On the Effects of Individual Labor-flexibility on Rational Portfolio Allocation: Making a Case for Businessman Risk and Employer Stock Ownership*

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Abstract

This paper studies the impact of labor flexibility on optimal life-cycle portfolio decisions, particularly the ability to change industries or firms within industries. The model addresses two frequently observed portfolio behaviors that are seemingly inconsistent with rational portfolio choice. The first is the tendency of workers and entrepreneurs to hold their company's stock. The second is the propensity of workers to limit their equity holdings through time.

The explanation offered here for both of these behaviors lies in the option to switch jobs when one's company does poorly. This is equivalent to holding put options on one's own company stock and call options in the other company's stock, where both options must be exercised at the same time. Given these initial undiversified implicit financial holdings, workers need to allocate a relatively large share of their regular financial assets to their own company's stock and a relatively small share to the stock of their alternative employment simply to restore overall portfolio balance.

I consider the case of an option to switch between a relative safe and a relatively risky job. I find that when workers start their career in the safer job, they optimally hold a positive amount in their company's stock (under some conditions almost half of their financial wealth). However, if workers start out in the riskier job, they short their company's stock – just as in the absence of job-switching options – but they short less.

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1 Introduction

The concept of *businessman risk* holds that among those with similar financial wealth, young businessmen should invest considerably more in risky assets than older widows. Since Samuelson's (1969) questioning of its validity, many others have extensively explored it and much debated whether to reject it or save it. [1]

In parallel, Markowitz (1952) and Sharpe (1964)'s economic prescriptions have been challenged by the evidence that individuals seemingly deviate from the principles of portfolio diversification and invest in the "familiar." Allocation strategies driven by the investor's geographical or professional proximity to a particular stock are generally conceptualized in the term *familiarity*, "the tendency of households' portfolios to be concentrated, of employees [...] to own their employers' stocks in their retirement accounts and [...] of home country bias in the international arena."[2] For example, Coca-Cola employees allocate to company stock 76 percent of their discretionary contributions to the plan assets (Benartzi, 2001) and the seemingly financially savvy employees of J.P. Morgan invest 19 percent of their 401(k) plan money in Morgan stock (Huberman, 2001). Altogether, investors do not hedge, invest in stocks correlated to their nonfinancial income and, in many cases, even increase their exposure to risky assets with age.[3]

Curcuru, Heaton, Lucas and Moore (2004) exhaustively summarize the evidence on household portfolio composition and re-examine some of the theories that have been proposed to account for portfolio heterogeneity across individuals. Within the traditional utility maximizing framework, labor and entrepreneurial income, transaction costs, borrowing constraints and other life-cycle considerations seem to explain some aspects of the observed cross-sectional variation in portfolio holdings. The lack of diversification in some unconstrained individual portfolios, however, remains a challenge for quantitative theories.

This paper studies the impact of labor flexibility on optimal life-cycle portfolio decisions, particularly the ability to change industries or firms within industries. The model addresses both the tendency of workers and entrepreneurs to hold their company's stock and their propensity to limit equity holdings through time.

The explanation offered here for both of these behaviors lies in the *option* to switch jobs when one's company does poorly. Prior theoretical models have investigated optimal portfolio decisions under the assumption of a lifetime employment at the same employer. However, since labor mobility is an essential determinant of human capital, my analysis provides new insights into the relation between financial and human wealth. Moreover,

[1] For instance, Brown (1990, p. 905) concludes "The middle-aged by comparison have a high tolerance for risk and a low marginal propensity to consume relative to the young; the middle-aged with significant savings is willing to take on the businessman's risk." Similarly, Malkiel (2007, p. 101) writes "According to one well-known theory, the bigger the swings – relative to the market as a whole – in an individual company's stock prices, the greater the risk. [...] A nonswinger gets the *Good Housekeeping* seal for "widows and orphans." [...] On the other hand a "flyer" is a businessman's risk."

[2] Huberman, 2001, p. 659.

[3] "One trend that almost everyone applauds is the attempt to prevent employees from investing too heavily in company stock, in some cases by restricting the amount of company stock that employees can buy," said Stephen P. Utkus of Vanguard's Center for Retirement Research.

(Source: http://www.washingtonpost.com/wp-dyn/content/article/2007/10/20/AR2007102000141.html) labor mobility is empirically relevant: over the 1968-1997 period, Kambourov and Manovskii (2006) document variations in U.S. industry mobility[4] from 7% to 12% at the one-digit level, from 8% to 13% at the two-digit level, and from 10% to 13% at the three-digit level.[5]

The term *option* is telling because the ability to change jobs represents the implicit holding of a financial option, namely a put option written on one's current company. To see this, assume a worker's wage is perfectly correlated with her employer's stock. In this case, her human capital represents an implicit employer stock holding, but of a special kind. If the value of her employer's stock falls, she can avoid a loss on her implicit employer stock holding simply by changing jobs. Consequently, her human capital constitutes the holdings not just of her employer's stock, but also of put options on that stock, albeit rather exotic put options, called *spread* options. The return on these implicit spread options is higher both when one's own company does poorly and when one's alternative company of employment does well. Consequently, holding these spreads is very similar to holding call options in the other company's stock and puts on one's own company, where both options must be exercised at the same time. Given these initial undiversified implicit financial holdings, workers need to allocate a relatively large share of their regular financial assets to their own company's stock and a relatively small share to the stock of their alternative employment simply to restore overall portfolio balance. This is not simply a theoretical curiosum, but, as shown here, a factor of potentially major import for assessing the suitability of workers' financial decisions.

I consider the case of an option to switch between a relative safe and a relatively risky job. I find that when workers start their career in the safer job, they optimally hold a positive amount in their company's stock (under some conditions almost half of their financial wealth). However, if workers start out in the riskier job, they short their company's stock – just as in the absence of job-switching options – but they short less. Thanks to mobility costs, the extent to which the ability to switch jobs constitutes a valuable spread option varies with age. As one approaches retirement, the spread option's value goes to zero leaving workers more exposed to their company's performance. This leads them to diversify away from their own company's equity holdings and, indeed, from equity holdings in general – a result that recalls Bodie, Merton, Samuelson's (1992) prescription to reduce equities with age, but that arises from a different source namely the implicit reduction through time in the amount of equity insurance provided by the job-switching put.

The structure of this paper is as follows. Section 2 describes the basic model, derives the solution of the optimal consumption–portfolio–job-regime plan and presents the baseline calibration for the case of employment in a specific industry versus the rest of the economy. In Section 3 the model's predictions are compared to the previous findings on portfolio allocation. Section 4 offers a sensitivity analysis of the findings to changes in various benchmark parameter values. Section 5 is dedicated to the comparison of the model's predictions with the findings of empirical studies on household portfolio selection. Section 6 examines how the model's predictions can be applied to other puzzles in the financial literature. These are

[4]Industry mobility is defined as the fraction of currently employed individuals who report a current industry different from their most recent previous report (Kambourov and Manovskii, 2006 p. 2).

[5]Data from the Panel Study of Income Dynamics.

the private equity premium puzzle documented by Moskowitz and Vissing-Jorgensen (2002) and the motion of wealth-to-income ratios with age studied by Gentry and Hubbard (2000). Section 7 summarizes the findings of the paper and briefly touches on future research based on the present methodology. Proofs and some details on the implementation of the numerical algorithm are collected in Appendix A. Appendix B illustrates how the present approach easily extends to the case of imperfect correlation between financial and nonfinancial income.

2 Model Specification and Calibration

2.1 The Investment Opportunity Set

This Section describes the operating markets accessible to the individual for trading in financial assets, her preferences over the life-cycle and affordability constraints.

I posit a financial market comprised of a riskless asset and two risky dividend-paying assets. The dynamics of the riskless security satisfy

$$dB_t = r_f B_t dt,\ B_0 > 0 \textit{ given}, \tag{1}$$

in which $r_f\ (> 0)$ is the instantaneous market rate of interest, and is constant over time. The two risky security prices, $S\ (= S_i, S_j)$, follow the two-dimensional Itô process

$$dS_t = S_t \left[(\mu_S - \delta_S)\, dt + \sigma_S dz_t \right],\ t \in [0, T],\ S_0 > 0 \textit{ given}, \tag{2}$$

where δ_S is a vector of dividend yields, μ_S is a vector of instantaneous expected rates of return and σ_S is a two-by-two diagonal matrix of instantaneous volatility coefficients. z_t is a Brownian Motion process defined in $\mathbb{R}^2$ and T is the individual's finite fixed planning horizon. The implied market price of risk is denoted by $\theta \equiv \sigma_S^{-1} (\mu_S - r_f \mathbf{1}_2)$, where $\mathbf{1}_2$ is the two-dimensional unit vector, and the state-price density process by $\xi_t \equiv e^{\left\{-r_f t - \frac{\theta\prime\theta}{2} t - \theta' z_t\right\}}$.

Financial markets are complete[6] and frictionless. Assets are traded continuously in the absence of transaction costs and both borrowing[7] and short-selling are allowed without restrictions.

2.2 The Investor's Preferences

Let $c_t \geq 0$ denote the individual's rate of consumption at time t. Preferences are posited to be an instantaneous time-separable power utility function over consumption

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},\ t \in [0, T], \tag{3}$$

[6] This is equivalent to assuming that there is a unique risk-neutral probability measure associated with the model.

[7] This assumption is supported by market completeness: in the present model, as in Bodie *et al.* (1992), any risk exposure borne by the individual can be hedged continuously in time by dealing in stocks.

in which $\gamma \geq 1$ is the constant coefficient of relative risk aversion – the elasticity of intertemporal substitution of consumption is $\frac{1}{\gamma}$.[8]

2.3 The Labor Income Process

This Subsection presents the labor income process and the cost of switching careers.

The bulk of life-cycle models analyzes optimal consumption and portfolio decisions under the assumption of a lifetime employment at the same employer. Here I explicitly allow for labor mobility by providing the individual with a real option to switch jobs during her working years. Consequently, along with optimal consumption and portfolio policies, the individual must select the optimal time for a change in her career. The option to leave one employment for another is especially valuable when the evolution of prices and wages in financial and labor markets is uncertain.

The finite-lived individual of this model finances consumption and investments by earning a per period salary, along with earnings on accumulated financial assets. The vector of wage flows offered in each of the two jobs, $w_t \equiv [w_{i,t}, w_{j,t}]'$, has dynamics given by

$$dw_t = w_t \left[\alpha dt + \sigma dz_t\right], \ t \in [0, \tau_2], \ w_0 > 0 \ given.^9 \tag{4}$$

The instantaneous expected wage growth rates are collected in the two-dimensional vector $\alpha > 0$, while σ is a two-by-two invertible matrix of wage volatility coefficients, whose entry $\sigma_{i,j}$ denotes the volatility of wage i with respect to the j^{th} dimension of $z_t \in \mathbb{R}^2$. Retirement is modeled as an irreversible labor income state beginning at time τ_2, $\tau_2 \in [0, T]$. Notice that the present analysis reflects job market turnovers only partially since the individual's decision is confined to just two jobs and the job-switching option can be exercised at most once prior to the retirement date.

The modeling structure of Eqs. (2) and (4) posits perfect correlation between labor income innovations and stock returns in each job.[10,11] Because the correlation between con-

[8] The individual is not permitted to bequeath wealth to her descendants at death: an extension of the current model specification could introduce a bequest motive component to Eq. (3) to avoid complete wealth dissaving by the end of the individual's life. This would be equivalent to lengthening the individual's planning horizon. Gomes and Michaelides (2005), Cocco, Gomes and Maenhout (2005) and Polkovnichenko (2007) demonstrate that bequests increase individual savings by curbing wealth accumulation after retirement. This effect is essential to match observed wealth-accumulation profiles over the life-cycle.

A more general description of individual preferences would also include an index of past consumption to capture the notion that individuals develop habits. Bodie *et al.*, (2004) derive solutions for optimal consumption, labor supply and financing portfolio in a life-cycle model with habit formation. Polkovnichenko (2007) demonstrates that, in the context of a life-cycle model with uninsurable labor income, additive and endogenous habit formation preferences can generate more conservative portfolios for younger than for middle-aged households .

[9] The wage distribution of Eq. (4) is non-stationary. An alternative specification with wage distributions independent of wage rates is offered by Van Der Berg (1992).

[10] This assumption is cardinal in justifying desired sizeable equity premia in response to stock and labor market losses resulting from financial crises.

[11] Appendix B demonstrates how an economy with less than perfectly correlated labor and financial markets (or, correspondingly, correlated industry wages) can be transformed to restore the canonical model specifica-

sumption and stock returns is intensified by the correlation of stocks and wages, positive correlation coefficients generate more conservative portfolios.

Letting $\tau_1 \in S_{0,\tau_2-\Delta}$ be the optimal time of a job change in the set of feasible stopping times $S_{0,\tau_2-\Delta}$, and $\Delta \geq 0$ represent a deterministic transition phase at the onset of a new job, Eq. (5) characterizes the static budget constraint of an individual leaving employment i for employment j. Individual *total wealth* is comprising both *financial wealth* – with initial value $W_0 > 0$ and current value W_t, $t \in (0, T]$, echoing past saving and investing – and *human capital wealth* – the present value of future labor income.[12] The lifetime budget constraint can be written as

$$E\left[\int_0^T \xi_t c_t dt\right] \leq W_0 + \sup_{\tau_1 \in S_{0,\tau_2-\Delta}} E\left[\int_0^{\tau_1} \xi_t w_{i,t} dt + \int_{\tau_1+\Delta}^{\tau_2} \xi_t w_{j,t} dt - \xi_{\tau_1} TU\right]^+, \qquad (5)$$

where E denotes the time-zero expectation operator that encompasses the probability distribution of all states of the world over the individual's planning horizon. $TU > 0$ is a fixed cost, e.g., the payment of school tuitions, which the individual faces as she leaves the workforce to acquire additional skills necessary to her new career.[13,14] The left-hand side quantifies the present value of consumption affordable to the individual, conditional on personal total lifetime resources consisting of initial financial wealth and labor earnings, the right-hand side.

The budget constraint of Eq. (5) internalizes the fact that young workers' future income streams derive from wages forthcoming in their middle age, while older workers rely increasingly more on financial savings.[15]

The Subsection to follow focuses on the individual's security holdings in relation to her implicit human capital risk exposures. This analysis disentangles mean-variance, job-specific

tion in this section of the paper. I propose a three-step procedure to convert a given set of parameter values for stocks and wages into corresponding figures that can be directly inputted in the solution algorithm illustrated in Appendix A. Specifically, this technology creates an artificial economy that is formally isomorphic to the actual one and that reproduces the dynamics of the original model.

[12] Bodie, Merton, and Samuelson, 1992, p. 428.

[13] Note that, for points in time sufficiently close to τ_2, it is optimal to reject all job change opportunities since insufficient time remains to recover tuition expenses. This reasoning justifies feasible stopping times in the closed interval $[0, \tau_{2-\Delta}]$.

[14] The significant growth in the number of education programs in the U.S., primarily due to increased public expenditures on schools and other non-monetary benefits accruing to schooling, persuaded me to elaborate on schooling as a mode of human capital enrichment ensuing from labor mobility. However, this is only an instance of mobility costs: housing, pension claims, fringe benefits, psychological and social costs, and age are at the core of a vast literature examining impediments to labor mobility.

Representative contributions are Mincer (1991) and Groot and Verberne (1997). The former uses information on total annual costs of job training in the U.S. for 1958, 1976, and 1987 to compute rates of return on training investments. The latter analyzes job mobility patterns and their evolution with age and tenure disentangling the effects of wage differentials, mobility costs, and the shadow price for physically arduous labor.

[15] This argument is also presented in Constantinides, Donaldson, and Mehra (2002), who study equilibrium demand for equities and bonds in the context of a stationary, overlapping-generation economy with borrowing constraints.

and hedging components that form the optimal investment policies. Malliavin calculus methods are implemented in the derivation of the policy equations.

2.4 Optimal Consumption, Portfolio and Job-Regime Plans

The solution methodology for optimal policies is based on Cox and Huang (1989), Detemple, Garcia, and Rindisbacher (2003) and Nualart (2006). Stochastic integration methods yield the closed-form expression $HC_{(\cdot),t,v} \equiv w_{(\cdot),\tau_1}\frac{e^{\left\{\eta_{(\cdot)}(v-\tau_1)\right\}}-e^{\left\{\eta_{(\cdot)}(t-\tau_1)\right\}}}{\eta_{(\cdot)}}$, in which $\eta_{(\cdot)}$ is the algebraic sum $\alpha_{(\cdot)} - r_f - \left(\sigma_{(\cdot),i}\theta_i + \sigma_{(\cdot),j}\theta_j\right)$, $i \neq j$. Although the functional form taken by $HC_{(\cdot),t,v}$ is rather complicated, the idea behind its definition is intuitive: by accounting for the present value of future labor earnings, $HC_{(\cdot),t,v}$ quantifies the individual's human capital wealth while employed in $(\cdot) = i, j$ over the time interval $[t, v]$. Equation (5) can be re-expressed in the more compact form

$$E\left[\int_0^T \xi_t c_t dt\right] \leq W_0 + w_{i,0}\frac{e^{\{\eta_i\tau_2\}}-1}{\eta_i} + \underbrace{\sup_{\tau_1\in S_{0,\tau_2-\Delta}} E\left[\xi_{\tau_1}\left(HC_{j,\tau_1+\Delta,\tau_2} - HC_{i,\tau_1,\tau_2} - TU\right)^+\right]}_{\equiv V_0(w_i,w_j,\tau_1)}. \tag{6}$$

The right-hand side of Eq. (6) disentangles the individual's initial financial wealth, her discounted stream of earnings conditional on maintaining job i till retirement and the value of having the flexibility to engage in the alternative job, j. The last term has the structure of a contingent claim written on the spread between the present value of payments received from jobs i and j, whose cost upon exercise amounts to TU. By inspection, this is the payoff structure of an American *spread* option.[16]

Because labor supply does not enter the individual's preferences,[17] Fisher's Separation Theorem obtains and the solution of the optimal consumption–portfolio–job-regime program for the individual can be described as taking place in two steps. The first is to select the optimal timing for exercising the life-cycle option so as to maximize lifetime wealth. The second is to use financial markets to allocate wealth so as to achieve the most-preferred, feasible consumption plan. The former optimization is embodied in the condition

$$V_0\left(w_i, w_j, \tau_1\right) \equiv \sup_{\tau_1\in S_{0,\tau_2-\Delta}} E\left[\xi_{\tau_1}\left(HC_{j,\tau_1+\Delta,\tau_2} - HC_{i,\tau_1,\tau_2} - TU\right)^+\right],$$

[16] A call spread option is a contingent claim written on the spread between two prices, S_i and S_j, with payoff $(\max(S_j - S_i, 0) - K)^+ = (S_j - S_i - K)^+$ upon exercise. When the exercise price K is null the spread option becomes an option to exchange asset i for asset j. Detemple (2006), pp. 121-124, provides key properties of a spread option exercise region.

