Employment Adjustment and Labor Utilization *

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Abstract

Labor adjustment costs determine the extent to which firms vary their employment in response to exogenous shocks. The standard models that are used to formalize and assess the impact of labor adjustment costs only allow firms to change the size of their workforce (extensive margin) and not the number of hours that each worker works (intensive margin). The goal of this study is to relax this assumption and to propose a dynamic general equilibrium model that introduces labor adjustment on both intensive and extensive margins, and subsequently test the model using firm-level data. The model also allows for an on-the-job search that generates different vacancy filling and attrition rates across firms. Calibrated to fit data from a unique matched employer-employee panel of Danish firms, the model captures the negative co-movement between hours and employment at the firm level; that growth in hours precedes growth in employment; and the relation between firm size, wages, and productivity. I find that the average cost of hiring a new worker is equal to about one month of wages. The parameterized model is then used to simulate an increase in hiring costs, which generates a substantial rise in the unemployment rate and a fall in the job-finding rate. I show that ignoring general equilibrium channels by leaving vacancy filling and quit rates unchanged leads to a twice as large increase in the unemployment rate.

Keywords: adjustment cost, labor utilization, worker flows, on-the-job search
JEL Classification: J23, J31, J63, J64, L11

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1 Introduction

The extent to which firms and consumers respond to changes in their environment depends mainly on the costs of adjustment. Thus labor adjustment costs – such as recruiting costs, costs of screening and training new employees, layoff notice periods, and mandated severance pay – play a prominent role in determining both the timing and the extent of employment variation in response to exogenous shocks. To the extent that adjustment costs hinder firms from exploiting profitable opportunities and responding to productivity changes, they can be detrimental to the reallocation of labor resources from less to more productive firms, and to economic growth in general[1]. Since a significant part of the labor adjustment costs is related to various forms of labor market regulations, that has motivated a by-now extensive literature that examines the impact of job security policies on employment dynamics.

In the short run, firms can respond to productivity fluctuations by varying their labor utilization. Consequently, hours of work become an important channel through which firms can modify their use of labor resources and economize on hiring and firing costs. The standard models that are used to formalize and assess the impact of labor adjustment costs only allow firms to change the number of workers they employ (extensive margin) and not the amount that each worker works (intensive margin). The goal of this study is to relax that assumption and to propose a dynamic model, which includes both intensive and extensive margins of labor adjustment, and subsequently test the model using firm-level data.

Due to the scarcity of high-frequency micro data on work hours, the existing empirical literature on labor adjustment that exploits information on work hours is limited to industry-level data or establishment-level data that pertain to the US manufacturing sector and that are more than three decades old[2]. The empirical analysis in this paper is based on a unique dataset drawn from a matched employer-employee panel of Danish administrative firm data that contains all private firms in the economy for the period of 1999-2006. This dataset includes information on quarterly total work hours and monthly employment, from which an average work hours series can be obtained for each firm. Moreover, time-consistent firm and person identifiers ensure that accurate monthly hiring and separation flows can be constructed. Using this information, I examine and document the labor adjustment patterns at the firm level on both intensive and extensive margins, the relative importance of the two channels, and the interaction between them.

The empirical evidence suggests that firms use variation in hours to mitigate changes in the number of workers. The growth rates of average hours worked and employment are negatively correlated at the firm-level; moreover, lagged changes in hours are positively correlated with changes in workforce[3]. To explain these empirical facts, I introduce hours margin of labor adjustment in the firm's employment decision. The exact mechanism works through a different timing of hours and employment responses. For example, suppose that it takes time to hire a new worker. Then, in the event of a positive shock to the firm's profitability, the firm raises average hours of work instantaneously, while adjustment in employment is sluggish. As the firm's labor force builds up to its new desired level, the average work hours fall. Hence, the theory predicts that growth in hours and employment are inversely related. Furthermore, the changes in average hours precede changes in the number of workers, thus leading employment growth.

In this paper, I develop a general equilibrium search model that includes a non-trivial theory of

[1] More than half of aggregate productivity growth in the US appears to be driven by reallocation (see for instance Foster, Haltiwanger and Krizan (2006)).

[2] See for instance Nadiri and Rosen (1973), Hamermesh and Pfann (1996) for applications based on the industry-level data. Cooper, Haltiwanger and Willis (2007) build their analysis on the Longitudinal Research Database, which includes quarterly work hours information at the establishment level for a sample of manufacturing firms for the period of 1972-1980.

[3] These facts have been documented for the US labor market by Cooper, Haltiwanger and Willis (2007).

a producer with multiple jobs, as opposed to a standard search theory, in which a firm employs one worker at a time. The driving force of the model is idiosyncratic profitability shocks, which firms can accommodate by changing the work hours of their existing employees, instead of (or jointly with) varying the size of their labor force. The presence of frictions in the labor market means that matching workers with vacant jobs takes time and uses resources. On the other hand, raising hours is expensive since the firm has to compensate its employees for an increase in utility costs arising from working longer hours. Hence, the firm faces a trade-off between changes in average work hours and the number of workers. The different timing of hours and employment adjustment generates the dynamic interaction between these two adjustment margins that is the focal point of this paper.

Hours of work and compensation are defined through the bargaining process between the firm and its employees. In order for firms to be willing to trade off the number of workers with the amount of hours each individual works, I abandon a commonly used assumption that the production (or revenue) function exhibits constant returns to scale in labor input. Then, the decreasing marginal revenue means that an outcome of the bargaining with one worker affects all other employees in the firm. Given search frictions in the market, it takes time to replace workers and, therefore, employment is considered to be predetermined at the bargaining stage. This setup provides a natural environment for a Stole and Zwiebel (1996) individual bargaining framework within multi-worker firms, in which the firm and its employees bargain over the current output. This bargaining framework allows for workers to quit the job if wages fall too low. Therefore, in the event of a negative profitability shock, the firm does not necessarily incur dismissal costs.

Another contribution of this study is to extend the standard adjustment model to allow for on-the-job search. Job-to-job transitions generate different vacancy filling and attrition rates across firm types, which has important implications for the firm's optimal employment policy in the presence of labor adjustment costs. That is, the firm faces an increase in the quit rate in the event of a negative shock and, vice versa, the firm finds it easier to fill vacancies and keep its current employees in the event of a positive shock. Many of the existing studies essentially ignore this channel by assuming constant quit rates and thus making no distinction between net and gross employment changes. Ample empirical evidence suggests that, by focusing on the net employment changes, we disregard a substantial worker turnover at the firm level (see for instance Burgess, Lane and Stevens (2001)). Moreover, quit rates are found to be different across firms, depending on their size, age, wages, etc. To reconcile the theory with these empirical facts, I allow for endogenous and stochastic quits by incorporating on-the-job search into the model.

The model is calibrated to fit the Danish firm data and appears to be quite successful in capturing the overall features of the data. It is able to match the rich patterns of labor adjustment at the firm level, including the distribution of hiring and separation rates by net employment growth, firm size, and wages. On-the-job search is a crucial component that enables the model to capture most of the characteristics of the data related to worker and job flows. Regarding the changes in labor utilization, the simulation reproduces the negative co-movement between hours and employment, though it falls short with respect to magnitude of the relationship observed in the data. In addition, the model's predictions are consistent with stylized facts on the association between employment, wages, and productivity.

Given the calibrated parameters, I find that the average cost of hiring a new worker is equal to about one month of wages. The parameterized model is then used to analyze the effect of changes in adjustment costs on unemployment and work hours. First, I simulate an increase in vacancy creation costs. Doubled vacancy posting costs generate an increase in the unemployment rate of about four percentage points. Moreover, firms substitute towards the intensive margin of adjustment in that the variation in hours growth rises, while the variation in employment growth falls. Second, I show that a hiring subsidy in the amount of two weeks of wages is effective in reducing unemployment by 1.5 percentage points. I also demonstrate the importance of general equilibrium effects by performing the same experiments in a partial equilibrium framework; that is, I increase the vacancy posting costs (or introduce a hiring subsidy) while keeping the vacancy filling rate and the quit rate at the same level as in the benchmark model. In the partial equilibrium case, the effect of changes in hiring costs on the unemployment rate is almost doubled. These results suggest that using partial equilibrium models to evaluate the impact of adjustment costs may significantly overestimate the quantitative effect of the costs on unemployment.

In contrast to other studies of labor adjustment, I find that firing costs have virtually no effect on changes in labor resources. Allowing for wage bargaining and on-the-job search is at the heart of this result, since most of the reduction in the workforce in the event of an adverse profitability shock is achieved though quits (as opposed to layoffs). The main reason for that is high rates of job-to-job transitions in Denmark and a fairly low unemployment duration. One can imagine that in countries with limited worker mobility, as well as in economic downturns (as they are associated with lower quit rates - see for instance Nagypal (2008)), workers will be less willing to quit the firm in the event of a negative shock, thus making firing constraints binding for employers. Although this finding is preliminary, in that more work is needed to examine the effect of dismissal costs in the presence of aggregate shocks, it indicates that bringing endogenous quits into labor adjustment models may produce different results and, ultimately, different policy implications regarding changes in labor market regulations.

The numerical simulation in this paper is a step towards the structural estimation of the model. The simulation of the model is used to establish a mapping between structural parameters and moments observed in actual data. From relations between wages, labor productivity, hours, and employment, I can identify the fundamental model parameters. Estimating the model by indirect inference approach (see Gourieroux, Monfort and Renault (1993)), which essentially minimizes a distance criterion between key moments from actual data and simulated data, and recovering structural parameters is an anticipated task on my research agenda. The estimation of the model allows for quantifying the adjustment costs and evaluating their impact on employment, firms' profits, and workers' welfare. Using the obtained structural parameters, I can perform policy experiments of such changes in labor market policy as mandatory overtime premium, severance pay, employment subsidies, etc.

There are several strands of literature related to this paper. This study is obviously linked to empirical and theoretical work on labor adjustment costs and their impact on labor demand (see Hamermesh and Pfann (1996) for a comprehensive survey)[4]. Recent papers that attempt to quantify the adjustment costs find considerable hiring and firing costs (see among others Rota (2004), Goux, Maurin and Pauchet (2001), and Kramarz and Michaud (2004)). Most of these studies, though, tend to focus on changes in workforce and ignore the intensive margin of employment adjustment.

Previous research that accounts for labor utilization in the adjustment cost models includes Caballero, Engel and Haltiwanger (1997). They use variation in hours to identify the (unobserved) gap between the actual and target levels of employment, which, in turn, determines the hazard rate of employment adjustment[5]. Cooper and Willis (2009) estimate a structural model (based on the "gap" methodology

[4] The literature on labor adjustment costs is closely related to the investment literature (see among others Caballero and Engel (1991; 1992)) and studies on factor demand in general (see Nadiri and Rosen (1973)). Bond and Van Reenen (2007) provide a survey of econometric research on adjustment processes for both capital and labor that uses micro data. Recent papers of Bloom (2009) and Mertz and Yashiv (2007) model capital and labor adjustment costs jointly.

[5] As Caballero, Engel and Haltiwanger (1997) note, their estimation procedure may yield biased estimates since the error term in the identifying equation is likely to be correlated with changes in hours. That is, a positive shock to profitability may induce the plant to increase both hours and the desired level of employment. They argue that this problem is partially alleviated by looking at periods of large adjustments, so that the changes in hours and employment overcome the changes

of Caballero et al. (1997)), in which the relationship between variation in hours and employment and shocks is specified explicitly. These papers model labor adjustment in a partial equilibrium framework and abstract from the labor supply side effects. Cooper, Haltiwanger and Willis (2007) examine variation in hours, employment, and vacancies based on a general equilibrium search model with frictions in hiring and firing[6]. I extend their analysis to introduce wage bargaining and on-the-job search. That produces stochastic and endogenous quits in the model, which play an important role for optimal employment policies of firms and, in fact, generate different results in terms of the significance of adjustment costs for labor dynamics.

This paper is also related to literature on worker turnover and job creation and destruction at the firm level. Empirical studies have documented that worker flows and job flows are quite distinct - most employers are simultaneously hiring and facing separations; moreover, quit rates are found to be increasing at contracting firms (see for instance Burgess, Lane and Stevens (2001), Davis, Faberman and Haltiwanger (2006)). Until recently, theoretical work on employment adjustment has commonly assumed constant quit rates and focused on net employment growth. The only study I am aware of that links the theory of quits at the firm level with empirical evidence is a recent paper by Faberman and Nagypal (2008). They introduce a notion of replacement hiring, thus explicitly distinguishing the cost of creating a new job from the cost of replacing a worker. In contrast to their model, I assume a decreasing returns to scale revenue function to account for the hours-workers trade-off in the firm's labor demand. That has different implications for wage bargaining; moreover, it enables the model to produce firm size distribution that is in line with the data.

In many aspects of the methodology, this paper is linked to standard random search models (see for instance Mortensen and Pissarides (1994), Mortensen (2003)) and more recent work that introduces a theory of multi-worker firms into the equilibrium search model (see for instance Lentz and Mortensen (2009)).

The paper proceeds as follows. Section 2 presents empirical evidence on employment and hours adjustment using Danish firm data. Section 3 introduces and describes the model. Section 4 shows the calibration of the model and its fit to the data. It then demonstrates the effect of an increase in adjustment costs on the unemployment rate and the job-finding rate in the partial and general equilibrium framework. Section 5 summarizes the findings and outlines directions for future work. The Appendix provides details on the data sources used in this paper, as well as on the numerical simulation procedure.

2 Data

This section documents empirical facts on labor adjustment that provide motivation for this paper. Section 2.1 presents key moments that pertain to firm-level hours and workforce dynamics. The results reported below suggest that to adjust their labor resources firms use variation in hours, which is then followed by changes in the number of workers. In Section 2.2, I examine gross and net employment change patterns at the firm level and document the differences between them.

The empirical analysis in this paper is based on a matched employer-employee panel of Danish firms. It includes all businesses in the economy for 1999-2006. This dataset is unique in that it provides high-

in error.

[6] For general equilibrium business cycle model with labor-market frictions see for instance Andolfatto (1996). Compared to standard RBC models, incorporating search frictions delivers a substantial implovement in terms of matching relative variation in hours, employment, and productivity in the aggregate data. These models, however, predict that hours and employment move in the same direction (consistent with the macro data) and cannot be used to explain the negative correlation between growth in hours and employment found in the firm-level data.

frequency (monthly or quarterly) detailed information on employment changes and work hours of firms' employees, which is ideal for investigation of the dynamic interaction of adjustment on both intensive and extensive margins. Moreover, the data are drawn from administrative records and therefore are more precise than survey-based datasets used in the existing empirical studies (see for instance Cooper, Haltiwanger and Willis (2007)).

The available data come from two major sources (a full data description is contained in Appendix A). The first dataset is a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly number of workers for each firm is obtained as a head count of all individuals employed in that firm. Quarterly employment is derived as the average of three months employment. The particular structure of this dataset enables me to construct monthly hires and separations for each firm. Moreover, high-quality longitudinal links ensure that the constructed worker and job flows series are accurate.

A work hours series is derived from the second data source, which contains firm mandatory pension contributions data collected on a quarterly basis. In Denmark, firms are required to pay pension contributions for each employee according to her weekly hours of work. In particular, the rule is as follows:

- full amount of contribution is paid for an employee working more than 27 hours a week[7];
- 2/3 of the full amount is paid for an employee working between 18 and 27 hours a week;
- 1/3 of the full amount is paid for an employee working between 9 and 18 hours a week;
- zero contribution is paid for an employee working less than 9 hours a week.

For each firm, the sum of quarterly contributions over the full set of its employees is reported. Given the proportionality of the contribution schedule, I construct a work hours measure by dividing the total sum of payments by the payment amount for a full-time employee and multiplying by 27 hours a week. That is, I implicitly assign the left boundary of each of the 9-hour intervals to all workers; therefore, the hours variable constructed in this manner represents a lower bound on weekly hours of work. The empirical moments below refer to this hours measure, unless stated otherwise[8].

Combining total work hours data derived from the pension contributions with quarterly employment, I compute a quarterly average hours series for each firm, which I use as a measure of labor utilization. It is important to bear in mind that this hours variable, due to its interval nature, may mask some of the variation in actual hours that happens in response to changes in demand, input prices, etc. This is particularly true in the event of a positive shock since it is impossible to identify overtime. Most of the observed variation in hours comes from employees moving between the 9-hour intervals. For instance, if some workers switch from part-time to full-time jobs then an increase in labor utilization will be reflected in the data. Therefore, the observed hours variation is thought of as a lower bound on actual hours variation.

The empirical analysis is carried out based on private companies. To reduce the noise in the data, I exclude from the analysis all firms that employ fewer than five employees for six consecutive months

[7] Full contribution was 670.95 DKK in 1999-2005 and 731.70 DKK in 2006 per quarter.

[8] Alternatively, I construct an upper bound measure of work hours (see Appendix A for details), which gives very similar results for most of the empirical relations presented in this paper. The only case in which the two measures produce different results is the relationship between wages and hours (discussed in detail in Section 4). Thus, I report moments based on the lower bound measure of work hours for all relations but wages-hours correlation.

(they comprise 53.7% of all firm-quarter observations, but only 6.3% of total employment). The resulting dataset has 120,058 firms that are observed for 14 quarters on average[9].

2.1 Hours and Employment

Two key questions to address are (i) do firms vary average work hours of their employees and (ii) is the observed dynamic interaction between hours and employment consistent with the model of adjustment costs. If hiring is impeded by search frictions then average work hours overshoot in response to a positive shock and start falling, as the firm's labor force builds up to its new desired level. In that case, we expect to see growth in hours and employment moving in different directions in the data. Moreover, the changes in average hours precede changes in number of workers, thus leading employment growth. Likewise, a negative shock in combination with mandated advanced layoff notice produces an immediate hours response and a more sluggish employment drop, thus generating a similar negative co-movement between these two variables.

The dataset underlying the empirical moments contains quarterly observations of total hours and monthly employment, from which I construct quarterly employment (N_t) and hours per employee (H_t) series. In the analysis below, I focus on the growth rates of hours and employment expressed as first differences of log variables. The results reported here are based on the raw series, as well as employment share-weighted moments. In addition, given that in the model I abstract from aggregate demand shocks, I remove aggregate time effects from the original series and explore the cross-sectional variation in growth rates of hours and employment.

Figure 1 depicts the histograms of growth rates of both variables. First, we observe a significant inaction region in both hours and employment. A spike at zero change in average work hours is not surprising given the interval nature of the hours variable. The growth of employment is measured more precisely and thus is more informative about the region of zero employment adjustment: about 18% of firms employ the same number of workers in any two consecutive quarters. Much lower magnitudes of employment changes have been reported in empirical studies that look at other European countries. Varejao and Portugal (2007), for instance, find that employment remains unaltered over the course of a quarter for 74.7% of the establishments in a representative sample of Portuguese firms. This finding suggests that the Danish labor market, in comparison to other European countries (especially in continental Europe), is characterized by relatively low adjustment costs.

Second, despite the fact that hours are measured in intervals, there is a significant variation in hours growth. In fact, the standard deviation of hours and employment growth is about the same (see Table 1). This finding provides some evidence that firms use both intensive and extensive margins to make adjustments to their labor input.

Table 1 presents empirical moments that describe the relationship between hours and employment adjustment. Similarly to the results reported by Cooper, Haltiwanger and Willis (2007) for the US labor market, I find a negative correlation between hours and employment growth at the firm level. Figure 2 shows non-parametric regression of hours growth on workforce growth[10]. Hours-employment growth relationship is monotone and the negative correlation between the two series is observed for virtually

[9] According to the FIDA dataset (yearly matched employer-employee data that provides information on establishment level employment), more than 99.9% of firms are one-establishment units; while less than 0.1% of firms have more than one establishment. Therefore, the results in this paper are comparable to previous studies that have used establishment-level micro data.

[10] Non-parametric regressions in this paper are based on Nadaraya-Watson estimator (with Gaussian or Uniform kernels) and confidence intervals are obtained by bootstrapping (see Simonoff (1996) for theory and applications of kernel-based regressions and Horowitz (1997) for bootstrapping methods).

all values of employment growth. Moreover, changes in hours lead changes in employment: there is a positive association between employment growth this period and hours growth last period. Both of these findings call for a labor dynamic model that incorporates variation in both the number of employees and the amount of hours each employee works.

