Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy

(Preliminary draft)

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Abstract

As Latin American countries have become more open, their job turnover rates have risen, their informal sectors have become larger, and their wage distributions have become less equal. We develop a dynamic general equilibrium trade model that explains these phenomena. The model combines standard search frictions in labor markets with heterogeneous firms that experience ongoing productivity shocks. Each period, firms decide whether to exit or continue producing. Those firms that remain active choose their export volumes and adjust their employment levels through vacancy postings or lay-offs.

Openness matters in our model because it makes profits more sensitive to productivity shocks, as Rodrik (1997) argued. Thus when trade barriers are low, firms drawing negative shocks shed labor relatively rapidly (and perhaps exit), while firms drawing positive shocks acquire new workers relatively rapidly. Further, since openness decreases the rents of the former and increases the rents of the latter, it spreads the wage distribution. After fitting this model to Colombian micro data on establishments and households, we isolate the effects of trade frictions on labor market outcomes using counter-factual simulations. Preliminary results suggest that the mechanisms highlighted by our model can be important.
1 Introduction

In developing countries, globalization is often blamed for exacerbating wage inequality, reducing job security, and increasing the size of the informal sector. This has been particularly true in Latin America, where many countries that pursued trade liberalization programs subsequently experienced heightened job turnover, wage dispersion and/or informality.¹ But the extent to which trade liberalization is to blame remains an open question. Labor market outcomes reflect many factors besides foreign competition, and reduced-form regressions have not convincingly isolated causal relationships. To better isolate the effects of openness on developing countries’ workers, we formulate a dynamic structural model of trade with labor market frictions. Then we fit our model to plant-level panel data and household survey data from Colombia—a country that in many respects typifies Latin American experiences.

The mechanism that links openness and labor market outcomes in our model is similar to one described by Rodrik (1997). He argues that openness increases the elasticity of demand for goods, and thus makes firms’ labor demand functions more sensitive to idiosyncratic productivity shocks. Workers’ job security and bargaining power are therefore compromised as trade barriers come down. In our model, although the elasticity of demand for goods is parametrically fixed at home and abroad, openness likewise increases firms’ responsiveness to idiosyncratic productivity shocks, both in terms of their vacancy postings and in terms of their lay-offs. These adjustments in firm behavior drive associated changes in the equilibrium

¹ "Between the mid-1980s and the beginning of the 1990s, countries in Latin America began trade liberalization programs, with reductions of at least 15 percentage points in the average tariff rate, which fell from an average of 48.9 percent in the prereform years to 10.7 percent in 1999." (Inter-American Development Bank, 2004, p. 137). Haltiwanger et al (2004) document the association between job and openness in Latin America. Goldberg and Pavcnik (2007) survey the evidence linking openness to wage inequality and informality in Latin America and other developing regions.
turnover rate, wage distribution, and rate of self-employment/informality.

Our formulation also shares some features with a number of recent trade models that
describe the effects of openness on labor market outcomes (Felbermayr et al, 2007; Helpman
and Itskhoki, 2007; Helpman, et al, 2008; Egger and Kreickemeier, 2007; Davis and Harrigan,
2008; Amiti and Davis, 2008). In particular, it embodies Melitz’s (2003) basic insight that
openness compounds the advantage of relatively efficient firms by creating new exporting
opportunities for them, while it compounds the problems of relatively inefficient firms by
intensifying the competitive pressures they face. However we depart from this literature in
two ways. First, we assume that firms experience ongoing, idiosyncratic productivity shocks
(as in Hopenhayn, 1992, and Hopenhayn and Rogerson, 1993), inducing them to adjust their
vacancy postings, lay-offs and exit decisions. (as in Bertola and Caballero, 1994, and Bertola
and Garibaldi, 2001). Second, we fit our model to micro data and use it to perform numerical
experiments that quantify the effects of openness under alternative assumptions. Simulations
at plausible parameter values show that these effects can be important.

While we do not pretend to capture all of the channels through which openness can af-
fect labor market outcomes, our focus on firm-level entry, exit and idiosyncratic productivity
shocks is supported by existing empirical evidence on the sources of job turnover and wage het-
erogeneity. Studies of job creation and job destruction invariably find that most reallocation

\(^2\)Several less-related linkages between openness and labor market outcomes have been modeled in the recent
trade literature. One strand of this literature emphasizes the changes in skill-premia and/or unemployment
rates that result from trade-induced changes in the relative demand for different types of labor (e.g., Albrecht
and Vroman, 2002; Yeaple, 2005; Davidson et al, 2006). Another characterizes the adjustments in wages,
unemployment and labor turnover patterns that derive from trade-induced changes in sectoral output prices
(e.g., Kambourov, 2006; Artuc, Chaudhuri and McClaren, 2008). And finally, some studies have focussed on
cross-country differences in the flexibility of labor markets as a source of comparative advantage (Davidson et
al, 1999; Cunat and Melitz, 2007; Helpman and Itskhoki, 2008).

\(^3\)Other recent papers that study firm dynamics and labor market frictions in a closed economy context include Cooper et al (2007), Hobijn and Sahin (2007), and Lentz and Mortensen (2008).
is due to idiosyncratic (rather than industry-wide) adjustments (Davis et al, 1998; Roberts, 1996; Inter-American Development Bank, 2004). "This is true even in Latin America’s highly volatile macro environment," where producer entry and exit alone account for 30-40 percent of job creation and destruction (Inter-American Development Bank 2004, chapter 2). Further, as Goldberg and Pavcnik (2007) note, if openness has had a significant effect on job flows, it has mainly been through intra-sectoral effects: "Most studies of trade liberalization in developing countries find little evidence in support of [trade-induced labor] reallocation across sectors.” Finally, while cross-worker differences in wages are obviously partly due to differences in worker characteristics, much is attributable to labor market frictions and firm heterogeneity.4

2 Environment: The Closed Economy

For expositional clarity, we first develop our model for the case of a closed economy. This formulation extends Bertola and Cabellero (1994) and Bertola and Garibaldi (2001) to a general equilibrium setting with fully articulated product markets, arbitrary (stationary) Markov processes for productivity shocks, endogenous firm entry and exit, and risk of exogenous worker separation. Once we have characterized the interactions between the labor markets, product markets, and productivity shocks in this setting, it is straightforward to generalize the analysis to an open economy and allow for intra-sectoral trade.

There are two types of output in our economy—services and industrial goods. The former are non-traded while the latter are tradable, subject to transport costs. Services are supplied

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4 Studying data from France and the United States, Abowd, et al (1999) and Abowd, et al (2002) show that roughly half of the cross-worker variation in compensation in French workers is due to employer effects. The only study of employer-employee data in developing countries we are aware of is Menezes-Filho and Muendler (2007). This paper does not report results on sources of wage variation. (Double check)
by firms and, less efficiently, by unemployed workers engaged in home production. Regardless of their source, services are produced with labor alone, homogeneous across suppliers, and sold in competitive product markets. Firms who supply services use a common constant returns technology, and face no hiring or firing costs.

Industrial goods cannot be home-produced. They must be supplied by firms, which pay a sunk start-up cost to initiate production of a single variety of output. Each firm produces its output using labor alone, and competes in a monopolistically competitive product market. Unlike service sector firms, suppliers of industrial goods are subject to ongoing idiosyncratic productivity shocks, and they must create costly vacancies in order to attract new workers. The shocks can equally well be thought of as affecting firms’ relative product appeal.

Producer dynamics in the industrial sector resemble those in Hopenhayn and Rogerson (1993) in that firms react to their productivity shocks by optimally hiring, firing or exiting. Also, new firms enter whenever their expected future profit stream exceeds the entry costs they face. However, unlike Hopenhayn and Rogerson, we assume hiring in the industrial sector is subject to search frictions captured by a standard matching function. Further, workers make forward-looking decisions concerning which sector to work in and what job offers to accept.

Each worker decides whether to participate in the industrial labor market at the beginning of each period. Those who are already employed in the industrial sector can continue with their current job unless their employer lays them off or shuts down entirely (They can also quit in order to move to the service sector or to search for other industrial sector jobs, although in equilibrium none find it optimal to do so.) Those not yet employed in the industrial sector can forego certain employment with a service sector firm in order to search for a higher-
wage industrial sector job, but they risk remaining unemployed if they fail to match with an industrial sector producer.\textsuperscript{5} Those who end up unemployed subsist during the current period by using a relatively inefficient technology to home-produce services.

\section*{2.1 Production Technologies}

All service-sector firms exploit a common constant-returns technology to produce the homogeneous good. So with an appropriate choice of output units, we may write the total supply of services as

\[ S = L_S, \]

where \( L_S \) is labor hired in services.

In the industrial sector, output of producers with productivity level \( z \) is given by

\[ q(z, l) = z l^\alpha, \tag{1} \]

where \( l \) is the labor input and \( \alpha > 0 \). Productivity is firm-specific, independent across firms, and serially correlated. Its evolution is characterized by the transition density \( h(z'|z) \), which is common to all firms.

