Estimating Labor Market Rigidities with Heterogeneous Firms

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Abstract

A large literature in firm dynamics estimates models of labor adjustment costs. Most studies assume that wages are set competitively, an important assumption since wage flexibility reduces the incentive to adjust the workforce. I relax this assumption and introduce wage bargainning with multiple workers. A reduced-form empirical decomposition suggests that transitory shocks to sales have a strong effect on wages, but that permanent shocks have a very small effect on wages. I propose and estimate a structural model of firm behavior that can account for these dynamics: firms adjust relatively less the workforce in response to temporary shocks, leading to higher wages since the labor productivity goes up. The model is estimated using French panel data, through a new indirect inference procedure which allows for heterogeneity in firm parameters. The model fits the data well. Only a small amount of adjustment costs is needed to reproduce observed job reallocation and inaction rates. Permanent shocks are the most important source of output fluctuations. Ignoring measurement errors or wage flexibility lead to erroneous conclusions. The model with heterogeneous coefficients greatly improves the fit of the observed dispersion of labor productivity compared to a model with homogeneous coefficients.

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1 Introduction

The analysis of data on individual firms reveals enormous amount of heterogeneity in firm-level productivity and the importance of idiosyncratic shocks. Davis and Haltiwanger (1999) document that while over 10% of existing jobs are destroyed each year, approximately the same amount is created within the same year. More recently, Hsieh and Klenow (2009) find that equalizing the marginal productivity of labor and capital across plants would boost aggregate total factor productivity by around $30-40\%$ in the U.S. manufacturing sector and even more in China or India. These results suggest we should study in more details the impediments to reallocation of resources from high to low productivity firms.[1] The main impediments are adjustment costs. These costs may be technological, e.g. reduced efficiency during the period of adjustment, or they may be institutional, such as employment protection legislation.

While there is a substantial literature studying labor adjustment costs, most of this literature assumes that wages are not linked to idiosyncratic shocks. This is a critical assumption, since as noted by Bertola and Rogerson (1997) wage flexibility can reduce the incentives to adjust the workforce. Wages are only partially flexible because of regulations such as the minimum wage, because of union bargaining power and because firms provide some insurance to their workers.[2]

Besides the role of the relative flexibility of employment and wages, firms face both permanent and transitory shocks to business conditions and may well adopt different strategies depending on the persistence of the shocks.

As a first step, I conduct a reduced-form analysis using panel data from France. I obtain a new and surprising finding: transitory shocks to output have a strong effect on wages while permanent shocks have a very small effect on wages. To explain these patterns, I introduce and estimate a model which combines imperfect wage flexibility, standard labor adjustment costs, and permanent and transitory shocks. Wages are determined through Nash bargainning with multiple workers (modeled as in Stole and Zwiebel (1996)).

[1] This is related to the process of creative destruction. This concept attributed to Schumpeter (1942) refers to the incessant product and process innovation mechanism by which new production units replace outdated ones.

[2] Some degree of risk sharing is certainly efficient but there are many rationales for partial insurance and not full insurance. Notably, financial constraints prevent firms from fully diversifying their risks. Unobservability of effort is another obstacle (see Holmstrom and Milgrom (1987)).

The intuition for the observed dynamics is the following. A transitory shock only changes today's profits while a permanent shock changes both today's profits and expected future profits. The benefits of creating or destroying jobs following a transitory shock are on average small. Because of adjustment costs, the firm then decides not to create or destroy many jobs. As a result, wages change in reaction to the change in labor productivity. Permanent shocks lead to larger employment changes. Since there are decreasing returns to labor, the variations of labor productivity and thus the variations of wages are smaller for a permanent shock compared to a transitory shock of the same magnitude.

To assess this explanation quantitatively, I estimate the structural parameters, i.e. labor adjustments costs, worker bargaining power and the sources of dispersion of observed variables. Adjustment costs are used in many fields of economics to explain a wide range of facts.[3] In these models, the calibration procedure uses some measure of dismissal costs to assign values to adjustment costs parameters.[4] However, this practice is not entirely satisfactory because adjustment costs have an implicit component that is intrinsically difficult to measure (Hamermesh and Pfann (1996)) and because in most countries there exists many regulations[5] that can hardly be summarized by dismissal costs. And indeed the estimation results points out that relatively modest adjustment costs (less than a month of wages) can reproduce the data well which conflicts with the perceived sclerosis of the French labor market.

The estimated model allows for many sources of dispersion of observed variables, namely, permanent shocks, transitory shocks, measurement errors, adjustment costs and firm-specific coefficients. Allowing for firm-specific coefficients is a technical contribution of this paper which develop an estimation framework that allows firms to be heterogeneous ex-ante and ex-post. Heterogeneity ex-post comes from idiosyncratic shocks. Heterogeneity ex-ante means firm-specific coefficients. This is natural because in reality, different firms have different degrees of return to scale, face different levels of competition and pay different wages. To be tractable, I use a finite number of types. I consider

[3] A non-exhaustive list includes the behavior of gross job flows and aggregate employment over the business cycle (Campbell and Fisher (2000), Veracierto (2008)), the impact of firing costs on productivity and employment (Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993)), the micro-foundations of aggregate employment adjustment (King and Thomas (2006)).

[4] Measures of dismissal costs are reported for example in Heckman et al. (2000).

[5] See Appendix A for a concise description of labor market institutions in France.

an indirect inference estimator where unobserved heterogeneity is introduced in the auxiliary model via finite mixture modelling.

A key motivation for allowing for many sources of heterogeneity is the observed large dispersion in the average product of labor. In my dataset, the variance of the log productivity of labor is equal to 0.16.[6] Also, the between-firms differences account for three and a half as much dispersion as within-firms differences. I find that the estimated model with homogeneous coefficients can fit relatively well the within-firms variance of labor productivity, output and wages. However it cannot account for the between-firms variance in labor productivity. I then show that introducing 3-types of firms each using a different technology improves the capacity of the model to fit the between-firms variance in labor productivity.

Connections with Existing Literature The estimation of a structural model of labor demand dynamics links with the work of Cooper et al. (2005), Rota (2004) and Aguirregabiria and Alonso-Borrego (2009). Cooper et al. (2005) conclude that the quadratic adjustment cost model is unable to generate the observed negative correlation of hours and employment growth at the plant level while non-convex adjustment costs do. Rota (2004) estimates a labor demand model with fixed costs using Hotz and Miller (1993)'s estimator. Aguirregabiria and Alonso-Borrego (2009) focus on the impact of a reform of fixed-term contract. Some empirical papers investigate the relationship between wages and profits[7] but none of them consider this question simultaneously with that of employment flexibility.

Most models of firm dynamics so far have neglected the distinction between less and more persistent variations in firm productivity and it is customary to assume that firm productivity follows a stationary autoregressive process of order one. Despite the presence of some descriptive studies[8], with the exception of Gourio (2008) who focuses on investment, there are no other structural models. This contrasts with the various dynamic models for individual workers that have been proposed in the earnings dynamics literature.[9]

[6] This number is smaller than the variance in the entire economy because the data-set does not include entry and exit and because I focus on the manufacturing sector and firms with a size of between 100 and 500 employees.

[7] See Guiso et al. (2005) for example.

[8] For example, Franco and Philippon (2007) document the importance of permanent shocks for firm dynamics.

[9] See McCurdy (1982); Abowd and Card (1989); Meghir and Pistaferri (2004); Blundell et al. (2008).

The wage setting mechanism is borrowed from Stole and Zwiebel (1996), which generalizes the Nash bargainning solution to a setting with downward-sloping labor demand. This has recently been used in a similar context by Elsby and Michaels (2008) and Fujita and Nakajima (2009). However, these papers focus on aggregate shocks and the unemployment-volatility puzzle. I focus on the impact of idiosyncratic shocks and productivity dispersion and estimate the model, while they use calibration.

Finally, this paper relates to the literature on productivity dispersion. Bartelsman et al. (2006) report that the variance of labor productivity ranges from 0.3 to 1 across countries. Midrigan and Xu (2009) calibrate a model of firm dynamics with capital adjustment costs, financing frictions and uninsurable investment risk. They conclude that none of the aformentionned features can explain the dispersion of firm productivity. This paper shows that the introduction of firm-specific coefficients improves the capacity of the model to fit the between-firm variance of log labor productivity.

Organization of the Paper The remaining of the paper proceeds as follows. Section 2 conducts a reduced-form empirical investigation. Section 3 develops the theoretical framework. Section 4 solves the model numerically and examine the impact of transitory and permanent shocks. Section 5 provides sufficient conditions for parametric identification of the model. Section 6 develops the methodology for the structural estimation of the parameters. Section 7 presents the results of the structural estimation. Finally, Section 8 concludes.

2 Data

The data used is the BRN (Real Normal Profits). The BRN declarations are completed annually by firms with a turnover of more than 3.5 million francs (1992 threshold) liable for income tax in respect of BIC (Industrial and Commercial Profits). The BIC correspond to the profits declared by firms whose commercial, industrial or craft-work activity is carried out for lucrative purposes (60 % of the firms, 94 % of the turnover). The data cover the period 1994-2000. I focus on the manufacturing sector.

I drop firms with a level of employment below 100 employees. This is to avoid the effect of size-

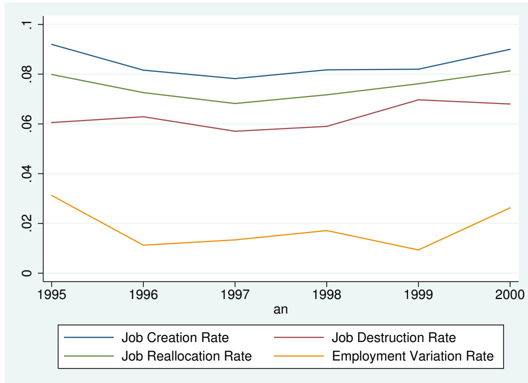

Figure 1: Job Turnover Statistics

dependent policies (especially important when the size of 10 and 50 employees are crossed) which result in threshold effects in labor demand. The empirical strategy cannot accomodate these effects and I leave this for future research. I trimmed all variables at the .99-quantile and the .01-quantile.

Average labor compensation per firm is calculated as the ratio of the total real wage bill and the average number of employees over the year in full-time equivalent.

2.1 Labor Adjustment

Figure 1 plots aggregate yearly rates of job turnover which are computed according to standard definitions (see Davis and Haltiwanger (1999)). As usually found in most industrial countries, aggregate job flows are large within the business cycle. Net employment growth is always much smaller than job creation or destruction.

Figure 2 shows the range of variation of production and employment. Throughout the period, on average, the rate of change in employment was zero or close to zero for about 15% of the firms. Hence employment growth rates display high spikes around zero, compared to the smooth patterns of sales variations observed. It reveals a considerable stickiness in employment.

Two facts stand out from the distribution of job reallocation: 1. there is a significant amount of relatively small net employment adjustment and 2. these small adjustments are complemented by significant bursts of job creation and destruction: almost 30% of firms either contract or expand employment by more than 10% in a given year.

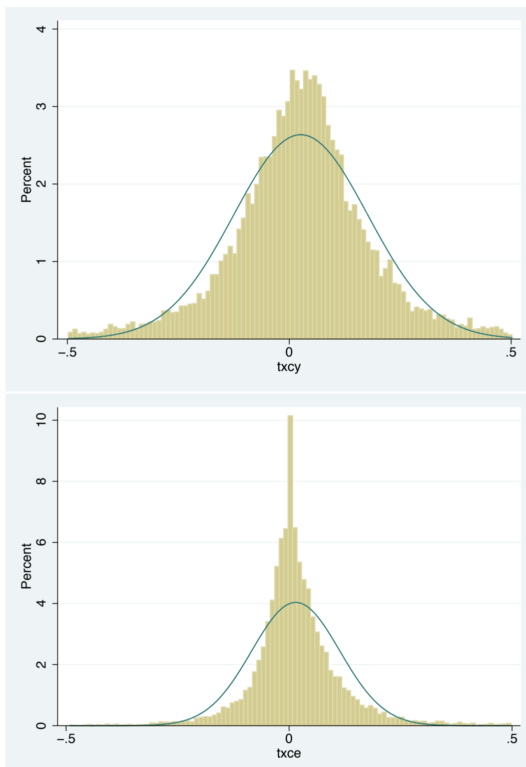

Figure 2: Output and Employment Variation

2.2 Modelling Shocks: Univariate Series

This section investigates the presence of permanent shocks, measurement errors and transitory shocks for output, employment and wages.

Suppose the logarithm of log value added, employment or wages , say $\log Y$, can be decomposed into a permanent component P and mean-reverting transitory component ν. The process for each firm i is

$$\log Y_{it} = Z_{it}\varphi_t + P_{it} + \nu_{it} \tag{1}$$

where t indexes time and Z is a set of characteristics observable. I allow for a calendar year effect.

The permanent P_{it} follows a martingale process of the form

$$P_{it} = P_{it-1} + \zeta_{it} \tag{2}$$

where ζ_{it} is serially uncorrelated, and the transitory component ν_{it}, follows an $MA(q)$ process, where the order q is to be established empirically:

$$\nu_{it} = \sum_{j=0}^{q} \theta_j \epsilon_{it-j} \tag{3}$$

with $\theta_0 = 1$. It follows that (unexplained) growth is

$$\Delta y_{it} = \zeta_{it} + \Delta\nu_{it} \tag{4}$$

where $y_{it} = \log Y_{it} - Z_{it}\varphi_t$ denotes the log of the variable net of predictable individual components.

Assume that ζ_{it} and ν_{it} are uncorrelated at all leads and lags.

Assume stationarity. The parameters to estimate are $\sigma_\zeta^2, \sigma_\epsilon^2, q, \theta_1, \cdots, \theta_q$.

If ν is an $MA(q)$ process, $cov(\Delta\nu_t, \Delta\nu_{t+s})$ is zero whenever $s \geq q+1$. Then those covariances identifies $q, \theta_1, \cdots, \theta_q$ and σ_ϵ^2.

Variables	Order	Estimate	p-value
Output	0	0.0414	0
	1	-0.0047	0
	2	-0.0009	.1223
Wage	0	0.0120	0
	1	-0.0047	0
	2	-0.0003	0.6411

Table 1: Test of the Hypothesis of Zero Autocovariances

The key moment condition that identifies the variance of the permanent shock is:

$$E\left[\Delta y_t \left(\sum_{j=-(1+q)}^{(1+q)} \Delta y_{t+j}\right)\right] = \sigma_\zeta^2 \tag{5}$$

It has been derived for the first time by Meghir and Pistaferri (2004). This exploits the structure of the MA process to cancel terms out. Identification of σ_ζ^2 rests on the idea that the variance of the variable growth coincides with the variance of the innovation to the permanent component, after removing the contribution of the mean revering component.

