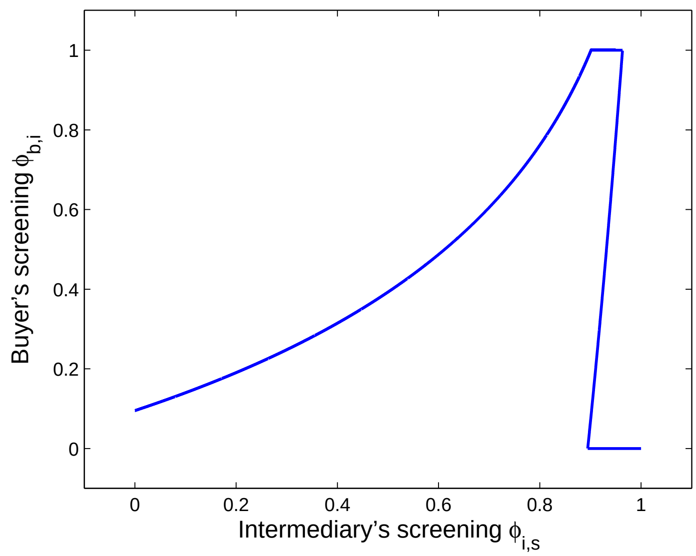


Figure 4: Buyer's best response

Lemma 5. *For any* $\epsilon > 0$, $\exists e_i^* > 0$ *such that any equilibrium satisfies* $|\phi_{b,s} - \hat{\phi}_{b,s}| < \epsilon$, $|\rho_s - \hat{\rho}_s| < \epsilon$, $|\mu_s - \hat{\mu}_s| < \epsilon$, $|\mu_b - \hat{\mu}_b| < \epsilon$, $|W - \hat{W}| < \epsilon$, $|S_H - \hat{S}_H| < \epsilon$, *for all* $e_i \in [0, e_i^*)$.

The Lemma allows us to treat these equilibrium objects as fixed in the spirit of a partial equilibrium exercise and treat only $\phi_{b,i}$, $\psi_{b,i}$, $\phi_{i,s}$, $\xi_{i,s}$, J, J_H, J_L, and ρ_i as endogenous.

In our discussion above we found that, if $\phi_{b,i}$ is larger than $\phi_{b,s}$, intermediaries can offer a higher speed of trade to sellers, but that intermediaries must receive a high enough share of the price for them to recover the screening cost. Figure 4 plots the buyer's equilibrium screening effort $\phi_{b,i}$ for a given value of the intermediaries' screening intensity $\phi_{i,s}$ and taking into account that all other buyers choose the same screening intensity $\phi_{b,i}$. That is, in equilibrium $\phi_{b,i}$ is a best response to both, $\phi_{i,s}$ and $\phi_{b,i}$. The curve is non-monotonic in $\phi_{i,s}$ because the buyer's value of information is non-monotonic in ρ_i. If all intermediaries screen, then $\rho_i = 1$ and additional information is worthless for buyers. Note that for lower values of $\phi_{i,s}$, buyers may be indifferent between screening and not screening. This is due to the selection effect in (17) through which buyers who screen lower the value of information to other buyers. For the similar reasons, the best response curve is backward bending for high values of $\phi_{i,s}$. When $\phi_{i,s}$ is sufficiently high, ρ_i is high enough to make uninformed buyers willing to trade. The value of information is therefore decreasing in ρ_i, so that, by acquiring information, buyers raise the value of information to other buyers. Hence, the backward bending (or Z-shaped) part of the best response curve mirrors the multiplicity of equilibria in Region F of Figure 3.

3.1 Equilibria with $\psi_{b,i} = 0$

First, let us focus on equilibria in which $\psi_{b,i} = 0$. For this to be the case, we need $\rho_i < \rho^*_{b,i}$ which holds as long as intermediaries do not screen too much. In these equilibria, the stationary match values J_H and J_L, the intermediaries' value of search J, and the quality composition ρ_i satisfy

$$J_H = \lambda\mu_b \frac{\phi_{b,i}(1-q)(H-W)}{r+\delta+\lambda\mu_b(1-q)\phi_{b,i}}, \tag{25}$$

$$J_L = 0, \tag{26}$$

$$J = \frac{\lambda\mu_s}{r+\delta+\lambda\mu_s}\alpha\rho_s J_H, \tag{27}$$

$$\rho_i = \frac{\delta\rho_s}{\delta\rho_s + (1-\phi_{i,s})(1-\rho_s)(\delta+\lambda\mu_b\phi_{b,i})} \tag{28}$$

Sellers participate whenever $J_H(1-\alpha) > S_H$, which we can now rewrite as

$$(1-\alpha)\lambda\mu_b \frac{\phi_{b,i}(1-q)(H-W)}{r+\delta+\lambda\mu_b(1-q)\phi_{b,i}} > \lambda\mu_b \frac{\phi_{b,s}(1-q)(H-W)}{r+\delta+\lambda\mu_b(1-q)\phi_{b,s}}.$$

This condition can be simplified to

$$\phi_{b,i} > \phi_{b,s} \frac{(r+\delta)}{(1-\alpha)(r+\delta) - \alpha\lambda\mu_b(1-q)\phi_{b,s}} \tag{29}$$

Given Proposition 2, intermediaries are indifferent between screening and remaining uninformed, or

$$c_i = (1-\rho_s)J$$

which can be rewritten as

$$c_i = (1-\rho_s)\alpha\rho_s \frac{\lambda\mu_s}{r+\delta+\lambda\mu_s}\lambda\mu_b \frac{\phi_{b,i}(1-q)(H-W)}{r+\delta+\lambda\mu_b(1-q)\phi_{b,i}} \tag{30}$$

Note that the right-hand side of (30) is increasing in $\phi_{b,i}$. Hence, there is a unique $\phi_{b,i}$ at which intermediaries are indifferent between screening and not screening.

The dashed line in the left panel of Figure 5 illustrates the intermediaries' screening choice as a best response to the buyers' screening policy when $\psi_{b,i} = 0$. In equilibrium, the buyers' and intermediaries' best-response correspondences must intersect. In general, there are at most two such intersections. One at $\phi_{i,s} = 0$ and one with $\phi_{i,s} \in (0,1)$. It is easy to see from (28) that for $\phi_{i,s} = 0$, $\rho_i < \rho_s$ so that $\phi_{b,i} < \phi_{b,s}$ and the sellers never participate. Hence, the only remaining candidate for an equilibrium is the interior intersection of the best-response correspondences. The equilibrium amount of screening by buyers is given by (30).

