Marriage, Labor Supply, and Home Production*

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January 26, 2017

*We benefited from comments by Elizabeth Caucutt, Nicole Fortin, Pierre-André Chiappori, Bernard Fortin, Guy Lacroix, Costas Meghir, Rob Shimer, Edoardo Ciscato as well as conference participants at Congrès annuel de la Société Canadienne de Sciences Economiques, Montebello, 2008, Annual CIRPEE Workshop, Mont-Orford), 2008, INED Workshop « Recent advances in the economics of the household », Paris, 2010, ANR-Cemmap workshop « Matching and sorting : theory and applications », Sciences-Po, Paris, 2010, ICREA-MOVE Conference on Family Economics, Autonoma Barcelona, 2011, Conference "Economic analysis using linked employer and employee data: bringing together theory and empirics", Porto University, 2011, SED Annual Meeting, Gent, 2011, the 2015 SOLE-EALE Annual Conference, and seminar participants at various seminars. We gratefully acknowledge financial support from ANR-GINDHILA. Jacquemet acknowledges financial support from the Institut Universitaire de France. Robin acknowledges financial support from the Economic and Social Research Council through the ESRC Centre for Microdata Methods and Practice grant RES-589-28-0001, and from the European Research Council (ERC) grant ERC-2010-AdG-269693-WASP.

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Abstract

We develop a search-matching-and-bargaining model where individuals draw utility from private consumption and leisure, and from a non-market good that is produced in the home using non-market time. We condition individual decisions on wages, education and an index of family attitudes. A match-specific, stochastic bliss shock induces variation in matching given wages, education and family values, and renegotiation and divorce. Using BHPS (1991-2008) data, we take as given changes in wages, education and family values by gender, and study their impact on marriage decisions and intrahousehold resource allocation. The model allows to evaluate how much of the gender division of labor results from gender wage and education differences, or gender differences in preferences and home production, or different family attitudes. We find that family attitudes is a dimension of preference heterogeneity that is nearly as powerful as gender-specific preferences in explaining the gender division of labor.

Keywords: Search-matching, bargaining, assortative mating, collective labor supply, social norms, gender identity, structural estimation.

JEL classification: C78, D83, J12, J22.

1 Introduction

In this paper, we develop a search-matching-and-bargaining model where individuals draw utility from the consumption of a market good, leisure, and a non-market good that is produced in the house using all household members' non-market time. The model is in many ways similar to Knowles (2013) or Greenwood, Guner, Kocharkov, and Santos (2016). However, as we focus on a more recent period (after 1990), we neglect equipment inputs into home production, and education (like wages) is exogenous. These simplifications allow us to sophisticate the model in other dimensions. Individuals can choose within a continuum of hours, and the distributions of individual characteristics by gender, as well as the function measuring compatibility between spouse types, are nonparametrically flexible. In other words, we put more effort in fitting cross-sectional distributions of individual time uses and characteristics than it has usually been the case. On the technical side, we provide a formal proof of identification and we develop numerical approximation techniques to implement the estimation and the simulation of the model.

Another important aspect of this paper is that we try to discipline the model as much as we can by assuming the minimal amount of unobserved shocks and individual heterogeneity. We introduce a match-specific, stochastic bliss shock. Initial shocks generate randomness in marriage choice given observables, and subsequent shocks generate randomness in resource allocation and infrequently trigger divorce. There is no unobserved heterogeneity, but we introduce a second source of observed heterogeneity besides wage and education picking up variation in family attitudes. For this we make use of various survey questions such as this one: On a 1-5 scale, would you say that the husband should earn and the wife stay at home? We use Principal Component Analysis to construct a Family Values Index.

We end up with a tightly specified behavioral model where individuals differ in wage, education and family values, and with a stochastic process of match-specific shock observed by the individuals but not by the econometrician. As far as we are aware, this is the first paper developing a framework for evaluating how much of the gender division of labor results from gender wage inequality, gender differences in preferences and home production technologies, or different family attitudes.

Our empirical study uses the British Household Panel Survey, 1991-2008. By 1990, the major trends described by Knowles and by Greenwood *et al.* are largely completed. The fraction of married households has stopped plummeting and assortative mating has stopped rising. The labor-force participation of married women soared after 1960 until the late 1980s. Yet, female education continued to rise until 2000, and the wages of married men and women increased faster than the wages of singles until the mid-2000s.

We take as given changes in wages, education and family values by gender, and study their impact on marriage decisions and intrahousehold resource allocation. At the risk of oversimplification, we could say that education determines preferences and home production quite similarly for men and women. Family values have little influence on preferences, but strongly determine home production, quite differently for men and women. The affinity between male and female types depends very much on one single complementarity, the one that makes individuals prefer marrying within their own education group. We quantify the share of income that is privately consumed by male and female spouses. We find that men capture 55 percent of total earnings for private consumption and leisure, and that rent sharing tends to reflect wage differences between spouses more over time.

The model matches the 1991-2008 trends and the cross-sectional distributions well. In this paper, we focus on two distributions, recently emphasized by Bertrand, Kamenica, and Pan (2015): the distribution of male and female spouses' wage ratios and the distribution of labor earnings ratios. Despite a fairly symmetric distribution of wages, the distribution of earnings is not symmetric. Men work more outside the home and women work more inside even if the wife's wage is above the husband's wage. This is all the more so for traditionalist couples. The models fits these empirical facts well.

Finally, we proceed to various counterfactual simulations. First, we eliminate the conditional gender wage gap given education and family values. Second, we keep the distribution of characteristics fixed and give all females in the sample the preferences and home production of males. Third, we consider an economy with only progressive individuals. This is a way of measuring how much of gender specialization in market and non-market work is related to wages, how much stems from gender-specific preferences and home production, and how much results from different family attitudes.

We find that abolishing gender wage differences increases female labor supply and reduces non market work by about 10%. Removing all gender differences in preferences and home production increases female labor supply by 8 hours (starting from a gender differential of 16 hours). Making individuals progressive is as effective as changing preferences and home production technology in boosting female labor supply. However, giving women the same preferences as men increases the marriage rate whereas making people more progressive decreases it. Lastly, equalizing preferences on top of making everybody progressive leads married women to supply 3 more hours of market labor. Yet the time use differences between men and women remain substantial (about 3 market hours and less than one non market hour) as a consequence of the gender wage gap.

The layout of the paper is as follows. After a brief review of the literature, we describe salient features of intrahousehold allocation of time and matching between 1991 and 2008. Then we construct the search-matching-and-bargaining model. The next section describes the empirical specification, studies identification and develops the estimation procedure. We then show the results. The appendix deals with technical details.

2 Relation to the literature

Lundberg and Pollak (1996) end their insightful survey on bargaining and distribution in marriage by stating that "bargaining models provide an opportunity for integrating the analysis of distribution within marriage with a matching or search model of the marriage market." They make this statement after arguing that policy interventions affecting the distribution of resources within the family can have very different short-run and long-run effects because of marriage market equilibrium feedback. Since then the search-matching-and-bargaining framework has been widely used to model household formation and decisions mostly in the perspective of understanding long term changes such as declining marriage rates or rising female labor supply.

The search-matching-and-bargaining project took time to take off in the applied microeconomic literature of the family. We think that this is because Chiappori (1988, 1992), Apps and Rees (1988) introduced the collective framework in the late 1980s. The collective model has been thereafter dominant.[1] Their point is that in order to model individual consumption and labor supply in situations where household resources are shared cooperatively, Nash bargaining is an unnecessary restriction. Pareto optimality is sufficient to derive testable restrictions. As long as researchers' interest was on modeling demand given a sharing rule, modeling marriage and divorce was not very useful. It was possible to show that the demand model could be identified in many contexts without endogenizing the sharing rule, for as long as the sharing rule could be instrumented.

However, Lundberg and Pollak's concern resurfaces as soon as the aim of the empirical work is not only to test a theory but to understand the long-term effects on individual labor supply of exogenous changes in individual characteristics such as education or wages, or the long-term effects of family and anti-poverty policies such as the Working Family Tax Credit program in the UK or the Earned Income Tax Credit in the USA. Collective models very successfully describe resource sharing within the family given a sharing rule. A matching model – that is an equilibrium model of the marriage market – is required in order to endogenize the distribution of powers in the family (Chiappori, 2012). For example, if education and wage differentials by gender change over time, this will have an effect on couples' labor supply for a given way of sharing resources, but this may also affect individuals' outside options and the sharing rule itself.

In the standard search model singles meet mates sequentially; they share resources according to Nash bargaining with bachelorhood as threat point; marriage is realized if there exists a jointly profitable resource allocation; the distributions of individual types by gender and marital status are calculated at the steady state equilibrium. Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, and Knowles (2000, 2003), Caucutt, Guner,

[1]Other important developments can be found in Browning, Bourguignon, Chiappori, and Lechene, 1994, Browning and Chiappori, 1998, Chiappori, Fortin, and Lacroix, 2002, Blundell, Chiappori, Magnac, and Meghir, 2007. A dynamic version of the collective model was explored by Mazzocco (2004, 2007).

and Knowles (2002), Gould and Paserman (2003), Fernandez, Guner, and Knowles (2005) are early examples of applications, and Knowles (2013), Greenwood, Guner, Kocharkov, and Santos (2016) are recent ones. They note the dramatic changes in relative wages and education of men and women and the technological innovations in home appliances leading to reduced equipment prices after 1960, and they analyze their long run effects on assortative mating and labor supply. Burdett, Dong, Sun, and Wright (2016) offer an original analysis of the role of home production and marriage for cash holding, and the correlative link between inflation and marriage. Shimer and Smith (2000), Atakan (2006), Lauermann and Nöldeke (2015) and Manea (2017) provide theoretical studies of equilibrium existence.

Search-matching is not the only way of modeling a marriage market. Long after the seminal work of Becker (1973, 1974, 1981), and Grossbard-Shechtman's (1984) discussion of the role of household production, Choo and Siow (2006) revive the perfect-information assignment framework of Shapley and Shubik (1971), and develop an econometric model of matching between heterogeneous men and women that is the stable-matching equivalent of Wong's (2003) empirical application of Shimer and Smith (2000).[2] A series of applications of this framework followed, including Chiappori and Oreffice (2008), Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanié, and Weiss (2016). Choo, Seitz, and Siow (2008) is a first attempt at introducing labor supply in the Choo and Siow model, and Chiappori, Costa Dias, and Meghir (2015) is a big step forward, introducing not only labor supply and endogenous education (like Greenwood, Guner, Kocharkov, and Santos 2016), but also savings and unobserved heterogeneity.

What is the best framework for a marriage market? There is still little literature on this subject. Adachi (2003) shows that, as search costs disappear, the set of equilibrium outcomes of a search matching model with non-transferrable utility converges to the set of stable equilibria of a Gale-Shapley marriage model. Recently, Lauermann and Nöldeke (2015) tone this prediction down by proving that it holds if and only if there is a unique stable matching in the underlying marriage market. With multiple stable matchings, sequences of equilibrium matchings converging to unstable, inefficient matchings can be constructed. We do not know of any result for transferrable utility models. Still, the search framework can be thought of as a tâtonnement mechanism, whose equilibrium outcomes are hopefully not too far from the stable matchings of the corresponding frictionless economy. Tâtonnement models are empirically appealing because they are usually easier to simulate, absent of coordination issues and strategic behavior, and they naturally generate the sluggishness that seem to be a general characteristic of many markets.

[2]Useful connections between the Choo-Siow model and optimal transportation theory were made by Galichon and Salanié (2012) and Dupuy and Galichon (2014).

3 Data and facts

BHPS data

We use the original British Household Panel Survey (BHPS) sample of 5,050 British households and 9,092 adults interviewed in the first wave (1991). The panel interviews all adult members of all households comprising either an original sample member or an individual born to an original sample member every year until 2008. It therefore remains broadly representative of the British population (excluding Northern Ireland and North of the Caledonian Canal) as it changes over time. We only keep individuals who are either single or married to (or cohabiting with) an heterosexual partner, and who are between 22 and 50 years of age at the time of interview. To reduce non response biases we use the Individual Respondent Weights provided in the survey.

We keep data on usual gross pay per month for the current job, the number of hours normally worked per week (including paid and unpaid overtime hours) and the number of hours spent in a week doing housework (core non market work excluding child caring and rearing, information not provided by the survey). Hourly wage is the usual gross pay per month divided by the number of hours normally worked per month (without overtime). Wages are deflated by the Consumer Price Index and computed in 2008 pounds.

In order to reduce the number of labor supply corners (zero market hours and missing wages) we replace current observations on wages and market hours by a moving average of past, present and future observations. Specifically, suppose that we observe wages $w_1, w_2, ...$ and hours $h_1, h_2, ...$ We replace w_t and h_t by

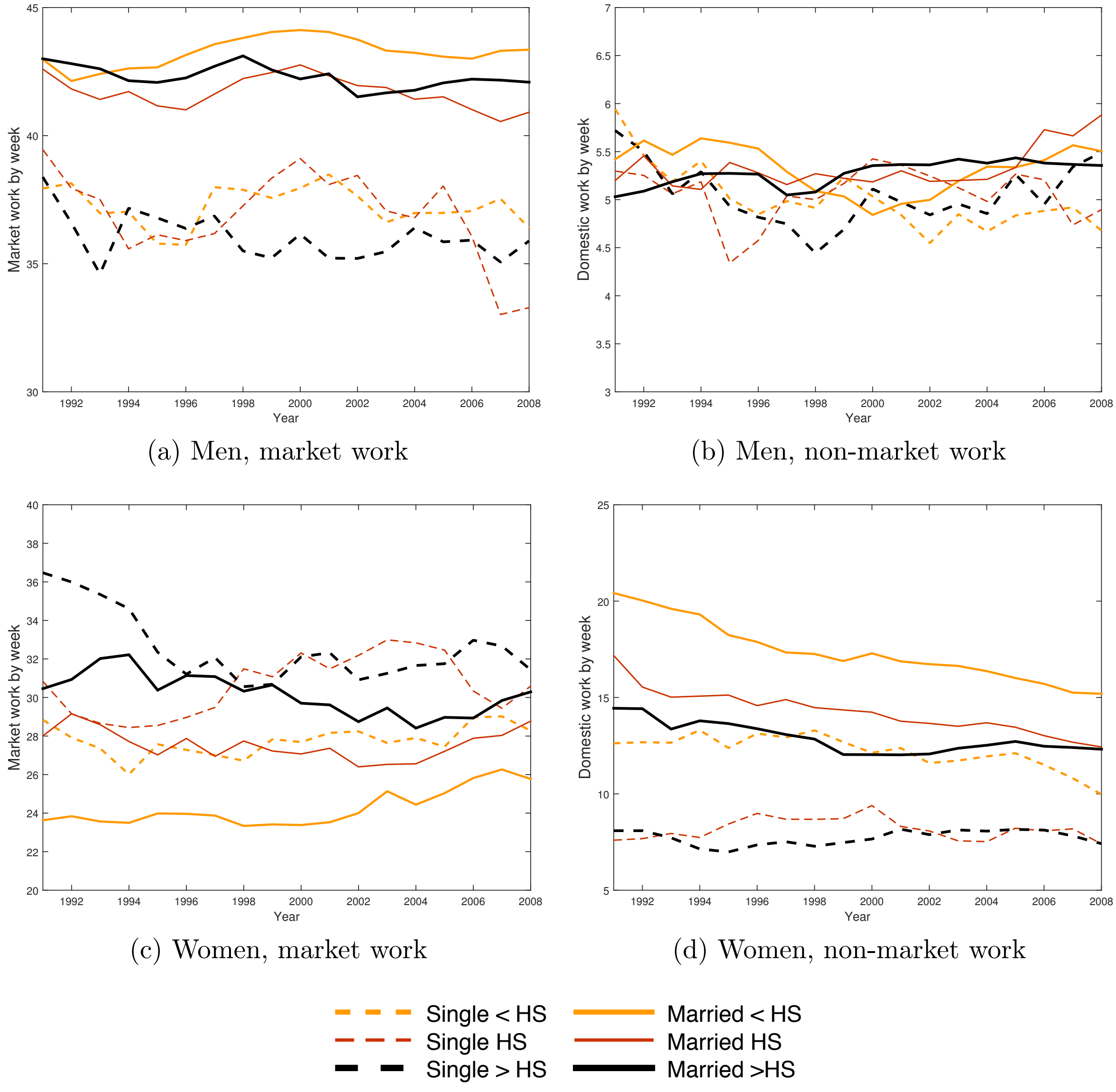
$$\widehat{w}_t = \frac{\sum_{\tau=-\infty}^{+\infty} w_{t+\tau}\mathbf{1}\{h_{t+\tau} \neq 0\}\phi\left(\tau/k\right)}{\sum_{\tau=-\infty}^{+\infty} \mathbf{1}\{h_{t+\tau} \neq 0\}\phi\left(\tau/k\right)}, \quad \widehat{h}_t = \frac{\sum_{\tau=-\infty}^{+\infty} h_{t+\tau}\phi\left(\tau/k\right)}{\sum_{\tau=-\infty}^{+\infty} \phi\left(\tau/k\right)},$$

where ϕ is the standard normal PDF and k is a smoothing parameter that we arbitrarily choose equal to 2 (after experimenting with other choices) yielding weights 1, 0.882, 0.607, 0.325, 0.135, 0.044, 0.011 for 0, 1, 2, 3, 4, 5, 6 years apart. Then we trim the 1% top and bottom tails of wage and time use variables. We thus obtain an unbalanced panel of 18 years (1991-2008), whose cross-sectional sizes remain roughly constant over the years with about 1000-1200 couples and 400 singles of each sex, the fraction of married men and women being remarkably stable at around 70%.

