Risk-Sharing and the Probability of Crises

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Abstract

In this paper I add heterogeneous agents and risk-sharing opportunities to a coordination game which represents deposit withdrawals from the banking system. I find that heterogeneity in risk aversion within the population amplifies the effect of the business cycle on the probability of a bank run. In particular, risk-sharing enhances the likelihood of bank runs during bad times. The novel insight is that when there is a risk-sharing motive, fundamentals drive not only individual behavior, but also which individuals are more relevant for the likelihood of a crisis. This mechanism has important consequences for the way we think about policy questions. In the paper I discuss three such implications in detail: (1) I show that a policy that facilitates access to banking for previously unbanked individuals generates externalities, and may even decrease welfare of the least risk averse group in the population. (2) I characterize the bias in the estimation of the probability of a banking crisis when heterogeneity is neglected. (3) I show how to correctly calculate the social value of deposit insurance when depositors differ in their risk aversion.

1 Introduction

How do bad outcomes propagate and evolve into a crisis? This question has been deemed equivalent to asking *how* bad fundamentals provoke individual actions which, when aggregated, produce the crisis. But, there exists a second component. Fundamentals also have a role in selecting *who* is more relevant for triggering a crisis. In this paper I study this selection role of fundamentals when there is a risk-sharing motive.

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I consider a bank run model in which depositors differ in their relative risk aversion. I show that the occurrence of a bank run depends on the joint cross-sectional distribution of risk aversion and wealth. Risk aversion is "hard-wired" into agents' preferences, but wealth is a choice variable. Therefore, I allow for the endogenous determination of wealth by allowing depositors to trade in state-contingent securities to insure against aggregate risk. Heterogeneity in risk aversion implies that agents mutually benefit from trading in this market and use it to effectively share risk. As a consequence, more risk averse depositors hold a larger fraction of the aggregate endowment in bad states. Since these depositors are also more likely to withdraw their deposits, bank failures become more likely in bad states. Further, there is feedback between the cross-sectional distribution of wealth and the individual decisions to run on the bank. Thus, ignoring one side of the story downplays the magnitude of the other.

This mechanism has important consequences for the way we think about policy questions. In the paper I discuss three such implications in detail.

First, I show that a policy that facilitates access to banking for previously unbanked individuals generates externalities which may be positive or negative. This result is relevant for the cost-benefit analysis of any policy that generates increased access to the banking system. The mere presence of depositors with a given level of risk aversion affects the likelihood of a bank run. Thus, the entrance of a group of new depositors may increase or decrease the probability of a bank crisis, depending on how their risk aversion compares to the group of pre-existing depositors.

Second, the estimation of the probability of a banking crisis is sensitive to the degree of heterogeneity among depositors. Consider the case in which a researcher observes the complete set of prices of state-contingent claims. Even though these prices incorporate the true probabilities of a bank run, I show that if the estimation does not allow for heterogeneity (for example by estimating a unique risk aversion parameter), then the resulting estimate is biased in a certain way: it underestimates the effect of the aggregate endowment on the probability of a crisis. The size of this bias depends on the degree of heterogeneity in the population.

Third, I show how the social value of deposit insurance is affected by endogenous risk-sharing. Again, even in the case in which a researcher observes the complete set of prices of state-contingent claims, the response of the probability of bank run to the business cycle is underestimated if heterogeneity is neglected. This error is compounded if a single risk aversion parameter is used to calculate the value of deposit insurance. The reason is that when depositors differ in their risk aversion and risk-sharing occurs, the "average" level of risk aversion that is obtained is overly sensitive to low values of risk aversion. Even the existence of a few close to risk neutral agents will introduce a sizable error in the calculation of how valuable deposit insurance is to society.

My paper advances on the theoretical side as well. To derive my results, in order to obtain a well-defined probability of a bank run, I use the tools popularized by Morris and Shin (1998). These tools have spawned a burgeoning "Global Games" literature. In this paper I extend the "Global Games" toolset to deal with an *ex-ante* heterogeneous population consisting of several types representing different risk aversion levels. I show that for homothetic preferences the level of fundamentals that triggers a crisis can be conveniently expressed as a nested double expectation of individual strategies. The first expectation is standard in "Global Games" models. Aggregate behavior for a certain type of agent is an expectation over equilibrium beliefs. The new result is that aggregate behavior for the whole population can be expressed as a second expectation over per-type aggregate behavior, where the measure of each type is given by its relative wealth. Standard stochastic-dominance results can then be used to derive comparative statics results. I also show that the condition for a unique equilibrium is more stringent when there is *ex-ante* heterogeneity in the population than the condition that must hold any single type considered in isolation. The reason is that fundamentals have an effect not only on the behavior for a given type but also how relevant each type is for the occurrence of a crisis.

1.1 Modeling Choices

A bank run is a crisis that arises because of a coordination problem. Although throughout the paper I focus on the case of a bank run, the model is general enough to deal with any crisis that is triggered by agents who take a destabilizing action fearing similar action by others.

I consider the case of agents who face a decision of how much of their deposits to leave in the bank. This problem has the features a coordination problem; specifically, the bank survives only if a sufficiently large fraction of wealth remains deposited in the bank. If agents believe that others will leave their deposits in the bank, it is in their own best interest to do so as well. If, on the other hand, they believe that others will withdraw, it is best for them to withdraw their funds as well.

The bank's survival is not exclusively dependent on the actions of depositors. The quality of its assets plays a role, as well. Higher quality assets are more liquid assets. A bank with a higher quality of assets is in a better position to face withdrawals.

Agents do not have access to contracts that are contingent on the survival of the bank. Apart from this market incompleteness, they live in a world with complete markets and perfect contracting.

In order to uncover the effect of risk-sharing on the probability of crises I must make two innovations. First, there must be diversity in the population. Hence, the assumption of homogeneous agents or a representative agent must be dropped. Since the occurrence of a bank run is uncertain, it is natural to focus on diversity in attitudes towards this uncertainty, and the opportunities for risk-sharing that this diversity causes. I model agents with constant but heterogeneous relative risk aversion. This is the only departure from what can be termed a standard economic model on the preferences side.

Secondly, I must be careful with how I define fundamentals. To study the risk-sharing component in the determination of crises it is important to distinguish between two phenomena that co-move in the business cycle: the quality of the bank's assets and resources available for general consumption. Hence, I decompose fundamentals into two separate components: the "quality of assets" variable and a variable representing the aggregate endowment. The "quality of assets" variable is common in the literature following Morris and Shin (1998) and is often simply called "fundamentals". This is understandable since aggregate risk plays no important role within a homogeneous agent framework.

To produce a well-defined probability of a crisis and cope with the multiplicity of equilibria, I use the tools of Carlsson and van Damme (1993) and Morris and Shin (1998). Agents are (realistically) only imperfectly informed about the true quality of assets. If private signals are relatively precise with respect to public information, then an agent's action does not depend too strongly on her beliefs about others. This reduction in the level of strategic complementarities allows for only a unique equilibrium to survive iterated elimination of dominated strategies. A unique equilibrium can be subjected to comparative statics exercises.

The intuition of the model can be summarized as follows. Agents receive imperfect but informative signals of the quality of the bank's assets. The information conveyed by the signals makes crises more likely in bad times (when a low quality of assets is expected). But agents with heterogeneous risk aversion differ in how they behave with respect to risk. A more risk averse depositor is more likely to choose a safe action such as withdrawing. The likelihood of the bank run depends on the proportion of wealth in the hands of the fraction of the population that plays the safe action. However, wealth is endogenous. It is used as an instrument to share risk across the population. Very risk averse agents tend to buy assets that pay off in bad times and less risk averse agents sell those assets, and buy assets that pay off in good times. This means that when bad times hit, the economy's wealth will be concentrated in the hands of the more risk averse segment of the population, which is also more likely to run on the bank.

1.2 Literature

1.2.1 Global Games

The success of the Global Games literature is tied to its ability to restore uniqueness in models that have multiple equilibria. Building on Carlsson and van Damme (1993), Morris and Shin (1998) construct a currency crisis model in which strategic risk and complementarities in payoffs coordinate heterogeneously informed agents on a unique equilibrium.

Assessing the likelihood of bank runs is an example of the usefulness of the Global Games apparatus. The multiple equilibria model of Diamond and Dybvig (1983) is silent about the likelihood of a bank run equilibrium. The model was subjected to the Global Games treatment by Rochet and Vives (2004) and Goldstein and Pauzner (2005). By delivering a unique equilibrium, these two models offer a way of calculating the probability of a bank run.

The models presented in Angeletos and Werning (2006), Hellwig, Mukherji, and Tsyvinski (2006) and Tarashev (2007) are similar to my model in that agents participate in a financial market before playing the global game. But those models are exclusively concerned with how the existence of a financial market affects the multiple vs. unique equilibrium question. Therefore, the model of the financial market is different from mine. They model an asset in positive net supply that serves the purpose of improving the amount of public information in the model. In their paper the asset traded in the financial market depends on the fundamentals. The equilibrium price therefore helps in the inference problem and eases coordination, increasing the degree of complementarities. In this way it helps in restoring multiplicity. In my model, the reason for having a financial market before participating in the global game is unrelated to the inference problem. The financial market is used exclusively for risk-sharing between *ex-ante* heterogeneous agents. The only assets needed are state-contingent securities which exist in zero net supply.

Goldstein and Pauzner (2004) construct a model in which the same agents participate in two consecutive coordination games. If a crisis occurs in the first game, then wealth decreases and the absolute risk aversion of all agents increases. This makes a crisis in the second game more likely. The main difference with my paper is that in the model by Goldstein and Pauzner (2004) agents are homogeneous even after the first global game. Therefore, the feedback that comes through wealth shifts is absent from their analysis.

The model which is closest to mine is that of Guimarães and Morris (2007). They deal with risk averse *ex-ante* homogeneous agents who are allowed to choose from a continuum of actions. Although they recognize that their model could easily accommodate *ex-ante* heterogeneous agents, they do not pursue that route. An immediate result when *ex-ante* heterogeneity is added is that the equilibrium depends on the cross-sectional distribution of wealth. Realizing this, I allow wealth to be determined endogenously by allowing risk averse agents to smooth consumption across states.