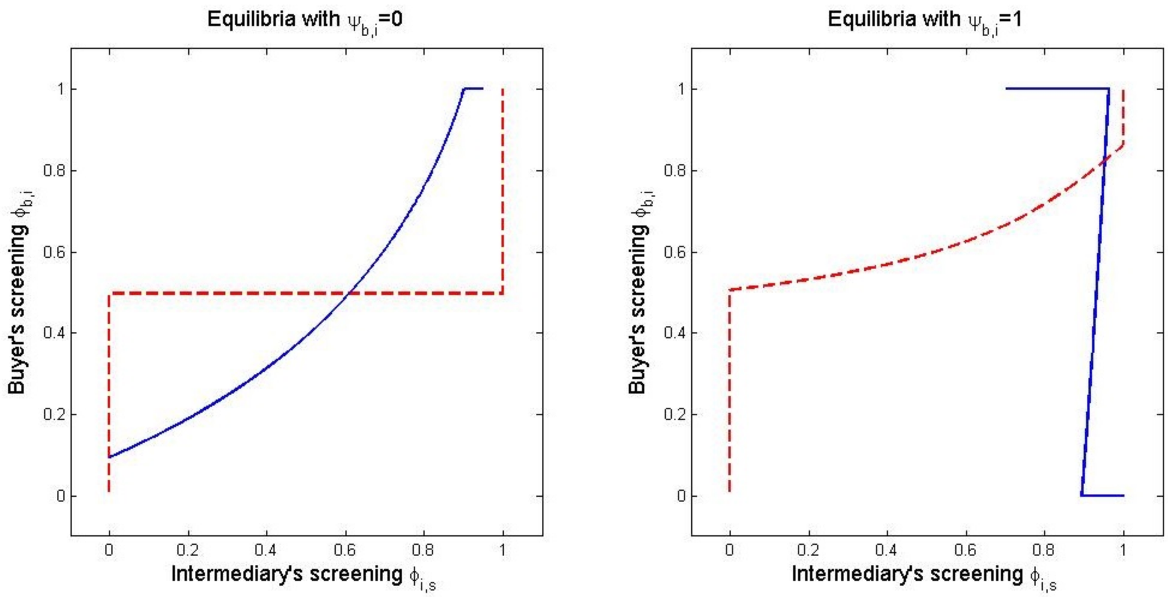


Figure 5: Equilibrium intermediation: Equilibria occur where the buyer's (solid line) and intermediary's (dashed line) best response correspondences intersect.

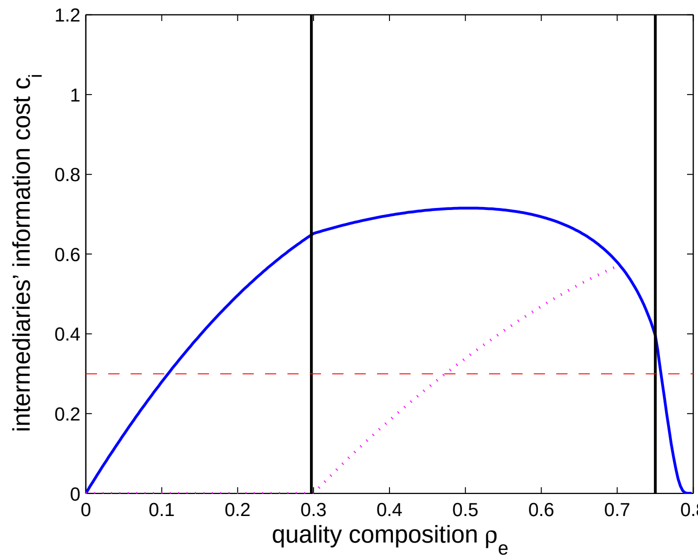
To establish the existence of an equilibrium, we need to verify two conditions. First, because we need to make sure that an interior intersection exists, so that the $\phi_{b,i}$ implied by (30) is between 0 and 1. This can be interpreted as the intermediary's incentive compatibility constraint. And second, we need to check that $\phi_{b,i}$ satisfies the participation constraint (29). Hence, an equilibrium with $\phi_{b,i} > 0$ and $\psi_{b,i} = 0$ exists if and only if

$$1 \geq \phi_{b,i} > \phi_{b,s} \frac{r+\delta}{(1-\alpha)(r+\delta) - \alpha\lambda\mu_b(1-q)\phi_{b,s}}, \tag{31}$$

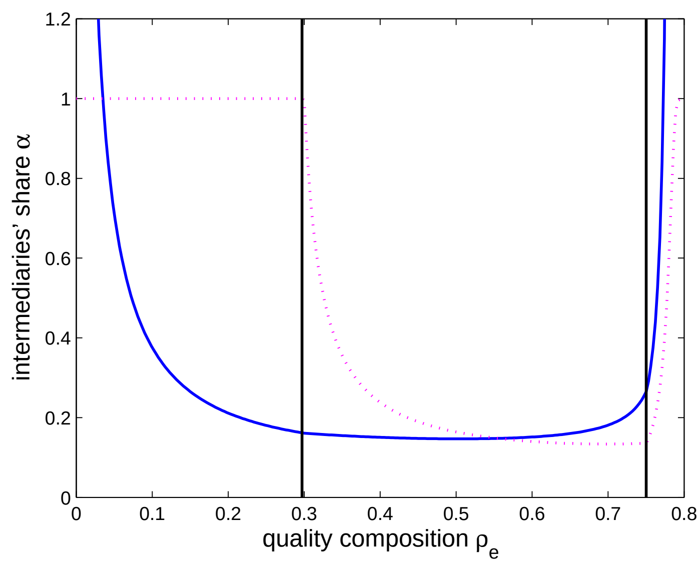
where

$$\phi_{b,i} = \frac{\frac{c_i}{\alpha}(r+\delta)}{\lambda\mu_b(1-q)\left[(1-\rho_s)\rho_s \frac{\lambda\mu_s}{r+\delta+\lambda\mu_s}(H-W) - \frac{c_i}{\alpha}\right]}. \tag{32}$$

