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Abstract

This paper considers the welfare effect of introducing a liquidity-saving mechanism (LSM) in a real-time gross settlement (RTGS) payment system. We study the planner's problem to get a better understanding of the economic role of an LSM and find that an LSM can achieve the planner's allocation for some parameter values. The planner's allocation cannot happen without an LSM, as long as some payments can be delayed without cost. We show that, in equilibrium with an LSM, there can be either too few or too many payments settled early compared with the planner's allocation, depending on the parameter values. Using Fedwire data to calibrate our model, we describe the equilibrium that would arise with an LSM and compare welfare with and without an LSM. Our results suggest that introducing an LSM could have significant benefits.

Key words: liquidity-saving mechanism, real-time gross settlement, large-value payment system

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1 Introduction

This paper studies the planner's solution of a model of a large value payment system and compares it with the equilibrium allocation(s). We also use Fedwire data to calibrate the model and measure the welfare benefits of a liquidity-saving mechanism. Studying the planner's problem allows us to deepen our understanding of the economic role of liquidity-saving mechanisms. Our calibration is the first attempt to quantify the possible welfare effects of alternative designs of a large value payment system.

In contrast to the standard theory of inter-temporal trade, where agents borrow and lend at a market price that they take as given, there is typically no market for short-term (intraday) credit in interbank settlement systems. Instead, under typical arrangements, banks can borrow reserves from the central bank at a fixed price.

Concerns about systemic and operational risk have made managing the flow of interbank payments an important concern of central banks in the last 25 years. Over this period, most central banks have abandoned settlement systems based on the netting of interbank claims and replaced them with systems where all payments are settled on a gross basis (see Bech and Hobijn (2007)). Net settlement systems were viewed as representing too big a risk of cascades of defaults or unexpected settlement demands on banks. Real-time gross settlement systems, also known as RTGS, have reduced the risk of cascades of defaults, but at the cost of an increased need for liquidity and an incentive for banks to delay their payments. In recent years, a number of countries have introduced, or are planning to introduce, liquidity-saving mechanisms (LSMs), as a complement to their RTGS systems.[1] LSMs aim to combine the strengths of net settlement and RTGS systems. The Federal Reserve is also studying the possibility of implementing an LSM for Fedwire, its large value payment system.

In an RTGS system, banks have the choice between sending a payment early or delaying the payment. Banks are required to hold positive reserves but can borrow reserves from the central bank. Banks have an incentive to delay their payment if they expect the cost of borrowing reserves to exceed the cost of delaying their payment. A bank's expected cost of sending a payment early increases as other banks delay more of their payments. This can

[1] For example, the ECB launched TARGET 2 in November 2007 and the Bank of Japan is planning to introduce an LSM in October 2008.

lead to undesirable surges of payments late in the day.

With an LSM, banks still have the option to send or delay a payment. In addition, banks can put a payment in a queue. The payment will be released from the queue according to some pre-specified rules. Typically, a payment is released from the queue if the bank has sufficient reserves in its account or if the payment is part of a bilaterally or multilaterally offsetting group of payments in the queue.

We first study the planner's allocation, with or without an LSM, in the model of Martin and McAndrews (2008). In that model, all banks are ex-ante identical. Banks receive shocks at the beginning of the economy and can be of six different types. Banks can either have to make a time-critical payment, which is costly to delay, or a non-time-critical payment, which can be delayed at no cost. In addition, banks can receive a positive, a negative, or no liquidity shock. We assume that the planner can choose the action of banks; delay or send in the case of RTGS, delay, queue, or send in the case of an LSM.

The planner maximizes the ex-ante welfare of all banks. In particular, the planner can direct banks to take a feasible action even if, ex-post, it is not in the best interest of that particular bank given its type. We can think of the planner as having a commitment technology. The planner can achieve allocations to which banks would like to commit. The planner knows the distribution of payments in the economy but does not know the identity or the type of the bank that will receive a given payment. Hence, the planner cannot choose the action of a sending bank conditional on the type or the identity of the bank that will receive the payment.

Unless liquidity shocks are large, the planner chooses the same allocation whether a LSM is available or not. With small liquidity shocks, the planner wants all banks to send their payments early. This shows that the ability to commit could make an LSM inessential. However, when liquidity shocks are large, the planner is able to achieve higher welfare with an LSM than without. Here, the LSM goes beyond the commitment technology by helping the planner find receiving banks which can reciprocate in delivering payments.

For some parameter values, the planner's allocation, where all banks make their payment early, can be achieved in equilibrium with an LSM. However, this cannot happen with an RTGS system only, provided some payments can be delayed without cost. With RTGS, banks always delay their non-time-critical payments.

When liquidity shocks are large, the planner can improve welfare by making banks with a negative liquidity shock delay their payments. Some of these banks will receive a payment which will offset the liquidity shock and reduce their borrowing cost. When some banks delay, an LSM can help the planner by allowing the release of some payments conditional on the receipt of an offsetting payment. This conditionality cannot be achieved with an RTGS system and has value in addition to the kind of commitment technology that is available to the planner.

These results show how the conditionality that an LSM provides can partially substitute for commitment, when commitment is unavailable, and can complement the commitment, when commitment is available. In addition, an LSM can improve welfare by offsetting payments inside the queue.

In the second part of the paper, we calibrate the model using Fedwire data. This allows us to provide some quantitative measure of the benefits of an LSM in the case of the US. Our data suggest that the size of liquidity shocks on Fedwire are relatively small. The benefits of an LSM are large in that region of the parameter space. Hence our calibration suggests that implementing an LSM for Fedwire would have important welfare benefits. Our calibration also allows us to describe the change in the timing of settlement that could be expected from implementing an LSM.

The remainder of the paper proceeds as follows. Section 2 presents some descriptive material concerning LSMs and discusses the related literature. Section 3 introduces the environment. Section 4 describes the planner's problem. Section 5 characterizes the solution to the planner's problem when an LSM is not available, while section 6 characterizes the solution when an LSM is available. Section 7 describes our calibration and section 8 concludes.

2 RTGS systems and liquidity-saving mechanisms

Modern banking systems use large-value transfer systems to settle payment obligations of commercial banks. The payment obligations can represent obligations of bank customers or obligations of the commercial banks themselves. Among the bank's own payments one can note three types. First, a bank often uses an RTGS for the return and delivery of money market loans. Second are payments to a special-purpose settlement system, such as a securities settlement system or a foreign-exchange settlement system.[2] For example, in the U.S. banks use the Federal Reserve System's RTGS system (Fedwire©) to make payments into and to receive payouts from CLS Bank, a special-purpose bank that settles foreign-exchange trades on its books. Finally, other types of payments made by banks on an RTGS include progress payments under a derivatives contract with another party and payments made on behalf of bank customers. The amounts of some of these payments may not be precisely known by the bank at the start of the business day.

Both customer-initiated payments and a bank's own payments may or may not be time-critical. Consider a payment to settle a real estate transaction of a customer, for which many people are gathered in a closing or settlement meeting. The customer's demand for the payment is highly time-critical. The considerations outlined in this paragraph and the previous one motivate the assumption that banks are subject to liquidity shocks and may have to make time-critical payments.

liquidity-saving mechanisms to be used in conjunction with RTGS systems are a fairly recent phenomenon.[3] At least in part, LSMs are one way to attempt to reduce the demands for liquidity in the RTGS system, while maintaining the flexibility to make timely payments.

There are many possible design alternatives for an LSM, but some features are common among all such LSMs. An LSM offers to the bank participating in the payment system two alternatives by which to submit payment orders. The first alternative (sometimes called the "express" route) is to submit the payment order for immediate settlement as though the system were a plain RTGS system. The second alternative is to submit payment orders to the LSM–a queue in which the payment order remains pending some event that will release the payment (this route is sometimes called the "limit order" route).

The types of events that could trigger the release of payment orders from the queue would

[2] In the U.S., many banks make payments into three private-sector special-purpose payment and settlement systems: CHIPS, CLS Bank, and the Depository Trust Company (DTC).

[3] See McAndrews and Trundle (2001) and Bank for International Settlements (2005) for a review and extensive descriptive material on LSMs.

be the arrival into the bank's account of sufficient funds so that the bank's balance rises above some threshold, or the appearance in another bank's queue of an offsetting payment, or the receipt by the bank of a payment equal in size to the pending payment order. In all these cases, the release of the payment order in the LSM queue is contingent on some state of the world. An LSM offers a new alternative, not available in RTGS, to make the settlement of payments state contingent in a particular way.

2.1 Relevant literature

Several papers examine the theoretical behavior in RTGS systems. Angelini (1998, 2000) considers the behavior of banks in an RTGS systems in which they face delay costs for payments as well as costly borrowing of funds. He shows that the equilibria of RTGS systems involve excessive delay of payments, as banks don't properly internalize the benefits to banks from the receipt of funds. Bech and Garratt (2003) carefully specify a game-theoretic environment in which they find that RTGS systems can be characterized by multiple equilibria, some of which can involve excessive delay. Mills and Nesmith (2008) study an environment similar to the one in this paper. Their approach is complementary to ours as they focus on the effect of risk, fees, and other factors on the incentives of banks to sent their payments early, or delay, in RTGS systems without LSMs.

Some recent work studying LSMs includes Roberds (1999), who compares gross and net payment systems with systems offering an LSM. He examines the incentives participants have to engage in risk-taking behavior in the different systems. Kahn and Roberds (2001) consider the benefits of coordination from an LSM in the case of CLS. Willison (2005) examines the behavior of participants in an LSM. Martin and McAndrews (2008 b) study two different designs for an LSM. Our paper uses the framework first presented in Martin and McAndrews (2008 a). They derive the equilibrium allocation that we use in this paper.

3 The environment

The environment is similar to the one in Martin and McAndrews (2008 a). The economy is populated by a continuum of mass 1 of risk neutral agents. These agents are called payment system participants or banks. There is also a nonstrategic agent, which is identified with settlement institutions.[4]

The economy lasts two periods, morning and afternoon. Each bank makes two payments and receives two payments each day. One payment is sent to another bank and is called the core payment. The other payment is sent to the nonstrategic agent and affects the bank's liquidity shock. Similarly, one payment is received from another bank and one is received from the nonstrategic agent. Core payments have size μ, while payments to and from the nonstrategic agent have size $1-\mu$. It is assumed that $\mu \geq \frac{1}{2}$.

Three factors influence the banks' payoff of sending, queuing, or delaying their core payment. First, banks must pay a cost to borrow from the central bank. Second, banks may need to send a time-critical payment. Third, banks may receive a liquidity shock.

Each bank starts the day with zero reserves. Reserves can be borrowed from the central bank at an interest cost of R.[5] In our welfare analysis, we think of R as representing both the private and the social cost of borrowing reserves. Banks that receive more payments than they send in the morning have excess reserves. It is assumed that these reserves cannot be lent to other banks so that banks receive no benefit from excess reserves. Payments received and sent in the same period offset each other. Hence, a bank only needs to borrow from the central bank if the payments it sends in the morning exceed the payments it receives in the morning.

Banks learn in the morning whether the payment they must make to another bank is time-critical. The probability that a payment is time-critical is denoted by θ.[6] If a bank fails to make a time-critical payment in the morning a cost γ is incurred. Delaying non-time-critical payments until the afternoon has no cost. Banks choose whether to send the payment in the morning before they know if they will receive a payment from another bank in the morning. Banks form rational expectations about the probability of receiving a payment from some other bank in the morning. Let π denote this expectation.

[4] One can think of the nonstrategic agent as aggregating several distinct institutions such as the CLS bank, CHIPS, and DTC, as well as the payment side of securities transactions.

[5] Evidence discussed in Mills and Nesmith (2008) suggests that the cost of intraday reserves can influence banks payment behavior.

[6] Throughout the paper it is assumed that if x represents the probability of an event occurring for a bank, then the fraction of banks for whom this event occurs is x as well. Hence, a fraction θ of banks must make a time-critical payment.

Banks learn in the morning when they receive a payment from the nonstrategic agent and when they must send an offsetting payment. The probability of receiving the payment in the morning is $\bar{\pi}$ and so is the probability of having to send the payment in the morning. Both events are independent. Payments to the nonstrategic agent cannot be delayed. Let $\sigma \equiv \bar{\pi}(1-\bar{\pi})$. A fraction σ of banks receive a payment from the nonstrategic agent in the morning and do not need to make a payment until the afternoon. These banks receive a positive liquidity shock. A fraction σ of banks must make a payment to the nonstrategic agent in the morning and do not receive an offsetting payment until the afternoon. These banks receive a negative liquidity shock. The remaining banks, a fraction $1-2\sigma$, make and receive a payment from the strategic agent in the same period, either in the morning or in the afternoon. These banks do not receive a liquidity shock.

Banks can be of six different types. A bank may receive a positive, a negative, or no liquidity shock. In addition, banks may or may not have to make a time-critical payment. The following table contains the definition of all parameters.