While all other models in the literature deal with *ex-post* heterogeneity, this paper is the only one with both *ex-ante* and *ex-post* heterogeneity.

1.2.2 General Equilibrium and Finance

The question of Pareto-efficient risk-sharing within a group of agents with heterogeneous risk aversion can be traced back to Wilson (1968).

Rubinstein (1974) and Constantinides (1982), building on work by Negishi (1960) and Wilson (1968), derive aggregation results for heterogeneous agents in complete market models, implying that assets can be priced off the intertemporal marginal rate of substitution of an agent who consumes the aggregate endowment.

Specifically, in the finance literature, Dumas (1989) studies a competitive financial market with two agents who differ in their risk aversion. Chan and Kogan (2002) study the asset pricing implications of assuming a heterogeneously risk-averse population.

Following Benninga and Mayshar (2000) and Wang (1996), who deal explicitly with the existence of a representative agent representation in such models, Kuzmics, Hara, and Huang

(2006) offer the most advanced treatment of efficient risk-sharing in a heterogeneous population.

Except for a few special cases, the equilibrium in my model is not Pareto-efficient. However, I am able to use methods similar to Benninga and Mayshar (2000) and Kuzmics, Hara, and Huang (2006) to characterize equilibrium risk-sharing. Moreover, I show how preferences of a stand-in representative agent need to be modified when agents face a coordination problem in addition to endowment risk.

2 Model

2.1 Environment

There are three dates $t = 0, 1, 2$. At date 0 agents are allowed to participate in a complete financial market in order to hedge against date 1 aggregate uncertainty. At date 1 the *ex-ante* uncertain level of aggregate endowment is realized and observed by everybody. After observing this realization and having the proceeds of their financial positions deposited into their bank account, agents must now make a decision of how much of their wealth to withdraw from the bank. Money withdrawn from the bank has a safe rate of return which is normalized to one. Money left in the bank has a payoff that depends on the survival of the bank. The payoffs of leaving the money in the bank are dependent on how much of their deposits everybody else leaves in the bank. If too much of deposits are withdrawn, a bank run is triggered and agents only obtain a fraction of their deposited money. If there is no bank run, then they earn an interest which is higher than what the safe asset delivers. The banking system's strength is higher during recessions when their assets are more likely to be non-performing than during boom times.

2.1.1 Aggregate risk

Let S be a set of possible states that can occur at date 1. From an *ex-ante* (date 0) perspective, a state $s \in S$ has density $p(s)$ and the corresponding level of aggregate endowment is $e(s) \in [\underline{e}, \overline{e}]$. It is convenient to think of endowments as an increasing function of the states.[1] At date 0, agents are allowed to trade in state contingent claims that pay off at date 1 in order to hedge against the aggregate risk.

2.1.2 Risky project

At date 1, agents choose how much to withdraw from the banking system. Deposits have a date 2 payoff that depends on the amount of deposits left in the system. Its payoff is binary. The bank either survives or fails. If it survives, deposits have a return of $R > 1$. If the bank fails, the payoff is $f < 1$ per dollar deposited.

Let D be the aggregate fraction of deposits withdrawn from the bank, and let θ be the quality of the bank's assets, whose value is unknown at the time of the withdrawal decision. The bank fails if too large of a fraction of its deposits are withdrawn

$$D \geq \theta \tag{1}$$

and survives otherwise.[2] A higher value the bank's assets makes a failure less likely. The quality of assets is unknown to agents at date 0 and is only realized at date 2.

2.2 Information Structure

Although agents do not know the true value of the bank's assets when they decide their date 1 investment, they do know the true distribution of the quality of assets θ given state s. This

[1] In this sense, one can think of the endowment levels as being the states themselves. I choose to maintain a distinction because in this model states of nature influence the quality of assets as well.

[2] More generally, the condition for bank failure could have a function $f(\theta)$ on the right hand side. As long as the strength of the bank is strictly increasing in the quality of assets, $f'(\theta) > 0$, results will be qualitatively the same.

distribution is common knowledge. Agents are able to further refine their prediction of θ by receiving an informative signal x which is correlated with the true value of θ.

2.2.1 Priors and Signals

The common prior about θ has a Normal distribution $\theta \sim N(\mu(s), \sigma_\theta^2)$. The prior depends on the state s only through the mean $\mu(s)$. This mean $\mu(s)$ is a measure of *ex-ante* expected quality of the bank's assets. As such, it should be increasing with the aggregate endowment. Given the ordering of endowment levels with respect to states, this implies $\frac{d\mu(s)}{ds} \geq 0$.

Agents receive imperfect signals about the value of θ. These signals are $x = \theta + \sigma_\varepsilon \varepsilon$, with ε distributed according to a standard Normal distribution $\varepsilon \sim N(0, 1)$. I assume that $Cov(\theta, \varepsilon) = 0$, meaning that ε is a well behaved error term. Further, the error terms are independent across individuals. Consequently, the bivariate vector (θ, ε) has the following joint distribution

$$\begin{bmatrix} \theta \\ \varepsilon \end{bmatrix} \sim N\left(\begin{bmatrix} \mu(s) \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & 1 \end{bmatrix} \right) \tag{2}$$

2.3 Population

2.3.1 Agents and Preferences

Agents differ in risk aversion. They have constant relative risk aversion (CRRA) preferences. This family of preferences is represented by an expression of the form

$$u(c; \rho) = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \text{if } \rho \neq 1 \\ \log(c) & \text{otherwise.} \end{cases} \tag{3}$$

Heterogeneity in risk aversion is modeled by allowing for different values of $\rho > 0$ in the population. Let there be a set of types I and a mass one of agents of each type. An agent of type $i \in I$ has CRRA preferences with relative risk-aversion parameter ρ_i taken from the interval $[\underline{\rho}, \overline{\rho}]$. The values $\underline{\rho}$ and $\overline{\rho}$ are, respectively, the lowest and highest level of risk aversion in the population. I use the notation $\rho(I)$ for the set of risk aversion levels actually present in the population.

Agents maximize expected utility over consumption $EU(\rho) = E[u(c;\rho)]$. Heterogeneity in risk aversion is the only source of *ex-ante* heterogeneity in the model. All agents share equally in the aggregate endowment. Their individual share is $e_i(s) = \frac{e(s)}{\#I}$.

2.3.2 Choices

Agents care only about date 2 wealth. In their transit from date 0 to date 2 they face two kinds of uncertainty: (1) aggregate endowment uncertainty between date 0 and date 1 and (2) strategic uncertainty between date 1 and date 2 stemming from the possibility of a bank run.

As a way of dealing with aggregate uncertainty, agents engage in risk-sharing through the use of state-contingent claims. The optimal date-0 trading strategy implies an optimal choice of wealth that each type of agent wishes to carry into each possible state. Agents belonging to a certain type i with risk aversion ρ_i are identical at this stage. Therefore, optimal wealth carried over can be represented as a function of risk aversion levels. Write $w : S \times \rho(I) \to R_+$ for the wealth choice.

The second source of uncertainty occurs after a certain state s has been realized. Agents own the amount of wealth contracted for this state $w(s, \rho_i)$, which may be different across different types. The choice that has to be made at date 1 is how much of this amount is left in the bank. Agents who share the same level of risk aversion ρ_i, although holding the same level of wealth, may observe different signals x on the quality of the bank, and therefore assess the likelihood of a bank failure differently. On the other hand, in equilibrium, agents who observe the same signal x must assign the same subjective probabilities to success and failure of the project, regardless of their level of risk aversion. They must hold the same beliefs since agents are Bayesian updaters who share a common prior and their conjecture about the play of opponents is correct in equilibrium. I use the notation π for the subjective probability that an agent assigns to the bank's survival (to the event $\{D(s) < \theta\}$) after observing signal x and state s, regardless of his type.

Let $d(s, \pi, \rho_i)$ stand for amount withdrawn from the bank at the individual level. It is the amount withdrawn by an agent of type i, acting under beliefs π with the knowledge that state s was realized. The amount left deposited in the bank by this agent is equal to $w(s, \rho_i) - d(s, \pi, \rho_i)$. Date 2 wealth for an agent of type i who acted under belief π is

$$d(s, \pi, \rho_i) + \left[\mathbf{1}_{(D(s)<\theta)} R + \mathbf{1}_{(D(s)\geq\theta)} f\right] (w(s, \rho_i) - d(s, \pi, \rho_i)) \tag{4}$$

The return on funds withdrawn from the bank is one. An agent obtains $d(s, \pi, \rho_i)$, the amount withdrawn from the risky project regardless of how the bank fares. The return on deposits depends on the aggregate choice of the whole population. I use indicator functions to write the two possible events of survival and failure of the bank.

Agents assign likelihoods to the bank run/no bank run events through their private belief π. I use the name *interim utility* to denote the expected utility of an agent with respect to this belief.

$$\begin{aligned} U_\pi(\rho) = \frac{1}{1-\rho} \Big\{ &\pi \left[Rw(s, \rho) - (R-1)d(s, \pi, \rho)\right]^{1-\rho} \\ &+ (1-\pi)\left[fw(s, \rho) + (1-f)d(s, \pi, \rho)\right]^{1-\rho} \Big\} \end{aligned} \tag{5}$$

It is this expression that an agent seeks to maximize when choosing how much to withdraw, $d(s, \pi, \rho_i)$.

Ex-ante expected utility is the expectation of *interim utility* over the random variable s. By the law of iterated expectations, $EU(\rho) = E_s\left[E[U_\pi(\rho)|s]\right]$.

