Financial Crises and Policy

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Abstract

I develop a quantitative macroeconomic model with heterogeneous agents and endogenously determined balance sheet constraints on financial intermediaries. I study the full equilibrium dynamics of the model which are characterized by two regimes: tranquil periods, during which financial intermediation is only indirectly affected by financial frictions; and occasional crises periods, during which balance sheet constraints on bankers are binding and the economy experiences prolonged periods of depressed economic activity. I then turn to policy analysis and study two types of unconventional policies: bailouts and credit market interventions. I address two key aspects of these types of intervention which gave rise to significant controversy: the tradeoff between the stimulatory and the redistributive impact, and the possibility that anticipation of such policies generates moral hazard. I find that credit market interventions are superior to bailouts both in terms of their stimulatory effect and in terms of their effect on depositors' welfare; moreover they can achieve significant Pareto improvements on the decentralized equilibrium allocation. I then conduct a more systematic study of the inefficiencies in the model, namely incomplete markets and pecuniary externalities, by solving for the constrained efficient allocation and find that the simple rule for credit market intervention achieves welfare gains that are close to those associated with the second best.

1 Introduction

Financial crises are rare episodes during which bank funding markets come under severe pressure, banks' balance sheets quickly deteriorate and the resulting disruption of financial intermediation leads to deep and prolonged recessions. Over the past few years, the U.S. economy has experienced the worst financial crisis of the post-war era. In the recent U.S. recession, losses in the subprime mortgage market triggered the collapse of the commercial paper market, on which financial institutions relied heavily to obtain funds for use in longer term investment. High bank leverage made investors reluctant to lend in this market, forcing banks to sell assets at fire sale prices and hence inducing further deterioration of banks' balance sheets. The significant financial disruption resulted in an increase in financing costs and a big contraction in durable goods consumption, output and employment.

One striking feature of the recent crisis has been the unprecedented nature and size of central bank intervention. Starting from late 2007 the central bank intervened to address the quick deterioration in bank funding conditions by increasing the supply of credit available to banks, issuing emergency loans to bailout institutions deemed to be "too big to fail" and eventually, engaging in massive purchases of mortgage backed securities.

In this work I develop a quantitative macroeconomic DSGE model that can replicate the salient features of financial crises described above and can be used as a framework to assess the implications of the types of policies observed during the recent crisis. While there is a rich and quickly growing body of literature on financial crises (see literature review below for a more detailed discussion), the novelty of my approach is that I study the full equilibrium dynamics of a quantitative macroeconomic model with two groups of heterogeneous utility maximizing agents, households and bankers, and endogenously determined balance sheet constraints that occasionally limit the amount of funds bankers are able to obtain from households. The combination of the quantitative nature of the analysis and the explicit specification of the model in terms of primitives is the main element that distinguishes this work from recent contributions that underscore the importance of studying the full equilibrium dynamics of models with financial frictions.

Apart from the intrinsic desirability of deriving a model from first principles, this allows me to ask two questions that have not been previously addressed in the literature: how big are the welfare costs of financial crises? what are the welfare implications of government interventions aimed at stimulating the economy by redistributing resources to the financial sector? In the rest of this introduction I briefly describe the key mechanisms at work in the model and the answers it provides to the above questions.

Financial intermediaries in the model are a separate group of agents with superior skills in lending to the productive sector. They fund themselves by issuing short term risk free debt to households and use these funds and inside equity to make loans to the productive sector. There is however an agency problem that endogenously determines a limit on the amount of funds that banks are able to obtain from households, linking banks' leverage to excess returns on banks' loans.

Financial crises in the model are periods in which this constraint on banks' borrowing is binding. The full equilibrium dynamics are characterized by two regimes: normal times and financial crises. When banks' balance sheets are healthy, the agency problem does not affect their borrowing decisions, i.e. the borrowing constraint does not bind, and banks are able to raise funds from households to finance all profitable investment opportunities. In this region, losses on banks' portfolios have only a mild effect on real economic activity, as well capitalized banks are able to finance a quick recovery.

As bankers' balance sheet position deteriorates, the amount of external funds they need in order to finance projects increases and eventually exceeds the limit imposed by their creditors. Binding financial constraints turn on the familiar financial amplification mechanism: losses in bankers' portfolios feed back into amplified losses in net worth as bankers are leveraged; the borrowing constraint tightens as a result, and reduces banks' ability to finance investment; lower demand puts downward pressure on asset prices, hence inducing further losses in net worth. The stabilizing force in this loop operates through increased excess returns on loans: a standard corporate finance argument implies that as excess returns on banks' projects increase, banks will be allowed to take on more leverage and the equilibrium is reached at a point where financial institutions' leverage and spreads on banks' loans are both higher. The increased costs of borrowing translate into lower investment levels and prolonged periods of depressed economic activity.

Two forces operate to make financial crises recurrent but rare. On the one hand, bankers' risk aversion gives them an incentive to avoid the big fluctuations in wealth associated with financial crises. On the other hand, bankers are assumed to be more impatient than depositors so that, unless they earn excess returns on their loans, they will have a tendency to deplete their capital buffers.

After describing the behavior of the decentralized equilibrium, I turn to policy analysis. In particular I address two main related concerns regarding the desirability of such massive interventions as the ones observed. First, insofar as such programs have entailed a redistribution of resources from tax-payers at large to the financial sector, an assessment of their overall impact needs to weigh their stimulatory effects against their direct redistributive effects. A second concern arises from the possibility that anticipation of future intervention will result in moral hazard behavior on the side of bankers, leading them to engage in riskier portfolio strategies and hence inducing more frequent crises.

In the model I consider two types of government intervention that are meant to capture the early programs established by the Fed before the rounds of purchases of MBS. These programs had two main purposes: easing credit conditions for banks (discount window lending, TAF, CPFF); and bailing out financial institutions (AIG, Bear Sterns). I assume that the central authority raises lump-sum taxes on households to either make transfers to bankers (bailouts) or allow them to borrow at more favorable terms than the ones they would face in the market (credit market intervention).[1]

I find that credit market interventions have both a stronger stimulatory effect on economic activity and a more beneficial effect on depositors' welfare than bailouts. The reason for this result is that with a credit market intervention the central bank directly addresses the main friction present in the model by affecting bankers' incentives to invest. Moreover, I demonstrate that a simple rule for credit market intervention can be designed to achieve significant Pareto improvements on the decentralized equilibrium allocation. The anticipation of future interventions does induce moral hazard on the side of bankers, increasing their leverage in tranquil periods and the probability of crises; on the other hand, it makes typical crises much less severe and allows bankers to intermediate larger amounts of capital in normal times, hence increasing average levels of economic activity.

To uncover the theoretical reasons that give rise to the possibility of a Pareto improving government intervention, I conduct a more systematic analysis of the inefficiencies in the model, namely incomplete markets and pecuniary externalities. I proceed in two steps: I first characterize and compute the complete market equilibrium of the model; and then turn to study the problem of a planner that internalizes how bankers' portfolio choices affect equilibrium asset prices and through prices the borrowing constraint. In particular, whenever the system reaches states in which the constraint is either binding or can be binding the following period, the planner's marginal cost of investment will be smaller/higher than the private marginal cost if financial conditions are expected to deteriorate/improve. The intuition is simple: when financial conditions deteriorate and are expected to worsen, the planner realizes that financial constraints will impede investment and prevents fire sale of assets in order to allow bankers to intermediate more in

[1] While, in fact, the central bank exploited its ability to issue risk free short term government debt to finance its programs, in the model bankers are assumed to be able to issue securities that are perfect substitutes for risk free government debt. Allowing the central authority to raise lump-sum taxes on households provides a simple way to model its superior ability to obtain funds from households and directly addressing the stimulatory/redistributive trade-off discussed above.

the subsequent period by increasing the supply of assets and hence the expected returns on bankers' investments. I then discuss implementation and compare the welfare gains associated with the second best with the ones achieved with the simple rule for credit market intervention and find that they are remarkably close.

This work contributes to a very rich body of literature on financial frictions in macroeconomic models. Traditional work in this area, building on the work of Bernanke and Gertler (1989) and Kiyotaki and Moore(1997), focused on the quantitative macroeconomic implications of the financial accelerator mechanism by studying how the presence of borrowing constraints affect the local response of the economy to different shocks around a deterministic steady state.

More recently, various contributions have stressed the importance of nonlinearities in the response of the economy to shocks that affect the balance sheet positions of borrowing constrained agents, both empirically and theoretically, e.g. He and Krishnamurthy (2014) (hereafter HK) Brunnermeier and Sannikov (2013) (hereafter BS) and Mendoza (2010). Here I follow BS and HK in considering financial constraints on banks while adopting the discrete time methodology of Mendoza (2010). Apart from minor differences, the main feature that distinguishes the analysis here from the one in BS is that I develop a quantitative model whereas their approach is purely qualitative. On the other hand HK develop a quantitative model, but they make ad-hoc assumptions about the preference specifications which make their approach unfit for policy analysis. Moreover, both of these papers assume an AK production technology while in my model employment plays an important role in generating deep and persistent recessions following a crisis.

Finally I contribute to the literature on pecuniary externalities in models with financial constraints. Papers in this literature, e.g. Lorenzoni (2008) and Bianchi (2011), showed how agents' failure to internalize the effect of their portfolio decisions on asset prices and through prices on financial constraints can result in overborrowing. This result constitutes an important theoretical argument in support of macroprudential regulation. Here, I exploit the same economic mechanism to derive the policy implications of pecuniary externalities for government intervention during a crisis.

2 The Model

The model is cast in discrete time. The economy is populated by four types of agents: depositors, bankers, final good producers and capital producers. The main departure from a standard real business cycle model lies in the way financial intermediation is modeled. In particular, bankers are the only agents that can invest in the productive sector and the intermediation of funds from households to firms is subject to an agency problem that, occasionally, limits the ability of bankers to raise deposits to their desired level. I will first describe the physical setup and the market structure and then turn to each agent's problem and describe the agency problem in detail.

2.1 Technology and Markets

There are two goods in the economy: a perishable consumption good and a durable capital good. There are a continuum of perfectly competitive firms each operating a constant returns to scale technology that employs capital and labor to produce consumption goods. Aggregate output Y_t as a function of aggregate capital K_t and aggregate labor L_t is given by:

$$Y_t = (\xi_t K_t)^{\alpha} L_t^{1-\alpha} \tag{1}$$

where ξ_t is a shock to the quality of capital which follows a Markov process. I will let ξ_t be the only exogenous source of economic fluctuations and describe its impact on the aggregate production possibility set in more detail below.

