

Are Voters Rational?

Turnout in Union Representation Elections

A Progress Report

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Plan of Analysis

- 1 Introduction and context.
- 2 The decision to hold an election.
- 3 Turnout in elections.

Voting behavior presents a puzzle for economists.

- Simplest economic model has individual voting if the expected utility from voting exceeds the expected utility from not voting.
- This utility difference is the product of the probability that the individual's vote will be pivotal (change the outcome) times the difference in utility from the two outcomes.
- Probability of being pivotal trivial in all but smallest elections \Rightarrow very low turnout.
- Solution of Sorts: Derive utility directly from the act of voting \Rightarrow act as if cost of voting is trivial for some fraction of the population.

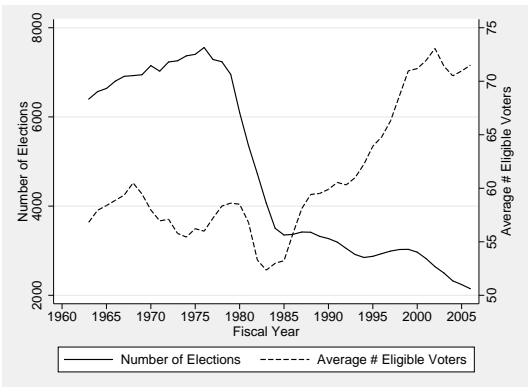
Use of Data from Small Elections

- The probability of being pivotal is higher in small elections.
- \Rightarrow Small elections can serve as a laboratory to study voting behavior.
- Question: Is there evidence that the probability of being pivotal affects voters' decisions regarding whether or not to vote?

NLRB Supervised Representation Elections

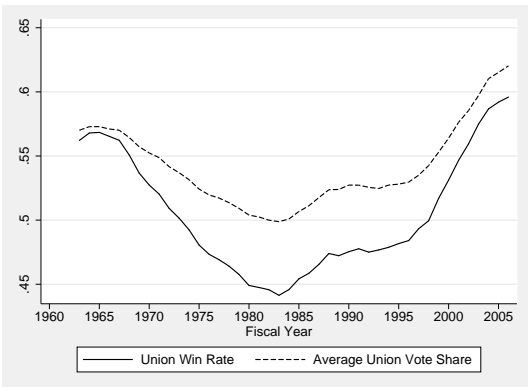
- Single question: Should the union be certified as the bargaining agent of the workers.
- Elections held in workplace \Rightarrow low cost of voting.
- My analysis sample has over 70,000 elections decided between 1973 and 2006.
- Most elections are small:
 - 37 percent have ≤ 10 eligible voters.
 - 60 percent have ≤ 20 eligible voters.
 - 72 percent have ≤ 30 eligible voters.
- Good laboratory to study voter behavior.

Number of Elections and Average Size of Elections, 5-year moving average



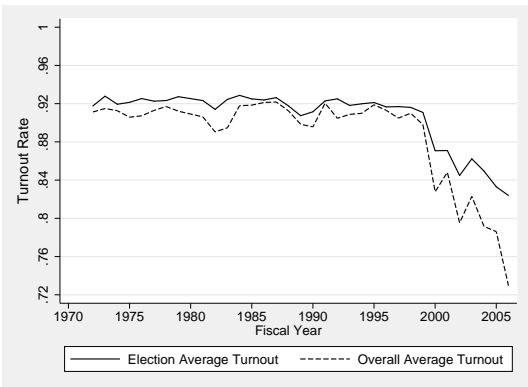
- Number of elections declined sharply since the mid 1970s.
- Average size of elections increased sharply since the time 1980s.
- Return to these facts later (when considering decision to hold an election).

Union Win Rate and Pro-Union Vote Share 5-year moving average



- Decline followed by increase in both win rate and pro-union vote share.
- Win rate moves more sharply than pro-union vote share.
 - Return to this below.

Turnout Rate in Union Representation Elections



- Turnout rate declining since the mid 1990s.
- This puzzle is only partly explained by introduction of mail ballots in some elections.

The decline in turnout presents a puzzle

- A small part of the decline reflects the introduction of mail ballots in the mid-1980s.
- Mail ballots used in 1.2 percent of elections between 1984 and 1990.
- Mail ballots used in 4.2 percent of elections between 1991 and 2000.
- Mail ballots used in 10.2 percent of elections between 2001 and 2006.
- Average turnout rate since 1984 was 91.1 percent in on-site elections and 73.2 percent in mail elections.
- Average turnout rate in on-site elections fell from 90.0 percent to 75.5 percent between 1999 and 2006.