Table 1: *Timeline*

Date 0	Date 1	Date 2
1. choose wealth $w(s, \rho)$ for each state $s \in S$	1. state s realizes 2. $w(s, \rho)$ deposited 3. observe signal x 4. calculate beliefs π 5. withdraw $d(s, \pi, \rho)$	1. θ realizes 2. bank survives or fails 3. consumption takes place

2.4 Strategies and Equilibrium

2.4.1 Timing

At date 0 agents choose the level of wealth $w(s, \rho_i)$ that they want to carry into every possible state of the world. At the beginning of date 1, a state s, one of the possible states in S, is realized. The realization of the state determines fundamentals: the aggregate endowment $e(s)$ and the expected strength of the bank $\mu(s)$. All agents share the common prior $\theta \sim N(\mu(s), \sigma_\theta)$. Agents privately observe their individual signal x, compute their belief π and then decide how much of their wealth to withdraw from the bank $d(s, \pi, \rho_i)$. At date 2 payoffs are made and consumption takes place.

2.4.2 Strategies

A strategy for player of type i is a pair of functions (w, d), consisting of a wealth allocation function $w : S \times \rho(I) \to R_+$, along with a function $d : S \times [0, 1] \times \rho(I) \to R_+$, such that $d(s, \pi, \rho_i) \in [0, w(s, \rho_i)]$ for all states in S.

2.4.3 Equilibrium Definition

An equilibrium consists of a strategy for each player such that agents choose function $d(s, \pi, \rho_i)$ to maximize interim payoffs, the expression in equation (5). Each player computes beliefs π by Bayesian updating using the common prior, the observed private signal and conjectured equilibrium strategies by other players. Beliefs are consistent with equilibrium play. Date 1 equilibrium choices imply a date 0 expected indirect utility function. Agents choose function $w(s, \rho_i)$ to maximize this indirect utility.

2.4.4 Outcome

An equilibrium implies an *ex-ante* probability of a bank run as a function of s. I denote this probability by a function $\psi : S \to [0, 1]$.

Uniqueness of equilibrium is of paramount importance for this probability. Under multiplicity of equilibria, there can be no meaningful way of expressing the probability of a bank run as a function of the state, as the same state may give to completely different probabilities of a bank run.

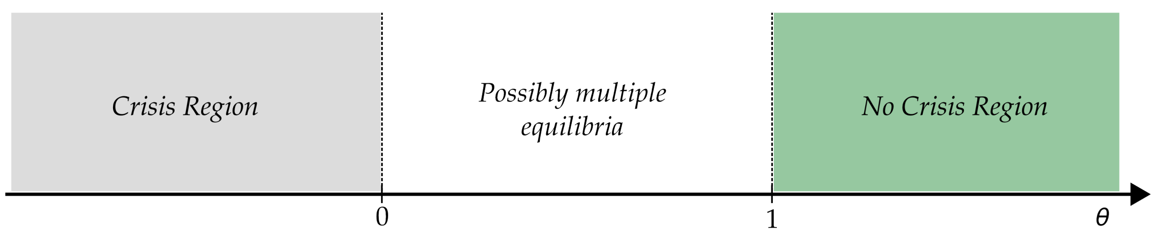

Figure 1: *Outcome Regions*

The model, as is common for models of self-fulfilling beliefs, may have belief-driven multiplicity of equilibria for some range of realizations of θ. There are, however, two regions in which realizations of θ determine the outcome regardless of beliefs. These two regions correspond to very good and very bad fundamentals. If the variable θ falls into either of these two regions, the fact that a bank run occurs or not is completely independent of how agents behave. If θ lies in the interval $(\infty, 0]$, then fundamentals are so bad that D will always exceed θ, as D is non-negative. If, on the other hand θ lies in $[1, +\infty)$, then fundamentals are so good that even if everybody plays the destabilizing action, the crisis does not occur. It is only in $[0, 1]$ that the occurrence of a crisis depends on equilibrium aggregate choices. An important task of solving the model is, therefore, to explain under which conditions on parameters the equilibrium is unique for all possible realizations of θ.

2.4.5 Roadmap

Section 3 provides a two-state two-type example. This example showcases many of the the features which are shown to hold for many states and many types in Section 4, where the model is solved.

3 Example

The main points can be grasped by focusing on a simplified example featuring only two states and two types of risk aversion in the population. There are two equally likely states, state H with a high endowment e_H and state L with a low endowment e_L. The population consists of only two types: a risk neutral type ($\rho = 0$) and an infinitely risk averse type ($\rho = +\infty$). Both types are entitled to one half of the aggregate endowment.

3.1 Bank run stage

The withdrawal behavior for risk averse types is trivial. They will always run on the bank as long as there is some chance of a failure. Aggregate withdrawals of the group of infinitely risk averse types is

$$D_\infty = 1. \tag{6}$$

Risk neutral types, on the other hand, maximize the expected return of their wealth. This means that they will leave the money in the bank if and only if the expected return from doing so exceeds 1, which is the return from withdrawing and storing the money until date 2. They will keep their deposits in the bank if and only if

$$\pi R + (1 - \pi) f > 1 \tag{7}$$

Their optimal withdrawal strategy, as a function of their belief π, is

$$\delta(\pi, 0) = \begin{cases} 1 & \text{if } \pi \leq \bar{\pi} \equiv \frac{1-f}{R-f} \\ 0 & \text{if } \pi > \bar{\pi} \equiv \frac{1-f}{R-f} \end{cases} \tag{8}$$

The value of π for an individual is a function of the signal she has observed, as well as her beliefs about the withdrawal decisions of other agents in equilibrium. The distribution of these beliefs across the population is, of course, and equilibrium object. In any case, since a fraction of the risk neutral types will receive signals that generate beliefs $\pi > \bar{\pi}$,

$$D_0 < 1 \tag{9}$$

Consider, for example, the case in which beliefs π are distributed uniformly across the population of risk neutral agents.[3] Withdrawals aggregated across risk neutral types is

$$D_0 = \int_{\pi=0}^{1} \delta(\pi, 0) d\pi = \int_{\pi=0}^{\bar{\pi}} 1 d\pi = \bar{\pi} < 1 \tag{10}$$

There will be a bank run if and only if

$$\frac{w(s,0)}{e(s)} D_0 + \frac{w(s,\infty)}{e(s)} D_\infty \geq \theta, \quad s \in \{H, L\} \tag{11}$$

or, equivalently, using the result that $D_\infty = 1$ from (6),

$$\left(1 - \frac{w(s,\infty)}{e(s)}\right) D_0 + \frac{w(s,\infty)}{e(s)} \geq \theta, \quad s \in \{H, L\} \tag{12}$$

The left hand side is a weighted average of $D_0 < 1$ and $D_\infty = 1$. It is increasing in the fraction of wealth in the hands of the infinitely risk averse group in the population. The probabilities of a bank run, $\psi(L)$ and $\psi(H)$, can be computed as the probability that realization of θ takes a value lower than the left hand side of (12). The quality of assets θ is a random variable

[3] In fact, it will be shown below that this actually corresponds to the distribution of beliefs in equilibrium for the limit case when $\sigma_\varepsilon \to 0$.

with a known distribution. A bank run will be more likely in the low endowment state if the left hand side in (12) is larger in state L than in state H. This depends on the outcome of risk-sharing between the two types.

3.2 Risk-sharing

The type with infinite risk aversion will be insured by the risk neutral type. The risk averse type desires to perfectly smooth consumption across states. Given the particular withdrawal strategy of the infinitely risk averse type, this is equivalent to perfectly smoothing wealth. This implies that the risk neutral agents will transfer wealth to the risk averse type in the low state in exchange for wealth in the high endowment state.

The argument can be made graphically as follows. Consider the Edgeworth box in figure 2. The lower left corner represents the origin for the infinitely risk averse type. The opposite corner is that of the risk neutral type. Trade in state-contingent claims necessarily produces an allocation on the contract curve. Given that indifference curves for the infinitely risk averse type are of the Leontief, this implies that, in this case, the contract curve is the 45° line in figure 2. Since both types initially start by owning one half of the aggregate endowment, trade must deliver an allocation in the upper left quarter of the Edgeworth box. In consequence, the following relationships hold.

$$\frac{w(H,\infty)}{e(H)} < \frac{1}{2} < \frac{w(H,0)}{e(H)} \quad \text{and} \quad \frac{w(L,\infty)}{e(L)} > \frac{1}{2} > \frac{w(L,0)}{e(L)} \tag{13}$$

3.3 Putting it all together

Trade in state-contingent claims generates a lower probability of a bank run in the state with the high endowment. Recall the expression on left hand side of (12) and denote it as θ_s^*.

$$\theta_s^* = \left(1 - \frac{w(s,\infty)}{e(s)}\right) D_0 + \frac{w(s,\infty)}{e(s)}, \quad s \in \{H, L\} \tag{14}$$

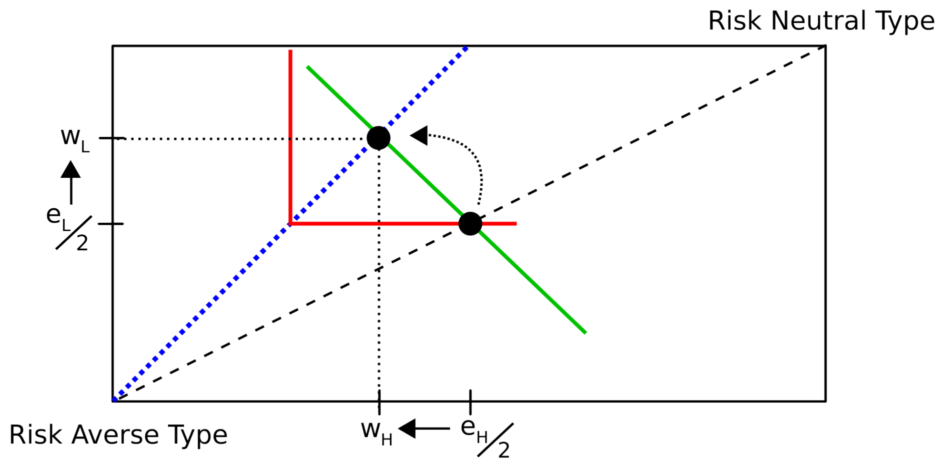

Figure 2: *The Edgeworth box is drawn for two states H and L. The horizontal axis measures wealth transferred into state H, the vertical axis measures wealth transferred into state L. Trade in state-contingent claims will always produce an allocation that is in the upper left quarter of the Edgeworth box. The type with infinite risk aversion will be insured by the risk neutral type. This implies that in state L, the more risk averse agent will own a higher fraction of the aggregate endowment. In state H this will be reversed and the less risk averse agent is entitled to a majority stake in the aggregate endowment.*

Using the relationships in (13),

$$\theta_L^* > \frac{1}{2}D_0 + \frac{1}{2} > \theta_H^* \tag{15}$$

The probability of a bank run is equal to the probability that the quality of assets is lower than the threshold values θ_L^* and θ_H^*. Therefore,

$$\psi(L) = \Pr[\theta \leq \theta_L^*] > \Pr[\theta \leq \theta_H^*] = \psi(H) \tag{16}$$

Notice that this result does not use the fact that the distribution of θ is shifted towards lower values in state L. The result holds even if the expected quality of assets does not differ between the two states, i.e. even if $\mu(L) = \mu(H)$.