Capital good producers can linearly refurbish depreciated units of capital and units made unproductive by the capital quality shock at a unitary real cost, but they face convex costs in producing new units of capital or disposing of existing ones. The total amount of investment expenditure needed to produce K_{t+1} capital goods with $(1-\delta)\,\xi_t K_t$ units of undepreciated capital is given by:

$$\psi\left(K_{t+1}, K_t, \xi_t\right) = K_{t+1} - (1-\delta)\,\xi_t K_t + \frac{\kappa}{2}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2 K_t \tag{2}$$

Inspection of equations (1) and (2) clarifies the physical interpretation of the capital quality shock. When $\xi_t < 1$, (I will assume that $\xi_t \leq 1$), a proportion $1-\xi_t$ of the capital stock turns out to be unproductive at time t; these units of capital will need to be refurbished just as the depreciated units in order to be (potentially) productive at time $t+1$.

I make two critical assumptions about the market structure which I discuss in turn: limited market participation and incomplete financial markets.

In the model, depositors can't directly hold securities issued by firms so that their only source of financial wealth consists of securities issued by bankers. While stark, this assumption is meant to capture the idea that bankers have superior skills in lending to the productive sector and specialize in intermediating assets that cannot be absorbed by other agents. As a case in point, in the recent crisis banks were highly exposed to mortgage backed securities, and while losses on their positions triggered sales of these assets among different segments of the financial sector, households did not directly step in this market to purchase MBS.

I assume that agents can only trade in a market for short term risk free assets. In equilibrium, banks will issue risk free assets to finance capital asset's purchases so that banks' financing relies on inside equity and risk free debt which I will refer to as deposits in what follows. In section 4.1 I describe the implications of completing asset markets.

2.2 Firms

2.2.1 Consumption Good Producers

As explained above, consumption good producers can issue perfectly state contingent securities to capital owners, i.e. bankers. At each time t their problem is static: they choose labor to maximize the profits that they return to bankers. Given the constant returns to scale technology I can focus on the problem of the representative firm:

$$\max_{L_t} (\xi_t K_t)^\alpha L_t^{1-\alpha} - W_t L_t + \bar{Q}_t K_t$$

where $\bar{Q}_t$ is the price at which firms sell capital to capital goods producers. The first order condition for this representative firm yields the labor demand schedule

$$(1-\alpha)\frac{Y_t}{L_t} = W_t \tag{3}$$

The time t return to capital are therefore given by

$$R_t^k = \frac{\alpha Y_t + \bar{Q}_t K_t}{Q_{t-1}} \tag{4}$$

where Q_{t-1} is the price of capital at $t-1$.

2.2.2 Capital Good Producers

After production has taken place, capital goods producers buy capital from firms at price $\bar{Q}_t$ and combine it with final goods to produce new capital available for production in the subsequent period; they then sell this capital back to firms at price Q_t. The problem of a representative capital good producer is

$$\max_{K_{t+1},K_t} Q_t K_{t+1} - \psi\left(K_{t+1}, K_t, \xi_t\right) - \bar{Q}_t K_t$$

Under the homogeneity assumption on their technology, capital goods' producers make zero profit in equilibrium. Optimality conditions are given by:

$$Q_t = 1 + \alpha\left(\frac{K_{t+1} - K_t}{K_t}\right) \tag{5}$$

$$\bar{Q}_t = Q_t - \left(1 - (1-\delta)\,\xi_t\right) + \frac{\left(Q_t - 1\right)^2}{2\alpha}$$

2.3 Depositors

There is a continuum of depositors that consume, supply labor and hold banks' liabilities. Let $V_t^h\left(d_{t-1}\right)$ be the optimal value to a depositor of entering time t with an amount of deposits d_{t-1}. $V_t^h\left(d_{t-1}\right)$ solves the following functional equation

$$V_t^h\left(d_{t-1}\right) = \max_{c_t^h, d_t, l_t} \log\left(c_t^h - \chi\frac{l_t^{1+\varphi}}{1+\varphi}\right) + \beta E_t V_{t+1}^h\left(d_t\right)$$

$$s.t.$$

$$c_t^h + \frac{d_t}{R_t} \leq d_{t-1} + l_t W_t$$

where R_t is the interest paid on deposits. Depositors' preferences are time separable with flow utility of the GHH form. This preference specification allows aggregation and has the important property of eliminating the wealth effect on labor supply, thus helping both in matching the cyclical behavior of employment and in reducing computational complexity. The first order conditions for the representative depositor are

$$\beta E_t \Lambda_{t,t+1}^h R_t = 1 \tag{6}$$

$$\chi L_t^{\varphi} = W_t \tag{7}$$

where $\beta E_t \Lambda^h_{t,t+1}$ is the intertemporal marginal rate of substitution

$$\beta E_t \Lambda^h_{t,t+1} = \beta E_t \frac{\left(C^h_t - \chi \frac{L_t^{1+\varphi}}{1+\varphi}\right)}{\left(C^h_{t+1} - \chi \frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)}$$

2.4 Bankers

If firms were able to issue equity directly to households, the model would reduce to a standard business cycle model. As explained above, in order to introduce a meaningful role for financial intermediation, I assume that households are not capable of channelling funds directly to the productive sector, but rather, their savings need to be intermediated by bankers who can then lend to firms without any friction. There is however an agency problem between depositors and bankers that limits the ability of bankers of raising deposits from households. In particular, after paying out dividends and raising deposits, bankers can divert a proportion θ of the goods they have available to invest in firms. If they decide to divert, they are permanently excluded from financial markets and live in autarky thereafter. In autarky, bankers have only access to a storage technology that yields one unit of consumption at $t+1$ for each unit invested at time t.

There are a continuum of bankers in the economy each operating his own bank. Let $V^b_t(\bar{n}_t)$ denote the optimal value to a banker with a total amount of resources available $\bar{n}_t$ and $V^{aut}(\theta q_t k_{t+1})$ denote the value to a banker of going to autarky with an amount of resources $\theta q_t k_{t+1}$. $V^b_t(\bar{n}_t)$ solves:

$$V^b_t(\bar{n}_t) = \max_{c^b_t, n_t, d_t, k_{t+1}, \bar{n}_{t+1}} \log\left(c^b_t\right) + \sigma E_t V^b_{t+1}(\bar{n}_{t+1})$$

$$s.t.$$

$$\bar{n}_{t+1} \leq q_t k_{t+1} R^k_{t+1} - d_t \tag{8}$$

$$c^b_t + n_t \leq \bar{n}_t \tag{9}$$

$$q_t k_{t+1} \leq n_t + \frac{d_t}{R_t} \tag{10}$$

$$\sigma E_t V_{t+1}(\bar{n}_{t+1}) \geq \sigma V^{aut}(\theta q_t k_{t+1}) \tag{11}$$

Equation (8) is the evolution of bankers' wealth; it says that $\bar{n}_t$ is the cum dividend value of capital held by bankers at time t net of bankers' indebtedness. The banker decides how much to consume, c^b_t, out of this wealth and how much to save, n_t ; he can then use his savings plus the amount he raises by issuing deposits, $\frac{d_t}{R_t}$, to invest in capital. Equations (9) and (10)represent the budget constraints associated with the consumption saving decision and the portfolio choice. Finally, equation (11), is the incentive constraint: it says that after consuming and raising deposits the banker must have the incentive to continue operating the bank rather than diverting the funds and going to autarky.

Banker's value from diverting funds and going to autarky is:

$$V^{aut}\left(\theta q_t k_{t+1}\right) = \max_{c,s} \log\left(c\right) + \sigma V^{aut}\left(s\right)$$

$$s.t.$$

$$c + s \leq \theta q_t k_{t+1}$$

It is straightforward to show that the above value function takes the form:

$$V^{aut}\left(\theta q_t k_{t+1}\right) = \frac{\log\left(\left(1-\sigma\right)\theta q_t k_{t+1}\right)}{1-\sigma} + \frac{\sigma \log\left(\sigma\right)}{\left(1-\sigma\right)^2}$$

In the Appendix I show that the value function exists and is unique. Moreover, letting $\Lambda^b_{t,t+1}$ denote the intertemporal marginal rate of substitution for bankers and ϕ_t the leverage ratio, i.e.

$$\Lambda^b_{t,t+1} = \sigma \frac{c^b_t}{c^b_{t+1}}$$

$$\phi_t = \frac{q_t k_{t+1}}{n_t}$$

the first order conditions for the banker's problem are given by the budget constraints at equality, an Euler equation for consumption

$$E_t \left\{ \Lambda^b_{t,t+1} \frac{\bar{n}_{t+1}}{n_t} \right\} = 1 \tag{12}$$

and then, if the incentive constraint is not binding

$$E_t \left\{ \Lambda^b_{t,t+1} \left(R^k_{t+1} - R_t \right) \right\} = 0 \tag{13}$$

otherwise

$$E_t V_{t+1}\left(\bar{n}_{t+1}\right) = V^{aut}\left(\theta q_t k_{t+1}\right) \tag{14}$$

$$\phi_t E_t \left\{ \Lambda^b_{t,t+1} \left(R^k_{t+1} - R_t \right) \right\} = \frac{\rho_t}{\left(1+\rho_t\right)} > 0 \tag{15}$$

where ρ_t is the Lagrange multiplier attached to (11).

Equations (12) - (15) describe the implications of the agency problem on bankers' policies. In particular, given the logarithmic specification for preferences, the incentive problem does not *directly* affect their optimal consumption saving decision as shown by equation (12) and so it is standard to show that the policy function for consumption is given by

$$c_t^b = (1-\sigma)\,\bar{n}_t$$

On the other hand, the agency problem imposes a direct limit on bankers' leverage ratio. If the incentive constraint is not binding bankers must be indifferent between investing in capital and holding deposits in another bank. In what follows I will refer to the price of capital implied by equation (13) as the unconstrained price.[2] Whenever, at the unconstrained prices, the banker would have an incentive to divert funds, households limit the amount of deposits they are willing to hold in a bank until bankers' incentive constraint is satisfied at equality (see section 3.1).

As I show in the Appendix equation (11) can be written as:

$$E_t \log\left(\frac{\left[\phi_t\left(R_{t+1}^k - R_t\right) + R_t\right]}{\phi_t}\right) \geq a - (1-\sigma)\,E_t \sum_{i\geq 0} \sigma^i \log\left(\sigma^i R_{t+1,t+1+i}^n\right) \tag{16}$$

where a is a constant depending on parameters only, and $R_{t+1,t+1+i}^n$ is the compounded return on net worth from time $t+1$ to $t+1+i$, i.e.

$$R_{t+1,t+1+i}^n = \prod_{j=1}^{i} \left\{\left[\phi_{t+j}\left(R_{t+1+j}^k - R_{t+j}\right) + R_{t+j}\right]\right\}$$

Equation (16), together with equation (13), make clear that individual banker's leverage only depends on aggregate variables so that leverage is constant across bankers and I can write aggregate demand for capital as

$$Q_t K_{t+1} = \phi_t \sigma \bar{N}_t \tag{17}$$

where the leverage ratio is determined either by (13) if the constraint is not binding and by (16) otherwise.

2.5 Equilibrium

I am now ready to define a recursive competitive equilibrium for the economy.

[2] See the Appendix for a precise definition of the unconstrained price level.

