Who Cares About The Great Moderation?[*]
Job Market Paper

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Abstract

In this paper I compute the welfare cost of the Great Moderation, using a consumption based asset pricing model. The Great Moderation is modeled according to the data properties of the stationary component of consumption, which displays a reduction of the volatility at high frequencies, and an unchanged volatility at medium frequencies. The theoretical model, calibrated to match the average asset pricing variables in the data, relies on the evolution of the habit stock, which depends on the lower frequencies of consumption. These two features generate a modest welfare gain of the Great Moderation (0.6 percent). I show that this result depends mainly on the medium frequency properties of consumption.

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1 Introduction

The term Great Moderation describes the reduction in macroeconomic volatility perceived by macroeconomists to have occurred after the early 1980s. The Great Moderation has received an enormous amount of attention in the literature, much of it devoted to assessing a range of possible causal factors.[1] Relatively little research, however, has addressed whether the Great Moderation is important in terms of improving household welfare. In this paper I calculate the welfare improvement caused by the Great Moderation, and conclude that it is more than likely modest, equivalent to roughly a 0.6 percent increase in household consumption.

The procedure I follow to measure the welfare gain from the moderation is characterized by two important features. First, I show that computed welfare gains depend crucially on the assumed laws of motion of consumption before and after the Great Moderation. Specifically, welfare calculations are sensitive to the spectral shape of consumption fluctuations. Therefore, a careful accounting of how macroeconomic volatility changed at different frequencies is required in order to assess changes in welfare. Additionally, because macroeconomic fluctuations are a source of risk for households, it seems natural that we should assess the gain from reducing this risk using a model which has empirically reasonable asset pricing implications[2]. After all, these observed prices are our best measures of how actual agents value risk. A second feature of my analysis, therefore, is to pay close attention to the asset pricing implications of the models used in my analysis.

The vast literature on the Great Moderation focuses mainly on the significant reduction in the variance of either the growth rates of macroeconomic variables, or the business cycle components of these time series, isolated using a variety of filters. I show, however, that when a wider range of frequencies is considered, there was no apparent reduction in the variance after 1983. The decline in volatility at the relatively high frequencies studied in the literature actually coincided with a modest increase in volatility at medium to low frequencies. This finding is important, because depending on the preferences used to measure agents' welfare, we might actually expect to find no gain, or even a reduction in utility in the post-1983 period.

The following example serves to illustrate the importance of linking welfare calculations to asset prices. Lucas (1987) assessed the welfare cost of business cycle fluctuations using a simple representative-agent consumption-based asset pricing model with time separable constant relative risk aversion (CRRA) preferences. In his calibrated example the implied welfare gain of eliminating fluctuations is equivalent to a 0.01 percent increase in steady state consumption. Lucas' model, however, implies a negligible equity premium. If one calibrates the preference parameters, instead, so that the equity premium in the model is 6 percent (its average value in the post-war period),

[1] Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Blanchard and Simon (2001) are among the pioneers of the literature on the Great Moderation. A survey of this literature can be found in Stock and Watson (2002).

[2] Alvarez and Jermann (2004) also analyze the relation between welfare cost of cycles and asset pricing in a "model-free" enviroment.

the welfare gain from eliminating fluctuations rises to 7.5 percent of steady state consumption.

With these considerations in mind, I proceed as follows. To model the Great Moderation, I estimate a fourth order autoregressive model of real U.S. per capita consumption for both the pre-1983 and post-1983 periods. This model is sufficiently rich that I am able to capture both the high and medium frequency behavior of the data in the two subsamples. To measure welfare gains, I use an endowment economy framework with habit formation preferences. The model parameters are calibrated such that the model is able to match key asset pricing moments (average risk-free return, average risky asset-return, average equity premium) across the two subsamples. The ability of the habit model to match the asset pricing moments is a consequence of the assumption that the agent's utility depends upon the consumption surplus, the distance between consumption and the habit stock. Since the latter is a smooth function of the past values of consumption, even a small degree of volatility in consumption results in significant volatility of the stochastic discount factor, and this generates a significant risk premium.