To describe the structure of transitory shocks, I estimate the autocovariances of Δy_{it} using standard methods (Abowd and Card (1989)). The test statistic equals squared-autocovariance divided by their respective variance. It is distrbuted as a Chi-Square with a degree of freedom equal to the number of time periods available for estimation. Table 1 reports the estimated autocovariances up to order two along with the test of zero restrictions for the null hypothesis that $cov(\Delta y_t, \Delta \nu_{t+s}) = 0$ with $1 \leq s \leq 2$.

Residuals wages and outputs growth rates appear only correlated up to the first order. Autocovariances at the second order and beyond are small and statistically insignificant or of boderline significance. The statistical implication is that $q = 0$. Then for both wage and output the transitory component follows a $MA(0)$: it is i.i.d.

To test for the absence of permanent shocks, the test statistic is equal to the pooled estimate of the variance of the permanent shock divided by its standard error. It is asymptotically (for large N) distributed standard normal. The standard error is computed using the block bootstrap procedure (see Hall and Horrowitz (1996)). In this way I account for serial correlation of arbitrary form,

	Output	Wage
Pooled Estimate	0.0294	0.0015
	(0.0010)	(0.0002)
Variance Decomposition		
Permanent	75.77%	13.76%
Transitory	24.23%	86.24%

Table 2: Permanent Shock

heteroskedasticity, as well as for the use of pre-estimated residuals.

The results are reported in Table 2. The variance of the permanent shock is estimated to be .0294 (with a bootstrap standard error of .0010) for output, and .0015 (with a bootstrap standard error of .0002) for wages. The hypothesis of no permanent shock is strongly rejected in both case.

2.3 Joint Dynamics of Output and Wages

Building on the previous section (unexplained) growth of output is

$$\Delta y_{it} = \zeta_{it}^y + \Delta\nu_{it}^y + \Delta r_{it}^y \tag{6}$$

Assume (unexplained) growth of wage is

$$\Delta w_{it} = \tau_1\zeta_{it}^y + \tau_2\Delta\zeta_{it}^y + \phi\Delta\nu_{it}^y + \Delta r_{it}^w + \zeta_{it}^w \tag{7}$$

where ζ_{it}^w is a permanent wage shock independent of output, r_{it}^w is wage measurement error, permanent productivity shocks ζ_{it}^y have a permanent (transitory) impact on wage with a loading factor of $\tau_1(\tau_2)$, transitory productivity shocks ν_{it}^y have an impact on wage with a loading factor of $\phi \in [0,1]$. Theoretical Moments are derived in Appendix B.

It is possible to point identify $\sigma^2_{\zeta^y}, \sigma^2_{\zeta^w}, \tau_1$ and τ_2:

$$
\begin{aligned}
\sigma^2_{\zeta^y} &= E\left[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right] \\
\tau_1 &= \frac{E\left[\Delta w_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right]}{E\left[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right]} \\
\tau_2 &= \frac{E(\Delta w_t \Delta y_{t+1}) - E(\Delta w_{t+1} \Delta y_t)}{E\left[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right]} \\
\sigma^2_{\zeta^w} &= E\left[\Delta w_t(\Delta w_{t-1} + \Delta w_t + \Delta w_{t+1})\right] - \tau_2^2 \sigma^2_{\zeta^y}
\end{aligned}
$$

It is not possible to separately identify measurement error in output from transitory shock to ouput. Only the sum is identified:

$$
\sigma^2_{\nu^y} + \sigma^2_{r^y} = -E(\Delta y_t \Delta y_{t+1})
$$

ϕ is not point-identified. A lower bound is:

$$
\begin{aligned}
\phi &\geq \frac{E(\Delta w_t \Delta y_{t+1})}{E(\Delta y_t \Delta y_{t+1})} \\
&= \phi \frac{\sigma^2_{\nu^y}}{\sigma^2_{\nu^y} + \sigma^2_{r^y}}
\end{aligned}
$$

Table 3 displays the pooled estimate and the equally weighted minimum distance (EWMD) estimate. It also displays the variance decomposition of wage growth. The estimate of τ_1 and τ_2 are economically small (0.0076 and 0.0155) and not statistically significant. The hypothesis of insensitivity of wages to permanent output shock cannot be rejected. Conversely, wages strongly respond to transitory shock to value added. Because of the presence of measurement error, I can only provide a lower bound at that stage. But the effect is larger than 57%.

To check the robustness of the result, I perform a similar decomposition using labor productivity instead of output. Table 4 reports the results.

The impact of transitory shock to productivity on wage is estimated to be larger than 0.4290.

	Estimate		
	τ_1	τ_2	ϕ (lower bound)
Pooled	0.0076	0.0155	0.5708
	(0.0101)	(0.0105)	(0.0713)
EWMD	0.0073	0.0161	
	(0.0106)	(0.0108)	

	Variance Decomposition		
	$\tau_1^2\sigma_{\zeta^y}^2 + 2\tau_2^2\sigma_{\zeta^y}^2$	$\phi^2\sigma_{\nu^y}^2 + \sigma_{r^w}^2$	$\sigma_{\zeta^w}^2$
	0.15%	86.38%	13.47%

Table 3: Joint Dynamics of Wage and Output

	Estimate		
	τ_1	τ_2	ϕ (lower bound)
Pooled	0.0975	0.0227	0.4290
	(0.0147)	(0.0141)	(0.0288)
EWMD	0.0967	0.0228	
	(0.0146)	(0.0141)	

	Variance Decomposition		
	$\tau_1^2\sigma_{\zeta^y}^2 + 2\tau_2^2\sigma_{\zeta^y}^2$	$\phi^2\sigma_{\nu^y}^2 + \sigma_{r^w}^2$	$\sigma_{\zeta^w}^2$
	1.5%	85.2%	13.3%

Table 4: Joint Dynamics of Wage and Labor Productivity

	Data
Pooled Variance of Log Productivity	0.1670
$\frac{90th-Productivity}{10th-Productivity}$	3.2529
Within-Time Variance of Log Productivity	0.0008
Between Firm Variance of Log Productivity	0.1291
Within-Firm Variance of Log Productivity	0.0372

Table 5: Dispersion of Labor Productivity

Regarding permanent shocks, the coefficients are now significant and have a larger economic level. Yet, the transmission of permanent output shock to wage only explains 1.5% of the variance of wage growth.

2.4 Firm Heterogeneity

Table 5 documents the dispersion in labor productivity. When I pool all firm-year observations, the variance of the average product of labor is equal to 0.16. The ratio of the labor productivity of the 90th centile producer to the 10th centile producer is about 2.45. Hence the 90th centile producer is more than twice more productive that the 10th centile producer.

I next decompose the dispersion in the log of average product of labor into temporal variation, between-firm variation (that remain constant over time) within-firm variation (that vary over time and units) effects. I consider a simple variance components model of the form:

$$\log p_{it} = \mu + \varphi_t + \eta_i + \nu_{it}$$

where μ is an intercept, $\varphi_t \sim I.I.D(0, \sigma_\varphi^2), \eta_i \sim I.I.D(0, \sigma_\eta^2), \nu_{it} \sim I.I.D(0, \sigma_\eta^2)$, and φ_t, η_i and ν_{it} are independent of each other. The total variance of $\log p_{it}$ is given by $\sigma_\varphi^2 + \sigma_\eta^2 + \sigma_\nu^2$. σ_φ^2 corresponds to differences that remain constant over unit (aggregate shocks), σ_η^2 corresponds to differences that remain constant over time (permanent differences between firms), σ_ν^2 are differences that vary randomly over time and units (residual variance).

The between-firm variance is estimated to be 0.1291, the within-firm variance is estimated to be 0.0372. Between firm differences in log labor productivity account for three and half as much dispersion as within firm differences. The aggregate variation in labor productivity is negligible.

3 A Model of Labor Demand

The simplest (and widely used) dynamic model of labor demand[10] is a so-called partial adjustment model[11] in which the firm adjust the level of employment (say n_t) to a target (say n_t^*). Formally,

$$\log(n_t) - \log(n_{t-1}) = \lambda(\log(n_t^*) - \log(n_{t-1})) \tag{8}$$

The change in employment is proportional to the difference between the previous level of employment and a target where λ parameterizes how quickly the gap is closed. Hence employment adjusts smoothly and continuously to shocks. The popularity of the quadratic adjustment cost structure reflects it tractability but conflicts with evidence of inactivity and bursts at the firm level. In a seminal paper, Hamermesh (1989) examines monthly data on output and employment between 1983 and 1987 across seven manufacturing plants. For each plant, output fluctuates substantially over the sample. Employment exhibits long periods of constancy broken by infrequent and large jumps at times roughly coinciding with the largest output fluctuations.To mimic these microeconomic facts, I consider a constant cost to create and destroy jobs which is simple but enough to reproduce observed patterns of employment at the firm level.

3.1 Framework

Time is discrete and indexed by t. Time horizon is infinite. Consider a risk neutral firm that produces an homogeneous good.

Each firm has a production function Bn^a in productivity B and employment n where others inputs are assumed to be maximized out. The firm faces an iso-elastic demand curve with elasticity η: $CP^{-\eta}$. The firm's monopoly power decreases when η rises.

These can be combined into a revenue function: $BC^{1-\frac{1}{\eta}}n^{a(1-\frac{1}{\eta})}$. To alleviate notation, I define

[10] see Hamermesh (1993) and Bond and Reenen (2006) for exhaustive surveys of the literature on, respectively, labor demand and micro-econometric models of investment and employment

[11] Sargent (1978) shows that more elaborated version of this model may be derived as the solution to a firm's dynamic profit maximization problem under the assumption that there are quadratic costs of adjusting the workforce.

$A^{1-\alpha} = BC^{1-\frac{1}{\eta}}$ and $\alpha = a(1-\frac{1}{\eta})$. I add a purely transitory shock to productivity, denoted ϵ^T. Combined with the wage function $w(A, \epsilon^T, n)$, it gives the following profit function as a function of A, ϵ^T and n:

$$\pi(A, \epsilon^T, n) = \left(e^{\epsilon_t^T} A_t\right)^{1-\alpha} n^\alpha - w(A, \epsilon^T, n)n \tag{9}$$

$\alpha < 1$ reflects decreasing returns-to-scale and/or market power. Variations in profitability A could reflect variations in product demands or variations in the productivity of inputs. Thus a firm may have a high productivity because it has market-power and/or because of the higher quality if its production. I do not have information on output prices so that I can not disentangle these two effects.

There is a constant cost $\overline{c}$ that is paid for every job destroyed and simliarily a constant cost $\underline{c}$ for every job created. I consider net adjustment costs not brut: adjustment costs are associated with job destruction but not with workers flows. Thus adjustment costs here are unrelated to the identity of the workers who fill these positions. This choice is entirely determined by data availability. I do not consider fixed adjustment costs since it creates many technical difficulties (see Roys (2007)) and it requires informations on plants (and not firms). I do not consider convex adjustment costs since they generates smoothing which do not appear in the data. The model also assumes no entry or exit. This is for tractability and because I do not have data on entry/exit.

At the beginning of each period. the timing of events is:

- At the beginning of the period, the manager knows his past employment level (n_t) , current level of profitability (A_t) and the transitory shock ϵ^T.
- Given $\left(A, \epsilon^T, n\right)$ the manager creates or destroys jobs (d_t) which eventually contribute to current period production.
- Firm and workers bargain over current period wage $w(A, \epsilon^T, n + d)$
- Production takes place

State variables obey the following laws of motion:

$$\log A_{t+1} = \log A_t + \epsilon_t^P \tag{10}$$

$$n_{t+1} = n_t + d_t \tag{11}$$

The manager problem is to choose a state contingent sequence of employment to maximize the present discounted value of current and future profits. The parameter β represents the rate at which the agent discounts utility at future periods, and it belongs to the interval $(0,1)$. The decision problem of a firm at time t is to maximize the present discounted value of current and future profits, given the previous level of employment and the current state of the profitability and the transitory shock. Define the value function at period 0, $V\left(A_0, n_{-1}, \epsilon_0^T\right)$ as the present discounted value of current and future profits given initial productivity, A_0, lagged employment, n_{-1}, and initial transitory shock, ϵ_0^T. It writes:

$$\sup_{n_t, t\geq 0} E\left\{\sum_{t=0}^{\infty}\beta^t\left[\left(e^{\epsilon_t^T}A_t\right)^{1-\alpha}n_t^{\alpha} - w_t(A_t,\epsilon_t^T,n_t) - \underline{c}(n_t-n_{t-1})^+ - \overline{c}(n_t-n_{t-1})^-\right]|A_0,n_{-1}\right\} \tag{12}$$

where $x^+ = x$ if x is positive and zero otherwise and $x^- = -x$ if x is negative and zero otherwise.

Under regularity conditions described in the appendix, $V(A, \epsilon^T, n)$ is the unique solution to Bellman's equation:

$$\begin{aligned} V(A,\epsilon^T,n) &= \max_d\Big\{e^{(1-\alpha)\epsilon^T}A^{1-\alpha}(n+d)^{\alpha} - w(A,\epsilon^T,n+d)(n+d) - \underline{c}d^+ - \overline{c}d^- \\ &\quad +\beta\int V(Ae^{\epsilon^P},\epsilon^{T'},n+d)d\Phi(\epsilon^P,\epsilon^{T'})\Big\} \end{aligned} \tag{13}$$

3.2 Employment Policy

Given the wage function determined later on and given (A, ϵ^T, n), the optimal choice d must statisfy the first order conditions:

$$\alpha e^{(1-\alpha)\epsilon^T} A^{1-\alpha}(n+d)^{\alpha-1} - w(A,\epsilon^T,n+d) - w_n(A,\epsilon^T,n+d)\,(n+d) - \underline{c} + \beta E(V_n(Ae^{\epsilon^{P'}},\epsilon^{T'},n+d)) \leq 0 \tag{14}$$

with equality if $d > 0$, and

$$\alpha e^{(1-\alpha)\epsilon^T} A^{1-\alpha}(n+d)^{\alpha-1} - w(A,\epsilon^T,n+d) - w_n(A,\epsilon^T,n+d)\,(n+d) + \overline{c} + \beta E(V_n(Ae^{\epsilon^{P}},\epsilon^{T},n+d)) \leq 0 \tag{15}$$

with equality if $d < 0$.