Definition 1 *Let* $s_t = \left(K_t, \bar{N}_t, \xi_t\right)$ *denote the state of the economy at time* t*. A recursive competitive equilibrium for a given initial state* $s_0 = \left(K_0, \bar{N}_0, \xi_0\right)$ *is given by pricing functions* $\left\{Q(s_t), R(s_t), W(s_t), R^k(s_t, s_{t+1})\right\}$ *a conjectured evolution for the state* $\Gamma(s_t)$ *, value functions* $\left\{V^h(s_t), V^b(s_t)\right\}$ *and policy functions* $\left\{C^h(s_t), C^b(s_t), L(s_t), Y(s_t), K_{t+1}(s_t), \phi(s_t), \bar{N}_{t+1}(s_t)\right\}$ *such that*

1. *Given* $\{q(s_t), R(s_t), \Gamma(s_t)\}$ *the policy functions solve the representative depositor's banker's firm's and capital good's producer's problem and* $\left\{V^h(s_t), V^b(s_t)\right\}$ *are the associated value functions*

2. *Markets clear (notice that market clearing conditions for labor and investment are implicitly imposed by denoting demand and supply by the same variable and market for deposits clears by Walras law)*

$$Y(s_t) = C^h(s_t) + C^b(s_t) + \psi\left(K_{t+1}(s_t), K_t, \xi_t\right)$$

3. *The conjectured evolution and the actual evolution coincide*

3 Financial Crises and Policy

3.1 Financial Crises

There are two related reasons why bankers' wealth affects the dynamics of aggregate variables in the economy: the market structure and the agency problem.

The combination of limited market participation and market incompleteness imply that in equilibrium bankers will bear all the risk associated with investing in the capital stock. When bankers' suffer losses on their portfolio, in order to undertake a given level of investment they will have to increase the amount they borrow from households. The increase in leverage magnifies the riskiness of bankers' portfolios and results in lower asset prices and investment levels. This risk channel is active both in the constrained region and in the unconstrained region. Rewriting equation (13) as

$$E_t\left\{\frac{R^k_{t+1} - R_t}{\phi_t\left(R^k_{t+1} - R_t\right) + R_t}\right\} = 0$$

makes clear that as leverage increases, the covariance between excess returns and total returns on equity increases, resulting in higher risk premia and lower investment. As explained below, this effect is further amplified when incentive constraints are binding.

The second channel through which bankers' wealth affects investment operates through the incentive constraint. As bankers' balance sheet position deteriorates, their continuation values from operating the bank decrease. When bankers' level of indebtedness is high enough, the economy enters the constrained region in which bankers' leverage is pinned down by the incentive constraint rather than by the indifference condition above.

To describe the behavior of the economy in regions where the constraint is binding it is useful to inspect the incentive constraint as expressed in Equation (16), which I report below for convenience

$$E_t \log \left(\frac{\left[\phi_t \left(R_{t+1}^k - R_t\right) + R_t\right]}{\phi_t} \right) \geq a - (1-\sigma)\, E_t \sum_{i \geq 0} \sigma^i \log \left(\sigma^i R_{t+1,t+1+i}^n\right) \tag{18}$$

Notice that the LHS of this inequality is a decreasing function of leverage so that, from an individual banker's perspective, this constraint imposes a limit on the amount of leverage that he will be allowed to take on. However, this limit on leverage endogenously depends on excess returns on assets which, in equilibrium, depend on aggregate investment. In particular, lower levels of investment imply a lower price of capital and lower interest rates, thus relaxing the incentive constraint by increasing excess returns and making the continuation value from operating the bank higher. Hence, for given future random paths for prices and interest rates, $\left\{R_{t+1+i}^n\right\}_{i\geq 1}$, the above inequality implicitly defines an upper bound for the leverage ratio that individual bankers will be allowed to take on, that is decreasing in the realized price of capital at time $t : \bar{\phi}_t \left(q; \left\{R_{t+1+i}^n\right\}_{i\geq 1}\right)$. On the other hand, for any given aggregate level of (high enough[3]) indebtedness and capital, a decrease in the price of capital feeds back into amplified losses in bankers' wealth through the familiar financial amplification mechanism, and hence higher aggregate leverage. Figure 1 shows how the two forces interact to determine equilibrium leverage when the economy is in the constrained region.

At the unconstrained level of prices,[4] the leverage implied by market clearing in the market for capital goods, $\phi\left(q^{uc}\right)$, is strictly above the maximum leverage consistent with the incentive constraint, $\bar{\phi}\left(q^{uc}\right)$. As reduced demand for investments reduces the market

[3] For the financial amplifiaction mechanism to be active debt needs to be high enough. In practice the threshold level of debt that makes aggregate leverage a decreasing function of prices is far below the infimum of the support of the equilibrium ergodic distribution of debt levels.

[4] As explained in the Appendix the unconstrained level of q is found by solving the system of equilibrium restrictions, neglecting the incentive constraint and keeping expections of future variables at their equilibrium values. Notice that such expectations would not be rational given this different price level.

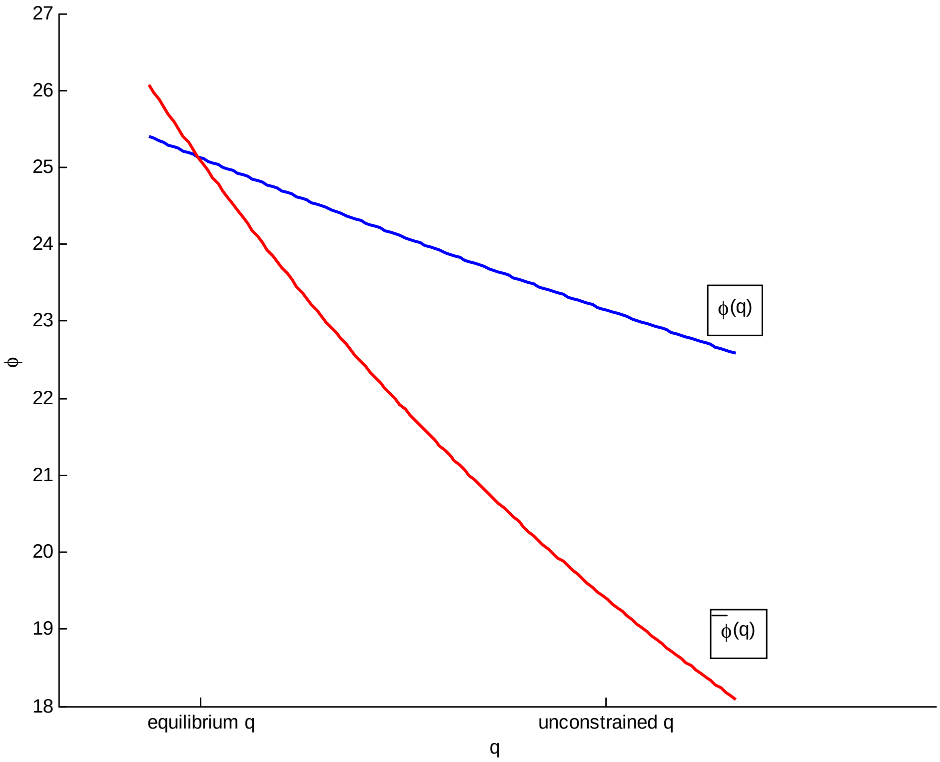

Figure 1: Effect of binding constraint on price of capital and leverage

clearing price of capital, excess returns increase and depositors allow a higher leverage. Equilibrium is reached at a point where the price is lower and both leverage and spreads are higher.

Although in equilibrium the adjustment is instantaneous, this process by which individual banks' creditors' attempts to enforce a lower leverage ratio result in a drop in asset prices and an increase in leverage caused by magnified losses on net worth, is reminiscent of the "margin spirals" experienced during the recent financial crisis as described by, e.g., Brunnermeier (2009).

As will be shown below, given the higher degree of impatience of bankers, crises will be recurrent in the model. On the other hand, while in the non-stochastic steady state the constraint must bind, the risk channel discussed above implies that crises will happen only rarely in the full equilibrium dynamics of the model.

3.2 Calibration

There are 9 parameters in the model. Five of them are calibrated externally $\beta, \kappa, \delta, \varphi, \chi$. The parameters that are specific to the model are σ, that measures the degree of impatience of bankers' and θ the seizure rate. I calibrate them to match an average leverage ratio of 10 and a probability of a financial crisis of 1%. Finally, the standard deviation and serial correlation of the shocks are set to generate a variance of annual consumption of 2.5%.

β	depositor's discount factor	.99
σ	banker's discount factor	.98
θ	seizure rate	.23
κ	inverse elasticity of investment	1
χ	labor weight in GHH	.18
δ	depreciation	.025
φ	inverse Frisch elasticity	1
ρ_ξ	serial correlation of capital quality shock	.3
σ_ξ	std of capital quality shock	.01

3.3 Nonlinearities and crisis experiment

I now turn to illustrating some properties of the behavior of the economy. Figure 2 demonstrates how financial frictions induce significant nonlinearities in the response of the economy to a shock to the quality of capital. The figure shows the recovery of some key real and financial variables after a 1% shock to the quality of capital followed by a sequence of realization at the mean. The two lines show the different behavior of the economy depending on the initial financial position of bankers: the initial state for the solid blue line is $s_0 = (E(K_t), E(D_t), .986)$ while for the crossed red line it is $s_0^t = \left(E(K_t), \bar{D}(E(K_t), .986), .986\right)$, where $\bar{D}(E(K_t), .986)$ is the threshold level for deposit above which bankers' incentive constraints are binding.

When financial constraints are not binding the economy recovers quickly as bankers are able to finance the increased investment demand of firms. Financial variables such as leverage and spreads are hardly affected, investments drop only very slightly on impact but recover in the very next quarter. On the other hand, if the same shock hits the economy when bankers' balance sheet position is weak, their ability to intermediate

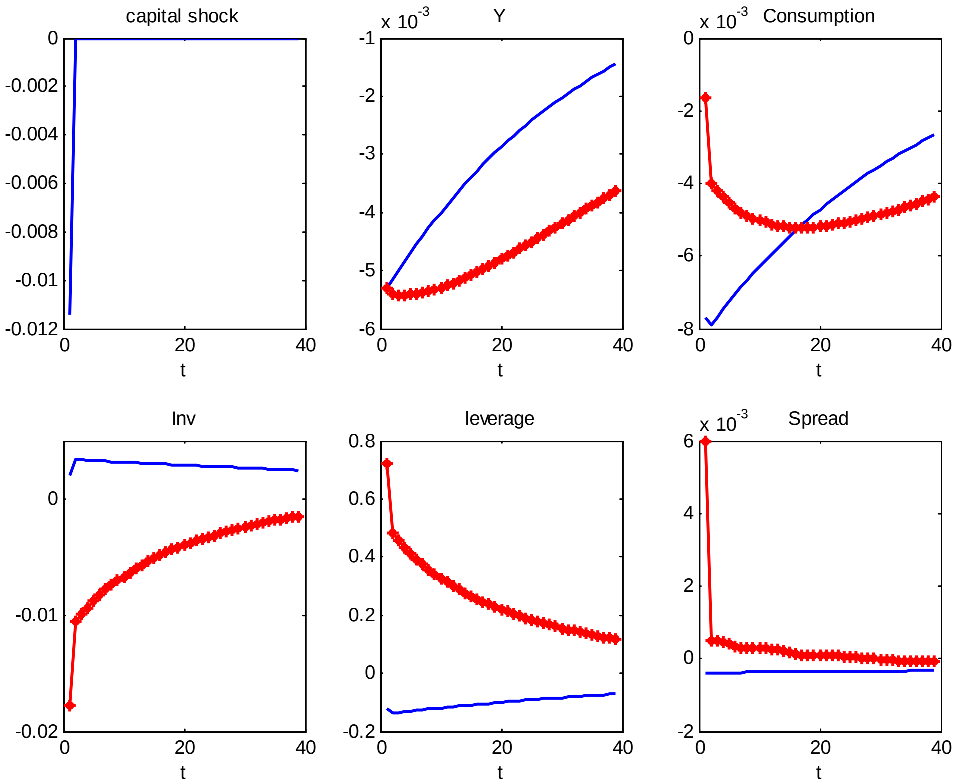

Figure 2: Recovery from $(E(K), E(D), .986)$, (blue solid line); $\left(E(K), \bar{D}(E(K), .986), .986\right)$, (red crossed line).

assets is impaired and the initial drop in output is followed by a much more persistent recession as investments remain depressed for a prolonged period of time. Spreads and leverage spike and the bigger output losses eventually lead to lower consumption levels.

