A New Perspective on "The New Rule"of the Current Account[1]

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Abstract

In an influential contribution that predates the recent renewed interest in portfolio choice models of international capital flows, Kraay and Ventura (2000) offer a "new rule" for the current account that puts portfolio choice at the center of the analysis. The new rule says that in response to a change in saving, the change in the current account is equal to the change in saving times the ratio of net foreign asset to wealth. We show that while the focus on portfolio choice is well placed, the inferences in terms of the international allocation of savings are misleading. Using a simple two-country general equilibrium model with portfolio choice, we show that the "new rule" does not hold; most of an increase in a country's saving will be invested abroad. We also show that the empirical evidence presented in Kraay and Ventura (2000) in favor of the "new rule" is consistent with an expression for the current account that holds in the steady state of almost any model. The "new rule" does not necessarily follow as an implication.

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1 Introduction

In an influential 2000 article in the Quarterly Journal of Economics, Aart Kraay and Jaume Ventura developed the "new rule"for the current account, which states that in response to a transitory income shock "the current account response equals the saving generated by the shock times the country's share of foreign assets in total assets." Their work is important as they both develop empirical evidence in support of the new rule and a simple model focusing on portfolio choice that can account for the new rule. In addition their work is several years ahead of the recent renewed interest in portfolio choice models in open economy macro and in analyzing international capital flows from a portfolio choice perspective.

In this paper we offer a different interpretation for the empirical findings in Kraay and Ventura (2000, hereinafter KV), which is fundamentally different from the "new rule". We develop a two-country general equilibrium model encompassing portfolio choice to both shed light on the empirical evidence in KV and to analyze the impact on the current account of a rise in saving resulting from a temporary income shock. We find that while the empirical evidence in KV is consistent with the model, the new rule does not necessarily follow as an implication. Specifically, the empirical evidence presented in KV is consistent with an expression for the current account that holds exactly in the steady state of the model. In terms of the short-run dynamics following an income shock however, the model implies that the extra saving is mostly invested abroad, in sharp contrast to the prediction by the new rule.

Before further developing these insights it is useful to first be more precise about the empirical exercise in KV, as well as the follow-up paper Kraay and Ventura (2003). In both papers they regress the current account on a term equal to the share of net foreign assets in total wealth times saving. In a panel regression they find that the regression coefficient is close to 1. They find that this result is primarily due to the cross-section aspect of the data. In a cross-section regression the coefficient is not only close to 1 but the empirical fit is also very accurate with an R^2 of about 0.85. In contrast, the equation has very little explanatory power based on the time series aspect of the data. When subtracting long-term-averages from both sides of the regression for each country, they find an R^2 close to 0.02 and acknowledge that "the new rule explains essentially none of the year-to-year within country differences in current accounts."

KV interpret the cross-section regression evidence as being consistent with the new rule. The logic is as follows. Assume that a change in saving does not lead to a change in expected returns, implying that the portfolio allocation across alternative assets is unchanged. The resulting capital outflow is then equal to the change in saving times the share of net foreign assets in total wealth. KV develop a model where a change in saving has little impact on portfolio shares, as long as asset return risk is large and diminishing returns to capital are weak. In order to explain that the new rule does not hold well in the short-run, Kraay and Ventura (2003) introduce adjustment costs to investment, which implies that most of an increase in saving is invested abroad in the short-run. Their model implies that in the long-run, when adjustment costs no longer play a role, the new rule will hold again. They argue that a model with adjustment costs can therefore account for both the cross-section and time-series evidence.

Our interpretation of the evidence is different. First, we interpret the cross-section evidence as reflecting a steady state phenomenon. In steady state the ratio of a country's net foreign asset position to its total wealth must be equal to its equivalent in terms of financial flows, namely the ratio of the current account to saving. It follows that the current account must be equal to saving times the ratio of net foreign assets to wealth, a result that we formally develop. Second, we show that the time-series evidence is a natural implication of our model even in the absence of adjustment costs. Even in the context of a partial equilibrium small open economy model along the line of KV, an increase in saving is entirely invested abroad to a first-order, and one needs an unrealistic high degree of asset return risk to get a result resembling the new rule. We then move to a two-country general equilibrium setup, and find that the model is even further apart from the new rule.

The importance of general equilibrium analysis in this context can be seen as follows. The new rule holds if and only if a change in saving does not lead to a change in the ratio of net foreign assets to total wealth. In general equilibrium it is hard to see how that can be the case. Consider that there are two countries, Home and Foreign, so that the net foreign asset position of Home and Foreign add up to zero. It is then easy to see that when relative wealth of the two countries changes due to an increase saving in Home, the ratio of net foreign assets to wealth must change in at least one of the countries, and the new rule cannot hold for both countries. Consider a temporary income shock that raises saving in Home but not in Foreign and that the net foreign asset position in Home is positive. The new rule implies that the current account rises in Home and does not change in Foreign, a clear violation of the aggregate identity that the current accounts sum to zero.

Moreover, the ratio of net foreign assets to total wealth is not a portfolio share once we think in general equilibrium terms. Again consider the example of two countries and assume that they invest in each other's equity markets. The Home country does not decide how much the Foreign country wishes to invest in Home's stock market. That is a portfolio decision by the Foreign country. Now assume that both countries hold their portfolio shares constant (fraction invested in both equity markets). Then the ratio of the net foreign asset position to total wealth for Home is equal to Home's portfolio share invested abroad minus an interaction term that is equal to Foreign's portfolio share invested in Home times the ratio of Foreign wealth relative to Home wealth. This example illustrates that even when portfolio shares are held constant, the ratio of net foreign assets to wealth will change with changes in relative wealth, which in turn is affected by saving.

The paper is organized as follows. Section 2 considers a partial equilibrium small open economy model, allowing us to connect as closely as possible to a similar framework used by KV to develop the theoretical underpinnings for the new rule. The full general equilibrium analysis will be conducted in section 3. Section 4 concludes.

2 A Small Open Economy Model

Consider a small open economy in which investors can buy claims on both domestic capital and a foreign asset. For the purpose of this section it does not matter what the foreign asset is (e.g. a stock or a bond). The setup is a partial equilibrium model in that the large foreign country cannot buy claims on the small country's capital. While other details of the model differ somewhat from the model in KV, the partial equilibrium small open economy setup is key to their analysis.

2.1 Production and Investment

There is one good that is produced in both countries. Production in the small country uses a constant returns to scale technology combining labor and capital:

$$Y_t = A_t K_t^{1-\omega} N_t^{\omega}$$

Y is output, A is an exogenous stochastic productivity term, K is the capital input and N the labor input.

