Hot and Cold Seasons in the Housing Market*

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Abstract

Every year during the second and third quarters (the "hot season") housing markets in the U.K. and the U.S. experience systematic above-trend increases in both prices and transactions. During the fourth and first quarters (the "cold season"), housing prices and transactions fall below trend. A similar seasonal cycle is observed in other developed countries. We present a search-and-matching model that can quantitatively mimic the seasonal fluctuations in transactions and prices observed in the U.K. and the U.S. The model features "thick-market" effects that can generate substantial differences in the volume of transactions and prices across seasons, with the extent of seasonality in prices depending positively on the bargaining power of sellers. As a by-product, the model sheds new light on the mechanisms governing fluctuations in housing markets and can be adapted to study lower-frequency movements in prices and transactions.

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1 Introduction

A rich empirical and theoretical literature has been motivated by dramatic boom-to-bust episodes in regional and national housing markets.[1] Booms are typically defined as times when prices rise and there is intense trading activity, whereas busts are times when prices and trading activity fall below trend.

While the boom-to-bust episodes motivating the extant work are relatively infrequent and of unpredictable timing, this paper shows that in several housing markets, booms and busts are just as frequent and predictable as the seasons. In particular, in all regions of the U.K. and the U.S., as well as other continental European countries, every year a housing boom of considerable magnitude takes place in the second and third quarters of the calendar year (the "hot season"), followed by a bust in the fourth and first quarters (the "cold season"). The predictable nature of housing prices fluctuations is furthermore confirmed by U.K. estate agents, who in conversations with the authors observed that during winter months there is less activity and owners tend to sell at a discount. And, perhaps more compelling, publishers of house price indexes go to great lengths to produce seasonally adjusted versions of their indexes, usually the index that is published in the media. As stated by the publishers:

"House prices are higher at certain times of the year irrespective of the overall trend. This tends to be in spring and summer, when more buyers are in the market and hence sellers do not need to discount prices so heavily in order to achieve a sale." and *"...we seasonally adjust our prices because the time of year has some influence. Winter months tend to see weaker price rises and spring/summer see higher increases all other things being equal."* (From Nationwide House Price Index Methodology.)

"Houses prices are seasonal with prices varying during the course of the year irrespective of the underlying trend in price movements. For example, prices tend to be higher in the spring and summer months when more people are looking to buy." (From Halifax Price Index Methodology.)

The first contribution of this paper is to systematically document the existence, quantitative importance, and cross-regional variation of these seasonal booms and busts.[2]

[1] See for example Stein (1995), Muellbauer and Murphy (1997), Genesove and Mayer (2001), Krainer (2001), Ortalo-Magne and Rady (2005), Brunnermeier and Julliard (2008), and the contributions cited therein.

[2] Studies on housing markets have typically glossed over the issue of seasonality. There are a few exceptions, albeit they have been confined to only one aspect of seasonality (e.g., either quantities or prices) and/or to a relatively small geographical area. In particular, Goodman (1991) documents pronounced seasonality in *moving patterns* in

The surprising size and predictability of seasonal fluctuations in housing prices poses a challenge to standard models of durable-good markets. In those models, anticipated changes in prices cannot be large: If prices are expected to be much higher in May than in December, then buyers will shift their purchases to the end of the year, narrowing down the seasonal price differential. More formally, in the absence of risk, the asset-market equilibrium condition states that the one-period rental value of a house plus its appreciation should equal the one-period gross cost of housing services.[3] Calling p_t and d_t the real price of housing and rental services, respectively, and assuming that the gross real service cost is a (potentially changing) proportion c_t of the property price, the equilibrium asset-market condition is:

$$d_{t+1} + (p_{t+1} - p_t) = c_t \cdot p_t \tag{1}$$

where c_t is the sum of the (potentially time-varying) depreciation rate, maintenance and repair expenditure rate, property tax rate, and the tax-adjusted interest rate.[4] The arbitrage condition thus states that the seasonals in real prices must be accompanied by seasonals in the cost of housing services c_t or in the rental service flow d_t. Rents, however, display no seasonality, implying a substantial and, as we shall argue, unrealistic degree of seasonality in service costs c_t.[5,6,7]

It is important to remark that no-arbitrage conditions such as (1) may become irrelevant when

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the U.S., Case and Shiller (1989) find seasonality in prices in Chicago and—to a lesser extent—in Dallas, and Hosios and Pesando (1991) find seasonality in prices in the City of Toronto; the latter conclude "that individuals who are willing to purchase against the seasonal will, on average, do considerably better."

[3]For an early asset-market approach to the housing market, see Poterba (1984).

[4]The effective interest rate is a weighted average between mortgage interest rate plus the opportunity cost of housing equity, where the weights are given by the loan-to-value ratio.

[5]The paucity of (good) data on rents (and particularly, new rents) may be one reason why we do not find seasonality in the data. Note, however, that one should observe extremely high levels of seasonality in rents (together with extremely high discount rates) for rents to be driving seasonality in prices; this is because prices should in principle reflect the present discounted value of a presumably long stream of rental services. Data on rentals, however, display no discernible pattern of seasonality.

[6]For example, the degree of price seasonality observed in the U.K. implies that service costs should be at least 300 percent higher in the cold season than in the hot season—see Appendix 7.1. This seems unlikely, particularly because interest rates and tax rates, two major components of c_t, display no seasonality.

[7]The seasonal in housing markets does not seem to be driven by seasonal differences in liquidity related to overall income. Income is typically high in the last quarter, a period in which housing prices and the volume of transactions tend to fall below trend. Beaulieu and Miron (1992) and Beaulieu, Miron, and MacKie-Mason (1992) show that in most countries, including the U.K., income peaks in the fourth quarter of the calendar year. There is also a seasonal peak in output in the second quarter, and seasonal recessions in the first and third quarters. Housing price

transaction costs are very high, as it is likely to be the case in housing markets.[8] Still, the question remains as to why (presumably informed) buyers systematically tend to buy in the high-price season. The seasonal behavior of housing prices and the failure of *a priori* appealing explanations, thus poses a challenge to models of the housing market based on standard asset-pricing conditions. This paper offers a novel explanation by resorting to a search-and-matching framework.

The model starts from the premise that every house is a little different and families have different housing needs. In that context, buyers are more likely to find a better-quality match (and thus their willingness to pay is more likely to increase) when there are more houses for sale.[9] Hence, in a thick market (or hot season), sellers can charge higher prices. Because prices are higher, more houses are put up for sale, better matches are formed, and so on. This self-reinforcing dynamics leads to higher number of transactions and prices in the hot season.

In the baseline model, we distinguish seasons by differences in the ex-ante propensity to move. These differences may arise, for example, from the school calendar: Families may prefer to move in the summer, before sending their children to new schools;[10] good weather may also make the search more convenient in the summer. We show that a higher *ex-ante* probability of moving in a given season can trigger thick-market externalities that make it appealing to a large number of agents to buy and sell during that season. This amplification mechanism can create substantial seasonality in transactions; the extent of seasonality in prices increases with the bargaining power of sellers. The calibrated model can quantitatively account for most of the seasonal fluctuations in transactions and prices in the U.K. and the U.S.[11]

The contribution of the paper can be summarized as follows. First, it systematically documents seasonal booms and busts in housing markets; it argues that the predictability and high extent of seasonality in prices observed in some of them cannot be quantitatively reconciled with the standard asset-pricing equilibrium condition embedded in most models of housing markets (or

seasonality is not in line with income seasonality: prices are above trend in the second and third quarters. Finally, mortgage rates and tax rates do not display any seasonal pattern.

[8]See, for example, Englund and Quigley ().

[9]The labor literature distinguishes the thick-market effects due to faster arrival of offers and those due to the quality of the match. See for example Diamond (1981) and Petrongolo and Pissarides (2006). Our focus is entirely on the quality effects.

[10]See Goodman (1991) and Harding, Rosenthal, and Sirmans (2003).

[11]Our focus on these two countries is largely driven by the reliability and quality of the data.

consumer durables, more generally). Second, it develops a search-and-matching model that can quantitatively account for the seasonal patterns of prices and transactions observed in the U.K. and the U.S., shedding new light on the mechanisms governing fluctuations in housing markets. As a by-product, the model can be adapted to study lower-frequency movements in prices and transactions.

The paper is organized as follows. Section 2 presents the empirical evidence and discusses different potential (though ultimately unsuccessful) explanations; it discusses why, given the evidence, we need to deviate from the standard asset-pricing approach to housing markets. Section 3 presents the model. Section 4 presents a quantitative analysis of the model and confronts it with the empirical evidence. Section 5 presents extensions of the baseline model; in particular, it studies the role of seasonal transaction and moving costs as alternative drivers of seasonality in housing markets. Section 6 presents concluding remarks. Analytical derivations and proofs are collected in the Appendix.

2 Hot and Cold Seasons

This section documents the behavior of housing prices across what we refer to as the two main seasonal terms: the summer term (second and third quarters of the calendar year) and the winter term (first and fourth quarters) in different countries and regions within a country.

2.1 Data

In the analysis we shall pay particular attention to the housing-market records of the U.K. and the U.S., the countries for which the data are of highest quality. Below is a brief description of the data on prices and transactions in these two countries. A description of the data sets and sources for other countries studied in this Section is available in the Data Appendix.

U.K.

In the U.K. there are two main data sets providing quality-adjusted non-seasonally adjusted prices: the Halifax House Price Index, derived from the data collected by Halifax, one of the country's largest mortgage lenders, and the price index produced by the Office of the Deputy

Prime Minister (ODPM).[12]

Halifax reports regional indexes on a quarterly basis for the 12 standard planning regions of the U.K., as well as for the U.K. as a whole. The indexes calculated are 'standardized' and represent the price of a typically transacted house. The standardization is based on hedonic regressions that control for a number of characteristics, including location, type of property (house, sub-classified according to whether it is detached, semi-detached or terraced, bungalow, flat), age of the property, tenure (freehold, leasehold, feudal), number of rooms (habitable rooms, bedrooms, living-rooms, bathrooms), number of separate toilets, central heating (none, full, partial), number of garages and garage spaces, garden, land area, and road charge liability. Accounting for these characteristics allows to control for the possibility of seasonal changes in the composition of the set of properties (for example, shifts in the location or sizes of properties). The index reports transaction prices based on mortgages to finance house purchase at the time the mortgage is approved; re-mortgages and further advances are excluded.

The ODPM index is based on the same method as is the Halifax index, and differs only in two respects. First, it collects information from a large sample of all mortgage lenders in the country.[13] And second, it reports the price at the time of completion, rather than approval. Completion might take on average three to four weeks after the agreement, due generally to paper-work delays. The ODPM index goes back to 1963, though only after 1993 does it include all mortgage lenders (before that time, prices are based on Building Societies reports).

To compute real price indexes, we later deflate the housing price indexes using the non-seasonally adjusted retail price index (RPI) including "All items except housing" provided by the U.K. Office for National Statistics.

[12] Other price indices, like Nationwide Building Society, report quality adjusted data but they are already seasonally adjusted (the NSA data are not made publicly available). Nationwide Building Society, however, reports in its methodology description that June is generally the strongest month for house prices (with prices typically 1.3% above their SA level) and January is the weakest (with prices 1.9% below their SA level), differences that are comparable to the numbers we reported in Figure 1; this justifies the SA they perform in the published series. In a somewhat puzzling paper, Rosenthal (2006) argues that seasonality in Nationwide Building Society data is elusive; we could not, however, gain access to the NSA data to asses which of the two conflicting assessments (Nationwide Building Society's or Rosenthal's) was correct. We should perhaps also mention that Rosenthal (2006) finds different results from Muellbauer and Murphy (1997) with regards to lower-frequency movements.

The Land Registry data reports average prices, without adjusting for quality.

[13] The Halifax index uses all the data from Halifax mortgages.

As an indicator of the number of transactions, we use the number of mortgages advanced for home purchases; the data are collected by the ODPM through the Survey of Mortgage Lenders and are disaggregated by region.

U.S.

The non-seasonally adjusted price index for the U.S. comes from the Office of Federal Housing Enterprise Oversight (OFHEO), which in turn builds its index from data provided by Fannie Mae and Freddie Mac, the biggest mortgage lenders; this is a repeat-sale, purchase-only index (and hence, barring depreciation, quality is kept constant). The index is calculated for the whole of the U.S. and also disaggregated by state (the 50 states and the District of Columbia) and by the 379 metropolitan statistical areas defined by OFHEO. We also study the Case-Shiller index carried out by Standard & Poor's for 20 big cities (and a composite of the 20 cities); this index is also a repeat-sale, purchase-only index.

To compute real price indexes, we use the non-seasonally adjusted consumer price index (CPI) including "All items less shelter" provided by the U.S. Bureau of Labor Statistics.[14]

Data on the number of transactions come from the National Association of Realtors, and correspond to the number of sales of existing single-family homes. The data are disaggregated into the four major Census regions.

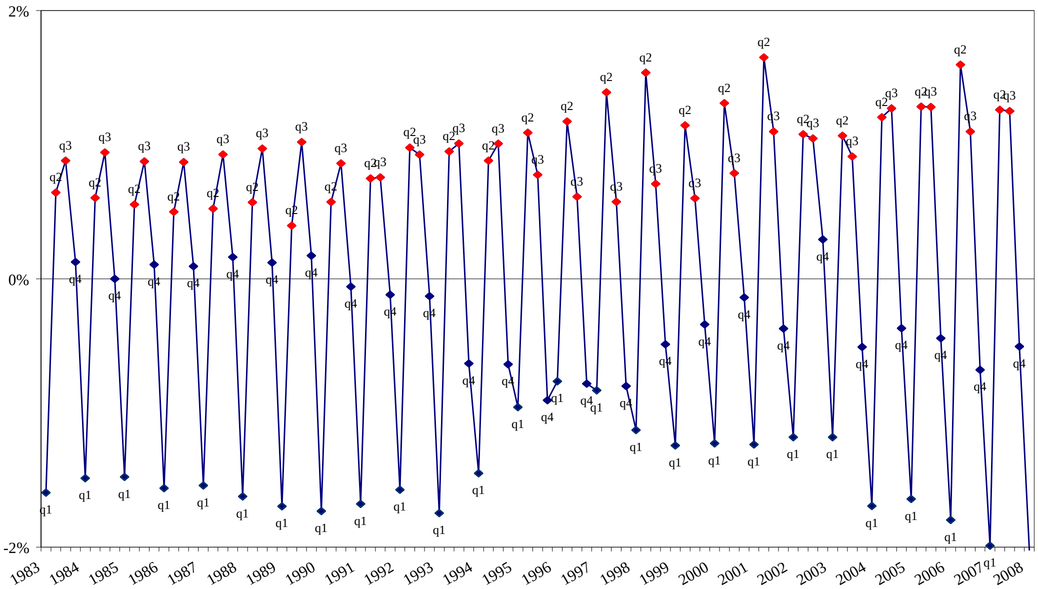
2.2 The Cross-Country Evidence

This Section briefly summarizes the cross-country evidence on seasonal fluctuations in housing prices and transactions. We start, by way of motivation, by showing the price data for the U.K. and the U.S. as a whole. Figures 1 and 2 illustrate the price patterns for the U.K. and the U.S. Figure 1 shows the (log) of the non-seasonally adjusted (NSA) housing price series, P_t, relative to the seasonally adjusted (SA) series, P_t^*, from 1983:01 to 2008:01, $\left\{\ln \frac{P_t}{P_t^*}\right\}$, in the U.K. Both the NSA and the SA series come from Halifax and correspond to the U.K. as a whole. Figure 2 shows the corresponding figures for the U.S.'s OFHEO purchase-only index from 1991:01 through 2008:01 (both the NSA and SA series come from OFHEO). The start of the sample, in both cases is dictated by data availability.

