The Bond Market's q^*

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Abstract

I propose an implementation of the q-theory of investment using bond prices instead of equity prices. Credit risk makes corporate bond prices sensitive to future asset values. I construct a measure of q using information from the bond market, and I show that this new measure performs well in standard investment equations. With post-war aggregate U.S. data, the bond market's q delivers high R^2, drives out cash flows, and reduces the implied adjustment costs by more than an order of magnitude. It also appears useful in firm level investment equations.

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I propose a new implementation of the q-theory of investment using bond prices instead of equity prices. According to the theory, investment should be an increasing function of marginal q, defined as the net present value of the future marginal product of capital. Hayashi (1982) shows that, with constant returns to scale, marginal and average q are the same. Therefore, the ratio of the market value of unlevered equity over the existing capital stock should be a sufficient statistic for the investment rate. Unfortunately, the implementation of the theory using equity has not been successful. Summers (1981) finds low explanatory power and implausible adjustment costs using aggregate data in levels. Hassett and Hubbard (1997) and Caballero (1999) review the evidence, micro and macro, in levels and in first differences: they conclude that q-theory does not explain investment well.[1]

There are several explanations for the failure of the theory: adjustment costs might not be convex (Caballero and Engle (1999)), returns to scale might not be constant, competition might be imperfect, firms might be credit constrained (Fazzari, Hubbard, and Petersen (1988), Bernanke and Gertler (1989)) and there could be measurement errors and aggregation biases in the capital stock or the rate of investment. All these reasons are plausible, but none is overwhelming. The evidence for constant returns and price taking seems quite strong (Hall (2003)). Heterogeneity and aggregation, over time or over firms, do not seem to create strong biases (Hall (2004)). Moreover, aggregation limits the influence of non convexities (Thomas (2002)). At the end of the day, much empirical research points towards issues with the market value of equity (Gilchrist and Himmelberg (1995), Erickson and Whited (2000), Cumins, Hasset, and Oliner (2006), Erickson and Whited (2006)). Recent research shows that models based on estimated equity value instead of observed equity value perform better. However, when the proxies are based on accounting information or analysts' forecasts, which are themselves based on accounting information, we are left without a theory linking market prices to investment. This is definitely not satisfactory.

The contribution of this paper is to show that q-theory can be implemented with bond prices, and that this implementation performs better that the usual one. Just like Hayashi

[1] Cochrane (1996) studies the joint dynamics of physical and financial returns and finds a fairly strong correlation between stock returns and the growth rate of the aggregate capital stock, but **?** argue that it is driven by the correlation with residential investment, not corporate investment. By contrast, the bond market's q developed here forecasts corporate investment.

(1982), I use a model with constant returns to scale and convex costs of adjustment. Thus, in my setup, marginal and average q are the same. The novelty is that I use the corporate bond market, as opposed to the equity market, to construct the proxy for q. Just like equity prices, corporate bond prices react to changes in the risk-adjusted discount rate and to news about future profitability. This is clear, for instance, in the model of Merton (1974). In general, there exists a non-linear function linking q to bond prices, conditional on leverage and volatility. I calibrate a quantitative model using firm level and aggregate data, and I derive this non linear function.

The implementation of the investment equation using aggregate data and the new measure of q is successful: it accounts for more than half of the volatility of investment in post-war US data, it drives out cash flow variables, and it reduces the implied adjustment costs by more than an order of magnitude. At the firm level, I use credit risk ratings to construct a proxy for bond prices. The coefficient estimated in the cross section is fairly similar to the one estimated in the time series. Recent work by Gilchrist and Zakrajsek (2007) also provides evidence that firm specific interest rates forecast firm level investment. At the firm level, however, cash flows (and equity) remain significant.

The remaining of the paper is organized as follows. In section 1, I present the setup of the model. In section 2, I study a simple case where, to a first order, bond spreads are proportional to q. In section 3, I develop the general case and I explain the calibration. In section 4, I present evidence based on aggregate data. In section 5, I present evidence based on firm level data. Section 6 discusses the interpretation of the results.

1 Model

1.1 Program of the firm

Consider the standard partial equilibrium model of production and investment. The generic firm operates a production technology with constant returns to scale under perfect competition in all markets. All factors of production, except physical capital k_t, can be freely adjusted. This implies that, once the other inputs have been chosen optimally in period t, the profit function of the firm can be written as $p_t k_t$, where the profit rate p_t does not depend on k_t. The constant return to scale function $\gamma^k (k_t, k_{t+1})$ captures the cost of adjusting the level of capital from k_t to k_{t+1} between time t and $t+1$. The total market value of a firm at time 0 is

$$V_0 = E \sum_{t=0}^{\infty} m_{0,t} \left[p_t k_t - (k_{t+1} - (1-\delta) k_t) - \gamma^k (k_t, k_{t+1}) \right] \tag{1}$$

where $m_{0,t}$ is the pricing kernel between 0 and t, and δ is the depreciation rate of physical capital. It is convenient to distinguish discounting over time and across states. I therefore define β_t as the price of a one period real bond with unit face value

$$\beta_t \equiv E_t [m_{t,t+1}],$$

and the risk neutral probabilities measure π. Pricing a bond is like pricing a derivative, and risk neutral probabilities are therefore going to simplify both the notations and the algebra. With these notations, we can write the discounted value of any random variable, say p_{t+1} for instance, as

$$E_t [m_{t,t+1} p_{t+1}] = \beta_t E_t^{\pi} [p_{t+1}].$$

The program of the firm is to maximize (1) with respect to the non-negative sequence $\{k_t\}_{t\geq 1}$. I assume that the solution to this program is finite and strictly positive for all initial conditions. Since the technology exhibits constant returns to scale, it is convenient to work with the scaled value function and the growth rate of capital:

$$v_t \equiv \frac{V_t}{k_t} \text{ and } x_t \equiv \frac{k_{t+1} - k_t}{k_t}$$

I assume that the state of the firm can be characterized by a Markov process $\{\omega_t\}_{t\geq 0}$. The profit rate p_t, the one period discount rate β_t, as well as the risk neutral probabilities, are functions of ω_t. The function $v(.)$ solves the Bellman equation:

$$v(\omega) = \max_{x>-1} \left\{ p(\omega) - \gamma(x) + (1+x) \beta(\omega) E^{\pi} \left[v(\omega') | \omega \right] \right\}, \tag{2}$$

where $\gamma(.)$ is the normalized version of $\gamma^k(.,.)$. I assume that $\lim_{x\to -1} \gamma(x) = \infty$ and $\lim_{x\to\infty} \gamma(x) = \infty$, so that the solution is always interior. The transversality condition is

$$\lim_{\tau\to\infty} E [m_{t,\tau} (1+x_t) .. \times .. (1+x_\tau) v_\tau | s_t, \eta_t] = 0. \tag{3}$$

The optimal investment rate $x(\omega)$ satisfies

$$\frac{\partial \gamma}{\partial x} (x(\omega)) = \beta(\omega) E^{\pi} \left[v(\omega') | \omega \right]. \tag{4}$$

Equation (4) defines the q-theory of investment: it says that the marginal cost of investment is equal to the expected discounted marginal product of capital. The most important practical issue is the construction of the right-hand side of equation (4), which I discuss in the next section.

1.2 Debt, equity and q

The firms issues debt and equity, and I assume that the Modigliani-Miller assumptions hold. Default can happen, but default does not entail any deadweight loss. In case of default, the creditors take over the assets, and the firm continues to operate. In the Modigliani-Miller world, debt policy does not affect firm value or investment. It does, however, affect bond prices, so I must specify debt dynamics before I can use bond prices to estimate q.