\section*{2.2 Preferences}

Worker-consumers in the economy are homogenous and their measure is normalized to unity. Each has lifetime utility given by

\[ U = \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t u(s_t^c, q_t^c), \]

\textsuperscript{5}The notion that workers trade job security in a low wage sector for the opportunity to search in a higher wage sector traces back at least to the Harris and Todaro (1970) model.
where \( r \) is the rate of time preference, \( s^c_t \) is consumption of services, and \( q^c_t \) is an index of differentiated good consumption. The momentary utility function \( u \) takes the form

\[
u(s^c, q^c) = \frac{(s^c)^{1-\gamma} (q^c)^\gamma}{(1-\gamma)^{1-\gamma} \gamma},\]

where \( \gamma \in (0, 1) \) and our index of industrial goods consumption is

\[
q^c = \left( \int_0^N q^c(n)^{\frac{\sigma-1}{\sigma}} f(n)dn \right)^{\frac{\sigma}{\sigma-1}}.
\]

Here \( N \) is our measure of differentiated varieties, \( q^c_t(n) \) is consumption of variety \( n \), and \( \sigma > 1 \) is the elasticity of substitution between varieties.

The price of services is our numeraire, and given our representation of preferences above, the exact price index for the composite good \( q^c \) is

\[
P = \left( \int_0^N p(n)^{1-\sigma} f(n)dn \right)^{\frac{1}{1-\sigma}},
\]

where \( p(n) \) is the price of variety \( n \).

Letting \( I_i \) be the income of worker \( i \) and disallowing savings, the period-by-period budget constraint is

\[
I_i = s^c_i + P q^c_i.
\]

Welfare maximization implies that consumer \( i \) spends a fraction \( \gamma \) of her income on the composite industrial good and her demand for variety \( n \) is

\[
q^c_i(n) = \frac{\gamma I_i}{P} \left( \frac{p(n)}{P} \right)^{-\sigma} = D_i p(n)^{-\sigma}, \quad (2)
\]
where $D_i = \gamma I_i P^{\alpha-1}$. Finally, since worker-consumers are risk neutral, consumer $i$ enjoys momentary indirect utility

$$W_i = I_i P^{-\gamma}. \quad (3)$$

2.3 Labor Markets and the Matching Technology

The service sector labor market is frictionless so, given that the price of services unity, the service sector wage is $w_s = 1$. Search frictions make things more complicated in the industrial sector. Each period the number of new matches between unemployed workers and vacancy posting firms is given by

$$M(V, U) = \frac{V U}{(V^m + U^m)^{1/m}},$$

where $U$ is the measure of unemployed workers searching in industrial sector and $V$ is the measure of vacancies in industry.\(^6\) Consequently, industrial firms fill each vacancy they post with probability

$$\phi^f(V, U) = \frac{M(V, U)}{V} = \frac{U}{(V^m + U^m)^{1/m}},$$

while unemployed workers searching for industrial jobs find matches with probability

$$\phi^w(V, U) = \frac{M(V, U)}{U} = \frac{V}{(V^m + U^m)^{1/m}}.$$

At the beginning of each period, all workers choose whether to work in industry or services. Workers who choose services are employed with certainty at the wage $w_s$, but workers who choose industry may or may not find employment. Those who already hold industrial sector jobs can continue with them so long as they neither experience an exogenous separation nor lose their jobs because their employers contract or shut down. Those who are not already

\(^6\)The functional form of the matching function follows den Haan et al. (2000). It is constant returns to scale, and increasing in both arguments. The advantage relative to the standard Cobb-Douglas form is that it has no scale parameter and is naturally bounded between zero and one.
employed in the industrial sector can begin working there if they successfully match with a vacancy-posting producer. The probabilities of these different events shape workers’ sectorial choices, as well as firms’ employment policies. We start by describing the latter.

2.4 Incumbent Firm’s Problem

The demand function (2) and the production function (1) imply that any producer with productivity $z$ who chooses employment level $l$ will earn revenue

$$r(z, l) = D^\frac{z}{l} (z^\frac{z-1}{l^2}),$$

(4)

where $D = \int_0^1 D_d di$ is aggregate demand for differentiated goods.

When choosing employment levels, firms weigh this payoff against wage costs, the effects of changes in $l$ on the their continuation value, and hiring costs. To characterize the latter, let the cost of posting $v$ vacancies for a firm of size $l$ be

$$C_h(l, v) = \frac{c_h}{\lambda} \left( \frac{v}{l} \right)^\lambda.$$

where $c_h$ and $\lambda > 1$ are positive parameters.\(^7\) Also assume that firms are large in the sense that cross-firm variation in realized arrival rates is ignorable. (That is, all firms fill the same fraction $\phi^f$ of their posted vacancies.) It follows that expansion from $l$ to $l'$ simply requires the posting of $v(l, l') = \frac{l'-l}{\phi^f}$, vacancies, and the cost of expanding from $l$ to $l'$ workers is

$$C_h(l, l') = \frac{c_h}{\lambda} \frac{1}{(\phi^f)^\lambda} \left( \frac{l'-l}{l} \right)^\lambda.$$

\(^7\)As discussed in Bertola and Caballero (1994) “convexity is necessary to obtain a well-defined vacancy-posting equilibrium when productivity is heterogeneous across firms, as firms with high productivity and low employment levels would want to post infinitely many vacancies for arbitrarily short intervals of time if such policies were not made prohibitively costly.”
Finally, assume that each firm bargains with its workers individually and continuously, ensuring, as in Stole and Zwiebel (1996) and Cahuc and Wasner (2000), that all workers at a given firm are paid the same wage.

To derive firms’ optimal employment policies, we first specify the sequencing events within each period (see Figure 1). An incumbent firm enters the current period with the productivity $z_{-1}$ and work force $l_{-1}$, which were determined at the end of the previous period. Immediately the firm decides whether to stay in business or to exit. If it stays, it proceeds to an *interim stage* in which it observes its current-period productivity realization, $z$, and exogenous separations shrink its labor force to $l = (1 - \delta)l_{-1}$. Then, taking stock of its updated state, $(z, l)$, the relevant wage schedule, and the economy-wide worker arrival rate, $\phi^f$, it chooses its current period work force, $l'$. The firm can decide to hire ($l' > l$) or fire ($l' \leq l$) workers. If it hires workers, they are immediately available to produce output in the current period.
Finally, revenues accrue and wages and other costs are paid at the end of the period.

Given the presence of search frictions, workers at hiring firms generate rents, and as we will detail shortly, these are bargained over to determine wages. However, since firms can shed workers costlessly, the marginal worker at a firing firm creates no rents and has no bargaining power. Hence expanding and contracting firms face different wage schedules, and current operating profits depend upon both \( l \) and \( l' \). More precisely, defining \( w_h(z, l) \) to be the wage function faced by a hiring firm and \( w_f(z, l) \) to be the wage function faced by a firing firm, profits are

\[
\pi(z, l, l') = \begin{cases} 
  r(z, l') - w_h(z, l')l' - c_f & \text{if } l' > l \\
  r(z, l') - w_f(z, l')l' - c_f & \text{otherwise.}
\end{cases}
\]  

(5)

where \( c_f \), the per-period fixed cost of operation, is common to all firms.

Using (5), the value of a firm in state \((z, l)\) at the interim stage is given by

\[
V(z, l) = \max_{l'} \frac{1}{1 + r} \left\{ \pi(z, l, l') - C(l, l') + \max (E_{z'}[V(z', (1 - \delta)l')|z], 0) \right\},
\]  

(6)

where

\[
C(l, l') = \begin{cases} 
  C_h(l, l'), & \text{if } l' > l, \\
  0, & \text{otherwise.}
\end{cases}
\]

In turn, the solution to (6) implies an employment policy function, hereafter denoted \( l' = L(z, l) \). It also implies an indicator function that distinguishes hiring firms from others,

\[
T^h(z, l) = \begin{cases} 
  1, & \text{if } L(z, l) > l, \\
  0, & \text{otherwise.}
\end{cases}
\]  

(7)

and an indicator function that summarizes firm’s beginning-of-period continuation/exit policy

\[
T^e(z_{-1}, l_{-1}) = \begin{cases} 
  1, & \text{if } E_z[V(z, l)|z_{-1}] > 0, \\
  0, & \text{otherwise.}
\end{cases}
\]  

(8)

where \( l = (1 - \delta)l_{-1} \).
2.5 Entry

In the steady state, a constant (endogenous) fraction $\mu_{exit}$ of firms exits the industry. There is an infinite pool of potential entrants and actual entrants replace this mass of exiting firms by paying a sunk entry cost of $c_e$ in terms of the service sector good. Entrants draw their initial productivity from the density function denoted by $f_e(z)$ with support $[\bar{z}, \overline{z}]$, and immediately hire $l_e > 0$ workers at no additional cost. After observing $z$, entrants behave exactly like the incumbent firms in the interim stage, with their interim state given by $(z, l_e)$ – see Figure 1. Free entry implies that

$$\mathcal{V}_e = \int_{\bar{z}}^{\overline{z}} \mathcal{V}(z, l_e) f_e(z) dz \leq c_e,$$

(9)

which holds with equality if there is a positive mass of entrants $M$.