$\mu \in [0.5, 1]$	Size of payment to other banks
$R > 0$	Cost of borrowing
$\theta \in [0, 1]$	Probability of having to make a time-critical payment
$\gamma > 0$	Cost of delay
$\sigma \in [0, 0.25]$	Probability of a liquidity shock

Table 1: Parameters of the model

The timing of events in summarized in figure 1. First, nature chooses the banks that receive a liquidity shock and the banks that must make time-critical payments. Next, morning payments to and from the nonstrategic agent are made and banks can choose to send their payment to another bank, delay their payment, or–if an LSM is available–put their payment in queue. At the end of the morning period, banks that must borrow from the central bank incur a borrowing cost while banks that did not send time-critical payments incur a delay cost. All remaining payments are made in the afternoon.

The role played by the different frictions in the model can be summarizes as follows. The cost of borrowing provides an incentive to bunch payments. Indeed, absent liquidity

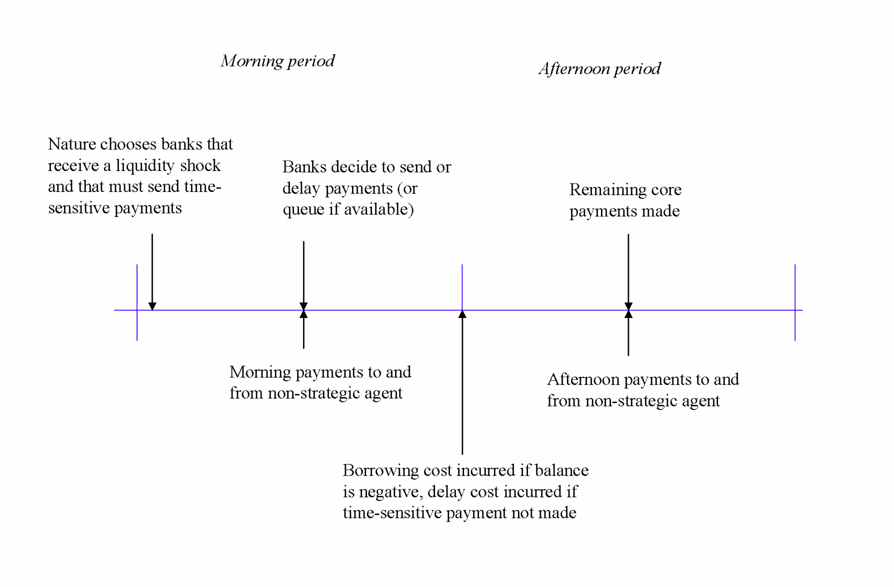

Figure 1: Timeline of events.

shocks banks could avoid borrowing if either all payments are sent in the morning or all payments are delayed. Time-critical payments provide an incentive for some banks to send their payment early in order to avoid the delay cost. In contrast, banks have an incentive to delay their payment to avoid having to borrow from the central bank.

3.1 The settlement system design

We consider two types of settlement system designs: Real-time gross settlement (RTGS) and an RTGS augmented by a liquidity-saving mechanism (LSM). We consider the settlement system as part of the physical environment.

With an RTGS system, banks have the choice between sending their core payment in the morning or delaying that payment until the afternoon. With an LSM banks get an additional option. Banks can put their payment in a queue and the payment will be settled under some pre-specified conditions.

With an LSM, the number of payments settled depends on the underlying pattern of those payments. This pattern affects the number of payments that are released from the queue. In this paper, we consider two cases, illustrated by figure 2.

In the left panel of figure 2, all payments form a unique cycle so that any proper subset of payments in the queue cannot be multilaterally offsetting. With this pattern of payments,

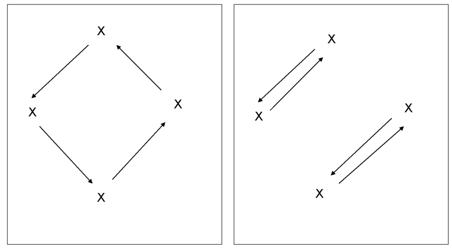

Figure 2: Pattern of payments. Left panel: one cycle. Right panel: multiple unique cycles.

no offsetting occurs in the queue unless all payments are queued. In the right panel of figure 2, payments are matched in bilaterally offsetting pairs. This pattern of payments maximizes the amount of offsetting that occurs in the queue.

3.1.1 The queue

A payment in the queue may belong to a cycle having the property that all other payments in the cycle are also in the queue, as illustrated in the left panel of figure 3. In this case the payments are released by the queue since they offset multilaterally (or bilaterally if the cycle is of length 2). A payment in the queue may also be part of a cycle having the property that at least one payment in the cycle is not in the queue, as illustrated in the right panel of figure 3. In this case, the payment belongs to a path (within the queue).[7] Payments in a path cannot offset multilaterally. However, it is possible that the bank who must make the "first" payment in the path receives a payment from outside the queue. In that case, the first payment in the path is released, creating a cascade of settlements until eventually a payment is made to someone outside the queue. We denote by χ the probability that a payment in the queue is part of a cycle and $1 - \chi$ the probability that it is part of a path.

We consider the value of χ for the two extreme cases described above. We use λ_e to denote the fraction of banks who send their payment early, λ_q to denote the fraction of banks who put their payments in the queue, and λ_d to denote the fraction of banks who delay their payments. Clearly, $\lambda_e + \lambda_q + \lambda_d = 1$.

[7] Of course, a queue could contain both payments in cycles and payments in paths.

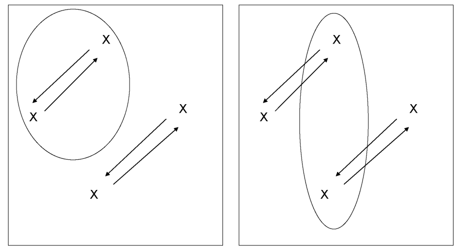

Figure 3: Left panel: queued payments in a cycle. Right panel: queued payments in paths.

If all payments form a unique cycle, then the probability that a payment in the queue is in a cycle is zero unless all banks put their payment in the queue. Formally, $\chi = 0$ if $\lambda_q < 1$ and $\chi = 1$ if $\lambda_q = 1$. Under this assumption, the queue releases the fewest payments. This case is also interesting because the role of the queue is only to allow banks to send their payment conditionally on receiving another payment. The queue no longer plays the role of settling multilaterally offsetting payments.

If all payments are in cycles of length 2, then the probability that a payment in the queue is in a cycle is λ_q. With this pattern of payments, the planner would be able to figure out the types of all pairs of banks. We do not let the planner condition on this information. Hence, the planner can assign an action to a bank conditional on that bank's type, but not on the type of the bank that receives the payment.

Next, we can derive the expressions for π^o, the probability of receiving a payment conditionally on not putting the payment in the queue, and π^q, the probability of receiving a payment conditionally on putting a payment in the queue. The latter probability is equivalent to the probability that a payment in the queue is released.

Suppose that there are no payments in the queue. Then, the probability of receiving a payment is given by the mass of banks who send a payment outright divided by the total mass of banks. Formally, $\pi^o = \lambda_e/(\lambda_e + \lambda_d)$. It turns out that the expression for π^o does not change when there are payments in the queue. Indeed, note that every payment made early by some bank outside the queue to a bank inside the queue must ultimately trigger a payment from a bank inside the queue to a bank whose payment is outside the queue. From the perspective of banks outside the queue, this is the same as if the payment had been made directly from a bank outside the queue. For this reason, we can ignore the queue. In summary, the expression for π^o is

$$\pi^o \equiv \frac{\lambda_e}{\lambda_e + \lambda_d} = \frac{\lambda_e}{1 - \lambda_q}. \tag{1}$$

If a bank puts a payment in the queue, the payment will be in a cycle with probability χ, in which case it is released for sure. With probability $1 - \chi$, the payment is in a path. The probability that a payment in a path is released is equal to the probability of receiving a payment from outside the queue. This probability is equal to π^o. So the expression for π^q is given by

$$\pi^q \equiv \chi + (1 - \chi)\frac{\lambda_e}{\lambda_e + \lambda_d} = \chi + (1 - \chi)\pi^o. \tag{2}$$

Under our "long-cycle" assumption, $\chi = 0$ if $\lambda_q < 1$ so that $\pi^o = \pi^q = \lambda_e/(\lambda_e + \lambda_d)$. If $\lambda_q = 1$, then $\pi^o = 0$ and $\pi^q = 1$, since all the payments are put in the queue. Under our "short-cycles" assumption, $\chi = \lambda_q$ so that

$$\pi^q = \lambda_q + (1 - \lambda_q)\frac{\lambda_e}{\lambda_e + \lambda_d} = \lambda_q + (1 - \lambda_q)\pi^o.$$

4 The planner's problem

In this section, we describe the planner's problem. The planner knows the type of each bank and chooses an action for each bank. The feasible actions are pay early, delay, and, in the case of an LSM, queue. The planner's objective is to maximize a weighted average of the welfare of all banks in the economy, where the weights are given by the population sizes. Equivalently, the planner's objective is to maximize the expected utility of a bank before the bank's type is known.

The planner's allocation corresponds to the allocation that banks would choose if they could commit to a type-conditional action before they know their type. After knowing their type, banks may have an incentive to deviate from the action prescribed by the planner because banks do not take into account the effect their actions have on the probability that other banks will receive a payment early.

Let the set of types be $I = \{s+, s0, s-, r+, r0, r-\}$. We use s to denote banks with time-critical payments and r to denote banks with a non-time-critical payment. Let λ_i^j denote the fraction of participants of type $i \in I$ who choose action $j \in J = \{e, q, d\}$, where e means that the payment is sent early, q means that the payment is queued, and d means that the payment is delayed. We have the following restriction:

$$\lambda_i^e + \lambda_i^q + \lambda_i^d = 1, \forall i \in I. \tag{3}$$

Also, when the settlement system is RTGS, then $\lambda_i^q \equiv 0$, for all i.

The expression for π^o can be rewritten as

$$\pi^o \equiv \frac{\sigma\left[\theta\left(\lambda_{s+}^e + \lambda_{s-}^e\right) + (1-\theta)\left(\lambda_{r+}^e + \lambda_{r-}^e\right)\right] + (1-2\sigma)[\theta\lambda_{s0}^e + (1-\theta)\lambda_{r0}^e]}{\sigma\Gamma + (1-2\sigma)\Sigma}, \tag{4}$$

where

$$\Gamma \equiv \theta\left(\lambda_{s+}^d + \lambda_{s-}^d + \lambda_{s+}^e + \lambda_{s-}^e\right) + (1-\theta)\left(\lambda_{r+}^d + \lambda_{r-}^d + \lambda_{r+}^e + \lambda_{r-}^e\right), \tag{5}$$

$$\Sigma \equiv \theta\left(\lambda_{s0}^d + \lambda_{s0}^e\right) + (1-\theta)\left(\lambda_{r0}^d + \lambda_{r0}^e\right). \tag{6}$$

Recall that π^q is given by equation (2). We can write χ as

$$\chi = \sigma\left[\theta\left(\lambda_{s+}^q + \lambda_{s-}^q\right) + (1-\theta)\left(\lambda_{r+}^q + \lambda_{r-}^q\right)\right] + (1-2\sigma)\left(\theta\lambda_{s0}^q + (1-\theta)\lambda_{r0}^q\right),$$

in the short-cycles case. In the long-cycle case $\chi = 0$ if any of the banks do not put their payments into queue, i.e.

$$\sigma\left[\theta\left(\lambda_{s+}^q + \lambda_{s-}^q\right) + (1-\theta)\left(\lambda_{r+}^q + \lambda_{r-}^q\right)\right] + (1-2\sigma)\left(\theta\lambda_{s0}^q + (1-\theta)\lambda_{r0}^q\right) < 1,$$

and $\chi = 1$ otherwise.

In the case where the settlement system is RTGS, χ is trivially equal to zero. The expression for the probability of receiving a payment in the morning is given by equation (4).

Welfare is given by

$$\begin{aligned} W = & -\sigma\left[(\theta\lambda^e_{s+} + (1-\theta)\lambda^e_{r+})(1-\pi^o)(2\mu-1)R\right] && (7)\\ & -\sigma\theta\lambda^q_{s+}(1-\pi^q)\gamma \\ & -\sigma\theta\lambda^d_{s+}\gamma \\ & -(1-2\sigma)\left[(\theta\lambda^e_{s0} + (1-\theta)\lambda^e_{r0})(1-\pi^o)\mu R\right] \\ & -(1-2\sigma)\theta\lambda^q_{s0}(1-\pi^q)\gamma \\ & -(1-2\sigma)\theta\lambda^d_{s0}\gamma \\ & -\sigma\left[(\theta\lambda^e_{s-} + (1-\theta)\lambda^e_{r-})(1-\mu\pi^o)R\right] \\ & -\sigma\left[\theta\lambda^q_{s-}(1-\pi^q)\gamma + (\theta\lambda^q_{s-} + (1-\theta)\lambda^q_{r-})(1-\mu)R\right] \\ & -\sigma\left[\theta\lambda^d_{s-}\gamma + \left(\theta\lambda^d_{s-} + (1-\theta)\lambda^d_{r-}\right)(1-\pi^o)(1-\mu)R\right], && (8) \end{aligned}$$

Lemma 1 *W is convex in λ^j_i, for all $i \in I, j \in J$.*

The proof is provided in the appendix. A consequence of lemma 1 is that the planner always assigns the same action to all banks of the same type. Hence, we can limit our attention to action profiles of the type $\{a_i\}_{i\in I}$, with $a_i \in J$, in the case of an LSM and $a_i \in \{e, d\}$, in the case of an RTGS system.