The optimal choice for next period employment $n'(A,\epsilon^T,n)$ given the state (A,ϵ^T,n) reads

$$n'(A,\epsilon^T,n_{-1}) = \begin{cases} \overline{n}(A,\epsilon^T) & \text{if } n_{-1} > \overline{n}(A,\epsilon^T) \\ n_{-1} & \text{if } \underline{n}(A,\epsilon^T) < n_{-1} < \overline{n}(A,\epsilon^T) \\ \underline{n}(A,\epsilon^T) & \text{if } n_{-1} < \underline{n}(A,\epsilon^T) \end{cases} \tag{16}$$

where the two targets function $\overline{n}$ and $\underline{n}$ are defined as:

$$\begin{aligned} & \alpha\left(e^{\epsilon^T}A\right)^{1-\alpha}\overline{n}(A,\epsilon^T)^{\alpha-1} + \beta E(V_n(Ae^{\epsilon^P},\epsilon^T,\overline{n}(A,\epsilon^T))) \\ = \; & w(A,\epsilon^T,\overline{n}(A,\epsilon^T)) + w_n(A,\epsilon^T,\overline{n}(A,\epsilon^T))\overline{n}(A,\epsilon^T) + \overline{c} \end{aligned}$$

$$\begin{aligned} & \alpha\left(e^{\epsilon^T}A\right)^{1-\alpha}\underline{n}(A,\epsilon^T)^{\alpha-1} + \beta E(V_n(Ae^{\epsilon^P},\epsilon^T,\underline{n}(A,\epsilon^T))) \\ = \; & w(A,\epsilon^T,\underline{n}(A,\epsilon^T)) + w_n(A,\epsilon^T,\underline{n}(A,\epsilon^T))\underline{n}(A,\epsilon^T) + \underline{c} \end{aligned}$$

Optimality requires the firm to create and destroy jobs as needed to keep the marginal value of labor in the closed interval $[-\underline{c},\overline{c}]$. The optimal decision rule consists of two targets that verify an Euler equation. If the level of employment at the beginning period lies between the two targets, it is not worth hiring/firing and the firm stays put until the next period. The optimization problem has a sequential nature: 1. choosing a target for employment and 2. whether to hire/ fire or stay put. And there is no smoothing: if the manager decides to adjust, he directly jumps to the target without additional smoothing and independently of lagged employment. Figure 3 plots optimal employment given shocks.

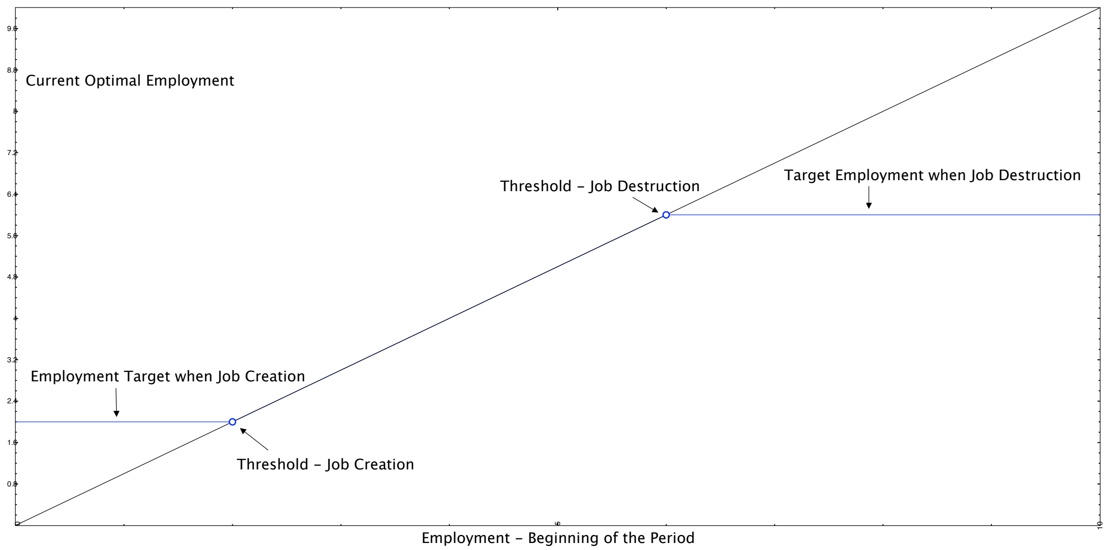

Figure 3: Optimal Decision rule conditional on Shocks

Labor productivity is higher in expansion than in contraction. This is because the firm is not destroying (creating) as much jobs as it would in a frictionless labor market. This is here not coming from lagging effects of employment.

The effect of adjustment costs on labor productivity is ambiguous. Following a positive (negative) shock, the firm hires (fires) relatively less which lead to a higher (lower) productivity of labor in expansion (recession) period.

3.3 Bargaining and Wage

I adopt the wage bargaining solution of Stole and Zwiebel (1996) which generalizes the Nash solution to a setting with downward-sloped labor demand. This has been used recently in a similar context by Elsby and Michaels (2008) and Fujita and Nakajima (2009). This sub-section borrows from these references.

The firm cannot commit to long term contracts and costless renegotiation takes place every period. Stole and Zwiebel (1996) assume that the current wage is the outcome of a sequence of bilateral negotiations with its employees where each is regarded as the marginal worker. Wages are then the outcome of a Nash bargaining between the firm and its workers over the marginal surplus.

I consider homogeneous workers and no aggregate uncertainty. If unemployed, the job-seeker achieves a value U. Upon finding a job, she receives a wage contract with present value of $W(A, \epsilon^T, n)$ that depends on firm state (A, ϵ^T, n).

The worker and the firm receive given fraction γ and $1-\gamma$ of the surplus. The job-seekers threat point is the value achieved during the prospective employment period by disclaiming the current job opportunity and continuing to search, that is, the unemployment value U. Wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage-setting. The firm threat point is the value achieved by destroying the job, that is, the cost of destroying the job $-\bar{c}$.

$$(1-\gamma)\left[W(A, \epsilon^T, n) - U\right] = \gamma\left[V_n(A, \epsilon^T, n) + \bar{c}\right]$$

While employed, a worker receives a flow payoff equal to the bargained wage minus taxes $w(A, \epsilon^T, n)(1-\tau)$ where $\tau \in (0,1)$ is a constant taxe rate that captures the difference between labor costs for the firm and the actual wage perceived by the worker. She losts her job with some probability s next period[12]:

$$W(A, \epsilon^T, n) = (1-\tau)\, w(A, \epsilon^T, n) + \beta E\left[sU + (1-s)W(A', \epsilon^{T'}, n')\right]$$

where the expectation is taken over the distribution of the permanent and transitory shock.

An unemployed workers receive flow payoff b, which represents unemployment benefits, the value of leisure and home production. She finds a job with probability f.

$$U = b + E\left[(1-f)\, U + fW(A, \epsilon^T, n)\right]$$

where the expectation is taken over the distribution of the permanent and transitory shock and over the stationary distribution of employment and productivity.

The wage function writes:

[12] It will turn out to be unecessary to characterize s because the value of working in a firm that is downsizing will be equal to the value of unemployment.

$$
\begin{aligned}
w(A,\epsilon^T,n) &= \frac{1}{1-\tau}\left((1-\gamma)\,b+\overline{c}\gamma\,(1-\beta\,(1-f))+\underline{c}\beta\gamma f\right) \\
&\quad +\frac{\gamma\alpha}{1-\tau-\gamma(1-\alpha)}\left(e^{\epsilon^T}A\right)^{1-\alpha}n^{\alpha-1}
\end{aligned}
$$

Wages are a function of labor productivity: $w(A,\epsilon^T,n)$ is homogeneous of degree 0 in (A,n). Wages are increasing in adjustment costs because adjustment costs increase the surplus from the existing relationship.

The relationship between wages and the curvature of the prodit function α is ambiguous. The less concave are demand and the production technology, the lower the rate of diminishing returns as workers are added and, as a result, workers are able to negotiate a larger share of the surplus. On the other hand, the less concave are demand and the production technology, the higher the size of the firm and the lower the productivity of labor.

4 Model Solution and Simulation

4.1 Numerical Solution

The full details of the numerical approach are in the Appendix. I use a collocation method (see for a detailed exposition Judd (1998)) and B-splines to approximate the unknown functions. The Approximation of the derivatives of the value function $V_n(A,n+d,\epsilon^T)$ is difficult. It has two kinks at the unknown thresholds. However that to find the optimal policy and simulate the model, it is only necessary to know the expected values $E\left[V_n(Ae^{\epsilon^P},n,\epsilon^T)\right]$ as a function of (A,n) and where the expectation is taken with respect to the joint distribution of permanent and transitory shocks. And this expectation is a smooth function of (A,n): the convolution of any integrable function and a normal density is analytic (see Lehmann (1959)).

Figure plots $E\left[V_n(Ae^{\epsilon^P},n+d,\epsilon^T)\right]$ as a function of n and for a fixed A. For low (high) values of employment, the firm creates (destroys) jobs and the marginal value of the firm is equal to the costs of creating (destroying) jobs. For intermediate values, the firm is inactive and the marginal

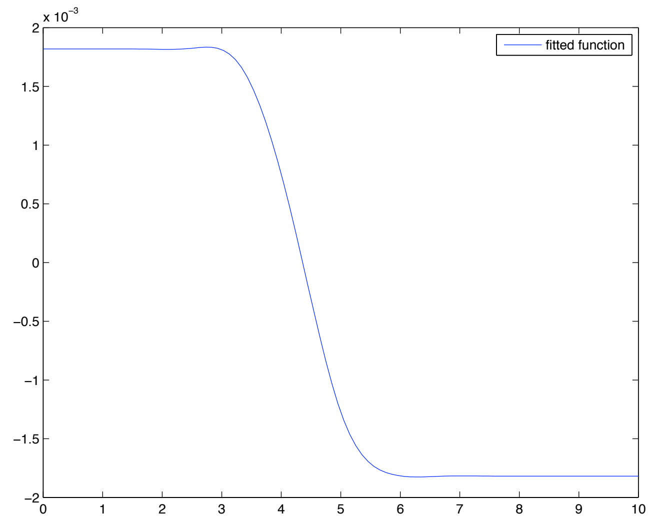

Figure 4: $E\left[V_n(Ae^{\epsilon^P}, n+d, \epsilon^T)\right]$ as a function of n and for a fixed A

	Baseline	Costless Adjustment
α	0.1964	
b	0.0420	
c	0.0094	0
γ	0.4469	
σ_{ϵ^P}	0.1759	
σ_{ϵ^T}	0.0805	
β	0.95	
f	0.35	
τ	0.38	

Table 6: Parameter Values

value of employment is a decreasing function of employment. The figure displays clearly that the kinks at the threshold values are smoothed-out by the expectation operator.

4.2 The Differential Impact of Transitory and Permanent Shocks

I list all the parameter values in Table 6 and discuss how I obtained them in Section 7. I set an equal variance for transitory and permanent shocks. This is to isolate the differential impact of transitory and permanent shocks from the magnitude effect.

I consider the response of employment, wage, labor productivity and output to permanent and transitory shocks to shocks of different magnitude. Figure 5 displays the average values of wages, productivity, employment and output at different values of permanent and transitory shocks. This is

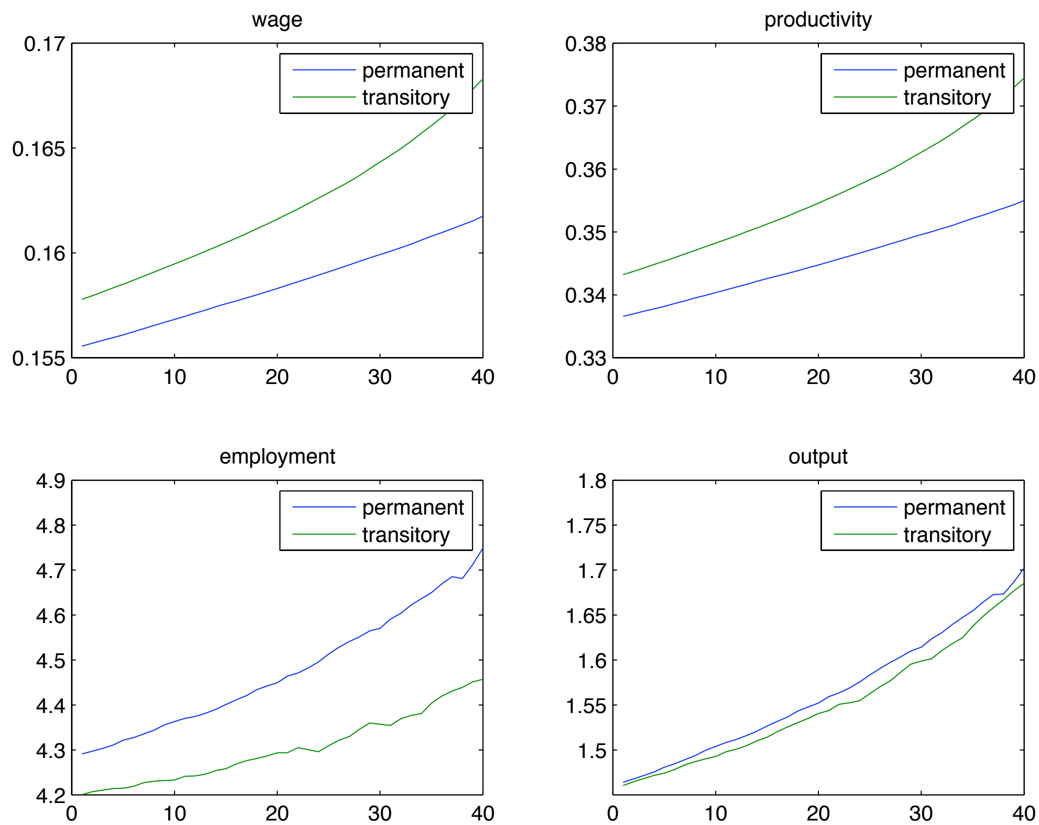

Figure 5: Response to Shocks - Baseline Calibration

an average over the distribution of other shocks and over the stationary distribution of employment across firms.[13] Figure 6 plots the inaction rate at different values of permanent and transitory shocks.