3.4 Feedback through state prices

The price ratio $\frac{q_L}{q_H}$ measures the relative value of transferring wealth into the two states. This price ratio must equal the marginal rate of substitution of the risk neutral type. It seems intuitive that, since both states are equally likely, the price ratio must be unity always. This is not true, however. The reason is that agents care about *consumption*, not *wealth*. Therefore, it is necessary to consider the use that the risk neutral agent makes with wealth in the bank run stage.

Suppose that the price ratio is unity. It has been established, that trade will move the allocation into the upper left quarter of the Edgeworth box. This raises the probability of a crisis in the low endowment state and lowers it in the high endowment state. But agents are forward-looking. As a consequence, the value of transferring wealth into state L is lower than that of transferring wealth into the high endowment state. Therefore, the price of a claim that pays one dollar in state L must be lower than the price of a claim that pays in H, i.e. $\frac{q_L}{q_H} < 1$. This, in turn, hurts the risk neutral type but is a positive pure income effect for the risk averse type, who is buying cheap claims to wealth in state L and selling expensive claims to wealth in state H. She is now able to obtain an even higher fraction of the endowment in state L, thus raising the probability of a bank run.

Figure 3 illustrates the change in the relative price in the form of a clockwise rotation of the indifference curve of the risk neutral type. This clockwise rotation also corresponds to the change in the budget sets of both agents.

In this simple example, there exists feedback between the choice of wealth and the probability of a bank run in the low endowment state. This feedback is also present in the more general model, as will be shown in the next section. It has important policy implications, as it increases the sensibility of the probability of a bank run to the aggregate endowment.

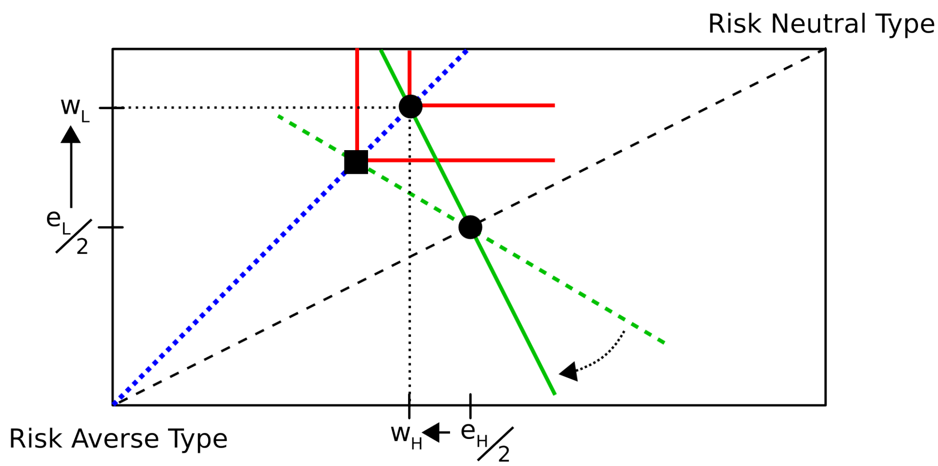

Figure 3: *Trade in state-contingent claims affects the probability of a bank run. This, in turn lowers the value of transferring wealth into state L relative to state H. This favors the risk averse type who is buying claims to wealth in state L and selling claims to wealth in state H. The positive income effect of the price change allows the risk averse type to consume even more in the low state. In the picture, the indifference curve of the risk neutral agent rotates clockwise. The new equilibrium is higher up on the 45° line from the risk averse type's perspective.*

4 Solution of the Model

4.1 The bank run stage

It is convenient to approach the bank run stage in two steps. First I solve for the optimal choice at the individual level for any given set of beliefs. Then I deal with the determination of equilibrium beliefs.

4.1.1 Optimal withdrawals as a function of beliefs

Treating $w(s,\rho)$ as predetermined, an agent with beliefs π maximizes interim utility

$$\max_{d(s,\pi,\rho)\in[0,w(s,\rho)]} \quad \frac{1}{1-\rho}\Big\{\pi\left[Rw(s,\rho)-(R-1)d(s,\pi,\rho)\right]^{1-\rho} \\ +(1-\pi)\left[fw(s,\rho)+(1-f)d(s,\pi,\rho)\right]^{1-\rho}\Big\} \tag{17}$$

The CRRA utility function belongs to the class of homothetic utility functions. This means that the optimal choice of consumption, both in case of a bank run, or with the bank surviving, is linear in wealth. This implies that withdrawals, as the vehicle used to transfer consumption between these two outcomes, has to be linear in wealth too. The following lemma characterizes the solution and provides comparative statics with respect to beliefs π and risk aversion ρ.

Lemma 1 *The amount withdrawn from the bank is linear in wealth*

$$d(s, \pi, \rho) = \delta(\pi, \rho)w(s, \rho)$$

The function $\delta(\pi, \rho)$ is the fraction of wealth withdrawn from the bank. This function is continuous and almost everywhere differentiable. Further, there exist values π_L and $\pi_H(\rho)$ with the following properties

(i) $0 < \pi_L < \pi_H(\rho) < 1$,

(ii) $\delta(\pi, \rho) = 1$ *if and only if* $\pi \in [0, \pi_L]$,

(iii) $\delta(\pi, \rho) = 0$ *if and only if* $\pi \in [\pi_H(\rho), 1]$.

(iv) For $\pi \in (\pi_L, \pi_H(\rho))$, $\delta(\pi, \rho)$ is strictly decreasing in π and strictly increasing in ρ. The function is constant in both arguments if π is outside that interval.

Agents choose corner solutions for both low and high values of π. The corner solution when $\pi \in [0, \pi_L]$, for which $\delta(\pi, \rho) = 1$, occurs for beliefs of bank survival that are sufficiently low for it to be dominated by the safe choice of withdrawing. The cutoff value at which agents start leaving money in the bank, π_L, is the same for all types and coincides with the cutoff belief at which a risk-neutral agent would switch from withdrawing to leaving deposits in the bank. The reason for this is that, regardless of their level of risk aversion, all agents are locally risk-neutral when they withdraw the totality of their money from the bank. Evaluating marginal interim utility at $\delta(\pi, \rho) = 1$ shows that it is proportional to the marginal interim utility of a risk-neutral agent.

$$\left.\frac{\partial U^{\pi}}{\partial d(s,\pi,\rho)}\right|_{\delta(\pi,\rho)=1} = \left[-\pi(R-1)+(1-\pi)(1-f)\right] w(s,\rho)^{-\rho} \tag{18}$$

The term in brackets is the marginal interim utility of a risk neutral agent. Hence, the sign of marginal interim utility is the same for all types as it does not depend on the value of ρ.

As already argued in the example in Section 3, the condition for a risk-neutral agent to choose withdrawal is that the return of leaving the money in the banks is dominated in expected return by the safe return of 1, which is equivalent to requiring that the term in brackets is positive.

$$\pi R + (1-\pi)f < 1 \quad \Leftrightarrow \quad -\pi(R-1)+(1-\pi)(1-f) > 0 \tag{19}$$

The exact value of π_L is calculated by taking (19) with equality and solving for π.

Whenever $\delta(\pi,\rho) < 1$, and especially at the other corner, at which $\delta(\pi,\rho) = 0$, the sign of marginal interim utility does in fact depend on the value of ρ.

4.1.2 Inference problem

The inference problem is the same for agents of all types. It depends only on the signal x that each agent privately observes. Their other characteristics, such as level of risk aversion or wealth does not go into this computation. Agents will use their signal to update beliefs about the quality of the banks assets, θ. They will individually calculate the conditional CDF of θ given that signal x was observed.

$$H_{\mu}(\theta|x) \equiv \Pr\{\tilde{\theta} \leq \theta | \tilde{x} = x\} \tag{20}$$

$H_{\mu}(\theta|x)$ measures the probability that the quality of assets lies in the region below a given value θ given that a signal of value x was observed.

4.1.3 Threshold Equilibrium: Existence and Uniqueness

A threshold equilibrium is characterized by a critical value θ_s^* such that a bank run occurs if and only if $\theta < \theta_s^*$. In this equilibrium, agents who observe signal x must attach probability

$$\pi = 1 - H_{\mu(s)}(\theta_s^*|x) \tag{21}$$

to the bank's survival. From this calculation type ρ withdraws a fraction $\delta(\pi, \rho)$ of wealth. By the Law of Large Numbers, the total amount withdrawn by the continuum of agents of risk aversion ρ is equal to their expected behavior, where the expectation is with respect to the distribution of signals.

In a threshold equilibrium there is a one-to-one relationship between signals x and beliefs π. Equation (21) can be inverted to express signals as a function of beliefs in a unique way. This allows to write the fraction of wealth withdrawn as an expectation with respect to π.

