# Taxing Cash? 

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January 2015


#### Abstract

We present a bargaining model of tax evasion where a buyer offers a price discount to a seller who does not ask for the receipt and pays cash, easing tax evasion. We show that a tax on cash withdrawals (TCW), which imposes a cost on the buyers who pay cash, is effective at reducing evasion only if it is set sufficiently high, and it must be higher the higher the tax evasion in the country and the larger the mass of individuals that typically pays in cash. We discuss the implementation problems and suggest how to partially overcome them.


JEL: O17, H21

Keywords: collaborative tax evasion; tax on cash

[^0]
## 1 Introduction

The first economic model of tax evasion by Allingham and Sandmo (1972) explains evasion as the result of a cost benefit analysis by perfectly rational individuals, who choose to evade if the expected cost of the sanction is lower than the tax payments. A great deal of economic literature followed their pioneering work, adding many elements to their baseline framework (Sirinvasan 1973; Yitzhaki 1974; Baldry 1979; Marrelli 1984; Reinganum and Wilde 1985; Usher 1986; Marrelli and Martina 1988; Andreoni 1992). However, what is in general missing in this literature, besides few exceptions, is the role of the buyer: asking for a receipt of the transaction makes tax evasion more difficult, while paying cash without asking for a receipt facilitates it. Since the buyers have the power to ease or impede tax evasion, it is plausible that some sellers will try to induce a cooperative behavior from them, for instance offering a price discount. When the two parties reach an agreement, a form of "collaborative tax evasion" takes place.

Our goal is to study the effects of two policy instruments designed to stifle this collaborative tax evasion: a tax deduction for the buyers who keep the receipt of the transaction and a tax on cash withdrawals (TCW henceforth) that imposes a cost on the buyers who pay cash.

The tax deduction, or tax credit, is a standard policy instrument, embedded in many tax codes around the world, sometimes very creatively: in Taiwan, China, Puerto Rico and in the city of Sao Paulo, for instance, the receipt of the transaction can be used to claim a lottery ticket (Marchese 2009, Fabbri 2013). The purpose of this tax deductions is to reduce evasion by rewarding honest taxpayers, rather than punishing dishonest ones, and many experimental studies suggest that this strategy can be effective (among others, Alm et al. 1992 and Berhan and Jerkins 2005). Similarly to the tax rebates, the purpose of the TCW should be to reduce the incentives of the buyer to cooperate with the seller. The argument goes as follows: since evasion is facilitated, if not allowed, by the use of cash, then making cash more expensive should induce less cooperation and less evasion. We are aware of only two countries that implemented this tax, Pakistan in 2001 and India from 2005 to 2009 (the so called Banking Cash Transaction Tax or BCCT). In both cases, however, the official reason for the introduction of the tax was not to directly reduce tax evasion nor to increase tax revenues, but rather to provide information for the tax enforcing authorities to
better guide the audits ${ }^{1}$.
We present a model where price taking sellers enter in a bargaining round with their customers, offering a price discount in exchange for not issuing the receipt. A deal forces the customer to pay cash, since debit cards, credit cards, bank transfers, checks and other non-cash instruments leave a trace of the transaction that impedes evasion. Conversely, there is no discount for the customer if there is no deal, but this leaves him free to choose between cash and non-cash instruments. We model heterogenous sellers with respect to their honesty or tax morale, and heterogeneous buyers with respect to two dimensions: their tax morale and their cost of managing non-cash payment instruments. The government commits to a policy before the bargaining game, choosing an income tax rate, a sale (or value added) tax rate, a tax rebate and a TCW rate.

If the buyers and the sellers as risk neutral, we have an analytical solution for the model equilibrium, which allows us to study the effects of the policy instruments on tax evasion, government revenue and welfare. Nevertheless, since some of the net effects of the policies are ambiguous, we also consider a numerical solution. We calibrate the model to a fictitious "prototype economy" that features empirically plausible values for the deep parameters and for the calibration targets, but that is not representative of a specific real world country. We choose this approach to highlight the principles that should guide the anti-evasion policy in general, for a large set of countries. We also make an effort to study the robustness of our results to a wide range of alternative assumptions (risk aversion), parameter values and calibration targets, which allow us to generalize our results.

We show that a small tax deduction is effective at both reducing tax evasion and increasing government revenue and welfare. We also show that the deduction must be higher the higher the tax evasion and the higher the statutory tax rate. The reason is that a tax rebate is a transfer from the government to the (already) honest taxpayers, which means that the cost of using the rebate to fight tax evasion is higher the smaller the tax evasion rate. In other words, it is not optimal to fight small levels of tax evasion using a tax rebate.

As for the TCW, we find that its introduction can actually increase evasion, especially in economies where the use of cash is widespread. The reason is that the individuals with high costs

[^1]of using non-cash instruments prefer to use cash even if they don't want to favor tax evasion. For these individuals, the TCW actually makes cooperation more attractive: a collaborative buyer pays the TCW on the price of the good net of the discount, while a non-collaborative buyer pays it on the full price. Nevertheless, the higher the TCW, the smaller the measure of individuals that prefer to use cash. We show that the first effect prevails for a small TCW rate, while the second prevails for high rates. We conclude that taxing cash is effective at reducing evasion (and increasing government revenue) only if its rate is high enough, where "high enough" depends on the characteristics of the economy. We also find that the TCW must be higher the bigger the measure of individuals with high costs of using non-cash payment instruments and the higher the tax evasion rate. The gain in government revenue determined by the TCW, on the other hand, is higher the higher the tax evasion rate and the higher the percentage of individuals with high costs of non-cash instruments. Aggregate welfare, unfortunately, is always decreasing in the TCW.

For the calibrated version of the model, we can also isolate the optimal policies numerically. We consider two different policy objectives: the first is maximization of total welfare conditional on raising a given amount of government revenue; the second is the maximization of government revenue conditional on reducing tax evasion below a certain threshold. The main result from this exercise is that, with an appropriate policy mix, it is possible to curb tax evasion and, at the same time, increase government revenue and welfare.

The main problem with our policy instruments is the implementation of the TCW. First, the TCW can foster the emergence of a parallel cash economy: firms and consumers can use whatever cash they have for the transactions, bypassing the banking system (Morse et al. 2005). Second, there is the possibility of a bank run at the moment of the announcement of the tax and before its introduction. Third, the TCW might have important redistributive effects in favor of the banking sector. Fourth, the TCW should be ideally implemented in all the countries of a currency area and, to avoid arbitrage, the rate should be equal or, at least, not very different. We propose a thoroughly discussion of these issues, suggesting how to (partially) overcome them.

The rest of the paper is organized as follows. Section 2 briefly summarizes the related economic literature. Section 3 describes the model and the analytical results. Section 4 illustrates the numerical results. In Section 5 we discuss the implementation problems of the TCW. Section 6 offers some concluding remarks. In Appendix A we provide the proofs of the analytical results.

Appendix A. 1 discusses the optimal policy for a real world country, Italy.

## 2 Related literature

The paper follows the quite abundant economic literature on tax evasion which has already been extensively reviewed (Andreoni, Erard and Feinstein 1998, Slemrod and Yitzhaki 2002, Cowell 2004, Marchese 2004, Sandmo 2005, Slemrod 2007 and Franzoni 2008).

The first work on collaborative tax evasion is Gordon (1990), who suggests that under-thecounter cash sales at a discount price, on which the seller evades taxes, can be used as a price discrimination tool. Namely, all the after-sale services hinge on the possession of a receipt of the transaction, including the possibility of returning a defective item and the possibility of suing the seller. Thus the customers that are not interested in this services, for various reasons, are willing to pay a lower price. A second work is Boadway, Marceau and Mongrain (2002), who model evasion as collusion between a buyer and a seller. They assume that joint evasion efforts can reduce the detection probability more than individual efforts, which gives an incentive to cooperate. They show that, under these assumption, tax evasion might actually increase after an increase in sanctions, since the gain from cooperation is higher. Finally, Chang and Lai (2004) also model collaborative tax evasion as a bargaining game between a seller and a buyer but study a different question, that is how social norms shape the incentives of the agents. In particular, evading taxes induces psychological costs, associated to the feelings of guilt and shame, but these costs are higher the higher the social sanction of evasion, that is, the lower the tax evasion rate. Differently from all these previous works on collaborative tax evasion, instead of focusing the attention on fines and enforcement, we study the effects of two different policy instruments, the tax rebate and the $\mathrm{TCW}^{2}$.

The idea of taxing currency dates back to the work of Gesell (1916) and it has been discussed by Goodfriend (2000), Buiter and Panigirtzoglou (2003), Buiter (2009), Mankiw (2009) and Rogoff (2014). The main focus of all this works, however, is how to overcome the zero bound on interest rates faced by the central bank, which is actually a consequence of the existence of paper currency:

[^2]if only bank deposits and electronic payments were available, there would be the possibility to charge negative interest rates, which is akin to taxing currency. Goodfriend (2000) and Buiter and Panigirtzoglou (2003), in particular, carefully explain how to implement this Gesell (1916) "carry tax " on currency by setting a nominal interest rate on the base money below the interest rate on the non monetary instruments. This negative nominal interest rate, in turn, allows a better monetary policy response to deflationary shocks and it can also be used to avoid deflation expectations. In this work we consider a different tax which applies to bank teller or ATM withdrawals of bank notes, rather than on cash balances, and whose objective is to limit the customers' incentive to pay cash instead of improving the transmission of monetary policy.

Our work is also related to the literature on the inflation tax (Friedman 1969; Phelps 1973; Chamley 1985; Woodford 1990, among others). Like the inflation tax, the TCW reduces the purchasing power of the consumers. Unlike the inflation tax, however, the TCW, which is paid once and for all, disincentivize the withdrawing of cash but does not disicentivize hoarding it. In this respect, the inflation tax is complementary to the TCW, because it eases its implementation (on this see Section 5). ${ }^{3}$ Nicolini (1998) and Koreshkova (2006) discussed the role of the inflation tax as a way to raise revenue from tax evaders and from the underground sector. With respect to their work, in this paper we take a different perspective, since we study how to reduce evasion rather than how to extract revenue from evaders.

## 3 The Model

The economy is composed by price taking, risk neutral, sellers, by risk neutral buyers and by the government. We model heterogeneous sellers with respect to their honesty or tax morale (Gordon 1989; Andreoni, Erard and Feinstein 1998; Feld and Frey 2002): honest sellers will always issue the receipt, while less honest ones will bargain with the buyers. The buyers, on the other hand, are heterogenous along two dimensions. The first is tax morale, as for the sellers: honest buyers will always ask for a receipt, preventing tax evasion, while less honest ones will bargain with the

[^3]sellers. ${ }^{4}$ The second is the cost of managing non-cash instruments such as credit cards, debit cards, bank transfers, cheques etc.: some individuals find it easy to manage them while some others, like the elderly (Humphrey et al. 2003) or the less financially educated, do find it very cumbersome. Moreover, some consumers are uncomfortable with the idea that their purchases will be tracked and they are ready to pay a price to have an anonymous payment (Garcia Swartz et al. 2006).

The government sets an income tax for the sellers and a sale tax for the buyers and enforces them with random audits and fines. We assume that there can be tax evasion only if the seller does not issue the receipt of the transaction and if the buyer pays cash. This last requirement is neede because non-cash payments leave a trace of the transaction and that, as a consequence, impairs or precludes evasion. In this setting, which is indeed similar to what happens in many real world situations (doctors, contractors, plumbers, etc.), a negotiation between the seller and the buyer is likely: the seller might offer a price discount to the buyer in exchange for not issuing the receipt, but forcing the buyer to pay cash. If the buyer and the seller do not reach a deal, no tax evasion is possible, and the buyer is free to choose between cash and non-cash payments.

Together with income and sale tax, the government has two other policy instruments: a tax rebate for the customers who keep the receipt of the transaction and a tax on cash withdrawals (TCW) from ATM or bank tellers. We assume that government commits to a policy before the bargaining between the seller and the buyer takes place. Moreover, we study three different government objectives: the reduction of tax evasion, the maximization of social welfare for given revenue and the maximization of tax revenue. After observing the policy, one buyer and one seller are randomly matched for a single transaction and they bargain over the price discount. If they reach a deal, there is collaborative tax evasion.