[14] As it turns out, there is little seasonality in the U.S. CPI index, a finding first documented by Barsky and Miron (1989), and hence the seasonal patterns in nominal and real housing prices coincide.

Both Figures seem to show a consistent pattern: housing prices in the second and third quarters tend to be above trend (captured by the NSA adjusted series), and prices in the fourth, and particularly in the first quarter, tend to be in general at or below trend. (Later on we show that this general pattern is also observed at finer levels of geographical aggregation for both countries). The Figures also make it apparent that the extent of price seasonality is more pronounced in the U.K. than in the U.S.

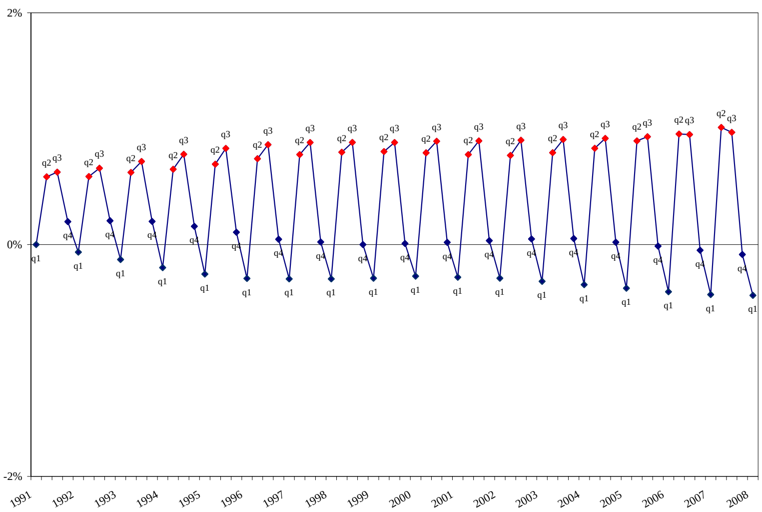
Figure 1: Seasonal Component of Housing Prices in the U.K.: $\left\{\ln \frac{P_t}{P_t^*}\right\}$.
Halifax NSA and SA Indices 1983:01-2008:01.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$, where P_t is the NSA price index and P_t^* is the corresponding SA index

Figure 2: Seasonal Component of Housing Prices in the U.S.: $\left\{ \ln \frac{P_t}{P_t^*} \right\}$.

OFHEO-purchase only NSA and SA Indices 1991:01-2008:01.



Note: The plot shows $\left\{ \ln \frac{P_t}{P_t^*} \right\}$, where P_t is the NSA price index and P_t^* is the corresponding SA index

Given the patterns above, we posit the following econometric model to summarize the extent of seasonality and make quantitative comparisons across countries and (later on, with Given the patterns above, we study more systematically whether the relative prices (and transactions) at the end of the broadly defined summer term, that is, at the end of the third quarter (p_3), relative to the prices (and transactions) at the end of the winter, that is, at the end of the first quarter (p_1), $\frac{p_3}{p_1}$ are significantly different from the ratio of winter to summer prices, $\frac{p_{1'}}{p_3}$ where $p_{1'}$ indicates the end-of-winter price in the following year. More specifically, and to focus on the seasonal variation, we study the difference between growth rates in summers relative to winters $\ln\left(\frac{p_3}{p_1}\right)$ and in winters relative to summers $\ln\left(\frac{p_{1'}}{p_3}\right)$. In the Tables that follow, we report the results from a test on the mean difference between these two growth rates (or relative prices):

$$Diff. = \ln\left(\frac{p_3}{p_1}\right) - \ln\left(\frac{p_{1'}}{p_3}\right) \tag{2}$$

Table 1 displays a uniform pattern of signs for housing prices, with countries in the northern hemisphere displaying a positive summer effect and countries in the southern hemisphere displaying a negative summer effect (note that the austral summer takes place in the fourth and first quarters and hence the negative signs in the southern hemisphere). However, the statistical and economic significance varies across countries. Belgium, France, the U.K. and the U.S. display strongly significant summer effects in housing prices; Ireland, Sweden, and South Africa exhibit a less marked, though still significant summer effect; and finally, Denmark, Norway, Australia, and New Zealand show no statistically significant summer effect.[15]

Table 1: Average Difference in Annualized Housing Price Growth (nominal and real) between Second-Third Quarters and Fourth-First Quarters, by Country

	Nominal price inflation		Real price inflation		
Northern Hemisphere	Coef.	Std. Error	Coef.	Std. Error	Observations
Belgium	14.447**	(1.507)	13.695**	(1.740)	95
Denmark	1.085	(2.074)	1.029	(2.072)	51
France	12.459**	(1.200)	12.198**	(1.220)	34
Ireland	6.076*	(2.934)	4.456	(2.999)	35
Netherlands	2.723	(1.537)	3.234	(1.701)	48
Norway	3.072	(3.333)	4.628	(3.224)	52
Sweden	4.504	(2.270)	5.484*	(2.187)	76
United Kingdom	8.233**	(2.325)	6.105*	(2.354)	91
United States	3.640**	(0.891)			
Southern Hemisphere					
Australia	-1.163	(2.389)	-0.796	(2.415)	73
New Zealand	-1.516	(1.775)	-2.148	(1.808)	146
South Africa	-5.816*	(2.618)	-6.112	(3.129)	120

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Similarly, Table 2 suggests a strong and positive "summer" effect in all countries for which non-seasonally adjusted data on housing transactions are available.

Table 2: Average Difference in the Annualized Growth Rate of the Number of Transactions between Winters and Summers, by Country

Country	Coef.	Std. Error	Observations
Belgium	61.675**	(15.008)	51
Ireland	47.834**	(17.936)	120
Sweden	194.489**	(35.106)	75
United Kingdom	130.277**	(20.738)	124
United States	162.354**	(19.369)	149

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

2.3 The Within-Country Regional Evidence

The size of countries (and hence the number of potential regional housing markets) varies substantially in the sample studied before. In particular, for large countries, it is in principle inappropriate

[15] While the time span differs across countries, a sensitivity analysis performed by the authors shows that the period covered does not significantly affect the extent of seasonality. Still, results should be read with the caveat that not all countries perform quality adjustments, as discussed in the data Appendix. This is why the paper focuses attention on the U.S. and the U.K., for which the data are quality-adjusted.

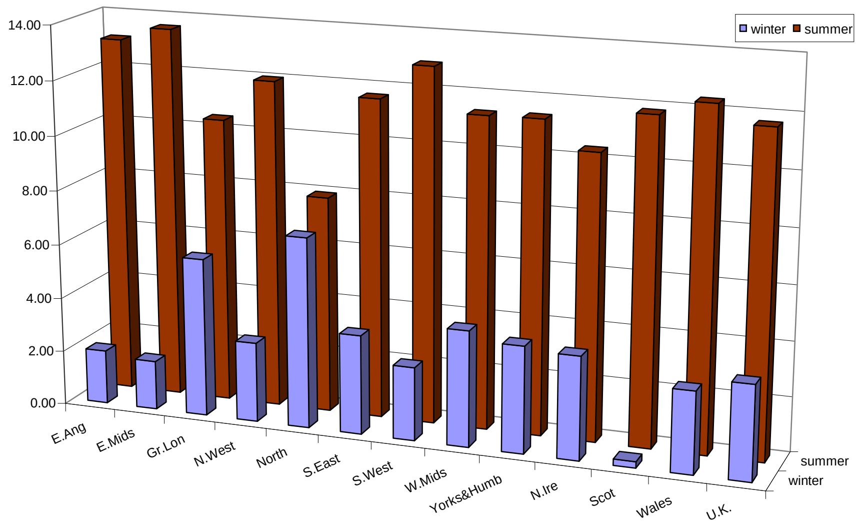
to talk about a single national housing market. The finding of no seasonal patterns in prices at the aggregate level, for example, might mask different seasonal behaviors at more disaggregated levels. Conversely, the existence of a seasonal pattern in the aggregate might reflect some aggregation anomalies. It is hence of importance to study the behavior of prices (and transactions) at a more disaggregated level. We do so in this Section, starting with the U.K. and the U.S., the countries with highest-quality data; we also document the behavior of rentals and interest rates for these two countries. Finally, we describe the seasonal patterns for prices in different regions of Belgium and France.

Housing Market Seasonality in the U.K.

Nominal Housing Price Changes

Figure 3 reports the average annualized price growth rates in the summer term (that is, the price growth from the end of the first quarter to the end of the third quarter) and the winter term (the price growth from the end of the third quarter to the end of the first quarter in the following year) over the period 1983 through 2005 using the Halifax index. As shown in the graph, the differences in price growth rates across the two big seasons are generally very large and economically significant. During the period analyzed, the average price increases in the winter term were below 4 percent in all regions except for West Midlands (4.8 percent), Greater London (5.4 percent) and the North region (6.6 percent). In the summer term, the average growth rates were above 11 percent in all regions, except for the North (9 percent).

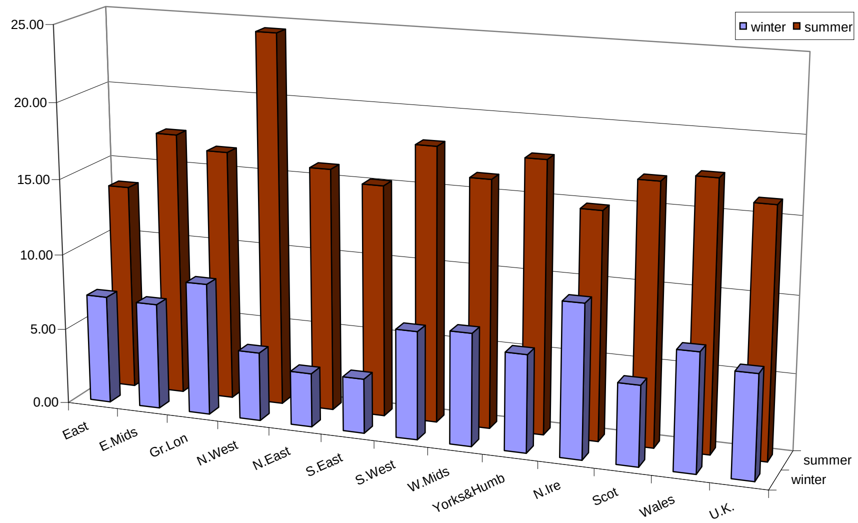
Figure 3: Average annualized housing price growth in summers and winters. Halifax Index 1983-2005.



Note: Annualized (quality-adjusted) price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. Halifax, 1983-2005.

Figure 4 shows the results from the ODPM index, starting in 1983 (for comparability with the Halifax Index). The patterns are similar to those reported using Halifax. The annualized average price growth during the summer term is above 12 percent in all cases, whereas the increase during the winter term is systematically below 6 percent, except for Greater London and Northern Ireland. The relatively small quantitative differences between the two indexes might be explained by the lag between approval and completion, which, as we mentioned, is a key difference between the two indices. As noted before, the ODPM index goes back to 1968 for most regions. The average difference in growth rates between summers and winters during the longer period (not shown for the sake of brevity), are of the same order of magnitude, roughly above 8 percent.

Figure 4: Average annualized housing price growth in summers and winters. ODPM Index 1983-2005.



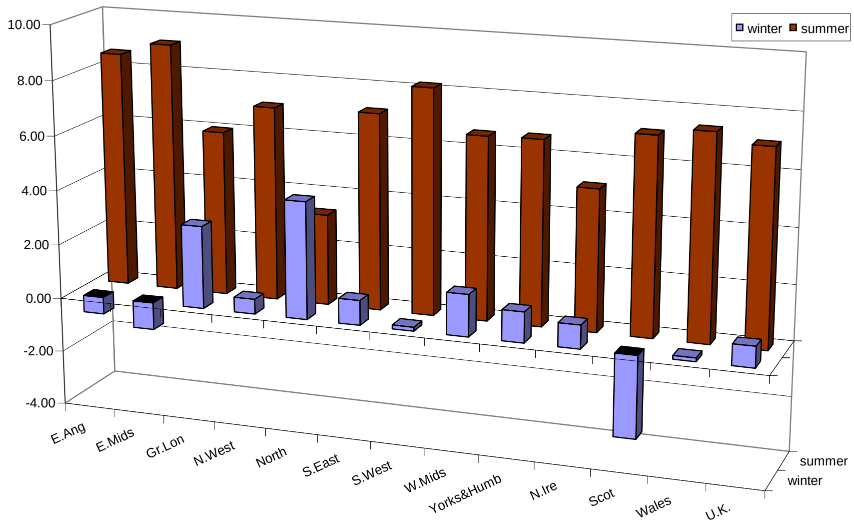
Note: Annualized (quality-adjusted) price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. ODPM index 1983-2005.

Real Housing Price Changes

The previous Figures showed the seasonal pattern in nominal housing price inflation. The seasonal pattern of real housing prices (that is, housing prices relative to the overall non-seasonally-adjusted price index) depends of course on the seasonality of overall inflation. In the U.K. overall price inflation displays a slightly seasonal pattern. In particular, over the period 1983 through 2005, the average annualized non-seasonally-adjusted inflation rate in the summer term has been 4.7 percent, whereas the corresponding figure in the winter term has been 2.8 percent. The difference of 2 percent can hardly "undo" the differences of over 8 percent in nominal housing price inflation, implying a significant seasonal in real housing prices. This is illustrated in Figure 5. The graph is based on the Halifax index, but the results are similar for the ODPM index, not shown in the interest of space. Netting out the effect of overall inflation reduces the differences in growth rates between winters and summers to a country-wide average just above 6 percent.

We should note in addition that non-seasonally adjusted indexes of inflation are rarely used in practice (indeed it is even hard to find them), so they are unlikely to serve in contracts as financial means to "hedge" part of the seasonal nominal housing price fluctuations.

Figure 5: Average annualized real housing price growth in summers and winters. Halifax Index 1983-2005.

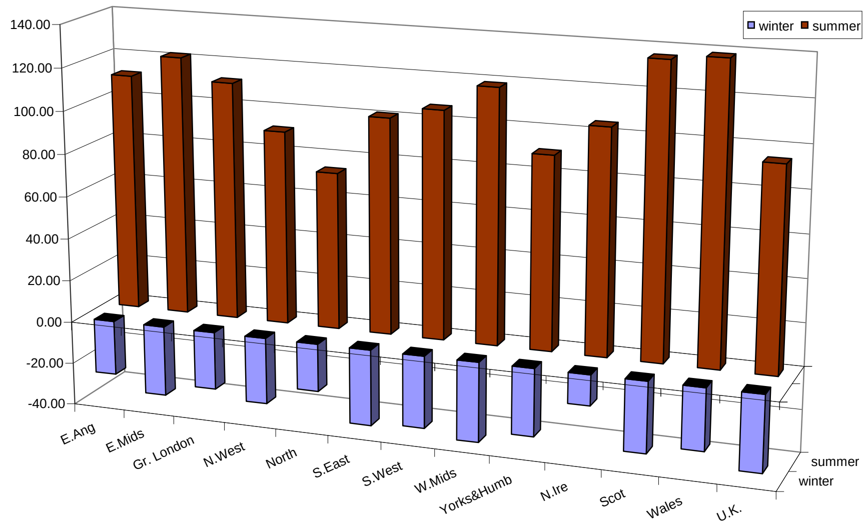


Note: Annualized (quality-adjusted) real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. Halifax, 1983-2005.

