Central Bank Preferences, Distribution Forecasts and Economic Stability in a Small Open-economy

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Abstract

This paper relates the central bank preferences on the inflation index to the quality of its forecasts and the perturbing impact of real and financial shocks. The framework is a Markov jump-linear-quadratic New Keynesian model, where the central bank searches for the optimal policy facing uncertainty on the behaviour of households and firms. Comparing CPI and domestic inflation targeting, the paper shows that the latter implies considerably less variability in the distribution forecast of the economic dynamics. Furthermore, domestic inflation targeting stands out for much less sensitiveness to interest rate smoothing, as well as for resulting in less economy-wide perturbation after several shocks. A relevant policy implication is that by choosing domestic inflation targeting an open-economy can significantly improve the prediction accuracy of the interest rate behaviour.

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Key Words: Inflation targeting; additive and multiplicative uncertainty; Markov jump linear quadratic systems; small open-economy; optimal monetary policy; central bank preferences.

1 Introduction

Forecasting the evolution of the economy and buffering unforeseen shocks are key issues in modern monetary policy. The relevance of central bank forecasting became
apparent with the advent of Inflation Targeting. This new monetary policy highlighted how variable and uncertain lags in the transmission of the monetary policy along with the exposure to exogenous disturbances require an operating procedure based on distribution forecast targeting. A call for central bank proficiency at forecasting also matters to shape the expectations of the private sector, this enhancing monetary policy effectiveness. This is the so called expectations channel. Its relevance in the monetary policy transmission mechanism is well captured by the growing view that monetary policy is the management of the private sector expectations as Woodford put it. Theoretically, the New-Keynesian model embedding agents’ forward looking behaviour shows the major role played by the expectations channel. In practice, last decade new trend in central banks transparency, in particular the publication of the internal distribution forecasts, signals the importance attributed to this channel.

This paper relates the accuracy of the central bank distribution forecasts and the perturbing impact of unforeseen shocks to the preferences of a small open-economy central bank. It shows that the stabilization of CPI inflation is inversely related to both the accuracy of the distribution forecasts of the other macrovariables and the perturbing impact of several shocks. Importantly, this trade-off is such that a small reduction in CPI inflation stabilization allows a large benefit in the accuracy of forecasting the evolution of the economy and in the ability to buffer unforeseen shocks. The policy implication is that the central bank can exploit this trade-off by shifting its preferences from CPI to domestic inflation. In this way it can improve its forecasting ability and reduce the perturbing impact of the exogenous shocks.

This finding is related to three strands in the monetary policy literature: i. the choice of the inflation measure to target, ii. central bank transparency and the publication of its distribution forecasts in particular for future policy intentions, and iii. optimal monetary policy with model uncertainty and exogenous shocks.

Which inflation measure to target is an open question in monetary policy. All over the world, inflation targeting central banks choose the CPI as the index to target. Yet this is more and more a contentious choice as argued in an increasing number of contributions from central banks practice and academic literature. Clearly, the CPI bears the advantage to be an index the private sector is more sensitive and familiar with. Thus targeting CPI inflation favours central bank accountability. This statistic, however, has various downsides. One problem pointed to by Batini, Levine and Pearlman (2005) is that policy rules which include the CPI may lead to economic indeterminacy. A second problem is that the CPI index is quite exposed to shocks turning out to be temporary. In this case the central bank tends not to react because interventions in presence of lags between the instrument and the goal can increase, rather than reduce the variability of CPI. Heikensten (1999) and Rosemberg (2004) discuss how this made difficult for the Riksbank to explain its behaviour to the private
sector requiring sometimes motivating policy decisions using other price indexes less exposed to temporary shocks. Rosemberg also notes that in some occasions the actual monetary policy has de facto been based on a different index. Similarly, Macklem (2001) maintains that while the Bank of Canada’s inflation-control target is specified in terms of CPI inflation, operationally, the Bank uses a measure of trend or "core" inflation as short term guide for its monetary policy actions. Further along the line, Young Ha (2002) and Guender (2003) introduced a case for choosing domestic inflation as it is less exposed to temporary shocks.

Adopting a welfare perspective various scholars reached contrasting conclusions on the inflation measure to target. Aoki (2001) and Benigno (2004) examine a model with two sectors that differ in their degree of price stickiness and show that monetary policy should target inflation in the sticky-price sector. In an open economy this prescription suggests to target domestic inflation as it tends to be stickier than CPI inflation. Gali and Monacelli (2005) submit that domestic inflation targeting dominates both CPI inflation targeting and an exchange-rate peg. They base their argument on the "excess smoothness" induced in the exchange rate by CPI targeting or an exchange rate peg which prevents relative prices from adjusting sufficiently fast. Similar results are obtained by Clarida et al. (2001) while Kirsanova et al. (2006) Benigno and Benigno (2004), De Paoli (2004) and Pappa (2002) find that the preferences of the central bank should include the terms of trade gap together with the output gap and domestic price inflation.

These contrasting findings can be explained by different assumptions at the basis of the private sector behaviour. From a central bank operational perspective, however, it is difficult to assess the most appropriate assumptions to model the behaviour of the private sector. This is due to considerable uncertainty on the true model of the economy. Furthermore, the relationship between optimal monetary policy for small open-economy and welfare in presence of realistic transmission lags is still largely unexplored. It is arguably premature to use directly welfare models for policy prescriptions. Moving from this remark, the current work adopts an operational perspective. It assesses alternative price indexes in terms of overall forecasting accuracy and amplitude of business cycle fluctuations. Regarding economic stability, the current work relates to Mankiw and Reis (2002) which in an elegant one-period model show that output stability is increasing in the weight given to targeting inflation in the stickier prices sector.

Aiming to portray a real-world monetary policy scenario, the framework encompasses both additive and multiplicative uncertainty. In particular, I consider the distribution forecasts of the macrovariables determined by the optimal monetary policy response to several shocks in presence of model uncertainty. Clearly, forecasting the evolution of the economy requires specifying a correct economic dynamics. Accordingly, this paper uses a monetary policy transmission mechanism with realis-
tic lags and inertia in the private sector behaviour. In this set up, the current work unveils domestic inflation targeting as the central bank preferences that perform best at forecasting accuracy and shocks buffering.

Furthermore, the possibility to increase the overall forecasting accuracy, in particular for the interest rate, favours the idea that central bank distribution forecasts are potentially very powerful to shape the private sector expectations. Thus the current paper supports central bank transparency and the publication of the internal forecasts in particular the one for the interest rate.

Finally, the paper relates to the literature on optimal monetary policy and model uncertainty. Starting with the Brainard (2007) seminal contribution, some authors among which Söderstrom (2002) have investigated how the optimal monetary policy response to the state variables of the economy attenuates or increases in presence of model uncertainty. It has not been investigated, however, if and to what extent the presence of model uncertainty may lead to reconsider the preferences of the central bank. Adopting this new perspective, the current paper contributes to the literature on monetary policy and model uncertainty by showing that the preferences on the inflation measure significantly affect central bank forecast accuracy and economic stability.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 shows the result using distribution forecasts of the economy with alternative central bank preferences. Section 4 discusses the paper’s finding in relation to transparency in monetary policy and the publication of future policy intentions. Conclusions are in section 5. Finally, the Appendix provides details on the derivation of the model.

2 The framework

The model consists of a linear-quadratic setup for optimal monetary policy nested into a non-certainty equivalence framework. As to the agents’ behavior, preferences and constraints are modeled to have a realistic transmission mechanism of the monetary policy. This is a necessary condition to have a proper dynamics and, consequently, realistic forecasts of the economy. Non-certainty equivalence, the second component of the framework, is a necessary condition to study how multiplicative uncertainty affects the optimal monetary policy. Since the model has also forward looking variables, non-certainty equivalence is modeled by using the general approach presented in Svensson and Williams (2007). This approach is based on Markov jump-linear-quadratic systems; the advantage lies in handling many forms of model uncertainty using a linear-quadratic setup which allows also for forward looking variables.

The characterization of the behaviour of the private sector follows Flamini (2007)\(^1\)

\(^1\)The description of the private sector behaviour is reported here as it allows presenting clearly
and can be summarized in five main assumptions. First, the economy is populated by four optimizing agents: a representative firm both for the sector that produces and retails domestic goods and for the sector that imports and retails foreign goods (henceforth the domestic and the import sector respectively), a representative household and a central bank. Second, the domestic and import sectors are connected. Indeed, the domestic one employs import goods as intermediate input and the import sector, in turn, may employ domestic goods to retail foreign goods creating incomplete pass-through. Third, both sectors are characterized by monopolistic competition and sticky prices, the latter assumption with respect to the import sector determines delayed pass-through. Fourth, realistic persistence in the behavior of the firms and households is captured, respectively, by inflation indexation and habit formation in consumption. Fifth, in line with the empirical evidence observed by central banks, a two-period lag for monetary policy to affect domestic inflation and a one-period lag to affect the aggregate demand are introduced, respectively, by predetermined pricing and consumption decisions.