The Decision to Hold an Election: A Complicating Factor

- Unions only contest elections where it is in their interest to do so.
- This implies that the set of elections actually held (and for which I have data) is a selected subset of possible organizing targets.
- This selection is clearly related to expected voter behavior and election outcomes.
- A model of the union organizing decision is required along with a model of voter behavior.

The Union's Decision to Hold an Election

- An economically rational labor union will contest elections only where there is a positive expected value associated with the election. This is based on several factors:
 - 1 the per-worker benefit to the union of a union victory (R),
 - 2 the per-worker cost to the union (net of union dues) of negotiating a contract and administering a unionized workplace (C_a),
 - 3 the per-worker cost to the union of the organization effort (C_o), and
 - 4 the probability of a union victory in an election (θ).

The Probability of a Union Victory

- The union wins the election when the the majority of votes cast are cast in favor.
- The probability of a union victory is

$$\theta = Pr(n_p > n_c) = \sum_{n_a=0}^{N-1} \sum_{n_p=1}^{N-n_a} \sum_{n_c=0}^{n_p-1} \frac{N!}{n_p! n_c! n_a!} p_p^{n_p} p_c^{n_c} p_a^{n_a}$$

- n_p = number of pro-union votes.
- n_c = number of anti-union votes.
- n_a = number of non-votes.
- N = Number of eligible voters.
- p_p = probability pro-union.
- p_c = probability anti-union.
- p_a = probability not vote.

The Critical Value of θ

- The per-worker expected value to the union of contesting an election at target i is

$$V_i = \theta_i(R_i - C_{ai}) - C_{oi}$$

- The union will undertake to organize the target if $V_i > 0$.
- The condition for an election to be held is that

$$\theta_i > \frac{C_{oi}}{(R_i - C_{ai})}$$

- The critical value for the probability of a union victory is

$$\theta_i^* = \frac{C_{oi}}{(R_i - C_{ai})}$$

- Unions will contest elections where $\theta_i > \theta_i^*$.

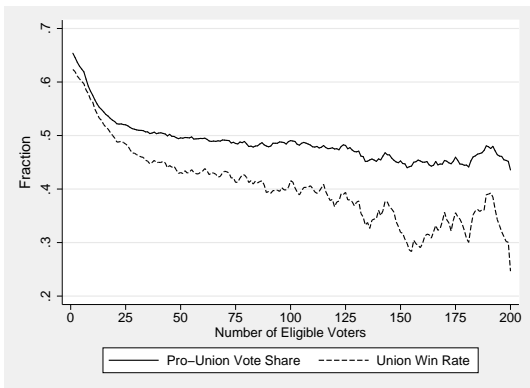
The Role of Election Size

- Direct effect on the probability of a union victory.
- A union victory in a large election could have important positive spillovers for the union in terms of bargaining leverage and “marketing” value in other organizing campaigns ($\frac{\partial R_i}{\partial N_i} > 0$).
- There may be decreasing costs per worker of holding the organizing drive ($\frac{\partial C_{oi}}{\partial N_i} < 0$) and/or decreasing costs per member of servicing a bargaining unit once there is a union victory ($\frac{\partial C_{ai}}{\partial N_i} < 0$).
- \Rightarrow Critical value for the probability of a union victory is decreasing in election size ($\frac{\partial \theta_i^*}{\partial N_i} < 0$).

The Relationship Between Election Size and the Probability of a Union Victory

- Unions will contest larger elections where they have a smaller chance of winning.
- Strong cross-sectional prediction that observed union win rates will be negatively related to the number of eligible voters.
- The key lesson to take away from this model is that any analysis of voting behavior and election outcomes must take into account the union selection process regarding where to contest elections.

Union Win Rate and Pro-Union Vote Share, by Election Size (5-year moving average)



- Union win rate and pronoun vote share decline sharply with election size, as predicted by the model.
- The win rate declines more sharply than the pro-union vote share.
 - This is a natural implication of binomial distributions.

Theoretical Framework for Turnout

There are three relevant groups of actors in determining the outcomes of representation elections.

- 1 Labor unions who decide which groups of workers to attempt to organize through the election process.
- 2 Non-union workers in workplaces who can vote if an election is held.
- 3 Employers of non-union workers who can affect both the likelihood of an election being held and the outcome of elections that are held through their treatment of workers and actions during an organizing drive.
 - I do not consider employer behavior directly.