2.6 Worker’s Problem

Figure 2 presents the intra-period timing of events for workers. Consider first a worker who at the beginning of the current period is employed by an industrial sector firm in state $(z_{-1}, l_{-1})$. This worker learns immediately whether her firm will exit and, if not, whether she will be separated from her job for exogenous reasons. If either event occurs she joins the pool of industrial job seekers (enters state $u$) in the interim stage. If neither event occurs, she enters the interim stage as an employee of the same firm that she worked for in the previous period. (No one voluntarily quits because, in equilibrium, firms always pay their workers at least their reservation wage.) Her firm then realizes its new productivity level $z$ and enters interim state $(z, l)$, with its labor force from the previous period depleted by exogenous separations: $l = l_{-1}(1 - \delta)$. At this point her firm decides whether to hire or fire workers. In the former
case it expands its workforce to $l' > l$, and she earns $w_h(z, l')$. She is then positioned to start
the next period in state $(z, l')$. In the latter case, she loses her job and reverts to state $u$ with
probability $p_f = (l - l')/l$. Finally, with probability $1 - p_f$ she retains her job, earns $w_f(z, l')$
during the current period, and starts next period in state $(z, l')$.

Workers in state $u$ search for an industrial job. They are hired by entering and expanding
firms that post vacancies. If they are matched with a firm, they receive the same wage as those
who were already employed by the firm. If they are not matched, they remain unemployed in
the current period, and support themselves by home-producing $b \in [0, 1)$ units of the service
good. At the start of the next period, they can choose to work in the service sector (enter
state $s$) or look for a job in the industrial sector (remain in state $u$). Likewise, workers who
start the current period in the service sector choose between continuing to work at the service
wage $w_s = 1$ and entering the pool of industrial job-seekers. As these workers have the option
to choose either labor market, they are said to be in state $o$.

We now specify the value functions for the workers in the interim stage. Going to the
service sector generates an end-of-period income of 1 and returns a worker to the $o$ state at
the beginning of next period. Accordingly, the interim value of this choice is

$$J^s = \frac{1}{1 + r} (1 + J^o),$$

(10)

Searching in the industrial sector exposes workers to the risk of spending the period unem-
ployed, supporting themselves by home-producing $b$ units of the service good. But it also
opens the possibility of landing on a high-value job. Given the probability of finding a match
is $\phi^w$, the interim value of searching for an industrial job is given by

$$J^u = \frac{1}{1 + r} \left[ (1 - \phi^w) (b + J^o) + \phi^w E J^e_h \right],$$

(11)
where $EJ_h^e$ is the expected value of being employed in a hiring firm. Given these two values, the value of the sectorial choice is $J^o = \max\{J^s, J^u\}$. Since both sectors must exist in equilibrium (given consumer preferences),

$$J^o = J^s = J^u. \quad (12)$$

The expected value of matching to an industrial job, $EJ_h^e$, depends only on the distribution of hiring firms and the value of the jobs they offer. For workers who match with a firm that is in state $(z, l)$ before hiring, the interim period value is given by

$$J_h^e(z, l) = \frac{1}{1 + r} \left[ w_h(z, l') + J^e(z, l') \right], \quad (13)$$

where $l' = L(z, l)$ and $J^e(z, l')$ is the value of being employed at an industrial firm in state $(z, l')$, which may or may not be hiring, at the start of the next period.
The expected value of being employed in a hiring firm depends on the density of vacancies across firm. This density is given by

\[ g(z, l) = \frac{v[l, l'(z)]\tilde{f}(z, l)\mathcal{I}^{h}(z, l)\mathcal{I}^{l}(z, l)}{\int_{z} \int_{l} \tilde{f}(z, l)\mathcal{I}^{h}(z, l)v(z, l)dldz}, \]  

(14)

where \( v[l, l'(z)] = [l'(z) - l] / \phi \) and \( \tilde{f}(z, l) \) is the unconditional density of firms over \((z, l)\) in the interim. Accordingly, the expected value of a match for a worker, as perceived at the interim stage, is

\[ EJ^c_h = \int_{z} \int_{l} J^c_h(z, l)g(z, l)dldz. \]  

(15)

It remains to specify the unconditional value of starting the period matched with an industrial firm, \( J^c(z_{-1}, l_{-1}) \), which appears in (13). This object can be written as a weighted average of the value of exogenous separation (which occurs with probability \( \delta \)) and the value of continuing with the firm, and is given by

\[ J^c(z_{-1}, l_{-1}) = \delta J^u + (1 - \delta) \int_{z} \tilde{J}^c[z, (1 - \delta)l_{-1}]h(z|z_{-1})dz, \]  

(16)

where, since firing firms pay workers their reservation wage, the continuation value is

\[ \tilde{J}^c(z, l) = \begin{cases} \frac{1}{1+\tau}[w_h(z, l') + J^c(z, l')], & \text{if } l' \geq l, \\ J^u, & \text{otherwise}. \end{cases} \]

Given the wage schedules, firms employment policies, and the distribution of firms, the system of equations described in this section determines workers’ payoffs and their associated behavior.
2.7 Wage Schedules

All firms that decide to hire—either to replace workers who were lost due to exogenous separations or to expand their workforce—post vacancies. After the matching takes place, the firm bargains with its workers simultaneously and on a one-to-one basis, treating each worker as the marginal one. Hiring firms cannot begin bargaining with their workers until they have finished posting their current-period vacancies and matching has taken place. As a result, vacancy posting costs are already sunk and workers who walk away from the bargaining table cannot be replaced in the current period. These timing assumptions create rents to be split between the firm and the worker.

As detailed in Appendix 1, it follows that the wage schedule for expanding/replenishing firms is given by

$$w_h(z, l) = (1 - \beta)U + \Gamma(\alpha, \beta, \sigma) D^{\frac{1}{\sigma - 1}} z \left[ \frac{\sigma - 1}{\sigma + (1 - \alpha)} \right]$$

where \( \Gamma(\alpha, \beta, \sigma) = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)} \). Note that if \( \beta = 0 \), i.e. all the bargaining power is with the firm, the firm pays reservation wages equal to the flow value of the outside option, \( U \).

The marginal worker at a contracting firm generates no rents, so the firing wage just matches their reservation wage. It is given by (see Appendix 1):

$$w_f(z, l') = (1 + r) J^u - J^e(z, l').$$

Note that, unlike \( b + J^o \), \( w_f(z, l') \) varies across firms, reflecting their different future prospects. It can be less than the value of home production, as firing firms may draw a better \( z \) values in the future and raise their wages.
3 Closed Economy Equilibrium

Five basic conditions characterize our equilibrium. First, the distribution of firms over \((z, l)\) states reproduces itself each period through the Markov processes on \(z\), the policy functions (including hiring, firing, entry and exit), and the productivity draws firms receive upon entry. Second, supply matches demand for services and for each differentiated good, where supplies are determined by employment levels in each type of good. Third, the flow of workers into unemployment matches the flow of workers out of unemployment—that is, the Beveridge condition holds. Fourth, aggregate income matches aggregate expenditure. And finally, workers optimally choose the sector in which they are working or seeking work. Appendix 2 provides the algebraic details of these conditions, and Appendix 5 summarizes the numerical solution algorithm we use to find the associated equilibrium.

4 Open Economy Equilibrium

4.1 Symmetric Countries

Now consider an open economy version of our model in which two symmetric countries trade industrial goods with one another, subject to iceberg transportation costs and fixed exporting costs. Transport costs are parameterized by \(\tau\), which denotes the amount of each good that must be shipped in order for one unit to arrive at its foreign destination. Fixed costs are denoted by \(c_x\), and must be paid once per period by each exporting firm, regardless of its total shipments. These fixed costs keep firms with high labor costs and/or low productivity levels out of foreign markets.

Demand for an exporting firm comes from domestic and foreign consumers. By symmetry,
product-specific demand functions have the same intercept $D$ in both markets. However trade costs induce cross-country asymmetries in sales. If a firm in state $(z, l)$ exports some fraction $\eta$ of its output, its export revenues are:

$$r_x(z, l, \eta) = D^{\frac{1}{\sigma}} \left[ \frac{\eta}{\tau}zl^{\alpha} \right]^\alpha,$$

and its domestic revenues are

$$r_d(z, l, \eta) = D^{\frac{1}{\sigma}} [(1 - \eta)zl^{\alpha}]^\alpha.$$

where $a = \frac{\alpha - 1}{\sigma}$. Accordingly, if the firm chooses $\eta$ optimally, its total revenue (net of fixed exporting costs) is:

$$r(z, l) = \max_{\eta \in [0,1]} \left( r_d(z, l, \eta) + r_x(z, l, \eta) - c_xI^x \right) \quad (17)$$

$$= \max \left\{ D^{\frac{1}{\sigma}} (zl^{\alpha})^\alpha \left[ (\eta^*/\tau)^a + (1 - \eta^*)^a \right] - c_xI^x \right\} \quad (17)$$

where $\eta^* = \frac{1}{1 + \tau - \tau}$ and $I^x = 1_{\eta>0}$ is an indicator function that equals one if the firm exports a non-zero quantity of its output. Clearly, since $\eta^* > 0$, firms can always increase their revenues by exporting and there is a threshold value of $zl^{\alpha}$ above which firms do best to export. Once the closed economy revenue function (4) has been replaced with (17), and we have redefined our price and quantity indices in the usual way for intra-industry trade (see Appendix 3), the analysis proceeds exactly as before.