Number of Transactions

The seasonal differences in housing prices are mirrored by the patterns exhibited by the number of loans for housing purchases, which are a good proxy for the number of transactions. The data are collected by the National Survey of Mortgage Lenders and go back to 1974. For comparability with the price sample, Figure 6 shows the growth rate in the number of loans for mortgage completions in the U.K. from 1983 to 2005. (The 1974-2005 pattern is qualitatively and quantitatively similar to the one depicted in the Figure.) As the Figure shows, the number of transactions increases sharply in the summer term and declines in the winter term. Similar results are obtained by detrending the data using a linear trend (not shown).

Figure 6: Annualized growth rate of the number of loans in summers and winters.



Note: Annualized growth rate of the number of loan transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions, 1983-2005.

Statistical Significance of the Differences between Summers and Winters

This Section reports on the statistical significance of the results displayed in the previous Figures, as well as the characteristics of the houses and buyers involved in the transactions, by way of region-by-region tests on the difference in means, as expressed in (2). The data are based on the Halifax series, although similar results are obtained from the ODPM data (results available on request). Table 3 summarizes the results. The first two columns show the mean difference and standard errors for the data based on prices for all houses and buyers. They show that the differences in housing price inflation are statistically significant at standard levels in all regions, except the North.

The following four columns show the corresponding figures for the prices of existing houses and new houses. The figures indicate that seasonal differences are mainly driven by the prices of existing houses, though new houses also display a fair amount of seasonality in some regions. In particular, new houses' inflation rates display a strong seasonal pattern in Greater London, Scotland, Northern Ireland and West Midlands. Note that, while economically sizeable, however, the seasonal differences are in many cases not statistically significant; the lower precision might be due to the fact that new houses represent a very small share of the market (due mostly to stringent construction restrictions), and hence the test on mean differences across seasons unavoidably displays lower significance levels. Another explanation might be differences in repair and maintenance costs across the two seasons. To the extent that repair costs are smaller in the summer (because good weather and the time of the owners are important inputs in construction), sellers will take this into account and post accordingly higher prices in the market. If differences in seasonal repair costs are behind the differences in prices, then, insofar as new houses need less repair and the potential buyers can ask the developers to tailor the final touches of the house to their needs, we should observe less seasonality in the prices of new houses than in those of existing houses. Though qualitatively possible, yet, the question remains as whether plausible differences in repair costs alone can quantitatively match the seasonal variation in the data, a point to which we come back later.

Table 3: Average Difference in Annualized Housing Price Inflation Between Summer and Winters, by Region and Type of House or Buyer

	All Houses (All buyers)		Existing houses (All buyers)		New houses (All buyers)		Former owner occupiers (All houses)		First-time buyer (All houses)	
	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error
E. Anglia	10.770**	(3.509)	10.028**	(3.727)	5.513	(6.878)	12.201**	(3.453)	5.663	(4.385)
E.Midlands	12.125**	(3.607)	12.905**	(3.651)	1.849	(5.814)	13.637**	(3.847)	9.496*	(3.699)
Gr. London	6.291*	(2.865)	6.624*	(2.898)	18.970*	(9.316)	5.357*	(2.658)	6.355*	(3.086)
N. West	8.629**	(2.813)	9.915**	(2.871)	-1.164	(7.051)	10.168**	(3.026)	5.675+	(2.950)
North	1.864	(3.224)	2.319	(3.333)	1.559	(5.606)	0.742	(3.295)	3.294	(3.897)
S. East	7.675**	(2.908)	8.061**	(2.889)	3.112	(4.066)	8.775**	(2.900)	4.301	(2.952)
S. West	10.961**	(3.439)	11.202**	(3.556)	8.004	(4.945)	11.895**	(3.549)	6.530+	(3.907)
W. Midlands	7.380+	(3.766)	7.126+	(3.799)	14.721+	(8.072)	8.160*	(3.965)	6.257+	(3.606)
Yorkshire&Humb	7.477*	(3.137)	8.249*	(3.194)	2.561	(6.449)	8.203*	(3.121)	7.340*	(3.506)
N. Ireland	9.253**	(3.425)	11.172**	(4.055)	10.977+	(6.082)	7.319	(4.524)	10.237*	(5.014)
Scotland	11.028**	(2.604)	13.627**	(2.895)	15.305*	(7.130)	12.591**	(2.673)	6.257*	(3.046)
Wales	9.332*	(3.721)	9.255*	(3.726)	1.146	(7.924)	9.943**	(3.729)	6.902+	(3.938)
U.K.	8.233**	(2.325)	8.896**	(2.364)	5.674*	(2.484)	9.114**	(2.348)	5.809**	(2.196)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

The last four columns of Table 3 show the mean differences and standard deviations corresponding to the data on prices paid by former-owner occupiers and first-time buyers. The distinction between former-owner occupiers and first-time buyers is interesting as some might a priori hypothesize that repeated buyers have more information on the seasonal patterns of the housing market and will hence be able to time their purchases to get better prices. On the other hand, first-time buyers might be less dependent on chains (that is, they do not need to sell a house before buying) and can thus better arbitrage across seasons. The results tend to point to slightly stronger seasonality in prices paid by former-owner occupiers, favouring the second hypothesis, though as before, the results can also be driven by the natural loss of precision caused by the relatively small number of first-time buyers in the market.

Table 4 shows the corresponding numbers for average differences in *real* housing price growth.

Since the average difference in overall inflation rates across summers and winters is around 2 percent, the average difference in real housing price growth is roughly equivalent to the difference in nominal housing price inflation minus 2 percent.

Table 4: Average Difference in Annualized Real Housing Price Growth Between Summer and Winters, by Region and Type of House or Buyer

	All Houses (All buyers)		Existing houses (All buyers)		New houses (All buyers)		Former owner occupiers (All houses)		First-time buyer (All houses)	
	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error
E. Anglia	8.597*	(3.589)	7.787*	(3.780)	3.114	(6.815)	10.160**	(3.531)	3.444	(4.483)
E.Midlands	10.148**	(3.675)	10.854**	(3.716)	-0.027	(5.989)	11.766**	(3.951)	7.495+	(3.772)
Gr. London	4.161	(3.006)	4.435	(3.034)	15.296	(9.526)	3.585	(2.803)	4.115	(3.275)
N. West	6.224*	(2.784)	7.620**	(2.847)	-4.022	(7.140)	7.456*	(3.012)	3.764	(2.905)
North	-0.224	(3.238)	0.284	(3.356)	-0.637	(5.747)	-1.315	(3.327)	1.446	(3.910)
S. East	5.677+	(3.015)	6.084*	(2.990)	0.756	(4.211)	6.854*	(3.001)	2.259	(3.109)
S. West	8.569*	(3.579)	8.863*	(3.701)	4.188	(4.997)	9.567*	(3.687)	3.869	(4.012)
W. Midlands	5.291	(3.800)	4.983	(3.823)	14.448+	(8.201)	6.02	(4.004)	4.285	(3.656)
Yorkshire&Humb	5.468+	(3.113)	6.195+	(3.169)	0.53	(6.536)	6.155+	(3.132)	5.521	(3.467)
N. Ireland	7.422*	(3.580)	9.976*	(4.186)	11.885*	(5.813)	4.701	(4.544)	8.936+	(5.216)
Scotland	9.305**	(2.462)	12.317**	(2.695)	12.163+	(7.260)	11.010**	(2.544)	4.476	(3.021)
Wales	6.895+	(3.723)	6.818+	(3.749)	-1.32	(8.084)	7.659*	(3.743)	5.021	(3.957)
U.K.	6.105*	(2.354)	6.788**	(2.393)	3.444	(2.579)	7.016**	(2.387)	3.760+	(2.255)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

The behavior of prices is mimicked by that of the number of transactions. Table 5 shows the average differences in growth rates in the number of transactions between summers and winters. The Table reports the mean differences across seasons and their standard errors (**??**) corresponding to each region. The annualized difference in growth rates is roughly 120 percent. Northern Ireland and the North region show the smallest average difference, which is roughly 100 percent. As the Table shows, the difference is stronger for former-owner occupiers than for first-time buyers, consistent with the price patterns observed before. (Unfortunately, the data are not disaggregated by type of house).

Table 5: Average Difference in Annualized Growth Rates in the Number of Transactions Between Summer and Winters, by Region and Type of Buyer

	All Houses (All buyers)		Former owner occupiers (All houses)		First-time buyer (All houses)	
	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error
E. Anglia	137.066**	(22.313)	214.294**	(38.983)	136.538**	(29.901)
E.Midlands	154.761**	(44.188)	215.595**	(58.098)	204.546*	(89.538)
Gr. London	138.723**	(40.132)	204.390**	(71.944)	112.855**	(28.587)
N. West	121.901**	(17.117)	155.872**	(19.788)	105.037**	(21.158)
North	95.811**	(16.419)	183.704**	(35.753)	82.895*	(37.257)
S. East	136.708**	(16.753)	164.647**	(18.295)	102.878**	(15.453)
S. West	140.322**	(24.109)	182.283**	(27.215)	109.224**	(21.898)
W. Midlands	155.984**	(29.471)	207.046**	(37.535)	112.131**	(24.538)
Yorkshire&Humb	121.736**	(20.539)	171.579**	(31.494)	106.622**	(22.217)
N. Ireland	118.920**	(38.895)	172.178*	(74.599)	119.912**	(41.468)
Scotland	169.156**	(42.906)	320.131**	(67.460)	84.948**	(25.485)
Wales	167.241**	(39.668)	184.066**	(38.418)	158.468**	(40.656)
U.K.	130.277**	(20.738)	168.636**	(22.563)	102.730**	(19.682)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Put together, the data point to a strong seasonal cycle, with a large increase in transactions and prices during the summer relative to the winter term. Also, the seasonal patterns are similar across regions, except for the North, which tends to display less seasonality in prices.

Rents

Data on rents are not documented in as much detail as the data on prices. The series available corresponds to the aggregate of the U.K. and comes from ODPM; the data are not disaggregated by region. We run regressions using as dependent variables both the rent levels and the log of rents on the dummy variable S_t, which takes the value 1 in the second and third quarter and 0 otherwise. We also include, where indicated, a trend term. The results are summarized in Table 6, which shows that there is virtually no seasonality in rents for the U.K. as a whole. This is in line with anecdotal evidence suggesting that rents are fairly sticky. Given the paucity of data on rents, there is little we can say with high confidence. Still, note that for rents to be the driver of price seasonality, one would need an enormous degree of seasonality in rents (as well as a high discount rate), since prices should in principle, according to the standard asset-pricing approach, reflect the present values of all future rents (in other words, prices should be less seasonal than rents). The lack of even small discernible levels of seasonality in the data suggest that we need another explanation for the seasonality in prices.

Table 6: Summer Differentials in Rents in the U.K.

	Rents		log(Rent)	
Summer-dummy S_t	-47.90833	12.53771	-0.01406	0.00743
	(255.798)	(29.529)	(0.091)	(0.010)
Trend		61.67964**		0.02194**
		(1.276)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Mortgage Rates

Interest rates in the U.K. do not seem to exhibit a seasonal pattern. The evidence is summarized in Table 7, which shows the summer dummy coefficients for different interest rate series provided by the Bank of England. The first column shows the results for the quarterly average of the repo (base) rate; the second column shows the corresponding results for the average interest rate charged by 4 U.K. major banks (Barclays Bank, Lloyds Bank, HSBC, and National Westminster Bank); and the third column shows the results for the weighted average standard variable mortgage rate from Banks and Building Societies. The first two series cover the period 1978 through 2005, whereas the third goes from 1994 through 2005.

As the Table shows, none of the interest rate measures appears to be different, on average, during the summer term.

Table 7: Summer Differentials in Interest Rates in the U.K.

	Repo rate	Bank-4 Rate	Mortgage Rate
Summer-dummy S_t	-0.163	-0.144	0.018
	(0.701)	(0.696)	(0.310)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Housing Market Seasonality in the U.S.

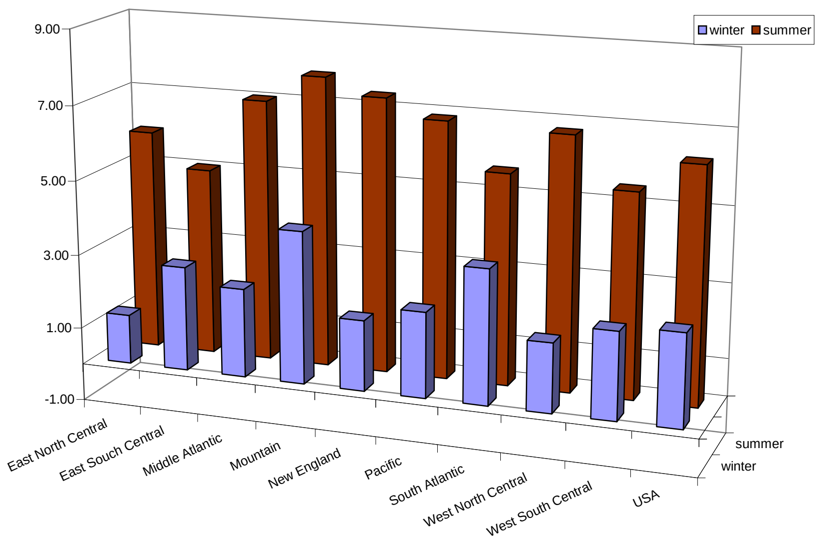
Housing Price Changes

As noted before, the U.S. aggregate price index displays a consistent seasonal behavior, albeit the degree of seasonality is generally smaller than that in the U.K.

Figures 7 and 8 illustrate, correspondingly, the degree of nominal and real housing price seasonality at the Census-level Division. The NSA CPI index for the US is used to deflate the nominal housing price series; the CPI display a relatively low degree of seasonality: inflation rates during the period considered averaged 2.9 in the summer term and 2 percent in the winter term. We also examined the differences across summers and winters at finer levels of disaggregation; we omit the

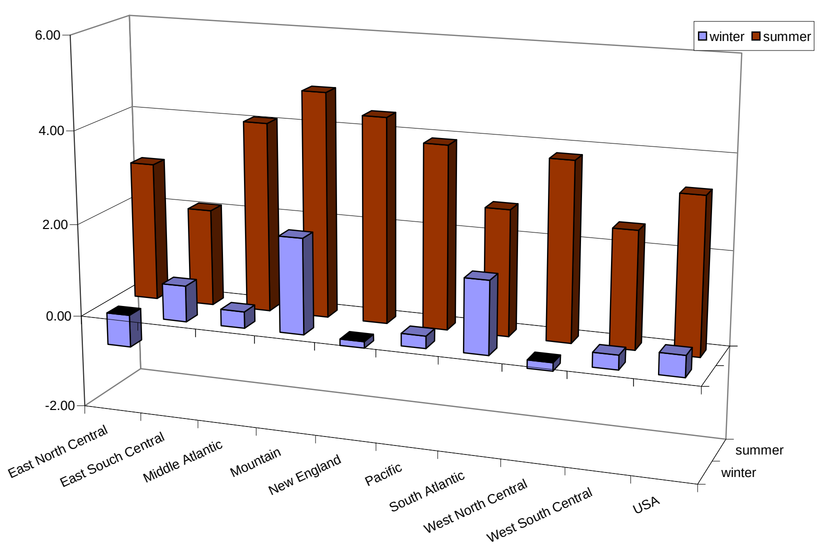
Figures for the sake of space, but report the results from a test on the mean difference in annualized growth rates across seasons, as in (2), using different data sets. Specifically, Table 8 shows the results based on the Figures, at the Census Division level, Table 8*a* shows the results using state-level data, and Tables 8*b* and 8*c* show the results using city-level data. Tables 8, 8*a* and 8*b* are based on the OFHEO NSA indices and Table 8*c* is based on the Standard'Poor's Case-Shiller index. The latter displays more seasonality than the OFHEO-city index.