The Worker's Voting Decision

Borrows from Coate, Conlin, and Moro (2006)

- μ – The expected fraction of workers who are pro-union.
- $(1 - \mu)$ – The expected fraction of workers who are anti-union.
- $b_p > 0$ – The benefit to pro-union workers if the union wins the election.
- $b_c < 0$ – The “benefit” to anti-union workers if the union wins the election.
- Assume $b_p = -b_c = b$.

The Cost of Voting – C_i

- C_i is the cost of voting to worker i net of the direct benefit worker i receives from the act of voting itself
- C_i is independent of any expected benefit that comes from the possibility that his vote would alter the election outcome.
- C_i may well be negative.
- C_i varies independently across workers and is distributed with CDF $G(\cdot)$.

Pro-Union Workers

- The NLRA specifies that the union is certified as the bargaining agent of the workers if and only if a majority of those voting vote in favor. \Rightarrow unions lose ties.
- \Rightarrow A pro-union worker's vote will be pivotal only if the election would be tied without his vote.
- Denote the probability that the vote would be tied without his vote by ΔW_+ .
- The change in a pro-union worker's expected utility from voting is $b\Delta W_+ - C_i$.
- \Rightarrow vote if $C_i \leq b\Delta W_+$
- The probability that a randomly selected worker votes for the union is

$$p_p = \mu G(b\Delta W_+)$$

Anti-Union Workers

- The NLRA specifies that the union is certified as the bargaining agent of the workers if and only if a majority of those voting vote in favor. \Rightarrow unions lose ties.
- \Rightarrow An anti-union worker's vote will be pivotal only if the union would win by a single vote without his vote.
- Denote the probability that the union would win by a single vote without his vote by ΔW_- .
- The change in an anti-union worker's expected utility from voting is $b\Delta W_- - C_i$.
- \Rightarrow vote if $C_i \leq b\Delta W_-$
- The probability that a randomly selected worker votes against the union is

$$p_c = (1 - \mu)G(b\Delta W_-)$$

The Turnout Rate and the Distribution of Votes

- The turnout rate in the election is

$$p_v = p_p + p_c = \mu G(b\Delta W_+) + (1 - \mu)G(b\Delta W_-)$$

- The probability that a worker does not vote (the abstention rate) is

$$p_a = 1 - p_v = 1 - \mu G(b\Delta W_+) + (1 - \mu)G(b\Delta W_-)$$

- The number of pro-, anti-, and non-votes has a multinomial distribution:

$$Pr(n_p, n_c, n_a) = \frac{N!}{n_p!n_c!n_a!} p_p^{n_p} p_c^{n_c} p_a^{n_a}$$

Probability of Being Pivotal

- Pro-Union Worker:

$$\Delta W_+ = Pr(n_p = n_c) = \sum_{i=0}^{INT(n/2)} \frac{n!}{i!(n-2i)!} p_p^i p_c^i p_a^{n-2i}$$

- Anti-Union Worker

$$\Delta W_- = Pr(n_p = n_c + 1) = \sum_{i=0}^{INT((n-1)/2)} \frac{n!}{(i+1)!i!(n-2i-1)!} p_p^{i+1} p_c^i p_a^{n-2i-1}$$

- $n = N - 1$, the number of eligible voters less one
- $INT(\cdot)$ returns the truncated integer value of its argument.

Implications of Model for the Likelihood of Being Pivotal

- 1 The probability that a worker's vote is pivotal falls with the number of eligible voters.
- 2 Holding election size fixed, the probability of a tie is maximized when $p_p = P_c$. \Rightarrow The probability that a worker's vote is pivotal varies directly with the gap between P_p and P_c (approximate for anti-union).
- 3 The marginal effect of a change in the gap between P_p and P_c on the probability of being pivotal falls with election size. (A second-order prediction)
- 4 Pro-union workers are more likely to be pivotal when the total number of eligible voters is odd than when it is even.
- 5 Anti-union workers are more likely to be pivotal when the total number of eligible voters is even than when it is odd.

Empirical Predictions

- 1 Turnout will fall as the cost of voting increases.
- 2 Turnout will fall with election size.
- 3 Holding election size fixed, turnout will increase with the expected closeness of *ex ante* preferences for and against union representation.
- 4 The marginal effect on turnout of an increase in expected closeness of preferences will fall with election size.
- 5 The pro-union vote share will be larger in elections with an odd number of eligible voters.
- 6 The anti-union vote share will be larger in elections with an even number of eligible voters.