We assume that productivity follows a simple i.i.d. process:

$$A_t = 1 + \varepsilon_t \tag{1}$$

where ε_t has a $N(0, \sigma_a^2)$ distribution. While one could easily allow for persistence of productivity shocks, this obscures the analysis of the new rule. A persistent positive technology shock raises investment independent of its effect on saving. The new rule is instead about the impact of a change in saving on domestic investment and therefore the current account. It is about the non-separability between saving and investment decisions that results from portfolio choice. As we will discuss later, a transitory positive technology shock will raise saving in the model. We will then analyze the impact of this increase in saving on domestic investment.

Labor input is fixed and normalized to unity. The wage is equal to the marginal product of labor:

$$W_t = \omega A_t K_t^{1-\omega} \tag{2}$$

The dynamics of the capital stock reflect investment and depreciation:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{3}$$

We assume that there are no adjustment costs associated with investment, so that the consumption and capital good are the same and have a relative price of 1. As explained in the introduction, Kraay and Ventura (2003) introduce adjustment costs in order to explain why the new rule does not hold in the short-run as reflected in time series data. In their model the new rule will hold in the long-run when adjustment costs no longer play a role. They therefore interpret the cross-section regression results as the new rule holding in the long run. We will cast doubt on this interpretation as we will show that the new rule does not hold in the short run even without adjustment costs.

2.2 Two Assets

In the absence of adjustment costs the gross return on domestic capital is

$$R_{t+1} = 1 - \delta + (1 - \omega) A_{t+1} K_{t+1}^{-\omega} \tag{4}$$

The last term on the right hand side is the marginal product of capital. From a national accounting point of view this is the income part of the return (the dividend). In addition to investment in domestic capital with return (4), investors can also buy a claim on foreign capital with an exogenous return of R^*_{t+1}. Analogous to the claims on the small country's capital, we have

$$R_{t+1} = 1 - \delta + D^*_t \tag{5}$$

where D^*_t if the dividend on foreign capital. From the point of view of the small country D^*_t is exogenous. We assumed that D^*_t is equal to a constant D^* plus a dividend innovation ϵ^D_{t+1} that has a $N(0, \sigma^2_d)$ distribution.[1]

Investment in the foreign asset does entail a cost. Specifically, when investors in the small country invest abroad they receive the foreign return times $1 - \tau$, where τ is a second-order constant (proportional to the variance of model innovations). τ is an iceberg cost that captures the hurdles of investing outside the domestic country. It does not generate a loss in resources as it is a fee paid to a broker. As a technicality, brokers are assumed to simply consume these fees right away. The cost τ can generate portfolio home bias as seen in the data. With substantial portfolio home bias the new rule implies that an increase in saving is mostly invested domestically and therefore generates relatively small capital outflows.

The portfolio return from t to $t+1$ of small country investors is

$$R^p_{t+1} = z_t R_{t+1} + (1 - z_t)(1 - \tau) R^*_{t+1} \tag{6}$$

where z_t is the fraction of wealth invested in domestic capital.

2.3 Consumption and Portfolio Choice

We adopt a simple OLG structure to ensure that there is a well-defined steady state wealth distribution. Agents live two periods. They earn the wage W_t in (2) when young. They consume in both periods, with the consumption in the second period financed by the return on the portfolio. A young agent at time t maximizes:

$$\frac{(C_{y,t})^{1-\gamma}}{1-\gamma} + \beta E_t \frac{(C_{o,t+1})^{1-\gamma}}{1-\gamma}$$

[1] One could assume that the expectation of D^*_t is time-varying, being a function of a time-varying state space. This complication does not change the results that follow.

subject to:

$$C_{o,t+1} = (W_t - C_{y,t})R^p_{t+1} \tag{7}$$

and the portfolio return (6).

The first order conditions with respect to $C_{y,t}$ and z_t are:

$$(C_{y,t})^{-\gamma} = \beta E_t \left(C_{o,t+1}\right)^{-\gamma} R^p_{t+1} \tag{8}$$

$$E_t \left(R^p_{t+1}\right)^{-\gamma} \left(R_{t+1} - (1-\tau)R^*_{t+1}\right) = 0 \tag{9}$$

The Euler equation (9) is a standard arbitrage condition for portfolio choice, which says that the expected product of the asset pricing kernel and asset return is the same across all assets.

As the small country investors are the only ones who can purchase claims on domestic capital, the small country asset market clearing condition is

$$K_{t+1} = (W_t - C_{y,t})z_t \tag{10}$$

2.4 Savings and the current account

Aggregate saving in the small economy is the sum of saving by young and old agents. The saving of the former is simply the difference between their wage income and their consumption. The income of old agents is the dividend component of the return on their portfolio, while their consumption reflects the overall return, including the sale of the depreciated capital. Aggregate saving is then:

$$S_t = W_t - C_{y,t} - (1-\delta)\left(W_{t-1} - C_{y,t-1}\right)$$

The current account is saving net of investment: $CA_t = S_t - I_t$.

Turning to the stocks of financial assets, the overall wealth is simply the saving of young agents, as old agents have exited asset markets. The net foreign asset position is a share $1 - z_t$ of wealth.

2.5 Solution Method

The solution of the simple small open economy model is not affected by the technical difficulties that emerge in general equilibrium models of portfolio choice. In these models the standard first and second-order solution methods need to be adjusted because portfolio choice is not well-defined in a deterministic environment. This issue only emerges in a setup with heterogeneous investors, such as the one described in the next section where domestic and foreign investors choose different portfolios, with a portfolio home bias. Solving such a general equilibrium model with portfolio choice and heterogeneous agents requires an extension of standard first and second-order solution methods recently developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2007).

In the partial equilibrium model described here all agents in the small country choose the same portfolio allocation. We can then use the standard solution method. When all agents choose the same portfolio, steady state portfolio shares are simply determined by steady state asset supplies. In our model the portfolio share z_t follows directly from (10) as the ratio between the capital stock K_t and wealth $W_t - C_{y,t}$.

The key insights can be obtained from a simple first-order solution of the model. It will turn out to be useful though, in particular in comparison to KV, to obtain further precision by also computing the second and third-order solutions of the model. The order of variables is defined as follows. The zero-order component is the level of a variable when standard deviations approach zero (deterministic steady state). The first-order component is proportional to standard deviations of model innovations or innovations themselves. The second-order component is proportional to the variance of model innovations (or the product of model innovations), and so on. We can always write a variable as the sum of its components of various orders, which we will write as $x_t = x(0) + x_t(1) + x_t(2) + ...$ for any variable x.

