Portfolio Choices with Near Rational Agents: A Solution to Some International-Finance Puzzles*

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Abstract

A dynamic model of consumption and portfolio decisions is analyzed in which agents seek robust choices against some misspecification of the model probability distribution. This near-rational environment can at the same time explain an imperfect international portfolio diversification and break the link between cross-country consumption correlation and real exchange rate as it is usually implied by standard preference specifications. Portfolio decisions imply moment restrictions on asset prices that are useful to extract information on the degree of near-rationality present in the data.

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Home bias in equities is one of the most consistent observation in international finance. Investors hold a disproportionate share of their wealth in domestic securities more than what would be dictated by the share of these securities in the world market.[1] Per se, this is not a puzzle. A growing body of the literature has proposed portfolio models that can account for a partially diversified portfolio. The current explanations range from the existence of information frictions to trade costs in goods and asset markets, home bias in consumption, sticky prices, terms of trade movements.[2] Absence of diversification in portfolio choices does not imply lack of international risk sharing. Indeed, these analyses start from the complete-market allocation and build the optimal portfolio shares to mimic that allocation. However, full risk sharing implies a strong connection between stochastic discount factors and the nominal exchange rate and, given standard preference specification, a counterfactual relation between cross-country consumption differentials and real exchange rates. The model implies consumption to fall in one country relative to the other while the real exchange rate appreciates. In the data cross-country consumption differential and real exchange rate are weakly correlated. This is the Backus-Smith anomaly.[3]

In his Ohlin Lecture Obstfeld (2006) has pointed out that one of the key challenges in international finance is the integration of portfolio theories with the Backus-Smith anomaly.[4] Needless to say, preference specifications currently used, in general and partial equilibrium open-economy models, are unable to match other asset price moments as the high and volatile returns on equities and the shape and volatility of the yield curve.

This paper attempts to solve these issues in a near-rational environment. Near rationality is modelled as the possibility that decision makers fear some misspecification of the model probability distribution like in the robustness literature developed by Hansen and Sargent (2005).[5] The degree of irrationality is bounded by the fact that the model distrust is statistically difficult to distinguish in finite sample.

I write down a simple two-country two-asset representative agent model, in which

[1] See Bertaut and Grever (2004).

[2] An incomplete list of successful papers includes: Coeurdacier (2005), Cole and Obstfeld (1991), Devereux and Sutherland (2006), Engel and Matsumoto (2006), Heatcote and Perri (2004), Kollman (2006).

[3] Backus and Smith (1993).

[4] See also Obstfeld and Rogoff (2001).

[5] I borrow the term near rationality from Woodford (2006) for it captures in a better way the economic content of the application that I am interested in.

near-rational decision makers choose optimally their intertemporal consumption profile together with the portfolio allocation. In the limiting case of rational expectations, the model implies complete portfolio diversification. By equally investing in the two securities agents can completely share the movements in their nominal expenditure once evaluated in the same currency achieving the full risk-sharing allocation. Near rationality allows a departure from this allocation whose direction towards home bias is a question of empirical relevance of certain covariances and variances. Near rationality modifies the stochastic discount factor by a multiplicative term that translates the fears of misspecification in fears of bad news on the expected discounted value of consumption. In this case, there is a close parallel with the stochastic discount factor of non-expected utility models discussed in Hansen et al. (2005) and Piazzesi and Schneider (2006).[6]

The reasons for why the model with near-rational agents generates home bias in portfolio choices can be understood in the following way. News on current and future real exchange rate appreciation are bad news for the expected consumption profile. Investors would like to invest more in securities that provide a good hedge against this risk. If this happens to be the case for the return on the domestic asset, investors would desire to hold more of this asset. I find that the data validates this theory.

The multiplicative component, that near-rationality adds to the nominal discount factor, serves for the purpose of breaking the link between cross-country consumption differentials and the real exchange rate as it happens when there is a preference shock, with the important difference that this is now observable. I perform a test similar to the one used by Kocherlakota and Pistaferri (2007) to evaluate whether this preference specification can account for the Backus-Smith anomaly. The model is successful for a wide range of parameter values.

Having expressed the stochastic discount factor as a function of observable variables, I can estimate the degree of near-rationality embedded in the moment restrictions that characterize the portfolio decisions of investors. The values obtained compare well with the ones used in the literature on recursive non-expected utility models.

Finally, this paper shows that it is not necessarily the case that perturbations to the model probability distribution have only second-order effects on the equilibrium allocation. The reason is that they have first-order effects on the stochastic

[6] See Barillas et al. (2006) for an equivalent mapping in terms of indirect utility.

discount factor which contributes to the first-order dynamics of consumption and wealth through the direct effect on the steady-state portfolio allocations. In an international context (or in an heterogenous-agent model), first-order perturbations to the stochastic discount factors have a first-order effect on the cross-country (cross-agent) consumption differentials and wealth distribution.

I work in discrete time. There is a related continuos-time literature on portfolio choices under ambiguity. Maenhout (2004) develops a modification of the continuos-time robust-control literature to study portfolio and consumption choices in a partial-equilibrium dynamic model. To get a closed-form solution he adopts a transformation of the objective function of the decision makers that changes the penalization of entropy from a constant Lagrange multiplier into a function of the value function. This modification deeply changes the nature of the approach proposed by Hansen and Sargent (2005) in a way that it is not comparable with the one proposed here.[7] Epstein and Miao (2005) develops a two-person (two-country) continuos-time dynamic general equilibrium model in which an alternative approach to model ambiguity is used based on recursive multiple priors. In contrast to this paper, they focus on a complete-market allocation. Most important, their conclusion for asset home bias depends on imposing the external assumption that agents have more ambiguity in the foreign asset return. Instead, in this paper, near rationality creates a departure from full portfolio diversification that can go in either directions, to justify more or less home bias. The answer depends on data covariances.

This work is structured as follows. Section 1 presents the model under rational expectations. Section 2 discusses the near-rationality model while Section 3 approximates its solution. Section 4 studies whether the data validate the theory proposed. Section 5 estimates the degree of near-rationality. Section 6 discusses the Backus-Smith anomaly. Section 7 concludes.

1 Model

In this section, I describe the model under rational expectations. I consider two countries, domestic and foreign. The representative agent in the domestic economy

[7] See the discussion in Pathak (2002).

maximizes the expected present discounted value of the utility flow

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\ln C_t\right\} \tag{1}$$

where β is the discount factor, with $0 < \beta < 1$. The utility flow is logarithmic in a consumption basket C. Preferences are similar for the representative agent in the foreign economy, except for the fact that variables are denoted with an asterisk.[8] In both countries agents can invest their nominal wealth in two risky assets denominated respectively in each of the two currencies. There are no transaction costs. With S_t I denote the nominal exchange rate as the price of foreign currency in terms of domestic currency. At time t each asset has a price in the respective currency of denomination given by V_t and V_t^* and delivers dividends D_t and D_t^*.

The flow budget constraint for the agent in the domestic economy is given by

$$V_t x_t + S_t V_t^* y_t = (V_t + D_t)x_{t-1} + S_t(V_t^* + D_t^*)y_{t-1} - P_t C_t \tag{2}$$

where x_t and y_t denote respectively the shares of the domestic and foreign assets held by the domestic agent at time t; P_t is the price of the domestic consumption good. The flow budget constraint of the foreign agent is given by

$$\frac{V_t}{S_t}x_t^* + V_t^* y_t^* = \frac{(V_t + D_t)}{S_t}x_{t-1}^* + (V_t^* + D_t^*)y_{t-1}^* - P_t^* C_t^*,$$

where P_t^* is the price of the foreign consumption good. I assume that there can be deviations from purchasing power parity and thus real exchange rate movements, but for what follows I do not need to specify the sources.[9] Equilibrium in the asset markets requires portfolio shares to sum to one

$$x_t + x_t^* = 1, \tag{3}$$

$$y_t + y_t^* = 1, \tag{4}$$

[8] For the analysis that follows, I do not need to specify the composition of the consumption basket nor I need to detail the differences between the two countries. I follow a partial-equilibrium analysis, althought the results would be consistent with a properly written general equilibrium model.