While the above experiment clearly illustrates how the financial amplification mechanism interacts with the real economy to generate slower economic recoveries, it fails to capture the actual behavior of typical crises in the model. This is because the level of banks' deposits is chosen endogenously in equilibrium and bankers will adjust their borrowing so that financial crises realize only after a big enough deterioration in the quality of their assets. To address this point I simulate the economy for 50000 periods and use these simulated paths to study how typical crises materialize in the model and how the economy recovers from such episodes.

Figure 3 is a scatterplot of the endogenous states that are visited in the simulation.

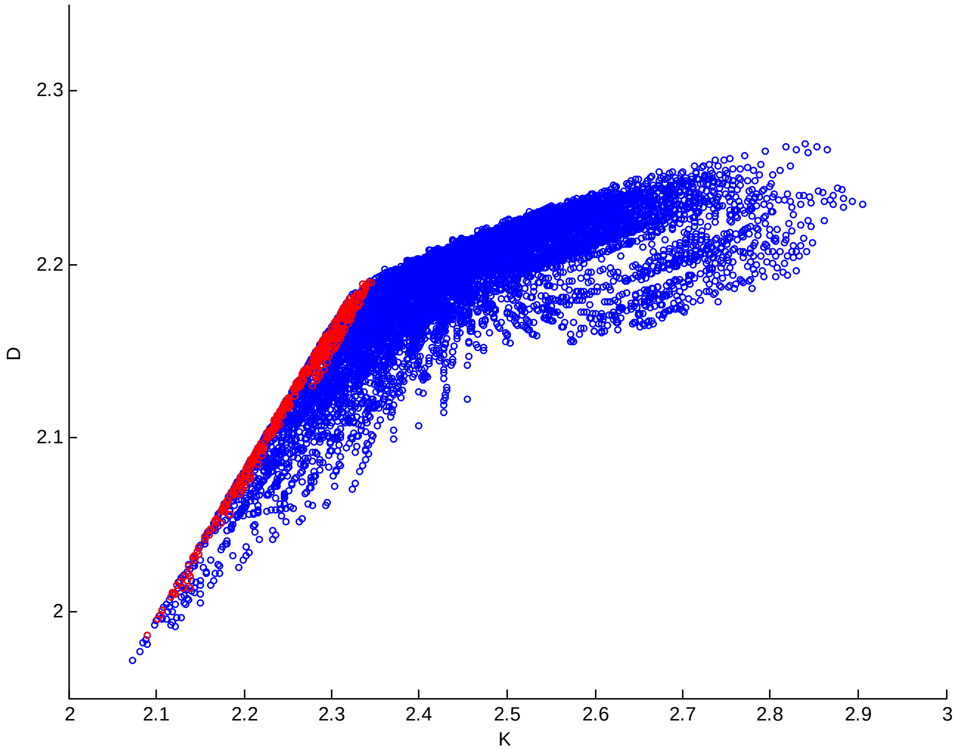

Figure 3: Scatterplot of endogenous state variables

On the $x-$axis I have the level of capital and on the $y-$axis the amount of deposits. The red points are points where the financial constraint is binding. From this figure it is apparent that when financial constraints become binding the maximum amount of deposits that banks will be able to issue decreases at a much faster rate as households limit the amount of deposits they are willing to hold in order to ensure that the incentive not to divert is satisfied.

Figure 4 depicts the typical path of the economy into a crisis and the subsequent recovery. The paths depicted are the result of the following experiment: I pick all periods in the simulation in which the constraint is binding (i.e. the red points in Figure 2) and for each ten year period I isolate only one of these periods; I then plot the median across all paths from 20 quarters before each of these periods to 20 quarters afterwards.

Around 10 quarters before the peak of the crisis, banks start suffering progressively bigger losses on their investments as the quality of capital deteriorates. This results

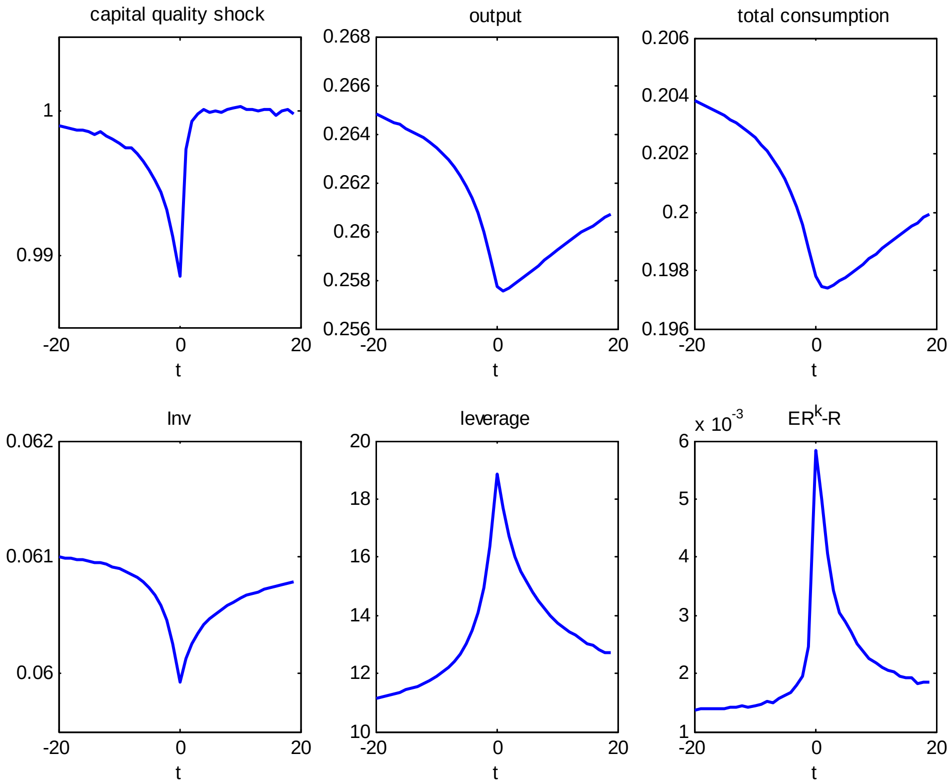

Figure 4: Typical crisis in simulation

in a steep increase in leverage, which almost doubles, making banks very exposed to variations in the returns from capital. Once the system reaches a state in which banks' balance sheets are fragile enough, a negative realization of the capital quality shock pushes the system into a crisis. The recovery from the crisis is very slow: the very high level of leverage translates into high risk-premia, as explained above, low asset prices and depressed investments. Five years after the peak of the crisis output is still 1.5% below average.

3.4 Government Intervention

During the recent financial crisis the Fed has stepped in with several unconventional measures. Such measures have entailed various forms of intervention whereby the Fed exploited its ability to borrow from households by issuing government debt and used these resources to ease credit conditions faced by banks and other borrowers, and to bailout some financial institutions. Here I will focus on all those programs that the Fed put in place before starting to purchase MBS and directly bearing private sector risk. I will distinguish between two broad types of intervention: bailouts and credit market interventions.

In particular, as discussed in the introduction, three facilities were used in order to ease bank funding conditions: discount window lending; the term auction facility (TAF); and the commercial paper funding facility (CPFF). During the same period the central bank issued emergency loans to bailout some financial institutions deemed to be "too interconnected to fail" (Bear Sterns, AIG). Such programs accounted for roughly half of the overall doubling of the federal reserve's balance sheet during 2008.

I model bailouts simply as lump sum transfers from households to bankers. To model credit market interventions I assume that the central bank borrows from households at a market determined rate and lends funds to banks at a lower rate, financing the cost of the intervention by raising lump-sum taxes on households. The budget constraint for the central bank in the case of a credit market intervention is given by[5]

$$\frac{D_t}{R_t} + T_t^h = \frac{D_t}{R_t}\left(1 + \tau_t^b\right) \tag{19}$$

where T_t^h are lump-sum taxes and τ_t^b measures the reduction of borrowing costs associated with the intervention.

I assume that the central bank adjusts the size of its intervention according to the degree of deterioration in financial conditions, using as a gauge the level of leverage of banks. In particular, for both types of intervention I assume that the total amount of lump-sum taxes raised on households is determined by the following simple rule

$$T_t^h = \max\left\{\tau\left(\phi_t - \phi^*\right), 0\right\}$$

where ϕ^* is a target level of leverage and $\tau > 0$. Bailouts simply transfer this amount as a lump-sum to bankers, while with a credit market intervention the same amount of resources is used to offer lower borrowing rates to bankers as implied by equation (19).[6]

[5] With bailouts the budget constraint simply equates lump-sum taxes and tansfers. Throughout I am assuming that the government balances its budget at each period, an interesting extension that I am not pursuing here would be to consider the interaction of private and public debt in this economy.

[6] As bailouts are implemented through lump-sum transfers, the agents' optimality conditions are unaffected under this type of policy and the only equilibrium object that is modified is the one determining the evolution of bankers' wealth. On the other hand, by affecting bankers' borrowing costs, credit market interventions influence the optimality conditions characterizing bankers' policies, so that in this case,

A credit market intervention implies that as bankers' balance sheet position deteriorates, the central bank intervenes to intermediate funds from households to bankers allowing them to obtain credit at more favorable conditions than the ones they face in the market. Notice that, since bankers are able to issue risk free debt, the model does not capture the real world advantage that the central bank has in raising funds at a cheaper rate by issuing risk-free short term debt. Accordingly, in the model, in order to offer advantageous lending conditions to bankers, the central banker needs to raise lump-sum taxes on households.