Condition (31) implies that, for sufficiently high values of $\phi_{b,s}$, we can rule out an intermediation equilibrium with $\psi_{b,i} = 0$. Moreover, $\phi_{b,i}$ is decreasing in μ_b because, if we lower μ_b, each buyer must provide more screening to keep the intermediaries indifferent. This means that for sufficiently low levels of e_b, intermediation can also be ruled out. In other words, for intermediation to take place, a certain market size is required. Finally, note that $\phi_{b,i}$ is strictly increasing in $\frac{c_i}{\alpha}$ and that the participation threshold in (31)



(a) Conditions on c_i: Each point in the graph represents an economy. The vertical lines separate the regions for which $\phi_{b,s} = 0$, $\phi_{b,s} \in (0, 1)$, and $\phi_{b,s} = 1$. The horizontal dashed line indicates economies for which $c_i = c_b$. The two other curves represent the two inequalities in (31). Economies below the upper bound satisfy the condition $\phi_{b,i} \leq 1$. Economies above the dotted upward-sloping line satisfy the sellers' participation constraint. Therefore, economies between the two lines have an intermediation equilibrium.



(b) Conditions on α: Each point in the graph represents an economy. The vertical lines separate the regions for which $\phi_{b,s} = 0$, $\phi_{b,s} \in (0, 1)$, and $\phi_{b,s} = 1$. The intermediary's information cost is held constant at $c_i = c_b$. The two curves represent the two inequalities in (31). Economies above the solid bound satisfy the condition $\phi_{b,i} \leq 1$. Economies below the dotted line satisfy the sellers' participation constraint. Therefore, economies between the two lines have an intermediation equilibrium with $\psi_{b,i} = 0$.

Figure 6: Conditions for an intermediation equilibrium with $\psi_{b,i} = 0$

is increasing in α. Hence, the two conditions together imply upper and lower bounds on c_i and α within which an intermediation equilibrium exists. Figure 6a illustrates the two conditions for different values of the parameters ρ_e and c_i. The horizontal dashed line marks the buyer's information cost so that, for some values of ρ_e, intermediation may take place even if $c_i > c_b$. Figure 6b plots the two conditions for different values of the splitting rule α and ρ_s, while keeping the intermediaries' cost fixed at $c_i = c_b$.

3.2 Equilibria with $\psi_{b,i} = 1$

To find conditions for an intermediation equilibrium in which uninformed buyers are willing to trade, we proceed as in the preceding Section. An equilibrium is again a situation in which buyers' and sellers' screening decisions are mutually best responses, as illustrated by the right panel of Figure 5. One key difference between the two sides of Figure 5 is that now the intermediary's best response is no longer a simple cutoff strategy. When uninformed buyers are willing to trade, the cutoff at which an intermediary is indifferent between screening and remaining uninformed depends on other intermediaries' screening effort. This is due to the fact that uninformed buyers offer the price

$$qJ_H + (1-q)\left[\rho_i H + (1-\rho_i)L - W\right],$$

which depends on $\phi_{i,s}$ through the quality share ρ_i. If one intermediary decides to screen more, he marginally raises ρ_i (see equation (17)). Due to this improvement in the expected quality of matched sellers, uninformed buyers are willing to pay a higher price, and being matched with a lemon seller is less bad for an intermediary (αJ_L increases). The value of information to an intermediary is the expected value of avoiding a lemon seller, $(1-\rho_s)(J-\alpha J_L)$, which decreases in turn.

The second difference is that for any given value of $\phi_{i,s}$ an intermediary's cutoff is higher in the right panel than in the left. This is because, when uninformed buyers are willing to trade, obtaining a lemon is less costly for the intermediary, so that the value of information is lower.

When $\phi_{b,i} = 1$, the decision of uninformed buyers, $\psi_{b,i}$, does not matter for the intermediary's decision problem. Therefore, the upper bound on c_i for an intermdiation equilibrium to exist is the same as in the preceding section. On the other hand, Figure 5 implies that the seller's participation constraint is less restrictive when $\psi_{b,i} = 1$ for two reasons. First, keeping $\phi_{b,i}$ fixed, a high quality seller's speed of trade is increasing in $\psi_{b,i}$. And second, due to the upward shift of the intermediary's best response (the dashed line), the equilibrium screening level $\phi_{b,i}$ is higher when $\psi_{b,i} = 1$. Therefore, whenever there exists an intermediation equilibrium with $\psi_{b,i} = 0$, there also exists an equilibrium with $\psi_{b,i} = 1$.

More formally, we can solve for the stationary values to obtain

$$
\begin{aligned}
J_H &= \lambda\mu_b(1-q)\frac{H-W-(1-\phi_{b,i})(1-\rho_i)(H-L)}{r+\delta+\lambda\mu_b(1-q)}, \\
J_L &= \lambda\mu_b(1-\phi_{b,i})\frac{(1-q)\left(\rho_i H+(1-\rho_i)L-W-J_H\right)+J_H}{r+\delta+\lambda\mu_b(1-\phi_{b,i})}, \\
J &= \frac{\lambda\mu_s\alpha}{r+\delta+\lambda\mu_s}\left[\rho_s J_H+(1-\rho_s)J_L\right],
\end{aligned}
$$

and

$$
\rho_i = \frac{\rho_s}{\rho_s+(1-\rho_s)\frac{(1-\phi_{i,s})(\delta+\lambda\mu_b)}{\delta+\lambda\mu_b(1-\phi_{b,i})}}. \tag{33}
$$

The intermediaries' cutoff rule, and hence their best response correspondence is given by

$$
c_i = (1-\rho_s)\left[J-\alpha J_L\right], \tag{34}
$$

while the buyers' cutoff rule is

$$
c_b = (1-\rho_i)q(J_H+W-L). \tag{35}
$$

Hence, the equilibrium strategies $\phi_{b,i}$ and $\phi_{i,s}$ and the quality composition ρ_i are jointly determined by (33)-(35). To find parameters such that the participation constraint holds in the steady state, we need to make sure that $(1-\alpha)J_H > S_H$, or

$$
(1-\alpha)\frac{H-W-(1-\phi_{b,i})(1-\rho_i)(H-L)}{r+\delta+\lambda\mu_b(1-q)} > \frac{\phi_{b,s}(H-W)}{r+\delta+\lambda\mu_b(1-q)\phi_{b,s}}. \tag{36}
$$

Figure 7 adds condition (36) to Figure 6a. As discussed above, the sellers' participation constraint in an equilibrium with $\psi_{b,i} = 1$ is always below the participation constraint when $\psi_{b,i} = 0$, so that the two intermediation equilibria can coexist.