These ingredients map into aggregate demands and supplies for the two sectors and an uncovered interest parity relation. Finally, the model is closed with an intertemporal loss function modelling the preferences of the central bank and exogenous relations to capture the behaviour of the rest of the world.

2.1 The household

The economy is populated by a continuum of unit mass of consumers/producers indexed by \( j \in [0, 1] \) sharing the same preferences and living forever. Intertemporal utility for the representative agent is given by

\[
E_t \sum_{\tau=0}^{\infty} \delta^\tau U \left( C_{t+\tau}, C_{t+\tau-1} \right),
\]

where \( \delta \) is the intertemporal discount factor, \( C_t \) is total consumption of consumer \( j \), and \( \hat{C}_t \) is the total aggregate consumption. Preferences over total consumption feature habit formation which is modeled in the style of Abel (1990) by the following instantaneous utility function

\[
U \left( C_{t+\tau}, \hat{C}_{t+\tau-1} \right) = \begin{cases} 
\left( \frac{C_{t+\tau}}{C_{t+\tau-1}} \right)^{1-\frac{1}{\sigma}} & \sigma \neq 1, \\
\ln \left( \frac{C_{t+\tau}}{\hat{C}_{t+\tau-1}} \right) & \sigma = 1
\end{cases},
\]

where \( \iota \geq 0 \) captures the willingness to “keep up with the Joneses” and \( \sigma > 0 \) is the intertemporal elasticity of substitution.
Total consumption, $C_t$, is a Cobb-Douglas function of two subindices for consumption of the domestic good, $C^d_t$, and import good, $C^i_t$,

$$C_t = C^d_t(1-w) + C^i_t w,$$  \hspace{1cm} (3)  

where $w$ determines the steady state share of imported goods in total consumption and $C^d_t$, $C^i_t$ are Dixit-Stiglitz aggregates of continuum of differentiated domestic goods and import goods (henceforth indexed with $d$ and $i$ respectively),

$$C^h_t = \int \left( C^h_t(j) \right)^{1-\frac{1}{\vartheta}} dj^{\frac{1}{\vartheta}} , \hspace{1cm} h = d, i ,$$

where $\vartheta > 1$ is the elasticity of substitution between any two differentiated goods and, for sake of simplicity, is the same in both sectors$^2$.

The flow budget constraint for consumer $j$ in any period $t$ is given by

$$\frac{B_t}{1+I_t} + \frac{B^*_t}{1+I^*_t} S_t + P_t C_t = B_{t-1} + B^*_{t-1} S_t + D^d_t + D^i_t ,$$

where $B$ and $B^*$ are two international bonds issued on a discount basis and denominated in domestic and foreign currency with interest rates $I_t$ and $I^*_t$ respectively, $S_t$ is the nominal exchange rate, expressed as home currency per unit of foreign currency. $D^d_t$ and $D^i_t$ are the dividends distributed by the domestic and the import sector and, finally, $P^c$ is the overall Dixit-Stiglitz price index for the minimum cost of a unit of $C_t$ and is given by

$$P^c_t = \frac{P^{w^d} t^{(1-w)}}{w w^d (1-w) (1-w)} ,$$  \hspace{1cm} (4)  

with $P^d$, $P^i$ denoting, respectively, the Dixit-Stiglitz price index for goods produced in the domestic and import sector.

To rule out “Ponzi schemes”, I assume that in any period $t$ the consumer chooses the value of the portfolio in $t+1$ such that his borrowing is no larger than the present value of all future dividends

$$B_{t+1} + B^*_{t+1} S_{t+1} \geq - \sum_{T=t+1}^{\infty} (1 + I_T)^{-1} \left( D^d_T + D^i_T \right) ,$$

and that the present value of future dividends is finite.

Utility maximization subject to the budget constraint and the limit on borrowing gives the Euler equation and the uncovered interest rate parity, respectively

$$c_t = \beta c_{t-1} + (1-\beta) c^i_{t+1|t} - (1-\beta) \sigma \left( i_t - \pi^c_{t+1|t} \right) , \hspace{1cm} \beta \equiv \frac{\iota \left( 1-\sigma \right)}{1 + \iota (1-\sigma)} < 1 ,$$  \hspace{1cm} (5)  

$^2$As in Corsetti and Pesenti (2004) I assume that the intratemporal elasticity of substitution between domestic and import goods is equal to one. This assumption ensures the stationarity of the model.
\[ i_t - i_t^* = s_{t+1|t} - s_t + \phi_t, \quad (6) \]

where for any variable \( x \), the expression \( x_{t+\tau|t} \) stands for the rational expectation of that variable in period \( t + \tau \) conditional on the information available in period \( t \) and, by means of a log-linearization, the variables \( c_t, \pi^e_t, i_t, \phi_t, (s_{t+1|t} - s_t) \) and \( \phi_t \) are log-deviations from their respective constant steady state values; finally, \( c_t \) denotes total aggregate consumption, obtained considering that in equilibrium total consumption for agent \( j \) is equal to total aggregate consumption, i.e. \( C_t = \tilde{C}_t \), \( \pi^e_t \) denotes CPI inflation (measured as the log deviation of gross CPI inflation from the constant CPI inflation target), and \( \phi_t \) is a risk premium shock added to capture financial market volatility.

### 2.1.1 Domestic consumption of goods produced in the domestic sector

Preferences captured by equation (3) imply that the (log deviation of the) domestic demand for goods produced in the domestic sector, \( c^d_t \), is given by

\[ c^d_t = c_t - \left( p^d_t - p^e_t \right), \]

which, considering the (log-linearized version of the) price index equation (4), can be rewritten as

\[ c^d_t = c_t + w q_t, \quad (7) \]

where \( q_t \equiv p^d_t - p^e_t \) is the (log-deviation of the) terms of trade.

Then, solving equation (5) for \( c_t \) and combining it with equation (7) I obtain

\[ c^d_t = -\sigma (1 - F_1 L)^{-1} \rho_t - \sigma (1 - F_1 L)^{-1} w q_t + w q_t, \quad (8) \]

where \( F_1 < 1 \) is the smaller root of the characteristic polynomial of equation (5) and

\[ \rho_t = \sum_{\tau=0}^{\infty} \left( i_{t+\tau|t} - \pi^d_{t+\tau+1|t} \right) \quad (9) \]

can be interpreted as the long real interest rate\(^3\).

### 2.1.2 Aggregate demand for goods produced in the domestic sector

Total aggregate demand for the good produced in the domestic sector is

\[ \tilde{Y}^d_t = C^d_t + Y^d,d_t + Y^d,i_t + C^*d_t, \quad (10) \]

\(^3\)Under the expectations hypothesis and considering a zero-coupon bond with a finite maturity, the variable \( \rho_t \) is approximately the product of the long real bond rate and its maturity; for further discussion see Svensson (2000).
where $Y^{d,d}_t$, $Y^{d,i}_t$ and $C^{*d}_t$ denote the quantity of the (composite) domestic good which is used as an input in the domestic sector, as an input in the import sector and which is demanded by the foreign sector, respectively.

I assume that both sectors share the same Leontief technology and each one features a continuum of unit mass of firms, indexed by $j$, that produce differentiated goods $Y^{d}_t(j)$ and $Y^{i}_t(j)$ in the domestic and import sector respectively. Furthermore, I assume that sectors differ for the input used: the domestic sector uses a composite input consisting of the domestic (composite) good itself and the (composite) import good produced in the import sector; the import sector uses a composite input consisting of the foreign good $Y^{*}_t$ and the domestic (composite good). Thus the production functions in the domestic and import sector are given respectively by

$$Y^{d}_t(j) = f \left( A^{d}_t \min \left\{ \frac{Y^{d,d}_t}{1 - \mu}, Y^{i,d}_t \right\} \right), \quad Y^{i}_t(j) = f \left( A^{i}_t \min \left\{ \frac{Y^{*,i}_t}{1 - \mu}, Y^{*,i}_t \right\} \right), \quad \mu, \mu^{i} \in [0, 1],$$

(11)

where $f$ is an increasing, concave, isoelastic function, $A_t$ is an exogenous (sector specific) economy-wide productivity parameter, $(1 - \mu)$ and $\mu$ denote, respectively, the shares of domestic good and import good in the composite input required to produce the differentiated domestic good $j$, and $(1 - \mu^{i})$ and $\mu^{i}$ denote, respectively, the shares of foreign good and domestic good in the composite input required to produce the differentiated import good $j$.