Table 1: The relationship between hours and employment growth rates.

	Non-weighted	Emp.share weighted	Emp.share-weighted no time effects
$Std.dev\,(\Delta \log N_t)$	0.242	0.215	0.215
$Std.dev\,(\Delta \log H_t)$	0.248	0.222	0.221
$Corr\,(\Delta \log N_t, \Delta \log H_t)$	-0.340	-0.464	-0.469
$Corr\,(\Delta \log N_t, \Delta \log H_{t-1})$	0.071	0.109	0.115

Source: Author's tabulations from the Danish firm data, 1999-2006.

Table 2 reports the correlation between hours and employment growth by broad industry groups (in accordance with the standard Statistical Classification of Economic Activities in the European Union - NACE). The results show that hours-employment relationship is weaker in Hotels and Restaurants, Fishing and Construction sectors. These industries are associated with relatively low-skilled labor and presumably with lower adjustment costs. On the other hand, Real Estate and Business Activities and Transport and Telecommunication demonstrate stronger association between growth rates of hours and employment.

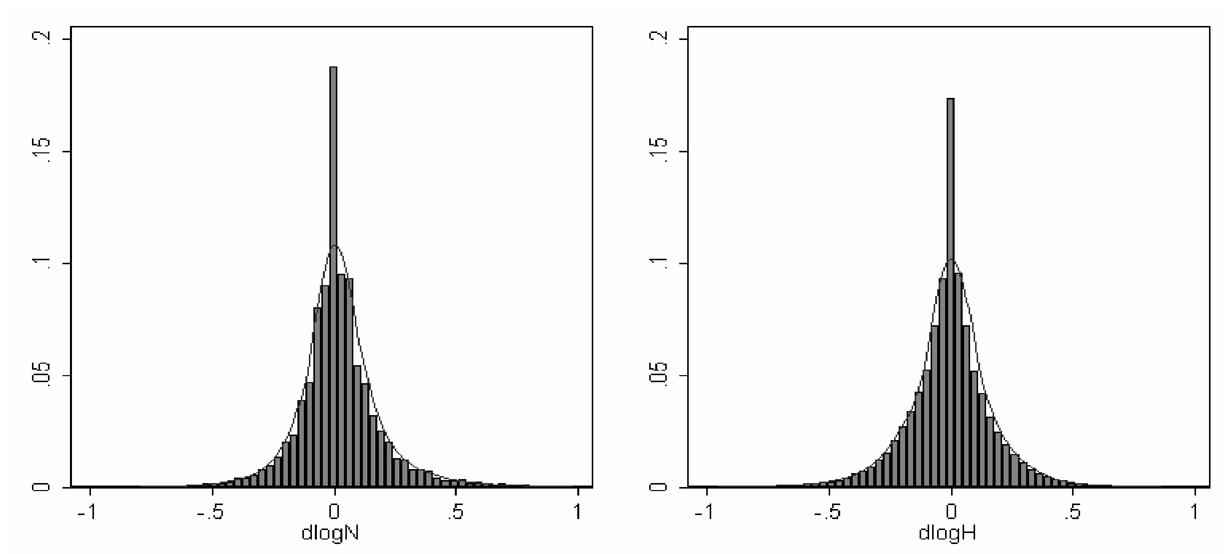
Table 2: Correlation between growth rates of average work hours and employment, by industry.

	Non-weighted	Emp.share weighted	Weighted, no time effects	Emp. share,%
Agriculture	-0.292	-0.369	-0.379	1.47
Fishing	-0.242	-0.241	-0.261	0.08
Mining and quarrying	-0.199	-0.290	-0.303	0.15
Manufacturing	-0.345	-0.509	-0.521	27.22
Electricity, gas and water supply	-0.776	-0.365	-0.361	0.56
Construction	-0.161	-0.257	-0.281	8.90
Wholesale and retail trade	-0.346	-0.402	-0.398	23.80
Hotels and restaurants	-0.190	-0.180	-0.185	3.78
Transport and communication	-0.339	-0.549	-0.551	6.87
Financial intermediation	-0.620	-0.340	-0.332	5.37
Real estate and business activities	-0.420	-0.599	-0.597	14.79
Education, health and social work	-0.420	-0.391	-0.395	2.07
Other social and personal services	-0.574	-0.569	-0.573	4.88
Activity not stated	-0.403	-0.280	-0.285	0.06

Source: Author's tabulations from the Danish firm data, 1999-2006.

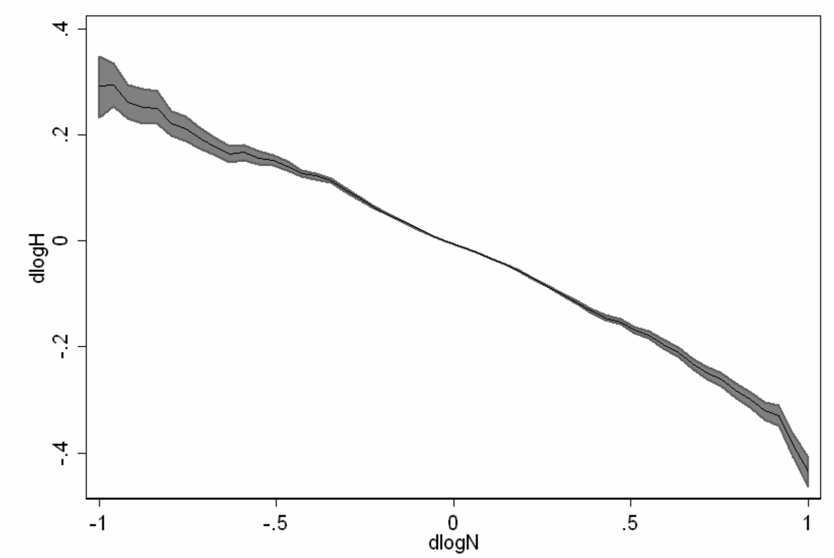
Previous studies on labor adjustment costs that use micro level data pertain to the manufacturing sector only (see Cooper, Haltiwanger and Willis (2007)). One of the advantages of using Danish firm data is the possibility of comparing the manufacturing industry to the overall population of firms. According to Table 2, the manufacturing sector (that comprises 27% of overall employment in Denmark) is characterized by a more negative correlation coefficient than overall economy (-0.52 and -0.47, respectively). Although the gap between the two coefficients is statistically significant at 1% level; the magnitude of this difference is fairly small (especially if compared to the difference between some of the other industries, for instance Hotels and Restaurants, and overall economy).

Figure 1: Growth rates of employment (left panel) and average work hours (right panel)



Note: Vertical axis shows a fraction of firm-quarter observations. Density estimation is based on Uniform kernel with bandwidth of 0.1. Source: Author's calculations from the Danish firm data, 1999-2006.

Figure 2: Non-parametric regression of hours growth on employment growth



Note: Estimates are based on Gaussian kernel with bandwidth of 0.025. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

2.2 Worker Flows

It is important to recognize that beyond hires and layoffs firms can adjust their labor force through modifying their attrition rates. Most of the existing models that formalize the effect of labor adjustment costs on employment dynamics focus on net employment change. Consequently, they do not distinguish between firms hiring new workers or devoting more resources to keep their existing employees. Yet, these channels have different implications in terms of labor adjustment costs, especially considering recruiting and training costs. In this subsection, I examine employment growth at the firm level in detail.

Here, I follow the existing literature in constructing and analyzing job flows and worker flows at the firm level (see for instance Davis, Faberman and Haltiwanger (2006), Burgess, Lane and Stevens (2001) and the references therein). Monthly hires are computed as the number of individuals that are working in a given firm during month t but not during month $t-1$. Separation flows are equal to the number of workers that are employed in a given firm during month $t-1$ but not during month t. Job flows are defined as the number of jobs created in growing firms (job creation) and the number of jobs destroyed in contracting firms (job destruction) within month t. The corresponding rates are expressed in flows divided by the average employment in month t and $t-1$. This procedure yields growth rates in the interval $[-2, 2]$ with endpoints corresponding to births and deaths (for more details on the properties of this rate measure see Davis, Haltiwanger and Schuh (1996)).

The data at hand indicate that there is a fair amount of job and worker mobility in the Danish labor market (see Table 3)[11]. Monthly hiring and separation rates average about 9% of employment. Job destruction and job creation rates are about 5-6% of employment (4% for continuing firms), more than twice the rates in the US labor market (see Davis, Faberman and Haltiwanger (2006)). That is, one of every 20 jobs on average is being destroyed from one month to the next.

Table 3: Average monthly job flow and worker flow rates.

	Non-weighted	Emp. share-weighted	Emp. share-weighted, continuing firms
Hires	0.180	0.097	0.078
Separations	0.152	0.091	0.076
Job Creation	0.158	0.061	0.041
Job Destruction	0.129	0.054	0.039
Net employment change	0.029	0.006	0.002
Churning	0.046	0.074	0.075

Note: Sample includes all private firms. Source: Author's tabulations from the Danish firm data, 1999-2006.

To highlight the difference between job flows and worker flows, I construct worker churning rates, defined as the sum of hiring and separation rates less the absolute value of the net growth rate in employment (see Burgess, Lane and Stevens (2001) for more details on this measure). The churning rate refers to worker flows in excess of job flows. The fact that firms churn workers indicates that contracting businesses still hire workers and workers leave growing firms. The Danish economy is characterized by a quite high average churning rate of 7.5%. On average over the period of 1999-2006, job creation constitutes 32.2% of all (size-weighted) hires; while 30.6% of all separations are associated with job destruction.

Table 4 shows the relationship between monthly worker flows and employment adjustment, size-weighted by employment share. Firms are split into five groups according to their net employment

[11] In order to be able to include firm's entry and exit, I do not restrict the sample to firms with 5 or more employees. In fact, the data set used for this analysis contains all firms in the private sector regardless of their size. Overall, the sample contains more than 10 million firm-month observations.

growth rate. Firms that represent 50.3% of employment have monthly net employment growth between -2.5% and 2.5%. Contracting firms reduce their labor force mostly through separations; while growing firms increase their employment mostly through hiring. However, even contracting firms are hiring at a 4.5% rate. These results appear to be qualitatively similar to those found for the US and Dutch labor markets (see Davis, Faberman and Haltiwanger (2006) and Hamermesh, Hassink and van Ours (1996), respectively), but are in contrast to the behavior of French firms reported by Abowd, Corbel and Kramarz (1999). The latter paper finds that employment variation in France is made predominantly through the hiring margin; that is, establishments are changing their labor force primarily by reducing entry and not by varying their separation rates.

Table 4: Average monthly hiring and separation rates, by net employment growth rate.

Net Emp. Growth	Hires	Sep.	Net	Emp. Share, %
Less than -0.10	0.045	0.496	-0.450	10.1
-0.10 to -0.025	0.040	0.093	-0.053	13.7
-0.025 to 0.025	0.035	0.035	0.000	50.3
0.025 to 0.10	0.093	0.040	0.053	14.8
More than 0.10	0.504	0.044	0.460	11.1
Total	0.097	0.091	0.006	100.0

Note: Sample includes all private firms. Source: Author's tabulations from the Danish firm data, 1999-2006.

Compared to the US labor market, there is more labor adjustment in the Danish economy. For instance, Cooper, Haltiwanger and Willis (2007) document that about 8% of workers are employed at the establishments that had reported net employment growth rate of more than 10% by absolute value. In Denmark, the corresponding figure is 21.2%. The inaction region is larger in the US in terms of employment share of firms that do not change their number of workers from one month to the next (20.6% in Denmark vis-à-vis 32% in the US).

An important question to address regarding employment change in the presence of adjustment costs is temporary layoffs. Presumably it is less costly for a firm to re-hire a worker who has been laid off temporarily than hire a new worker. In that case, we expect the hours adjustment margin to be less important in firm's labor demand decisions. In order to examine worker flows during a quarter, I construct two measures of hires and separations. The first measure is derived by contrasting employment in the last month of two consecutive quarters; while the second measure sums monthly flows over a quarter. Comparing the two measures, it appears that about two fifths of all hires and separations arise in connection with employment relationships lasting less than a quarter. Moreover, about 30% of all individuals hired during a quarter were employed at the same firm during the previous quarter. Hence, the fact that the relationship between employment and hours growth is found to be so strong in the economy with fairly frequent temporary layoffs is reassuring about the importance of hours as a channel of labor adjustment.

The model developed in this paper is motivated by the empirical facts outlined above. First, firms vary their labor input on both extensive and intensive margins: the standard deviation of employment and average hours growth series is about the same. Second, the movements in hours and employment are negatively related, which is consistent with the idea of a fast response of hours to demand shocks and a more sluggish response of employment. Furthermore, hours growth leads employment growth. In addition, this section presents empirical evidence that worker flows and job flows are quite distinct. In fact, only about 30% of monthly hires and separations arise in connection with job creation and destruction. Different implications of net and gross employment changes in terms of adjustment costs call for a theory that explicitly models hiring and separation decisions of firms.

3 Model

The model is an alternative formulation of Mortensen (2009) continuous time matching model of multi-worker firms with heterogenous profitability. I modify the production process by adding the intensive margin and allowing for workers and employers to bargain over a wage contract that specifies hours schedule and compensation. The driving process in the model is firm-specific profitability shocks that the firm can accommodate by adjusting its workforce and/or the number of hours that its employees work. Hiring a worker is impeded by search frictions; hence, employment cannot be adjusted instantaneously. Incorporating on-the-job search ensures that the vacancy filling and attrition rates vary with the firm's profitability.

I start with an overview of the model and then discuss the components in detail. I proceed in three steps. First, I show how work hours and wages are determined within each period as a result of bargaining between firms and workers. Second, I look at the optimal employment policies of workers and firms in the dynamic context. The employment path within the firm depends on both the firm's and its workers' decisions: the worker's problem determines the quit rate to unemployment and the rate of job-to-job transitions, while the firm's problem defines optimal hiring and firing decisions. Third, I derive steady state conditions for aggregate variables and the distribution of firms.

3.1 Overview

A final good Y is produced by a continuum of intermediate inputs $x(j)$ and is sold by many suppliers in a competitive output market at price P. Intermediate product suppliers face downward-sloping demand for their products, $p(x(j))$. The production technology for the intermediate good is linear in labor input, in particular,

$$x = qhn, \tag{1}$$

where x is the number of units supplied, q is firm's productivity, and hn is total labor input, the product of the number of workers n and average work hours h.

Each firm supplies one intermediate product. Firms differ in their productivity level q that at each point in time is subject to the shock that arrives at Poisson rate μ. In the event of a shock, a new productivity level is drawn from distribution $\Phi(\cdot)$, defined on support $\left[\underline{q}, \overline{q}\right]$, independently of the current productivity level. Existing firms are subject to the exogenous destruction risk and die at rate δ. Market entry rate is exogenous and is equal to η.

Firms and workers are brought together pairwise through a sequential and random matching process. To recruit, firms post vacancies v and incur hiring cost of $c(v)$ per unit of time, where $c(\cdot)$ is strictly increasing and convex. Reflecting search frictions, the offer arrival rate and the vacancy filling rate are exogenous to workers and firms but are determined in equilibrium. A job separation occurs if a worker quits or gets laid off. Firing a worker is assumed to be costless[12].

There is a continuum of infinitely lived identical workers, with a mass normalized to one, that supply labor to intermediate product firms. Individuals derive utility from consuming the aggregate good and incur disutility from working. Worker's utility function is assumed to be separable in aggregate good

[12] The baseline model has zero dismissal costs. In the numerical simulation of the model, I introduce firing costs and show that they have minimal effect on firms' employment policies.

consumption and work hours:

$$\tilde{u}(y,h) = y - g(h),$$

where y is the amount of the aggregate good consumed, h is hours of work, and $g(\cdot)$ is a strictly increasing convex function with $g(0) = g'(0) = 0$. Worker's consumption flow equals to real wage $\frac{w}{P}$ when employed and equals to b when unemployed[13]. Here, b can be viewed as unemployment insurance benefit that is indexed by the aggregate price level. Alternatively, b can be regarded as the value of home production of the aggregate good less the disutility from producing it. Hours of work and compensation are defined through the bargaining process. Finally, workers search while employed and unemployed.

3.2 Intra-firm Wage Bargaining

Hours of work and compensation are defined through the bargaining process between the firm and its employees. The hiring costs imply that it takes time to replace workers, and therefore employment is considered to be predetermined at the bargaining stage. This setup provides a natural environment for Stole and Zwiebel (1996) individual bargaining framework within multi-worker firms, in which firms engage in pairwise negotiations with workers over the current output[14]. The key assumption of their setup is that firms and their employees cannot commit to long-term employment and wage contracts. When a worker joins the firm, wages are renegotiated individually with all workers; therefore, the firm's outside option is not to remain idle, but rather producing with one worker less. Note that in the original Stole-Zwiebel bargaining problem the production function exhibits diminishing returns to scale. Here, although the production technology is linear in the number of workers, the fact that each firm faces downward-sloping demand leads to a decreasing marginal revenue product.

First, the firm chooses work hours schedule that maximizes total surplus for a given number of workers n, i.e.

$$\max_{h \geq 0} \left\{ \frac{R\,(qnh)}{P} - g\,(h)\,n \right\} = \max_{h \geq 0} \left\{ \frac{p\,(qnh)}{P} qhn - g\,(h)\,n \right\}.$$

Assuming an interior solution, the optimal number of hours h^* satisfies

$$\left(p'\,(qnh^*)\,qnh^* + p\,(qnh^*)\right)\frac{q}{P} = g'\,(h^*)\,. \tag{2}$$

Next, the firm and its workers bargain over wage taking hours schedule as given.

Following Hall and Milgrom (2008), the outside option of a worker is assumed to be the value of a delay, i.e. the value of home production b. Then, based on a generalization of Stole and Zwiebel (1996) bargaining problem for continuous employment n and bargaining power of workers $\beta \gtreqless \frac{1}{2}$, a wage contract solves the following problem:

$$\begin{aligned} & \max_{w(n)} \left(\frac{\pi'\,(n)}{P}\right)^{1-\beta}\left(\frac{w(n)}{P} - g(h\,(n)) - b\right)^{\beta} \\ = & \max_{w(n)} \left(\frac{R'\,(n)}{P} - \frac{w'\,(n)}{P}n - \frac{w\,(n)}{P}\right)^{1-\beta}\left(\frac{w(n)}{P} - g(h\,(n)) - b\right)^{\beta}, \end{aligned} \tag{3}$$

subject to the participation constraint for both parties in a sense that the continuation value is no less

[13] Linear utility in consumption implies risk neutrality; therefore, there is no savings motive in the worker's decisions.

[14] See also Cahuc, Postel-Vinay and Robin (2006), Cahuc and Wasmer (2001), and Ebell and Haefke (2003) for a similar application of Stole-Zwiebel wage bargaining setup, in which firms and their employees bargain over the match surplus instead of the current output. The paper of Cahuc, Marque and Wasmer (2008) further extends the original Stole and Zwiebel's problem to account for the case when workers have different bargaining power parameters.

than that of searching for a new partner. Note that this bargaining problem is equivalent to the one, in which workers bargain over both wage and hours of work simultaneously (see Appendix B.1 for details).

Solving for the first order condition of (3) leads to a first-order linear differential equation in wage:

$$w(n) = \beta R'(n) + (1-\beta) P [g(h(n)) + b] - \beta w'(n) n, \tag{4}$$

which implies the following equation for the wage function (see Appendix B.2 for a solution method):

$$\frac{w(n)}{P} = n^{-\frac{1}{\beta}} \int_0^n z^{\frac{1-\beta}{\beta}} \left(\frac{R'(z)}{P} + \frac{(1-\beta)}{\beta} g(h(z)) \right) dz + (1-\beta) b. \tag{5}$$

The solution has to satisfy the participation constraint for both parties: the value of working should be no less than that of quitting and searching for a new employer; moreover, the value of hiring that worker for the firm is non-negative.