5 Welfare in the case of RTGS

This section characterizes the solution to the planner's problem with an RTGS settlement system. We show that, depending on the parameter values, the planner may choose to make all banks make their core payment early, or all banks except banks of type $r-$ pay early, or all banks except banks of type $r-$ and $s-$ pay early.

Inspection of equation (7) shows that, everything else being equal, a reduction in the fraction of payments settled in the morning, π^o, reduces welfare. There can be welfare gains from delaying a payment, however, as it may reduce the need of some banks to borrow. Consider for example a bank that receives a negative liquidity shock. If this bank delays its core payment, it will receive a payment with probability $1-\pi^o$, in which case it will not need to borrow. Even if that bank does not receive a payment, the amount it needs to borrow is only $(1-\mu)$ if it delays its core payment instead of 1 if it sends the payment early. The delay of this bank's payment will have a cost for the bank that was supposed to receive the payment. The size of this cost will depend on whether the would-be receiving bank has a positive, a negative, or no liquidity shock, and whether that bank delays its payment.

The expected cost to other banks is independent of the delaying bank's type because we have assumed that the type of the recipient of a payment is not correlated with the sender's type. In contrast, different types of banks have different benefit from delaying their payment. The next lemma compares the benefit of delay depending on a bank's liquidity shock.

Lemma 2 *The benefit of delay, relative to sending a payment early, is greater for a bank that receives a negative liquidity shock than for a bank that receives no liquidity shock. It is greater for a bank that receives no liquidity shock than for a bank that receives a positive liquidity shock.*

Proof. A bank with a positive liquidity shock has an expected borrowing cost of $(1-\pi^o)(2\mu-1)R$ if it does not delay, and does not need to borrow if it delays. So the expected benefit from delaying is $(1-\pi^o)(2\mu-1)R$. A bank with no liquidity shock has an expected borrowing cost of $(1-\pi^o)\mu R$ if it does not delay, and does not need to borrow if it delays. So the expected benefit from delaying is $(1-\pi^o)\mu R$. A bank with a negative liquidity shock has an expected borrowing cost of $(1-\pi^o)R+\pi^o(1-\mu)R$ if it does not delay, and an expected cost of $(1-\pi^o)(1-\mu)R$ if it delays. So the expected benefit from delaying is $(1-\pi^o)\mu R+\pi^o(1-\mu)R$. It can easily be verified that

$$(1-\pi^o)(2\mu-1)R \leq (1-\pi^o)\mu R \leq (1-\pi^o)\mu R+\pi^o(1-\mu)R.$$

■

When comparing banks with the same liquidity shock, the planner always prefers to delay the payment of a bank that must make a non-time-critical payment.

Let W_0 denote welfare when all banks make their core payment early, W_{r-} denote welfare when only banks of type $r-$ delay their core payment, and W_- denote welfare when banks with a negative liquidity shock delay their core payments.

$$W_0 = -\sigma(1-\mu)R, \tag{9}$$

$$W_{r-} = -(1-\theta)\sigma\left[\sigma(2\mu-1)+(1-2\sigma)\mu+(1-\theta)\sigma(1-\mu)\right]R \tag{10}$$

$$-\sigma\theta\left[(1-\theta)\sigma+(1-(1-\theta)\sigma)(1-\mu)\right]R, \tag{11}$$

$$W_{-} = -\sigma\left[\sigma(2\mu-1)+(1-2\sigma)\mu+\sigma(1-\mu)\right]R-\theta\sigma\gamma. \tag{12}$$

Proposition 1 *Depending on parameter values, the solution to the planner's problem can take one of three forms: 1) all banks pay early, 2) only bank of type $r-$ delay, 3) only banks of types $r-$ and $s-$ delay.*

Proof. We will show that, depending on the parameter values, either W_0, or W_{r-}, or W_- can be largest. We also show that the planner will never choose to make a bank with no liquidity shock delay. Since the benefit of delay is even lower for banks with a positive liquidity shock, the planner will not make such banks delay.

First, we compare W_0 and W_{r-}:

$$W_0 - W_{r-} = \left[\mu(2+2\theta\sigma-\sigma)-1-\sigma\theta\right](1-\theta)\sigma R,$$

so $W_0 < W_{r-}$ if and only if

$$\mu(2+2\theta\sigma-\sigma) < \sigma\theta+1.$$

Since $\theta \geq 0$ and $\sigma \leq \frac{1}{4}$, this inequality can hold only if $\mu < \frac{4}{7}$.

Next, we compare W_{r-} and W_-:

$$W_{r-} - W_- = \theta\sigma\gamma + \theta\sigma R\left[(2\mu-1)(1-\sigma)-\mu\sigma(1-\theta)-\theta\sigma(1-\mu)\right].$$

This expression is positive if $\frac{\gamma}{R}$ is large enough. Indeed, if the cost of delay is large, the planner will never choose to make banks with time-critical payment delay. However, if μ is close enough to $\frac{1}{2}$ and $\frac{\gamma}{R}$ is small, this expression is negative. We already know that if μ is small then W_{r-} can be larger than W_0.

We check that the planner would never choose to delay banks of type $r0$. The welfare when all banks of type $r-$ and $r0$ delay is given by

$$\begin{aligned}W_{r0} &= -(1-\theta)(1-\sigma)\left[\sigma(2\mu-1)+\theta(1-2\sigma)\mu\right]R\\&\quad-\theta\sigma\left[1-\mu(\theta+\sigma-\theta\sigma)\right]R\\&\quad-(1-\theta)\sigma\left[(1-\theta)(1-\sigma)\right](1-\mu)R.\end{aligned}$$

Since

$$W_{r-}-W_{r0}=(1-2\sigma)(\mu-\sigma+\mu\sigma)(1-\theta)\theta R\geq 0,$$

the planner never chooses to delay banks of type $r0$.

Finally, we check that the planner would never choose to delay banks of type $s0$ and $r0$. The welfare when all banks of type $s-$, $s0$, and $r-$ and $r0$ delay is given by:

$$W_{s0,r0}=-\sigma(1-\sigma)\mu R-(1-\sigma)\theta\gamma$$

Since

$$W_{-}-W_{s0,r0}=\theta\gamma(1-2\sigma)\geq 0,$$

the planner will never choose to delay banks of type $r0$ and $s0$. ■

5.1 Comparison with equilibria

Equilibria for this model are characterized in Martin and McAndrews (2008 a).

Proposition 2 *Four equilibria can exist. For all equilibria non-time-critical payments are delayed. In addition,*

1. *If $\gamma\geq\left[\mu-\theta(2\mu-1)\right]R$, it is an equilibrium for all time-critical payments to be sent early.*

2. *If $\left\{\mu-\theta\left(1-\sigma\right)\left(2\mu-1\right)\right\}R>\gamma\geq\left[1-\theta\left(1-\sigma\right)\right]\mu R$, then it is an equilibrium for banks of type $s-$ to delay time-critical payments while other banks pay time-critical payments early.*

3. *If $(1-\sigma\theta)\mu R > \gamma \geq (1-\sigma\theta)(2\mu-1)R$, it is an equilibrium for only banks of type $s+$ to send time-critical payments early.*

4. *If $(2\mu-1)R > \gamma$, then it is an equilibrium for all banks to delay.*

These equilibria can coexist, as shown in Martin and McAndrews (2007).

Proposition 3 *It is not possible to achieve the planner's allocation with an RTGS settlement system, as long as some payments can be delayed without cost.*

Proof. If some payments are non-time-critical, then the planner never chooses to make banks of type $r+$ or $r0$ delay. If all payments are time-critical, then the planner's allocation can be achieved if the cost of delay is high enough. For a high cost, neither the planner, nor the banks in equilibrium, wish to delay. ■

Proposition 4 *With RTGS, there are at least as many payments settled early under the planner's allocation as in equilibrium.*

Proof. The planner will always have $s+$ and $s-$ banks send their payments early. If, in equilibrium, $s-$ banks delay their payments then there will be no more payments sent early than in the planner's allocation.

If $s-$ banks send their payments early in the equilibrium allocation, then θ payments will be sent early. This is the largest fraction of payments that can possibly be sent early in the equilibrium allocation. The condition for $s-$ banks to send their payments early, in equilibrium, is

$$\frac{\gamma}{R} \geq \mu - \theta(2\mu - 1). \tag{13}$$

Under the planner's allocation, at least $1-\sigma$ payments are sent early. These represent the payments of all banks of types $s+$, $s0$, $r+$, and $r0$.

So we need to show that if $\theta \geq 1-\sigma$, then the planner will prefer to make all banks pay early rather than only banks of types $s+$, $s0$, $r+$, and $r0$.

The condition for $W_0 \geq W_-$ is

$$\frac{\gamma}{R} \geq \frac{\sigma\mu - (2\mu - 1}{\theta}. \tag{14}$$

We need to show that the condition given by equation (13) implies the condition given by equation (14) when $\theta \geq 1-\sigma$. This amounts to showing that

$$\theta\mu + (1-\theta^2)(2\mu - 1) \geq \sigma\mu,$$

whenever $\theta \geq 1-\sigma$. This must be true since $1-\sigma > \sigma$. ■

One interpretation of this result is that there are typically too few payments settled early in equilibrium. Hence, this proposition can help justify measures taken by central banks aimed at providing incentives for early submission of payments. For example, the Bank of England imposes "throughput requirements" on CHAPS, its RTGS payment system. Under such requirements, banks must send a certain fraction of their daily payments before a given time. This limits the banks ability to delay. The Swiss National Bank provides incentives by charging a higher price for payments sent late than payments sent early.

6 Welfare in the case of an LSM

In this section we characterize the solution to the planner's problem when an LSM is available. The planner may find it beneficial to use the LSM because it allows the settlement of a payment to be made conditional on the receipt of another payment. This conditionality reduces the amount, and thus the total cost, of borrowing.

As in the RTGS case, the planner will always choose the same action for all banks of the same type. However, since each of the six types of banks can, in principle, be assigned any of three actions, this leaves $6^3 = 216$ cases to be checked. We can eliminate some cases outright.

It is equivalent for the planner to require all banks to make their core payment early, or to queue all core payments. In either case, all core payments are settled in the morning. However, there is no benefit from making some banks delay their payments unless some banks send their payment early. If all other banks queue, delaying a payment has no benefit since the probability that the delaying banks will receive a payment in the morning is zero.

Lemma 3 *The planner will never choose to make banks with a positive or zero liquidity shock delay their payments.*

Proof. Queuing is always preferable to delay for such banks. Indeed, if a bank with positive or zero liquidity shock does not receive a payment in the morning, queueing a payment will be equivalent to delaying. If a bank with positive or zero liquidity shock does receive a payment, then the queue will release the outgoing payment, but the bank's balance will remain nonnegative. ■

Lemma 4 *The planner will never choose to make banks of type $r-$ queue or send early, unless all other payments are being sent early.*

Proof. Suppose that banks of type $r-$ are queueing or sending their payments early while some other banks are delaying their payments. Let Δ denote the set of these other banks. Recall that the benefit of receiving a payment is independent of the type of the sending bank. If the planner switches the actions of some of the banks of type $r-$ that were not delaying and an equal mass of the banks in Δ, this leaves the welfare of all other banks unchanged, since the mass of banks that delay, queue, or send early has not changed. However, because the benefit of delay is greater for banks of type $r-$ than for any other banks, from lemma 2, welfare must have increased. ■

The next lemma compares the cost of sending a payment early depending on a bank's liquidity shock.

Lemma 5 *The cost of sending a payment early, relative to queuing or delaying, is smaller for banks with a positive liquidity shock than for banks with no liquidity shock. It is smaller for banks with no liquidity shock than for banks with a positive liquidity shock.*

Proof. Banks that have a positive or no liquidity shock never need to borrow if they queue or delay their payment. If they pay early, banks with a positive liquidity shock face an expected cost of $(1-\pi^o)(2\mu-1)$, while banks with no liquidity shock face an expected cost of $(1-\pi^o)\mu \geq (1-\pi^o)(2\mu-1)$. Banks with a negative liquidity shock face an expected cost of $1-\pi^o\mu$ if they pay early, $1-\mu$ if they queue, and $(1-\pi^o)(1-\mu)$ if they delay. Hence the cost of sending a payment early, relative to queuing of delaying, is at least as large or greater for banks with a negative liquidity shock than for banks with no liquidity shock. ■

Lemma 6 *The planner will never choose to have banks of type $s+$ delay or queue payments while other banks send payments early.*

Proof. Let us suppose banks of type $s+$ are queueing or delaying payments while some other banks are sending payments early. Let Δ denote the set of these other banks. As in the proof of the previous lemma, the planner can switch the actions of some of the banks of type $s+$ and some of the banks in Δ, leaving all other banks unaffected. However, because the cost of sending a payment early is smaller for banks of type $s+$ than for any other banks, from lemma 5, welfare must have increased. ■

Lemma 6 shows that the planner always chooses for banks of type $s+$ to make their payment early.[8] Lemma 4 shows that unless all banks pay early, the planner makes banks of type $r-$ delay. Lemma 3 shows that banks of types $s+$, $s0$, $r+$, and $r0$ never delay. This brings the number of options the planner will consider down to 25 $(1 + 2 \cdot 2 \cdot 2 \cdot 3)$.