Thus the quantities of the (composite) domestic good used as an input in the domestic and import sector are

$$Y^{d,d}_t = \frac{1}{A^{d}_t} (1 - \mu) f^{-1} \left( \hat{Y}^{d}_t \right), \quad Y^{d,i}_t = \frac{1}{A^{i}_t} \mu^{i} f^{-1} \left( \hat{Y}^{i}_t \right),$$

(12)

where $\hat{Y}^{i}_t$ denotes the demand of the import good. Finally, log-linearizing equation (10) around the steady state values yields

$$\hat{y}^{d}_t = \kappa_1 (\mu^{i}) c^{d}_t + \kappa_2 (\mu^{i}) \hat{y}^{i}_t + \kappa_3 (\mu^{i}) c^{*d}_t,$$

(13)

where $\kappa_{1,t} (\mu^{i})$, $\kappa_{2,t} (\mu^{i}) < 0$ and $\kappa_{3,t} (\mu^{i}) > 0$, (see the appendix in Flamini (2007) for details).

Next, I assume, as in Svensson (2000), that $c^{*d}_t$ is exogenous and given by

$$c^{*d}_t = \beta y^* + \theta^* w^* q_t,$$

(14)

where $c^*_t$ denotes (log) foreign real consumption, $\theta^*$ and $w^*$ denote, respectively, the foreign atemporal elasticity of substitution between domestic and foreign goods and the share of domestic goods in foreign consumption. Furthermore, I define the output-gap in the domestic sector $y^{d}_t$ as

$$y^{d}_t \equiv \hat{y}^{d}_t - y^{d,n}_t,$$
where \( y_{d,t}^{d,n} \) denotes the log deviation of the natural output in the domestic sector from its steady state value, and I assume that in both sectors the log-deviation of the natural output from its steady state value is exogenous, stochastic and follows

\[
y_{h,t+1}^{h,n} = \gamma_{y}^{h,n} y_{t}^{h,n} + \eta_{h,t+1}^{h,n}, \quad 0 \leq \gamma_{y}^{h,n} < 1, \quad h = d, i,
\]

where \( \eta_{h,t+1}^{h,n} \) is a serially uncorrelated zero-mean shock to the natural output level (a productivity shock). Finally, in line with the central banks’ view of the approximate one-period lag necessary to affect aggregate demand, I assume that consumption decisions are predetermined one period in advance. Accordingly, repeating the same derivation with preferences maximized on the basis of one period ahead information I obtain the aggregate demand in the domestic sector. This relation, expressed in terms of the output-gap, is given by

\[
y_{d,t+1} = \beta_y y_{t+1}^{d} - \beta_p y_{t+1}^{d} q_{t} + \beta_y y_{t+1}^{d} q_{t+1} + \beta_y y_{t+1}^{d} q_{t+1} n_{t+1} + \eta_{d,t+1} - \eta_{d,t+1}, \quad (15)
\]

where \( \eta_{d,t+1} \) is a serially uncorrelated zero-mean demand shock. In (16) all the time-varying coefficients are positive and functions of the structural parameters of the model.

### 2.1.3 Aggregate demand of goods produced in the import sector

Aggregate demand for import goods is given by

\[
\hat{Y}_{t}^{i} = C_{t}^{i} + Y_{t}^{i,d}
\]

where \( Y_{t}^{i,d} \) denotes the amount of the import good used as an input in the domestic sector. Log-linearizing (17) around the steady state results in

\[
\hat{y}_{t}^{i} = (1 - \kappa) c_{t}^{i} + \kappa \hat{y}_{t}^{d}.
\]

Finally, the same assumptions used to derive the aggregate demand for the domestic sector goods yield

\[
y_{t+1}^{i} = \beta_y y_{t+1}^{i} - \beta_p y_{t+1}^{i} q_{t} + \beta_y y_{t+1}^{i} q_{t+1} + \beta_y y_{t+1}^{i} q_{t+1} n_{t+1} + \eta_{t+1}^{i} - \eta_{t+1}^{i,n}, \quad (19)
\]

where all the time-varying coefficients are positive and depend on the structural parameters of the model, and \( \eta_{t+1}^{i} \) is a serially uncorrelated zero-mean demand shock.

### 2.2 Firms

In both sectors, aggregate supply is derived according to the Calvo (1983) staggered price model and inflation inertia is introduced as in Christiano, Eichenbaum and Evans (2005) and also by the presence of the terms of trade as shown in Benigno (2004). Beyond the use of different inputs, the two sectors differ in the firms decision timing.
2.2.1 Domestic sector

In the domestic sector, the representative consumer/producer $j$ produces the variety $j$ of the domestic good, $Y^d_t(j)$, with a composite input whose price is $W_t$. Since all the varieties use the same technology, there is a unique input requirement function for all $j$ given by $\frac{1}{A^d_t} f^{-1}[Y^d_t(j)]$ and the variable cost of producing the quantity $Y^d_t(j)$ is $W_t \frac{1}{A^d_t} f^{-1}[Y^d_t(j)]$. Furthermore, since there is a Dixit-Stiglitz aggregate of domestic goods, the demand for variety $j$ is

$$Y^d_t(j) = \hat{Y}^d_t \left( \frac{P^d_t(j)}{P^d_t} \right)^{-\vartheta},$$

where $P^d_t(j)$ is the nominal price for variety $j$ and $\vartheta$ is the elasticity of substitution between different varieties. As shown in equation (11), the composite input is a convex combination of both aggregates of domestic and import goods. Thus the price of the input is given by $W_t \equiv (1 - \mu) P^d_t + \mu P^i_t$.

Then, I assume (i) that the consumer/producer chooses in any period the new price with probability $(1 - \alpha)$ or keeps the previous period price indexed to past inflation with probability $\alpha$, and (ii) that the price at period $t + 2$ is chosen 2 periods in advance. The latter assumption is motivated by the fact that domestic sector firms take both production and retailing decisions. The implication is that monetary policy needs a two-period lag to affect domestic inflation. This is in line with the central banks’ experience of an approximate two-period lag (where the period is the year) for monetary policy to have the highest impact on inflation. It follows that the decision problem for firm $j$ at time $t$ is

$$\max_{\tilde{P}^d_t} \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \tilde{\lambda}^d_{t+\tau+2} \left\{ \tilde{P}^d_{t+2} \left( \frac{P^d_{t+\tau+1}}{P^d_{t+1}} \right)^\zeta \tilde{Y}^d_{t+\tau+2} \left( \frac{P^d_{t+2+\tau}}{P^d_{t+1+\tau}} \right)^{-\vartheta} \right\}$$

$$f^{-1} \left[ \frac{W_{t+\tau+2}}{P^d_{t+\tau+2}} \right] A^d_{t+\tau+2},$$

where $\tilde{\lambda}^d_t$, $\tilde{P}^d_t$ and $\zeta$ denote, respectively, the marginal utility of domestic goods, the new price chosen in period $t$ for period $t + 2$ and the degree of indexation to the previous period inflation rate$^4$. Following Svensson (2000), I set $\delta = 1$ to ensure the natural-rate hypothesis. Finally, assuming that the purchasing power parity holds in

$^4$Recalling that consumption decisions are predetermined one period in advance, the marginal
the long run, the log-linearized version of the Phillips curve for the domestic sector turns out to be

\[
\pi^d_{t+2} = \frac{1}{1 + \zeta} \left[ \zeta \pi^d_{t+1} + \pi^d_{t+3|t} + \frac{(1 - \alpha)^2}{\alpha (1 + \omega \vartheta)} \left( \omega y^d_{t+2|t} + \mu q_{t+2|t} \right) \right] + \varepsilon_{t+2} \tag{21}
\]

\[
= \phi_{\pi} \pi^d_{t+1} + (1 - \phi_{\pi}) \pi^d_{t+3|t} + \phi_{y} y^d_{t} + \phi_{q} q_{t+2|t} + \varepsilon_{t+2}, \tag{22}
\]

where \( \omega \) in (21) is the output elasticity of the marginal input requirement function and \( \varepsilon_{t+2} \) is a zero-mean i.i.d. cost-push shock. In (22) all the implicitly defined coefficients are positive.

### 2.2.2 Import sector

Similarly to the domestic sector, variety \( j \) of the import goods, \( Y^i_t(j) \), is produced by the representative consumer/producer \( j \) with a composite input whose price is \( F_t \). Since the input requirement function is \( \frac{1}{A^i_t} f^{-1} \left[ Y^i_t(j) \right] \), the variable cost of producing the quantity \( Y^i_t(j) \) is \( F^i_t \frac{1}{A^i_t} f^{-1} \left[ Y^i_t(j) \right] \). Furthermore, considering that the input is a convex combination of the aggregate of domestic goods and of the foreign good, with price \( P^*_t S_t \), where \( P^*_t \) is the price in foreign currency of the foreign good, it follows that \( F_t \equiv \mu^i P^d_t + (1 - \mu^i) P^*_t S_t \).