There are two tradeoffs associated with both types of interventions. First, since these policies are redistributive in nature, their beneficial impact on the investment friction needs to be weighed against their direct effect on households' wealth. The second tradeoff arises from the anticipation effects that these policies induce on bankers' decisions. On the one hand, anticipating central bank's intervention, bankers' need to self insure against losses on their portfolios is lower so that crises become more frequent. This is the moral hazard effect that has received considerable attention throughout the recent financial crisis. On the other hand, the insurance scheme implemented by these policies allows bankers to intermediate a larger amount of capital during tranquil periods, hence increasing average economic activity.

Figure 5 shows the result of the same simulation experiment conducted for the baseline model under a credit market intervention. I fix the target level of leverage to be equal to the average level in the economy without intervention and pick the level of τ that maximizes households' expected welfare given this level of ϕ^*. The table below reports some summary statistics that help compare the effect of the two policies.

		Baseline	CMI	Bailout
$E(Y)$	mean output	.267	.276 (+3.37%)	.27 (+1.12%)
$E(C)$	mean consumption	.201	.205 (+2%)	.202 (+.5%)
$E(\phi)$	median leverage	9.6	10.75	9.1
$E(\phi \mid c)$	median leverage in crisis	20.3	15.3	-
	Households' gain	-	1.56%	.1%
	Bankers' gain	-	.3%	5.8%

Comparing Figure 5 and Figure 4, the effect of government intervention is apparent. As bank leverage rises, bankers can borrow at more favorable terms and hence they are able to keep capital losses from inducing too severe losses in output. The total drop is

both bankers' optimality conditions and their wealth evolution need to be modified to incorporate the lower cost of credit $\frac{R_t}{\left(1+\tau_t^b\right)}$.

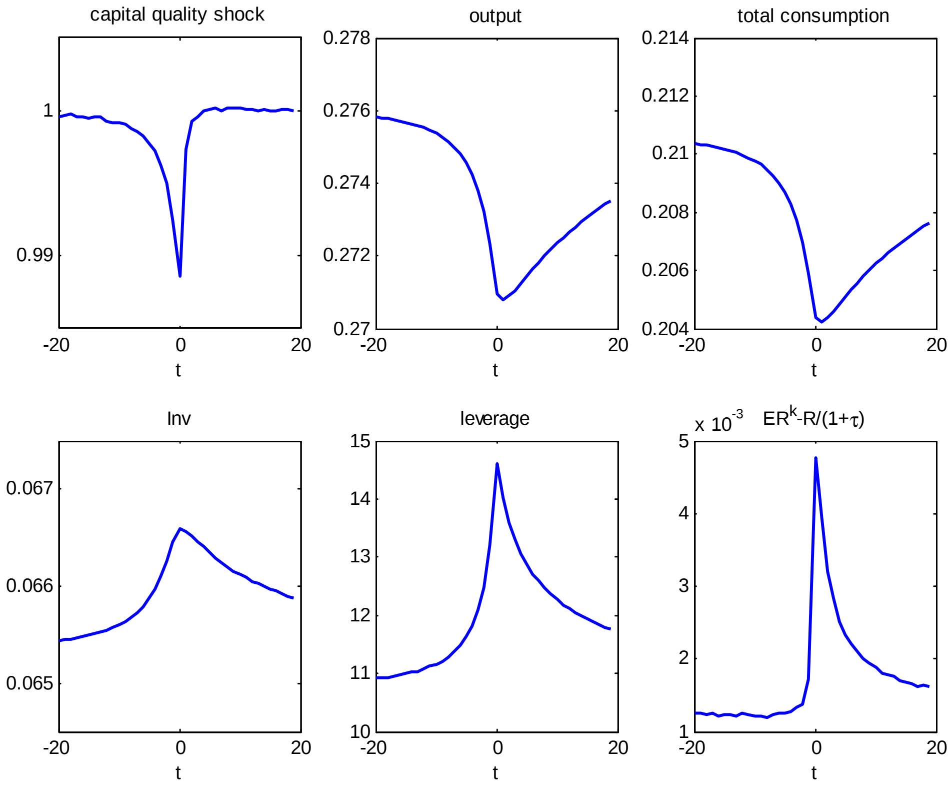

Figure 5: Crises with government intervention

1.7% against 2.6% in the case without government intervention. The table further helps clarify the effect of credit market interventions and compare the two policies. There are two important points to note from inspection of the values reported in the table: first, credit market interventions have a much stronger effect on output and depositors' welfare then bailout policies; second, both types of interventions can improve on the decentralized equilibrium allocation in a Pareto sense.

The agency problem and market incompleteness combine to reduce the level of intermediation that bankers undertake in the decentralized equilibrium below the optimal level. Credit market interventions directly address the need to facilitate financial intermediation, as they increase both bankers' willingness to undertake investment, by affecting the costs of borrowing that they face, and their ability to raise funds, by relaxing their incentive constraints. This implies that with a credit market intervention, enhanced incentives will allow bankers to increase investment and economic activity even without an average increase in their wealth, which allows households to reap most of the benefits associated with the intervention. Bailouts, on the other hand, do not directly affect bankers' incentives to invest, so that their effect on economic activity only depends on bankers' increased wealth, which explains why bankers will be the main beneficiary of this type of intervention.

Notice that under credit market interventions, anticipation effects induce moral hazard as bankers' leverage increases by about 10%. On the other hand, the reduced severity of crises has beneficial effects both directly and indirectly by allowing bankers to increase average levels of intermediation and hence average output. The net beneficial effects on household welfare are reported in the last line. They amount to a 1.56% increase in period consumption net of work effort.

4 Constrained Efficient Allocation

While in the previous section I have demonstrated the beneficial effects of central bank's credit market intervention using a simple exogenous policy rule, one might wonder whether a policymaker with access to the same sets of instruments could actually implement the second best. It turns out that using only lump sum taxes on households and debt subsidies this is not possible. However, if instead of using credit subsidies/taxes the planner can use a subsidy/tax on leverage, then the second best can be implemented. In this section I formalize this argument. In the economy there are two sources of inefficiency: market incompleteness and pecuniary externalities. As their net worth gets depleted the value to bankers of insurance against bad shocks realization increases. With complete markets households provide such insurance in return for higher returns in normal times. Even when markets are complete agents in the model fail to internalize the effect of their portfolio choices on asset prices and through asset prices on the incentive constraint.

I proceed in two steps to disentangle the effect of each source of inefficiency by first characterizing and computing the complete market equilibrium and then turning to constrained efficiency. I then show that a government with access to the same fiscal instruments as in the previous section can implement the constrained efficient allocation. The simple rule for government intervention, however, achieves welfare gains which are close to those associated with the constrained efficient allocation.

4.1 Complete Markets

In the complete market economy, at each time t agents can trade a complete set of Arrow-securities paying one unit of the consumption good contingent on the realization of the exogenous shock ξ_{t+1} at time $t+1$.Let $d_t\left(\xi_{t+1}\right)$ denote the amount of the $\xi_{t+1}-$ Arrow-security purchased by an agent at time t and $m_t\left(\xi_{t+1}\right)$ its price. The budget constraints of households, the portfolio constraints of bankers and their wealth evolution become respectively

$$c_t^h + \sum d_t\left(\xi_{t+1}\right) m_t\left(\xi_{t+1}\right) \leq d_{t-1}\left(\xi_t\right) + W_t l_t$$

$$q_t k_{t+1} \leq n_t - \sum d_t\left(\xi_{t+1}\right) m_t\left(\xi_{t+1}\right)$$

$$\bar{n}_{t+1} = \left(n_t - \sum_{\xi_{+1}} d_t\left(\xi_{t+1}\right) m_t\left(\xi_{t+1}\right)\right) R_{t+1}^k + d_t\left(\xi_{t+1}\right)$$

The representative household's Euler equation (6) is now replaced by the system (the superscript co denote variables in the complete market allocation)

$$\Pr\left(\xi_{t+1}\left|\xi_t\right.\right)\beta\frac{U_c^{h,co}\left(\xi_{t+1}\right)}{U_c^{h,co}(t)} = m\left(\xi_{t+1}\right) \quad \forall\xi_{t+1}\in\Theta \tag{20}$$

which gives the familiar expression for the vector of state prices $m_t\left(\xi_{t+1}\right)$.

Using this to substitute $m_t\left(\xi_{t+1}\right)$ in the optimality conditions for the representative banker's portfolio choices of Arrow-securities yields

$$\frac{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}\right)} = \frac{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}'\right)}{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}'\right)} \quad \forall\xi_{t+1},\xi_{t+1}'\in\Theta \tag{21}$$

$$\begin{array}{ll} V_t^{b,co}\left(\bar{N}_t^{co}\right) = V^{aut}\left(\theta Q_t^{co} K_{t+1}^{co}\right) & \frac{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}\right)} > 1 \\ \frac{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}\right)} = 1 & V_t^b\left(\bar{N}_t^{co}\right) \geq V^{aut}\left(\theta Q_t^{co} K_{t+1}^{co}\right) \end{array} \tag{22}$$

Equations (21) state that by trading in a complete set of Arrow securities agents equalize the ratio of their stochastic discount factors across realizations of the capital quality shock in the next period, thus exhausting all mutually beneficial trades across possible state realizations at time $t+1$.

On the other hand, equations (22) show that, when the constraint is binding, bankers' intertemporal marginal rate of substitution is strictly smaller than households', so they would find it strictly profitable to transfer wealth from time $t+1$ to time t at a rate that would keep households indifferent. The incentive problem, however, limits their ability to engage in such profitable trades and, just as in the incomplete market model, puts a constraint on the amount of funds they can obtain from households.

It is straightforward to show that the incentive constraint must bind with positive probability in the ergodic distribution. This is because bankers' higher degree of impatience imply that they would eventually disappear from the first best economy as can be seen by rewriting equation (22) when the constraint is not binding as[7]

$$\frac{C_{t+1}^{b,co}}{\left(C_{t+1}^{h,co} - \chi \frac{L_{t+1}^{co1+\varphi}}{1+\varphi}\right)} = \frac{\sigma}{\beta} \frac{C_t^{b,co}}{\left(C_t^{h,co} - \chi \frac{L_t^{co1+\varphi}}{1+\varphi}\right)}$$

and noticing that the denominator is bounded above.

Finally, equation (12) is now

$$\phi_t^{co} \left(\sigma E_t \Lambda_{t,t+1}^{b,co} R_{t+1}^{k,co} - \frac{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}\right)} \right) + \frac{\Lambda_{t,t+1}^{b,co}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{h,co}\left(\xi_{t+1}\right)} = 1 \tag{23}$$

Rewriting equation (12) as

$$\sigma E_t \Lambda_{t,t+1}^{b} \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] = 1$$

shows that the corresponding condition in the complete market economy is obtained by replacing the ratio of expected stochastic discount factors with the time t determined ratio of stochastic discount factors. Notice also that the indifference condition (13) that determines leverage in the unconstrained region of the incomplete market economy is already embedded in equation (23).