We can use lemma 5 to eliminate some more action profiles, as shown in the appendix. For example, we know that the planner would not choose for banks of type $s0$ to queue, while banks of type $s-$ pay early. After such profiles are discarded, we are left with 14 possible profiles. These profiles are listed in the table 2, where E denotes that a payment is sent early, Q that a payment is queued, and D that a payment is delayed.

[8] Recall that all action profiles such that no bank delays produce the outcome that all payments are settled in the morning. For simplicity, we assume that the planner always chooses for all banks to send their payment early instead of an alternative action profile with the same outcome.

Type	s+	s0	s-	r+	r0	r-
1	E	E	E	E	E	E
2	E	E	E	E	Q	D
3	E	Q	Q	E	Q	D
4	E	Q	D	E	Q	D
A	E	E	E	E	E	D
B	E	E	E	Q	Q	D
C	E	E	Q	E	E	D
D	E	E	Q	E	Q	D
E	E	E	Q	Q	Q	D
F	E	E	D	E	E	D
G	E	E	D	E	Q	D
H	E	E	D	Q	Q	D
I	E	Q	Q	Q	Q	D
J	E	Q	D	Q	Q	D

Table 2: Fourteen strategy profiles

Proposition 5 *Depending on parameter values, the planner will choose profiles 1, 2, 3, or 4. This is true both for the long cycle and for the short cycles assumption.*

The proof is provided in the appendix. We also describe the parts of the parameter space where profiles 1, 2, 3, and 4 are chosen.

As in the RTGS case, the planner chooses for all banks to send early if the size of the liquidity shock is not too large. If liquidity shocks are large and the cost of delaying time-critical payments, γ, is also large, the planner can choose profile 2. In this case, no time-critical payments are delayed. Profile 2 is chosen when θ is small and/or σ is large. When γ is small, and for moderate values of θ, profiles 3 or 4 are chosen.

We mentioned before that the planer will choose to send all payments early if the size of the shock is small. In the following proposition we to put a bound on what we mean by "small".

Proposition 6 *In both the RTGS and LSM case, for $\mu > \frac{2}{3}$, the planner will choose to have* *all payments sent early. This implies that, for $\mu > \frac{2}{3}$, the planner will do no better with a LSM, relative to RTGS.*

The proof is provided in the appendix.

In the long cycles scenario, no strict subset of payments is offsetting. In this scenario, the benefits the planner accrues by having payments sent early conditional on the bank having sufficient balances are minimized. For this reason, the planner is more likely to have the planner send payments outright in the long cycles scenario.

Proposition 7 *If, for a certain set of parameters, the planner chooses to delay some payments in the long-cycles case, then the planner will also choose to delay some payments in the short-cycles case*

The proof is provided in the appendix.

6.1 Comparison with equilibria

As in the RTGS case, the equilibria with an LSM are characterized in Martin and McAndrews (2008 a).

Proposition 8 *Under the long-cycle assumption, the following equilibria exist:*

1. *If $\gamma < (2\mu - 1)R$, then all banks queue their payment*
2. *If $\gamma \geq (2\mu - 1)R$ and $\mu \geq 2/3$, then*
   (a) *If $\gamma \geq \mu R$, then all time-critical payments are sent early. Banks with a negative liquidity shock delay non-time-critical payments and non-time-critical payments from other types of banks are queued.*
   (b) *If $\mu R > \gamma \geq (2\mu - 1)R$, then only banks with a positive liquidity shock send time-critical payment early. Banks with a negative liquidity shock delay non-time-critical payments and all others queue their payment.*
3. *If $\gamma \geq (2\mu - 1)R$ and $\mu < 2/3$, then*

(a) *If $\gamma \geq \mu R$, the equilibrium is the same as under 2a.*

(b) *If $\mu R > \gamma \geq (1-\mu)R$, the equilibrium is the same as under 2b.*

(c) *If $(1-\mu)R > \gamma \geq (2\mu-1)R$, then banks that receive a negative liquidity shock delay their payment. Banks that receive a positive liquidity shock send their time-critical payment early. All other payments are queued.*

The same types of equilibria arise in the short-cycles case, but for different parameter values. Also, while it is always an equilibrium for all banks to queue their payments, this equilibrium is not robust when other equilibria exist, as shown in Martin and McAndrews (2008 a).

Contrary to the RTGS case where the planner's allocation cannot be achieved in equilibrium, the planner's allocation can be achieved in the LSM case for some parameter values. If $\frac{\gamma}{R}$ is not too large, all banks queue in equilibrium, leading to all payments being settled early. This is the allocation chosen by the planner when the liquidity shock is not too large.

With an LSM there can be too little or too much early settlement, in equilibrium. With RTGS, there can never be too much early settlement. There is too little early settlement if the liquidity shock is not too large and the cost of delay, $\frac{\gamma}{R}$, is large. In that case, the planner would like all core payments to settle in the morning, but this will not occur in equilibrium. In contrast, if the liquidity shock is large, but the cost of delay is small, there can be too much early settlement with an LSM. Because $\frac{\gamma}{R}$ is small, all banks queue in equilibrium and all core payments are settled early. However, if the liquidity shock is large, the planner would like to redistribute some liquidity between banks with positive and negative liquidity shock. So the planner would prefer if not all payments were settled in the morning. For instance, when $\mu = 0.55, \frac{\gamma}{R} = 0.1, \theta = 0.4$ and $0.25 > \sigma > 0.1$, all payments are settled in the morning period, in equilibrium. Under the planner's solution $r-$ banks delay their payment, $s-$ banks either queue or delay, and $s0$ and $r0$ banks queue their payment.

7 Calibrating the Model using data from Fedwire

In this section, we use Fedwire data to calibrate the equilibrium of the model described in section 3, with and without an LSM. We will first describe our data, then calibrate values for σ and μ. With values for σ and μ in hand we compare the welfare under an LSM to the welfare in an RTGS system. We then venture further, calibrating values for θ, R, and γ in order to put a rough dollar figure on the welfare improvement of an LSM, relative to RTGS.

7.1 The data

The Fedwire Funds Service is an RTGS system that is owned and operated by the Federal Reserve. In the first and second quarters of 2007 approximately 534 thousand payments worth $2.48 trillion were made each day via Fedwire Funds. For this calibration exercise we use a transaction-level dataset that includes information on the sending and receiving institutions, the size of each transaction, the time at which the transactions are settled over Fedwire Funds, and a variable which differentiates transactions between overnight money market deliveries, returns of overnight money market loans, settlement transactions, third-party transactions, and interbank transactions.

Playing the role of our model's non-strategic agent are three settlement systems: CHIPS, CLS, and DTC. Each of these settlement systems maintain accounts with the Federal Reserve and may send and receive funds using Fedwire Funds. We also include the payment side of securities transactions made through the Fedwire Securities Service as part of our model's non-strategic agent. These transfers are usually initiated by the seller of the security and represent a large part of the daylight overdrafts of some banks.

CHIPS (Clearing House Interbank Payment System) is a privately-owned payment system that processes payments worth approximately $2 trillion each day. At the beginning of the day banks send funds from their account at the Federal Reserve to CHIPS. These funds are then used for the banks' activity in CHIPS. Near the end of the day, shortly after 5:00 p.m., banks will either need to send additional funds from their Federal Reserve account to CHIPS or will receive funds from CHIPS.

CLS (Continuous Linked Settlement) Bank is an industry-owned payment system that settles foreign exchange transactions, worth $3.5 trillion in the average day of our sample period. Its hours of operation are 1:00 a.m. to 6:00 a.m..

DTC (Depository Trust Company) is a securities settlement system, settling transactions worth almost $900 billion each day. DTCs final settlement occurs between 4:00 p.m. and

4:40 p.m.. Banks with negative positions with DTC send a Fedwire Funds payment to DTC's account at 4:35. At 4:40, DTC sends a Fedwire Funds transaction to banks with positive positions at DTC. Similar to CHIPS, DTC is a net receiver of funds in the morning and returns funds to banks in the late afternoon.

The Fedwire Securities Service, also owned and operated by the Federal Reserve, is a delivery versus payment securities settlement system. In the second quarter of 2007, the average daily cash value of the transactions over Fedwire Securities was approximately $1.83 trillion. We include the cash side of securities transactions until 12:00 p.m. in our calculation of the morning liquidity shock. More than 60% of the cash value of the day's transactions occurred before noon of the average day in the sample period.

7.2 Calibrating σ and μ

The proportion of banks that receive a positive liquidity shock is σ. It is also the proportion of banks that suffer a negative liquidity shock. In our model the aggregate shock is zero in both the morning and afternoon periods. Unlike our model, the distribution of actual shocks is not symmetric around zero. More banks send, rather than receive, payments to the settlement institutions in the morning. The difference between model and reality is partially offset by the fact that banks hold positive reserves at their account with the Federal Reserve, while our model assumes banks starts the day with zero balances. Banks need some of these reserves because they anticipate their morning pay-ins to CHIPS and DTC.

In our model the size of a bank's shock is equal to its net morning position with the nonstrategic agent divided by the sum of its net morning position with the nonstrategic agent and its other payments throughout the day:

$$1 - \mu = \frac{\text{Net Position with Nonstrategic Agent}}{\text{Net Position with Nonstrategic Agent} + \text{Gross Payments Throughout the Day}} \tag{15}$$

We define the shock for a bank on a particular day to be the position of the bank with CHIPS, CLS, DTC, and Fedwire Securities at 12:00 p.m., relative to the sum of its position with CHIPS, CLS, DTC and Fedwire Securities along with the sum of its time-critical payments sent over Fedwire Funds on that day. We choose not to incorporate non-time-critical payments in the "Gross Payments Throughout the Day" term since non-time-critical payments do not factor into a bank's decision making process on whether to delay or send payments early. We also ignore payments the bank makes to DTC, CLS, and CHIPS over Fedwire Funds in the second summand of the denominator.

Our calibrated value for μ will depend on which payments we classify as time-critical. If we classify more payments as non-time-critical, the denominator of the right hand side of equation 15 will decrease and our calibrated value for μ will decrease, as well. Since we cannot directly observe which payments are time-critical, we need to make an assumption concerning the categories of payments that are time-critical. In our model, only non-time-critical payments are always delayed. In our data, only deliveries of overnight money market loans are always made late in the day; 93% of these transactions are settled after 2:30 p.m.. Other types of transactions occur both early and late in the day. Transactions made on behalf of a third-party are settled, on average, earliest in the day. When calibrating μ we will consider the case where only deliveries of money market loans are non-time-critical and the case where all but third-party transactions are non-time-critical.

In figure 4, we draw the distribution of shocks across banks, under the assumption that only deliveries of overnight money market loans are non-time-critical. As can be seen in figure 4, a few large banks receive securities over the Fedwire Securities Service every morning. These banks consistently incur a small negative shock on their balances. Conversely a large number of smaller banks send securities and thus enjoy a positive liquidity shock.

To compute σ, we calculate the proportion of banks that have a shock that is far away from the mean shock. The size of σ is inversely related to our choice of what constitutes "far away." Our choice of "far away" is related to how we calculate μ. The size of the shock, $1-\mu$ should be greater than the cutoff between positive and zero liquidity shock banks. It would not make sense for a bank to receive a shock greater than $1-\mu$ and still be classified as a zero liquidity shock bank. We calculate $1-\mu$ as the mean distance from the average shock. The average shock is represented by the middle vertical line in figure 4 at -0.5%. Because the shock in our model is symmetric, we focus on one side of the distribution and extrapolate. We focus on the right hand side of the distribution since small changes in the

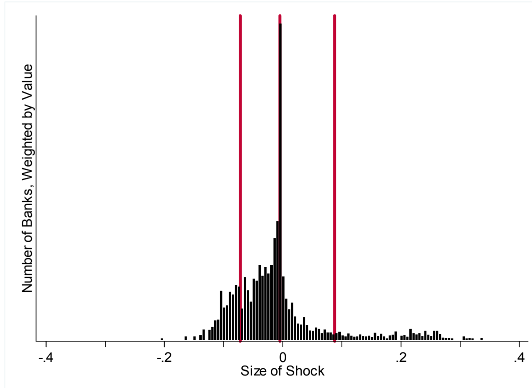

Figure 4: Distribution of shocks. The three vertical lines give the mean shock of the left hand side of the distribution, the mean shock, and the mean shock of the right hand side of the distribution.

cutoff we choose will only have minor effects on our calibration of the shock.[9] The mean of the right hand side of the distribution is 8.8%, which means that our calibrated value for $1-\mu$ is 9.3%. Thirteen percent of the banks have a shock that is greater than 0.088. Thus, our calibration of σ is 0.13. To have a symmetric shock, the cutoff for a negative liquidity shock must be set at -8.5%. This is slightly less than the mean of the left hand side of the distribution, which is -7.2%.