### 3.1 Sellers and Buyers

We assume that the parties of the transaction can either evade the full amount or nothing. ${ }^{5}$ The seller's utility in case of tax evasion, which requires cooperation from the buyer, is the following:

[^4]\[

$$
\begin{align*}
v_{s}^{1} & =(1-\pi)\left[p\left(1-t_{s}\right)+p t_{s}-d-v\right]+\pi\left[p\left(1-t_{s}\right)-d-p t_{s} f_{s}-v\right]  \tag{1}\\
& =p\left(1-t_{s}\right)+p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-d-v
\end{align*}
$$
\]

where $p$ is the price of the good or service (taken as given by the seller), $t_{s}$ is the income tax for the seller, $d$ is the discount bargained with the buyer, $\pi$ is the audit probability, $f_{s}$ is the fine and $v$ is the individual cost of tax evasion, which reflects differences in honesty between sellers. In case of no audit, with probability $1-\pi$, the seller earns the evaded amount $p t_{s}$. In case of audit, the seller is forced to pay the full amount of taxes plus a fine, which is computed on the evaded amount $p t_{s} f_{s} .{ }^{6}$ The cost of tax evasion, which is higher the higher is tax morale, is distributed according to the $\operatorname{cdf} G_{v}$, whose pdf is $g_{v}$. If the buyer and the seller do not reach a deal, the utility is simply equal to $v_{s}^{0}=p\left(1-t_{s}\right)$. Comparing $v_{s}^{0}$ with $v_{s}^{1}$, we notice that the cost of cheating is $d+v$ while the benefit is the evaded amount minus the expected sanction. To make the analysis interesting, we assume that $1-\pi\left(1+f_{s}\right)>0$, so that a trade off exist. This assumption implies that there must be an upper bound to the audit probability $\pi$ and to the fine $f_{s}$, which is reasonable. The utility of a buyer, in case of collaborative tax evasion, is the following:

$$
\begin{equation*}
v_{b}^{1}=u-(p-d)(1+\vartheta)-\pi p t_{b}\left(1+f_{b}\right)-s \tag{2}
\end{equation*}
$$

where $u$ is the utility from purchasing the good or service, $t_{b}$ is the tax paid by the buyer, $\vartheta$ is the TCW, $f_{b}$ is the fine and $s$ is the cost of tax evasion or tax morale. The tax paid by the buyer can be interpreted as a sale tax or, if the seller does not buy any intermediate goods from other suppliers, as a value added tax (VAT) ${ }^{7}$. We assume that $s$ is distributed according to the cdf $G_{s}$, whose pdf is $g_{s}{ }^{8}$. Since, in order to evade, the transaction must be paid in cash, the buyer must

[^5]pay $\vartheta$ on the negotiated effective amount of the transaction $(p-d)$. In case of audit, the buyer is forced to pay the tax plus a fine computed on the evaded amount $p t_{b}\left(1+f_{b}\right)$. If the buyer does not cooperate, he must still choose whether to use cash or an alternative payment instrument, such as a credit or debit card. In the former case, the utility of the buyer is
\[

$$
\begin{equation*}
v_{b}^{0}(c a s h)=u-p\left[1+t_{b}-\tau+\left(1+t_{b}\right) \vartheta\right] . \tag{3}
\end{equation*}
$$

\]

Instead, if he chooses a non-cash payment instrument, the utility becomes

$$
\begin{equation*}
v_{b}^{0}(\text { card })=u-p\left(1+t_{b}-\tau\right)-c \tag{4}
\end{equation*}
$$

where $\tau$ is the tax rebate and $c$ is the cost associated with non-cash payment instruments. For simplicity, we normalize the cost of using cash to zero while, in practice, it is not, since it must be withdrawn from ATM machines, stored and protected from theft, not to mention its loss in value due to inflation. We also disregard individuals who benefit from the use of non-cash payment instruments (negative $c$ ), since their behavior is identical to that of individuals with zero cost. We denote the cdf of the distribution of the cost $c$ by $G_{c}$ and the associated pdf by $g_{c}$. We also assume that the distributions of $c$ and $s$ are independent. If the buyer chooses not to cooperate with the seller and asks for a receipt, he receives a tax rebate on the full amount of the transaction $p$. In this case, since he is free to choose among different payment instruments, he will choose the one with the lower cost. More formally, cash is preferred to non-cash instruments if and only if $c \geq p\left(1+t_{b}\right) \vartheta$. From now on we define $\Upsilon=p\left(1+t_{b}\right) \vartheta$. Then, the utility of a non cooperating buyer is

$$
\begin{equation*}
v_{b}^{0}=u-p\left(1+t_{b}-\tau\right)-\min \{\Upsilon, c\} . \tag{5}
\end{equation*}
$$

Note that the tax rebate and the TCW affect the buyer's incentive to cooperate rather than the terms of the gamble faced by the seller. However, both instruments indirectly affect the behavior of the seller through the bargained discount.
authorities can also audit the accounts of the sellers, which is obviously not possible for the buyers. We make this assumption for simplicity and because the auditing probability has only a modest effect on the conclusions of the analysis.

### 3.2 Bargaining

We model the negotiation as a Nash bargaining. The solution is defined by

$$
\begin{align*}
d^{*}= & \arg \max _{d}\left(v_{s}^{1}-v_{s}^{0}\right)^{\beta}\left(v_{b}^{1}-v_{b}^{0}\right)^{1-\beta}  \tag{6}\\
& \text { s.t. } \quad v_{s}^{1} \geq v_{s}^{0}, v_{b}^{1} \geq v_{b}^{0}
\end{align*}
$$

where $\beta$ is the bargaining power of the seller. The solution for the discount is

$$
\begin{equation*}
d^{*}(v, s, c)=\beta \frac{p\left(\tau+\vartheta-t_{b}\right)+\pi p t_{b}\left(1+f_{b}\right)+s-\min \{\Upsilon, c\}}{1+\vartheta}+(1-\beta)\left\{p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-v\right\} \tag{7}
\end{equation*}
$$

for all $v$ such that $v_{s}^{1} \geq v_{s}^{0}$, and for all the couples $s$ and $c$ such that $v_{b}^{1} \geq v_{b}^{0}$, i.e.

$$
\begin{gather*}
v \leq p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-d^{*}(v, s, c)  \tag{8}\\
s \leq d^{*}(v, s, c)(1+\vartheta)-p\left(\tau+\vartheta-t_{b}\right)-\pi p t_{b}\left(1+f_{b}\right)+\min \{\Upsilon, c\} . \tag{9}
\end{gather*}
$$

Conversely, there is no evasion and the optimal discount is zero in case conditions (8) and (9) do not hold.

### 3.3 Tax evasion

By plugging the optimal discount (equation 7) into (9) we find

$$
\begin{equation*}
s \leq(1+\vartheta)\left\{p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-v-\frac{p\left(\tau+\vartheta-t_{b}\right)+\pi p t_{b}\left(1+f_{b}\right)-\min \{\Upsilon, c\}}{1+\vartheta}\right\} \tag{10}
\end{equation*}
$$

We then use condition (10) to compute the equilibrium level of tax evasion. First, we consider the buyers with $c \leq \Upsilon$ to obtain a threshold value $\tilde{s}_{1}(v, c)$ such that all the buyers of type $c \leq \Upsilon$, with an honesty lower than $\tilde{s}_{1}(v, c)$, cooperate. Next, we define the level of (seller) honesty $\tilde{v}_{1}$ that makes no buyer willing to collaborate, that is such that $\tilde{s}_{1}\left(\tilde{v}_{1}, c\right)=0$. Doing the same for $c \geq \Upsilon$, we obtain a second threshold $\tilde{s}_{2}(v)$ (which does not depend on $c$ and coincide with $\tilde{s}_{1}(v, c)$
in $c=\Upsilon$ ), such that all the buyers of type $c \geq \Upsilon$, with honesty lower than $\tilde{s}_{2}(v)$, collaborate. We also define the level of (seller) honest $\tilde{v}_{2}$ such that $\tilde{s}_{2}\left(\tilde{v}_{2}\right)=0$. Using the previously defined thresholds it is immediate to get the following expression for total tax evasion $E^{9}$ :

$$
\begin{equation*}
E=\int_{0}^{\Upsilon} E_{c}(c) g_{c} d c+\left[1-G_{c}(\Upsilon)\right] E^{c} \tag{11}
\end{equation*}
$$

where $E_{c}(c)=\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} g_{s} d s\right) g_{v} d v$ is the mass of evaders with low $c$, while $E^{c}=$ $\int_{0}^{\tilde{c}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} g_{s} d s\right) g_{v} d v$ is the mass of evaders with high $c$. We are interested in the effect of the government policy $\mathcal{P}=\left\{t_{s}, t_{b}, \tau, \vartheta\right\}$ on the total amount of tax evasion $E$.

Imposing taxes generates tax evasion of an amount that is increasing in the tax rates $t_{s}$ and $t_{b}$. This result is different from the original Allingham and Sandmo model, where an increase in the tax rate would reduce evasion, but perfectly in line with most of the empirical literature (Clotfelter 1983; Crane and Nourzad 1992; Alm 2012). The difference between our result and Allingham and Sandmo comes from the fact that we essentially look at the decision to evade or not (extensive margin) and the higher the tax rate the higher the mass of evaders. Conversely, in the Allingham and Sandmo model, there is an intensive margin effect with the evaded amount that is a decreasing function of the tax rate. In Section 4.4 we show that, if we relax the hypothesis of risk neutrality, we have an interior solution for the optimal level of tax evasion, which is decreasing in the tax rate. This implies that the response of evasion to the tax rate is first increasing (the extensive margin prevails) and then decreasing (the intensive margin prevails).

As for the effect of the tax deduction, it actually reduces the buyers' incentives to collaborate, decreasing tax evasion. The effect of the TCW $\vartheta$ on evasion is instead ambiguous, as shown by the next derivative:

$$
\begin{equation*}
\frac{\partial E}{\partial \vartheta}=\int_{0}^{\Upsilon} \frac{\partial E_{c}(c)}{\partial \vartheta} g_{c} d c+\left[1-G_{c}(\Upsilon)\right] \frac{\partial E^{c}}{\partial \vartheta} \tag{12}
\end{equation*}
$$

since

$$
\begin{equation*}
\frac{\partial E_{c}(c)}{\partial \vartheta}=\int_{0}^{\tilde{v}_{1}} g_{s}\left(\tilde{s}_{1}(v, c)\right)\left\{p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-v-p\right\} g_{v} d v<0 \tag{13}
\end{equation*}
$$

[^6]while
\[

$$
\begin{equation*}
\frac{\partial E^{c}}{\partial \vartheta}=\int_{0}^{\tilde{v}_{2}} g_{s}\left(\tilde{s}_{2}(v)\right)\left\{p t_{s}\left[1-\pi\left(1+f_{s}\right)\right]-v+p t_{b}\right\} g_{v} d v>0 . \tag{14}
\end{equation*}
$$

\]

The threshold $\tilde{s}_{1}(v, c)$, such that all the buyers of type $c \leq \Upsilon$, with tax morale lower than $\tilde{s}_{1}(v, c)$, collaborate, is decreasing in $\vartheta$. Instead, the threshold $\tilde{s}_{2}(v)$, for types $c \geq \Upsilon$ is increasing. The reason is that, if the buyer does not collaborate, he must still choose whether to use cash, which is better if and only if $c \geq \Upsilon$. In other words, buyers with high $c$ prefer to use cash even if they do not want to cooperate with the seller. Therefore the TCW does not impose an extra cost on them, but it actually makes cooperation more attractive: a collaborative buyer pays $\vartheta$ on the price net of the discount $p-d$, while a non collaborative buyer pays it on the full price $p$. Conversely, buyers with low $c$, who do not want to collaborate with the seller, prefer to bear this cost and to use the non-cash instruments. Thus an increase in the TCW rate makes cooperation relatively more costly for them. Since the derivative of tax evasion with respect to the TCW is positive for low values of $\vartheta$ and negative for high values, an increase in $\vartheta$ is more likely to decrease tax evasion the larger is $\vartheta$. In other words, a tax on cash withdrawals is an effective tool to fight tax evasion only if it is sufficiently high ${ }^{10}$. We summarize the previous analysis with the following proposition:

Proposition 1. Tax evasion is increasing in the tax rates $t_{s}$ and $t_{b}$ and decreasing in the tax rebate $\tau$; The $T C W \vartheta$ is an effective tool to fight tax evasion only if set sufficiently high.

### 3.4 Welfare

Total welfare is given by the following expression:

$$
\begin{align*}
W= & \int_{0}^{\Upsilon}\left\{\left(v_{s}^{0}+v_{b}^{0}\right)\left(1-E_{c}(c)\right)+\int_{0}^{\tilde{v}_{1}} \int_{0}^{\tilde{s}_{1}(v, c)}\left(v_{s}^{1}+v_{b}^{1}\right) g_{s} d s g_{v} d v\right\} g_{c} d c+  \tag{15}\\
& {\left[1-G_{c}(\Upsilon)\right]\left\{\left(v_{s}^{0}+v_{b}^{0}\right)\left(1-E^{c}\right)+\int_{0}^{\tilde{v}_{2}} \int_{0}^{\tilde{s}_{2}(v)}\left(v_{s}^{1}+v_{b}^{1}\right) g_{s} d s g_{v} d v\right\} }
\end{align*}
$$

[^7]The first line is the utility of sellers and buyers with low $c$ : the first term is the utility of non-evaders while the second term is the utility of sellers and buyers who collaborate to evade taxes. The second line is the utility of sellers and buyers with high $c$ and again the first term refers to non-evaders while the second to evaders.