If workers and firms bargain over the value of a match then the separations are bilaterally efficient. Here, given that the bargaining takes place over the current output, this is not necessarily true. The firm may choose to fire workers even if the value of employment at that firm is higher than the value of unemployment, and vice versa, the worker may quit even if the firm's value of employing that worker is positive. The implicit assumption of this bargaining process is that if the participation constraint binds for one party then there is no renegotiation and the match is dissolved.

3.3 CES Production Function

This subsection assumes the functional forms for aggregate production and disutility of working and uses the results obtained in the previous subsection to derive the optimal hours choice, worker's wage, and firm's profit.

The final output is determined by the (Dixit-Stiglitz) CES production function:

$$Y = \left[\int_0^K x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 0,$$

where $x(j)$ is the quantity of product j, σ represents the elasticity of substitution between any two intermediate goods, and K is the total measure of inputs available. The final good is produced by many competitive suppliers; therefore, the profit maximizing demand for each input is given by

$$x(j) = \left(\frac{P}{p(j)} \right)^{\sigma} Y, \ j \in K, \tag{6}$$

where P is the price of the final good, and $p(j)$ is the price of an input j. Zero-profit condition for final good producers implies that the aggregate price index is derived as

$$P = \left[\int_0^K p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}.$$

The intermediate good is produced according to the linear technology given in (1).

Assuming that the disutility of working $g(h)$ takes the following functional form:

$$g(h) = \chi h^{\xi}, \ \xi > 1, \ \chi > 0, \tag{7}$$

and using equation (6) for the firm's demand, the optimal number of hours that solves equation (2) is equal to

$$h\left(q,n\right)=\left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Y\frac{q^{\sigma-1}}{n}\right]^{\frac{1}{(\xi-1)\sigma+1}}, \tag{8}$$

which is increasing in the firm's productivity q and decreasing in the number of employees n.

The firm's revenue at the optimal number of hours expressed in units of the aggregate good reads

$$\frac{R\left(q,n\right)}{P}=\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\frac{\sigma-1}{(\xi-1)\sigma+1}}\left(Yq^{\sigma-1}\right)^{\frac{\xi}{(\xi-1)\sigma+1}}n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1)\sigma+1}}. \tag{9}$$

Revenue rises with productivity and with the number of workers if the elasticity of substitution between any too intermediate goods σ is higher than one. Hence, the condition $\sigma>1$ is imposed herein.

Using the expressions for the revenue and disutility from working functions specified above, evaluated at the optimal work hours (8), the solution to the bargaining problem defined in equation (5) leads to the following real wage curve equation:

$$\frac{w\left(q,n\right)}{P}=\chi\left(1+\beta\left(\xi-1\right)\frac{\left(\xi-1\right)\sigma+1}{(\xi-1)\sigma+1-\beta\xi}\right)\left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Y\frac{q^{\sigma-1}}{n}\right]^{\frac{\xi}{(\xi-1)\sigma+1}}+\left(1-\beta\right)b. \tag{10}$$

Note that $\frac{\partial w(q,n)}{\partial n}$ is negative. Stole and Zwiebel (1996) were the first to point out this hiring externality: as the number of workers per firm increases the bargained wage declines. Furthermore, from equation (4) it follows that the wage that the worker gets net of disutility of working is lower than her contribution to the total surplus.

Given the wage curve equation above, the firm's profit is derived as

$$\begin{aligned}\frac{\pi\left(q,n\right)}{P} &= \left(\frac{R\left(q,n\right)}{P}-\frac{w\left(q,n\right)}{P}n\right) \qquad (11)\\ &= \left(1-\beta\right)\left[\frac{\chi\left((\xi-1)\sigma+1\right)^{2}}{\left(\sigma-1\right)\left((\xi-1)\sigma+1-\beta\xi\right)}\left(\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Y\frac{q^{\sigma-1}}{n}\right)^{\frac{\xi}{(\xi-1)\sigma+1}}-b\right]n.\end{aligned}$$

Solving for the first order condition, the maximum profit is achieved at

$$n^{*}\left(q\right)=\left[\frac{\left((\xi-1)\sigma+1\right)\left(\xi-1\right)}{(\xi-1)\sigma+1-\beta\xi}\frac{\chi}{b}\right]^{\frac{(\xi-1)\sigma+1}{\xi}}\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Yq^{\sigma-1}, \tag{12}$$

and is equal to

$$\frac{\bar{\pi}\left(q\right)}{P}=\frac{\left(1-\beta\right)\xi}{\left(\sigma-1\right)}\left[\frac{\chi\left((\xi-1)\sigma+1\right)}{(\xi-1)\sigma+1-\beta\xi}\right]^{\frac{(\xi-1)\sigma+1}{\xi}}\left(\frac{(\xi-1)}{b}\right)^{\frac{(\xi-1)(\sigma-1)}{\xi}}\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Yq^{\sigma-1}. \tag{13}$$

Therefore, the firm's profit is a continuous function of employment and productivity, increasing in productivity, and bounded from above for a given q by $\bar{\pi}\left(q\right)$.

In Stole and Zwiebel (1996) original problem, n^{*}, as defined in equation (12), is the maximum number of workers that the firm would employ. This is a straightforward implication of the bargaining problem in which the firm has an incentive to hire additional workers to decrease their bargaining power down to the point where the marginal profit is zero and workers are paid their reservation wage. Here, however, the firm may choose to employ more than n^* workers in anticipation of future positive profitability shocks, as long as the expected change in the firm's value is positive.

Finally, the worker's utility is equal to

$$\begin{aligned}\tilde{u}(q,n) &= \left(\frac{w(q,n)}{P} - g\left(h\left(q,n\right)\right)\right) \\ &= \beta\left(\xi-1\right)\frac{\chi\left(\left(\xi-1\right)\sigma+1\right)}{(\xi-1)\sigma+1-\beta\xi}\left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma} Y\frac{q^{\sigma-1}}{n}\right]^{\frac{\xi}{(\xi-1)\sigma+1}} + (1-\beta)\,b,\end{aligned} \quad (14)$$

that is decreasing in employment and increasing in productivity.

Returning to the motivation for this exercise, the driving force in this model is idiosyncratic shocks to firm's productivity q[15]. The optimal hours function, defined in equation (8), generates the trade-off between the number of employees and the amount that each employee works. In particular, it guarantees that a positive shock to productivity q will produce an increase in average hours if there is no (or little) change in employment. As the number of workers starts growing the average hours decline. Hence, the model can capture the negative relationship between hours and employment growth observed in the data if the response of employment is slow enough. The employment decisions of firms and workers are discussed in detail in the following two subsections.

3.4 Worker's Problem

In this subsection, I describe the worker's problem taking as given all equilibrium objects that are outside of the worker's control, such as labor market tightness, distribution of offers and workers across firm types, as well as the optimal employment decisions of firms. Although the value of unemployment and the value of working at a firm with productivity q and employment n depend on the aggregate variables, they are not listed as arguments for notational simplicity.

When unemployed, the worker obtains consumption flow b by means of home production, and she has an option of finding a job. Hence, the value of unemployment expressed in terms of the final output, U, solves the continuous time Bellman equation

$$rU = b + \lambda\left(\theta\right)\int\left(\max\left\{W,U\right\} - U\right)dF\left(W\right), \quad (15)$$

where r is the common firm's and worker's discount rate; $\lambda\left(\theta\right)$ is the job arrival rate, and θ is market tightness; $F\left(W\right)$ is the cumulative distribution function of job vacancies posted by firms that provide workers with the value of employment of at most W.

The job arrival rate λ is derived from a matching function that is assumed to be increasing, concave, and homogenous of degree one in both arguments, vacancies and job seekers[16]. Given the matching function properties, $\lambda\left(\theta\right)$ is increasing and concave in market tightness θ, which is the ratio of the aggregate number of vacancies posted to individuals searching for a job, the variable that is determined endogenously in equilibrium.

[15] Note that I refer to the shock to q as a productivity shock. However, it can be thought of as a firm-specific demand shock or more generally as a profitability shock. For instance, consider an alternative specification where the aggregate demand function is defined as

$$Y = \left[\int_0^K \alpha\left(j\right)x(j)^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}},$$

where $\alpha\left(j\right)$ is a firm-specific demand shock, and production technology for the intermediate good is $x = hn$. This specification is equivalent to the current formulation of the production side of the market if $q = \alpha^{\frac{\sigma}{\sigma-1}}$.

[16] See Pissarides (2000) and Petrongolo and Pissarides (2001) for details on the concept of a matching function.

The value of employment at a firm with n workers and productivity q, $W_n(q)$, satisfies the following Bellman equation

$$rW_n(q) \tag{16}$$

$$= \left\{ \begin{array}{c} \tilde{u}_n(q) + (\delta + s_0)(U - W_n(q)) + \lambda(\theta)\kappa \int (\max\{W', W_n(q)\} - W_n(q))\, dF(W') \\ +H_n(q)(\max\{W_{n+1}(q), U\} - W_n(q)) + s_n(q)(n-1)(\max\{W_{n-1}(q), U\} - W_n(q)) \\ +\mu \int\limits_{\underline{q}}^{\overline{q}} \left(\begin{array}{c} \mathbf{1}[f_n(q') > 0]\left(f_n(q')U + (1 - f_n(q'))\max\left\{U, W_{\bar{n}^F(q')}(q')\right\}\right) \\ +\mathbf{1}[f_n(q') = 0]\max\{U, W_n(q')\} - W_n(q) \end{array} \right) \phi(q')dq' \end{array} \right\},$$

where $\tilde{u}_n(q)$ is the utility flow expressed in final output terms as defined in equation (14). The worker becomes unemployed at constant Poisson rate $\delta + s_0$, where s_0 represents the exogenous component of the quit rate and δ refers to the destruction shock. The worker receives an alternative job offer at rate $\lambda(\theta)\kappa$, where $\kappa \geq 0$ represents the search intensity when employed relative to the search intensity when unemployed (if $\kappa = 1$ then workers search with the same intensity regardless of their employment status; $\kappa = 0$ means no on-the-job search). Hence, the next term on the right-hand side of equation (16) is attributed to the option value of moving to a better employment position.

The following two terms are related to the expected change in the value of employment in the event of the firm adjusting its labor force. In particular, at rate $H_n(q)$ the firm hires another worker and at rate $s_n(q)(n-1)$ one of the other $(n-1)$ workers separates from the firm. These rates are determined endogenously in equilibrium and are defined in the firm's problem below.

The last term on the right-hand side of equation (16) embodies the expected change in the value attributable to the shock to the firm's productivity q that arrives at rate μ. A new productivity is drawn from distribution $\Phi(\cdot)$ with a corresponding density, $\phi(\cdot)$. Recall that separations are not necessarily mutually efficient; therefore, the value function accounts for the possibility that when hit by the productivity shock the firm may find it optimal to fire workers. Let $\bar{n}^F(q)$ be the maximum labor force size that the firm is willing to employ given its current productivity level q (it will be defined more precisely in the firm's problem below). Then, $f_n(q)$ is the firing probability equal to $\frac{n - \bar{n}^F(q)}{n}$ if $n > \bar{n}^F(q)$ and equal to zero otherwise. That is, it is assumed in the model that the firm fires workers randomly if its labor force exceeds its maximum size $\bar{n}^F(q)$.

Proposition 1. A unique continuous solution for the value of employment at a firm with n workers and productivity level q, $W_n(q)$, and for the value of unemployment, U, exists if hiring and separation rates, $H_n(q)$ and $s_n(q)$, are finite and continuous in q and n.

Proof: Equation (16) has a unique solution which is a fixed point of the contracting mapping $[W_n(q), U]_{t+1} = T[W_n(q), U]_t$ defined by equations (15) and (16). Given that $\tilde{u}_n(q)$ is positive and bounded from above by $\tilde{u}_1(q)$, T maps the set of non-negative, continuous, and bounded from above functions into itself. Given this set is compact under the sup norm, one can apply Blackwell's sufficient conditions to show that T is a contraction mapping (see Stokey and Lucas (1989)). Obviously, the mapping is monotone; furthermore, T discounts, i.e.

$$T\begin{bmatrix} W_n(q) + z \\ U + z \end{bmatrix} = T\begin{bmatrix} W_n(q) \\ U \end{bmatrix} + \begin{bmatrix} \frac{\delta + s_0 + \lambda(\theta)\kappa + H_n(q) + s_n(q)(n-1) + \mu}{r + \delta + s_0 + \lambda(\theta)\kappa + H_n(q) + s_n(q)(n-1) + \mu} \\ \frac{\lambda(\theta)}{r + \lambda(\theta)} \end{bmatrix} z,$$

which completes the proof that T is indeed a contraction. Then by Contraction Mapping Theorem, there exists a unique continuous solution $W_n(q)$ and U.■

Define $\bar{n}^W(q)$ as the lowest employment level, beyond which worker's participation constraint binds, i.e. $W_n(q) \leq U$. If search on the job is at least as efficient as when unemployed, i.e. $\kappa \geq 1$, then the participation constraint never binds for $n \leq n^*(q)$, where $n^*(q)$ is defined in equation (12). To see it, subtract equation (15) from equation (16) to get

$$W_n(q) - U = \frac{\left\{\begin{array}{c} \tilde{u}_n(q) - b + \lambda(\theta) \int (\kappa \max\{W' - U, W_n(q) - U\} - \max\{W' - U, 0\})\, dF(W') \\ +H_n(q) \max\{W_{n+1}(q) - U, 0\} + s_n(q)(n-1) \max\{W_{n-1}(q) - U, 0\} \\ +\mu \int\limits_{\underline{q}}^{\overline{q}} \left(\begin{array}{c} \mathbf{1}\left[n > \bar{n}^F(q')\right] \frac{\bar{n}^F(q')}{n} \max\left\{W_{\bar{n}^F(q')}(q') - U, 0\right\} + \\ \mathbf{1}\left[n \leq \bar{n}^F(q')\right] \max\{W_n(q') - U, 0\} \end{array}\right) \phi(q')dq' \end{array}\right\}}{r + \delta + s_0 + \lambda(\theta)\kappa + H_n(q) + s_n(q)(n-1) + \mu}$$

which is nonnegative for all $\tilde{u}_n(q) \geq b$ due to the option value of the firm getting a positive productivity shock and/or adjusting its labor force. This result implies that as long as the worker's utility is higher than or equal to b, she will never quit to unemployment. However, for all $n > n^*(q)$, as well as for other n when $\kappa < 1$, I need to verify that worker's participation constraint $W_n(q) - U \geq 0$ is satisfied.

The value of working at a given firm is not necessarily monotone in the firm's workforce, even though wages and utility are monotonically decreasing in employment, for a given value of productivity level. The intuition for this result resides in the difference between the effect that a rise in the number of workers has on current and future wages. An increase in employment unambiguously lowers current wages. On the other hand, it raises the prospects that some of the existing workers separate from the firm and that fewer workers are hired, thus lowering the probability of a further decrease in wages in the future. Therefore, contrary to the standard search theory, this model can potentially generate job-to-job transitions that are associated with wage cuts since a change in the value of employment may still be positive even if a change in wages is not. This prediction of the model is similar in spirit to Postel-Vinay and Robin (2002). Based on a different wage-setting mechanism, they show that a worker is willing to accept wage cuts to trade a lower share of the total rent today for a larger share tomorrow.

3.5 Firm's Problem

This subsection provides details on employment changes within a firm. Firms can adjust their labor force by recruiting and firing workers. To hire a worker, the firm needs to post vacancies that are then randomly matched with job seekers. In addition, the employment decision of the firm is affected by the separation rate, at which existing workers quit to unemployment or move to a better job. Both hiring and separation rates depend on aggregate market tightness, unemployment, and overall distribution of vacancies and workers across firm types. As before, for notational simplicity the aggregate variables are not listed as arguments in the firm value function, as well as in hiring and separation rates.

For the firm's problem, it is useful to write hiring and separation rates explicitly. The rate at which each worker separates from a firm with productivity q and employment n is the sum of the exogenous quit rate to unemployment and the job-to-job transition rate, i.e.

$$s_n(q) = s_0 + \lambda(\theta)\kappa\left[(1 - F(W_n(q))\right], \tag{17}$$

where $F(W_n(q))$ is the fraction of vacancies posted by firms that provide workers with the value of employment of at most $W_n(q)$.

The probability that any offer is acceptable to a randomly contacted worker is

$$a_n(q) = \begin{cases} \frac{u+(1-u)\kappa G(W_n(q))}{u+(1-u)\kappa}, \text{ if } W_n(q) - U \geq 0 \\ 0, \text{ otherwise} \end{cases} \tag{18}$$

where u is the fraction of unemployed workers and $G(W_n(q))$ is the fraction of employed workers who gain the value of employment of at most $W_n(q)$. Employed job seekers are weighted by their search intensity, κ. If worker's participation constraint is binding then no worker would accept the firm's offer.

Then, the hiring rate is equal to $H_n(q) = a_n(q)\,\upsilon_n(q)\,\omega(\theta)$, where $\omega(\theta)$ is the rate at which vacancies are matched with workers and $\upsilon_n(q)$ is the number of vacancies posted by the firm with n employees and productivity level q. The rate $\omega(\theta)$, that is exogenous to the firm, is derived from the matching function.

The value of the firm with productivity q and employment n, $V_n(q)$, expressed in final output terms, solves the following Bellman equation:

$$\begin{aligned} &(r+\delta)\,V_n(q) \\ = \quad &\max \left\{ \begin{array}{c} \frac{\pi_n(q)}{P} + \max\limits_{\upsilon \geq 0} \{a_n(q)\,\omega(\theta)\,\upsilon\,[V_{n+1}(q) - V_n(q)] - c(\upsilon)\} \\ +s_n(q)n\,[V_{n-1}(q) - V_n(q)] + \mu \int\limits_{\underline{q}}^{\bar{q}} (V_n(q') - V_n(q))\,\phi(q')dq' \end{array},\ (r+\delta)\,V_{n-1}(q) \right\}, \end{aligned} \tag{19}$$

under the assumption that firing a worker is costless and the worker's participation constraint is satisfied. If the worker's constraint binds for some n then workers quit randomly until $n = \bar{n}^W(q)$ and $V_n(q) = V_{\bar{n}^W(q)}(q)$.

The first term on the right-hand side of equation (19) is the firm's profit flow expressed in final good terms, as defined in equation (11). The second term refers to the capital gain that is obtained from the possibility of hiring an additional worker, given the optimally chosen vacancy posting decision. The third term is the expected capital loss related to the possibility that any worker quits. The last term accounts for the expected change in the value of the firm caused by a shock to the firm's productivity q.

Here, the advantage of modeling in continuous time, as opposed to discrete time, becomes evident. Discrete time models require a careful specification of the timing of events. For instance, in discrete time I would have to decide on one of these alternatives: (i) at the start of the period some of the existing employees separate from the firm, after that new workers are recruited; (ii) at the start of the period new workers are hired, after that any of the workers may quit; (iii) separations happen after new workers are hired, but new workers are not allowed to quit during the same period when they were hired. These three alternatives have different implications for the optimal employment policy of the firm, given that hiring and separation rates depend on the number of workers. On the other hand, by building the model in continuous time I avoid making any (often arbitrary) assumptions on the timing of events.

Proposition 2. Equation (19) has a unique continuous solution, $V_n(q)$, if $c(\upsilon)$ is strictly increasing, convex and $c(0) = c'(0) = 0$, and $a_n(q)$ and $s_n(q)$ are continuous in n and q.