[17] The preference structure of Eq. (3) admits no disutility from working or labor-leisure flexibility. This is consistent with Hansen's (1985) argument that real-world workers cannot continuously adjust their working time – committing to work full time or not working at all.

which was previously stated in Eq. (6). The latter optimization is stated as follows

$$\begin{aligned} &\max_{c_t \geq 0} E\left[\int_0^T e^{\{-\beta t\}} \frac{c_t^{1-\gamma}}{1-\gamma} dt\right],\ t \in [0,T] \\ &s.t. \quad E\left[\int_0^T \xi_t c_t dt\right] \leq W_0 + w_{i,0} \frac{e^{\{\eta_i \tau_2\}}-1}{\eta_i} + V_0\left(w_i, w_j, \tau_1\right), \end{aligned} \tag{7}$$

in which $\beta > 0$ is the constant subjective discount factor. Solving the unconstrained analog to Eq. (7) yields the optimal contingent consumption allocation

$$c_t^* = (\lambda^*)^{\frac{-1}{\gamma}} e^{\left\{\frac{-\beta}{\gamma} t\right\}} \xi_t^{\frac{-1}{\gamma}}, \tag{8}$$

where λ^*, defined below, is the constant multiplier attached to the static budget constraint of Eq. (7)

$$(\lambda^*)^{\frac{-1}{\gamma}} \equiv \mathcal{X} \frac{W_0 + w_{i,0} \frac{e^{\{\eta_i \tau_2\}}-1}{\eta_i} + V_0\left(w_i, w_j, \tau_1\right)}{1 - e^{\{-\mathcal{X} T\}}},\ \mathcal{X} \equiv \frac{\gamma - 1}{\gamma}\left(r_f + \frac{\beta}{\gamma - 1} + \frac{\theta_i^2 + \theta_j^2}{2\gamma}\right). \tag{9}$$

The static budget constraint also serves as a statement of the individual's desired risk exposure for the sustainment of her lifelong consumption

$$\underbrace{\xi_t c_t^* \frac{1 - e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}}}_{desired\ risk} = \underbrace{\xi_t W_t}_{financial\ risk} + \underbrace{\xi_t w_{i,t} \frac{e^{\{\eta_i(\tau_2 - t)\}} - 1}{\eta_i} + \xi_t V_t\left(w_i, w_j, \tau_1\right)}_{endowed\ risk}. \tag{10}$$

The right-hand side of Eq. (10) distinguishes between financial risk linked to the funds invested at time zero, whose performance is formalized in Eq. (11), and endowed risk implicit in both current and future wage payments. The optimal portfolio policy guarantees the match of the individual's financial risk exposure to her *net* desired risk exposure, an instance of personal liability-driven investing.

$$dW_t = \pi_{i,t}\sigma_{S_i}\left(\theta_i dt + dz_{i,t}\right) + \pi_{j,t}\sigma_{S_j}\left(\theta_j dt + dz_{j,t}\right) + W_t r_f dt + w_{(\cdot),t} dt - c_t dt,\ W_0\ given. \tag{11}$$

The optimal holding of risky security $(\cdot)$, either i or j, in dollars, is denoted by $\pi_{(\cdot),t}$ and it amounts to the right-hand side of Eq. (12). The derivation of $\pi_{(\cdot),t}$ is presented in Appendix A.

$$\pi_{(\cdot),t} \equiv \frac{\sigma_{S_{(\cdot)}}^{-1}\theta_{(\cdot)}\left(1 - e^{\{-\mathcal{X}(T-t)\}}\right)}{\gamma \mathcal{X}} c_t^* - \frac{\sigma_{S_{(\cdot)}}^{-1}\sigma_{i,(\cdot)}\left(e^{\{\eta_i(\tau_2 - t)\}} - 1\right)}{\eta_i} w_{i,t} - \sigma_{S_{(\cdot)}}^{-1}\mathfrak{D}_{(\cdot),t}\left[V_t\left(w_i, w_j, \tau_1\right)\right]. \tag{12}$$

The amount allocated to asset $(\cdot)$ is the algebraic sum of three components. The first one replicates the standard mean-variance efficient portfolio; the second one resembles Bodie *et* *al.* (1992)'s dollar-value equivalent exposure to the risky asset;[18] the last one corresponds to the option's delta in Bermin (2003, p. 77, Eq. 4.1).

In the remainder of this Section, I report the baseline calibration.

2.5 Calibration

Solving the individual's optimization problem in the presence of multiple job choices, as well as stochastic nonfinancial income, is technically complex. The numerical algorithm used here relies on a two-dimensional binomial lattice that extends Broadie and Detemple (1996)'s routine to produce optimal exercise boundaries and portfolio policies. The procedure is summarized in Appendix A. However, the key to obtaining meaningful consumption and portfolio policies is an appropriate model parameter calibration. An issue that I have not addressed so far is the qualification of jobs i and j.[19] The flexibility of the present framework permits to consider employments in alternative firms within the same industry, alternative industries or a specific industry vis-a-vis the rest of the economy. I examine the last instance. As a matter of fact, defining the individual's alternative employment as the set of all other industries in the economy, avoids limiting the job choice to two industries only. Since job-switching options are especially valuable to individuals able to pursue many careers within rather dissimilar industries or companies, this specification appears to be the most appropriate.

2.5.1 Asset Return Dynamics

The calibration of the dynamics of risky security j, the market index, is that of Cocco, Gomes, and Maenhout (2005): the mean equity premium is 4 percent and the standard deviation of innovations to the market portfolio is set to its historical value of 15.7 percent. The risk-free rate of interest is 2 percent, conforming to the historical average on constant-maturity Treasury Inflation-Protected Securities.[20] The dividend yield is set at 1.64 percent, the most recent rate available from Shiller 's (2000) updated stock market data. I set the parameters of risky security i, the industry, to match key average statistics on stock returns. I study both lower and higher than market-level volatility industries. In the former calibration, industry volatility is equal to 12.5 percent, in line with the estimate of Campbell *et al.* (2001); in the latter, it is set to 21.0 percent. The last figure is determined using value-weighted annual returns of 49 industry portfolios.[21] The choice of industry-level volatility

[18] Equation (18) on page 444 of Bodie, Merton, and Samuelson's article is comparable to Eq. (12); their problem, however, is founded on a single state variable, a risky asset modeled as a geometric Brownian motion.

[19] In a recent paper, Ruffino and Treussard (2007) investigate optimal initial career selection in the presence of occupational risks differentials and study the implications of the individual's human capital risk management for cumulative mobility probabilities, lifetime earning profiles, and gains (or losses) from voluntary and involuntary mobility.

[20] Source: FRED, http://research.stlouisfed.org/fred2/categories/82.

[21] Source: K. French Data Library, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. To construct 49 industry portfolios, each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio at the end of June of year t based on its four-digit Standard Industrial Classification code at that

is crucial: as will be seen, the measure of the optimal portfolio policies is quite sensitive to this parameter. Comparative statics disclose lower risky security holdings associated with increasing industry-specific risk. Rosenberg and Guy's (1976) industry betas dictate the sizes of the risk premia. The low- (high-)volatility industry stock offers a risk premium equal to 3.2 (5.1) percent.[22] Dividend yields are unchanged at 1.64 percent. To circumvent time-independent standard deviations and risk premia, variations in the fractions of financial wealth allocated to risky securities could be expressed in terms of variations in the price of risk. If one believes that the Sharpe ratio is intertemporally more stable than the risk premium (or the standard deviation), then, for a given interest rate and Sharpe ratio, one can solve for the fraction allocated to equities times the standard deviation of equity returns – a change of dimension units. Once this fraction is determined, variations in portfolio shares can be expressed in terms of variations in the standard deviation.[23] Although all these items are foremost, especially within a life-cycle model that necessarily features a long investment horizon, hereafter I consider time-independent parameters.

2.5.2 Preference Structure

The discount factor and the risk aversion coefficient defining the individual's preference structure are equal to 0.03 and 6, respectively. I further assume a one-year vocational training spell. The estimated annual expenses of attending school are set to $9,000, within the range published by ITT Educational Services.[24]

2.5.3 Labor Income Dynamics

The last four parameters to be ascribed are those of the wage processes. The expected rate of wage growth over the individual's working years is equal to 2 percent based on the analysis of McCue (1996, Table 1, p. 182, and Table 3, p. 185). The wage volatility associated with both the low-volatility industry and the market is set at 4.5 percent. The wage volatility corresponding to the high-volatility industry is 8.5 percent in accordance with Davis and Willen's (2000) occupational components of wage innovations.[25] Lastly, Bodie, Treussard,

time. Returns from July of year t to June of year $t+1$ are then computed.

Among all return series, I select "Electrical Equipment," industry 22, for which Davis and Willen (2000) provide corresponding wage-volatility values. See Appendix B for more details.

[22] Since the correlation between financial and nonfinancial income in industry 22 is equal to $0.45 (< 1)$, its parameter values need to be transformed as explained in Appendix B.

[23] This argument is also presented in Merton (1981) where the equilibrium expected excess return is given by the product of the reward-to-risk ratio and a known function of the observed standard deviation on the market return.

[24] ITT Educational Services, Inc. is a private college system focused on technology-oriented programs of study. According to U.S. Department of Education data, all of the ITT Technical Institutes combined granted the largest percentage (14.7 percent) of the total number of associate and bachelor degrees awarded in the U.S. in electronics and electronics-related programs in the 2000-2001 school year (the latest year for which statistics are available).

[25] Cocco, Gomes, and Maenhout (2005, Table 5, p. 507) also report estimates of the variance of both permanent and transitory labor income shocks for households in three different sectors, agriculture, construction and public administration.

and Willen (2006)'s annual earnings for 25-year-old men with a high school diploma dictate my choice of initial wage levels at $24,199 per year. I estimate initial financial wealth to be $20,000, circa 80 percent of the initial salary. This assumption is in line with Gentry and Hubbard's (2000) findings of average (median) household wealth-income ratios ranging between 1.2 (0.3) and 1.8 (1.4) for a subsample of non-entrepreneurs and for the entire population of individuals under age 35, respectively.[26] Retirement and death, the eventual planning horizon, are certain at age 65 and 90, respectively. The full set of parameters is listed in Table 1.

Table 1
Benchmark Parameters

Description	Parameter Value
Expected Stock Growth Rates (μ_S)	**0.052,**$^{(L)}$ **0.071,**$^{(H)}$ **.060**$^{(M)}$
Stock Volatilities (σ_S)	**0.125,**$^{(L)}$ **0.210,**$^{(H)}$ **0.157**$^{(M)}$
Stock Dividend Rates (δ_S)	**0.0164**
Risk-Free Rate (r_f)	**0.02**
Discount Factor (β)	**0.03**
Risk Aversion (γ)	**6**
Time to Build (Δ)	**1**
Tuition (TU)	**$9,000**
Expected Wage Growth Rates (α)	**0.02**
Wage Volatilities ($\sigma_{i,i}$,$\sigma_{j,j}$)	**0.045,**$^{(L)}$ **0.085,**$^{(H)}$ **0.045**$^{(M)}$
Initial Wages (w_0)	**$24,199**
Initial Financial Wealth (W_0)	**$20,000**
Retirement Age (τ_2)	**65**
Death Age (T)	**90**

Notes: $^{(L)}$ Low-volatility industry parameter value; $^{(H)}$ High-volatility industry parameter value; $^{(M)}$ Market parameter value.

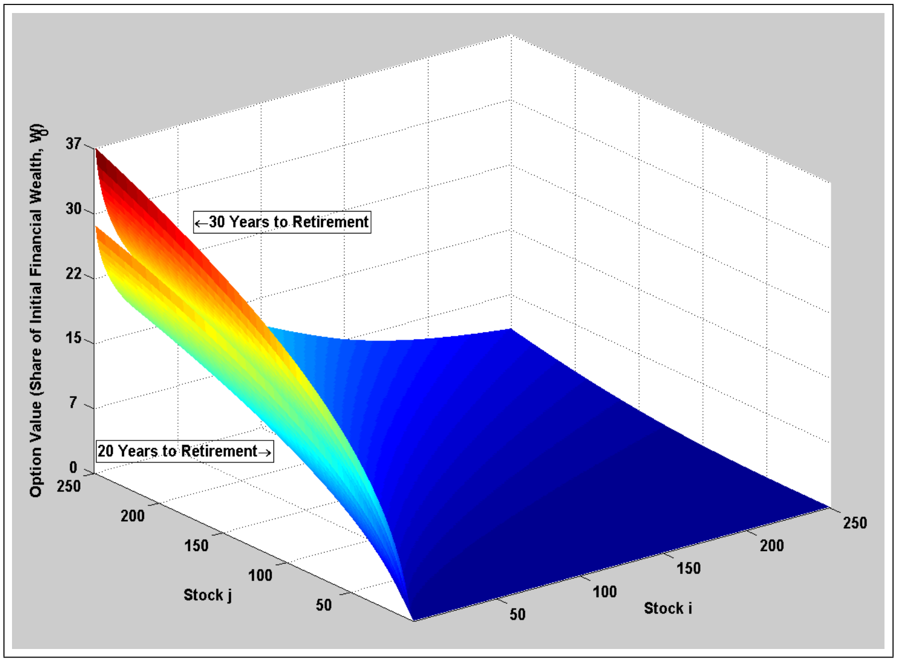
3 Simulation Results

In this Section I devote much attention to describing the model's predictions as they relate to the up-to-date portfolio literature. The set of figures thenceforth illustrates original results on the value of the life-cycle option and on the portfolio policies $\pi_{i,t}$ and $\pi_{j,t}$.

[26] Gentry and Hubbard (2000) use the 1989 Survey of Consumer Finances to construct household wealth-income ratios by age, education, income and entrepreneurial status.

3.1 Maturation of the Job-switching Option

Figure 1
Value of the Career Spread Option as a Function of the Expected Retirement Horizon



Source: Author's calculations: option to switch jobs from a low-volatility industry to the market.

Figure 1 photographs the maturation of the career option over the state space created by risky securities i and j.[27]

The spread option is very consequential for young workers endowed with rather little financial wealth, spiking up to 37 times their initial wealth floor, W_0.[28] Consequently, young decision-makers, who are likely to recoup mobility costs over the future working years, are more inclined towards immediate exercise of their life-cycle option, while the disposition of seasoned workers towards a late change of careers is much depressed by substantial costs. Accordingly, for any given pair (S_i, S_j), an indicator of industry i's wellness relative to the rest of the economy, benefits from swapping jobs diminish with working age as the payoff period declines – the option-value surface flattens.[29]

[27] Since the career option of Eq. (6) is completely characterized by the individual's human capital wealth, the present value of her future labor earnings, some algebraic manipulations are necessary to specify $V_0(w_i, w_j, \tau_1)$ over the state space created by the two risky assets. The new formula, which enters the numerical algorithm and eventually permits to produce the surfaces displayed in this Section, is derived in Appendix A.

[28] Allowing for a modest, though positive, correlation in wage innovations ($\rho_{i,j} = 0.03$) limits the option value to 13 times W_0. To the opposite, negative correlations ($\rho_{i,j} = -0.03$) enhance the option's value that reaches a maximum value of 85 times W_0. This conforms to the option's function to provide the individual with the opportunity to exchange her labor earnings in industry i with those offered by the rest of the economy. This flexibility is more precious to the worker the less alike her industry and the market are.

[29] In the present model, the individual's choice is narrowed to two jobs: industry i and the market economy, which comprises all industries but industry i. Because an option on a portfolio is less valuable than a portfolio

Additionally, at all ages, the option appreciates (depreciates) in the price of the risky stock that disciplines the alternative (current) industry of employment, a direct consequence of both the labor income processes and the American spread option's payoff structure.

3.2 Role of the Job-switching Option in Portfolio Decisions

To render the optimal investment policies more readily comparable with empirical findings and other available theoretical estimates, I construct shares of risky and risk-free investments as functions of accumulated financial wealth. Tables 2, 3, 4 and 5, which feature optimal asset holdings under various economic circumstances, compile these simulation results.

Table 2
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon
(Option to Switch Jobs from a Low-volatility Industry to the Market)

Fractions of Current Financial Wealth	**FW=TW**	**FW=TW-HW**	**FW=TW-HW-Option**
		30 Years to Retirement	
Own-Industry Risky Investment	**0.17**	**-3.05**	**0.38**
Cross-Industry Risky Investment	**0.16**	**49.47**	**4.26**
Risk-Free Investment	**0.67**	**-45.42**	**-3.64**
		20 Years to Retirement	
Own-Industry Risky Investment	**0.17**	**0.15**	**0.27**
Cross-Industry Risky Investment	**0.16**	**3.02**	**1.79**
Risk-Free Investment	**0.67**	**-2.17**	**-1.06**
		10 Years to Retirement	
Own-Industry Risky Investment	**0.17**	**0.30**	**0.31**
Cross-Industry Risky Investment	**0.16**	**0.78**	**0.68**
Risk-Free Investment	**0.67**	**-0.09**	**0.01**
		5 Years to Retirement	
Own-Industry Risky Investment	**0.17**	**0.32**	**0.33**
Cross-Industry Risky Investment	**0.16**	**0.45**	**0.42**
Risk-Free Investment	**0.67**	**0.22**	**0.25**

Source: Author's calculations.
Notes:
(1) FW stands for financial wealth.
(2) TW stands for total wealth, the mean-variance efficient portfolio.
(3) HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
(4) Option stands for option's delta.

Table 2 assesses the contribution of job-switching options in drawing optimal investment policies. The model's predictions are contrasted with the optimal intertemporal allocations originating from a model without human wealth (column 1) and a model with risky human wealth and without the job-switching option (column 2).

Samuelson (1969) and Merton (1969) demonstrate that, ignoring human wealth, an individual whose preferences are rationalized by a power utility function, and whose investment

of options (Merton, 1973, Theorem 7, p. 48), the figure reported above undervalues the job-switching option. If the individual could choose among many jobs, the spread option would be even more consequential. Valuation formulas for American options on multiple assets can be found in Broadie and Detemple (1997).

opportunity set is non-varying, invests in each risky asset a fraction of wealth that only depends on risk aversion and on asset's excess return moments. These shares are constant over time and independent of her financial wealth. The first column of Table 2 reports the results of these calculations using the calibration of Section 2.5. Since stock i is less volatile than stock j, its return in excess of the risk-free rate per unit of risk is higher than the one provided by the market index. As a result, the share of stock i is larger than the share of stock j and risk-free investments, which are derived by difference, constitute the bulk of the individual's assets. The composition of the initial portfolio remains unchanged till retirement.

Columns 2 and 3 examine the role of human capital in determining the individual's optimal investment decisions without and with the option to change jobs, respectively. In order to make the comparison between the two cases meaningful, I choose the initial wage level in the permanent employment model so as to equal the present value of human wealth under both specifications. I find that the individual invests considerably more in her own-industry stock when she is endowed with the option to change jobs. This result, which holds at all ages, reflects the extra "diversification on-the-job" embedded in the career option and it is particularly striking for young investors. After 10 years into her working life an individual who is compelled to keep the same job till retirement should sell short her industry stock. On the contrary, if she had the option to change jobs, she would allocate to her industry stock almost 40% of her financial wealth. Moreover, 5 years away from retirement, this percentage would still be as high as 33%.[30] In reference to numerous empirical studies documenting that investors do not hedge but invest more heavily in stocks closely related to them geographically or professionally, these figures are particularly appealing. Massa and Simonov (2006) rationalize this phenomenon in terms of "familiarity." They investigate its nature and find that the financial behavior of low-wealth investors, such as those I analyze in this context, is much driven by familiarity. My model's predictions agree with their, among others', evidence and shed new light on the importance of building sophisticated models that specialize human capital wealth along the life-cycle.

Finally, possessing the option to switch jobs reduces the need to borrow at all horizons to retirement: this is a consequence of the greater compensation that the individual receives conditional on her exercising the option.

All estimates in Table 2 depend on the assumption that employment begins in a low-volatility industry. Table 3 compiles a corresponding set of predictions that obtain when the individual starts her career in a high-volatility industry.