As expected, the calibrated value for μ decreases as we classify more payments as non-time-critical. If we classify only third-party transactions as time-critical, the mean shock, given by equation 15, is -1.0%. The mean value of the right hand side of the distribution of shocks is 16.5%, resulting in 82.6% as our calibrated value of μ (since $1-\mu = 17.5\%$). Seventeen percent of the banks have a shock that is greater than 0.165, giving us a calibrated value of 0.17 for σ.

Since μ is greater than $\frac{2}{3}$, the welfare under an LSM will be at least as high as the welfare under an RTGS (see proof of proposition 10 of Martin and McAndrews 2008 a). Thus, our results suggest that welfare would increase if an LSM were added to Fedwire. To get an idea of the possible welfare benefits we compute $\rho = \frac{\text{Aggregate Cost under RTGS}}{\text{Aggregate Cost with an LSM}}$ for $\mu \in \{0.80, 0.805, ...0.945\}$ and $\sigma \in \{0.10, 0.11, ..., 0.20\}$. Since we have not yet attempted to calibrate θ, R, and γ, we allow these parameters to take a wide range of values. We take

[9]Focusing on the left hand side of the distribution of shocks leads to little change in the calibrated values for σ and μ.

$\frac{\gamma}{R} \in \{0.10, 0.13....3.10\}$ and $\theta \in \{0.025, 0.075, ...0.975\}$. For these values, the median value of ρ is 3.11. For greater than 90% of the parameter combinations, ρ is greater than 1.38.

Furthermore, as we have shown in the proof of proposition 1, for μ greater than $\frac{2}{3}$ the planner will always choose to have all payments sent early. For the same set of parameter values for $\mu, \sigma, \frac{\gamma}{R}$, and θ given above $\frac{\text{Aggregate Cost under RTGS}}{\text{Aggregate Cost under a Planner's Solution}}$ has a median value of 8.09, and is always at least 1.0625.

We can provide an estimate of the welfare benefits of an LSM in dollar terms if we calibrate θ, γ, and R. This is what we do in the remainder of this section. When calibrating these parameters we consider the case where only money market deliveries are classified as non-time-critical payments as well as the case where deliveries on overnight money market loans, returns on overnight money market loans, and interbank transactions are non-time-critical payments.

7.3 Calibrating θ

In our model each bank must make one payment that is either time-critical or not. The fraction of banks that must make a time-critical payment is θ.

Martin and McAndrews (2008 a) show that 4 types of equilibria can occur in an RTGS system. Either all time-critical payments are delayed, or only banks with a positive liquidity shock make their time-critical payments early, or banks with a positive liquidity shock and banks with no liquidity shock make their time-critical payments early, or all time-critical payments are made early. Given that the data suggest a calibrated value of σ between 0.13 and 0.17, we should observe that the fraction of time-critical payments that are delayed is either 0%, in the range $13 - 17\%$, in the range $83 - 87\%$, or 100%, depending on the type of equilibrium. If we choose our cutoff time between morning and afternoon to be at noon and assume that only deliveries of overnight money market loans are non-time-critical, then 82% of the time-critical payments in our model are delayed. This would drop to 72% if the cutoff was 2:00 p.m.. We get similar values (81% for noon and 68% for 2:00 p.m.) when only transactions made on the behalf of a third-party are classified as time-critical. In either case, given these proportions, we conclude that Fedwire is in an equilibrium in which only banks that receive a positive liquidity shock make their time-critical payments early. All other payments are delayed.

If only positive liquidity shock banks make time-critical payments then the total borrowed from the central bank is equal to $\sigma\theta(2\mu-1)(1-\sigma\theta)+\sigma(1-\sigma\theta)(1-\mu)$. This expression corresponds to the share of banks with a positive liquidity shock, θ, multiplied by the amount these banks have to borrow if they do not receive a payment in the morning, $2\mu-1$, multiplied by the probability that they will not receive a payment in the morning period, $1-\sigma\theta$, plus the share of banks with a negative liquidity shock, σ, that do not receive a payment in the morning, $1-\sigma\theta$, and must borrow $1-\mu$. We choose θ so that the average amount borrowed in Fedwire is consistent with the amount predicted by our model, given the values we calibrated for μ and σ in the previous subsection.

In the first two quarters of 2007 the average daily amount paid in overdraft fees was $508 thousand per day. This means that, on the average day, the number of overdraft-hours is 1.219 trillion.[10] Since the average overdraft was 3 hours and 59 minutes in the first half of 2007, approximately $305 billion were borrowed each day from the Federal Reserve in this period. Since the total value sent over Fedwire was $4.31 trillion per day, the amount borrowed represents 7.1% of the gross value sent. Setting $\sigma\theta(2\mu-1)(1-\sigma\theta)+\sigma(1-\sigma\theta)(1-\mu)$ equal to 0.071 gives us combinations of σ, θ, and μ which are consistent with our model. These values are plotted in figure 5. We also plot the two (μ,σ) pairs calibrated in the previous subsection. Values for $\theta\in[\frac{1}{3},\frac{2}{3}]$ are consistent with our model and previously calibrated values of μ and σ.

In the previous subsection, we made two different assumptions regarding which payments were time-critical. The assumption which classified fewer payments as time-critical yielded calibrated values of $\mu=0.825$ and $\sigma=0.17$. In figure 5, we see that this calibration results in a smaller value for θ. This accords with our intuition: since θ is defined as the fraction of banks with time-critical payments, we would expect that fewer time-critical payments would lead to a smaller calibrated value of θ.

[10] Banks pay 36 basis points, at an annual rate, on their average intraday overdraft. If banks pay $ 508 thousand per day, then their aggregate intraday overdraft is $\frac{\text{\$ 508 thousand}}{\text{36 basis points/year*1year/360 days*1 day/24 hours}}=$ 1.219 trillion dollar-hours.

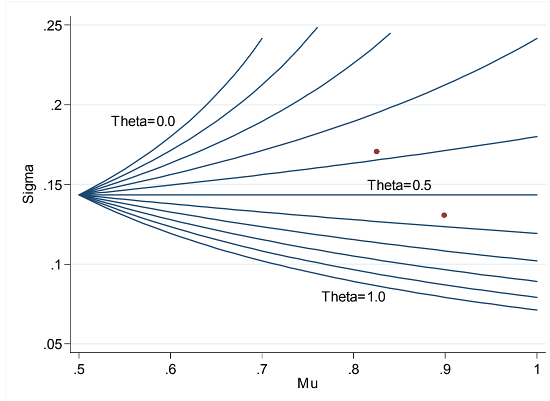

Figure 5: Values of θ, μ, and σ for which the data are consistent with the model. The two (μ, σ) pairs, as calibrated in the previous subsection are given in red.

7.4 Calibrating $\frac{\gamma}{R}$

Using the calibrated values of μ, θ, and σ, we can calibrate $\frac{\gamma}{R}$, the ratio of the cost of delay to the daylight overdraft fee. We observe the overdraft fee directly but may not observe other implicit costs of borrowing. Also, we do not observe the cost of delay. Our strategy is to choose a value of $\frac{\gamma}{R}$ such that the timing of settlement given by the model is consistent with the timing of settlement in the data. In our model, all non-time-critical payments are delayed. Banks will choose to send or delay time-critical payments depending on the magnitude of $\frac{\gamma}{R}$ and on their liquidity shock.

If only banks with a positive liquidity shock make their time-critical payments early, as we argued in the previous subsection, then we must have $(1 - \sigma\theta)\mu > \frac{\gamma}{R} \geq (1 - \sigma\theta)(2\mu - 1)$. Plugging in 0.90, 0.13 and 0.60 for μ, σ, and θ implies that $0.830 > \frac{\gamma}{R} \geq 0.738$. Taking 0.825, 0.17, and 0.40 for μ, σ, and θ results in $\frac{\gamma}{R}$ falling between 0.606 and 0.769.

7.5 Welfare Implications

We can do a couple of welfare comparisons. First, we compare welfare under an RTGS to welfare with an LSM. This gives us a sense of the welfare gains that could be achieved by implementing an LSM on Fedwire. Second, we can compare the equilibrium welfare with an RTGS to the welfare achieved by the planner's allocation with an RTGS. This give us a sense of the welfare loss arising from the lack of commitment.

First we compare equilibrium welfare with an RTGS and with an LSM. Given the value of our calibrated parameters, two equilibria could occur with an LSM, as shown in Martin and McAndrews (2008). If $\frac{\gamma}{R} < 2\mu - 1$ then all payments will be sent early in equilibrium. In this case, the equilibrium solution is identical to the planner's solution. This equilibrium is consistent with calibrated parameter values in the following ranges: $0.66 \geq \theta \geq 0.5$, $0.85 \geq \frac{\gamma}{R} \geq 0.73, 0.20 \geq \sigma \geq .10$, and $0.92 \geq \mu \geq 0.88$. For such parameter values, the ratio of the total costs with RTGS to the total costs with an LSM is always at least 8.66. If $\frac{\gamma}{R} \geq 2\mu - 1$, then the equilibrium is such that banks with a positive liquidity shock make time-critical payments early, non-time-critical payments are delayed by banks with a negative liquidity shock, and all other payments are queued. This equilibrium is consistent with calibrated parameter values in the following ranges: $0.50 \geq \theta \geq 0.33$, $0.77 \geq \frac{\gamma}{R} \geq 0.60, 0.20 \geq \sigma \geq .10$, and $0.85 \geq \mu \geq 0.80$. For these parameters, the ratio of the total costs in the RTGS scenario to the total costs with an LSM is always greater than 4.97.

In order to put a dollar amount on this increase in welfare, we must calibrate either R or γ. Fedwire participants pay 36 basis points to the Federal Reserve on their average daylight overdrafts. Since the average length of an overdraft is 3 hours and 59 minutes, our calibrated value of R is 6.0 $(=36\frac{3.99}{24})$ basis points. With $\frac{\gamma}{R} = 0.79$, $\mu = 0.90$, $\sigma = 0.13$ and $\theta = 0.60$, the loss in equilibrium with RTGS is \$3.67 million per day, compared to \$99 thousand when all payments are made early, as is the case with an LSM. Notice how the cost of borrowing from the central banks only makes up approximately 14%(=\$508 thousand /\$3.54 million) of the total costs of participating in the payment system. If $\frac{\gamma}{R}$ is 0.83, with the same values for μ, σ, R, and θ as before, only banks with a positive liquidity shock send payments early with an LSM in equilibrium. Negative liquidity shock banks delay their time-critical payments and all other payments are queued. Still, the aggregate cost with an LSM (\$1.56 million) is considerably smaller than with RTGS (\$3.79 million). It is higher than the cost under the planner's solution (\$99 thousand).

Similar results hold under the assumption of fewer time-critical payments. With $\frac{\gamma}{R} = 0.68$, $\mu = 0.82, \sigma = 0.17$ and $\theta = 0.40$, the aggregate cost with RTGS is \$2.67 million, the aggregate cost with an LSM is \$1.35 million, and the aggregate cost under the planner's solution is \$225 thousand.

8 Conclusion

In this paper, we studied the planner's solution of a model of a large value payment system introduced by Martin and McAndrews (2008 a) and calibrated that model to evaluate the potential impact of introducing an LSM. Studying the planner's problem deepens our understanding of the economic role of an LSM. We have seen that a planner would use an LSM for some parameter values because an LSM allows the settlement of payments conditional on the receipt of an offsetting payment. For other parameter values, the planner wants all payments to be made early and achieves the same welfare whether an LSM is available or not. For these parameters, Martin and McAndrews show that higher welfare can be achieved in equilibrium if an LSM is available.

Our calibration is the first attempt to use data to evaluate the potential impact of an LSM. We use Fedwire data to calibrate the parameters of our model. We then use these calibrated parameters to determine what an equilibrium would look like with an LSM. By comparing the two equilibria, we can derive the potential welfare benefit from adopting and LSM. Our results suggest that the welfare benefits of an LSM can be significant.

9 Appendix

Proof of lemma 1

The proof is an application of the fact that the product of two strictly increasing and weakly convex functions is convex. First, we can use equation (3) to eliminate λ_i^d, for all i. Notice that the denominator of π^o becomes

$$\Omega = 1 - \sigma\left[\theta(\lambda_{s+}^q + \lambda_{s-}^q) + (1-\theta)(\lambda_{r+}^q + \lambda_{r-}^q)\right] + (1-2\sigma)(\theta\lambda_{s0}^q + (1-\theta)\lambda_{s0}^q),$$

so that π^o is linear in the λ_i^es. Inspection of W reveals that the only terms that are not linear in λ_i^e are of the form $\lambda_i^e\pi^o$. Since this is the product of two increasing and weakly convex functions of λ_i^e, it must be a convex function of λ_i^e. A similar argument can be used to show that W is convex in λ_i^d, for all i.