In the rest of this section we discuss the comparative statics (computations can be found in Appendix A) of this measure of welfare that includes both evaders and non-evaders. Since the inclusion of evaders' welfare in the government objective is questionable, we also discuss the effect on non-evaders' welfare only.

We start with the effect of the income tax:

$$
\begin{equation*}
\frac{\partial W}{\partial t_{s}}=-p+p\left(1-\pi\left(1+f_{s}\right)\right) E+\vartheta(1-\beta) p\left(1-\pi\left(1+f_{s}\right)\right) E \tag{16}
\end{equation*}
$$

An increase in the income tax $t_{s}$ decreases the utility of non-evaders (first term) and increases the utility of evaders (second term), but the negative effect prevails. The bargained discount is higher and this decreases the amount $(p-d)$ that the collaborative buyers pay cash. Since the TCW paid to finalize the illegal transaction $(p-d) \vartheta$ decreases (last term), the utility of the collaborative buyers increases. The effect of an increase in the sale tax $t_{b}$ on welfare is similar to the increase in $t_{s}$, as summarized by the next derivative:

$$
\begin{equation*}
\frac{\partial W}{\partial t_{b}}=-p\left(1+\left(1-G_{c}(\Upsilon)\right) \vartheta\right)+p\left(1-\pi\left(1+f_{b}\right)\right) E-\frac{\vartheta \beta p}{1+\vartheta}\left(\left(1-\pi\left(1+f_{s}\right)\right) E+\left(1-G_{c}(\Upsilon)\right) \vartheta E^{c}\right) \tag{17}
\end{equation*}
$$

The main difference with respect to the income tax is that the sale tax decreases the discount, lowering (also) the utility of cooperating buyers (last term), so that the overall effect is negative.

As for the tax rebate, an increase in $\tau$ increases the utility of non-evaders and, since it increases the discount, it also increases the utility of the collaborative buyers (second term). The analytical expression of the derivative is

$$
\begin{equation*}
\frac{\partial W}{\partial \tau}=p(1-E)+\frac{\vartheta \beta p}{1+\vartheta} E \tag{18}
\end{equation*}
$$

In contrast to the previous results, the effect of the TCW $\vartheta$ on welfare is in general ambiguous.

We summarize the previous analysis with the following proposition:

Proposition 2. i) Total welfare is decreasing in the sale tax $t_{b}$ and increasing in the tax rebate $\tau$; ii) For low levels of the TCW, total welfare is decreasing in the income tax $t_{s}$; iii) The effect of the $T C W$ on total welfare is ambiguous.

### 3.5 Net Government Revenue

Total government revenue is:

$$
\begin{aligned}
G & =\int_{0}^{\Upsilon}\left\{\left[p \pi\left(t_{s}\left(1+f_{s}\right)+t_{b}\left(1+f_{b}\right)\right)+\left(p-d^{*}(v, s, c)\right) \vartheta\right] E_{c}(c)+p\left(t_{s}+t_{b}-\tau\right)\left(1-E_{c}(c)\right)\right\} g_{c} d c \\
& +\left[1-G_{c}(\Upsilon)\right]\left\{\left[p \pi\left(t_{s}\left(1+f_{s}\right)+t_{b}\left(1+f_{b}\right)\right)+\left(p-d^{*}(v, s)\right) \vartheta\right] E^{c}+\left(p\left(t_{s}+t_{b}-\tau\right)+\Upsilon\right)\left(1-E^{c}\right)\right\} .
\end{aligned}
$$

The first line is the revenue from transactions with low $c$ buyers. These buyers are either matched with evading sellers (first term) or not (second term). In case of tax evasion and audit, both the seller and the buyer are forced to pay the full amount of the tax plus a fine, computed on the evaded amount $p \pi\left[t_{s}\left(1+f_{s}\right)+t_{b}\left(1+f_{b}\right)\right]$. In addition, since there is a cash payment, the buyer also pays the TCW, which amounts to $\vartheta$ times the negotiated amount of the transaction net of the discount $(p-d)$. When the matching does not lead to tax evasion, the revenue for the government amounts to the taxes net of the rebate for the buyer, $p\left(t_{s}+t_{b}-\tau\right)$.

The second line is the revenue from transactions with high $c$ buyers. In case of tax evasion, the government cashes in exactly the same amount as in the case of low $c$ buyers. Conversely, the revenue is $p\left(t_{s}+t_{b}-\tau\right)+\Upsilon$ when the matching does not lead to tax evasion, because the government collects the TCW also from the non-collaborative buyers who prefer to use cash. Indeed, the TCW levied on those individuals is a pure transfer to the government and it should be reimbursed to leave the buyers' purchasing power unchanged.

Importantly, the TCW imposes the cost $c$ also on the non-collaborative buyers (with $c \leq \Upsilon$ ) who opt for non-cash payments. This cost is not a transfer, but a loss for society as a whole, and it is equal to

$$
\begin{equation*}
\int_{0}^{\Upsilon} c\left(1-E_{c}(c)\right) g_{c} d c \tag{19}
\end{equation*}
$$

Since $c$ is measured in monetary equivalents, it is possible to subtract it from the government revenue, to obtain what we call the "Net Government Revenue", denoted $G_{n} .{ }^{11}$ This measure gives a better idea of the positive net effects of the introduction of the TCW for the government.

An increase in the income tax $t_{s}$ increases the mass of evaders (extensive margin) and the revenue from both evaders and non-evaders (intensive margin). Since an increase in $t_{s}$ increases the equilibrium discount, the revenue collected with the TCW decreases. The cost imposed on the collaborative buyers who pay cash similarly goes down. If $\tau=\vartheta=t_{b}=0$, only the first two effects are left. When $t_{s}$ is low, the first negative effect is small compared to the positive one: an increase in $t_{s}$ increases the revenue. The opposite happens if $t_{s}$ is high. In other words, we have a standard Laffer curve type of result.

The sale tax $t_{b}$ acts in a similar way, with the only difference that a tax on the seller decreases the equilibrium discount.

An increase in tax rebate $\tau$ decreases the mass of evaders. The overall effect on revenue is ambiguous, unless $t_{s}$ and $t_{b}$ are sufficiently high, in which case it is positive. There is no effect of an increase in $\tau$ on the revenue from evaders but a negative effect on the revenue from nonevaders. Since the equilibrium discount increases with the tax rebate, there is a lower revenue from the TCW and a higher cost imposed on the non-collaborative buyers who opt for non-cash payments. Summarizing, low statutory taxes, a large mass of honest individuals $(1-E)$ and a high TCW rate make a tax rebate undesirable.

The comparative statics with respect to $\vartheta$ is quite complicated. In fact, an increase in $\vartheta$ increases the mass of evaders with high $c$ and decreases the mass of evaders with low $c$. Moreover, it increases the revenue at the intensive margin, both from evaders and non-evaders and it increases the discount for evaders with high $c$ and decreases the discount for evaders with low $c$. In general, the effect of the TCW on government revenue is ambiguous.

We summarize the previous analysis with the following proposition:
Proposition 3. i)If $\tau=\vartheta=t_{b}=0$ there is a Laffer curve for the response of government revenue

[^8]to $t_{s} ;$ i)If $\tau=\vartheta=t_{s}=0$ there is a Laffer curve for the response of government revenue to $t_{b}$; iii) The tax rebate $\tau$ can decrease the government revenue in case of low tax rates or in case there are many honest individuals; iv) The effect of the TCW on Government revenue is ambiguous.

## 4 Numerical Analysis

We now consider the numerical solution of a calibrated version of the model. We proceed as follows. In the next section (4.1) we define what we call a "Prototype Economy", that features reasonable values for the parameters and for the calibration targets, but that it is not representative of a real world country. We choose this approach to provide the widest possible perspective on these issues, rather than focusing on a specific country, making an effort to generalize the analysis as much as possible. In Section 4.2 and 4.3 we summarize the results. In Section 4.4 we review their robustness. In Section 4.5 we briefly comment on the efficiency issues raised by the TCW. In Section 4.6 we identify the optimal policy. To complete the analysis, we discuss an example of the optimal policy for a real world country, Italy, in (A.1) in the appendix.

### 4.1 The Prototype Economy

We start the numerical analysis by choosing a set of parameters and calibration targets that define the baseline, fictitious, prototype economy. We fix the income tax rate $t_{s}$ at $30 \%$ and the sale tax rate $t_{b}$ at $10 \%$. For the enforcement probability, we choose $\pi=0.01$ and we set the fine to $f_{s}=f_{b}=0.5$. Admittedly, we take a shortcut with the assumption of a constant auditing probability that does not depend on the seller's characteristics and on the evaded amount. In practice, a big firm that evades $90 \%$ of its profits faces a higher audit probability than a small, less visible, business that seldom evades a small $10 \%$ (Yitzhaki 1987). We also abstract from congestion effects in law enforcement (Galbiati and Zanella 2012), which imply that, for a fixed amount of government resources devoted to enforcement, the individual audit probability decreases the higher the number of individuals that evade. For all these reasons, we perform some robustness tests on the auditing probability: different probabilities reflect differences in the size and characteristics of the firms, with higher probabilities corresponding to bigger, more visible, businesses or to firms that evaded in the past. We do not consider the auditing probabilities as a policy instruments,
since we do not have the cost of enforcement in the model and since there is no easy way of introducing one ${ }^{12}$.

We set $\vartheta=\tau=0$ in the benchmark simulation. For the distribution of the cost $c$, we consider a probability distribution with a mass at zero, representing the individuals that can manage noncash instrument without a cost (perhaps also with a gain), and with a small probability mass at high costs, which reflects the individuals that find it very cumbersome to use non-cash instruments such as the elderly and the less financially educated. In particular, we chose the following mixture exponential distribution:

$$
g_{c}(x, \lambda)= \begin{cases}0 & \operatorname{Prob} \lambda  \tag{20}\\ \lambda e^{-\lambda x} & \operatorname{Prob} 1-\lambda\end{cases}
$$

We set $\lambda=0.2$ in the benchmark simulation, which entails assuming that, in the absence of the TCW, $20 \%$ of the population does not use cash for the transaction. In Section 4.3 we explore the robustness of the results to alternative values of $\lambda$.

We set $\beta=0.5$, since we have no particular reason to assign a higher bargaining power to the buyer or to the seller, but we discuss robustness in Section 4.4. As we discuss in section 4.5, if $u$ is not sufficiently higher than the price $p$, a TCW might discourage the buyer from purchasing the good. In this baseline parametrization, we rule out this possibility by choosing a high value for $u$. Other than that, the values of $p$ and $u$ are just scalings: different values will only deliver different calibrated parameters, but the same results. We set $p=10$ and $u=1.5 p$ for convenience.

Actually the mass of individuals that prefer to use non-cash instruments is increasing in the transaction price. Thus the cost $c$ should be decreasing in $p$, but such a model is very difficult to calibrate, because we don't have information on card use by transaction value. Nevertheless, we do account for this variability in a reduced form, varying the fraction $\lambda$ : a higher (lower) $\lambda$ is more likely in sectors with higher (lower) average transactions values. Therefore we can interpret the robustness of our results with respect to different values of $\lambda$ also as a robustness across different sectors of the economy with different transaction values. Notice that, since the price $p$ is just a scaling, we do not change it when we perform these robustness tests: as already noticed, changing

[^9]it will only deliver different calibrated parameters but exactly the same results.
For the distribution of tax morale, we consider an extremely versatile distribution that assigns values in an interval, the Kumaraswamy, which is essentially a Beta distribution but with a different parametrization. The pdf is the following:
\[

$$
\begin{equation*}
g(x ; a, b, \bar{x})=\frac{a b}{\bar{x}}\left(\frac{x}{\bar{x}}\right)^{a-1}\left[1-\left(\frac{x}{\bar{x}}\right)^{a}\right]^{b-1} \quad 0<x<\bar{x} \tag{21}
\end{equation*}
$$

\]

Depending on the value of the parameters, we can have an increasing pdf with most of the probability mass corresponding to high values of tax morale, a decreasing pdf, where the opposite is true, or a peak corresponding to intermediate values. We consider the same distribution of tax morale for both the buyer and the seller $(\bar{s}=\bar{v})$. In fact the occupational choice might be also driven by the opportunity to evade taxes, so that individuals who are more prone to tax evasion, because of moral reasons, would choose to work where it is easier to evade taxes (Pestieau and Possen 1991). We decided to abstract from these issues since there is no robust empirical evidence that confirms this hypothesis (Parker 2003).

To choose the parameters of the distribution of tax morale, we use data from the World Value Survey (WVS henceforth). This survey is part of an ongoing worldwide research project whose goal is to compare several aspects of culture among different countries. Among the questions administered to a significant number of individuals, there is one that is specifically related to tax morale, namely "Do you consider justifiable cheating on taxes?" Respondents are asked to pick a number between 0 and 10 , where 0 means always justifiable while 10 never justifiable. We consider the average frequencies of the responses to the question, where the average is with respect to all participating countries (not weighted). The shape of the empirical distribution of the answers is similar across countries: a big mass of individuals that never justifies evading and a rapidly declining probability mass.