3.3 Discussion

The finding that the model can have multiple intermediation equilibria is directly related to the assumption that buyers and intermediaries both face a choice of costly information acquisition. Due to this feature, there is a complex coordination problem. For sellers to participate, buyers need to screen with a sufficiently high probability. Through ρ_i, each individual buyer's screening effort is determined by both the intermediaries' and the other buyers' average screening choice. The more intermediaries screen, the more the buyers will screen in the future. And the more buyers screen in the future, the more intermediaries want to screen today. In previous work on intermediation in dynamic lemons markets buyers have either no access to information (Biglaiser (1993)) or they can observe the true quality at no cost with some probability (Li (1998)), so that

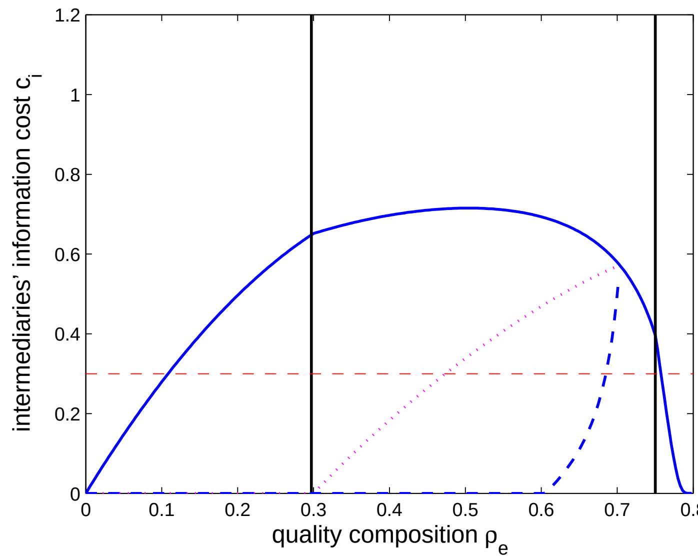


Figure 7: Conditions for intermediation equilibria: Each point in the graph represents an economy. The vertical lines separate the regions for which $\phi_{b,s} = 0$, $\phi_{b,s} \in (0, 1)$, and $\phi_{b,s} = 1$. The horizontal dashed line indicates economies for which $c_i = c_b$. The three other lines represent the inequalities in (31) and (36). Economies below the upper bound satisfy the condition $\phi_{b,i} \leq 1$. Economies above the dotted bound satisfy the sellers' participation constraint when $\psi_{b,i} = 0$. Economies above the dashed upward-sloping line have satisfy the sellers' participation constraint when $\psi_{b,i} = 1$.

there can be no such interaction between buyers' and intermediaries' screening decisions.

The result that the average screening intensity is increasing in the intermediary's information cost comes from the strategic complementarity in information choices, together with the fact that intermediaries need to be indifferent between screening and not screening (see Proposition 2). In the discussion of Lemma 3, which ruled out $\phi_{i,s} = 1$, it was pointed out that, if all intermediaries were to screen, buyers would just trust them and refrain from screening on their own. This argument relied on the assumption that intermediaries are short-lived and cannot build a reputation for selling only high quality goods, so that they can never be fully trusted. Neither can they credibly reveal their information to buyers. Similar assumptions are made in Biglaiser (1993) and Li (1998). In a related paper, Lizzeri (1999) finds that, even if they can reveal some of their information to buyers, intermediaries have little incentives to do so.

While individual reputation is absent from the model, the quality share ρ_i captures the reputation of matched intermediaries as a group in the sense of (Coate and Loury (1993) or Tirole (1996)). By applying more scrutiny, each intermediary marginally improves his group's reputation. However, because this externality does not enter their private value of information, intermediaries screen too little. As in Coate and Loury (1993), each matched intermediary's membership in the group is observable by buyers, but not their individual type. Hence, the resulting statistical discrimination and the multiplicity of equilibria are reminiscent of similar findings in other contexts.

4 General Equilibrium Effects

This section is a comparative statics exercise in which I gradually allow more potential intermediaries to enter the economy. When there are more intermediaries in the market, and when sellers participate in the intermediation contract, intermediaries have an effect on μ_b, μ_s, ρ_s, and on the buyers' and sellers' value functions, which in turn influence $\phi_{b,s}$, the outcome of bilateral meetings. I explore how this influence affects the existence region in Figure 6a. Then, in the next section, I discuss two numerical examples to study the impact of intermediation on the trade volume in the market. Note that while (32) did not rely on our previous assumption that e_i is small. Therefore, the existence region is still given by these conditions. However, some of the elements used in the conditions do depend on the intermediaries' behavior and hence on their masses, so that the size and position of the region change if e_i grows.

We have seen above that, for smaller values of e_i, intermediation can take place only if $\hat{\phi}_{b,s} < 1$. In that case, buyers are indifferent between screening and not trading at all, so that their value in the equilibrium without intermediation is $\hat{W} = 0$. When there is intermediation and $\psi_{b,i} = 0$, the buyers' value of search is still $W = 0$. On the other hand, when $\psi_{b,i} = 1$, W is strictly positive. The value of search to sellers is

$$S_H = \lambda\mu_b\phi_{b,s}(1-q)\frac{H-W}{r+\delta+\lambda\mu_b(1-q)\phi_{b,s}+\lambda\mu_{i,0}} + \lambda\mu_{i,0}\frac{(1-\alpha)J_H}{r+\delta+\lambda\mu_b(1-q)\phi_{b,s}+\lambda\mu_{i,0}}. \tag{37}$$

In a steady state, (16) becomes

$$\rho_s = \frac{\rho_e}{\rho_e + (1-\rho_e)\frac{\delta+\lambda\mu_b\phi_{b,s}+\lambda\mu_{i,0}}{\delta+\lambda\mu_{i,0}(1-\phi_{i,s})}}. \tag{38}$$