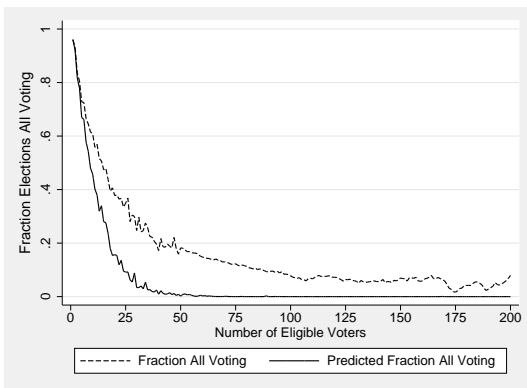
A Statistical Description of Turnout Rates

- The simplest statistical model of the turnout rate is a binomial model that is derived from the multinomial model of the pro-union, anti-union, abstain vote decision.
- p_j – the probability that a worker in election j votes.
- V_j – the number of votes cast in election j
- N_j – the number of eligible voters
- $Pr(V_j) = \frac{N_j!}{V_j!(N_j - V_j)!} p_j^{V_j} (1 - p_j)^{N - V_j}$.
- $p_j = \frac{\exp(X_j\beta)}{1 + \exp(X_j\beta)}$ (logistic function linking p_j to X 's).
- p_j – expected turnout in election j .

Heterogeneity across elections

- The binomial model may fit mean turnout rates quite well.
- If there is heterogeneity across elections in p_j , then the model will underpredict dispersion across election in turnout rates.
- Example: Let \hat{p}_j be an unconstrained function of the number of eligible voters.
 - This is equivalent to the MLE of binomial model where $p_j = \frac{\exp(X_j\beta)}{1+\exp(X_j\beta)}$ and X is a complete set of year dummies.
 - \hat{p}_j is the ratio of total votes cast to the number eligible in elections with given number eligible.
 - Fits average turnout by number eligible perfectly

Probability of Full Turnout, by Total Number of Eligible Voters.



- Predicted full turnout virtually zero in elections with more than 50 eligible voters.
- Yet many of these elections have full turnout (fully 20% of elections with 50 eligible voters).

- Predicted $Pr(V_t = N_t) = (\bar{p}_j | N)^N$

Solution to Over-dispersion: Beta Distribution for p_j

- Convenient parameterization of the beta distribution:

$$g(p; m, \alpha) = \frac{\Gamma(\alpha)}{\Gamma(m\alpha)\Gamma((1-m)\alpha)} p^{m\alpha-1}(1-p)^{(1-m)\alpha-1}.$$

- The expected value of p is m
- The variance of p is $\sigma^2 = \frac{m(1-m)}{(1+\alpha)}$.
- Applying Bayes Theorem, the unconditional distribution of number of votes cast (v) is beta-binomial

$$f(v) = \binom{N}{v} \frac{\Gamma(\alpha)\Gamma(m\alpha + v)\Gamma((1-m)\alpha + N - v)}{\Gamma(m\alpha)\Gamma((1-m)\alpha)\Gamma(N + \alpha)}.$$

- Over-dispersion is captured by the parameter α .
- As $\alpha \rightarrow \infty$, the variance of p goes to zero.
- Smaller values of α imply positive variance in the expected fraction voting across elections.

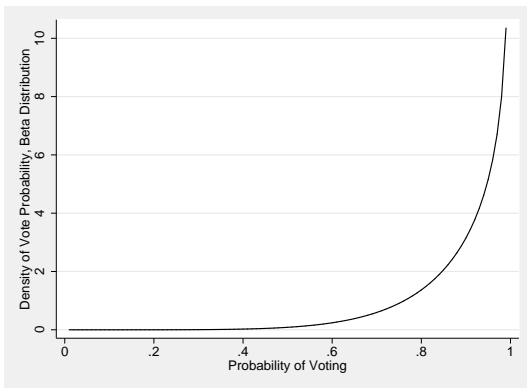
Evaluation of the Beta-Binomial Model

- Estimate binomial and beta-binomial models of turnout rate at the election level where $p_j = \gamma_0 + \gamma_1(1/\sqrt{N_j})$.
- Estimate this model using the sample of 72,892 elections with 2-200 eligible voters and at least two votes cast.