Proof: Equation (19) has a unique solution that is a fixed point of the mapping

$$[TV](q,n)=\max\left\{\max_{\upsilon\geq 0}\frac{\left(\begin{array}{c}\frac{\pi_n(q)}{P}+a_n(q)\upsilon\omega(\theta)V_{n+1}(q)-c(\upsilon)\\ +s_n(q)nV_{n-1}(q)+\mu\int_{\underline{q}}^{\overline{q}}V_n(q')\phi(q')dq'\end{array}\right)}{r+\delta+a_n(q)\upsilon\omega(\theta)+s_n(q)n+\mu},V_{n-1}(q)\right\}. \tag{20}$$

Given that (i) the profit function is bounded from above by $\bar{\pi}(q)$ defined in equation (13), (ii) $c(\upsilon)$ is strictly increasing and convex and $c(0)=c'(0)=0$, and (iii) firing a worker is costless, T maps the set of non-negative continuous functions bounded from above by $\bar{V}(q)=\frac{\bar{\pi}(q)/P}{r+\delta+\mu}+\frac{\mu}{r+\delta+\mu}\int_{\underline{q}}^{\overline{q}}\frac{\bar{\pi}(q')/P}{r+\delta}\phi(q')dq'$ into itself. As this set is compact under the sup norm, I need only to confirm that the map satisfies Blackwell's sufficient conditions for a contraction. First, note that T is monotone. Moreover,

$$T[V_n(q)+z]=T[V_n(q)]+\frac{a_n(q)\upsilon_n(q)\omega(\theta)+s_n(q)n+\mu}{r+\delta+a_n(q)\upsilon_n(q)\omega(\theta)+s_n(q)n+\mu}z,$$

where $\upsilon_n(q)=\arg\max_{\upsilon\geq 0}\{a_n(q)\omega(\theta)\upsilon[V_{n+1}(q)-V_n(q)]-c(\upsilon)\}$. The map T discounts if $\upsilon_n(q)<\infty$, a condition that holds under the assumption that $c(\upsilon)$ is strictly increasing and convex and the fact that $0\leq V_{n+1}(q)-V_n(q)\leq\bar{V}(q)$ due to boundedness and non-negativity of V function. Hence, T is indeed a contraction mapping. Therefore by Contraction Mapping Theorem, there exists a unique solution to equation (20). ■

Since profit falls without a bound as firm's labor force increases, there exists an upper limit on employment, $\bar{n}^F(q)$, beyond which the firm fires workers. That is, $\bar{n}^F(q)$ is the lowest number of workers for which $V_{\bar{n}^F(q)+1}(q)=V_{\bar{n}^F(q)}(q)$. If worker's participation constraint is binding and $\bar{n}^W(q)<\bar{n}^F(q)$ then workers quit, in which case the maximum labor force $\bar{n}(q)$ is determined by the worker's problem. Therefore, the maximum employment is defined as the minimum of the two threshold values, i.e.

$$\bar{n}(q)=\min\left\{\bar{n}^F(q),\bar{n}^W(q)\right\}.$$

Note that $\bar{n}(q)$ is also the lowest level of employment for which firms post zero vacancies.

Proposition 3. $V_n(q)-V_{n-1}(q)$ is strictly positive for all $n\leq n^*(q)$, that is $\bar{n}^F(q)\geq n^*(q)$[17].

Proof: Note that costless firing implies $V_n(q)-V_{n-1}(q)\geq 0$. To show that the difference is strictly positive for $n\leq n^*(q)$, I apply proof by induction. Differencing equation (19) leads to

$$\begin{aligned}&(r+\delta+\mu+a_{n-1}(q)\omega(\theta)\upsilon_{n-1}+s_n(q)n)(V_n(q)-V_{n-1}(q)) \\ =\;&\max\left\{\begin{array}{c}\frac{\pi_n(q)-\pi_{n-1}(q)}{P}+c(\upsilon_{n-1})+\max\limits_{\upsilon\geq 0}\{a_n(q)\omega(\theta)\upsilon[V_{n+1}(q)-V_n(q)]-c(\upsilon)\}\\ +s_{n-1}(q)(n-1)[V_{n-1}(q)-V_{n-2}(q)]+\mu\int_{\underline{q}}^{\overline{q}}(V_n(q')-V_{n-1}(q'))\phi(q')dq'\end{array},0\right\}.\end{aligned} \tag{21}$$

First, note that the assertion holds for $n=1$; that is, $V_1(q)-V_0(q)>0$ since $c(\cdot)$ is increasing and

[17] In the proof of Proposition 3, I implicitly assume that $n^*(q)\leq\bar{n}^W(q)$. However, the same logic applies if $n^*(q)>\bar{n}^W(q)$, then $V_n(q)-V_{n-1}(q)>0$ for all $n\leq\bar{n}^W(q)$.

$c(0) = 0$, the benefit from posting vacancies is non-negative, and $\pi_1(q) > 0$. Then under the assumption that $V_{n-1}(q) - V_{n-2}(q) > 0$, also $V_n(q) - V_{n-1}(q) > 0$ given that profit is increasing in n for all $n \leq n^*(q)$. That completes the induction proof. ■

Proposition 3 shows that, similarly to the original work of Stole and Zwiebel (1996), firms recruit workers up to the level of employment that maximizes per period profit flow. Contrary to their work, however, labor hoarding effect may arise in this model in a sense of $\bar{n}(q) > n^*(q)$. Intuitively, in the economy with search frictions, the firm might choose to hire workers above the profit-maximizing employment level in anticipation of a positive productivity shock.

3.6 Steady State Conditions

(a) **Size distribution**

Firms are identical ex ante and their type is revealed upon the entry. The distribution of potential entrants is assumed to be the same as the distribution of existing types. That is, under the assumption that shocks to q are drawn from the same distribution $\Phi(\cdot)$, the productivity distribution at entry is preserved among existing firms. Then the steady state number of firms conditional on type is derived by equating the market entry and exit

$$\delta K(q) = \eta\phi(q), \tag{22}$$

where η is the exogenous entry rate, $\phi(q)$ is the density of entrants of type q, and δ is the proportion of existing firms that become obsolete and exit the market.

Denote by $K_n(q)$ the aggregate number of products supplied by the set of firms of type q with employment n. Then steady state mass of firms conditional on firm's type is derived by equating inflows into and outflows from $K_n(q)$. First, for all $n \in [1, \bar{n}(q) - 1]$ the following relationship must hold

$$\begin{aligned} & H_{n-1}(q) K_{n-1}(q) + s_{n+1}(q)(n+1) K_{n+1}(q) + \phi(q)\mu \int_{\underline{q}}^{\overline{q}} K_n(q')\, dq' \\ = \quad & H_n(q) K_n(q) + s_n(q) n K_n(q) + \delta K_n(q) + \mu K_n(q), \end{aligned} \tag{23}$$

where the first two terms on the left-hand side represent the expected hiring and separation flows; whereas, the next term is the average proportion of all firms, which are hit by the idiosyncratic shock and become $q-$type firms. The outflow consists of the transition flows from n to $n+1$ workers due to new hires and to $n-1$ workers due to quits, of the firm destruction, $\delta K_n(q)$, and of becoming a different productivity type, $\mu K_n(q)$.

For $n = 0$, equation (23) includes an additional term that accounts for the entry of new firms of type q, $\eta\phi(q)$. Thus, the steady state relationship for $n = 0$ reads

$$\begin{aligned} & s_1(q) K_1(q) + \phi(q)\mu \int_{\underline{q}}^{\overline{q}} K_0(q')\, dq' + \eta\phi(q) \\ = \quad & H_0(q) K_0(q) + \delta K_0(q) + \mu K_0(q). \end{aligned} \tag{24}$$

In addition, equation (23) has to be modified for $n = \bar{n}(q)$ to incorporate the possibility of firing workers in the event of an adverse productivity shock, i.e.

$$
\begin{aligned}
& H_{\bar{n}(q)-1}(q) K_{\bar{n}(q)-1}(q)+\phi(q) \mu \int_{\underline{q}}^{\bar{q}} \sum_{n=\bar{n}(q)}^{\bar{n}\left(q^{\prime}\right)} K_n\left(q^{\prime}\right) d q^{\prime} \\
= \; & s_{\bar{n}(q)}(q) \bar{n}(q) K_{\bar{n}(q)}(q)+\delta K_{\bar{n}(q)}(q)+\mu K_{\bar{n}(q)}(q) .
\end{aligned} \tag{25}
$$

Note that the last equation uses the fact that $\upsilon_{\bar{n}(q)}(q)=0$ and $K_n(q)=0$ for all $n>\bar{n}(q)$ since there are no firms with employment that exceeds $\bar{n}(q)$. Therefore, $K_n(q)=0$ for all $n>\bar{n}(q)$ serves as a boundary condition for a second order difference equation defined in equations $(23)-(25)$.

The steady state distribution of firms of type q is

$$
K(q)=\sum_{n=0}^{\bar{n}(q)} K_n(q) .
$$

Summing equation (23) over $n \in[1, \bar{n}(q)-1]$ and adding equations (24) and (25), I recover equation (22).

The steady state number of workers employed by all firms of type q is equal to

$$
N(q)=\sum_{n=1}^{\bar{n}(q)} n K_n(q) . \tag{26}
$$

(b) **Aggregate output, vacancies, unemployment and market tightness**

In equilibrium, total output produced by intermediate firms is equal to aggregate demand, i.e.

$$
\begin{aligned}
Y & =\left[\int_{\underline{q}}^{\bar{q}} \sum_{n=1}^{\bar{n}(q)}\left(q h_n(q) n\right)^{\frac{\sigma-1}{\sigma}} K_n(q) d q\right]^{\frac{\sigma}{\sigma-1}} \\
& =\left(\frac{\sigma-1}{\chi \xi \sigma}\right)^{\frac{1}{\xi-1}}\left[\int_{\underline{q}}^{\bar{q}} q^{\frac{\xi(\sigma-1)}{(\xi-1) \sigma+1}} \sum_{n=1}^{\bar{n}(q)} n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1) \sigma+1}} K_n(q) d q\right]^{\frac{(\xi-1) \sigma+1}{(\xi-1)(\sigma-1)}} .
\end{aligned} \tag{27}
$$

Total number of vacancies posted by all firms is

$$
\upsilon=\int_{\underline{q}}^{\bar{q}} \sum_{n=0}^{\bar{n}(q)} \upsilon_n(q) K_n(q) d q . \tag{28}
$$

Aggregate market tightness is defined as the ratio of total vacancies to job seekers:

$$
\theta=\frac{\upsilon}{u+(1-u) \kappa} . \tag{29}
$$

The matching function is assumed to have constant returns to scale in job seekers and vacancies, then the job-finding rate can be expressed as a function of θ, $\lambda(\theta)$, and the worker meeting rate can be written as $\omega(\theta)=\frac{\lambda(\theta)}{\theta}$.

The unemployment rate can be derived from the labor market clearing condition that states that in equilibrium labor supplied to the market is equal to the total employment across all firms:

$$1-u=\int_{\underline{q}}^{\bar{q}} N(q)dq. \tag{30}$$

(*c*) $F\left(W_n\left(q\right)\right)$ **and** $G\left(W_n\left(q\right)\right)$ **distribution functions**

The distribution of job offers $F\left(W_n\left(q\right)\right)$ is merely the proportion of all vacancies that are posted by firms that provide workers with the value of employment of at most $W_n\left(q\right)$, i.e.

$$F\left(W_n\left(q\right)\right)=\frac{\int_{\underline{q}}^{\bar{q}}\sum_{n=0}^{\bar{n}(q')}\mathbf{1}\left[W_{n'}\left(q'\right)\leq W_n\left(q\right)\right]\upsilon_{n'}\left(q'\right)K_{n'}\left(q'\right)dq'}{\upsilon}. \tag{31}$$

Similarly, steady state distribution of workers $G\left(W_n\left(q\right)\right)$ refers to the fraction of total workforce in the economy employed at jobs that guarantee workers the value of employment of at most $W_n\left(q\right)$, i.e.

$$G\left(W_n\left(q\right)\right)=\frac{\int_{\underline{q}}^{\bar{q}}\sum_{n=0}^{\bar{n}(q')}\mathbf{1}\left[W_{n'}\left(q'\right)\leq W_n\left(q\right)\right]n'K_{n'}\left(q'\right)dq'}{\int_{\underline{q}}^{\bar{q}}N\left(q\right)dq}. \tag{32}$$

Combining equations $(23)-(25)$ with the definitions of $F\left(W_n\left(q\right)\right)$ and $G\left(W_n\left(q\right)\right)$, after some manipulations leads to a familiar condition for the steady state unemployment rate that equates flows into and out of unemployment. That is, the steady state unemployment rate solves the following equation:

$$\lambda\left(\theta\right)u=\left(1-u\right)\left[\delta+s_0+\mu\int_{\underline{q}}^{\bar{q}}\phi\left(q\right)\frac{\int_{\underline{q}}^{\bar{q}}\sum_{n=\bar{n}(q)}^{\bar{n}(q')}\left(n-\bar{n}\left(q\right)\right)K_n\left(q'\right)dq'}{\int_{\underline{q}}^{\bar{q}}N\left(q\right)dq}dq\right], \tag{33}$$

where the left-hand side refers to the outflow from unemployment due to finding a job and the right-hand side includes the inflow into the pool of unemployed workers due to three reasons: the destruction shock, the exogenous quit, and the layoff in the event of an adverse productivity shock[18]. The last term is computed as the product of the rate at which productivity shock arrives, μ, and the average proportion of workers that are laid off when the firm's productivity changes.

Denote the expected proportion of laid-off workers by

$$l=\int_{\underline{q}}^{\bar{q}}\phi\left(q\right)\frac{\int_{\underline{q}}^{\bar{q}}\sum_{n=\bar{n}(q)}^{\bar{n}(q')}\left(n-\bar{n}\left(q\right)\right)K_n\left(q'\right)dq'}{\int_{\underline{q}}^{\bar{q}}N\left(q\right)dq}dq$$

[18]Here, a term 'layoff' is used loosely since maximum labor force size may be determined by the worker's problem, i.e. if $\bar{n}\left(q\right)=\bar{n}^W\left(q\right)$ then workers quit.

then the steady state unemployment rate can be written as

$$u = \frac{\delta + s_0 + \mu l}{\delta + s_0 + \mu l + \lambda(\theta)}. \tag{34}$$

3.7 Equilibrium

Definition: A steady state market equilibrium is a set of numbers (θ, u, U, Y), a set of functions defined on a state space $(W_n(q), V_n(q), \upsilon_n(q), K_n(q), G(W_n(q)), F(W_n(q))) : [\underline{q}, \overline{q}] \times I_+ \rightarrow R_+$, a set of functions defined on firm types $(K(q), N(q)) : [\underline{q}, \overline{q}] \rightarrow R_+$ and $\bar{n}(q) : [\underline{q}, \overline{q}] \rightarrow I_+$, that satisfy equations $(15), (16), (19), (22) - (32)$.

To find a steady state equilibrium, I look for a fixed point of the mapping where the worker's and firm's problems are solved given aggregate market tightness, unemployment, aggregate demand, and distribution functions of vacancies and workers across firm types. Then, the aggregate objects and steady state distribution of firms are updated using the optimal employment adjustment decisions of firms. Appendix C.1 provides details on steady state equilibrium solution algorithm used in the numerical simulation procedure described below.

4 Calibration

This section shows the fit of the model to Danish firm data. The focus of the calibration exercise is to demonstrate how well the model can replicate labor input adjustment patterns, as well as relationships between wages, employment, and productivity observed in the data. The empirical evidence is based on a matched employer-employee dataset that is drawn from a panel of Danish administrative firm data. In addition to work hours and employment records described in Section 2, the data contain information on quarterly payroll costs for the period of 1999-2006 and purchases and sales records of all VAT-liable businesses for the period of 2002-2006 (see Appendix A for details). Combining these data with employment and hours series, I construct hourly wages and labor productivity variables.

The model is simulated under the assumption that the economy is in steady state (recall that the aggregate time effects are removed from the data series). In order to obtain employment paths for each firm, I first solve for type conditional equilibrium hiring and separation rates, $H_n(q)$ and $s_n(q)$, respectively, as well as the maximum labor force size, $\bar{n}(q)$. Given Poisson arrival rates for the destruction shock, the productivity shock, hires, and separations, the waiting time until the next occurrence of any of these events is distributed exponentially with parameter $H_n(q) + s_n(q)n + \delta + \mu$. Hence, employment histories of firms are simulated as random draws from exponential distribution and then aggregated into monthly series. Note that the continuous time nature of the model eliminates the need to make any (arbitrary) assumptions on the timing of events.

In the calibration exercise below I simulate the economy with 1000 firms for 300 months. Monthly employment is defined as all individuals who were on payroll in a given month; that is, it includes workers who have been hired during the month, as well as workers who have separated in the same month. Work hours, wages, and value added are aggregated to quarterly series to mimic the reporting frequency in the data sources used in this paper. Moreover, the hours measure is constructed according to the pension contribution payment rule, which is based on the 9-hour intervals, in parallel to the hours measure observed in the data.

In the following subsections, I first discuss the parameter choice for the simulation. Second, I present the main predictions of the model and compare them with Danish firm data. Overall, the model performs fairly well matching the labor adjustment patterns and empirical relationships between firm size, wages, and labor productivity. The model is capable of reproducing the trade-off between changes in work hours and employment qualitatively, though it underestimates the magnitude of the association. Third, using the calibrated parameters I show the effect of changes in labor adjustment costs on unemployment, the job-finding rate, and average work hours. I perform two experiments: (i) doubling vacancy creation costs and (ii) introducing a hiring subsidy. I compare the effects obtained in a general equilibrium framework to that of partial equilibrium.

4.1 Parameter Choice

Here, I discuss the parameters of the model and summarize them in Table 5 (further details can be found in Appendix C.2). Panel A of Table 5 presents parameters (and the corresponding empirical moments) that are calibrated to match the observed relationship in the data; whereas, Panel B shows parameters that are chosen to be consistent with the previous literature or to fit empirical regularities qualitatively.

Table 5: Parameter values and corresponding moments.

Parameter	Value	Empirical moment	Target	Value
A. Calibrated parameters				
Value of home production, b	12.0	Unemployment rate	4.80%	4.86%
Scale of vacancy cost, c_0	6.0	Job-finding rate	0.20	0.21
Curvature of vacancy cost, c_1	1.5	Relation between hiring rate and employment, $corr(HR_t, N_{t-1})$	-0.049	-0.084
Exogenous quit rate, s_0	$1.5e^{-3}$	Relation between separation rate and employment, $corr(SR_t, N_{t-1})$	-0.027	-0.012
Relative search intensity, κ	0.7	Mean hiring rate, $E(HR_t)$	7.8%	7.8%
Scale of utility cost, χ	$6e^{-5}$	Mean hours per week	33.7	33.2
Curvature of utility cost, ξ	2.5	Wage-hours relation, $corr\left(\frac{W_t}{N_t H_t}, H_t\right)$	0.209	0.182
Worker bargaining power, β	0.35	Labor share, $\frac{E[W_t]}{E[R_t]}$	0.47	0.49
Shock arrival rate, μ	0.004	Productivity persistence, $corr\left(\frac{R_{t-1}}{N_{t-1}H_{t-1}}, \frac{R_t}{N_t H_t}\right)$	0.64	0.68
Productivity distr. (Gen.Pareto)				
mean, $E(q)$	2.1	Mean employment	19.6	21.4
st.dev, $St.dev(q)$	5.5	St.dev. of employment	29.6	26.8
shape parameter	0.45	Median employment	9	13
B. Fixed parameters				
Monthly interest rate, r	0.004			
Destruction rate, δ	0.001			
Entry rate, η	$1.33e^{-4}$			
Elasticity of substitution, σ	2.3			
Efficiency of matching, m	0.4			
Curvature of matching, ζ	0.5			

Note: Empirical moments are size-weighted by employment share. Correlation between hourly wages and hours is based on the upper bound measure of hours (for more details see Section 4.2.4.)

For the numerical simulation, I use the following specification of the model. The vacancy posting cost is parameterized as $c(v) = c_0 v^{c_1}$, with $c_0 > 0$ and $c_1 > 1$. The scale parameter c_0 is chosen such that the job-finding rate is equal to 0.2, which corresponds to the average unemployment duration of 5 months[19]. The convexity of vacancy costs parameter c_1 determines the sensitivity of hires to employment

[19] The average distribution of unemployed workers over unemployment duration during the period of 1999-2006 was the

and productivity changes. This value ensures that the correlation coefficient between the hiring rate and lagged employment in the simulation is close to that in the data. The degree of convexity, $c_1 = 1.5$, may seem low compared to some estimates found in previous studies, for example Mertz and Yashiv (2007) report that labor adjustment costs are approximately cubic. However, these values are sensitive to both time and cross-section aggregation, in that the results based on more aggregated data produce higher estimates of convex costs than those of non-convex costs because observed labor variation patterns are more smooth in the aggregate data (Bloom (2009), for instance, illustrates how adjustment costs estimates change depending on time aggregation and within-firm aggregation over different production units). Thus, while Mertz and Yashiv (2007) find that a cubic specification for adjustment costs fits the data well on the quarterly basis, $c_1 = 1.5$ is a reasonable value for monthly worker flows.