The core of the calibration procedure entails choosing the parameters $a, b$, together with upper bound $\bar{s}=\bar{v}$, to match the empirical shape of the distribution of the answers and to match the observed level of tax evasion. We run a simple grid search procedure: for each upper bound of tax morale $\bar{s}=\bar{v}$ we divide the interval between 0 and $\bar{s}=\bar{v}$ into 9 equally spaced subintervals. We consider the threshold values of these intervals as corresponding to the 1-10 scale of the answers
of the WWS. We then take couples of $a$ and $b$ and, for each couple, we compute the value of the model-based distribution at the threshold values. We then compute, for each couple, the sum of square distances between the model based distribution and the empirical distribution, which is equal to the observed average relative frequencies from the empirical answers to the questionnaire. We choose $a, b$ and $\bar{s}=\bar{v}$ to minimize this sum of square residuals for the target calibrated level of tax evasion, so to have the closest possible match between the model and the data. For a target evasion level of $30 \%$, we end up with $b=1, a=5.93$ and $\bar{s}=\bar{v}=2.31$.

The $30 \%$ baseline choice for the evasion level is in line with what Pissarides and Weber (1989) find in the UK for self employed individuals, but it is sensibly smaller than the $57 \%$ tax evasion on business income for self-employed individuals in the US (Slemrod 2007, using data from the Tax Compliance Measuring Program implemented by the IRS). As robustness, checks we also consider two alternative scenarios of, respectively, high tax evasion (50\%), and low tax evasion (15\%).

### 4.2 Comparative statics by levels of tax evasion

Figures 1 and 2 report the comparative statics with respect to the tax rebate and to the TCW for different calibrated evasion levels, $15 \%, 30 \%$ and $50 \%$. We report the tax evasion rate in the upper left panel and the normalized government revenue in the upper right panel. This last measure is equal to 100 in the benchmark model parametrization for each evasion level, so that, subtracting 100 from the value of the net government revenue, we have their percentage variation with respect to the benchmark. A drawback of this normalization is that the three lines in the picture are not expressed in the same unit, which means that they are not directly comparable. In the lower left panel we report the normalized total welfare, while in the lower right panel the normalized welfare of non-evaders. Both welfare measure are normalized to be equal to 100 in the benchmark model parametrization.

Figure 1 reports the effect of a tax rebate $\tau$. Evasion decreases with $\tau$ because a higher tax rebate reduces the incentives of the buyer to cooperate with the seller. The effect on government revenue is the result of two contrasting effects: on the one hand, the decreasing evasion increases revenue; on the other, the higher the tax rebate, $\tau$, the higher the transfer from the government to non-cooperating buyers. The upper right panel shows that there is a threshold level for $\tau$ such
that the first effect prevails before it, with an increasing net revenue, while the second prevails after it, with a decreasing revenue. In other words, the numerical analysis confirms the analytical results. But the picture also shows that this threshold value, which is also the one that maximizes the government revenue, is higher the higher the prevailing tax evasion rate. For a calibrated tax evasion level of $30 \%$, the optimal tax rebate is $\tau=5 \%$. For a $50 \%$ evasion, instead, the optimal $\tau$ is $7 \%$, while it is only $3 \%$ in case of a $15 \%$ evasion rate. The reason is that, since an increase in the rebate is an increase in the total transfers from the government to the (already) honest taxpayers, the cost to fight tax evasion with the rebate is higher the smaller the tax evasion rate. The important consequence of this result is that it is not desirable to fight small levels of tax evasion using a tax rebate. Consistently with Proposition $2, \tau$ increases the aggregate welfare and the welfare of non-evaders.

Figure 2 reports the effect of the TCW $\vartheta$. As previously stressed, increasing $\vartheta$ increases the incentive to cooperate for the individuals with high cost $c$ (the cash users), increasing tax evasion. For the individuals with low cost $c$ (the non cash users), instead, increasing $\vartheta$ decreases the incentive to cooperate, decreasing tax evasion. In addition, the number of non-cash users increases with the TCW while the number of cash users decreases. The picture shows that evasion is first increasing in the TCW and then decreasing. In other words, there must be a sufficiently high number of non-cash users for the second effect to prevail and, therefore, a sufficiently high TCW rate to increases such a number. This confirms the analytical result in Proposition 1. In addition, we also find that the higher the prevailing tax evasion rate, the smaller the TCW rate above which tax evasion is decreasing. This is because there is a big mass of non-collaborative buyers who are cash users in case tax evasion is low.

The effect of the TCW on the net government revenue is twofold: on the one hand, an increase in the TCW affects the cooperation rate and, therefore, the level of tax evasion; on the other, it affects the total cost of non-cash instruments that must be subtracted from the gross revenue. For low levels of the TCW, cooperation and, therefore, evasion, is increasing, which translates in a decreasing gross revenue. However, since more individuals are using cash because of the increased evasion, there is a lower total cost of non-cash instruments. Therefore the net revenue can be increasing even in case of an increase in tax evasion. Viceversa, for high values of the TCW, tax evasion is decreasing, but the net government revenue can be decreasing because of the higher total
cost of non cash-instruments. Tax evasion is lower than the benchmark for a very high TCW, but the net government revenue might be lower or higher than the benchmark for such values. As shown in the upper right panel of Figure 2, the response of the net revenue to the TCW has an inverse $u$ shape. Thus the numerical analysis solves the analytical ambiguity summarized in Proposition 2. In addition, we also found that the TCW that maximizes the net government revenue is higher the higher the baseline tax evasion. In particular, for the baseline $30 \%$ tax evasion, $\vartheta=0.25$ maximizes the net revenue, while $\vartheta=0.13$ maximizes revenue for the $15 \%$ calibrated evasion. The intuition, once again, is that the higher the tax evasion level, the smaller the mass of non-collaborative cash users to compensate, since most of them evade. Similarly to the tax rebate, there might be no gain at all from the introduction of the TCW if the starting evasion rate is sufficiently low.

Total welfare (lower left panel), on the other hand, is always decreasing in the TCW rate, although the welfare of non-evaders (lower right panel) might be increasing. This last result is due to the increase in the mass of honest individuals (decrease in evasion) and not to an increase of the utility of each individual, that is instead decreasing in the TCW.

Summing up, the numerical analysis confirmed the results in Propositions 1 to 3, but also highlighted some additional results, summarized in the following proposition:

Proposition 4. The net government revenue is first increasing and then decreasing in the TCW rate. Total welfare is decreasing in the TCW rate. The higher the baseline tax evasion: i) The higher the optimal tax rebate that maximizes net government revenue; iii) The higher the TCW rate that maximizes net government revenue; and iv) The smaller the TCW rate above which tax evasion is decreasing.

### 4.3 Comparative statics by use of non-cash instruments

Figure 3 summarizes the comparative static results with respect to the TCW for different levels of $\lambda$ but for the same $30 \%$ baseline evasion rate. The main goal of this exercise is to get insights into what should be the rate of the TCW adopted in countries characterized by different development of the electronic means of payments. For instance, the value of $\lambda$ used in the appendix for Italy is a very low 0.12 (see Section A.1). An interesting result of the analysis is that if the electronic means of payments are sufficiently diffused ( $\lambda$ high) the introduction of the TCW does not increase
evasion. For instance with $\lambda=0.5$, tax evasion is always decreasing in $\vartheta$ (see the upper left panel). The intuition is that if there is a higher mass of individuals with a small cost $c$, the government requires a smaller TCW to prevent evasion. Reduced evasion, in turn, raises the tax revenues both from income and the sale tax and decreases the tax revenue from the TCW, but the first effect prevails.

The higher value of $\lambda$ implies that the welfare of non-evaders is increasing in the TCW also for the benchmark $30 \%$ evasion, since the extensive margin effect (increased number of non-evaders for increasing TCW) prevails. Total welfare is still decreasing in the TCW, although the effect is quantitatively smaller.

Finally, as we already stressed, different levels of $\lambda$ also describe differences in the incidence of the TCW in different sectors. Specifically, higher values of $\lambda$ correspond to economic sectors with higher average transaction value, where the use of cash is less common.

The previous results are summarized in the following proposition:

Proposition 5. The larger the mass of individuals with high cost of using non-cash payment instruments (the smaller $\lambda$ ), the higher the optimal TCW rate.

### 4.4 Robustness

Changing the bargaining power of the seller $\beta$ results in a different distribution of the gains from evasion, but in a similar effect of the policies on the equilibrium quantities. As for the enforcement probability, we tried a rather extreme value, $\pi=0.3$. In this scenario, the comparative static results are qualitatively similar, except that the optimal levels of $\tau$ and $\vartheta$ that maximize the net government revenue, everything else equal, are smaller. Therefore, we concluded that enforcement is a substitute for these two policies. However, since enforcement is costless in our model, we cannot really evaluate the impact of enforcement on the government revenue and, therefore, we cannot single out the optimal enforcement level. As for the effect on welfare, we found that enforcement worsen the evaders welfare but improves the non evaders welfare, with a negative, but quantitatively small, net effect. We also considered the comparative static results with respect to the fine $f$. Overall, a steeper fine results in a smaller level of tax evasion and in a higher government revenue, but the quantitative effect is very small. If $f=3$, six times bigger than the
baseline value, evasion is only $1 \%$ lower than the benchmark and the revenue only $2.5 \%$ higher.
We also relaxed the assumption of risk neutrality, assuming a CRRA utility function with a risk aversion parameter $\eta=3$ for both sellers and buyers. The main difference with the baseline model is that, with risk aversion, we do not have anymore a corner solution with full evasion for the seller. The optimal level of tax evasion for each seller is instead:

$$
\begin{equation*}
e^{*}=\frac{(1-k)\left[p\left(1-t_{s}\right)-d-v\right]}{t_{s}(k+f)} \tag{22}
\end{equation*}
$$

where

$$
\begin{equation*}
k=\left[\frac{\pi f}{1-\pi}\right]^{\frac{1}{\eta}} \tag{23}
\end{equation*}
$$

This implies that the evaded amount is decreasing in the level of $t_{s}$. Following an increase in the income tax rate, there is now an intensive margin effect (decreased evasion amount) that goes in the opposite direction of the extensive margin one (increased number of evaders). The result is an inverse $u$ shape relationship: evasion is increasing in the tax rate for low levels of the income taxes, but decreasing for high rates. Apart from this difference, all other results still hold: the net government revenue has an inverse $u$ shape, similarly to the baseline model, and the comparative statics with respect to $t_{b}, \tau$ and $\vartheta$ are qualitatively unchanged.

The previous results are summarized in the following proposition:
Proposition 6. i) The tax rebate and the TCW are a substitute for enforcement; ii) Risk aversion affects the response of tax evasion to the tax rate, which has now an inverse $u$ shape, but not the other results.

### 4.5 Efficiency

One potential side effect of the TCW is the reduction in the volume of trade resulting from the increased transaction costs for the buyers. In this section we study how to set the TCW to avoid this efficiency loss.

First of all, if the seller does not engage in tax evasion, his utility is simply equal to $v_{s}^{0}=p\left(1-t_{s}\right)$, which is always positive for any $t_{s}<1$. Therefore we do not have to worry about the effect of
our policy on the sellers' willingness to supply the good or service. Indeed, our policy will only decrease the seller utility from tax evasion $v_{s}^{1}$ through the discount, while still leaving the payoff from not engaging in tax evasion $\left(v_{s}^{0}\right)$ unaffected.

Conversely, the policy does affect the buyer gains from trade, both in case of collaborative tax evasion $\left(v_{b}^{1}\right)$ and, more importantly, in case he does not collaborate $\left(v_{b}^{0}\right)$. To guarantee that the buyer has always an incentive to trade, we must have that $v_{b}^{0}=u-p(1-\tau)-\min \{\Upsilon, c\} \geq 0$ for any possible cost $c$ and for any policy $\{\tau, \vartheta\}$. A sufficient condition is that $u-p(1-\tau)-\Upsilon \geq 0$ for any policy $\{\tau, \vartheta\}$. We can rewrite the condition as:

$$
\begin{equation*}
u-p \geq p \vartheta\left(1+t_{b}\right)-p \tau \text { for any }\{\tau, \vartheta\} \tag{24}
\end{equation*}
$$

In words, the consumer surplus must be at least equal to the difference between what the buyer pays because of the TCW and what he gets from the tax rebate. Then, a sufficient condition for any level of consumer surplus $(u \geq p)$ is

$$
\begin{equation*}
\vartheta\left(1+t_{b}\right) \leq \tau \tag{25}
\end{equation*}
$$

In what follows, we consider this constraint when computing the optimal policy.