Variable	Binomial	Beta-Binomial
m	(1)	(2)
Constant	0.8815 (0.0016)	0.8784 (0.0011)
$1/\sqrt{N}$	0.1382 (0.0053)	0.1407 (0.0033)
α		7.04 0.103
Log L	-200596.2	-121220.8

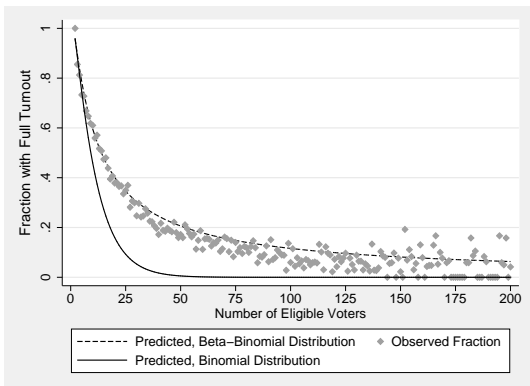
- Virtually no difference in the estimates of the coefficients of the mean probability of voting function.
- Estimate of α and substantial improvement in the log-likelihood implies significant variation in p_j across elections.
- At \bar{p} of 0.9, $\hat{\alpha}$ implies standard deviation of p_j is 0.061.

Beta Density Function of Vote Probability



- Value for m of 0.9 corresponds to an election with 42 eligible voters based on the beta-binomial estimates.
 - Illustrates that there are many elections with very high expected vote probabilities.
 - The 75th percentile is 0.97, and the 90th percentile is 0.99.
- Based on $m = 0.9$, $\alpha = 7.04$

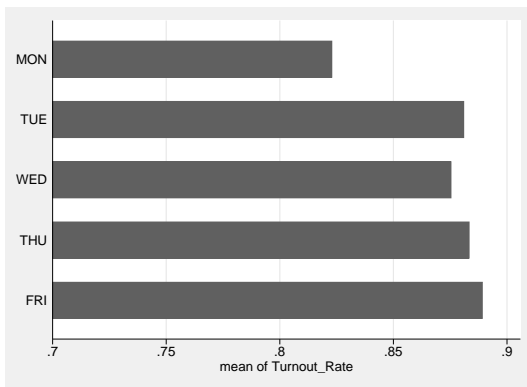
Predicted Probability Full Turnout, by Number Eligible



- The binomial model seriously under-predicts the likelihood of full turnout.
- The beta-binomial model tracks the observed probability of full turnout much more closely.

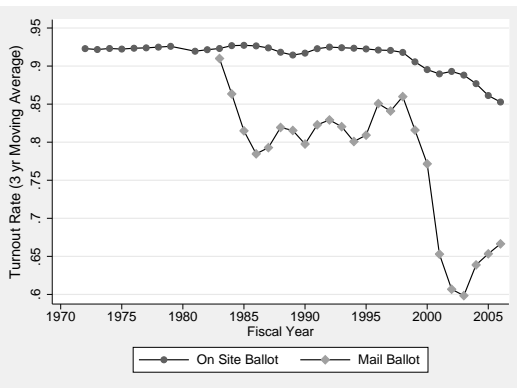
- I proceed using the beta-binomial model.

Cost of Voting and Turnout – Day of Week



- Turnout Rate, by Day of Week Election Held, 2000-2006.
- Turnout Rate Lower on Mondays
- If absentee rate lower on Mondays, then this is consistent with turnout inversely related to cost of voting.

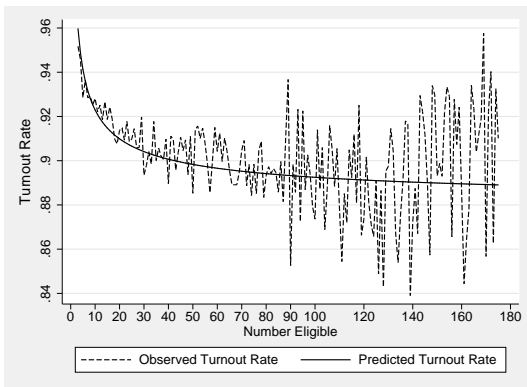
Cost of Voting and Turnout – Type of Ballot



- Turnout Rate by Type of Ballot

- There is a drop-off in the average turnout rate in on-site elections from over 90 percent through the mid-1990s to 85 percent in 2006.
- The turnout rate in on-site elections is substantially higher than the turnout rate mail elections.
- I proceed using only on-site elections.