As it is commonly assumed in the literature, the matching function exhibits constant returns to scale in job seekers and vacancies, i.e.

$$M\left(v, u + (1-u)\,\kappa\right) = m v^{\zeta} \left(u - (1-u)\,\kappa\right)^{\zeta - 1},$$

with $0 < \zeta < 1$ and a scaling parameter $m > 0$. The parameter m represents the efficiency of a matching process. The job-finding rate, defined as the ratio of matches to job seekers, can be expressed as a function of market tightness only, i.e. $\lambda(\theta) = m\theta^{\zeta}$. Similarly, the worker meeting rate, defined as the ratio of matches to posted vacancies, can be written as $\omega(\theta) = m\theta^{\zeta-1}$. Without the data on vacancies, the matching function parameters m and ζ cannot be identified separately. For the purpose of this simulation, the elasticity of the matching function with respect to vacancies, ζ, is set to 0.5 that is within the range of estimates found in the literature (for instance, Shimer (2005) reports the estimate of 0.38; while Hall (2005) finds the estimate of 0.765).

The monthly interest rate r is equal to 0.4%, which is equivalent to a yearly interest rate of about 5%. Unemployment benefit b, or alternatively the value of home production, is set to match the unemployment rate of 4.8%, the average unemployment rate during the period of 1999-2006 (OECD Economic Outlook 2007). The implied replacement ratio, defined as the ratio of unemployment benefit to the average monthly wage, is about 40%.

Worker and job flows data identify the parameters that govern job-to-job transitions in the model. To be consistent with the job-finding rate and the unemployment rate according to equation (34), the exogenous quit rate s_0 has to be less than 1%. Then exogenous quits alone are insufficient to generate the magnitude of separation rates observed in the data. Thus, job-to-job transitions are required to match the data, that is the relative search intensity κ has to be positive. I set $\kappa = 0.7$ to fit average monthly separation rate in the data. Note also that a higher s_0 increases the correlation between separation rate and employment. Intuitively, under the assumption of an exogenous and constant quit rate only, the separation rate is independent of employment (or positively related since the probability of layoffs is rising with the firm's workforce). In the data, however, this relationship is slightly negative. Thus, allowing for endogenous quits (and decreasing s_0) is crucial for matching the correlation between separation rate and employment.

The elasticity of substitution between intermediate goods σ governs the relationship between output and labor productivity, which is found to be rather strong and positive in the data. Worker's bargaining power parameter β affects the amount of rent-sharing in the model and is set to 0.35 to match the labor

following: 23.1% of workers were unemployed for less than one month; 18.4% - for 1 to 3 months; 19.5% - for 3 to 6 months; 17.6% - for 6 to 12 months, and 21.4% were unemployed for more than a year (Economic Outlook 2007n.d.). The median duration is between 3 and 6 month. Thus, I consider five month unemployment duration to be a reasonable target.

share in total revenues in the data. This value is consistent with the estimates reported in Bagger, Christensen and Mortensen (2009). The scale parameter χ of the utility cost arising from variation in work hours is chosen to reproduce average actual work hours (as opposed to the lower bound hours measure) to 34 hours a week (OECD Economic Outlook 2007). The curvature parameter ξ of the utility cost is equal to 2.5, which is within the range of values estimated by Cooper and Willis (2009) and Bloom (2009). This value ensures that the correlation between hourly wages and hours predicted by the model is close to that observed in the data. The aggregate price level P equates average monthly wages in the model and in the data.

The underlying productivity is assumed to follow a Generalized Pareto Distribution with the parameters chosen to fit the size distribution of firms (the following subsection provides thorough discussion on that). The persistence of the shock process, in terms of arrival rate μ, determines the persistence of labor productivity in the model and hence is chosen to fit the autocorrelation of productivity at the firm level[20].

The magnitude of the monthly firm exit rate found in the data implies a value for the destruction rate that, according to equation (34), is too high to be consistent with the unemployment rate of 4.8% and the job-finding rate of 0.2. Given that firm entry and exit are not the main focus of the model, I choose not to match those rates.

4.2 Model Fit

4.2.1 Employment Distribution

A well-established fact in the existing literature is that the size distribution of firms is highly skewed to the right with a very long right tail. That is, most of the firms in the data are small with a few firms that have much larger than average workforce. In order to replicate the empirical size distribution, a highly skewed distribution for the underlying productivity q is required. In that case, the simulation is able to reproduce the overall shape of the distribution, but not the right tail[21]. To remedy this problem, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process.

It is natural to think of large firms as multi-product entities. Lentz and Mortensen (2008) develop a model, in which the number of product lines for each firm is a result of costly innovation process and product destruction. In their model, more productive firms innovate more frequently and in steady state supply a larger share of product varieties. Here, I assume for simplicity that the distribution of products across firms is exogenous and independent of the firm's productivity q. In that case, all steady state equilibrium equations hold and we can think of $K_n(q)$ as a mass of product lines, instead of a mass of firms.

I use three parameter (location, scale and shape) Generalized Pareto distribution for underlying firm productivity. The location parameter is such that the firm with the lowest productivity would hire at least one worker; the scale and shape parameters are set to match the observed dispersion and median to mean ratio of firm size. The number of product lines per firm is drawn randomly from Poisson distribution at the start of the simulation and evolves stochastically as a birth-death process with the birth rate η and the death rate δ. The ratio of the entry to exit rate, η/δ, determines the total mass of products, and through that, the average employment per product line.

[20] This Poisson arrival shock process is equivalent to a discrete time mean-reverting AR(1) process with autocorrelation coefficient of $e^{-\mu}$. Thus, lower μ implies higher persistence of the underlying productivity process.

[21] Moreover, allowing for firms of larger size increases the time required to solve the model exponentially.

Under these assumptions, the model is able to replicate the employment distribution observed in the data fairly well (see Figure 3). Table 6 presents mean, median, and standard deviation of the firm's employment in the model and in the data (the empirical moments exclude the top one percent of firms). The simulated size distribution shows a lower dispersion and a higher median than the actual distribution. However, it successfully captures the fact that there is a significant size dispersion and that the average firm employs about twice as many workers as the median firm.

Table 6: Employment distribution in the model and in the data.

	Model	Data
Mean	21.4	19.1
Median	13	9.7
Standard deviation	26.8	29.9

Note: Empirical moments exclude firms that employ fewer than 5 workers for six consecutive months, as well as the top one percent of firms. Source: Author's tabulation from the Danish firm data and simulated data.

4.2.2 Hours and Employment Adjustment

In the model, the firm responds to a positive shock in profitability by increasing labor utilization and posting more vacancies. Given search frictions in the labor market, it takes time to recruit new workers; therefore, as vacancies start filling up, hours of work begin to fall. The mechanism is slightly different in the event of a negative shock. In the aftermath of the shock, the firm reduces work hours of its existing employees and, if the firm's productivity falls too low, lays off some of its workers. Following the initial cut in employment, the firm mainly relies on a now-higher attrition rate to drive its workforce further down. Average hours, on the contrary, start rising. Hence, the model can capture the mechanism, through which firms trade off changes in the number of workers and work hours as a response to either a positive or a negative profitability shock.

There are a few parameters that are important for matching the dynamic interaction between employment and hours adjustment. First, more persistent shock process, in terms of a lower arrival rate of profitability shocks, μ, strengthens the dynamic interaction between average work hours and the number of workers. The firm has a stronger incentive to respond to changes in profitability by adjusting its labor force size if the shock lasts longer. On the contrary, if the shocks were white noise then the firm will be more likely to keep its workforce at the same level and adjust labor input only on the hours margin.

Second, a higher vacancy cost parameter induces firms to create fewer vacancies and therefore slows down the recruiting process. However, it has a countervailing effect of increasing the vacancy filling rate and thus raising the return on vacancies. Therefore, the effect of a higher cost on the timing of employment adjustment may be non-monotone. Instead, I pin down vacancy posting costs to match the job-finding rate and use the efficiency of matching parameter m to essentially make a recruiting process more sluggish. The worker contact rate declines in m, given a fixed job-finding rate; thus lowering the vacancy posting rate of firms.

Third, the elasticity of substitution between intermediate goods σ affects the response of the firm to profitability shocks. A lower value of σ implies that profit is less sensitive to productivity level q, which weakens the incentive to hire new workers in the event of a positive shock. On the other hand, it also dampens the association between firm's value added and labor productivity that is found to be fairly strong and positive in the data. Here, I choose the value of σ such that the latter relationship is captured by the model.

Beyond the parameter choice, time aggregation and measurement issues appear to be important when trying to replicate the relationship between growth of hours and employment observed in the data. These will be discussed below. I start by reporting the variation in employment and average hours growth rates in the model and then proceed to examine the co-movement of the two series.

Figure 4 displays the distribution of quarterly labor adjustment series in the simulated data. The model captures the variation in employment growth quite well, but underestimates the variation in average hours growth. Recall that changes in actual hours are reflected in the data only when some of the workers move between the 9-hour intervals. In the model, on the other hand, all individuals are identical, which means that all employees at a given firm work the same number of hours. Hence, for the variation in labor utilization to be captured in the simulated data, all workers have to move to a different 9-hour interval. Therefore, worker heterogeneity could be one of the explanations for the difference in hours growth dispersion between the data and the model.

Table 7 contrasts empirical and simulated moments. In terms of matching the trade-off between hours and workers adjustment, the model does fairy well at monthly frequency: the correlation coefficient between the two series is -0.432. However, when the simulated data are aggregated to a quarterly series the coefficient of interest increases to -0.087, which is well above the level reported in the data (-0.340 for raw series and -0.469 for size-weighted series). The model is more successful in reproducing an important feature of the data that hours growth is leading growth in workforce: the correlation between employment growth and lagged hours growth is 0.076 in the model, compared to 0.071 in the data (0.115 for the size-weighted coefficient).

Table 7: Hours and employment growth rates in the model and in the data.

	Model		Data: Quarterly freq.	
	Monthly freq.	Quarterly freq.	Raw series	Weighted, no time effects
Std.dev ($\Delta \log N_t$)	0.183	0.283	0.242	0.215
Std.dev ($\Delta \log H_t$)	0.152	0.098	0.248	0.221
Corr ($\Delta \log N_t, \Delta \log H_t$)	-0.432	-0.087	-0.340	-0.469
Corr ($\Delta \log N_t, \Delta \log H_{t-1}$)	0.252	0.076	0.071	0.115

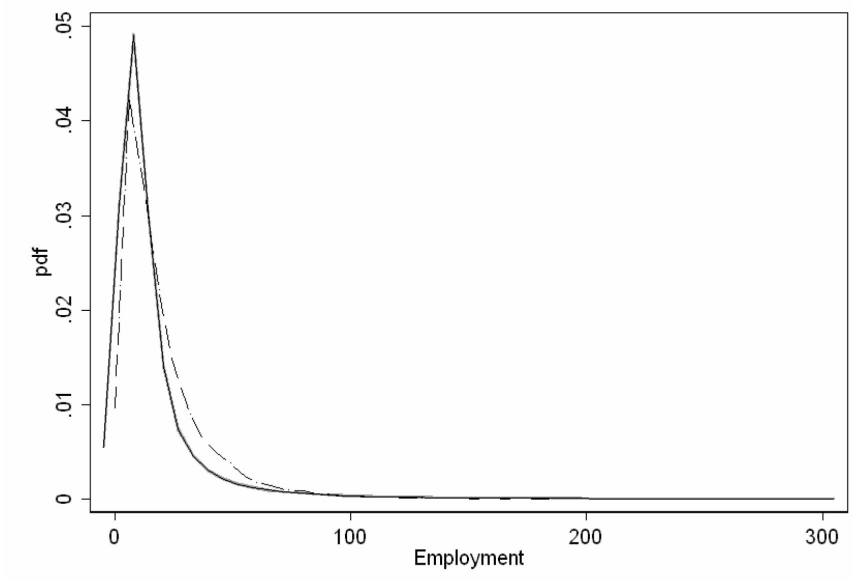
Note: Empirical moments exclude firms with fewer than 5 employees for six consecutive months. Source: Author's tabulation from the Danish firm data (1999-2006) and the simulated data.

In general, time aggregation and reporting frequency are important for observing the negative association between labor adjustment on intensive and extensive margins. Note that the correlation increases also in the data although to a much smaller extent (from -0.340 at quarterly frequency to -0.310 and -0.284 at semi-annual and annual frequency, respectively).

Furthermore, the timing of profitability shocks and of employment adjustment matters for replicating the negative correlation between growth rates in work hours and in the number of workers. Intuitively, if a shock arrives in the middle of the period (as opposed to the beginning of the period) the observed change in hours is smaller, given that we average the before-shock and after-shock levels of work hours over the period. Most of the labor adjustment models that use a discrete time framework implicitly synchronize the timing of the shock arrival with the variation in labor resources. This assumption, however, is not innocuous and tends to generate more negative relationship between changes in labor input on extensive and intensive margins. To illustrate this point, Figure 12 in Appendix C.3 shows an example of how a different timing of the shock arrival process affects the observed workforce and hours adjustment patterns.

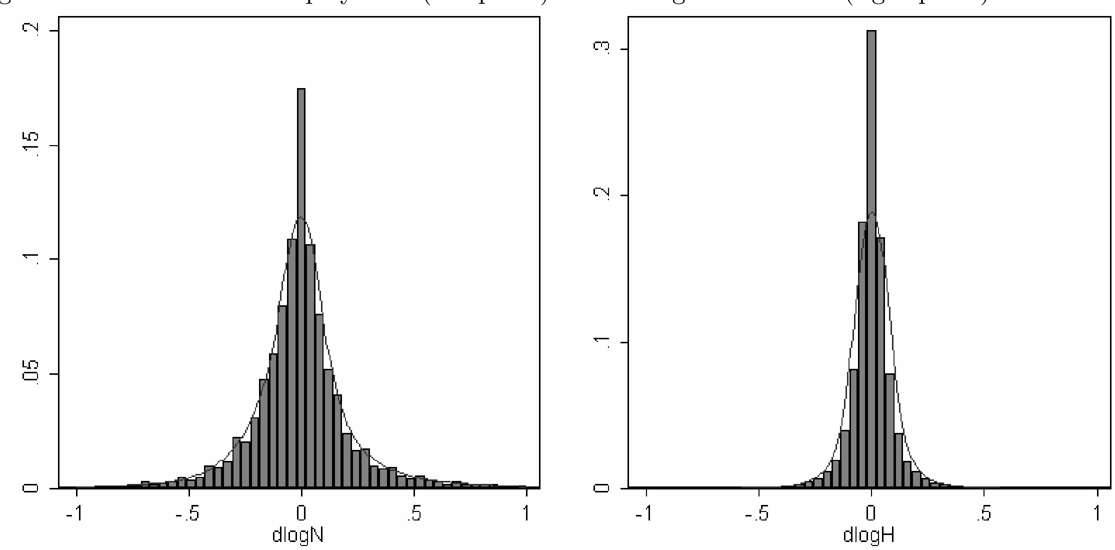
According to the results shown above, it appears that in the model most of the adjustment in the

Figure 3: Size distribution in the data (solid line) and in the model (dashed line)



Note: Density estimation is based on Gaussian kernel with bandwidth of 1. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

Figure 4: Growth rate of employment (left panel) and average work hours (right panel) in the model.



Note: Vertical axis shows a fraction of firm-quarter observations. Density estimation is based on Uniform kernel with bandwidth of 0.1.

workforce is completed within a quarter, so that convex vacancy posting costs and search frictions do not produce enough inactivity in employment. The standard solution in the literature is to introduce non-convex labor adjustment costs. That, however, does not remedy the problem. To see why, consider adding a fixed component to vacancy posting costs, i.e. $c(v) = c_0 v^{c_1} + 1[v > 0] FC$, where $FC > 0$. That is, if vacancies are positive then the firm pays a fixed component $FC > 0$, in addition to the convex part of the cost, $c_0 v^{c_1}$, and pays zero otherwise. Introducing fixed hiring cost component results in the inaction region for all n greater than some threshold value $n'(q)$, for a given value of q. However, this threshold increases in q so that in the event a positive shock the firm still raises vacancies and through that increases its labor force size. Using the plausible values for the fixed cost of hiring, the correlation coefficient between changes in hours and employment falls down to about -0.12, which is still above the value found in the data.

Alternatively, I introduce fixed firing costs into the model. Firing costs appear to be ineffective in producing enough inactivity in employment adjustment. Since steady state distribution of firms in equilibrium is concentrated at low values of employment, the mass of firms for which firing costs are important is negligible. Moreover, under the assumption that wages are not re-negotiated if either worker's or firm's participation constraint is binding, the distinction between layoffs and quits (and correspondingly whether the firm is sustaining firing costs or not) is somewhat arbitrary. In fact, given the parameter values used in this simulation, maximum employment is determined mostly by the worker's problem. Therefore, in the event of an adverse shock workers prefer to leave the firm, making firing costs non-binding for the employer. Note that in the case of downward wage rigidity firing costs are likely to become more important for firms' employment policies.

One possible way to slow down the response in employment is to introduce a recognition lag when the firm is uncertain about how large the shock is or whether it is temporary or permanent. Adding the recognition lag to the model greatly complicates it by making value functions of workers and firms non-stationary. Understanding the factors influencing the negative correlation between growth rates of hours and employment (and trying to bring the model closer to the data in that respect) remains an area for future work.

4.2.3 Worker and Job Flows

On-the-job search is a necessary component that enables the model to capture most of the characteristics of the data related to worker and job flows. Allowing for workers to search while employed means that both the quit rate and the offer acceptance rate depend on firm's type: more productive firms face lower attrition rates and are able to attract workers faster than their less productive counterparts. These features of the model are consistent with empirical evidence reported in earlier studies. Faberman and Nagypal (2008), for instance, document that the vacancy yield (the number of hires per vacancy) increases in employment growth. This result is in line with the prediction of the model that more productive firms have higher acceptance rate of their vacancies and grow faster. The model also predicts that a sizable workforce reduction can be brought about through quits in the case of an adverse profitability shock. Similarly, Davis, Faberman and Haltiwanger (2006) find that quits account for a bigger portion of separations than layoffs for firms that shrink by less than 12% during the month; furthermore, the quit rate is higher in contracting firms than in growing firms.

Table 8 shows empirical and simulated moments concerning worker and job flows. For these results, I restrict attention to continuing firms since the model does not have a rich theory of entry and exit (and I do not match firm entry and exit in the data). The average monthly job and worker flow rates in the data are matched almost precisely by the model. The assumption that the economy is in steady state implies zero net employment change that imposes symmetry on hires and separations, as well as on job creation and job destruction. For the model to be able to fit average hiring and separation rates, the search intensity for employed workers is required to be less than for unemployed workers, that is κ has to be less than one.

Table 8: Monthly job and worker flow rates in the data and in the model.

	Model	Data
Hires	0.078	0.078
Separations	0.078	0.076
Job Creation	0.044	0.041
Job Destruction	0.044	0.039
Net employment change	0.000	0.002
Churning	0.068	0.075

Note: Empirical moments are size-weighted and refer to continuing firms only. Source: Author's tabulations from the Danish firm data (1999-2006) and the simulated data.

Figure 5 illustrates the relation between hiring and separation rates and the firm's size[22]. The worker turnover is more prominent in smaller firms: both hiring and separation rates decrease as employment rises. The model is capable of replicating this relationship, although it predicts a somewhat sharper fall for the hiring rate.

Table 9 presents monthly worker flows in connection with net employment adjustment. The model captures employment growth patterns fairly well. The empirical finding that contracting firms reduce their labor force mostly through separations, while growing firms increase their employment mostly through hiring, is consistent with the model predictions. Furthermore, contracting firms in the model still exhibit positive hiring rates, albeit lower than those observed in the data (see Table 4). In general, the model performs well matching firms with employment adjustment between -10% and 10%, but underestimates worker turnover in firms that grow or contract by more than 10%.