### 4.6 Optimal Policy

We now look numerically at the optimal policy, considering two different policy objectives. Following the economic literature on optimal taxation, the first objective is the maximization of total welfare subject to keeping the government revenue above a certain level. In addition, since one of the main goals of fighting tax evasion is to increase the tax proceedings, we also consider the maximization of net government revenue. However, as noted by Slemrod and Yitzhaki (1987), the social benefit of a tax evasion reduction is not well measured by the tax revenue increases only. Additional benefits include, among others, reduced risk bearing, increased efficiency and better competition among businesses. Therefore we add a further constraint to this second objective: we consider the maximization of net government revenue conditional on keeping tax evasion below $1 \%$. We call this policy objective maximin, since it entails a maximization of the net revenue and
a simultaneous minimization of tax evasion.
For both policy exercises, we stress the gain in gross government revenue and welfare with respect to the corresponding benchmark with fixed tax rates, no tax rebates and no TCW. Notice that this gives an idea of the possibility of the government to compensate honest taxpayers for the side effects introduced by the TCW, which we discuss later. In this section we look at the optimal policy for the benchmark prototype economy, while Section A. 1 in the appendix looks at an example for a real world country, Italy.

The welfare maximizing policy subject to keeping revenue at least as high as in the benchmark entails increasing the income tax rate, decreasing the sale tax rate, a high tax rebate and a small TCW rate, $t_{s}=35 \%, t_{b}=7.5 \%, \tau=16 \%$ and $\vartheta=2 \%$. The level of tax evasion corresponding to this policy is below one percent, total welfare is $9.5 \%$ bigger and the non-evaders welfare $59 \%$ bigger. If we consider the welfare maximizing policy that increases the gross revenue by $20 \%$ above the benchmark, we end up with $t_{s}=35 \%, t_{b}=5 \%, \tau=11 \%$ and $\vartheta=7 \%$. In this case tax evasion is $1.5 \%$, while total welfare is $4 \%$ bigger than the benchmark and non-evaders welfare is $50 \%$ bigger. Thus it is possible to fight tax evasion and also raise an additional amount of government revenue while, at the same time, increasing total welfare.

The optimal policy to maximize net revenue conditional on keeping evasion below one percent entails lowering both tax rates, $t_{s}=25 \%, t_{b}=5 \%$, a small tax rebate $\tau=2 \%$ and a big TCW $\vartheta=18 \%$. For this policy the gross revenue is $38 \%$ bigger than the benchmark: tax evasion can be almost eliminated while generating an hefty amount of extra tax revenue. Total welfare, however, is $2.7 \%$ smaller than the benchmark, although non-evaders welfare is $40 \%$ bigger: evaders will lose at this policy, while non-evaders will gain substantially. We also consider the optimal maximin policy that entails no loss of efficiency, imposing condition (25). The result is $t_{s}=30 \%, t_{b}=12.5 \%$, $\tau=16 \%$, and $\vartheta=14 \%$. The gain in gross government revenue is $26 \%$, total welfare is $1.8 \%$ bigger than the benchmark and non-evaders welfare is $47 \%$ bigger. In this case is possible to both fight tax evasion and raise more than $25 \%$ more government revenue without any welfare cost and without any loss of efficiency. However, it is not possible to raise more than this amount without either an efficiency loss or an aggregate welfare loss.

We also look at the optimal policy when fixing the tax rates at their benchmark levels or, in other words, when the government can freely adjust only the tax rebate and the TCW. The
motivation behind this further exercise is that, since we are considering an extremely stylized microeconomic model, we are not able to capture all the possible micro and macroeconomic effects of a change in the tax rate. The policy that maximizes welfare subject to keeping gross revenue at least as high as in the benchmark entails $\tau=14 \%$ and $\vartheta=3 \%$, with evasion below one percent, a $9 \%$ bigger total welfare and a $58 \%$ bigger non-evaders welfare. If we slightly change the constraint and consider the welfare maximizing policy conditional on increasing the government revenue by $20 \%$, we obtain $\tau=11 \%$ and $\vartheta=7 \%$. The tax evasion rate at this policy is around $1 \%$, total welfare is $3.8 \%$ bigger than the benchmark and non-evaders welfare is $49 \%$ bigger.

The optimal maximin policy for fixed taxes is instead $\tau=13 \%$ and $\vartheta=14 \%$. At this policy the government revenue is $27 \%$ bigger than the benchmark and total welfare is $1 \%$ higher, with a $46 \%$ bigger non-evaders welfare. This maximin policy entails a potential loss of efficiency, since condition (25) is not satisfied. Imposing this further constraint, we find that the optimal policy is $\tau=13 \%$ and $\vartheta=10 \%$, with a $20 \%$ bigger government revenue, a $3.8 \%$ bigger total welfare and a $50 \%$ bigger non-evaders welfare.

The following proposition summarizes the main result of this exercise:

Proposition 7. An appropriate mix of tax rebate and tax on cash can curb tax evasion and raise additional tax revenue while, at the same time, increasing welfare. However, there is a limit to the amount of revenue that can be raised without any loss in efficiency and welfare.

In all the previous numerical exercises, we abstracted from all the issues associated with the implementation of the policies. The problem is that the maximin policy entails a fairly big value of the TCW, which is arguably difficult to implement, and the more difficult the higher its rate (see Section 5 for a discussion). Therefore we perform an additional exercise: we look for the maximin policy conditional on keeping the TCW rate below $5 \%$. For fixed tax rates, we end up with $\tau=12 \%$ and $\vartheta=5 \%$, with a $12 \%$ bigger revenue, a $6 \%$ bigger total welfare and a $53 \%$ bigger non-evaders welfare. Using also the tax rates, we found the optimal policy to be $t_{s}=20 \%$, $t_{b}=15 \%, \tau=6 \%$ and $\vartheta=5 \%$, with a $16 \%$ bigger government revenue, a $5 \%$ bigger total welfare and a $51 \%$ bigger non-evaders welfare. For both policies the constraint on the TCW and the efficiency constraint are binding. The conclusion is that Proposition 7 still holds with a upper bound to the TCW with the only difference of a lower amount of additional tax revenue that can
be raised without welfare or efficiency losses.

## 5 Cash Tax Implementation

In this section, we discuss some challenges to the implementation of the TCW and we propose solutions to (partially) overcome them. Since the TCW is a tax on withdrawals from a bank account, it can be naturally implemented by banks. The bank can collect the tax from the public and then transfer the proceedings to the tax administration. Importantly, the TCW scheme that we propose is different from the carry tax on currency described by Buiter and Panigirtzoglou (2003) and originally proposed by Gesell (1916), which consists in a lower interest rate on base money with respect to other monetary instruments. The TCW consists, in fact, in a fee that applies only to the withdrawals of bank notes (and coins) from ATM machines and from bank tellers, and not to the cash balances held in the accounts.

The first, and foremost, challenge to the implementation of the TCW is the possible emergence of a parallel cash economy: the sellers might start hoarding cash to pay their suppliers and their employees, which will in turn use this cash for their purchases and so on (Morse et al. 2009).

A first possibility to stifle the cash economy is obviously inflation. In particular, if there are interest bearing bank accounts, with an interest rate that partially or totally compensates for the loss due to inflation, there will be a lower incentive for the sellers to accept cash payments, which cannot be deposited in the bank account without triggering a tax audit. In this sense, the inflation tax is complementary to the TCW, because it eases its implementation. As already explained above the complementarity between the TCW and the inflation tax derives from the fact that the TCW, which is paid once and for all, disincentivize the withdrawing of cash but does not disicentivize hoarding it. While the opposite is true for the inflation tax. ${ }^{13}$

A second possibility to fight the emergence of a parallel cash economy is the proposal by Mankiw (2009) to have a lottery on the actual banknotes in circulation. This lottery, based on the serial numbers, would make the "winners" worthless. Curbing the incentive to hoard cash, exactly as the inflation tax, this proposal would lower the incentive for the sellers to accept cash payments easing the implementation of the TCW.

[^10]A third possibility to downsize the parallel cash economy is to fix a small TCW rate, which will not justify the high costs of operating a cash economy. As already shown, even with this constraint, it is still possible to reduce tax evasion and raise additional tax revenue, although less than in the unconstrained case.

A fourth possibility is the introduction of a ban on cash transactions above a certain threshold, both for financial and non-financial products. This will obviously reduce the incentives for the sellers to accept cash payments, because it will make it more difficult to use it later.

Another challenge related to the TCW concerns the dynamic of its introduction: if the tax is announced and then implemented, it is likely that a bank run will take place, with individuals withdrawing cash to avoid paying the tax in the future. Once again, the probability associated with these scenario is higher the higher the TCW: it is unlikely that a massive bank run will take place as a consequence of a small TCW, because the cost of managing all the payments in cash, including the potential loss due to theft, will most likely be higher than the cost imposed by the tax. In addition, operating exclusively in the cash economy hampers or precludes the possibility of obtaining mortgages or even short term financing for the sellers, imposing a very high cost on them (Straub 2005, Antunes and Cavalcanti 2007, Gordon and Li 2009, Capasso and Jappelli 2013). Also, some credit card issuers already charge a small fee for ATM withdrawals, but many individuals use them anyway.

A further problem is finding a way to compensate honest taxpayers. This is a crucial element since proper compensation will boost the acceptability of the TCW. If the policy was perceived as unfair, on the other hand, it would most likely be ineffective, since it will strengthen a social norm against it (Bordignon 1993, Falkinger 1995, Torgler 2003, Slemrod 2003). In the paper we showed that the introduction of a TCW can actually increase tax revenue, and this means that there is, in principle, the possibility to compensate buyers and sellers, for instance subsidizing the use of non-cash instruments. However, it is particularly challenging to design the compensations for those who pay a cost when using non-cash instruments over and above the banking fees, say because of psychological or cognitive reasons. A possibility would be to link a monetary compensation to observable characteristics such as old age, disability and low education, but these are obviously imperfect proxies of the costs.

The last implementation problem is specific to currency areas: to avoid arbitrage, the TCW
rate should be the same in all countries or, at a minimum, not very different. The problem is that this will hamper the possibility of tailoring the policy to the specific needs of a country. One plausible argument against this objection is that the arbitrage possibilities are limited. For instance, for the individuals that typically live and work in one country only, there is not really the possibility to open a bank account in another country, or to travel across borders just to withdraw cash. Most likely, the cost of these operations, which includes travel expenses and banking fees, will reduce the gain from arbitrage, at least for a small TCW. However there still is a problem for the individuals that live close to a border, which is difficult to deal with.

## 6 Conclusion

We presented a model of collaborative tax evasion where a buyer negotiates a price discount with a seller in exchange for not asking the receipt and paying cash, facilitating tax evasion. We studied how a tax rebate for the buyer and a tax on cash withdrawals affect tax evasion, government revenue and welfare. A small tax deduction can reduce tax evasion, increase government revenue and increase welfare and its rate must be higher the higher the tax evasion rate and the higher the statutory tax rate. The TCW is effective at reducing evasion and increasing revenue only if set sufficiently high, and its rate must be higher the higher the tax evasion rate and the bigger the mass of individuals using cash. We found that an appropriate mix of tax rebates and TCW can curb tax evasion while, at the same time, raising additional revenue and increasing aggregate welfare. However, there is a limit to the additional amount of revenue that can be raised without a welfare or efficiency loss. The additional revenue generated by the TCW can also be used to subsidize the use of non-cash instruments, to foster public support.

In other words, we showed that an abrupt and forced move towards a cashless economy, realized with the help of a tax on cash withdrawals, seems to be an effective way to fight tax evasion and to increase government revenue.

Alternatively to the TCW, the government can also enforce a ban on cash transactions to reduce the use of cash, as suggested in Buiter (2009), perhaps establishing a very low threshold value below which it is permitted to use cash. However, this alternative has many drawbacks: first, because it would be costly to enforce the ban; second, it would entail a generalized loss of
privacy; third, imposing the use of credit cards, cheques or bank transfers, even for transactions of small amount, can be too cumbersome and might reduce the number of transactions. In this perspective, the TCW can be seen as putting a price on privacy and transaction ease: it allows to use non-traceable payment instruments to ensure privacy or to speed up the transaction, but it transfers to the users the costs of these benefits.

An alternative policy to reduce tax evasion entails decreasing the cost to manage non-cash instruments, since a decrease in $c$, in the model, has the same effect of an increase of $\vartheta$. However, while the TCW increases the government revenue, decreasing $c$ is costly and, therefore, infeasible for financially constrained governments. Furthermore, it is extremely difficult to reduce the cost $c$ for some individuals, regardless of the magnitude and type of government expenditure: some of these costs are, in fact, fees charged by banks, that can be simply compensated with a subsidy, but some of them are psychological (loss of privacy) and cognitive costs (financial literacy), which are difficult, if not impossible, to reduce or eliminate.