Now relaxing the assumption that pricing decisions are predetermined and keeping all the remaining assumption used to derive the Phillips curve in the domestic sector results in

\[
\pi^i_t = \frac{1}{1 + \zeta} \left[ \zeta \pi^i_{t-1} + \pi^i_{t+1|t} + \frac{(1 - \alpha^i)^2}{\alpha^i (1 + \omega^i \vartheta)} \left( \omega^i y^i_t + q^i_t \right) \right] \tag{23}
\]

\[
= \phi_{\pi} \pi^i_{t-1} + (1 - \phi_{\pi}) \pi^i_{t+1|t} + \phi_{y} y^i_t + \phi_{q} q^i_t, \tag{24}
\]

where \( \alpha^i \) has the same meaning of its analogous variable in the domestic sector, \( q^i_t \) denotes (the log deviation of) the price of the composite input in the import sector expressed in terms of the import goods price, \( p^i_t \), and is defined as

\[
q^i_t \equiv (1 - \mu^i) (s_t + p^*_t) + \mu^i p^d_t - p^i_t, \tag{25}
\]

where \( p^*_t \) is the (log) foreign price level. Furthermore, relaxing the assumption of predetermined pricing decisions is here motivated by the fact that the import sector only acts as a retailer sector for the foreign goods and, in practice, retailers do not set their price before they take effect as much as producers do.

utility of domestic goods \( \tilde{\lambda}^d_t \) is obtained by the following first-order condition with respect to \( C^d_{t+1} \)

\[
E_t U_d \left( C^d_{t+1}, C^i_{t+1} \right) = E_t \left[ \lambda_{t+1} P^d_{t+1} \right] \equiv E_t \tilde{\lambda}^d_{t+1},
\]

where \( \lambda_t \) is the marginal utility of nominal income in period \( t \).
2.3 CPI inflation and the uncovered interest parity

CPI-inflation, \( \pi^c_t \), is given by

\[
\pi^c_t = (1 - w) \pi^d_t + w \pi^i_t,
\]

(26)

where \( w_t \) is the steady state share of imported goods in total consumption and determines the degree of openness of the economy. In order to eliminate the non-stationary nominal exchange rate, it is convenient to express the Uncovered Interest Parity in terms of \( q^i_t \) obtaining

\[
q^i_{t+1|t} - q^i_t = (1 - \mu^i) r_t - (1 - \mu^i) \left( \pi^c_t - \pi^d_{t+1|t} \right) - \left( \pi^i_{t+1|t} - \pi^d_{t+1|t} \right) - (1 - \mu^i) \varphi_t,
\]

(27)

where \( r_t \) is the short term real interest rate defined as \( r_t \equiv i_t - \pi^d_{t+1|t} \).

2.4 The public sector and the rest of the world

The behavior of the central bank consists of minimizing the following loss function:

\[
E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \mu^c \pi^c_{t+\tau}^2 + \mu^d \pi^d_{t+\tau}^2 + \lambda y_{t+\tau}^2 + \nu \left( i_{t+\tau} - i_{t+\tau-1} \right)^2 \right],
\]

(28)

where \( \mu^c, \mu^d, \lambda \) and \( \nu \) are weights that express the preferences of the central bank for CPI and domestic inflation targets, the output stabilization target, and the instrument smoothing target, respectively. As to the rest of the world, I assume stationary univariate AR(1) processes for the exogenous variables foreign inflation and income

\[
\pi^c_{t+1} = \gamma^c_\pi \pi^c_t + \varepsilon^c_{t+1},
\]

(29)

\[
y^c_{t+1} = \gamma^y y^c_t + \eta^c_{t+1},
\]

(30)

where the coefficients are non-negative and less than unity and the shocks are white noises, and also that the foreign sector sets the monetary policy according to the following Taylor rule

\[
i^*_t = f^*_\pi \pi^c_t + f^*_y y^c_t + \xi^*_t,
\]

(31)

where the coefficients are positive, and \( \xi^*_t \) is a white noise. Finally, the financial sector is modeled with a stationary univariate AR(1) process for the risk premium

\[
\varphi_{t+1} = \gamma^\varphi \varphi_t + \xi^\varphi_{t+1}
\]

2.5 Optimal monetary policy with structural parameters uncertainty

I now assume that the central bank is uncertain on the persistence in the behaviours of firms and households, the degree of price stickiness and the speed and completeness of the pass-through. This assumption is modeled assuming that the central bank has a prior belief on the probability distribution of the deep parameters underlying these
phenomena. A similar approach is followed by Kimura and Kurozumi (2007) who show in a forward-looking model how deep parameter uncertainty can lead to a more aggressive optimal monetary policy in a closed economy.

The uncertainty on the persistence in the household behavior is modeled by assuming uncertainty over habit formation in consumption preferences, captured by the parameter \( \iota \) in equation (2) that now becomes \( \iota_t \). This choice allows considering how the uncertainty on a basic feature of the household behavior impacts on many coefficients in the aggregate demands. Indeed, uncertain habit formation leads to aggregate demands where, for any period \( t \), not only the coefficient of the previous period output gap becomes uncertain, i.e. \( \beta_{y,t} \), but also several other coefficients become uncertain. Specifically, these are the coefficients for the previous period terms of trade, \( \beta_{q_{-1},t} \) and \( \beta_{q_{-1}y,t} \), foreign output \( \beta_{y^*,t} \) and \( \beta_{y^*y,t} \), and the natural output in the domestic sector and import sectors, \( \beta_{y^n,t} \) and \( \beta_{y^n^*,t} \).

The remaining sources of multiplicative uncertainty of the model are located in the supply side. Here the setup features sticky prices à la Calvo (1983) and indexation to the previous period inflation rate for the firms that cannot optimally update the price in the current period.

In this framework, by assuming that the central bank is uncertain on the firms’ degree of backward-looking indexation, \( \zeta \), that now becomes \( \zeta_t \), in (20), the central bank turns out to be uncertain on the degree of inertia and forward-looking behaviour in the inflation process. This assumption is motivated by a fair amount of disagreement in empirical evidence and theoretical works. Importantly, uncertain inertia in the firms behaviour, turns out to affect all the coefficients of the aggregate supplies and adds to the uncertainty on the degree of price stickiness. The latter is modeled by introducing an uncertain probability of optimally updating the prices in the current period, i.e. replacing \( (1 - \alpha) \) and \( (1 - \alpha^t) \) in (21) and (23) with the stochastic variables \( (1 - \alpha_t) \) and \( (1 - \alpha^t_t) \). With uncertainty on \( \zeta, \alpha, \alpha^t \), the central bank is uncertain in any period \( t \) about the slope of the aggregate supply in both sectors, \( \phi_{yt} \) and \( \phi_{yt^*} \). The slope of the Phillips curve, i.e. the response of inflation to fluctuations in resource utilization, is a relationship which seems difficult to pin down in a statistically significant way (Holmberg 2007). Furthermore, the last two decades point to a flattening of the Phillips curve whose causes are not yet fully understood. Anchoring inflation expectations via better monetary policy seems a prominent candidate to explain this phenomenon (Mishkin 2007, Boivin and Giannoni 2006, and Roberts 2006), yet changes in the price-setting behaviour could also be important and dependent on the level and variability of inflation. Bean (2007) also put forward the view that the flattening of the Phillips curve is observationally equivalent to a downward sloping Phillips curve shifting to the left as the natural rate of unemployment fell.

---

5 See Kimura and Kurozumi (2007) as to the contrasting results disseminated in the literature.
with monetary policy simultaneously ensuring that inflation remained stable. This implies that the uncertainty about the natural rate of unemployment makes it hard to pin down the slope of the Phillips curve. All in all, these factors surround the slope of the Phillips curve with a fair amount of uncertainty.

Also, uncertainty on $\zeta$, $\alpha$ and $\alpha'$ makes uncertain the impact of the terms of trade, $\phi_{q,t}^{d}$, on domestic inflation, and the impact of the input price in the import sector, $\phi_{q,t}^{i}$, on import inflation. These uncertain impacts capture the imperfect knowledge of the central bank on how the exchange rate affects the economy. As to the input price in the import sector, which is a function of the exchange rate, the uncertain coefficient $\phi_{q,t}^{i}$ determines the uncertainty on the speed of the pass through. Finally, uncertainty on the completeness of the pass-through is modelled by assuming that $\mu_t$ is a random variable. The relevance of the pass-through uncertainty is commonly known. For example, Cassino, Drew and McCaw (1999) point out that the pass-through has been quite variable over time in New Zealand. More in general, the impact of the exchange rate on the economy tends to be fairly uncertain for the policymakers\(^7\).