Embedded in our general equilibrium model, this standard revenue function delivers a number of desirable features. First, for any given $(z, l)$, it implies that the marginal revenue product of labor is larger when the economy is open. This is the underlying reason that productivity shocks induce larger adjustments in vacancy postings or firings when foreign markets are accessible. Second, since larger revenues at a given $(z, l)$ mean more surplus to
bargain over, it is also the reason that the wage paid by a firm that exports in state \((z, l)\) is higher than what it is in the closed economy equilibrium. This result is consistent with the empirical finding that, controlling for employment, exporters pay higher wages (Bernard and Jensen, 1999). Third, combined with the fact that search frictions make marginal costs vary across firms with identical \(z\) values, it explains why productive efficiency is a noisy predictor for exporting status.\(^8\) Finally, re-interpreting \(z\) shocks to be product appeal indices rather than productivity indices, it explains why exporters manage to be larger than non-exporters, even though they charge higher prices and pay higher wages.\(^9\)

### 4.2 The Small Country Case

The model can be modified to represent a small open economy such that domestic conditions do not affect the foreign price index, the extensive margin of imports and foreign income level. We normalize the price of the composite import good to 1. Since we have a domestic and a foreign numeriare now, there is an exchange rate \(\varepsilon_x\) which determines their parity. Iceberg trade costs are still denoted by \(\tau\) for exports, and there is an ad-valorem import tariff \(\tau_m\).

The domestic price of imports is thus \((1 + \tau_m)\varepsilon_x\).

Let \(D^F_F\) and \(D_F = \varepsilon_x D^F_F\) be foreign income in foreign and domestic currencies respectively. Then export revenues for a firm in state \((z, l)\) are given by

\[r_x(z, l, \eta) = D^F_F \left[ \frac{\eta}{\tau} z l^\alpha \right]^\alpha.\]

As above, the firm solves a static problem of exporting or not, and what fraction of output to

---

\(^8\)Hallak and Sivadasan (2008) explain this fact by assuming that (1) firms differ in terms of both their quality and their productivity efficiency, and (2) exporting requires that firms meet a minimum quality standard.

\(^9\)Kugler and Verhoogen (2008) interpret this pattern to be due to complementarities in production between worker ability and product quality, arguing that it is difficult to otherwise reconcile the data with optimizing behavior.
be exported, hence their revenue function is

\[ r(z, l) = \max_{\eta \in [0, 1]} (r_d(z, l, \eta) + r_x(z, l, \eta) - c_x I^x) \]

\[ = \max \left\{ \left[ D_H^\frac{1}{\sigma} (1 - \eta)^{\alpha} + D_F^\frac{1}{\tau} \left( \frac{\eta}{\tau} \right)^{\alpha} \right] (zl^\alpha)^{\alpha} - c_x I^x, \right. \]

\left. \left. D_H^\frac{1}{\alpha} (zl^\alpha)^{\alpha}. \right\} \right. \tag{18} \]

Conditional on exporting, firms choose the fraction of output exported optimally

\[ \eta^* = \frac{1}{1 + \frac{D_H}{D_F} \tau-\sigma - 1}. \]

The domestic price index can be written as

\[ P = \left[ \left( (1 + \tau_m)\varepsilon_x \right)^{1-\sigma} + \tilde{p}_d^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \]

where \( \tilde{p}_d^{1-\sigma} = N \int \int p_d(z, l)^{1-\sigma} f(z, l) dz dl \) is the price index of domestic sales of domestic varieties. In turn, import demand is

\[ q_m = \left[ \frac{\gamma I}{P} \right] \left( \frac{(1 + \tau_m)\varepsilon_x}{P} \right)^{-\sigma} = D_H \left[ (1 + \tau_m)\varepsilon_x \right]^{-\sigma}, \]

where \( D_H = \gamma IP^{\sigma-1} \) is the domestic demand intercept as before. Domestic expenditure on imports is \( E_m = q_m (1 + \tau_m)\varepsilon_x \). Because of the tariff wedge, this figure is higher than what is actually paid to foreigners (in domestic currency). Denoting the latter by \( R_m \),

\[ R_m = \frac{E_m}{1 + \tau_m} = \frac{D_H \left[ (1 + \tau_m)\varepsilon_x \right]^{1-\sigma}}{1 + \tau_m} = D_H (1 + \tau_m)^{-\sigma} \varepsilon_x^{1-\sigma}. \]

Total tariff revenues, \( T_m = R_m \tau_m \), are rebated back to consumers.

Total export revenue of the country is given by

\[ R_x = N \int \int_{l_e} r_x(z, l) I^x(z, l) f(z, l) dz dl. \]

Trade balance holds when

\[ R_m = R_x. \]
In Appendix 3, we show that if all other market clear, trade balance holds by Walras’ Law. Next we turn to the empirical implementation of the model which is based on the small open economy environment.

5 Quantitative Analysis

5.1 An Application to Colombia

To explore the implications of the small-country version of our model, we use a combination of econometric estimation and calibration techniques to fit it to Colombia. This country suits our purposes for several reasons. First, Colombia underwent a significant trade liberalization during the late 1980s and early 1990s, reducing its average nominal tariff rate from 21.5 percent to 11 percent (Table 1). Second, despite stable average unemployment rates, these trade reforms were associated with an increase in job turnover rates from 18.4 percent to 23 percent, and an increase in the wage inequality, controlling for observable worker characteristics (Table 1). These patterns are typical of Latin American experiences. Finally, although Colombia did experience some macro shocks during the period of interest, they were relatively mild. Thus the consequences of Colombia’s liberalization are relatively likely to come through in its data.

---

10Wage inequality figures are based on the residuals from a Mincerian regression of log wages on education, occupation, age, sectoral and occupational dummies. The data set pools biennial household survey data from Colombia’s national statistical agency (DANE) for the period 1986-98. Coefficients on all variables are allowed to shift through time in order to exclude changing skill premiums as a source of dispersion.
Table 1: Trade Reforms and Labor Market Outcomes – Colombia

<table>
<thead>
<tr>
<th>Variable</th>
<th>pre-liberalization</th>
<th>post-liberalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Import Tari$s</td>
<td>21.50</td>
<td>11.30</td>
</tr>
<tr>
<td>Job Turnover</td>
<td>18.43</td>
<td>22.95</td>
</tr>
<tr>
<td>Unemployment</td>
<td>9.99</td>
<td>9.87</td>
</tr>
<tr>
<td>90-10 Wage Inequality</td>
<td>4.16</td>
<td>4.50</td>
</tr>
</tbody>
</table>

5.2 The Revenue Function and Productivity Process

The job turnover and wage inequality documented in Table 1 help us to calibrate our model, as we will discuss shortly. But the parameters that characterize the revenue function and the productivity process can be estimated econometrically using Colombia’s annual industrial survey. Note that the revenue function (18) and CES preferences imply that log revenues (gross of fixed exporting costs) can be written as a function of employment, productivity and an index of market-wide demand, \( d_H = \ln[D_H^{\frac{1}{\sigma}}(1 - \eta^*)^a] \), an index of the percentage increase in total demand associated with exporting, \( d_F = \ln[D_F^{\frac{1}{\sigma}}(\eta^*/\tau)^a \cdot e^{-d_H} + 1] \), and an indicator for whether firm \( i \) is an exporter, \( I_x^i \):

\[
\ln r_{it} = d_H + I_x^i d_F + \frac{\sigma - 1}{\sigma} \ln z_{it} + \alpha \frac{\sigma - 1}{\sigma} \ln l_{it} \tag{19}
\]

Further, assuming that \( \ln(z) \) follows an exogenous AR(1) process,

\[
\ln z_{it} = \rho \ln z_{it-1} + \epsilon_{it} \tag{20}
\]


\[12\]It is noteworthy that this expression implies exporting status affects total revenue for a given amount of labor and a given productivity level. Thus, it provides an alternative interpretation for the common finding that measured productivity (i.e., deflated revenue per unit input bundle) is higher among exporters. The reason this result emerges is that labor market frictions prevent firms from freely adjusting their size as exporting opportunities come and go.
equation (19) can be restated as:

\[
\ln r_{it} = (d_H + \mathcal{I}_{it}^x \cdot d_F) - \rho (d_H + \mathcal{I}_{it-1}^x d_F) + \rho \ln r_{it-1} \\
- \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it-1} + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it} + \frac{\sigma - 1}{\sigma} \epsilon_{it},
\]

If one could obtain consistent estimates of the coefficients that appear on the right-hand-side observable variables, these would allow one to infer consistent estimates of \( \rho, \alpha, \) and \( \sigma, \) and the variance of the error term would allow one to infer \( \sigma_\varepsilon, \) the standard deviation of error terms in (20). However, selection bias and simultaneity bias prevent us from consistently estimating (21) with ordinary least squares. The former problem occurs because firms choose whether exit the market partly on the basis of their \( \epsilon_{it} \) realizations, so the \( \epsilon_{it} \) realizations observed for active producers are not random draws from the unconditional distribution of \( \epsilon \)'s. The latter problem occurs because firms' current exporting decisions and employment levels are chosen after the current realization on \( \epsilon \) is observed, so \( \epsilon_{it} \) is correlated with both \( \mathcal{I}_{it}^x \) and \( \ln l_{it}. \) Appendix 4 develops a generalized method of moments (GMM) estimator related to Olley and Pakes’ (1996) that deals with both problems.