[9] See Rogoff (1996) for a discussion of the possible explanations of the deviations from purchasing power parity.

for each of the two assets. I define the return on the two assets as $R_t \equiv (V_t + D_t)/V_{t-1}$ and $R_t^* \equiv (V_t^* + D_t^*)/V_{t-1}^*$. Starting with the domestic agent I write a more compact representation of the budget constraint by defining total nominal wealth, W_t, as

$$W_t \equiv V_t x_t + S_t V_t^* y_t \tag{5}$$

where the shares of wealth invested in the domestic and foreign assets are defined by

$$\alpha_{1,t} \equiv \frac{V_t x_t}{W_t}, \tag{6}$$

and

$$\alpha_{2,t} \equiv \frac{S_t V_t^* y_t}{W_t}, \tag{7}$$

respectively.

Given these definitions I can write the flow budget constraint (2) as

$$W_t = R_{p,t} W_{t-1} - P_t C_t \tag{8}$$

where the domestic-currency portfolio return is defined as

$$R_{p,t} \equiv \alpha_{1,t-1} R_t + \alpha_{2,t-1} \frac{S_t}{S_{t-1}} R_t^*. \tag{9}$$

Given an initial condition on wealth, W_{t_0-1}, and the sequence of asset returns, the domestic agent chooses consumption and portfolio shares to maximize (1) under the flow budget constraint (8), given (9) and an appropriate no-Ponzi game condition.

Similar steps for the foreign investor deliver a flow budget constraint of the form

$$W_t^* = R_{p,t}^* W_{t-1}^* - P_t^* C_t^*$$

where

$$W_t^* \equiv \frac{V_t}{S_t} x_t^* + V_t^* y_t^*,$$

$$R_{p,t}^* \equiv \alpha_{1,t-1}^* R_t \frac{S_{t-1}}{S_t} + \alpha_{2,t-1}^* R_t^*. \tag{10}$$

This rewriting implies that equation (3) is equivalent to

$$\alpha_{1,t} W_t + \alpha_{1,t}^* S_t W_t^* = V_t, \tag{11}$$

while (4) is equivalent to

$$\alpha_{2,t} \frac{W_t}{S_t} + \alpha_{2,t}^* W_t^* = V_t^*. \tag{12}$$

The optimization problem for the domestic and foreign agents has a simple solution. Logarithmic utility implies that nominal expenditure is proportional to nominal wealth

$$P_tC_t = \frac{(1-\beta)}{\beta}W_t, \qquad P_t^*C_t^* = \frac{(1-\beta)}{\beta}W_t^*, \tag{13}$$

where nominal wealth evolves according to

$$W_t = \beta R_{p,t}W_{t-1} \qquad W_t^* = \beta R_{p,t}^*W_{t-1}^*, \tag{14}$$

for the domestic and foreign agent, respectively. Optimal portfolio decisions replicate the complete-market allocation and imply full portfolio diversification.[10] First, consider the nominal discount factors given by M_{t+1} and M_{t+1}^*

$$M_{t+1} = \beta\frac{P_tC_t}{P_{t+1}C_{t+1}} \qquad M_{t+1}^* = \beta\frac{P_t^*C_t^*}{P_{t+1}^*C_{t+1}^*}. \tag{15}$$

The complete-market allocation requires to equate the nominal stochastic discount factors to the between-period changes in the nominal exchange rate

$$\frac{M_{t+1}}{M_{t+1}^*} = \frac{S_t}{S_{t+1}}. \tag{16}$$

Using (13), (14) and (15), condition (16) implies equal portfolio returns once evaluated in the same currency

$$R_{p,t+1} = R_{p,t+1}^*\frac{S_{t+1}}{S_t}.$$

By inspection of (9) and (10), this requirement is satisfied with symmetric portfolio choices, i.e. $\alpha_{1,t} = \alpha_{1,t}^*$ and $\alpha_{2,t} = \alpha_{2,t}^*$. Portfolio decisions are fully diversified and complete markets achieved. In this model agents would like to share the risks of movements in their nominal expenditures, once equated in the same currency. Indeed (16) implies that $P_tC_t \sim S_tP_t^*C_t^*$ at each point in time which is then achievable by taking identical portfolio choices. The model with near-rational agents, which is going to be detailed in the next sections, differs in a slight but non-innocuous way. It will be shown that nominal stochastic discount factors are given by

$$M_{t+1} = \beta\frac{P_tC_t}{P_{t+1}C_{t+1}}g_{t+1} \qquad M_{t+1}^* = \beta\frac{P_t^*C_t^*}{P_{t+1}^*C_{t+1}^*}g_{t+1}^* \tag{17}$$

[10] These two characteristics do not have necessarily to coincide.

where the additional terms g_t and g_t^* depend, in a log-linear approximation, on the revisions in the expected consumption path as it follows

$$\ln g_{t+1} = -\frac{1}{\theta}\sum_{j=0}^{\infty}\beta^j(E_{t+1}\ln C_{t+j+1} - E_t \ln C_{t+j+1})$$

and

$$\ln g_{t+1}^* = -\frac{1}{\theta^*}\sum_{j=0}^{\infty}\beta^j(E_{t+1}\ln C_{t+j+1}^* - E_t \ln C_{t+j+1}^*).$$

The parameters θ and θ^* measure the departures from rational expectations nested under the assumption that θ and θ^* approach infinity. Even in this model, (16) represents the risk-sharing objective. However, nominal stochastic discount factors are now perturbed by an additional term. Agents would like to share not only the risk of idiosyncratic movements in nominal expenditure but also that of shocks revealing bad or good news for the expected consumption path. Inspection of equilibrium conditions (13) and (14) shows that prices and real exchange rate developments are important in creating idiosyncratic differences in the consumption path across countries. In their portfolio decisions agents would like to invest in assets that hedge against such idiosyncratic movements. This is the motif for the lack of full portfolio diversifications in the model with near-rational investors whose theoretical construction and empirical validity will be the subject of the next sections.

2 The model with near-rational agents

I analyze departures from rational expectations in the form of a distrust that the agent has with respect to the model probability distribution. Agents fear model misspecification and surround the true model with a set of perturbations that are statistically difficult to distinguish in finite samples. I borrow this apparatus from the literature on robustness developed in economics by Hansen and Sargent (2005). The distorted probability distributions are built using martingale representations. Let $\mathcal{I}_t$ be the information set of a generic agent at time t and G_t a non-negative $\mathcal{I}_t$-measurable martingale. Define $g_{t+1} \equiv G_{t+1}/G_t$ if $G_t > 0$ and $g_{t+1} = 1$ if $G_t = 0$, then $G_{t+1} = g_{t+1}G_t$. It follows that $E_t(g_{t+1}) \equiv E(g_{t+1}|\mathcal{I}_t) = 1$. Hansen and Sargent (2005, 2006) use G_t to generate a distorted probability measure under which the expectation of a generic random variable X_{t+1} is given by $\hat{E}X_{t+1} = EG_{t+1}X_{t+1}$ while the distorted conditional expectation is given by $\hat{E}_t X_{t+1} = E_t g_{t+1} X_{t+1}$.

Preferences of near-rational agents are assumed to be of the form

$$U_{t_0} \equiv E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln C_t\right\} + \theta E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \beta E_t (g_{t+1} \ln g_{t+1})\right\}. \tag{18}$$

The utility of the decision maker is composed by the sum of two present discounted values. The first one is the "traditional" present discounted value of the utility from consumption (in expected terms) which is now perturbed by nearby probability distributions using the martingale distortion The second term represents a discounted measure of entropy discussed in Hansen and Sargent (2005). The distrust that the agent has in the model probability distribution is penalized by the expected log likelihood ratios (or relative entropies) of the alternative models.[11] The parameter θ, with $\theta > 0$, measures departures from rational expectations. In the literature on robust control agents seek decisions which are robust with respect to the possible model misspecification and in particular with respect to the worst-case misspecified scenario. The model misspecification is parametrized by the choice of the sequence $\{g_t\}$ while the agent is choosing sequences of consumption and portfolio shares $\{C_t, \alpha_{1,t}, \alpha_{2,t}\}$ taking as given prices and returns $\{P_t, S_t, R_t, R_t^*\}$. The near-rational agent is going to choose consumption and portfolio choices in a "robust" way to solve the following problem

$$\max_{\{C_t, \alpha_{1,t}, \alpha_{2,t}\}} \min_{\{g_t\}} U_{t_0}.$$

This maxminimizer allocation can be obtained as a part of a Nash equilibrium of a two-player game with commitments.[12] In this game on one side the "traditional" agent maximizes (18) under the flow budget constraint (8) by choosing the sequences of consumption and portfolio shares $\{C_t, \alpha_{1,t}, \alpha_{2,t}\}$ taking as given the choice of the other agent in terms of the sequence $\{g_t\}$ for given prices and returns $\{P_t, S_t, R_t, R_t^*\}$; on the other side the "malevolent" agent minimizes the same utility by choosing the stochastic sequence $\{g_t\}$ given the constraint

$$G_{t+1} = g_{t+1} G_t \tag{19}$$

[11] Althought entropy is additive in the overall utility, why it is a penalization is going to be clarified later in the text.