Consider first the case of short term debt. The firm issues at time t a one period bond with face value $d_t k_{t+1}$, where book leverage d_t can depend on the Markov state ω_t. The price of the bond is then $B_t = b_t d_t k_{t+1}$, where the pricing function solves

$$b(\omega) = \beta(\omega) E^{\pi}\left[\min\left(v\left(\omega'\right)/d(\omega), 1\right)|\omega\right]. \tag{5}$$

Note that the above equation assumes $v > 0$ in all states of the world. The ex-dividend value of equity, scaled by assets, is simply

$$e(\omega) = \beta(\omega) E^{\pi}\left[\max\left(v\left(\omega'\right) - d(\omega), 0\right)|\omega\right]. \tag{6}$$

Combining (5) and (6), we obtain a version of equation (4) that has empirical content:

$$\frac{\partial \gamma}{\partial x} = e(\omega) + b(\omega) d(\omega). \tag{7}$$

The marginal cost of investment is equal to the ratio of the market value of the firm, debt plus equity, over the replacement cost of the firm's assets.

The most natural way to test the theory is therefore to use equation (7). Unfortunately, it has not worked well in practice (Summers (1981), Caballero (1999)). Equation (7) has been estimated using aggregate and firm level data, in levels or in first differences, and usually with the book value of debt or even just equity on the right-hand-side. It fails most spectacularly with aggregate data in levels. At the firm level, it leaves large unexplained residuals correlated with cash flows.

There are several potential explanations for the lack of success of the investment equation. First, the adjustment costs might not be convex. In fact, they are certainly not convex at the plant level (Dixit and Pindyck (1994), Caballero and Engle (1999)). In the aggregate, the assumption of convex costs might be more sensible, but this is still a controversial issue (Thomas (2002), Bachmann, Caballero, and Engel (2006)). Second, the assumption of constant returns to scale and perfect competition might not hold. Third, firms might be credit constrained. Gomes (2001) considers both decreasing returns and credit constraints and shows that cash flow sensitivity can arise without constraints, while Tobin's q captures investment dynamics well even when the constraints bind. Fourth, there could be measurement errors and aggregation biases in the capital stock or the rate of investment. As argued in the introduction, none of these explanations is definitive.

My goal in this paper is to investigate equation (7) using bond prices instead of equity prices. It is not obvious why the bond market should perform better than the equity market. Indeed, bond prices are sensitive to future asset values only to the extent that default is possible. By focusing on bond prices, I will mostly use information from the left tail of the distribution of future values, and one might wonder how this could improve the matter. The answer lies in the recent research suggesting that "measurement errors" seem to be responsible for the failure of q-theory (Gilchrist and Himmelberg (1995), Erickson and Whited (2000), Cumins, Hasset, and Oliner (2006), Erickson and Whited (2006)). It is therefore legitimate to test the theory using other asset prices. The contribution of this paper is to show that q-theory can be implemented with bond prices.

2 Simple case

In this section, I study a particular case, and I derive approximate expressions for aggregate q as a function of bond yields. The case is special in two dimensions: the firm issues only short term debt, keeps a constant book leverage, and firm level shocks are *iid*. These assumptions are relaxed later. The special case provides the intuition for the results of the more complex model.

2.1 Aggregate and firm level shocks

I distinguish between firm level and aggregate shocks: the state $\omega = (s, \eta)$ contains both aggregate and firm level information. The variable $s_t \in [1, 2.., S]$ captures the aggregate state, which evolves according to a discrete stationary Markov chain. The aggregate state s determines the discount factor $\beta(s)$ and the average profit rate $a(s)$. Firms are also subject to idiosyncratic shocks η. The profit rate is of a firm in state (s, η) is:

$$p(s, \eta) = a(s) + \eta. \tag{8}$$

For simplicity, I assume that the support of η is bounded below in such a way that the value function $v(s, \eta)$ is always positive. The shocks η are distributed according to the density function $\bar{\zeta}(.)$ and normalized to have a mean of zero each period:

$$\int \eta \zeta(\eta)\, d\eta = 0. \tag{9}$$

I also assume in this section that the shocks are independent over time

$$\zeta(\eta' | \eta) = \zeta(\eta'). \tag{10}$$

The results presented in this section would also be valid with auto-correlated shocks, but the algebra would be tedious, and would not bring new insights. Define the value function that would prevail in an economy without idiosyncratic shocks as the solution to the following Bellman equation:

$$v(s) = \max_{x > -1} \left\{ a(s) - \gamma(x) + (1 + x)\beta(s) E^{\pi}\left[v^a(s') | s\right] \right\}. \tag{11}$$

Lemma 1 *When firm level shocks are independent over time*

$$v(s, \eta) = v(s) + \eta \tag{12}$$

Proof. From (11), it is clear that

$$v(s, \eta) = \max_{x > -1} \left\{ a(s) + \eta - \gamma(x) + (1 + x)\beta(s) E^{\pi}\left[v^a(s') | s\right] \right\}$$

From (10) and (8), it is clear that $v(s, \eta)$ solves (2) for $\omega = (s, \eta)$. QED. ■

A direct implication of the independence of firm level shocks over time is that all firms choose the same investment rate: x is independent of η. In this section, I rely on first order approximations to make the results more transparent. Define $\bar{v} \equiv E^{\pi}[v(s)]$ as the risk-neutral average value, and $\bar{\beta}$ as the average price of the real bond. Let $\hat{x}$ and $\hat{v}$ be the deviations for the corresponding risk-neutral averages. The first order approximation to the investment equation is:

$$\gamma_2 \hat{x}(s) \approx \hat{\beta}(s)\bar{v} + \bar{\beta} E^{\pi}\left[\hat{v}\left(s'\right)|s\right], \tag{13}$$

where γ_2 is the curvature of the adjustment cost function estimated at the average investment rate. Note that all the averages are taken under the risk neutral measure π.

Equation (13) makes it clear that we need to evaluate two terms. The first term captures the effect of real interest rates on investment. It depends on nominal interest rates and expected inflation (see Abel and Blanchard (1986) for an estimation of this first term). The issue of inflation for assets prices is a delicate one, that requires much specific attention. For instance, Piazzesi and Schneider (2006) discuss the role of disagreement about inflation across age groups. Equation (13) has little to say about real rates and inflation. I focus on the second term and I show that it can be estimated using the relative price of corporate and government bonds.

2.2 Short term debt with constant book leverage

I consider here the case of short term debt with constant book leverage. I consider later the case of long term debt. The appendix contains extension to time varying leverage and time varying idiosyncratic shocks. Let d be the (constant) book leverage. The bond pricing equation becomes $b(s) = \beta(s) E^{\pi}[\min(v(s') + \eta', d) \mid s]$. To price default risk, we must compute the losses per unit of face value that an investor holding the aggregate portfolio of bonds suffers if state s' is realized next period:

$$L\left(s'\right) \equiv \frac{1}{d}\int_{-\infty}^{d-v(s')} \left(d - v\left(s'\right) - \eta'\right)\bar{\zeta}\left(\eta'\right) d\eta'. \tag{14}$$

Lemma 2 *The first-order approximation of the credit loss function $L(.)$ is*

$$L\left(s, s'\right) \approx \bar{L} - \theta\ \hat{v}\left(s'\right)/d,$$

where $\bar{L}$ is the average loss and θ is the average default rate

$$\theta \equiv \int_{\eta'<d-\bar{v}} \zeta\left(\eta'\right) d\eta'.$$

Proof. See appendix ■

The price of corporate bonds relative to treasuries is given by:

$$\frac{b\left(s\right)}{\beta\left(s\right)} = 1 - \sum_{s'\in S} \pi_{s,s'} L\left(s'\right)$$

It is now clear that the relative price of corporate bonds contains valuable information regarding the future marginal product of capital. The relative price of corporate bonds captures the effect of expected macroeconomic conditions on expected credit losses. Expected future values are scaled by the risk-neutral likelihood, because θ is precisely the likelihood that future values become relevant for current bond prices. This is the sense in which we are using the left tail of the distribution.