2.6 Impact of a Temporary Income Shock

The first-order solution of the model follows directly from linearization around the zero-order component (deterministic steady state) of all variables. We focus on the main results, with a detailed solution presented in Appendix A.

The first-order component of savings is:

$$S_t(1) = (1 - \bar{c})(\varepsilon_t - (1 - \delta)\varepsilon_{t-1}) \tag{11}$$

where $\bar{c} = C_y(0)/W(0)$. A positive income shock at time t raises saving as some of the resulting increase in labor income is saved for consumption when old. Also notice that in this model a positive income shock during the previous period lowers saving as the old are consuming the principal value of their increased wealth from the previous period. From now on we will simply set $\varepsilon_{t-1} = 0$ as we will focus on the impact of a temporary increase in income during the current period that raises current saving.

The impact on the current account can be derived from the Euler equation (9) for portfolio choice together with the definition of the domestic return R_{t+1} in (4). The first-order component of the Euler equation gives

$$E_t R_{t+1}(1) = E_t R^*_{t+1}(1) = 0 \tag{12}$$

The first equality in (12) shows that to the first-order the expected return is the same for domestic capital as for the foreign asset, as optimal portfolio shares would otherwise be infinite. The second equality in (12) follows from our assumption that the return on the foreign asset is i.i.d., so that it's predictable component is fully captured by the zero-order term. (4) then implies that to a first-order the domestic capital stock and investment are not affected by the temporary income shock: $K_{t+1}(1) = I_t(1) = 0$. Intuitively, the exogenous return on the foreign asset ties down the expected return on domestic capital to a first order, which in turn ties down the capital stock to the first-order. The first-order component of the current account then immediately follows as $CA_t(1) = S_t(1) - I_t(1) = S_t(1)$. The increase in saving is therefore entirely invested abroad and the new rule clearly does not hold.

It is useful to compare this result to that of the small open economy model in KV. While their model is a bit different, with a continuous time setup where agents solve an infinite horizon consumption and portfolio decision problem, the essence of a partial equilibrium small open economy model is the same. Agents can invest in domestic capital, a risk-free foreign asset and a risky foreign asset (called foreign capital). They consider the impact of a temporary income shock that raises wealth and saving. Their key equation (7) describes the impact of the shock on investment. In their own notation, the change in investment is equal to the change in saving times

$$\frac{(1-\eta^2)\sigma^2}{(1-\eta^2)\sigma^2 - \frac{\partial \pi}{\partial k} a} \frac{k}{a} \tag{13}$$

Here k is the capital stock, a is wealth, π is the net return on domestic capital (corresponding to $R_{t+1} - 1$ in our notation), η is the correlation between the return on domestic capital and foreign capital, and σ^2 is the standard deviation of the return on domestic capital.

In terms of order notation (13) shows that the change in investment in response to the shock is of order 3 and higher. Specifically, the change in saving is of order one and higher. Thee share (13) is of order two and higher. The reason is that the denominator it is dominated by $\partial\pi/\partial k$ which has a well-defined zero-order component, both in KV and in our own model.[2] The change in investment is then third-order as it is the product of a first-order and a second-order term. The result in KV is therefore consistent with our finding that to the first-order investment does not change and the entire increase in saving is invested abroad.

How then is it possible that KV conclude that their model can account for the new rule? This can be seen by making asset return risk very high. Consider letting σ^2 approach infinity in (13). Then this expression converges to k/a. The increase in investment is then equal to the increase in saving times the fraction of wealth invested at home. This implies that the current account is equal to the change in saving times the fraction of wealth invested abroad, which is the new rule.

The question is then whether it is meaningful to let σ become very large. The impact of the shock on investment is of third and higher-order, which is generally very small. In order to check exactly how small is small, it is useful to quantify the expression in (13). For illustrative purposes we will assume that the capital to wealth ratio, k/a, is 1, so that the new rule implies that all of the increase in saving is invested at home. KV allow the return on domestic capital to be a general function of the capital stock. If we adopt the specific expression (4) for the return in domestic capital in our own model, which follows from a standard Cobb-Douglass production function, it follows that $(\partial\pi/\partial k)a$ is equal to $-\omega(\pi+\delta)$.

We use equity returns to calibrate the model. Note that if anything this will overstate the magnitude of third and higher-order components since equity claims are residual claims whose return volatility is considerably higher than total claims on a country's capital stock. Jorion and Goetzmann (1999) report moments based on stock return data for 39 countries from 1921 to 1999. Focusing on the 13 countries for which continuous data are available starting in the 1920s,[3] we set π equal to the average real annual return of 0.033. We set σ equal to the average

[2]Specifically, the zero-order component in our model is $-\omega(1-\omega)K(0)^{-\omega-1}$.

[3]The countries are the United States, Canada, Austria, Belgium, Denmark, France, Italy, Netherlands, Norway, Spain, Sweden, Switzerland and United Kingdom.

standard deviation of 0.174. It can be shown that $(1 - \eta^2)\sigma^2$ is equal to σ^2 minus the variance of the world return. The standard deviation of the global return reported by Jorion and Goetzmann (1999) is 0.121. Finally, we set the annual depreciation rate δ equal to 0.1, which is a standard assumption in calibrations, and the labor share ω equal to 0.7.

These parameters imply that the coefficient in (13) is equal to 0.14. This implies that 86% of the increase in saving is invested abroad and 14% at home, a sharp contrast to the new rule, which says that 100% is invested at home. In this case the 0.14 number is a second-order term that multiplies the increase in saving. This number is only as large as it is because of the very large stock return risk, with an average standard deviation of 0.174. As pointed out above, it would be even smaller if we had used a measure of the overall return on domestic capital, which would have much lower risk.

The fact that the impact of a change in saving on domestic investment is third-order in KV is also consistent with the model in this paper. In the Technical Appendix that is available on request we also solve the second and third-order components of model variables, from respectively the second and third-order components of model equations. Consistent with KV, the first and second-order components of investment are zero, but its third-order component is positive.

Leaving the algebraic details to the Technical Appendix, it is useful to explain why there is a third-order increase in investment. We have already seen that to the first-order all of the increase in saving is invested abroad. This leads to a first-order drop in the fraction z_t invested at home. This is immediately evident from (10) as the domestic capital stock does not change to the first-order but there is a first-order increase in wealth. This reflects the supply side of domestic capital. The demand side can be obtained from the portfolio Euler equation (9). A third-order expansion of this equation shows that the first-order component of the optimal portfolio share invested at home depends on the third-order component of the expected excess return, divided by the variance of the excess return. Therefore a relatively small drop in the third-order component of the excess return on domestic capital will generate a first-order drop in the portfolio share invested at home. This is needed to equate demand and supply of home capital. A third-order drop in the expected excess return on domestic capital is achieved by a third-order rise in the capital stock and therefore investment. This explains why the rise in investment is third-order and therefore small.