[12] See Osborne and Rubinstein (1994).

with $G_{t_0} = 1$ and

$$E_t g_{t+1} = 1, \tag{20}$$

considering as given the choices of the "traditional" agent in terms of the sequence $\{C_t, \alpha_{1,t}, \alpha_{2,t}\}$. When $\theta \to \infty$ the objective is minimized when $g_t = 1$ for each t from which the rational expectations model follows.

Starting with the problem of the "traditional" agent, preferences for robustness do not change the implication that, with logarithmic preferences, the consumption-wealth ratio is constant. Note that the consumption Euler equation requires

$$\frac{1}{P_t C_t} = \beta E_t \left\{ g_{t+1} R_{p,t+1} \frac{1}{P_{t+1} C_{t+1}} \right\}$$

which is indeed satisfied by (13) and (14) given (20). The above equation is equivalent to the condition

$$E_t\{M_{t+1} R_{p,t+1}\} = 1. \tag{21}$$

In a similar way, I can write

$$E_t\{M^*_{t+1} R^*_{p,t+1}\} = 1, \tag{22}$$

for the foreign country where the nominal discount factors M and M^* are defined in (17).

The constancy of the consumption-wealth ratio allows a lot of tractability for the analysis that follows. Where robustness matters for the consumption and wealth profile of each agent is in affecting the overall portfolio return through endogenous portfolio choices. I now move to analyze how these decisions are taken.

2.1 Optimal portfolio decisions

The optimal allocation of wealth between the two assets depends on standard no-arbitrage conditions. With respect to the domestic asset, these conditions imply that

$$E_t\{M_{t+1} R_{t+1}\} = 1, \tag{23}$$

$$E_t \left\{ M^*_{t+1} R_{t+1} \frac{S_t}{S_{t+1}} \right\} = 1, \tag{24}$$

for the domestic and foreign agent, respectively.

With respect to the foreign asset, they instead require

$$E_t \left\{ M_{t+1} R^*_{t+1} \frac{S_{t+1}}{S_t} \right\} = 1, \tag{25}$$

$$E_t \{ M^*_{t+1} R^*_{t+1} \} = 1, \tag{26}$$

for the domestic and foreign agent, respectively.

The equilibrium restrictions (21)–(26) are equivalent to the set of equilibrium conditions composed by (21) and (22) together with

$$\alpha_1 + \alpha_2 = 1 \tag{27}$$

$$\alpha^*_1 + \alpha^*_2 = 1 \tag{28}$$

$$E_t \left\{ \left(M_{t+1} - M^*_{t+1} \frac{S_t}{S_{t+1}} \right) \left(R_{t+1} - R^*_{t+1} \frac{S_{t+1}}{S_t} \right) \right\} = 0. \tag{29}$$

In particular (29) is an orthogonality condition between the excess return in domestic currency and the difference in the nominal stochastic discount factors evaluated in the same currency. When markets are complete, condition (16) replaces (29). Finally, note that (23) and (26) imply

$$V_t = E_t \{ M_{t+1} (V_{t+1} + D_{t+1}) \} \tag{30}$$

$$V^*_t = E_t \{ M^*_{t+1} (V^*_{t+1} + D^*_{t+1}) \} \tag{31}$$

which determine asset prices for given nominal stochastic discount factors and dividend process.

2.2 Decisions of the "malevolent" agent

I endogeneize the path of g_t which has been considered as given so far. To this purpose I analyze the optimal choice of the "malevolent" agent which commits to choose the sequence $\{g_t\}$ in order to minimize (18) under the constraints (19) and (20) taking as given the optimal choice of the other agent in terms of the sequences $\{C_t, \alpha_{1,t}, \alpha_{2,t}\}$. The first-order conditions of this problem require that

$$\ln C_t + \beta\theta E_t g_{t+1} \ln g_{t+1} + \lambda_t - \beta E_t \{ \lambda_{t+1} g_{t+1} \} = 0 \tag{32}$$

$$\beta\theta G_t (1 + \ln g_{t+1}) + \mu_t - \beta \lambda_{t+1} G_t = 0, \tag{33}$$

where λ_t is the Lagrange multiplier associated with the constraint (19) and μ_t the Lagrange multiplier associated with (20).

3 Approximated solution

While it is possible to solve for the consumption path and the dynamic of wealth in closed form for given portfolio returns, analytical solutions are not available for portfolio shares. The problem can be solved using approximations around the steady state. In the deterministic steady state, there is no concern for a possible misspecification of the model so that $g_t = G_t = 1$. I assume that steady-state inflation rates are zeros in both countries. Moreover $M_t = M_t^* = \beta$ and $R_t = R_t^* = \beta^{-1}$ while $\bar{C} = (1-\beta)\beta^{-1}\bar{W}/\bar{P}$ and $\bar{C}^* = (1-\beta)\beta^{-1}\bar{W}^*/\bar{P}^*$. The steady-state Lagrange multipliers are given by $\bar{\lambda} = (1-\beta)^{-1}\ln \bar{C}$ and $\bar{\mu} = \beta\bar{\lambda} - \beta\theta$. Furthermore $R_t = R_t^* = \beta^{-1}$ implies that $\bar{V}/\bar{D} = \bar{V}^*/\bar{D}^* = (1-\beta)^{-1}\beta$ in equations (30) and (31). In the steady state, equation (11) implies that

$$\bar{\alpha}_1\bar{W} + \bar{\alpha}_1^*\bar{S}\bar{W}^* = \bar{V}. \tag{34}$$

However, the initial distribution of wealth is not determined, as it is usually the case in open-economy models. I choose $\bar{W}$ and $\bar{W}^*$ to be equalized once evaluated in the same currency $\bar{W} = \bar{S}\bar{W}^*$. I can further normalize $\bar{D}$ and $\bar{D}^*$ in a way that $\bar{V} = \bar{W}$ and $\bar{V} = \bar{W}^*$. It follows that $\bar{S} = \bar{V}/\bar{V}^* = \bar{W}/\bar{W}^*$. Equation (34) implies that

$$\bar{\alpha}_1 + \bar{\alpha}_1^* = 1 \tag{35}$$

which together with (27) and (28) represent a set of three independent equations.[13] This is not enough to determine the steady-state portfolio shares. I follow Devereux and Sutherland (2006) to obtain, by continuity from the stochastic model, the last restriction needed.[14] Equilibrium condition (29) holds in a non-stochastic steady state as well as in the neighborhood of the deterministic steady state. In the limiting case when the randomness vanishes, variances and covariances vanish at the same rate in a way to still imply a restriction on the steady-state portfolio shares. This is a bifurcation point, as discussed in Judd and Guu (2001). To obtain this restriction, one needs to take a second-order approximation of (29) obtaining

$$cov_t(\hat{M}_{t+1} - \hat{M}_{t+1}^* - \Delta\hat{S}_{t+1}, \hat{R}_{t+1}^* + \Delta\hat{S}_{t+1} - \hat{R}_{t+1}) = 0, \tag{36}$$

[13] Note that the normalization used in the steady state implies that $\bar{\alpha}_1 = \bar{x}$, $\bar{\alpha}_2 = \bar{y}$, $\bar{\alpha}_1^* = \bar{x}^*$ and $\bar{\alpha}_2^* = \bar{y}^*$.