Applying this estimator to the set of all Colombian manufacturing plants observed for at least three years during the pre-liberalization period 1982 and 1991, we obtain the results summarized in Table 1 below.\(^{13} \) Since \( \sigma \) proved to be poorly identified, we fixed this parameter at several values typical of the literature: \( \sigma = 5 \) and \( \sigma = 8. \) All parameters are estimated with considerable precision and fall in plausible ranges. Note also that the results are not very sensitive to \( \sigma, \) although the lower \( \sigma \) value is associated with somewhat smaller estimates

\(^{13} \)The data are annual observations on all manufacturing firms with at least 10 workers. They were collected by Colombia’s National Statistics Department (DANE) and cleaned as described in Roberts (1996). Given that fixed capital and intermediate inputs do not appear in our model, we define revenue to be the value of output net of intermediate input and capital costs. Annual capital costs are 10 percent of the book value of firms’ capital stocks.
for $\alpha$ and $\rho$, and a modestly larger estimate for $\sigma_\varepsilon$. We will use the $\sigma = 8$ estimates for our baseline calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\sigma = 8$ estimate</th>
<th>std. error</th>
<th>z-ratio</th>
<th>$\sigma = 5$ estimate</th>
<th>std. error</th>
<th>z-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.785</td>
<td>0.005</td>
<td>165.573</td>
<td>0.725</td>
<td>0.004</td>
<td>169.914</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.814</td>
<td>0.023</td>
<td>35.975</td>
<td>0.581</td>
<td>0.021</td>
<td>27.239</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.546</td>
<td>0.008</td>
<td>69.373</td>
<td>0.582</td>
<td>0.008</td>
<td>74.283</td>
</tr>
<tr>
<td>$d_H$</td>
<td>0.214</td>
<td>0.004</td>
<td>51.409</td>
<td>0.215</td>
<td>0.004</td>
<td>51.418</td>
</tr>
<tr>
<td>$d_F$</td>
<td>0.214</td>
<td>0.019</td>
<td>11.030</td>
<td>0.180</td>
<td>0.019</td>
<td>9.318</td>
</tr>
</tbody>
</table>

**5.3 Remaining Parameters**

Results in Table 2 provides us with consistent estimates of $\alpha, \rho, \sigma_\varepsilon$, and $d_F$, and determine our choices for $\sigma$. Some of the remaining parameters can be selected based on observable aggregates. First, the real borrowing rate in Colombia fluctuated around 15% between late 1980s and early 2000s, so we set $r$ to be 15% (Bond et al, 2008). Second, the share of tradables in total consumption expenditure in Colombia was about 40% in 2005 (Wold Bank, 2008, Table 11, p. 134).\(^{14}\) This allows us to set $\gamma$ to be 0.4.

Several other parameters can be taken from the existing literature. Following den Haan, Ramey and Ramey (2000), we set the elasticity of the matching function, $m$, to be 1.27. And following Yashiv (2000), we take the curvature of labor adjustment costs, $\lambda$, to be 4. Finally, as a benchmark we give equal bargaining power to firms and workers, i.e. $\beta = 0.5$, assume that entrants have 1 workers, i.e. $l_e = 1$, and shut down exogenous worker separation, i.e.

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\(^{14}\)In order to find the expenditure share on tradables we sum the expenditure shares for food and nonalcoholic beverages, alcoholic beverages and tobacco, clothing and footwear. We also added half of furnishing, household equipment and maintenance and half of other consumption items as tradable. This leaves housing, water, electricity, health, transportation, communication, recreation and culture, education, restaurants and hotels as non-tradable items.
$\delta = 0.0^{15}$

**Table 3: Parameter Values**
(parameters that are set before simulations)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.785</td>
<td>production function</td>
<td>Estimates from Table 2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.814</td>
<td>persistence of $z$ process</td>
<td>Estimates from Table 2</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.546</td>
<td>standard deviation of shocks to $z$</td>
<td>Estimates from Table 2</td>
</tr>
<tr>
<td>$D_F$</td>
<td>6.87</td>
<td>foreign demand level</td>
<td>Estimates from Table 2</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>8</td>
<td>elasticity of subs. between dif. goods</td>
<td>Various studies</td>
</tr>
<tr>
<td>$r$</td>
<td>0.15</td>
<td>interest rate</td>
<td>Bond, Utar, Tybout (2008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4</td>
<td>share of Q sector in utility</td>
<td>World Bank (2005)</td>
</tr>
<tr>
<td>$m$</td>
<td>1.27</td>
<td>elasticity of matching function</td>
<td>den Haan, Ramey and Ramey (2000)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>4</td>
<td>curvature of labor adjustment</td>
<td>Yashiv (2000)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>bargaining power</td>
<td>-</td>
</tr>
<tr>
<td>$l_e$</td>
<td>1</td>
<td>employment at the entering firm</td>
<td>-</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0</td>
<td>exogenous worker separation rate</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3 collects the parameters discussed thus far, and implies that six parameters remain to be determined: fixed cost of operation $c_f$, vacancy posting cost $c_h$, fixed cost of exporting $c_x$, the value of home production $b$, iceberg trade costs $\tau$, and the value of entry $c_e$. The value of entry is determined endogenously in the model to satisfy the free entry condition. The rest of parameters are calibrated to match an equal number of targets. Specifically, to pin down $c_f$, $c_h$, $c_x$, and $b$, we select four natural targets: firm exit rate, job turnover rate, the fraction of firms that export and the unemployment rate. The firm exit rate and the fraction of firms that exit are calculated from Colombian plant level data for the pre-liberalization period, 1978-91. The job turnover rate is calculated from Inter-American Development Bank Job Flows Data Set for 1978-1992 period. The unemployment rate is taken from Inter-American Development Bank (2004).

Finally, the estimates in Table 1 determines $D_H/(D_H + D_F)$, which is the ratio of domestic demand to world demand for differentiated goods. Assuming that all countries spend the same

\(^{15}\)Although $l_e = 1$, by the time firms begin production they will have adjusted their initial labor force, and their observed employment will reflect their initial productivity.
fraction of their income on differentiated goods, this ratio can be calculated as the ratio of Colombia’s GDP to the GDP of its trade partners.\footnote{We take the GDP numbers from Penn World Tables. We calculate GDP of Columbia’s trade partners as a trade weighted average of their GDPs. We take the trade data from Feenstra et al (2005) data set, available at http://cid.econ.ucdavis.edu/data/undata/undata.html} Our calculations show that this ratio is about 0.0617 so we choose $\tau$ to generate the same ratio. Table 4 shows the calibrated parameters while Table 5 reports the targeted versus simulated values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f$</td>
<td>0.54</td>
<td>fixed cost of operation</td>
</tr>
<tr>
<td>$c_h$</td>
<td>21.54</td>
<td>vacancy posting cost</td>
</tr>
<tr>
<td>$c_x$</td>
<td>0.19</td>
<td>fixed cost of exporting</td>
</tr>
<tr>
<td>$b$</td>
<td>0.1</td>
<td>value of household production</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.813</td>
<td>iceberg trade costs</td>
</tr>
<tr>
<td>$c_e$</td>
<td>2.29</td>
<td>entry cost</td>
</tr>
</tbody>
</table>

Table 5: Calibration Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit Rate</td>
<td>0.1098</td>
<td>0.1181</td>
</tr>
<tr>
<td>Job Turnover</td>
<td>0.1843</td>
<td>0.1816</td>
</tr>
<tr>
<td>Export Rate</td>
<td>0.1142</td>
<td>0.1165</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0859</td>
<td>0.0936</td>
</tr>
<tr>
<td>Relative Demand</td>
<td>0.0617</td>
<td>0.0617</td>
</tr>
</tbody>
</table>

5.4 Simulated effects of openness: small country case

We are now prepared to examine the effects of trade reforms in our calibrated model. To do so we reduce the iceberg trade costs from 1.813 to 1.65. This reduction generates a rise in the fraction of firms that export, from 12.2 percent to 21.2 percent, which is in line with Colombian liberalization experience.

Table 6 shows how key labor market statistics change with the trade reforms. Note first that trade liberalization increases job turnover by 1.9 percentage points, which is about half
of the increase in the data. Thus, while our model does not explain the entire 4.5 percentage point increase that Colombia experienced, it accounts for a significant fraction of the change. That is, increased sensitivity to productivity shocks among exporters appears to be a significant phenomenon.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \tau = 1.813 )</th>
<th>( \tau = 1.65 )</th>
<th>Difference (model)</th>
<th>Difference (data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Firms that Export</td>
<td>12.2</td>
<td>21.2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Job Turnover</td>
<td>18.16</td>
<td>20.07</td>
<td>1.91</td>
<td>4.52</td>
</tr>
<tr>
<td>Unemployment</td>
<td>9.21</td>
<td>9.34</td>
<td>0.13</td>
<td>-0.12</td>
</tr>
<tr>
<td>90-10 Wage Inequality</td>
<td>1.97</td>
<td>2.11</td>
<td>0.14</td>
<td>0.34</td>
</tr>
</tbody>
</table>

It is worth noting that our model filters all shocks to the prices of imported goods through a general price index, and thus does not allow idiosyncratic shocks to foreign suppliers to affect different Colombian firms differently. If we were to use a nested demand system in which different domestic producers compete with different foreign exporters, the effects of openness on job turnover would be magnified. And since all producers compete with imports (while only 10 to 20 percent export), the effects could be dramatic.