Figure 7: Average annualized nominal housing price growth in summers and winters. OFHEO-Purchase-only, Census Division level 1991-2008.



Note: Annualized growth rates of nominal housing price growth in summers and winters in the U.S. and its regions (OFHEO-repeat sale, purchase-only index 1991:01-2008:01.)

Figure 8: Average annualized real housing price growth in summers and winters. OFHEO-Purchase-only, Census Division level 1991-2008.



Note: Annualized growth rates of *real* housing price growth in summers and winters in the U.S. and its regions, 1975-2005. (OFHEO-repeat sale, purchase-only index 1991:01-2008:01.)

We can see this in Table 8 at the Census Division level, Table 8a at the State level, and Tables 8b and 8c at the city level. Tables 8, 8a and 8b are based on the OFHEO NSA indices and Table 8c is based on the Case-Shiller index. The latter displays more seasonality than the OFHEO-city index.

Complete this discussion

Table 8: Average Difference in Annualized Housing Price Growth between Summers and Winters, by US Census Divisions

City	Coef.	Std. Error
East North Central	4.607**	(0.776)
East Souch Central	2.171**	(0.606)
Middle Atlantic	4.733**	(1.286)
Mountain	3.641**	(1.081)
New England	5.599**	(1.672)
Pacific	3.808*	(1.883)
South Atlantic	1.752	(1.105)
West North Central	4.852**	(0.784)
West South Central	3.007**	(0.535)
USA	3.640**	(0.891)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 8*a*: Average Difference in Annualized Housing Price Growth between Summers and Winters, by US State.

State	Coef.	Std. Error	State	Coef.	Std. Error
AK	4.075**	(1.429)	MT	4.659**	(1.458)
AL	2.493**	(0.750)	NC	1.658*	(0.650)
AR	2.645**	(0.844)	ND	4.974**	(1.300)
AZ	2.146	(2.309)	NE	4.224**	(1.040)
CA	4.808	(2.812)	NH	5.200**	(1.932)
CO	4.753**	(1.215)	NJ	4.832**	(1.685)
CT	6.540**	(1.701)	NM	3.272*	(1.330)
DC	10.851*	(4.447)	NV	4.807	(2.654)
DE	3.367*	(1.642)	NY	5.026**	(1.429)
FL	2.061	(2.167)	OH	4.234**	(0.853)
GA	2.357**	(0.791)	OK	3.556**	(0.754)
HI	1.408	(3.244)	OR	4.685**	(1.359)
IA	4.868**	(0.735)	PA	4.661**	(1.108)
ID	5.712**	(1.486)	RI	4.282	(2.383)
IL	4.405**	(0.810)	SC	1.403	(0.762)
IN	3.724**	(0.790)	SD	4.313**	(1.516)
KS	3.573**	(0.870)	TN	2.122**	(0.713)
KY	1.987**	(0.713)	TX	3.151**	(0.619)
LA	2.520**	(0.896)	UT	2.795	(1.630)
MA	5.495**	(1.817)	VA	2.311	(1.516)
MD	4.358*	(2.016)	VT	6.298**	(2.223)
ME	4.643*	(1.979)	WA	4.234**	(1.263)
MI	4.963**	(1.256)	WI	5.383**	(0.727)
MN	5.939**	(1.245)	WV	4.327*	(1.707)
MO	4.701**	(0.824)	WY	5.069**	(1.376)
MS	1.798	(0.979)			

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 8*b*: Average Difference in Annualized Housing Price Growth between Summers and Winters, by US City–OFHEO Index.

City	Coef.	Std. Error
Boston	6.876**	(2.089)
Chicago	4.044**	(0.885)
Denver	3.740**	(1.380)
Las Vegas	4.038	(2.800)
Los Angeles	5.213	(2.718)
Miami	0.349	(2.194)
New York	4.324*	(1.709)
San Diego	3.228	(2.776)
San Francisco	4.429	(2.835)
Washington DC	3.509	(2.346)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 8c: Average Difference in Annualized Housing Price Growth between Summers and Winters, by US City-Case-Shiller index

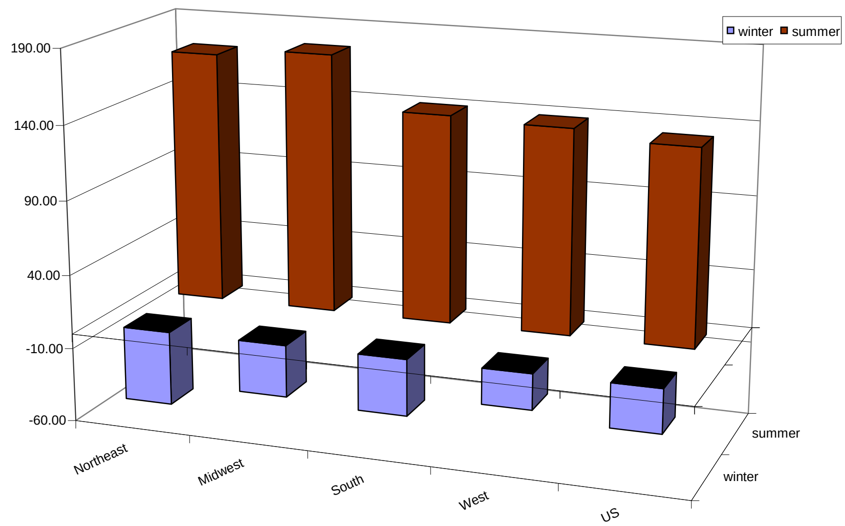
City	Coef.	Std. Error
Atlanta	3.523**	(0.519)
Boston	9.475**	(1.147)
Charlotte	2.987**	(0.667)
Chicago	5.013**	(0.996)
Cleveland	7.203**	(0.651)
Dallas	5.151**	(1.227)
Denver	4.698**	(0.837)
Detroit	5.046**	(0.811)
Las Vegas	5.497**	(1.931)
Los Angeles	8.286**	(1.706)
Miami	1.808	(1.252)
Minneapolis	5.289**	(0.879)
New York	6.144**	(1.074)
Phoenix	3.524*	(1.676)
Portland	5.648**	(1.020)
San Diego	7.043**	(1.681)
San Francisco	7.070**	(1.756)
Seattle	5.906**	(1.432)
Tampa	2.308	(1.192)
Washington	7.119**	(1.305)
Composite-20 cities	5.329**	(1.360)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Number of Transactions

As already observed, the U.S. as a whole displays a strong seasonality in the number of transactions. This remains true across all four major regions of the U.S. (state-level data are not available). The growth rates in the number of transactions in summers and winters are plotted in Figure 9. The average difference across seasons, together with the standard errors are summarized in Table 9.

Figure 9: Annualized growth rate of the number of transactions in summers and winters in the U.S. and its regions



Note: Annualized growth rates of the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions, 1975-2005. (Data for the U.S. as a whole corresponds to 1968-2005.)

Table 9: Average Difference in Annualized Growth Rates in the Number of Transactions Between Summer and Winters, by Regions in the U.S.

Region	Coef.	Std. Error
Northeast	220.718**	(19.762)
Midwest	210.968**	(27.558)
South	179.038**	(21.219)
West	162.818**	(25.816)
United States	162.354**	(19.369)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Rents

Data on rents for the U.S. come from the Bureau of Labor Statistics (BLS); as a measure of rents we use the non-seasonally adjusted series of owner's equivalent rent and the non-seasonally adjusted rent of primary residence; both series are produced for the construction of the CPI and correspond to averages over all cities. For each series, we run regressions using as dependent variables both the rent levels and the log of rents on the summer-term dummy. we also include, where indicated, a trend term. The results are summarized in Tables 10 (owner's equivalent rent) and 11 (rent of primary residence). Both Tables show that there is no discernible pattern of seasonality in rents for the U.S. as a whole. To reiterate, if seasonality in rents were the driver of seasonality in prices, we should observed substantial seasonality in rents to justify the seasonality in prices according to the standard approach, since prices reflect the present discounted flow of future rents. In the model we present later, we will work under the constraint that rents are not seasonal.

Table 10: Summer Differential in Rents in the U.S.: Owner's Equivalent Rent

	Rents		log(Rent)	
Summer-dummy S_t	-0.19638	-0.19638	-0.00102	-0.00102
	(8.133)	(0.269)	(0.051)	(0.006)
Trend		1.45183**		0.00905**
		(0.005)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 11: Summer Differential in Rents in the U.S.: Rent of Primary Residence

	Rents		log(Rent)	
Summer-dummy S_t	-0.16594	-0.16594	-0.00098	-0.00098
	(7.120)	(0.638)	(0.047)	(0.005)
Trend		1.26671**		0.00827**
		(0.012)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Mortgage Rates

Interest rates in the U.S. do not exhibit a seasonal pattern (Barsky and Miron, 1989). Since housing service costs are of particular interest here, we summarize In Table 12 the summer effect (or lack thereof) in mortgage rates. The data come from the Board of Governors of the Federal Reserve and correspond to contract interest rates on commitments for fixed-rate first mortgages; the data are quarterly averages beginning in 1972; the original data are collected by Freddie Mac. As the Table shows, mortgage rates do not appear to be higher on average during the summer term, consistent with the findings in Barsky and Miron (1989).

Table 12: Summer Differential in Mortgage Rates in the U.S.

	Mortgage Rate
Summer-dummy S_t	0.104
	(0.477)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Housing Market Seasonality in Belgium and France

Tables 13 and 14 show the housing-price mean differences across reasons and their corresponding standard errors for Belgium and France, disaggregated by regions with available data. As the Tables show, in both countries all regions display a strong seasonal pattern, comparable to that reported for the country as a whole. Data on transactions at the regional level are not available. As noted in the Data Appendix, the housing price indexes for these countries are not quality adjusted and hence seasonal variation in prices might mask variation in the quality of the houses on the market; this is why we emphasize throughout the paper the results from the U.K. and the U.S.

Table 13: Average Difference in Annualized Housing Price Growth between Second-Third Quarters and Fourth-First Quarters in Belgium, by Region.

Region	Coef.	Std. Error
Great Brussels	13.242**	(3.039)
Flanders	10.753**	(1.746)
Wallonia	19.329**	(1.903)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 14: Average Difference in Annualized Housing Price Growth between Second-Third Quarters and Fourth-First Quarters in France, by Region.

Region	Coef.	Std. Error
Ile-de-France	9.275**	(2.294)
Province (All regions except Ile-de-France)	17.347**	(1.906)
Provence-Alpes-Côte d'Azur	10.915**	(2.624)
Rhône-Alpes	11.977**	(2.648)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

2.4 Further Discussion

We have argued before that the predictability and size of the seasonal variation in housing prices in some countries pose a puzzle to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal service cost. In Appendix 7.1 we carry out back-of-envelope calculations to assess to what extent seasonality in service costs might be driving the seasonality in prices.

The exercise makes clear that a standard asset-pricing approach that relies on perfect arbitrage leads to implausibly large levels of seasonality in service costs.[16] The findings suggests that there

[16] Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 328 and 209 percent of those in the summer. Depreciation and repair costs might be seasonal, being potentially lower during the summer. But income-tax-adjusted interest rates and property taxes, two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the U.K. is even larger. Assuming, quite conservatively, that the a-seasonal component accounts

are important frictions in the market that impair the ability of investors to gain from any seasonal arbitrage and call for a deviation from the simple asset-pricing model.[17] A possible explanation for why the asset-pricing condition fails is of course that transaction costs are very high. Still, even if one takes that view, there still remain some puzzling observations: Why do potential buyers systematically buy in the high-price season? Could they be better-off waiting? Why do we observe a systematic seasonal pattern? (The lack of scope for seasonal arbitrage does not necessarily imply that most transactions should be carried out in one season nor does it implies that prices and transactions should be correlated.) In the next Section, we develop a search and matching model for the housing market that provides an answer to these questions.

3 A Search-and-Matching Model for the Housing Market

The model economy is populated by a unit measure of infinitely lived agents, who have linear preferences over a non-durable consumption good and housing services. Each period agents receive a fixed endowment of the consumption good which they can either consume or use to buy housing services. Agents can only enjoy housing services from living in one house a time, i.e. they can only be "matched" to one house at a time. A matched agent is a "homeowner" and an unmatched agent is a "buyer". There is a unit measure of housing stock. Each period houses can also be matched or unmatched. The matched house delivers a flow of housing services of quality ε to its homeowner, which we assume to be constant over time. The unmatched house is "for sale" and is owned by a "seller". The seller receives a flow of asset values u from an unmatched house he owns. Houses

for only 50 percent of the service costs in the summer the implied ratio of depreciation and repair costs between summers and winters for rent-to-price ratios in the range of 2 through 5 percentbe between 557 and 318 percent. (If the a-seasonal componentaccounts for 80 percent of the service costs, the corresponding values are 1542 and 944 percent.

[17] The need to deviate from the asset-market approach has been acknowledged, in a different context, among others, by Stein (1995). While static in nature, Stein's model is capable of generating unexpected booms and busts in prices (and transactions) in a rational-expectation setting. In a dynamic setting with forward-looking agents, however, predictably large changes in prices cannot be sustained: Expected price increases in the next season will actually be priced in the current season (or, in other words, sellers will refuse to sell at lower prices today given the perspective of higher prices in the next season); similarly, prospective buyers will benefit from waiting (at most a few months) and paying a significantly lower price. Even when agents are both sellers and buyers, if they are aware of the differences in prices, in a dynamic setting they will seek to sell in the summer and to buy in the winter; the excess supply in the summer will then push prices down, while the excess demand in the winter will push them up.

and agents are ex-ante identical. The asset flow value u of a house is common to all sellers. The quality of housing services ε, however, is match-specific, and it captures the quality of a match between a house and its homeowner. In other words, for any vacant house, the potential housing services are idiosyncratic to the match between the house and the buyer. Hence, ε is not the type of the house (or of the seller who owns a particular house); there is only one representative house in our model, but the utility derived from living in the house is idiosyncratic. This is consistent with our data, which are adjusted for houses' characteristics, such as size and location, but not for the (unobserved) quality of a match.[18] Since this is the key element of our model, we will first discuss in detail how we model it.

3.1 Match-specific Quality

The model embeds the intuitively appealing notion that in a market with many houses on sale a buyer can find a house closer to her ideal and hence her willingness to pay increases. We model this idea by assuming that a buyer draws the quality of a potential match, ε, from a distribution $F(\varepsilon, v)$ with positive support and finite mean, where v denotes the stock of vacant houses, and $f(\varepsilon, v)$ is the corresponding probability density function. Our notion of a "thick-market" is captured by the following assumptions:

Assumption 1 $F(., v)$ is decreasing in v.