Table 1 reports the uncertain structural parameters along with the coefficients or the variables they have a direct impact upon.

<table>
<thead>
<tr>
<th>Structural parameter</th>
<th>Coefficients/variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_t$</td>
<td>$\beta_{y,t}$ $\beta_{y,-1,t}$ $\beta_{q,t}^{i}$ $\beta_{y,t}^{i}$ $\beta_{y,n,t}$ $\beta_{y,n,t}$ $\beta_{y,n,t}$</td>
</tr>
<tr>
<td>$\zeta_t$</td>
<td>$\phi_{x,t}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>$\phi_{y,t}$ $\phi_{y,t}$ $\phi_{y,t}$ $\phi_{y,t}$ $\phi_{y,t}$ $\phi_{y,t}$ $\phi_{y,t}$</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>$\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$ $\phi_{q,t}^{d}$</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>$q_t$</td>
</tr>
</tbody>
</table>

Table 1

2.6 Certainty non-equivalence and parameters uncertainty

To illustrate the introduction of uncertainty on structural parameters in a non-certainty equivalence environment, it is convenient to rewrite the model in State-space form. From the central bank standpoint the problem is to find the expected interest rate path that minimizes its loss given the law of motion of the economy:

$$
\text{Min}_{\{i_{t+r}|t\}} \sum_{r=0}^{\infty} E_t \beta_{t+r} Y_{t+r} K Y_{t+r}
$$

subject to

$$
\begin{bmatrix}
X_{t+1} \\
x_{t+1|t}
\end{bmatrix} =
\begin{bmatrix}
A_{11,t+1} & A_{12,t+2} \\
A_{21,t} & A_{22,t}
\end{bmatrix}
\begin{bmatrix}
X_t \\
x_t
\end{bmatrix} +
\begin{bmatrix}
B_{1,t+1} \\
B_{2,t}
\end{bmatrix} i_t +
\begin{bmatrix}
B_{1,t+1}^1 \\
B_{2,t}^2
\end{bmatrix} \varepsilon_{t+1|t} +
\begin{bmatrix}
\varepsilon_{t+1|t} \\
0
\end{bmatrix},
$$

$$
Y_t \equiv C_{x,t} \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + C_{i,t} i_t.
$$

\(^7\)See Leitemo and Soderstrom (2007) for the impact on optimal monetary policy of the policymakers' fear of mispecification in the determination of the exchange rate.
where the target variables, the predetermined variables, and the forward looking variables are, respectively

\[
Y_t = \left( \pi_t^c, \pi_t^d, y_t^d, i_t - i_{t-1} \right)', \quad X_t = \left( \pi_{t+1}^d, \pi_{t-1}^d, \pi_t^i, y_t^i, y_t^d, i_t^*, y_t^{d,n}, y_t^{i,n}, i_{t-1}, q_{t-1}, q_{t-1}^i, \varphi_t \right)', \quad x_t = \left( \pi_t^d, q_t^i, \rho_t, \pi_{t+2}^d \right)',
\]

and where \( K \) captures the central bank preferences, a diagonal matrix with the diagonal \( \pi; \pi^d; ; \) and off-diagonal elements equal to zero, and following the approach developed by Svensson and Williams (2007) I assume that the matrices

\[
A_{11,t}, A_{12,t}, B_{1,t}, B_{1,t}^1, A_{21,t}, A_{22,t}, B_{2,t}, B_{2,t}^1, C_{Z,t}, C_{i,t}, \tag{32}
\]

are random, each free to take \( n_j \) different values in period \( t \) corresponding to the \( n_j \) modes indexed by \( j_t \in \{1, 2, ..., n\} \). This means that, for example, \( A_{11,t} = A_{11,j_t} \). The modes are drawn initially from a discrete stationary distribution which is assumed to be uniform. A uniform distribution captures the assumption that the central bank only knows a band for each uncertain deep parameter. To study the impact of different level of uncertainty on the optimal monetary policy, the analysis is carried for different variances of the distribution. Accordingly, for each uncertain parameter, say \( \xi \), a benchmark value is chosen, \( \bar{\xi} \), and the lower and upper bound of the support of the distribution are set equal to \( \bar{\xi} - x\bar{\xi} \) and \( \bar{\xi} + x\bar{\xi} \) respectively, where the coefficient \( x \) modules the variance of the distribution and therefore the amount of uncertainty. Table 2 reports the benchmark values for the uncertain parameters\(^8\).

<table>
<thead>
<tr>
<th>Structural parameter</th>
<th>Benchmark value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\pi} )</td>
<td>0.7</td>
</tr>
<tr>
<td>( \bar{\zeta} )</td>
<td>0.66</td>
</tr>
<tr>
<td>( \bar{\alpha} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \bar{\alpha^i} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \bar{\mu^i} )</td>
<td>0.35</td>
</tr>
</tbody>
</table>

\[ \text{Table 2} \]

After the initial draw from the stationary distribution the modes follow a Markov process with constant transition probabilities given by

\[
P_{jk} = \Pr \{ j_{t+1} = k | j_t = j \} = \frac{1}{n}, \quad j, k \in \{1, 2, ..., n\}.
\]

Arguably, between any two periods each uncertain deep parameter should not change much, at least in normal times. To capture this idea and check for robustness it is

---

\(^8\)This calibration follows Smets and Wouters (2005) as to \( \bar{\tau} \), Svensson (2000) as to \( \bar{\pi} \) and Flamini (2007) as to \( \bar{\alpha^i} \). Regarding \( \bar{\tau} \) and \( \bar{\mu^i} \) their values have been chosen so that the lower and upper bound of the support of their distribution are realistic for any uncertainty level considered in the analysis.
also considered the case where over any two periods the probability that the mode keeps the same value is 0.5, the probability that it jumps to adjacent value is 0.3, and the probability that skipping the adjacent values it jumps to the closer remaining values is 0.1. In both cases, however, results are similar.

Furthermore, I assume that model uncertainty and shocks to the economy are independent so that modes $j_t$ and innovations $z_t$ are independently distributed. Finally, I assume that the central bank does not know how the structural parameters co-move together, should they be dependent. Thus, also the modes are assumed to be independently distributed. As to the central bank knowledge before choosing the instrument-plan $\{i_{t+\tau}\}_{\tau=0}^{\infty}$ at the beginning of period $t$, the information set consists of the probability distribution of $z_t$, the transition matrix $[P_{jk}]$, the $n_j$ different values that each of the matrices can take in any mode, and finally the realizations of $X_t, j_t, z_t, X_{t-1}, j_{t-1}, z_{t-1}, x_{t-1}, \ldots$.

At this point, following Svensson and Williams (2007), I find the equilibrium in presence of multiplicative uncertainty and both forward and backward looking variables under commitment in a timeless perspective (see Woodford (2003) and Svensson and Woodford (2005)).

3 Central bank preferences and distribution forecasts

3.1 An overview with a cost push shock

Figures 1-2 illustrate the unconditional distribution forecasts of the impulse responses to a cost-push shock under the optimal policies of domestic and CPI inflation targeting (respectively the first and the second column). Assuming an uncertainty level of the 20% on all the uncertain parameters, they have been generated by drawing an initial mode of the Markov chain from its stationary distribution, simulating the chain for 16 periods forward, and then repeating this procedure for 1000 simulations runs. Thus these figures display mean (dashed line), and quantiles (grey bands), of the empirical distribution. In particular, the dark, medium and light grey band show

\[ P = \begin{bmatrix}
0.5 & 0.3 & 0.1 & 0.1 & 0 \\
0.15 & 0.5 & 0.15 & 0.1 & 0 \\
0.1 & 0.15 & 0.5 & 0.15 & 0 \\
0 & 0.1 & 0.15 & 0.5 & 0.15 \\
0 & 0.1 & 0.15 & 0.5 & 0.15 \\
\end{bmatrix} \]

and its implied stationary distribution is

\[ \pi = [0.1598 \quad 0.2371 \quad 0.2062 \quad 0.2371 \quad 0.1598] \]
the 30%, 60%, and 90% probability bands, respectively. Figures 1-2 consider the two extremes of the range of acceptable values for the interest rate smoothing preferences in equation (28), i.e. $\nu = 0.1$ and $\nu = 0.005$.