A transitory shock only increases today's profits while a permanent shock increases both today profits and future profits. This can be seen formally by looking at the first-order conditions for a firm creating jobs:

$$\alpha e^{(1-\alpha)\epsilon^T} A^{1-\alpha}(n+d)^{\alpha-1} - w(A,\epsilon^T,n+d) - w_n(A,\epsilon^T,n+d)\,(n+d) + \beta E(V_n(Ae^{\epsilon^{P'}},\epsilon^{T'},n+d)) = \underline{c}$$

The transitory shock ϵ^T only appears on the derivative of the one-period profit function. Con-

[13] Because of permanent shocks a stationary distribution of employment does not exist. To solve this issue, I introduce an exogenous rate of exit. Alternatively, I could have plotted the stationary distribution of job creation rate and destruction rate which have both a stationary distribution even without exit. See Appendix E.2 for details.

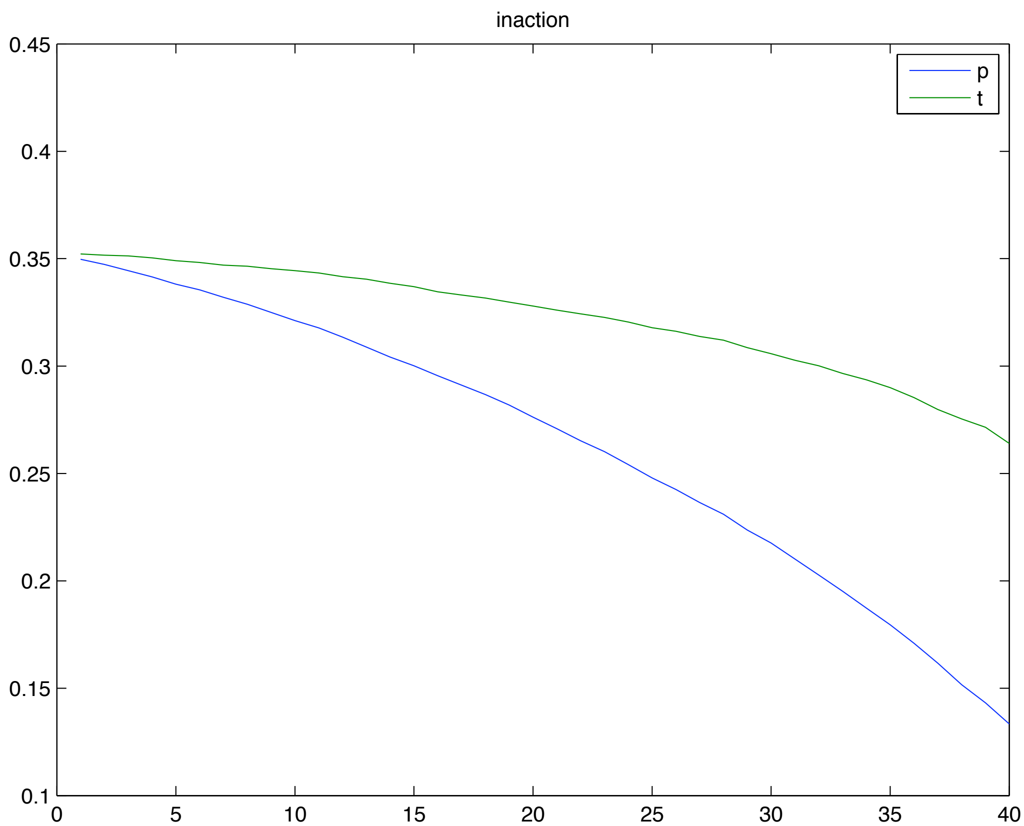

Figure 6: Inaction Rate

versely, the permanent level of profitability A appears in the derivative of both the one-period profit function and the value function. Hence the benefits of creating / destroying jobs following a transitory shock are on average small. Because of adjustment costs, the firm then decides not to create or destroy many jobs. Similarily, permanent shocks generate on average less inaction than transitory shocks.

Since there are decreasing returns to labor, the variations of labor productivity are smaller for a permanent shock compared to a transitory shock of the same magnitude. Following a positive transitory shock, labor productivity is high because employment does not adjust. Since wages are a function of labor productivity, wages react more to transitory shock than permanent one. This provides an explanation for the finding in Section 2. Finally, ouput reacts similarily to both kind of shocks. It comes from the assumption of constant-return-to scale with respect to (A, n).

Figure 7 shows the effects without adjustment costs. Because employment fully adjusts to shocks, labor productivity is constant. Wage being a function of labor productivity, it is constant. Employment reacts similarily to both kind of shocks.

4.3 The Reallocation Rate and the Structural Parameters

The average number of job created and destroyed is a central outcome of the model. Figure 8 describes the impact of the structural parameters of the model. The effects are obtained by simulating the model. Parameters values are listed in Table 6. A grid for each parameter is constructed by taking a deviation of 20% from the mean.

Obviously, high adjustment costs slowdown the process of reallocating of jobs. A volatile environment, represented by a high variance of transitory or permanent shocks, pushes the firm to create and destroy more jobs because business conditions change more often. These are standard results.

The impact of b and γ are new and due to the impact of wage setting on employment dynamics. The outside option of workers is assumed constant and equal to b.

The Nash bargainning parameter γ is an indicator of wage flexibility. At the limit when $\gamma = 0$, worker wage is constant and equal to b. A higher γ means that workers see their wage more

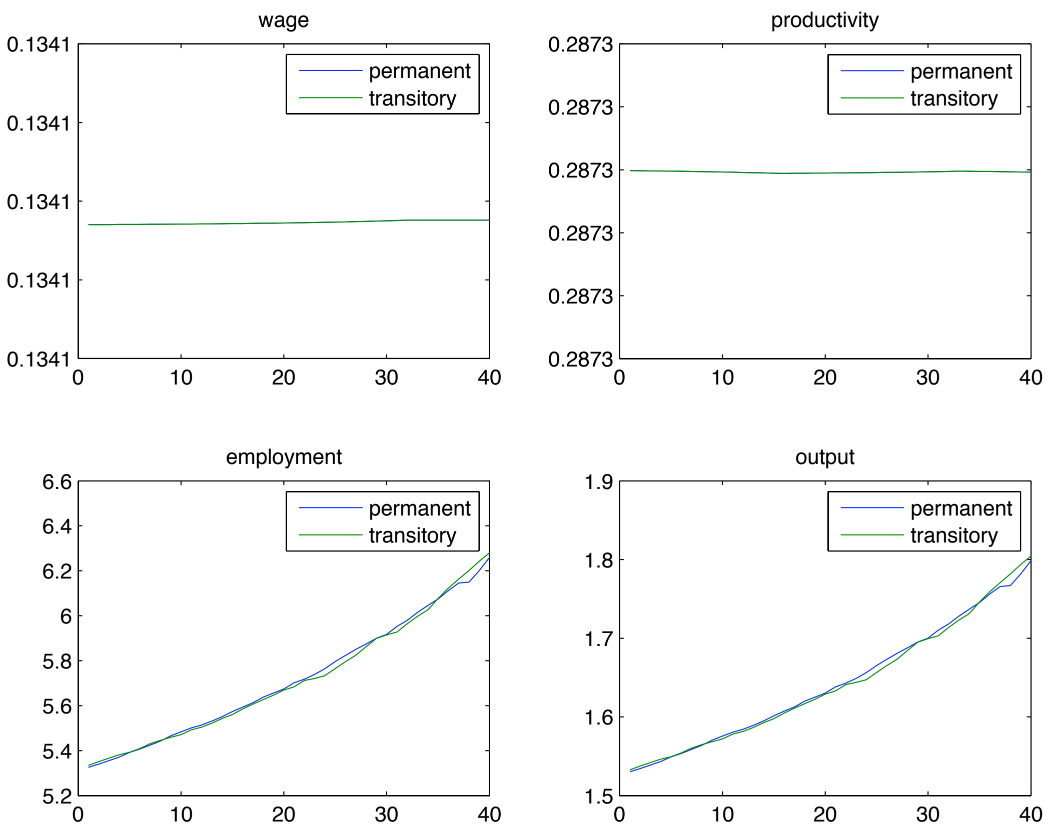

Figure 7: Response to Shocks - No Adjustment Costs

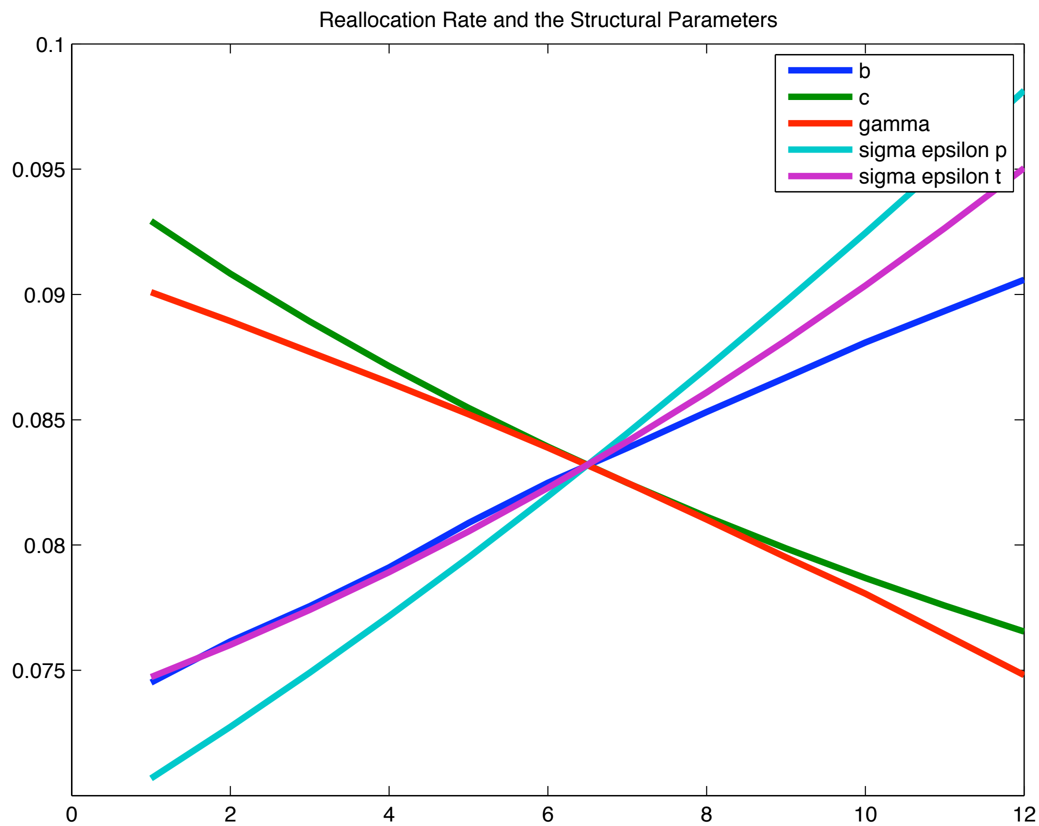

Figure 8: Reallocation Rate and the Structural Parameters

dependent on business conditions. Wages decrease more upon realization of a negative labor-demand shock, and increase more upon realization of a positive shock. Hence, more wage flexibility means that firm will need to create less jobs and destroy less jobs. This intuition was already in Bertola and Rogerson (1997).

The parameter b can be view as a proxy for unemployment benefits. With more generous unemployment benefits, the ouside option of workers binds more often. Thus, wages are less-dependent on business conditions.

5 Identification

As will be described below, I use indirect inference to estimate the structural parameters. This estimator needs observed moments that are a well-behaved function of the structural parameters. I show in this section how the parameters of the model affect features of employment, output and wages at the firm level leaving the exact description of the estimator to the next section.

An indirect inference estimator considers a set of auxiliary parameters that are statistics of the data. It defines a binding function $b(\theta)$ that maps the structural parameters θ to the auxiliary parameters. The model is identifed if the Jacobian of the binding function $\nabla b(\theta)$ is of full-rank. The problem is not trivial because b is unknown. I use the recent approach of Browning and Roys (2009). It involves imposing sign restrictions on $\nabla b(\theta)$ that are sufficient to insure that it is of full-rank. A sign $+$ $(-)$ indicates a positive (negative) relationship. No sign does not mean that the effect is exactly null but that it is negligible. Most sign restrictions are obtained by simulating the model.

I present a special case before turning to the general model.

5.1 Example

As an example consider the model with a constant wage and without transitory schocks: $\sigma^2_{\epsilon^T} = \gamma = 0$. As a results $w = b$. Table 7 reports the sign restrictions.

The time-series average of labor productivity and wages identify the parameter of the production function α and the value of home-production b. To understand the mechanism at work, consider

	$E(p)$	$E(w)$	$E(\mid \Delta \log n \mid)$	$E(I\{\log n = \log n_{-1}\})$	$E(\Delta \log p)^2$
α	$-$				
b	$+$	$+$			
c			$-$	$+$	$+$
$\sigma^2_{\epsilon^P}$			$+$	$-$	$+$

Table 7: Structural Parameters and Observed Moments: An Example

a firm facing no adjustment costs. Its decision consists in equalizing at every period the marginal productivity of labor with wage b. Simple calculations show that labor productivity is then:

$$p = \frac{O}{n} = \frac{A^{1-\alpha} n^{\alpha}}{n} = \frac{b}{\alpha}$$

Obviously, a high wage rate reduces employment and increases labor productivity. In a dynamic framework, this is still true but labor productivity follows a non-trivial dynamics due to adjustment costs.

To identify c and σ, consider the reallocation rate $E(\mid \Delta \log n \mid)$, which measures the number of jobs created or destroyed, and the inaction rate $E(I\{\log n = \log n_{-1}\})$, which counts periods where employment stays constant. Adjustment costs decrease the propensity to create and destroy jobs: reallocation rate should be low when adjustment costs are high. Similarily, adjustment costs increase inaction. The volatility of shocks has an opposite effect on those two moments. A highly volatile environment triggers more job reallocation and less inaction. From those two moments alone, identification may fail. Fortunately, a third moment achieves identification: labor productivity variance $E(\Delta \log p)^2$ increases when either c or σ increase.