Assumption 1 states that $F\left(., v'\right)$ stochastically dominates $F(., v)$ if and only if $v' > v$. In words, when the stock of houses v is bigger, a random draw of match quality ε from $F(\varepsilon, v)$ is likely to be higher. One useful implication of Assumption 1 is that higher v shifts up the expected surplus of quality above any threshold x :

$$h(x, v) \equiv [1 - F(x, v)]\, E(\varepsilon - x \mid \varepsilon > x) \text{ is increasing in } v. \tag{3}$$

To see this rewrite $h(x, v)$ using integration by part,

$$h(x, v) = \int_x^{\bar{\varepsilon}} (\varepsilon - x)\, dF(\varepsilon, v) = \bar{\varepsilon} - x - \int_x^{\bar{\varepsilon}} F(\varepsilon, v)\, d\varepsilon = \int_x^{\bar{\varepsilon}} [1 - F(\varepsilon, v)]\, d\varepsilon,$$

which is increasing in v from Assumption 1. Note that this expression also implies that the conditional surplus is:

$$E(\varepsilon - x \mid \varepsilon > x) = \frac{\int_x^{\bar{\varepsilon}} [1 - F(\varepsilon, v)]\, d\varepsilon}{1 - F(., v)}.$$

[18] Neither repeat-sale indices nor hedonic price indices can control for the quality of a match.

Thus, the conditional surplus is also increasing in v under the following assumption:

Assumption 2 $\frac{\int_x^{\bar{\varepsilon}}[1-F(\varepsilon,v)]d\varepsilon}{1-F(x,v)}$ is increasing in v.

Assumption 2 states that for any threshold x, as v increases, the increase in the integral of $[1-F(.,v)]$ for any ε above x is at least as large as the increase in $[1-F(x,v)]$. This will be true if higher v shifts up $[1-F(.,v)]$ evenly for all ε.

As will be shown later, a buyer is matched to a house (i.e. the transaction is successful) when ε exceeds some reservation level ε^r. Assumption 1 states that higher v shifts up the probability function of a successful transaction for any given level of ε^r. As is well-known, a sufficient condition for ε^r to be unique is to assume $f(\varepsilon,.)$ is log-concave, which will be shown to be true in our model as well, hence we assume[19]:

Assumption 3 $f(\varepsilon,.)$ is log-concave, i.e. $\ln f$ is concave in ε.

One way to interpret our assumptions is to use order statistics under the assumption that potential buyers sample a number of vacant houses before buying. Let the potential match quality between a buyer and any house in the *entire housing stock* be randomly distributed according to a distribution $G(.)$, and let $g(.)$ be the corresponding probability density function. Suppose the buyer samples n units of vacant houses. Let $(\varepsilon_1,\varepsilon_2,..\varepsilon_n)$ denote an *iid* random sample from the continuous distribution $G(.)$. Let ε be the maximum ε_i; then the distribution of ε is $F(.,n) = [G(.)]^n$, which is decreasing in n. Intuitively, as the sample size increases, the maximum becomes "stochastically larger." Let $f(.,n)$the corresponding probability density function. It follows that $\frac{f(.,n)}{1-F(.,n)}$ is also straightly decreasing in n.[20]Finally, if g is log-concave, then f is also log-concave. Therefore, if n is an increasing function of the stock of vacant houses v, then all our assumptions follow from assuming g to be log-concave. (Examples of distributions with log-concave density function include the uniform and normal distributions.) We postpone the discussion of the exact functional form between n and v until we specify the functional form for $F(.,v)$. For now, the interpretation is that any buyer can sample the *entire stock of vacant houses* (e.g. by searching

[19]As shown in Burdett (1996), this assumption implies both $[1-F(\varepsilon^r)]/f(\varepsilon^r)$ and $E(\varepsilon-\varepsilon^r \mid \varepsilon > \varepsilon^r)$ are decreasing in ε^r, which are the two key elements for showing the reservation ε^r is unique.

[20]More explicitly, $\frac{1-F}{f} = \left(\frac{1-F}{F}\right)\frac{F}{f} = \left(\frac{1-F}{F}\right)\frac{g}{nG} = \left(\frac{G^{-n}-1}{n}\right)\frac{g}{G}$. Note that for $G\in[0,1)$, the function $(G^{-x}-1)/x$ is straightly increasing in x for any positive x. To see this, derive its slope as $\frac{(G^x-\ln G^x-1)}{x^2G^x}$. Note that $H(y) = y-1-\ln y>0$ for any $y\in[0,1)$ and $H(1)=0$ because $\lim_{y\longrightarrow 0} H(0)>0$ and $H'(y)<0$.

online or through newspapers).[21] After sampling the stock, the buyer chooses the house that ranks first and makes contact with the seller, i.e. we assume that each period a buyer visits only one house—her best house. Given the *iid* assumption, it follows that the best house is different for each buyer and, as a result, a house is visited by only one buyer. This assumption implies that the seller "negotiates" a price for each house he owns *independently* with each different buyer, that is, the price of a house is determined between one buyer and one seller.

3.2 Seasons and Timing

There are two seasons, $j = s, w$ (for summer and winter); each model period is a season, and seasons alternate. At the beginning of a period, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, and the house is for sale. The homeowner becomes a buyer and seller simultaneously. In our baseline model, the parameter ϕ^j is the only (*ex ante*) difference between the seasons (determined, for example, by the school calendar or summer marriages, among other factors). We focus on periodic steady states with constant v^s and v^w, where v^j is the stock of vacant (unmatched) houses in season $j = s, w$. We call b^j be the stock of buyers (unmatched agents) in season $j = s, w$. Since a match is between one house and one agent, and there is a unit measure of agents and a unit measure of houses, it is always the case that $v^j = b^j$.

The sequence of events is as follows. At the beginning of season j, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, adding to the stock of vacant houses and buyers. The buyer observes ε (drawn from $F(., v^j)$) for her best house out of the available stock v^j and meets with the seller of this house. If the transaction goes through, the buyer pays a price (discussed later) to the seller, and starts enjoying the housing services from the same season j. If the transaction does not go through, the buyer looks for a house again next season, the seller receives the asset value flow in season j and puts the house up for sale again next period. An agent can hence be a homeowner, a buyer, a seller, both a seller and a homeowner, and both a buyer and a seller. Also, sellers may have multiple houses to sell.

[21] This is different from the stock-flow literature (see e.g. Coles and Smith, 1998), where new buyers can only draw from the stock of old vacant houses, and the stock of old buyers can only draw from the stock of new vacant houses. We do not draw a distinction here between old and new buyers.

3.3 The Homeowner

To study pricing and transaction decisions, we first derive the value of living in a house if a transaction goes through. The value function for a homeowner who lives in a house with quality ε in season s is given by:

$$H^s(\varepsilon) = \varepsilon + \beta\phi^w H^w(\varepsilon) + \beta(1-\phi^w)[V^w + B^w],$$

where $\beta \in (0,1)$ is the discount factor. With probability $(1-\phi^w)$ he receives a moving shock and becomes both a buyer and a seller (putting his house up for sale), with continuation value $(V^w + B^w)$, where V^j is the value of a vacant house to its seller and B^j is the value of being a buyer in season $j = s, w$, defined below. With probability ϕ^w he keeps receiving housing services of quality ε and stays in the house. (Notice that the formula for $H^w(\varepsilon)$ is perfectly isomorphic to $H^s(\varepsilon)$; in the interest of space we omit here and throughout the paper the corresponding expressions for season w.) The value of being a homeowner can be therefore re-written as:

$$H^s(\varepsilon) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon + \frac{\beta(1-\phi^w)(V^w+B^w) + \beta^2\phi^w(1-\phi^s)(V^s+B^s)}{1-\beta^2\phi^w\phi^s}, \tag{4}$$

which is straightly increasing in ε.

3.4 Market Equilibrium

In any season $j = s, w$, the buyer visits a house with match quality ε, drawn from the distribution $F^j(\varepsilon) \equiv F(\varepsilon, v^j)$. The buyer meets with the seller of this house to "negotiate" a price. We focus on the case in which the seller also observes ε and discuss the case in which he does not observe ε in the Appendix. If the transaction goes through, the buyer pays the price to the seller, and starts enjoying the housing services flow from the same season j. If the transaction does not go through, the buyer receives zero housing services and looks for a house again next season. This can be the case, for example, if buyers searching for a house pay a rent equal to the utility they derive from the rented property; what is key is that the rental property is not owned by the same potential seller with whom the buyer meets. On the seller's side, when the transaction does not go through, he receives the asset flow value u from season j and puts the house for sale again next season. The flow value u can be interpreted as a net rental income received by the seller. Again, what is key is that the tenant is not the same potential buyer who visits the house.

As in the search-and-matching literature, the realized transaction yields positive surplus. We assume that the surplus is shared according to Nash bargaining. Let $S_v^s(\varepsilon)$ and $S_b^s(\varepsilon)$ be the surpluses of a transaction to the seller and the buyer when the match quality is ε and the price is $p^s(\varepsilon)$:

$$S_v^s(\varepsilon) \equiv p^s(\varepsilon) - (u + \beta V^w), \tag{5}$$

$$S_b^s(\varepsilon) \equiv H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w. \tag{6}$$

The price maximizes the Nash product:

$$\max_{p^s(\varepsilon)} \left(S_v^s(\varepsilon)\right)^{\theta} \left(S_b^s(\varepsilon)\right)^{1-\theta}$$
$$s.t. \quad S_v^s(\varepsilon), S_b^s(\varepsilon) \geqslant 0;$$

where θ denotes the bargaining power of the seller. The solution implies

$$\frac{S_v^s(\varepsilon)}{S_b^s(\varepsilon)} = \frac{\theta}{1-\theta}, \tag{7}$$

and a transaction goes through as long as the total surplus $S^s(\varepsilon)$ is positive,

$$S(\varepsilon) \equiv S_v^s(\varepsilon) + S_b^s(\varepsilon) = H^s(\varepsilon) - [\beta(B^w + V^w) + u] \tag{8}$$

Given $H^s(\varepsilon)$ is increasing in ε, a transaction goes through if $\varepsilon \geqslant \varepsilon^s$, where the reservation ε^s is defined by

$$\varepsilon^s =: H^s(\varepsilon^s) = \beta(B^w + V^w) + u, \tag{9}$$

and $1 - F^s(\varepsilon^s)$ is thus the probability that a transaction is carried out.

The value functions for being a seller and a buyer in season s are, respectively:

$$V^s = \beta V^w + u + [1 - F^s(\varepsilon^s)] E^s[S_v^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{10}$$

$$B^s = \beta B^w + [1 - F^s(\varepsilon^s)] E^s[S_b^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{11}$$

where $E^s[.]$ indicates the expectation is taken with respect to the distribution $F^s(.)$. The Nash solution (7) implies (see Appendix):

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + (1-\theta)\frac{u}{1-\beta}, \tag{12}$$

which is a weighted average of the housing value and the present discounted value of the flow value u. So the price guarantees the seller his alternative usage of the house $(\frac{u}{1-\beta})$ and a θ fraction of the social surplus generated by the transaction $\left[H^s(\varepsilon) - \frac{u}{1-\beta}\right]$.

The average price of a transaction is:

$$P^s \equiv E\left[p^s(\varepsilon) \mid \varepsilon > \varepsilon^s\right] = (1-\theta)\frac{u}{1-\beta} + \theta E\left[H^s(\varepsilon) \mid \varepsilon > \varepsilon^s\right], \tag{13}$$

which is increasing in the conditional expected surplus of housing value for transactions exceeding the reservation ε^s.

We next derive the reservation quality ε^s. Observe from (8) and (9) that

$$S^s(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}(\varepsilon - \varepsilon^s), \tag{14}$$

Using the value functions (10) and (11), the sum of value for the buyer and the seller is:

$$B^s + V^s = H^s(\varepsilon^s) + \left[1 - F^s(\varepsilon^s)\right] E^s\left[S^s(\varepsilon) \mid \varepsilon > \varepsilon^s\right], \tag{15}$$

which is the sum of the housing value $H^s(\varepsilon^s)$ of the marginal transaction and the expected surplus from a transaction above the reservation ε^s. Using the definition of $S^s(\varepsilon)$ and ε^s in (8) and (9), and the expression in (14), the sum of values is:

$$B^s + V^s = \beta(B^w + V^w) + u + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left[1 - F^s(\varepsilon^s)\right] E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s\right]. \tag{16}$$

Solving this explicitly, we derive:

$$B^s + V^s = \frac{u}{1-\beta} + \frac{(1+\beta\phi^w)\, h^s(\varepsilon^s) + \beta(1+\beta\phi^s)\, h^w(\varepsilon^w)}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)}, \tag{17}$$

where $h^s(\varepsilon^s) \equiv h(\varepsilon^s, v^s) = \left[1 - F^s(\varepsilon^s)\right] E\left[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s\right]$ is the expected surplus of quality above threshold ε^s as described in (3). Using the definition of ε^s in (9) and the expression (4), we derive the reservation quality:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s = u - \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(B^s + V^s) + \frac{1-\beta^2\phi^s}{1-\beta^2\phi^w\phi^s}\beta\phi^w(B^w + V^w), \tag{18}$$

Note that (17) and (18) together imply that the reservation quality ε^s is independent of θ. This is because a transaction will go through as long as the total surplus is positive which is independent of how the surplus is divided among the buyer and the seller.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $(P^j, P^j, \varepsilon^j, \varepsilon^j)$ in season $j = s, w$ through two channels, as shown in (13) and (18): the unconditional expected surplus of quality above reservation ε^j, $h^j(\varepsilon^j)$, which affects the outside option $B^s + V^s$ as shown in (17); and the condition surplus of quality above reservation ε^j, $E^j\left[\varepsilon - \varepsilon^j \mid \varepsilon > \varepsilon^j\right]$, which affect the expected total surplus of a transaction $E^j\left[S^j(\varepsilon) \mid \varepsilon > \varepsilon^j\right]$ as in (14).

3.5 Stock of vacant houses

In any season s, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$v^s = (1 - \phi^s)\left[v^w\left(1 - F^w\left(\varepsilon^w\right)\right) + 1 - v^w\right] + v^w F^w\left(\varepsilon^w\right)$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = 1 - \phi^s + v^w F^w\left(\varepsilon^w\right)\phi^s \tag{19}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality ε^w and on the distribution $F^w(.)$.

An equilibrium is a vector $(B^s + V^s, B^w + V^w, \varepsilon^s, \varepsilon^w, v^s, v^w, P^s, P^w)$ that jointly satisfies equations (13), (17),(18), and (19), with $H^s(\varepsilon)$ given by (9), (14), and (17).

4 Model-generated Seasonality of Prices and Transactions

4.1 Qualitative Results

We now derive the extent of seasonality in prices and transactions generated by the model. The driver for seasonality in the model is the probability of a moving shock, which we assume to be higher in the summer: $1 - \phi^s > 1 - \phi^w$. Using (19), the stock of vacant houses in season s is given by:

$$v^s = \frac{1 - \phi^s + \phi^s F^w\left(\varepsilon^w\right)\left(1 - \phi^w\right)}{1 - F^s\left(\varepsilon^s\right)F^w\left(\varepsilon^w\right)\phi^s\phi^w}. \tag{20}$$

(The expression for v^w is correspondingly isomorphic). The *ex ante* higher probability of moving in the summer ($1 - \phi^s > 1 - \phi^w$) clearly has a direct positive effect on v^s.[22]

Given $v^s > v^w$, the thick-market effect implies $h^s(\varepsilon^s) > h^w(\varepsilon^w)$ as in (3). It then follows from (17) that $(B^s + V^s) > (B^w + V^w)$, and finally from (9) that $H^s(\varepsilon^s) < H^w(\varepsilon^w)$. In other words, the housing value of the marginal transaction is lower in the hot season.