Trends

The rather short period of time between 1991 and 2008 has produced some remarkable changes in time uses, wages and education by gender and marital status. We now briefly review some of them.[3]

[3] See Ramos (2005) and Gimenez-Nadal and Sevilla (2012) for a more detailed description.



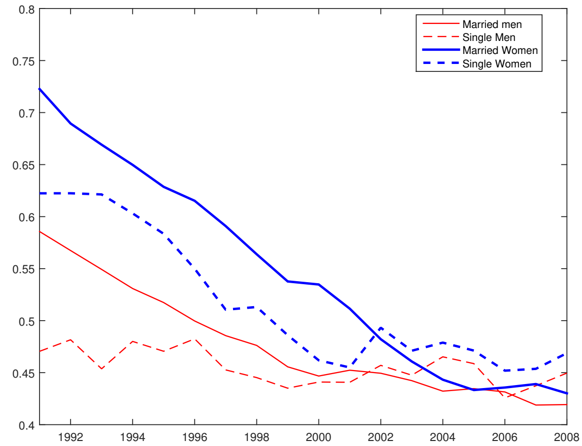
(a) Men, market work

(b) Men, non-market work

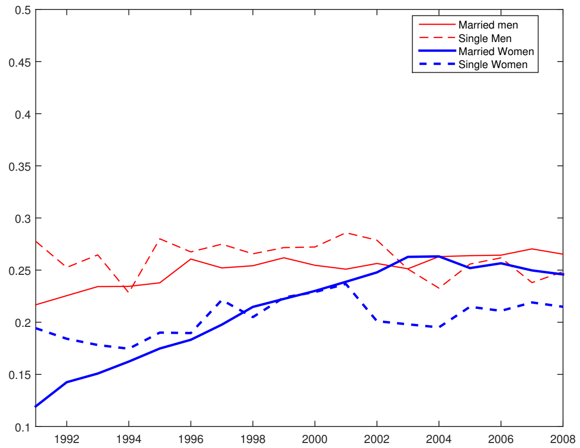
(c) Women, market work

(d) Women, non-market work

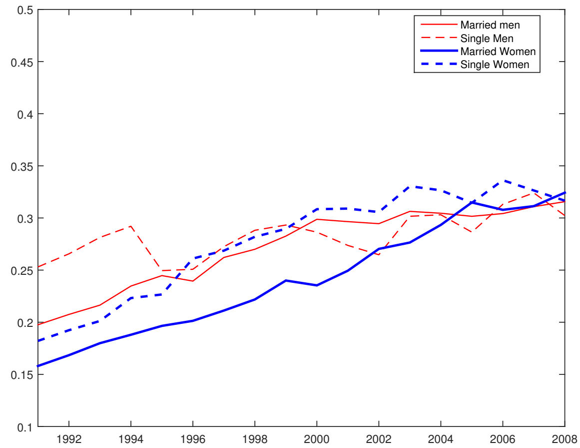
Figure 1: Time use trends



(a) Share with O-level or less

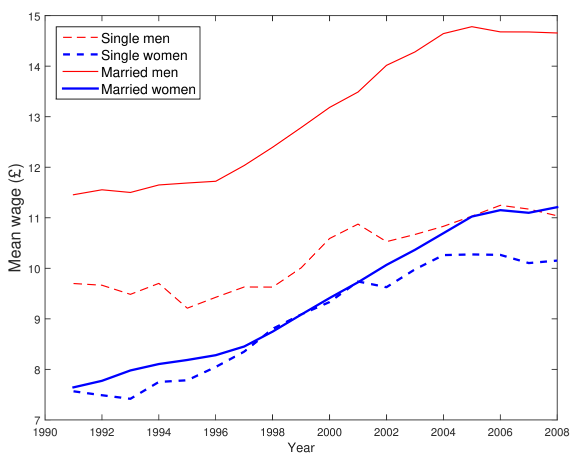


(b) Share of high school graduates (A-level)

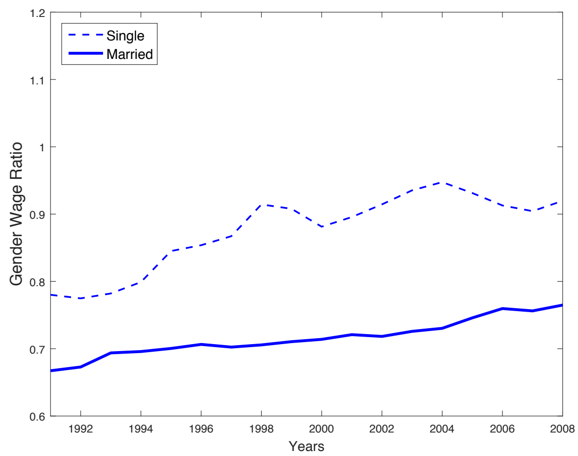


(c) Share of higher educated

Figure 2: Composition changes in education



(a) Mean wages



(b) Female wage ratio

Figure 3: Composition changes in wages

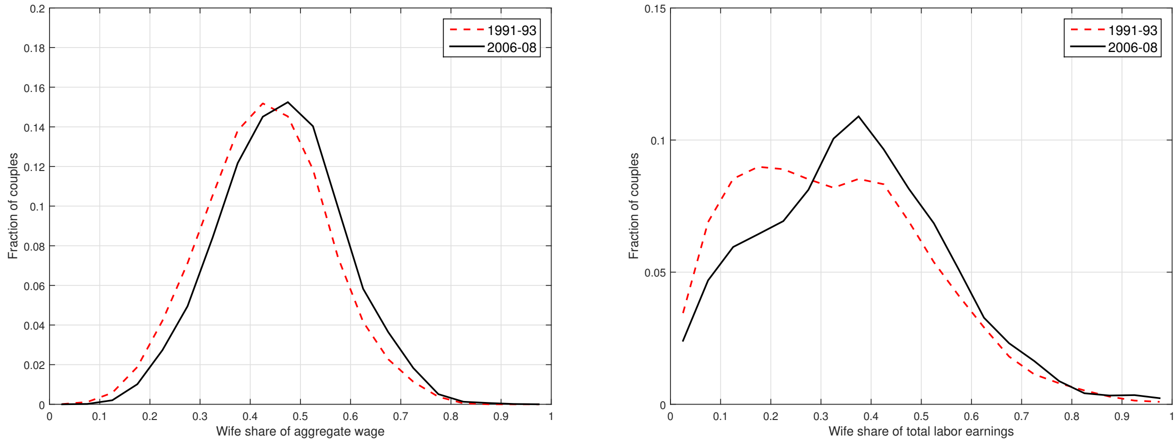
Figure 1 confirms well known facts about market and non market work. Men work more paid hours than women, married men more so than single men, and all men, married and single, devote the same amount of time (little) to home production. Married and low-educated women work fewer hours outside the home and more inside than single and higher-educated women. Education is not a key determinant for men; it is for women. Male hours are remarkably stable over time, while female differences by education and marital status are narrowing down.

Interesting composition changes are also observable in that period (Figure 2). All groups are converging to the educational norm of single men. In 1991, about 45% of single men did not have A-levels, much less than the corresponding 60% for married men and single women, and 70% for married women. By 2004 all groups have reduced the gap to 45%. Wages do not display the same convergence pattern (Figure 3). Married people's wages increase both in absolute terms and with respect to the wages of singles. The gender wage gap is smaller for singles than it is for married individuals and slowly shrinks over time for both married and single individuals. Yet, in 2008, married men still earn 25% more than married women, and single men about 10% more than single women.

Family values

In a recent paper, Bertrand, Kamenica, and Pan (2015) observe that "among married couples in the US, the distribution of the share of household income earned by the wife drops sharply at 1/2." Figure 4a shows the distribution of the female wage share ($\frac{w_f}{w_m+w_f}$) in the BHPS data. It is symmetric with a mode between 0.4 and 0.5. The distribution moves a little to the right over time, becoming a bit more equal (i.e. symmetric around 0.5). The distribution of the female share of labor earnings ($\frac{w_f h_f}{w_m h_m+w_f h_f}$) is however similar to Bertrand et al.'s US estimate. Figure 4b closely resembles their Figure II. The density puts more mass to the left of the distribution than to its right, and is steeply sloping down beyond 0.4. Females specialize less in market work than males when they are paid more than their husband.

So, the gender education gap and, to a lesser extent, the gender wage gap become smaller over time but the division of labor inside the family is adjusting much less rapidly. Bertrand et al. reckon that this is because "slow-moving identity norms are an important factor that limits further convergence in labor market outcomes." With this interpretation in mind, in addition to wages and education, we looked for independent information on social attitudes influencing time uses. Using the responses to various survey questions about children, marriage, cohabitation and divorces we constructed an index of family values by Principal Component Analysis. Table 1 lists the questions used for this construction and displays the corresponding factor loadings. Given the signs of factor loadings, our Family Values Index (FVI) is a measure of traditionalism.

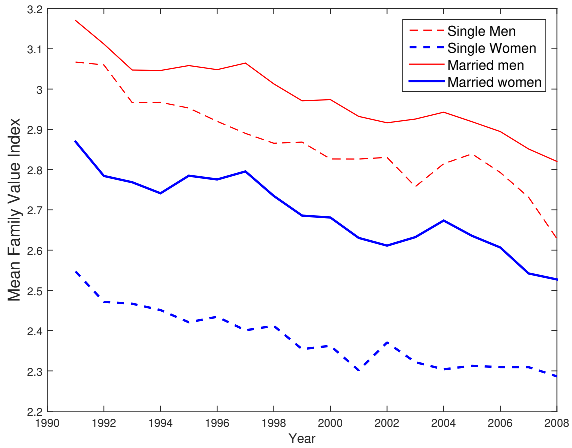


(a) Wage ratio, $\frac{w_f}{w_f+w_m}$ (b) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f+w_m h_m}$

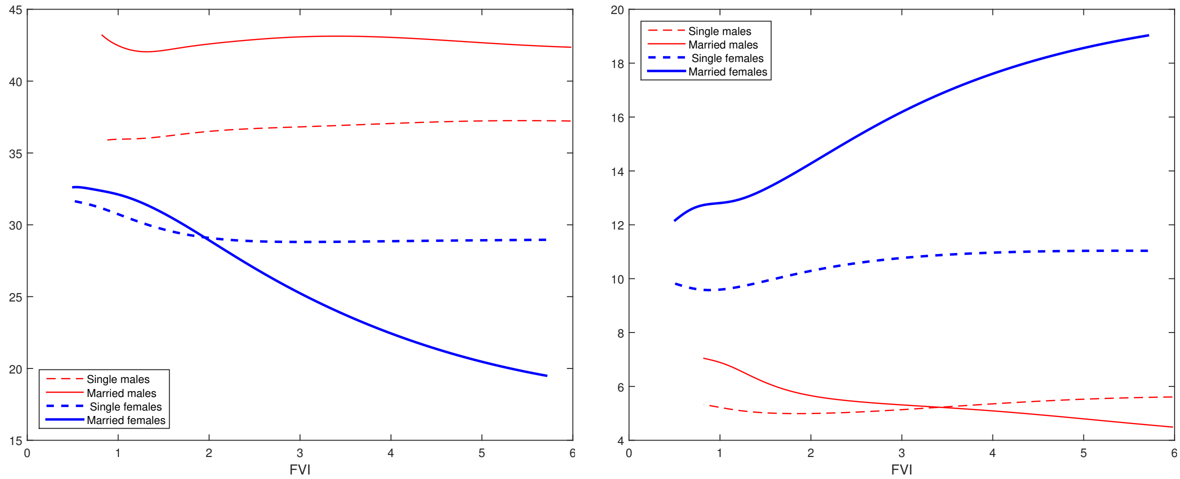
Figure 4: Distribution of wages and earnings within couples

Table 1: Family Values Index

Question	Loading
Pre-school child suffers if mother works	-0.24
Family suffers if mother works full-time	-0.25
Woman and family happier if she works	0.16
Husband and wife should both contribute	0.14
Full time job makes woman independent	0.12
Husband should earn, wife stay at home	-0.21
Children need father as much as mother	-0.05
Employers should help with childcare	0.12
Single parents are as good as couples	0.17
Adult children should care for parents	-0.07
Divorce better than unhappy marriage	0.12
Attendance at religious services	-0.07
Cohabiting is always wrong	-0.16



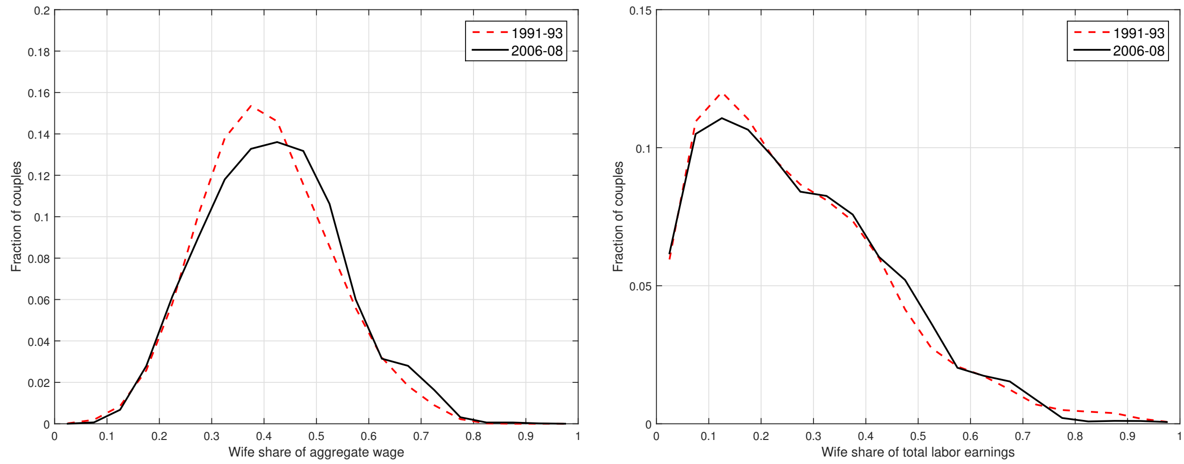
Notes: The Family Values Index is a weighted sum of all responses (1: strongly agree; 2: agree; 3: neither agree nor disagree; 4: disagree; 5: strongly disagree). The variable "Attendance at religious service" is coded: 1: Once a week or +; 2: At least once per month; 3: At least once per year; 4: Practically never; 5: Only weddings etc. After 1998, the item "Cohabiting is always wrong" became "Cohabiting is alright". So we changed it as "6 - item_answer" to maintain coherency over the years. Weights are estimated by Principal Component Analysis and are displayed in the table. The figure shows the evolution of the mean FVI over time by marital status and gender.



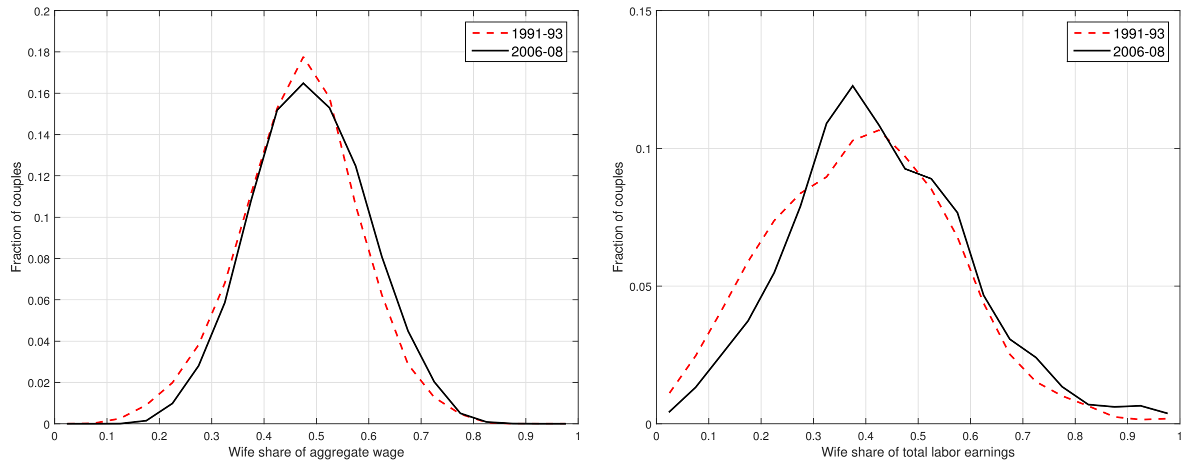
(a) Market hours by FVI (b) Non-market hours by FVI

Figure 5: Time uses and family values

(1) Wage ratio, $\frac{w_f}{w_f+w_m}$ (2) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f+w_m h_m}$



(a) Traditionalist wives and husbands



(b) Progressive wives and husbands

Figure 6: Distributions of wage and earnings ratios by family values

One may worry that the responses to the BHPS questions may just be another way of measuring time uses. However, our family values index is aggregating lots of different attitudes. Besides, had we introduced a discrete unobserved heterogeneity distribution in the model, we would have classified individuals ex post according to time-use data. Instead, we postulate that by the time men and women have reached the age of looking for a partner and of choosing an organization of the household, childhood and adolescence have imprinted representations in their minds that these simple survey questions allow us to measure. In a second step, we quantify the link between time uses and attitudes.