[30] Since the benefits from swapping jobs lower with age (Figure 1), the radical difference in initial portfolio compositions eventually disappears and the exposure to equities narrows as labor income substitutes for risk-free asset holdings.

Table 3
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon
(Option to Switch Jobs from a High-volatility Industry to the Market)

Fractions of Current Financial Wealth	FW=TW	FW=TW-HW	FW=TW-HW-Option
		30 Years to Retirement	
Own-Industry Risky Investment	**0.13**	**-4.41**	**-2.46**
Cross-Industry Risky Investment	**0.16**	**3.35**	**1.18**
Risk-Free Investment	**0.71**	**2.06**	**2.28**
		20 Years to Retirement	
Own-Industry Risky Investment	**0.13**	**-0.78**	**-0.43**
Cross-Industry Risky Investment	**0.16**	**0.78**	**0.36**
Risk-Free Investment	**0.71**	**1.00**	**1.06**
		10 Years to Retirement	
Own-Industry Risky Investment	**0.13**	**-0.16**	**-0.10**
Cross-Industry Risky Investment	**0.16**	**0.34**	**0.26**
Risk-Free Investment	**0.71**	**0.82**	**0.84**
		5 Years to Retirement	
Own-Industry Risky Investment	**0.13**	**-0.02**	**-0.01**
Cross-Industry Risky Investment	**0.16**	**0.24**	**0.22**
Risk-Free Investment	**0.71**	**0.78**	**0.79**

Source: Author's calculations.
Notes:
(1) FW stands for financial wealth.
(2) TW stands for total wealth, the mean-variance efficient portfolio.
(3) HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
(4) Option stands for option's delta.

Absent any human wealth, the portfolio composition is tilted more towards stock j than stock i. Indeed, because stock i has a much greater standard deviation than in the calibration of Table 2, its low reward-to-variability ratio makes it less desirable. Furthermore, the fraction of financial wealth apportionated to risk-free assets has grown with industry risk from 67% to 71%.

Similarly, higher own-industry volatility increases a worker's appetite for safe assets. Irrespective of the option, borrowing is never optimal and the individual's initial conservative positions are gradually adjusted in favor of a more balanced equity/risk-free asset mix (columns 2 and 3). As it is also detectable in Table 2, the final allocation results from continuously increasing one's position in own-industry stock while reducing cross-industry exposure. The difference between electing a risky initial job and possibly switching to a safer one (Table 3, column 3) and vice versa (Table 2, column 3) is that, in the former case, the extra flexibility supplied by the option moderates, but does not overturn, the basic logic of portfolio construction with a nontraded asset. The intuition behind this effect is as follows. In order to build a balanced portfolio suitable to her risk preferences, the worker needs to trade away the endowment of her nontraded assets. Without job-switching options, this implies that her share of own-industry stock is negative as long as the stock has a small weight in the market index. With job-switching options, the worker's human capital constitutes the holdings not just of her employer's stock, but also of put options on that stock. Because a synthetic put option would involve shorting a fraction, say $\Delta_{S_{i,t}}$, of the underlying stock, the worker would optimally sell $\left(1 - \Delta_{S_{i,t}}\right)$ shares of stock. If $\left(\Delta_{S_{i,t}} < 1\right)$, her negative position in own-industry stock is lessened but not reversed; if $\left(\Delta_{S_{i,t}} > 1\right)$, her exposure becomes positive.

3.3 Unexercised vs. Exercised Job-switching Options

Table 4 displays disentangled investments in securities i and j – the right-hand-side of Eq. (12) – along with the resulting net policies, as they apply to a worker who is employed in the same low-volatility industry till retirement, letting her career option expiring unexercised. This decomposition permits to highlight the main forces at work and to gain intuition on the determinants of portfolio choices.

Table 4
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon[31]
(Permanent Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	FW =	TW −	HW −	Option
	30 Years to Retirement			
Own-Industry Risky Investment	0.38	5.75	5.69	-0.32
Cross-Industry Risky Investment	4.26	4.55	0.00	0.30
Risk-Free Investment	-3.64	7.18	10.66	0.15
	20 Years to Retirement			
Own-Industry Risky Investment	0.27	2.32	2.11	-0.05
Cross-Industry Risky Investment	1.79	1.84	0.00	0.05
Risk-Free Investment	-1.06	2.90	3.94	0.02
	10 Years to Retirement			
Own-Industry Risky Investment	0.30	0.85	0.56	0.00
Cross-Industry Risky Investment	0.68	0.68	0.00	0.00
Risk-Free Investment	0.03	1.07	1.04	0.00
	5 Years to Retirement			
Own-Industry Risky Investment	0.33	0.53	0.21	0.00
Cross-Industry Risky Investment	0.42	0.42	0.00	0.00
Risk-Free Investment	0.25	0.66	0.40	0.00

Source: Author's calculations.
Notes:
(1) FW stands for financial wealth.
(2) TW stands for total wealth, the mean-variance efficient portfolio.
(3) HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
(4) Option stands for option's delta.

The mean-variance portfolio component (TW) embodies the investment in risky financial assets derived solely from the need to finance future consumption. It decreases as the individual ages because the remaining consumption stock erodes near death. Such diminution, however, does not lessen the proportion of individual savings in securities i and j, which is

[31] Row 1, column 1, 0.38 = financial wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 2, 5.75 = total wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 3, 5.69 = human wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 4, -0.32 = option's delta invested in own-industry stock as a fraction of accumulated total financial wealth.

constant over time. The ratio of consumption-driven investments readily obtains from the right-hand-side of Eq. (12) and takes the form

$$\frac{TW_{i,t}}{TW_{j,t}} \equiv \frac{\sigma_{S_i}^{-1}\theta_i}{\sigma_{S_j}^{-1}\theta_j} = \frac{\frac{\mu_{S_i}-r_f}{\sigma_{S_i}^2}}{\frac{\mu_{S_j}-r_f}{\sigma_{S_j}^2}}, \tag{13}$$

equal to 1.26 implementing the baseline calibration of Subsection 2.5. The minor exposure to industry i mirrors the individual's aversion towards the risk associated with her job relative to the market. The aggregate mean-variance risk exposure – the percentage invested in industry i and the market – ranges from 10.30 to 0.95 times current financial wealth as the individual nears retirement. Aggregate shares significantly above 1 at young ages reflects optimal consumption policies in excess of financial wealth accumulated by saving and investing.

The third column, HW, presents the Bodie-Merton-Samuelson implicit investment in securities i and j: it is an estimate of the individual exposure to risky assets of the capitalized value of wage flows. Accordingly, high risk exposures ought to relate to the industry of employment and low levels of exposure to the alternative one. Because of its negative sign in the optimal investment formula, the HW component reduces the individual's overall risky position by lowering her mean-variance share. Furthermore, the HW investment is decreasing over time, mirroring labor income substitution for risk-free asset holdings. In Table 4, the HW percentage reduces to almost one thirtieth of its initial value relative to industry i, while it never drifts from zero relative to the market because the individual elects to keep her job till the end.

The Option factor, last column of Table 4 (Option), obtains from the numerical evaluation of the last component of Eq. (12). Since this component is signed negatively, a negative hedge conduces to larger holdings while a positive one moderates the individual's positions. The idea behind this mechanism is intuitive. Because the individual's career option depreciates in the price of the risky stock that disciplines her current job – Figure 1 – her behavior in face of potential losses arising from higher value of stock i, all else constant, is to take on broader long positions in stock i. A symmetric argument applies to stock j. After 10 years of work, the option's delta accounts for 32% of the individual's financial wealth contribution to stock i and 30% of that she deposits into the market index. Both Option shares diminish over time in absolute values, becoming eventually negligible as the option matures. As a matter of fact, the individual would optimally decline all job change opportunities within few years to retirement since insufficient time would remain to recover tuition expenses.[32]

Presenting the first column of Table 4 is a complex task since its shares are a composite of the three factors analyzed above. I propose a dichotomized reading of the intertemporal dynamics developing as the worker ages. In what follows, I refer to the gradual increase in financial risk-taking resulting from the depreciation of risky human capital as the *aging effect*. On the other hand, a mature worker who has grown very entrenched in her industry tends to concentrate her holdings in the alternative one. This investment strategy connotes

[32] Equivalently, feasible stopping times must belong to the closed interval $[0, \tau_2 - \Delta]$.

what I identify as the *settling effect*. What is key to my representation, is that while the former effect depends purely on the length of the individual's working experience, irrespective of her sector of employment, the latter effect is contingent on her past career choices. Both effects are quantifiable.

The aging effect, which plays no role in the intertemporal allocation of the market index, dominates the time pattern of own-industry holdings after 20 years of employment: the share of financial wealth invested in stock i rises from 27% to 33%. The settling effect is also operative in that the percentage of equities in the market index consistently outpaces that in industry i where the individual is employed: $\left(\frac{\pi_{j,t}}{\pi_{i,t}+\pi_{j,t}}\right)$ ranges from circa 92% during the individual's early career to 56% near retirement. By difference, the individual tilts her portfolio towards her industry stock, which is perfectly positively correlated to her wage income, initially committing to it 8% of her total equity exposure and raising it later to as much as 44%.

Abstracting now from the absolute size allocated to each risky asset, the proportion of financial wealth invested in equities decreases over time. Towards retirement, the individual prefers safer investments and contributes up to 25% of her accumulated wealth to the risk-free asset.

Table 5 summarizes the investment strategy of an individual who switches jobs within the first 10 years of work in industry i. Since the most robust fact in the empirical labor literature is the sharp decline of labor mobility with age, studying the intertemporal financial allocation of a young mover is particularly relevant and fruitful.

Table 5
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon[33]
(Switch to the Market within the First 10 Years of Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	FW =	TW −	HW −	Option
	30 Years to Retirement			
Own-Industry Risky Investment	-0.18	0.79	0.97	0.00
Cross-Industry Risky Investment	1.00	1.00	0.00	0.00
Risk-Free Investment	0.18	1.25	-0.22	1.29
	20 Years to Retirement			
Own-Industry Risky Investment	-0.09	0.67	0.76	0.00
Cross-Industry Risky Investment	0.85	0.85	0.00	0.00
Risk-Free Investment	0.24	1.07	-0.15	0.98
	10 Years to Retirement			
Own-Industry Risky Investment	0.08	0.46	0.38	0.00
Cross-Industry Risky Investment	0.59	0.59	0.00	0.00
Risk-Free Investment	0.33	0.74	-0.01	0.00
	5 Years to Retirement			
Own-Industry Risky Investment	0.18	0.36	0.18	0.00
Cross-Industry Risky Investment	0.45	0.45	0.00	0.00
Risk-Free Investment	0.37	0.56	0.07	0.12

[33] Row 1, column 1, -0.18 = financial wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 2, 0.79 = total wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 3, 0.97 = human wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 4, 0.00 = option's delta invested in own-industry stock as a fraction of accumulated total financial wealth.

The individual's early exercise of her career option materializes in the third column of Table 5: both Option factors are null. Additionally, the TW components and the HW share invested in own-industry stocks are significantly positive and lessening over time.

Aside from the relative importance of the TW fractions, which I already showed to be constant over time, the size of each share is much reduced with respect to those of Table 4 at all investment horizons. Indeed, the exercise of the option has enriched the individual with a greater compensation, softening her positions in equities for the sole purpose of guaranteeing future consumption. The TW share allocated to own- (cross-)industry stock falls from 79% (100%) to 36% (45%) if the individual quits her job versus 575% (455%) and 53% (42%) if she stays.

Finally, special attention ought to be warranted to explaining the magnitude, the sign and the intertemporal variations of the optimal net policies. Recall that the settling effect designates an asset-holding position in which a worker who is much anchored to her industry invests heavily in the alternative one. Once more, the motive supporting this strategy is hedging: investors hold risky financial assets to offset their labor income risk. The first column of Table 5 validates this principle: 30 years ahead of the expected retirement date the share of equities in cross-industry stock equals 122%; 5 years away from retirement it is still as high as 71%. This, in turn, implies that investments in own-industry stock rise with age from -22% to 29%. These percentages summarize very effectively how a change of jobs when young causes large redistributions of individual financial wealth over the life-cycle. Additionally, an individual who has changed job allocates a lower fraction of equities to her current employment than she would had she not left her initial position.[34] The explanation lies in the loss of the extra "diversification-on-the-job" embedded in the individual's option. Consequently, in order to reconstruct a well-diversified portfolio after switching job, the individual needs to tilt her assets away from her industry and towards the alternative one.

As I anticipated earlier in the paper, whether $\pi_{i,t}$ and $\pi_{j,t}$ inflate or deflate during the individual's working years depends on the rate of decay of her human wealth, the aging effect. As in the simulation results of Table 4, the aging effect governs the intertemporal allocation of own-industry stock, mirroring labor income substitution for risk-free asset holdings.[35] Nonetheless, this does suffice to generate an increasing pattern of aggregate equity holdings over time.

In conclusion, what unify optimal investment behaviors, with or without a job change, are (i) the extensive fraction of financial wealth invested in own-industry equities, even if restricted relative to its cross-industry complement, and (ii) the large absolute value of portfolio rebalancing over time. Large transfer transactions are economically very important especially in view of non-negligible costs of stock market participation, as those estimated by Vissing-Jorgensen (2002) and Polkovnichenko (2004).

Conversely, what differentiates the eventuality of permanent employment from job mobility is the fact that, in the former case, total equity holdings are positive and tangible

[34] The own-industry share of equities grows from -22% to 39% if the individual quits her job, versus 8% and 44% if she stays (first column of Tables 5 and 4, respectively).

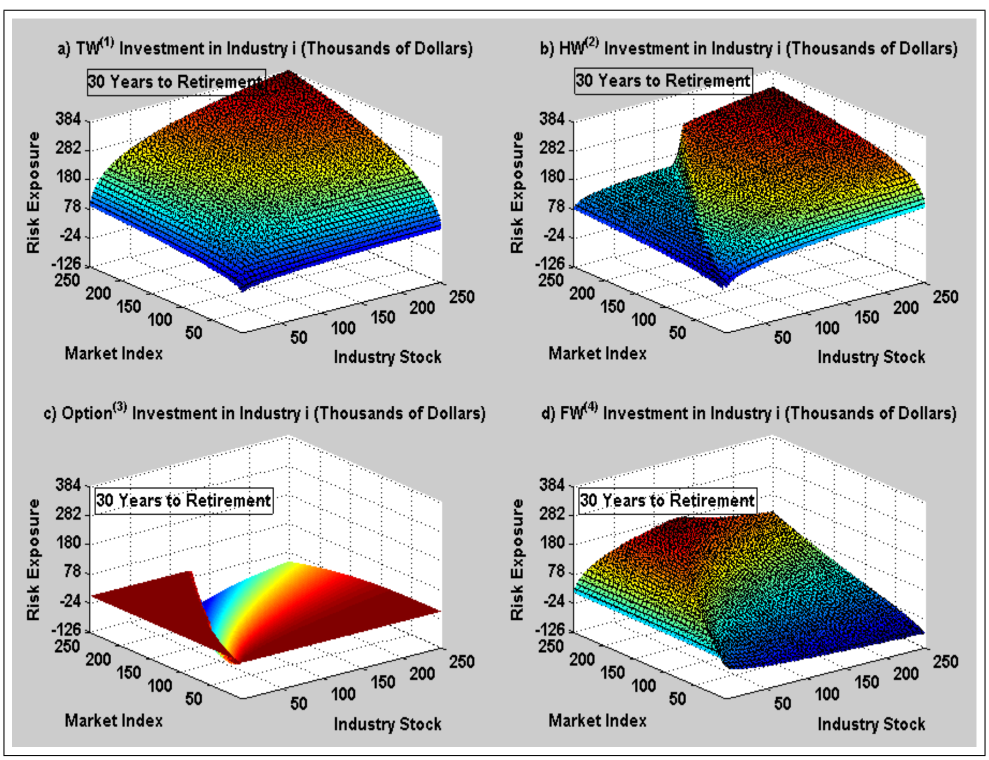
[35] Tracking $TW_{j,t}$ and $HW_{j,t}$ over time, it appears that $\pi_{j,t}$ – the fraction invested in own-industry stock upon switching job – is augmenting because the individual's human wealth depreciates at a higher speed than her demand for consumption.

while, in the latter case, they are positive but more limited due to negative initial positions in own-industry stock. De facto, as documented in Table 5, risk-free investments range between 18% and 37%, which are consistently lower numbers than those of Table 4. The prediction that, conditional on switching jobs, workers have no need of borrowing at any age is a very attractive feature of my life-cycle option model, particularly in the face of many borrowing constraints that investors face in reality and that prevent them from capitalizing future labor income.

3.4 Graphical Analysis

The next figure comprises four panels, picturing disentangled dollar investments in asset i – the right-hand-side of Eq. (12) – along with the resulting net policy. These simulation results apply to a worker with expected retirement date 30 years in the future. All regions colored in red feature high dollar-value equivalent exposures; those colored in blue detect low exposures. Optimal investments (in thousands of dollars) are calibrated to the state space created by risky securities i and j, the industry stock and the market index, satisfying the stochastic differential equation (2). The time-zero stock price, $S_{(\cdot),0}$, is set to 100: this is purely conventional, the initial value being only a scale factor.

Figure 2
Investment in Industry i: Relative Contributions of Total Wealth, Human Wealth and Option



Source: Author's calculations: option to switch jobs from a low-volatility industry to the market.

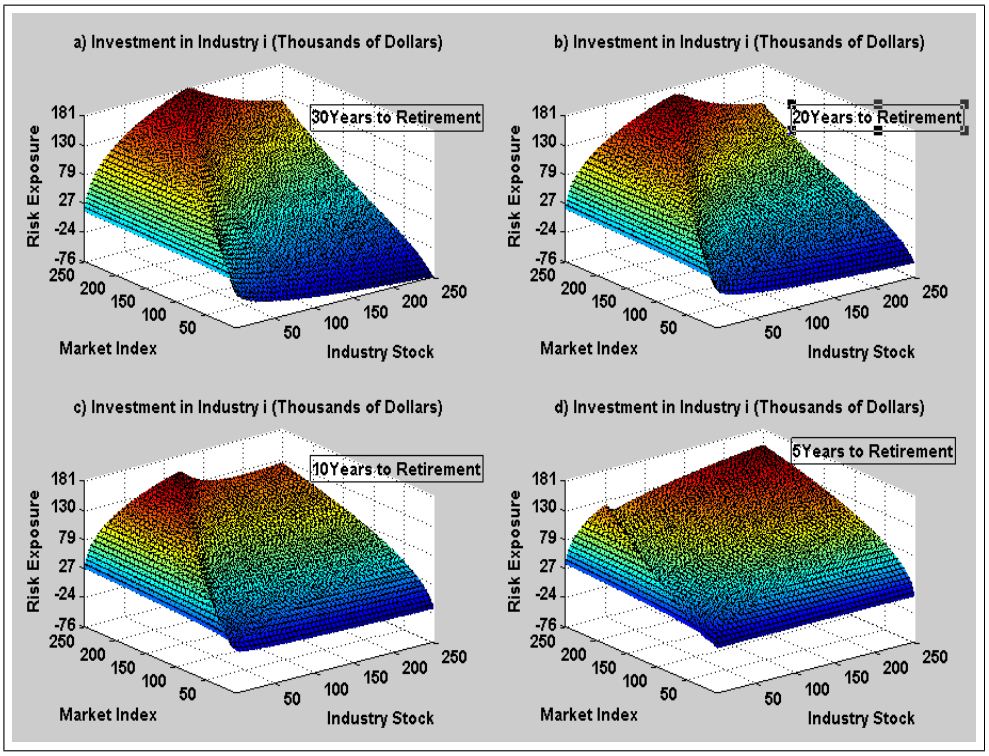
Panel a) displays the mean-variance (TW) portfolio component. Increases in the value of the industry stock (or the market index) entail cheaper consumption *ex ante* as well as higher wealth accruals *ex post*. Both effects render future consumption more desired and produce the smoothly increasing and concave surface depicted above. At first, a mean-variance investment attaining almost \$400,000 in industry i alone may seem an exorbitant exposure for an individual endowed with as little as \$20,000 at the beginning of her career. This figure, however, is rather conservative remembering that a 10-year experienced worker needs to finance her consumption expenditures for another 55 years before death.