Lemma 2 *In a threshold equilibrium, the fraction of wealth withdrawn from the bank by type ρ, $D(\mu, \rho, \theta_s^*, \theta)$, can be expressed as an expectation with respect to π*

$$D(\mu, \rho, \theta_s^*, \theta) = \int_{\pi=0}^{1} \delta(\pi, \rho) d\Gamma_\mu(\pi|\theta_s^*, \theta)$$

where $\Gamma_\mu(\pi|\theta_s^*, \theta)$ is the proportion of the population with beliefs π or less given a realization of θ and a conjectured threshold equilibrium with threshold θ_s^*. Further, $\Gamma_\mu(\pi|\theta_s^*, \theta)$ is strictly increasing in π and θ_s^*, and strictly decreasing in θ and μ.

The fraction of the endowment that is withdrawn under the conjectured equilibrium by agents of all types is

$$D(s, \theta_s^*, \theta) = \sum_{i \in I} \frac{w(s, \rho_i)}{e(s)} D(\mu(s), \rho_i, \theta_s^*, \theta) \tag{22}$$

Lemma 3 *For any state s, threshold θ_s^* is part of a threshold equilibrium if and only if the* *threshold equals withdrawals when the actual realization equals the threshold, i.e.*

$$\begin{aligned}\theta_s^* &= D(s, \theta_s^*, \theta_s^*) \\ &= \sum_{i \in I} \frac{w(s, \rho_i)}{e(s)} D(\mu(s), \rho_i, \theta_s^*, \theta_s^*)\end{aligned} \tag{23}$$

From now on I use hats above functions to suppress the double dependence on θ_s^*, i.e. I write $\hat{D}(\mu(s), \rho_i, \theta_s^*)$ instead of $D(\mu(s), \rho_i, \theta_s^*, \theta_s^*)$ and $\hat{\Gamma}_\mu(\pi|\theta_s^*)$ instead of $\Gamma_\mu(\pi|\theta_s^*, \theta_s^*)$.

Lemma 3, together with the result from Lemma 2, provides a very simple characterization of the thresholds θ_s^*. With the characterization for threshold equilibria found in Lemma 3 it is straightforward to prove existence and uniqueness of the equilibrium. Existence follows from the fact that the right hand side of equation (23) is continuous and bounded on the interval $[0, 1]$.

Proving uniqueness requires more work. Differentiate both sides of (23) with respect to θ_s^*

$$1 = \sum_{i \in I} \frac{d \frac{w(s,\rho_i)}{e(s)}}{d\theta_s^*} \hat{D}(\mu(s), \rho_i, \theta_s^*) + \sum_{i \in I} \frac{w(s, \rho_i)}{e(s)} \frac{\partial \hat{D}(\mu(s), \rho_i, \theta_s^*)}{\partial \theta_s^*} \tag{24}$$

For uniqueness, the right hand side has to be strictly less than one. This can be achieved by using an assumption on the accuracy of the private signal relative to the accuracy of public information. The second term on the right hand side of (24) can be shown to be bounded by $\frac{\sigma_\varepsilon}{\sigma_\theta^2} \frac{1}{\sqrt{2\Pi}}$. This is the same bound as the one derived in Morris and Shin (2004). A bound for the first term can be derived as well. It depends on R, f and $\rho(I)$. It could in principle be positive or negative, but is always finite.

In general, if no assumption is made about the composition of the population, a sufficient condition for uniqueness can be obtained by finding a more generous version of the bound, which can be written as a function of the highest and lowest level of risk aversion in the population and is equal to $\frac{\sigma_\varepsilon}{\sigma_\theta^2} \frac{1}{\sqrt{2\Pi}} \bar{B}(R, f, \underline{\rho}, \overline{\rho})$. As long as there is more than one type, this bound is always positive. The condition required for uniqueness then becomes

$$\frac{\sigma_\varepsilon}{\sigma_\theta^2}\frac{1}{\sqrt{2\Pi}}\left[1+\bar{B}(R,f,\underline{\rho},\overline{\rho})\right]<1 \tag{25}$$

As can be seen by taking the limit as $\frac{\sigma_\varepsilon}{\sigma_\theta^2}\to 0$, improving the informativeness of the private signal relative to the public information provides a sufficient condition for uniqueness.

Lemma 4 *If the condition in* (25) *holds, then there is a unique threshold equilibrium characterized by the threshold* $\theta_s^* \in [0,1]$ *that solves* (23)*. Further, there exist no other equilibria.*

The importance of Lemma 4 is that the threshold equilibrium is unique among all possible equilibria. And uniqueness is what makes it meaningful to compute the probability of a bank run, $\psi(s)$. With multiple equilibria it is impossible to calculate this probability without resorting to *ad-hoc* assumptions on the selection among equilibria. For the remainder of the paper I will assume that the condition in (25) holds.

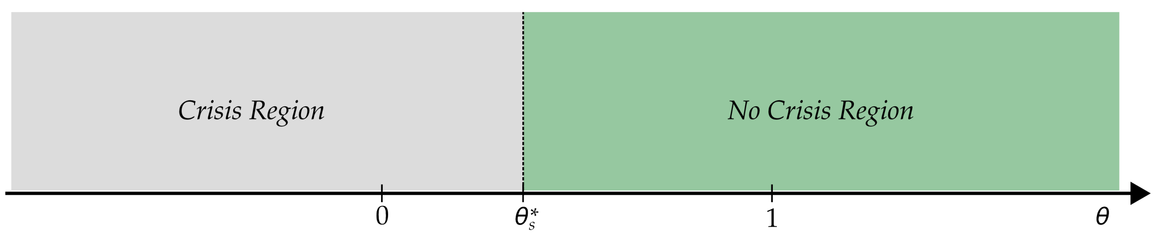

Figure 4: *Threshold Equilibrium*

4.1.4 Analysis

The threshold θ_s^* can be interpreted as a bank run index. The probability that θ lies below this equilibrium threshold is the *ex-ante* probability of a crisis in state s. This probability is increasing in θ_s^*. Therefore, higher values of θ_s^* correspond to higher *ex-ante* probabilities of a run on the bank. Formally, the probability is computed as

$$\psi(s) = \Pr(\{\text{Crisis in state } s\}) = \Phi\left(\frac{\theta_s^* - \mu(s)}{\sigma_\theta}\right)$$

The probability of a crisis depends on the expected value $\mu(s)$ as well, although in the opposite way. A lower value of $\mu(s)$ makes the event that θ is below θ_s^* more likely, thus boosting the probability of a crisis.

The interpretation of the threshold θ_s^* as a crisis index makes it interesting to analyze how this threshold is computed. Equation (23) is the key to understanding how the two channels act. The equation is repeated here with the new notation of adding hats.

$$\theta_s^* = \sum_{i \in I} \frac{w(s, \rho_i)}{e(s)} \hat{D}(\mu(s), \rho_i, \theta_s^*) \tag{26}$$

$$\hat{D}(\mu, \rho, \theta) = \int_{\pi=0}^{1} \delta(\pi, \rho) d\hat{\Gamma}_\mu(\pi|\theta) \tag{27}$$

Focus on the term $\hat{D}(\mu, \rho, \theta)$. From Lemma 2, the CDF $\Gamma_\mu(\pi|\theta, \theta)$ can be ranked according to first-order stochastic dominance. By Lemma 1*(iv)*, the function $\delta(\pi, \rho)$ is globally weakly decreasing in π and strictly decreasing in π on an interval $[\pi_L, \pi_H(\rho)]$. First-order stochastic dominance with respect to μ allows to compare the magnitudes of $\hat{D}(\mu_1, \rho, \theta)$ and $\hat{D}(\mu_2, \rho, \theta)$.

$$\mu_1 < \mu_2 \quad \Leftrightarrow \quad \hat{D}(\mu_1, \rho, \theta) > \hat{D}(\mu_2, \rho, \theta) \tag{28}$$

Now, looking across individuals, Lemma 1*(iv)* states that the function $\delta(\pi, \rho)$ is weakly increasing in ρ and strictly increasing in ρ on the interval $[\pi_L, \pi_H(\rho)]$. Consequently, $\hat{D}(s, \rho, \theta)$ is an increasing function of ρ for any value of θ.

$$\rho_i < \rho_j \quad \Leftrightarrow \quad \hat{D}(\mu, \rho_i, \theta) < \hat{D}(\mu, \rho_j, \theta) \tag{29}$$

The following proposition collects the findings.

Proposition 1 *The unique equilibrium in the date 1 game is a threshold equilibrium indexed by θ_s^*. If the realization of θ falls below this threshold then there is a bank run. Otherwise, no bank run occurs. The values of θ_s^* satisfy a system of equations, one for each state $s \in S$.*

$$\theta_s^* = \sum_{i \in I} \frac{w(s,\rho_i)}{e(s)} \hat{D}(\mu(s), \rho_i, \theta_s^*)$$

The term $\hat{D}(\mu,\rho,\theta)$ has the following comparative statics

$$\frac{\partial \hat{D}(\mu,\rho,\theta)}{\partial \mu} < 0, \quad \frac{\partial \hat{D}(\mu,\rho,\theta)}{\partial \rho} > 0, \quad 0 < \frac{\partial \hat{D}(\mu,\rho,\theta)}{\partial \theta} < 1$$

This proposition contains all the knowledge about the probability of a bank run that can be gained purely from the bank run stage of the game. States with higher μ will tend to decrease the bank run index. States in which risk averse agents are weighted stronger will tend to increase the bank run index. The next section closes the analysis by providing the part played by equilibrium risk-sharing that is missing from the analysis so far.