The response of ρ_s and $\phi_{b,s}$ to the entry of additional intermediaries is jointly determined by the buyers' indifference condition $c_b = q\rho_s\left(H - W - S_H\right)$, which I rewrite as

$$\frac{c_b}{q(H - S_H(\phi_{b,s}) - W)} = \rho_s(\phi_{b,s}) \tag{39}$$

The left-hand side of (39) is increasing in $\phi_{b,s}$ and the right-hand side is decreasing. Figure 8 illustrates how condition (39) determines the response of ρ_s and $\phi_{b,s}$ to a change in $\mu_{i,0}$. The two curves representing the two sides of (39) both shift to the left as additional intermediaries enter. Due to the selection effect, the equilibrium quality share ρ_s is decreasing in $\mu_{i,0}$ for a given level of $\phi_{b,s}$. And due to the opportunity cost effect, S_H and W are increasing in $\mu_{i,0}$, so that the left-hand side of (39) is also increasing in $\mu_{i,0}$. Both effects imply that an inflow of additional middlemen unambiguously leads to a lower level of screening $\phi_{b,s}$. Intermediation crowds out bilateral trade.

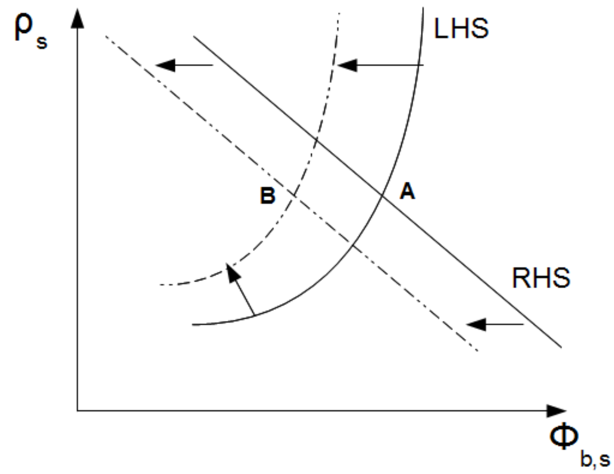


Figure 8: Illustration of condition (39) and the response of ρ_s and $\phi_{b,s}$ to a larger inflow of intermediaries. Both curves shift to the left and the economy moves from point A to B.

The effect on ρ_s, however, is ambiguous. On the one hand, the additional intermediaries and their screening activity reduce ρ_s, according to (38). On the other hand, the response of $\phi_{b,s}$ tends to raise ρ_s. The net effect depends on the relative importance of the selection and opportunity cost effect.

Given this discussion of condition (39), an increase in e_i affects condition (31) in two ways. First, because $\phi_{b,s}$ is smaller, the sellers' participation constraint is relaxed, which means that the existence region in Figure 6a expands to the right. Second, in (32), the buyers' strategy $\phi_{b,i}$ depends negatively on ρ_s if $\rho_s < \frac{1}{2}$ and positively otherwise. Assuming that ρ_s is small, this means that the effect of additional intermediaries on $\phi_{b,i}$ depends on q. If q is small, then an increase in e_i leads to an increase in $\phi_{b,i}$, which shifts the entire existence region in Figure 6a to the right. If we take an economy that has an intermediation equilibrium and let more potential intermediaries enter, then at some point the equilibrium disappears. Such a scenario is studied in Figure 9. On the other hand, if we take an economy with no intermediation equilibrium, an equilibrium with intermediation may appear if we augment the inflow of middlemen. Note also that, if the left-hand side of (39) responds more strongly, the region in which an intermediation equilibrium exists gets larger as e_i increases, because, through ρ_s, $\phi_{b,i}$ is decreasing in e_i.