Lemma 3 *The price of corporate bonds relative to treasuries is proportional to the expected, risk-adjusted product of capital*

$$\frac{b\left(s\right)}{\beta\left(s\right)} - \frac{\bar{b}}{\bar{\beta}} \approx \frac{\theta}{d}\, E^{\pi}\left[\hat{v}\left(s'\right)|s\right] \tag{15}$$

I can now state the bond market's q theorem, using interest rates instead of bond prices for convenience. Let r be the real risk free rate, let $r^{\$}$ be the nominal risk free rate, and let $y^{\$}$ be the nominal yield on corporate bonds. The appendix contains extension to time varying leverage and idiosyncratic risk. What is most important, however, is to consider long term debt.

Proposition 1 *bond market's q with constant short term book leverage. When aggregate shocks are small, aggregate q can be constructed using only information from the bonds market:*

$$q_t \approx \frac{\bar{v}}{1+r_t} + \frac{d}{\bar{\theta}\left(1+\bar{r}\right)} \frac{1+r_t^{\$}}{1+y_t^{\$}} + cte \tag{16}$$

Proof. With one period debt, the relative price of corporate bonds is

$$\frac{b(s)}{\beta(s)} = \frac{1 + r^{\$}(s)}{1 + y^{\$}(s)}$$

Substituting (15) into (13) delivers the result. QED. ■

The proposition sheds light on existing empirical studies, such as Bernanke (1983) and Stock and Watson (1989), showing that corporate bond spreads have predicting power for future output. Ignoring changes in real risk free rates, the proposition says that corporate bond spreads are, to a first order, proportional to Tobin's q.

3 General case

I now turn to the numerical solution for the case of long term debt and realistic firm level shocks. I therefore drop the assumption that firm level shocks are independent over time. The shocks η follow a continuous Markov chain with transition function $\zeta(\eta'|\eta)$ and ergodic distribution $\bar{\zeta}(\eta)$. As a consequence, the investment rate $x(s,\eta)$ depends on both aggregate and firm level shocks and the value function $v(s,\eta)$ is not additively separable in s and η.

3.1 Long term debt

There are many different types of long term liabilities, and my goal here is not to study all of them, but rather to consider a tractable model of long term debt. To do so, I use a version of the exponential, model introduced in Leland (1994) and Leland (1998). Corporate debt is similar to a sinking fund. Consider a bond issued at time t with coupon rate c and principal normalized to 1. A fraction ϕ of the bond is called each period at par value.[2] The promised cash flows of a bond issued at time t_0 are:

$$\begin{array}{lllll} t_0+1 & t_0+2 & .. & t & .. \\ c+\phi & (1-\phi)(c+\phi) & .. & (1-\phi)^{t-t_0-1}(c+\phi) & .. \end{array}$$

In case of default at any time t, the remaining value is split according to the relative face values of bonds outstanding at the beginning of the period. Let F_t be the total face value of all the outstanding bonds. The bond issued at time t_0 receives $(1-\phi)^{t-t_0-1} V_\tau / F_\tau$. Let $\Theta_{t,\tau}$ be the cumulative default rate between t and τ, given survival up to the end of time

[2] Two interpretation are equivalent here. A share ϕ of each bond is called, of a fraction ϕ of all bonds is randomly called.

t, and let θ_τ be the marginal default rate during period τ. By definition, $\Theta_{t,t} = 0$ and the default rates satisfy the recursive structure: $(1 - \Theta_{t,\tau}) = (1 - \Theta_{t,\tau-1})(1 - \theta_\tau)$. Since all bonds outstanding at the beginning of a period are treated equally in case of default during that period, we need only to compute the price of one unit of principal outstanding, without specifying when this principal was issued. The value of one unit of principal outstanding at the end of time t is:

$$b_t = E_t\left[\sum_{\tau=t+1}^{\infty} m_{t,\tau}(1 - \Theta_{t,\tau-1})(1-\phi)^{\tau-1-t}\left((1-\theta_\tau)(c+\phi) + \theta_\tau V_\tau / F_\tau\right)\right].$$

In Leland (1994) and Leland (1998), there is no investment, and book assets and principal outstanding are simply constant. The equivalent assumption in my setup is that the firm chooses its debt policy so as to leave the book leverage ratio F_t/k_t equal to some constant d. The following proposition characterizes the pricing function for long term debt.

Proposition 2 *The price of long term debt, per unit of principal, solves the equation:*

$$b(s,\eta) = \beta(s)\, E^{\pi}\left[\min\left(c+\phi+(1-\phi)\, b(s',\eta');\frac{v(s',\eta')}{d}\right) \mid s,\eta\right] \tag{17}$$

Proof. See appendix. ■

Note a few special cases. Short term debt corresponds to $\phi = 1$ and $c = 0$. The difference between short- and long-term debt is the presence of the pricing function on both sides of equation (17).[3] A perpetuity corresponds to $\phi = 0$, and, more generally, $1/\phi$ is the average maturity of the debt. The value of a risk free bond with the same coupon and maturity structure would be

$$b^f(s) = \beta(s)\, E^{\pi}\left[c+\phi+(1-\phi)\, b(s') \mid s\right].$$

With a constant risk free rate b^f is simply equal to $(c+\phi)/(\phi+r)$. When the risk free rate is not constant, the bond price is a weighted average of various zero coupon bond prices.

[3] Note that to keep a constant book leverage ratio d the firm might have to buy back some of its debt from time to time, if ϕ is small and the stock of physical capital shrinks very fast. But this buy-back happens at market prices, so the bond investors are indifferent. It is not the same as calling the bonds at par-value.

3.2 Benchmark Calibration

In the benchmark calculation, I set the book leverage to $d = 0.5$, the average maturity to 10 years ($\phi = 0.1$). I use a quadratic adjustment cost function with curvature of $\gamma = 8$ (annually). There is much disagreement about this parameter in the literature. Shapiro (1986) estimates γ around 2.2, and Hall (2004) finds even smaller adjustment costs.[4] On the other hand, Gilchrist and Himmelberg (1995) find values around 20, and estimates from macro data are often much higher (Summers (1981)), and implausible. I pick a value in the middle of the existing estimates, but it turns out that the mapping from bond yields to q is not very sensitive to this parameter.

I assume a constant annual (real) risk free interest rate of $r = 4\%$ in this benchmark calculation because it simplifies the exposition, but I take into account changes in the real interest rate over time in the empirical section below. With a constant real rate, the 'relative price' of corporate bonds is simply $(0.1 + r) / (0.1 + \bar{y}(s))$, where $\bar{y}(s)$ is the average yield across corporate bonds in aggregate state s. As in Leland (1998), I choose the coupon rate so that the average bond price is equal to one.[5]

I approximate aggregate and firm level dynamics with AR(1) processes:

$$\eta_t = \lambda_\eta \eta_{t-1} + \varepsilon_t^\eta, \tag{18}$$

and

$$a_t - \bar{a} = \lambda_a (a_{t-1} - \bar{a}) + \varepsilon_t^a. \tag{19}$$

I estimate the processes using aggregate and firm level data. For firms, I use profit and book assets data from Compustat. I define $a + \eta$ as operating income divided by property plants and equipment for each firm that has a bond rating in Compustat. I exclude firms in finance and real estate. I include firm fixed effects in the panel regression. I find $\lambda_\eta = 0.466$ and $\sigma_\eta = 15\%$. These values are consistent with previous studies.[6]

[4] Shapiro (1986) estimates between 8 and 9 using quarterly data, which corresponds to 2 to 2.2 at annual frequencies.

[5] I choose c so that $\int ds \int d\eta b(s,\eta)\bar{\zeta}(\eta)\bar{\pi}(s) = 1$, where $\bar{\zeta}$ and $\bar{\pi}$ are the ergodic distributions. Of course the pricing function b also depends on the dynamics of $a(s)$ and η so all the parameters are jointly determined.