2.7 Two-Way Asset Trade

An odd feature of the model considered so far is the asymmetric structure of asset markets, as only agents of the small country can invest abroad. An alternative setup considers that the world is divided up into many small countries and all agents can invest abroad. It follows that from the perspective of each of the countries the rest of the world is also investing in their domestic capital. In the next section we study a two-country general equilibrium model where investors from both countries can buy claims on each other's capital. The relative size of the countries will be a free parameter. But even in the context of the small open economy setup developed so far we can introduce this key general equilibrium feature by allowing the large country to buy claims on the small country's capital.

If we let m_{t+1} be the asset pricing kernel of the large country and we assume the same cost τ of investing abroad by the large country, the portfolio Euler equation for the large country is

$$E_t m_{t+1}(1-\tau)R_{t+1} = E_t m_{t+1}R^*_{t+1} \tag{14}$$

Since the stochastic processes of m_{t+1} and R^*_{t+1} are exogenously given from the perspective of the small open economy, it follows that a shock in the small country cannot affect $E_t m_{t+1}R_{t+1}$. This implies that the capital stock K_{t+1} cannot change and therefore investment I_t will not change. Otherwise the stochastic process of the return R_{t+1} would change and so will $E_t m_{t+1}R_{t+1}$. Note that investment will not change to any order, not even third or higher order. The infinite relative size of the large country completely ties down the expected return in the small country.

In this case the entire increase in saving in the small country will be invested abroad. This is now the case to *any* order of approximation. Introducing this key general equilibrium feature clearly breaks down the new rule further. It now does not even matter how large the standard deviation is of the return on the assets. Note that the small country investors will not change their portfolio allocation in response to the shock. Even with a constant portfolio share of the small country, the ratio of net foreign assets to GDP will still change in the small country. It will increase as the entire increase in saving is invested abroad.

In the partial equilibrium model above where foreign investors cannot invest in the small country, as well as in KV, the ratio of net foreign assets to GDP is a portfolio share from the perspective of the small country. But this is no longer the case in the present general equilibrium setting as the net foreign asset position of the small country is also affected by claims of the large country on the small country. In this example the capital outflow from the small country is equal to the increase in saving times the steady state fraction invested abroad, so it's first-order component is $(1 - z(0))S_t(1)$. But since there is no change in investment, we already know that the first-order component of net capital outflows is $S_t(1)$. Therefore the first-order component of capital inflows must be equal to $-z(0)S_t(1)$. In other words, capital inflows will go down. As long as the return on domestic capital does not change, foreign investors are indifferent between the domestic and foreign asset. In this case they reduce their claims on the small country's capital so that overall demand for domestic capital remains unchanged. This is consistent with no change in the capital stock of the small country and therefore no change in the stochastic process for R_{t+1}.

3 Two-Country General Equilibrium Model

The analysis in the last subsection already foreshadows the full general equilibrium analysis that will be conducted in this section. The model in this section extends the two-country partial equilibrium model of the previous section in several directions. First, as in the last subsection, agents from both countries can buy claims on the capital of the other country. Second, the relative size of the two countries is a free parameter. Third, we allow for an exogenous positive growth rate of the population in both countries in order to allow steady state saving rates to be non-zero. Finally, we allow for different time-discount rates across countries. This will lead to different steady state saving rates across the two countries and a non-zero steady state net foreign asset position relative to GDP.

3.1 Production and Investment

The two countries produce the same good. We call the countries Home and Foreign, denoted with superscripts H and F. The production function in country i $(= H, F)$ is

$$Y_t^i = A_t^i (K_t^i)^{1-\omega} (N_t^i)^{\omega}$$

We assume that

$$N_t^H = n(1+g)^t \tag{15}$$
$$N_t^F = (1-n)(1+g)^t \tag{16}$$

The relative size of the two countries is measured by the relative population size. A fraction n of the world population lives in the Home country and a fraction $1-n$ in the Foreign country. The population in both countries grows at the constant rate g. We will use lower case letters to denote the ratio of a variable relative to the young population in the country. For example, $k_t^i = K_t^i/N_t^i$.

Productivity in both countries follows a simple i.i.d. process

$$A_t^i = 1 + \varepsilon_t^i \tag{17}$$

where ε_t^i has a $N(0, \sigma_a^2)$ distribution. Without loss of generality we assume that productivity shocks are uncorrelated across the two countries. Wages are equal to the marginal product of labor:

$$W_t^i = \omega A_t^i (k_t^i)^{1-\omega} \tag{18}$$

The dynamics of the capital stock reflects investment and depreciation:

$$K_{t+1}^i = (1-\delta)\, K_t^i + I_t^i$$

3.2 Two Assets

Countries trade claims on each other's capital. The gross return on country i capital is

$$R_{t+1}^i = 1 - \delta + (1-\omega) A_{t+1}^i \left(k_{t+1}^i\right)^{-\omega} \tag{19}$$

When investing abroad each country receives the gross return times $1-\tau$, where τ is again a second-order constant iceberg cost that captures the hurdles of investing abroad. Country i invests a fraction z_t^i in Home capital. The portfolio returns from t to $t+1$ of investors from both countries are then

$$R_{t+1}^{p,H} = z_t^H R_{t+1}^H + (1 - z_t^H)(1-\tau) R_{t+1}^F \tag{20}$$
$$R_{t+1}^{p,F} = z_t^F (1-\tau) R_{t+1}^H + (1 - z_t^F) R_{t+1}^F \tag{21}$$

3.3 Consumption and Portfolio Choice

We adopt the same OLG structure as in the small open economy model, allowing for different time-discount rates in the two countries. A young agent in country i at time t maximizes:

$$\frac{\left(C^i_{y,t}\right)^{1-\gamma}}{1-\gamma} + \beta^i E_t \frac{\left(C^i_{o,t+1}\right)^{1-\gamma}}{1-\gamma}$$

subject to:

$$C^i_{o,t+1} = (W^i_t - C^i_{y,t}) R^{p,i}_{t+1} \tag{22}$$

and the portfolio return (20) or (21).