[14] See also Tille and Van Wincoop (2006).

where variables with hats represent log-deviations with respect to the steady state. To evaluate (36), it is sufficient to take a first-order approximation of the equilibrium conditions in which the portfolio shares appear as a subset of the coefficients in the linear expansion. The derivations are left in the appendix. As a first step, note that, under near rationality, a log-linear approximation to the nominal discount factor of the home agent reads as

$$\hat{M}_{t+1} = \hat{C}_t - \hat{C}_{t+1} - \pi_{t+1} - \frac{1}{\theta(1-\beta)} \sum_{j=0}^{\infty} \beta^j (E_{t+1}\Delta\hat{C}_{t+j+1} - E_t\Delta\hat{C}_{t+j+1}).$$

Since the nominal discount factor measures the appetite for receiving nominal wealth in future states of nature, it would be more desirable to have such wealth either when future nominal expenditure is low or when there are bad news on future consumption growth. With logarithmic utility, the discount factor with near-rational agents is equivalent to that obtainable when agents treat differently the intertemporal distribution of risk as in the recursive utility representation of Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989). This mapping has been already expounded in Barillas et al. (2006) and Hansen and Sargent (2006) where it has been related to the stochastic discount factor of Tallarini (2000) which features additional terms expressed in terms of indirect utility. Here I show a direct relation with the stochastic discount factor derived in Hansen et al. (2005), Piazzesi and Schneider (2006) and Restoy and Weil (2004) in which the additional terms are expressed in terms of observable variables.[15] The parallel is one-to-one if the parameter θ is related to their risk-aversion coefficient γ as follows

$$\theta = \frac{1}{(1-\beta)(\gamma-1)}. \tag{37}$$

However, the interpretation is substantially different. With Kreps-Porteus preferences γ is a measure of the agent aversion toward risk. Instead, with near-rational agents θ measures (in an inverse way) the departure from the rational expectations environment and in particular the difficulties to distinguish the near-rational allocation from the true model. As discussed in Barillas et al. (2006) relatively high values

[15]Note that in Hansen et al. (2006) and Piazzesi and Schneider (2006), there are additional terms not present in my analysis since they are of second-order magnitude. They will be evaluated in later sections when needed.

of γ, low value of θ, are associated with relatively low detection error probabilities which depend on the model true probability distribution.

In the appendix it is shown that (36) implies

$$\bar{\alpha}_2 = \frac{1}{2} - \frac{1}{2}\frac{1}{\theta(1-\beta)+1}\frac{cov_t(\sum_{j=0}^{\infty}\beta^j[E_{t+1}\Delta\hat{Q}_{t+1+j} - E_t\Delta\hat{Q}_{t+1+j}], er_{t+1})}{var_t(er_{t+1})}, \qquad (38)$$

which then determines the home share of foreign assets.[16] In (38), I have defined the domestic-currency excess return between the two assets

$$er_{t+1} = (\hat{R}^*_{t+1} + \Delta\hat{S}_{t+1} - \hat{R}_{t+1}).$$

The rational expectations model follows directly when $\theta \to \infty$ where I have shown that there is full international portfolio diversification. Near rationality permits an important departure from this result even in the direction of implying home bias in portfolio choices– the empirical relevant case. This is possible if the excess return between the two investment opportunities evaluated in domestic currency covaries in a positive way with the surprises in the real exchange rate. Given a positive covariance, the higher is the degree of misspecification the higher is the home bias. As discussed in section 2 there are now two reasons for risk sharing. The first one is the traditional desire to insure nominal expenditure as discussed in the previous section. The second additional motif depends on the possible model misspecification feared by the agent. This is captured by the fact that agents would like to insure themselves against receiving bad news on future consumption. For the same portfolio return, an increase in the domestic price level relative to the foreign, i.e. an appreciation of the real exchange rate or a fall in Q, is a bad news for future domestic consumption because reduces current and future real wealth. Domestic assets are a good hedge with respect to this risk insofar as they pay well when those bad news are received. A positive relation between the above defined excess return and the surprises in the real exchange rate serves indeed for this interest. The relevance of this explanation is then a matter of empirical analysis. Since (38) is expressed in terms of observable variables, I can use data for these variables to study its significance.[17]

[16] Conditional variances and covariances do not vary over time in a second-order approximation so that (38) indeed determines a steady-state portfolio share.

[17] Van Wincoop and Warnock (2006) have criticized current general equilibrium models that explain home bias in the asset market for they imply variances and covariances not consistent with the data.

4 A look at the data

In the previous section, I have shown that departures from rational expectations can justify home bias in portfolio choices depending on the relation between some observable variables. In this section I investigate if there is some empirical support for the theory. I look at data for two countries United Kingdom and United States to analyze the bilateral relation UK-US where I consider the US as the home country. Data are described in the appendix. They are at a quarterly frequency for the sample 1970:Q1 to 2005:Q4 and corresponds to the following variables Δc, Δc^*, π, π^*, r, r^*, Δs, where small-case variables represent the log of the capital-case variables of the previous section and $\pi_t \equiv \ln P_t/P_{t-1}$, $\pi_t^* \equiv \ln P_t^*/P_{t-1}^*$. This set of variables is also sufficient to build a data analogous for the excess return, er, and the real exchange rate changes, Δq. Table 1 shows the sample means and standard deviations of the variables of interest

Table 1: Data moments

	mean* (%)	s.d.* (%)
Δc	3.29	2.66
Δc^*	2.74	4.57
π	4.10	2.60
π^*	6.39	5.40
r_t	12.06	35.59
r_t^*	15.71	44.05
Δs	-0.93	19.57
Δq	1.35	19.65
er	2.72	37.79

*Annual percent

UK data are characterized by lower consumption growth and higher inflation rate than in the US and by higher nominal and real returns on the stock market. The volatilities of all these variables are higher in UK than US. Quarterly changes of the nominal exchange rate are on average negative. Real exchange rate changes are on average positive. Both variables have high volatility. Nominal excess returns are positive meaning that once evaluated in the same currency nominal returns are higher for the UK stock market than in the US. These excess returns are quite volatile. Table 2 presents some correlations of interest. In what follows, $\mu(\cdot)$, $std(\cdot)$ and $corr(\cdot, \cdot)$ are the operators for the following statistics: mean, standard deviation and correlation, respectively.

Table 2: Data correlations

	US-UK
$corr(\Delta c, \Delta c^*)$	0.22
$corr(\pi, \pi^*)$	0.76
$corr(r, r^*)$	0.66
$corr(\Delta s, \Delta q)$	0.98
$corr(\Delta s, er)$	0.45
$corr(\Delta q, er)$	0.47

Consumption growth is not as correlated across countries as are inflation and stock market returns. Real and nominal exchange rates are highly correlated even for quarterly changes confirming the findings of Mussa (1986). The correlation that matters in (38) is that between excess returns and current and future revisions in the real exchange rate growth. In the data, changes in the real and nominal exchange rates are highly correlated with the excess return. This is not surprising. Since stock market returns are highly correlated excess returns are mainly driven by nominal exchange rate movements. Similarly, nominal and real exchange rates are highly correlated because inflation rates do not move much and are highly correlated across countries. It follows that real exchange rate movements and excess returns are positively correlated. At a first look, this positive correlation goes in the right direction to explain portfolio home bias using the above presented model. However, to evaluate (38) conditional moments should be evaluated so that a forecasting model is needed.

Piazzesi and Schneider (2006) have used a simple state-space representation for consumption growth and inflation to match empirical moments of the US term structure with Krep-Porteus stochastic discount factor. A minimalist two-country replica of their model requires using the following variables, Δc, Δc^*, π, π^*, Δq where the real exchange rate matters for capturing relative price adjustments. I further enlarge this set of variables by including the real returns for each of the stock markets, $r - \pi$ and $r^* - \pi^*$ to obtain a reasonable forecasting model with the minimum set of variables needed to evaluate (38). As in Piazzesi and Schneider (2006), having defined the vector $z_t \equiv (\Delta c_t, \Delta c_t^*, \pi_t, \pi_t^*, \Delta q_t, r_t - \pi_t, r_t^* - \pi_t^*)$, I assume a state-space model

$$z_{t+1} = \mu_z + x_t + \xi_{t+1}$$

$$x_{t+1} = \Phi x_t + \Gamma \xi_{t+1}$$

where $\xi_{t+1} \sim N(0, \Omega)$ and where x and ξ are vectors of the same dimension as z, μ_z is a vector and Φ and Γ are matrices. Conditional on this representation, I analyze two alternative scenarios: one with β equal to 0.995 and another with β equal to 0.998. I build the present discounted value of the revisions in the expected path of real exchange rate changes as well as the other conditional moments of interest in (38). I evaluate (38) by varying γ (which is related to θ as shown in equation 37). Results are presented in Table 3.