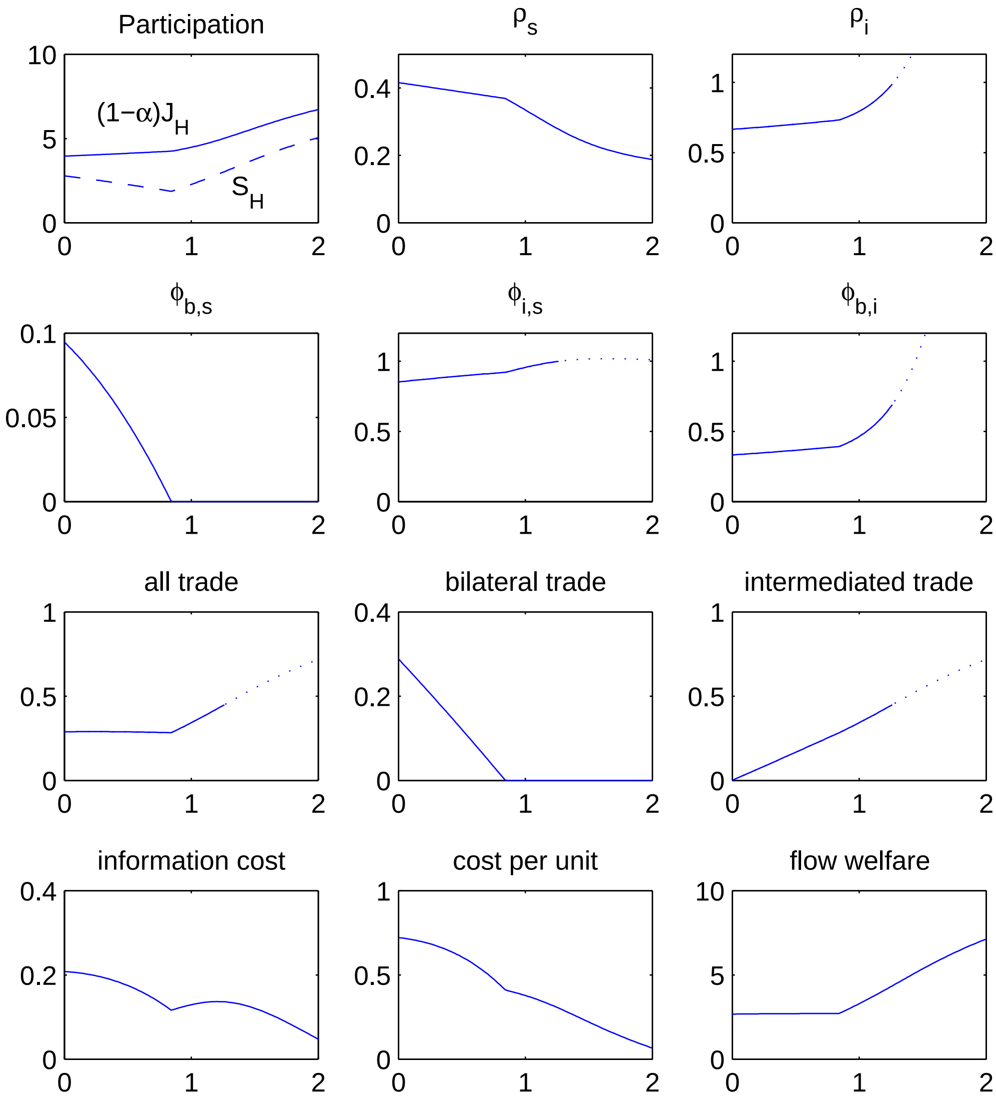


Figure 9: Comparative statics: increasing e_i: The equilibrium without intermediation corresponds to the case when e_i is 0. When e_i becomes larger, the intermediation equilibrium eventually disappears (as indicated by the dotted lines).

5 Examples

5.1 The Selection Effect

In the first numerical example, I consider an economy that has an intermediation equilibrium when e_i is small. Moreover, in the equilibrium without intermediation, we have $\phi_{b,s} \in (0,1)$. The equilibrium without intermediation is further characterized in Figure 9, for $e_i = 0$. I then undertake a comparative statics exercise similar to the previous section, in which I gradually increase e_i. Consistent with the analysis above, the participation constraint holds throughout the exercise (first panel of Figure 9). However, as we let e_i grow, $\phi_{i,s}$, ρ_i, and $\phi_{b,i}$ also grow up to the point where the economy leaves the existence region.

In this example, the more intermediaries are allowed into the economy, the less trade is conducted bilaterally. Due to the selection effect, screening by intermediaries drives down the quality share ρ_s, so that buyers value information less when meeting a seller and $\phi_{b,s}$ decreases. The crowding-out in bilateral trade is more than offset by the growth in intermediated trade. Not only are there more intermediaries as we increase e_i, but these additional intermediaries also induce buyers to screen more. In addition to the additional trade, intermediaries also reduce the amount of resources used up in the production of information. Hence, in the present example, intermediaries unambiguously lead to a higher steady-state level of welfare even though they have no technological advantage over buyers and sellers. Note that the crowding out in the present example is exclusively driven by the selection effect. Buyers are indifferent between trading and not trading so that their value of search W is always zero and the opportunity cost effect does not come to bear.

5.2 The Opportunity Cost Effect

When the mass of sellers is very large, the masses of buyers or intermediaries are too small to have an impact on the quality composition in the pool of sellers, so that $\rho_s \to \rho_e$. In this example, I therefore assume that $e_s \to \infty$ to shut down the selection effect. Let $\rho_e > \frac{c}{qH}$, so that in the equilibrium without intermediation, all buyers are willing to screen ($\phi_{b,s} = 1$). Due to the large mass of sellers, the equilibrium trade flow is e_b. Buyers enter and find a seller immediately.

In an intermediation equilibrium, when $e_s \to \infty$ and $\phi_{b,s} = \psi_{b,s} = 0$, and $\phi_{i,s} > 0$, we obtain

$$\begin{aligned}
\mu_b &\to \frac{e_b}{\delta + \lambda \mu_{i,1} \rho_i}, \\
\mu_s &\to \infty, \\
\mu_{i,0} &\to 0, \\
\mu_{i,1} &\to \frac{e_i}{\delta + \lambda \mu_b \rho_i}.
\end{aligned}$$

Because buyers don't trade immediately when entering the economy, the trade flow in the gatekeeper equilibrium is smaller than in the equilibrium with bilateral trade only.

$$\lambda\mu_b\mu_{i,1}\rho_i = e_b - \delta\mu_b < e_b.$$

To make sure that this is indeed an equilibrium, we need to verify that buyers are willing to wait for an intermediary instead of trading immediately with an unmatched seller. The value of deviating is

$$\rho_e qH - c_b,$$

whereas the value of waiting is

$$W = \lambda\mu_{i,1}\frac{q\rho_i H - c_b(1+\lambda\mu_b\frac{1-q}{r+\delta})}{r+\delta+\lambda\mu_b(1-q)+\lambda\mu_{i,1}q\rho_i},$$

so that buyers have no incentive to deviate whenever

$$\lambda\mu_{i,1}\frac{q\rho_i H - c_b(1+\lambda\mu_b\frac{1-q}{r+\delta})}{r+\delta+\lambda\mu_b(1-q)+\lambda\mu_{i,1}q\rho_i} \geq \rho_e qH - c_b. \tag{40}$$

Obviously, to obtain an intermediation equilibrium, the inflow of intermediaries has to be sufficiently high. Figure 10 shows the impact of more intermediaries on the intermediation equilibrium when the mass of sellers is very large. For values of e_i high enough, the buyer's value of search becomes larger than the value of trading screening the seller immediately. The quality share among unmatched sellers barely changes as e_i increases. Even though on average intermediaries help buyers save some information cost, the overall welfare gains from intermediation are negative due to the much smaller trade volume.

6 Conclusion

This paper investigates the role of intermediaries in markets with quality uncertainty and arrives at two main results. First, intermediaries can earn rents even if they have no advantage in terms of their search or screening technologies. In the model, intermediaries distinguish themselves from buyers through their preferences rather than technologies. The fact that intermediaries are not able or willing to buy the goods for themselves and that they therefore face no downside risk from low quality assets implies that they have different incentives to acquire information than buyers. Intermediaries who screen divide the market into two pools of goods so that buyers can engage in statistical discrimination when they encounter sellers with and without intermediaries. While the intuition of Rubinstein and Wolinsky (1987) that intermediaries provide