4.2 Risk Sharing

4.2.1 Indirect Utility

To study risk-sharing the indirect utility function involving equilibrium behavior at date 1 has to be constructed. Before observing x, an agents values a payoff of $w(s,\rho)$ at date 1 by correctly anticipating the distribution of π given the equilibrium value of θ_s^*, and using the knowledge of the distribution over θ. Indirect utility is the expectation with respect to the distribution of π given θ_s^* and the distribution of θ.

$$\begin{aligned} u^1(s,\rho) = E_{(\pi,\theta)} \Bigg[\frac{1}{1-\rho} \Big\{ & \pi \left[Rw(s,\rho) - (R-1)d(s,\pi,\rho) \right]^{1-\rho} \\ & + (1-\pi) \left[fw(s,\rho) + (1-f)d(s,\pi,\rho) \right]^{1-\rho} \Big\} \Big| \theta_s^* \Bigg] \end{aligned} \tag{30}$$

The result in Lemma 1 states that $d(s, \pi, \rho)$ is linear in wealth. Since wealth is predetermined and therefore not dependent on π, it can be taken out of the expectation. As a consequence, indirect utility has the familiar CRRA form modified by a multiplicative constant.

$$u^1(s, \rho_i) = V(s, \rho)\frac{w(s, \rho)^{1-\rho}}{1-\rho} \tag{31}$$

The multiplicative constant is both state- and type-dependent. It depends on the type through the function $\delta(\pi, \rho)$ and the exponent ρ. It depends on the state through $\mu(s)$ and the threshold θ_s^*, but not through $e(s)$.

$$V(s, \rho) = E_{(\pi,\theta)}\left[\pi\left[R - (R-1)\delta(\pi, \rho)\right]^{1-\rho} + (1-\pi)\left[f + (1-f)\delta(\pi, \rho)\right]^{1-\rho}\middle|\theta_s^*\right] \tag{32}$$

Ex-ante indirect utility is the expectation of indirect utility over all possible states $s \in S$.

$$U^0(\rho) = \int_S p(s)V(s, \rho)\frac{w(s, \rho)^{1-\rho}}{1-\rho}ds \tag{33}$$

Agents decide on their risk-sharing behavior by maximizing this *ex-ante* indirect utility. They take both $\mu(s)$ and θ_s^* as given when doing so. Therefore, they take $V(s, \rho)$ as given. They are limited in their choices of $w(s, \rho_i)$ by the budget constraint, which is common to all types. Letting $q(s)$ stand for the state-prices and recalling that types share equally in the aggregate endowment $e_i(s) = \frac{e(s)}{\#I}$, the budget constraint is given by

$$\int_S q(s)w(s, \rho)ds \leq \int_S q(s)\frac{e(s)}{\#I}ds \tag{34}$$

4.2.2 Equilibrium wealth distribution

The characteristics of an equilibrium allocation are well known for the special case in which the term $V(s, \rho)$ is absent. The solution to the risk-sharing problem in this case is that risk averse agents carry wealth predominantly into good states and less risk averse agents carry wealth into bad states.

These features of an equilibrium allocation are true in the case with $V(s,\rho)$, as well. The reason is that, although $V(s,\rho)$ works as a shifter of marginal utility, the curvature of the utility function is not affected. In fact, the way in which marginal utility is shifted exacerbates the features of an equilibrium allocation.

Proposition 2 states the main result of this section. In this section, I discuss the intuition behind the result. A thourough treatment is performed in Appendix A.

Proposition 2 *The cross sectional composition of wealth holdings changes across states. In states with low aggregate endowment wealth is predominantly in the hands of the most risk averse population.*

There are two steps in the derivation of the result in Proposition 2. First I prove that the allocation in an equilibrium must be constrained efficient. Then, using the First Welfare Theorem, I characterize equilibrium allocations as a function of the state s. This second step shows that the aggregate endowment $e(s)$ has an effect on the cross-sectional distribution of wealth that is enhanced by the presence of $V(s,\rho)$.

To understand why the existence of the term $V(s,\rho)$ enhances risk-sharing behavior in the second step, it is useful to study how the presence of $V(s,\rho)$ modifies the usual risk-sharing problem. The expression $V(s,\rho)$ represents the value by which utility in state s is modified because of the availability of the risky investment. Inspection of the functional form of $V(s,\rho)$ reveals that $V(s,\rho)$ is greater than 1 for types who are less risk averse than logarithmic and less than 1 for types who are more risk averse. The availability of this investment increases utility for all types.[4] Also, states with a higher expected quality of assets increase the value of the risky investment, which implies that $V(s,\rho)$ is increasing in $\mu(s)$ if $\rho < 1$ and decreasing if $\rho > 1$. By the same token, $V(s,\rho)$ is decreasing or increasing in θ_s^* for $\rho < 1$ and $\rho > 1$, respectively.

[4] Since agents are allowed to opt out of the bank and use the safe storage technology, they can never be worse off. On the contrary, they must be strictly better off because they voluntarily choose to leave money in the bank for a sufficiently high belief of success. Utility is negative for types with $\rho < 1$. Therefore, $V(s,\rho) < 1$ represents an increase in utility for these types.

Next, turn to how the presence of $V(s,\rho)$ affects risk-sharing. Denote the Lagrange multiplier for the budget constraint as λ_ρ^B, then, an (interior) optimal choice of $w(s,\rho)$ requires

$$p(s)V(s,\rho)w(s,\rho)^{-\rho} = \lambda_\rho^B q(s) \quad \forall s \in S, \rho \in \rho(I) \tag{35}$$

Now, ask what happens if a bad shock hits. A bad shock will lower $\mu(s)$ and in equilibrium it will raise $q(s)$ and θ_s^*. The return of investing in the bank worsens. There are two effects. On the one hand, agents have a desire to substitute away from bank deposits and towards the safe storage investment. On the other hand, they want to smooth consumption. This second effect is stronger for agents with a higher risk aversion, since risk aversion is the inverse of the elasticity of substitution between states.[5]

An agent will optimally increase of decrease wealth in a bad state depending on which effect dominates. Clearly, the desire to smooth is strongest for more risk averse types. The cutoff in risk aversion for which an agent chooses to carry more wealth into bad states is $\rho = 1$. For a type with $\rho < 1$, a bad state (a fall in $\mu(s)$ and an increase in θ_s^*) will lower $V(s,\rho)$. This agent will carry less into the bad state than originally planned. For an agent with $\rho > 1$ the effect is the opposite. Even though the value of deposits worsens, the agents wishes to carry more wealth into bad states.

By raising wealth in low endowment states for risk averse agents and lowering it in good states for these same agents, the presence of $V(s,\rho)$ adds to the original risk-sharing behavior, and thus reinforces it.

4.3 Putting it all together

The *ex-ante* probability of a bank run is equal to the probability that θ lies below the threshold θ_s^*. Since θ has a Normal distribution with mean $\mu(s)$ and standard deviation σ_θ, this

[5]The inverse of relative risk aversion is sometimes called relative risk tolerance. This name is used, for example, in the literature following Wilson (1968).

probability is

$$\psi(s) = \Phi\left(\frac{\theta_s^* - \mu(s)}{\sigma_\theta}\right) \tag{36}$$

where $\Phi(\cdot)$ is the standard normal CDF. Mathematically, the probability of a bank run changes with the state according to

$$\frac{d\psi(s)}{ds} = -\frac{1}{\sigma_\theta}\phi\left(\frac{\theta_s^* - \mu(s)}{\sigma_\theta}\right)\frac{d\mu(s)}{ds} + \frac{1}{\sigma_\theta}\phi\left(\frac{\theta_s^* - \mu(s)}{\sigma_\theta}\right)\frac{d\theta_s^*}{ds} \tag{37}$$

The first term arises because the state determines where the distribution is centered through the mean $\mu(s)$. An increase in $\mu(s)$ reduces the probability mass below any given value for θ_s^*. The second term appears because θ_s^*, the threshold value defining the limit between the crisis region and the non-crisis region, shifts as well. The movement in θ_s^* itself is in part due to the change in $\mu(s)$, as a shift in fundamentals increases the expected value of the risky project, and thus lowers θ_s^*. The second reason of why θ_s^* shifts with the state is that $e(s)$ increases. In equilibrium, an increase in the aggregate endowment puts relatively more wealth into the hands of less risk averse types, lowering θ_s^*.

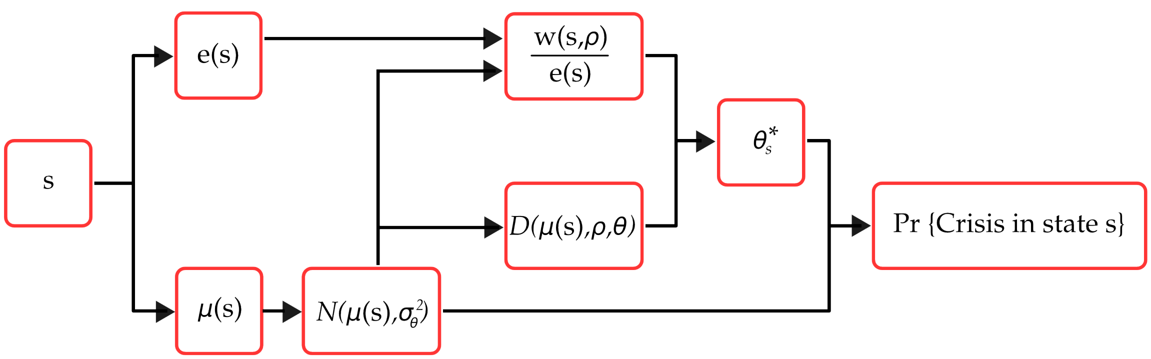

Figure 5: *Relationship between states of nature and the probability of a crisis.*

The diagram in Figure 5 represents the complex relationship between states of nature and the probability of a crisis. Without *ex-ante* heterogeneity, the bank run index θ_s^* would be determined exclusively by the expected quality of assets $\mu(s)$ through the size of withdrawals $D(\mu(s), \cdot)$. The change in probability of a bank run would incorporate the effect of $\mu(s)$ on θ_s^*

and the shift in the distribution through the distribution $N(\mu(s), \sigma_\theta^2)$.

With heterogeneity, the cross sectional distribution of $\frac{w(s,\rho)}{e(s)}$ becomes important. The aggregate endowment affects this distribution. A high level of risk aversion relative to the population average moves an agent to hold relatively more wealth in states with a low endowment. High endowments will produce wealth in the hands of less risk averse agents. The quality of assets variable $\mu(s)$ enhances this risk-sharing motive by affecting the indirect utility derived from equilibrium behavior in the bank run stage.