5.2 The General Case with Measurement Errors

The variable employment in the dataset is defined as full-time equivalent employment that is total hours worked divided by average annual hours worked in full-time jobs. The observed variable includes some part-time and temporary workers. Further, heavy rounding can be expected. Output is measured by value-added which is defined in the dataset as the difference between production and intermediate consumptions net of all variations in stocks. From the literature on structural estimates of productivity (see for example the recent survey of Ackerberg et al. (2007)), conflating

	$E(p)$	$E(w)$	$E(\mid \Delta \log n \mid)$	$P(o,o)$	$P(w,o)$	$T(p,w)$	$T(o,o)$	$T(w,w)$	$T(p,p)$
α	$-$								
b	$+$	$+$	$+$		$-$				
c			$-$		$+$				$+$
σ_{ϵ^P}			$+$	$+$	$+$			$+$	$+$
σ_{ϵ^T}			$+$		$-$	$-$	$+$	$+$	$+$
γ		$+$	$-$		$+$	$-$		$+$	
σ_{MRO}							$+$		$+$
σ_{MRW}								$+$	
σ_{MRN}			$+$						$+$

Table 8: Structural Parameters and Observed Moments: An Example

true variation in productivity and measurement error in output would overestimate the degree of profitability dispersion over both time and firm.

I explicity introduce measurement error into the simulated moments to mimic the bias these impute into the actual data moments. I incorporate measurement error in employment, output and wages into the simulation by multiplying these variables by, respectively, $mn_{it}, mo_{it}, mw_{it}$ that are i.i.d over firm and time and follow a log-normal distribution with mean zero and standard deviations, respectively, $\sigma_{MRN}, \sigma_{MRO}, \sigma_{MRW}$.

Table 8 summarizes the arguments for identification.

The value of unemployment b and the bargainning power γ of workers have a positive effect on average wages but an opposite effect on the reallocation rate (see Section 4.3 for the intuition).

To separately identify the variance of permanent and transitory shocks, I use the idea from Section 2.3. I define $P(x,y) = E\left(\Delta x \sum_{i=-1}^{1} \Delta y_i\right)$. It captures the correlation between x and y that comes from permanent shocks and not transitory shocks. I define $T(x,y) = -E\,(xy)$. It captures the correlation between x and y that comes from transitory shocks and not permanent shocks.

How to separately identify measurement error in output σ_{MRO} from true transitory variation in profitability σ_{ϵ^T}? Measurement errors in output do not affect the reallocation rate nor the correlation between wages and labor productivity while transitory shocks to profitability do. The approach is similar for measurement errors in employment and wages.

6 Structural Estimation: Framework

Estimating the parameters using Euler equation techniques is not adequate here because corner solutions are relatively important: around 15% of the observations correspond to zero adjustment. This means that selection bias is likely to be severe. Pakes (1994) proposed to estimate the structural parameters using some modified Euler equations which take into account the number of periods between two consecutive interior solutions. But there remains several limitations of the Euler equations. For example in the consumption literature, the estimation using Euler equation techniques has been very disappointing (see the discussion in Alan and Browning (2009)). The problems identified are manifold but the most important seem to be the paucity of long panels and the substantial measurement error in observed variables. The data-set used here is relatively short $T = 7$. Measurement errors issues are discussed above.

Exploiting the discrete decision (whether to adjust employment level of not) would be another possibility. For example, Rota (2004) estimate a labor demand model with fixed costs using Hotz and Miller (1993)'s estimator. The former is suited for discrete decision processes (the decision to adjust or not). The continuous decision (how much to invest) is estimated non-parametrically. Three important limitations appear here. First, it heavily relies on the conditional independence assumption (Rust (1987)) which essentially states that once we condition on observables, there are no serially correlated unobservables. But the dataset used contains a very large number of firms with a limited number of variables. There is considerable heterogeneity among the observed variables. Second, identification of the structural parameters would entirely rely on the discrete decision. Important measurement errors can be expected in recorded employment. It is an average of the number of employees in the firm that ignore the flows during the year. Lastly, I have information on firms and not plants.

As an alternative, I use indirect inference.

6.1 The Auxiliary Model

The model endogenously account for different scale of operations that come from heterogenous initial conditons and from permanent shocks. The former are relevant from Abbring and Campbell

(2005) who show that heterogeneity in firms' pre-entry scale decisions accounts for most of firms' heterogeneity. Evidence for the latter has been presented in Section 2.2.

In its current form, the model generates a dispersion of labor productivity because of adjustment costs, shocks and measurement errors. This will prove insufficient to explain the observed variance of labor productivity and in particular the between-firms differences in labor productivity (cf. Section 2.4).

For this very reason, I choose an auxiliary model which is a finite mixture model that accounts for a finite number of firms' type whose respective proportions are estimated. The basic idea is that data comes from a population with several subpopulations. The overall population is a mixture of subpopulations each having its own model. Let Z_i denote a random vector whose components are the time series average for firm i of moments related to employment, production and wages dynamics. $f_Z(z)$ is its density function. Not all the moments used for estimation are informative for clustering firms. I partition $Z_i = \left(Z_i^1, Z_i^2\right)$ where Z_i^1 are the clustering variables and Z_i^2 are the non-clustering variables.

The density function of $z^1, f_{Z^1}\left(z^1\right)$, is a finite mixture of gaussian:

$$f_{Z^1}(z_i^1; \Psi) = \sum_{k=1}^{K} \lambda_k \phi^1(z_i^1; \mu_k^1, \Sigma_k^1) \tag{17}$$

where the $\phi^1(z_i; \mu_k, \Sigma_k)$ are multivariate gaussian densities. For each $k \in 1, ..., K, \lambda_k$ is the proportion of the population in the group k. The λs are nonnegative quantities that sum to one.

The density function of z^2 is a multivariate gaussian distribution $\phi^2\left(z_i^2, \mu^2, \Sigma^2\right)$. Under the assumptions that Z_i are iid and that $\left(Z_i^1, Z_i^2\right)$ are independent, the log-likelihood of the auxiliary model writes:

$$\log L(\Psi; z) = \sum_{i=1}^{N} \log\left(\sum_{k=1}^{K} \lambda_k \phi_k(z_i; \mu_k, \Sigma_k)\right) + \sum_{i=1}^{N} \log \phi^2\left(z_i^2, \mu^2, \Sigma^2\right) \tag{18}$$

where $\Psi = (\mu_1^1, ..., \mu_K^1, \Sigma_1^1, ..., \Sigma_K^1, \lambda_1^1, ..., \lambda_{K-1}^1, \mu^2, \Sigma^2)$.

The parameters of the density of z^1 are estimated using the EM Algorithm (see Dempster et al. (1977) and McLachlan and Peel (2000)). I describe the details of the algorithm in Appendix F. It is well-known that asymptotic theory does not provide reliable standard errors unless the number of observations is very large. For that reason, I use non-parametric bootstrap for standard errors.

Determining the number of components is an important but difficult problem which has not been completely resolved. The BIC is a commonly used criteria for choosing the number of components. Montecarlo experiments show that it does a poor job at determining the number of types. I adopt a more heuristic approach. I increase the number of types until the confidence bands overlap between two different groups in each dimension. It has some degree of arbitrariness though.

6.2 Indirect Inference

I describe how I recover the structural parameters $\theta = (\theta_1, ..., \theta_K, \theta_c)$ where $\theta_k = (\alpha_k, b_k, \lambda_k)$ is type-specific and $\theta_c = (c, \gamma, \sigma_{\epsilon^P}, \sigma_{\epsilon^T}, \sigma_{MRN}, \sigma_{MRO}, \sigma_{MRW})$ is common across firms. The proportion of each type λ_k are used to determined the number of firms of each type used in the estimation. I simulated an economy population with a number of firms for each type equal to $N_k = round(\lambda_k N)$ where N is the sample size.

Given assumed values of the structural parameters, I use the structural model to generate S statistically independent simulated data set $z^s(\theta)$. I then choose $\Psi^s(\theta)$ such that:

$$\hat{\Psi}^s(\theta) = \arg\max_{\Psi} \sum_{s=1}^{S} \log L(\Psi; z^s(\theta))$$

where the likelihood function associated with the auxiliary model is evaluated using the simulated data.

The estimated value of the structural parameters $\hat{\theta}$ minimizes the difference between the constrained and unconstrained values of the likelihood function of the auxiliary model using the observed data:

$$\hat{\theta} = \arg\min_{\theta} \left[\log L\left(\hat{\Psi}; z\right) - \log L\left(\hat{\Psi}^s(\theta); z\right)\right]$$

7 Structural Estimation: Results

Some parameters are not estimated and set off the model. The probability of finding a job f is set to 0.35 which is an average of the annual transition from unemployment to employment observed over the period of observations (the data are taken from the "Enquete Emploi", a survey of about 1/300th of the French population, conducted annually by INSEE, the French National Statistical Institute). The ratio between the labor costs for the employer and the remuneration perceived by the worker $1-\tau$ is set to 0.62. The discount factor β is set to 0.95 so that the annual real interest rate is 5 percent.

7.1 Auxiliary Model

The moments I use to estimate the model are listed in Table 9. Some moments were discussed in Section 2. The variance of labor productivity is much larger than the variance of output and wages (0.0092 against 0.0047). The structural model will then answer whether this is explained by high adjustment costs or whether it comes from measurement errors: labor productivity is computed as the ratio of output to employment and thus contains measurement error in output and employment. Wages are as volatile as output. This may be because of measurement error: wages are computed as the ratio of labor costs and employment. Or it may be because the variations in productivity transmit into wages which means a high γ in the structural model.

I estimate a mixture for the joint distribution of wages and labor productivity. I consider 3 groups of firms. Table 10 reports type-specific means, the proportion of each type and standard errors. Figure 9 plots the data and reports the most likely type of each firm. Labor productivity and wages are strongly positively correlated across firms.

I consider firms that differ in terms of the structural parameters α and b and show that it translates in terms of differences in labor productivity and average wages. I could also have considered firms that differ in terms of volatility, bargaining power,... I experimented the estimation of a mixture with a larger set of moments for each firm without finding much evidence of clustering on those extra dimensions. Hence, not all the moments used for estimation are informative for clustering firms. For example, I use the estimated group appartenance probabilities $P(i \in k)$ that estimate

Moments	Data	Homogeneous Model	3-Types Model
$E(p)$	0.3152	0.3186	
$E(w)$	0.1501	0.1480	
Reallocation	0.0725	0.0757	0.0737
Inaction	0.1717	0.1712	0.1745
$E\left(\Delta \log o \sum_{-1}^{1} \Delta \log o_i\right)$	0.0294	0.0235	0.0278
$E\left(\Delta \log w \sum_{-1}^{1} \Delta \log p_i\right)$	0.0015	0.0037	0.0034
$E\left(\Delta \log w \Delta \log p_{-1}\right)$	-0.0031	-0.0038	-0.0037
$\frac{E(\Delta \log w \Delta \log p_{-1})}{E(\Delta \log p \Delta \log p_{-1})}$	0.4690	0.4691	0.4539
$E\left(\Delta \log p \Delta \log p_{-1}\right)$	-0.0092	-0.0081	-0.0083
$E\left(\Delta \log o \Delta \log o_{-1}\right)$	-0.0047	-0.0057	-0.0064
$E\left(\Delta \log w \Delta \log w_{-1}\right)$	-0.0047	-0.0046	-0.0050

Table 9: Average Moments - Auxiliary Model

Type	Variable	Mean	Standard Deviation
1	Proportion	0.4071	0.0344
	Wages	0.1719	0.0023
	Productivity	0.4012	0.0094
2	Proportion	0.4354	0.0478
	Wages	0.1414	0.0023
	Productivity	0.2753	0.0061
3	Proportion	0.1574	0.0395
	Wages	0.1177	0.0037
	Productivity	0.2026	0.0093

Table 10: Type-Specific Means for Labor Productivity and Wages

the probability that firm $i \in N$ belongs to group $k \in K$ to compute an estimate of the variance of the permanent shock for each group of firms $E\left(\Delta \log o \sum \Delta \log o_i \mid i \in k\right)$. I find an estimated variance of $0.0272, 0.0294$ and 0.0286. The differences are thus not very important.

7.2 Structural Parameters

Table 11 reports the results of the estimation of the homogeneous model and the model with 3 types of firms with type-specific parameters for the curvature of the profit function α and for the value of home-production b. Column 2 and Column 3 in Table 9 reports the simulated moments for, respectively, the homogeneous Model and the 3-Types Model. Last Column in Table 10 reports

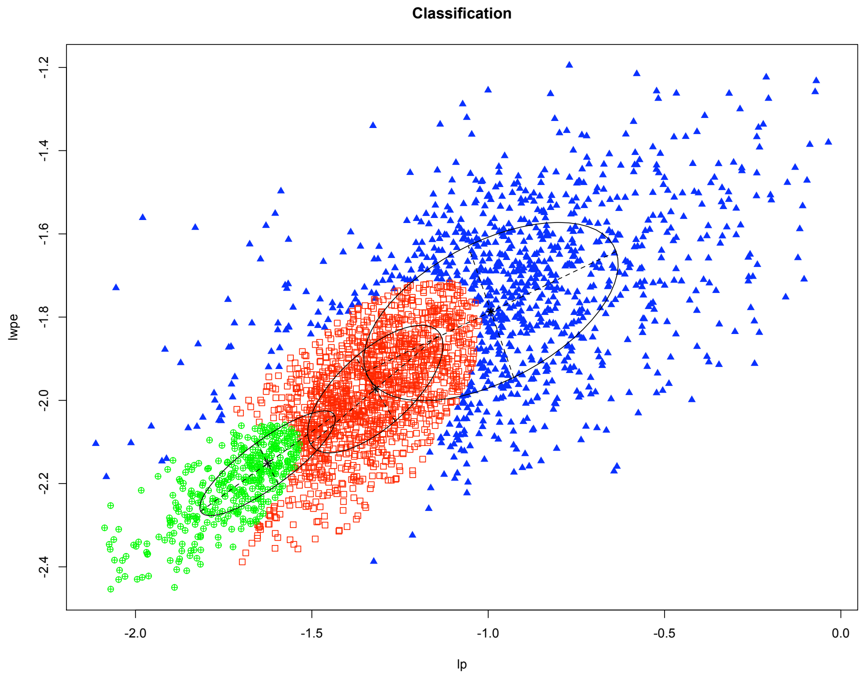

Figure 9: Classification

the simulated type-specific moments for the 3-Types Model.

The estimated value of α is within the interval $0.16 - 0.26$. This is lower than estimates typically obtained from production functions estimation. Using a related structural model Rota (2004) find even lower coefficients in the range $0.11 - 0.13$ with a panel of Italian firms. This is not implausible given that the capital stock is treated as a component of the firm idiosyncratic shock and it is assumed to follow an exogenous process.