Panel b) presents the Bodie-Merton-Samuelson implicit investment in security i (HW). The ridge observable in the implicit dollar exposure corresponds to the *immediate exercise boundary* – the level of relative wage compensation or, equivalently, of relative wellness of the two economies, that renders career switching optimal. The region to the right of the boundary is the *continuation region*, in which the individual has yet to change jobs, and the one to the left of the boundary is the *immediate exercise region* indicative of a career change. Accordingly, the surface in Panel b) depicts high risk exposures in the initial employment, topping \$300,000, and low levels of exposure in the subsequent one, which never reaches beyond \$130,000 .

The Option factor illustrated in Panel c) is the Malliavin derivative of the career option with respect to the i^{th} dimension of the Brownian motion $z_t \in \mathbb{R}^2$, $\mathfrak{D}_{(\cdot),t}\left[V_t\left(w_i, w_j, \tau_1\right)\right]$, scaled by the volatility of the underlying risky security i. Its definition was originally provided in Eq. (12) where it appears negatively signed. While discussing the results of Table 4, I have already emphasized that the negative sign preceeding the derivative is connotative of the hedging feature of the option. Gathering that the diversification properties implicit in the individual's option permit her to sustain larger financial positions in her initial industry of employment than she would had she not been provided with the option (as it is shown in Tables 2 and 3), is crucial especially towards an explanation of households's portfolio selection mechanisms. According to the data, and in stark contrast to the seminal work of Markowitz (1952) and Sharpe (1964) who predict that households should hold well-diversified portfolios, the allocation to employer stock reaches a third of the assets in large retirement-saving plans and about a quarter of employees' discretionary contributions (Benartzi, 2001). Career options seem to provide means to rationalize – if only partially – these well-established empirical facts.

Figure 3 presents the individual's holdings in asset i at various horizons to retirement. It is instructive in respect of both portfolio rebalancing and relocation of the immediate exercise boundary over time.

Figure 3
Investment in Industry i as Functions of the Expected Retirement Horizon



Source: Author's calculations: option to switch jobs from a low-volatility industry to the market.

From Panel a) to Panel d), the trigger boundary is shown to move counterclockwise in time, meaning that younger workers are facilitated in switching jobs. This line of reasoning may motivate steeper slopes in wage profiles during the early years of the individual's career, a reflection of industries' negotiating higher wages for retention purposes.[36] The motion of the boundary is further accelerated near the retirement date, especially in view of positive and not negligible discounting.[37]

Tracking the individual's exposure to her industry near the boundary is particularly insightful. Because the TW component, Panel a) in Figure 2, is independent of the individual's sector of employment – its surface displays no ridge – only the HW and the Option terms contribute to reshape the surface of the overall investment in the proximity of the boundary. While closer to the switch the HW element falls drastically, the Option surface spikes up.[38] The huge exposure of Panel a) demonstrates that the career option is so worthy to rule the individual's optimal investments during most of her working life. Only a few years

[36] As a matter of fact, Ruffino and Treussard (2007) predict empirically documented concave logarithmic wage patterns resulting from the introduction of career options in a model featuring prototypical constant wage parameters.

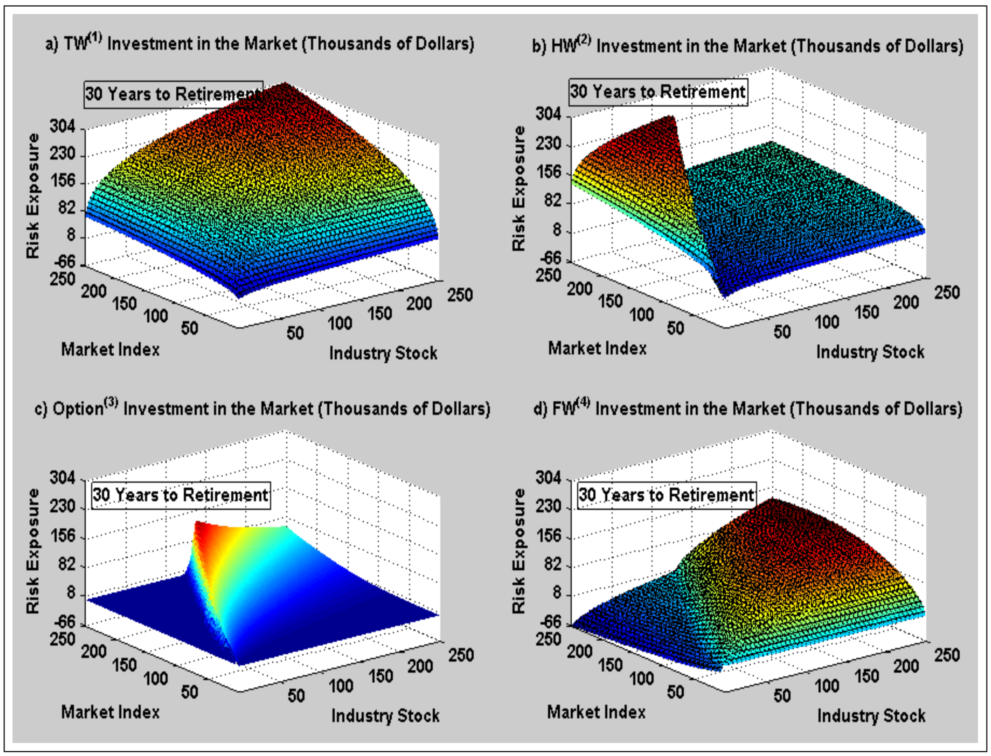
[37] In the set of simulations that I have conducted imposing negative correlations in the wage functions, the boundary does not relocate notably over time. See Appendix B for more details.

[38] Recall that in Eq. (12), which defines the optimal investment policies, Option is negatively signed.

ahead of retirement the worker's human wealth implicit in her HW risk exposure offsets her Option factor. Close to the boundary, optimal dollar investments diminish over time from \$180,000 to circa \$120,000. A final remark is that large stock values, i.e., booming economic conditions both in industry i and in the market, designate loci where dollar-equivalent risk exposures increase as the worker grows older (Tables 2 and 3). This is a direct consequence of rapidly accumulating financial wealth.

Figure 4 presents the decomposition of the individual's holdings in the market index.

Figure 4
Investment in the Market: Relative Contributions of Total Wealth, Human Wealth and Option



Source: Author's calculations: option to switch jobs from a low-volatility industry to the market.

The mean-variance portfolio surface of Panel a) reproduces the shape of its correspondent in Figure 2: individual savings needed to sustain future consumption grow in the prices of the industry stock and the market index and diminish as the individual ages because the remaining consumption stock erodes near death. The volume of the TW exposure to the market index can be either estimated substituting the optimal consumption policy into its definition, Eq. (12), or evaluated indirectly from TW$_{i,t}$ recalling that $\frac{TW_{i,t}}{TW_{j,t}} = 1.26$, which can be rewritten as $TW_{j,t} = \frac{TW_{i,t}}{1.26}$. Its range of variation extends from \$0 to \$304,000 $\left(= \frac{\$384{,}000}{1.26}\right)$, a fairly reasonable measure in proportion to the individual's planning horizon.

The human wealth implicit investment, Panel b), displays a symmetric behavior with respect to that of Figure 2: it is lower during the initial employment, inside the continuation region, than during the subsequent one, to the left of the trigger boundary inside the immediate exercise region. More specifically, a worker who is 30 years from retirement deposits to the market fund circa \$100,000, if she is yet employed in industry i, and as much as \$250,000, if she has elected another job. This is consistent with Bodie *et al.*'s (1992) characterization of implicit risk exposure borne by the individual and attached to her choice of industry.

Panel c) illustrates the Option factor. The optimal share invested in the market index via the career option systematically lowers the overall risky investment in the index. Formally, this effect derives from a significantly positive hedge that is negatively signed in the policy equation; intuitively, it originates from the option's appreciation in the price of the risky security that controls the alternative industry, S_j. While appreciations in S_j procure conspicuous gains, as pictured in Figure 1, unexpected drops translate in a more costly exercise of the option, especially close to the boundary. To hedge against losses arising from falls in the value of the market index, holding S_i unchanged, the individual would optimally short it. Proving that job mobility is perceived as a less alluring prospect when old, the Option component, initially equal to \$131,000, reduces to \$95,000, then further to \$30,000 and eventually to less than \$10,000 at 20, 10 and 5 years to retirement, respectively.[39] This is not inconceivable in light of the substantial mobility costs that the worker faces as she switches jobs – in my calibration these expenses amount to \$9,000 per year (Table 1).

In spite of the negative exposure imparted by Option, which sums to the negative HW hedge, the individual's net dollar exposure to the market index is not incontrovertibly negative because of the mean-variance factor. Recall that, technically, Option is the Malliavin derivative of the career option: it measures the marginal change in the option's value associated with an innovation in wages. Panel d) unveils that, when employed in industry i, the individual invests in the market primarily to finance her consumption expenditures and in response to unexpected labor income shocks. To the opposite, when employed in any other industry, consumption expenditures and human wealth justify her very modest risk exposures. For any pair (S_i, S_j) featuring weak economic conditions both in industry i and in the rest of the economy, dollar-equivalent risk exposures are negative, meaning that the worker should optimally short the market index. To sum up, prior to the switch, investments in the market are approximately ranging from \$10,000 to \$140,000. After the switch, they never rise above \$8,000 and they are often negative. Finally, as observed in Figure 3, for a young worker about to change jobs, the option's delta investment is very consequential, both in absolute value and in its contribution to her total investment. Approaching the trigger boundary from the right, the option's delta increases sharply and the net exposure slides correspondingly.[40]

In the next Section I further detail the model's implications that I have described so far by conducting a sensitivity analysis of my findings to several changes in the benchmark parameter values.

[39] I will gladly share with interested readers these and other results that I refer to in the manuscript and that, for reasons of space only, I am compelled not to report in tables or figures.

[40] It is worth mentioning that, irrespective of asset markets behavior, negative correlations in wage innovations afford additional diversification to the portfolio and produce more steady policies over time. See Appendix B for more details.

4 Comparative Static Analysis

The first set of comparative statics involves the individual's degree of risk aversion. Table 6 compiles own- and cross-industry portfolio shares, along with risk-free asset investments, as functions of the expected retirement horizon and for coefficients of relative risk aversion equal to 4, 6 (baseline), 8 and 10. Each share is the algebraic sum of TW, HW and Option.

Table 6
Optimal Portfolio Shares as Functions of the Degree of Relative Risk Aversion
(Permanent Employment in a Low-volatility Industry)

	Coefficient of Risk Aversion			
	4	**6**	**8**	**10**
Fractions of Current Financial Wealth				
	30 Years to Retirement			
Own-Industry Risky Investment	**1.68**	**0.38**	**-2.31**	**-6.89**
Cross-Industry Risky Investment	**3.01**	**4.26**	**6.39**	**8.07**
Risk-Free Investment	**-3.69**	**-3.64**	**-3.08**	**-0.18**
	20 Years to Retirement			
Own-Industry Risky Investment	**1.23**	**0.27**	**-0.33**	**-0.70**
Cross-Industry Risky Investment	**2.19**	**1.79**	**1.44**	**1.13**
Risk-Free Investment	**-2.42**	**-1.06**	**-0.11**	**0.58**
	10 Years to Retirement			
Own-Industry Risky Investment	**0.72**	**0.30**	**0.09**	**-0.04**
Cross-Industry Risky Investment	**1.00**	**0.68**	**0.51**	**0.40**
Risk-Free Investment	**-0.73**	**0.03**	**0.40**	**0.63**
	5 Year to Retirement			
Own-Industry Risky Investment	**0.59**	**0.32**	**0.19**	**0.11**
Cross-Industry Risky Investment	**0.63**	**0.42**	**0.31**	**0.25**
Risk-Free Investment	**-0.22**	**0.26**	**0.50**	**0.64**

Source: Author's calculations.
Notes: Benchmark coefficient of relative risk aversion equal to 6.

Raising risk aversion from 4 to 10 lowers the fraction of net worth allocated to equities from 4.69 (1.22) to 1.18 (0.36) when retirement is expected in 30 (5) years. Correspondingly, at all horizons to retirement, low risk averse individuals short the risk-free asset while high risk averse individuals hold large amounts of it in their portfolios.[41] For instance, two individuals who are 10 years from retirement, identical in all respects but their degree of risk aversion, equal to 4 and 10 respectively, would invest -73% and 63% in the risk-free option.

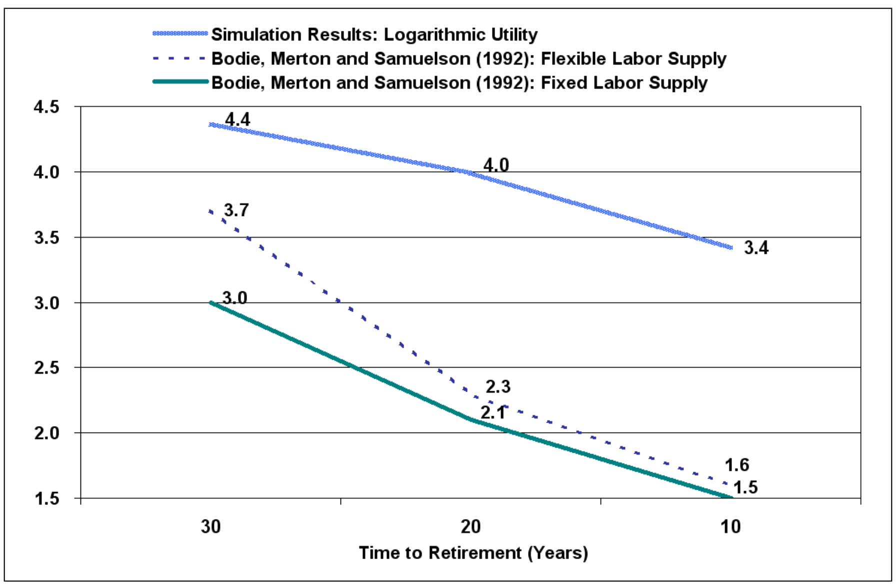
In addition, the reduction in equity risk exposures that accompanies higher coefficients of risk aversions is characterized by a relatively more substantial decrease in own-industry investments. These shares become negative for degrees of risk aversion equal to 8 and 10 indicating strong hedging motives. Note that, irrespective of her degree of risk aversion, the individual invests a positive and considerable fraction of her accumulated wealth to the cross-industry stock – the settling effect introduced earlier. The share of total equity held

[41] For $\gamma = 10$ risk-free asset investments are initially negative, -0.18, and grow rapidly to large positive amounts.

in own-industry stock by a low risk averse individual ($\gamma = 4$) raises with age from 0.36 to 0.48 times the level of current financial wealth. The comparable shares held by a high risk averse individual ($\gamma = 10$) are -5.84 and 0.30, respectively.

The next numerical exercise that I conduct consists of repeating my simulations for the case of logarithmic utility over consumption, i.e., imposing risk aversion equal to 1. This further analysis is aimed at comparing the individual's optimal proportion in risky assets predicted by the model with the life-cycle behavior illustrated by Bodie *et al.* (1992) in their Section 3. My reasoning for pursuing this study is that, even though Bodie *et al.*'s specification differs from mine in many ways,[42] both models provide the individual with an implicit insurance contract against adverse investment outcomes. In the present context, this insurance takes the form of a career option, in theirs it materializes in flexible labor supply.

Figure 5
Proportion in Risky Assets (Multiple of Financial Wealth):
Simulation Results (Logarithmic Utility) vs. Bodie *et al.*'s (1992) Life-Cycle Allocation



Source: Author's calculations; Bodie, Merton and Samuelson (1992), Table 2, p. 439.

In Section 3, I explained how the opportunity to change jobs induces the individual to take on greater risks in her investment portfolio. Similarly, Bodie *et al.* find that the ability to vary labor supply ex post entails larger riskier positions ex ante. The concept behind their result is intuitive: an individual who can freely decide how much or how long she will work later in her life favors portfolio allocations that are more inclined towards risky securities than she would in the absence of such flexibility. Because young investors benefit the most from variable labor supply, the proportion of wealth invested in equities is plausibly very large at the onset of their working life and it diminishes thereafter. Figure 5 shows that

[42]From a formal viewpoint, Bodie et al.'s problem is founded on one state variable only.

both in mine and in Bodie *et al.*'s model, under either labor supply regime, the individual borrows to finance her investment in the risky asset: her degree of leverage is greater early in her life and, in all instances, the optimal share allocated to equities remains above 1 till at least 10 years from retirement. Imposing logarithmic preferences, the simulated equity shares exceed Bodie *et al.*'s at all horizons to retirement and decrease less sharply over time. In the absence of labor/leisure choices, a young worker who has recently changed job holds 4.4 times her net worth level in equities. Without the job-switching option, but with flexible (fixed) labor supply, her share equals 3.7 (3.0) times her wealth. As the retirement date nears, the individual's proportion in equities falls to 3.4 and 1.6 (1.5), respectively.

Table 7 summarizes optimal investment policies as I vary industry i wage risk, $\sigma_{i,i}$.

Table 7
Optimal Portfolio Shares as Functions of Industry i Wage Volatility
(Permanent Employment in a Low-volatility Industry)

	Industry i Wage Volatility			
	2%	**4.5%**	**6%**	**10%**
Fractions of Current Financial Wealth				
	30 Years to Retirement			
Own-Industry Risky Investment	**1.84**	**0.38**	**-1.46**	**-8.34**
Cross-Industry Risky Investment	**2.45**	**4.26**	**5.26**	**6.67**
Risk-Free Investment	**-3.29**	**-3.64**	**-2.80**	**2.67**
	20 Years to Retirement			
Own-Industry Risky Investment	**1.30**	**0.27**	**-0.22**	**-0.59**
Cross-Industry Risky Investment	**1.70**	**1.79**	**1.59**	**0.99**
Risk-Free Investment	**-2.00**	**-1.06**	**-0.37**	**0.60**
	10 Years to Retirement			
Own-Industry Risky Investment	**0.63**	**0.30**	**0.14**	**0.07**
Cross-Industry Risky Investment	**0.70**	**0.68**	**0.63**	**0.46**
Risk-Free Investment	**-0.33**	**0.03**	**0.22**	**0.47**
	5 Year to Retirement			
Own-Industry Risky Investment	**0.44**	**0.32**	**0.26**	**0.15**
Cross-Industry Risky Investment	**0.43**	**0.42**	**0.41**	**0.38**
Risk-Free Investment	**0.13**	**0.26**	**0.34**	**0.47**

Source: Author's calculations.
Notes: Benchmark coefficient of industry wage volatility equal to 4.5%.