Table 9: Simulated average monthly hiring and separation rates, by net employment growth rate.

Net Emp. Growth	Hires	Sep.	Net	Emp. Share, %
Less than -0.10	0.022	0.271	-0.251	9.11
-0.10 to -0.025	0.033	0.088	-0.055	20.10
-0.025 to 0.025	0.044	0.044	0.000	39.53
0.025 to 0.10	0.089	0.034	0.055	21.13
More than 0.10	0.249	0.029	0.220	10.12

Note: Simulated moments are size-weighted by employment share.

Table 9 also shows that about 39.5% of workers are employed at firms with monthly employment change of less than 2.5% by absolute value; whereas in the data the corresponding number is 50.3% (see Table 4). A common claim in the relevant literature is that non-convex adjustment costs are necessary to match the large region of inactivity observed in the data (see for instance Cooper and Willis (2009)). This model is capable of generating a significant share of firms with zero employment growth with just convex adjustment costs[23]. The reason for that is the presence of search frictions in the market: although firms

[22] In the simulation, there are only a few observations with employment greater than 150 workers. Thus, I present the results for firms with employment of less than 150 workers, that correspond to 97.7% of firm-quarter observations in the data.

[23] In Danish data, about half of all firms have zero monthly net employment change. However, most of these firms are small - they represent one fifth of total employment. In the model, on average 36.4% of firms have zero net employment

post vacancies continuously, they may have zero hires if these vacancies are not matched with workers.

Earlier studies (see for instance Christensen, Lentz, Mortensen, Neumann and Werwatz (2005)) have reported that the separation rate is higher in low-pay jobs. Although in this paper hourly wage per se is not a sufficient statistics for the separation rate, the model admits the negative association between average hourly wages and separations: the correlation coefficient between quarterly (cumulative) separation rates and average firm wages is -0.35 in the model and -0.23 in the data.

4.2.4 Wages and Hours

The trade-off between changes in hours and employment in firm's labor demand comes from two sources: labor adjustment costs that halt the adjustment in the number of workers and costs of changing hours of work. If variations in hours are inexpensive then firms would make all the adjustment on the intensive margin and not through hires and layoffs. In the model, it is the convexity of disutility of working that generates the increasing marginal cost of employing a worker for an extra hour. The existing studies support the claim that variations in hours are expensive in the data: ample empirical evidence for the negative part-time/full-time wage premium has been documented in the literature (see Blank (1990) for a review)[24]. Then, an important question to address is whether hourly wages are increasing in Danish data.

The hourly wage measure is constructed as the ratio of total payroll cost paid in a given quarter to the total work hours, where hours are measured, as before, at their lower bound (see formula (LB) in Appendix A). Wages defined in this manner represent an upper bound on actual hourly wages. The correlation between the two series, wages and work hours, turns out to be slightly negative in the data. One explanation for this counterintuitive finding is the mismeasurement of wages: if wages are overestimated relatively more for low values of hours then we expect to see a decline in hourly wages as hours rise. For that reason, I construct an alternative (upper bound) measure of average weekly work hours that assumes the right boundary of each 9-hour interval for all employees with positive pension contributions; moreover, it assigns 9 hours to workers with zero contributions (see Appendix A for more details on how this variable is constructed). Then, the hourly wage series that is derived based on this alternative measure of hours represents a lower bound on actual wages. The difference between the two wage measures is more prominent for low values of hours, mainly due to fact that the latter measure accounts for employees that work less than 9 hours.

Figure 6 displays the relationship between two measures of wages and work hours. The upper bound on wages displays a marked drop in wages for low values of hours. On the contrary, the lower bound is undoubtedly increasing in work hours. Hence, I conclude that the negative association between wages and hours is attributable mainly to the noise in measurement. Most importantly, the model is capable of reproducing the empirical relationship between wages and hours. The upper bound on wages (constructed in the same way as in the data) is non-monotone for low values of hours; while the lower bound on wages in increasing for the whole range of hours.

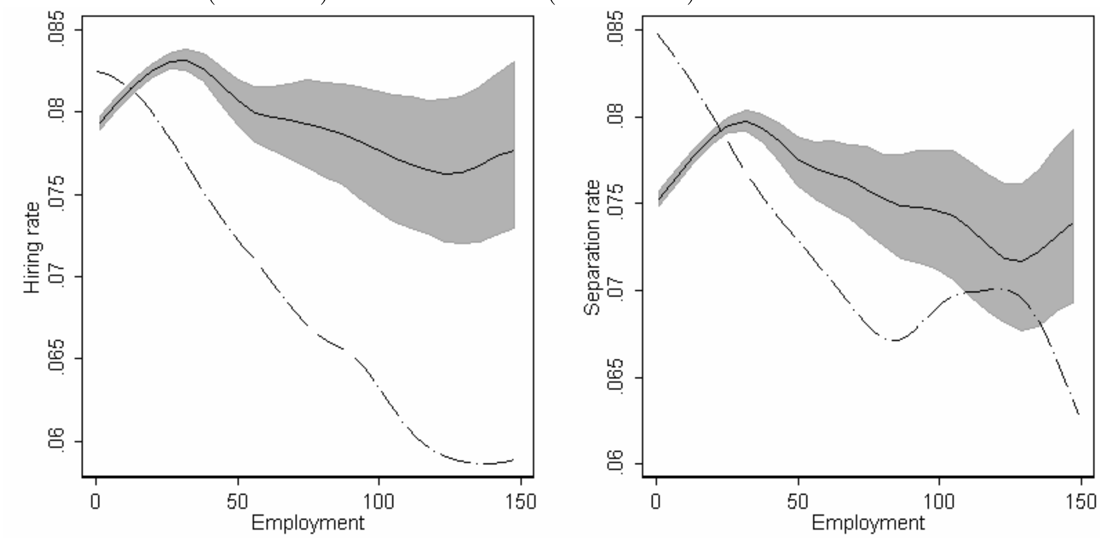
4.2.5 Wages and Labor Productivity

Significant firm and labor productivity differentials have been reported by various studies. The upper panel of Figure 7 illustrates the cross section distribution of hourly wages and labor productivity (con-

adjustment and they employ 22.5% of all workers.

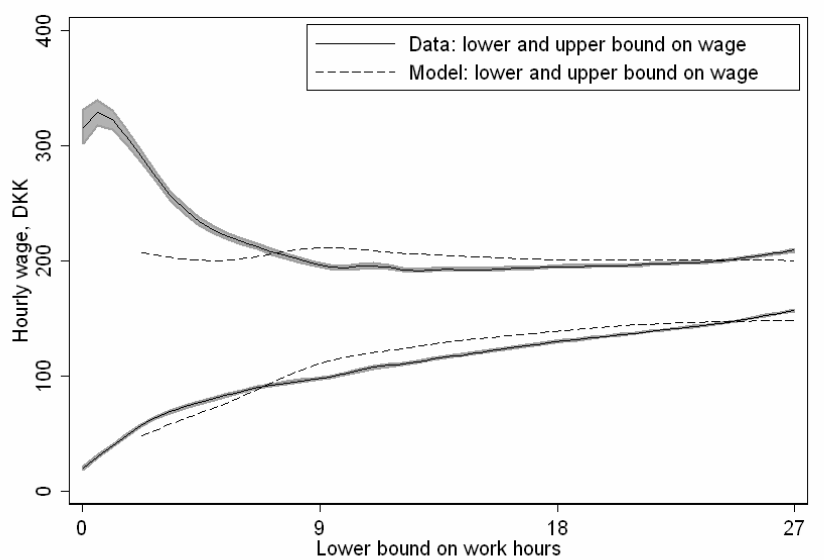
[24] Aaronson and French (2007), for instance, document a 25% wage penalty for men who cut their work week from 40 to 20 hours; however, they reports no such effect on women.

Figure 5: Non-parametric regression of monthly hiring and separation rates on average two-month employment in the data (solid line) and in the model (dashed line)



Note: Estimates are based on Gaussian kernel with bandwidth of 10. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Based on continuing firms. Source: Author's calculations based on the Danish firm data, 2006.

Figure 6: Non-parametric regression of hourly wages on average work hours in the data (solid line) and in the model (dashed line).



Note: Estimates are based on Gaussian kernel with bandwidth of 0.5. Shaded area is 90% pointwise bootstrap confidence interval around upper and lower bound on hourly wage (clustered by firm ID). Wages are deflated by the quarterly CPI. Source: Author's calculations from the Danish firm data, 2006.

structed as value added per work hour) found among firms: both distributions are significantly dispersed and skewed to the right. The interquantile range to median ratio for labor productivity distribution is 0.86; while the ratio of the 90th to the 10th percentile is about 6 to 1. For the wage distribution, the corresponding statistics are 0.39 for the interquantile range to median ratio and 2 to 1 for the 90/10 ratio. These observations are in line with empirical facts found in the other datasets[25].

Hourly wages and productivity distributions in the model are shown in the lower panel of Figure 7. The correlation between wages and labor productivity implied by the theory is almost one; hence, the simulated distributions follow each other very closely. Although the model can reproduce the overall shape of the wage and productivity distributions fairly well, the simulated distributions are less skewed to the right than in the data. The interquantile range to median ratio is 0.16 for labor productivity and 0.12 for wages.

To find empirical support for the wage bargaining assumption in the model, I examine the association between wages and labor productivity in the data. Figure 8 depicts non-parametric regression of hourly wages on hourly labor productivity. More productive firms pay higher wages on average, the finding that has been established for other countries and time periods (see for instance Dunne, Foster, Haltiwanger and Troske (2002) for labor productivity and Baily, Hulten and Campbell (1992) for TFP measure of productivity). This result is consistent with rent-sharing between workers and employers where workers in more productive firms are able to extract higher wages. The correlation between the two series is 0.36 (0.29 for size-weighted series) in the data. The relationship appears to be concave: an increase in wages is more pronounced for a lower part of productivity distribution. In the model, the relationship between labor productivity and wages is almost linear with the correlation coefficient close to one.

Mean wages appear to be increasing in firm employment, the finding that has been well-documented in many other studies (see for instance Oi and Idson (1999), Moscarini and Postel-Vinay (n.d.)). Left panel of Figure 9 shows that hourly wages are higher in larger firms. Despite the fact that Stole-Zwiebel bargaining process implies that wages are decreasing in employment, the model reproduces the positive relationship between wages and firm size. The reason for that is a change in type composition of firms as their workforce rises: larger firms tend to be more productive on average, which offsets the negative effect arising from an increase in the number of workers.

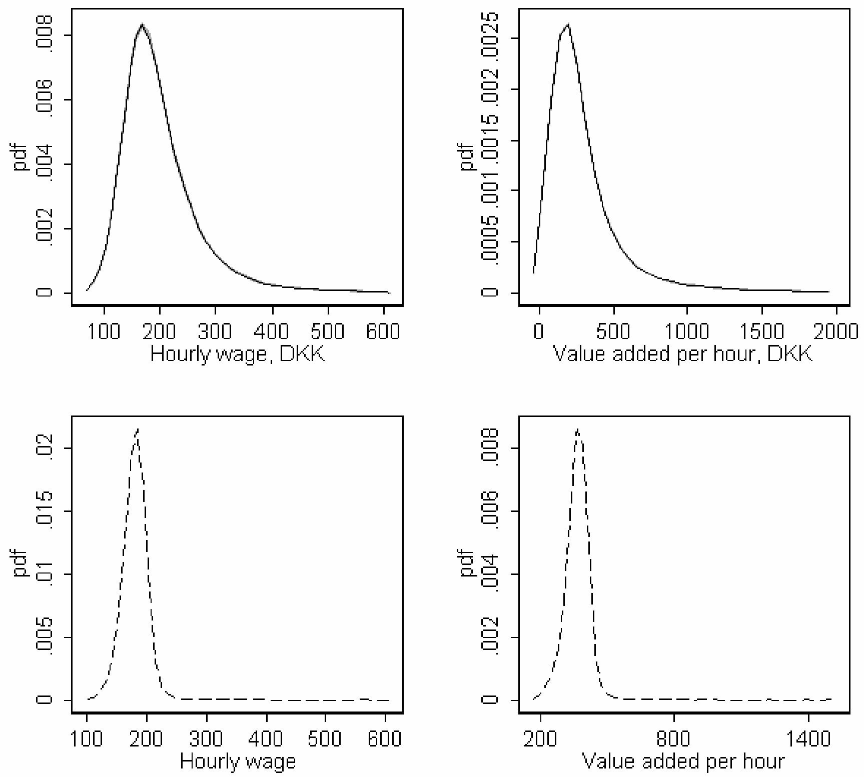
Right panel of the same figure depicts a positive association between firm employment and average work hours: larger firms employ workers for longer hours in the data and to a lesser extent in the model. The level of hours is different for the actual and simulated series[26]. Moreover, the model does not produce enough variation in hours: the number of work hours is virtually the same for all levels of employment. The reason for that is the model predicts that in the absence of search frictions each firm would hire $n^*(q)$ workers and would employ them for the same number of hours regardless of the firm's productivity level q, that is $h(n^*(q))$ as defined in equation (8) is independent of q. Therefore, the variation in hours in the model is driven only by the deviations of employment from the optimal level due to profitability shocks and the presence of hiring costs.

A positive correlation between output and labor productivity has been previously documented in the literature. Baily, Bartelsman and Haltiwanger (2001), for instance, find that the correlation between the two series is 0.29 in the unbalanced panel of the US manufacturing firms. Likewise, the relationship between value added and productivity is found to be rather strong and positive in the Danish firm data

[25] See for instance Mortensen (2003) on firm wage differentials; and Bartelsman and Doms (2000) for a review on productivity related results.

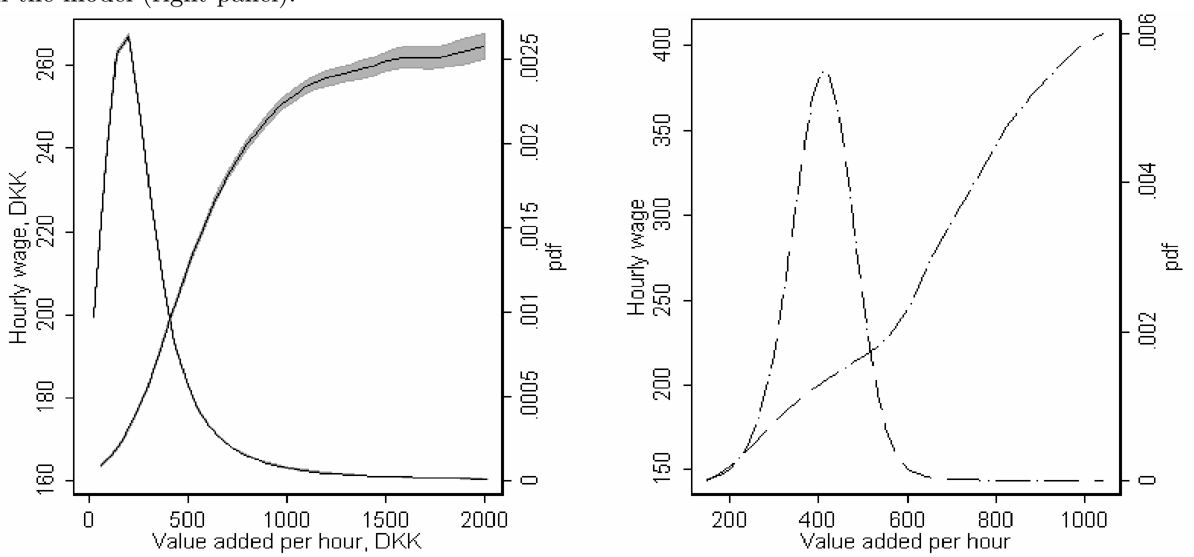
[26] In the simulation, I chose to match average actual hours of work to 34 hours a week, instead of matching the mean of the lower bound measure of work hours observed in the data.

Figure 7: Wage and productivity distribution in the data (upper panel) and in the model (lower panel).



Note: Density estimation is based on Gaussian kernel with bandwidth of 5. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2006.

Figure 8: Non-parametric regression of hourly wages on labor productivity in the data (left panel) and in the model (right panel).



Note: Estimates are based on Gaussian kernel with bandwidth of 100. Shaded area is 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2002-2006 and more so for size-weighted series (see Table 10). Figure 10 displays the association between the two series in the data (left panel) and in the model (right panel). The model fits the relationship between output and productivity fairly well[27].

Table 10: Correlation coefficient between labor productivity and size in the model and in the data.

	Model	Data: non-weighted	Data: no time effects, emp. share-weighted
	$\frac{R_t}{N_t H_t}$	$\frac{R_t}{N_t H_t}$	$\frac{R_t}{N_t H_t}$
Value added, R_t	0.5425	0.0984	0.2970
Employment, N_t	0.4506	0.0005	0.0012
Total work hours, $N_t H_t$	0.4494	0.0008	0.0091

Source: Author's tabulations from the Danish VAT statistics data, 2002-2006.

On the other hand, the model shows discrepancy with the data with respect to the relationship between productivity and labor input. As Lentz and Mortensen (2008) point out, if technological progress is capital augmenting or neutral then we would expect to see more productive firms employing more people. However, the data seems to be on odds with this prediction as employment, and more generally labor input, is virtually uncorrelated with the firm's productivity (Table 10). The model, on the contrary, predicts a strong positive relationship between labor productivity and the number of workers. Some form of labor saving productivity shocks is required to replicate this feature of the data (see Lentz and Mortensen (2008)).

4.3 Counterfactual Experiment

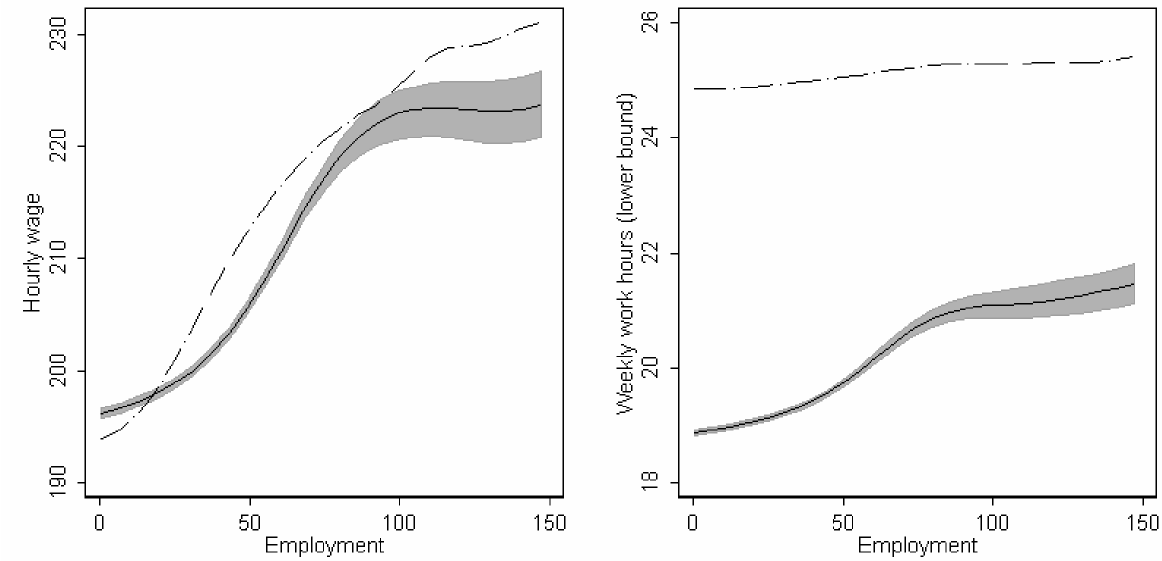
Given the calibrated parameter values described above, I use the model to analyze the implications of adjustment costs for employment policies of firms. First, I perform a counterfactual experiment of doubling the vacancy creation costs and analyze the impact of an increase in costs on unemployment, work hours, and firm value. I compare the effect obtained in a general equilibrium framework to that of partial equilibrium. Second, I simulate the model with the employment subsidy of two weeks of wages per hire.