Adjustment costs per worker are estimated to be equal to $6,5\%$ of the average worker wage. Combined with an observed job reallocation rate of $7,25\%$, the estimate implies that adjustment costs incurred by firms average about $47,5\%$ of the annual wage bill per worker. It says that the firm has to pay an extra-year of wage for every fifteen job created or destroyed. Rota (2004) estimates the median level of fixed (not linear) costs to be around 15 months' labor cost in Italy. Using compustat, Bloom (2009) estimate linear adjustment costs of about 1.8% of annual wages, and a fixed cost of around 2.1% of annual revenue with no quadratic adjustment costs. Those numbers are lower than what could be expected given the stringent labor market regulation in France. An

Parameters	Homogeneous Model	3-Types Model
α	0.1964	
	(0.0020)	
b	0.0420	
	(0.0034)	
α_1		0.1669
		(0.0187)
α_2		0.2283
		(0.0106)
α_3		0.2650
		(0.0142)
b_1		0.0425
		(0.0036)
b_2		0.0426
		(0.0034)
b_3		0.0334
		(0.0037)
c	0.0094	0.0082
	(0.0031)	(0.0010)
γ	0.4469	0.4504
	(0.0034)	(0.0122)
σ_{ϵ^P}	0.1759	0.1716
	(0.0025)	(0.0029)
σ_{ϵ^T}	0.0805	0.0863
	(0.0026)	(0.0031)
σ_{MRN}	0.0183	0.0193
	(0.0008)	(0.0016)
σ_{MRO}	0.0531	0.0561
	(0.0022)	(0.0028)
σ_{MRW}	0.0446	0.0452
	(0.0025)	(0.0017)

Table 11: Structural Parameters Estimates - Standard-Errors are given in parentheses below the point estimates

examination of the data on worker flows provides an explanation. Almost 60% of the exit from employment are attributed to the end of fixed-duration contracts which are by definition flexible. Less than 7% of the exit from employment are made through a layoff procedure and less than 1% through a layoff for economic reasons. Hence by using fixed-duration contracts, French firms destroy jobs at relatively low costs.

I estimate worker bargainning power to be equal to 0.44. Using the same firm dataset matched with a worker dataset and a different methodology, Cahuc et al. (2006) find a bargainning power in the range $0.15 - 0.62$ depending on skills categories. They warn though that these numbers are biased upward because they do not account for on-the-job-search and between-firm competition for employed workers.

I estimate the value of unemployment to be between 500 euros and 700 euros per month which is close but lower than minimum wage during that period (about 900 euros per month).

Measurement errors in output is important as usually reported in the literature on the structural estimation of production functions (see Ackerberg et al. (2007) for a survey). Typically, it is assumed in this literature a unique source of variations of profitability. I consider here both transitory and permanent shocks to output. Nevertheless, measurement errors in output remain important.

To estimate measurement errors in employment, Bloom (2009) uses the following approach. Assume firm log wage-bill can be decomposed into: $\log(wb_{it}) = \log(n_{it}) + \eta_t + u_i + \omega_{it}$ where η_t is a time dummy, u_i is a firm specific wage rate, n_{it} is observed employment and ω_{it} is a residual. If this decomposition is correct, the coefficient on $\log(n_{it})$ is $\frac{\sigma_n^2}{\sigma_n^2+\sigma_{mn}^2}$ where σ_n^2 is the variance in log employment and σ_{MRN}^2 is the variance of the measurement error in log employment. I find a coefficient (standard error) on $\log(n_{it})$ of 0.882(0.003). This means a standard deviation of 0.0247. This is reassuringly similar to the estimated value here.

Measurement errors in wages are large. At first, this number is surprising because wages are typically a better recorded variable. In the model, the only source of variations in wages is labor productivity. There is probably many other sources of wages variations that are identified by the job-search literature (on-the-job search, mismatch,...) that I do not consider here and that may explain the results.

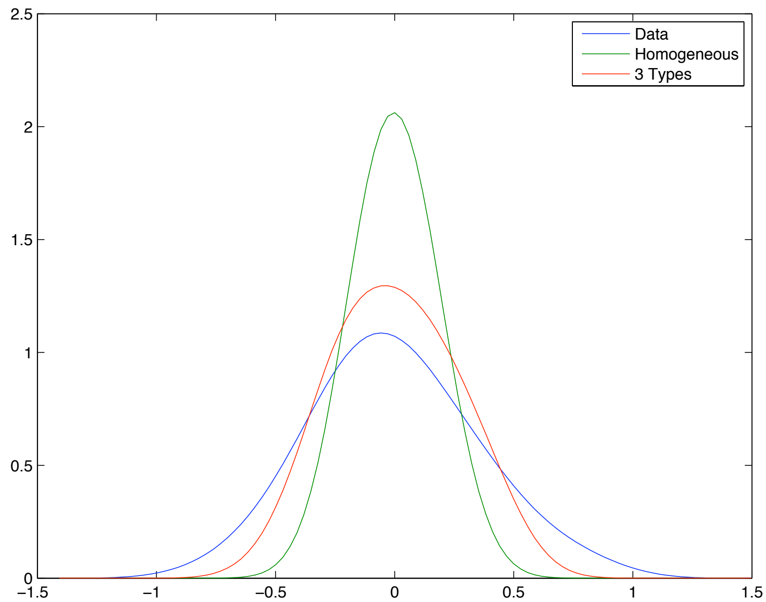

Figure 10: Distribution of Labor Productivity

The estimates of the parameters which are not type-specific are quite similar for both the homogeneous and the 3-types model. Yet the 3-types model greatly improves the capacity of the model to fit the observed dispersion of labor productivity. Figure 10 plots the observed distribution of labor productivity, the predicted distribution for the homogeneous and the 3-types model.

8 Conclusion

This paper offers a structural framework to analyze the impact of permanent and transitory shocks to profitablity on wages and employment at the firm-level.

Firms face permanent and transitory shocks to business conditions. There are frictions that result in both wage rigidities and employment rigidities. Creating and destroying jobs is costly. Wages are only partially flexible. There are decreasing-returns to labor and wages are linked to labor productivity via Nash-Bargaining.

The benefits of creating / destroying jobs following a transitory shock are on average small. Because of adjustment costs, the firm then decides to not create or destroy much jobs. As a result, wages change through the change in labor productivity. Permanent shocks lead to more employment changes. Since there are decreasing returns to labor, the variations of labor productivity and thus the variations of wages are smaller for a permanent shock compared to a transitory shock of the same magnitude.

I find support for this mechanism using a panel of french firms. I find that transitory shocks to output have a strong effect on wages but that permanent shocks have a very small effect on wages.

I provide a structural estimation of the model that allows for firm-specific parameters. I consider 3 types of firms that have a specific production-function and their workers have a different opportunity cost of employment. The model fits the data well and the introdution of 3-types of firm greatly improves the fit of the observed dispersion of labor productivity. The estimation results points out that relatively modest adjustment costs (less than a month of wages) can reproduce the data well which conflicts with the perceived sclerosis of the French labor market. Permanent shocks are the most important source of output fluctuations.

An important extension is to allow for worker heterogeneity. This is natural because employment and compensation of heterogeneous workers may fluctuate differently for two important reasons: the specificity of human capital and insurance motives. On the former, for instance, a firm is probably more reluctant to fire a worker with a highly specific and valuable human capital. Following a negative shock, the worker and the firm may then agree on a temporary wage cut to avoid human capital losses. On the latter, the level of insurance may varies with worker characteristics. A central result of the principal-agent model is the trade-off between incentive and insurance. The more the principal wants to incentivize an agent the less the agent will be insured. Turning to the labor contract, we should therefore expect that the firm offers less insurance to those employees whose effort is more relevant to performance. For instance, managers see their income changing with firm performance much more than unskilled production workers do. I leave this for future research.

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A Labor Market Institutions in France

This section briefly introduces the main features of the French labor market that are relevant for the interpretation of the results.

Employment protection may be described as restrictions placed on the ability of the employer to utilize labor. It covers dismissals protection (procedural inconveniences, notice and severance payments, penalties for unfair dismissals), limitations on the uses of fixed-term and temporary work agency contracts and the regulation of working hours (maximum weekly/annual normal hours, minimum rest periods, limits on overtime, restrictions on weekend and night work,...). They comprise both hiring costs (from screening an training new workers) and firing costs (due to notice periods, bureaucratic procedures and explicit firing costs like severance payments).

There are two types of regular employment contracts in France: indefinite-term contracts (CDI) and fixed-duration contracts (CDD). Although their use is formally restricted, CDDs are the most common method of hiring: more than 2/3 of all hires are through CDD.

Employment Protection Legislation heavily regulates the termination of CDI. Firms can layoff a worker for personal reasons, in which case they have to show that the worker cannot do the job she was hired for. Firm can also layoff a worker for economic reasons in which case the firm must prove that it needs to reduce its employment. In both case, the firm must observe a mandatory waiting notice period and pay a severance payment. The notice period depends on seniority. In the absence of a specific contract between unions and firms, the amount of severance pay set by law is also modest, typically 1/10 of a month per year of work, plus 1/15 of a month for years above 10 years. Sectoral agreements typically set higher amounts. Severance packages offered by firms in exchange for a quick resolution are typically much more generous than the legal or the contractual minimum.

The legislation also differs depending on whether it is an individual termination or a collective termination (dismissal of at least 10 workers during a 30 days period). Collective terminations are typically more expensive than individual terminations (Abowd and Kramarz (2003)).

The French system of wage setting is complex because it depends simultaneously on state level wage policies and collective bargaining at other levels. The minimum wage (Salaire Minimum

Interprofessional de Croissance, or SMIC) is set by the government. Administrative procedures are used to adjust the SMIC each July to reflect both consumer price increases and real wage increases in the hourly wages of manual workers. In addition, the government has sometimes enacted additional increases in the minimum wage. There are strong limits on the scope for derogations and for direct negotiations by social partners. Around 12% of the workers are paid at the minimum wage during the period of observations. This number is nowadays closer to 15%.

Aside the minimum wage, the French system is a largely decentralized and uncoordinated system of collective bargaining because wage bargaining only take place at the branch and enterprise levels.

B Theoretical Moments for Section 2.3

$$\begin{aligned}
E\left[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right] &= \sigma^2_{\zeta^y} \\
E(\Delta y_t \Delta y_t) &= \sigma^2_{\zeta^y} + 2\left(\sigma^2_{\nu^y} + \sigma^2_{r^y}\right) \\
E(\Delta y_t \Delta y_{t+1}) &= -\left(\sigma^2_{\nu^y} + \sigma^2_{r^y}\right)
\end{aligned}$$

and $E(\Delta y_t \Delta y_{t+s}) = 0, s \geq 2$. Similarly,

$$\begin{aligned}
E\left[\Delta w_t(\Delta w_{t-1} + \Delta w_t + \Delta w_{t+1})\right] &= \tau_2^2 \sigma^2_{\zeta^y} + \sigma^2_{\zeta^w} \\
E(\Delta w_t \Delta w_t) &= \tau_1^2 \sigma^2_{\zeta^y} + 2\left(\tau_2^2 \sigma^2_{\zeta^y} + \phi^2 \sigma^2_{\nu^y} + \sigma^2_{r^w}\right) + \sigma^2_{\zeta^w} \\
E(\Delta w_t \Delta w_{t+1}) &= -\left(\tau_2^2 \sigma^2_{\zeta^y} + \phi^2 \sigma^2_{\nu^y} + \sigma^2_{r^w}\right)
\end{aligned}$$

and $E(\Delta w_t \Delta w_{t+s}) = 0, s \geq 2$. The covariance between output growth and wage growth is:

$$\begin{aligned}
E(\Delta w_t \Delta y_t) &= \tau_1 \sigma^2_{\zeta^y} + \tau_2 \sigma^2_{\zeta^y} + 2\phi \sigma^2_{\nu^y} \\
E(\Delta w_t \Delta y_{t+1}) &= -\phi \sigma^2_{\nu^y} \\
E(\Delta w_{t+1} \Delta y_t) &= -\left(\phi \sigma^2_{\nu^y} + \tau_2 \sigma^2_{\zeta^y}\right) \\
E\left[\Delta w_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})\right] &= \tau_1 \sigma^2_{\zeta^y}
\end{aligned}$$

and $E(\Delta w_t \Delta y_{t+s}) = 0, |s| \geq 2$.