Men are found to be more traditionalist than women, and couples are more traditionalist than singles (see the figure next to Table 1).[4] There is a common negative trend, but it is not extremely pronounced (less that half a point in 18 years on a 1-to-5 scale). Figure 5 shows how family values determine the market and non-market hours of married men and women. The effect is stronger for married women's labor supply and for married men and women's work in household. For singles there is no sizable effect.

The relative distributions of wages and earnings between male and female spouses also vary with family values. In Figure 6 we display the kernel densities of within-couple wage and earnings ratios conditional on whether male and female FVIs are above or below their respective medians. All wage ratios are symmetric, but couples with both spouses traditionalist have a lower and more dispersed wage ratio. Their distribution of earnings ratios is also much more concentrated to the left. At the other extreme, couples with both spouses progressive have a perfectly symmetric distribution of wage ratios, more concentrated around a mode that is closer to 0.5, and their distribution of earnings ratios is the most symmetric of all.

4 Model

In this section we develop a model of marriage and intrahousehold decisions, building on the equilibrium search-bargaining model of Shimer and Smith (2000), which we enrich with labor supply and home production decisions.

4.1 The marriage market

Men and women are fully characterized by a vector of characteristics that comprises the market wage, the education level and the family values index, and that we call a *type*. Conventionally, we use the label i for male types and j for female types. Let $\ell_m(i)$ and $\ell_f(j)$ denote the density functions of male and female types in the whole population at a given point in time (there is an implicit indexation on time), with $L_m = \int \ell_m(i)\,\mathrm{d}i$ and $L_f = \int \ell_f(j)\,\mathrm{d}j$ denoting the total numbers of men and women in the population. Let

[4]See Fortin (2005) for a study of gender attitudes and labor market outcomes across OECD countries.

$n_m(i), n_f(j), N_m, N_f$ be the corresponding notations for the sub-populations of single men and women. Let $m(i,j)$ and $M = \iint m(i,j)\,\mathrm{d}i\,\mathrm{d}j$ denote the density function of couples' types and the total number of couples. Measures ℓ_m, ℓ_f are exogenous and measures n_m, n_f, m are endogenous.

Assume that only singles search for a partner, ruling out search for an alternative spouse during marriage (voluntary and involuntary). Let λ be the number of meetings between singles per unit of time divided by $N_m N_f$ (the number of potential meetings). The rates at which male and female singles meet a potential partner are equal to λN_f and λN_m. This meeting parameter λ is endogenous and is a function of N_m and N_f. With quadratic returns, as in Shimer and Smith (2000), λ is constant; but a more reasonable assumption may be constant returns to scale in the meeting function $\lambda N_m N_f$, for example Cobb-Douglas.

When a female single of type j and a male single of type i meet for the first time, a match-specific bliss shock z is drawn from a distribution G. The willingness to marry ($\mathit{WtM}(i,j,z) = 1$ if yes, $= 0$ if no) depends of all three variables (i,j,z). Let $\alpha_{ij} = \Pr\{\mathit{WtM}(i,j,z) = 1|i,j\}$ denote the probability of marriage given i,j upon meeting.

We also assume that the match-specific bliss shock z is subject to infrequent updates. For simplicity we assume that in every period, with probability δ, a new value z' is drawn from G independently of the current value z. By changing δ we can make the stochastic process of shocks (indexed by calendar time) more or less persistent. Divorce occurs if $\mathit{WtM}(i,j,z') = 0$, and $1 - \alpha_{ij}$ is the probability of divorce given a bliss shock occurrence. Matches with a higher probability of marriage thus also have a lower probability of divorce. Conversely, a marriage that results from an exceptionally large realization of z (love at first sight) should break faster than a marriage based on solid fundamentals.

4.2 Private preferences

Individuals draw utility from private consumption c (the numeraire), private leisure e, and a public good q that is produced in the house. Let $U_i(c,e,q)$ denote the utility function of an individual with exogenous characteristics i (or j if female). For later use, we also define the conditional indirect utility function

$$\psi_i(R,q) = \max_{c,e} U_i(c,e,q) \text{ s.t. } c + w_i e \leq R, \tag{4.1}$$

for a given total private income R and public good q, and where w_i is the wage that is associated to the individual type i. The demands for consumption and leisure follow from the indirect utility function by application of Roy's identity.

Singles have access to a household production technology that requires domestic time as single input: $q = F_i^0(d)$. For married couples, the home production inputs are the hours spent on domestic chores by both spouses (d_m, d_f).[5] Domestic production of couples varies by spouses' types (i, j) and bliss shock z as $q = zF^1_{ij}(d_m, d_f)$. We normalize to one the total amount of time available per week to any individual. So labor supply is $h = 1-e-d$, and w_i is therefore both the wage rate and the total income available to an individual, to be shared between market work, non-market work and leisure. In this paper, we only consider interior solutions with e, d, h strictly between 0 and 1.

In addition we assume that couples' households are subject to a living cost (or benefit) C_{ij} that is a function of exogenous characteristics but not wages. Actual in-work benefit schemes such as US EITC or UK WFTC make C_{ij} a function of family income and of the number of children. We leave to future work this extension as well as other extensions such as aging, endogenous fertility, or female labor market non-participation.[6]

For a single of type i, total private income R in equation (4.1) is equal to $w_i(1-d)$. For a married couple of male-female type (i, j), we introduce intra-household transfers t_m, t_f such that

$$c_m + w_i e_m = w_i(1-d_m) - t_m \equiv R_m, \quad c_f + w_j e_f = w_j(1-d_f) - t_f \equiv R_f.$$

The household budget is balanced if $c_m + c_f + C_{ij} = w_i h_m + w_j h_f$, which implies that $t_m + t_f = C_{ij}$.

4.3 Marriage contracts

We assume that individuals do not make long term commitments and can walk away from the negotiation at any time. A marriage contract between a male of type i and a female of type j, for a current match-specific shock z, specifies a utility level for both spouses, u_m and u_f. Let $V^1_m(z')$ and $V^1_f(z')$ denote the promised continuation values for any realization z' of the next match-specific shock if the match continues, and let V^0_i and V^0_j denote the values of being single.

The present values W_m and W_f of a marriage contract to the male and female spouses for any given choice of (u_m, u_f) follow from the Bellman equations. For example,

$$rW_m = u_m + \delta \int \left[\max\left\{V^0_i, V^1_m(z')\right\} - W_m\right] \mathrm{d}G(z'), \tag{4.2}$$

where r is the time discount rate. The second term of the right-hand side is the option value of divorce after a shock to the match-specific component. If a bliss shock z' accrues,

[5] As far as notations are concerned, we index by i and j the variables that are exogenously given, like the wage (w_i is the component of i that is the wage), and we index by m and f the variables of the decision problem.

[6] In a previous version of this paper, C_{ij} was an input of the home production function and was endogenously chosen. However we estimated a zero elasticity of home production to market expenditure, meaning, given the Stone-Geary specification, a nil effect on home production and preferences. This is why we now have C_{ij} simply as a cost or benefit.

then either the match continuation value $V_m^1(z')$ is greater than the value of bachelorhood V_i^0 and the match continues, or it is lower and there is a divorce.

Marriage utilities u_m, u_f depend on controls c, d_m, d_f, t_m, t_f as

$$u_m = \psi_i \left[w_i(1-d_m) - t_m, q\right], \quad u_f = \psi_j \left[w_j(1-d_f) - t_f, q\right], \quad q = zF_{ij}^1(d_m, d_f), \tag{4.3}$$

and these controls are chosen so as to maximize the Nash bargaining criterion

$$\left[W_m - V_i^0\right]^\beta \left[W_f - V_j^0\right]^{1-\beta}, \tag{4.4}$$

subject to the feasibility constraint $t_m + t_f = C_{ij}$ and the participation constraint

$$\mathit{WtM}(i,j,z) = \mathbf{1} \Leftrightarrow V_m^1(z) - V_i^0 \geq 0 \text{ and } V_f^1(z) - V_j^0 \geq 0.$$

Individuals draw bargaining power both from their outside options (V_i^0, V_j^0) and from their bargaining coefficients β and $1-\beta$.

Next, without commitment, the continuation values should align with personal interest. Hence $W_m = V_m^1(z)$, and $V_m(z)$ must satisfy the intertemporal constraint

$$(r+\delta)\left[V_m^1(z) - V_i^0\right] = u_m + \delta \int \left[V_m^1(z') - V_i^0\right]^+ \mathrm{d}G(z') - rV_i^0, \tag{4.5}$$

denoting $x^+ \equiv \max\{x, 0\}$, and with a symmetric expression for $V_f^1(z)$.[7] The equilibrium value of a marriage contract is a function of types i, j and z. We shall use the notation $V_m^1(i,j,z), V_f^1(i,j,z)$ whenever necessary.

Lastly, the present value of bachelorhood satisfies the Bellman equation,

$$rV_i^0 = u_i^0 + \lambda \iint \left[V_m^1(i,j,z) - V_i^0\right] \mathit{WtM}(i,j,z)\, \widehat{n}_f(j)\, \mathrm{d}j, \tag{4.6}$$

where $u_i^0 = \max_d \psi_i[w_i(1-d), F_i^0(d)]$ and where $\widehat{n}_f(j)$ denotes singles' expectations about type distributions in the future. With anticipated probability $\lambda\widehat{n}_f(j)$ a male single of type i meets a female single of type j. The match-specific bliss shock is z with probability $\mathrm{d}G(z)/\,\mathrm{d}z$. The marriage is consummated if $\mathit{WtM}(i,j,z) = 1$; in which case the male individual i enjoys net continuation value $V_m^1(i,j,z) - V_i^0$ from next period onward.

4.4 Steady state

Calculating the value of being single requires forecasting the chance of meeting a partner of any type in the future. Assuming that the economy is in a steady state easily solves

[7] Notice that the solution to the bargaining problem in c, d_m, d_f, t_m, t_f is invariant to an affine change of indirect utility functions (ψ_i, ψ_j) into $(a_i+b_i\psi_i, a_j+b_j\psi_j)$. Utilities u_m, u_f and flow values $rV_m^1(z), rV_f^1(z)$ are then subject to the same affine transformations.

the expectation formation problem. In steady state, flows in and out of the stocks of married couples of each type must exactly balance each other out. This means that, for all (i, j),

$$\delta\,(1-\alpha_{ij})\,m(i,j) = \lambda\, n_m(i)\, n_f(j)\, \alpha_{ij}. \tag{4.7}$$

The left-hand side is the flow of divorces. A fraction δ of the $m(i, j)$ married couples draw a new bliss shock. Divorce occurs with probability one minus the marriage probability α_{ij}. The right-hand side is the flow of new (i, j)-marriages. It has two components: a single male of type i, out of the $n_m(i)$ identical ones, meets a single female of type j with probability $\lambda n_f(j)$; the marriage is consummated with probability α_{ij}.

Now, making use of the accounting restrictions,

$$\ell_m(i) = n_m(i) + \int m(i,j)\,\mathrm{d}j, \quad \ell_f(j) = n_f(j) + \int m(i,j)\,\mathrm{d}i, \tag{4.8}$$

and replacing $m(i, j)$ by its value from (4.7),

$$m(i,j) = \frac{\lambda}{\delta}\frac{\alpha(i,j)}{1-\alpha(i,j)} n_m(i) n_f(j), \tag{4.9}$$

the equilibrium measures of singles, $n_m(i), n_f(j)$, are solutions to the following fixed-point system:

$$n_m(i) = \frac{\ell_m(i)}{1+\frac{\lambda}{\delta}\int n_f(j)\frac{\alpha(i,j)}{1-\alpha(i,j)}\,\mathrm{d}j}, \quad n_f(j) = \frac{\ell_f(j)}{1+\frac{\lambda}{\delta}\int n_m(i)\frac{\alpha(i,j)}{1-\alpha(i,j)}\,\mathrm{d}i}. \tag{4.10}$$

4.5 Equilibrium solution with transferable utility

We now assume that the indirect utility is linear in both arguments:

$$\psi_i(R,q) = q\frac{R-A_i}{B_i}, \tag{4.11}$$

where $A_i \equiv A_i(w_i)$ and $B_i \equiv B_i(w_i)$ are differentiable, increasing and concave functions of the wage w_i. We normalize the denominator as $B_i(1) = 1$. It is an individual-specific price index and the numerator $q(R - A_i)$ can therefore be interpreted as a nominal utility level.

Under this assumption we show that the equilibrium satisfies the following two properties:

1. **Separability**. Domestic production inputs are determined independently of transfers and continuation values.

2. **Transferability**. There exists a match surplus that is shared between spouses, and matching requires positive surplus.

Separability

The first order conditions of the Nash bargaining problem with respect to domestic production are

$$\frac{1}{w_i}\frac{\partial \ln F^1_{ij}(d_m,d_f)}{\partial d_m} = \frac{1}{w_j}\frac{\partial \ln F^1_{ij}(d_m,d_f)}{\partial d_f} = \frac{1}{X_{ij}(d_m,d_f)},$$

where $X_{ij}(d_m,d_f) = w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - C_{ij}$ is the *net total private expenditure*, i.e. what is left of total family income $w_i + w_j$ to be spent on private consumption and leisure after spending $w_i d^1_m + w_j d^1_f + C_{ij}$ on home production, above and beyond the minimal expenditures $A_i + A_j$.

These conditions deliver two functions $d^1_m(i,j), d^1_f(i,j)$ of observable match characteristics.[8] Let us simply write X_{ij} and F^1_{ij} as the equilibrium values $X_{ij}(d^1_m,d^1_f)$ and $F^1_{ij}(d^1_m,d^1_f)$. The function $(i,j,z) \mapsto zF^1_{ij}X_{ij}$ is the equilibrium value of aggregate nominal utility $B_i u_m + B_j u_f$.[9] It is the joint flow value that both spouses can split between themselves in every period.

Transferability

In Appendix A we show that the first-order conditions of the Nash bargaining problem with respect to transfers imply the following rent sharing conditions,

$$(r+\delta)B_i\left[V^1_m(z) - V^0_i\right] = \beta S_{ij}(z), \quad (r+\delta)B_j\left[V^1_f(z) - V^0_j\right] = (1-\beta)S_{ij}(z), \tag{4.12}$$

where the (nominal) match surplus $S_{ij}(z)$ solves

$$S_{ij}(z) = zF^1_{ij}X_{ij} - B_i r V^0_i - B_j r V^0_j + \frac{\delta}{r+\delta}\int S_{ij}(z')^+ \,\mathrm{d}G(z'). \tag{4.13}$$

The match surplus is equal to the difference of utility flows in marriage and bachelorhood, $zF^1_{ij}X_{ij} - B_i r V^0_i - B_j r V^0_j$, plus a supplement incorporating the expected effect of a change to the match-specific bliss shock.

Let $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \,\mathrm{d}G(z')$ denote the integrated surplus. Let $\mathcal{G}(s) \equiv \int (z-s)^+ \,\mathrm{d}G(z) = \int_s^{+\infty} z\,\mathrm{d}G(z) - s[1-G(s)]$. The function $\mathcal{G}$ is decreasing and invertible on the support of G, with $\mathcal{G}' = -(1-G)$. By integrating equation (4.13) we can show that $\overline{S}_{ij}$ solves

$$\overline{S}_{ij} = F^1_{ij}X_{ij}\,\mathcal{G}\left(\frac{B_i r V^0_i + B_j r V^0_j - \frac{\delta}{r+\delta}\overline{S}_{ij}}{F^1_{ij}X_{ij}}\right). \tag{4.14}$$

[8] These demand functions are independent of z because the production function for couples is proportional to z. This is a simplifying condition that is not essential for separability.

[9] Indeed, $B_i u_m + B_j u_f = q\left[w_i(1-d_m) - t_m + w_j(1-d_f) - t_f\right] = qX_{ij}$.

The matching probability becomes

$$\begin{aligned}\alpha_{ij} &= \Pr\{S_{ij}(z) > 0\} \\ &= 1 - G\left(\frac{B_i r V_i^0 + B_j r V_j^0 - \frac{\delta}{r+\delta}\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right) = 1 - G\left[\mathcal{G}^{-1}\left(\frac{\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right)\right]. \end{aligned} \tag{4.15}$$

The second equality results from equation (4.13) and the last one uses equation (4.14). Moreover, equation (4.6) for the value of singlehood yields

$$B_i r V_i^0 = B_i u_i^0 + \frac{\lambda\beta}{r+\delta}\int \overline{S}_{ij} n_f(j)\,\mathrm{d}j, \quad B_j r V_j^0 = B_j u_j^0 + \frac{\lambda(1-\beta)}{r+\delta}\int \overline{S}_{ij} n_m(i)\,\mathrm{d}i, \tag{4.16}$$

respectively for single men and women.

Lastly, we also show in Appendix A that equilibrium transfers $t_m(i,j,z)$ and $t_f(i,j,z)$ are a way of sharing net total instantaneous utility $B_i u_m + B_j u_f = zF_{ij}^1 X_{ij}$. That is

$$B_i u_m = B_i r V_i^0 + \beta\left[zF_{ij}^1 X_{ij} - B_i r V_i^0 + B_j r V_j^0\right]. \tag{4.17}$$

Alternatively, dividing by zF_{ij}^1, we obtain transfers as a way of sharing net total private expenditure X_{ij} :

$$w_i(1-d_m^1) - t_m - A_i = \beta_{ij}(z)\,X_{ij}, \quad w_j(1-d_f^1) - t_f - A_j = [1-\beta_{ij}(z)]\,X_{ij}, \tag{4.18}$$

where the *collective sharing rule* $\beta_{ij}(z)$ can be derived from equation (4.17) as

$$\beta_{ij}(z) = \beta + \frac{(1-\beta)B_i r V_i^0 - \beta B_j r V_j^0}{zF_{ij}^1 X_{ij}}. \tag{4.19}$$

The great advantage of this expression vis-à-vis equation (4.17) is that in equation (4.17) surplus sharing proceeds from both the bargaining coefficient β and the threat points $B_i r V_i^0, B_j r V_j^0$. In equation (4.19) a single parameter, $\beta_{ij}(z)$, suffices to characterize rent sharing given match type (i,j,z).