To show that W is convex in λ_i^q, for all i, we need to show that π^q is convex in λ_i^q. First note that the derivative of Ω with respect to λ_i^q is a constant, for all i. The first and second derivatives of π^o, with respect to the λ_i^qs are given by

$$\frac{\partial\pi^o}{\partial\lambda_i^q} = -\pi^o\frac{\partial\Omega}{\partial\lambda_i^q}\frac{1}{\Omega}\text{and}\frac{\partial^2\pi^o}{\partial\lambda_i^{q2}} = 2\pi^o(\frac{\partial\Omega}{\partial\lambda_i^q})^2(\frac{1}{\Omega})^2.$$

Now we can take the partial derivative of π^q with respect to λ_i^q. Note that the partial derivative of χ with respect to λ_i^q is a constant. We get

$$\begin{aligned}\frac{\partial\pi^q}{\partial\lambda_i^q} &= \frac{\partial\chi}{\partial\lambda_i^q} + (1-\chi)\frac{\partial\pi^o}{\partial\lambda_i^q} - \frac{\partial\chi}{\partial\lambda_i^q}\pi^o,\\ \frac{\partial^2\pi^q}{\partial\lambda_i^{q2}} &= (1-\chi)\frac{\partial^2\pi^o}{\partial(\lambda_i^q)^2} - 2\frac{\partial\chi}{\partial\lambda_i^q}\frac{\partial\pi^o}{\partial\lambda_i^q}.\end{aligned}$$

Since $\partial\pi^o/\partial\lambda_i^q < 0$, $\partial^2\pi^q/\partial\lambda_i^{q2} > 0$, which completes the proof.

Ruling out more action profiles

Of the 25 action profiles that cannot be eliminated by lemmas 3, 4, and 6, 14 are in table 2. Here we show how the remaining 11 profiles can be ruled out. We use the notation of table 2 to describe a profile. Some profiles can be ruled out by the observation that, everything else being equal, it is more costly to queue or delay a time-critical payment than a non-time-critical payment. These profiles are:

Type	s+	s0	s-	r+	r0	r-
	E	Q	E	E	E	D
	E	Q	E	Q	E	D
	E	Q	Q	E	E	D
	E	Q	Q	Q	E	D
	E	Q	D	E	E	D

Other profiles can be eliminated using lemma 5:

Type	s+	s0	s-	r+	r0	r-
	E	E	E	Q	E	D
	E	E	Q	Q	E	D
	E	Q	E	E	Q	D
	E	Q	E	Q	Q	D
	E	E	D	Q	E	D
	E	Q	D	Q	E	D

In each of these profiles, a type of bank for which it is relatively costly to send payments early sends early, while a type of bank for which it is relatively less costly to send early queues.

Proof of proposition 5

Let W_i, $i \in \{1, ..., 4, A, ..., J\}$, denote welfare when banks adopt the action corresponding to their type in row i of table 2. Then we have

$$W_1 = -(1-\mu)R\sigma, \tag{16}$$

$$W_2 = \frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[-\theta+\mu(2\theta-1)\right]}{2\sigma(1-\theta)+\theta}, \tag{17}$$

$$W_3 = -\frac{\mu R\sigma(1-2\theta)+\theta\left[R\sigma+\gamma(1-\sigma)(1-\theta)\right]}{2-\theta}, \tag{18}$$

$$W_4 = -\frac{1}{2}(\mu R\sigma+\gamma\theta), \tag{19}$$

$$W_A = -R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right], \tag{20}$$

$$W_B = -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta}, \tag{21}$$

$$W_C = -\frac{\sigma\left\{\left[R(1-\sigma)+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta}, \tag{22}$$

$$W_D = -\frac{\sigma\left\{\mu R(2\theta-1)\left[\sigma(2\theta-1)-\theta\right]+\theta\left[R(\sigma+\theta(1-2\sigma)+\gamma\sigma(1-\theta)\right]\right\}}{\theta+\sigma(2-3\theta)}, \tag{23}$$

$$W_E = -\frac{\sigma\left\{\gamma\sigma(1-\theta)\theta+R\left[\theta(\mu+\theta-2\mu\theta)+\sigma(\mu(\theta+\theta^2-1)+1-2\theta)\right]\right\}}{\theta+\sigma(1-2\theta)}, \tag{24}$$

$$W_F = -\sigma(\mu R(1-\sigma)+\theta\gamma), \tag{25}$$

$$W_G = -\sigma\left[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})\right], \tag{26}$$

$$W_H = -\frac{\sigma\left\{\theta\gamma(\sigma+\theta-\sigma\theta)+R\left[\mu\theta-\sigma(-1+\mu+\theta)\right]\right\}}{\sigma(1-\theta)+\theta}, \tag{27}$$

$$W_I = -R\sigma\left[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)\right]-\gamma\theta(1-\sigma)(1-\theta), \tag{28}$$

$$W_J = -\frac{R\sigma(1-\mu-\theta+2\theta\mu)+\gamma\theta(1-\sigma+\theta\sigma)}{1+\theta}. \tag{29}$$

Now we show that the planner will never choose action profiles A to J in table 2.

A) The planner will never choose profile A, because aggregate welfare under profile 2 is always higher than profile A. $W_2 \geq W_A$ holds if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta}+R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right] \geq 0.$$

This expression can be simplified to yield

$$\frac{(\theta+\mu-2\mu\theta)(1-\theta)^2(1-2\sigma)R\sigma^2}{2\sigma(1-\theta)+\theta} \geq 0.$$

This inequality always holds since $\theta+\mu-2\mu\theta \geq 0$ is equivalent to $1 \geq \mu$.

B) The planner will never choose profile B, because either the welfare under profile 1 is higher than the welfare under profile B or the welfare under profile 2 is higher. If the shock is small, the planner prefers to send all core payments early. First we show that $W_1 \geq W_B$ if $\mu \geq \frac{1+\sigma}{2+\sigma}$. $W_1 \geq W_B$ holds if

$$-(1-\mu)R\sigma \geq -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta},$$

which can be simplified to $\frac{R\sigma[-1-\sigma+\mu(2+\sigma)](1-\theta)\theta}{\sigma(1-\theta)+\theta} \geq 0$. The numerator is positive whenever $\mu \geq \frac{1+\sigma}{2+\sigma}$.

Next we show that $W_2 \geq W_B$ if $\mu \leq \frac{1+\sigma}{2+\sigma}$. $W_2 \geq W_B$ holds if

$$\frac{R\sigma^2(1-\theta)^2\left\{\theta-\mu\theta+\sigma\left[2-3\theta+\mu(4\theta-3)\right]\right\}}{2\sigma^2(1-\theta)^2+3\theta\sigma(1-\theta)+\theta^2} \geq 0.$$

The numerator is positive if $\mu \leq \frac{\theta+2\sigma-3\sigma\theta}{\theta+3\sigma-4\sigma\theta}$. It can be checked that $\mu \leq \frac{1+\sigma}{2+\sigma}$ implies $\mu \leq \frac{\theta+2\sigma-3\sigma\theta}{\theta+3\sigma-4\sigma\theta}$.

C) The planner will never choose profile C. If $\gamma\theta \leq R\theta(1-2\mu)+\mu R$ then the planner will choose profile D over profile C. Otherwise, the planner will choose profile A over profile C. $W_D \geq W_C$ holds if

$$\begin{aligned} &-\frac{\sigma\left\{\mu R(2\theta-1)\left[-\theta+\sigma(-1+2\theta)\right]+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)} \\ &\quad+\frac{\sigma\left\{\left[R-R\sigma+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} \geq 0. \end{aligned}$$

This can be simplified to yield

$$\frac{\sigma^2(1-\theta)^2(1-2\sigma)[R\theta-\gamma\theta+\mu R(1-2\theta)]}{[\theta+\sigma(2-3\theta)](1-\sigma\theta)} \geq 0.$$

The numerator will be positive if $R\theta-\gamma\theta+\mu R(1-2\theta) \geq 0$, which is equivalent to $\gamma\theta \leq R\theta(1-2\mu)+\mu R$.

$W_A \geq W_C$ is equivalent to

$$\begin{aligned} &-R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right] \\ &+\frac{\sigma\left\{\left[R-R\sigma+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} \geq 0. \end{aligned}$$

Simplifying gives us

$$\frac{\sigma^2(1-\theta)\theta[\gamma-R\sigma(\theta+\mu-2\mu\theta)]}{1-\sigma\theta} \geq 0.$$

The numerator is positive if $\gamma\theta \geq R\sigma\theta(\theta+\mu-2\mu\theta)$, which occurs provided $\gamma\theta \geq R\theta(1-2\mu)+\mu R$.

D) The planner never chooses action profile D as it provides lower welfare than either 2 or 3, depending on parameter values. $W_3 \geq W_D$ if $\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\mu\theta) \geq 0$.

$W_3 \geq W_D$ if

$$-\frac{\mu R\sigma(1-2\theta)+\theta\left[R\sigma+\gamma(1-\sigma)(1-\theta)\right]}{2-\theta} + \frac{\sigma\left\{\mu R(2\theta-1)\left[-\theta+\sigma(-1+2\theta)\right]+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This can be simplified to

$$\frac{(1-2\sigma)(1-\theta)\theta\left[\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta)\right]}{(2-\theta)\left[\theta+\sigma(2-3\theta)\right]} \geq 0,$$

which must hold if $\gamma\left[2\sigma(-1+\theta)-\theta\right]+R\sigma(\mu+\theta-2\mu\theta) \geq 0$.

Next we show that $W_2 \geq W_D$ if $\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta) \leq 0$. $W_2 \geq W_D$ if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta} + \frac{\sigma\left\{\mu R(2\theta-1)(-\theta+\sigma(-1+2\theta))+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This can be simplified to

$$-\frac{(1-\theta)\sigma^2\theta\left[\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta)\right]}{\left[2\sigma(1-\theta)+\theta\right]\left[\theta+\sigma(2-3\theta)\right]} \geq 0,$$

which must hold if $\gamma\left[2\sigma(-1+\theta)-\theta\right]+R\sigma(\mu+\theta-2\mu\theta) \leq 0$.

E) The planner will never choose action profile E because if E provides higher welfare than action profile 1, then welfare under action profile C is higher than under E. $W_1 \geq W_E$ if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \leq 0$. $W_1 \geq W_E$ holds if

$$-(1-\mu)R\sigma + \frac{\sigma\left(\gamma\sigma(1-\theta)\theta+R\left\{\theta(\mu+\theta-2\mu\theta)+\sigma\left[1-2\theta+\mu(-1+\theta+\theta^2)\right]\right\}\right)}{\theta+\sigma(1-2\theta)} \geq 0.$$

This simplifies to

$$-\frac{\sigma(1-\theta)\theta\left\{R\left[1-\mu(2-\sigma)\right]-\gamma\sigma\right\}}{\theta+\sigma(1-2\theta)} \geq 0,$$

which must hold if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \leq 0$.

$W_C \geq W_E$ if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \geq 0$. $W_C \geq W_E$ holds if

$$\frac{\sigma\left\{\left[R(1-\sigma)+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} + \frac{\sigma\left(\gamma\sigma(1-\theta)\theta+R\left\{\theta(\mu+\theta-2\mu\theta)+\sigma\left[1-2\theta+\mu(-1+\theta+\theta^2)\right]\right\}\right)}{\theta+\sigma(1-2\theta)} \geq 0.$$

This can be simplified to

$$\frac{\sigma^2(1-\theta)^2\left\{R\left[1-\mu(2-\sigma)\right](1-\theta)+\gamma(1-\sigma)\theta\right\}}{(1-\sigma\theta)\left[\theta+\sigma(1-2\theta)\right]}\geq 0,$$

which must hold since $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma\geq 0$ implies $R\left[1-\mu(2-\sigma)\right](1-\theta)+\gamma(1-\sigma)\theta\geq 0$.

F) The planner will never choose action profile F because action profile G always provides higher welfare. $W_G \geq W_F$ holds if

$$-\sigma\left[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})\right]+\sigma\left[\mu R(1-\sigma)+\theta\gamma\right]\geq 0,$$

which must hold since

$$\frac{\mu R\sigma^2(1-2\sigma)(1-\theta)}{2\sigma(1-\theta)+\theta}\geq 0.$$

G) The planner will never choose action profile G. Action profile 4 provides higher welfare if $\mu R\sigma \geq \gamma(2\sigma(1-\theta)+\theta)$ and action profile 2 provides higher welfare otherwise. $W_4 \geq W_G$ holds if

$$-\frac{1}{2}(\mu R\sigma+\gamma\theta)+\sigma\left[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})\right]\geq 0.$$

This simplifies to

$$\frac{(1-2\sigma)\theta\{\mu R\sigma-\gamma[2\sigma(1-\theta)+\theta]\}}{4\sigma(1-\theta)+2\theta}\geq 0,$$

which must hold when $\mu R\sigma \geq \gamma(2\sigma(1-\theta)+\theta)$.

$W_2 \geq W_G$ holds if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta}+\sigma\left[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})\right]\geq 0.$$

This simplifies to

$$-\frac{\theta\sigma\left[(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta-\gamma(2\sigma(1-\theta)+\theta)\right]}{2\sigma(1-\theta)+\theta}\geq 0.$$

$(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta \leq \gamma(2\sigma(1-\theta)+\theta)$ will be true if $(1-\mu)R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$. This inequality must hold when $\mu R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$ since $\mu \geq 1-\mu$.