Figure 1 features a high preference for interest rate smoothing. Here visual inspection shows that the volatility of the macrovariables distribution and the perturbing impact of the shock tend to be higher under CPI inflation targeting. In Figure 2, switching to interest rate smoothing indifference the previous result is strongly amplified: domestic inflation targeting implies much less volatility of the projections of the economy, in particular of the interest rates, and a surprisingly better ability to absorb the cost-push shock. As we would have expected, under CPI inflation targeting the optimal monetary policy attempts to absorb the cost-push shock using the exchange rate. This is reflected in the initial decrease of import inflation, $\pi^i$, shown in the sixth row, second column. What is interesting here is that this manoeuvre manages to better absorb the impact of the shock on CPI inflation only in a negligible way as one can see in the last row comparing the distribution forecasts in both cases. Furthermore, in terms of volatility of the distribution forecast, CPI inflation, $\pi^c$, does not seem to be less volatile under CPI inflation targeting. Thus, with a cost-push shock, shifting from CPI inflation targeting to domestic inflation targeting would have a negligible cost in terms of higher CPI inflation versus a large benefit in terms of the volatility of the distribution forecast of all the other variables and the perturbing impact of the shock. This result is in line with Cassino, Drew and McCaw (1999) as to output and nominal interest rate, yet it differs with respect to CPI inflation and the real exchange rate, which in their model feature lower variability under CPI inflation targeting\(^\text{10}\).

It is also worth noting that switching from high to low interest rate smoothing, which corresponds to passing from a mild to an aggressive policy, the overall volatility of the economy does not increase much with domestic inflation while it gets huge with CPI inflation. Thus, preferences on domestic inflation stand out also for much less sensitiveness to abrupt changes of the interest rate.

### 3.2 Second moment of the $i$ and $y^d$ distributions with a cost-push shock

On the basis of the previous analysis with two extreme interest rate smoothing values, a natural question to ask is whether volatility is monotonous in the preferences for smoothing. Furthermore, one can wonder to what extent, if any, the amount of uncertainty affects this relationship. To address these questions, figures 3-4 focus on the nominal interest rate and figure 5-6 on the output-gap in the domestic sector. These variables have been selected as they stand out among the other macrovariables

\(^{10}\)Further analysis will be devoted to ascertain if reasons other than the different methodology used in these two works may account for this difference.
for the higher sensitivity of their distribution forecasts to the preferences of the central bank. Explaining these figures, each row presents four graphs which refer to the same uncertainty case but differ in the uncertainty level (from 10% to 40% for $x$ moving from 0.1 to 0.4 respectively). Each graph, in turn, displays the standard deviation of the distribution forecast of a variable for the considered period and for interest rate smoothing values in the admissible range. The four cases consider uncertainty on (i) the pass-through, (ii) the persistence of the households’ behaviour, (iii) the degree of price flexibility in the domestic sector, and (iv) general uncertainty which encompasses them all.

A first feature of figures 3-4 is that either the CPI inflation targeting surface is always and significantly above the domestic inflation targeting surface (in the uncertainty on the pass-through and general uncertainty cases, first row of figure 3 and second row of figure 4 respectively). Or the two surfaces tend to overlap with the domestic inflation targeting one slightly above the CPI one for small preferences on interest rate smoothing (in the cases of uncertainty on the persistence in the household and firms behaviour and uncertainty on the slope of the Phillips curve in the domestic sector, second row of figure 3 and first row of figure 4 respectively). This shows that under the pass-through and general uncertainty cases the CPI inflation targeting policy results systematically in a larger standard deviation for the distribution forecast of the interest rate than domestic inflation targeting. Instead, when we consider the cases of uncertainty on the persistence of the households behavior and on the degree of price flexibility in the domestic sector (slope of the Phillips curve), the standard deviations associated to these surface tend to be similar. Moving to the variability of the distribution forecast of the output gap in the domestic sector (figure 5-6), visual inspection reveals that in the pass-through, agents behaviour inertia and general uncertainty cases, CPI inflation targeting policy surface is always above the domestic inflation targeting policy one.

Second, the volatility of the distribution forecasts of the interest rate and the output gap are monotonically increasing in the preference for not smoothing the interest rate. Yet, it is interesting to note that decreasing interest rate smoothing the volatility under CPI inflation targeting tends to increase more than under domestic inflation targeting.

The relevance of these results lies in unveiling domestic inflation targeting as a policy that leads to less variability of the distribution forecasts of the interest rate and the output gap. Also, it is less sensitive to interest rate smoothing than the CPI inflation targeting policy. Since the interest rate smoothing preference is inversely linked with the preferences for the other target variables\textsuperscript{11}, it follows that with domestic inflation targeting the central bank can stabilize the output gap and inflation with a lower cost in terms of a rough path of the interest rate\textsuperscript{12}.

\textsuperscript{11}To see this, just scale the weighs in the loss function.

\textsuperscript{12}For an overview of the costs associated with a rough interest rate see Appendix C in Alcidi,
In order to compare the policies associated to the two surfaces it is informative to compute the ratio of the means (along all the smoothing preferences values and the periods considered) of the standard deviations in the two policy cases, i.e.

\[
R^\sigma = \frac{\text{mean}_{i,t} (i_{std}^c_{\nu,t})}{\text{mean}_{i,t} (i_{std}^d_{\nu,t})},
\]

where \(i_{std}^c_{\nu,t}\) and \(i_{std}^d_{\nu,t}\) denote the standard deviation of the interest rate distribution forecast for period \(t\), and smoothing preferences value \(\nu\), and \(c\) and \(d\) denote CPI and domestic inflation targeting, respectively. Table 3 presents the statistics \(R^\sigma\) when \(T = 15\) for various uncertainty types and levels and considering a cost-push shock.

<table>
<thead>
<tr>
<th>Uncertainty type</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass-through</td>
<td>1.65</td>
<td>2.14</td>
<td>2.48</td>
<td>2.51</td>
</tr>
<tr>
<td>Private sector inertia</td>
<td>0.65</td>
<td>0.66</td>
<td>0.72</td>
<td>0.79</td>
</tr>
<tr>
<td>Price flexibility in domestic sector</td>
<td>0.79</td>
<td>0.80</td>
<td>0.82</td>
<td>0.84</td>
</tr>
<tr>
<td>General</td>
<td>1.68</td>
<td>2.22</td>
<td>2.58</td>
<td>2.59</td>
</tr>
</tbody>
</table>

**Table 3.** \(R^\sigma\) for the nominal interest rate.

This analysis shows that the uncertainty cases where domestic inflation preferences dominates CPI inflation preferences are more relevant than the other cases of at least one order of magnitude. Furthermore when we focus on the more representative case of general uncertainty, which includes all the previous cases, domestic inflation targeting dominates CPI inflation targeting. The \(R^\sigma\) statistic relative to the distribution forecast of the domestic sector output gap are presented in table 4.

<table>
<thead>
<tr>
<th>Uncertainty type</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass-through</td>
<td>2.26</td>
<td>2.17</td>
<td>2.20</td>
<td>2.15</td>
</tr>
<tr>
<td>Private sector inertia</td>
<td>1.27</td>
<td>1.26</td>
<td>1.26</td>
<td>1.25</td>
</tr>
<tr>
<td>Price flexibility in domestic sector</td>
<td>0.97</td>
<td>0.99</td>
<td>1.02</td>
<td>1.03</td>
</tr>
<tr>
<td>General</td>
<td>1.40</td>
<td>1.51</td>
<td>1.63</td>
<td>1.68</td>
</tr>
</tbody>
</table>

**Table 4.** \(R^\sigma\) for the output gap in the domestic sector.

Here domestic inflation targeting tends to dominate CPI inflation targeting in all the uncertainty cases. In particular, the general uncertainty case shows that the average variability of the distribution forecast for the output gap with the CPI policy is 1.5 times larger than with the other policy.

### 3.3 Different policies and the perturbing impact of a cost push shock on \(i\) and \(y^d\)

An interesting question to ask is how central bank preferences rank in terms of the perturbing impact of exogenous shocks on the economy. The medians of the dis-
tribution forecasts provide a *prima facie* answer. Figures 7-8 and 9-10 illustrate the medians of the distribution forecasts of the nominal interest rate and the domestic output gap, respectively. These medians for a range of values of interest rate smoothing preferences and different periods are described by two surfaces for CPI and domestic inflation targeting policies. Denoting these surfaces as median surfaces, the *distance* of the median surface from zero provides a measure of the median perturbing impact of the shock. Visual inspections shows that the distance of the median surface from zero under CPI inflation targeting is always larger than under domestic inflation targeting. Recalling that the model variables are log deviations from their steady state values, this results shows that preferences on domestic inflation rather than CPI inflation allow the nominal interest rate and the output gap to deviate less from the long-run equilibrium after a cost push shock.

In order to quantitatively compare the distance of the two surfaces from zero it is informative to introduce the ratio of the mean of the medians in the two policy cases for all the smoothing preferences values and the periods considered, that is

\[
R_M = \sum_{\nu \in N} \sum_{t=0}^{T} \frac{|\text{median}^\nu_{c,t}|}{|\text{median}^\nu_{r,t}|},
\]

Tables 5 and 6 report \(R_M\) with respect to the nominal interest rate and the output gap respectively.