Increased wage volatility engenders at least three noteworthy effects for intertemporal optimal asset allocations: (i) the fraction of financial wealth invested in own-industry stock decreases monotonically, (ii) the share of risk-free investments grows, and (iii) highly volatile wages produce increasing equity patterns over time.

More volatile labor earnings affect the proportion of financial wealth in equities in a similar manner to higher degrees of risk aversion: the individual invests more cautiously in equities, notably reducing the relative exposure to her industry stock. 30 years ahead of retirement, she invests in industry i 1.84 or -8.34 times her accumulated wealth, depending on $\sigma_{i,i}$ being fairly low, 2%, or very steep, 10%. As she ages, she contributes less financial wealth to her own-industry stock, eventually apportioning to it 44% or 15% in the lowest and highest wage volatility scenarios, respectively. Similarly, her exposure to the cross-industry stock, the market index, declines steadily over time to 43% and 38% in correspondence to the lowest and the highest $\sigma_{i,i}$ values, respectively.

Two elements distinguish the predictions deriving from higher risk aversion and greater uncertainty in the wage functions. First, despite the lessening in own-industry *relative* risk exposure that characterizes both comparative statics, the *absolute* proportion allocated to the own-industry stock becomes more negative when $\sigma_{i,i}$ grows than when γ grows. Second, while all risk-aversion calibrations carried out deliver monotonically declining equity investments with age, workers with more uncertain jobs (column 4 in Table 7) exhibit increasing equity patterns over time. Investors whose stream of future earnings is highly uncertain, and whose overall equity exposure is initially very limited, abandon their conservative positions in favor of a more balanced equity/risk-free asset mix. On the contrary, young workers receiving relatively safe labor income payments borrow massively and invest all in equities. As they grow older, they reduce their exposure to equities and shift their assets towards safer options.[43] Levels of wage volatility as low as 2%, 4.5% or 6% produce decreasing equity time patterns (first three columns of Table 7). A level of wage volatility equal to 10% is sufficient to invert the pattern (column 4 in Table 7).[44]

The analyses of Tables 6 and 7 exemplify how sensitive portfolio allocation rules are to different assumptions about utility and the stochastic process for labor income. With respect to the baseline calibration, trends are generally not inverted – the qualitative implications of the model are robust to the variations in parameter values examined up to now – but the optimal equity/risk-free asset mix often rebalances substantially.

The next Section is dedicated to contrast the model's predictions with the major findings of perhaps the most frequently cited and trustworthy empirical studies on household portfolio selection. Because career options are a novelty within the life-cycle portfolio literature, I believe it is extremely important to verify that their inclusion really permits to better represent individual portfolio choices. Hence, the importance of meticulously choosing case studies to which the results that I have presented so far may be compared.

5 Matching Actual Portfolio Selection Criteria

The first data source that I consider is the annual report on 401(k) plan asset allocation, account balances and loan activity by VanDerhei, Holden, Copeland and Alonso (2007)

[43] Ibbotson (2006) exemplifies this point writing "As we've mentioned, a person's human capital is very bond-like. But it's more like a junk bond than a government bond or even a high-grade corporate bond – and some people's human capital is "junkier" than others. The risk profile of an individual's human capital depends on job stability, savings rate, income stability and physical health. Changing jobs, dropping out of the workforce, falling ill, etc., all impact the amount of risk in your human-capital portfolio. So, for example, someone who works on commission, or in the type of job that is being outsourced, has a more risky human-capital profile than someone who may work for the government and is entitled to a pension. All of these people may be the same age, but the commissioned stock broker should have a more conservative portfolio than the government mail carrier, all else being equal."

[44] Predictions that originate from highly volatile labor payments are strongly supported by the data indicating that most households hold significant amounts of low-risk assets in their portfolios and increase their share of stocks with age.

for the Employee Benefit Research Institute (EBRI). EBRI and the Investment Company Institute (ICI) have developed the most comprehensive database on 401(k) plan participants yet assembled: their multi-source longitudinal database provides information on participant-level decisions with respect to participation, contributions, and asset allocation.

At year-end 2006, the EBRI/ICI database included statistical information about 20.0 million 401(k) plan participants in 53,931 employer-sponsored 401(k) plans holding $1.228 trillion in assets. The EBRI/ICI database covers 40 percent of the universe of 401(k) plan participants, 12 percent of plans, and 46 percent of 401(k) plan assets. Its investment options are grouped into eight categories: these do not comprise the number of distinct investment options presented to a given participant, but rather the types of options presented to her. Equity funds consist of pooled investments primarily invested in stocks such as equity mutual funds, bank collective trusts, life insurance separate accounts, and other pooled investments. Similarly, bond funds are pooled accounts primarily invested in bonds, and balanced funds are pooled accounts invested in both stocks and bonds. Company stock is equity in the plan's sponsor (the employer). Money funds consist of those funds designed to maintain a stable share price. Stable value products, such as guaranteed investment contracts (GICs) and other stable value funds, mainly insurance company products that guarantee a specific rate of return on the invested capital, are reported as one category. The last two categories are residual for other investments, such as real estate funds, and for funds that could not be identified.

Figure 6 quantifies own- and cross-industry stock ownerships, as well as risk-free asset ownership, in the model and in the EBRI/ICI database. A few clarifications on how I have aggregated VanDerhei *et al.*'s original figures into those of the histograms below are in order.

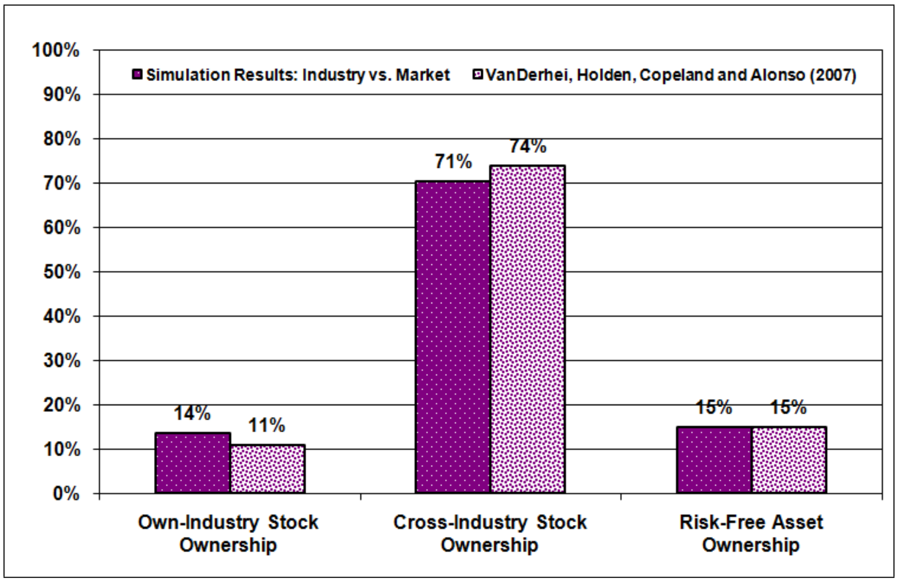
First, own-industry stock holdings as a fraction of accumulated financial wealth correspond to the equity percentage in the plan's sponsor.

Second, cross-industry stock holdings collect all equity investments, but the fraction in the company stock, and other risky securities. These include equity funds, balanced funds and risky bonds. GICs and money market accounts constitute risk-free investments. Therefore, in my reclassification of financial securities, risk-free investment generally indicates less risky assets.

Third, the EBRI/ICI database records asset allocation information for retirement assets only, not for all individual financial assets. Since distinct average shares for non-retirement and retirement assets are unavailable, I assume that retirement assets alone form individual portfolios. While I would not expect the share invested in own-industry stock to grow with more detailed information, the fraction of wealth allocated to cross-industry stocks and risk-free assets could vary towards heavier low-risk investments.

Lastly, my simulation results are equal-weighted averages over the individual's working life and VanDerhei *et al.*'s calculations are averages across age groups. To my knowledge no data set containing time series on household portfolio allocations is lengthy enough to allow for an exhaustive investigation of the individual's saving and investing decisions over the life-cycle. This compels me to make use of cross-sectional average shares that may differ significantly from lifetime averages if birth cohort effects systematically impact optimal behaviors. However, this is not too much of a concern in light of my simulation results in Table 2: as previously uncovered by Ameriks and Zeldes (2004), decreasing equity shares with age – in conformity with my predictions in Section 3 – appear in the data if age and time effects only are included in the specification, but cohort effects are not. Figure 6 shows that simulated portfolio shares approximate exceptionally well the percentages from the EBRI/ICI database.

Figure 6
Own- and Cross-Industry Stock Ownerships and Risk-free Asset Ownership: Simulation Results (Baseline Calibration) vs. Actual Portfolio Allocation



Source: Author's calculations, VanDerhei, Holden, Copeland and Alonso (2007), Figure 20, p. 26.

Besides the very close match of own-industry stock holdings to the documented empirical evidence, the histograms underline that the greater part of total assets is invested in stocks: more than two-thirds of 401(k) participants' assets are invested in equity securities through equity funds, the equity portion of balanced funds, and company stock.

In their report, VanDerhei *et al.* stress that the share of 401(k) accounts invested in company stock has been shrinking over the past decade, falling by 2 percentage points in the sole 2006 and continuing a steady decline that started in 1999. Their analysis persuaded me to examine previous EBRI/ICI reports as well. The share of assets allocated to own-industry stocks equaled 19% in 1996 and 1999 and it started decreasing afterwards reaching 16% in 2002 and 13% in 2005.[45] This suggests changes in both plan design and participants' behaviors. VanDerhei *et al.* focus on recently hired participants to draw out information about the impact of current plan design and other factors on individual participants' decisions. They find that not only are fewer recent hires holding own-industry stock, but fewer recent hires are holding high concentrations of own-industry stock. For example, among recently hired participants, 4.5% percent held more than 90 percent of their account balance

[45] Although some of the decline since 2000 reflects a drop in stock prices during the 2000-2002 bear market, the share of assets held in company stock continued to drop during the stock market's rebound.

in own-industry stock in 2006 (VanDerhei *et al.* (2007), Figure 38, p. 36). Among the comparable group in 1998, 12.4% percent had such concentration. Irrespective of this trend in usage and concentration, observed shares invested in own-industry stock did not distance significantly from the optimal policies and remained very close to the predicted average of 14%. Risk-free investments ranged from a pre stock-market bubble minimum of 15% in 1999 to a post bubble maximum of 22% in 2002.

Overall, the model's predictions are robust and the explanatory power of life-cycle options for portfolio selection mechanisms is unquestionable. Background risk exposures, transaction costs and facilitated/better information on a particular stock have been often advocated as possible explanations for the puzzling concentrated holdings in employer's stock.[46] The additional diversification afforded by the individual's career option justifies her taking large financial positions in her industry stock. As a matter of fact, one could argue that, even though both channels are feasible, job mobility may provide a more easily implementable means of diversification than active trading in financial markets. Because labor markets have grown more flexible, and because career options do not entail a premium while operating in financial markets still commands a price[47] and demands some degree of financial knowledge, overinvestments in own-industry stocks could be anticipated.

As I have mentioned at the beginning of this manuscript, the present model lends itself to various qualifications of jobs i and j. What I intend to do next is to further exploit this flexibility by examining employment in a specific firm vis-a-vis the rest of the economy.

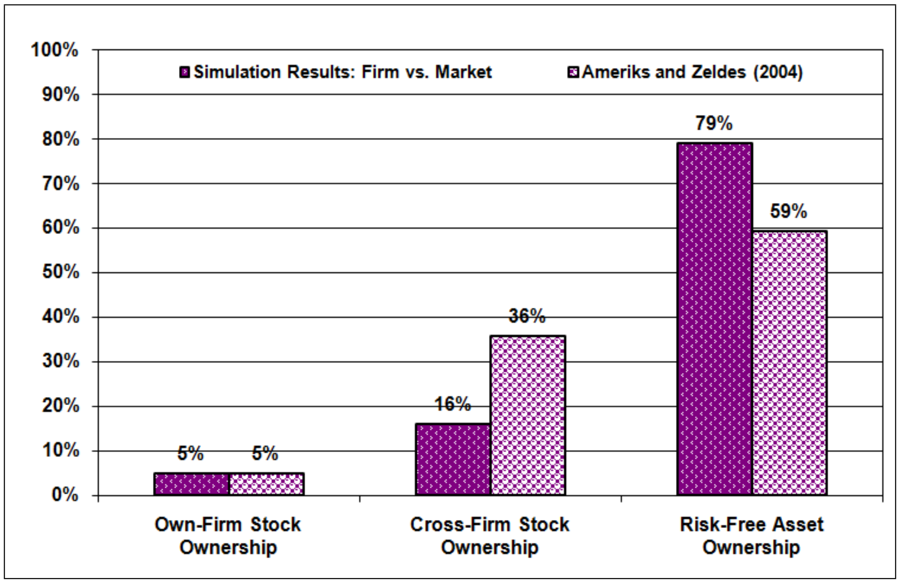
Key parameters for the market index are unchanged relative to those of Table 1. I set firm-level volatility equal to 30 percent, an upper bound to the 25 percent average estimated by Campbell *et al.* (2001), and risk premium and dividend yield equal to 4 and 1.64 percent, respectively. Campbell, *et al.* (2001) find a large and significant positive trend in firm-level variances, more than doubling between 1962 and 1997 and motivating my adopting an even higher historical average as of today. The equivalence between the risk premium on stock i and the one on the market index follows from a CAPM equation in which beta equals 1, a plausible assumption since beta coefficients of stocks drift towards 1 over time. The statistical explanation for this phenomenon is that the average beta over all securities is 1, my best estimate for the firm coefficient.[48] Figure 7 compares average shares obtaining from this alternative calibration to the empirical analysis of Ameriks and Zeldes (2004).

[46] Cohen (2005) determines that workers' company loyalty, broadly defined as "an emotional tie," (p. 1), helps explaining large proportions of employee pension wealth invested in own company stock.

[47] The relevance of transaction costs for individual investors is studied by Vissing-Jorgensen (2002) and Polkovnichenko (2004). Vissing-Jorgensen estimates that a per period stock market participation cost of just \$50 is sufficient to explain the choices of half of stock market nonparticipants. Similarly, Polkovnichenko concludes that participation costs of less than 1 percent of per capita labor income support equilibria with no trading in equities for 70% of the population.

[48] There are many sources for regression results on estimated beta coefficients, the most widely-used being the Security Risk Evaluation published by the Research Computer Services Department of Merrill Lynch (http://newarkwww.rutgers.edu/RBS/ALPHA_Jan05.pdf).

Figure 7
Own- and Cross-Firm Stock Ownership and Risk-free Asset Ownership:
Simulation Results (Alternative Calibration) vs. Actual Portfolio Allocation



Source: Author's calculations; Ameriks and Zeldes (2004), Table 2, p. 59.

Ameriks and Zeldes use pooled cross-sectional data from the 1998 Survey of Consumer Finances that includes information on assets both inside and outside of retirement accounts as well as demographic information. Using these data for the investigation of portfolio allocation, however, presents some disadvantages. Ameriks and Zeldes highlight that, for instance, the survey responses to questions regarding the allocation of assets held in mutual funds or in retirement accounts are categorical in nature, adding noise to the data on household portfolio shares. Another disadvantage is that the survey does not follow the same set of households over time.

Financial assets are classified into four categories: stocks, bonds, cash, and "other". The data show that the average portfolio share invested in stocks through retirement accounts is roughly 57% and that, on average, households hold in retirement accounts circa 30% of their net worth. This does not suffice to quantify individual holdings in the firm's stock. For this purpose I apply Benartzi's (2001) estimated firm stock ownership, as a percentage of the employee *voluntary* contributions, to Ameriks and Zeldes' findings. When the plan requires the employer match to be invested in the firm stock, this share is equal to 29% (Benartzi (2001), Table II, p.1753), which translates into the 5% average share reported in Figure 7 ($5 \cong 57 \cdot 0.29 \cdot 0.3$). Cross-firm holdings are pooled investments of other stocks and risky bonds while risk-free investments include cash and "other."

The histograms show that the own-firm investment predicted by the model is equal to that of the data, a very positive result that supports the goodness of the model. The aggregate share allocated to risky equities, however, differs considerably between the two. The model engenders limited positions in equities, 21% of total financial wealth, while the bulk of total assets is allocated to risk-free investments. The data reveals instead equity shares on the order of 40%. I attribute this controversial finding to the measure of firm-specific volatility: as documented in Table 3, optimal portfolio policies are very sensitive to this parameter and high levels of risk largely reduce equity holdings. The same argument explains the radical change in the relative weight allocated to risky versus risk-free securities when the model is calibrated to a specific firm rather than a specific industry. The optimal equity–risk-free asset mix is 85%–15% according to the baseline calibration, Figure 6, and almost reverse, 20%–80%, according to the alternative one, Figure 7.

These numbers clearly illustrate how the individual's perception of her being employed in a particular firm, or belonging to a particular industry, affects the model's calibration and substantially alters the resulting optimal policies. In a similar manner, the model could be adapted to a setting featuring new employment in another firm rather than the rest of the economy. Then, depending on the two firms being part of the same industry, the individual would need to acquire little new skills at her new job, if any, and mobility costs would be minor. All these examples underline once more how rich the present model is and how, if properly calibrated, it can predict optimal behaviors for a great variety of labor and financial market conditions.

Sections 3, 4 and 5 demonstrate the importance of the interaction between risky labor income and career options for life-cycle portfolio choices. The next Section considers two interesting channels for relating the model's predictions to other somewhat puzzling facts in the financial literature. These are the private equity premium puzzle documented by Moskowitz and Vissing-Jorgensen (2002) and the motion of wealth-to-income ratios with age studied by Gentry and Hubbard (2000).

6 Further Discussion

6.1 The Private Equity Premium Puzzle

The acknowledgment of a worldwide transition from defined benefit pension plans to defined contribution saving plans has bred renovated interest in insufficient asset diversification and its long-run costs to individual investors. Home country bias, geographic bias of domestic funds and employer stock investments are just a few examples of lack of diversification. The perhaps most challenging empirical fact to be explained is the extreme concentration of entrepreneurial investments. Moskowitz and Vissing-Jorgensen (2002) define the willingness to invest substantial amounts in a single privately held firm, with a worse risk-return trade-off than a publicly traded one, as the "private equity premium puzzle."

Collecting all U.S. private firm values and profits, Moskowitz and Vissing-Jorgensen construct an index of private equity returns by which they estimate arithmetic and geometric average annual returns and standard deviations. In Table 6, p. 765, they report the returns to private equity for Proprietors and Partnerships from the Federal Reserve Board's Flow of Funds Accounts and the National Income and Product Accounts over the period 1953-1999. The arithmetic average of annualized private equity returns is equal to 13.1%, the geometric average is 12.8% and the estimated standard deviation is 6.9%.[49] Arithmetic and geometric averages and standard deviation for a value-weighted index of public equity returns are available from the Center for Research in Security Prices. These amount to 14%, 12.7% and 17%, respectively.

I use Moskowitz and Vissing-Jorgensen's key average statistics for private and public firm returns to re-calibrate the stochastic process of the underlying risky asset prices S_i and S_j. This exercise is aimed at testing whether the career option model provides new insights into the tendency of entrepreneurs to hold most of their investment in the same private firm in which they work. Weighting households using the 1989, 1992, 1995, and 1998 Survey of Consumer Finances weights, Moskowitz and Vissing-Jorgensen find that 11% of respondents have some ownership in a non-publicly traded firm. Also, the average household, among all households with some private equity holdings and positive net worth, invests 41% of her of net worth in private equity. Absent any correlation in wage innovations, the newly parameterized career option model predicts private equity holdings ranging from 23.66% to 55.95% depending on the individual's choice to remain in the non-publicly traded firm or to switch to the publicly traded one, respectively.[50] Thus, Moskowitz and Vissing-Jorgensen's estimate lies between the lower and upper bound figures derived from the model.