A less obvious possibility consists in supporting an explicit cost-based pricing of payment instruments. Van Hove (2004) argues that, under current banking pricing schemes, the fees charged to consumers for cash withdrawals do not cover the full cost of cash, which is recovered through cross-subsidization. These costs include, among others, the warehousing, distribution, transportation and the protection from theft. "In this way, infrequent cash-users de facto subsidize those who make heavy use of cash (including those active in the underground economy) "Van Hove, 2004, p. 80.

Buiter (2009) proposes an alternative way to reduce the use of currency: the introduction of two different currencies, one that has the numeraire function and another that has the medium of exchange function, with a variable exchange rate between the two. However this alternative solution, although theoretically appealing, is extremely difficult to implement, because it is cumbersome to quote prices in a different currency from the one currently used for transactions.

The substitution of paper currency with electronic currency, which is one of the potential effects of the introduction of our TCW, beside the benefits of a reduced tax evasion and of a potentially more effective monetary policy, has also several costs, as discussed in Rogoff (2014): a potential decline in the demand for debt, more volatile inflation expectations and a system of payments more vulnerable to cyber attacks, power blackouts etc. In this respect our analysis is incomplete,

# since we considered just a limited sets of costs and benefits of the introduction of the TCW. 

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# Appendices not for publication 

## A Proofs

Proof of Proposition 1: The amount of tax evasion is increasing in the tax $t_{s}$ :

$$
\begin{equation*}
\frac{\partial E}{\partial t_{s}}=\int_{0}^{\Upsilon} \frac{\partial E_{c}(c)}{\partial t_{s}} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) \frac{\partial E^{c}}{\partial t_{s}}>0 \tag{26}
\end{equation*}
$$

since

$$
\begin{equation*}
\frac{\partial E_{c}(c)}{\partial t_{s}}=\int_{0}^{\tilde{v}_{1}} g_{s}\left(\tilde{s}_{1}(v, c)\right) p\left(1-\pi\left(1+f_{s}\right)\right)(1+\vartheta) g_{v} d v \tag{27}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial E^{c}}{\partial t_{s}}=\int_{0}^{\tilde{v}_{2}} g_{s}\left(\tilde{s}_{2}(v)\right) p\left(1-\pi\left(1+f_{s}\right)\right)(1+\vartheta) g_{v} d v \tag{28}
\end{equation*}
$$

are both positive. The same is true for an increase in the sale tax $t_{b}$ :

$$
\begin{equation*}
\frac{\partial E}{\partial t_{b}}=\int_{0}^{\Upsilon} \frac{\partial E_{c}(c)}{\partial t_{b}} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) \frac{\partial E^{c}}{\partial t_{b}}>0 \tag{29}
\end{equation*}
$$

since

$$
\begin{equation*}
\frac{\partial E_{c}(c)}{\partial t_{b}}=\int_{0}^{\tilde{v}_{1}} g_{s}\left(\tilde{s}_{1}(v, c)\right) p\left(1-\pi\left(1+f_{b}\right)\right) g_{v} d v \tag{30}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial E^{c}}{\partial t_{b}}=\int_{0}^{\tilde{v}_{2}} g_{s}\left(\tilde{s}_{2}(v)\right) p\left[1-\pi\left(1+f_{b}\right)\right] g_{v} d v \tag{31}
\end{equation*}
$$

are both positive. For the effect of the tax deduction on evasion, we have:

$$
\begin{equation*}
\frac{\partial E}{\partial \tau}=\int_{0}^{\Upsilon} \frac{\partial E_{c}(c)}{\partial \tau} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) \frac{\partial E^{c}}{\partial \tau}<0 \tag{32}
\end{equation*}
$$

since

$$
\begin{equation*}
\frac{\partial E_{c}(c)}{\partial \tau}=-\int_{0}^{\tilde{v}_{1}} p g_{s}\left(\tilde{s}_{1}(v, c)\right) g_{v} d v \tag{33}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial E^{c}}{\partial \tau}=-\int_{0}^{\tilde{v}_{2}} p g_{s}\left(\tilde{s}_{2}(v)\right) g_{v} d v \tag{34}
\end{equation*}
$$

are both negative. The effect of the TCW $\vartheta$ on the total amount of evasion was studied in the main text.

Proof of Proposition 2: We rewrite (15) as:

$$
\begin{align*}
W= & \int_{0}^{\Upsilon}\left(\left(v_{s}^{0}+v_{b}^{0}\right)+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)}\left(\left(v_{s}^{1}-v_{s}^{0}\right)+\left(v_{b}^{1}-v_{b}^{0}\right)\right) g_{s} d s\right) g_{v} d v\right) g_{c} d c+  \tag{35}\\
& \left(1-G_{c}(\Upsilon)\right)\left(\left(v_{s}^{0}+v_{b}^{0}\right)+\int_{0}^{\tilde{v}_{2}} \int_{0}^{\tilde{s}_{2}(v)}\left(\left(v_{s}^{1}-v_{s}^{0}\right)+\left(v_{b}^{1}-v_{b}^{0}\right)\right) g_{s} d s g_{v} d v\right) .
\end{align*}
$$

Define with $R_{h c}=v_{s}^{0}+v_{b}^{0}$ the total utility of those non-evaders agents who have a low $c$ and with $R_{h}^{c}=v_{s}^{0}+v_{b}^{0}$ the same utility for agents with high $c$. Moreover, define with $\Delta R_{d c}=\left(v_{s}^{1}-v_{s}^{0}\right)+\left(v_{b}^{1}-v_{b}^{0}\right)$ the increase in utility from evading taxes both for sellers and buyers with low $c$ and with $\Delta R_{d}^{c}=\left(v_{s}^{1}-v_{s}^{0}\right)+\left(v_{b}^{1}-v_{b}^{0}\right)$ the same increase in utility for agents with high $c$. Then, the derivative of total welfare with respect to $t_{s}$ is:

$$
\begin{align*}
& \int_{0}^{\Upsilon}\left(-p+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \frac{\partial \Delta R_{d c}}{\partial t_{s}} g_{s} d s+\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right) g_{s}\left(\tilde{s}_{1}(v, c)\right) \frac{\partial \tilde{s}_{1}(v, c)}{\partial t_{s}}\right) g_{v} d v\right) g_{c} d c+  \tag{36}\\
& \left(1-G_{c}(\Upsilon)\right)\left(-p+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \frac{\partial \Delta R_{d}^{c}}{\partial t_{s}} g_{s} d s+\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right) g_{s}\left(\tilde{s}_{2}(v)\right) \frac{\partial \tilde{s}_{2}(v)}{\partial t_{s}}\right) g_{v} d v\right)+ \\
& (1-\beta) p\left(1-\pi\left(1+f_{s}\right)\right) \vartheta E .
\end{align*}
$$

Notice that $\frac{\partial \Delta R_{d c}}{\partial t_{s}}=\frac{\partial \Delta R_{d}^{c}}{\partial t_{s}}=p\left(1-\pi\left(1+f_{s}\right)\right)$ and $\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right)=\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right)=0$. The latter is true since by definition $\tilde{s}_{1}(v, c)$ is such that all the buyers of type $c \leq \Upsilon$, with a tax morale lower than $\tilde{s}_{1}(v, c)$, cooperate and $v_{b}^{1}=v_{b}^{0}\left(\tilde{s}_{1}(v, c)\right)$. Then, $\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right)=v_{s}^{1}-v_{s}^{0}=$ $p t_{s}\left(\pi\left(1+f_{s}\right)-1\right)-d^{*}(v, s, c)-v$. By substituting $d^{*}(v, s, c)$ from (7) it is easy to check that $\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right)=0$. A similar reasoning proves that $\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right)=0$. Then, expression (36) simplifies to:

$$
\begin{equation*}
\frac{\partial W}{\partial t_{s}}=-p+p\left(1-\pi\left(1+f_{s}\right)\right) E+\vartheta(1-\beta) p\left(1-\pi\left(1+f_{s}\right)\right) E \tag{37}
\end{equation*}
$$

which is negative for a small TCW. Similarly, the derivative of total welfare with respect to $t_{b}$ is:

$$
\begin{align*}
& \int_{0}^{\Upsilon}\left(-p+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \frac{\partial \Delta R_{d c}}{\partial t_{b}} g_{s} d s+\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right) g_{s}\left(\tilde{s}_{1}(v, c)\right) \frac{\partial \tilde{s}_{1}(v, c)}{\partial t_{b}}\right) g_{v} d v\right) g_{c} d c+  \tag{38}\\
& \left(1-G_{c}(\Upsilon)\right)\left(-p-p \vartheta+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \frac{\partial \Delta R_{d}^{c}}{\partial t_{b}} g_{s} d s+\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right) g_{s}\left(\tilde{s}_{2}(v)\right) \frac{\partial \tilde{s}_{2}(v)}{\partial t_{b}}\right) g_{v} d v\right)+ \\
& g_{c}(\Upsilon) p \vartheta\left(u-p\left(t_{s}+t_{b}-\tau\right)+\Upsilon+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \Delta R_{d c}\left(p\left(1+t_{b}\right) \vartheta\right) g_{s} d s\right) g_{v} d v\right)+ \\
& -g_{c}(\Upsilon) p \vartheta\left(u-p\left(t_{s}+t_{b}-\tau+\Upsilon+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \Delta R_{d}^{c} g_{s} d s\right) g_{v} d v\right)+\right. \\
& -\frac{\vartheta \beta p}{1+\vartheta}\left(\left(1-\pi\left(1+f_{s}\right)\right) E+\left(1-G_{c}(\Upsilon)\right) \vartheta E^{c}\right) .
\end{align*}
$$

Notice that: $\frac{\partial \Delta R_{d c}}{\partial t_{b}}=\frac{\partial \Delta R_{d}^{c}}{\partial t_{b}}=p\left(1-\pi\left(1+f_{b}\right)\right)$ and evaluated at $c=\Upsilon$ we have that $\Delta R_{d c}(\Upsilon)=\Delta R_{d}^{c}$. The latter also implies that $\tilde{s}_{1}(v, c)=\tilde{s}_{2}(v)$ and $\tilde{v}_{1}=\tilde{v}_{2}$. Then, expression (38) can be rewritten as:

$$
\begin{equation*}
\frac{\partial W}{\partial t_{b}}=-p\left(1+\left(1-G_{c}(\Upsilon)\right) \vartheta\right)+p\left(1-\pi\left(1+f_{b}\right)\right) E-\frac{\vartheta \beta p}{1+\vartheta}\left\{\left(1-\pi\left(1+f_{s}\right)\right) E+\left(1-G_{c}(\Upsilon)\right) \vartheta E^{c}\right\} \tag{39}
\end{equation*}
$$

which is always negative. For the rebate we have:

$$
\begin{align*}
& \int_{0}^{\Upsilon}\left(p+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \frac{\partial \Delta R_{d c}}{\partial \tau} g_{s} d s+\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right) g_{s}\left(\tilde{s}_{1}(v, c)\right) \frac{\partial \tilde{s}_{1}(v, c)}{\partial t_{s}}\right) g_{v} d v\right) g_{c} d c+  \tag{40}\\
& \left(1-G_{c}(\Upsilon)\right)\left(p+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \frac{\partial \Delta R_{d}^{c}}{\partial \tau} g_{s} d s+\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right) g_{s}\left(\tilde{s}_{2}(v)\right) \frac{\partial \tilde{s}_{2}(v)}{\partial t_{s}}\right) g_{v} d v\right)+\frac{\vartheta \beta p}{1+\vartheta} E
\end{align*}
$$