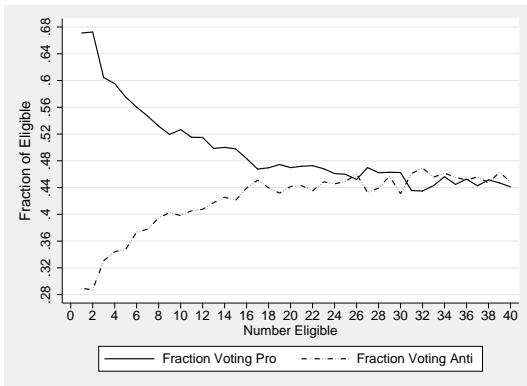
Election Size and Turnout



- Turnout Rate by Number Eligible
- Predicted based on $\hat{\alpha} = 7.04$,
 $\hat{m} = 0.8784 + 0.1407 \cdot 1/\sqrt{N_j}$

- Turnout falls with election size as workers are less likely to be pivotal.
- Turnout high even in relatively large elections.
- \Rightarrow Most vote despite very low probability of being pivotal.
- There is a margin of voters who may be considering the likelihood that their vote will be pivotal,

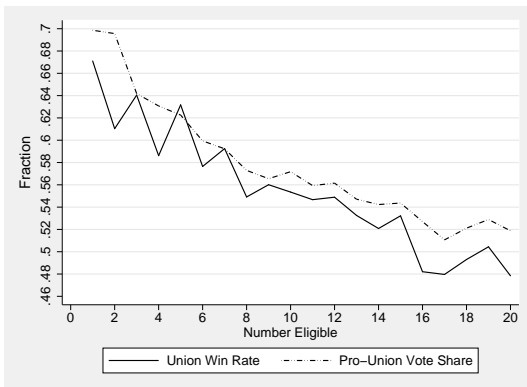
The Odd-Even Distinction: Pro- and Anti- share by Eligible



- Prediction: Pro-union more likely to vote in odd elections.
- Prediction Anti-union more likely to

- No evidence of relationship of vote shares with “even-ness”.
- The pronoun share is decreasing and the anti-union share is increasing in election size.
- Consistent with hypothesis that unions are more likely to contest larger elections where they have a smaller chance of winning.

The Odd-Even Distinction: Vote Share and Win Rate by Eligible



- Win rate is smaller when eligible is even in small elections..
- No difference in pro-union vote share.

- Suggests that unions do not completely offset the unions-lose-ties bias in the NLRA when selecting targets for organization.
- Unions may not be able to predict precisely how many workers will be eligible to vote on election date.
- Workers may not understand the odd-even distinction in making their voting decisions.

Turnout and Election Closeness

- Ignore the odd/even distinction in what follows.
- Assume pro- and anti- union workers vote with the same probability.
- μ represents the fraction pro-union among eligible.
- Even split when $\mu = 0.5$.
- μ varies across elections.
- Proxy for μ_j – Posterior mean of the distribution of μ given the observed pro-union vote share in election j .

The Distribution of Pro-Union Votes

- The unconditional distribution of the number of pro-union votes:

$$h(s|\mu) = \binom{n}{s} \mu^s (1 - \mu)^{n-s}$$

- Assume that μ is distributed as beta such that

$$g(\mu; \rho, \nu) = \frac{\Gamma(\nu)}{\Gamma(\rho\nu)\Gamma((1-\rho)\nu)} \mu^{\rho\nu-1} (1-\mu)^{(1-\rho)\nu-1}.$$

- $E(\mu) = \rho$ $Var(\mu) = \sigma^2 = \frac{\rho(1-\rho)}{(\nu+1)}$.
- The unconditional distribution of the number of pro-union votes:

$$f(s) = \binom{n}{s} \frac{\Gamma(\nu)\Gamma(\rho\nu + s)\Gamma((1-\rho)\nu + n - s)}{\Gamma(\rho\nu)\Gamma((1-\rho)\nu)\Gamma(n + \nu)}.$$

The Posterior Distribution of μ Given s Pro-Union Votes Among n Votes

- By the Bayes theorem, the distribution of μ conditional on observing s pro-union votes among n votes cast is

$$f(\mu|s) = \frac{h(s|\mu)g(\mu)}{f(s)}$$

- The posterior distribution of μ is

$$f(\mu|s) = \frac{\Gamma(n + \nu)}{\Gamma(s^*)\Gamma(n + \nu - s^*)} \mu^{s^*-1} (1 - \mu)^{n-s^*+\nu-1}$$

where $s^* = s + \rho\nu$.

The Posterior Mean of μ Given s Pro-Union Votes Among n Votes

- The posterior mean of μ given s is

$$E(\mu|s) = \left[\frac{n}{n + \nu} \right] \left(\frac{s}{n} \right) + \left[\frac{\nu}{n + \nu} \right] \rho.$$

- This is a weighted average of the observed vote share and the prior mean.
- The weight depends on the election size (n) and the parameter ν that controls the variance of the Beta distribution for μ .
- As the sample grows, the weight shifts from the prior mean (ρ) to the observed pro-union vote share (s/n).