4.3.1 Hiring Costs

Since a significant part of labor adjustment costs is related to various forms of labor market regulations, the effect of these costs on labor market dynamics has been widely studied by labor economists. In fact, it is the current consensus in this literature that legal impediments to firing workers are responsible for the relatively sluggish employment growth and high levels of unemployment that many European countries have experienced in recent decades (see for instance Goux, Maurin and Pauchet (2001) and Kramarz and Michaud (2004) for empirical studies, Bentolila and Bertola (1990) for a theoretical model). That has led economists to promote labor market reforms aimed at lowering the cost of dismissals, a policy change that many European countries have considered or implemented. In Denmark, firing costs are considered to be

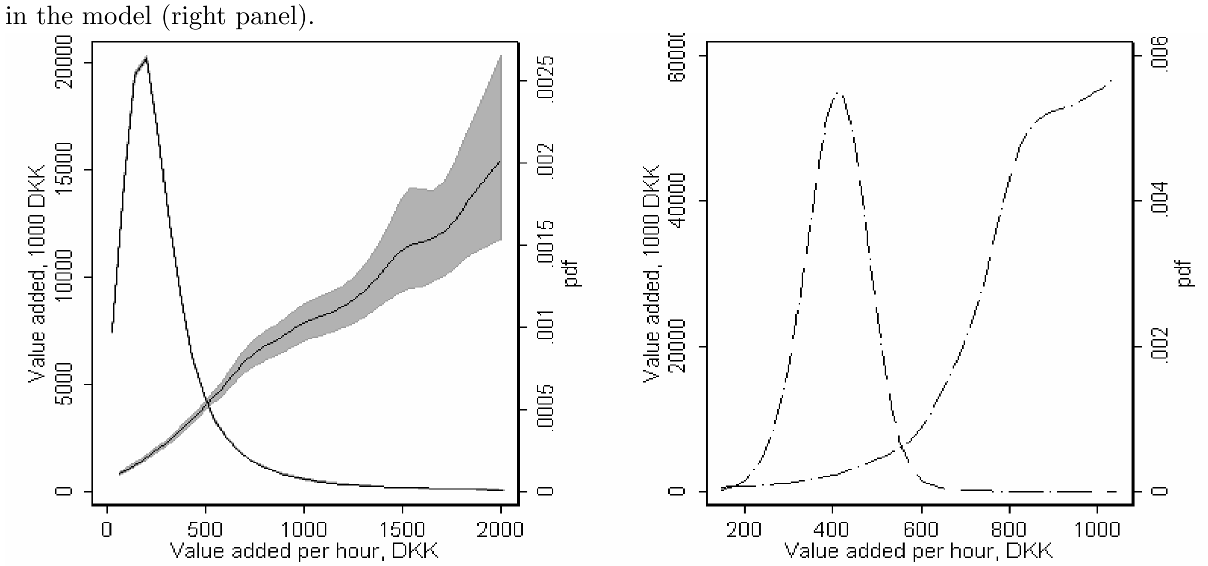
[27] There is a trade-off between matching size-productivity relationship and hours-employment growth relationship. Allowing for a long right tail of the underlying productivity distribution weakens the correlation coefficient between employment and hours growth rates. On the other hand, having a skewed to the right productivity distribution is required to replicate the positive association between output and labor productivity (so that the positive type composition effect offsets the negative effect of employment). For instance, a lognormal distribution was not capable of reproducing the positive output-productivity correlation.

Figure 9: Non-parametric regression of hourly wages and hours on employment in the data (solid line) and in the model (dashed line).



Note: Estimates are based on Gaussian kernel with bandwidth of 20. Shaded area is 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2006

Figure 10: Non-parametric regression of value added on labor productivity in the data (left panel) and in the model (right panel).



Note: Estimates are based on Gaussian kernel with bandwidth of 100. Shaded area is 90% pointwise bootstrap confidence intervals. Source: Author's calculations based on the Danish firm data, 2002-2006 virtually non-existent[28]. As mentioned in section 4.2.2, the costs of dismissals are essentially ineffective in the model as most of the reduction in the workforce is achieved through quits[29]. The main reason for that is high rates of job-to-job transitions in Denmark and a fairly low unemployment duration, so that workers leave troubled firms and move to other businesses or quit to unemployment. Here, I focus on the effect of hiring costs on the unemployment rate, the job-finding rate, and the trade-off in the variation in hours and employment. The results in this section are relevant for marginal employment subsidies programs (aimed at reducing recruiting costs).

The parameters of the benchmark model imply that the average cost of hiring a new worker is equal to about one month of wages. Shimer (2005) calibrates hiring costs of a similar magnitude for the US labor market[30]. In general, it is not easy to obtain information on the sources and sizes of adjustment costs because many of these costs are implicit, in that they result in lost output and are thus not measured and reported. Recent works of Abowd and Kramarz (2003) and Kramarz and Michaud (2004) estimate firing and hiring costs directly based on survey data for a representative sample of French firms. They find considerable costs of hiring and separation, with separation costs exceeding hiring costs. Rota (2004), based on annual firm level data from the Italian manufacturing industry, reports an estimate of fixed adjustment costs of 15 months of labor costs. Mertz and Yashiv (2007) find that marginal costs of hiring is roughly equivalent to two quarters of wage payments. Bloom (2009) estimates per capita labor adjustment costs of 1.8% of annual wages together with a fixed component of 2.1% of annual sales. Also note that many of the current estimates of labor adjustment costs pertain to net employment changes, as opposed to gross worker flows, and therefore are not directly comparable to this study.

To evaluate the effect of an increase in hiring costs on employment policies of firms, I consider the following model specifications. First, I simulate the model with doubled vacancy creation costs; that is, I set $c_0 = 12$ vis-à-vis the benchmark case of $c_0 = 6$. In that case, the convexity of the cost structure is preserved. Many studies of labor adjustment advocate non-convex adjustment costs (see for instance Cooper and Willis (2009)). To examine the difference implied by the cost structure, I add a fixed component to vacancy posting costs, i.e. $c(v) = c_0 v^{c_1} + 1[v > 0] FC$. The fixed cost is set to be equal to mean monthly wages, so that the average hiring cost is about twice as high as in the benchmark model.

The first three columns of Table 11 compare the two experiments with the baseline model. Doubling the costs increases the unemployment rate from 4.9% to about 9% and reduces the job-finding rate by about 2 percentage points. Higher adjustment costs lead to a decrease in employment growth variation. In the case of higher convex costs of hiring ($c_0 = 12$) firms substitute towards the intensive margin of adjustment in that the variation in hours growth rises, while the variation in employment growth falls. Also note that both average hours per person and standard deviation of hours increase compared to the benchmark case.

Non-convex vacancy posting costs reduce the variation in both employment and hours growth. The intuition behind this result is that fixed hiring costs induce firms not to make small employment adjustments. Then, in the event of a positive shock, employment builds up faster and hours deviate from their

[28] Here, I refer to measures of labor market flexiblity developed by Botero, Djankov, Porta, de Silanes and Shleifer (2004). Their original data have been extended by the World Bank and are available at http://www.doingbusiness.org/ExploreTopics/EmployingWorkers/. Difficulty of firing index in Denmark is 0 out of 100 compared to, for instance, 30 in France, 40 in Italy, and an average of 22.6 for OECD countries (as downloaded on October 12, 2009).

[29] Introducing fixed firing costs of about three months of wages increases unemployment by 0.4 percentage points, while the job finding rate remains at about the same level.

[30] In his model, the cost of hiring a worker is equal to the flow cost of sustaining an open vacancy (0.213) multiplied by an average duration of a vacancy (1/1.35 of a quarter), which corresponds to about two weeks of wages. Note that the total share of labor income in his model is about twice as high as in this simulation (0.993 and 0.49, respectively). Thus, the hiring costs here are of about the same magnitude in terms of output per worker, and not in terms of wages.

Table 11: The effect of doubling the vacancy posting cost.

	Benchmark	Doubled cost			
	Model	General Equilibrium		Partial Equilibrium	
	$c_0 = 6$	$c_0 = 12$	$FC = 30$	$c_0 = 12$	$FC = 30$
Unemployment rate	4.9%	9.1%	8.7%	13.7%	14.3%
Job-finding rate	0.214	0.191	0.192	0.172	0.151
Average hours per person	33.2	33.6	33.1	34.4	34.0
Standard deviation of hours	3.62	3.82	3.63	3.84	3.70
Change relative to benchmark:					
std.dev($\Delta \log N_t$)	—	-2.8%	-2.6%	-0.7%	-0.9%
std.dev($\Delta \log H_t$)	—	9.4%	-4.6%	16.8%	6.0%
Corr ($\Delta \log H_{LB,t}, \Delta \log N_t$)	-0.087	-0.038	-0.107	-0.061	-0.107

optimal level for a shorter period of time. That reduces the variation in hours growth. Non-convex hiring costs also produce a more negative correlation coefficient between variation in hours and employment than convex costs do.

Most of the existing models that analyze the impact of labor adjustment costs on the firm's labor demand use partial equilibrium framework (see for instance Bentolila and Bertola (1990)). If we were to use the results of these studies to draw policy conclusions, it is natural to think of extending these models to account for labor supply effects. To illustrate the importance of general equilibrium effects for the analysis of labor adjustment costs, I perform the same experiment of increasing vacancy posting costs in a partial equilibrium framework. A rise in hiring costs causes a decline in vacancies posted by all firms. In general equilibrium, a fall in total vacancies raises the worker contact rate through the matching process. That in turn increases the return on vacancies, thus mitigating the initial negative effect of higher costs. In the experiment, I shut down the feedback coming from the labor supply side and aggregate distribution of vacancies, that is I keep the separation rate, the offer acceptance rate, and the vacancy filling rate fixed at the same level as in the benchmark simulation, and solve for the firm's problem with higher vacancy costs.

The results of the second experiment are shown in the last two columns of Table 11. Eliminating general equilibrium channels generates a much higher unemployment rate and a lower job-finding rate: unemployment rises to 14% (by 5 percentage points higher than in the general equilibrium case) and the implied unemployment duration increases to 6 - 7 months[31]. In addition, in partial equilibrium firms employ fewer workers on average but for longer work hours. Given the calibrated parameter values, the average loss in the firm's value arising from doubling the cost parameter c_0 is 0.94 million DKK in general equilibrium, compared to 1.21 million DKK in the partial equilibrium framework. These results caution against using partial equilibrium models to evaluate aggregate implications of the labor adjustment costs.

4.3.2 Employment Subsidy

The model in this paper can be used to evaluate the effect of a hiring subsidy on the unemployment rate and the job-finding rate. One of the main concerns of policymakers during recessions is to find an effective way to stimulate job creation. Among the proposed solutions is the introduction of a new jobs

[31] The difference between partial and general equilibrium results is robust to changes in parameters. Using different parameter values (e.g. a lower dispersion of underlying productivity), I obtain a somewhat smaller increase in the unemployment rate due to doubling the vacancy posting costs (unemployment rises by 2 percentage points instead of current 4 percentage points). However, an increase in the unemployment rate is about two times higher in partial equilibrium than in general equilibrium.

tax credit, which is a tax credit for businesses that expand their employment (see Bishop (2008) for a discussion of the current policy debate on the effectiveness of tax credits for employment growth). To examine the impact of such a policy, I simulate the model with the employment subsidy received by the firm for every new hire in the amount of two weeks of wages. Table 12 compares the results of the two experiments with the baseline model. The hiring subsidy reduces the unemployment rate from 4.9% to 3.4% and increases the job-finding rate by about 1 percentage point. Average work hours decrease as firm hire more workers. Again, in the partial equilibrium case the effect of the hiring subsidy on the unemployment rate is almost twice as large as in the general equilibrium case.

Table 12: The effect of the hiring subsidy.

	Benchmark	Hiring subsidy ($HS = 15$)	
	Model	General Equilibrium	Partial Equilibrium
Unemployment rate	4.9%	3.4%	1.8%
Job-finding rate	0.214	0.221	0.232
Average hours per person	33.24	33.11	32.80
Standard deviation of hours	3.63	3.55	3.60

5 Conclusion

This study is motivated by the observation that firms use variation in work hours of their employees extensively to adjust their labor demand in response to exogenous shocks. The goal of this paper has been to develop a model of labor adjustment that introduces variation in labor resources on both intensive and extensive margins. The model with search frictions in the labor market appears to be a natural candidate for formalizing firms' recruiting strategies and analyzing factors that hinder employment adjustment.

In this paper, I build a general equilibrium theory of heterogeneous multi-worker firms that choose their hiring and firing policies optimally in the economy with search frictions. The driving force of the model is idiosyncratic profitability shocks that firms can accommodate by varying work hours of their existing employees and/or adjusting their workforce. Wages are determined through the bargaining process. In addition, allowing for on-the-job search delivers a rich theory of quits that enables the model to capture most of the features of the data regarding worker flows.

The model is calibrated to assess its fit to the Danish firm data and appears to be quite successful in capturing the overall characteristics of the data. The numerical simulation does an outstanding job of reproducing employment variation at the firm level. It matches hiring and separation rates, job creation and job destruction rates, the distribution of firms by net employment growth, and difference in worker flows depending on the firm size and wages paid. In addition, the empirical relationships between firm size, wages, and productivity at the firm level are reproduced in the simulated data.

Regarding labor adjustment on the intensive margin, the model underestimates the variation in average work hours. The negative association between changes in number of workers and hours as found in the data is replicated in the simulation at the monthly frequency. At the quarterly frequency, however, the correlation rises above the level observed in the data, reflecting the importance of time aggregation for dynamic interaction between hours and employment. It seems that the model requires additional frictions (such as uncertainty on the nature of the shock, whether it is temporary or permanent) to slow down the response of employment. On the other hand, the empirical fact that changes in hours lead changes in employment is consistent with model predictions.

Given the calibrated parameters, the average cost of hiring a new worker in the model is equal to about one month of wages. In stark contrast with other labor adjustment models, I find that firing costs have virtually no effect on changes in labor resources. Allowing for wage bargaining and on-the-job search is at the heart of this result since most of the reduction in the workforce in the event of an adverse profitability shock is achieved though quits. Using the numerical model simulation, I then show that doubled hiring costs raise the unemployment rate from 4.9% to about 9% and reduce the job-finding rate by about 2 percentage points. Moreover, the same increase in hiring costs in a partial equilibrium framework (when the worker contact rate, the offer acceptance rate, and the separation rate are held at the same level as in the baseline model) generates a twice higher increase in the unemployment rate. These results caution against using partial equilibrium models to evaluate aggregate implications of the labor adjustment costs. I also show that a hiring subsidy in the amount of two weeks of wages (or about a half of average hiring costs) is effective in reducing unemployment by 1.5 percentage points.

The next step is to estimate the model using indirect inference approach. Then, using the estimated structural parameters, I can examine the effect of hiring costs and search efficiency on firms' optimal employment policies, firms' profit, and workers' welfare. Moreover, I can perform such policy experiments as changes in over-time premium, introduction of advanced layoff notice, mandated work week, etc. Likewise, I can evaluate the importance of hours channel in labor input adjustment at the firm level by running the counterfactual experiment that shuts down the hours margin.

In addition, extending the model to include aggregate shocks may be important for the analysis of adjustment costs on employment dynamics and the interaction between labor adjustment on intensive and extensive margins. Intuitively, aggregate shocks will have different implications for firm employment strategies than idiosyncratic shocks do. First, in the event of a positive shock, in addition to posting more vacancies, the firm finds it easier to retain workers. If all firms experience an increase in their profitability at the same time the attrition rate will remain unchanged; hence, firms have to post more vacancies to reach the desired level of employment. Second, in the event of a negative shock workers will be less willing to quit the firm (since the value of unemployment is lower in recessions due to a lower job-finding rate), thus making firing costs binding for employers. The structural estimation of the model and further possible extensions, such as introduction of aggregate shocks into the model, remain an area for future work.

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Appendix

A. Data Sources

The empirical analysis in this paper is based on Danish firm data drawn from administrative records for the period of 1999-2006. They come from four major sources. First, the detailed information on employment changes is obtained from a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly employment is constructed as a head-count of all individuals employed in a given firm. Quarterly number of employees is derived as an average of three months employment. The particular structure of these data makes it possible to construct hires and separations series for each firm.

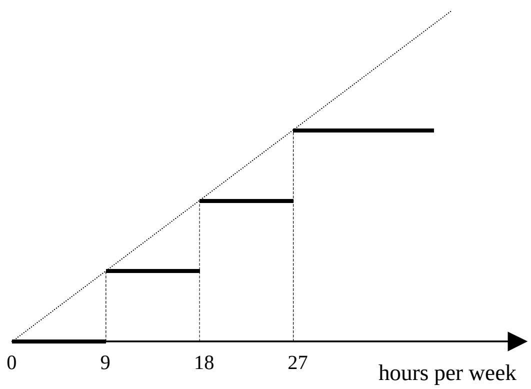
Second, a full-time equivalent (FTE) employment series, and correspondingly work hours series, is derived from the firms' mandatory pension contribution data collected on a quarterly basis. In Denmark, firms are required to pay pension contributions for each 16-66 year old employee according to her weekly hours of work. The rule for the pension contribution (depicted in Figure 11) is as follows:

- full amount of contribution (670.95 DKK in 1999-2005 and 731.70 DKK in 2006 per quarter) is paid for an employee working more than 27 hours a week;
- 2/3 of the full amount is paid for an employee working between 18 and 27 hours a week;
- 1/3 of the full amount is paid for an employee working between 9 and 18 hours a week;
- zero contribution is paid for all employees working less than 9 hours a week.

It is important to note that the mandatory pension contributions are differentiated in accordance with the collective wage agreements: for some employees in the public sector full contributions (A-type) are paid, while for other employees B, C, or D contributions are paid. These rates of contributions make up, respectively, about 40%, 60%, and 48% of the ordinary contributions (further details on pension contribution rule can be found on Statistics Denmark website www.dst.dk and ATP Pension Fund website www.atp.dk). The exact distribution of employees by type of the pension plan is known only on a yearly basis; therefore, if some workers switch to a different plan within a year then the reported FTE measure will be incorrect. Given that B, C, or D contributions are paid primarily in the public sector,the empirical analysis is restricted to private firms. After the exclusion of the public sector, there are about 0.6% of firm-quarter observations that report having paid other than A-type contributions. These observations are removed from the analysis to avoid the FTE measurement inaccuracy.

The available data contain the sum of pension contributions paid by the firm for all of its employees in a given quarter. Then, the FTE measure reported by the Danish Central Statistical Office is constructed

Figure 11: Mandatory pension contribution scheme.



as the total amount of quarterly pension contributions divided by the payment norm for a full-time employee (where full-time refers to working more than 27 hours per week). Given the proportionality of the schedule, the average weekly hours of work can be derived by dividing the total FTE measure, N^*, by the number of employees, N, and multiplying by 27 hours a week, i.e.

$$H_{LB} = 27\frac{N^*}{N}. \tag{LB}$$

This approach implicitly assumes the least work hours (i.e. left boundary of each 9-hour interval) for all employees and therefore represents the lower bound on weekly hours of work.

In order to construct the upper limit of average work hours, I consider the right boundary point for each of the 9-hour intervals in Figure 11. The right boundary of the upper interval is assumed to be 36 hours a week. This assumption, albeit not very realistic, preserves the proportionality of the hours schedule. Also, recall that the FTE measure exclude employees that work less than 9 hours per week. Therefore, if the number of workers in a given firm is higher than the number of full-time employees, I allocate 9 hours of work to those extra workers. In sum, the upper bound on weekly work hours per employee is defined as

$$H_{UB} = \frac{36N^* + 9(N - N^*)\mathbf{1}\left[N > N^*\right]}{N}. \tag{UB}$$

The third dataset is drawn from the VAT statistics for the period of 2002-2006. It provides information on purchases and sales of all VAT-liable businesses on a quarterly basis, measured in Danish Kroner (DKK). In Denmark a business enterprise must register for VAT if its annual turnover is expected to exceed 50,000 DKK. The VAT declaration frequency depends on the annual turnover: firms report monthly if their annual turnover exceeds 15 million DKK, quarterly if their turnover is between 1 million DKK and 15 million DKK, and semi-annually if it is below 1 million DKK. Hence, the empirical moments on value added and labor productivity in this paper refer to businesses with annual turnover above 1 million DKK (in total, 13.9% of firm-quarter observations are excluded due to missing quarterly information). Lastly, I use only data on firms with positive value added.