Equilibrium

The equilibrium is fully characterized by the following functions of individual types, $\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0, n_m(i), n_f(j), \alpha_{ij}$. They are obtained by iterating the fixed-point operator defined by equations (4.10), (4.14), (4.15) and (4.16) for $\lambda = \lambda(N_m, N_f)$ with $N_m = \int n_m(i)\,\mathrm{d}i$ and $N_f = \int n_f(j)\,\mathrm{d}j$. Proving equilibrium existence is difficult (see Shimer and Smith 2000, Atakan 2006, Lauermann and Nöldeke 2015, Manea 2017) and the equilibrium is likely not unique.

To calculate the equilibrium numerically we discretize the value and density functions on a Chebyshev grid and we use Clenshaw-Curtis quadrature to approximate the integrals. Then, the equilibrium is obtained by iterating the fixed point operator (see Appendix D for details).

5 Specification, identification and estimation

In this section we specify parametrically the utility functions and the production functions, and explain how we let them depend on individual types. Then, we develop a constructive identification proof and explain the estimation procedure.

5.1 Parametric specification

The way parameters depend on exogenous variables (gender g_i, education Ed_i, wage w_i and family values index FVI_i) is specified as follows. Let $x_i = (Ed_i, FVI_i)$ be the vector of observed socio-demographics.

Meeting rates

The meeting function is Cobb-Douglas: $\lambda(N_m, N_f) = \xi(N_m N_f)^{-1/2}$.

Preferences

Males' indirect utility for consumption and leisure is such that

$$A_i = a_{0i} + a_{1i}w_i + \frac{1}{2}a_{2i}w_i^2, \quad \ln B_i = b_i \ln w_i,$$

where all parameters may depend on x_i: $a_{0i} = a_{0m}(x_i), a_{1i} = a_{1m}(x_i), a_{2i} = a_{2m}(x_i)$ and $b_i = b_m(x_i)$.[10] Leisure expenditure follows as

$$w_i e_m = a_{1i}w_i + a_{2m}w_i^2 + b_i(R_m - A_i), \quad R_m = w_i(1 - d_m) - t_m, \tag{5.1}$$

given domestic time d_m and transfer t_m ($t_m = 0$ for singles). Consumption is then

$$c_m = R_m - w_i e_m = a_{0i} - \frac{1}{2}a_{2m}w_i^2 + (1 - b_i)(R_m - A_i). \tag{5.2}$$

For women we obtain similar expressions with j and f in place of i and m.

[10] The consumption good is the numeraire. With a non-unitary price p for c, A and B become $A = a_0 p + a_1 w + \frac{1}{2}a_2 w^2/p$ and $\ln B = b \ln w + (1 - b)\ln p$. Concavity of the cost function implies $a_2 < 0$.

Domestic production

The domestic production functions are Stone-Geary:

$$[couples] \quad F^1_{ij}(d_m, d_f) = Z_{ij}\left(d_m - D^1_i\right)^{K^1_m}\left(d_f - D^1_j\right)^{K^1_f}, \tag{5.3}$$

$$[singles] \quad F^0_i(d_m) = (d_m - D^0_i)^{K^0_m}, \quad F^0_j(d_f) = (d_f - D^0_j)^{K^0_f}, \tag{5.4}$$

Note the absence of TFP parameter in front of the home production for singles. This is a normalizing constraint that is rendered necessary by the ordinal nature of preferences (see footnote 7). The TFP parameter Z_{ij} measures the quality of the public good that is produced in the household. We let the D-parameters be general functions of characteristics: $D^{0,1}_i = D^{0,1}_m(x_i), D^{0,1}_j = D^{0,1}_f(x_j)$. Moreover, home production expenditure C_{ij} and quality Z_{ij} are functions of both spouses' characteristics: $C_{ij} = C(x_i, x_j)$, $Z_{ij} = Z(x_i, x_j)$. In the estimation, to make the evaluation of complementarities easier to interpret, we will exclude interactions between x_i and x_j from C_{ij}. We could also let elasticities $K^{0,1}_{m,f}$ depend on x_i, x_j.

The times used in home production are therefore, for singles,

$$d^0_m = D^0_i + \tfrac{K^0_m}{1+K^0_m}\left(1 - D^0_i - A_i/w_i\right), \quad d^0_f = D^0_j + \tfrac{K^0_f}{1+K^0_f}\left(1 - D^0_j - A_j/w_j\right), \tag{5.5}$$

and for couples,

$$d^1_m = D^1_i + K^1_m X_{ij}/w_i, \quad d^1_f = D^1_j + K^1_f X_{ij}/w_j, \tag{5.6}$$

with net total private expenditure being

$$X_{ij} = \frac{w_i(1 - D^1_i) + w_j(1 - D^1_j) - C_{ij} - A_i - A_j}{1 + K^1_i + K^1_j}. \tag{5.7}$$

The equilibrium domestic productions are, for singles,

$$F^0_i = \left[\tfrac{K^0_m}{1+K^0_m}\left(1 - D^0_i - A_i/w_i\right)\right]^{K^0_m}, \quad F^0_j = \left[\tfrac{K^0_f}{1+K^0_f}\left(1 - D^0_j - A_j/w_j\right)\right]^{K^0_f}, \tag{5.8}$$

and for couples,

$$F^1_{ij} = Z_{ij}\left(\frac{K^1_m}{w_i}\right)^{K^1_m}\left(\frac{K^1_f}{w_j}\right)^{K^1_f} X_{ij}^{K^1_m + K^1_f}. \tag{5.9}$$

This specification is quite flexible. Suppose that some exogenous factor increases D^1_j (say family values). Then the wife will increase her non-market hours and the husband will reduce his own input to home production (because of the induced decrease of X_{ij}). Minimal inputs (D^1_i, D^1_j) thus govern home-production specialization. At the same time, domestic output F^1_{ij} should fall following the decrease of X_{ij}. It is therefore important to allow the factors determining minimal inputs to determine public good quality Z_{ij} or living cost C_{ij} at the same time in order to decouple the level of home production from the division of labor in home production.

Lastly, the distribution of match-specific shocks z is assumed to be log-normal: $G(z) = \Phi(\ln z/\sigma)$, where Φ is the standard normal CDF and σ is the standard deviation of z. We then have

$$\mathcal{G}(s) = \int (z-s)^+ \, \mathrm{d}G(z) = -s\Phi\left(-\frac{\ln s}{\sigma}\right) + e^{\frac{\sigma^2}{2}}\Phi\left(-\frac{\ln s}{\sigma} + \sigma\right).$$

5.2 Identification

Although identification may hold under far less restrictive assumptions, we only discuss identification under the preceding parametric restrictions. We first show that rates λ and δ determining meetings and divorce risk and the conditional matching probability α_{ij} are identified from marriage and divorce flows. Then we show that the parameters of preferences and home production are identified in much the same way as in standard labor supply model, the other spouse's earnings acting as a source of non-earned income allowing to identify income effect parameters. For this to work, we need the sharing rule to exhibit residual variation conditional on spouses' socio-demographic characteristics and wages. The source of variation is here the match-specific bliss variable z, which is unobserved to the econometrician. Third, the bargaining parameter β and the variance of match-specific shocks are identified from the first and second-order moments of married couples' leisure demands. Lastly, the public good quality parameter Z_{ij} is identified from the structural link between public good quality and matching probability.

Step 1: Meetings and matching probability

The number of new marriages (or cohabitations) of type (i, j) per unit of time is

$$MF(i,j) = \lambda \, n_m(i) \, n_f(j) \, \alpha_{ij}. \tag{5.10}$$

We observe the flows $MF(i,j)$ and the stocks $n_m(i), n_f(j)$. We are thus faced with the usual inference problem of disentangling λ from α_{ij}, the meeting rate from the matching probability.

The solution lies in the common mechanism linking marriage and divorce flows. Divorce occurs when the last draw of z (occurring at rate δ) ceases to satisfy the matching rule (with probability $1-\alpha_{ij}$). It follows that the flow of divorces per unit of time is

$$DF(i,j) = m(i,j) \, \delta \, (1-\alpha_{ij}). \tag{5.11}$$

Define $MR(i,j) \equiv \frac{MF(i,j)}{n_m(i) n_f(j)}$, the marriage rate (by potential match type), and $DR(i,j) \equiv$

$\frac{DF(i,j)}{m(i,j)}$, the divorce rate. Eliminating α_{ij} from equations (5.10) and (5.11) yields

$$\frac{MR(i,j)}{\lambda} + \frac{DR(i,j)}{\delta} = 1, \tag{5.12}$$

Under the assumption that α_{ij} is not a constant independent of couple characteristics, $MR(i,j)$ and $DR(i,j)$ are not colinear, and this equation identifies $1/\lambda$ and $1/\delta$. Then, α_{ij} follows as

$$\alpha_{ij} = \frac{\delta MR(i,j)}{\delta MR(i,j) + \lambda DR(i,j)}. \tag{5.13}$$

Step 2: Preferences and home production

Fix socio-demographics (x_i, x_j). Given x_i, x_j all parameters are thus only gender-specific or constant. All remaining variation is due to wages (w_i, w_j).

First consider time uses for singles:

$$w_j e_f^0 = a_{1f} w_j + a_{2f} w_j^2 + b_f \left[w_j(1 - d_j^0) - A_j \right], \quad A_j = a_{0f} + a_{1f} w_j + \tfrac{1}{2} a_{2f} w_j^2, \tag{5.14}$$

$$w_j d_f^0 = w_j D_f^0 + \tfrac{K_f^0}{1+K_f^0} \left[w_j(1 - D_f^0) - A_j \right]. \tag{5.15}$$

The leisure equation is identified if there is exogenous variation in non market time d_f^0. This is the usual identification issue with female labor supply models, which are identified if a source of non-earned income is observed, usually husband's earnings. We are going to show that this argument still applies (in a way) in our context.

Consider married couples. Domestic time expenditures are

$$w_i d_m^1 = w_i D_m^1 + K_m^1 X_{ij}, \quad w_j d_f^1 = w_j D_f^1 + K_f^1 X_{ij}, \tag{5.16}$$

with

$$X_{ij} = \frac{w_i(1 - D_m^1) + w_j(1 - D_f^1) - C - A_i - A_j}{1 + K_m^1 + K_f^1}. \tag{5.17}$$

The leisure expenditures of husband and wife are

$$w_i e_m^1 = a_{1m} w_i + a_{2m} w_i^2 + b_m \beta_{ij}(z) X_{ij}, \tag{5.18}$$

$$w_j e_f^1 = a_{1f} w_j + a_{2f} w_j^2 + b_f [1 - \beta_{ij}(z)] X_{ij}, \tag{5.19}$$

The sharing rule $\beta_{ij}(z)$ is a function of wages w_i, w_j and z given observed spouse types x_i and x_j. Eliminating $\beta_{ij}(z)$ out of equations (5.18) and (5.19), we obtain the following structural link between the spouses' leisure supplies:

$$w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1 = a_{1m} w_i + a_{2m} w_i^2 + \frac{b_m}{b_f} \left(a_{1f} w_j + a_{2f} w_j^2 \right) + b_m X_{ij}. \tag{5.20}$$

We show in Appendix B that all preference and home production parameters are identified from equations (5.15) and (5.14) (singles) and (5.16) and (5.20) (couples). In particular,

1. The match-specific source of variation in the sharing rule $\beta_{ij}(z)$, namely z, allows to identify b_m/b_f by regressing $w_i e_m^1$ on $w_j e_f^1$, keeping wages constant.

2. Preference and home production parameters a_{1m}, a_{1f}, a_{2m}, a_{2f}, b_m, b_f, D_m^1, D_f^1, K_m^1, K_f^1 and C are identified from the nonlinear link between between expenditures $w_i d_m^1$, $w_j d_f^1$, $w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1$ and wages w_i, w_j.

3. Parameters K_m^0, K_f^0 and a_{0m}, a_{0f} are identified from the time uses of singles.

Step 3: Sharing rule and the distribution of match-specific shocks

At this stage the sharing rule $\beta_{ij}(z)$ is identified from leisure expenditures using equations (5.18) or (5.19). One can thus calculate conditional means and variances of $\beta_{ij}(z)$ across married couples given their type (i,j), say

$$\mathbb{E}[\beta_{ij}(z)|i,j,S_{ij}(z)\geq 0]=\overline{\beta}_{ij} \quad \text{and} \quad \mathbb{V}[\beta_{ij}(z)|i,j,S_{ij}(z)\geq 0]=\sigma_{ij}^2.$$

The match formation condition is $S_{ij}(z)\geq 0$. However, the matching probability α_{ij} is already identified. Therefore, the matching condition on z can be also written as $z\geq G^{-1}(1-\alpha_{ij})$. Next, remember that

$$\beta_{ij}(z)=\beta+\frac{1}{z}\frac{(1-\beta)B_i r V_i^0-\beta B_j r V_j^0}{F_{ij}^1 X_{ij}}.$$

Hence,

$$\overline{\beta}_{ij}=\beta+\mathbb{E}\left[\frac{1}{z}|z\geq G^{-1}(1-\alpha_{ij})\right]\frac{(1-\beta)B_i r V_i^0-\beta B_j r V_j^0}{F_{ij}^1 X_{ij}}, \tag{5.21}$$

and

$$\sigma_{ij}^2=\mathbb{V}\left[\frac{1}{z}|z\geq G^{-1}(1-\alpha_{ij})\right]\left(\frac{(1-\beta)B_i r V_i^0-\beta B_j r V_j^0}{F_{ij}^1 X_{ij}}\right)^2. \tag{5.22}$$

where the conditional mean and variance of $1/z$ can easily be calculated under the log-normal assumption (or any other parametric specification).[11]

[11]Specifically, for $\ln z\sim\mathcal{N}(0,\sigma^2)$,

$$\mathbb{E}\left(\frac{1}{z}|z>s\right)=e^{\frac{\sigma^2}{2}}\Phi\left(-\frac{\ln s}{\sigma}-\sigma\right)/\Phi\left(-\frac{\ln s}{\sigma}\right),$$
$$\mathbb{E}\left(\frac{1}{z^2}|z>s\right)=e^{2\sigma^2}\Phi\left(-\frac{\ln s}{\sigma}-2\sigma\right)/\Phi\left(-\frac{\ln s}{\sigma}\right).$$

Hence, the following restriction must hold for all (i,j),

$$\overline{\beta}_{ij} = \beta + \frac{\mathbb{E}\left[1/z|z \geq G^{-1}(1-\alpha_{ij})\right]}{\sqrt{\mathbb{V}\left[1/z|z \geq G^{-1}(1-\alpha_{ij})\right]}}\sigma_{ij}. \tag{5.23}$$

This is a nonlinear equation that should identify β and σ. This statement is difficult to prove because of the nonlinearity of this equation with respect to σ^2 that ensues from marriage selection. However, suppose that there exists a set of match types (i,j) such that α_{ij} is close to one, then, under the log-normal assumption,

$$\frac{\mathbb{E}\left[1/z|z \geq G^{-1}(1-\alpha_{ij})\right]}{\sqrt{\mathbb{V}\left[1/z|z \geq G^{-1}(1-\alpha_{ij})\right]}} \simeq \frac{e^{\sigma^2/2}}{e^{2\sigma^2}-e^{\sigma^2}}$$

is a decreasing function of e^{σ^2}. In this region, equation (5.23) identifies β and σ.

Step 4: Public good quality

The last parameter to identify is public good quality Z_{ij}. For this we need to dig into the model more deeply. By substituting out $F_{ij}^1 X_{ij}$ from equations (4.14) and (4.15), one can show that

$$\frac{\delta}{r+\delta}\overline{S}_{ij} = \left(B_i r V_i^0 + B_j r V_j^0\right)\theta_{ij}, \tag{5.24}$$

with

$$\theta_{ij} \equiv \frac{\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}{\frac{r+\delta}{\delta}G^{-1}(1-\alpha_{ij}) + \mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}.$$

Parameter θ_{ij} is identified as α_{ij} and G are already identified.