Table 3: Portfolio shares

	$\beta = 0.995$ $\bar{\alpha}_2$	$\beta = 0.998$ $\bar{\alpha}_2$
$\gamma = 1$	0.50	0.50
$\gamma = 2$	0.45	0.45
$\gamma = 5$	0.42	0.42
$\gamma = 10$	0.41	0.41
$\gamma = 20$	0.40	0.40
$\gamma = 50$	0.40	0.40
$\gamma = 100$	0.40	0.40

When $\gamma = 1$, the model collapses to the rational expectations case in which there is full international portfolio diversification. As soon as we depart from this benchmark case the share of foreign assets held by the domestic agents move in the direction to explain home bias. The estimated covariances have the right sign. Moreover a small departure from rational expectations is sufficient for this purpose. Near-rationality can play a non-negligible role in explaining asset home bias, although it should be complemented with other explanations to achieve higher degrees of home bias.[18] But what is a reasonable value to assume for the degree of near rationality?

5 Estimation of the degree of near-rationality

The portfolio model presented in the previous sections provides some important moment restrictions on asset returns that can be estimated to get inference on the degree of near-rationality. In particular the moment restrictions of interest are given by equations (23), (24), (25) and (26). In the analysis that follows, I do not restrict θ to be equal across countries. Let $\boldsymbol{\theta} = (\theta, \theta^*)$. Starting with (23) I note that I can write

$$E_t[\beta e^{-\Delta c_{t+1} - \pi_{t+1} + \ln g_{t+1} + r_{t+1}}] = 1$$

where all the variables are observable except for $\ln g_{t+1}$. However, I have shown that g can be approximated and expressed in terms of observable variables, the higher the order of approximations the better is the approximation of g. I choose a second-order approximation to write that

$$\ln g_{t+1} = -\frac{1}{\theta(1-\beta)}\mathcal{S}_{t+1}\Delta c_{t+1} - \frac{1}{2\theta^2(1-\beta)^2}var_t(\mathcal{S}_{t+1}\Delta c_{t+1}),$$

where I define the operator $\mathcal{S}_{t+1}$ applied to a generic variable x_{t+1} as

$$\mathcal{S}_{t+1}x_{t+1} \equiv \sum_{j=0}^{\infty}\beta^j(E_{t+1}x_{t+j+1} - E_t x_{t+j+1}).$$

The forecasting model of the previous section can be used to compute g_t. I can then write more compactly

$$E_t p(\boldsymbol{\theta}, \beta, \mathbf{y}_{t+1}) = 0$$

for an appropriate vector $\mathbf{y}_{t+1}$ of observable variables and an appropriate function $p(\cdot)$. In a similar way, I can write the other arbitrage restrictions (24), (25) and (26) in a way to obtain a set of moment restrictions of the form

$$E_t\left\{\mathbf{p}(\boldsymbol{\theta}, \beta, \mathbf{y}_{t+1}) \otimes \mathbf{d}_t\right\} = 0$$

[18] Indeed, the analysis has started from a benchmark case of full diversification.

for an appropriate vector of functions $\mathbf{p}(\cdot)$ and set of instruments $\mathbf{d}$. I assume β equal across countries under two alternative parametrizations: $\beta = 0.995$ or $\beta = 0.998$. I use two sets of moments restrictions. In the first one, called *benchmark* case, I use just the moments of the model, equations (23), (24), (25) and (26), and a vector of instruments $\mathbf{d}_t = [\Delta c_t \ \pi_t \ \Delta c_t^* \ \pi_t^*, \ r_t \ \ r_t^*]$. In the second set of moment restrictions, the *extended* case, I enrich the previous model with the additional four moments restrictions, together with the orthogonality conditions, when arbitrage restrictions that apply to domestic and foreign risk-free assets are also considered. In this case the set of instruments is given by $\mathbf{d}_t = [\Delta c_t \ \pi_t \ \Delta c_t^* \ \pi_t^*, \ r_t \ \ r_t^*, \ r_t^f \ \ r_t^{f*}]$ where r_t^f and r_t^{f*} are nominal risk-free interest rates in US and UK, respectively. Within each set of moment restrictions, I perform two kinds of GMM estimations: one in which I use the identity weighing matrix and the other using the "efficient" matrix.[19] Table 4

[19] As discussed in Cochrane (2001), it is not obvious that the efficient estimate should be preferred when asset returns are considered.

shows the results.[20]

Table 4: GMM estimation

	$\beta = 0.995$				$\beta = 0.998$			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
θ	22.22	31.04	21.94	23.85	49.63	71.35	51.07	61.63
	(18.12)	(7.46)	(11.54)	(1.90)	(44.40)	(18.17)	(24.62)	(7.34)
θ^*	50.26	69.07	51.95	49.58	110.79	154.78	112.05	126.72
	(30.85)	(16.35)	(33.86)	(4.47)	(72.63)	(37.84)	(75.68)	(18.94)
γ	10.00	7.44	10.11	9.38	11.07	8.00	10.78	9.11
	(7.34)	(1.54)	(4.79)	(0.66)	(9.01)	(1.78)	(4.71)	(0.96)
γ^*	4.98	3.89	4.84	5.03	5.51	4.23	5.46	4.94
	(2.44)	(0.68)	(2.50)	(0.36)	(2.95)	(0.78)	(3.01)	(0.58)
$J - stat$	61.94	63.79	2239	2257	60.05	62.98	2511	2588

(1) One-step GMM with identity weighing matrix, benchmark case

(2) Two-step GMM with efficient weighing matrix, benchmark case

(3) One-step GMM with identity weighing matrix, extended case

(4) Two-step GMM with efficient weighing matrix, extended case

Estimates are presented for θ and θ^* and for the respective γ and γ^* computed using (37).[21] Focusing on the estimates for γ and γ^*, Table 5 shows that they are similar across the different specifications and alternative moment restrictions used: for the US, γ ranges from 7 to 10, while for UK, γ^* ranges from 4 to 6. The data seems to

[20] The initial value for the parameters in the GMM estimation is obtained by grid search on the minimum of the GMM objective function when the weighing matrix is the identity matrix. Moments are de-meaned, the efficient matrix is computed using the Newey-West method, with lags $T^{1/3}$. Results are similar with no de-meaned moments except for lower values for the J statistics.

[21] Standard deviations for γ and γ^* are computed using the delta method.

confirm the presence of an irrational component which corresponds to 'reasonable' values for the risk-aversion coefficient in non-expected utility models. Moreover, the estimates are consistent with explaining the most of the home bias in assets that the model can. (see Table 3)

6 Backus-Smith anomaly

When asset markets are complete there is a unique and positive stochastic discount factor that enables the pricing of the securities. Discount factors of securities denominated in different currencies are related through the condition (16). When discount factors are derived from preferences, then (16) has strong implications for the relation between observable macro variables. Assuming agents with rational expectations and log utility, (16) implies that the consumption-growth differential should be equal to the changes in the log of the real exchange rate

$$\Delta c_{t+1} - \Delta c^{*}_{t+1} = \Delta q_{t+1}. \tag{39}$$

According to (39), nominal expenditure growths should be equalized across countries once evaluated in the same currency, which means that countries with faster growth in real consumption should experience a depreciation in the real exchange rate. Condition (39) has strong implications for the data: 1) the mean and 2) the volatility of the consumption growth differential should be equal to the mean and volatility, respectively, of the real exchange rate; 3) consumption growth differential should be perfectly correlated with real exchange rate movements.

Table 5: Other data moments

	US-UK
$\mu(\Delta c - \Delta c^{*})$	0.54%
$\mu(\Delta q)$	1.35%
$\mu(\Delta c - \Delta c^{*} - \Delta q)$	-0.81%
$sd(\Delta c - \Delta c^{*})$	4.75%
$sd(\Delta q)$	19.65%
$sd(\Delta c - \Delta c^{*} - \Delta q)$	20.3%
$corr(\Delta c - \Delta c^{*}, \Delta q)$	-0.11

Annual rates

Table 5 shows the relevant statistics in percent and at annual rates for the US-UK relation. The mean of the changes in the real exchange rate is three times higher than the mean of the consumption-growth differential. Real exchange rate changes are more volatile than the consumption-growth differential in the order of four times.

The correlation between the consumption growth differential and the real exchange rate is slightly negative. There is no evidence of relation (39) holding in the data. This is the Backus-Smith anomaly. Log preferences are an important restriction in equation (39). However, departures from the unitary elasticity of substitution (and risk aversion coefficient) do not explain the data, as discussed further in Backus and Smith (1993).