Another peculiarity of this set of simulations is that, irrespective of cross-industry risk, a large portion of net worth is allocated to risk-free securities. When investors are offered three investment options, the private firm stock in which they work, a value-weighted market index of publicly traded firms and a risk-free asset, they only exploit their firm stock and the risk-free asset. The share of financial wealth held in the market index is never significantly above zero and in fact, in very few cases, it even takes negative values. These results are certainly relevant and contribute to enrich Moskowitz and Vissing-Jorgensen's discussion on the size of investments in private equity. While they cite non-pecuniary benefits, a preference for skewness, and overestimated probability of survival as potential motives for a large portion of aggregate savings in privately held firms, the present model succeeds at accounting for such savings via a spread option that permits a career change over the working life of the individual.

Lastly, in relation to what I have discussed just now, I want to briefly reason about the extent to which low private equity premia should be considered puzzling. Kambourov and Manovskii (2004) document significant changes in the volatility of labor earnings from the early 1970s to the 1990s. This empirical fact engenders the question of whether greater labor risk has markedly lowered workers' welfare. Ruffino and Treussard (2007) show that the answer to this question is not necessarily affirmative if labor markets have become flexible and have begun to supply insurance against wage risk. Furthermore, a gradual completion of financial markets has facilitated risk sharing. Positing that the volatility of corporate assets is fairly steady over time, variations in the risk of labor liabilities must be fully

[49] Moskowitz and Vissing-Jorgensen (2002) extensively discuss the difficulties of estimating the overall risk of private equity. They argue that private firm specific risk implies that the index of aggregate returns overestimates average returns to each entrepreneur.

[50] Allowing for positive cross-industry risk, the corresponding shares of net worth equal 19.62% and 42.08%, respectively.

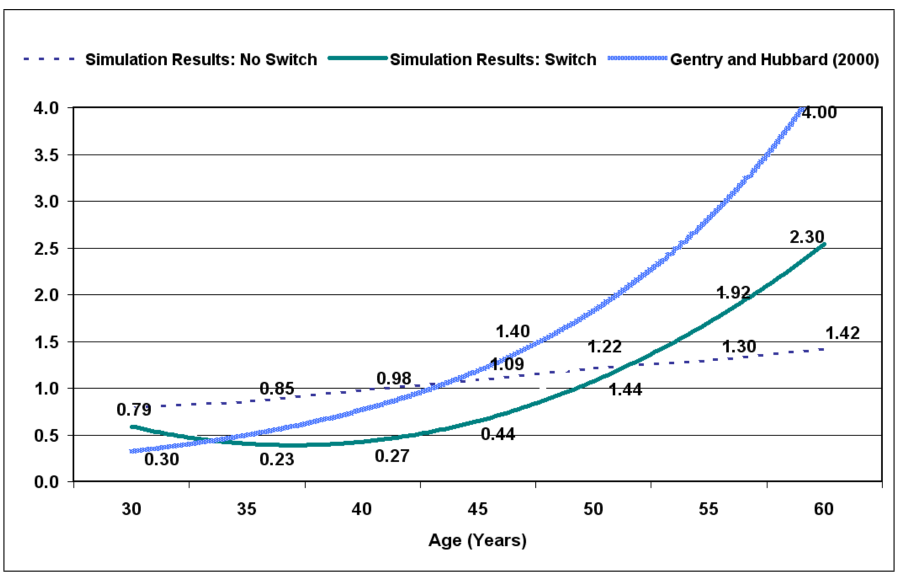
absorbed by opposite variations in corporate equity risk.[51] Consequently, in parallel with the increase in the variability of productivity shocks to occupations established by Kambourov and Manovskii (2004), low levels of risk borne by equity claimants of the firm's assets, and low equity premia, ought to be expected. This analysis is only qualitative but, in fact, shifts in income risk between equity owners and workers could be quantified to test its validity .

6.2 Household Savings: Life-Cycle Wealth-To-Income Ratios

Gentry and Hubbard's (2000) focus is on entrepreneurial household saving and investing decisions. Using information from the cross-section of households in the 1989 Survey of Consumer Finances, they identify entrepreneurs as individuals endowed with "entrepreneurial skills" who engage in up-front business investments seeking economic profits. In 1989, 8.7% of U.S. households fit their Schumpeter-inspired definition of entrepreneurs.

Gentry and Hubbard classify households by age in three groups: under age 35, between 35 and 54, and 55 or older. Based on data on 3,110 households with positive income, they find that *non*-entrepreneurs are 93.7% of households with heads under age 35. This percentage decreases to 86.6% of households with heads between the ages of 35 and 54 and then rises to 94% of households with heads over age 54. Overall, *non*-entrepreneurs own 62.3% of assets and 61% of net worth. Figure 8 contrasts variations in simulated wealth-to-income ratios with Gentry and Hubbard's median values for non-entrepreneurs.

Figure 8
Wealth-To-Income Ratios by Age:
Simulation Results vs. Actual Figures



Source: Author's calculations; Gentry and Hubbard (2000), Table 3, p. 48.

[51] For these dynamics to obtain, the participation shares of equity holders and workers in the firm must remain constant, which may result from immediate renegotiations.

Figure 8 shows that all ratios of wealth to income grow as the individual approaches retirement. Such increase is monotonic both in Gentry and Hubbard's findings and in the case of permanent employment in industry i, the "No Switch" line. On the contrary, observing more closely the "Switch" curve, a slight decrease in wealth-to-income ratios appears between age 30 and 35. This is the time frame during which the individual leaves industry i and, after one year of unpaid vocational training, starts working for her new employer. The solid line interpolates on the grid of simulated values via a polynomial of order 2, thus smoothing the drop in the ratio due to the time-to-build. Afterwards, the ratio of wealth to income picks up and grows steadily. Although the trends generated by the option model are consistent with the predictions of frictionless permanent-income saving models, all ratios in Figure 8, including Gentry and Hubbard's, use per annum income instead of permanent income.

Gentry and Hubbard report median non-entrepreneurs wealth-to-income ratios equal to 0.3 under age 35, 1.4 between 35 and 54, and 4.0 above 55. The model's predictions are below these figures at any age, reaching maximum values of 1.42 and 2.3 for the instances of "no switch" and "switch," respectively. Finally, individuals who endogenously elect to remain in industry i in spite of being endowed with the career option, often referred to as *stayers*, do not exhibit systematically lower ratios than *movers*. Indeed, it is only after 15 years from the switch that a mover's wealth-to-income ratio outpaces that of a stayer. This outcome should be well understood within the option model: relative to a young mover, a stayer who chooses not to exercise her career option must benefit from a stochastic path of high and growing industry wages that she is not willing to forgo. However, 5 years prior to retirement the mover's wealth-to-income ratio is 70% higher than the stayer's one and the gap widens even further till age 65.

In conclusion, while the career option model reproduces the shape of documented actual wealth-to-income ratios over time, it does not succeed at matching the magnitude of the averages observed in reality. Partially, this lack of correspondence may be attributed to the fact that information on population wealth and income from the Surveys of Consumer Finances tends to oversample higher-wealth individuals.

7 Summary

In this paper I study household portfolio decisions over the life-cycle. I extend the theoretical literature on optimal intertemporal asset allocation positing that the finite-lived individual of this economy is enabled to change job during her working years.

I find that the individual optimally bears more own-industry risk in her financial portfolio than if she did not have the extra "diversification on-the-job" embedded in her option to change career. This effect contributes to explain (i) the investors' tendency to choose "familiar" stocks, and (ii) the young businessmen pouring of money into risky investments versus the conservative strategies of old widows. Another remarkable result characterizing the individual's optimal investment behavior is the large absolute value of portfolio rebalancing over time.

Increasing the individual's aversion to risk or the degree of uncertainty of her wage function does not generally invert observed trends – the qualitative implications of the model are robust to the variations in parameter values – but the optimal equity/risk-free asset mix often rebalances substantially.

The model matches own-industry stock holdings documented by the empirical evidence very closely and, because its specification lends itself to various qualifications of "jobs," if properly calibrated, it can predict optimal policies for a great variety of labor and financial market conditions.

This framework also enables a wide range of future research. It could be adapted, for example, to analyze optimal portfolio policies within an economy in which the individual can repeatedly alternate between her initial job and the later one. No-borrowing constraints, no-short-sale constraints, participation and trading costs could also be included. I am currently developing an incomplete market economy with less than perfect correlation between innovations in wages and stocks. This model features four state variables: two risky securities, accumulated cash-on-hand and job status of the individual. Contrarily to the present framework, the multi-dimensionality of the state space entails exclusively numerical methods for solving the optimization problem.

8 Appendix A

8.1 The Option to Switch Careers

8.1.1 Specification Over the State Space

Since the career option of Eq. (6) is completely characterized by the individual's human capital wealth, the present value of her future labor earnings, some algebraic manipulations are necessary to specify $V_0(w_i, w_j, \tau_1)$ over the state space created by the two risky assets. The new formula enters the numerical algorithm and eventually permits to produce surfaces as those displayed in Section 3. Letting $\mathfrak{S}_{(\cdot),0,t} \equiv \frac{S_{(\cdot),t}}{S_{(\cdot),0}}$, I re-express wage earnings in terms of current stock values. Equations (2) and (4) yield

$$\frac{w_{i,t}}{w_{i,0}} = e^{\{\gamma_i t\}} \left(\mathfrak{S}_{i,0,t}\right)^{\frac{\sigma_{i,i}}{\sigma_{S_i}}} \left(\mathfrak{S}_{j,0,t}\right)^{\frac{\sigma_{i,j}}{\sigma_{S_j}}}, \tag{14}$$

$$\frac{w_{j,t}}{w_{j,0}} = e^{\{\gamma_j \tau_1\}} \left(\mathfrak{S}_{i,0,t}\right)^{\frac{\sigma_{j,i}}{\sigma_{S_i}}} \left(\mathfrak{S}_{j,0,t}\right)^{\frac{\sigma_{j,j}}{\sigma_{S_j}}} \tag{15}$$

where γ_i and γ_j are defined by

$$\gamma_i \equiv \alpha_i - \frac{\sigma_{i,i}^2}{2} - \frac{\sigma_{i,j}^2}{2} - \frac{\sigma_{i,i}}{\sigma_{S_i}}\left(\mu_{S_i} - \delta_{S_i} - \frac{\sigma_{S_i}^2}{2}\right) - \frac{\sigma_{i,j}}{\sigma_{S_j}}\left(\mu_{S_j} - \delta_{S_j} - \frac{\sigma_{S_j}^2}{2}\right), \tag{16}$$

$$\gamma_j \equiv \alpha_j - \frac{\sigma_{j,j}^2}{2} - \frac{\sigma_{j,i}^2}{2} - \frac{\sigma_{j,j}}{\sigma_{S_j}}\left(\mu_{S_j} - \delta_{S_j} - \frac{\sigma_{S_j}^2}{2}\right) - \frac{\sigma_{j,i}}{\sigma_{S_i}}\left(\mu_{S_i} - \delta_{S_i} - \frac{\sigma_{S_i}^2}{2}\right). \tag{17}$$

Equations (14) and (15) illustrate that, in the presence of strictly positive cross-industry volatilities, a rise in any security price generates a positive wealth effect. However, because my baseline calibration assumes $\sigma_{i,i} > \sigma_{i,j}\,(\sigma_{j,j} > \sigma_{j,i})$, wages in industry $i\,(j)$ are driven primarily by the evolution of stock $i\,(j)$, conducing to disproportionate increases in each job's labor income and to a potential substitution effect of the current career for the alternative one. Employing Eqs. (14) and (15), the value of the job switching option assumes the more convenient form

$$\begin{aligned} V_0(w_i, w_j, \tau_1) \equiv & \sup_{\tau_1 \in S_{0,\tau_2-\Delta}} E\left[\xi_{\tau_1}\left(w_{j,0} e^{\{\gamma_j \tau_1\}} \left(\mathfrak{S}_{i,0,t}\right)^{\frac{\sigma_{j,i}}{\sigma_{S_i}}} \left(\mathfrak{S}_{j,0,t}\right)^{\frac{\sigma_{j,j}}{\sigma_{Sj}}} \frac{e^{\{\eta_j(\tau_2-\tau_1)\}} - e^{\{\eta_j \Delta\}}}{\eta_j}\right)\right. \\ & \left. -\xi_{\tau_1}\left(w_{i,0} e^{\{\gamma_i \tau_1\}} \left(\mathfrak{S}_{i,0,t}\right)^{\frac{\sigma_{i,i}}{\sigma_{S_i}}} \left(\mathfrak{S}_{j,0,t}\right)^{\frac{\sigma_{i,j}}{\sigma_{S_j}}} \frac{e^{\{\eta_i(\tau_2-\tau_1)\}} - 1}{\eta_i} + TU\right)\right], \end{aligned} \tag{18}$$

or, under the proper risk-neutral measure,

$$V_0(w_i, w_j, \tau_1) \equiv \sup_{\tau_1 \in S_{0,\tau_2-\Delta}} \tilde{E}\left[e^{\{-r\tau_1\}}\left(w_{j,0}e^{\{\gamma_j\tau_1\}}\left(\tilde{\mathfrak{S}}_{i,0,t}\right)^{\frac{\sigma_{j,i}}{\sigma_{S_i}}}\left(\tilde{\mathfrak{S}}_{j,0,t}\right)^{\frac{\sigma_{j,j}}{\sigma_{S_j}}}\frac{e^{\{\eta_j(\tau_2-\tau_1)\}}-e^{\{\eta_j\Delta\}}}{\eta_j}\right)\right.$$
$$\left. - e^{\{-r\tau_1\}}\left(w_{i,0}e^{\{\gamma_i\tau_1\}}\left(\tilde{\mathfrak{S}}_{i,0,t}\right)^{\frac{\sigma_{i,i}}{\sigma_{S_i}}}\left(\tilde{\mathfrak{S}}_{j,0,t}\right)^{\frac{\sigma_{i,j}}{\sigma_{S_j}}}\frac{e^{\{\eta_i(\tau_2-\tau_1)\}}-1}{\eta_i}+TU\right)\right]. \qquad (19)$$

8.1.2 Numerical Methods

To solve the individual's optimization problem I rely on a two-dimensional binomial lattice for the underlying risky securities. Broadie and Detemple (1996, Appendix B) suggest a computationally efficient binomial routine for the pricing of American options on a single underlying asset. I extend their routine to the present two-dimensional environment in a way that allows me to produce a time series of optimal exercise boundaries in addition to the pricing of the career switching spread option. I begin by succinctly describing the peculiarities of the lattice method. The efficient routine of Broadie and Detemple (1996) does not require to store the entire tree in memory: only the information related to the current time step is required. I determine the step amplitude via Hull and White's (1988) equations adjusted for dividends as in Broadie and Detemple (1996). The range of an "up" movement in the binomial tree is expressed by

$$Up_i = \frac{tmp_i + \sqrt{tmp_i^2 - 4a_i^2}}{2a_i}, \qquad (20)$$

where $tmp_i = a_i^2 + b_i + 1$, $a_i = e^{\{(r-\delta_{S_i})dt\}}$, and $b_i = a_i^2\left(e^{\{\sigma_{s_i}^2 dt\}} - 1\right)$. The down movement $Down_i$ is set so that $Up_i Down_i = 1$. The risk-neutral probabilities for security i, $P_i^{Up} = \frac{(a_i - Down_i)}{(Up_i - Down_i)}$ and $P_i^{Down} = 1 - P_i^{Up}$, determine the state prices over the four potential outcomes at each node. These equal $AD^{uu} = e^{\{-rdt\}}P_i^{Up}P_j^{Up}$, $AD^{ud} = e^{\{-rdt\}}P_i^{Up}P_j^{Down}$, $AD^{du} = e^{\{-rdt\}}P_i^{Down}P_j^{Up}$and $AD^{dd} = e^{\{-rdt\}}P_i^{Down}P_j^{Down}$. Discounting risk-neutral probabilities initially to obtain Arrow-Debreu prices reduces the computational burden by saving a multiplication at each node. Finally, the stock price ladders are computed recursively via the formula $S_{i,0}Up^l Down^{n-l}$, where l represents the position in the ladder relative to the smallest possible realization of security i at step n. This sidesteps the need of relatively time-consuming power functions. The routine requires to input the tuition cost, TU, the wages parameters, α_i, α_j, $\sigma_{i,i}$, $\sigma_{M,M}$, $\sigma_{i,M}$, and $\sigma_{M,i}$, and the stocks parameters μ_{S_i}, μ_{S_M}, δ_{S_i}, δ_{S_M}, σ_{S_i}, σ_{S_M}. Initial values for the stock prices, $S_{i,0}$ and $S_{M,0}$, as well as for the wages in both industries, $w_{i,0}$ and $w_{M,0}$, also need to be specified. Finally, the user must enter the risk-free rate of interest, r, and the individual's time to retirement, τ_2, along with the spell during which the individual reinvests in human capital, Δ. The algorithm returns the value of the spread option, along with the optimal exercise boundary at each point in time in terms of positions in the binomial ladder, stock prices, and wages.