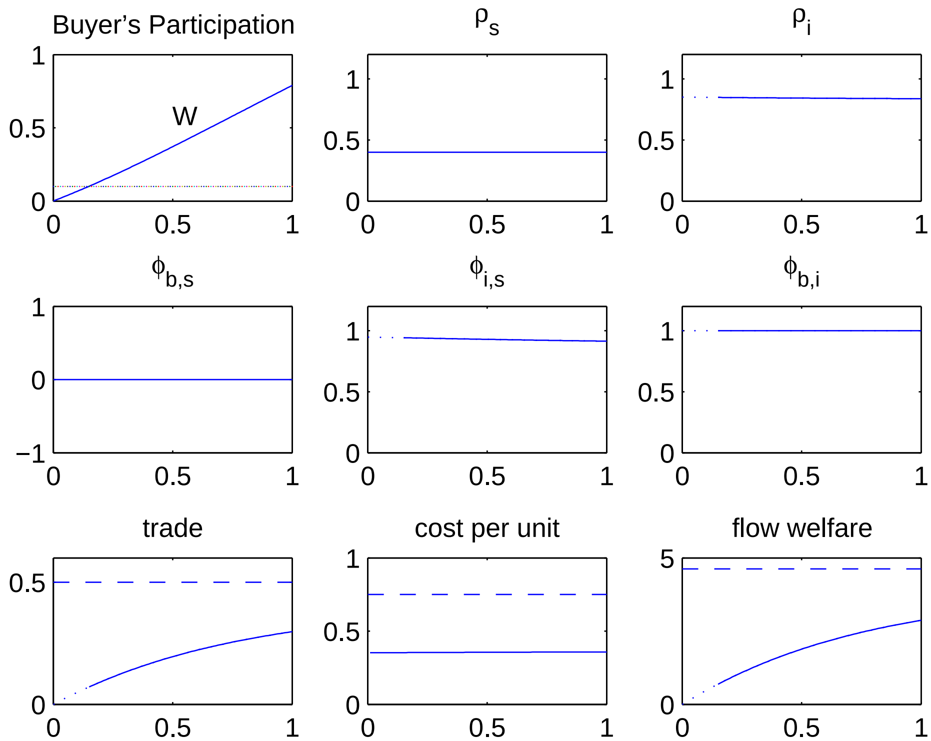


Figure 10: Comparative statics in Example 2: increasing e_i from 0 to 1. The first panel illustrates the buyers' participation constraint (40). The bottom row compares trade volumes, information cost and flow welfare in the intermediation equilibrium (solid line) with the equilibrium without intermediation (dashed line).

a faster speed of trading is still valid, the speeds of intermediated and bilateral trade are endogenous under asymmetric information. The second result is that the presence of intermediaries has important general equilibrium effects, due to which the welfare implications of intermediation are ambiguous. The numerical examples demonstrate that intermediaries can be both, facilitators and bottlenecks.

References

[1] Gara Afonso and Ricardo Lagos. Trade dynamics in the market for federal funds. *FRB of New York Staff Report*, (549), 2012.

[2] George A Akerlof. The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, pages 488–500, 1970.

[3] Gary Biglaiser. Middlemen as experts. *The RAND journal of Economics*, pages 212–223, 1993.

[4] John H Boyd and Edward C Prescott. Financial intermediary-coalitions. *Journal of Economic Theory*, 38(2):211–232, 1986.

[5] Braz Camargo and Benjamin Lester. Trading dynamics in decentralized markets with adverse selection. *Unpublished Manuscript*, 2011.

[6] Jonathan Chiu and Thorsten V Koeppl. Trading dynamics with adverse selection and search: Market freeze, intervention and recovery. 2011.

[7] Stephen Coate and Glenn C Loury. Will affirmative-action policies eliminate negative stereotypes? *The American Economic Review*, pages 1220–1240, 1993.

[8] Tri Vi Dang. Bargaining with endogenous information. *Journal of Economic Theory*, 140(1):339–354, 2008.

[9] Tri Vi Dang, Gary Gorton, and Bengt Holmström. Financial crises and the optimality of debt for liquidity provision. *Unpublished working paper. Yale School of Management*, 2010.

[10] Douglas W Diamond. Financial intermediation and delegated monitoring. *The Review of Economic Studies*, 51(3):393–414, 1984.

[11] Darrell Duffie, Nicolae Gârleanu, and Lasse Heje Pedersen. Over-the-counter markets. *Econometrica*, 73(6):1815–1847, 2005.

[12] Erik Durbin. Mcdonald's or the michelin guide? revealing quality through private-sector certification. Technical report, 1999.

[13] Emmanuel Farhi and Jean Tirole. Information, tranching and liquidity. Technical report, Institut d'Économie Industrielle (IDEI), Toulouse, 2012.

[14] Veronica Guerrieri and Robert Shimer. Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. Technical report, National Bureau of Economic Research, 2012.

[15] Veronica Guerrieri, Robert Shimer, and Randall Wright. Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862, 2010.

[16] Klaus-Peter Hellwig and Shengxing Zhang. Market runs: Liquidity and the value of information. 2013.

[17] Kory Kroft, Fabian Lange, and Matthew J Notowidigdo. Duration dependence and labor market conditions: Evidence from a field experiment*. *The Quarterly Journal of Economics*, 2013.

[18] Ricardo Lagos and Guillaume Rocheteau. Liquidity in asset markets with search frictions. *Econometrica*, 77(2):403–426, 2009.

[19] Yiting Li. Middlemen and private information. *Journal of Monetary Economics*, 42(1):131–159, 1998.

[20] Alessandro Lizzeri. Information revelation and certification intermediaries. *The RAND Journal of Economics*, pages 214–231, 1999.

[21] Ben Lockwood. Information externalities in the labour market and the duration of unemployment. *The Review of Economic Studies*, 58(4):733–753, 1991.

[22] Ariel Rubinstein and Asher Wolinsky. Middlemen. *The Quarterly Journal of Economics*, 102(3):581–93, August 1987.

[23] Robert Shimer and Lones Smith. Matching, search, and heterogeneity. *Advances in Macroeconomics*, 1(1):1010–1029, 2001.

[24] Jean Tirole. A theory of collective reputations (with applications to the persistence of corruption and to firm quality). *The Review of Economic Studies*, 63(1):1–22, 1996.

[25] Robert M Townsend. Intermediation with costly bilateral exchange. *The Review of Economic Studies*, 45(3):417–425, 1978.

[26] Randall Wright and Yuet-Yee L Wong. Buyers, sellers and middlemen: Variations on search-theoretic themes. Technical report, Working paper, 2011.