H) The planner will never choose profile H. Either profile 1 will produce higher welfare or profile G will. If the shock is small, the planner will always want all payments to be made early. $W_1 \geq W_H$ holds if

$$-(1-\mu)R\sigma+\frac{\sigma\{\theta\gamma(\sigma+\theta-\sigma\theta)+R[\mu\theta-\sigma(-1+\mu+\theta)]\}}{\sigma(1-\theta)+\theta}\geq 0.$$

This simplifies to

$$\frac{\sigma\theta[R(1-2\mu+\mu\sigma)-\gamma\sigma-\gamma\theta+\gamma\theta\sigma]}{\sigma(-1+\theta)-\theta}\geq 0$$

$R(1-2\mu+\mu\sigma)-\gamma\sigma-\gamma\theta+\gamma\theta\sigma\leq 0$ will be true if $(1-2\mu+\mu\sigma)\leq 0$ which will occur if $\mu \geq \frac{2}{3}$.

$W_G \geq W_H$ holds if

$$-\sigma[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma + \theta - 2\theta\sigma})] + \frac{\sigma\{\theta\gamma(\sigma + \theta - \sigma\theta) + R[\mu\theta - \sigma(-1 + \mu + \theta)]\}}{\sigma(1-\theta) + \theta} \geq 0.$$

This simplifies to

$$\frac{R\sigma^2(1-\theta)[\theta(1-\mu)(1-2\sigma) + \sigma(2-3\mu)]}{2\sigma^2(1-\theta)^2 + 3\sigma(1-\theta)\theta + \theta^2} \geq 0.$$

This occurs if $\theta(1-\mu)(1-2\sigma) + \sigma(2-3\mu) \geq 0$, which holds if $\mu \leq \frac{2}{3}$.

I) The planner will never choose profile I. If $\mu \geq \frac{2}{3}$, profile 1 will produce higher welfare. Otherwise, profile 3 will. $W_1 \geq W_I$ holds if

$$\begin{aligned} &-(1-\mu)R\sigma + \gamma\theta(1 - \sigma - \theta + \sigma\theta) + \\ &+ R\sigma[1 - 2(1-\theta)\theta - \mu(1 + 3(-1+\theta)\theta)] + \gamma\theta(1 - \sigma - \theta + \sigma\theta)] \geq 0. \end{aligned}$$

This simplifies to $((3\mu - 2)R\sigma + \gamma(1-\sigma))(1-\theta)\theta \geq 0$, which holds if $\mu \geq \frac{2}{3}$.

$W_3 \geq W_I$ is true if

$$\begin{aligned} &-\frac{\mu R\sigma(1-2\theta) + \theta[R\sigma + \gamma(1-\sigma)(1-\theta)]}{2-\theta} \\ &+ R\sigma[1 - 2(1-\theta)\theta - \mu(1 + 3(-1+\theta)\theta)] + \gamma\theta(1 - \sigma - \theta + \sigma\theta)] \geq 0. \end{aligned}$$

This simplifies to

$$\frac{(1-\theta)^2[(2-3\mu)R\sigma(1-\theta) + \gamma(1-\sigma)\theta]}{2-\theta} \geq 0,$$

which occurs if $(2-3\mu)R\sigma(1-\theta) + \gamma(1-\sigma)\theta \geq 0$. This holds if $\mu \leq \frac{2}{3}$.

J) The planner will never choose profile J. Either profile 1 will produce higher welfare or profile 4 will. $W_1 \geq W_J$ if

$$-(1-\mu)R\sigma + \frac{R\sigma(1 - \mu - \theta + 2\theta\mu) + \gamma\theta(1 - \sigma + \theta\sigma)}{1+\theta} \geq 0.$$

This inequality is true if

$$\frac{[(3\mu - 2)R\sigma + \gamma(1 - \sigma + \sigma\theta)]\theta}{1+\theta} \geq 0,$$

which is true for $\mu \geq \frac{2}{3}$.

$W_4 \geq W_J$ if

$$-\frac{1}{2}(\mu R\sigma + \gamma\theta) + \frac{R\sigma(1 - \mu - \theta + 2\theta\mu) + \gamma\theta(1 - \sigma + \theta\sigma)}{1+\theta} \geq 0,$$

which simplifies to

$$\frac{(1-\theta)[(2-3\mu)R\sigma + \gamma(1-2\sigma)\theta]}{2(1+\theta)} \geq 0.$$

This occurs if $(2-3\mu)R\sigma + \gamma(1-2\sigma)\theta \geq 0$, which is true if $\mu \leq \frac{2}{3}$.

Short Cycles Case:

Let W_i, $i \in \{1, ..., 4, A, ..., J\}$, denote welfare when banks adopt the action in row i of table 2, this time for the short cycles scenario. We have

$$W_1 = -(1-\mu)R\sigma, \tag{30}$$

$$W_2 = \frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[-\theta+\mu(2\theta-1)\right]}{2\sigma(1-\theta)+\theta}, \tag{31}$$

$$W_3 = -\frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta+\mu R(1-2\theta)\}}{2-\theta}, \tag{32}$$

$$W_4 = -\frac{1}{2}\mu R\sigma - 2\gamma(1-\sigma)\sigma\theta, \tag{33}$$

$$W_A = -R\sigma[1+\sigma(\theta-1)][\theta+\mu(1-2\theta)], \tag{34}$$

$$W_B = -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta}, \tag{35}$$

$$W_C = -\frac{\sigma\{\mu R(1-\sigma)(1-2\theta)+\theta[R-R\sigma+\gamma\sigma(1-\theta)(1-\sigma\theta)]\}}{1-\sigma\theta}, \tag{36}$$

$$W_D = -\frac{\sigma\{\mu R(2\theta-1)[\sigma(2\theta-1)-\theta]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)}, \tag{37}$$

$$W_E = -\frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)]+R[\theta(\mu+\theta-2\mu\theta)+\sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)}, \tag{38}$$

$$W_F = -\sigma[\mu R(1-\sigma)+\theta\gamma)], \tag{39}$$

$$W_G = -\sigma[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})], \tag{40}$$

$$W_H = -\frac{\sigma\{\theta\gamma(\sigma+\theta-\sigma\theta)+R[\mu\theta-\sigma(-1+\mu+\theta)]\}}{\sigma(1-\theta)+\theta}, \tag{41}$$

$$W_I = -\sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\}, \tag{42}$$

$$W_J = -\frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta}. \tag{43}$$

Notice that, for profiles such that no banks with time-critical payments are queueing their payments, the welfare under the long cycles scenario is the same as the welfare in the short cycles scenario. Thus W_1, W_2, W_A, W_B, W_F, W_G, and W_H, are the same for the long and short cycles scenarios. Hence action profiles A, B, F, G, and H will not be chosen by the planner in the short cycles case.

C) The planner will never choose profile C. The welfare under profile D is always higher than the welfare under profile C. $W_D \geq W_C$ is equivalent to

$$-\frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)}$$
$$+\frac{\sigma\{\mu R(1-\sigma)(1-2\theta)+\theta[R-R\sigma+\gamma\sigma(1-\theta)(1-\sigma\theta)]\}}{1-\sigma\theta} \geq 0.$$

This simplifies to $\frac{R\sigma^2(1-2\sigma)(1-\theta)^2(\mu+\theta-2\mu\theta)}{(1-\theta\sigma)(\theta+2\sigma-3\theta\sigma)} \geq 0$, which is always true since $\mu+\theta-2\mu\theta \geq 0$.

D) Likewise, the planner will never choose profile D. Either profile 2 or profile 3 will produce higher welfare than the welfare under profile D.
$W_3 \geq W_D$ holds if

$$-\frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta+\mu R(1-2\theta)\}}{2-\theta} + \frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This simplifies to

$$\frac{[-R\theta+\mu R(-1+2\theta)+\gamma(2-\theta)(2\sigma+\theta-3\sigma\theta)]\theta(1-\theta)(1-2\sigma)\sigma}{(2-\theta)(\theta+\sigma(2-3\theta))} \leq 0.$$

The numerator will be negative provided $-R\theta+\mu R(-1+2\theta)+\gamma(2-\theta)(2\sigma+\theta-3\sigma\theta) \leq 0$.

$W_2 \geq W_D$ is equivalent to

$$-\frac{R\sigma[\sigma(-1+\theta)-\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta} + \frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This simplifies to

$$\frac{\sigma^2(1-\theta)\theta(-R\sigma\theta+\mu R\sigma(-1+2\theta)+\gamma(2\sigma(1-\theta)+\theta)(2\sigma+\theta-3\sigma\theta))}{(2\sigma(1-\theta)+\theta)(\theta+\sigma(2-3\theta))} \geq 0.$$

The numerator is positive provided $-R\theta\sigma+\mu R\sigma(-1+2\theta)+\gamma(2\sigma-2\theta\sigma+\theta)(2\sigma+\theta-3\sigma\theta) \geq 0$. Since $-2\theta\sigma+\theta \geq -\theta\sigma$, the inequality from the last sentence is true if $-R\theta\sigma+\mu R\sigma(-1+2\theta)+\gamma(2\sigma-\theta\sigma)(2\sigma+\theta-3\sigma\theta) \geq 0$. Dividing by σ, we see that the last inequality is equivalent to $-R\theta+\mu R(-1+2\theta)+\gamma(2-\theta)(2\sigma+\theta-3\sigma\theta) \geq 0$.

E) The planner will never choose profile E. If the size of the shock is small, the planner will choose to have all payments sent early. Otherwise, the planner would choose profile D over profile E. $W_D \geq W_E$ holds if

$$-\frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} + \frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)]+R[\theta(\mu+\theta-2\mu\theta)+\sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)} \geq 0.$$

This simplifies to $\frac{R\sigma^2(1-\theta)^2(\theta-\mu\theta+\sigma(2-4\theta+\mu(5\theta-3)))}{\sigma(3-5\theta)\theta+\theta^2+\sigma^2(2-7\theta+6\theta^2)} \geq 0$. Now $\sigma(3-5\theta)\theta+\theta^2+\sigma^2(2-7\theta+6\theta^2)$ will be nonnegative for $\theta \in [0,1]$ and $\sigma \in [0, 0.25]$. The numerator is nonnegative provided $\theta-\mu\theta+\sigma(2-4\theta+\mu(5\theta-3)) \geq 0$, which is equivalent to $\theta+2\sigma-4\sigma\theta \geq \mu(\theta-5\theta\sigma+3\sigma)$. Since $\frac{2}{3} \leq \frac{.5}{.75-.25\theta} \leq \frac{\theta+2\sigma-4\theta\sigma}{\theta+3\sigma-5\theta\sigma}$, $W_D \geq W_E$ will hold for all $\mu \leq \frac{2}{3}$.

$W_1 \geq W_E$ occurs if

$$
\begin{aligned}
&-(1-\mu)R\sigma \\
&+\frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)]+R[\theta(\mu+\theta-2\mu\theta)+\sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)} \geq 0.
\end{aligned}
$$

This inequality simplifies to

$$
\frac{\sigma\theta(-1+\theta)\{R[1+\mu(-2+\sigma)]+\gamma\sigma[-\theta+\sigma(-1+2\theta)]\}}{\theta+\sigma(1-2\theta)} \geq 0.
$$

The numerator will be positive provided $R[1+\mu(-2+\sigma)]+\gamma\sigma[-\theta+\sigma(-1+2\theta)] \leq 0$, which will occur for $\mu \geq \frac{1}{2-\sigma} - \frac{\gamma\sigma(\theta+\sigma(1-2\theta))}{2R-\sigma R}$. This inequality holds true for $\mu \geq \frac{2}{3}$.

G) The planner will never choose profile G. If the size of the shock is small and the penalty for delaying time-critical payments is large the planner will choose profile 2. Otherwise the planner will choose profile 4 over profile G. The only difference between these two profiles is that zero liquidity shock banks with time-critical payments are putting their payments in queue under profile 4, but sending them outright under profile G. $W_4 \geq W_G$ is equivalent to

$$
-\frac{1}{2}\mu R\sigma - 2\gamma\theta\sigma(1-\sigma)+\sigma[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})]) \geq 0,
$$

which can be written as $\frac{\sigma(1-2\sigma)[\mu R-2\gamma(2\sigma(1-\theta)+\theta)]\theta}{4\sigma(1-\theta)+2\theta} \geq 0$. The numerator is positive if $\frac{\mu R}{2} \geq \gamma(2\sigma(1-\theta)+\theta)$.

$W_2 \geq W_G$ is equivalent to

$$
-\frac{R\sigma[\sigma(-1+\theta)-\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta}\sigma[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})] \geq 0
$$

Combining terms we get

$$
\frac{\theta\sigma((1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta-\gamma(2\sigma(1-\theta)+\theta))}{2\sigma(-1+\theta)-\theta} \geq 0,
$$

which occurs if $(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta-\gamma(2\sigma(1-\theta)+\theta) \leq 0$. This is equivalent to $(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta \leq \gamma(2\sigma(1-\theta)+\theta)$ which occurs if $(1-\mu)R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$. This is true if $\frac{\mu R}{2} \leq \gamma(2\sigma(1-\theta)+\theta)$ since $(1-\mu) \leq \mu$ and $\sigma \leq \frac{1}{4} < \frac{1}{2}$.