This in turn implies that $\left(B_i r V_i^0, B_j r V_j^0\right)$ is the solution to the Fredholm equation of the second kind:

$$B_i r V_i^0 = B_i u_i^0 + \frac{\lambda}{\delta}\beta\int\left(B_i r V_i^0 + B_j r V_j^0\right)\theta_{ij}\, n_f(j)\,\mathrm{d}j, \tag{5.25}$$

$$B_j r V_j^0 = B_j u_j^0 + \frac{\lambda}{\delta}(1-\beta)\int\left(B_i r V_i^0 + B_j r V_j^0\right)\theta_{ij}\, n_m(i)\,\mathrm{d}i. \tag{5.26}$$

Assuming a unique solution for $\left(B_i r V_i^0, B_j r V_j^0\right)$, equation (5.24) identifies $\overline{S}_{ij}$ and equation (4.15) then identifies $F_{ij}^1 X_{ij}$. The identification of Z_{ij} finally results from equilibrium home production F_{ij}^1:

$$F_{ij}^1 = Z_{ij}\left(\frac{K_m^1}{w_i}\right)^{K_m^1}\left(\frac{K_f^1}{w_j}\right)^{K_f^1} X_{ij}^{K_m^1+K_f^1}. \tag{5.27}$$

Now, there will be a unique solution for $\left(B_i r V_i^0, B_j r V_j^0\right)$ to equation (5.25)-(5.26) if the right-hand side's linear integral operator is contracting. A sufficient condition for that is $\lambda\beta\int\theta_{ij}\, n_f(j)\,\mathrm{d}j < \delta$ and $\lambda(1-\beta)\int\theta_{ij}\, n_m(i)\,\mathrm{d}i < \delta$. We have $0 \leq \theta_{ij} \leq 1$. We will estimate meeting rates $\lambda N_m \simeq \lambda N_f \simeq 0.15, \delta = 0.038$ and $\beta \simeq 0.45$. So $\lambda N_m \beta/\delta \simeq 2$. We will estimate the mean of θ_{ij}, averaged across i and j among singles, below $1/2$. This is sufficient for identification.

5.3 Estimation strategy

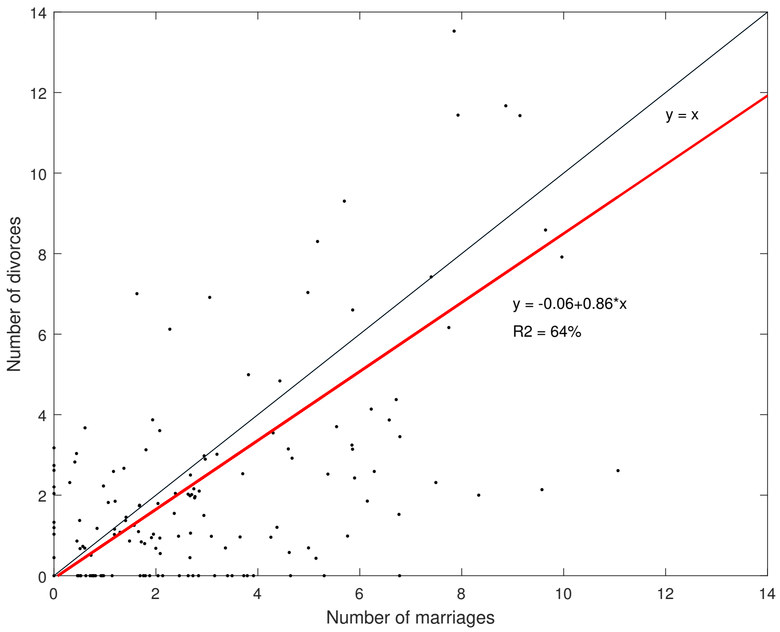
We use household data on time uses, gender, wages, family values and education covering the period 1991-2008. The index i now refers to an observation unit of the sample of male singles, j refers to female singles, and (i, j) refers to couples. For singles, we observe domestic time use d_i^0, labor supply h_i^0, and education $Ed_i \in \{L, M, H\}$ (O-level, A-level and higher education), wages w_i and family values indices FVI_i. For couples, the corresponding time use observations are $d_{mij}^1, d_{fij}^1, h_{mij}^1$ and h_{fij}^1. Leisure is $e = 1 - d - h$. The estimation procedure is iterative and goes through the following steps.

Step 1: Meetings and matching probability

We split the whole sample into 6 three-year periods: 91-93, 94-96, 97-99, 00-02, 03-05, 06-08. For each sub-period, pooling the three cross-sections, we compute kernel density estimates of $n_m(i)$, $n_f(j)$ and $m(i, j)$ from the stocks of male singles, female singles and married couples on a grid of values for i and j comprising three education categories, 16 Chebyshev nodes for wages and 8 Chebyshev nodes for the family values index. As explained in Appendix D, interpolating smooth functions on bounded sets between Chebyshev points by Discrete Cosine Transform (DCT) allows approximating the functions by very high-order multivariate Chebyshev polynomials (of order the number of grid points minus one).

We then calculate marriage and divorce flows $MF(i, j), DF(i, j)$ by education only because of the small sample sizes of flows. For example, we calculate the number of new marriages for 1994 as the number of singles in 1994 who are married in 1995, and the flow of divorces for 1994 as the number of singles in 1994 who were married in 1993. Then we add the numbers for 1994, 1995 and 1996 to get the aggregate flows for the period 1994-1996. Using equation (5.12) and the Cobb-Douglas specification of the meeting rate $\lambda = \xi(N_m N_f)^{-1/2}$, we estimate $1/\xi$ and $1/\delta$ by OLS, pooling all four intermediate sub-periods 94-96, 97-99, 00-02, 03-05 together and eliminating the two extreme periods (91-93 and 06-08) because of boundary issues (no divorce flow can be calculated in 1991 and no marriage flow in 2008).[12] We use this approach to estimating rates instead of a duration model because of censoring (right-censoring for marriage and left- and right-censoring for singlehood).

[12]Note also that running the regression with or without weighting observations (i, j) by their sample size $\ell_m(i)\, \ell_f(j)$ gives close estimates.



Notes: 1) To make this graph, we calculated marriage and divorce flows by education (3 categories) and FVI (2 categories: above or below the median FVI), and we considered 4-year subperiods (92-95, 96-99, 00-03, 04-07). We obtain $3 \times 3 \times 2 \times 2 \times 4 = 144$ points. 2) The regression line is obtained by regressing $DF(i,j)$ on $MF(i,j)$, weighting each point (i,j) by their sample representativeness, measured as $\ell_m(i)\,\ell_f(j)$.

Figure 7: Link between new marriages and divorces by periods and match types

Then we could estimate a different matching probability α_{ij} for each sub-period using equation (5.13). However, the model assumes that households anticipate steady-state equilibrium distributions. If we impose equal inflows and outflows, $MF(i,j) = DF(i,j)$, matching probabilities follow from equation (4.7) as

$$\alpha_{ij} = \frac{\delta m(i,j)}{\delta m(i,j) + \lambda n_m(i) n_f(j)}. \tag{5.28}$$

Equation (5.28) is not only consistent with the way households are supposed to form expectations, it also makes the estimation of matching probabilities much more precise as stock samples are much bigger than flow samples.[13] Figure 7 displays the empirical link between marriage and divorce flows $MF(i,j)$ and $DF(i,j)$. There is clearly a lot of noise and the regression line is a bit off the 45 degree line. But there is no clear structure that would indicate that equation (5.28) is not a good approximation of the true marriage probability.

[13]Estimates of $1/\lambda$ and $1/\delta$ rely on flows; there is no other way. But they are more precisely estimated than matching probabilities because the regression procedure averages over types (i,j), which smoothes out small sample errors.

Step 2: Preferences, home production and sharing

We then estimate the parameters of preferences and domestic productions, as well as the Nash bargaining parameter β and match dispersion σ, by nonlinear least squares. The full list of residuals (derived from first and second order moments of leisure and domestic times) is provided in Appendix C. We restrict parameters a_{0i}, a_{1i} and b_i to be linear functions of characteristics x_i, and we set a_{2m} constant given gender. We also constrain C_{ij} further to be linear in x_i and x_j, whereas Z_{ij} is estimated as a flexible high-order polynomial in (x_i, x_j) allowing for all interactions.

6 Empirical results

In this section, we first describe parameter estimates and the fit. Then we discuss the implied sharing rule. We end the section by various counterfactual analyses.

6.1 Parameter estimates and fit

Meeting rates

We estimate $\xi = 0.1515$ and nearly identical meeting rates for singles ($\lambda N_m = \xi(N_m/N_f)^{1/2}$ is close to $\lambda N_f = \xi(N_f/N_m)^{1/2}$ as $N_m \simeq N_f$). This means a median duration between two consecutive datings of about 4.5 years. This may seem a long time but remember that we pool together all ages between 22 and 50 years. "Serious" datings between singles willing to ask themselves whether they could embark in a long term relationship may be a rare event. We estimate the yearly probability of a bliss shock to be $\delta = 0.0378$, or a median duration between two consecutive shocks of 18 years.[14]

Marriage probabilities

We start with the marriage probabilities estimated using equation (5.28). Table 2 displays mean matching probabilities by education, *FVI* and wages quartiles for three periods. The first observation is that the sorting patterns are very stable. Secondly, the sorting patterns are quite different by education, family values or wages. Yet, all three patterns express different forms of positive assortative mating.

The matching probability matrix for education is symmetric with a strong main diagonal. One could change the ordering of rows and columns without changing the matrix very much. It is nearly bisymmetric. This is the usual form of homophily.

[14] There is a bit of attrition across time in the panel. If we keep in the stocks only singles who are still in the panel the following year, and couples who were already there the year before, we estimate $\xi = 0.1634$ and $\delta = 0.0391$. The results are not substantially affected.

Table 2: Sorting by education and wages

		Female								
		1991-1993			2000-2002			2006-2008		
		<HS	HS	>HS	<HS	HS	>HS	<HS	HS	>HS
Male	<HS	0.49	0.31	0.21	0.49	0.42	0.23	0.47	0.46	0.26
	HS	0.34	0.35	0.23	0.41	0.42	0.26	0.44	0.47	0.32
	>HS	0.21	0.17	0.39	0.24	0.32	0.42	0.18	0.33	0.43

(a) By education

		Female											
		1991-1993				2000-2002				2006-2008			
		Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1
Male	Q4	0.51	0.49	0.41	0.30	0.59	0.47	0.42	0.31	0.58	0.49	0.46	0.36
	Q3	0.50	0.48	0.42	0.32	0.49	0.44	0.41	0.34	0.49	0.46	0.44	0.36
	Q2	0.45	0.44	0.41	0.35	0.45	0.42	0.40	0.35	0.45	0.44	0.43	0.36
	Q1	0.37	0.37	0.38	0.37	0.38	0.38	0.38	0.36	0.39	0.39	0.39	0.35

(b) By *FVI*

		Female											
		1991-1993				2000-2002				2006-2008			
		Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1
Male	Q4	0.50	0.49	0.48	0.42	0.52	0.50	0.48	0.44	0.59	0.55	0.52	0.44
	Q3	0.43	0.44	0.43	0.39	0.50	0.49	0.48	0.45	0.51	0.49	0.47	0.40
	Q2	0.41	0.42	0.41	0.37	0.40	0.42	0.42	0.41	0.41	0.42	0.41	0.35
	Q1	0.29	0.31	0.31	0.30	0.25	0.28	0.30	0.31	0.27	0.30	0.30	0.27

(c) By wage quartile

Notes: Q1 is the interval below the first quartile, Q2 between first and second quartile, etc.

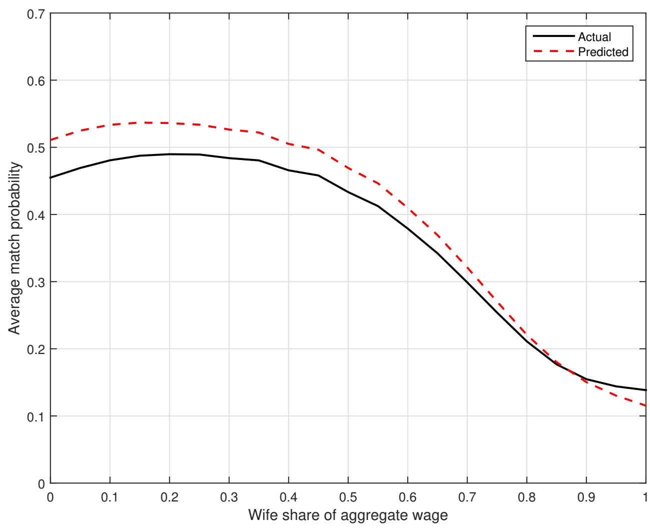


Figure 8: Match probability conditional on woman wage ratio – mean over 1991-2008

The matching probability matrix for FVI is also symmetric, but it is far from being bisymmetric. The Q1 row and the Q1 columns are identical, with nearly identical entries. This means that progressive individuals are rather indifferent to the family values of their partner. On the other end, the Q4 row and the Q4 column are also similar, but with decreasing entries. Traditionalist individuals value traditionalist partners.

For wages it is yet again different. There is no symmetry. To really push the point, we could say that female wage does not matter for matching (though richer women marry richer men more often). Male wage comparatively matters a lot; males with a high wage (in the Q4 interval) marry twice more frequently than males with a low wage (Q1).

This is interesting in relation to the social norm discussed by Bertrand, Kamenica, and Pan (2015). Figure 8 (solid line) plots the estimated matching probability α_{ij} as a function of the female wage ratio ($\frac{w_f}{w_m+w_f}$). The matching probability is decreasing with the female wage ratio for values of this ratio above 0.5, whereas it is more or less constant for values less than 0.5. However, matching probability estimates indicate that this is likely because low-wage males are just not attractive in general, and not because of a social norm preventing marriages with the husband earning less than the wife.

Preferences and home production estimates

We now comment on the preference and home production parameter estimates displayed in Table 3. The effect of education on preferences and home production (a, b, D) do not differ much between men and women. More educated individuals value consumption more than leisure (parameter a_0). Education also increases the income effect of leisure (parameter b). There is little effect of education on home production.

Family values, on the contrary, have little effect on preferences. The only significant parameter is $a_{1f}[FVI]$: traditionalist females supply less market work; but even for $FVI = 5$ the effect is lower than 0.1 hours. However, family values determine home production quite differently for men and women. Traditionalist women demand more leisure (parameter a_{1f}) and home production time ($D_f^{0,1}$), while traditionalist men (when married) want to spend less time in home production (D_m^1).

There is thus a correlation between family attitudes, as they can be measured by the questionnaire, and task specialization, some individuals being more prone than others to adopt the traditional family model, where the wife specializes in home-production and the husband in market work.

Public good quality, sorting and complementarities

The standard deviation of the logged match-specific bliss shock is estimated at 0.2684 (with a standard error of 0.1114), which is far from negligible. It moves home production in a $[-41\%, +69\%]$ interval around its equilibrium value F_{ij}^1 with 95% probability.

Table 3: Estimated preference and home-production parameters

Preferences			Home production, singles		
$a_{0f}[Ed = L]$	-21.4598	*(7.8024)*	$D_f^0[Ed = L]$	0.0524	*(0.0177)*
$a_{0m}[Ed = L]$	-26.6340	*(8.0581)*	$D_m^0[Ed = L]$	0.0467	*(0.0188)*
$a_{0f}[Ed = H]$	-12.7428	*(5.0074)*	$D_m^0[Ed = H]$	0.0481	*(0.0142)*
$a_{0m}[Ed = H]$	-13.5792	*(5.5035)*	$D_f^0[Ed = H]$	0.0363	*(0.0142)*
$a_{0f}[FVI]$	0.7455	*(0.7355)*	$D_f^0[FVI]$	0.0096	*(0.0041)*
$a_{0m}[FVI]$	0.6168	*(0.6101)*	$D_m^0[FVI]$	0.0031	*(0.0039)*
$a_{1f}[Ed = L]$	0.4403	*(0.0175)*	K_f^0	0.0177	*(0.0070)*
$a_{1m}[Ed = L]$	0.3939	*(0.0197)*	K_m^0	0.0002	*(0.0034)*
$a_{1f}[Ed = H]$	0.4350	*(0.0215)*			
$a_{1m}[Ed = H]$	0.3812	*(0.0258)*	Home production, couples		
$a_{1f}[FVI]$	0.0184	*(0.0046)*	$D_f^1[Ed = L]$	0.0701	*(0.0105)*
$a_{1m}[FVI]$	-0.0001	*(0.0050)*	$D_m^1[Ed = L]$	0.0660	*(0.0095)*
			$D_f^1[Ed = H]$	0.0564	*(0.0094)*
a_{2f}	-0.0031	*(0.0007)*	$D_m^1[Ed = H]$	0.0708	*(0.0085)*
a_{2m}	-0.0008	*(0.0005)*	$D_f^1[FVI]$	0.0159	*(0.0029)*
			$D_m^1[FVI]$	-0.0073	*(0.0025)*
$b_f[Ed = L]$	0.0303	*(0.0119)*			
$b_m[Ed = L]$	0.0345	*(0.0122)*	$C[constant]$	37.1368	*(10.0483)*
$b_f[Ed = H]$	0.0721	*(0.0248)*	$C[FVI_f]$	-1.2509	*(0.7866)*
$b_m[Ed = H]$	0.0940	*(0.0340)*	$C[FVI_m]$	-3.8962	*(1.1802)*
$b_f[FVI]$	-0.0023	*(0.0020)*	$C[Ed_f = L]$	-4.0234	*(2.3276)*
$b_m[FVI]$	-0.0000	*(0.0021)*	$C[Ed_m = L]$	-9.1052	*(3.7425)*
			K_f^1	0.0183	*(0.0038)*
			K_m^1	0.0056	*(0.0026)*

Notes: Standard errors in parentheses. $Ed = H$ indicates "high-school graduate" and $Ed = L$ indicates an education level that is less that A-level.