Note next that openness also causes more wage inequality in our model. This is not a skill-premium effect; rather it reflects the fact that exporters who experience positive productivity shocks enjoy relatively large rents when trade costs are low. Thus expanding exporters pay larger wage premiums to their workers than they would have in a more closed economy. The left-hand tail of the wage distribution is also stretched by openness, but for a more subtle reason. Large and productive firms get occasional negative shocks, so they shed labor. However, the option value of remaining in such a firm is higher in an open environment because when the firm experiences a positive shock it’s going to rehire and pay higher wages. So workers who stay accept lower wages. As with our turnover results, the model does not
explain the entire increase in the interquantile wage spread that we observe in the data; similar comments apply here concerning the understated role of import competition in our model.

Finally, observe that despite the higher job turnover and greater wage inequality that our model predicts, it does not suggest that aggregate unemployment rates should respond much to openness. If one equates self-employment in our model with unemployment in the official statistics, this is consonant with Colombian (and Latin American) experiences. On the other hand, if one equate self-employment with working in the informal sector, our model under-predicts responses to globalization in this respect.

Having solved for the stationary equilibrium in a relatively open and a relatively closed environment, it is interesting to ask how different types of workers fare under the different regimes. Figure 1 depicts the change in the value of being a worker at firms located throughout
(\(z, l\)) space, going from \(\tau = 1.65\) to \(\tau = 1.813\). Not surprisingly, workers fortunate enough to be employed by high productivity firms do substantially better in the relatively open environment. This is particularly true at the largest productive firms, which have the most rents to share with their workers.

6 Summary

In Latin America and elsewhere, globalization has been associated with less job security, more wage inequality, and more informality. We have formulated a dynamic structural model that explains this association as a consequence of heightened firm responsiveness to idiosyncratic productivity shocks, and we have shown that this mechanism alone could account for a substantial fraction of the heightened volatility and wage dispersion in Colombia.

Our paper makes several methodological contributions to the literature. First, although Rodrik (1997) postulated a linkage between labor markets and openness related to the one we model, previous attempts to quantify this phenomenon have been limited to simple tests for structural shifts in the demand for labor equation. Second, we have generalized the representation of labor markets developed by Bertola and Caballero (1994) to an open economy setting with fully articulated product markets, and we have developed a means to solve the model numerically. Finally, we have developed a means to characterize plant-level productivity processes that does not require us to observe a measure of physical output, and is robust with respect to simultaneity bias and selection bias.
Hiring Wages

In order to characterize wages in hiring firms, we first determine the total surplus for a the firm and a worker that are matched in state \((z, l)\). The surplus that the marginal worker generates for a firm is given by

\[
\Pi^{\text{firm}} = \frac{1}{1 + r} \left[ \frac{\partial \pi(z, l)}{\partial l} + \mathcal{I}^c(z, l) \int \mathcal{I}^h(z', l') \frac{\partial V(z', l')}{\partial l'} \frac{\partial V(z', l')}{\partial l} h(z'|z) dz' \right],
\]

where \(l' = (1 - \delta)l\). The surplus that a marginal worker generates consists of two parts: the current increase in the firm's profits, i.e., marginal revenue product net of wages, and being in state \((z, l')\) at the start of the next period. If the firm does not exit next period, i.e. if \(\mathcal{I}^c(z, l) = 1\), the marginal worker will have a positive value for the firm only if the firm decides to hire, i.e. \(\mathcal{I}^h(z', l') = 1\). As exit and firing are costless, if the firm exists or fires workers, his expected marginal value from its current marginal hire is zero.

Similarly, the surplus for the marginal worker who is matched by a hiring firm in state \((z, l)\) is

\[
\Pi^{\text{work}} = \frac{1}{1 + r} \left[ w_h(z, l) + J^e(z, l) \right] - \frac{b + J^o}{1 + r},
\]

where the worker enjoys \(w_h(z, l)\) in the current period, and starts next period in a firm with beginning of the period state \((z, l)\).

The worker and firm split the total surplus by Nash bargaining where the bargaining power of the worker is given by \(\beta\). Wages are thus determined by

\[
\beta \left[ \frac{\partial \pi(z, l)}{\partial l} + (1 - \delta)\mathcal{I}^c(z, l) \int \mathcal{I}^h(z', l') \frac{\partial V(z', l')}{\partial l'} h(z'|z) dz' \right] = (1 - \beta) \left[ w_h(z, l) + J^e(z, l) - (b + J^o) \right].
\]

(22)
Extending (16) to account for the exit decision of firms,

\[ J^c(z,l) = J^c(z,l) \left[ \delta J^u + (1 - \delta) \int z \tilde{J}^c[z',l'] h(z'|z)dz \right] + [1 - J^c(z,l)] J^u, \]

and noting that \( J^u = J^o \) in equilibrium, the right-hand side of (22) can be written as

\[ (1 - \beta)w_h(z,l) - (1 - \beta)U + (1 - \beta)(1 - \delta)I^c(z,l) \int (\tilde{J}^c[z',l'] - \frac{b + J^o}{1 + r}) h(z'|z)dz' \]

(23)

where

\[ U = I^c(z,l)(1 - \delta) \left[ b + r \left( \frac{b + J^o}{1 + r} \right) \right] - b \]

is the value of unemployment taking into account the exit decision of the firm and the probability of exogenous separation.\(^{17}\) The last term in (23), \( (1 - \delta)I^c(z,l) \int (\tilde{J}^c[z',l'] - \frac{b + J^o}{1 + r}) h(z'|z)dz' \), is the expected capital gain or loss next period for a worker employed in this firm. Correspondingly, the expected capital gain of employing the marginal worker to the firm is given by the term \( (1 - \delta)I^c(z,l) \int I^h(z',l') \frac{\partial V(z',l)}{\partial l} h(z'|z)dz' \). We assume that the firm and the worker split the expected surplus according to the same rule.\(^{18}\)

\[ (1 - \beta)(1 - \delta)I^c(z,l) \int (\tilde{J}^c[z',l'] - \frac{b + J^o}{1 + r}) h(z'|z)dz' \]

\[ = \beta(1 - \delta)I^c(z,l) \int I^h(z',l') \frac{\partial V(z',l')}{\partial l'} h(z'|z)dz'. \]

So the forward-looking terms cancel in (22), leaving

\[ \frac{\partial w_h(z,l)}{\partial l} \beta l + w_h(z,l) - \beta \frac{\partial r(z,l)}{\partial l} - (1 - \beta)U = 0, \]

\(^{17}\)Since hiring is effective within the same period, we cannot rule out the case where a firm hires and exits in the following period. For \( \delta = 0 \) and \( I^c(z,l) = 1 \), this is equal to \( r \left( \frac{b + J^o}{1 + r} \right) \), flow value of the outside option in a continuing firms.

\(^{18}\)Since the parties bargain every period, this ex-ante surplus sharing being renegotiation-proof is not an issue.
which is the same as Bertola and Garibaldi (2001)’s equation (10). Using

\[
\frac{\partial r(z,l)}{\partial l} = \alpha \sigma^{-1} D^{\alpha \sigma} z^{\alpha (\alpha - 1)} l^{\alpha (\alpha - 1)} ,
\]

the wage schedule for expanding/replenishing firms is given by

\[
w_h(z,l) = (1 - \beta)U + \Gamma(\alpha, \beta, \sigma)D^{\frac{1}{\alpha}} z^{\frac{\alpha - 1}{\sigma}} l^{-[\frac{\sigma}{\sigma + (1 - \alpha)}]},
\]

where \(\Gamma(\alpha, \beta, \sigma)\) is a function of the parameters of the problem

\[
\Gamma(\alpha, \beta, \sigma) = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)}.
\]

**Firing Wages**

To derive the firing wage schedule, we begin by writing the value of employment at a firing firm as

\[
J_f^e(z,l) = \frac{1}{1 + r} \left[ p_f(z,l)((1 + r)J^u) + (1 - p_f(z,l)) (w_f(z,l') + J^e(z,l')) \right],
\]

where \(l' = L(z,l)\). This expression reflects the possibility of losing one’s job, \(p_f(z,l)\), which we assume occurs at firing firms with probability

\[
p_f(z,l) = \frac{l - l'}{l}.
\]

It also reflects the fact that workers who are not fired are paid just enough to retain them.