The “Closeness” Index

- The index I use to measure the *a priori* closeness of an election is the expectation of the squared difference between pro-union vote preference given the observed vote share and 0.5.
- After some algebra, this is

$$E((\mu - 0.5)^2 | s) = 0.25 - \left[\frac{n + \nu}{n + \nu + 1} \right] E(\mu | s)(1 - E(\mu | s))$$

- Calculation of the index for each election requires data on the total number of votes cast and the number of pro-union votes cast along with estimates of the parameters (ρ and ν) of the beta distribution of the fraction pro-union (μ).

Estimation of the Parameters of Distribution of μ

- The unconditional distribution of the number of pro-union votes is beta-binomial:

$$f(s) = \binom{n}{s} \frac{\Gamma(\nu)\Gamma(\rho\nu + s)\Gamma((1-\rho)\nu + n - s)}{\Gamma(\rho\nu)\Gamma((1-\rho)\nu)\Gamma(n + \nu)}.$$

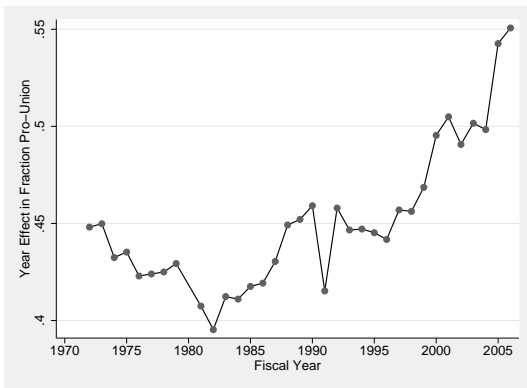
- Use this distribution to derive a likelihood function for the vote counts.
- Specify the mean of the distribution of the pronoun vote share in election i in year t to be a linear function of the inverse square root of the number eligible and a set of fiscal year fixed effects:

$$\rho_{it} = \beta_0 + \beta_1 \cdot (1/\sqrt{N_{it}}) + \theta_t.$$

- Estimate using data on the 73,223 on-site elections with 200 or fewer eligible voters and no challenged ballots.

Estimated Year Effects in Mean of Pro-Union Share

$(\hat{\beta}_0 + \hat{\theta}_t)$



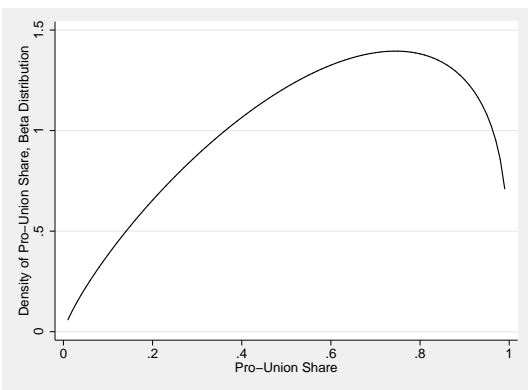
- Show a steady increase from 0.40 to about 0.55 since the mid-1980s.
- Mirrors the trend in the pro-union vote share in the raw data.

The distribution of the pro-union Share

- $\hat{\beta}_1 = 0.391$ (s.e. = 0.008)
 - $\hat{\theta}_t = 0.5$ and $n = 4 \Rightarrow E(\mu = 0.7)$
 - $\hat{\theta}_t = 0.5$ and $n = 100 \Rightarrow E(\mu = 0.54)$
 - Substantial decline in μ with election size.
- $\hat{\nu} = 3.10$ (s.e. = 0.025)
 - Implied S.D of $\mu = \sqrt{\frac{\rho(1-\rho)}{\nu+1}}$
 - $\rho = 0.59$ ($\hat{\rho}$ where $n = 20$ and $\beta_0 + \theta_t = 0.5$) \Rightarrow S.D. of $\mu = 0.242$.
 - Substantial variation in μ across elections of a given size.

Beta Density Function of Pro-Union Share

Based on $\rho = 0.59$, $\nu = 3.10$ – Corresponding to an Election with 20 Eligible Voters and $\beta_0 + \theta_t = 0.5$.