<table>
<thead>
<tr>
<th>Uncertainty type</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass-through</td>
<td>2.79</td>
<td>2.74</td>
<td>2.58</td>
<td>2.23</td>
</tr>
<tr>
<td>Private sector inertia</td>
<td>2.34</td>
<td>2.41</td>
<td>2.49</td>
<td>2.56</td>
</tr>
<tr>
<td>Price flexibility in domestic sector</td>
<td>2.32</td>
<td>2.44</td>
<td>2.65</td>
<td>2.79</td>
</tr>
<tr>
<td>General</td>
<td>2.87</td>
<td>3.01</td>
<td>3.14</td>
<td>2.95</td>
</tr>
</tbody>
</table>

*Table 5. \(R_M\) for the nominal interest rate.*

<table>
<thead>
<tr>
<th>Uncertainty type</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass-through</td>
<td>1.51</td>
<td>1.59</td>
<td>1.64</td>
<td>1.69</td>
</tr>
<tr>
<td>Private sector inertia</td>
<td>1.46</td>
<td>1.46</td>
<td>1.48</td>
<td>1.51</td>
</tr>
<tr>
<td>Price flexibility in domestic sector</td>
<td>1.47</td>
<td>1.54</td>
<td>1.64</td>
<td>1.72</td>
</tr>
<tr>
<td>General</td>
<td>1.54</td>
<td>1.76</td>
<td>2.17</td>
<td>2.95</td>
</tr>
</tbody>
</table>

*Table 6. \(R_M\) for the output gap in the domestic sector.*

Table 5 shows that if the central bank chooses the CPI inflation targeting policy, the nominal interest rate is expected to be perturbed by a cost-push shock approximately 2/3 times more than if it chooses the domestic inflation targeting policy.
Similarly, Table 6 shows that the domestic output gap is 1.5 times more perturbed under CPI inflation targeting than under domestic inflation targeting.

### 3.4 Extending the analysis to other relevant shocks and macrovariables

To obtain a more complete picture of the relation between the preferences on the inflation index, distribution forecasts accuracy and economic stability I extend the analysis to other macrovariables and shocks. The further macrovariables considered are CPI and domestic inflation, $\pi^c$ and $\pi^d$ respectively, the short term real interest rate, $r$, and the real exchange rate, $q$. Further shocks considered are a shock to the aggregate demand in the domestic sector, to the foreign interest rate, and to the risk premium. Summarising the results for the general uncertainty case\(^{13}\), the analysis shows that

1. CPI inflation, $\pi^c$, is less volatile and perturbed by any shock considered with CPI inflation targeting, while domestic inflation, $\pi^d$, is less volatile and perturbed with domestic inflation targeting. Clearly, this result reflects the preferences of the central bank in the two cases. Interestingly, with domestic inflation targeting the prediction of $\pi^d$ is more accurate than the prediction of $\pi^c$ with the inflation targeting policy;

2. with respect to the output gap, $y^d$, domestic inflation targeting is dominant in terms of less volatility and sensitivity to a cost-push and a foreign interest rate shock, while for the aggregate demand and risk premium shocks the results are not conclusive;

3. with respect to the short nominal and real interest rates, $i$ and $r$ respectively, domestic inflation targeting is dominant in terms of less volatility and sensitivity to any shock but the aggregate demand one. This is clear as the presence of import inflation in CPI inflation buffers part of the demand shock requiring a less active intervention of the central bank under CPI inflation targeting. It is worth noting that with the foreign interest rate shock the impact of the shock is much higher with CPI inflation targeting than with domestic inflation targeting;

4. with respect to the real exchange rate, $q$, domestic inflation targeting is dominant under a cost-push shock in terms of volatility and shock sensitivity and under an aggregate demand shock in terms of shock sensitivity. For the other cases, CPI inflation targeting tends to be the dominant policy. Indeed risk

\(^{13}\)The complete set of results in terms of the ratio of the means of the standard deviation associated to the policy surfaces, $R^{s}$, and the ratio of the mean of the medians associated to the policy surfaces, $R^{M}$, is available upon request.
premium and foreign interest rate shocks impact on the nominal exchange rate via the uncovered interest parity. Then, if the central bank does not react, the shock propagates to CPI inflation. Thus with CPI inflation targeting the central bank responds to this shock resulting in less variability of the nominal exchange rate, which in turn implies less variability of CPI inflation and finally less variability of the real exchange rate. Yet, the central bank may not be willing to react to shocks that affect the nominal exchange rate. Leitemo and Söderstrom (2005) maintain that it should not. Their argument is that there is uncertainty about how the exchange rate is determined and the effect of exchange rate movements on the economy. This implies that rules with the exchange rate are more sensitive to model uncertainty. Thus a monetary policy developed in the context of one exchange rate model could perform poorly if that model is incorrect. Evidence in this respect is not conclusive. Lubik and Schorfheide (2007) find that Australia and New Zealand did not react to movements in the exchange rate while Canada and UK did.

3.5 Interpreting the results

This paper shows overall that CPI inflation targeting leads to more economic instability and less accurate distribution forecasts than domestic inflation targeting. Two key differences between CPI and domestic inflation provide an explanation for this result. The first is that domestic and import inflation differ in their degree of predeterminism in pricing decisions which affects price flexibility. In fact, the domestic sector produces and retail domestic goods while the import sector only retails foreign goods. Clearly, production decisions take more time to be implemented than retailing decisions. This motivates the assumption that in the domestic sector pricing decisions are predetermined two-period in advance and in the import sector they are not predetermined\textsuperscript{14}. Predeterminess in pricing decisions affects inversely price flexibility and, as in Mankiw and Reis (2002), targeting inflation in the sector with less flexible prices results in more economic stability.

The second difference is that CPI inflation is strongly related to the nominal exchange rate while domestic inflation it is not\textsuperscript{15}. This link adds the monetary policy exchange rate channels to the aggregate demand channel. As a result, in a flexible inflation targeting framework, the more the central bank attempts to stabilize the

\textsuperscript{14}These assumptions are consistent with the central bank conventional wisdom on the transmission mechanism according to which monetary policy needs a couple of period to have its largest impact on domestic inflation and a shorter time span to affect import inflation via the exchange rate.

\textsuperscript{15}Domestic inflation depends on the exchange rate because production in the domestic sector uses intermediate import goods. Yet, the main input in this sector is the domestic composite good so that \( \pi_d \) depends on a small extent on the exchange rate. In particular, the share of the import good in the composite input required to produce the differentiated domestic good \( j \), i.e. \( \mu \) in equation (11) is set equal to 0.1.
output gap, the more it can generate exchange rate volatility that feeds back via the exchange rate channels mostly to CPI inflation. Volatility in the exchange rate can also be generated by foreign shocks via the uncovered interest parity. In this case to avoid that exchange rate volatility leads to CPI inflation volatility the central banks has to intervene. Thus under CPI inflation targeting, in presence of foreign shocks there is a trade-off between interest rate and CPI inflation volatility.

4 Distribution forecast accuracy and future policy intentions

The current paper shows that an inflation targeting central bank that chooses domestic instead of CPI inflation stabilization improves the quality of its forecasts in particular for the interest rate. Importantly, this result relates to the recent debate on the instrument-rate assumption underlying projections of target variables. The debate arises from two alternative strategies facing monetary policy: either publishing the optimal instrument-rate plan and the corresponding projections of the economy, or publishing the projections of the economy based on a specific assumption on the interest rate, e.g. the assumption of constant interest rate or an interest rate path given by market expectations. The first alternative has been pioneered by the Reserve Bank of New Zealand and recently been adopted by the Norges Bank and the Riksbank. The choice among these alternatives can be related to central bank transparency and implies a positive externality for monetary policy efficiency via the management of the private sector expectations. In fact, the more accurate and reliable the central bank distribution forecasts, the more the central bank can affect the private sector expectations if it chooses to be transparent and disclose future policy intentions. Hence, by showing the existence of a relationship from the central bank preferences to its distribution forecasts, and that this mechanism could be exploited to improve the quality of the forecasts, this paper favours the alternative of publishing the distribution forecast of the optimal instrument-rate plan and the corresponding projections of the other macrovariables.

5 Conclusions

This paper presents a novel relationship from the central bank preferences to the quality of its forecasts and to the size of the economy-wide perturbation following various shocks in presence of general model uncertainty.

It first shows that domestic inflation targeting tends to result in considerably less variability of the distribution forecast of the economy’s dynamics. In particular, the

\footnote{For a discussion of these alternatives, see for example Goodhart (2005), Honkapohja and Mitra (2005), Qvigstad (2006), Svensson (2005) and Woodford (2005).}
variability of the distribution forecasts for the interest rates, domestic inflation, and the output gap are much larger with CPI inflation targeting for almost all the shocks considered.