Next we note that, since workers are indifferent between staying and leaving, which gives us

\[
(w_f(z,l') + J^e(z,l')) = (1 + r)J^u,
\]

and defines the wage schedule faced by firing firms as

\[
w_f(z,l') = (1 + r)J^u - J^e(z,l').
\]
Appendix 2: Steady State Equilibrium

In steady state equilibrium, the aggregate variables \{N, M, P, Q, I, L_S, u, \phi^f, \phi^w, \mu_{exit}\}, the value functions and associated policy functions \{V(z, l), L(z, l), T^h(z, l), T^c(z, l), J^o, J^u, J^s, J^e\}, the wage schedules \{w_h(z, l), w_f(z, l)\}, and end-of-the period and interim distributions \{f(z, l), \tilde{f}(z, l)\} satisfy the following conditions:

1. **Steady State Distributions:** Because of the transitions that occur within the period, we have to distinguish the distributions at relevant points in time. Let \(f(z, l)\) and \(\tilde{f}(z, l)\) be the stationary probability distributions over \((z, l)\) at the end and interim respectively. In equilibrium, these distributions reproduce themselves through the Markov processes on \(z\), the policy functions and the productivity draws upon entry. The interim distribution is defined as

\[
\tilde{f}(z, l) = \begin{cases} 
    h(z|z_{-1})f(z_{-1}, l_{-1}) & \text{if } l = (1 - \delta)l_{-1} \text{ and } T^c(z_{-1}, l_{-1}) = 1, \\
    0, & \text{otherwise.}
\end{cases}
\]

In turn, the end-of-the period distribution is

\[
f(z, l') = \begin{cases} 
    \tilde{f}(z, l) & \text{if } T^h(z, l)L(z, l) + \left[1 - T^h(z, l)\right]L(z, l) = l', \\
    0, & \text{otherwise.}
\end{cases}
\]

2. **Market Clearance in Sector \(S\):** Demand for the \(S\)-sector comes from two sources: consumers spend a \((1 - \gamma)\) fraction of aggregate income \(I\) on it, and firms demand it to pay their fixed operation costs, labor adjustment and entry costs. Define the fraction of hiring firms as \(\mu_h = \int_z \int_l T^h(z, l)\tilde{f}(z, l)dl dz\). Average labor adjustment cost is given by

\[
\bar{c} = \int_z \int_l C[l, L(z, l)]T^h(z, l)\frac{\tilde{f}(z, l)}{\mu_h}dl dz.
\]
Market clearance condition in this sector is

\[ L_S + b \cdot (uL_Q) = (1 - \gamma)I + N\bar{c} + Nc_f + Mc_e, \]

where \( L_S \) and \( L_Q \) are the size of the workforce in the two sectors, and \( u \) is the unemployment level within the \( Q \)-sector.

3. **Labor Market:** With a normalized measure of workers, the size of the workforce in the \( Q \)-sector is \( L_Q = 1 - L_S \). Total production employment in the differentiated good sector is given by

\[ E_Q = N \int_0^1 \int_l f(z, l) dldz = (1 - u)L_Q. \]

The measure of unemployed workers is then

\[ U = L_Q - E_Q = uL_Q. \]

The equilibrium condition for the labor market in the \( Q \)-sector is that flows out of employment equal the flows into employment. Every period, a fraction \( k \) of workers in that sector are laid off due to exits and downsizing. The equilibrium flow condition is

\[ uL_Q\phi^w = (1 - u)L_Qk, \]

which yields the usual Beveridge curve

\[ u = \frac{k}{k + \phi^w}. \]

Aggregate number of vacancies in this economy is

\[ V = N \int_0^1 \int_l v(z, l)I^h(z, l) \frac{\bar{f}(z, l)}{\mu_h} dldz. \]

which, together with \( U \), determines matching probabilities \( \phi^f(V, U) \) and \( \phi^w(V, U) \) that firms and workers take as given.
4. **Firm turnover:** In equilibrium, there is a positive mass of entry $M$ every period so that the free entry condition (9) holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,

$$\mu_{\text{exit}} = \int_z \int_l [1 - T_c(z, l)] f(z, l) dl dz,$$

and measure of exits equals that of entrants,

$$M = \mu_{\text{exit}} N.$$

5. **Income and Market Clearance for the $Q$-sector:** The composite good $Q$ and its price are given by:

$$P = \left( N \int_z \int_l p(z, l)^{1-\sigma} f(z, l) dl dz \right)^{\frac{1}{1-\sigma}},$$

and

$$Q = \left( N \int_z \int_l q(z, l) \frac{\sigma-1}{\sigma} f(z, l) dl dz \right)^{\frac{\sigma}{\sigma-1}}.$$  

Aggregating over the revenue functions (4) across producers, total revenues earned by the differentiated good sector is a fraction $\gamma$ of total income in the economy:

$$\gamma I = PQ = R.$$  

By Walras’ Law, market clearance in the labor market and the $S$-sector implies the clearance of the $Q$-sector. We show that by writing aggregate income in the closed economy:

$$I = L_S + b \cdot (uL_Q) + W_Q + \Pi,$$

where $L_S$ is employment (and income earned) in the $S$-sector and $uL_Qb$ is the income earned by the unemployed through home production. Let $\mathbb{T}^h(z, l)$ be an indicator function which equals one if a firm in state $(z, l)$ at the end of a period achieved this state.
by hiring in the interim. $\Pi$ is total profits net of entry, vacancy and firing costs,

$$
\Pi = N \int_z \int_l \left[ \tilde{\tau}^h(z, l) \{ r(z, l) - w_h(z, l) \cdot l \} + \left[ 1 - \tilde{\tau}^h(z, l) \right] \{ r(z, l) - w_f(z, l) \cdot l \} \right] f(z, l) dldz
$$

(25)

and $W_Q$ is the total wage bill in the $Q$-sector

$$
W_Q = N \int_z \int_l \left\{ \tilde{\tau}^h(z, l) w_h(z, l) \cdot l + \left[ 1 - \tilde{\tau}^h(z, l) \right] w_f(z, l) \cdot l \right\} f(z, l) dldz.
$$

(26)

Using (24), (25) and (26),

$$
\gamma I = N \int_z \int_l \left[ \tilde{\tau}^h(z, l) r(z, l) + \left[ 1 - \tilde{\tau}^h(z, l) \right] r(z, l) \right] f(z, l) dldz.
$$

Right-hand of that equation is total revenue $R$ earned by the differentiated good sector.

6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching a job in industrial sector.

$$
J^o = J^u = J^u.
$$

(27)
Appendix 3: Open Economy Equilibrium

Equilibrium conditions

The equilibrium definition in an open economy is similar to that in a closed economy with the addition of the export policy function $\pi_x(z, l)$, exchange rate $\varepsilon_x$, fraction of firms exporting $\mu_x$ and the trade balance condition. Here we show that when all markets clear, trade balance condition follows from Walras’ Law.

By definition of income as before,

\[
I = L_s + b \cdot (uL_Q) + W_Q + \Pi + T_m
\]

\[
I = L_s + b \cdot (uL_Q) + W_Q + R_x + R_d - W_Q - N\widetilde{c} - Nc_f - Mc_e + T_m
\]

where $N\widetilde{c}, Nc_f$ and $Mc_e$ are aggregate hiring costs, overhead and entry costs respectively.

Market clearance for $S$ sector is

\[
L_s + b \cdot (uL_Q) = (1 - \gamma) I + N\widetilde{c} + Nc_f + Mc_e
\]

which implies

\[
\gamma I = R_x + R_d + T_m.
\]

On the expenditure side, a fraction $\gamma$ of income is spent on differentiated goods, foreign and domestic.

\[
\gamma I = E_m + E_d.
\]

By domestic market clearance, $E_d = E_m$ which implies

\[
R_x + T_m = E_m.
\]
Payment to foreigners is given by \( R_m = E_m/(1 + \tau_m) \). Substituting \( E_m \) and cancelling tariff revenues \( T_m = R_m\tau_m \) leaves us with the trade balance condition:

\[
R_m = R_x.
\]

**Price and quantity indices**

To re-state the price and quantity indices for the open economy case, let \( N \) now denote total varieties sold in each country, let \( N_D \) denote the number of varieties that each country produces domestically, and let \( \mu_x \) be the equilibrium fraction of firms in each country that export. (By symmetry, \( N = (1 + \mu_x)N_D \)). Then, the composite good \( Q \) is the following weighted average of foreign and domestic quantities:

\[
Q = N \frac{-1}{\sigma} \left( \frac{\mu_x}{1 + \mu_x} Q_F^{\frac{\sigma-1}{\sigma}} + \frac{1}{1 + \mu_x} Q_D^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},
\]

where

\[
Q_F = \left( \int_z^{\infty} \int_{l_e}^{\infty} \frac{\eta^*}{\tau} \left[ z^{l^\alpha} \right]^{\frac{\sigma-1}{\sigma}} \mathcal{I}_x(z, l) \frac{f(z, l)}{\mu_x} dldz \right)^{\frac{\sigma}{\sigma-1}},
\]

and

\[
Q_D = \left( \mu_x Q_{D,x}^{\frac{\sigma-1}{\sigma}} + (1 - \mu_x)Q_{D,nx}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.
\]

Also the domestic index \( Q_D \) is itself an aggregate across domestic sales of exporters given by \( Q_{D,x} \) and sales of domestic only producers represented by \( Q_{D,nx} \):

\[
Q_{D,x} = \left( \int_z^{\infty} \int_{l_e}^{\infty} [(1 - \eta^*)z^{l^\alpha}]^{\frac{\sigma-1}{\sigma}} \mathcal{I}_x(z, l) \frac{f(z, l)}{\mu_x} dldz \right)^{\frac{\sigma}{\sigma-1}},
\]

and

\[
Q_{D,nx} = \left( \int_z^{\infty} \int_{l_e}^{\infty} (z^{l^\alpha})^{\frac{\sigma-1}{\sigma}} [1 - \mathcal{I}_x(z, l)] \frac{f(z, l)}{1 - \mu_x} dldz \right)^{\frac{\sigma}{\sigma-1}}.
\]
The price index for $Q$ follows from a similar aggregation. The export indicator function $T^x(z, l)$ and the fraction of exporting firms $\mu_x$ are additions to the equilibrium definition in the open economy case. All conditions in the closed economy equilibrium are valid with the additional demand for the homogenous good resulting from fixed exporting costs $\mu_x N_D f_x$ and the modified aggregate profit function to account for export revenues and costs.
Appendix 4: Estimating the Revenue Function and the Productivity Process

The Revenue Function

From (21), the equation we wish to estimate is:

\[
\ln r_{it} = \ln r_{it-1} + (d_H + T^x_{it} \cdot d_F) - \rho (d_H + T^x_{it-1} \cdot d_F) + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it} - \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it-1} + \left( \frac{\sigma - 1}{\sigma} \right) \epsilon_{it},
\]

(A3.1)

Selection bias and simultaneity bias prevent us from consistently estimating this expression with ordinary least squares. The former problem occurs because firms choose whether to exit the market partly on the basis of their \( \epsilon_{it} \) realizations, and the latter problem occurs because firms’ current exporting decisions \( (T^x_{it}) \) and employment levels \( (l_{it}) \) depend upon their current productivity levels.