Noticing that $\frac{\partial \Delta R_{d c}}{\partial \tau}=\frac{\partial \Delta R_{d}^{c}}{\partial \tau}=-p$ we can rewrite (40) as:

$$
\begin{equation*}
\frac{\partial W}{\partial \tau}=p-p E+\frac{\vartheta \beta p}{1+\vartheta} E \tag{41}
\end{equation*}
$$

which is always positive. Finally, for the effect of the TCW on welfare we have:

$$
\begin{align*}
& \int_{0}^{\Upsilon}\left(\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \frac{\partial \Delta R_{d c}}{\partial \vartheta} g_{s} d s+\Delta R_{d c}\left(\tilde{s}_{1}(v, c)\right) g_{s}\left(\tilde{s}_{1}(v, c)\right) \frac{\partial \tilde{s}_{1}(v, c)}{\partial t_{b}}\right) g_{v} d v\right) g_{c} d c+  \tag{42}\\
& \left(1-G_{c}(\Upsilon)\right)\left(-p\left(1+t_{b}\right)+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \frac{\partial \Delta R_{d}^{c}}{\partial \vartheta} g_{s} d s+\Delta R_{d}^{c}\left(\tilde{s}_{2}(v)\right) g_{s}\left(\tilde{s}_{2}(v)\right) \frac{\partial \tilde{s}_{2}(v)}{\partial t_{b}}\right) g_{v} d v\right)+ \\
& g_{c}(\Upsilon) p\left(1+t_{b}\right)\left(u-p\left(t_{s}+t_{b}-\tau\right)-\Upsilon+\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \Delta R_{d c}\left(p\left(1+t_{b}\right) \vartheta\right) g_{s} d s\right) g_{v} d v\right)+ \\
& -g_{c}(\Upsilon) p\left(1+t_{b}\right)\left(u-p\left(t_{s}+t_{b}-\tau\right)-\Upsilon+\int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \Delta R_{d}^{c} g_{s} d s\right) g_{v} d v\right)+ \\
& \int_{0}^{\Upsilon}\left(\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \vartheta \frac{\partial d^{*}(v, s, c)}{\partial \vartheta} g_{s} d s\right) g_{v} d v\right) g_{c} d c+\left(1-G_{c}(\Upsilon)\right) \int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \vartheta \frac{\partial d^{*}(v, s)}{\partial \vartheta} g_{s} d s\right) g_{v} d v .
\end{align*}
$$

where

$$
\begin{equation*}
\frac{\partial d^{*}(v, s)}{\partial \vartheta}=-\beta \frac{p \tau+\pi p t_{b}\left(1+f_{b}\right)+s}{(1+\vartheta)^{2}}<0 \tag{43}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial d^{*}(v, s, c)}{\partial \vartheta}=\beta \frac{p-p \tau+p t_{b}\left(1-\pi\left(1+f_{b}\right)\right)-s+c}{(1+\vartheta)^{2}}>0 \tag{44}
\end{equation*}
$$

since $s<p-p \tau+p t_{b}\left(1-\pi\left(1+f_{b}\right)\right)+c .{ }^{14} \quad$ Moreover, $\frac{\partial \Delta R_{d c}}{\partial \vartheta}=-\left(p-d^{*}(v, s, c)\right)$ while $\frac{\partial \Delta R_{d}^{c}}{\partial \vartheta}=-\left(p-d^{*}(v, s)\right)+p\left(1+t_{b}\right)$. Summing up (42) can be rewritten as:

$$
\begin{align*}
\frac{\partial W}{\partial \vartheta}= & -\left(1-G_{c}(\Upsilon)\right) p\left(1+t_{b}\right)+  \tag{45}\\
& -\int_{0}^{\Upsilon}\left(\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)}\left(p-d^{*}(v, s, c)\right) g_{s} d s\right) g_{v} d v\right) g_{c} d c+ \\
& \left(1-G_{c}(\Upsilon) \int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)}\left(d^{*}(v, s)+p t_{b}\right) g_{s} d s\right) g_{v} d v+\right. \\
& \int_{0}^{\Upsilon}\left(\int_{0}^{\tilde{v}_{1}}\left(\int_{0}^{\tilde{s}_{1}(v, c)} \vartheta \frac{\partial d^{*}(v, s, c)}{\partial \vartheta} g_{s} d s\right) g_{v} d v\right) g_{c} d c+ \\
& \left(1-G_{c}(\Upsilon)\right) \int_{0}^{\tilde{v}_{2}}\left(\int_{0}^{\tilde{s}_{2}(v)} \vartheta \frac{\partial d^{*}(v, s)}{\partial \vartheta} g_{s} d s\right) g_{v} d v,
\end{align*}
$$

[^11]whose sign is ambiguous.
Proof of Proposition 3: Define $R_{c}=p t_{s}\left(\pi\left(1+f_{s}\right)-1\right)+p t_{b}\left(\pi\left(1+f_{b}\right)-1\right)+\left(p-d^{*}(v, s, c)\right) \vartheta+p \tau$ and $R^{c}=p t_{s}\left(\pi\left(1+f_{s}\right)-1\right)+p t_{b}\left(\pi\left(1+f_{b}\right)-1\right)+\left(p-d^{*}(v, s)\right) \vartheta+p \tau-\Upsilon$. Then, the derivative of the net government revenue with respect to $t_{s}$ is:
\[

$$
\begin{align*}
\frac{\partial G_{n}}{\partial t_{s}}= & \int_{0}^{\Upsilon} R_{c} \frac{\partial E_{c}(c)}{\partial t_{s}} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) R^{c} \frac{\partial E^{c}}{\partial t_{s}}+p \pi\left(1+f_{s}\right) E+  \tag{46}\\
& +p(1-E)-(1-\beta) p\left(1-\pi\left(1+f_{s}\right)\right) \vartheta E+\int_{0}^{\Upsilon} c \frac{\partial E_{c}(c)}{\partial t_{s}} g_{c} d c .
\end{align*}
$$
\]

The first and second term (the extensive margin) show that a higher tax rate $t_{s}$ increases the mass of evaders (27 and 28). This effect on revenue is ambiguous, except if $\tau=\vartheta=t_{b}=0$, in which case the effect is negative, since $R_{c}=R^{c}=p t_{s}\left(\pi\left(1+f_{s}\right)-1\right)<0$. The third and fourth term (the intensive margin) measure the increase in revenue due to the increase in the tax rate, both for evaders and non-evaders. The fifth term is the decrease in revenue from the TCW, since an increase in $t_{s}$ increases the equilibrium discount. The last term measures the decrease in the cost imposed on those non-collaborative buyers who opt for non-cash payments. If $\tau=\vartheta=t_{b}=0$, only the first four effects are left. Moreover, when $t_{s}$ is low, the first two negative terms are small, while the third and fourth terms are large; in this case, an increase in $t_{s}$ increases the revenue. The opposite happens if $t_{s}$ is high. We have a standard Laffer curve type of result.

Similarly the derivative of net government revenue with respect to $t_{b}$ is:

$$
\begin{align*}
\frac{\partial G_{n}}{\partial t_{b}}= & \int_{0}^{\Upsilon} R_{c} \frac{\partial E_{c}(c)}{\partial t_{b}} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) R^{c} \frac{\partial E^{c}}{\partial t_{b}}+p \pi\left(1+f_{b}\right) E+  \tag{47}\\
& \int_{0}^{\Upsilon} p\left(1-E_{c}(c)\right) g_{c} d c+\left(1-G_{c}(\Upsilon)\right) p(1+\vartheta)\left(1-E^{c}\right)+ \\
& \frac{\beta \vartheta p}{1+\vartheta}\left(1-\pi\left(1+f_{b}\right)\right) \int_{0}^{\Upsilon} E_{c}(c) g_{c} d c+\frac{\beta \vartheta p}{1+\vartheta}\left(1-\pi\left(1+f_{b}\right)+\vartheta\right)\left(1-G_{c}(\Upsilon)\right) E^{c}+ \\
& \int_{0}^{\Upsilon} c \frac{\partial E_{c}(c)}{\partial t_{b}} g_{c} d c-\Upsilon\left(1-E_{c}(\Upsilon)\right) g_{c}(\Upsilon) p \vartheta+ \\
& g_{c}(\Upsilon) p \vartheta\left(R_{c}(\Upsilon) E_{c}(\Upsilon)+p\left(t_{s}+t_{b}-\tau\right)-R^{c} E^{c}-p\left(t_{s}+t_{b}-\tau\right)-\Upsilon\left(1-E^{c}\right)\right) .
\end{align*}
$$

Again a higher tax rate $t_{b}$ increases the mass of evaders (first and second term). This effect on revenue is ambiguous, except if $\tau=\vartheta=t_{b}=0$, in which case the effect is negative, since
$R_{c}=R^{c}=p t_{b}\left(\pi\left(1+f_{b}\right)-1\right)<0$. The third, fourth and fifth term (the intensive margin) measure the increase in revenue due to the increase in the tax rate, both for evaders (third) and non-evaders (fourth and fifth). The sixth and seventh term measure the increase in revenue from the TCW: differently from the tax on the seller an increase in $t_{b}$ decreases the equilibrium discount. The eighth and ninth term measure the change in the cost imposed on those non collaborative buyers who opt for non-cash payments. Finally, since (in $c=\Upsilon) R_{c}(\Upsilon)=R^{c}$ and $E_{c}(\Upsilon)=E^{c}$, the last term, that measures the decrease in revenue from the TCW due to the decrease in the mass of non-collaborative buyers using cash, simplifies to $-\Upsilon\left(1-E_{c}(\Upsilon)\right) g_{c}(\Upsilon) p \vartheta$. If $\tau=\vartheta=t_{s}=0$, only first, second, third and fifth term are left. Moreover, when $t_{b}$ is low, the first two negative terms are small, while the third and fifth term are large; in this case, an increase in $t_{b}$ increases the revenue. The opposite happens if $t_{b}$ is high. Again we have a standard Laffer curve type of result.

For the rebate we have:

$$
\begin{align*}
\frac{\partial G_{n}}{\partial \tau}= & \int_{0}^{\Upsilon} R_{c} \frac{\partial E_{c}(c)}{\partial \tau} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) R^{c} \frac{\partial E^{c}}{\partial \tau}+  \tag{48}\\
& 0-p(1-E)-\frac{\beta p \vartheta}{1+\vartheta} E+\int_{0}^{\Upsilon} c \frac{\partial E_{c}(c)}{\partial \tau} g_{c} d c .
\end{align*}
$$

An increase in $\tau$ decreases the mass of evaders (first and second term) with an ambiguous effect on revenue unless $t_{s}$ and $t_{b}$ are sufficiently high, in which case the effect is positive. There is no effect of an increase of $\tau$ on evaders (third term) and a negative effect on the revenue from non-evaders (fourth term). Moreover, an increase in $\tau$ decreases the revenue from the TCW, as a consequence of a higher equilibrium discount (fifth term), and increases the cost imposed on non-collaborative buyers who opt for non-cash payments (last term). In other words, low statutory taxes, a large mass of honest individuals $(1-E)$ and an high TCW are all factors that can make a tax rebate
undesirable. For the effect of the TCW on revenue we have:

$$
\begin{align*}
\frac{\partial G_{n}}{\partial \vartheta}= & \int_{0}^{\Upsilon} R_{c} \frac{\partial E_{c}(c)}{\partial \vartheta} g_{c} d c+\left(1-G_{c}(\Upsilon)\right) R^{c} \frac{\partial E^{c}}{\partial \vartheta}+  \tag{49}\\
& \int_{0}^{\Upsilon}\left(p-d^{*}(v, s, c)\right) E_{c}(c) g_{c} d c+\left(1-G_{c}(\Upsilon)\right)\left(p-d^{*}(v, s)\right) E^{c}+ \\
& p\left(1+t_{b}\right)\left(1-G_{c}(\Upsilon)\right)\left(1-E^{c}\right)+ \\
& -\vartheta \int_{0}^{\Upsilon} \frac{\partial d^{*}(v, s, c)}{\partial \vartheta} E_{c}(c) g_{c} d c-\vartheta\left(1-G_{c}(\Upsilon)\right) \frac{\partial d^{*}(v, s)}{\partial \vartheta} E^{c}+ \\
& \int_{0}^{\Upsilon} c \frac{\partial E_{c}(c)}{\partial \vartheta} g_{c} d c-\Upsilon\left(1-E_{c}(\Upsilon)\right) g_{c}(\Upsilon) p\left(1+t_{b}\right)+ \\
& g_{c}(\Upsilon) p\left(1+t_{b}\right)\left(R_{c}(\Upsilon) E_{c}(\Upsilon)+p\left(t_{s}+t_{b}-\tau\right)-R^{c} E^{c}-p\left(t_{s}+t_{b}-\tau\right)-\Upsilon\left(1-E^{c}\right)\right)
\end{align*}
$$

The comparative statics with respect to $\vartheta$ is quite complicated since an increase in $\vartheta$ increases the mass of evaders with high $c$ (14) and decreases the mass of evaders with low $c$ (13). An increase in $\vartheta$ also increases the revenue at the intensive margin, both from evaders (third and fourth term) and non-evaders (fifth term). Moreover, it increases the discount for evaders with high $c$ (sixth term see 44) and decreases the discount for evaders with low $c$ (seventh term see 43). The eigth and ninth term measure the increase in the cost imposed on non-collaborative buyers who opt for non-cash payments. Finally, the last term, which simplifies to $-\Upsilon\left(1-E_{c}(\Upsilon)\right) g_{c}(\Upsilon) p\left(1+t_{b}\right)$, measures the decrease in revenue from the TCW due to the decrease in the mass of non-collaborative buyers using cash. We conclude that the effect of the TCW on government revenue is ambiguous.