Replacing equations (6),(12)-(15) in the system of equilibrium restrictions with incomplete markets with (20)-(23), yields the system of equilibrium restrictions with complete markets.

4.2 Constrained Efficiency

As explained above, in the complete market economy the incentive constraint will continue to affect the equilibrium allocation. Here I study the problem of a planner that internalizes how bankers' portfolio decisions affect the equilibrium price of capital and hence bankers' incentive to divert funds. In particular, when choosing the aggregate

[7] See the Appendix for a formal proof.

level of capital for the subsequent period, K_{t+1}, the planner internalizes the effect of this choice on Q_t through the first order condition for capital good producers (5). Letting $Q^*(K_{t+1}, K_t)$ denote the equilibrium pricing function and V_t the promised expected utility to the representative banker the planner's problem is:

$$W^{pl}(V_t, K_{t,}\xi_t) = \max_{C_t^h, L_t, C_t^b, K_{t+1}, V(\xi_{t+1})} \log\left(C_t^h - \chi\frac{L_t^{1+\varphi}}{1+\varphi}\right) + \beta E_t W\left(V_{t+1}, K_{t+1,}\xi_{t+1}\right)$$

$$s.t.$$

$$C_t^h + C_t^b + \psi(K_{t+1}, K_t) \leq K_t^{\alpha} L_t^{1-\alpha} \tag{24}$$

$$\log\left(C_t^b\right) + \sigma E_t V_{t+1} \geq V_t \tag{25}$$

$$E_t V_{t+1} \geq V^{aut}\left(\theta Q^*(K_{t+1}, K_t) K_{t+1}\right) \tag{26}$$

where (24)-(26) are, respectively, the resource constraint, the promise keeping constraint and the incentive constraint.

As shown in the Appendix, the first order conditions for this problem are identical to the equilibrium restrictions in the complete market equilibrium apart from equation (23) that is replaced by the following Euler equation for capital accumulation:

$$\frac{\sigma C_t^{b,pl}}{(1-\sigma) Q_t^{pl} K_{t+1}^{pl}}\left(\sigma E_t \Lambda_{t,t+1}^{b,pl} R_{t+1}^{k,pl} - \frac{\Lambda_{t,t+1}^{b,pl}(\xi_{t+1})}{\Lambda_{t,t+1}^{h,pl}(\xi_{t+1})}\right) + \frac{\Lambda_{t,t+1}^{b,pl}(\xi_{t+1})}{\Lambda_{t,t+1}^{h,pl}(\xi_{t+1})} = 1 - E_t \Upsilon\left(\rho_t, \rho_{t+1}\right) \tag{27}$$

where the superscript *pl* is used to denote a variable in the planner's allocation and ρ_t is the Lagrange multiplier on 26.:

$$\rho_t = \frac{C_t^{b,pl}}{\left(C_t^{h,pl} - \chi\frac{L_t^{pl1+\varphi}}{1+\varphi}\right)}\left(\frac{\Lambda_{t,t+1}^{h,pl}}{\Lambda_{t,t+1}^{b,pl}} - 1\right)$$

Notice that in the complete market equilibrium

$$\phi_t^{co} = \frac{\sigma C_t^{b,co}}{(1-\sigma) Q_t^{co} K_{t+1}^{co}}$$

therefore, comparing equations (27) and (23), the only difference between the system determining the complete market equilibrium allocation and the planner's optimal allocation is the presence of the extra term $E_t \Upsilon\left(\rho_t, \rho_{t+1}\right)$ in the Euler equation for capital accumulation. The expression for this extra term is given by:

$$E_t \Upsilon\left(\rho_t, \rho_{t+1}\right) = \left[\sigma E_t \left\{ \left(\frac{\Lambda^{h,pl}_{t+1,t+2}}{\Lambda^{b,pl}_{t+1,t+2}} - 1 \right) \eta^{q_{t+1}}_{k_{t+1}} \right\} - \left(1 - \frac{\Lambda^{b,pl}_{t,t+1}}{\Lambda^{h,pl}_{t,t+1}} \right) \eta^{q_t}_{k_{t+1}} \right] \tag{28}$$

where $\eta^{q_{t+i}}_{k_{t+j}}$ is the elasticity of the price of capital at time $t+i$ with respect to K_{t+j}.

Equations (27) and (28) describe how the presence of pecuniary externalities introduces a wedge between the privately optimal choices of investment and the socially optimal one. In particular the planner's marginal cost of investment, the RHS of equation (27), will internalize both the simultaneous and the expected effect of investment on prices and through prices on the incentive constraint. The planner internalizes that increasing investments at time t will relax time $t+1$ incentive constraints by expanding the supply of capital and reducing the incentives to divert at any level of investment at $t+1$, this is captured by the term $\left(\frac{\Lambda^{h,pl}_{t+1,t+2}}{\Lambda^{b,pl}_{t+1,t+2}} - 1 \right) \eta^{q_{t+1}}_{k_{t+1}}$. On the other hand, increasing investments at time t increases the contemporaneous incentives to divert. When the incentives constraint is binding at time t, this force counterbalances the previous one as captured by the second term $\left(1 - \frac{\Lambda^{b,pl}_{t,t+1}}{\Lambda^{h,pl}_{t,t+1}} \right) \eta^{q_t}_{k_{t+1}}$. The net effect of pecuniary externalities on the social marginal cost of investing depends on whether financial conditions are expected to get worse or better, as measured by the tightness of financial constraints: when financial conditions are expected to get worse the social marginal cost of investing is lower than the private one and vice versa.

4.3 Implementation

I now show that a central authority with access to lump-sum taxes/transfers on households and a distortionary tax/subsidy on banker's debt cannot implement the second best allocation while it can if it uses a tax/subsidy on investment rather than a on debt. Intuitively, in order to implement the second best, even when the incentive constraint is not binding the planner might want to change the incentives of bankers' to invest if the constraint is binding in the following period. Doing so by using a debt subsidy however necessarily introduces a wedge between the banker's stochastic discount factor and the household's, which can be avoided by subsidizing capital purchases since given limited market participation only the banker's stochastic discount factor is used to price capital.

Proposition 2 *A central authority with access to lump-sum taxes/transfers on households T^h_t and a tax/subsidy on banker's debt τ^b_t, cannot implement the second best allocation. If instead of using a credit market intervention the central authority directly* *subsidizes asset purchases the second best can be implemented with a subsidy/tax on bankers' investments*

$$\tau_t^k = \frac{1}{\phi_t^{pl}} \frac{E_t \Upsilon\left(\rho_t, \rho_{t+1}\right)}{\frac{\Lambda_{t,t+1}^b(\xi_{t+1})}{\Lambda_{t,t+1}^h(\xi_{t+1})}}$$

Proof. See Appendix. ■

The table below reports welfare gains associated with the second best allocation and compares it to the simple rule for policy intervention showing that the simple rule achieves welfare gains that are very close to those associated with the second best.

	simple rule	*Second Best*
consumption equivalent gain	1.57%	1.8%

5 Conclusions

One of the key challenges facing academics and policymakers in the aftermath of the recent financial crisis is the development of a coherent theory that can explain the events that led to the Great Recession and can be used to understand how policy should be shaped to avoid the recurrence or mitigate the severity of these episodes. In this work I make a step towards this end by developing a quantitative macroeconomic model that can generate rare episodes of disruption in financial intermediation that share many of the characteristics of financial crises. I use the model to address two main related concerns regarding the desirability of such massive interventions as the ones we observed during the recent crisis: their redistributive effects and the possibility that they generate moral hazard behavior. I find that a simple rule for credit market intervention has quantitatively significant beneficial effects both on economic activity and depositors' welfare. The anticipation of future bailouts does induce moral hazard on the side of bankers, increasing their leverage in tranquil periods and the probability of crises. On the other hand, it makes typical crises much less severe. I then conduct a more systematic study of the inefficiencies in the model, namely incomplete markets and pecuniary externalities, by solving for the constrained efficient allocation and find that the simple rule for credit market intervention achieves welfare gains that are close to those associated with the second best.

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6 Appendix

6.1 Banker's problem

Noticing that all budget constraints must be satisfied at equality at the optimum and introducing bannker's leverage

$$\phi_t = \frac{q_t k_{t+1}}{n_t}$$

I can rewrite the banker's problem as

$$V_t^b(\bar{n}_t) = \max_{c_t, n_t, \phi_t} \log(c_t) + \sigma E_t V_{t+1}^b \left(n_t \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] \right)$$

$$s.t.$$

$$c_t + n_t \leq \bar{n}_t$$

$$\sigma E_t V_{t+1}^b \left(n_t \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] \right) \geq V^{aut}(\theta n_t \phi_t) \tag{29}$$

Letting ρ_t be the Lagrange multiplier attached to (29), the first order conditions for this problem are given by :

$$c_t + n_t = \bar{n}_t$$

$$\sigma E_t \left\{ \frac{1}{c_{t+1}^b} \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] \right\} (1 + \rho_t) = \frac{1}{c_t^b} + \frac{\sigma}{1-\sigma} \rho_t \frac{1}{n_t} \tag{30}$$

$$\sigma E_t \frac{1}{c_{t+1}^b} \left(R_{t+1}^k - R_t \right) (1 + \rho_t) = \frac{\sigma}{1-\sigma} \rho_t \frac{1}{\phi_t n_t}$$

$$\left(\sigma E_t V_{t+1}^b \left(n_t \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] \right) - V^{aut}(\theta n_t \phi_t) \right) \rho_t = 0$$

$$\rho_t \geq 0$$

$$\left(\sigma E_t V_{t+1}^b \left(n_t \left[\phi_t \left(R_{t+1}^k - R_t \right) + R_t \right] \right) - V^{aut}(\theta n_t \phi_t) \right) \geq 0$$

I guess a policy function

$$c_t^b = (1-\sigma) \bar{n}_t$$

Equation 30 then becomes equation 12.