[6] For instance, Gomes (2001) uses a volatility of 15% and a persistence of 0.62 for the technology shock. Hennessy, Levy, and Whited (2007) report a persistent of the profit rate of 0.51 and a volatility of 11.85%, which they match with a persistence of 0.684 and a volatility of 11.8% for the technology shocks. Note that

I need to estimate three parameters for the aggregate shocks: $\bar{a}$, λ_a and σ_a. The practical difficulty is that the model uses the risk neutral measure, which is different from the historical measure of the data. The most important consequence is that one should calibrate $\bar{a}$ and σ_a using bond prices, and not using historical profit rates and default rates. The persistence parameter can also be different under the risk neutral and historical measure, but I take the historical persistence to be a good benchmark. Using annual NIPA data on corporate profits and the stock on non-residential capital over the post-war period, I estimate $\lambda_a = 0.7$. I choose the other parameters to match the mean and standard deviation of the spread of Baa bonds over Treasuries. The data used in the calibration are described in more details below, and are summarized in Table 1. Choosing simultaneously the coupon rate so that the average bond price is one, I find $\bar{a}/r = 0.92$ and $\sigma\left(\varepsilon_t^a\right) = 3.5\%$.

The calibrated model implies a relationship between bond prices and Tobin's q, as shown on Figure 1. Tobin's q is an increasing and convex function of the relative price of corporate bonds, $\left(0.1 + r\right) / \left(0.1 + \bar{y}\left(s\right)\right)$. This figure extends Proposition 1 to the case of long term debt, persistent firm level shocks, and (potentially) large aggregate shocks.

4 Aggregate Evidence

In this section, I construct a new measure of q using only data from the bond market. I then compare this measure to the usual measure of q, and I assess their respective performance in the aggregate investment equation.

4.1 Data and measurement issues

Bond yields

In equation (16), the key variable is the price of corporate bonds relative to treasuries. I use Moody's Baa index, y_t^{Baa} as my main measure of the yield on risky corporate debt. Moody's index is the equal weighted average of yields on Baa-rated bonds issued by large non financial corporations. To be included in the index, a bond must have a face value of at least 100 million, an *initial* maturity of at least 20 years, and most importantly, a liquid secondary market. Beyond these characteristics, Moody's has some discretion on the

in both of these papers, firms operate a technology with decreasing returns, while here the technology has constant returns to scale. This affects the relationship between the observed profit rate and the theoretical driving force.

selection of the bonds. The number of bonds included in the index varies from 75 and 100 in any given year. The main advantages of Moody's measure are that it is available since 1919, and that it is broadly representative of the U.S. non financial sector, since Baa is close to the median among rated companies.

For the risk free nominal yield, I follow the literature and use the 10-year treasury yield. Both r_t^{10} and y_t^{Baa} are obtained from FRED.[7] The issue with using the 10-year treasury bond is that it incorporates a liquidity premium relative to corporate bonds. To adjust for this, it is customary to use the LIBOR/swap rate instead of the treasury rate as a measure of risk free rate (see Hull for instance), but these rates are only available for relatively recent years. I add 30 basis points to the risk free rate in order to adjust for liquidity (see Almeida and Philippon (2005) for a discussion).

Expected inflation and real rate

In theory, one should model jointly the dynamics of the real rate and the dynamics of corporate default. In practice, however, real rates dynamics do not matter for pricing risky bonds: the theoretical credit spreads do not depend much on the persistence and volatility of the real rate. Indeed, most theoretical models assume a constant real interest rate (Leland (1998)). I therefore model real interest rates and the relative price of corporate bonds separately: I study β_t on the one hand, and b_t/β_t on the other hand. I use the Livingston survey to construct expected inflation and the yield on the 10-year treasury to construct the ex-ante real interest rate.

Idiosyncratic risk and leverage

The model described in section 3 allows me to construct q from the relative price of corporate bonds, conditional on book leverage and idiosyncratic risk. To implement the model in the time series, I must take into account that leverage and idiosyncratic risk have evolved over time. The average book leverage in Compustat has increased from 0.4 to 0.55 over the post-war period. The spread of corporate bonds over treasuries has also increased over this period, and Campbell and Taksler (2003) argue that this is due to a secular increase in the volatility of publicly traded companies.[8] I incorporate the trends in

[7] Federal Reserve Economic Data: http://research.stlouisfed.org/fred2/

[8] See Comin and Philippon (2005) for a discussion, and Davis, Haltiwanger, Jarmin, and Miranda (2006) for evidence on privately held companies.

leverage and volatility by constructing a theoretical mapping similar to the one in Figure 1, but in three dimension: the bond market's q is a function of b_t/β_t, d_t and $\sigma_{\eta,t}$. In the data, I measure book leverage from Compustat (debt over assets), and idiosyncratic volatility as the estimated standard deviation of residuals from equation (18). There are other ways to define idiosyncratic risk at the firm level – for instance with the standard deviation of the growth rate of sales or the idiosyncratic component of stock returns – but they produce similar trends, and, once mapped into their theoretical counterparts, have similar implications for the construction of the bond market's q (see Comin and Philippon (2005) for a comparison of the various measures of firm volatility).

Classic measure of Tobin's q

Finally, I construct the usual measure of Tobin's q using the flow of funds, as in **?**: the ratio of the value of ownership claims on the firm less the book value of inventories to the reproduction cost of plant and equipment.

Table 1 displays the summary statistics.

4.2 Results

Figure 2 shows the two measures of q. Tobin's q is constructed from the flow of funds. The average value of the bond market's q is normalized to 1.5: the model does not pin down this value because any linear term in the cost function appears as a constant in the marginal cost equation. Notice that the scales are different for the two series. Tobin's q (the left scale), is much more volatile and persistent. The bond market's q is approximately stationary despite the upward trend in bond spreads documented in Campbell and Taksler (2003), because the calibration takes into account the evolution of idiosyncratic volatility and book leverage, as explained above.

Figure 3 shows the bond market's q and the investment rate, in levels. The corresponding regressions are reported in the upper panel of Table 2. They are based on quarterly data. The investment rate in structure and equipment is regressed on the two measures of q, lagged one quarter:

$$x_t = \phi^b q_{t-1}^{bond} + \phi^e q_{t-1}^{usual} + \varepsilon_t$$

The standard errors control for auto-correlation in the error terms up to four quarters. The bond market's q alone accounts for nearly 60% of aggregate variations in the investment rate. The standard measure of Tobin's q, in contrast, account for only 14% of aggregate variations. Moreover, once the bond q is included, the standard measure has no additional explanatory power. The bond q is more correlated with the investment rate, hence the better fit of the estimated equation, but it is also less volatile than the standard measure of q. As a result, the elasticity of investment to q is more than 10 times higher with this new measure, which is an encouraging result since the low elasticity of investment to q has long been a puzzle in the academic literature. Of course, as Erickson and Whited (2000) discuss, there are many theoretical and empirical reasons why the inverse of the estimated coefficient is likely to underestimate the true elasticity, so one would not expect to obtain a coefficient of 0.25 or 0.5 in any case. Looking at Figure 3, the fit of the investment equation is uniformly good, except during the 1991 recession, where the drop in the investment rate exceeds the one predicted by the bond market. This is perhaps consistent with the credit crunch interpretation

Figure 4 shows the 4 quarter difference of the investment rate, a measure used by Hassett and Hubbard (1997) among others, because of the high auto-correlation of the series in levels. The corresponding regressions are presented in the bottom panel of Table 2. The fit of the equation in difference is almost as good as the fit in levels. In the third regression, the change in corporate cash flows over capital is added to the right hand side of the equation, but the additional explanatory power is very small.