The increase in medium-frequency volatility experienced in the post-1983 period is due to increased persistence in the endowment process, while the decrease in high-frequency volatility is due to a decrease in the volatility of innovations to consumption. Consequently, in response to a negative shock to consumption, a representative agent expects his level of consumption to be close to the habit stock for several periods. Welfare losses stemming from this undesirable (from the perspective of the agent) feature of the post-1983 endowment process offset the welfare gain associated with the decline in high-frequency volatility. This explains my modest estimate of the welfare gain brought about by the Great Moderation: about 0.6 percent of steady-state consumption. To show that the medium-frequency behavior of consumption is indeed responsible for this small estimate of the welfare improvement, I consider a counterfactual scenario in which the variance of consumption is assumed to decline at all frequencies. In this experiment the persistence parameters are held fixed at their pre-1983 values, while the variance of the innovation is set equal to its post-1983 value (which is 47 percent lower). In the counterfactual scenario, agents experience a bigger welfare gain, equal to about 2 percent of steady state consumption. Finally, using a bootstrap procedure I compute a 95 percent confidence band for my estimate of the welfare improvement: $(-2.8, 3.7)$ percent. The wide range of possibilities captured within the confidence set reflects the difficulty of precisely estimating the parameters governing the low-frequency properties of consumption. Consistent with my findings, Reis (2009) confirms that the persistence of the consumption process is a crucial determinant of the welfare cost and that the point estimate of this persistence is associated with large confidence bands.

My model of habit formation bears some similarity to the model proposed by Campbell and Cochrane (1999). Despite their model's attractive asset pricing qualities, I depart from it for important conceptual reasons. In their model, the habit stock depends non-linearly on past consumption. The particular form of the nonlinearity is problematic when computing the welfare change associated with a change in the law of motion of the endowment. In particular, the parameters of the law of motion of consumption implicitly affect the preference parameters that determine the sensitivity of the agent to consumption fluctuations. The lower the variance of consumption is, the more the habit stock responds to an endowment shock of a given magnitude. This mechanism plays an important role in the model's ability to match specific asset pricing facts, namely the first and second moments of the risk-free rate and the equity premium. However, unfortunately it obscures welfare calculations, because it is not possible to isolate the effects of the changes in the exogenous process while holding the preference parameters fixed[3]. On the other hand, in my linear habit model preference parameters are independent of the law of motion consumption, which allows me to study the effects of the Great Moderation on welfare.

My model matches several asset pricing facts when calibrated to the full sample (1947–2007). However, the pre and post-1983 processes for consumption present a problem for my model, if I consider their separate asset pricing implications. In a time separable consumption-based model, the law of motion of consumption has similar implications for both the equity premium and welfare. A change in the law of motion leading to a big increase in welfare also significantly reduces the equity premium. Small changes in welfare are associated with small changes in the equity premium. However, in my habit model, even though there is only a small decline in welfare in the post-1983 period, there is a significant predicted decline in the average equity-premium. This is because in the habit model the equity premium displays more sensitivity to the behavior of the high frequency component of consumption than does welfare. This result is in line with Otrok (2001) and Otrok et al. (2002), which separately analyze the effects of habits on utility and on the equity premium. The predicted decline in the equity premium is at odds with the data, in which there is no significant change in the mean of the equity premium or other key asset pricing moments in the post-1983 period. While I could solve this problem by allowing for a change in the preference parameters, this solution would lead to the same criticism of my model that I gave of the Campbell and Cochrane model.

To assess whether other models predict a low gain from the Great Moderation, I consider two additional models in the recent macro-finance literature that have been shown to successfully match key asset pricing facts: the rare disaster model[4] and the long-run risk model[5]. In these models only a small fraction of the equity premium depends on the high-frequency properties of the consumption process, whereas it is in large part due to the probability and magnitude of rare disasters, and the importance of the very long-run component of consumption growth. If the Great Moderation is assumed to have left these features of the law of motion of consumption unchanged, there is little predicted change in the moments of financial variables, and only a very small welfare gain.

The paper is organized as follows. Section 2 presents the empirical analysis of the effect of

[3]Ljungqvist and Uhlig (2009) also examines some related implications of the Campbell and Cochrane model for welfare calculations.

[4]Rietz (1988), Barro (2005, 2009) among others.