Fortunately, it is possible to disentangle the roles played by the aggregate endowment and the expected expected profitability of the leaving deposits in the bank.

$$\frac{d\psi(s)}{ds} = \underbrace{-\frac{1}{\sigma_\theta}\phi(\cdot)\frac{d\mu(s)}{ds}}_{\text{Conventional channel}} + \underbrace{\frac{1}{\sigma_\theta}\phi(\cdot)T_1\frac{d\mu(s)}{ds}}_{\text{Information channel}} + \underbrace{\frac{1}{\sigma_\theta}\phi(\cdot)T_2\frac{de(s)}{ds}}_{\text{Risk-sharing channel}} \tag{38}$$

The probability of a bank run reacts to fundamentals through three channels: the *conventional* shift in the distribution, the *information channel* and the *risk-sharing channel.*

Proposition 3 *The informational channel and the risk-sharing channel are both negative and reinforce.*

This result follows directly from Propositions 1 and 2. The formal proof of Proposition 3 is contained in Appendix B. The rest of this section intuitively discusses the result.

4.3.1 Discussion

The effect of information

For a moment, disregard the risk-sharing channel. Then, the probability of a crisis evolves according to

$$-\frac{1}{\sigma_\theta}\phi\left(\frac{\theta_s^* - \mu(s)}{\sigma_\theta}\right)M\frac{d\mu(s)}{ds}, \quad M > 1 \tag{39}$$

This equation can be interpreted as a generalized version of a result in Morris and Shin (2004).[6] It is generalized in two dimensions. If a representative agent is assumed, then the expression simplifies considerably. If, in addition, this representative agent is risk neutral, then the same expression as in Morris and Shin (2004) is obtained.

If the diversity in the population is reduced by assuming the existence of only one type, this effect can be shown to be smaller. The reason is that a negative term in the denominator of M vanishes.

The risk-sharing channel

This channel has a negative effect on the slope of the probability. This effect is stronger in bad states and and weaker in good states. The intuition for the overall negative effect is that good states are those in which the more risk-averse segment of the population chooses to hold very little of their wealth. Since destabilizing behavior is increasing in the level of risk-aversion, the probability decreases with the state.

4.3.2 Illustration of both channels at work

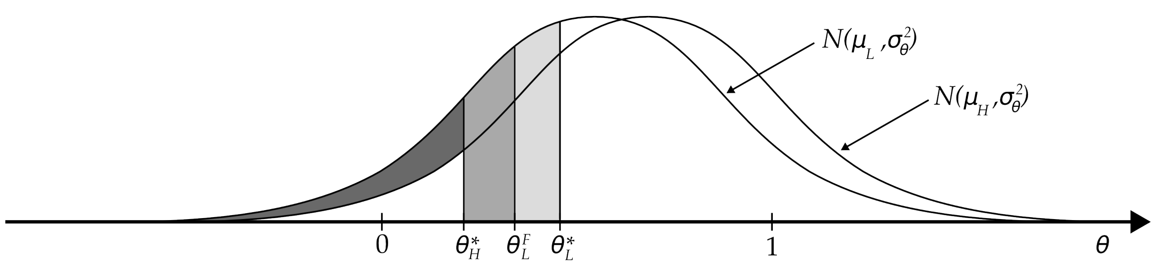

Figure 6: *The graph shows the difference in the probability of a crisis between a state with a high (H) and a low (L) aggregate endowment. The region between the two tails and with the darkest shade corresponds to the change in probability of a crisis that is due purely to the shift in the prior distribution over "strength". The second darkest area corresponds to the change in the probability due to a movement in $\mu(s)$. The lightest shade measures the change in probability due to the risk-sharing channel.*

Figure 6 decomposes the movement of the bank run index θ_s^*, and the consequent change in the probability of a crisis, into the components described in the diagram of Figure 5. The figure

[6](Morris and Shin, 2004, equation (17))

is constructed for two states. A good state with a high aggregate endowment (H) and a bad state with a low aggregate endowment (L). Expected quality of assets is μ_H or μ_L, respectively. Consider what happens when the economy moves from the good state into the bad state. Even if the threshold value did not move and remained fixed at θ^*_H, there would be an increase in the probability of the bank run simply because the *ex-ante* distribution of quality shifts to the left. This difference in probability is the distance between the tails for the two distributions

The movement from μ_H to μ_L increases the fraction of wealth that each type of agent withdraws from the bank. Without considering the change in the distribution of $\frac{w(s,\rho)}{e(s)}$ across types, this by itself raises the bank run index to the point θ^F_s. The probability of a crisis increases by the area below the distribution and between θ^*_H and θ^F_L.

The remaining movement, from θ^F_L to θ^*_L, is produced by the change in the cross-sectional distribution of $\frac{w(s,\rho)}{e(s)}$. Moving to a lower state means moving to a lower aggregate endowment. More risk averse, choose to carry more wealth into those states, and their safe choice of withdrawing is therefore weighted more heavily. The bank run index increases by an additional amount. The probability increase due to the risk-sharing channel is the area below the density and between θ^F_L to θ^*_L.

5 Implications

5.1 Bankarization

Several countries pursue active policies to integrate the unbanked population into the banking system. In the case of developing countries, where sometimes more than 50% of the population is unbanked, "bankarization" is seen as aiding development. The reason is that having a checking account is the first step towards financial literacy and access to credit necessary for an efficient allocation of credit.[7]

[7] Claessens (2005) and Beck and de la Torre (2006) provide good expositions of the issues involved. The actual implementation of such policies in Latin America is discussed in de la Torre, Gozzi, and Schmukler (2007).

Even developed countries, like the US pursue policies aimed at providing subsidized bank accounts to the unbanked. In 1999, under the Clinton administration, the US Treasury started offering subsidies for bank to set up inexpensive accounts for Social Security receivers. These accounts are called Electronic Transfer Accounts (ETA). According to Claessens (2005), "the switch would benefit the government as supplier (lower costs), but in the end could also help the recipient by giving him access to financial services".

My model implies that adding agents to the banking system may have redistributive consequences that need to be taken into account in the cost benefit analysis of such policies.

If the unbanked population differs from the banked population with respect to risk aversion, there is an externality that must be taken into account when in a cost-benefit analysis of a policy promoting wider access to banking services. Consider, for example, the case in which the unbanked are more risk averse than the rest of the population. In this case, introducing them as bank clients increases the probability of a bank run. This has a negative welfare effect on the less risk averse individuals, who now face a banking system with a higher probability of bank runs. If the population entering were less risk averse, the result would be reversed. The probability of bank runs would decrease.

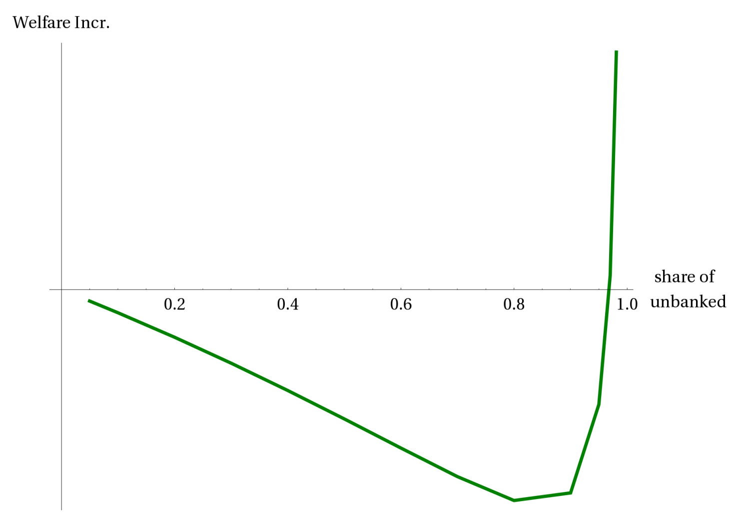

Figure 7: *Percentage change in the welfare of the risk neutral type as a function of the size of the risk averse group introduced as depositors of the bank.*

Figure 7 illustrates the welfare for the risk neutral type of the example in Section 3. It graphs the change in welfare for a risk neutral type when a risk averse type is introduced, as a function of the size of this group. In this example, although welfare decreases for risk neutral agents, it increases for the group entering the banking system.

My model implies that, in general, a "bankarization" policy will benefit a subset of the population but harm another.[8] Evaluating the desirability of such a policy must deal with this redistributive effect.

5.2 Effect of the aggregate endowment on the probability of a bank run

In the model, the aggregate endowment has a larger effect on the probability of a bank run. Since wealth is determined endogenously, a low aggregate endowment state has most aggregate wealth owned by the most risk averse segment of the population, who are more propense to run on the bank. Even less risk averse agents, realizing that the risk averse will withdraw their deposits, update their beliefs about a bank run accordingly, and switch to a running strategy for a larger set of fundamentals.

Figure 8 is qualitatively different from what would be the case in a representative agent model. The response of the probability of a bank run to the aggregate endowment is larger than that of any single type. The reason for this is clear. The threshold value of fundamentals underlying the probability of a bank run is a weighted average of the thresholds that would prevail if any single type were alone in the economy. The wealth shares weigh the most risk averse agents more for low endowments and less for high endowments. The resulting probability looks more similar to the one by the more risk averse types for low endowments and more like less risk averse for high endowments.

Consider the case in which a researcher observes the complete set of prices of state-contingent claims. Even though these prices incorporate the true probabilities of a bank run, the estimation does not allow for heterogeneity (for example by estimating a unique risk aversion

[8] The only exception is when the unbanked are less risk averse than the rest of the population.