Back to Voter Turnout

- Use the estimates of the beta-binomial distribution of the pro-union vote share to compute measure of closeness

$$E((\mu - 0.5)^2 | s) = 0.25 - \left[\frac{n + \hat{v}}{n + \hat{v} + 1} \right] E(\mu | s)(1 - E(\mu | s))$$

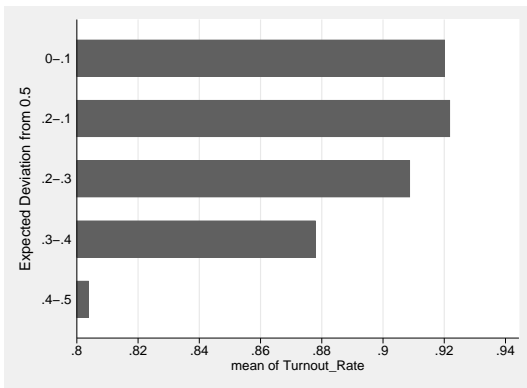
$$E(\mu | s) = \left[\frac{n}{n + \hat{v}} \right] \left(\frac{s}{n} \right) + \left[\frac{\hat{v}}{n + \hat{v}} \right] \hat{\rho}.$$

- Unconditional distribution of the number of votes cast is

$$f(v) = \binom{N}{v} \frac{\Gamma(\alpha)\Gamma(m\alpha + v)\Gamma((1 - m)\alpha + N - v)}{\Gamma(m\alpha)\Gamma((1 - m)\alpha)\Gamma(N + \alpha)}.$$

$$m_j = \frac{\exp(X_j\beta)}{1 + \exp(X_j\beta)}$$

Turnout Rate, by Square Root of Expected Squared Deviation of Union Share from 0.5



- Turnout rate drops off substantially as this deviation exceeds 0.2.
- Consistent with a worker's vote/no-vote decision being positively related to the probability of being pivotal.

MLE of Beta-Binomial Model Model of Voter Turnout

Determinants of m	(1)	(2)
Constant	0.8861 (0.0033)	0.8767 (0.0025)
$1/\sqrt{N}$	0.1593 (0.0030)	0.1633 (0.0025)
$E((\mu - 0.5)^2 s)$	-0.3083 (0.0087)	-0.7563 (0.0154)
$E((\mu - 0.5)^2 s)/\sqrt{N}$		1.6345 (0.0358)
Year FE's	Yes	Yes
α	7.4848 (0.0741)	7.2933 (0.0726)
Log L	-120336.9	-120004.4

- Strong inverse relationship between turnout and size.
- Strong positive relationship between turnout and expected closeness.
- Marginal effect of closeness *increases* with election size (contrary to expectation).

- $N=72,892$ on-site elections with no challenged ballots, between 2 and 200 eligible voters, and at least 2 votes cast. Total of 1,945,204 eligible voters. Asymptotic standard errors in parentheses.

Turnout and the Interaction of Election Size and Election Closeness

- The usual statistical properties imply that there is more variation around the mean in small samples than in large samples.
- \Rightarrow For a given expected pro-union vote share (μ), the likelihood that a voter will be pivotal will fall with election size.
- \Rightarrow The marginal effect of the deviation of the expected pro-union vote share from 0.5 on turnout will fall in absolute value
- I find the opposite.

Implications

- At least some workers understand in general terms how likely it is that they will be pivotal in deciding whether or not to vote. Reflected in:
 - 1 Lower turnout rates in larger elections
 - 2 Higher turnout rates in close elections.
- However, it does not appear that workers take into account the precise calculus regarding being pivotal when deciding whether or not to vote.
 - 1 They do not appear to consider the second order interaction between election size and closeness.
 - 2 They do not appear to consider the rules that lead to asymmetry between even and odd elections.
 - Lack of understanding of rules?
 - Lack of information on whether election is odd or even?

Is the Glass Half Empty or Half Full?

- Even in very large elections that are not at all close, over 85 percent of workers vote with probability one.
 - If $N = 100$ and $E(\mu|s) = 0.1$
 - $\Rightarrow \hat{m} = 0.89 + 0.16 \cdot .1 - 0.31 \cdot 0.15 = 0.86$
- One way to understand this is that about 85 percent of workers, on average, vote without consideration of the economic calculus.
 - They may get substantial utility from the act of voting.
- The remaining 15 percent or so do consider (at least in a rough way) the likelihood that they would be pivotal.

Applicability of Results – External Validity

- The specific numbers are relevant for union representation elections.
 - These elections are low cost (voting at the workplace).
 - These elections are very salient in that they involve the conditions of employment.
- Would expect quantitatively different results in larger political elections
- Likely to find less support for the economic model in larger political elections.