The yield-based measure is successful because yield spreads of corporate bonds over treasuries forecast investment. It is well-known (Bernanke (1983) and Stock and Watson (1989)) that credit spreads have forecasting power for output, but there has been much disagreement about how one should interpret this fact. The results above suggest a straightforward interpretation: this is just q-theory. In particular, it does not require that firms be credit constrained.[9]

The conclusions from the empirical investigation on aggregate data are the following:

- The fit of the investment equation with the bond market's q is good, both in levels and in differences;

[9] Bond q forecasts output through corporate investment, not consumption or residential investment (regressions are available upon request).

- The estimated elasticity of investment to q is more than 10 times higher than the one estimated with the usual measure of q;
- Corporate cash flows do not have significant explanatory power once the bond market's q is included in the regression.

5 Firm Level Evidence

It is difficult to test the theory at the firm level because there are no readily available panel data set on corporate bond prices, certainly nothing comparable to the Compustat and CRSP data for equity pricing and accounting information. To test the model in a cross section of firms, rather than in the time series, I create a panel of corporate bond yields by matching the rating of a company in a given year, available from Compustat, to the average yield in the same year and rating category. Thus, my measure of yields is rating-year specific, not firm-year specific.

I obtain data on corporate yields from Citigroup's yield book, which covers the period 1985-2004. For bonds rated A and BBB, these data are available separately for maturities 1-3, 3-7, 7-10, and 10+ years and I use the 10+ maturity. For bonds rated BB and below, these data are available only as an average across all maturities. The firm level data come from Compustat and are entirely standard in the literature. All data are annual. The data are described in Table 3.

To test the theory in the cross section, I construct the bond market's q using the same model as with the aggregate data, but ignoring time variation in the real interest rate, leverage and volatility. Instead, I include a full set of year dummies and I estimate a panel regression with firm-fixed effects. The time dummies capture any change in the real interest rate and any common trend in leverage or risk. The firm fixed effect capture any permanent heterogeneity in credit risk at the firm level.[10] These controls offer a test of the model that is entirely orthogonal to the one performed earlier in the time series. The construction of the bond market's q at the firm level follows the procedure described in the benchmark

[10] I include fixed effects rather than estimating firm level measures of risk because, while the trends in average or median volatility are consistent across a wide variety of measures (see Comin and Philippon (2005)), the cross sectional dispersion of the measures is quite large. In any case, the combination of fixed effects and time dummies actually accounts for most of the variations in idiosyncratic risk in the panel of rated firms.

calibration at the end of section 3.

I estimate the following equation:

$$x_{it} = \alpha_i + \xi_t + \phi^b q_{t-1}^{bond}(rating_{it}) + \phi^u q_{i,t-1}^{usual} + \varepsilon_{it}. \tag{20}$$

I write explicitly $q_{i,t-1}^{bond}$ as a function of the rating to emphasize that all firms with the same rating in the same year have the same imputed yield. By contrast, $q_{i,t-1}^{usual}$ and might miss some important information.

Table 4 presents the results. For $q_{i,t-1}^{bond}$, the results are consistent with the ones obtained in the time series. The coefficients are significant and of a similar order of magnitude. For $q_{i,t-1}^{usual}$, the results are better. The regression coefficients are significant, and larger than the ones obtained in the time series. Relative stock market values are consistently linked to relative investment rates, even though the relation is weak in the aggregate. Finally, in contrast with the macro data, cash flow variables retain significant explanatory power at the firm level, even after the inclusion of $q_{i,t-1}^{bond}$ and $q_{i,t-1}^{usual}$.

The results are consistent, both qualitatively and quantitatively, with the ones obtained by Gilchrist and Zakrajsek (2007). Gilchrist and Zakrajsek (2007) build a large panel data set of firm level bond prices. They regress the investment rate on a firm specific measure of the cost of capital, based on industry specific prices of capital, and firm level bond yields. They find a strong negative relationship between the investment rate and the corporate yields.

6 Conclusion

I have shown that it is possible to construct q using only bond prices, and that this new measure explains aggregate investment reasonably well. I now discuss the interpretations and remaining puzzles. At the aggregate level, q-theory seems to work fine, provided that one uses the bond market instead of the equity market to construct q. It is therefore unlikely that non-convexities, aggregation biases, measurement errors in accounting data, or credit constraints explain the failures of the usual approach with aggregate data. They probably explain the failures with firm level data, however.

The results shed light on the role of risk premia in macroeconomics. Abel and Blanchard (1986) construct a time series for marginal q that takes into account changes in the risk free rate, while abstracting from changes in risk premia. In this setup, changes in aggregate investment are driven either by changes in the risk free rate, or by news about future aggregate profitability. Compared to the approach in Abel and Blanchard (1986), the new measure has the advantage of taking into account changes in risk premia over time.[11]

The results, like recent work cited in the introduction, point to issues with the market value of equity. The bond market could be useful for several reasons. Adverse selection might limit the use of equity financing. If, at the margin, large firms finance their investment with debt, the price of debt might be more relevant than the price of equity. The presence of bubbles in equity prices, but not in bond prices, is another potential explanation. If bubbles are driven by investors with non standard beliefs, behavioral theories usually predict more biases when returns are more skewed. The bond market might therefore be less biased than the equity market.

Large growth options, coupled with specific adjustment costs, might also contribute to the results. A small probability of a large increase in the present value of assets would have a small impact on bond prices, and a large impact on equity prices. If, in addition, firms must wait for the growth options to materialize before they can actually invest, this could explain the low correlation between equity prices and investment. It would be as if firms faced two types of adjustment costs, the ones for normal investment, well captured by convex costs, and the ones for growth options, highly non-convex.[12] Each potential explanation deserves further research.

[11] Lettau and Ludvigson (2002) also emphasize the role of time varying risk premia, and risk premia are also central in the bond pricing literature. In the macroeconomic literature, however, the leading papers focusing on credit constraints, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), assume that investors are risk neutral. In this case, it is the objective probability of financial distress that matters for investment. This objective probability, however, is much smaller that the risk adjusted one, as shown in Berndt, Douglas, Duffie, Ferguson, and Schranz (2005). Thus, irrespective of whether firms are indeed constrained or not, one should not neglect the effects of investor risk aversion. Similarly, models of non-convex adjustment costs in the tradition of Caballero and Engle (1999) also rely on risk neutral investors for tractability. To the extent that one is concerned with explaining firm level investment dynamics, the assumption of risk neutral investors is probably fine, because much of the volatility at the firm level reflects cash flow news, not discount rate shocks, as emphasized for instance in Vuolteenaho (2002). In the aggregate, however, discount rate shocks are much more important. My results suggest that understanding the nature of these macroeconomic shocks could be necessary and sufficient to obtain a successful theory of aggregate investment.

[12] This would seem to be consistent with the fact that IPOs are well explained by equity market valuations, while the bulk of capital expenditures is well explained by bond market valuations.