The first order conditions with respect to $C^i_{y,t}$ and z^i_t are:

$$\left(C^i_{y,t}\right)^{-\gamma} = \beta^i E_t \left(C^i_{o,t+1}\right)^{-\gamma} R^{p,i}_{t+1} \quad i = H, F \tag{23}$$

$$E_t \left(R^{p,H}_{t+1}\right)^{-\gamma} \left(R^H_{t+1} - (1-\tau) R^F_{t+1}\right) = 0 \tag{24}$$

$$E_t \left(R^{p,F}_{t+1}\right)^{-\gamma} \left((1-\tau) R^H_{t+1} - R^F_{t+1}\right) = 0 \tag{25}$$

The asset market clearing conditions are

$$K^H_{t+1} = (W^H_t - C^H_{y,t}) N^H_t z^H_t + (W^F_t - C^F_{y,t}) N^F_t z^F_t \tag{26}$$

$$K^F_{t+1} = (W^H_t - C^H_{y,t}) N^H_t (1 - z^H_t) + (W^F_t - C^F_{y,t}) N^F_t (1 - z^F_t) \tag{27}$$

The budget constraints together with asset market clearing conditions imply that the world goods market equilibrium condition is satisfied as well.

3.4 Solution Method

As indicated in section 2, the solution method for a general equilibrium portfolio choice model is somewhat different than the standard first and higher order solution method. The standard solution method solves the zero-order component of all variables (deterministic steady state) from the zero-order component of all equations, and the same for first, second and higher orders. This does not work in general equilibrium models with portfolio choice. For example, the deterministic steady state of the model ties down average portfolio shares, as they must equate steady state relative asset supplies, but not the difference across countries in portfolio shares.

Even the zero-order component of the difference across countries in portfolio shares depends on second moments, such as variances and covariances involving asset returns. For example, it is well known that in a two-country, two-asset portfolio choice problem the optimal portfolio share depends on a covariance between the excess return and the real exchange rate, divided by the variance of the excess return. The ratio of these two second moments is of order zero. One can only get at these second moments from a second-order expansion of the portfolio Euler equations. That is why the zero-order component of portfolio allocation can only be computed by using the second-order component of portfolio Euler equations.

Similarly, the first-order component of portfolio shares depend on the third-order component of portfolio Euler equations. Intuitively, in the example above the optimal portfolio depends on a covariance divided by a variance, cov_t/var_t. Then first-order changes in portfolio shares depend on changes in these second moments. When the covariance is time varying, the portfolio will depend on the change in cov_t, divided by var_t. As discussed in Tille and van Wincoop (2007), changes in second moments are of third and higher order. One can only get at them by using a third-order approximation of portfolio Euler equations.[4]

We will adopt here the solution method developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2007). The method distinguishes between the difference across countries in portfolio shares and all "other variables". Similarly, it distinguishes between the difference across countries in portfolio Euler equations and all "other equations". The solution involves two steps. The first step solves the zero-order component of the difference across countries in portfolio shares jointly with the first-order component of all other variables. This uses the second-order component of difference across countries in portfolio Euler equations together with the first-order component of all other equations.

The solution involves a fixed point problem, which is seen as follows. One can solve the first-order component of all "other variables" from the first-order component of all "other equations" by using the standard first-order solution method. However, the solution will be conditional on the zero-order component of the dif-

[4] In a two-asset setup optimal portfolio shares also depend on the expected excess return divided by the variance of the excess return. Therefore a third-order change in the expected excess return leads to a first-order change in portfolio shares. This again illustrates that one needs to use the third-order component of portfolio Euler equations in order to compute the first-order component of portfolio allocation.

ference across countries in portfolio shares. For example, if there is portfolio home bias, then a relatively high return on the Home assets leads to an increase in wealth in the Home country relative to the Foreign country, which affects (to the first-order) relative consumption and other variables. At the same time, the zero-order component of the difference across countries in portfolio shares depends on second moments, which in turn depend on the first-order solution of the model (e.g. they depend on the first-order response of asset returns to shocks in the model).

To summarize, the zero-order component of the difference across countries in portfolio shares depends on the first-order solution for the "other variables", which in turn depends on the zero-order component of the difference across countries in portfolio shares. This leads to a fixed point problem for the latter. The next step of the solution method proceeds along similar lines, but one order higher. It involves a fixed point problem for the first-order component of the difference across countries in portfolio shares. A nice contribution of Devereux and Sutherland (2006) is to show that these fixed point problems have a simple analytical solution in a broad class of models. The implication is that even large scale general equilibrium models with portfolio choice can be solved very easily using standard first and second-order solution methods, combined with the analytical solution of the fixed point problem for the difference across countries in portfolio shares.

Both steps of the solution method are needed to solve the first-order component of gross capital flows. The solution is described in a Technical Appendix available on request. The first step of the solution method is outlined in Appendix B, and is sufficient to solve for net capital flows. Intuitively, one of the "other variables" is the average portfolio share, whose first-order component is solved from the first step described above. It is the *average* portfolio share that drives *net* capital flows. Whether Home investors shift their portfolio towards Foreign assets or Foreign investors do so, either way there will be a net portfolio shift towards Foreign assets that leads to net capital outflows from the perspective of the Home country. It will nonetheless be of interest to go through the second step of the solution method as well as it tells us how the change in the current account breaks down into changes in capital inflows and outflows. It also tells us whether and how a change in saving leads to changes in portfolio shares.

Our assumptions of an overlapping generation setup, a unique good, and i.i.d. investment returns further simplifies the solution. While the zero-order component of the difference across countries in portfolio shares affects the first-order solution of relative consumption across the two countries, it does not affect the first-order solution of asset returns. Moreover, since there is only one good in the model, it does not affect the real exchange rate and therefore a hedge against real exchange rate fluctuations in optimal portfolio shares. While this is not a general result, in this particular model the second moments that affect the zero-order component of the difference across countries in portfolio shares will therefore not be affected by the zero-order component of the difference in portfolio shares itself. The zero-order component of the difference across countries in portfolio shares can be solved completely from the second-order component of the difference across countries in portfolio Euler equations.

In addition, it turns out that this zero-order component of the difference in portfolio shares plays no role in the impact of a change in saving on the current account. When there is portfolio home bias, the income of the old generation in the Home country rises relative to that in the Foreign country when a shock raises the relative return on Home assets. But since the old generation consumes all their income, this has no effect on saving or the current account.