The value function takes the form

$$\begin{aligned} V_t(\bar{n}_t) &= E_t \sum_{i\geq 0} \log\left((1-\sigma)\,\bar{n}_t \sigma^i R^n_{t,t+i}\right) = \\ &= \frac{\log((1-\sigma)\,\bar{n}_t)}{1-\sigma} + E_t \sum_{i\geq 0} \sigma^i \log\left(\sigma^i R^n_{t,t+i}\right) \end{aligned}$$

the value from autarky is

$$V^{aut}(\theta q_t k_{t+1}) = \frac{\log((1-\sigma)\,\theta\phi_t\sigma\bar{n}_t)}{1-\sigma} + \frac{\sigma\log(\sigma)}{(1-\sigma)^2}$$

therefore

$$\begin{aligned} & E_t V_{t+1}(\bar{n}_{t+1}) - V^{aut}(\theta q_t k_{t+1}) \\ = \;& E_t \frac{\log\left((1-\sigma)\,\sigma\bar{n}_t R^n_{t+1}\right)}{1-\sigma} + E_t \sum_{i\geq 0} \sigma^i \log\left(\sigma^i R^n_{t+1,t+1+i}\right) - \frac{\log(\theta\phi_t\sigma\bar{n}_t)}{1-\sigma} - \frac{\sigma\log(\sigma)}{(1-\sigma)^2} \\ = \;& E_t \frac{\log\left((1-\sigma)\,\frac{R^n_{t+1}}{\phi_t}\right)}{1-\sigma} + E_t \sum_{i\geq 0} \sigma^i \log\left(\sigma^i R^n_{t+1,t+1+i}\right) - \frac{\log(\theta)}{1-\sigma} - \frac{\sigma\log(\sigma)}{(1-\sigma)^2} \\ = \;& \frac{1}{1-\sigma}\left(E_t \log\left(\frac{R^n_{t+1}}{\phi_t}\right) + (1-\sigma)\,E_t \sum_{i\geq 0} \sigma^i \log\left(\sigma^i R^n_{t+1,t+1+i}\right) + \log(1-\sigma) - \log(\theta) - \frac{\sigma\log(\sigma)}{(1-\sigma)}\right) \end{aligned}$$

so that the incentive constraint can be written as 16 where the constant a is

$$a = \log(\theta) + \frac{\sigma\log(\sigma)}{(1-\sigma)} - \log(1-\sigma)$$

Therefore, under the guess, the banker has a concave objective and a convex set of constraints and the first order conditions are sufficient. Subsituting the guess in the first order conditions verifies that it satisfies the optimality conditions.

6.2 Planner problem

$$W(V_t, k_{t,}\xi_t) = \max_{c_t, l_t, c^b_t, i_t, k_{t+1}, V_{t+1}(\xi_{t+1})} \log\left(c_t - \chi\frac{l_t^{1+\varphi}}{1+\varphi}\right) + \beta E_t W\left(V_{t+1}, k_{t+1,}\xi_{t+1}\right)$$

$$s.t.$$

$$c_t + c_t^b + \psi\left(k_{t+1}, k_t, \xi_t\right) \leq f\left(\xi_t k_t, l_t\right)$$

$$\log\left(c_t^b\right) + \sigma E_t V_{t+1} \geq V_t$$

$$E_t V_{t+1} \geq V^{aut}\left(\theta q\left(k_{t+1}, k_t, \xi_t\right) k_{t+1}\right)$$

c^h :

$$U_c\left(t\right) = \lambda_t$$

c^b

$$\lambda_t = \mu_t \frac{1}{c_t^b}$$

n

$$\frac{U_l\left(t\right)}{U_c\left(t\right)} = f_l\left(t\right)$$

k_{t+1}

$$\beta E_t W_k\left(t+1\right) = U_c\left(t\right) q_t + \rho_t V_{k_{t+1}}^{aut}\left(t\right)$$

ν_{t+1}

$$\beta \mu_{t+1} = \sigma \mu_t + \rho_t$$

W_k

$$W_k\left(t\right) = U_c\left(t\right)\left(a f_k\left(t\right) - \psi_{k_t}\left(k_{t+1}, k_t, \xi_t\right)\right) - \rho_t V_{k_t}^{aut}\left(t\right)$$

where

$$\psi_{k_{t+1}}\left(k_{t+1}, k_t, \xi_t\right) = q\left(k_{t+1}, k_t, \xi_t\right)$$

$$\psi_{k_t}\left(k_{t+1}, k_t, \xi_t\right) = -\bar{q}\left(k_{t+1}, k_t, \xi_t\right)$$

$$V^{aut}\left(k_{t+1}, k_t, \xi_t\right) = \frac{\log\left(q\left(k_{t+1}, k_t, \xi_t\right) k_{t+1}\right)}{1-\sigma} + z$$

$$\begin{aligned} V_{k_{t+1}}^{aut}\left(t\right) &= \frac{1}{1-\sigma}\left(\frac{1}{k_{t+1}} + \frac{\frac{dq}{dk}}{q}\right) = \\ &= \frac{1}{1-\sigma}\frac{1}{k_{t+1}}\left(1 + \eta_{k_{t+1}}^{q_t}\right) \end{aligned}$$

$$V_{k_t}^{aut}\left(t\right) = \frac{1}{1-\sigma}\frac{\eta_{k_t}^{q_t}}{k_t}$$

therefore

$$\mu_t = \frac{U_c(t)}{U_c^b(t)}$$

$$\rho_t = \beta\mu_{t+1} - \sigma\mu_t = \beta\frac{U_c(t+1)}{U_c^b(t+1)} - \sigma\frac{U_c(t)}{U_c^b(t)}$$

$$W_k(t) = U_c(t)\left(f_k(t) - \psi_{k_t}(k_{t+1}, k_t, \xi_t)\right) - \rho_t V_{k_t}^{aut}(t)$$

$$\beta E_t\left\{U_c(t+1)\left(\alpha\frac{y_{t+1}}{k_{t+1}} + \bar{q}_{t+1}\right) + \rho_{t+1}V_{k_{t+1}}^{aut}(t+1)\right\} = U_c(t)\,q_t + \rho_t V_{k_{t+1}}^{aut}(t)$$

Substituting the above expressions

$$\begin{aligned}
& \beta E_t\left\{U_c(t+1)\left(\alpha\frac{y_{t+1}}{k_{t+1}} + \bar{q}_{t+1}\right) + \rho_{t+1}\frac{1}{1-\sigma}\frac{1}{k_{t+1}}\eta_{k_{t+1}}^{q_{t+1}}\right\} \\
= \quad & U_c(t)\,q_t + \rho_t\left(\frac{1}{1-\sigma}\frac{1}{k_{t+1}} + \frac{1}{1-\sigma}\frac{1}{k_{t+1}}\eta_{k_{t+1}}^{q_t}\right)
\end{aligned}$$

After some algebra equation 27 is obtained

$$\frac{\sigma C_t^b}{(1-\sigma)\,Q_t K_{t+1}}\left(\sigma E_t\Lambda_{t,t+1}^b R_{t+1}^k - \frac{\Lambda_{t,t+1}^b(\xi_{t+1})}{\Lambda_{t,t+1}^h(\xi_{t+1})}\right) + \frac{\Lambda_{t,t+1}^b(\xi_{t+1})}{\Lambda_{t,t+1}^h(\xi_{t+1})} = 1 - E_t\Upsilon(\rho_t, \rho_{t+1})$$

therefore the system determining the planner's allocation is given by

$$\frac{U_l(t)}{U_c(t)} = f_l(t)$$

$$c_t + c_t^b + \psi(k_{t+1}, \xi_t k_t) = f(\xi_t k_t, l_t)$$

$$\log\left(c_t^b\right) + \sigma E_t V_{t+1} = V_t$$

$$\frac{\sigma C_t^b}{(1-\sigma)\,Q_t K_{t+1}}\left(\sigma E_t\Lambda_{t,t+1}^b R_{t+1}^k - \frac{\Lambda_{t,t+1}^b(\xi_{t+1})}{\Lambda_{t,t+1}^h(\xi_{t+1})}\right) + \frac{\Lambda_{t,t+1}^b(\xi_{t+1})}{\Lambda_{t,t+1}^h(\xi_{t+1})} = 1 - E_t\Upsilon(\rho_t, \rho_{t+1})$$

$$\begin{array}{ll}
\frac{\beta}{\sigma}\frac{\mu_{t+1}}{\mu_t} = 1 & if\ E_t\nu_{t+1} > \nu^d(k_{t+1}) \\
E_t V_{t+1} = V^{aut}(k_{t+1}) & \frac{\beta}{\sigma}\frac{\mu_{t+1}}{\mu_t} > 1
\end{array}$$

6.3 Implementation

I briefly describe how the introduction of credit subsidies/taxes and subsidies/taxes on investment changes the first order conditions of the banker in the incomplete amrket model.

$$V_t^b(\bar{n}_t) = \max_{c_t^b, n_t, d_t, k_{t+1}, \bar{n}_{t+1}} \log\left(c_t^b\right) + \sigma E_t V_{t+1}^b(\bar{n}_{t+1})$$

$$s.t.$$

$$\bar{n}_{t+1} \leq q_t k_{t+1} R_{t+1}^k - d_t \tag{31}$$

$$c_t^b + n_t \leq \bar{n}_t \tag{32}$$

$$\left(1-\tau_t^k\right) q_t k_{t+1} \leq n_t + \frac{d_t}{R_t}\left(1+\tau_t^b\right) \tag{33}$$

$$\sigma E_t V_{t+1}(\bar{n}_{t+1}) \geq \sigma V^{aut}(\theta q_t k_{t+1}) \tag{34}$$

The incentive constraint clarifies that the government's transfers can be stolen by the banker just as deposits. Rewriting in terms of leverage yields

$$V_t^b(\bar{n}_t) = \max_{c_t^b, n_t, d_t, k_{t+1}, \bar{n}_{t+1}} \log\left(c_t^b\right) + \sigma E_t V_{t+1}^b\left(n_t\left[\phi_t\left(R_{t+1}^k - \left(1-\tau_t^k\right)\frac{R_t}{1+\tau_t^b}\right) + \frac{R_t}{1+\tau_t^b}\right]\right)$$

$$s.t.$$

$$c_t^b + n_t \leq \bar{n}_t \tag{35}$$

$$\sigma E_t V_{t+1}\left(n_t\left[\phi_t\left(R_{t+1}^k - \left(1-\tau_t^k\right)\frac{R_t}{1+\tau_t^b}\right) + \frac{R_t}{1+\tau_t^b}\right]\right) \geq \sigma V^{aut}(\theta \phi_t n_t) \tag{36}$$

The first order conditions are

$$E_t\left\{\Lambda_{t,t+1}^b\left[\phi_t\left(R_{t+1}^k - \left(1-\tau_t^k\right)\frac{R_t}{1+\tau_t^b}\right) + \frac{R_t}{1+\tau_t^b}\right]\right\} = 1 \tag{37}$$

and then, if the incentive constraint is not binding

$$E_t\left\{\Lambda_{t,t+1}^b\left(R_{t+1}^k - \left(1-\tau_t^k\right)\frac{R_t}{1+\tau_t^b}\right)\right\} = 0 \tag{38}$$

otherwise

$$E_t V_{t+1}(\bar{n}_{t+1}) = V^{aut}(\theta q_t k_{t+1}) \tag{39}$$

$$\phi_t E_t\left\{\Lambda_{t,t+1}^b\left(R_{t+1}^k - \left(1-\tau_t^k\right)\frac{R_t}{1+\tau_t^b}\right)\right\} > 0 \tag{40}$$

Proof. I now turn to proving Proposition 1. To do this it is convenient to change the state of the system to be (d_{t-1}, k_t, ξ_t) . The equilbrium conditions are exactly the same apart from the evolution of bankers' wealth which is replaced by

$$\left(1-\tau_t^k\right) q_t k_{t+1} = \left((\eta y_t + \xi_t k_t \bar{q}_t) - d_{t-1} - c_t^b + \frac{d_t}{R_t}\left(1+\tau_t^b\right)\right)$$