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A Short term debt

A.1 Proof of Lemma 2

I prove directly the case where book leverage depends on the state,

$$d(s) = d_0 \left(1 + \hat{d}(s)\right),$$

where $E^{\pi}\left[\hat{d}(s)\right] = 0$. Note that the leverage process can be quite general. It accommodates all the cases where firms increase or decrease their leverage based on current market conditions. Using Leibniz's formula

$$\begin{aligned} L(s, s') &= \int_{v(s',\eta')<d(s)} \left(1 - \frac{v(s',\eta')}{d(s)}\right) \zeta(\eta')\, d\eta' \\ &\approx \int_{\bar{v}+\eta'<d_0} \left(1 - \frac{\bar{v}+\eta'}{d_0} - \frac{\hat{v}(s')}{d_0} + \frac{\bar{v}+\eta'}{d_0}\hat{d}(s)\right) \zeta(\eta')\, d\eta' \\ &\approx \bar{L} - \frac{\hat{v}(s')}{d_0} \int_{\bar{v}+\eta'<d_0} \zeta(\eta')\, d\eta' + \hat{d}(s) \int_{\bar{v}+\eta'<d_0} \frac{\bar{v}+\eta'}{d_0} \zeta(\eta')\, d\eta' \end{aligned}$$

where

$$\bar{L} \equiv \int_{\bar{v}+\eta'<d_0} \left(1 - \frac{\bar{v}+\eta'}{d_0}\right) \zeta(\eta')\, d\eta'$$

Now define the default rate:

$$\bar{\theta} \equiv \int_{\bar{v}+\eta'<d_0} \zeta(\eta')\, d\eta'$$

and the recovery rate:

$$\bar{\rho} = \frac{1}{\bar{\theta}} \int_{\bar{v}+\eta'<d_0} \frac{\bar{v}+\eta'}{d_0} \bar{\zeta}(\eta')\, d\eta'$$

to get

$$L(s, s') \approx \bar{L} + \bar{\theta}\ \left(\bar{\rho}\ \hat{d}(s) - \hat{v}(s')/d_0\right)$$

.QED

Lemma 4 *When book leverage changes, the bond market's q formula becomes*

$$q_t \approx \frac{\bar{v}}{1+r_t} + \frac{d_0}{1+\bar{r}} \left(\frac{1}{\bar{\theta}} \frac{1+r_t^{\$}}{1+y_t^{\$}} + \bar{\rho}\hat{d}_t\right) + cte$$

A.2 Time varying idiosyncratic risk

Suppose that the distribution of shocks at time $t+1$ depends on the aggregate state at t

$$\zeta(\eta', s) = \bar{\zeta}(\eta') + \hat{\zeta}(\eta', s)$$

and define

$$\hat{L}(s) \equiv \int_{\eta'<d_0-\bar{v}} \left(1 - \frac{\bar{v}+\eta'}{d}\right) \hat{\zeta}(s, \eta')\, d\eta'$$

Then the following proposition shows how one can construct q

Lemma 5 *When the conditional distribution of idiosyncratic shocks varies over time,*

$$q_t \approx \frac{\bar{v}}{1+r_t} + \frac{d}{\bar{\theta}\,(1+\bar{r})} \left(\frac{1+r_t^{\$}}{1+y_t^{\$}} + \hat{L}_t \right) + cte$$

The case of time varying idiosyncratic volatility is more than a theoretical curiosity, since the volatility of publicly traded companies has indeed changed over the post-war period (Campbell, Lettau, Malkiel, and Xu (2001), Comin and Philippon (2005)). Not surprisingly, changes in idiosyncratic risk are mirrored by changes in corporate bond spreads.

B Proof of proposition 2

The price of debt at time $t+1$ is

$$b_{t+1} = E_{t+1}\left[\sum_{\tau=t+2}^{\infty} m_{t+1,\tau}\left(1-\Theta_{t+1,\tau-1}\right)(1-\phi)^{\tau-2-t}\left(\left(1-\theta_\tau\right)(c+\phi)+\theta_\tau V_\tau/F_\tau\right)\right]$$

So we can write

$$\begin{aligned} b_t &= E_t\left[m_{t,t+1}\left(\left(1-\theta_{t+1}\right)(c+\phi)+\theta_{t+1}V_{t+1}/F_{t+1}\right)\right] \\ &\quad + E_t\left[\sum_{\tau=t+2}^{\infty} m_{t,\tau}\left(1-\Theta_{t,\tau-1}\right)(1-\phi)^{\tau-1-t}\left(\left(1-\theta_\tau\right)(c+\phi)+\theta_\tau V_\tau/F_\tau\right)\right] \end{aligned}$$

but since

$$(1-\phi)\left(1-\theta_{t+1}\right) m_{t,t+1} b_{t+1} = E_{t+1}\left[\sum_{t'=t+2}^{\infty} m_{t,\tau}\left(1-\Theta_{t,\tau-1}\right)(1-\phi)^{\tau-1-t}\left(\left(1-\theta_\tau\right)(c+\phi)+\theta_\tau V_\tau/F_\tau\right)\right]$$

we get

$$b_t = E_t\left[m_{t,t+1}\left(\left(1-\theta_{t+1}\right)\left(c+\phi+(1-\phi)\,b_{t+1}\right)+\theta_{t+1}V_{t+1}/F_{t+1}\right)\right]$$

Now assume that the firms chooses its debt policy so as to leave the ratio F_{t+1}/k_{t+1} equal to some constant d. In this case, we obtain

$$b(\omega) = \beta(\omega)\, E^{\pi}\left[\left(\left(1-\theta\left(\omega'\right)\right)\left(c+\phi+b\left(\omega'\right)\right)+\theta\left(\omega'\right)\frac{v\left(\omega'\right)}{d}\right) \mid \omega\right]$$

Finally, assuming that default happens when equity value drops to 0, we get

$$b(\omega) = \beta(\omega)\, E^{\pi}\left[\min\left(c+\phi+(1-\phi)\,b\left(\omega'\right); \frac{v\left(\omega'\right)}{d}\right) \mid \omega\right]$$

Table 1: Aggregate Summary Statistics

Quarterly Aggregate Data. 1953:2-2005:3.					
	Obs.	Mean	St. Dev.	Min	Max
I / K	210	0.103	0.009	0.082	0.123
E(inflation)	210	0.038	0.025	-0.016	0.113
y^{Baa}	210	0.082	0.031	0.035	0.170
r^{10}	210	0.065	0.027	0.023	0.148
$y^{Baa} - r^{10}$	210	0.017	0.007	0.004	0.037
Classic Tobin's Q	210	2.005	0.848	0.821	4.989
Bond Market's Q	210	1.500	0.115	1.168	1.727

Investment and replacement cost of capital are from NIPA. Expected inflation is from Livingston survey. Yields on 10-year Treasuries and Baa bonds are from FRED. Classic Tobin's Q is computed from the flow of funds, following Hall (2001).

Table 2: Aggregate Invesment and Q

	Equation in Levels: I/K(t)		
Bond Q (t-1)	**0.0631**		**0.0601**
s.e.	*0.0058*		*0.0064*
Tobin's Q (t-1)		**0.0042**	0.0011
s.e.		*0.0014*	*0.0009*
N	209	209	209
R^2	0.5920	0.1396	0.5998
	Estimation in Changes: I/K(t) - I/K(t-4)		
Δ(Bond Q) (t-5,t-1)	**0.0490**		**0.0417**
s.e.	*0.0054*		*0.0064*
Δ(Tobin's Q) (t-5,t-1)		**0.0071**	0.0020
s.e.		*0.0017*	*0.0013*
Δ(Profit Rate) (t-5,t-1)			0.1159
s.e.			*0.0574*
N	205	205	205
R^2	0.5593	0.1208	0.5886

Fixed private non-residential capital and investment series are from the BEA. Quarterly data, 1953:3 to 2005:3. Tobin's Q is constructed from the Flow of Funds, as in Hall (2001). Bond Q is constructed by applying the structural model to Corporate and Treasury yields, expected inflation, book leverage and idiosyncratic firm volatility. Newey-West standard errors with autocorrelation up to 4 quarters are below the coefficients, in italics. Bold coefficients are significant at the 1% level or higher.

Table 3: Firm Level Summary Statistics

Annual Firm Level Data. 1985-2004					
	Obs.	Mean	St. Dev.	Min	Max
Capital Expenditures / Prop. Plant & Equip.	17085	0.193	0.129	-0.031	1
Yield Spread (corporate-treasury)	17085	0.029	0.031	0.005	0.337
Market Value over Book Assets	17085	1.590	0.926	0.445	10
Operating Income over Book Assets	17085	0.131	0.077	-0.799	0.918

Notes: Capital expenditures, income, market and book values are from Compustat. Corporate yields are from Citibank's Yieldbook, and 10-year treasury yields are from FRED.