Table 4: Fit of matching probabilities (R^2 of regression of actual α_{ij} on predicted)

	1991	1994	1997	2000	2003	2006	Mean
Nonparametric estimation of Z_{ij}	0.72	0.83	0.86	0.87	0.82	0.78	0.81
Quadratic projection of $\ln Z_{ij}$	0.68	0.77	0.80	0.80	0.75	0.72	0.75
Quadratic projection of $\ln Z_{ij}$ without interactions	0.38	0.51	0.60	0.62	0.57	0.53	0.54
Quadratic projection of $\ln Z_{ij}$ with no interactions but...							
$w_m * w_f$	0.43	0.54	0.63	0.65	0.59	0.56	0.57
$FVI_m * FVI_f$	0.37	0.51	0.61	0.63	0.57	0.54	0.54
$Ed_m * Ed_f$	0.66	0.72	0.76	0.75	0.69	0.67	0.71
$w_m * Ed_f$	0.43	0.55	0.64	0.66	0.61	0.57	0.58
$w_f * Ed_m$	0.53	0.61	0.68	0.67	0.63	0.58	0.62
$w_m * FVI_f$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$w_f * FVI_m$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$FVI_f * Ed_m$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$FVI_m * Ed_f$	0.37	0.52	0.60	0.63	0.59	0.55	0.54

Next, we look at how the model fits marriage probabilities. Given parameters, including the nonparametrically estimated public good quality parameter (Z_{ij}), we predict marriage probabilities as indicated in Section 4.5 and compare them with the matching probabilities that we described at the beginning of this section (the "actual" ones). In Table 4 we calculate the measure of fit: $R^2 = 1 - \mathrm{Var}(\alpha - \widehat{\alpha})/\mathrm{Var}(\alpha)$, where α and $\widehat{\alpha}$ are actual and predicted matching probabilities, calculated for each couple given their characteristics. We obtain a R^2 close to 80% in all years. Figure 8 (dotted line) shows how the model fits the matching probability by male-female wage ratio.

Table 4 also shows the fit obtained with the quadratic projection of the nonparametric estimates of $\ln Z_{ij}$. By using the quadratic approximation the fit is reduced by about 7 percentage points. This shows that higher-order polynomial terms could be neglected. Then we predict marriage probabilities with a quadratic projection of Z_{ij} without male-female interaction terms. Removing male-female complementarities reduces the fit by 33%. So complementarities do matter. Interestingly, only one complementarity really seems to count, education homophily, by which individuals strongly prefer to marry within their own education group. Also, despite important changes in women's educational attainment since the early 1990s, the strength of education homophily does not seem to vary over time in any specific way.

Fit of selection and time uses

We next show how our model fits selection, i.e. the dynamics of wages and individual types by marital status. We take the changes in the distributions of wages, family values and education by gender as given and we simulate different steady state equilibria for the 6 time periods, keeping the model parameters fixed.

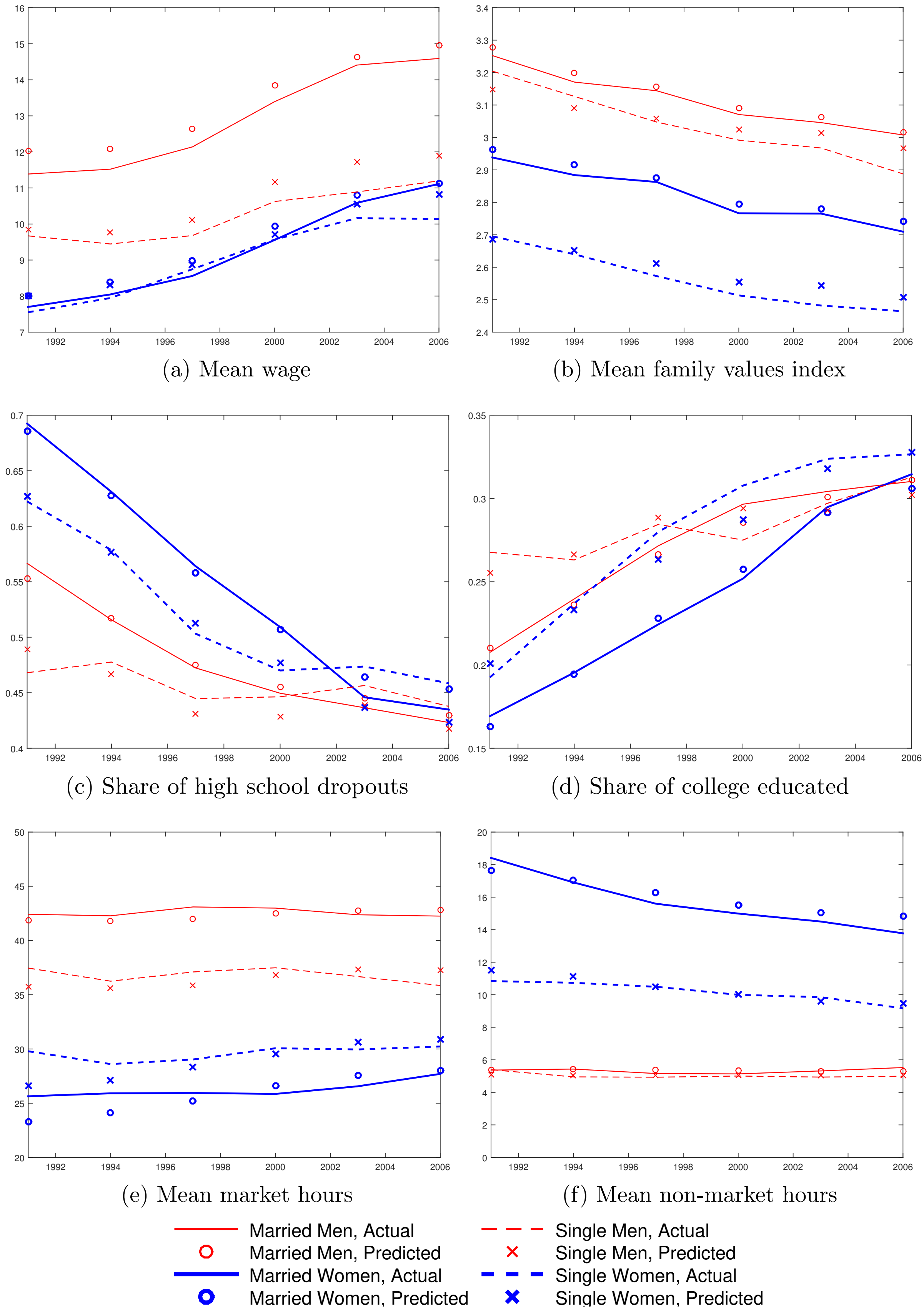
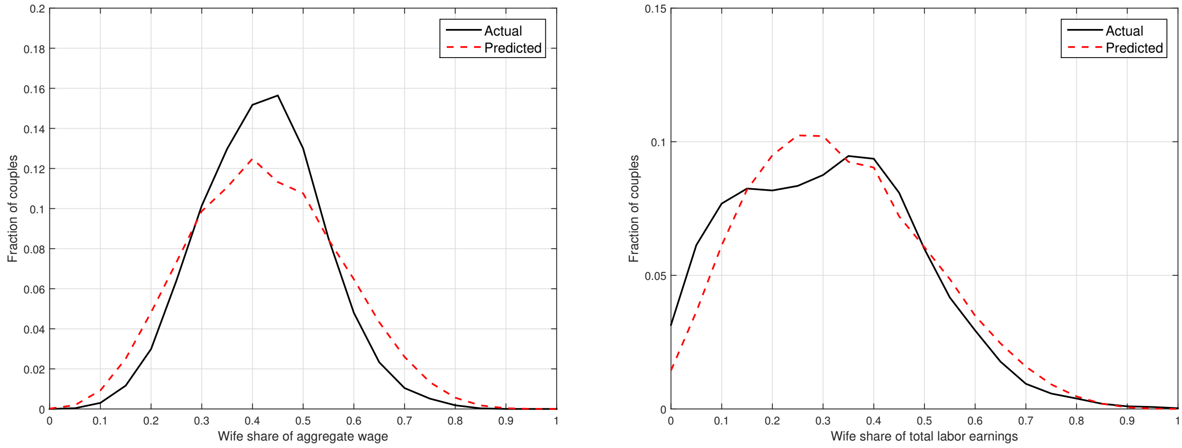
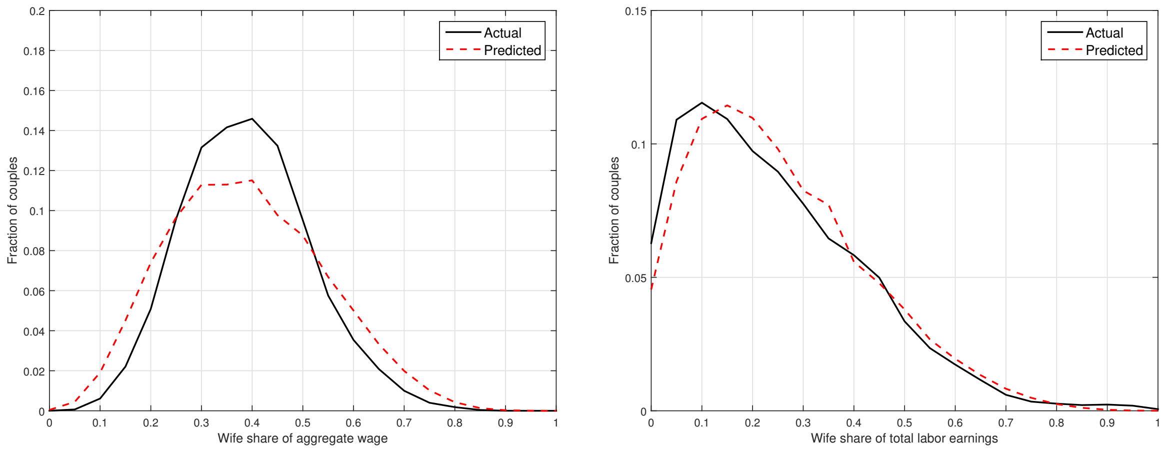


Figure 9: Fit of hours and selection

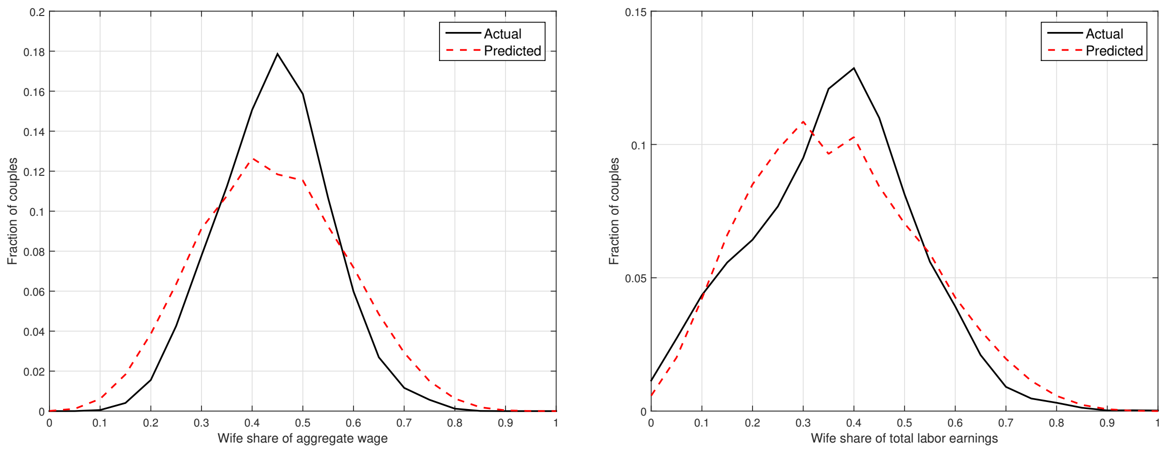
(1) Wage ratio, $\frac{w_f}{w_f+w_m}$ (2) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f+w_m h_m}$



(a) All couples



(b) Traditionalist wives and husbands



(c) Progressive wives and husbands

Figure 10: Fit of the distributions of wages and earnings ratios, 2000-2002

We first note that the difference between the actual number of married couples and its prediction is never greater than 4%, and the errors on the number of singles are correspondingly small. Then the first two rows of Figure 9 show how the model fits selection. The wages of married men are slightly overestimated and the wages of single men are slightly underestimated. Yet the different marriage wage gaps (married *vs* single) of men and women are well replicated by the model. The fit of family values indices and education by marital status over time is very good. The last row of Figure 9 shows the fit of time use trends. The trends in the market hours of married and single women are slightly overestimated, the model wanting a bit more change than is actually observed. The marriage differentials of market and non market hours are very well fitted.

Lastly, Figure 10 shows how the model fits the distributions of wage and earnings ratios (within couples) for the middle period 2000-2002, first for all couples, then for traditionalist couples (both spouses' FVIs below their medians), and finally for progressive couples (FVIs above medians). The fit is a bit worse for wages than for labor earnings but the model reproduces the observed patterns well, both overall and by family values.

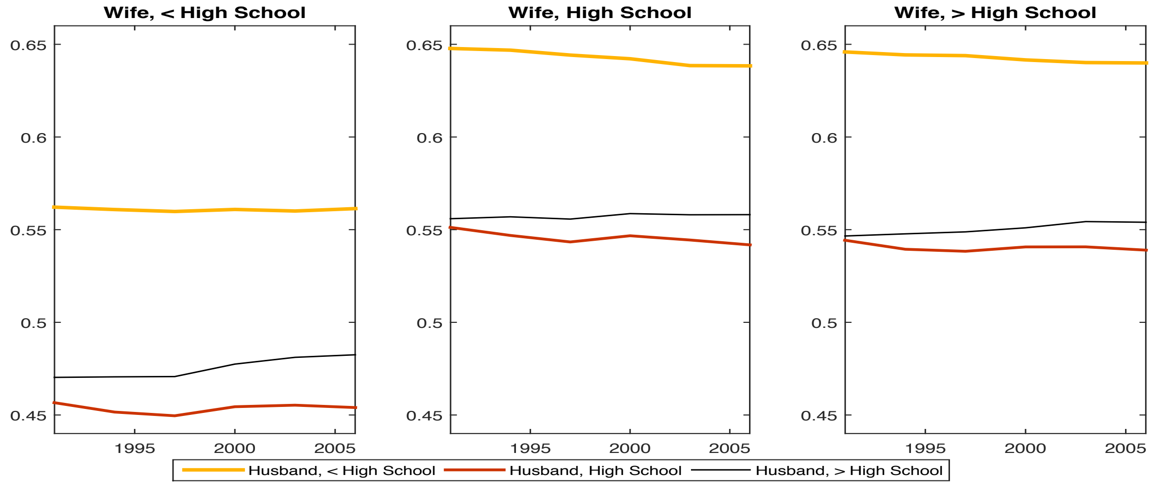
6.2 The sharing rule

The bargaining power parameter β is estimated to be 0.4482 with a standard deviation of 0.1020. However this does not imply that men get less than half of the surplus. The share of the surplus that each spouse reclaims also depends on his/her outside option (value of singlehood). The sharing rule $\beta_{ij}(z)$, i.e. the share of the net total private expenditure X_{ij} that is appropriated by the husband, is a better indicator of market power.

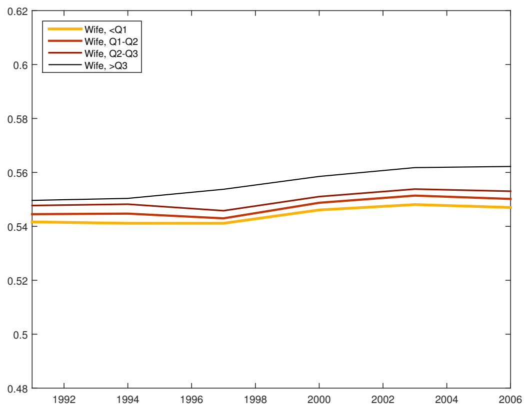
Its estimated mean value across all couples' types is remarkably stable—between 0.55 and 0.56 throughout the whole period. Hence, better outside options compensate for a bargaining coefficient that is estimated slightly less than 0.5 and give the husband a share of the rent slightly bigger than 0.5. Nevertheless, the division of labor in the family is gender-specific, but the total income that is left for private consumption and leisure is split between spouses in a way that is quite egalitarian.

In Figure 11 we show the evolution of the sharing rule over the period, separately for various household types. The husband gets a bigger share of the rent if he is less educated and if the wife is more educated. Educated females always get less than the fair share. The only case where the wife gets more than the husband is for uneducated females married to educated males. For some reason, uneducated males thus seem to be systematically compensated for this "handicap". Family values have little effect on income sharing. They affect work in household but not private income sharing.