[5]Bansal and Yaron, (2006).

the Great Moderation on macroeconomic variables and asset pricing. Section 3 illustrates the relationship between welfare cost, asset prices, and law of motion of consumption. Section 4 and 5 presents the asset prices model and its solution method. Section 6 illustrates the computed welfare costs of the Great Moderation. Section 7 discuss the effects of the medium frequency on the asset prices. Section 8 presents alternative models. Section 9 concludes with some remarks.

2 Great Moderation: Stylized Facts on Macroeconomic Variables and Asset Pricing

The extensive literature on the Great Moderation has mainly analyzed the stabilization of the high frequency volatility of macroeconomic variables by documenting the reduced variance of either the growth rate or the business cycle component of each series. In this section I extend the analysis of the Great Moderation in two directions. First, I analyze the behavior of some of the most relevant macroeconomic variables, namely consumption, output, and investment at the medium frequencies, in addition to the higher frequencies studied in the literature. I show that the large decline in volatility at high frequencies during the Great Moderation does not coincide with a reduction of the volatility at medium frequencies. Second, I investigate whether the Great Moderation affected any of the key moments of some asset prices, such as the average values of the risk-free rate, the equity premium, and the price-dividend ratio.

2.1 Macroeconomic Variables

Consider the following three U.S. macroeconomic variables measured in real per capita terms: aggregate consumption, measured as non-durable goods plus services, output, and investment. The dataset includes observations from the period 1947Q1-2007Q4. I will refer to the subperiod 1947Q1-1982Q4 as Sample 1 (the period before the Great Moderation), and the subperiod 1983Q1-2007Q4 as Sample 2 (the period of the Great Moderation). The choice of 1983 as the break date for the beginning of the Great Moderation is in line with the large literature on this topic (see Stock and Watson, 2002).

Although many papers document the decline of the volatility of aggregate macroeconomic variables at the business cycle frequencies in the last twenty years, Pancrazi (2009) shows that the Great Moderation phenomenon disappears when medium frequencies are taken into account. In this section, I report some stylized facts that confirm the absence of moderation for the three macroeconomic variables when medium frequencies are considered.

For this purpose, consistent with Pancrazi (2009), I decompose a stochastic process as follows[6]:

[6] Comin and Gertler (2006) also analyze the medium-cycle properties of some economic series. However their definition of medium-cycle includes fluctuations with periodicity up to 50 years.

Definition 1 *Given a time series x_t, its High-Frequency component (HF), x_t^{HF}, corresponds to the cyclical fluctuations of x_t included in the period between 2 and 32 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $\left[\pi, \frac{\pi}{16}\right]$.*

Definition 2 *Given a time series x_t, its Medium-Frequency component (MF), x_t^{MF}, corresponds to the cyclical fluctuations of x_t included in the period between 32 and 80 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $\left[\frac{\pi}{16}, \frac{\pi}{40}\right]$.*

Definition 3 *Given a time series x_t, its High-to-Medium frequency component (HM), x_t^{HM}, corresponds to the cyclical fluctuations of x_t included in the period between 2 and 80 quarters. For quarterly data, in the frequency domain, these fluctuations belong to the interval $\left[\pi, \frac{\pi}{40}\right]$.*

Figures 1, 2, 3 plot the three components defined above for consumption, output, and investment, respectively. I use a band-pass filter (Christiano and Fitzgerald, 2003) to isolate the frequencies of interest in the data. The high-frequency components (solid line) display lower volatility in the post-1983 period compared to the pre-1983 period. This reduction of volatility at high-frequencies is a well-established fact in the literature about the Great Moderation. However, the analysis of the behavior of the three series at lower frequencies leads to some interesting and less familiar evidence. First, the magnitude of the fluctuations at medium-frequencies (dotted line) exceeds that of the high-frequencies fluctuations throughout the whole sample, as measured by the peak-to-trough distances. This suggests that a considerable part of the variability of consumption, output, and investment depends on fluctuations beyond the business cycle. Second, the Great Moderation period does not display any evident decline of the volatility of the medium-frequencies with respect to the previous subsample. These two facts explain why the fluctuations of the high-to-medium frequency component (dashed line) are mainly due to the medium-frequencies, especially in the Great Moderation period.