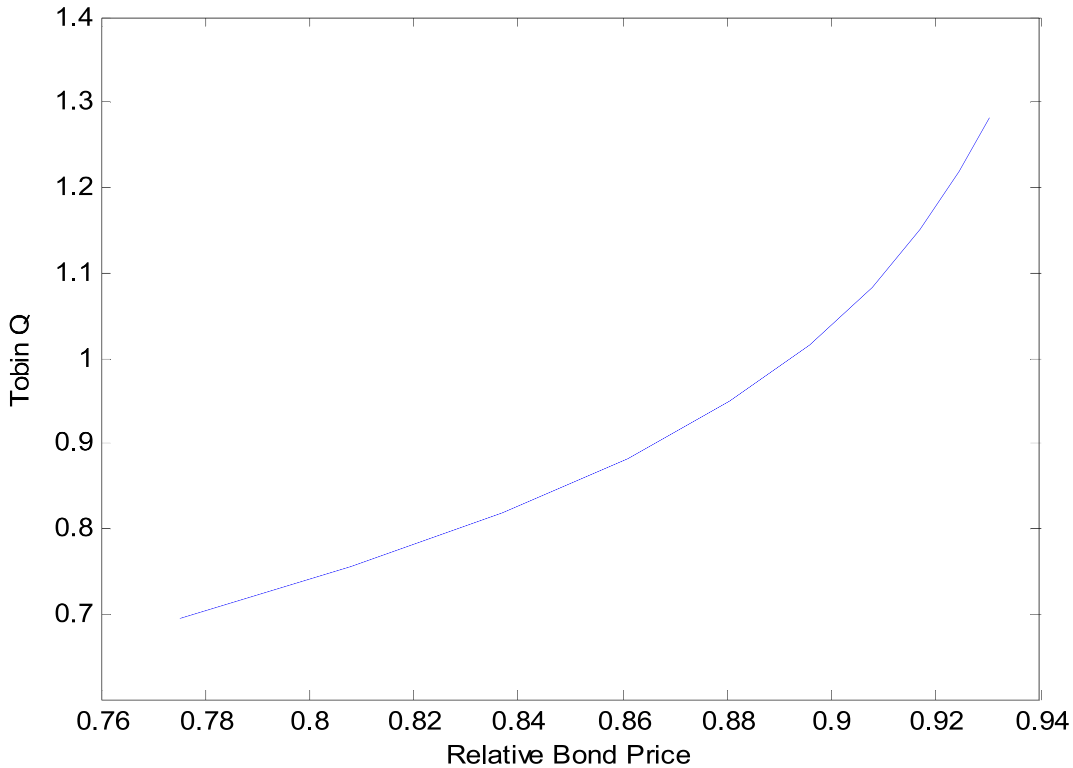
Table 4: Firm Investment and Q

Dependent variable is Capital Expenditure over lagged Property, Plants and Equipment. Panel regressions with fixed effects and robust standard errors.

Bond Q (lagged one year)	**0.0676**	**0.0579**
s.e	*0.0054*	*0.0054*
Market Value over Book Assets (lagged one year)	**0.0301**	**0.0243**
s.e	*0.0019*	*0.002*
Operating Income over Book Assets (lagged one year)		**0.2165**
s.e		*0.0206*
Firm Fixed Effects	yes	yes
Year Dummies	yes	yes
N	14,536	14,536

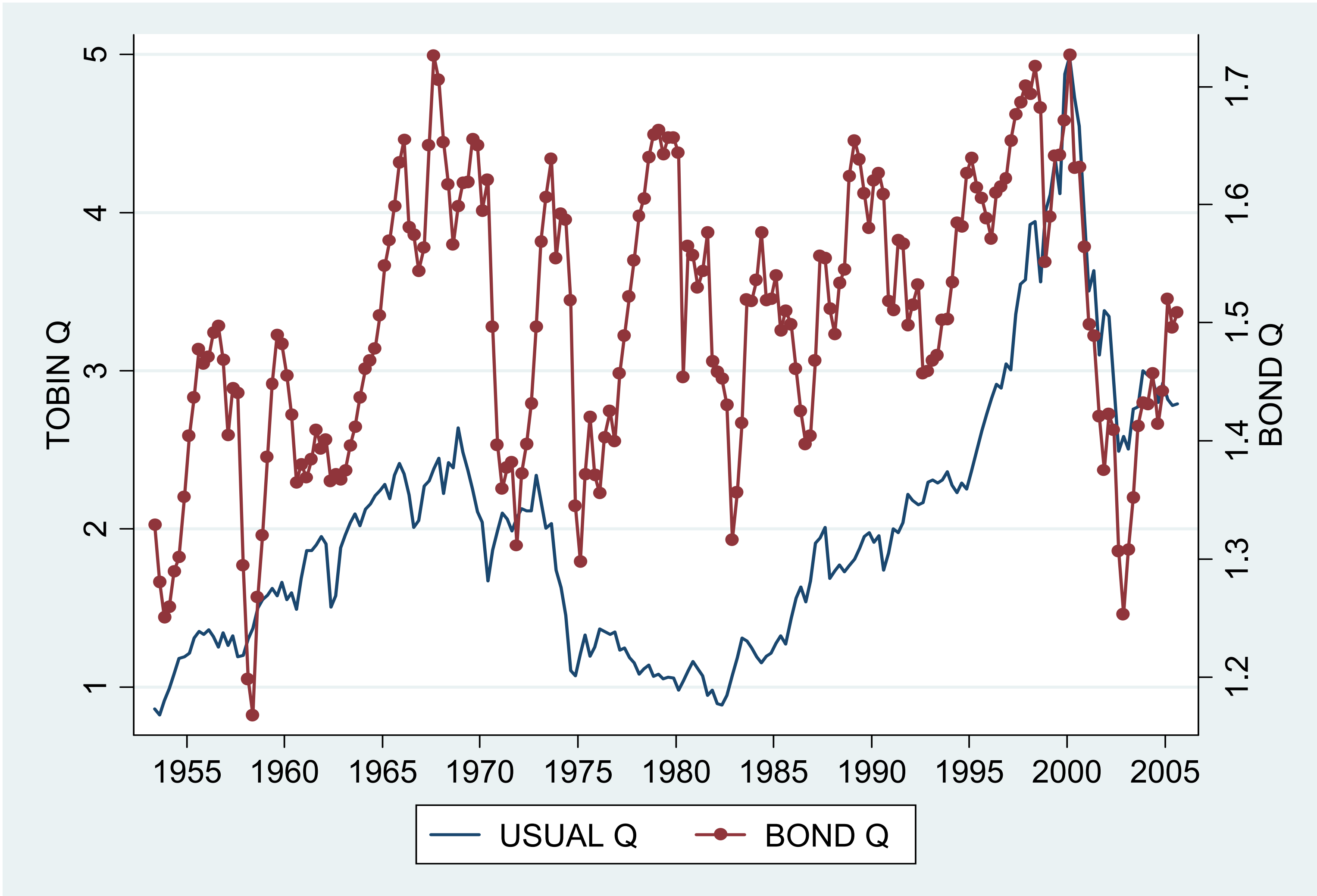
Annual data 1985-2004. Bond Q constructed from corporate yields and treasury yields. Standard errors corrected for firm level clustering. Bold coefficients are significant at the 1% level of higher. Sources: firm level data from Compustat, yields by rating from Citibank's yieldbook.

Figure 1: Aggregate Tobin's Q and Relative Prive of Corporate Bonds.