Incomplete markets, instead, represent an important departure for why (39) might not hold. When assets markets are incomplete, the nominal discount factor is not unique but there exists a unique projection of the nominal stochastic discount factors on the space generated by the asset payoffs.[22] These projections are not necessarily positive but are such that (16) does hold. The nominal stochastic discount factor derived from preferences coincides with this unique projection having included an additional component of uninsurable risk. In particular, with rational expectations

[22] See Brandt et al. (2005) and Cochrane (2001) for a discussion.

and log preferences, (39) still holds with the addition of a component υ_{t+1} such that $E_t\upsilon_{t+1} = 0$

$$\Delta c_{t+1} - \Delta c^*_{t+1} = \Delta q_{t+1} + \upsilon_{t+1}.$$

Properties of υ_{t+1} in relation with the other observable variables can explain the data. The failure of condition (39) points toward several explanations: *i*) measurement and sampling errors; *ii*) incompleteness of financial markets; *iii*) incorrect preference specification; *iv*) a combination of *i*) $-$ *iii*). Recently Kocherlakota and Pistaferri (2007) have proposed a test for the joint hypothesis of complete markets and correct preference specification. By defining the variable χ_t as

$$\chi_t \equiv \Delta q_t - \Delta c_t + \Delta c^*_t,$$

they suggest that a regression of χ_t on Δq_t should have a slope equal to zero for the null hypothesis to be true. Repeating their test, I confirm a rejection of the standard model, because the slope coefficient is 1.02 with a standard deviation of 0.02 and the constant in the regression is -0.001 with a standard deviation of 0.0009. Kocherlakota and Pistaferri (2007) proposes a model with heterogeneity of consumers at the country level to pass this test. A similar test can be performed also for the model presented in this paper having defined

$$\chi_t(\gamma, \gamma^*) \equiv \Delta q_t - (\Delta c_t - \Delta c^*_t) + \ln g_t(\gamma) - \ln g^*_t(\gamma^*), \tag{40}$$

where

$$\ln g_t(\gamma) = -(\gamma - 1)\mathcal{S}_t\Delta c_t - \frac{(\gamma-1)^2}{2} var_{t-1}(\mathcal{S}_t\Delta c_t),$$

and

$$\ln g_t(\gamma^*) = -(\gamma^* - 1)\mathcal{S}_t\Delta c^*_t - \frac{(\gamma^*-1)^2}{2} var_{t-1}(\mathcal{S}_t\Delta c^*_t).$$

I regress $\chi_t(\gamma, \gamma^*)$ on Δq_t for different values of γ and γ^*.[23] Figure 1 reports the p-values of a Wald test in which the null hypothesis is that both the coefficient and the slope of the regression are zeros.

[23] The parameter β also affects $\chi_t(\gamma, \gamma^*)$ and is calibrated to 0.998.

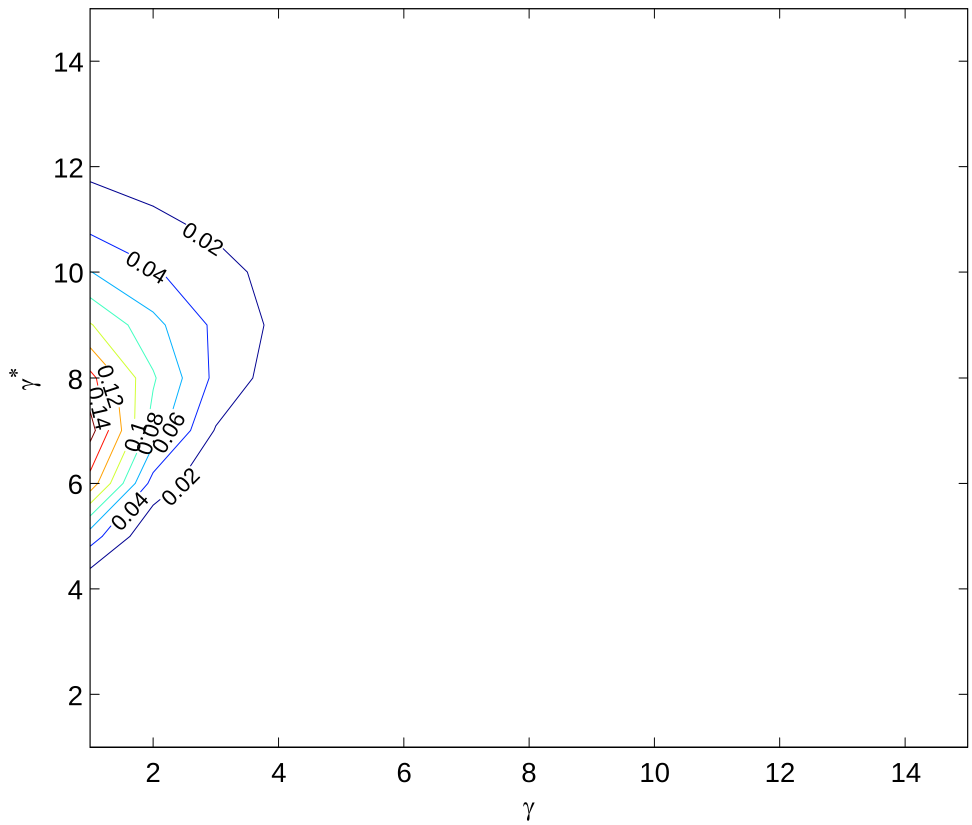

Figure 1: p-value of the Wald test on the null hypothesis that the intercept and slope of the regression of $\chi_t(\gamma, \gamma^*)$ (defined in 40) on Δq_t are both zeros.

It is possible to find values for γ and γ^* such that the test is not 'easily' rejected. This involves low values of γ, below 2, and values of γ^* around 7. It is encouraging that the preference specification used here surpasses for some values the test used by Kocherlakota and Pistaferri (2007), but those values are incompatible with the estimates of Table 4 where γ is found to be higher than γ^*. This points towards a possible misspecification of the preferences used or to some degree of market incompleteness. An important restriction that I have kept so far is the unitary elasticity of substitution. Indeed Vissing-Jœrgensen and Attanasio (2003) have estimated values of the elasticity of substitution close to one with some variability across different estimation strategies. With a non-unitary elasticity of substitution (ψ), the definition

(40) changes to

$$\chi_t(\gamma, \gamma^*) \equiv \Delta q_t - \frac{1}{\psi}(\Delta c_t - \Delta c_t^*) + \ln g_t(\gamma) - \ln g_t^*(\gamma^*), \tag{41}$$

where now

$$\ln g_t(\gamma) = -(\gamma - 1)\mathcal{S}_t \Delta c_t - \frac{1}{2}(\gamma - 1)(1 - \beta)\left(1 - \frac{1}{\psi}\right)\mathcal{S}_t c_t^2 - \frac{1}{2}(\gamma - 1)^2 var_{t-1}(\mathcal{S}_t \Delta c_t),$$

and

$$\ln g_t(\gamma^*) = -(\gamma^* - 1)\mathcal{S}_t \Delta c_t^* - \frac{1}{2}(\gamma^* - 1)(1 - \beta)\left(1 - \frac{1}{\psi}\right)\mathcal{S}_t c_t^{*2} - \frac{1}{2}(\gamma^* - 1)^2 var_{t-1}(\mathcal{S}_t \Delta c_t^*).$$

I regress the newly defined $\chi_t(\gamma, \gamma^*)$ on Δq_t for different values of γ and γ^* under two alternative assumptions on ψ: i) $\psi = 0.8$ and ii) $\psi = 1.2$.[24] Figures 2 and 3 present the results. The ranges for γ and γ^* for which the test is rejected are larger than before and the power of the test is even higher. The approach taken here can complement well other explanations of the Backus-Smith anomaly, as the within-country heterogeneity discussed in Kocherlakota and Pistaferri (2007).

[24] I keep the rewriting of θ in terms of γ as in (37), although γ has not the same interpretation as in the case in which $\psi = 1$, since near-rational preferences do not generically correspond to Kreps-Porteus preferences when $\psi \neq 1$.