C Theoretical Model

C.1 The Firm's Problem

Bellman equation writes:

$$
\begin{aligned}
V(A, \epsilon^T, n) \quad = \quad & \max_d \Big\{ \left(e^{\epsilon^T} A\right)^{1-\alpha} (n+d)^\alpha - w(A, \epsilon^T, n+d)(n+d) - \underline{c}d^+ - \overline{c}d^- \\
& + \beta \int V(Ae^{\epsilon^P}, \epsilon^{T'}, n+d) d\Phi(\epsilon^P, \epsilon^{T'}) \Big\} \qquad (19)
\end{aligned}
$$

Define $D(A, n) = \int V(Ae^{\epsilon^P}, n+d, \epsilon^{T'}) d\Phi(\epsilon^P, \epsilon^{T'})$ and accordingly $D_n(A, n)$. Decomposing V and V_n,

$$
V_n(A, n, \epsilon^T) = \begin{cases} \underline{c} & \text{if } n < \underline{n}(A, \epsilon^T) \\ \alpha \left(e^{\epsilon^T} A\right)^{1-\alpha} n^{\alpha-1} - w(A, \epsilon^T, n) - w_n(A, \epsilon^T, n)n + \beta D_n(A, n) & \text{if } \underline{n}(A, \epsilon^T) < n < \overline{n}(A, \epsilon^T) \\ -\overline{c} & \text{if } n > \overline{n}(A, \epsilon^T) \end{cases}
$$

C.2 Bargaining

Let $W(A, \epsilon^T, n)$ be the value of employment in a firm of size n with state (A, ϵ^T, n). Due to Nash sharing, the worker's surplus in an expanding firm reads:

$$
\begin{aligned}
W\left(A, \epsilon^T, \underline{n}(A, \epsilon^T)\right) - U \quad &= \quad \frac{\gamma}{1-\gamma}\left(V_n(A, \epsilon^T, \underline{n}(A, \epsilon^T)) + \overline{c}\right) \\
&= \quad \frac{\gamma}{1-\gamma}\left(\underline{c} + \overline{c}\right)
\end{aligned}
$$

Similarily, in a contracting firm:

$$
\begin{aligned}
W\left(A, \epsilon^T, \overline{n}(A, \epsilon^T)\right) - U \quad &= \quad \frac{\gamma}{1-\gamma}\left(V_n(A, \epsilon^T, \overline{n}(A, \epsilon^T)) + \overline{c}\right) \\
&= \quad 0
\end{aligned}
$$

Upon finding a job, the new job must be in a firm which is creating jobs. The value to a worker of unemployment reads:

$$U = b + \beta\left\{(1-f)U + f\int\int\left[\int W\left(A, \epsilon^T, \underline{n}(Ae^{\epsilon^P}, \epsilon^T)\right)\frac{d\Phi\left(\epsilon^P, \epsilon^T \mid n < \underline{n}(A\epsilon^P, \epsilon^T)\right)}{P(n < \underline{n}(Ae^{\epsilon^P}, \epsilon^T)|A, n)}\right]dH(A,n)\right\}$$

where $H(A,n)$ is the distribution of (A,n) in the economy. Then,

$$(1-\beta)U = b + \beta f\frac{\gamma}{1-\gamma}\left(\underline{c} + \overline{c}\right)$$

The value of employment reads:

$$\begin{aligned}
W(A, \epsilon^T, n) &= (1-\tau)w(A, \epsilon^T, n) \\
&\quad + \beta\int W\left(A\epsilon^{P'}, \epsilon^{T'}, \underline{n}(A\epsilon^{P'}, \epsilon^{T'})\right)d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid n < \underline{n}(A\epsilon^{P'}, \epsilon^{T'})\right) \\
&\quad + \beta\int W\left(A\epsilon^{P'}, \epsilon^{T'}, n\right)d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid \underline{n}(A\epsilon^{P'}, \epsilon^{T'}) < n < \overline{n}(A\epsilon^P, \epsilon^T)\right) \\
&\quad + \beta\int U d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid n > \overline{n}(A\epsilon^P, \epsilon^T)\right) \\
&= (1-\tau)w(A, \epsilon^T, n) \\
&\quad + \beta P\left(n < \underline{n}(A\epsilon^{P'}, \epsilon^{T'})\right)\left(U + \frac{\gamma}{1-\gamma}\left(\underline{c} + \overline{c}\right)\right) \\
&\quad + \beta\int\left(U + \frac{\gamma}{1-\gamma}V_n\left(A\epsilon^{P'}, \epsilon^{T'}, n\right)\right)d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid \underline{n}(A\epsilon^{P'}, \epsilon^{T'}) < n < \overline{n}(A\epsilon^P, \epsilon^T)\right) \\
&\quad + \beta\int U d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid n > \overline{n}(A\epsilon^P, \epsilon^T)\right)
\end{aligned}$$

The last term is because when a firm is contracting, its workers get the same surplus whether they remain employed or not.

Rearranging terms, gives:

$$\begin{aligned}
W(A, \epsilon^T, n) &= (1-\tau)w(A, \epsilon^T, n) + \beta U + \beta\frac{\gamma}{1-\gamma}P\left(n < \underline{n}(A\epsilon^{P'}, \epsilon^{T'})\right)\left(\underline{c} + \overline{c}\right) \\
&\quad + \beta\frac{\gamma}{1-\gamma}\int\left[V_n\left(A\epsilon^{P'}, \epsilon^{T'}, n\right) + \overline{c}\right]d\Phi\left(\epsilon^{P'}, \epsilon^{T'} \mid \underline{n}(A\epsilon^{P'}, \epsilon^{T'}) < n < \overline{n}(A\epsilon^P, \epsilon^T)\right)
\end{aligned}$$

It follows that workers surplus is:

$$\begin{aligned} W(A,\epsilon^T,n)-U &= (1-\tau)w(A,\epsilon^T,n)-b+\beta\frac{\gamma}{1-\gamma}\left(P\left(n<\underline{n}(A\epsilon^{P'},\epsilon^{T'})\right)-f\right)(\underline{c}+\overline{c}) \\ &\quad +\beta\frac{\gamma}{1-\gamma}\int\left[V_n\left(A\epsilon^{P'},\epsilon^{T'},n\right)+\overline{c}\right]d\Phi\left(\epsilon^{P'},\epsilon^{T'}\mid\underline{n}(A\epsilon^{P'},\epsilon^{T'})<n<\overline{n}(A\epsilon^{P},\epsilon^{T})\right) \end{aligned}$$

This must equal $\frac{\gamma}{1-\gamma}\left[V_n(A,\epsilon^T,n)+\overline{c}\right]$. Then,

$$\begin{aligned} & (1-\tau)w(A,\epsilon^T,n)-b+\beta\frac{\gamma}{1-\gamma}\left(P\left(n<\underline{n}(A\epsilon^{P'},\epsilon^{T'})\right)-f\right)(\underline{c}+\overline{c}) \\ & +\beta\frac{\gamma}{1-\gamma}\int\left[V_n\left(A\epsilon^{P'},\epsilon^{T'},n\right)+\overline{c}\right]d\Phi\left(\epsilon^{P'},\epsilon^{T'}\mid\underline{n}(A\epsilon^{P'},\epsilon^{T'})<n<\overline{n}(A\epsilon^{P},\epsilon^{T})\right) \\ = & \frac{\gamma}{1-\gamma}\alpha\left(e^{\epsilon^T}A\right)^{1-\alpha}n^{\alpha-1}-\frac{\gamma}{1-\gamma}w(A,\epsilon^T,n)-\frac{\gamma}{1-\gamma}w_n(A,\epsilon^T,n)n \\ & +\beta\frac{\gamma}{1-\gamma}\left[-\overline{c}P(n>\overline{n}(Ae^{\epsilon^P},\epsilon^T)|A,n)+\underline{c}P(n<\underline{n}(Ae^{\epsilon^P},\epsilon^T)|A,n)\right] \\ & +\beta\frac{\gamma}{1-\gamma}\int V_n(A\epsilon^P,n,\epsilon^T)d\Phi\left(\epsilon^P,\epsilon^T\mid\underline{n}(A\epsilon^P,\epsilon^T)<n<\overline{n}(A\epsilon^P,\epsilon^T)\right) \\ & +\frac{\gamma}{1-\gamma}\overline{c} \end{aligned}$$

which simplifies to,

$$\begin{aligned} (1-\tau)w(A,\epsilon^T,n) &= (1-\gamma)\,b \\ &\quad +\gamma\alpha\left(e^{\epsilon^T}A\right)^{1-\alpha}n^{\alpha-1}-\gamma w_n(A,\epsilon^T,n)n \\ &\quad +\overline{c}\gamma\left(1-\beta\left(1-f\right)\right)+\underline{c}\beta\gamma f \end{aligned}$$

The solution writes:

$$\begin{aligned} w(A,\epsilon^T,n) &= \frac{1}{1-\tau}\left((1-\gamma)\,b+\overline{c}\gamma\left(1-\beta\left(1-f\right)\right)+\underline{c}\beta\gamma f\right) \\ &\quad +\frac{\gamma\alpha}{1-\tau-\gamma(1-\alpha)}\left(e^{\epsilon^T}A\right)^{1-\alpha}n^{\alpha-1} \end{aligned}$$

$w(A,\epsilon^T,n)$ is homogeneous of degree 0 in (A,n).

C.3 Homogeneity

V is homogeneous of degree 1 in A and n. Define $y = \frac{n}{A}$. Consider $\lambda \geq 0$.

$$\begin{array}{rc} & V(\lambda A_0, \epsilon^T, \lambda n_{-1}) \\ = & \lambda A_0 \sup_{y_t, t\geq 0} E \sum_{t=0}^{\infty} \beta^t \frac{A_t}{A_0} \left\{ e^{\epsilon_t^T (1-\alpha)} y_t^\alpha - w(y_t, \epsilon_t^T) y_t - \underline{c} \left(y_t - \frac{y_{t-1}}{e^{\epsilon_t^P}} \right)^+ - \overline{c} \left(y_t - \frac{y_{t-1}}{e^{\epsilon_t^P}} \right)^- |\lambda A_0, \lambda n_{-1} \right\} \\ = & \lambda V(A_0, \epsilon^T, n_{-1}) \end{array}$$

with the constraint that $\frac{y_{-1}}{e^{\epsilon_0^P}} = \frac{n_{-1}}{A_0}$. The result holds because $\frac{y_{-1}}{e^{\epsilon_0^P}}$ and $\frac{A_t}{A_0}$ for any t are not modified by such a transformation.

Define $v(x, \epsilon^T) = V(1, \frac{n_{-1}}{A}, \epsilon^T)$ with $x = \frac{n_{-1}}{A}$. v satisfies the Bellman equation:

$$\begin{aligned} v(x, \epsilon^T) \quad = \quad & \max_y \left\{ e^{(1-\alpha)\epsilon^T} y^\alpha - w(y, \epsilon^T) y - \underline{c}(y-x)^+ - \overline{c}(y-x)^- \right. \\ & \left. + \beta \int e^{\epsilon^P} v(ye^{-\epsilon^P}, \epsilon^{T'}) d\Phi(\epsilon^P, \epsilon^{T'}) \right\} \end{aligned} \tag{20}$$

C.4 Properties of v and v_y

Define

$$d(y) = \int e^{\epsilon^P} v(ye^{-\epsilon^P}, \epsilon^{T'}) d\Phi(\epsilon^P, \epsilon^{T'})$$

$d(y)$ is written as the convolution of a normal density and a integrable function. Such a convolution is analytic from a well known property of the exponential family of distributions (see Theorem 9 in Lehmann (1986)). Therefore $d(y)$ has derivatives of all order.

$$d_y(y) = \int v_y(ye^{-\epsilon^P}, \epsilon^{T'}) d\Phi(\epsilon^P, \epsilon^{T'})$$

Denote:

$$K_w = \frac{1}{1-\tau} \left((1-\gamma) b + \overline{c}\gamma \left(1 - \beta \left(1-f\right)\right) + \underline{c}\beta\gamma f \right)$$

Compute the following:

$$w(y,1,\epsilon^T)+w_y(y,1,\epsilon^T)y = K_w + \frac{\gamma\alpha^2}{1-\tau-\gamma(1-\alpha)}e^{(1-\alpha)\epsilon^T}y^{\alpha-1}$$

The FOC reads:

$$\begin{aligned}
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)}e^{(1-\alpha)\epsilon^T}\underline{y}(\epsilon^T)^{\alpha-1} - K_w - \underline{c} + \beta\int v_y(\underline{y}(\epsilon^T)e^{-\epsilon^P},\epsilon^{T'})d\Phi(\epsilon^P,\epsilon^{T'}) &= 0 \\
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)}e^{(1-\alpha)\epsilon^T}\overline{y}(\epsilon^T)^{\alpha-1} - K_w + \overline{c} + \beta\int v_y(\overline{y}(\epsilon^T)e^{-\epsilon^P},\epsilon^{T'})d\Phi(\epsilon^P,\epsilon^{T'}) &= 0
\end{aligned}$$

and it translates in the optimal policy:

$$y(x,\epsilon^T) = \begin{cases} \underline{y}(\epsilon^T) & \text{if } x < \underline{y}(\epsilon^T) \\ x & \text{if } \underline{y}(\epsilon^T) < x < \overline{y}(\epsilon^T) \\ \overline{y}(\epsilon^T) & \text{if } x > \overline{y}(\epsilon^T) \end{cases} \tag{21}$$

also:

$$v_y(x,\epsilon^T) = \begin{cases} \underline{c} & \text{if } x < \underline{y}(\epsilon^T) \\ \frac{\alpha(1-\gamma)}{1-\tau-\gamma(1-\alpha)}e^{(1-\alpha)\epsilon^T}x^{\alpha-1} - K_w + \beta d_y(x) & \text{if } \underline{y}(\epsilon^T) < x < \overline{y}(\epsilon^T) \\ -\overline{c} & \text{if } x > \overline{y}(\epsilon^T) \end{cases}$$

Then,

$$\begin{aligned}
d_y(y) &= \int v_y(ye^{-\epsilon^P}, \epsilon^{T'}) d\Phi(\epsilon^P, \epsilon^{T'}) \\
&= \underline{c} P\left(ye^{-\epsilon^P} < \underline{y}(\epsilon^{T'})\right) \\
&\quad -\overline{c} P\left(ye^{-\epsilon^P} > \overline{y}(\epsilon^{T'})\right) \\
&\quad +\frac{\alpha(1-\gamma)}{1-\tau-\gamma(1-\alpha)} y^{\alpha-1} \int e^{(1-\alpha)\left(\epsilon^{T'}+\epsilon^P\right)} d\Phi\left(\epsilon^P, \epsilon^{T'} \mid \underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right) \\
&\quad +K_w P\left(\underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right) \\
&\quad +\beta \int d_y\left(ye^{-\epsilon^P}\right) d\Phi\left(\epsilon^P, \epsilon^{T'} \mid \underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right)
\end{aligned}$$

I examine the different components of d_y.

$$\begin{aligned}
P\left(ye^{-\epsilon^P} < \underline{y}(\epsilon^{T'})\right) &= \int \bar{\Phi}_{\epsilon^P}\left(\log \frac{y}{\underline{y}(\epsilon^{T'})}\right) d\Phi_{\epsilon^T}(\epsilon^{T'}) \\
P\left(ye^{-\epsilon^P} > \overline{y}(\epsilon^{T'})\right) &= \int \Phi_{\epsilon^P}\left(\log\left(\frac{y}{\overline{y}(\epsilon^{T'})}\right)\right) d\Phi_{\epsilon^T}(\epsilon^{T'})
\end{aligned}$$

Define the function

$$\begin{aligned}
\lambda(a,b,c) &= \int_a^b e^{c\epsilon^P} d\Phi_{\epsilon^P}(\epsilon^P) \\
&= e^{\sigma^2_{\epsilon^P}\frac{c(c-1)}{2}}\left[\Phi_{\sigma^2_{\epsilon^P}\frac{c-1}{2},\sigma^2_{\epsilon^P}}(b) - \Phi_{\sigma^2_{\epsilon^P}\frac{c-1}{2},\sigma^2_{\epsilon^P}}(a)\right]
\end{aligned}$$

Then,

$$\begin{aligned}
&\int e^{(1-\alpha)\left(\epsilon^{T'}+\epsilon^P\right)} d\Phi\left(\epsilon^P, \epsilon^{T'} \mid \underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right) \\
= &\int e^{(1-\alpha)\epsilon^{T'}}\left[\int e^{(1-\alpha)\epsilon^P} d\Phi_{\epsilon^P}\left(\epsilon^P \mid \underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right)\right] d\Phi_{\epsilon^T}(\epsilon^T) \\
= &\int e^{(1-\alpha)\epsilon^{T'}} \lambda\left(\log\left(\frac{y}{\overline{y}(\epsilon^{T'})}\right), \log\left(\frac{y}{\underline{y}(\epsilon^{T'})}\right), 1-\alpha\right) d\Phi_{\epsilon^T}(\epsilon^T)
\end{aligned}$$

Summarizing,

$$
\begin{aligned}
d_y(y) \quad = \quad & \int \Big\{ \underline{c}\bar{\Phi}_{\epsilon^P}\left(\log \frac{y}{\underline{y}(\epsilon^{T'})}\right) - \bar{c}\Phi_{\epsilon^P}\left(\log\left(\frac{y}{\overline{y}(\epsilon^{T'})}\right)\right) \\
& + \frac{\alpha(1-\gamma)}{1-\tau-\gamma(1-\alpha)} y^{\alpha-1} e^{(1-\alpha)\epsilon^{T'}} \lambda\left(\log\left(\frac{y}{\overline{y}(\epsilon^{T'})}\right), \log\left(\frac{y}{\underline{y}(\epsilon^{T'})}\right), 1-\alpha\right) \\
& - K_w\left[\Phi_{\epsilon^P}\left(\log\frac{y}{\underline{y}(\epsilon^{T'})}\right) - \Phi_{\epsilon^P}\left(\log\left(\frac{y}{\overline{y}(\epsilon^{T'})}\right)\right)\right] \\
& + \beta\left[\int d'\left(ye^{-\epsilon^P}\right) d\Phi_{\epsilon^P}\left(\epsilon^P \mid \underline{y}(\epsilon^{T'}) < ye^{-\epsilon^P} < \overline{y}(\epsilon^{T'})\right)\right]\Big\} d\Phi_{\epsilon^T}(\epsilon^T)
\end{aligned} \qquad (22)
$$

D Solving the Model

I use a collocation method as described in Judd (1998).