[22] More specifically, the numerator is a weighted average of 1 and $F^w(\varepsilon^w)(1 - \phi^w)$ with $1 - \phi^s$ being the weight assigned to 1. Therefore, as along as $F^w(\varepsilon^w)(1 - \phi^w)$ is sufficiently smaller than one (which is true when we calibrate ϕ^j to match the average duration of staying in a house), we have $v^s > v^w$.

4.1.1 Seasonality in Prices

From the price equation (13), since the flow value u is a-seasonal, housing prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. The following result follows:

Result 1 *When sellers have some bargaining power ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in θ.*

To see this, note that from (36) the equilibrium price P^s is the discounted sum of the flow value $(\frac{u}{1-\beta})$ plus a positive surplus from the sale. The surplus $E^s\left[\left(H^s(\varepsilon) - \frac{u}{1-\beta}\right) \mid \varepsilon > \varepsilon^s\right]$ is seasonal. The price is higher in the hot season if the average quality of transacted houses is higher, i.e. if $E^s[H^s(\varepsilon) \mid \varepsilon > \varepsilon^s] > E^s[H^w(\varepsilon) \mid \varepsilon > \varepsilon^w]$. In words, if the quality of matches goes up in the summer (and hence the total surplus of a transaction), then sellers can obtain a higher surplus in the summer.

Recall from (14) that

$$E^s[H^s(\varepsilon) \mid \varepsilon > \varepsilon^s] = H^s(\varepsilon^s) + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} E[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s]. \tag{21}$$

There are two opposite forces on the average quality of transacted houses: a negative one from the lower housing value of marginal transaction and a positive one from the conditional expected surplus of quality above the reservation (see **??**). Since $\beta^2\phi^w\phi^s$ is close to 1, the positive effect dominates. Given that θ affects P^s only through the equilibrium vacancies (recall the reservation quality ε^s is independent of θ), it follows that the extent of seasonality in prices is increasing in θ. Since (13) holds independently from the steady state equation for v^s and v^w, Result 1 holds independently of what drives $v^s > v^w$. Finally, the effect of the flow-value u on the seasonality of prices is as follows:

Result 2 *The extent of seasonality in prices is decreasing in the flow value u.*

Result 2 follows from the fact that the extent of seasonality in prices decreases as the a-seasonal component—the outside option u—increases.

We next turn to the degree of seasonality in transactions.

4.1.2 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s(1 - F^s(\varepsilon^s)). \tag{22}$$

(An isomorphic expression holds for Q^w). Seasonality in transactions stems from three sources. First, a bigger stock of vacancies in the summer, $v^s > v^w$, tends to increase transactions in the summer. Second, the thick-market effect shifts up the probability of a transaction for any given ε^s. The final effect is due to fact that the seasonality in sellers' and buyers' outside options, tends to reduce the cutoff ε^s in the hot season. This is because the outside option in the hot season s is linked to the sum of values in the winter season: $B^w + V^w$. To see this negative effect more explicitly, rewrite (18) as

$$\begin{aligned} & \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s \\ = \quad & u + \frac{\beta\phi^w(1-\beta)(1+\beta\phi^s)}{1-\beta^2\phi^w\phi^s}(V^w + B^w) + \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(V^w + B^w - V^s - B^s), \qquad (23) \end{aligned}$$

which makes clear that $(B^w + V^w) > (B^s + V^s)$ has a negative effect on $\varepsilon^s/\varepsilon^w$.

It is important to note that the amplification mechanism present in the model: for any given level of seasonality in vacancies, the thick-market effect through the first-order stochastic dominance of $F^s(.)$ over $F^w(.)$ can generate *higher* seasonality in transactions.

Given ε^s is independent of θ, we can summarize the results as follow:

Result 3 Transactions are more seasonal than vacancies. The extent of seasonality is independent of the seller's bargaining power θ.

To see this, note that the outside option for both the buyer and the seller in the hot season is to wait and transact in the cold season. This makes both buyers and sellers less demanding in the hot season, yielding a larger number of transactions. In other words, the "counter-seasonality" in outside options increases the seasonality in transactions. Finally, the effect of the flow value u on the seasonality of transactions is as follows:

Result 4 The extent of seasonality of transactions is decreasing in the rental flow u.

Result 4 follows from the fact that the extent of the seasonality of outside options for buyers and sellers is decreasing in u. Hence, as u increases, transactions become less seasonal.

In the Appendix we derive the case when the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer and $1-\theta$ as the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[23] The

[23] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

main difference between the observable and unobservable is the following. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon's$ are not sufficiently high enough (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that Results 1,2, and 4 continue to hold, while Result 3 is slightly modified so that the seasonality in transactions is decreasing in θ. This is because when ε is unobservable there is a second effect through the seller's surplus on the seasonality of reservation quality which is opposite to the effects from the seasonality of outside option described above. The intuition is the following. When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions. The second effect through the seller's surplus is increasing in θ (and disappears when $\theta = 0$).

4.2 Calibration of the model

4.2.1 Parameter values

We now calibrate the model to study its quantitative implications. We set the discount factor β so that the implied annual real interest rate is 5 percent.

We set the average probability of staying in the house $\phi = (\phi^s + \phi^w)/2$ to match survey data on the average duration of stay in a given house, which in the model is given by $\frac{1}{1-\phi}$). The median duration in the U.S. from 1993 through 2005, according to the American Housing Survey, was 18 semesters; the median duration in the U.K. during this period, according to the Survey of English Housing was 26 semesters. The implied (average) moving probabilities ϕ per semester are 0.056 and 0.039 for the US and the UK, respectively. These two surveys also report the main reasons for moving. Around 30 percent of the respondents report that living closer to work or to their children's school and getting married are the main reasons for moving.[24] These factors

[24] Using monthly data on marriages from 1980 through 2003 for the U.K. and the U.S., we find that marriages are highly seasonal in both countries, with most marriages taking place between April and September. (The difference in annualized growth rates of marriages between the broadly defined "summer" and "winter" semesters are 200 percent in the U.S. and 400 percent in the U.K.). Results are available from the authors.

are of course not entirely exogenous, but they can carry a considerably exogenous component; in particular, the school calendar is certainly exogenous to housing market movements (see Tucker, Long, and Marx (1995)'s study of seasonality in children's residential mobility[25]). In all, the survey evidence supports our working hypothesis that the *ex ante* probability to move is higher in the summer (or, equivalently the probability to stay is higher in the winter).

We calibrate the asset flow value, u, to match the implied average (de-seasonalized) rent-to-price ratio *received by the seller.* In the UK, the average *gross* rent-to-price ratio is roughly around 5 percent per year, according to *Global Property Guide.*[26] For the US, Davis et. al. (2008) argue that the ratio was around 5 percent prior to 2000 when it fell by 1.5 percent. The u/p ratio in our model corresponds to the *net* rental flow received by the seller after paying taxes and other relevant costs. It is accordingly lower than the *gross* rent-to-price ratio. As a benchmark, we choose u so that u/p is equal to 3 percent per year (equivalent to paying a 40 percent income tax on rent).[27] To do so, we use the equilibrium equations in the model without seasonality, that is, the model in which $\phi^s = \phi^w = \phi$. From (13) and (18), the average price and the reservation quality ε^d in the absence of seasonality in moving probabilities are (see Appendix 7.2.2):

$$P = \frac{u}{1-\beta} + \theta \frac{\left(1 - \beta F\left(\varepsilon^d\right)\right) E\left[\varepsilon - \varepsilon^d \mid \varepsilon > \varepsilon^d\right]}{(1-\beta)(1-\beta\phi)}, \tag{24}$$

and

$$\frac{\varepsilon^d}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\int_{\varepsilon^d}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon)}{1 - \beta\phi F\left(\varepsilon^d\right)}. \tag{25}$$

To obtain a calibrated value for u, we substitute $u = 0.03 \cdot p$ for $\theta = 1/2$, (when sellers and buyers have the same bargainining power) and find the equilibrium value of p given the calibrated values for β and $F(.)$. We then use the implied value of $u = 0.03 \cdot p.$ as a parameter.[28]

To be completed

[25]See also Harding, Rosenthal, and Sirmans (2003), who find that families with school-age children are willing to accept less favorable terms of trade during July through September. In other words, everything equal, they are more likely to move during school vacation.

[26]Data for the U.K. and other European countries can be found in http://www.globalpropertyguide.com/Europe/United-Kingdom/price-rent-ratio

[27]In principle, other costs can trim down the 3-percent u/p ratio, including maintenance costs, and inefficiencies in the rental market that lead to a higher wedge between what the tenant pays and what the landlord receives; also, it might not be possible to rent the house immediately, leading to lower average flows u. Note, however, that lower values of u/p lead to even higher seasonality in prices and transactions for any given level of seasonality in moving shocks. In that sense, lower u/p-ratios make it "easier" for our model to generate seasonality in prices.

[28]We also calibrated the model using different values of u for different θ (instead of setting $\theta = 1/2$), keeping the

5 Efficiency

This Section discusses the efficiency of equilibrium in the decentralized economy under both a seller's and a buyer's market scenarios. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy

To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by h_{t-1}, as well as the beginning of period's stock of vacant houses, v_t. Thus, taking as given the initial levels h_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0...}$, which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, h_t, v_t\}_{t=0...}\right) \equiv \sum_{t=0}^{\infty} \beta^t \left[h_t + uv_t F\left(\varepsilon_t; v_t\right)\right] \tag{26}$$

subject to the law of motion for h_t :

$$h_t = \phi_t h_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \tag{27}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1} = v_t \phi_{t+1} F\left(\varepsilon_t; v_t\right) + 1 - \phi_{t+1}, \tag{28}$$

and the inequality constraint:

$$0 \leqslant \varepsilon_t \leqslant \bar{\varepsilon}\left(v_t\right). \tag{29}$$

Intuitively, the planner faces two types of trade-offs when deciding the optimal reservation quality ε_t : a static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacant houses in period t in the objective function (26). The utility per person generated from vacant houses is the rental income per person, captured by $uv_t F\left(\varepsilon_t\right)$. The utility generated by occupied houses in period t is captured by h_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in

ratio u/p constant. Results are not significantly different under this procedure, but the comparability of results for different values of θ becomes less clear, since u is not kept fixed.

period t. The utility h_t, which follows the law of motion (27), is the sum of the pre-existing expected housing utility h_{t-1} that survives the moving shocks and the expected housing utility from the new matches. By increasing ε_t, the expected housing value h_t decreases, while the utility generated by vacant houses increases (since $F(\varepsilon_t)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacant houses in (28). By increasing ε_t (which in turn decreases h_t), the number of transactions in the current period decreases; this leads to more vacant houses in the following period, v_{t+1}, and consequently to a thicker market in the next period.

Assuming the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons, the optimal reservation quality, ε^j, $j = s, w$, in the periordic steady state is (see Appendix 7.3):

$$\varepsilon^j \left(\frac{1 + \beta \phi^{j'}}{1 - \beta^2 \phi^j \phi^{j'}} \right) = \frac{\left(1 + \beta \phi^{j'} A^{j'}\right) u + \beta^2 \phi^j \phi^{j'} A^{j'} D^j + \beta \phi^{j'} D^{j'}}{1 - \beta^2 \phi^j \phi^{j'} A^j A^{j'}}, \tag{30}$$

where

$$A^j \equiv F^j\left(\varepsilon^j\right) - v^j T_1^j; \qquad D^j \equiv \frac{1 + \beta \phi^{j'}}{1 - \beta^2 \phi^j \phi^{j'}} \left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j} \varepsilon dF^j(\varepsilon) + v^j T_2^j \right), \tag{31}$$

and the stock of vacant houses, v^j, $j = s, w$, satisfies (68) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j \equiv \frac{\partial}{\partial v^j} \left[1 - F^j\left(\varepsilon^j\right)\right] > 0$ and $T_2^j \equiv \frac{\partial}{\partial v^j} \int_{\varepsilon^j}^{\bar{\varepsilon}^j} \varepsilon dF^j(\varepsilon) > 0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1 - F^j(\varepsilon)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (30) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j < v^j$, $j = s, w$. However, when there are no seasons, $\phi^s = \phi^w$, it follows immediately from (68) that the solution is interior, $\varepsilon < v$. On the other hand, when the exogenous difference in moving propensities across seasons is large enough, the Planner might find it optimal to close down the market in the cold season. Before we turn to such situation, it is helpful to understand the sources of inefficiency in the decentralized economy when there are no seasons.

Abstracting from seasonality for the moment, the inefficiency in the decentralized economy is due to the fact that the optimal decision rules of buyers and sellers take the stock of houses in each period as given, thereby ignoring the effects of their decision rules on the stock of vacant houses in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high for any given stock of vacant houses. More specifically, setting $\phi^s = \phi^w = \phi$ in (30) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}} \varepsilon dF\left(\varepsilon\right) + vT_2\right)}{1-\beta\phi F\left(\varepsilon^p\right) + \beta\phi v T_1}. \tag{32}$$

Comparing (32) with (25), the thick-market effect, captured by $T_1$and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1-\beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome.[29]

We now return to the planner's problem in the case in which it is optimal to close down the market during the cold season. In this case, the solution implies setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$ in the planner's problem. The optimal reservation quality, ε^s, in the periodic steady state is (see Appendix 7.3):

$$\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s} = \frac{u + \frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} \varepsilon dF^s\left(\varepsilon\right) + v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right) - v^s T_1^s\right]}, \tag{33}$$

which is similar to the Planner's solution with no seasons in (32) with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

6 Concluding Remarks

This paper documents seasonal booms and busts in housing markets and argues that the predictability and high extent of seasonality in prices observed in some of them cannot be quantitatively reconciled with standard asset-pricing equilibrium conditions.

To explain the empirical patterns, the paper presents a search-and-matching model that can quantitatively account for most of the empirical puzzle. As a by product, the model sheds new light on interesting mechanisms governing fluctuations in housing markets that can potentially be useful in a study of lower-frequency movements. In particular, the model highlights the roles of thick-market externalities as an important determinant of the extent of housing markets' fluctuations.

In future work, the authors plan to adapt the model presented in the paper to study lower frequency movements in the housing markets.

[29] This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

7 Appendix

7.1 A back-of-the-envelope calculation

We argued before that the predictability and size of the seasonal variation in housing prices in some countries pose a puzzle to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[30]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j} \tag{34}$$

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real *gross* (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[31] As in Poterba (1984), we make the following simplifying assumptions so that service-cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): *i*) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, also constant for a given season. *ii*) The income-tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; they are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, they are actually constant across seasons; we come back to this point below).[32] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$,

[30] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

[31] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[32] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume that the opportunity cost of funds equals the cost of borrowing.

for $j = s, w$.