To quantify the stylized facts presented above, Tables 1, 2, and 3 report the standard deviations of the three macroeconomic variables at different intervals of frequencies in the two subsamples. The standard deviations of consumption, output, and investment at high-frequencies (2-32 quarters) declined by 44, 53, and 35 percent, respectively, during the Great Moderation. Similar results are obtained if the high-frequency component is defined using the first difference filter, rather than the bandpass filter.

However, once the medium-frequencies are taken into account, there is no apparent reduction in volatility during the Great Moderation. In fact, the standard deviation of the medium-frequency component (32-80 quarters) in the post-83 period actually increases by 25 percent for consumption, declines slightly, by 15 percent, for output, and more than doubles for investment, with respect to the pre-83 period. However, the standard errors suggest that these changes before and after the

Great Moderation are not statistically significant. These results suggest that the impact of the Great Moderation on the spectrum of the macroeconomic variables is not homogenous throughout all the frequencies, since it has largely reduced their spectral mass only at high frequencies.

As the plots of the cycles show, the medium frequency component determines a substantial part of the total volatility of the three macroeconomics series. Therefore, it is not surprising that the behavior of the high-to-medium frequency component (2-80 quarters) during the Great Moderation is greatly affected by the medium frequency properties. The standard deviation of the high-to-medium frequency component of consumption decreased by only 6 percent during the Great Moderation, since the large reduction of high frequency volatility is offset by the increase of its medium frequency counterpart. The standard deviation of the high-to-medium frequency component of output declined by about 30 percent during the Great Moderation period, a value smaller than for the high frequency component. Finally, the standard deviation of the high-to-medium frequency component of investment increased by 15 percent during the Great Moderation.

These facts lead to another interesting consequence of the Great Moderation. Since the volatility at high frequencies for the macroeconomic variables was reduced by a large amount and since the same reduction did not happen at lower frequencies, the percentage contribution of the medium frequency component to the total variance of the macroeconomic variables significantly increased in the second subsample. In fact, the medium frequency component accounts for 47 percent of the variance of the high-to-medium frequency component of consumption in the pre-1983 sample, 52 percent for output, and only 21 percent for investment. During the Great Moderation period, the medium frequency component accounts for more than 80 percent of the high-to-medium frequency variance for the three variables.

Given my analysis of the medium frequencies during the Great Moderation I draw two important conclusions: the stabilization of the fluctuations of macroeconomic variables is less evident at these frequencies, and the relative importance of medium frequency fluctuations rose considerably.

2.2 Asset Prices

In contrast to the vast literature on the stabilization of macroeconomic variables during the Great Moderation, relatively little attention has been paid to changes in the behavior of financial variables[7]. In this section I analyze some key moments of asset prices before and during the Great Moderation, to assess whether the reduction in the volatility of macroeconomic variables at high frequencies coincides with changes in the moments of financial variables. In particular, I first consider three time series: the real annualized return of a risk-free asset, measured as the return of 3-month Treasury bills, the annual real return of equity, measured using the value-weighted market return defined by Fama and French[8], and the risk premium, measured as the difference between

[7] Lettau, Ludvigson, and Wacther (2008) analize the increase of the price-divendend ration in the 1990s.

[8] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the risk-free return and the equity return.

Table 4 shows some moments of these variables in the two subsamples. The average returns of the assets rose during the Great Moderation, by 2.3 percentage points for equities, and 1.6 percentage points for risk assets. As a result, the mean of the equity risk-premium grew slightly in the post-1983 period, by 0.7 percentage points. However, the standard errors and the Chow (1960) test statistic suggest that this small increase in the average risk premium is not statistically significant. Thus, I infer that the risk-premium did not change as a result of the Great Moderation. Although in this paper I focus mainly on the level of the risk premium, it is worth noting that its volatility was also approximately unchanged across the two subsamples.