3.5 Steady State Analysis

We assume without loss of generality that $\beta^H > \beta^F$, so that the Home country is relatively patient (gives more weight to future consumption). Appendix B shows that the steady state level of consumption by young agents in country i is

$$C_y^i(0) = \frac{W(0)R(0)}{R(0) + (\beta^i)^{1/\gamma} R(0)^{1/\gamma}} \tag{28}$$

where $W(0)$ and $R(0)$ are the zero-order components of wages and asset returns, which are the same for both countries. The patient Home country is clearly characterized by lower consumption by the young generation, which translates into higher aggregate savings. Appendix B shows that the zero-order component of national savings, scaled by the young population, is:

$$s^i(0) = \frac{g+\delta}{1+g}(W(0) - C_y^i(0)) \tag{29}$$

The steady state level of investment (relative to the labor force) is the same in both countries. The portfolio Euler equations imply that the zero-order component of asset returns must be the same for both countries. From the definition of asset returns in (19) it then follows that the steady state capital-labor ratios are the same. The steady state ratio of investment to labor is $(g + \delta)k(0)$, which is therefore also the same across countries. Since the current account is saving minus investment, the analysis so far implies that the Home country will have a steady state current account surplus and the Foreign country a steady state current account deficit. Appendix B shows that the steady state ratio of the Home country current account relative to the labor force is

$$ca^H(0) = \frac{g+\delta}{1+g}(1-n)\left(C_y^F(0) - C_y^H(0)\right) = -\frac{1-n}{n}ca^F(0) \tag{30}$$

The steady state current account surplus in the Home country also leads to a positive steady state ratio of the net foreign asset position relative to labor supply:

$$nfa^H(0) = (1-n)\left(C_y^F(0) - C_y^H(0)\right) = -\frac{1-n}{n}nfa^F(0) \tag{31}$$

Define $x^i(0)$ as the steady state ratio of net foreign assets to wealth. Steady state wealth per unit of labor is $W(0) - C_y^i(0)$. It is then easily seen from the expressions above that

$$ca^i(0) = x^i(0)s^i(0) \tag{32}$$

Multiplying both sides by $(N/Y)^i(0)$, this can also be expressed as

$$(CA/Y)^i(0) = x^i(0)(S/Y)^i(0) \tag{33}$$

In their cross-sectional empirical work KV regress $(CA/Y)^i$ on x^i times $(S/Y)^i$, with all variables averaged over 23 years for 13 OECD countries. They find a coefficient close to one and an excellent fit with an R^2 of 0.68. Kraay and Ventura (2003) use an even longer sample of 32 years for 21 OECD countries and obtain an R^2 of 0.85. By averaging over such a long span of data, the results should be reasonably close to the steady state. These results are not surprising as our analysis above shows that the relationship holds exactly in steady state.

This result is much more general than the specifics of our model. Another way to see why (33) holds in steady state is to consider any set of economies that grow at a constant steady state rate g. Therefore

$$\frac{d(Wealth)}{Wealth} = g \quad ; \quad \frac{d(NFA)}{NFA} = g \tag{34}$$

where $Wealth$ is national wealth. Using that $d(Wealth) = S$ is national saving and $d(NFA) = CA$, it follows that

$$CA = \frac{NFA}{Wealth} S = xS \tag{35}$$

This simply holds as a matter of identity. One only needs non-zero growth, so that savings and the current account are non-zero. In addition there must obviously be a source of asymmetry across countries that leads to cross-sectional variation in steady state saving rates and current accounts, which we model by allowing the time discount rates to be different across countries.

KV interpret a unitary coefficient of a regression of CA^i on $x^i S^i$ as consistent with the new rule, an interpretation that is fundamentally different from ours. The logic behind their interpretation is as follows. From the definition of x it follows that $NFA = x * Wealth$. Therefore

$$CA = d(NFA) = x * d(Wealth) + Wealth * d(x) = x * S + Wealth * d(x) \tag{36}$$

The new rule is defined in the context of the current account response to a temporary income shock: "the current account response is equal to the saving generated by the shock multiplied by the country's share of foreign assets in total assets". This last ratio is what we called net foreign assets relative to wealth, which is x. KV go on to argue that the new rule holds when a change in saving does not lead to a change in x. It is indeed immediate from (36) that when changes in x are orthogonal to saving, a regression of the current account on $x * S$ should have a coefficient of 1.

We fully agree with this reasoning. However, the empirical evidence in favor of the new rule in KV relates to a cross-section regression. If we run a cross-section regression of $(CA/Y)^i$ on the product of x^i times $(S/Y)^i$ we are comparing the level of saving rates and current accounts across different countries. The new rule is instead about the dynamic response of the current account to a change in saving.

To clarify matters further, consider that a country experiences a shock that permanently raises its saving rate. In our model this is simply done by lowering the time discount rate. It takes two periods in the model to reach a new steady state. In the process x will change (it will rise), as can be checked from the equations above. Therefore the new rule does not hold during the adjustment phase. But once we reach a new steady state, x will no longer change. We then have $d(x) = 0$ and it follows from (36) that once again $CA = xS$. A cross-section regression reflects to a large extent the steady state, after the adjustment to shocks has already taken place. This is especially the case when taking averages over several decades as KV and Kraay and Ventura (2003) do.

In our view therefore the cross-section evidence reported in these papers is reflective of the steady state rather than the dynamic new rule. To further strengthen this position we now turn to an analysis of the current account response to a rise in saving resulting from a temporary income shock. The analysis confirms that the new rule is at odds with the model, as was the case for the partial equilibrium model discussed in section 2.

3.6 Impact of a Temporary Income Shock

To be written

4 Conclusion

To be written

Appendix

A Solution Small Country Model

In this Appendix we first describe the steady state solution (zero-order components of variables) and then use linear expansions of all equations around the zero-order components of variables to compute the first-order solution.

Zero-order component of variables

The model is driven by nine relations. The exogenous processes are domestic productivity (1) and the return on the foreign asset (5). These are completed by the wage (2), the dynamics of domestic capital (3), the return on domestic capital (4), the portfolio return (6), the Euler equation for consumption (8), the optimal portfolio condition (9) and the asset market clearing condition (10).