8.2 Optimal Portfolio Policies: Formal Derivations

Let me now consider the optimal life-cycle portfolio. The wealth process is given by

$$\begin{aligned} dW_t &= \pi_{i,t}\left[\mu_{S_i}dt + \sigma_{S_i}dz_{i,t}\right] + \pi_{j,t}\left[\mu_{S_j}dt + \sigma_{S_j}dz_{j,t}\right] + \left(W_t - \pi_{i,t} - \pi_{j,t}\right)rdt - c_t dt \\ &= \pi_{i,t}\sigma_{S_i}\left(\theta_i dt + dz_{i,t}\right) + \pi_{j,t}\sigma_{S_j}\left(\theta_j dt + dz_{j,t}\right) + W_t r dt + w_{(\cdot),t}dt - c_t dt \end{aligned}$$

and

$$d\xi_t = -\xi_t\left(rdt + \theta_i dz_{i,t} + \theta_j dz_{j,t}\right)$$

so that

$$\begin{aligned} d\left(W_t\xi_t\right) &= d\left(W_t\right)\xi_t + d\left(\xi_t\right)W_t + d\left\langle W,\xi\right\rangle_t \\ &= \phi_{i,t}dz_{i,t} + \phi_{j,t}dz_{j,t} - \xi_t c_t^* dt \end{aligned}$$

I denote $\phi_{i,t} \equiv \xi_t\pi_{i,t}\sigma_{S_i} - \xi_t W_t\theta_i$. In addition, the dynamic budget constraint is

$$\underbrace{\xi_t W_t}_{financial\ risk} = E_t\left[\underbrace{\int_t^T \xi_v c_v^* dv}_{desired\ risk} - \underbrace{\int_t^{\tau_2} \xi_v w_{i,v} dv}_{endowed\ risk}\right] - \underbrace{\xi_t V_t\left(w_i, w_j, \tau_1\right)}_{endowed\ risk},$$

in which I solve the conditional expectations in closed forms to obtain

$$\begin{aligned} \xi_t W_t &= \xi_t c_t^* \frac{1 - e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}} - \xi_t w_{i,t}\frac{e^{\{\eta_i(\tau_2 - t)\}} - 1}{\eta_i} \\ &\quad -\xi_t V_t\left(w_i, w_j, \tau_1\right). \end{aligned} \tag{21}$$

The dynamic budget constraint serves not only as a balance sheet equality in value, but also as a risk balance sheet equality. Its right-hand side expresses the individual's desired risk exposures implied in the optimal consumption flow (i.e., the individual's liability) net of endowed risk exposures from initial wage income and job switching opportunities (i.e., the individual's non-financial assets). The optimal portfolio policies are those that match the financial risk exposures of the individual's portfolio (the left-hand side of the budget constraint) with her net desired risk exposures. Thus, the optimal investment policy may be interpreted as resulting from liability driven investing procedures at the level of the individual. Based on the Clark-Ocone formula,

$$\phi_{i,t} = \mathfrak{D}_{i,t}\left[\xi_t W_t\right].$$

I proceed by deriving the Malliavin derivative of the first two terms in Eq. (21). The Malliavin derivative for the first term is

$$\begin{aligned}
& \mathfrak{D}_{(\cdot),t}\left[E_t\left[\int_t^T \xi_v c_v^* dv\right]\right] \\
= & -\theta_i \frac{R-1}{R}\left[\xi_t c_t^* \frac{1-e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}}\right] \\
= & -\theta_i \frac{R-1}{R} E_t\left[\int_t^T \xi_v c_v^* dv\right] \qquad (22)
\end{aligned}$$

and that for the second term is

$$\mathfrak{D}_{(\cdot),t} E_t\left[\int_t^{\tau_2} \xi_v w_{i,v} dv\right] = \left(\left(\sigma_{i,(\cdot)} - \theta_{(\cdot)}\right) \frac{e^{\{\eta_i(\tau_2-t)\}}-1}{\eta_i}\right) \xi_t w_{i,t} = \left(\sigma_{i,(\cdot)} - \theta_{(\cdot)}\right) E_t\left[\int_t^{\tau_2} \xi_v w_{i,v} dv\right]. \qquad (23)$$

As regards the third term in Eq.(21), no closed form for the Malliavin derivative can be obtained due to the presence of a stochastic stopping time. I will approximate the Malliavin derivative for the spread option via numerical methods.

Substituting Eqs. (22) and (23) into the Clark-Ocone formula, I find

$$\begin{aligned}
\phi_{(\cdot),t} = & -\theta_{(\cdot)} \frac{R-1}{R} E_t\left[\int_t^T \xi_v c_v^* dv\right] - \left(\sigma_{i,(\cdot)} - \theta_{(\cdot)}\right) E_t\left[\int_t^{\tau_2} \xi_v w_{i,v} dv\right] \\
& -\mathfrak{D}_{(\cdot),t}\left[\xi_t V_t\left(w_i, w_j, \tau_1\right)\right].
\end{aligned}$$

From the definition of $\phi_{i,t}$ above, I determine the dollar portfolio policy invested in stock i to support the optimal consumption stream in Eq. (8).

Using the expression for W_t in Eq. (11), I find

$$\begin{aligned}
\pi_{(\cdot),t} = & W_t \sigma_{S_{(\cdot)}}^{-1} \theta_{(\cdot)} + \xi_t^{-1} \sigma_{S_{(\cdot)}}^{-1} \phi_{(\cdot),t} \\
= & \left(\xi_t^{-1} E_t\left[\int_t^T \xi_v c_v^* dv - \int_t^{\tau_2} \xi_v w_{i,v} dv\right] - \xi_t^{-1} \xi_t V_t\left(w_i, w_j, \tau_1\right)\right) \sigma_{S_{(\cdot)}}^{-1} \theta_{(\cdot)} \\
& -\xi_t^{-1} \sigma_{S_{(\cdot)}}^{-1} \theta_{(\cdot)} \frac{R-1}{R} E_t\left[\int_t^T \xi_v c_v^* dv\right] - \xi_t^{-1} \sigma_{S_{(\cdot)}}^{-1}\left(\sigma_{i,(\cdot)} - \theta_{(\cdot)}\right) E_t\left[\int_t^{\tau_2} \xi_v w_{i,v} dv\right] \\
& -\xi_t^{-1} \sigma_{S_{(\cdot)}}^{-1} \mathfrak{D}_{(\cdot),t}\left[\xi_t V_t\left(w_i, w_j, \tau_1\right)\right] \\
= & \frac{\sigma_{S_{(\cdot)}}^{-1} \theta_{(\cdot)}}{R} E_t\left[\int_t^T \xi_{t,v} c_v^* dv\right] - \sigma_{S_{(\cdot)}}^{-1} \sigma_{i,(\cdot)} E_t\left[\int_t^{\tau_2} \xi_{t,v} w_{i,v} dv\right] - \sigma_{S_{(\cdot)}}^{-1} \mathfrak{D}_{(\cdot),t}\left[V_t\left(w_i, w_j, \tau_1\right)\right], \qquad (24)
\end{aligned}$$

or, using the closed-form expressions from Eq. (21), $\pi_{(\cdot),t} = \frac{\sigma_{S_{(\cdot)}}^{-1} \theta_{(\cdot)}\left(1-e^{\{-\mathcal{X}(T-t)\}}\right)}{R\mathcal{X}} c_t^* - \frac{\sigma_{S_{(\cdot)}}^{-1} \sigma_{i,(\cdot)}\left(e^{\{\eta_i(\tau_2-t)\}}-1\right)}{\eta_i} w_{i,t} - \sigma_{S_{(\cdot)}}^{-1} \mathfrak{D}_{(\cdot),t}\left[V_t\left(w_i, w_j, \tau_1\right)\right].$

9 Appendix B

This appendix extends my study on the impact of labor flexibility on optimal life-cycle portfolio decisions. The model built in the text of the paper assumes that a worker's wage is perfectly correlated with her employer's stock – her human capital constitutes the holdings of her employer's stock and of put options on that stock – and that correlation between industry wages is null. Because the return on these implicit options is higher both when one's own industry does poorly and when one's alternative industry does well, studying the instance of zero correlation between wages is particularly insightful. This case shows the value of introducing the option mechanism and it provides the economic intuition behind it.[52] To the contrary, in the instance of two highly correlated companies, the benefit from holding the job-switching option is modulated and, in the limit case of perfect positive correlation, the option is worthless.[53]

This appendix demonstrates how an economy with less than perfectly correlated labor and financial markets (or, correspondingly, correlated industry wages) can be transformed to restore the canonical model specification in the text of the paper. I propose a three-step procedure to convert a given set of parameter values for stocks and wages into corresponding figures that can be directly inputted in the solution algorithm illustrated in Appendix A. Specifically, this technology creates an artificial economy that is formally isomorphic to the actual one and that reproduces the dynamics of the original model.

The Section to follow summarizes labor and financial markets accessible to the individual.

9.1 Original Model Specification

In the baseline model I posit a financial market comprised of a risk-less asset and two risky dividend-paying assets. The two risky security prices, S_i, and S_j, follow Itô processes

$$\begin{array}{l} \frac{dS_{i,t}}{S_{i,t}} = \left(\mu_{S_i} - \delta_{S_i}\right) dt + \sigma_{S_i} dz_{i,t} \quad t \in [0,T];\ S_{i,0} > 0 \ given \\ \frac{dS_{j,t}}{S_{j,t}} = \left(\mu_{S_j} - \delta_{S_j}\right) dt + \sigma_{S_j} dz_{j,t} \quad t \in [0,T];\ S_{j,0} > 0 \ given \end{array} ,^{54} \ dz_{i,t} dz_{j,t} = 0 \qquad (25)$$

where δ_{S_i} and δ_{S_j} are dividend yields, μ_{S_i} and μ_{S_j} are instantaneous expected rates of return and σ_{S_i} and σ_{S_j} are instantaneous volatility coefficients. $z_{i,t}$ and $z_{j,t}$ are Brownian motion processes and T is the individual's finite fixed planning horizon. The implied market prices

[52] A befitting example is that of an individual with a Master of Business Administration (MBA). The commitment of MBA programs is to help individuals to acquire new skills that can be exercised throughout the course of their career. Education and training in a wide range of disciplines enhance the individual's ability to pursue many careers and her flexibility to move among rather dissimilar industries or companies. To this individual the job-switching option is most consequential.

[53] Intuitively, positing perfect positive correlation in wages is equivalent to assuming a unique source of risk impacting both companies. Under this condition, the individual is indifferent between being employed in either industry and her job-switching option is valueless.

[54] Equation (25) corresponds to Eq. (2) in the text of the paper.

of risk are denoted by $\theta_i \equiv \sigma_{S_i}^{-1}\left(\mu_{S_i} - r_f\right)$ and $\theta_j \equiv \sigma_{S_j}^{-1}\left(\mu_{S_j} - r_f\right)$, where $r_f\,(>0)$ is the instantaneous market rate of interest, and it is constant over time.

The finite-lived individual of this economy finances consumption and investments by earning a per period salary, along with earnings on accumulated financial assets. The wage rates offered in industry i and in the rest of the economy are given by

$$\begin{array}{ll} \frac{dw_{i,t}}{w_{i,t}} = \alpha_i dt + \sigma_{i,i} dz_{i,t} & t \in [0,\tau_2],\ w_{i,0} > 0 \text{ given} \\ \frac{dw_{j,t}}{w_{j,t}} = \alpha_j dt + \sigma_{j,j} dz_{j,t} & t \in [0,\tau_2],\ w_{j,0} > 0 \text{ given} \end{array},^{55}\ dz_{i,t}dz_{j,t} = 0. \tag{26}$$

The instantaneous expected wage growth rates are indicated by $\alpha_i\,(>0)$ and $\alpha_j\,(>0)$, while $\sigma_{i,i}$ $(\sigma_{j,j})$ is the volatility of wage $i\,(j)$ with respect to the i^{th} $\left(j^{th}\right)$ Brownian motion. Retirement is an irreversible labor income state beginning at time τ_2, $\tau_2 \in [0,T]$.

Lastly, given the dynamics in Eq. (25), $dz_{i,t}dz_{j,t} = 0$ is a necessary and sufficient condition for the two stocks to be uncorrelated. The same argument applies to Eq. (26).

9.2 Construction of the Artificial Economy

Consider now a given economy whose stocks and wages' dynamics depart from those of Eqs. (25) and (26). This Section exposes a procedure to "translate" the set of equations characterizing the actual economy into new dynamic equations consistent with the specification of Section 9.1. This technology is implementable via a three-step process.

1. Determine stock and wage parameters of the actual economy.
2. Re-establish orthogonality between wages. This is achieved by decomposing the wage rate received under the alternative job in two independent components: a measure of the current wage rate and a residual wage increment.
3. Re-establish perfect correlation between stocks and wages. This requires to construct two portfolios of the original assets whose optimal shares are chosen to guarantee perfect correlation with the wage rates defined in Step 2.

9.2.1 Defining the Actual Economy

The first step consists of identifying the parameter values that fully describe the given actual economy. These include dividend yields, instantaneous expected growth rates and volatility coefficients of both stock returns as well as instantaneous expected wage growth rates and volatilities. While the investment opportunity set of the actual economy needs be identical to that of the original economy – the individual can freely trade in the same two risky securities and risk-free asset – the dynamics of the labor income rates are assumed to satisfy

$$\begin{array}{l} \frac{dw'_{i,t}}{w'_{i,t}} = \alpha_i dt + \sigma_{w'_i} dz'_{i,t}, \quad \sigma_{w'_i} dz'_{i,t} \equiv \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t} \\ \frac{dw'_{j,t}}{w'_{j,t}} = \alpha_j dt + \sigma_{w'_j} dz'_{j,t}, \quad \sigma_{w'_j} dz'_{j,t} \equiv \sigma_{j,i} dz_{i,t} + \sigma_{j,j} dz_{j,t} \end{array},\ dz_{i,t}dz_{j,t} = 0. \tag{27}$$

[55] Equation (26) corresponds to Eq. (4) in the text of the paper.

Equation (27) states that each industry's wage rate is impacted by two sources of risk: the former originates from the individual's industry of employment (own-industry risk, $\sigma_{i,i}$ and $\sigma_{j,j}$); the latter originates from the alternative industry of employment (cross-industry risk, $\sigma_{i,j}$ and $\sigma_{j,i}$).

The effect of adding a cross-industry component of risk is twofold. First, in spite of the orthogonality between $z_{i,t}$ and $z_{j,t}$, $w'_{i,t}$ is correlated with $w'_{j,t}$. Specifically, indicating $dz'_{i,t}dz'_{j,t} = \rho_{i',j'}dt$, the correlation coefficient $\rho_{i',j'}$ derives from

$$\frac{dw'_{i,t}}{w'_{i,t}}\frac{dw'_{j,t}}{w'_{j,t}} = \sigma_{w'_i}\sigma_{w'_j}\rho_{i',j'}dt = \left(\sigma_{i,i}\sigma_{j,i} + \sigma_{i,j}\sigma_{j,j}\right)dt$$

and equals $\rho_{i',j'} = \frac{\sigma_{i,i}\sigma_{j,i}+\sigma_{i,j}\sigma_{j,j}}{\sigma_{w'_i}\sigma_{w'_j}} = \frac{\sigma_{i,i}\sigma_{j,i}+\sigma_{i,j}\sigma_{j,j}}{\sqrt{\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)\left(\sigma_{j,i}^2+\sigma_{j,j}^2\right)}} \neq 0.$[56]

Second, each industry's stock and wage processes are no longer perfectly correlated. Defining $\rho_{i,i'}$ and $\rho_{j,j'}$ according to $dz_{i,t}dz'_{j,t} = \rho_{i,i'}dt$ and $dz_{j,t}dz'_{j,t} = \rho_{j,j'}dt$, Eqs. (25) and (27) permit to calculate the correlation between stock and wage functions, namely

$$\frac{dS_{i,t}}{S_{i,t}}\frac{dw'_{i,t}}{w'_{i,t}} = \sigma_{S_i}\sigma_{w'_i}\rho_{i,i'}dt = \sigma_{S_i}\sigma_{i,i}dt, \;\; \rho_{i,i'} = \frac{\sigma_{S_i}\sigma_{i,i}}{\sigma_{S_i}\sigma_{w'_i}} = \frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \neq 1$$

$$\frac{dS_{j,t}}{S_{j,t}}\frac{dw'_{j,t}}{w'_{j,t}} = \sigma_{S_j}\sigma_{w'_j}\rho_{j,j'}dt = \sigma_{S_j}\sigma_{j,j}dt, \;\; \rho_{j,j'} = \frac{\sigma_{S_j}\sigma_{j,j}}{\sigma_{S_j}\sigma_{w'_j}} = \frac{\sigma_{j,j}}{\sqrt{\sigma_{j,i}^2+\sigma_{j,j}^2}} \neq 1.$$

Only a financial asset with returns generated by stochastic fluctuations in both $z_{i,t}$ and $z_{j,t}$ would permit to re-establish perfect correlation.[57]

9.2.2 Restoring Orthogonality Between Wages

The second step consists of retrieving orthogonality between wages. Re-expressing the wage process in the market as a function of that in industry i yields

[56] The last equality in the formula of $\rho_{i',j'}$ follows from the definition of $\sigma_{w'_i}dz'_{i,t}$ and $\sigma_{w'_j}dz'_{j,t}$ in Eq. (??).

[57] Assume a fictitious risky asset, S, whose rate of return follows the Itô process

$$\tfrac{dS_t}{S_t} = \left(\mu_S - \delta_S\right)dt + \sigma_{S_i}dz_{i,t} + \sigma_{S_j}dz_{j,t} \quad t\in[0,T];\; S_0 > 0 \; given.$$

In this case, perfect correlation between $\frac{dS_t}{S_t}$ and $\frac{dw'_{i,t}}{w'_{i,t}}$ could be re-introduced by simply scaling the dynamics of S to satisfy

$$\tfrac{dS^*_t}{S^*_t} = \left(\mu_{S^*} - \delta_{S^*}\right)dt + \upsilon_{S^*}\left(\sigma_{i,i}dz_{i,t} + \sigma_{i,j}dz_{j,t}\right) \quad t\in[0,T];\; S^*_0 > 0 \; given,$$

in which υ_{S^*} is the proper scale factor.

While this example is purely illustrative, it provides the intuition behind the mechanism of restoring perfect correlations between stocks and wages. Applying the same line of reasoning, the third step of the procedure presented here demonstrates how perfect correlations can be re-obtained by constructing fictitious securities deriving their returns from optimal combinations of the existing risky and risk-free assets.

$$\frac{dw'_{j,t}}{w'_{j,t}} \equiv \psi_j dt + \beta_j \frac{dw'_{i,t}}{w'_{i,t}} + \sigma_\varepsilon d\varepsilon_{j,t},\ \sigma_{\varepsilon_j} d\varepsilon_{j,t} \equiv \lambda_i dz_{i,t} + \lambda_j dz_{j,t}, \tag{28}$$

where $\varepsilon_{j,t}$ is a Brownian motion process with λ_i and λ_j chosen to satisfy

$$d\varepsilon_{j,t} \frac{dw'_{i,t}}{w'_{i,t}} = \frac{1}{\sigma_{\varepsilon_j}} \left[\lambda_i dz_{i,t} + \lambda_j dz_{j,t}\right] \left[\alpha_i dt + \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t}\right] = 0. \tag{29}$$

Equation (28) defines the new parameters of the model, which will serve to restore orthogonality between stocks and wages in Step 3. Substituting the definitions both of industry i's wage rate and of the newly defined Brownian motion $\varepsilon_{j,t}$ in Eq. (28) yields

$$\begin{aligned}
\frac{dw'_{j,t}}{w'_{j,t}} &= \psi_j dt + \beta_j \left[\alpha_i dt + \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t}\right] + \lambda_i dz_{i,t} + \lambda_j dz_{j,t} \\
&= \left(\psi_j + \beta_j \alpha_i\right) dt + \left(\beta_j \sigma_{i,i} + \lambda_i\right) dz_{i,t} + \left(\beta_j \sigma_{i,j} + \lambda_j\right) dz_{j,t}.
\end{aligned} \tag{30}$$

The unknown parameters in Eq. (30) derive from solving the following system of equations

$$\begin{cases} \psi_j + \beta_j \alpha_i = \alpha_j \\ \beta_j \sigma_{i,i} + \lambda_i = \sigma_{j,i} \\ \beta_j \sigma_{i,j} + \lambda_j = \sigma_{j,j} \\ \lambda_i \sigma_{i,i} + \lambda_j \sigma_{i,j} = 0 \end{cases}. \tag{31}$$

Few algebraic manipulations provide the values

$$\begin{aligned}
\psi_j &= \alpha_j - \tfrac{\sigma_{i,i}\sigma_{j,i} + \sigma_{i,j}\sigma_{j,j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \alpha_i \\
\beta_j &= \tfrac{\sigma_{i,i}\sigma_{j,i} + \sigma_{i,j}\sigma_{j,j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \\
\lambda_i &= \tfrac{\sigma_{i,j}(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \\
\lambda_j &= \tfrac{\sigma_{i,i}(\sigma_{i,i}\sigma_{j,j} - \sigma_{i,j}\sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}.
\end{aligned} \tag{32}$$

[58]

I indicate the portion of wage j that is independent of wage i by $\frac{dw''_{j,t}}{w''_{j,t}}$, such that

$$\frac{dw''_{j,t}}{w''_{j,t}} = \frac{dw'_{j,t}}{w'_{j,t}} - \beta_j \frac{dw'_{i,t}}{w'_{i,t}} = \psi_j dt + \sigma_{\varepsilon_j} d\varepsilon_{j,t}, \tag{33}$$

[58] These parameters permit to compute the volatility of $\varepsilon_{j,t}$, $\sigma_{\varepsilon_j} = \sqrt{\lambda_i^2 + \lambda_j^2} = \frac{|\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}$.

in which $\frac{dw'_{i,t}}{w'_{i,t}}\frac{dw''_{j,t}}{w''_{j,t}} = 0$,[59] according to Eq. (29). The size of $w''_{j,t}$ – the wage increment earned in the market economy – determines whether the individual should exercise her career option and leave industry i.[60]

This second step has transformed the wage processes of a given general model to fit the equations of the canonical specification. One last step is necessary to reinstate perfect correlations between stocks and wages so as to reaffirm the original economy.