I) The planner will never choose profile I. If the size of the shock is small the planner would rather have everyone pay early rather than choose profile I. If the size of the shock is large profile 3 is preferable to profile I. $W_1 \geq W_I$ if

$$
\begin{aligned}
&-(1-\mu)R\sigma \\
&+\sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\} \geq 0.
\end{aligned}
$$

This occurs provided $[(3\mu-2)R+\gamma(1-\sigma)]\sigma(1-\theta)\theta \geq 0$, which holds for $\mu \geq \frac{2}{3}$.

$W_3 \geq W_I$ is true when

$$-\frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta+\mu R(1-2\theta)\}}{2-\theta}$$
$$+\sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\}\geq 0.$$

holds. This is true if $\frac{(1-\theta)^3 R\sigma(2-3\mu)}{2-\theta} \geq 0$, which is true for $\mu \leq \frac{2}{3}$.

J) As in the previous case, when the size of the shock is small profile 1 leads to higher welfare than profile J. Otherwise, profile 4 is preferable to profile J. $W_1 \geq W_J$ is equivalent to

$$-(1-\mu)R\sigma+\frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta}\geq 0,$$

which simplifies to

$$\frac{\sigma\theta((3\mu-2)R+2\gamma(1-\sigma)(1+\theta))}{1+\theta}\geq 0.$$

The numerator is positive provided $\mu \geq \frac{2}{3}$.

$W_4 \geq W_J$ is positive if

$$-\frac{1}{2}\mu R\sigma-2\gamma\theta\sigma(1-\sigma)+\frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta}\geq 0.$$

This simplifies to

$$\frac{(2-3\mu)R\sigma(1-\theta)}{2(1+\theta)}\geq 0.$$

The numerator is positive if $\mu \leq \frac{2}{3}$

Conditions on Planner's Choices under BRLSM

Long Cycles Case:

$W_1 \geq W_2$

$\iff -(1-\mu)R\sigma \geq -\frac{R\sigma[\sigma(1-\theta)+\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta}$

$\iff \frac{R\sigma(1-\theta)[(3\mu-2)\sigma+(2\mu-1)(1-\sigma)\theta]}{2\sigma(1-\theta)+\theta} \geq 0$

$\iff (3\mu-2)\sigma+(2\mu-1)(1-\sigma)\theta \geq 0$

$\iff \mu[3\sigma+2\theta(1-\sigma)] \geq 2\sigma+(1-\sigma)\theta$

$\iff \mu \geq \frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$

$W_1 \geq W_3$

$\iff -(1-\mu)R\sigma \geq -\frac{\mu R\sigma(1-2\theta)+\theta[R\sigma+\gamma(1-\sigma)(1-\theta)]}{2-\theta}$

$\iff (-1+\theta)[(-2+3\mu)R\sigma-\gamma(-1+\sigma)\theta] \leq 0$

$\iff (-2+3\mu)R\sigma-\gamma(-1+\sigma)\theta \geq 0$

$\iff 3\mu R\sigma \geq 2R\sigma+\gamma(-1+\sigma)\theta$

$\iff \mu \geq \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$.

$W_1 \geq W_4$

$\iff -(1-\mu)R\sigma \geq -\frac{1}{2}(\mu R\sigma+\gamma\theta)$

$\iff R\sigma-\frac{1}{2}\gamma\theta \leq \frac{3}{2}\mu R\sigma$

$\iff \mu \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$

Note: Since $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$, $W_1 \geq W_3 \Longrightarrow W_1 \geq W_4$

$W_2 \geq W_3$

$\iff -\frac{R\sigma(\sigma(1-\theta)+\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta} \geq -\frac{\mu R\sigma(1-2\theta)+\theta(R\sigma+\gamma(1-\sigma)(1-\theta))}{2-\theta}$

$\iff \frac{(1-\sigma)(1-\theta)\theta\{\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\theta\mu)\}}{[2\sigma(1-\theta)+\theta](2-\theta)} \le 0$
$\iff (1-\sigma)(1-\theta)\theta\{\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\theta\mu)\} \le 0$
$\iff R\sigma(\mu-2\theta\mu) \le \gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta$
$\iff \mu \le \frac{\gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta}{R\sigma(1-2\theta)}$ if $\theta < \frac{1}{2}$
$\mu \ge \frac{\gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta}{R\sigma(1-2\theta)}$ if $\theta > \frac{1}{2}$
$W_2 \ge W_4$
$\iff -\frac{R\sigma(\sigma(1-\theta)+\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta} \ge -\frac{1}{2}(\mu R\sigma+\gamma\theta)$
$\iff \theta\gamma[2\sigma(-1+\theta)-\theta]+R\sigma\theta[\mu+2\sigma-4\mu\sigma+2(2\mu-1)(-1+\sigma)\theta] \le 0$
$\iff R\sigma\theta[\mu+2\sigma-4\mu\sigma+2(2\mu-1)(-1+\sigma)\theta] \le \theta\gamma(2\sigma(1-\theta)+\theta)$
$\iff R\sigma\mu(1-4\sigma+4(-1+\sigma)\theta) \le \gamma(2\sigma(1-\theta)+\theta)+2(-1+\sigma)R\sigma\theta-2R\sigma^2$
$\iff \mu \le \frac{\gamma[2\sigma(1-\theta)+\theta]+2(-1+\sigma)R\sigma\theta-2R\sigma^2}{R\sigma(1-4\sigma+4(-1+\sigma)\theta)}$ if $1-4\sigma+4(-1+\sigma)\theta > 0$
$\mu \ge \frac{\gamma[2\sigma(1-\theta)+\theta]+2(-1+\sigma)R\sigma\theta-2R\sigma^2}{R\sigma(1-4\sigma+4(-1+\sigma)\theta)}$ if $1-4\sigma+4(-1+\sigma)\theta < 0$
$W_3 \ge W_4$
$\iff -\frac{\mu R\sigma(1-2\theta)+\theta(R\sigma+\gamma(1-\sigma)(1-\theta))}{2-\theta} \ge -\frac{1}{2}(\mu R\sigma+\gamma\theta)$
$\iff \frac{\theta\{(3\mu-2)R\sigma+\gamma[2\sigma(1-\theta)+\theta]\}}{2(-2+\theta)} \le 0$
$\iff (3\mu-2)R\sigma+\gamma[2\sigma(1-\theta)+\theta] \ge 0$
$\iff 3\mu R\sigma \ge 2R\sigma-\gamma[2\sigma(1-\theta)+\theta]$
$\iff \mu \ge \frac{2R\sigma-\gamma[2\sigma(1-\theta)+\theta]}{3R\sigma}$
Short Cycles Case:
$W_1 \ge W_2$
Same as long cycles case.
$W_1 \ge W_3$
$\iff -(1-\mu)R\sigma+\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta} \ge 0$
$\iff \frac{\sigma(1-\theta)((3\mu-2)R+\gamma\theta(1-\sigma)(2-\theta))}{2-\theta} \ge 0$
$\iff (3\mu-2)R+\gamma\theta(1-\sigma)(2-\theta) \ge 0$
$\iff (3\mu-2) \ge -\frac{\gamma\theta}{R}(1-\sigma)(2-\theta)$
$\iff \mu \ge \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$
$W_1 \ge W_4$
$\iff -(1-\mu)R\sigma+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \ge 0$
$\iff -\frac{1}{2}\sigma((2-3\mu)R-4\gamma\theta(1-\sigma)) \ge 0$
$\iff (2-3\mu)R-4\gamma\theta(1-\sigma) \le 0$
$\iff 3\mu R \ge 2R-4\gamma\theta(1-\sigma)$
$\iff \mu \ge \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$
$W_2 \ge W_3$
$\iff -\frac{R\sigma(\sigma(-1+\theta)-\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta}+\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta} \ge 0$
$\iff \frac{(1-\sigma)\sigma(1-\theta)\theta(\gamma(2\sigma(-1+\theta)-\theta)(-2+\theta)-R\theta+\mu R(-1+2\theta))}{(2\sigma(1-\theta)+\theta)(2-\theta)} \ge 0$
$\iff \gamma(2\sigma(-1+\theta)-\theta)(-2+\theta)-R\theta+\mu R(-1+2\theta) \ge 0$
$\iff \mu R(2\theta-1) \ge \gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta$
$\iff \mu \ge \frac{\gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta}{R(2\theta-1)}$ if $\theta > \frac{1}{2}$
$\mu \le \frac{\gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta}{R(2\theta-1)}$ if $\theta < \frac{1}{2}$
$W_2 \ge W_4$
$\iff -\frac{R\sigma(\sigma(-1+\theta)-\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta}+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \ge 0$
$\iff \frac{1}{2}\mu R\sigma+\mu\frac{R\sigma(\sigma(1-\theta)+\theta)(2\theta-1)}{2\sigma(1-\theta)+\theta} \ge \frac{R\sigma\theta(\sigma(1-\theta)+\theta)}{2\sigma(1-\theta)+\theta}-2\gamma(1-\sigma)\sigma\theta$

$\iff \frac{1}{2}\mu R\sigma[2\sigma(1-\theta)+\theta]+\mu\{R\sigma[\sigma(1-\theta)+\theta](2\theta-1)\} \geq R\sigma\theta(\sigma(1-\theta)+\theta)-2\gamma(1-\sigma)\sigma\theta(2\sigma(1-\theta)+\theta)$

$\iff \mu(\frac{1}{2}R\theta\sigma(4\theta-1+4\sigma(1-\theta))) \geq \sigma\theta((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)$

$\iff \mu \geq \frac{2((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)}{R(4\theta-1+4\sigma(1-\theta))}$ if $4\theta-1+4\sigma(1-\theta)>0$

$\mu \leq \frac{2((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)}{R(4\theta-1+4\sigma(1-\theta))}$ if $4\theta-1+4\sigma(1-\theta)<0$

$W_3 \geq W_4$

$\iff -\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta}+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \geq 0$

$\iff \frac{\sigma\theta((3\mu-2)R+2\gamma(1-\sigma)(2-\theta)(1+\theta))}{2(2-\theta)} \geq 0$

$\iff (3\mu-2)R+2\gamma(1-\sigma)(2-\theta)(1+\theta) \geq 0$

$\iff 3\mu R \geq 2R-2\gamma(1-\sigma)(2-\theta)(1+\theta)$

$\iff \mu \geq \frac{2}{3}-\frac{2\gamma}{3R}(1-\sigma)(2-\theta)(1+\theta)$

Proof of proposition 6

Proof. We need to show that if the parameters are such that the planner does not choose profile 1, then $\mu<\frac{2}{3}$. Formally, for the BRLSM long cycle case, we need to show that $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$, and $\frac{2R\sigma-\gamma\theta}{3R\sigma}$ are all less than or equal to $\frac{2}{3}$. For the BRLSM short cycles case we need to show that $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, $\frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$, and $\frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$ are all less than or equal to $\frac{2}{3}$. For the RTGS case, we will have to show that $\frac{(1+\sigma\theta)}{2(1+\sigma\theta)-\sigma}$ and $\frac{R-\gamma\theta}{R(2-\sigma)}$ are both less than $\frac{2}{3}$.

1) , 4) $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)} \leq \frac{2}{3}$ holds if $6\sigma+4\theta(1-\sigma) \geq 6\sigma+3\theta(1-\sigma)$, which is true since $\theta(1-\sigma) \geq 0$.

2) $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}$ holds if $6R\sigma-3\gamma\theta(1-\sigma) \leq 6R\sigma$, which is true since $-3\gamma\theta(1-\sigma) \leq 0$.

3) $\frac{2R\sigma-\gamma\theta}{3R\sigma} \leq \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$.

5) $\frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta) \leq \frac{2}{3}$ since $\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta) \geq 0$.

6) $\frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma) \leq \frac{2}{3}$ since $\frac{4\gamma\theta}{3R}(1-\sigma) \geq 0$.

7) $\frac{(1+\sigma\theta)}{2(1+\sigma\theta)-\sigma} \leq \frac{2}{3}$ holds if $3+3\sigma\theta \leq 4+4\sigma\theta-2\sigma$, which must be true since $2\sigma \leq 1+\sigma\theta$.

8) $\frac{R-\gamma\theta}{R(2-\sigma)} \leq \frac{2}{3}$ holds if $3R-3\gamma\theta \leq 4R-2\sigma R$, which must be true since $-3\gamma\theta \leq 0 \leq R(1-2\sigma)$. ■

Proof of proposition 7

Proof. We will have proven is proposition if we can show that $W_1 \geq Wi$ for the short cycles case implies $W_1 \geq Wi$ for the long cycles case, for $i \in \{2,3,4\}$ To do this we just check that 1) $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)} \leq \frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, 2) $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$, and 3) $\frac{2R\sigma-\gamma\theta}{3R\sigma} \leq \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$:

1) Trivial.

2) $(2-\theta) \leq 2 \leq \frac{1}{\sigma}$

$\implies (2-\theta)\frac{\gamma\theta(1-\sigma)}{3R} \leq \frac{\gamma\theta(1-\sigma)}{3R\sigma}$

$\implies \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$

3) $4(1-\sigma) \leq \frac{1}{\sigma}$

$\implies \frac{4\gamma\theta}{3R}(1-\sigma) \leq \frac{\gamma\theta}{3R\sigma}$

$\implies \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma) \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$ ■

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