Let the initial state of the incomplete market economy be (d_{-1}, k_0, ξ_0) . Pick a point on the Pareto frontier $\left(V^{bpl}, W^{pl}\left(V^{bpl}, k_0, \xi_0\right)\right)$. To implement the planner allocation given the initial state $\left(V^{bpl}, k_0, \xi_0\right)$ with a subsidy/tax on banks' investments and lump-sum taxes transfers on households construct processes $\left\{\tau^k\left(d_{t-1}\left(\xi_0^{t-1}\right), k_t^{pl}\left(\xi_0^t\right), \xi_0^t\right)\right\}_{t\geq 0}$, $\left\{T_t^h\left(d_{t-1}\left(\xi_0^{t-1}\right), k_t^{pl}\left(\xi_0^t\right), \xi_0^t\right)\right\}_{t\geq 0}$, $\left\{R_t\left(d_{t-1}\left(\xi_0^{t-1}\right), k_t^{pl}\left(\xi_0^t\right), \xi_0^t\right)\right\}_{t\geq 0}$ and such that for each $t \geq 0$, $\left(d_{t-1}\left(\xi_0^{t-1}\right), k_t^{pl}\left(\xi_0^t\right), \xi_0^t\right)$

$$E_t \Lambda_{t,t+1}^{b,pl}\left[\phi_t^{pl}\left(R_{t+1}^{kpl} - \left(1-\tau_t^{k*}\right) R_t\right) + R_t\right] = 1 \tag{41}$$

$$\frac{1}{E_t \Lambda_{t,t+1}^{hpl}} = R_t \tag{42}$$

$$q_t^{pl} k_{t+1}^{pl} = \left(\left(\eta y_t^{pl} + \xi_t k_t^{pl} \bar{q}_t^{pl}\right) - d_{t-1} - c_t^{bpl} + \frac{d_t}{R_t}\right)$$

$$\tau_t^k q_t^{pl} k_{t+1}^{pl} = T_t^h$$

Obviously the planner allocation satisfies goods market clearing capital market clearing and labor market clearing. The only other conditions that need to be checked are the indifference condition for the banker when the constraint is not binding and positive excess returns when it is binding. These follow directly from the fact that when the incentive constraint is not binding the planner's allocation satisfies

$$\frac{\Lambda_{t,t+1}^{bpl}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{hpl}\left(\xi_{t+1}\right)} = 1 = \frac{E_t \Lambda_{t,t+1}^{b,pl}}{E_t \Lambda_{t,t+1}^{hpl}}$$

and hence equation 41 and 42 imply

$$E_t \Lambda_{t,t+1}^{b,pl}\left(R_{t+1}^{kpl} - \left(1-\tau_t^{k*}\right) R_t\right) = 0$$

Analogously, when the constraint is binding

$$\frac{\Lambda_{t,t+1}^{bpl}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{hpl}\left(\xi_{t+1}\right)} < 1$$

hence

$$\frac{E_t\Lambda_{t,t+1}^{bpl}}{E_t\Lambda_{t,t+1}^{hpl}} < 1$$

which using 41 and 42 once again yields

$$E_t\Lambda_{t,t+1}^{b,pl}\left(R_{t+1}^{kpl} - \left(1-\tau_t^{k*}\right)R_t\right) > 0$$

Notice that

$$\phi_t^{pl}\left(E_t\Lambda_{t,t+1}^{b}R_{t+1}^{k} - \frac{\Lambda_{t,t+1}^{b}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{h}\left(\xi_{t+1}\right)}\right) + \frac{\Lambda_{t,t+1}^{b}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{h}\left(\xi_{t+1}\right)} = 1 - E_t\Upsilon\left(\rho_t,\rho_{t+1}\right)$$

which together with equation 41

$$\frac{1}{\phi_t^{pl}}\frac{E_t\Upsilon\left(\rho_t,\rho_{t+1}\right)}{\frac{\Lambda_{t,t+1}^{b}\left(\xi_{t+1}\right)}{\Lambda_{t,t+1}^{h}\left(\xi_{t+1}\right)}} = \tau_t^{k*}$$

Finally following the same steps as above it is easy to show that with a credit tax/subsidy the second best is not attainable. Assume it is, than τ_t^b, $T^{h*}\left(\xi_0^t\right), R_t^*$ and d_t^* must solve

$$E_t\Lambda_{t,t+1}^{b,pl}\left[\phi_t^{pl}\left(R_{t+1}^{kpl} - \frac{R_t^*}{1+\tau_t^b}\right) + \frac{R_t^*}{1+\tau_t^b}\right] = 1$$

$$\frac{1}{E_t\Lambda_{t,t+1}^{hpl}} = R_t^*$$

$$q_t^{pl}k_{t+1}^{pl} = \left(\left(\eta y_t^{pl} + \xi_t k_t^{pl}\bar{q}_t^{pl}\right) - d_{t-1}^* - c_t^{bpl} + \frac{d_t^*}{R_t^*}\left(1+\tau_t^b\right)\right)$$

$$\tau_t^{b*}\frac{d_t^*}{R_t^*} = T_t^h$$

pick a state where the constraint is not binding at time t but it might be at $t+1$. Then

$E_t \Upsilon\left(\rho_t, \rho_{t+1}\right) > 0$ so that $\tau_t^b > 0$. But then this would imply

$$E_t \Lambda_{t,t+1}^{b,pl}\left(R_{t+1}^{kpl} - \left(1 - \tau_t^{k*}\right) R_t\right) > 0$$

which contradicts equation 13 that must hold when the constraint is not binding. ■

6.4 Computation

I aprroximate the equilibrium functions on a grid of points $G \subset R^3$ by standard iteration techniques.

Let $l\left(\bar{n}_t, \xi_t, k_t\right) = l\left(\xi_t, k_t\right)$ be given by

$$l = \left[\frac{(1-\eta)}{\iota}\left(\xi_t k_t\right)^{\eta}\right]^{\frac{1}{\varphi+\eta}}$$

and $y\left(\xi_t, k_t\right)$ accordingly

$$y = \left[\frac{(1-\eta)}{\iota}\left(\xi_t k_t\right)^{\eta}\right]^{\frac{1-\eta}{\varphi+\eta}}\left(\xi_t k_t\right)^{\eta}$$

Guess $c_o^h\left(\bar{n}_t, k_t, \xi_t\right), q_0\left(\bar{n}_t, k_t, \xi_t\right), \bar{n}_0'\left(\bar{n}_t, k_t, \xi_t\right), k_0'\left(\bar{n}_t, k_t, \xi_t\right), \nu_0\left(\bar{n}_t, k_t, \xi_t\right)$

Unconstrained solution: for any $\left(\bar{n}_t, k_t, \xi_t\right)$ let x^{uc} denote the value of variable x that solves the system of first order condititons and market clearing, under the guessed continualtion policies and find $\left(c^{huc}, q^{uc}, k'^{uc}, R^{uc}\right)$ from the soution of:

$$c^{huc} = y\left(\xi_t, k_t\right) - (1-\sigma)\,\bar{n}_t - \psi\left(k'^{uc}, k, \xi\right)$$

$$q^{uc} = 1 + \psi_{k'}\left(k'^{uc}, k, \xi\right)$$

$$\beta E_t\left\{\frac{\left(c^{huc} - \frac{l(\xi_t,k)^{1+\varphi}}{1+\eta}\right)}{c\left(\bar{n}_0'\left(s_t\right), k_0'\left(s_t\right), \xi_t\right) - \frac{l\left(\xi_{t+1},k_0(s_t)\right)^{1+\varphi}}{1+\varphi}}\right\} R^{uc} = 1$$

$$E_t\left\{\frac{\left(\frac{\eta\frac{y\left(\xi_{t+1},k_0(s_t)\right)}{k_0(s_t)} + \xi_{t+1}\bar{q}\left(\bar{n}_0'(s_t),k_0(s_t),\xi_t\right)}{q^{uc}} - R^{uc}\right)}{\bar{n}_0'\left(s_t\right)}\right\} = 0$$

It s trivial to show that the solution exists and is unique. The unconstrained price level that I refer to in the text is the q^{uc} found by solving this system under the equilibrium guess. Notice that when the constraint binds this guess would not be correct.

Check the constraint: if

$$E_t \nu_0 \left(\bar{n}_0' \left(s_t\right), k_0' \left(s_t\right), \xi_t\right) \geq V^d \left(\theta q^{uv} k_{t+1}^{uc}\right)$$

update

$$c_1^h \left(\bar{n}_t, k_t, \xi_t\right) = c^{huc} \left(\bar{n}_t, k_t, \xi_t\right)$$
$$q_1 \left(\bar{n}_t, k_t, \xi_t\right) = q^{uc} \left(\bar{n}_t, k_t, \xi_t\right)$$
$$k_1' \left(\bar{n}_t, k_t, \xi_t\right) = k'^{uc} \left(\bar{n}_t, k_t, \xi_t\right) \tag{43}$$

$$\begin{aligned} & \bar{n}_1' \left(\bar{n}_t, k_t, \xi_t\right) \\ = \quad & \sigma \bar{n}_t \left[\frac{q^{uc} \left(\bar{n}_t, k_t, \xi_t\right) k'^{uc} \left(\bar{n}_t, k_t, \xi_t\right)}{\sigma \bar{n}_t} \left(\frac{\eta \frac{y\left(\xi_{t+1}, k_0(s_t)\right)}{k_0(s_t)} + \xi_{t+1} \bar{q} \left(\bar{n}_0' \left(s_t\right), k_0 \left(s_t\right), \xi_t\right)}{q^{uc}} - R^{uc} \right) + R^{uc} \right] \end{aligned}$$

$$v_1 \left(\bar{n}_t, k_t, \xi_t\right) = \log \left(\left(1 - \sigma\right) \bar{n}_t\right) + E_t \nu_0 \left(\bar{n}_0' \left(s_t\right), k_0' \left(s_t\right), \xi_t\right)$$

Constrained solution: for any $\left(\bar{n}_t, k_t, \xi_t\right)$ at which the constraint is not satisified get q^c and $k^{'c}$ from

$$E_t \nu_0 \left(\bar{n}_0' \left(s_t\right), k_0' \left(s_t\right), \xi_t\right) = V^d \left(\theta q^c k_{t+1}^c\right)$$
$$q^c = 1 + \psi_{k'} \left(k'^c, k, \xi\right)$$

then get c^{hc} and R^c from

$$c^{hc} = y\left(\xi_t, k_t\right) - \left(1 - \sigma\right) \bar{n}_t - \psi \left(k'^c, k, \xi\right)$$

$$\beta E_t \left\{ \frac{\left(c^{hc} - \frac{l(\xi_t, k)^{1+\varphi}}{1+\eta}\right)}{c\left(\bar{n}_0' \left(s_t\right), k_0' \left(s_t\right), \xi_t\right) - \frac{l\left(\xi_{t+1}, k_0(s_t)\right)^{1+\varphi}}{1+\varphi}} \right\} R^c = 1$$

and update accordingly. Iterate until convergence.