9.2.3 Restoring Perfect Correlation Between Stocks and Wages

The third step consists of constructing two portfolios of the original assets that are perfectly correlated with $w'_{i,t}$ and $w''_{j,t}$, respectively. These portfolios, which we indicate as $S'_{i,t}$, $S'_{j,t}$, are defined and constructed by the dynamic strategies $x_{i,i}$, $x_{i,j}$, $x_{j,i}$, $x_{j,j}$ such that

$$\begin{aligned}\frac{dS'_{i,t}}{S'_{i,t}} &= x_{i,i}\left[\frac{dS_{i,t}}{S_{i,t}} - r_f dt\right] + x_{i,j}\left[\frac{dS_{j,t}}{S_{j,t}} - r_f dt\right] + r_f dt \\ &= \left[x_{i,i}\left(\mu_{S_i} - \delta_{S_i}\right) + x_{i,j}\left(\mu_{S_j} - \delta_{S_j}\right) + r_f\left(1 - x_{i,i} - x_{i,j}\right)\right] dt + \sigma_{S'_i} dz'_{i,t}, \qquad (34)\end{aligned}$$

where $\sigma_{S'_i} dz'_{i,t} \equiv x_{i,i}\sigma_{S_i} dz_{i,t} + x_{i,j}\sigma_{S_j} dz_{j,t}$, and

$$\begin{aligned}\frac{dS'_{j,t}}{S'_{j,t}} &= x_{j,i}\left[\frac{dS_{i,t}}{S_{i,t}} - r_f dt\right] + x_{j,j}\left[\frac{dS_{j,t}}{S_{j,t}} - r_f dt\right] + r_f dt \\ &= \left[x_{j,i}\left(\mu_{S_i} - \delta_{S_i}\right) + x_{j,j}\left(\mu_{S_j} - \delta_{S_j}\right) + r_f\left(1 - x_{j,i} - x_{j,j}\right)\right] dt + \sigma_{S'_j} dz'_{j,t} \qquad (35)\end{aligned}$$

[59] It is straightforward to verify that $\frac{dw'_{i,t}}{w'_{i,t}}\frac{dw''_{j,t}}{w''_{j,t}} = 0$. Defining $\rho_{i',j''}$ to satisfy $dz'_{i,t} d\varepsilon_{j,t} = \rho_{i',j''} dt$,

$$\frac{dw'_{i,t}}{w'_{i,t}}\frac{dw''_{j,t}}{w''_{j,t}} = \sigma_{w'_i}\sigma_{\varepsilon_j}\rho_{i',j''} dt = \left(\sigma_{i,i}\lambda_i + \sigma_{i,j}\lambda_j\right) dt$$

which is equivalent to obtaining

$$\begin{aligned}\rho_{i',j''} &= \frac{\sigma_{i,i}\lambda_i + \sigma_{i,j}\lambda_j}{\sigma_{w'_i}\sigma_{\varepsilon_j}} \\ &= \frac{\sigma_{i,i}\frac{\sigma_{i,j}(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2} + \sigma_{i,j}\frac{\sigma_{i,i}(\sigma_{i,i}\sigma_{j,j} - \sigma_{i,j}\sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}\frac{|\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\ &= \frac{\frac{\sigma_{i,i}\sigma_{i,j}(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}) - \sigma_{i,j}\sigma_{i,i}(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}\frac{|\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\ &= 0.\end{aligned}$$

[60] Notice that the wage increment $w''_{j,t}$ can be either positive or negative.

in which $\sigma_{S'_j} dz'_{j,t} \equiv x_{j,i}\sigma_{S_i} dz_{i,t} + x_{j,j}\sigma_{S_j} dz_{j,t}$.

The newly created $S'_{i,t}$ and $S'_{j,t}$ can be interpreted as artificial Exchange-Traded Funds (ETFs). Indeed, these securities would allow the individual to trade index portfolios of the original risky assets just as they do shares of stocks. The optimal combination of stock i and stock j in each ETF derives from solving the following system of equations

$$\begin{cases} x_{i,i}\sigma_{S_i} = \chi\sigma_{i,i} \\ x_{i,j}\sigma_{S_j} = \chi\sigma_{i,j} \\ x_{j,i}\sigma_{S_i} = \kappa\lambda_i \\ x_{j,j}\sigma_{S_j} = \kappa\lambda_j \\ \sigma_{S_i} = \sigma_{S'_i} \\ \sigma_{S_j} = \sigma_{S'_j} \end{cases} . \tag{36}$$

The first two equations ensure that $S'_{i,t}$ and $w'_{i,t}$ are perfectly correlated – they change jointly up to a scale factorχ. Similarly, the third and fourth equations guarantee a perfect correlation between $S'_{j,t}$ and $w''_{j,t}$, with corresponding scale factor κ. The last two equations, which are functional to determine the unknown scale factors, set the volatilities of the fictitious risky assets equal to those of the original assets.[61] Solving for the unknown parameters yields

$$\begin{aligned}
\chi &= \tfrac{\sigma_{S_i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
x_{i,i} &= \tfrac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
x_{i,j} &= \tfrac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
\kappa &= \tfrac{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}{|\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j}|} \\
x_{j,i} &= \left(1_{\{(\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j})>0\}} - 1_{\{(\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j})<0\}}\right) \tfrac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
x_{j,j} &= \left(1_{\{(\sigma_{i,i}\sigma_{j,j}-\sigma_{i,j}\sigma_{j,i})>0\}} - 1_{\{(\sigma_{i,i}\sigma_{j,j}-\sigma_{i,j}\sigma_{j,i})<0\}}\right) \tfrac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}},
\end{aligned}$$ [62]

in which κ, $x_{j,i}$ and $x_{j,j}$ utilize the optimal solution for λ_i and λ_j given in Eq. (32). The optimal strategies $x_{i,i}$, $x_{i,j}$, $x_{j,i}$, $x_{j,j}$ permit to verify that $\rho_{i',i'}$ and $\rho_{j',j''}$, defined according to $dz'_{i,t}dz'_{i,t} = \rho_{i',i'}dt$ and $dz'_{j,t}d\varepsilon_{j,t} = \rho_{j',j''}dt$, equal 1. In particular,

$$\frac{dS'_{i,t}}{S'_{i,t}}\frac{dw'_{i,t}}{w'_{i,t}} = \sigma_{S'_i}\sigma_{w'_i}\rho_{i',i'}dt = \left(x_{i,i}\sigma_{S_i}\sigma_{i,i} + x_{i,j}\sigma_{S_j}\sigma_{i,j}\right)dt,$$

[61] Notice that, since markets are complete, both portfolios can be freely rescaled given certain wage dynamics. In this sense, neither of the last two conditions is restrictive.

[62] $1_{\{A\}}$ denotes the indicator function assuming value 1 if event A occurs and 0 otherwise.

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$$\begin{aligned}
\rho_{i',i'} &= \frac{x_{i,i}\sigma_{S_i}\sigma_{i,i}+x_{i,j}\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i'}\sigma_{w_i'}} \\
&= \frac{\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_i}\sigma_{i,i}+\frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_j}\sigma_{i,j}}{\sqrt{\left(\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2+\left(\frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
&= \frac{\frac{\sigma_{S_i}^2\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}}{\sqrt{\frac{\sigma_{S_i}^2\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)}{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}} \\
&= 1.
\end{aligned}$$

Similarly,

$$\frac{dS'_{j,t}}{S'_{j,t}}\frac{dw''_{j,t}}{w''_{j,t}}=\sigma_{S_j'}\sigma_{\varepsilon_j}\rho_{j',j''}dt=\left(x_{j,i}\sigma_{S_i}\lambda_i+x_{j,j}\sigma_{S_j}\lambda_j\right)dt,$$

leading to

$$\begin{aligned}
\rho_{j',j''} &= \frac{x_{j,i}\sigma_{S_i}\lambda_i+x_{j,j}\sigma_{S_j}\lambda_j}{\sigma_{S_j'}\sigma_{\varepsilon_j}} \\
&= \frac{\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_i}\frac{\sigma_{i,j}(\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j})}{\sigma_{i,i}^2+\sigma_{i,j}^2}-\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_j}\frac{\sigma_{i,i}(\sigma_{i,i}\sigma_{j,j}-\sigma_{i,j}\sigma_{j,i})}{\sigma_{i,i}^2+\sigma_{i,j}^2}}{\sqrt{\left(\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2+\left(\frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2}\frac{\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}} \\
&= \frac{\frac{\sigma_{S_j}\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)(\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j})}{\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}}{\sqrt{\frac{\sigma_{S_j}^2\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)}{\sigma_{i,i}^2+\sigma_{i,j}^2}}\frac{\sigma_{i,j}\sigma_{j,i}-\sigma_{i,i}\sigma_{j,j}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}} \\
&= 1
\end{aligned}$$

Finally, consider the stochastic dynamic equations (27), (33), (34), (35). Since $\rho_{i',i'}$=1, $\rho_{j',j''}$=1 and $\rho_{i',j''}$=0, then $corr\left(\frac{dS'_{i,t}}{S'_{i,t}},\frac{dS'_{j,t}}{S'_{j,t}}\right)$=$\rho_{i',j'}$=0. Indeed, indicating $dz'_{i,t}dz'_{j,t}=\rho_{i',j'}dt$

$$\frac{dS'_{i,t}}{S'_{i,t}}\frac{dS'_{j,t}}{S'_{j,t}}=\sigma_{S_i'}\sigma_{S_j'}\rho_{i',j'}dt=\left(x_{i,i}x_{j,i}\sigma_{S_i}^2+x_{i,j}x_{j,j}\sigma_{S_j}^2\right)dt,$$

and

$$
\begin{aligned}
\rho_{i',j'} &= \frac{x_{i,i}x_{j,i}\sigma_{S_i}^2 + x_{i,j}x_{j,j}\sigma_{S_j}^2}{\sigma_{S_i'}\sigma_{S_j'}} \\
&= \frac{\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_i}^2 + \frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sigma_{S_j}^2}{\sqrt{\left(\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2 + \left(\frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2}\sqrt{\left(\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2 + \left(\frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2+\sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2}} \\
&= \frac{\frac{\sigma_{i,i}\sigma_{i,j}\sigma_{S_i}\sigma_{S_j}}{\sigma_{i,i}^2+\sigma_{i,j}^2} - \frac{\sigma_{i,i}\sigma_{i,j}\sigma_{S_i}\sigma_{S_j}}{\sigma_{i,i}^2+\sigma_{i,j}^2}}{\sqrt{\frac{\sigma_{S_i}^2\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)}{\sigma_{i,i}^2+\sigma_{i,j}^2}}\sqrt{\frac{\sigma_{S_j}^2\left(\sigma_{i,i}^2+\sigma_{i,j}^2\right)}{\sigma_{i,i}^2+\sigma_{i,j}^2}}} \\
&= 0.
\end{aligned}
$$

9.3 An Application to Positive Correlation in Wage Innovations

This Subsection implements the three-step methodology presented above to industry 22 in French's 49 industry portfolios, "Electrical Equipment."[63] To construct 49 industry portfolios, each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio at the end of June of year t based on its four-digit Standard Industrial Classification code at that time. Returns from July of year t to June of year $t+1$ are then computed. For industry 22, the standard deviation of annual returns over the period 1968-1994 equals 21.0 percent. In Rosenberg and Guy's (1976) analysis, "Electrical Equipment" has a beta of 1.27. Assuming a 6 percent market return, this implies an industry stock return, net of the dividend yield, equal to 3.46 percent. Accordingly, the dynamics of the two risky securities are given by

$$
\begin{array}{l}
\frac{dS_{i,t}}{S_{i,t}} = (0.051 - 0.0164)\,dt + 0.210 dz_{i,t} \quad t \in [0,T];\ S_{i,0} = 100 \ given \\
\frac{dS_{j,t}}{S_{j,t}} = (0.06 - 0.0164)\,dt + 0.157 dz_{j,t} \quad t \in [0,T];\ S_{j,0} = 100 \ given
\end{array}, \ dz_{i,t}dz_{j,t} = 0. \quad (37)
$$

The wage equations are characterized in conformity with Davis and Willen's (2000) study of occupation-level components of individual income innovations. In their paper, Davis and Willen analyze several properties of these innovations, including their covariance with aggregate equity returns, long-term bond returns and returns on selected industry-level equity returns. Using repeated cross sections of the Current Population Survey, they find that selected industry-level equity portfolios are significantly correlated with income innovations for several occupations. For each occupation they identify one or more industries that account for a large fraction of the occupation's employment: industry 2, "Electrical Equipment" is matched to "Electrical Engineers." For "Electrical Engineers," the correlation between income innovations and industry-level returns is 0.45[64] and the standard deviation of inno-

[63] Source: K. French Data Library, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[64] Source: Davis and Willen (2000), Table 7, p. 54.

vations to the occupation-level component of earnings equals 3.78 percent.[65] The stochastic wage process associated with "Electrical Engineers" is

$$\frac{dw'_{i,t}}{w'_{i,t}} = 0.02dt + 0.085dz'_{i,t}, \quad 0.085dz'_{i,t} \equiv 0.038dz_{i,t} + 0.076dz_{j,t} \ , \ dz_{i,t}dz_{j,t} = 0, \tag{38}$$

where $\sigma_{i,j} = \sqrt{\sigma^2_{i,i}\left(\frac{1}{\rho^2_{i,i'}} - 1\right)} = \sqrt{0.038^2\left(\frac{1}{0.45^2} - 1\right)} = 0.076$. Similarly,

$$\frac{dw'_{j,t}}{w'_{j,t}} = 0.02dt + 0.09dz'_{j,t}, \quad \sigma_{w'_j}dz'_{j,t} \equiv 0.078dz_{i,t} + 0.045dz_{j,t} \ , \ dz_{i,t}dz_{j,t} = 0, \tag{39}$$

where $\sigma_{j,i} = \sqrt{\sigma^2_{j,j}\left(\frac{1}{\rho^2_{j,j'}} - 1\right)} = \sqrt{0.045^2\left(\frac{1}{0.5^2} - 1\right)} = 0.078$.[66]

As illustrated in Subsubsection 9.2.2, the portion of wage j that is independent of wage i is equal to

$$\frac{dw''_{j,t}}{w''_{j,t}} = \psi_j dt + \sigma_{\varepsilon_j} d\varepsilon_{j,t}, \tag{40}$$

in which $\psi_j = 0.02 - \frac{0.038 \cdot 0.078 + 0.076 \cdot 0.045}{0.038^2 + 0.076^2} \cdot 0.02 = 0.002$ and $\sigma_{\varepsilon_j} = \frac{|0.076 \cdot 0.078 - 0.038 \cdot 0.045|}{\sqrt{0.038^2 + 0.076^2}} = 0.05$. This definition transforms the wage processes of the actual economy so to fit Eq. (26).

Finally, as shown in Subsubsection 9.2.3, the dynamics of the artificially constructed ETFs, $S'_{i,t}$ and $S'_{j,t}$, follow

$$\frac{dS'_{i,t}}{S'_{i,t}} = \left[x_{i,i}\left(\mu_{S_i} - \delta_{S_i}\right) + x_{i,j}\left(\mu_{S_j} - \delta_{S_j}\right) + r_f\left(1 - x_{i,i} - x_{i,j}\right)\right]dt + \sigma_{S'_i}dz'_{i,t}, \tag{41}$$

where $\sigma_{S'_i}dz'_{i,t} \equiv x_{i,i}\sigma_{S_i}dz_{i,t} + x_{i,j}\sigma_{S_j}dz_{j,t}$, and

$$\frac{dS'_{j,t}}{S'_{j,t}} = \left[x_{j,i}\left(\mu_{S_i} - \delta_{S_i}\right) + x_{j,j}\left(\mu_{S_j} - \delta_{S_j}\right) + r_f\left(1 - x_{j,i} - x_{j,j}\right)\right]dt + \sigma_{S'_j}dz'_{j,t}$$

in which $\sigma_{S'_j}dz'_{j,t} \equiv x_{j,i}\sigma_{S_i}dz_{i,t} + x_{j,j}\sigma_{S_j}dz_{j,t}$.

The optimal dynamic strategies defining the ETFs portfolios are $x_{i,i} = \frac{0.038}{\sqrt{0.038^2+0.076^2}} = 0.45$, $x_{i,j} = \frac{0.210 \cdot 0.076}{0.157 \cdot \sqrt{0.038^2+0.076^2}} = 1.20$, $x_{j,i} = \frac{0.157 \cdot 0.076}{0.210 \cdot \sqrt{0.038^2+0.076^2}} = 0.67$ and $x_{j,j} = \frac{-0.038}{\sqrt{0.038^2+0.076^2}} = -0.45$. Perfect correlation between stocks and wages is thus re-established and the newly created set of parameters can be used in the numerical algorithm for the derivation of op-

[65] Source: Davis and Willen (2000), Tables 2 and 4, pp. 48 and 50.

[66] $\rho_{j,j'}$ is the correlation between income innovations and market index returns. It is approximated by the arithmetic average of all correlation coefficients between income innovations and industry-level returns.

timal consumption and portfolio policies. Table 3 in the text reports simulation results for the case of industry 22.

9.4 Comparative Statics: An Algebraic Exploration

The analysis of Subsection 9.2 permits to measure algebraically the directional changes in optimal investment policies when transforming the actual economy into the artificial equivalent economy.

By definition of $\sigma_{w_i'}$, regardless of the sign of $\sigma_{i,j}$, $\sigma_{w_i'} > \sigma_{i,i}$. As shown in Section 4, higher wage volatilities reduce the fraction of financial wealth invested in own-industry stock and, if large enough, they may produce increasing equity patterns over time. In this sense, the effect of non-zero correlations between wage innovations is to generate more conservative portfolios. Additionally, all else constant, cross-industry risk can either inflate or deflate industry j's wage volatility. In particular,

$$\begin{array}{l} \text{if } \sigma_{i,j}\sigma_{j,i} < 0, \text{ then } \sigma_{\varepsilon_j} > \sigma_{j,j} \text{ iff } \left\{ \sigma_{j,j} > \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i} - \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \right\}; \\ \text{if } \sigma_{i,j}\sigma_{j,i} > 0, \text{ then } \sigma_{\varepsilon_j} > \sigma_{j,j} \text{ iff } \left\{ 0 < \sigma_{j,j} < \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i} + \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \text{ or } \sigma_{j,j} > \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i}} \right\}. \end{array} \tag{42}$$

Under these conditions, the individual would invest more cautiously in equities, both own- and cross-industry stock (Table 7, Section 4). On the contrary, for values of $\sigma_{j,j}$ outside the sets defined in Eq. (42), the artificially constructed economy features low uncertainty in the wage of the alternative industry leading to increased exposure to risky assets.

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