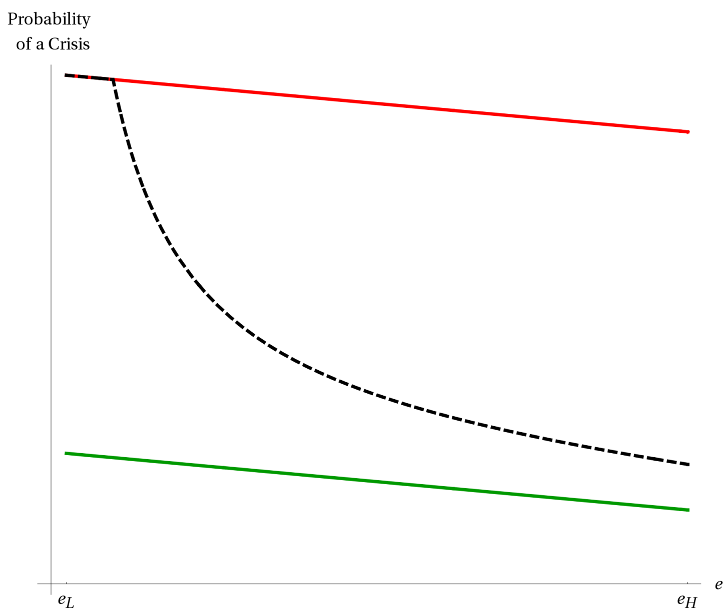

Figure 8: *Probability of a bank run as a function of the aggregate endowment. The solid red line is drawn for a population consisting of a single type with $\rho = 1$. The solid green line is drawn for a population consisting of a single risk neutral type ($\rho = 0$). The dashed line is the probability of a bank run in the economy where both of these types are present.*

parameter), then the resulting estimate is biased in a certain way: it underestimates the effect of the aggregate endowment on the probability of a crisis. The size of this bias depends on the degree of heterogeneity in the population.

Consider the case in which a researcher observes the complete set of prices of state-contingent claims. Prices incorporate the true probabilities of a bank run. However, if this hypothetical researcher attempts to either calibrate or estimate the relative risk aversion of of a representative CRRA utility function from this data in order to predict the likelihood of a crisis, the resulting estimate would underestomate the influence of the business cycle. The picture for the resulting estimate of the relationship between the aggregate endowment and the probability of a crisis would be as shallow as any of the two curves drawn for a single agent in Figure 8, when the true probability follows the dashed line in the figure. The representative agent model would do a decent job in explaining the average probability of a bank run, but would do far worse with respect to changes in this probability as the underlying state of the economy changes.

Policies based on this representative agent model would be off target by underestimating the importance of the state of the economy for the likelihood of a crisis.

Consider now, the converse problem. Imagine that the economist has directly observed data on the probability of a crisis, or has a proxy for it, and wishes to use this data to test the hypothesis that agents have CRRA preferences. The economist would reject the hypothesis of CRRA preferences because of the failure of the model to fit the data. This is because the variation in the probability of a bank run would be attributed in their entirety to changes in individual behavior and not to changes in which individual was more important in driving the bank run.

5.3 The value of deposit insurance

Heterogeneity and risk-sharing affect how valuable deposit insurance is to society. In the previous section I have shown how the response of the probability of bank run to the business cycle is underestimated if heterogeneity is neglected. This error is compounded if a single risk aversion parameter is used to calculate the value of deposit insurance. The reason is that when depositors differ in their risk aversion and risk-sharing occurs, the "average" level of risk aversion that is obtained is overly sensitive to low values of risk aversion. Even the existence of a few close to risk neutral agents will introduce a sizable error in the calculation of how valuable deposit insurance is to society.

I model deposit insurance as the absence of bank failures. In other words, the bank always pays R, regardless of the level of withdrawals. Obviously, no withdrawals will occur in equilibrium in this case. For each type of agent it is possible to calculate the percentage of consumption loss that makes type i indifferent between deposit insurance and no deposit insurance. Denote σ_i as the loss that equates utility in an equilibrium under a deposit insurance regime to utility in an equilibrium in which bank runs may occur.

$$E\left[u_i\left((1-\sigma_i)c_i^{DI}\right)\right] = E\left[u_i(c_i^{BR})\right] \tag{40}$$

In order to provide an example calculation of the value of deposit insurance, I use the following parameterization. I consider two equally likely states, with $e_L = 1$ and $e_H = 2$. I take $R = 1.05$ and $f = 0.8$. Incidentally, the value of f is the average loss to depositors during the US banking panics during the Great Depression. The distribution of the variable θ is chosen in such a way that the probability of a bank run is on average 5% in the low state and effectively zero in the good state. I consider two types of agents. One type has $\rho = 2$. For the other type I vary ρ between 2 and 8.

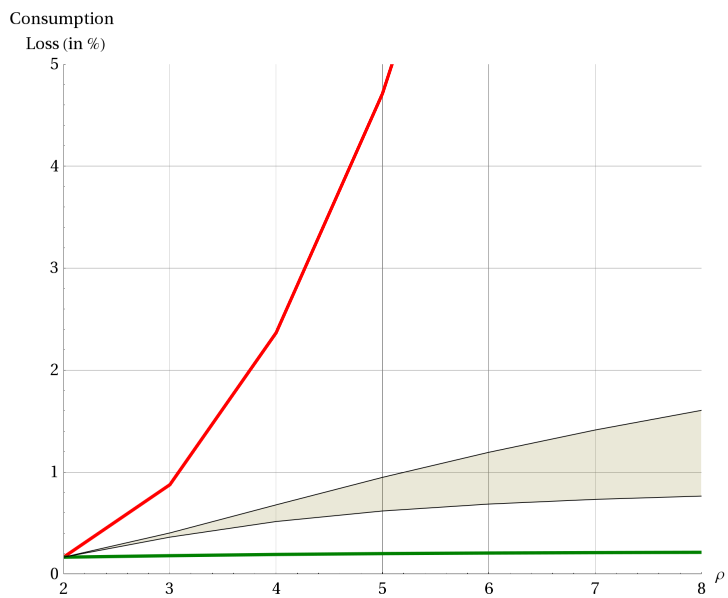

Figure 9: *Equivalent consumption loss that makes a depositor indifferent between living in an economy with deposit insurance and no deposit insurance.*

Figure 9 displays the loss in consumption that depositors would be willing to give up in order to enjoy the benefit of deposit insurance. The upper solid line in the figure is the relative consumption that the most risk averse agent would be willing to give up. The nearly horizontal solid line on the bottom is this same measure for the less risk averse type. By definition, the two curves intersect when both agents have a risk aversion of 2. The figure shows that the two values diverge as soon as some heterogeneity is introduced. If the prices of state-contingent securities are used to infer the (state-dependent) relative risk aversion of a hypothetical representative agent, then the valuation of deposit insurance by a homogeneous population with a risk aversion level equal to this hypothetical representative agent will be in the shaded are in Figure 9. The upper bound of the shaded area corresponds to the highest level of risk aversion that could be inferred for the representative consumer. The lower bound corresponds to the lowest level of risk aversion.

The calculation for a homogeneous populations is closer to the less risk averse agent. The reason is that the relative risk aversion of the representative consumer is calculated as a weighted harmonic mean of the levels of risk aversion of the two types (as shown by Wilson (1968)). The harmonic mean is very sensitive to low values.

5.4 Probability of a crisis vs. its magnitude

In my model, the threshold θ_s^* varies with the aggregate endowment. Guimarães (2007) devises a test of whether it is sunspots of thresholds that drive currency crises. The magnitude of θ_s^* is tied to the magnitude of a devaluation while the probability $\psi(s)$ is the probability of a devaluation. Standard global games have a threshold which is constant with respect to the aggregate endowment since there is no redistribution of wealth in the population. This causes the standard global game to predict a constant expected size of a devaluation, which is at odds with Brazilian data for the period leading up to that country's late-nineties devaluation of the *real*.

A model with heterogeneous risk aversion, like this one, but applied to currency crises, would fare better in the test performed by Guimarães (2007). Both the probability of a devaluation and the expected size of the devaluation would increase with bad economic fundamentals.

5.5 Extensions

Ever since Diamond and Dybvig (1983), bank runs are the archetypical story of a self-fulfilling crisis. This is why I use the bank run as the leading example in this paper. Some recent research breaks the multiplicity and is therefore closer to my model. See, for example, Rochet and Vives (2004) and Goldstein and Pauzner (2005).

There are, however, alternative interpretations of the model.

In the currency crisis story, the risky investment is equivalent to holding domestic currency; the safe asset corresponds to buying foreign currency. The "strength" variable can be thought of as a combination of the central bank's reserves and the willingness to defend a fixed exchange rate. If, relative to reserves and willingness to defend a fixed exchange rate, too many pesos are converted into foreign currency, a devaluation occurs. This is the version of the model that popularized the global games literature in Morris and Shin (1998).

For an (illiquidity-based) debt crisis, one can think of the risky investment as rolling over debt and the safe asset as requiring immediate payment or cashing collateral. The "strength" variable is the debtor's liquidity. If the fraction of debt that is rolled over is too low, a default occurs. This interpretation is taken up in Morris and Shin (2004).

6 Conclusion

This paper studies the effect of heterogeneity in risk aversion on the probability of a bank run. I add *ex-ante* heterogeneity to a coordination problem which represents the game played by depositors during a bank run. With a heterogeneous population, the cross-sectional distribution of wealth is a key factor in driving a bank run. Rather than taking the wealth distribution as exogenous, I endogenize it by allowing agents to participate in trade of state-contingent claims to insure against aggregate endowment shocks.

I find that heterogeneity in risk aversion increases the effect of the state of the economy on the probability of a bank run. The reason is that the endogenous determination of wealth acts as an amplification mechanism by putting more wealth in the hands of the most risk averse during economic downturns.

The model has implications for policy questions. The presence of risk-sharing in real life undermines the applicability of a representative agent framework for studying bank runs. A model that abstracts from wealth shifts will systematically underestimate the effect of the business cycle on the probability of bank runs. Thus, policies aimed at preventing bank runs that are tested in artificial economies inhabited by a representative agent will yield erroneous conclusions.

My model implies that there is an externality that must be taken into account when performing a cost-benefit analysis of a policy promoting wider access to banking services. In the discussion of the convenience of such policies, the costs are limited to the financial cost of setting up the program. If the target population is more risk averse than the banked population, then welfare of existing depositors is reduced by the ensuing greater instability of the banking system. Thus, it must be taken into account in the analysis on the desirability of "bankarization" policies. On the other hand, a less risk averse target population produces a positive externality by making the banking system safer.

Appendix

Appendix A and Appendix B are available online at `http://rolf.bol.ucla.edu/papers/jobmarket.pdf`.

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