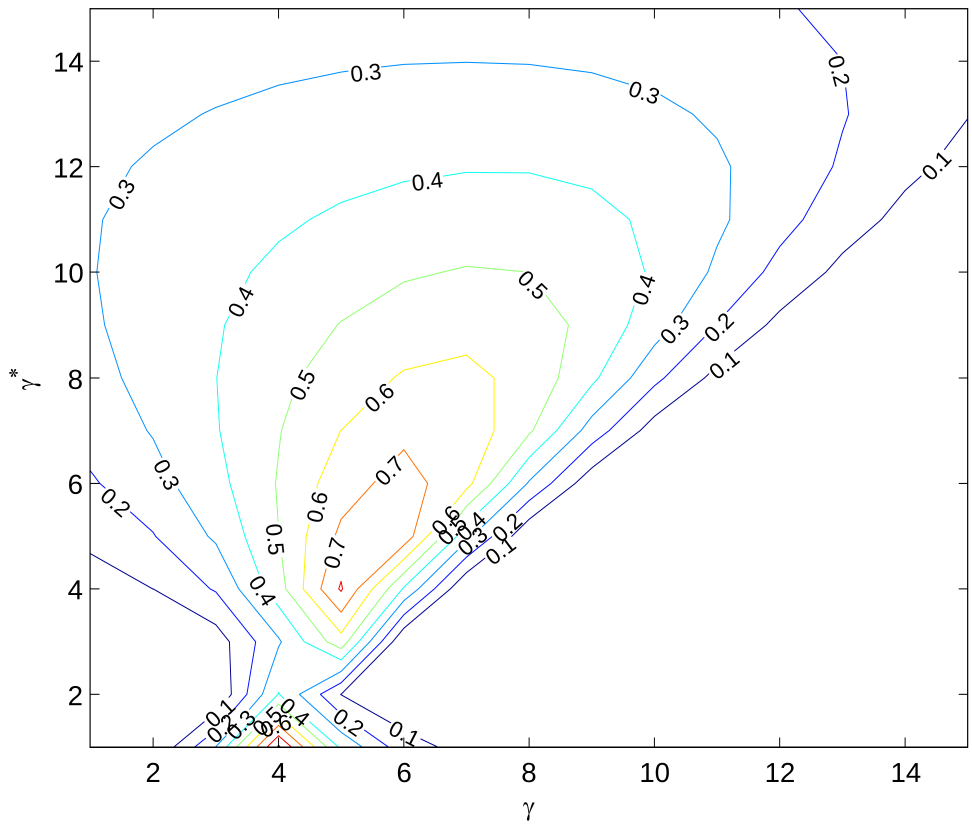

Figure 2: p-values of the Wald test on the null hypothesis that the intercept and slope of the regression of $\chi_t(\gamma, \gamma^*)$ (defined in 41) on Δq_t are both zeros, when $\psi = 0.8$.

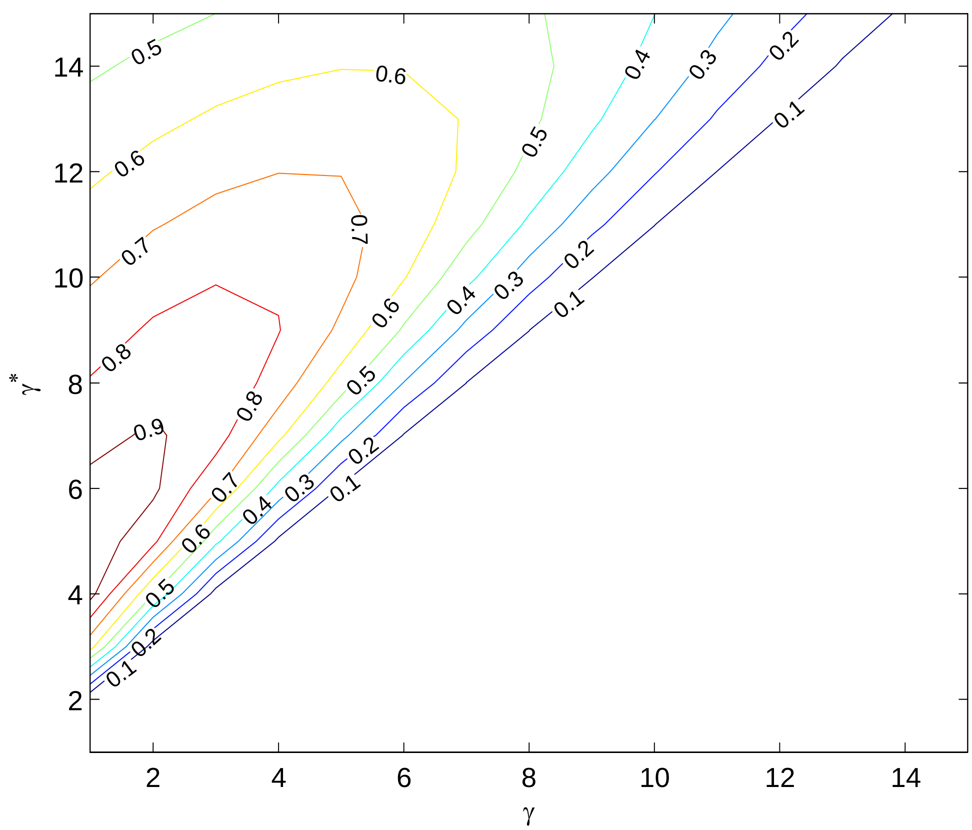

Figure 3: p-values of the Wald test on the null hypothesis that the intercept and slope of the regression of $\chi_t(\gamma, \gamma^*)$ (defined in 41) on Δq_t are both zeros, when $\psi = 1.2$.

7 Conclusion

This paper has shown that a model in which agents are near-rational can explain an imperfect degree of international portfolio diversification. At the same time this environment breaks the tight link between cross-country consumption differentials and the real exchange rate implied by standard preference specifications explaining the Backus-Smith anomaly. There are other puzzling features of the data that perhaps can be explored within this framework. Brandt et al. (2005), have argued that standard preferences fail to account for the high correlation of stochastic discount factors that would be implied by the prices of financial assets. Colacito and Croce

(2006) using Kreps-Porteus preferences have provided an explanation of this anomaly. Since there is a parallel between Kreps-Porteus preferences and the near-rational preferences of this paper with a unitary elasticity of substitution, it might be possible that this model can be also successful in this direction. I leave this analysis for future research. The model presented here enriches the stochastic discount factor with additional terms that have been found to be critical in explaining the equity premium puzzles (see Barillas et al, 2006) and the shape of yield curve (see Piazzesi and Schneider, 2006). It is possible that this model can represent also a first step to explain the forward-discount puzzle and the behavior of the nominal exchange rate.

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8 Appendix

Derivation of equation (38).

To derive (38), note that in a first-order approximation the stochastic discount factors are given by

$$\hat{M}_{t+1} = \hat{C}_t - \hat{C}_{t+1} + \hat{g}_{t+1} - \pi_{t+1},$$
$$\hat{M}^*_{t+1} = \hat{C}^*_t - \hat{C}^*_{t+1} + \hat{g}^*_{t+1} - \pi^*_{t+1},$$

where $\pi_t \equiv \ln P_t/P_{t-1}$ and $\pi^*_t \equiv \ln P^*_t/P^*_{t-1}$.

First-order approximations to (9), (13) and (14) imply that

$$\hat{C}_{t+1} = (1-\bar{\alpha}_2)\hat{R}_{t+1} + \bar{\alpha}_2(\hat{R}^*_{t+1} + \Delta\hat{S}_{t+1}) + \hat{W}_t - \pi_{t+1}$$

for the domestic agent and

$$\hat{C}^*_{t+1} = (1-\bar{\alpha}^*_2)(\hat{R}_{t+1} - \Delta\hat{S}_{t+1}) + \bar{\alpha}^*_2\hat{R}^*_{t+1} + \hat{W}^*_t - \pi^*_{t+1},$$

for the foreign agent.[25]

I can then write

$$\hat{M}_{t+1} - \hat{M}^*_{t+1} + \Delta\hat{S}_{t+1} = \hat{C}_t - \hat{C}^*_t - (\bar{\alpha}_2 - \bar{\alpha}^*_2)er_{t+1} - \hat{W}_t + \hat{W}^*_t + \hat{g}_{t+1} - \hat{g}^*_{t+1}. \quad (42)$$

To complete the evaluation of the nominal discount factor, I need to specify how departures from rational expectations affect it. In particular I need to solve for the optimal paths of g_t and g^*_t but just in a first-order approximation. To obtain the path of g_t it suffices to take a first-order approximation of (32) and (33) to get

$$\bar{\lambda}\hat{\lambda}_t = -\hat{C}_t + \beta\bar{\lambda}E_t\hat{\lambda}_{t+1}$$

and

$$\beta(\theta - \bar{\lambda})\hat{G}_t + \beta\theta\hat{g}_{t+1} + \bar{\mu}\hat{\mu}_t = \beta\bar{\lambda}\hat{\lambda}_{t+1}.$$