Appendix

A Equilibria without Intermediation

Proposition 3. *There exists a stationary equilibrium without intermediation. For any* $c_b > 0$*, there exists* $\hat{\rho}(c_b) \in (0,1)$ *such that,*

1. *For* $\rho_e < \hat{\rho}(c_b)$ *and* $c_b \geq q\rho_e H$*, the unique equilibrium without intermediation is* $(\phi_{b,s}, \psi_{b,s}) = (0,0)$.

2. *For* $\rho_e < \hat{\rho}(c_b)$ *and* $q\rho_e H > c_b > \kappa_1 \rho_e q H$*, the unique equilibrium without intermediation is* $\phi_{b,s} \in (0,1)$ *and* $\psi_{b,s} = 0$.

3. *For* $\rho_e < \hat{\rho}(c_b)$ *and* $c_b < \kappa_1 \rho_e q H$*, the unique equilibrium without intermediation is* $\phi_{b,s} = 1$

4. *For* $\rho_e > \hat{\rho}(c_b)$*, there exists an equilibrium without intermediation and* $(\phi_{b,s}, \psi_{b,s}) = (0,1)$

5. *For* $\rho_e > \hat{\rho}(c_b)$ *and* $\kappa_1 \rho_e q H < c_b < qH\frac{-L}{H-L}$*, there exist*
   - *an equilibrium without intermediation and* $\phi_{b,s} \in (0,1)$ *and* $\psi_{b,s} = 0$.
   - *an equilibrium without intermediation and* $\phi_{b,s} \in (0,1)$ *and* $\psi_{b,s} = 1$.

6. *For* $\rho_e > \hat{\rho}(c_b)$ *and* $c_b < \min(\kappa_1 \rho_e q H, \frac{q(1-\tilde{\rho}_s)}{r+\delta+\lambda\tilde{\mu}_b(1-q)+\lambda\tilde{\mu}_s q}\left[\tilde{\rho}_s(H-L)-(r+\delta)L\right])$*, there exist*
   - *an equilibrium without intermediation and* $\phi_{b,s} \in (0,1)$ *and* $\psi_{b,s} = 1$.
   - *an equilibrium without intermediation and* $\phi_{b,s} = 1$

where $\kappa_1 = \frac{\delta}{\delta\rho_e+(1-\rho_e)(\delta+\lambda\tilde{\mu}_b)}\frac{r+\delta}{r+\delta+\lambda\tilde{\mu}_b(1-q)}$ *and* $\tilde{\mu}_b$*,* $\tilde{\mu}_s$*,* and $\tilde{\rho}_s$ *are the stationary values of* μ_b*,* μ_s*, and* ρ_s *when* $\phi_{b,s} = 1$.

The proof is similar to Hellwig and Zhang (2013).

B Proofs

Proof of Proposition 1

Given a stationary strategy profile $\Gamma = (\phi_{b,s}, \psi_{b,s}, \phi_{b,i}, \psi_{b,i}, \phi_{i,s}, \xi_{i,s})$, we can define a mapping $\mathcal{M}(\Gamma)$ as the vector $(\mu_b, \mu_s, \mu_{i,0}, \mu_{i1}, \rho_s, \rho_i)$ that solves the system of equations obtained by setting the left-hand sides of (9)-(17) to zero. Similarly, there is a mapping $\mathcal{Q} : [0,1]^6 \times \mathbb{R}^4_+ \times [0,1]^2 \longrightarrow \mathbb{R}^6_+$ such that, given a strategy profile Γ and a distribution $M = (\mu_b, \mu_s, \mu_{i,0}, \mu_{i1}, \rho_s, \rho_i)$, the elements of $\mathcal{Q}(\Gamma; M)$ solve the system of equations implied by (18)-(23) when $(\dot{W}, \dot{J}, \dot{J}_H, \dot{J}_L, \dot{S}_H, \dot{S}_L) = \mathbf{0}$.

The optimal decision rules given by equations (1)-(8) imply a correspondence $-' : [0,1]^6 \longrightarrow [0,1]^6$, where $\Gamma'(\Gamma)$ is the vector of policies $(\phi'_{b,s}, \psi'_{b,s}, \phi'_{b,i}, \psi'_{b,i}, \phi'_{i,s}, \xi'_{i,s})$ that are optimal given the distribution $\mathcal{M}(\Gamma)$ and the valuations $\mathcal{Q}(\Gamma; \mathcal{M}(\Gamma))$. The correspondence Γ' is convex-valued and defined on a non-empty, closed, bounded, and convex subset of $\mathbb{R}^6_+$. It maps into $\mathbb{R}^6_+$ and has a closed graph. A stationary equilibrium is a fixed point of Γ' which exists by Kakutani's Fixed Point Theorem.

Proofs for Section 3

Lemma. *There is no stationary equilibrium with $\phi_{b,i} = 0$ and $\phi_{i,s} > 0$.*

Proof. Whenever $\phi_{b,i} = 0$, $J_H = J_L$, so that

$$J = \frac{\phi_{i,s}\left(\rho_s \alpha J_L - c_i\right) + (1 - \phi_{i,s})\alpha J_L}{\frac{r+\delta}{\lambda \mu_s} + \rho_s \phi_{i,s} + (1 - \phi_{i,s})} < \alpha J_L.$$

$J < \alpha J_L$ implies that the value of information, $(1 - \rho_s)(J - \alpha J_L)$ is negative. □

Lemma. *There is no stationary equilibrium with $\phi_{i,s} = 1$.*

Proof. whenever $\phi_{i,s} = 1$, it must be that $\rho_i = 1$. The buyers' best response to such a strategy would be $(\phi_{b,i}\psi_{b,i}) = (0, 1)$. But the intermediary's best response to this would also be $\phi_{i,s} = 0$. □

Lemma. *In any stationary equilibrium, $\rho_s \alpha J_H + (1 - \rho_s)\alpha J_L - J \geq 0$.*

Proof. Suppose that uninformed intermediaries do not want to trade, because $\rho_s \alpha J_H + (1-\rho_s)\alpha J_L - J < 0$. For this to be an equilibrium, we need $\rho_s < \frac{J - \alpha J_L}{\alpha J_H - \alpha J_L}$. But when intermediaries do not acquire information, there is no trade at all, so that $J = 0$, so that $\rho_s \geq \frac{J - \alpha J_L}{\alpha J_H - \alpha J_L}$. □