## A. 1 Optimal Policy: the Case of Italy

In this section we provide an example of the optimal policy for a real world country, Italy. For the income tax rate we choose $t_{s}=0.35$, which is a (rounded) weighted average of the different rates with weights equal to the percentage of income in the bracket. We also set sale tax to $t_{b}=0.2$, which was the main VAT rate in Italy before 2011 (raised to $22 \%$ in 2013). For the distribution of the cost of payment instruments other than cash $c$, we consider data on payment instruments from the ECB (gathered through the national central banks). We divide the sum of all transactions made with credit cards and debit cards by the consumption component of GDP (goods, including durable, and services). We obtain $\lambda=0.127$. We stick to the assumption of $\beta=0.5$ and to $p=10$
and $u=1.5 p$.
For the enforcement probability $\pi$, we divide the number of tax audits made in 2011 by the "Guardia di Finanza", the main tax enforcement authority in Italy, by the number of economic units (firms, entrepreneurs, individual professionals) operating in Italy in 2011. There are two kinds of audits implemented by this tax enforcement authority: a more in-depth one, which is less frequent but that detects evasion with certainty (a careful screening of all the fiscal documents, together with a detailed analysis of the economic activity) and a more superficial one, much more frequent but less effective (a simple spot control where the agents monitor the day to day activity and step in if there is a violation). In the first case we have $\pi=0.0067$ while in the second $\pi=0.172$. We present the results for the first value and we assume that all audits are random and independent from sellers subjective characteristics. While this is certainly true for the spot controls, it is not for the more thorough controls, which are typically the final step of some monitoring activity that takes into account the business characteristics, also based on the past tax reports. For the fine we use the value $f=0.3$ according to the Italian tax law that prescribes a fine from $6 \%$ to $30 \%$ to be paid on the evaded amount. In Italy, tax evasion is also subject to jail sentences, in addition to the fines, but only in extreme cases (very high amount), which makes them extremely unlikely. Thus we focus on pecuniary fines only.

To calibrate the model, we need a target value for tax evasion and we need a shape of the distribution of the tax morale to match. For the tax morale we take the values from the WWS website. For tax evasion, we consider two different sources. The first is the Study by EURES (2012), an Italian independent research institute. Total tax evasion in Italy is estimated to be between $16.3 \%$ and $17.5 \%$ of the GDP. Both numbers are obtained averaging over different sectors (32.8\% agriculture, $12.4 \%$ Industry and between $20 \%$ and $27 \%$ for services) and across different geographic areas. The other source is the ISTAT, the Italian statistical institute, that reports an average of $12.7 \%$, also obtained averaging over different sectors ( $22 \%$ agriculture, $6 \%$ industry, $11 \%$ construction, $14 \%$ services) and geographic areas ( $9 \%$ North, $11 \%$ center, $20 \%$ South). We set our benchmark close to the average of the two numbers at $15 \%$.

For the $15 \%$ benchmark tax evasion level, we end up with $b=1$ and $a=5.87$ and $\bar{s}=\bar{v}=3.451$.
The optimal welfare maximizing policy for Italy conditional on keeping gross revenue at least as high as in the benchmark entails an unchanged vat tax rate $t_{b}=20 \%$, a smaller income tax
rate $t_{s}=25 \%$, a small tax rebate $\tau=4 \%$ and a small TCW rate $\vartheta=6 \%$. Tax evasion is below one percent at this policy, total welfare is $7 \%$ bigger and non-evaders welfare is $28 \%$ bigger. The optimal welfare maximizing policy conditional on raising at least $20 \%$ revenue more than the benchmark entails the same tax rates but $\tau=1 \%$ and $\vartheta=16 \%$. The tax evasion rate at this policy is equal to $1.7 \%$, but total welfare is $4 \%$ smaller than the benchmark and non-evaders welfare $12 \%$ bigger. In case the constraint on the additional revenue to be raised is $10 \%$ instead of $20 \%$, we found that it is possible to reduce evasion at $1 \%$, raise $10 \%$ more revenue and increase total welfare by $2 \%$ fixing $t_{s}=25 \%, t_{b}=22.5 \%, \tau=4 \%$ and $\vartheta=9 \%$. In other words, Proposition 6 still holds for Italy, although the amount of extra government revenue that it is possible to raise without any welfare cost is more limited than in the prototype economy. This last policy, however, does not satisfy the efficiency constraint. If we add it, we find that the optimal welfare maximizing policy that raise $10 \%$ more revenue is $t_{s}=35 \%, t_{b}=22.5 \%, \tau=16 \%$ and $\vartheta=12 \%$, with a $1.7 \%$ total welfare gain and a $19 \%$ non-evaders welfare gain.

The optimal maximin policy for Italy entails a reduction in both tax rates, $t_{s}=25 \%$ and $t_{b}=17.5 \%$, a small tax rebate, $\tau=2 \%$ and a high TCW rate, $\vartheta=23 \%$. At this policy the revenue is $20 \%$ bigger than the benchmark, total welfare $5 \%$ smaller and non-evaders welfare $12 \%$ bigger. Imposing the efficiency constraint, the optimal maximin policy entails keeping the tax rates fixed, $\tau=15 \%$ and $\vartheta=12 \%$, with a $7 \%$ increase in revenue and a $3 \%$ bigger total welfare and a $22 \%$ bigger non-evaders welfare. We conclude that, in the case of Italy, it is not possible to eliminate tax evasion and raise a lot of extra revenue, in excess of around $10 \%$, without a cost in terms of total welfare.

Fixing the tax rates, the policy that maximizes total welfare conditional on raising as much revenue as in the benchmark is $\tau=16 \%$ and $\vartheta=9 \%$, with an almost $8 \%$ gain in total welfare and tax evasion below $1 \%$. If we consider a $20 \%$ higher target for the revenue, the optimal welfare maximizing policy for fixed taxes is $\tau=7 \%$ and $\vartheta=13 \%$. At this policy the tax evasion is $6 \%$, total welfare is $5 \%$ smaller than the benchmark and non-evaders welfare $6 \%$ bigger. Looking at the second policy objective, the maximin, the optimal policy for fixed taxes entails $\tau=16 \%=\vartheta$, with a $10 \%$ bigger government revenue, a $1 \%$ bigger total welfare and a $20 \%$ bigger non-evaders welfare.

If the maximum feasible cash tax rate is $5 \%$, the optimal welfare maximizing policy that raises
at least $10 \%$ more revenue is $t_{s}=25 \%, t_{b}=25 \%, \tau=1 \%$ and $\vartheta=4 \%$, with a $3.5 \%$ evasion rate, a $2 \%$ bigger total welfare and a $19 \%$ bigger non evaders welfare. The optimal maximin policy is instead $t_{b}=0.25, t_{s}=22.5 \%$ and $\tau=\vartheta=5 \%$, with a $1 \%$ bigger gross revenue, a $7 \%$ bigger total welfare a $27 \%$ bigger non evaders welfare. Therefore, even if it is unfeasible to have high TCW rates, it is still possible to reduce tax evasion and raise a small amount of additional tax revenue without welfare costs, although it looks like a high TCW rate above $5 \%$ is important to raise revenue.

Figure 1: Effect of the tax rebate by evasion level





Notes: Upper left panel: tax evasion in percentage terms. Upper right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ( $t_{s}=0.3, t_{b}=0.1, \tau=0$ and $\vartheta=0$ ) for each tax evasion level. Lower left panel: total welfare, scaled to be 100 in the baseline model specification for each tax evasion level. Lower right panel: welfare of non evading sellers and buyers, scaled to be 100 in the baseline model specification for each tax evasion level.

Figure 2: Effect of the TCW by evasion level


Notes: Upper left panel: tax evasion in percentage terms. Upper right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ( $t_{s}=0.3, t_{b}=0.1, \tau=0$ and $\vartheta=0$ ) for each tax evasion level. Lower left panel: total welfare, scaled to be 100 in the baseline model specification for each tax evasion level. Lower right panel: welfare of non evading sellers and buyers, scaled to be 100 in the baseline model specification for each tax evasion level.

Figure 3: Effect of the TCW by use on non-cash instruments


Notes: Upper left panel: tax evasion in percentage terms. Upper right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ( $t_{s}=0.3, t_{b}=0.1, \tau=0$ and $\vartheta=0$ ) for each tax evasion level. Lower left panel: total welfare, scaled to be 100 in the baseline model specification for each tax evasion level. Lower right panel: welfare of non evading sellers and buyers, scaled to be 100 in the baseline model specification for each tax evasion level. lam is the probability mass at zero cost of using non cash instruments.


[^0]:    *University of Salerno and CSEF and University of Naples Federico II and CSEF respectively. We thank Emilio Calvano, Elena Creti, Elena D'Agosto, Vincenzo Denicolò, Luigi Franzoni, Marco Pagano, Nicola Pavoni, Carmelo Petraglia, Emanuela Randon, Marisa Ratto, Harald Uhlig and seminar participants at the 10th CSEF-IGIER Symposium on Economics and Institutions, at the 3rd Shadow Economy Conference 2013 (Munster), Università Federico II (Naples), Universitè Dauphine (Paris), Universitè Paris Ouest Nanterre La Defense (Paris), SIDE 2013 (Lugano), Università di Bologna.

[^1]:    ${ }^{1}$ Indeed, consistently with this view, the tax was abolished in India in 2009 on the grounds of its irrelevance after the adoption of more sophisticated IT technologies to track down evaders.

[^2]:    ${ }^{2}$ See Santoro (2006), for an argument against the effectiveness of policy instruments meant to reduce the incentives to cooperate on tax evasion and Piolatto (2014) for an opposite argument in favour of the effectiveness of itemized deductions.

[^3]:    ${ }^{3}$ In addition, the introduction of the TCW, by reducing the number of transactions in cash, will most likely reduce the seigniorage revenue. Although this source of revenue is only of limited importance for low inflation, advanced, economies, it can be important for developing economies, which are not able to manage a tax collection system or that have high rates of irregular activity.

[^4]:    ${ }^{4}$ According to Gordon (1990), there is also another reason, beyond honesty, for which the buyers need a receipt: to have a formal guarantee on the product, so that, for instance, they can return a defective item. In what follows, we will ignore this feature, mainly because, as narrative evidence suggests, the sellers typically offer the same kind of customer service even when the fail to provide the receipt (in fact this is part of the argument they use to convince the buyers to go without receipt).
    ${ }^{5}$ This is without loss of generality for the risk neutral benchmark.

[^5]:    ${ }^{6}$ We follow Yitzhaki (1974) and set a penalty on the evaded amount rather than on the evaded tax as in Allingham and Sandmo (1972).
    ${ }^{7}$ In case there are intermediate producers, it is more complicated to model the VAT tax scheme, since the seller is itself a buyer that pays the VAT only on the difference in value. In addition, if there is an intermediate producer the negotiation between the final producer and the buyer depends also on the decision to evade or not at the intermediate stage, which in turn might be the result of a negotiation. We chose to abstract from these complications to keep the model tractable.
    ${ }^{8} \mathrm{We}$ assume that the enforcement probability for the buyer is the same as the enforcement probability for the seller or, in other words, that the enforcement is on the transaction. This is a shortcut since, in practice, the

[^6]:    ${ }^{9}$ By plugging the expression (7) into (8) instead of (9) we find exactly the same result.

[^7]:    ${ }^{10}$ Notice that for $\vartheta=0 \frac{\partial E}{\partial \vartheta}=\int_{0}^{\tilde{v}_{2}} g_{s}\left(\tilde{s}_{2}(v)\right) \frac{\partial \tilde{s}_{2}(v)}{\partial \vartheta} g_{v} d v>0$ while for $\vartheta=1 \frac{\partial E}{\partial \vartheta}=$ $\int_{0}^{1} \int_{0}^{\tilde{v}_{1}} g_{s}\left(\tilde{s}_{1}(v, c)\right) \frac{\partial \tilde{z}_{1}(v, c)}{\partial \vartheta} g_{v} d v g_{c} d c<0$.

[^8]:    ${ }^{11}$ Notice that our results do not change if we include a mass of buyers with negative costs of using non cash payment instruments, since they would have chosen those alternative means of payment irrespectively of the policy.

[^9]:    ${ }^{12}$ Reinganum and Wilde (1985) highlight the optimal auditing rule of the tax authority. Slemrod et al. (2001) and Kleven et al. (2011) study the effects of the threat of enforcement on reported income using field experiments.

[^10]:    ${ }^{13}$ The formal study of the interaction between inflation tax and TCW is left for future research.

[^11]:    ${ }^{14}$ From condition (9), we know that, in order to have tax evasion, $s$ must be smaller than $d^{*}(v, s, c)(1+$ $\vartheta)-p\left(\tau+\vartheta+t_{b}\right)-\pi p t_{b}\left(1+f_{b}\right)+\min \{\Upsilon, c\}$. Then, we can show that $p-p \tau+p t_{b}\left(1-\pi\left(1+f_{b}\right)\right)+c$ is larger than $d^{*}(v, s, c)(1+\vartheta)-p\left(\tau+\vartheta+t_{b}\right)-\pi p t_{b}\left(1+f_{b}\right)+\min \{\Upsilon, c\}$ (which is larger than $s$ ) or equivalently that $d^{*}(v, s, c)(1+\vartheta)-p \vartheta+\min \{\Upsilon, c\}<p+c$. This is always true. Indeed, if $\min \{\Upsilon, c\}=c$ we have $d^{*}(v, s, c)(1+\vartheta)<p(1+\vartheta)$. If, instead, $\min \{\Upsilon, c\}=\Upsilon$ we have $d^{*}(v, s, c)(1+\vartheta)+\Upsilon<p(1+\vartheta)+c$.