Subtracting (34) from the corresponding expression in the following season and using the condition that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}} \tag{35}$$

Considering the *real* differences in house price growth rates documented for the whole of the U.K., $\frac{p_s - p_w}{p_w} = 7.04\%$, $\frac{p_w - p_s}{p_s} = 0.75\%$, the left-hand side of (35) equals $6.3\% \approx 7.04\% - 0.75\% \cdot \frac{1}{1.0075}$. Therefore, $\frac{c_w}{c_s} = \frac{0.063}{c_s} + \frac{1}{1.0075}$. The value of c_s can be pinned-down from equation (34) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table 15, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.75\%$).

Table 15: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	459%
2.0%	328%
3.0%	267%
4.0%	232%
5.0%	209%
6.0%	193%

As the Table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 328 and 209 percent of those in the summer. Depreciation and repair costs ($\delta_j + \kappa_j$) might be seasonal, being potentially lower during the summer.[33] But income-tax-adjusted interest rates and property taxes ($r_j + \tau_j$), two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the U.K. is even larger. Assuming, quite conservatively, that the a-seasonal component ($r_j + \tau_j = r + \tau$) accounts for only 50 percent of the service costs in the summer ($r + \tau = 0.5c_s$), then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + 0.5c_s}{\delta_s + \kappa_s + 0.5c_s}$ implies that the ratio of depreciation and repair costs between summers

[33] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though it is key here that leisure is not too valuable for the owners.

and winters is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = 2\frac{c_w}{c_s} - 1$.[34] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 557 and 318 percent of those in the summer. (If the a-seasonal component $(r+\tau)$ accounts for 80 percent of the service costs $(r+\tau = 0.8c_s)$, the corresponding values are 1542 and 944 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for.

7.2 Derivation for the model with observable value

7.2.1 Model with seasons

Derive $p^s(\varepsilon)$ in (12)

Using the Nash solution,

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\theta$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u. \tag{36}$$

Using the value functions (10) and (11),

$$(1-\theta)V^s - \theta B^s = \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (36) to obtain (12).

7.2.2 The model without seasons

The value functions for the model without seasonality are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$.

[34] Call λ the asesonal component as a fraction of the summer service cost rate: $r+\tau = \lambda c_s$, $\lambda \in (0,1)$ (and hence $\delta_s + \kappa_s = (1-\lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{\delta_s+\kappa_s+\lambda c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w - \lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w+\kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

Using (13),(14) and (17), we can express the average price explicitly as:

$$P^s = \frac{u}{1-\beta} + \theta \left[\frac{\beta (1+\beta\phi^s) h^w (\varepsilon^w) + \left(1-\beta^2 F^s(\varepsilon^s)\right)(1+\beta\phi^w) E[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)} \right], \tag{37}$$

Using (18),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}(1-\beta)(V+B)$$

and $B+V$ from (17),

$$B+V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2} \left\{ \begin{array}{c} \frac{1-F}{1-\beta\phi} E[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} > \varepsilon] \\ +\beta \frac{1-F}{1-\beta\phi} E[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} > \varepsilon] \end{array} \right\}$$

which reduces to:

$$B+V = \frac{u}{1-\beta} + \frac{1-F(\varepsilon)}{(1-\beta)(1-\beta\phi)} E(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} > \varepsilon).$$

so

$$\varepsilon = u + \frac{\beta\phi}{1-\beta\phi}[1-F(\varepsilon)] E(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} > \varepsilon).$$

and the final equilibrium condition is:

$$v = \frac{1-\phi}{1-\phi F(\varepsilon)}.$$

7.3 Analytical derivations of the planner's solution

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,...}$ alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (h_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$\begin{aligned} W(h_{t-1}, v_t, \phi_t) &= \max_{\varepsilon_t} \left[h_t + u v_t F(\varepsilon_t; v_t) + \beta W\left(h_t, v_{t+1}, \phi_{t+1}\right)\right] \\ & \quad s.t. \\ h_t &= \phi_t h_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t), \\ v_{t+1} &= v_t \phi_{t+1} F(\varepsilon_t; v_t) + 1 - \phi_{t+1}. \end{aligned} \tag{38}$$

The first-order condition implies

$$\left(1+\beta\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial h_t}\right)v_t\left(-\varepsilon_t f\left(\varepsilon_t;v_t\right)\right)+\left(\beta\phi_{t+1}\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}+u\right)v_t f\left(\varepsilon_t;v_t\right)=0,$$

which simplifies to

$$\varepsilon_t\left(1+\beta\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial h_t}\right)=u+\beta\phi_{t+1}\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}. \tag{39}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W\left(h_{t-1},v_t,\phi_t\right)}{\partial h_{t-1}}=\phi_t\left(1+\beta\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial h_t}\right) \tag{40}$$

and

$$\begin{aligned}\frac{\partial W\left(h_{t-1},v_t,\phi_t\right)}{\partial v_t} &= \left(u+\beta\phi_{t+1}\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F\left(\varepsilon_t;v_t\right)-v_tT_{1t}\right)\\ &\quad+\left(1+\beta\frac{\partial W\left(h_t,v_{t+1},\phi_{t+1}\right)}{\partial h_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)+v_tT_{2t}\right)\end{aligned} \tag{41}$$

where $T_{1t}\equiv\frac{\partial}{\partial v_t}\left[1-F\left(\varepsilon_t;v_t\right)\right]>0$ and $T_{2t}\equiv\frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)>0$.

In the periodic steady state, the first order condition (39) becomes

$$\varepsilon^j\left(1+\beta\frac{\partial W^{j'}\left(h^j,v^{j'}\right)}{\partial h^j}\right)=u+\beta\phi^{j'}\frac{\partial W^{j'}\left(h^j,v^{j'}\right)}{\partial v^{j'}} \tag{42}$$

The envelope condition (40) implies

$$\frac{\partial W^j\left(h^{j'},v^j\right)}{\partial h^{j'}}=\phi^j\left[1+\beta\left(\phi^{j'}+\beta\phi^{j'}\frac{\partial W^j\left(h^{j'},v^j\right)}{\partial h^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(h^{j'},v^j\right)}{\partial h^{j'}}=\frac{\phi^j\left(1+\beta\phi^{j'}\right)}{1-\beta^2\phi^j\phi^{j'}} \tag{43}$$

Substituting this last expression into (41), we obtain:

$$\frac{\partial W^j\left(h^{j'},v^j\right)}{\partial v^j}=\left(u+\beta\phi^{j'}\frac{\partial W^{j'}\left(h^j,v^{j'}\right)}{\partial v^{j'}}\right)A^j+D^j,$$

where

$$A^j\equiv F^j\left(\varepsilon^j\right)-v^jT_1^j;\qquad D^j\equiv\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j}\varepsilon dF^j\left(\varepsilon\right)+v^jT_2^j\right).$$

Hence, we have

$$\frac{\partial W^{j}\left(h^{j'},v^{j}\right)}{\partial v^{j}}=\left\{u+\beta\phi^{j\prime}\left[\left(u+\beta\phi^{j}\frac{\partial W^{j}\left(h^{j'},v^{j}\right)}{\partial v^{j}}\right)A^{j'}+D^{j'}\right]\right\}A^{j}+D^{j},$$

which implies

$$\frac{\partial W^{j}\left(h^{j'},v^{j}\right)}{\partial v^{j}}=\frac{uA^{j}\left(1+\beta\phi^{j'}A^{j'}\right)+\beta\phi^{j'}D^{j'}A^{j}+D^{j}}{1-\beta^{2}\phi^{j}\phi^{j'}A^{j}A^{j'}}. \tag{44}$$

Substituting (43) and (44) into the first-order condition (42), we get:

$$\varepsilon^{j}\left(1+\beta\frac{\phi^{j'}\left(1+\beta\phi^{j}\right)}{1-\beta^{2}\phi^{j}\phi^{j'}}\right)=u+\beta\phi^{j\prime}\frac{uA^{j'}\left(1+\beta\phi^{j}A^{j}\right)+\beta\phi^{j}D^{j}A^{j'}+D^{j'}}{1-\beta^{2}\phi^{j}\phi^{j'}A^{j}A^{j'}}$$

simplify to (30).

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w=\bar{\varepsilon}_t^w$, the Bellman equation (38) can be rewritten as:

$$\begin{aligned}
W^{s}\left(h_{t-1}^{w},v_{t}^{s}\right) &= \max_{\varepsilon_t^s}\left[\begin{array}{c}\phi^{s}h_{t-1}^{w}+v_{t}^{s}\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_{t}^{s}\left(\varepsilon\right)+uv_{t}^{s}F_{t}^{s}\left(\varepsilon_{t}^{s}\right)\\ +\beta\left(h_{t+1}^{w}+u\left[v_{t}^{s}\phi^{w}F_{t}^{s}\left(\varepsilon_{t}^{s}\right)+1-\phi^{w}\right]\right)\\ +\beta^{2}W^{s}\left(h_{t+1}^{w},v_{t+2}^{s}\right)\end{array}\right] \\
& \quad s.t. \\
h_{t+1}^{w} &= \phi^{w}\left[\phi^{s}h_{t-1}^{w}+v_{t}^{s}\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_{t}^{s}\left(\varepsilon\right)\right], \\
v_{t+2}^{s} &= \phi^{s}\left[v_{t}^{s}\phi^{w}F_{t}^{s}\left(\varepsilon_{t}^{s}\right)+1-\phi^{w}\right]+1-\phi^{s}.
\end{aligned} \tag{45}$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(h_{t-1}^w,v_t^s\right)$, the state variables for next period are $\left(h_{t+1}^w,v_{t+2}^s\right)$, and the control variable is ε_t^s.

The first order condition:

$$\begin{aligned}
0 &= v_{t}^{s}\left(-\varepsilon_{t}^{s}f_{t}^{s}\left(\varepsilon_{t}^{s}\right)\right)+uv_{t}^{s}f_{t}^{s}\left(\varepsilon_{t}^{s}\right) \\
&\quad +\beta\left(\phi^{w}v_{t}^{s}\left(-\varepsilon_{t}^{s}f_{t}^{s}\left(\varepsilon_{t}^{s}\right)\right)+uv_{t}^{s}\phi^{w}f_{t}^{s}\left(\varepsilon_{t}^{s}\right)\right) \\
&\quad +\beta^{2}\left[\frac{\partial W^{s}}{\partial h_{t+1}^{w}}\left(\phi^{w}v_{t}^{s}\left(-\varepsilon_{t}^{s}f_{t}^{s}\left(\varepsilon_{t}^{s}\right)\right)\right)+\frac{\partial W^{s}}{\partial v_{t+2}^{s}}\left(\phi^{s}v_{t}^{s}\phi^{w}f_{t}^{s}\left(\varepsilon_{t}^{s}\right)\right)\right],
\end{aligned}$$

which simplifies to:

$$\begin{aligned}
0 &= -\varepsilon_{t}^{s}+u+\beta\left(-\phi^{w}\varepsilon_{t}^{s}+u\phi^{w}\right) \\
&\quad +\beta^{2}\left[\frac{\partial W^{s}\left(h_{t+1}^{w},v_{t+2}^{s}\right)}{\partial h_{t+1}^{w}}\left(-\phi^{w}\varepsilon_{t}^{s}\right)+\frac{\partial W^{s}\left(h_{t+1}^{w},v_{t+2}^{s}\right)}{\partial v_{t+2}^{s}}\phi^{s}\phi^{w}\right]
\end{aligned}$$

and can be written as:

$$\varepsilon_t^s\left[1+\beta\phi^w+\beta^2\phi^w\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial h_{t+1}^w}\right]=\left(1+\beta\phi^w\right)u+\beta^2\phi^w\phi^s\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{46}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s\left(h_{t-1}^w,v_t^s\right)}{\partial h_{t-1}^w}=\phi^s+\beta\phi^w\phi^s+\beta^2\phi^w\phi^s\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial h_{t+1}^w}, \tag{47}$$

and

$$\begin{aligned}
&\frac{\partial W^s\left(h_{t-1}^w,v_t^s\right)}{\partial v_t^s}\\
=\;&\left(1+\beta\phi^w\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_t^s\left(\varepsilon\right)+v_t^sT_{2t}^s\right)+\left(1+\beta\phi^w\right)u\left[F_t^s\left(\varepsilon_t^s\right)-v_t^sT_{1t}^s\right]\\
&+\beta^2\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial h_{t+1}^w}\phi^w\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_t^s\left(\varepsilon\right)+v_t^sT_{2t}^s\right)\\
&+\beta^2\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\left[F_t^s\left(\varepsilon_t^s\right)-v_t^sT_{1t}^s\right],
\end{aligned}$$

where $T_{1t}^s\equiv\frac{\partial}{\partial v_t^s}\left[1-F_t^s\left(\varepsilon^s\right)\right]>0$ and $T_{2t}^s\equiv\frac{\partial}{\partial v_t^s}\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_t^s\left(\varepsilon\right)>0$. This last expression can hence be written as:

$$\begin{aligned}
&\frac{\partial W^s\left(h_{t-1}^w,v_t^s\right)}{\partial v_t^s}\\
=\;&\left(1+\beta\phi^w+\beta^2\phi^w\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial h_{t+1}^w}\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s}\varepsilon dF_t^s\left(\varepsilon\right)+v_t^sT_{2t}^s\right)\\
&+\left(\left(1+\beta\phi^w\right)u+\beta^2\phi^s\phi^w\frac{\partial W^s\left(h_{t+1}^w,v_{t+2}^s\right)}{\partial v_{t+2}^s}\right)\left[F_t^s\left(\varepsilon_t^s\right)-v_t^sT_{1t}^s\right]
\end{aligned} \tag{48}$$

In steady state, (47) and (48) become

$$\frac{\partial W^s\left(h^w,v^s\right)}{\partial h^w}=\frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}, \tag{49}$$

and

$$\begin{aligned}
&\frac{\partial W^s\left(h^w,v^s\right)}{\partial v^s}\left(1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^sT_1^s\right]\right)\\
=\;&\left(1+\beta\phi^w+\beta^2\phi^w\frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s}\varepsilon dF^s\left(\varepsilon\right)+v^sT_2^s\right)\\
&+\left(1+\beta\phi^w\right)u\left[F^s\left(\varepsilon^s\right)-v^sT_1^s\right].
\end{aligned} \tag{50}$$

Substituting into the FOC (46),

$$\begin{aligned}
\varepsilon^s \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} &= (1+\beta\phi^w)\,u \\
&+\beta^2\phi^w\phi^s \frac{(1+\beta\phi^w)\,u\left[F^s(\varepsilon^s)-v^sT_1^s\right]+\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s}\varepsilon dF^s(\varepsilon)+v^sT_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s)-v^sT_1^s\right]}
\end{aligned}$$

which simplifies to (33).

7.4 Model with unobservable value

Assume that the seller does not observe ε. As shown by Samuelson (1984) in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: the seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta \in [0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. Broadly speaking, we can interpret θ as the "market power" of the seller. The setup of the model implies that the buyer accepts any offer p^{sv} if $H^s(\varepsilon)-p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i=b,v$:

$$S_v^{si} \equiv p^{si}-(u+\beta V^w), \tag{51}$$

$$S_b^{si}(\varepsilon) \equiv H^s(\varepsilon)-p^{si}-\beta B^w. \tag{52}$$

Note that the definition of S_v^{si} implies that

$$p^{sv} = S_v^{sv} + p^{sb} \tag{53}$$

i.e. price is higher when the seller is making an offer.

Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i=b,v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i=b,v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$H^s\left(\varepsilon^{si}\right)-p^{si}=\beta B^w. \tag{54}$$

$1-F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (4), the response of the reservation quality ε^{si} to a change in price is given by:

$$\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1-\beta^2\phi^w\phi^s}{1+\beta\phi^w}. \tag{55}$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{56}$$

7.4.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_p \left\{ \left[1-F^s(\varepsilon^{sv})\right]\left[p-\beta V^w - u\right] \right\}$$

The optimal price p^{sv} solves

$$\left[1-F^s(\varepsilon^{sv})\right] - \left[p-\beta V^w - u\right] f^s(\varepsilon^{sv}) \frac{\partial \varepsilon^{sv}}{\partial p^s} = 0. \tag{57}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv}-\beta V^w - u}{p^{sv}}}_{\text{mark-up}} = \underbrace{\left[\frac{p^{sv} f^s(\varepsilon^{sv}) \frac{\partial \varepsilon^s}{\partial p^s}}{1-F^s(\varepsilon^{sv})}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1-F^s(\varepsilon^s)$). The optimal decisions of the buyer (55) and the seller (57) together imply:

$$S_v^{sv} = \frac{1-F^s(\varepsilon^{sv})}{f^s(\varepsilon^{sv})} \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}. \tag{58}$$

Equation (58) says that the surplus to a seller generated by the transaction is higher when $\frac{1-F^s(\varepsilon^{sv})}{f^s(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.[35]

[35] When f is normal, $(1-F)/f$ is also called the Mills ratio, which is proportional to the area of the tail of a frequency curve.

Given the price-setting mechanism, in equilibrium, the value of a vacant house to its seller is:

$$V^s = u + \beta V^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}. \tag{59}$$

Solving out V^s explicitly,

$$V^s = \frac{u}{1-\beta} + \theta \frac{\left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}, \tag{60}$$

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv} = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta^2 F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{61}$$

7.4.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_v^{sb} = 0 \Leftrightarrow p^{sb} = u + \beta V^w$$

Using the value function V^w from (60), the price offered by the buyer is:

$$p^{sb} = \frac{u}{1-\beta} + \theta \frac{\beta^2 \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{62}$$

The buyer's value function is:

$$\begin{aligned} B^s &= \beta B^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s \left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\ &\quad + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s \left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right], \end{aligned} \tag{63}$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$\begin{aligned} B^s &= \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s \left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s \left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\ &\quad + \beta \left\{\theta \left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w \left[S_b^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w \left[S_b^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}. \end{aligned} \tag{64}$$

7.4.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i = v, b$, satisfies

$$H^s\left(\varepsilon^{si}\right) = S_v^{si} + u + V^w + \beta B^w, \tag{65}$$

which equates the housing value of a marginal owner in season s, $H^s(\varepsilon^s)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller ($\beta V^w + u$). Using (4), ε^{si} solves:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} = S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w+V^w\right) - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s+B^s\right). \tag{66}$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (60) and (64):

$$\begin{aligned} & B^s + V^s \\ = \; & \frac{u}{1-\beta} + \\ & \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]E^s\left[S^{sv}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{sv}\right] + (1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right]E^s\left[S^{sb}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{sb}\right] + \\ & \beta\left\{\theta\left(1-F^w\left(\varepsilon^{sv}\right)\right)E^w\left[S^{wv}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{wv}\right] + (1-\theta)\left[1-F^w\left(\varepsilon^{sb}\right)\right]E^w\left[S^{wb}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{wb}\right]\right\}, \end{aligned} \tag{67}$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon) + S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (66) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv}-\varepsilon^{sb}\right) = S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv}, p^{jb}, \varepsilon^{jv}, \varepsilon^{jb}\right)$ in season $j = s, w$ through two channels, as shown in (61), (62), and(66)): the conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j\left(\varepsilon^{jv}\right)}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1-F^j\left(\varepsilon^{ji}\right)\right)E^j\left[\varepsilon-\varepsilon^{ji}\mid\varepsilon>\varepsilon^{ji}\right]$, $i = b, v$. As shown in (58), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (61) and (62). Similarly as shown in (56), a higher expected surplus quality above ε^{jv} (follows from (3)) implies a higher expected surplus to the buyer $\left(1-F^j\left(\varepsilon^{ji}\right)\right)E^s\left[S_b^{si}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{si}\right]$, $i = b, v$. These two channels affect V^j and B^j in (60) and (64), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (18).

7.4.4 Stock of vacant houses

In any season s, the average probability that a transaction goes through is $\left\{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+(1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$\begin{aligned} v^s &= \left(1-\phi^s\right)\left\{v^w\left[\theta\left(1-F^w\left(\varepsilon^{wv}\right)\right)+(1-\theta)\left(1-F^w\left(\varepsilon^{wb}\right)\right)\right]+1-v^w\right\} \\ &\quad +v^w\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s=v^w \phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s, \tag{68}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv}, \varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv}, p^{sb}, p^{wv}, p^{wb}, B^s+V^s, B^w+V^w, \varepsilon^{sv}, \varepsilon^{sb}, \varepsilon^{wv}, \varepsilon^{wb}, v^s, v^w\right)$ that jointly satisfies equations (61),(64),(66), (67) and (68), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j=s, w$, derived as in (58), and (56).

Using (68), the stock of vacant houses in season s is given by:

$$v^s=\frac{\left(1-\phi^w\right) \phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s}{1-\phi^w \phi^s\left\{\theta F^s\left(\varepsilon^{sv}\right)+(1-\theta) F^s\left(\varepsilon^{sb}\right)\right\}\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}}. \tag{69}$$

Given $1-\phi^s>1-\phi^w$, as in the observable case, the equilibrium $v^s>v^w$.

7.4.5 Seasonality in Prices

Let

$$p^s \equiv \frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] p^{sv}+(1-\theta) p^{sb}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

be the average price observed in season s. Given $p^{sv}=S_v^{sv}+p^{sb}$, we can rewrite it as

$$p^s=p^{sb}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

using (62)

$$\begin{aligned} p^s &= \frac{u}{1-\beta}+\theta \frac{\beta^2\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}+\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{1-\theta F^s\left(\varepsilon^{sv}\right)} \\ &= \frac{u}{1-\beta}+\theta\left(\frac{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right] \beta^2+1-\beta^2}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}\right)\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}+\frac{\theta \beta\left[1-F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2} \end{aligned}$$

so finally,

$$p^s = \frac{u}{1-\beta} + \theta \left\{ \frac{\left[1-\theta\beta^2 F^s\left(\varepsilon^{sv}\right)\right]\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)} + \frac{\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2} \right\}. \tag{70}$$

Since the flow value u is a-seasonal, housing prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. As in Result 1, *when sellers have some "market power" ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ*. To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (58), is seasonal. Given $v^s > v^w$ and Assumption 2, the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv} > S_v^{wv}$. Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ. Since (70) holds independently from the steady state equation for v^s and v^w, Result 1 holds independently of what drives $v^s > v^w$. Finally, the effect of the flow-value u on the seasonality of prices is the same as Result 2.

7.4.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta\left(1-F^w\left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1-F^w\left(\varepsilon^{wb}\right)\right)\right]. \tag{71}$$

(An isomorphic expression holds for Q^w). A bigger stock of vacancies in the summer, $v^s > v^w$, tends to increase transactions in the summer. On the other hand, a relatively higher reservation quality in the hot season, $\varepsilon^{si} > \varepsilon^{wi}$, $i = b, v$, tends to decrease the number of transactions in the summer. As shown in (66), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. We have already shown that $S_v^{sv} > S_v^{wv}$ because of the thick market effect (Assumption 2). Using (3) and (56), the thick market effect also implies that the expected surplus to the buyer is higher in the hot season, so the expected total surplus is also higher in the hot season. It follows from (67) that $(B^s + V^s) > (B^w + V^w)$. The seasonality of S_v^{sv} implies a higher reservation value ε^{sv} in the hot season s (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). The seasonality in sellers' and buyers' outside options, on the other hand, tends to reduce the cutoff ε^{si} in the hot season for $i = b, v$. This is because the outside option in the hot season s is linked to the sum of values in the winter season: $B^w + V^w$. To see this negative effect more explicitly, rewrite (66) as

$$\begin{aligned} & \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} \\ = \; & S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta\right)\left(1+\beta\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^w+B^w\right) + \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^w+B^w-V^s-B^s\right), \end{aligned} \tag{72}$$

which makes clear that $(B^s + V^s) > (B^w + V^w)$ has a negative effect on $\varepsilon^{si}/\varepsilon^{wi}$. As in Result 3, transactions are seasonal but the extent of seasonality is decreasing in the seller's market power θ. To see this, note that the outside option for both the buyer and the seller in the hot season is to wait and transact in the cold season. This makes both buyers and sellers less demanding in the hot season, yielding a larger number of transactions. In other words, the "counter-seasonality" in outside options increases the seasonality in transactions. On the other hand, when the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions. Hence, the seasonality of outside options and of the seller's surplus (S_v^{sv}) have opposite effects on the seasonality of reservation quality. The second effect (through S_v^{sv}) is increasing in θ (and disappears when $\theta = 0$). Finally, the effect of the flow value u on the seasonality of transactions is the same as Result 4.

7.4.7 The model without seasons

The value functions for the model without seasonality are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$. for $i = b, v$,

Using (66),

$$\frac{\varepsilon^v}{1-\beta\phi} = S_v^v + u + \frac{\beta\phi}{1-\beta\phi}\left(1-\beta\right)\left(V+B\right) \tag{73}$$

where S_v^v follows from (58),

$$S_v^v = \frac{1-F\left(\varepsilon^v\right)}{f\left(\varepsilon^v\right)\left(1-\beta\phi\right)}.$$

and $B+V$ from (67),

$$\begin{aligned}
& (1-\beta)(B+V) \\
= \; & u+\frac{\theta\left[1-F\left(\varepsilon^{v}\right)\right]}{1-\beta\phi}\left\{\left[E\left(\varepsilon-\varepsilon^{v} \mid \varepsilon>\varepsilon^{v}\right)\right]+\frac{1-F\left(\varepsilon^{v}\right)}{f\left(\varepsilon^{v}\right)}\right\} \\
& +\frac{(1-\theta)\left[1-F\left(\varepsilon^{b}\right)\right]}{1-\beta\phi}\left[E\left(\varepsilon-\varepsilon^{b} \mid \varepsilon>\varepsilon^{b}\right)\right]
\end{aligned}$$

substitute into (73), we can jointly solve for ε^{v} and ε^{b}. More specifically, rewrite it as

$$\varepsilon^{b}=\varepsilon^{v}-\frac{1-F\left(\varepsilon^{v}\right)}{f\left(\varepsilon^{v}\right)} \tag{74}$$

and

$$\begin{aligned}
& \frac{\varepsilon^{b}}{1-\beta\phi}-\frac{\beta\phi}{1-\beta\phi}(1-\theta)\left[1-F\left(\varepsilon^{b}\right)\right]\left[E\left(\varepsilon-\varepsilon^{b} \mid \varepsilon>\varepsilon^{b}\right)\right] \\
= \; & u+\frac{\beta\phi}{1-\beta\phi}\theta\left[1-F\left(\varepsilon^{v}\right)\right]\left[E\left(\varepsilon-\varepsilon^{v} \mid \varepsilon>\varepsilon^{v}\right)+\frac{1-F\left(\varepsilon^{v}\right)}{f\left(\varepsilon^{v}\right)}\right].
\end{aligned} \tag{75}$$

Given f is log-concave, we know $E\left(\varepsilon-\varepsilon^{i} \mid \varepsilon>\varepsilon^{i}\right)$ is decreasing in ε^{i}, $i=b, v$ and $\frac{1-F(\varepsilon^{v})}{f(\varepsilon^{v})}$ is increasing in ε^{v}. Therefore, in the $\varepsilon^{v}-\varepsilon^{b}$ space, ε^{b} is increasing in ε^{v} for (74) and decreasing in ε^{v} for (75). Therefore the equilibrium is unique given v.

To compare to the Planner's problem, rewrite (75) as

$$\frac{\varepsilon^{b}}{1-\beta\phi}=\frac{u+\frac{\beta\phi}{1-\beta\phi}\left[(1-\theta) \int_{\varepsilon^{b}} x dF+\theta \int_{\varepsilon^{v}} x dF\right]}{1-\beta\phi\left[(1-\theta) F^{b}+\theta F^{v}\right]},$$

so there is an additional source of inefficiency when $\theta>0$ due to the asymmetric information. The positive term S_{v}^{sv} increases the reservation quality in the decentralized equilibrium with $\theta>0$ and hence lowers the number of transactions with respect to the efficient (Planner's) outcome. As in the observable case, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. Hence, the number of transactions can be too low or too high, depending ultimately on the shape of the distribution $F(.)$.

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To be Completed

8 Data Sources

For U.K. and U.S. data, see text.

Australia The housing price index comes from the Australia Bureau of Statistics (ABS); it is a weighted average for eight capital cities, available from 1986; the series is based on prices at settlement and are based on data provided to the land titles office; it is not quality adjusted. The CPI (non seasonally adjusted, NSA) also comes from the ABS and is a national index, not available at a disaggregated level; in what follows, for all countries, the price index considered in the analysis corresponds to the national index.

Belgium The housing price index comes from STADIM (*Studies & advies Immobiliën*) and covers Belgium and its three main regions from 1981; the series is based on the average selling prices of small and average single-family houses; apartments are not included; the data come from registered sales, and are not quality adjusted. The CPI (NSA) comes from the National Institute for Statistics.

Denmark The housing price index comes from the Association of Danish Mortgage Banks and corresponds to existing single-family homes (including flats and weekend cottages). The data come from the Land Registry, where all housing transactions are registered; they are not adjusted by quality and start in 1992. The CPI (NSA) comes from *Danmarks Statistik*.

France The housing price index comes from INSEE (National Institute for Statistics and Economic Studies) and corresponds to existing single-family homes. The data are not quality adjusted and start in 1994. The index covers all regions, and comes also disaggregated into 4 regions. The CPI (NSA) comes from the same source.

Ireland The housing price index comes from *Permanent TSB*, which accounts for about 20 percent of residential mortgage loans in the country, starting in 1996; the index is adjusted by the size of the property, dwelling type (detached, semi-detached, terrace, or apartment), and heating system. The number of transactions (loans) comes from the same source. The CPI (NSA) comes from the Central Statistical Office in Ireland.

Netherlands The housing price index comes from the Dutch Land Registry; it is a repeat-sale index, starting in 1993. The CPI (NSA) comes from the CBS (Statistics Netherlands).

New Zealand The housing price index comes from the Reserve Bank of New Zealand, starts in 1968, and is not adjusted by quality; the CPI (NSA) comes from the same source.

Norway The housing price index comes from Statistics Norway, starting in 1992; the data are not adjusted by quality as meticulously as in the U.K., however, the properties considered need to satisfy a set of broadly defined) characteristics to be included in the index; the CPI (NSA) comes from the same source.

South Africa The housing price index comes from ABSA, a commercial bank that covers around 53 percent of the mortgage market in South Africa. The data are recorded at the application stage of the mortgage lending process and the series starts in 1975. There is no quality adjustment, although the properties considered need to satisfy a set of (broadly defined) characteristics to be included in the index. The CPI (NSA) comes from Statistics South Africa.

Sweden The housing price index comes from *Statistika Centralbyrån*; the data correspond to one and two-dwelling properties and are not quality-adjusted; the series starts in 1986; data on transactions and CPI (NSA) come from the same source.