The fourth dataset contains information on total payroll costs that firms pay in a given quarter. Wages are measured in Danish Kroner (DKK) and are deflated using quarterly CPI with 2001 Q1=100.

In Denmark, there is no statutory national minimum wage since legal collective agreements are the main mechanism used for regulating low pay[32]. The hourly wages less than 80 DKK per hour are removed from the analysis. This figure is regarded as an estimate of the effective legal minimum wage. In addition, I exclude the wage rates of the top one percent of the observed distribution.

The empirical analysis is carried out based on private firms information. Furthermore, the sample is restricted to firms with at least five employees to reduce the noise in the data (these firms are largely excluded from the VAT statistics dataset). In order to avoid the creation of spurious exit and entry flows into the sample when employment falls below five workers in one quarter and exceeds it in the next quarter, I apply the following sampling rule: the firm is considered to have exited the sample if its employment has been fewer than 5 employees for six consecutive months. In total, there are 53.7% of firm-quarter observations that fall into the category of 'exiting' firms; however, they comprise only 6.3% of total employment. The resulting dataset has 120,058 firms that are observed in the data for 14 quarters on average. Given that the model does not account for aggregate shocks, time effects were removed from hourly wages and labor productivity series.

B. Wage Bargaining

Here, I show that in Stole-Zwiebel framework the bargaining over hours and wages simultaneously is equivalent to the bargaining over wages, when hours schedule is chosen by the employer prior to the bargaining and set at its optimal level. Then, I provide a solution to the differential equation in wages that arises as the outcome of the bargaining problem.

B.1 Bargaining over Hours Schedule

First, I extend the basic Stole and Zwiebel (1996) bargaining problem to include hours of work and continuous labor. Consider the bargaining problem for an individual i. Her wage and work hours have to maximize the following equation:

$$\max_{w_i,h_i}\left\{\begin{array}{c}\left(\frac{R\left(\bar{h}(n+\Delta n)n+h_i\Delta n\right)}{P}-\frac{\bar{w}(n+\Delta n)}{P}n-\frac{w_i}{P}\Delta n-\frac{R\left(\bar{h}(n)n\right)}{P}+\frac{\bar{w}(n)}{P}n\right)^{1-\beta}\times\\ \times\left(\left(\frac{w_i}{P}-g\left(h_i\right)-b\right)\Delta n\right)^{\beta}\end{array}\right\}, \tag{B1}$$

where $\bar{h}(n)$ is the average work hours chosen by other workers in the firm with employment n and $\bar{w}(n)$ is wages paid to other employees if total employment is n. The first order condition with respect to wage w_i reads

$$\begin{aligned}&\beta\left(\frac{R\left(\bar{h}\left(n+\Delta n\right)n+h_i\Delta n\right)}{P}-\frac{\bar{w}\left(n+\Delta n\right)}{P}n-\frac{w_i}{P}\Delta n-\frac{R\left(\bar{h}\left(n\right)n\right)}{P}+\frac{\bar{w}\left(n\right)}{P}n\right) \\ =\ &(1-\beta)\left(\frac{w_i}{P}-g\left(h_i\right)-b\right)\Delta n.\end{aligned} \tag{B2}$$

The number of hours for an employee i is chosen optimally to maximize the problem above, taking the work hours of other employees as given. The first order condition with respect to h_i determines the optimal choice of hours:

$$\frac{R'\left(\bar{h}\left(n+\Delta n\right)n+h_i\Delta n\right)}{P}=g'\left(h_i\right), \tag{B3}$$

[32] The percentage of employees covered by collectively agreed wages is estimated above 80% (see for instance Danish Confederation of Trade Unions website www.lo.dk).

where $R'(\cdot)$ refers to the derivative of total revenues with respect to total labor input, hn. Equation $(B3)$ is equivalent to equation (2), given the symmetry of the bargaining problem for all workers.

Next, note that the following limit can be rewritten as

$$\begin{aligned} & \lim_{\Delta n \to 0} \frac{R\left(\bar{h}(n+\Delta n)\,n + h_i(n+\Delta n)\,\Delta n\right) - R\left(\bar{h}(n)\,n\right)}{\Delta n} \\ = & \left(\begin{array}{c} \lim_{\Delta n \to 0} \frac{R\left(\bar{h}(n+\Delta n)n + h_i(n+\Delta n)\Delta n\right) - R\left(\bar{h}(n+\Delta n)n\right)}{h_i(n+\Delta n)\Delta n} h_i(n+\Delta n) \\ + \lim_{\Delta n \to 0} \frac{R\left(\bar{h}(n+\Delta n)n\right) - R\left(\bar{h}(n)n\right)}{\bar{h}(n+\Delta n)n - \bar{h}(n)n} \lim_{\Delta n \to 0} \frac{\bar{h}(n+\Delta n) - \bar{h}(n)}{\Delta n} n \end{array} \right) \\ = & R'(h(n)\,n)\left(h(n) + \frac{\partial h(n)}{n} n\right), \end{aligned}$$

again, under the symmetry assumption. Likewise,

$$\lim_{\Delta n \to 0} \frac{w(n+\Delta n)(n+\Delta n) - w(n)\,n}{\Delta n} = w'(n)\,n + w(n).$$

Dividing equation $(B2)$ by Δn and taking limits as $\Delta n \to 0$, I obtain the outcome of the bargaining problem that is identical to equation (3).

B.2 Solution of the Bargaining Problem

Solving for the first order condition of equation (3) leads to a first-order linear differential equation in wage

$$w(n) = \beta R'(n) + (1-\beta)\,P\,[g(h(n)) + b] - \beta w'(n)\,n. \tag{B4}$$

The solution of the homogenous equation $w'(n) + \frac{w(n)}{n\beta} = 0$ is equal to

$$w(n) = An^{-\frac{1}{\beta}}, \tag{B5}$$

where A is a constant of integration of the homogenous equation. Assuming that A is a function of n and substituting $(B5)$ into $(B4)$ I get

$$A'(n) = R'(n)\,n^{\frac{1-\beta}{\beta}} + \frac{(1-\beta)}{\beta} P\,[g(h(n)) + b]\,n^{\frac{1-\beta}{\beta}},$$

or, by integration

$$A(n) = \int_0^n z^{\frac{1-\beta}{\beta}} \left(R'(z) + \frac{(1-\beta)}{\beta} Pg(h(z))\right) dz + (1-\beta)\,Pbn^{\frac{1}{\beta}} + B,$$

where B is a constant of integration.

The last equation implies that the solution to $(B4)$ is

$$\frac{w(n)}{P} = n^{-\frac{1}{\beta}} \int_0^n z^{\frac{1-\beta}{\beta}} \left(\frac{R'(z)}{P} + \frac{(1-\beta)}{\beta} g(h(z))\right) dz + (1-\beta)\,b, \tag{B6}$$

where to pin down B, I assumed as in Stole and Zwiebel (1996) that wage is finite when $n \to 0$ which implies $B = 0$[33].

[33] Also, I assume as in Stole and Zwiebel (1996) that the conditions for the existence of the integral in $(B6)$ are satisfied.

C. Simulation

C.1 Solving for Equilibrium

Given the model parameters $(r,\ \beta,\ \sigma,\ \mu,\ \delta,\ \eta,\ \chi,\ \xi,\ b,\ s_0,\ m,\ \zeta,\ \kappa,\ c(\cdot), \Phi(\cdot))$, the solution algorithm can be described as a fixed point search of equilibrium variables (θ, u, Y) and distribution functions $F(W_n(q))$ and $G(W_n(q))$ through the mapping, which is defined in equations $(15)-(32)$. I solve for the equilibrium numerically applying iteration on this mapping. The iteration procedure turned out to be more stable than looking for a fixed point using minimum distance routines.

Note that the value functions and steady state equilibrium equations can be rewritten in terms of $\hat{q} = Y^{\frac{1}{\sigma-1}}q$. In this way, I reduce the number of equilibrium variables that I iterate over. Thus, I start with the distribution of $\hat{q}$ and solve for an equilibrium. Then I derive the aggregate output as

$$Y = \left[\frac{\sigma-1}{\xi\sigma}\right]^{\frac{\sigma-1}{(\xi-1)\sigma+1}} \int \hat{q}^{\frac{\xi(\sigma-1)}{(\xi-1)\sigma+1}} \sum_{n=0}^{\bar{n}(\hat{q})} n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1)\sigma+1}} K_n(\hat{q}) d\hat{q},$$

and recover the underlying productivity q from $q = \frac{\hat{q}}{Y^{\frac{1}{\sigma-1}}}$.

I discretize the state space in terms of productivity q and use Gaussian quadrature method to approximate the expected value of any function of $\hat{q}$. Here, I assume that the productivity $\hat{q}$ follows Generalized Pareto Distribution with the density

$$\frac{1}{\sigma_{GPD}}\left(1 + k\frac{\hat{q} - \underline{\hat{q}}}{\sigma_{GPD}}\right)^{-\frac{1}{k}-1},$$

where k is a shape parameter, σ_{GPD} is a scale parameter, and $\underline{\hat{q}}$ is a location parameter. The mean of the distribution is $\underline{\hat{q}} + \frac{\sigma_{GPD}}{1-k}$ for $k < 1$ and variance is $\frac{\sigma^2_{GPD}}{(1-k)^2(1-2k)}$ for $k < 1/2$.

First, I compute the firm's profit and the worker's utility from equations (11) and (14). Then, given the initial guess for the distribution functions $F(W_n(\hat{q}))$ and $G(W_n(\hat{q}))$, the market tightness θ, and the unemployment rate u, I construct the separation and offer acceptance rates. I apply value function iteration procedure to find the firm's value $V_n(\hat{q})$, the value of employment $W_n(\hat{q})$, and the value of unemployment U. Note that at each step we need to verify that worker's and firm's participation constraints are satisfied in the bargaining process.

The optimal vacancy posting rate $v_n(q)$ and the maximum labor force size $\bar{n}(q)$, derived from the firm's problem, are then used to find steady state distribution of products across types, $K_n(\hat{q})$. Given the state space is discretized in the numerical solution algorithm, the equations $(23)-(25)$ represent a linear programming problem that can be solved for directly. However, due to numerical approximation imprecisions, iteration over the product distribution turned out to perform better. Using equations (29)-(32), I update the initial guess for the distribution functions $F(W_n(\hat{q}))$ and $G(W_n(\hat{q}))$, the unemployment rate u, and market tightness θ. I then repeat the procedure until the convergence of equilibrium objects is achieved.

C.2 Simulation

The equilibrium hiring and separation rates, $H_n(\hat{q})$ and $s_n(\hat{q})$, as well as the maximum labor force size $\bar{n}(q)$, are the main variables that determine employment dynamics at the firm level. Given Poisson arrival rates, the waiting time until the next occurrence of any shock is distributed exponentially with parameter $x = \mu + \delta + H_n(\hat{q}) + s_n(\hat{q})\, n$. Thus, I generate a time path for each of the simulated firms as a random draw from an exponential distribution. Whether it is a destruction shock, a productivity shock, a new hire, or a separation is decided according to the relative probability of each event.

I simulate 1000 firms for 300 months and discard first 30 months under the assumption that the economy will converge to the steady state equilibrium within the first 30 periods. The simulated monthly employment series includes all workers who were employed in a given month. Quarterly employment is an average of three month employment. The average hours series is constructed according the pension payment contribution schedule to replicate the (interval) hours measure reported in the data. Wage, revenue and hours variables are aggregated over three months to generate the corresponding quarterly series.

Below, I describe in detail the parameter choice in the simulation.

1. The monthly interest rate r is equal to 0.4%, which corresponds to about a 5% yearly interest rate.

2. To fit the empirical size distribution of firms, in particular, a long right tail of the distribution, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process. The number of products per firm is exogenous and independent of firm productivity q. In that case, all steady state equilibrium equations hold and we can think of $K_n(q)$ as the distribution of product lines. The number of product lines for each firm is drawn from Poisson distribution (with the average number of products equal to 2.5) at the start of the simulation; subsequently, it evolves as a birth-death process with birth rate η and death rate δ.

   Potentially, the destruction rate parameter δ can be identified from the exit rate of firms. Average monthly entry and exit rates in the data over the period of 1999-2006 are above 4%. Most of the firm turnover, however, is related to small firms that have sporadic zeros in their employment: the employment share of entering firms is 1.4%, while exiting firms represent about 0.8% of the total workforce. To fit the exit rate observed in the data, the exogenous destruction rate δ needs to be higher than 2%, which is then inconsistent with the observed unemployment rate of 4.8% and the job-finding rate of 0.2 (according to formula (34)). Thus, I choose not to match the exit rate of firms observed in the data, instead I set δ to 0.001 to get a better fit on the hours-employment co-movement (a higher destruction rate appears to weaken the inverse relationship between growth rates in hours and the number of workers). The entry rate parameter η determines the total mass of product lines, and through it, the average employment per product line.

3. The distribution of underlying productivity is assumed to be a Generalized Pareto Distribution. To ensure that the model is able to capture the positive association between wages and employment, the productivity distribution needs to have a long right tail. The reason for that is that the positive type composition effect of larger firms being on average more productive has to offset the negative effect of employment on wages. Thus, a Pareto distribution is preferred over, for instance, a lognormal distribution. The scale and shape parameters are chosen to replicate the size distribution of firms observed in the data[34]. Similarly, the elasticity of substitution between intermediate goods σ governs the sensitivity of firms profits to profitability shocks. A higher value of σ increases the incentive to hire workers in response to a positive shock and thus magnifies the type composition effect in size-productivity relationship.

[34] I use a Generalized Pareto Distribution with parameters $\underline{\hat{q}} = 10$, $\sigma_{GPD} = 24$, and $k = 0.45$ for $\hat{q} = Y^{\frac{1}{\sigma-1}} q$ as described in Appendix C.1. I solve for the equilibrium aggregate output, Y, and use it to recover the underlying productivity, q. The implied distribution has a mean of 2.1 and a standard deviation of 5.5.

4. The persistence of the shock process (in terms of the arrival rate μ) determines the persistence of labor productivity in the model. I choose parameter μ such that on average a shock to the firm's productivity arrives once in ten months[35]. This value guarantees that the quarterly autocorrelation in labor productivity at the firm level in the model is close to that in the data (0.658 and 0.640, respectively).

5. The vacancy posting costs is parameterized as $c(v) = c_0 v^{c_1}$, with $c_0 > 0$ and $c_1 > 1$. Parameter c_0 is chosen such that the job-finding rate is about 0.2 that corresponds to the average unemployment duration of 5 months. The convexity of vacancy costs parameter c_1 determines sensitivity of hires to employment and productivity changes: the correlation coefficient between the hiring rate and lagged employment in the simulation is close to that in the model (-0.084 and -0.049, respectively).

6. As is commonly assumed in the literature, the matching function exhibits constant returns to scale in job seekers and vacancies, i.e.

$$M(v, u + (1-u)\kappa) = m v^{\zeta} (u - (1-u)\kappa)^{\zeta - 1},$$

   with $0 < \zeta < 1$ and a scaling parameter $m > 0$. The parameter m represents the efficiency of a matching process. Then, the job-finding rate can be expressed as $\lambda(\theta) = m\theta^{\zeta}$, whereas the worker contact rate can be written as $\omega(\theta) = m\theta^{\zeta-1}$. Note that without the data on vacancies, the matching function parameters m and ζ cannot be identified separately. The elasticity of the matching function with respect to vacancies ζ is set to 0.5 that is within the range of estimates found in the literature (for instance, Shimer (2005) reports the estimate of 0.38; while Hall (2005) finds the estimate of 0.765). The parameter m influences the efficiency of matching and through that the return on vacancies. A lower value of m decreases the correlation between hours and employment growth in the model. I choose $m = 0.4$.

7. Unemployment benefit, or alternatively the value of home production, b is set to match the unemployment rate of 4.8%, the average unemployment rate during the period of 1999-2006 (OECD Economic Outlook 2007). The implied replacement ratio, defined as the ratio of the unemployment benefit to average monthly wages, is 40%.

8. The worker and job flow data identify the parameters that govern job-to-job transitions in the model. The exogenous quit rate s_0 has to be consistent with the job-finding rate and the unemployment rate based on equation (34). This parameters affects the sensitivity of separations to employment and productivity and is set to fit the correlation between separation rate and lagged employment in the data. Note that exogenous quits alone are insufficient to generate separation rates of the magnitude observed in the data. Thus, the model is required to have job-to-job transitions. The relative search intensity κ is set to be less than one in order to match average separation rate.

9. The workers' bargaining power β determines the amount of rent-sharing in the model and is set to match the labor share in total revenues. In the data, the ratio of total wage bill to total value added is found to be about one half.

10. The scale parameter χ in the utility costs of working is chosen to match average actual work hours

[35] The arrival rate μ is equal to 0.04 which together with the average number of products per firm of 2.5 generates the average arrival rate of 0.1.

(as opposed to the lower bound hours measure) to 34 hours a week[36]. The curvature of the utility cost ξ plays a prominent role in the trade-off between changes in hours and employment in firm's labor demand through its effect on the cost of varying hours of work. I set $\xi = 2.5$ that is within the range of values estimated by Cooper and Willis (2009). This value ensures that the correlation between hourly wages and hours predicted by the model is close to that observed in the data: the size-weighted correlation coefficient is 0.209 in the data and 0.182 in the model (based on H_{UB} measure; recall that H_{LB} variable implies a negative wage-hours relation).

11. The aggregate price level P equates mean monthly wage in the model to its mean in the data.

C.3 Timing of Productivity Shocks

Most of the labor adjustment models use a discrete time framework to evaluate the impact of adjustment costs on employment policies of firms. These models implicitly synchronize the timing of the shocks and of the changes in labor resources. That is, it is commonly assumed in these models that shocks arrive at the beginning of the period; moreover, the firm adjusts its workforce at the same time. This assumption is not innocuous. The timing of profitability shocks and of employment adjustment turns out to be important for observing the negative correlation between work hours and employment growth rates.

Figure 12 demonstrates the difference between two cases: in one the profitability shock arrives at the beginning of the period and in another - in the middle of the period. The vertical dotted lines show the time periods when the firm is first hit by a positive shock, then by a negative shock. The dashed lines represent the true process of employment and average hours evolution in response to these shocks. In the event of the positive shock, for example, average work hours jump up instantaneously and start declining slowly, as employment builds up to a new level. Similarly, in response to the negative shock, hours fall initially and start rising, as employment declines. Thus, the adjustment on intensive and extensive margins is inversely related.

Then, the solid (marked) lines in Figure 12 illustrate the aggregation of employment and hours variables into monthly series. The left panel shows the case, in which the timing of shocks coincides with the beginning of the month, while the right panel shows the case, in which the shocks arrive in the middle of the month. In the data, monthly employment refers to all individuals on payroll in month t; that is, it includes new hires, as well as workers who have separated during that month. For instance, on the left panel the number of employees is 14 in month t_2, as four new workers have been hired, and it is 14 in month t_4, as workers, which have separated, are still considered to be on payroll during that month. The average hours series is constructed by dividing total work hours by the number of workers employed in that month. Therefore, the observed average hours series may differ from the actual hours process and may even move in the opposite direction (as hours in month t_2 do on the right panel).

Comparing the left and right panels of Figure 12, the observed growth rates of hours and employment are inversely related if shocks arrive at the beginning of the period (on the left) and seem to move in the same direction if shocks arrive in the middle of the period (on the right). The correlation between growth rates of average hours and workforce is -0.34 in the former case and 0.33 in the latter case. Labor adjustment models in a discrete time framework align the timing of shock arrival with the start of measurement period, which artificially generates more negative correlation between hours and employment adjustment.

[36] OECD Economic Outlook 2007 reports that average annual hours (defined as the total numbers of hours worked over the year divided by the average numbers of people in employment) was 1559 in Denmark over the period of 1999-2006. To get weekly hours, I then divide it by 46 weeks assuming there are 6 weeks of vacation.

Figure 12: Profitability shocks in the beginning of the period (left panel) and in the middle of the period (right panel).