Since in a first-order approximation $E_t\hat{g}_{t+1} = 0$, the above two equations can be combined to eliminate the Lagrange multipliers and express the variable g_{t+1} as a function of the revisions in the expected consumption profile at time $t+1$

$$\hat{g}_{t+1} = -\frac{1}{\theta}\sum_{j=0}^{\infty}\beta^j(E_{t+1}\hat{C}_{t+j+1} - E_t\hat{C}_{t+j+1})$$

[25] Note that the expansions in the alphas cancel out.

or in the expected consumption growth at time $t+1$ as

$$\hat{g}_{t+1} = -\frac{1}{\theta(1-\beta)}\sum_{j=0}^{\infty}\beta^j(E_{t+1}\Delta\hat{C}_{t+j+1} - E_t\Delta\hat{C}_{t+j+1}).$$

Similar steps yield to

$$\hat{g}^*_{t+1} = -\frac{1}{\theta^*(1-\beta)}\sum_{j=0}^{\infty}\beta^j(E_{t+1}\Delta\hat{C}^*_{t+j+1} - E_t\Delta\hat{C}^*_{t+j+1})$$

for the foreign agent.

An alternative representation of the stochastic discount factor can be obtained by substituting out consumption growth with the portfolio return using a first-order approximation to (13) and (14). In this case, I obtain

$$\hat{g}_{t+1} = -\frac{1}{\theta(1-\beta)}\sum_{j=0}^{\infty}\beta^j[E_{t+1}(\hat{R}_{p,t+j+1} - \pi_{t+j+1}) - E_t(\hat{R}_{p,t+j+1} - \pi_{t+j+1})].$$

I can further elaborate on this equation by noting that up to a first-order approximation $E_{t+1}er_{t+j+1} = 0$ for each $j > 0$, from which it follows that $E_{t+1}\hat{R}_{p,t+j+1} = E_{t+1}\hat{R}_{t+j+1}$ for each $j > 0$. Using these results and observing that $\hat{R}_{p,t+1} = (1 - \bar{\alpha}_2)\hat{R}_{t+1} + \bar{\alpha}_2(\hat{R}^*_{t+1} + \Delta\hat{S}_{t+1})$, I can write the above expression as

$$\begin{aligned}\hat{g}_{t+1} &= -\frac{1}{\theta(1-\beta)}[(1-\bar{\alpha}_2)(\hat{R}_{t+1} - E_t\hat{R}_{t+1}) + \bar{\alpha}_2(\hat{R}^*_{t+1} - E_t\hat{R}^*_{t+1} + \Delta\hat{S}_{t+1} - E_t\Delta\hat{S}_{t+1}) \\ &\quad -(\pi_{t+1} - E_t\pi_{t+1})] - \frac{1}{\theta(1-\beta)}\sum_{j=1}^{\infty}\beta^j[E_{t+1}(\hat{R}_{t+j+1} - \pi_{t+j+1}) - E_t(\hat{R}_{t+j+1} - \pi_{t+j+1})]\end{aligned}$$

Similarly steps imply that $\hat{g}^*_{t+1}$ is given by

$$\begin{aligned}\hat{g}^*_{t+1} &= -\frac{1}{\theta^*(1-\beta)}[(1-\bar{\alpha}^*_2)(\hat{R}_{t+1} - E_t\hat{R}_{t+1} - \Delta\hat{S}_{t+1} + E_t\Delta\hat{S}_{t+1}) + \bar{\alpha}^*_2(\hat{R}^*_{t+1} - E_t\hat{R}^*_{t+1}) \\ &\quad -(\pi^*_{t+1} - E_t\pi^*_{t+1})] - \frac{1}{\theta^*(1-\beta)}\sum_{j=1}^{\infty}\beta^j[E_{t+1}(\hat{R}^*_{t+j+1} - \pi^*_{t+j+1}) - E_t(\hat{R}^*_{t+j+1} - \pi^*_{t+j+1})].\end{aligned}$$

To further simplify the analysis, I assume that the degree of misspecification is the same across countries, i.e. $\theta^* = \theta$. In this case, it follows that

$$\begin{aligned}\hat{g}_{t+1} - \hat{g}^*_{t+1} &= -\frac{1}{\theta(1-\beta)}[(\bar{\alpha}_2 - \bar{\alpha}^*_2)(er_{t+1} - E_ter_{t+1})] - \\ &\quad -\frac{1}{\theta(1-\beta)}\sum_{j=0}^{\infty}\beta^j[E_{t+1}\Delta\hat{Q}_{t+1+j} - E_t\Delta\hat{Q}_{t+1+j}].\end{aligned}$$

Substituting this expression into (42), I can finally derive that

$$\begin{aligned}\hat{M}_{t+1} - \hat{M}^*_{t+1} - \Delta\hat{S}_{t+1} &= \hat{C}_t - \hat{C}^*_t - (\bar{\alpha}_2 - \bar{\alpha}^*_2)er_{t+1} - \hat{W}_t + \hat{W}^*_t + \\ &\quad -\frac{1}{\theta(1-\beta)}(\bar{\alpha}_2 - \bar{\alpha}^*_2)(er_{t+1} - E_t er_{t+1}) + \\ &\quad -\frac{1}{\theta(1-\beta)}\sum_{j=0}^{\infty}\beta^j[E_{t+1}\Delta\hat{Q}_{t+1+j} - E_t\Delta\hat{Q}_{t+1+j}].\end{aligned}$$

It is now possible to solve for the optimal portfolio choices by substituting the above equation into (36) noting that $\bar{\alpha}_2 + \bar{\alpha}^*_2 = 1$. The result is equation (38).

Data

The data used are constructed at quarterly frequency and on the sample 1970:Q1–2005:Q4. Consumption data are taken from DATASTREAM. Real consumption corresponds to the mnemonic USCNPER.D (US personal consumption expenditure) for the US and to UKCNHLD.D (UK final consumption expenditure) for the UK. The variables Δc and Δc^* are obtained after taking the log-difference of the respective series. Inflation data are from DATASTREAM. Price indexes are obtained by taking the ratio of nominal and real consumption (USCNPER.B and USCNPER.D for US and UKCNHLD.B and USCNPER.D for UK). The variables π and π^* are obtained by taking the log-difference of the price index data. Data on portfolio returns are constructed as in Campbell (1999). For the UK, the source is Morgan Stanley Capital Perspective and data have monthly frequency. Denoting with PI^*_t the stock market price index in local currency at time t, and with RI^*_t the return index. I construct the dividend yield as $DY^*_t = (RI^*_t/RI^*_{t-1})/(PI^*_t/PI^*_{t-1}) - 1$ and the dividend as $D^*_t = 1.33 \cdot DY^*_t \cdot PI^*_t$ where 1.33 enters because of a tax credit system of 33%; the quarterly return is computed as $r^*_{t-3,t} = (D^*_t + D^*_{t-1} + D^*_{t-2} + P^*_t)/P^*_{t-3} - 1$. I use the UK stock-market price index in dollars ($PI^{*\$}_t$) to build the dollar-pound nominal exchange rate as $S_t = PI^{*\$}_t/PI^*_t$, the variable Δs_t corresponds to the log-difference of the nominal exchange rate on a quarterly frequency. For the US stock market, data are from CRSP using the mnemonic WRETD and WRETX on a monthly frequency. The US stock-market price index is constructed as $PI_t = PI_{t-1} \cdot (WRETX_t + 1)$ with initial condition $PI_0 = 100$, while the dividend yield corresponds to $DY_t = (1+WRETD_t)/(1+WRETX_t) - 1$. Dividends are computed as $D_t = DY_t \cdot PI_t$ and the US stock-market return is computed as $r_{t-3,t} = (D_t + D_{t-1} + D_{t-2} + P_t)/P_{t-3} - 1$. The excess return is computed as $er_t = r_t^* + \Delta s_t - r_t$ and the real exchange rate as $\Delta q_t = \Delta s_t + \pi_t^* - \pi_t$. Short-term interest rates are taken from the International Financial Statistics of the IMF and they corresponds to the mnemonic 11260C..ZF... for the UK and 11160C..ZF... for the US. Data are taken at quarterly frequency to construct the variables r_t^f and r_t^{f*}.