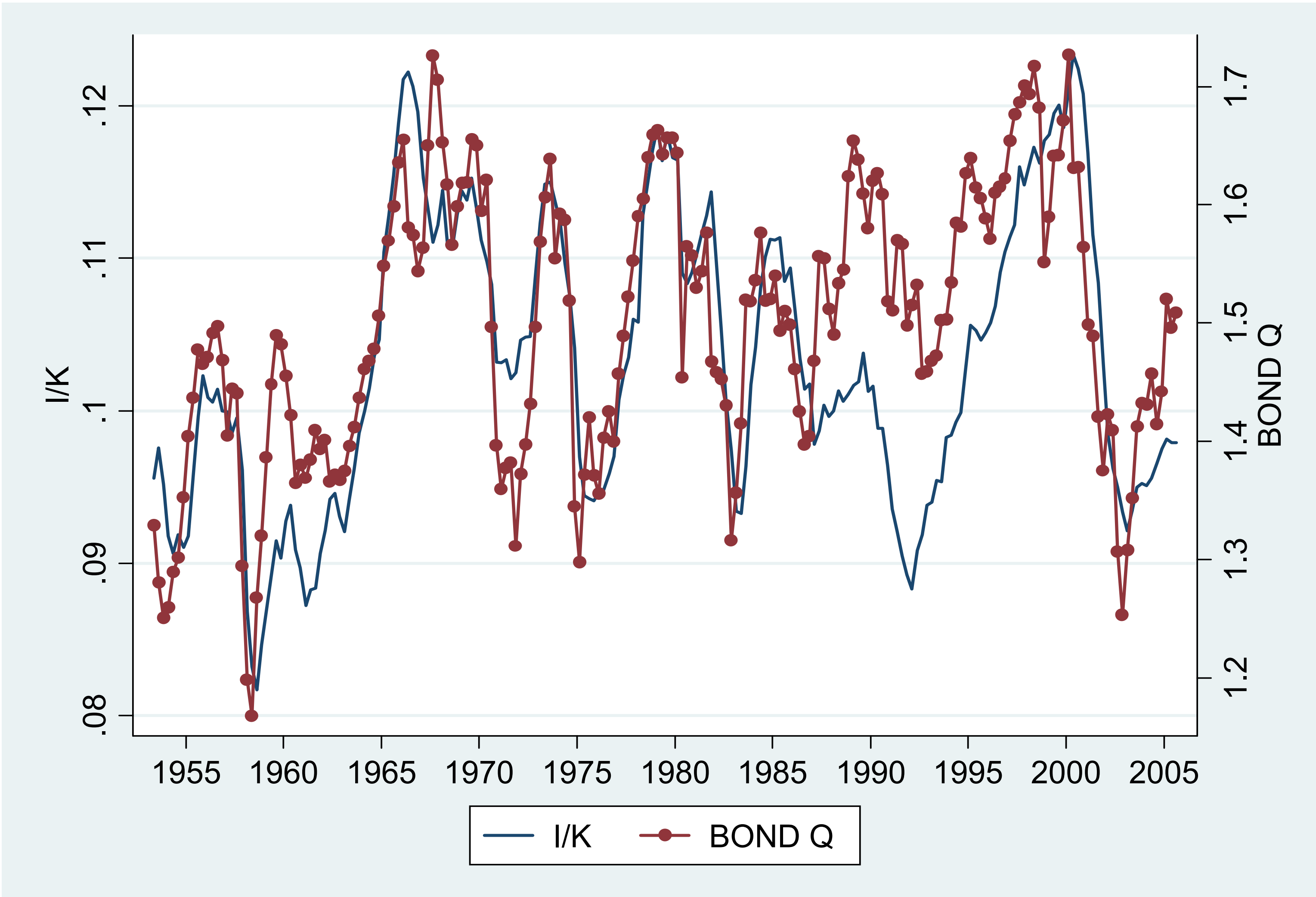
Note: Bonds are calibrated to an average maturity of 10 years. Corporate bonds are calibrated using the Baa yield, and risk free bonds using the 10-year treasury yield. The relative price of corporate bonds is defined as (0.1+r)/(0.1+y), where r is the risk free yield and y is the (average) corporate yield.

Figure 2: Usual Measure of Q and Bond Market's Q



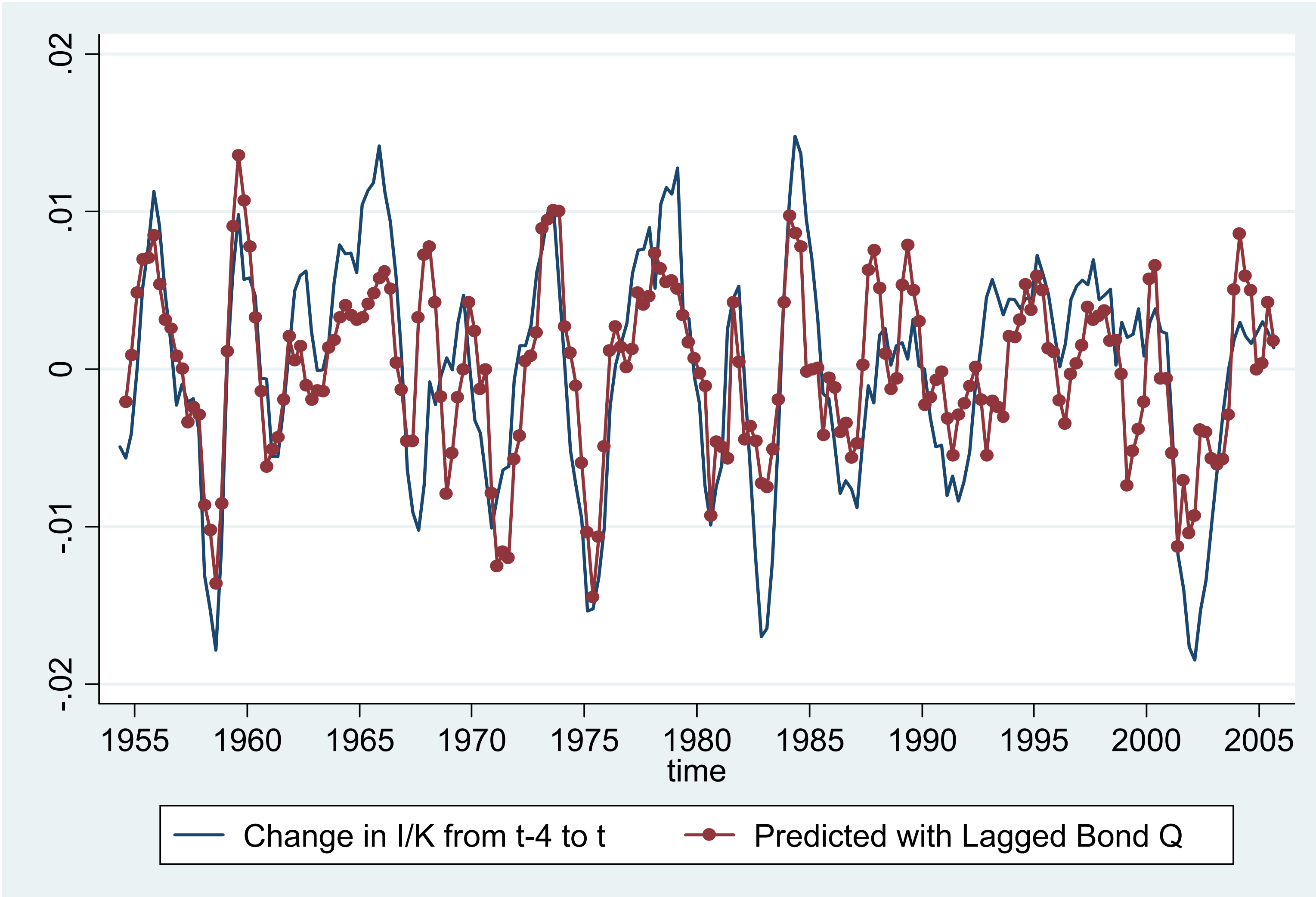
Notes: Tobin's Q is constructed from the Flow of Funds, as in Hall (2001). Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.

Figure 3: Bond Market's Q and Investment Rate (levels)



Notes: I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.

Figure 4: Investment Rate (4-quarter changes), actual and predicted with Bond Q.



Notes: I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.