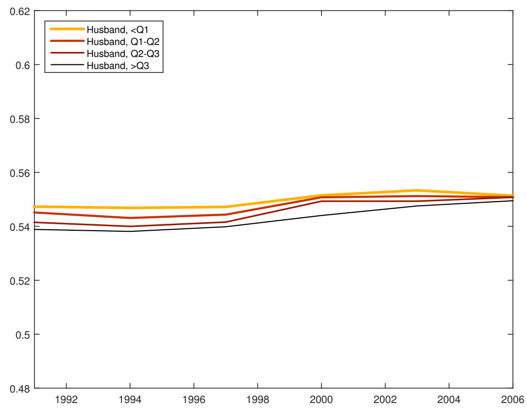
The husband gets more if his wage is higher; the wife also but to a lesser extent (first row of Figure 12). The variance of the sharing rule between spouses' wage quartiles is increasing over time for both females and males. These changes are parallel to the



(a) By spouses' education

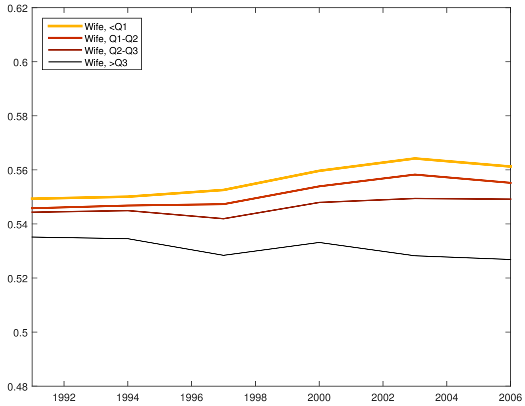


(b) By female spouse's FVI

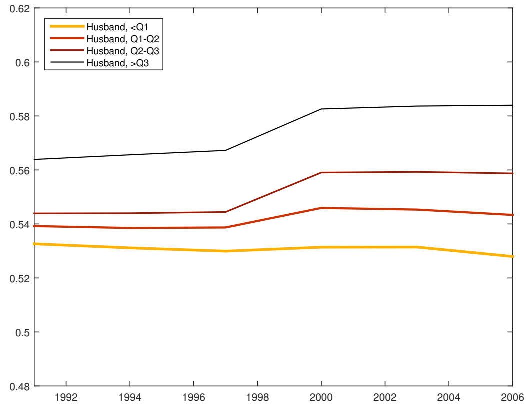


(c) By male spouse's FVI

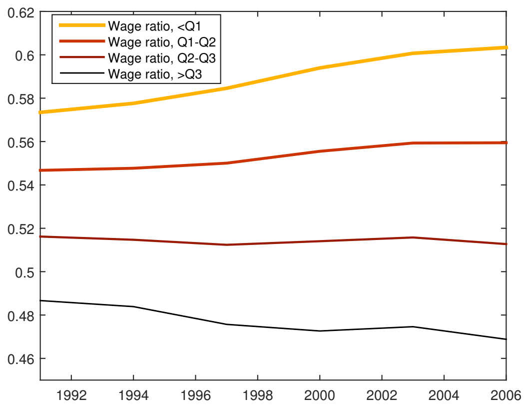
Figure 11: Mean sharing rule by education and family values



(a) By female spouse's wage



(b) By male spouse's wage



(c) By female wage ratio

Figure 12: Mean sharing rule by wages

observed changes in the wage differentials between married and single individuals (Figure 3). Lastly, Figure 12c summarizes the effect of within-household wage inequality on income sharing. The share of income that is appropriated by one spouse is higher if his/her wage ratio is higher. Moreover, this link is getting stronger over time. Hence, a consequence of the (slow) cultural change away from the traditional model of the family may be that there is progressively less redistribution between spouses.

6.3 Counterfactual changes

Wage elasticities

First, we calculate the effect of a marginal change in wages on time uses and marriages. We increase all wages in the 2000-02 sample by 10%, separately for men and women, and simulate changes in market and non market hours.[15] We first run the simulations with unchanged individual expectations of the distributions of individual types by gender and marital status, and then we simulate the new equilibrium including distributional changes. The results on hours are reported in Table 5. The first column displays the actual numbers. The second column displays the baseline simulation with the estimated parameters. The small discrepancies between the actual and baseline columns provide additional evidence on the fit of the model.

In the literature, estimates of uncompensated or Marshallian elasticities vary a lot across publications (see Blundell and Macurdy, 1999, Meghir and Phillips, 2016). Our estimates of female own-wage labor supply elasticities (0.38 for married and single women) are similar to the estimates of Blundell, Dias, Meghir, and Shaw (2016), who develop a dynamic labor supply model for female workers. They estimate an elasticity of participation rates (extensive margin) equal to 0.47 and an elasticity of hours worked (intensive margin) equal to 0.22. Our estimates of female wage elasticities of labor supply are more than twice as large as male elasticities. Cross elasticities are moderate, although women respond to an increase of husband wage by reducing their market hours and by increasing their non market hours. The elasticities of time used in home production are similar in absolute value to labor supply elasticities.

Finally we comment on the consequences of endogenizing marriage formation on the wage elasticities of hours. In comparing elasticity estimates with fixed distributions and with endogenous selection, we see that the general equilibrium effects of a change in female wage on hours are small because female wage is not a strong determinant of matching. A wage rise however renders males more attractive in the marriage market; there are more marriages and increased specialization in paid work for men and housework for women.

[15]We use this sample because it is in the middle of the period. It is also one of the biggest subsamples and by being away from the borders the moving averages of wages and hours are more accurate.

Table 5: Wage elasticities

	Actual	Baseline	Equilibrium distributions				Fixed distributions			
	2000-02	sim.	$\frac{\Delta w_m}{w_m} = 10\%$		$\frac{\Delta w_f}{w_f} = 10\%$		$\frac{\Delta w_m}{w_m} = 10\%$		$\frac{\Delta w_f}{w_f} = 10\%$	
Labor supply										
Married Men	42.99	42.49	43.02	1.3%	42.44	-0.1%	43.03	1.3%	42.39	-0.2%
Single Men	37.48	36.83	37.78	2.6%	36.78	-0.1%	37.70	2.4%	36.78	-0.1%
Married Women	25.86	26.63	26.46	-0.7%	27.64	3.8%	26.32	-1.2%	27.58	3.5%
Single Women	30.07	29.56	29.48	-0.3%	30.68	3.8%	29.89	1.1%	30.96	4.7%
Home production time										
Married Men	5.13	5.33	5.25	-1.5%	5.34	0.3%	5.24	-1.6%	5.35	0.4%
Single Men	5.00	5.04	5.04	-0.1%	5.04	0.0%	5.03	-0.2%	5.03	-0.1%
Married Women	14.99	15.52	15.64	0.8%	15.08	-2.9%	15.72	1.3%	15.11	-2.7%
Single Women	10.00	10.01	10.04	0.3%	9.73	-2.8%	9.88	-1.3%	9.61	-4.0%
Population										
# single men	1416	1468	1384	-5.7%	1466	-0.1%				
# single women	1452	1509	1458	-3.4%	1495	-0.9%				
# couples	3802	3745	3814	1.8%	3753	0.2%				

Counterfactual simulations

Finally, we simulate various structural changes to the economy. First, we consider a change in female wages suppressing the gender wage gap conditional on education and family values. Specifically, we multiply each female wage w_j by $\exp x_j(\gamma_m - \gamma_f)$, where γ_m, γ_f denote the regression coefficients of the regressions of log wages on education and FVI separately for men and women. Second, we keep the distribution of characteristics fixed and give all females in the sample the preferences and home production of males with the same characteristics. Third, we consider an economy with only progressive individuals ($FVI = 1$). Lastly, we implement the last two changes simultaneously.

Table 6 displays the results. First, wage equalization has very little demographic effects. This is because matching is essentially independent of female wages. Female market hours increase by $9, 10\%$ and non-market hours are reduced by $7, 7.5\%$ irrespective of marital status. The effect on male hours is negligible. Second, preference equalization has huge effects on both time uses and marriage rates. There are much fewer singles, and time use differences by gender are much reduced. Married men work less outside the home and married women work more. Third, changing attitudes also has dramatic effects. The number of singles increases, and married women very significantly substitute paid work to house work ($+8$ and -4 hours). Married men increase the number of hours devoted to home production by about one hour per week. Changing preferences on top of the counterfactual change in family values induces married women to supply three additional hours of market labor. Yet, because of the gender wage gap, the time use

Table 6: Counterfactual simulations

	Actual 2000-02	Baseline sim.	(1) No wage gap		(2) Same preferences		(3) $FVI = 1$ for all		(4) Same pref. & $FVI = 1$	
Labor supply										
Married Men	42.99	42.49	42.37	-0.3%	38.89	-8.5%	43.24	1.8%	40.96	-3.6%
Single Men	37.48	36.83	36.71	-0.3%	36.83	0.0%	36.49	-0.9%	36.21	-1.7%
Married Women	25.86	26.63	29.27	9.9%	34.00	27.7%	34.53	29.6%	37.65	41.3%
Single Women	30.07	29.56	32.29	9.3%	35.65	20.6%	30.64	3.7%	34.68	17.4%
Home production time										
Married Men	5.13	5.33	5.37	0.8%	6.08	14.0%	6.26	17.5%	6.71	26.0%
Single Men	5.00	5.04	5.04	-0.1%	5.09	1.0%	4.54	-9.8%	4.55	-9.8%
Married Women	14.99	15.52	14.35	-7.5%	6.97	-55.1%	11.45	-26.2%	7.32	-52.8%
Single Women	10.00	10.01	9.30	-7.1%	4.83	-51.7%	9.23	-7.8%	4.47	-55.3%
Population										
# single men	1416	1468	1475	0.5%	500	-65.9%	1586	8.0%	590	-59.8%
# single women	1452	1509	1498	-0.7%	598	-60.4%	1633	8.2%	642	-57.5%
# couples	3802	3745	3747	0.1%	4687	25.2%	3627	-3.2%	4620	23.4%

Notes: In simulation (1) (no gender wage gap) we replace each female wage w_j by $w_j \times e^{x_j(\gamma_m - \gamma_f)}$. In simulation (2) (same preferences for men and women) we give females the preferences and home production of males. In simulation (3) we make every man and woman in the sample progressive ($FVI = 1$). The last simulation does simulations (2) and (3) simultaneously. All percentage changes are with respect to baseline simulation.

differences between men and women remain substantial (about three market hours and less than one non market hour).

7 Conclusion

In this paper we build a model of marriage formation and within-household allocation of resources. By endogenizing marriage formation, we endogenize the sharing rule and its link to distribution factors. The model allows to explain how exogenous changes to wages, education and family attitudes by gender across time are transmitted to gender-specific time uses in the home and outside the home. The second important innovation is that in addition to standard wage and education characteristics we introduce a new variable measuring different attitudes regarding the family. We show that more traditional individuals indeed follow the traditional model of the family more than self-declared progressive individuals. Counterfactual analyses also show that this single variable captures a dimension of preference heterogeneity that is nearly as powerful as gender-specific preferences in explaining the gender division of labor.

Family values are a factor of gender identity (Akerlof and Kranton, 2000). They definitely shape the way economic outcomes such as labor supply or work in the household vary by gender. For lack of space, we could not consider all the outcomes studied by Bertrand, Kamenica, and Pan (2015). Nevertheless, whereas Bertrand et al. show that gender differences in labor supply or marriage duration cannot be explained without advocating some strong and slow-changing gender identity norm, we demonstrate that this norm is not uniformly shared by all individuals and households. The most progressive households behave very differently from the most conservative ones. Our structural model allows us to simulate a counterfactual economy where everybody would be progressive. The marriage rate would decline and married women would increase labor market participation very substantially—yet not to the point where married women would work as much as their husbands (if paid the same wages). So, it is likely that even the most progressive individuals still have some way to go to uphold full gender equality. How much?—In similar proportion to the difference in labor supply that is induced by the gender wage gap.

In future work it will be interesting to explore further classic family economics' topics, such as fertility and children, divorce laws, the evaluation of family tax credits, etc. In addition, our description of matching can and should be improved by introducing aging and the life cycle in the analysis. Allowing for other types of shocks and uncertainty, such as wage and unemployment shocks is also important. More generally, we should aim at understanding better the link between marriage and labor markets. For example, women who specialize in home production may be losing human capital.

APPENDIX

A Equilibrium solution with transferable utility

Match surplus. Spouses solve the Nash bargaining problem

$$\max_{c,d_m,d_f,t_m,t_f} \left[W_m - V_i^0\right]^\beta \left[W_f - V_j^0\right]^{1-\beta}$$

subject to $t_m + t_f = C_{ij}$, where

$$(r+\delta)\left[W_m - V_i^0\right] = u_m + \delta \int \left(V_m^1(z') - V_i^0\right)^+ \mathrm{d}G(z') - rV_i^0 \equiv u_m + v_m \text{ (say)},$$

$$(r+\delta)\left[W_f - V_j^0\right] = u_f + \delta \int \left(V_f^1(z') - V_j^0\right)^+ \mathrm{d}G(z') - rV_j^0 \equiv u_f + v_f,$$

with

$$u_m = q\frac{w_i(1-d_m) - t_m - A_i}{B_i}, \quad u_f = q\frac{w_j(1-d_f) - t_f - A_j}{B_j}, \quad q = zF_{ij}^1(d_m, d_f),$$

The first-order conditions for transfers t_m, t_f,

$$\frac{\beta}{(r+\delta)[W_m - V_i^0]} \frac{\partial u_m}{\partial R} = \frac{1-\beta}{(r+\delta)[W_f - V_j^0]} \frac{\partial u_f}{\partial R},$$

yield the rent sharing conditions

$$\frac{\beta}{u_m + v_m} \frac{q}{B_i} = \frac{1-\beta}{u_f + v_f} \frac{q}{B_j} \equiv \frac{q}{S_{ij}(z)} \quad \text{(say)}.$$

It follows that

$$S_{ij}(z) = B_i(u_m + v_m) + B_j(u_f + v_f) = B_i(\underline{u}_m + v_m) + B_j(\underline{u}_f + v_f) - qC_{ij},$$

using $t_m + t_f = C_{ij}$ and with

$$\underline{u}_m = q\frac{w_i(1-d_m) - A_i}{B_i}, \quad \underline{u}_f = q\frac{w_j(1-d_f) - A_j}{B_j}.$$

Domestic production inputs. The first-order condition for d_m, d_f is

$$\begin{aligned} \frac{w_i d_m}{\epsilon_m} = \frac{w_j d_f}{\epsilon_f} &= q\left[\frac{\partial u_m/\partial q}{\partial u_m/\partial R} + \frac{\partial u_f/\partial q}{\partial u_f/\partial R}\right] \\ &= w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - C_{ij}, \end{aligned}$$

for elasticities $\epsilon_m = \frac{\partial \ln F_{ij}^1(d_m, d_f)}{\partial \ln d_m}, \epsilon_f$. The optimal home-production inputs d_m, d_f and home-production output q are thus simple functions of match characteristics. Let X_{ij} be the equilibrium value of $w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - C_{ij}$ obtained from the above first-order conditions. Because home production depends on z multiplicatively, $q = zF_{ij}^1, d_m, d_f)$, X_{ij} only depends on i, j but not on z.

Continuation values. Making use of the promise keeping constraints, $W_m = V_m^1(z)$, we have

$$\begin{aligned} (r+\delta)\left[V_m^1(z) - V_i^0\right] &= u_m + v_m = \beta\frac{S_{ij}(z)}{B_i}, \\ (r+\delta)\left[V_f^1(z) - V_j^0\right] &= u_f + v_f = (1-\beta)\frac{S_{ij}(z)}{B_j}. \end{aligned}$$

Hence

$$\begin{aligned} v_m &\equiv \delta \int \left(V_m^1(z') - V_i^0\right)^+ \mathrm{d}G(z') - rV_i^0 = \frac{\delta\beta}{r+\delta}\frac{\overline{S}_{ij}}{B_i} - rV_i^0, \\ v_f &\equiv \delta \int \left(V_f^1(z') - V_j^0\right)^+ \mathrm{d}G(z') - rV_j^0 = \frac{\delta(1-\beta)}{r+\delta}\frac{\overline{S}_{ij}}{B_j} - rV_j^0, \end{aligned}$$

for $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \, \mathrm{d}G(z')$, and

$$S_{ij}(z) = B_i(\underline{u}_m - rV_i^0) + B_j(\underline{u}_f - rV_j^0) - qC_{ij} + \frac{\delta}{r+\delta}\overline{S}_{ij}.$$

Thus $S_{ij}(z)$ solves the integral equation

$$S_{ij}(z) = qX_{ij} - B_i r V_i^0 - B_j r V_j^0 + \frac{\delta}{r+\delta}\overline{S}_{ij}, \tag{A.1}$$

as

$$B_i u_m + B_j u_f = B_i \underline{u}_m + B_j \underline{u}_f - qC_{ij} = qX_{ij} = zF_{ij}^1 X_{ij}.$$

Transfers. Transfers follow from the above rent sharing equations. Indeed,

$$B_i u_m = \beta S_{ij}(z) - v_m = \beta\left[qX_{ij} - B_i r V_i^0 - B_j r V_j^0\right] + \beta\frac{\delta}{r+\delta}\overline{S}_{ij} - \frac{\delta\beta}{r+\delta}\overline{S}_{ij} - rB_i V_i^0.$$

Hence,

$$q\left[w_i(1-d_m) - t_m - A_i\right] = B_i r V_i^0 + \beta\left[qX_{ij} - B_i r V_i^0 - B_j r V_j^0\right], \tag{A.2}$$

with a similar expression for t_f. Note that we can then write

$$w_i(1-d_m) - t_m - A_i = \beta_{ij}(z)\,X_{ij}, \quad w_j(1-d_f) - t_f - A_j = \left[1-\beta_{ij}(z)\right]X_{ij},$$

for $\beta_{ij}(z) = \beta + \frac{B_i r V_i^0 - \beta\left[B_i r V_i^0 + B_j r V_j^0\right]}{zF_{ij}^1 X_{ij}}$.

Singles. It remains to work out the value for singles, i.e.

$$rV_i^0 = \max_d \psi_i\left[w_i(1-d), F_i^0(d)\right] + \beta\frac{\lambda}{r+\delta}\frac{1}{B_i}\iint S_{ij}(z)^+ \mathrm{d}G(z)\,n_f(j)\,\mathrm{d}j, \tag{A.3}$$

with a similar expression for females.