D.1 Parametrizing $d_y(y)$ and Approximation Grids

Let $\hat{d_y}(y;p)$ be the function used to approximate $d_y(y;p)$ with p a vector of parameters. I assume it can be written as linear combination of a set of P known linearly independent basis function $B_1, \ldots, B_n$,

$$\hat{d_y}(y;p) = \sum_{i=1}^{P} p_i B_i(y)$$

whose basis coefficients $p_1, \ldots, p_P$ are to be determined.

Consider Q Chebyshev interpolation nodes for y which has bounded support. The Q Chebyshev interpolation nodes on $[-1, 1]$ are:

$$z_q = -\cos\left(\frac{2q-1}{2Q}\pi\right)$$

Adjust the nodes to the $[y_{\min}, y_{\max}]$ interval:

$$y_q = (z_q + 1)\frac{y_{\max} - y_{\min}}{2} + y_{\min}$$

Consider the change of variable $q = \frac{\epsilon^T - \mu_{\epsilon^T}}{\sqrt{2}\sigma_{\epsilon^T}}$ such that $\epsilon^T = \sqrt{2}\sigma_{\epsilon^T} q + \mu_{\epsilon^T}$. Let q and h be a L-vectors of Gauss-Hermite nodes and weights for approximating the integral with respect to ϵ^T.

Consider the function $F(\epsilon^T)$ and the integral

$$\begin{aligned}\int F(\epsilon^T) d\Phi_{\epsilon^T}(\epsilon^T) &= \frac{1}{\sigma_T\sqrt{2\pi}} \int F(\epsilon^T) \exp\left(-\frac{\left(\epsilon^T-\mu_{\epsilon^T}\right)^2}{2\sigma^2_{\epsilon^T}}\right) d\epsilon^T \\ &= \pi^{-1} \int F\left(\sqrt{2}\sigma_{\epsilon^T} q + \mu_{\epsilon^T}\right) e^{-q^2} dq\end{aligned}$$

that can be approximated by

$$\pi^{-1} \sum_{l=1}^{L} h_l F\left(\sqrt{2}\sigma_{\epsilon^T} q_l + \mu_{\epsilon^T}\right)$$

For each $l \in L$, the following integral has to be approximated:

$$\begin{aligned}&\int d'\left(ye^{-\epsilon^P}\right) d\Phi_{\epsilon^P}\left(\epsilon^P \mid \underline{y}(\epsilon^T) < ye^{-\epsilon^P} < \overline{y}(\epsilon^T)\right) \\ = &\int_{\log \frac{\overline{y}(\epsilon^T)}{y}}^{\log \frac{\underline{y}(\epsilon^T)}{y}} d'\left(ye^{-\epsilon^P}\right) d\Phi_{\epsilon^P}\left(\epsilon^P\right)\end{aligned}$$

Let q^p and h^p be the L^P-vectors of Gauss-Legendre nodes and weights over the interval $[-1, 1]$. Consider the linear tranformation $q^p = 2\frac{\epsilon^p - a}{b-a}$ implying $dq^p = 2\frac{d\epsilon^p}{b-a}$ and $\epsilon^p = a + \frac{(q^p+1)(b-a)}{2}$.

Consider the function $F(\epsilon^P)$ and the integral

$$\begin{aligned}\int_a^b F(\epsilon^P) d\Phi_{\epsilon^P}(\epsilon^P) &= \frac{1}{\sigma_T\sqrt{2\pi}} \int_a^b F(\epsilon^P) \exp\left(-\frac{\left(\epsilon^P-\mu_{\epsilon^P}\right)^2}{2\sigma^2_{\epsilon^P}}\right) d\epsilon^P \\ &= \frac{1}{\sigma_T\sqrt{2\pi}} \frac{b-a}{2} \int_{-1}^{1} F\left(a + \frac{(q^p+1)(b-a)}{2}\right) \exp\left(-\frac{\left(a + \frac{(q^p+1)(b-a)}{2} - \mu_{\epsilon^P}\right)^2}{2\sigma^2_{\epsilon^P}}\right) dq^P\end{aligned}$$

that can be approximated by

$$\frac{1}{\sigma_T\sqrt{2\pi}} \frac{b-a}{2} \sum_{l^p=1}^{L^p} h^p_{l^p} F\left(a + \frac{\left(q^p_{l^p}+1\right)(b-a)}{2}\right) \exp\left(-\frac{\left(a + \frac{\left(q^p_{l^p}+1\right)(b-a)}{2} - \mu_{\epsilon^P}\right)^2}{2\sigma^2_{\epsilon^P}}\right)$$

D.2 Contraction mapping for d_y

The approximate contraction mapping condition for d_y is then

$$
\begin{aligned}
& \hat{d}_y(y;p) \\
= \quad & \pi^{-1}\sum_{l_1=1}^{L} h^T_{l_1} \times \left\{ \underline{c}\bar{\Phi}_{\epsilon^P}\left(\log \frac{y}{\hat{\underline{y}}(\epsilon^T_{l_1})}\right) - \overline{c}\Phi_{\epsilon^P}\left(\log\left(\frac{y}{\hat{\overline{y}}(\epsilon^T_{l_1})}\right)\right)\right. \\
& + \frac{\alpha(1-\gamma)}{1-\tau-\gamma(1-\alpha)} y^{\alpha-1} e^{(1-\alpha)\epsilon^T_{l_1}} \lambda\left(\log\left(\frac{y}{\hat{\overline{y}}(\epsilon^T_{l_1})}\right), \log\left(\frac{y}{\hat{\underline{y}}(\epsilon^T_{l_1})}\right), 1-\alpha\right) \\
& - K_w\left[\Phi_{\epsilon^P}\left(\log \frac{y}{\hat{\underline{y}}(\epsilon^T_{l_1})}\right) - \Phi_{\epsilon^P}\left(\log\left(\frac{y}{\hat{\overline{y}}(\epsilon^T_{l_1})}\right)\right)\right] \\
& + \beta\frac{1}{\sigma_T\sqrt{2\pi}} \frac{\log\frac{\underline{y}(\epsilon^T)}{y} - \log\frac{\overline{y}(\epsilon^T)}{y}}{2} \\
& \left. \times \sum_{l_2=1}^{L} h^P_{l_3} \exp\left(-\frac{\left(\epsilon^P_{l1l2}-\mu_{\epsilon^P}\right)^2}{2\sigma^2_{\epsilon^P}}\right)\hat{d}_y\left(ye^{-\epsilon^P_{l_1 l_2}};p\right) I\left[\log\left(\frac{y}{\hat{\overline{y}}(\epsilon^T_{l_1})}\right) \leq \epsilon^P_{l_1 l_2} \leq \log\left(\frac{y}{\hat{\underline{y}}(\epsilon^T_{l_1})}\right)\right]\right\}
\end{aligned}
$$

To solve for the unknown thresholds, k indexes values of ϵ^T on a grid of size L. I apply a non-linear equations solver to:

$$
\begin{aligned}
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)} e^{(1-\alpha)\epsilon^T_k} \underline{y}_k^{\alpha-1} - K_w - \underline{c} + +\beta\hat{d}_y(\underline{y}_k;p) &= 0 \\
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)} e^{(1-\alpha)\epsilon^T} \overline{y}_k^{\alpha-1} - K_w + \overline{c} + \beta\hat{d}_y(\overline{y}_k;p) &= 0
\end{aligned}
$$

with $\overline{y}_k = \overline{y}(\epsilon^T_k)$ and $\underline{y}_k = \underline{y}(\epsilon^T_k)$.

E Aggregation

E.1 The Economy

There is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L.

In order to hire unemployed workers, firms must post vacancies. There is a matching function

$M = M(U, V)$ that regulates the number of job creation, M that the economy can sustain given that there are V vacancies and U unemployed workers. Vacancies are therefore filled with probability $q = \frac{M}{V}$. There is no aggregate uncertainty, thus, $q < 1$ is a parameter taken as given by firms.

Using a law of large numbers, the cost of creating a job is deterministic. Hence, given the job filling probability, the firm can hire the exact number of workers it is willing to. The cost of creating jobs $\underline{c}$ can alternatively be interpreted as a cost of creating a vacancy. There is a flow cost per vacancy c that transform into job creation wih probability q. Then, the costs of creating a job is $\underline{c} = \frac{c}{q}$.

E.2 Stationary distribution of employment

A stationary distribution of productivity exists among incumbents. But because of permanent shocks, a stationary distribution of employment and output does not exists. It is necessary to introduce entry and exit to the model. Firms die with exogenous probability λ. Firms enter with a distribution H^e, no workers and at a constant flow δ. Let $\mu(A, n)$ be the stationary distribution of A, n.

For any measurable sets B, C:

$$\begin{aligned} \mu^{'}(B \times C) &= (1-\lambda) \int I \left\{ \sigma(Ae^{\epsilon^P}, \epsilon^T, n) \in B \right\} \times P(A^{\epsilon^P} \in C) \times \Phi(d\epsilon^T) \times \mu\left(dA, dn\right) \\ &\quad + \delta \times \int I \left\{ \sigma(Ae^{\epsilon^P}, \epsilon^T, 0) \in B \right\} \times P(A^{\epsilon^P} \in C) \times \Phi(d\epsilon^T) \times H^e(dA) \end{aligned}$$

The total mass of firms T evolves according to $T' = (1-\lambda)T + \delta$. To keep this number constant it must be: $\lambda T = \delta$. With $T = 1$, it means $\delta = \lambda$.

E.3 Aggregation and Steady State Equilibrium

The Number of unemployed is:

$$U = L - \int \sigma(Ae^{\epsilon^P}, \epsilon^T, n) d\Phi(\epsilon^P, \epsilon^T) \times d\mu(A, n)$$

The number of match is:

$$M = \int (n'(A, \epsilon^T, n) - n) I \left\{ n'(A, \epsilon^T, n) > n \right\} \times d\Phi(\epsilon^P, \epsilon^T) \times d\mu(A, n)$$

and accordingly, the number of vacancies is $V = qM$.

The job finding probability for a job-seeker writes: $f = \frac{M(V,U)}{U}$.

The number of separations is:

$$S = \int (n - n'(A, \epsilon^T, n)) I \left\{ n'(A, \epsilon^T, n)] < n \right\} \times d\Phi(\epsilon^P, \epsilon^T) \times d\mu(A, n)$$

At the steady state, it holds: $S = M$.

F The EM Algorithm

The EM algorithm is a general approach to maximum likelihood estimation for problems in which the data can be viewed as consisting as N multivariate observations (z_i, t_i) in which t_i is unobserved. For mixture models, z_i is observation for firm i and $t_i = (t_{i1}, ..., t_{iK})$ is defined as:

$$t_{ik} = \begin{cases} 1 & \text{if } z_i \text{belongs to group } k \\ 0 & \text{otherwise} \end{cases}$$

The t_i's are iid realizations from a multinomial distribution of one draw from K types with probability $\lambda_1, ..., \lambda_K$. The complete-data likelihood (say L_c) is then:

$$L_c(\Psi) = \prod_{i=1}^{N} \sum_{k=1}^{K} \left[\lambda_k \phi_k(z_i; \mu_k, \Sigma_k) \right]^{t_{ik}} \tag{23}$$

At each iteration j, givenΨ^j the EM algorithm alternate between two steps.

1. E Step: Compute the conditional expectation of the complete-data log-likelihood L_c given the observed data and the current parameter estimated Ψ^j. From the linearity of L_c in the

unobservables t_{ik}, it gives for every $k = 1, ..., K$ and $i = 1, ..., N$:

$$t_{ik}^j = \frac{\lambda_k^j \phi_k(z_i; \mu_k^j, \Sigma_k^j)}{\sum_{k'=1}^K \lambda_{k'}^j \phi_{k'}(z_i; \mu_{k'}^j, \Sigma_{k'}^j)} \tag{24}$$

2. M Step: Maximize the expected log-likelihood from the E step. Expressions are given explicitly by:

$$\lambda_k^{j+1} = \frac{1}{N} \sum_{i=1}^N t_{ik}^j \tag{25}$$

$$\mu_k^{j+1} = \frac{\sum_{i=1}^N t_{ik}^j z_i}{\sum_{i=1}^N t_{ik}^j} \tag{26}$$

$$\Sigma_k^{j+1} = \frac{\sum_{i=1}^N t_{ik}^j (z_i - \mu_k^{j+1})(z_i - \mu_k^{j+1})'}{\sum_{i=1}^N t_{ik}^j} \tag{27}$$