Selection Bias and Identification

To deal with these problems, let \( \chi_{it} \) be an indicator variable that takes a value of 1 if the \( i^{th} \) firm continues to operate in period \( t \), and 0 otherwise. Then, defining \( \xi_{it} = \epsilon_{it} - E \left[ \epsilon_{it} | \chi_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T^x_{it-1} \right] \), the revenue function can be re-formulated as:

\[
\ln r_{it} = \rho \ln r_{it-1} + d_H (1 - \rho) + d_F (I^X_{it} - \rho \cdot I^X_{it-1}) + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln \ell_{it} - \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln \ell_{it-1} + \left( \frac{\sigma - 1}{\sigma} \right) \xi_{it}.
\]

(A3.2)

The error term \( \xi_{it} \) has zero mean and is orthogonal to \( \ln r_{it-1}, \ln l_{it-1}, T^x_{it-1}, \) and \( E[\epsilon_{it} | \chi_{it} = 1, ...] \).

Further, although it is correlated with current exporting decisions \( (I^X_{it}) \), it is orthogonal to
$E \left[ T_{it}^x | \chi_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x \right]$. Hence, if $E \left[ \epsilon_{it} | \chi_{it} = 1, \ldots \right]$ and $E \left[ T_{it}^x | \chi_{it} = 1, \ldots \right]$ can be calculated, the orthogonality conditions $E(\xi_{it} \cdot r_{it-1}) = 0$, $E(\xi_{it} \cdot \ln \ell_{it-1}) = 0$, $E(\xi_{it} \cdot T_{it-1}^x) = 0$, $E(\xi_{it} \cdot E(\varepsilon_{it} | \chi_{it} = 1, \ldots)) = 0$, and $E(\xi_{it} \cdot E[T_{it}^x | \chi_{it} = 1, \ldots]) = 0$ provide the basis for a generalized method of moments (GMM) estimator that identifies the parameters of equation (A3.1). Further, the efficiency of this estimator can be improved by exploiting information on the ratio of exports to total sales—hereafter, $x_{it}$. Our model implies that this variable is related to the foreign demand shifter $d_F$ by $T_{it}^x(1 - e^{-d_F}) = x_{it}$, so treating $x_{it}$ as a noisy measure of true export intensity, the moment condition $E(T_{it}^x(1 - e^{-d_F}) - x_{it}) = 0$ can be incorporated into the analysis.

To implement this estimation strategy we need a way to calculate $E \left[ \epsilon_{it} | \chi_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x \right]$ and $E \left[ T_{it}^x | \chi_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x \right]$. Consider the former expression first. There is a threshold productivity level above which all firms with beginning-of-period employment level $\ell_{it-1}$ will continue operating. Denote this productivity level $g^*(\ell_{it-1})$, so that the continuation condition is $z_{it} = \rho z_{it-1} + \epsilon_{it} > g^*(\ell_{it-1})$. By (19), $z_{it-1} = \frac{\sigma}{\sigma - 1} \left[ \ln r_{it-1} - (d_H + T_{it-1}^x d_F) \right] - \alpha \ln l_{it-1}$, so continuation occurs when $\frac{\epsilon_{it}}{\sigma} > g(r_{it-1}, l_{it-1}, T_{it-1}^x)$, where

$$g(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x) = \left( g^*(\ell_{it-1}) - \frac{\sigma}{\sigma - 1} \left[ \ln r_{it-1} - (d_H + T_{it-1}^x d_F) \right] - \alpha \ln l_{it-1} \right) / \sigma_{\epsilon}. $$

Accordingly, letting $\epsilon_{it} \sim N(0, \sigma_{\epsilon}^2)$, it follows that the probability of continuation can be calculated as

$$p_{it} = 1 - \Phi \left[ g(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x) \right], \tag{A3.3}$$

---

Note that the dependence of $\ln \ell_{it}$ on $\epsilon_{it}$ does not prevent us from obtaining consistent estimates of these parameters because the coefficient on $\ln \ell_{it}$ can be inferred from the coefficients on $\ln \ell_{it-1}$ and $\ln r_{it-1}$.
where $\Phi()$ is the standard normal cumulative distribution. And once $p_{it}$ has been estimated with the probit function (A3.3), the object of interest can be calculated using well-known properties of the normal distribution (e.g., Maddala, 1983):\(^{20}\)

$$E[\epsilon_{it}|X_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x] = \sigma_\epsilon \cdot M_{it}$$
$$\text{var}[\epsilon_{it}|X_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x] = \sigma_\epsilon^2 \cdot (1 - M_{it} [M_{it} - \Phi^{-1}(p_{it})])$$

Here $M_{it} = \frac{\phi(\Phi^{-1}(p_{it}))}{p_{it}}$ is the relevant Mills ratio and $\phi() = \Phi'(\cdot)$.

It remains to discuss the endogeneity of exporting status, $I^X_{it}$. Since the probability of exporting in the current period depends upon $(z_{it-1}, \ell_{it-1})$, we can calculate:

$$E[T^X_{it}|X_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x] = p^X_{it} = 1 - \Phi [h(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x)] \ , \quad \text{(A3.4)}$$

where $h(r_{it-1}, l_{it-1}, T_{it-1}^x)$, like $g(\cdot)$, is a flexible function of its arguments.\(^{21}\) Thus to deal with the simultaneity between $I^X_{it}$ and $\epsilon_{it}$, we can use $p^X_{it}$ as an instrument for $I^X_{it}$.

Although $g(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x)$ and $h(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x)$ could, in principle, be estimated at the same time as the structural parameters that appear in (A1.1), doing so makes for a poorly-behaved objective function. We therefore obtain estimates of these probabilities by fitting the profit functions (A3.3) and (A3.4) in a preliminary stage; then we use the resulting $(p_{it}, p^X_{it})$ as input to the GMM estimator described below.\(^{22}\)

\(^{20}\)When estimating this probit, we use a flexible (translog) functional form for $g(r_{it-1}, l_{it-1}, T_{it-1}^x)$.

\(^{21}\)It is interesting that lagged exports help predict current exports here, even though we have assumed away sunk entry costs. The reason is that, (19), lagged exports help to explain lagged productivity.

\(^{22}\)Olley and Pakes (1996) pursue a similar strategy.
The Moment Conditions

We are now prepared to summarize our estimation strategy. Define the following three error terms:

\[
\begin{align*}
\xi_{it} &= \frac{\sigma}{\sigma - 1} \left[ \ln r_{it} - d_H (1 - \rho) - d_F (T_{it}^x - \rho T_{it-1}^x) - \rho \ln r_{it-1} \right] + \alpha \rho \ln \ell_{it-1} - \alpha \ln \ell_{it} - \sigma_\varepsilon \cdot M_{it} \\
\nu_{it}^\varepsilon &= \xi_{it}^2 - \sigma_\varepsilon^2 \cdot (1 - M_{it} \left[M_{it} - \Phi^{-1}(p_{it})\right]) \\
\nu_{it}^\tau &= T_{it}^\tau (1 - e^{-d_X}) - x_{it}
\end{align*}
\]

where \( x_{it} \) is the share of total revenues that firm \( i \) generated through exports in period \( t \).

Letting \( \ell \) be a vector ones, Our GMM estimator is based on the moment conditions:

\[
\begin{align*}
E[\xi_{it} \ln r_{it-1}] &= 0, \ E[\xi_{it} \ln \ell_{it-1}] = 0, \ E[\xi_{it} M_{it}] = 0, \ E[\xi_{it} T_{it-1}^x] = 0, \\
E[\xi_{it} P_{it}^X] &= 0, \ E[\xi_{it}] = 0, \ E[\nu_{it}^\varepsilon] = 0, \ E[\nu_{it}^\tau] = 0.
\end{align*}
\]

In principle, these conditions identify \( \rho, \alpha, \sigma_\varepsilon^2, d_X, d_H, \frac{\sigma-1}{\sigma} \). In practice, while \( \rho, \alpha, \sigma_\varepsilon^2, d_X \), and \( d_H \) can be estimated with some precision using this estimator, \( \frac{\sigma-1}{\sigma} \) is poorly identified. We therefore fix \( \frac{\sigma-1}{\sigma} \) at several alternative values taken from the existing literature, and generate corresponding sets of estimates for the remaining parameters. (Refer to Table 1 in the text.)

Our results proved not to be sensitive to the inclusion of time dummies in A1.1. Accordingly, since our theoretical model presumes that the macro environment is stable, we focus our attention on the case in which they are omitted.
Appendix 5: Numerical Solution Algorithm

[TO BE COMPLETED]
References