Second, the median of the distribution forecasts of the interest rates, domestic inflation, and the domestic output gap under almost all the shocks considered is always larger in absolute value under CPI inflation targeting then under domestic inflation targeting. This shows that the economy is more perturbed by shocks under CPI inflation targeting.

On the other hand, if the central bank has a special interest in reducing the volatility and the shock sensitivity of CPI inflation, and in some cases of the real exchange rate, then CPI inflation targeting seems a more proper policy. However, it is important to note that the gain in terms of CPI stabilization turn out to be small while the cost in terms of instability of the other macrovariables high.

Finally, the paper shows that preferences on smoothing the interest rate do not affect much the behavior of the economy under domestic inflation targeting, while they do affect it under CPI inflation targeting. Arguably, central banks may not have any preferences on smoothing the interest rate (see Rudebusch 2002, 2006). Yet, if they do, the relevance of this result lies in allowing the central bank to stabilize the output gap and inflation with a lower cost in terms of a rough path of the interest rate.

The policy implication of these results is that, all other things equal, switching to domestic inflation will improve the quality of the expectations as well as the expected deviation between the position of the economy after a shock and its steady state equilibrium. Possibly, the scope of these results is underestimated. Mankiw and Reis (2002) show that the greater the magnitude of idiosyncratic shocks in a sector, the less its inflation measure should be targeted for stabilizing purposes. Mankiw and Reis adopt a static setup. To the extent that their result carries over to the intertemporal framework and the import sector is more exposed than the domestic sector to exogenous shocks via the exchange rate, the scope of the results in the current paper would be larger.

These results also present an additional reason for central banks to publish the optimal instrument-rate plan and the corresponding projections of the economy. In fact, using domestic inflation instead of CPI inflation it is possible to obtain more accurate forecasts of the economy’s dynamics. If the accurateness of the forecasts increases the potential accountability of the central bank, it can also improve the effectiveness of the monetary policy via the management of the private sector expectations. Thus, these results break a lance on the alternative to publish the projections of the economy other than the ones corresponding to the optimal interest rate path expected by the central bank.

Further extensions left to future analysis consists of investigating the distribution
forecasts when the central bank has a preference on the stabilization of the real exchange rate, and considering the case where modes are not observable.

6 Appendix

To write the model in state-space form note that

\[
\begin{align*}
\pi^d_{t+2|t+1} &= \frac{\pi^d_{t+2|t} + \xi_t}{1 + \xi_t}, \\
\rho_{t+1|t} &= \rho_t - i_t + \pi^d_{t+1|t}, \\
q_{t+1|t} &= q_t + \pi^d_{t+1|t} - \pi^d_{t+1|t}.
\end{align*}
\]

(33) (34)

Then, take the expectation in period \(t\) of equation (21) and solve it for \( \pi^d_{t+3|t} \):

\[
\pi^d_{t+3|t} = (1 + \xi_t) \pi^d_{t+2|t} - \xi_t \left[ \omega y^d_{t+2|t} + \mu q_{t+2|t} \right].
\]

(35)

where \( \xi_t \equiv \frac{(1-\alpha_t)^2}{\alpha_t(1+\omega q)} \). Next, lead equation (16) one period and take the expectation in period \(t\). Then apply the same procedure to (33), (34) and (23) and substitute for \( \rho_{t+2|t}, q_{t+2|t} \) and \( \pi^i_{t+2|t} \) in the equation for \( y^d_{t+2|t} \). This gives

\[
y^d_{t+2|t} = \beta_y y^d_{t+1|t} - \beta_y \rho_{t+1|t} + \beta_y \pi^i_{t+1|t} - (\beta_y + \beta_q) \pi^d_{t+2|t} + (\beta_q - \beta_{q-1}) q_t + [\beta_q (2 + \xi_t) - \beta_{q-1}] \pi^i_{t+1|t} - (\beta_q - \beta_{q-1}) \pi^d_{t+1|t} - \beta_q \xi_t \pi^i_{t+1|t} - \beta_q \xi_t \omega y^d_{t+1|t} - \beta_q \xi_t y^i_{t+1|t} + \beta_y y^i_{t+1|t} + \beta_y y^d_{t+1|t},
\]

where \( \xi^i_t \equiv \frac{(1-\alpha_t)^2}{\alpha^i_t(1+\omega q)} \). Finally, substitute for \( y^d_{t+2|t} \) and \( q_{t+2|t} \) in (35). This gives

\[
\pi^d_{t+3|t} = [1 + \xi_t + \xi_t \omega (\beta_Y + \beta_q) + \xi_t \mu] \pi^d_{t+2|t} + \xi_t \mu - \xi_t + \xi_t \omega (\beta_q - \beta_{q-1})] \pi^d_{t+1|t} \\
+ (\xi_t \omega \beta_Y \xi^i_t + \xi_t \mu \xi^i_t) y^d_{t+1|t} - \xi_t [\omega (\beta_q (2 + \xi_t) - \beta_{q-1}) + \mu (1 + \xi_t) + \mu] \pi^i_{t+1|t} \\
- \xi_t \omega \beta_Y \xi^i_{t+1|t} + \xi_t \omega \beta Y \rho_{t+1|t} - \xi_t \omega \beta_Y \pi^i_{t+1|t} + \xi_t \omega \beta_q - \beta_{q-1} + \mu \pi^i_{t+1|t} \\
+ \omega \xi_t \xi^i_t (\omega \beta_q + \mu) y^d_{t+1|t} - \omega \xi_t \beta_Y y^i_{t+1|t} - \omega \xi_t \beta_Y y^d_{t+1|t} - \xi_t \omega (\beta_q - \beta_{q-1} + \mu) q_t.
\]

It follows that
where $e_j$, $j = 0, \ldots, n$ stands for a $1 \times n$ row vector that for $j = 0$ has all the elements equal to zero and for $j \neq 0$ has element $j$ equal to unity and all other elements equal to zero; $A_j$ stands for row $j$ of the matrix $A$ and

$$
A_n = [1 + \zeta_t + \xi_t \omega (\beta_p + \beta_q) + \xi_t \mu] e_n + [\xi_t \mu - \zeta_t + \xi_t \omega (\beta_q - \beta_{q-1})] A_1 + (\xi_t \omega \beta_q \xi_t + \xi_t \mu \xi_t) A_n X + \xi_t [\omega (\beta_q (2 + \xi_t) - \beta_{q-1} + \mu (1 + \zeta_t) + \mu] A_n X + 1
$$

Finally the vectors $B$ and $B^1$ are given by
\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & \beta_p & \beta_q & 0 & 0 & 0 & 1 & 0 & 0 & 0 & (1 - \mu_i^t) & -1 \\
-\xi_t \omega \beta_p (1 + \beta_y) + \xi_t \xi_t^i (\omega \beta_q + \mu) (1 - \mu_i^t) & + & \xi_t \xi_t^i \omega (\beta_q \omega + \mu) \beta_p^i \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad B^3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\xi_t \omega \beta_p \\
\end{bmatrix}
\]


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Figure 1. Unconditional impulse responses to a cost-push shock under domestic and CPI inflation targeting for $\nu = 0.1$ and assuming uncertainty on the speed and completeness of the pass-through.
Figure 2. Unconditional impulse responses to a cost-push shock under domestic and CPI inflation targeting for $v = 0.005$ and assuming uncertainty on the speed and completeness of the pass-through.
Figure 3. STD of the distribution forecast of $i$ for the cases of uncertainty on the pass-through (first row) and on the inertia in the households and firms behaviours (second row)
Figure 4. STD of the distribution forecast of $i$ for the cases of uncertainty on the degree of price stickiness in the domestic sector (first row) and for the general uncertainty case (second row).
Figure 5. STD of the distribution forecast of domestic $y$ for the cases of uncertainty on the pass-through (first row) and on the inertia in the households and firms behaviours (second row)
Figure 6. STD of the distribution forecast of domestic $y$ for the cases of uncertainty on the degree of price stickiness in the domestic sector (first row) and for the general uncertainty case (second row).
Figure 7. Median of the distribution forecast of $i$ for the cases of uncertainty on the pass-through (first row) and on the inertia in the households and firms behaviours (second row).
Figure 8. Median of the distribution forecast of $i$ for the cases of uncertainty on the degree of price stickiness in the domestic sector (first row) and for the general uncertainty case (second row)
Figure 9. Median of the distribution forecast of domestic $y$ for the cases of uncertainty on the pass-through (first row) and on the inertia in the households and firms behaviours (second row)
Figure 10. Median of the distribution forecast of domestic y for the cases of uncertainty on the degree of price stickiness in the domestic sector (first row) and for the general uncertainty case (second row)