The zero-order solution is computed by letting the standard deviation of shocks approach zero. (1) implies that $A(0) = 1$. From (9) and (6)

$$R(0) = R^p(0) = 1 - \delta + D^*$$

The level of domestic capital then follows from (4), the investment from (3), and the wage from (2):

$$K(0) = \left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad I(0) = \delta \left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad W(0) = \omega \left[\frac{1-\omega}{D^*}\right]^{\frac{1-\omega}{\omega}}$$

(8) gives the consumption of young agents, and the portfolio share is obtained from (10):

$$\begin{aligned} C_y(0) &= \frac{\omega}{1 + \beta^{\frac{1}{\gamma}} \left[1 - \delta + D^*\right]^{\frac{1-\gamma}{\gamma}}} \left[\frac{1-\omega}{D^*}\right]^{\frac{1-\omega}{\omega}} \\ z(0) &= \frac{1-\omega}{\omega} \frac{1}{D^*} \frac{1 + \beta^{\frac{1}{\gamma}} \left[1 - \delta + D^*\right]^{\frac{1-\gamma}{\gamma}}}{\beta^{\frac{1}{\gamma}} \left[1 - \delta + D^*\right]^{\frac{1-\gamma}{\gamma}}} \end{aligned}$$

A useful measure is the ratio between the zero-order component of young consumption and the zero-order component of the wage:

$$\bar{c} = \frac{C_y(0)}{W(0)} = \frac{1}{1 + \beta^{\frac{1}{\gamma}} \left[1 - \delta + D^*\right]^{\frac{1-\gamma}{\gamma}}}$$

Finally, the zero-order component of the budget constraint (7) gives

$$C_o(0) = (1 - \bar{c}) R(0) W(0)$$

In terms of international asset stocks and flows, the net foreign asset position, which is equal to a share $1 - z(0)$ of wealth, is:

$$NFA(0) = W(0) - C_y(0) - K(0)$$

Turning to flows, aggregate savings and the current account are given by:

$$\begin{aligned} S(0) &= \delta(W(0) - C_y(0)) \\ CA(0) &= \delta \left[W(0) - C_y(0) - K(0)\right] = \left[1 - z(0)\right] S(0) \end{aligned}$$

The current account is then equal to savings, times the share of foreign assets in total wealth.

First-Order Solution

We now take first-order Taylor expansions of the nine equations of the model around the zero-order allocation. The domestic productivity (1) and the return on the foreign asset (5) only have first order components:

$$A_t(1) = \varepsilon_t \qquad\qquad R^*_{t+1}(1) = \epsilon^D_{t+1}$$

The first-order component of the equations (5), (9) and (6) immediately imply that the expected first-order component of the return on the domestic and foreign capital, as well as on the portfolio, are zero:

$$E_t R_{t+1}(1) = E_t R^p_{t+1}(1) = E_t R^*_{t+1}(1) = 0$$

Taking an expectation of the first-order component of (4) we have

$$E_t R_{t+1}(1) = -\omega(1-\omega)K(0)^{-\omega-1}K_{t+1}(1)$$

Since $E_t R_{t+1}(1) = 0$, it follows that $K_{t+1}(1) = 0$.

As the domestic capital stock never changes, the realized wage is obtained from (2):

$$\frac{W_t(1)}{W(0)} = \varepsilon_t$$

Taking the expectation of the first-order component of the budget constraint (7), we have

$$E_t C_{o,t+1} = (W_t(1) - C_{y,t}(1))R(0) \tag{37}$$

The first-order component of the consumption Euler equation (8) is

$$-\gamma \frac{C_{y,t}(1)}{C_y(0)} = -\gamma E_t \frac{C_{o,t+1}(1)}{C_0(0)} + E_t \frac{R^p_{t+1}(1)}{R(0)}$$

Using (37) and $E_t R^p_{t+1}(1) = 0$, we can write this as

$$C_{yt}(1) = \bar{c}W_t(1) = \bar{c}W(0)\varepsilon_t \tag{38}$$

Finally, the first-order component of the asset market clearing condition (10) gives

$$z_t(1) = -z(0)\varepsilon_t$$

This again uses that $K_{t+1}(1) = 0$.

We can now also compute the first-order component of national saving. Saving is equal to income minus consumption. Aggregate consumption at time t in the small country is

$$C^H_{y,t} + (W_{t-1} - C_{y,t-1})R^{p,H}_t + \tau R^*_t(W_{t-1} - C_{y,t-1})(1 - z_{t-1})$$

The three components are consumption by the young, the old and the brokers that immediately consume the revenues from the fee τ on foreign returns. Aggregate income is

$$W_t + (W_{t-1} - C_{y,t-1})\left(z_{t-1}(R_t - 1 + \delta) + (1 - z_{t-1})(R^*_t - 1 + \delta)\right)$$

Therefore national saving is

$$S_t = (W_t - C_{y,t}) - (1 - \delta)(W_{t-1} - C_{y,t-1}) \tag{39}$$

The first-order component is

$$S_t(1) = (1 - \bar{c})W(0)(\varepsilon_t - (1 - \delta)\varepsilon_{t-1}) \tag{40}$$

B Solution General Equilibrium Model

Zero-order component of variables

The zero-order components of all equations other than the difference across countries in portfolio Euler equations gives is the zero-order component of all variables other than the zero-order component of the difference across countries in portfolio shares. Variables that grow at rate g will be divided by the country's labor supply.

From (17) we have $A^i(0) = 1$ for $i = H, F$. From the portfolio Euler equations we have

$$R^H(0) = R^F(0) \equiv R(0)$$

It then follows from (19) and (18) that

$$\begin{aligned} &k^H(0) = k^F(0) \equiv k(0) \\ &R(0) = 1 - \delta + (1 - \omega)k(0)^{-\omega} \\ &W_H(0) = W_F(0) = \omega k(0)^{1-\omega} \end{aligned}$$

From the capital accumulation equations (3.1) it follows that

$$i^H(0) = i^F(0) = (g+\delta)k(0)$$

From (22) and (24) we have

$$C_o^i(0) = (W(0) - C_y^i(0))R(0)$$
$$C_o^i(0) = C_y^i(0)(\beta^i)^{1/\gamma}R(0)^{1/\gamma}$$

It follows that

$$C_y^i(0) = \frac{W(0)R(0)}{R(0) + (\beta^i)^{1/\gamma}R(0)^{1/\gamma}} \equiv c^i(0)W(0)$$

The sum of the asset market clearing conditions (26)-(27) gives

$$(1+g)k(0) = n(W(0) - C_y^H(0)) + (1-n)(W(0) - C_y^F(0)) \tag{41}$$

Substituting the expressions for $W(0)$, $C_y^H(0)$ and $C_y^F(0)$ above then yields an implicit solution for $k(0)$. Finally, from the Home asset market clearing condition (26) we have

$$(1+g)k(0) = (W(0) - C^H(0))z^H(0) + (W(0) - C^F(0))z^F(0)\frac{1-n}{n} \tag{42}$$

This gives a solution for a weighted average of portfolio shares.