Solving for values. Let $\overline{S}_{ij} \equiv \int S_{ij}(z)^+ \mathrm{d}G(z)$ denote the integrated surplus. The following fixed-point equation thus defines $\overline{S}_{ij}$:

$$\overline{S}_{ij} = F_{ij}^1 X_{ij}\mathcal{G}\left(\frac{B_i r V_i^0 + B_j r V_j^0 - \frac{\delta}{r+\delta}\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right), \tag{A.4}$$

with $\mathcal{G}(s) \equiv \int (z-s)^+\,\mathrm{d}G(z) = \int_s^{+\infty} z\,\mathrm{d}G(z) - s[1-G(s)]$. Note that $\mathcal{G}'(s) = -[1-G(s)] \in (-1,0)$ for all interior point s. Hence $\mathcal{G}$ is a contracting operator.[16] Moreover,

$$B_i r V_i^0 = B_i u_i^0 + \frac{\lambda\beta}{r+\delta}\int \overline{S}_{ij} n_f(j)\,\mathrm{d}j, \tag{A.5}$$

and

$$B_j r V_j^0 = B_j u_j^0 + \frac{\lambda(1-\beta)}{r+\delta}\int \overline{S}_{ij} n_m(i)\,\mathrm{d}i. \tag{A.6}$$

These two equation define $\left(\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0\right)$ as a fixed-point of a contracting operator. Then $S_{ij}(z)$ follow.

[16] In the particular lognormal case of $G(s) = \Phi(\ln s/\sigma)$ and $G^{-1}(x) = e^{\sigma\Phi^{-1}(x)}$, then

$$\mathcal{G}(s) = -s\Phi\left(-\frac{\ln s}{\sigma}\right) + e^{\frac{\sigma^2}{2}}\Phi\left(-\frac{\ln s}{\sigma} + \sigma\right).$$

B Identification

Claim 1. The match-specific source of variation in the sharing rule $\beta_{ij}(z)$, namely z, allows to identify b_m/b_f by regressing $w_i e_m^1$ on $w_j e_f^1$, keeping wages constant.

Proof. Consider married couples. The leisure expenditures of husband and wife are

$$w_i e_m^1 = a_{1m} w_i + a_{2m} w_i^2 + b_m \beta_{ij}(z) X_{ij}, \tag{B.1}$$
$$w_j e_f^1 = a_{1f} w_j + a_{2f} w_j^2 + b_f [1 - \beta_{ij}(z)] X_{ij}, \tag{B.2}$$

with

$$X_{ij} = \frac{w_i(1 - D_m^1) + w_j(1 - D_f^1) - C - A_i - A_j}{1 + K_m^1 + K_f^1}. \tag{B.3}$$

The sharing rule $\beta_{ij}(z)$ is a function of wages w_i, w_j and z given observed spouse types x_i and x_j. Eliminating $\beta_{ij}(z)$ out of equations (B.1) and (B.2), we obtain the following structural link between the spouses' leisure supplies:

$$w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1 = a_{1m} w_i + a_{2m} w_i^2 + \frac{b_m}{b_f} \left(a_{1f} w_j + a_{2f} w_j^2 \right) + b_m X_{ij}. \tag{B.4}$$

This equation implies that conditional on all observable characteristics and wages, the right hand side of this equation is constant and residual variation with respect to z in leisure expenditures identifies b_m/b_f. □

Claim 2. Parameters $a_{2m}, a_{2f}, b_m, b_f, K_m^1, K_f^1, a_{1m}, a_{1f}, D_m^1, D_f^1$ are identified from the nonlinear link between expenditures $w_i d_m^1$, $w_j d_f^1$, $w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1$ and wages w_i, w_j.

Proof. Consider domestic time expenditures:

$$w_i d_m^1 = w_i D_m^1 + K_m^1 X_{ij}, \quad w_j d_f^1 = w_j D_f^1 + K_f^1 X_{ij}. \tag{B.5}$$

As D_i^1, D_j^1 and C_{ij} do not depend on wages, denoting $k_m^1 = \frac{K_m^1}{1+K_m^1+K_f^1}$,

$$\frac{\partial w_i d_m^1}{\partial w_i} = D_m^1 + K_m^1 \frac{\partial X_{ij}}{\partial w_i} = D_m^1 + k_m^1 (1 - D_m^1 - a_{1m} - a_{2m} w_i), \tag{B.6}$$
$$\frac{\partial w_i d_m^1}{\partial w_j} = K_m^1 \frac{\partial X_{ij}}{\partial w_j} = k_m^1 (1 - D_f^1 - a_{1f} - a_{2f} w_j). \tag{B.7}$$

The variation of these derivatives with respect to wages identifies $k_m^1 a_{2m}$ and $k_m^1 a_{2f}$. By symmetry, $k_f^1 a_{2m}$ and $k_f^1 a_{2f}$ are also identified (with $k_f^1 = \frac{K_f^1}{1+K_m^1+K_f^1}$).

Moreover, differentiating equation (B.4),

$$\begin{aligned}\frac{\partial w_i e_m^1}{\partial w_i} + \frac{b_m}{b_f} \frac{\partial w_j e_f^1}{\partial w_i} &= a_{1m} + 2 a_{2m} w_i + b_m \frac{\partial X_{ij}}{\partial w_i} \\ &= a_{1m} + 2 a_{2m} w_i + b_m \frac{1 - D_m^1 - a_{1m} - a_{2m} w_i}{1 + K_m^1 + K_f^1},\end{aligned}$$

which identifies $\left[2 - b_m(1 - k^1)\right] a_{2m}$, denoting $k^1 = k_m^1 + k_f^1$. By symmetry, i.e. differentiating with respect to the other wage, $\left[2 - b_f(1 - k^1)\right] a_{2f}$ is also identified. Since b_m/b_f, $k_m^1 a_{2m}, k_m^1 a_{2f}$ and $k_f^1 a_{2m}, k_f^1 a_{2f}$ are already identified, simple algebra shows that $k_m^1, k_f^1, a_{2m}, a_{2f}, b_m, b_f$ are identified.

Next, the intercept of $\partial w_j d_j^1/\partial w_i$ identifies $1 - D_m^1 - a_{1m}$ (equation (B.7)); $\partial w_i d_m^1/\partial w_i$ identifies D_m^1 (equation (B.6)); $\partial(w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1)/\partial w_i$ identifies a_{1m}. A symmetric argument applies for a_{1f} and D_f^1.

Finally, parameter X_{ij} is identified from equation (B.4), and parameter C will follow from equation (B.3) if we can identify a_{0m} and a_{0f}. We do that by proving the next claim. □

Claim 3. Parameters K_m^0, K_f^0 and a_{0m}, a_{0f} are identified from the time uses (leisure and home production) of singles.

Proof. The leisure equation for singles,

$$w_j e_f^0 = a_{1f} w_j + a_{2f} w_j^2 + b_f \left[w_j(1 - d_j^0) - A_j\right], \quad A_j = a_{0f} + a_{1f} w_j + \tfrac{1}{2} a_{2f} w_j^2, \tag{B.8}$$

identifies $b_f a_{0f}$, hence a_{0f}, all other parameters being known at this stage. The RHS of home production time,

$$w_j d_f^0 = w_j D_f^0 + \tfrac{K_f^0}{1+K_f^0} \left[w_j(1 - D_f^0) - A_j\right], \tag{B.9}$$

is a quadratic function of w_j, which identifies K_f^0 and D_f^0. □

C Estimation details

We first define the following residuals.

1. For single men, the residuals are

$$\begin{aligned} \varepsilon_i^0 &= d_i^0 - D_i^0 - K_m^0 X_i^0 / w_i, \\ \eta_i^0 &= e_i^0 - a_{1i} - a_{2m} w_i - b_i X_i^0 / w_i, \end{aligned}$$

   with $X_i^0 = \frac{w_i[1-D_i^0]-A_i}{1+K_m^0}$. Similar expressions can be obtained for single women.

2. For couples, the residuals for conditional means are

$$\begin{aligned} \varepsilon_{mij}^1 &= d_{mij}^1 - D_i^1 - K_m^1 X_{ij}/w_i, \\ \varepsilon_{fij}^1 &= d_{fij}^1 - D_j^1 - K_f^1 X_{ij}/w_j, \\ \eta_{mij}^1 &= e_{mij}^1 - a_{1i} - a_{2m} w_i - b_i \overline{\beta}_{ij} X_{ij}/w_i, \\ \eta_{fij}^1 &= e_{fij}^1 - a_{1j} - a_{2f} w_j - b_j (1 - \overline{\beta}_{ij}) X_{ij}/w_j, \end{aligned}$$

   where $\overline{\beta}_{ij}$ is calculated using equation (5.21). We solve the Fredholm equation for $B_i r V_i^0, B_j r V_j^0$ on the Chebyshev grids mentioned earlier. This allows to approximate the integrals by finite sums using Clenshaw-Curtis quadrature. The Fredholm equations (5.25) and (5.26) thus become a standard linear system that can be solved by matrix inversion. See Appendix D for details.

3. To identify σ we use second order moments for leisure. Specifically,

$$\begin{aligned} \nu_{mmij}^1 &= (\eta_{mij}^1)^2 - b_i^2 \sigma_{ij}^2 X_{ij}^2 / w_i^2, \\ \nu_{ffij}^1 &= (\eta_{fij}^1)^2 - b_j^2 \sigma_{ij}^2 X_{ij}^2 / w_j^2, \\ \nu_{mfij}^1 &= \eta_{mij}^1 \eta_{fij}^1 + b_i b_j \sigma_{ij}^2 X_{ij}^2 / w_i / w_j, \end{aligned}$$

   where σ_{ij}^2 is calculated using equation (5.22).

We minimize the sum of squared residuals ε, η and ν with respect to the parameters of preferences, home production, β and σ. Once all these parameters have been estimated, we estimate public good quality Z_{ij} from equation (5.27):

$$Z_{ij} = F_{ij}^1 X_{ij} \left[\left(\frac{K_m^1}{w_i} \right)^{K_m^1} \left(\frac{K_f^1}{w_j} \right)^{K_f^1} X_{ij}^{1+K_m^1+K_f^1} \right]^{-1}.$$

D Computational details

This appendix shortly describes the numerical tools used in estimation. The best reference here is Trefethen (2013).

D.1 Chebyshev nodes

We discretize continuous functions on a compact domain using Chebyshev grids. For example, let $[\underline{x}, \overline{x}]$ denote the support of male wages, we construct a grid of $n+1$ points as

$$x_k = \frac{\overline{x} + \underline{x}}{2} + \frac{\overline{x} - \underline{x}}{2} \cos \frac{k\pi}{n}, \ k = 0, ..., n.$$

D.2 Clenshaw-Curtis quadrature

Many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals:

$$\int_{\underline{x}}^{\overline{x}} f(x)\,\mathrm{d}x \simeq \frac{\overline{x} - \underline{x}}{2} \sum_{k=0}^{n} \omega_k f(x_j),$$

where the weights ω_j can be easily computed using Fast Fourier Transform (FFT). The following MATLAB code can be used to implement CC quadrature (Waldvogel, 2006):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-l;
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).

D.3 Integral equations

We need to solve functional fixed point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed point operator T involves integrals, we simply iterate the finite dimensional operator $\widehat{T}$ obtained by replacing the integrals by their approximations at grid points.

Example 1: equilibrium. Using the previous approximations, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1 + \rho \int_{\underline{x}}^{\overline{x}} u(y)\alpha(x,y)\,\mathrm{d}y}$$

becomes

$$\boldsymbol{u} = [u(x_k)]_{k=0,\dots,n} = \widehat{T}(\mathbf{u}) = \left[\frac{\ell(x_k)}{1 + \rho \sum_{\ell=0}^{n} \omega_\ell u(x_\ell)\alpha(x_k, x_\ell)}\right]_{k=0,\dots,n}.$$

It was sometimes necessary to "shrink" steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0,1]$. A stepsize $\theta < 1$ may help if T is not everywhere strictly contracting.

Example 2: singles' values. $\left(B_i r V_i^0, B_j r V_j^0\right)$ solve the inhomogeneous Fredholm system

$$\begin{aligned} B_i r V_i^0 &= B_i u_i^0 + \frac{\lambda}{r+\delta}\beta \int \left(B_i r V_i^0 + B_j r V_j^0\right) \theta_{ij}\, n_f(j)\,\mathrm{d}j, \\ B_j r V_j^0 &= B_j u_j^0 + \frac{\lambda}{r+\delta}(1-\beta) \int \left(B_i r V_i^0 + B_j r V_j^0\right) \theta_{ij}\, n_m(i)\,\mathrm{d}i. \end{aligned}$$

This is a linear system that can be solved for after discretizing the state space, or value function iteration. Suppose that i and j are discrete variables with weights ω_i, ω_j. Define the matrices

$$\Theta_m = -\frac{\lambda\beta}{r+\delta}\left[\theta_{ij}\, n_f(j)\,\omega_j\right]_{i\times j}, \quad \Theta_f = -\frac{\lambda(1-\beta)}{r+\delta}\left[\theta_{ij}\, n_m(i)\,\omega_i\right]_{i\times j}^{\top},$$

and

$$\Delta_m = 1 - \frac{\lambda\beta}{r+\delta}\,\mathrm{diag}\Big(\sum_j \theta_{ij}\, n_f(j)\,\omega_j\Big), \quad \Delta_f = 1 - \frac{\lambda(1-\beta)}{r+\delta}\,\mathrm{diag}\Big(\sum_i \theta_{ij}\, n_m(i)\,\omega_i\Big).$$

Then

$$\begin{bmatrix} B_i r V_i^0 \\ B_j r V_j^0 \end{bmatrix} = \begin{pmatrix} \Delta_m & \Theta_m \\ \Theta_f & \Delta_f \end{pmatrix}^{-1} \begin{bmatrix} B_i u_i^0 \\ B_j u_j^0 \end{bmatrix}.$$

D.4 Interpolation

We can interpolate functions very easily between points $y_0 = f(x_0), \dots, y_n = f(x_n)$ using Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^{n} Y_k \cdot T_k(x), \tag{D.1}$$

where Y_k are the OLS estimates of the regression of $y = (y_0, \dots, y_n)$ on Chebyshev polynomials

$$T_k(x) = \cos\left(k \arccos\left(\frac{x - \frac{\overline{x}+\underline{x}}{2}}{\frac{\overline{x}-\underline{x}}{2}}\right)\right),$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, \dots, y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
Y = real(fft(Y'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))...
*Y(1:n+1,1:n+1)...
*cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin)));
```

The fact that the grid $(x_0, ..., x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon).

Another advantage of DCT is that, having calculated $Y_0, ..., Y_n$, then polynomial projections of $y = (y_0, ..., y_n)$ of any order $p \le n$ are obtained by stopping the summation in (D.1) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},$$

and

$$\int \cos(k \arccos x)\, \mathrm{d}x = \begin{cases} x & \text{if } k = 0, \\ \frac{x^2}{2} & \text{if } k = 1, \\ \frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \ge 2. \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

$$\int_{\underline{x}}^{x} \mathbf{1}\{t \le x\} f(x)\, \mathrm{d}x \simeq \sum_{k=0}^{n} w_k \mathbf{1}\{t \le x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

D.5 Kernels

In the application we had to calculate the density of variables such as the wage ratio $y_{ij} = \frac{w_j}{w_i + w_j}$ across couples (i, j). A kernel density estimator of the distribution of M observations y_{ij} is

$$\widehat{f}(y) = \frac{1}{M} \sum_{(i,j)} K_h(y - y_{ij}),$$

for a kernel K_h with width h. When we use the model to predict this density, we use our estimate of the match distribution $m(w_m, w_f)$ (omitting socio-demographic characteristics for simplicity, which we average over), which we have tabulated it on the Chebyshev grid (using the same notation for the data and the grid points):

$$w_i = \frac{\overline{w}_m + \underline{w}_m}{2} + \frac{\overline{w}_m - \underline{w}_m}{2} \cos \frac{i\pi}{n}, \ i = 0, ..., n,$$
$$w_j = \frac{\overline{w}_f + \underline{w}_f}{2} + \frac{\overline{w}_f - \underline{w}_f}{2} \cos \frac{j\pi}{n}, \ j = 0, ..., n.$$

Then we calculate

$$\widehat{\widehat{f}}(y) = \frac{\overline{w}_m - \underline{w}_m}{2} \frac{\overline{w}_f - \underline{w}_f}{2} \sum_{i,j=0}^{n} K_h\left(y - \frac{w_j}{w_i + w_j}\right) m(w_i, w_j).$$

We do the same for earnings ratios.

To calculate the conditional mean matching probability given wage ratio, we use a Nadaraya-Watson estimator on data:

$$\widehat{\mathbb{E}}\left[\alpha_{ij} | \frac{w_j}{w_i + w_j} = y\right] = \frac{\sum_{(i,j)} \alpha_{ij} K_h\left(y - \frac{w_j}{w_i + w_j}\right)}{\sum_{(i,j)} K_h\left(y - \frac{w_j}{w_i + w_j}\right)}.$$

The prediction is

$$\widehat{\widehat{\mathbb{E}}}\left[\alpha(w_i, w_j) | \frac{w_j}{w_i + w_j} = y\right] = \frac{\sum_{(i,j)} \alpha(w_i, w_j) K_h\left(y - \frac{w_j}{w_i + w_j}\right) m(w_i, w_j)}{\sum_{(i,j)} K_h\left(y - \frac{w_j}{w_i + w_j}\right) m(w_i, w_j)},$$

where $\alpha(w_i, w_j)$ is the predicted matching probability evaluated on the Chebyshev grid.

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