This is the zero-order solution of the model. We can now also compute the implied zero-order components of saving and the current account. Saving is equal to income minus consumption. Aggregate consumption at time t in the Home country is

$$N_t^H C_{y,t}^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H R_t^{p,H} + \tau R_t^F(W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H(1 - z_{t-1}^H)$$

The three components are consumption by the young, the old and the brokers that immediately consume the revenues from the fee τ on foreign returns. Aggregate income is

$$W_t^H N_t^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H\left(z_{t-1}^H(R_t - 1 + \delta) + (1 - z_{t-1}^H)(R_t^F - 1 + \delta)\right)$$

Therefore national saving, which is income minus consumption, is

$$S_t^H = N_t^H(W_t^H - C_{y,t}^H) - N_{t-1}^H(1-\delta)(W_{t-1}^H - C_{y,t-1}^H) \tag{43}$$

Dividing by N_t^H and taking the zero-order component, we have

$$s^H(0) = \frac{g+\delta}{1+g}(W(0) - C_y^H(0))$$

Since the current account is saving minus investment, the previous results imply

$$ca^H(0) = \frac{g+\delta}{1+g}(W(0) - C_y^H(0)) - (g+\delta)k(0) \tag{44}$$

Substituting (41), this becomes

$$ca^H(0) = \frac{g+\delta}{1+g}(1-n)\left(C_y^F(0) - C_y^H(0)\right) \tag{45}$$

Finally we compute the steady state net foreign asset position of the Home country. We have

$$NFA_t^H = (W_t^H - C_{y,t}^H)N_t^H(1-z_t^H) - (W_t^F - C_{y,t}^F)N_t^F z_t^F \tag{46}$$

Dividing by N_t^H and taking the zero-order component, we have

$$nfa^H(0) = (W(0) - C_y^H(0))(1 - z^H(0)) - (W(0) - C_y^F(0))z^F(0)\frac{1-n}{n} \tag{47}$$

Substituting (42), this becomes

$$nfa^H(0) = (W(0) - C_y^H(0)) - (1+g)k(0) \tag{48}$$

which together with (41) becomes

$$nfa^H(0) = (1-n)\left(C_y^F(0) - C_y^H(0)\right) \tag{49}$$

Total wealth per unit of the labor force is $W(0) - C_y^H(0)$, so that the ratio of net foreign assets to wealth is

$$(1-n)\frac{C_y^F(0) - C_y^H(0)}{W(0) - C_y^H(0)} \tag{50}$$

This is exactly equal to the ratio of the current account to saving, so that $ca^H(0) = x^H(0)s^H(0)$ holds exactly with x^H the ratio of net foreign assets to wealth.

First-Order Solution

From (17) we have $A_t^i(1) = \varepsilon_t^i$ for $i = H, F$. From a first-order expansion of the portfolio Euler equations, and the definition of the portfolio return, we have

$$E_t R_{t+1}^H(1) = E_t R_{t+1}^F(1) = E_t R_{t+1}^{p,i}(1) \equiv E_t R_{t+1}(1)$$

It then follows from (19) and (18) that

$$
\begin{aligned}
k_{t+1}^H(1) &= k_{t+1}^F(1) \equiv k_{t+1}(1) \\
E_t R_{t+1}(1) &= -\omega(1-\omega)k(0)^{-\omega-1}k_{t+1}(1) \\
W_t^i(1) &= W(0)\varepsilon_t^i
\end{aligned}
$$

In the last equation we assume that $k_t(1) = 0$ as we will abstract from shocks in earlier periods. From the capital accumulation equations (3.1) it follows that

$$i_t^H(1) = i_t^F(1) = (g+\delta)k_{t+1}(1)$$

From (22) and (24) we have

$$
\begin{aligned}
E_t C_{o,t+1}^i(1) &= R(0)(W_t^i(1) - C_{y,t}^i(1)) + (W(0) - C_y^i(0))E_t R_{t+1}(1) \\
\frac{C_{y,t}^i(1)}{C_y^i(0)} &= E_t \frac{C_{o,t+1}^i(1)}{C_o^i(0)} - \frac{1}{\gamma}\frac{E_t R_{t+1}(1)}{R(0)}
\end{aligned}
$$

Combining these last two equations, after some algebra we obtain

$$C_{y,t}^i(1) = c^i W_t^i(1) + \left(1 - \frac{1}{\gamma}\right)(1 - c^i(0))\frac{E_t R_{t+1}(1)}{R(0)} \tag{51}$$

The sum of the asset market clearing conditions (26)-(27) gives

$$(1+g)k_{t+1}(1) = n(W_t^H(1) - C_{y,t}^H(1)) + (1-n)(W_t^F(1) - C_{y,t}^F(1)) \tag{52}$$

Substituting (51), this implies after a little bit of algebra that

$$k_{t+1}(1) = \frac{1}{d(0)}\left[nh^H(0)W_t^H(1) + (1-n)h^F(0)W_t^F(1)\right] \tag{53}$$

where

$$d(0) = 1 + g - \left(1 - \frac{1}{\gamma}\right)[n(1-c^H(0)) + (1-n)(1-c^F(0))]\omega(1-\omega)\frac{k(0)^{-\omega-1}}{R(0)}$$

Abstracting again from past shocks, the definition of saving (39) implies

$$s_t^i(1) = W_t^i(1) - C_{y,t}^i(1) = (1-c^i)(0)W_t^i(1) + \left(1 - \frac{1}{\gamma}\right)(1-c^i(0))\frac{\omega(1-\omega)k(0)^{-\omega-1}}{R(0)}k_{t+1}(1)$$

An income shock in the Home country at time t then implies

$$
\begin{aligned}
s_t^H(1) &= (1-c^H(0))\frac{1 + g - (1-\frac{1}{\gamma})(1-n)(1-c^F(0))\omega(1-\omega)\frac{k(0)^{-\omega-1}}{R(0)}}{d(0)}W_t^H(1) \\
s_t^F(1) &= (1-c^F(0))\frac{(1-\frac{1}{\gamma})n(1-c^H(0))\omega(1-\omega)\frac{k(0)^{-\omega-1}}{R(0)}}{d(0)}W_t^H(1)
\end{aligned}
$$

The current account is saving minus investment, which is

$$
\begin{aligned}
ca_t^H(1) &= (1-n)(1-c^H(0))\frac{1+g-(1-\frac{1}{\gamma})(1-c^F(0))\omega(1-\omega)\frac{k(0)^{-\omega-1}}{R(0)}}{d(0)}W_t^H(1) \\
ca_t^F(1) &= n(1-c^F(0))\frac{(1-\frac{1}{\gamma})(1-c^H(0))\omega(1-\omega)\frac{k(0)^{-\omega-1}}{R(0)}}{d(0)}W_t^H(1)
\end{aligned}
$$

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