Other asset pricing variables of interest are the price-dividend ratio and the price-consumption ratio[9]. Figure 4 displays these two series, where the first observation is normalized to unity. The averages of the two variables in the two subsamples are reported in Table 5. The price-dividend almost doubled during the Great Moderation period, whereas the price-consumption ratio increased by only 27 percent in the second subsample. The Chow tests suggest the presence of a structural break in the mean of the variables. It is interesting to compute the effect of the new-technology "bubble" in the late 1990s on the price-dividend and price-consumption ratios. To build a "bubble-free" scenario, I assume a linear pattern between 1995 and 2003 for the two variables. In this scenario, the average price-dividend ratio in the second subsample is 1.90, and the average price-consumption ratio is 1.89. When the "bubble" is eliminated, the Chow test cannot reject the null hypothesis of no changes in the mean of the price-consumption ratio, although the Chow test detects a break in the average price-dividend ratio.

3 Welfare and Asset Pricing

Lucas (1987) concludes that the welfare gain from eliminating business cycle fluctuations is negligible. In this section I demonstrate, however, that welfare calculations depend on two important features of a model: the specification of the exogenous consumption process and the asset pricing implications of the model structure.

3.1 Revisiting Lucas' Calculation

Lucas (1987) finds that the cost of business cycles is extremely low; an agent would agree to give up less than 0.04 percent of his consumption to avoid them entirely. However, to compute this cost Lucas (1987) uses two crucial assumptions: the logarithm of consumption is specified as an i.i.d. process around a linear trend, and CRRA utility is calibrated with a small coefficient of risk aversion. In this section I show that departures from these assumptions greatly affect the computed welfare cost. Specifically, I compute the welfare cost using the same CRRA utility specification as

[9] These variable are extracted from Robert Shiller's online database. http://www.econ.yale.edu/~shiller/data.htm

in Lucas, but I adopt the autoregressive process for consumption growth specified as in Mehra and Prescott (1985), which is fit to the 1889-1978 sample of U.S. data. I also calibrate the preference parameters so that the model matches the average equity premium and average risk-free rate in the Mehra and Prescott data set (1985).

To illustrate the sensitivity of welfare calculations to the model specification, in the first step, I assume that the preference parameters and the consumption process are specified as in Lucas (1987), i.e. the discount factor β is equal to 0.95, the coefficient of risk aversion is equal to 2, and the logarithm of consumption is i.i.d. around a linear trend, i.e.

$$\log\left(C_t\right) = gt + z_t \qquad z_t \overset{iid}{\sim} N\left(0, \sigma_z^2\right).$$

Following Lucas'(1987) calibration, the mean growth rate of consumption, g, is set equal to 0.03 and the standard deviation of the stationary component, σ, is set equal to 0.013. As the first column of Table 6 shows, the model predicts a negligible welfare cost from eliminating the fluctuations equal to 0.017 percent. Moreover, the model is not able to predict a significant equity premium, as Mehra and Prescott (1985) pointed out.

Similar results can be obtained maintaining the assumption of a CRRA utility function, but assuming that the growth rate of consumption is an i.i.d. normal random variable with mean μ and standard deviation σ, i.e.

$$\log\left(C_t\right) = \log\left(C_{t-1}\right) + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N\left(\mu, \sigma^2\right).$$

Calibrating $\mu = 0.03$ and $\sigma = 0.013$ as estimated in the post-war period, the second column of Table 6 shows that the model predicts an equity premium close to zero, and a low welfare cost from eliminating the fluctuations equal to 0.1 percent in consumption compensation. In both the trend stationary and difference stationary specifications of the consumption process, the model prediction of a low equity premium is associated with a low welfare cost of the fluctuations.

The link between asset pricing and welfare cost can be defined analytically in a basic consumption-based asset pricing model with time-separable CRRA utility, like the one considered for the previous calculations. In what follows, I show the tight relationship between the financial variables and welfare.

The representative agent maximizes the lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\right),$$

where E_0 denotes the conditional expectations given the information at time 0, $U\left(\cdot\right)$ denotes the instantaneous utility function, C_t denotes consumption at time t, and β is the discount factor. There is a competitive market for trading assets (trees) which pay dividends (fruits). Let P_t be the price of one unit of the asset and A_t be the agent's shareholding at time t, then the agent's budget constraint is

$$C_t + P_t A_{t+1} = (P_t + D_t) A_t,$$

where d_t denotes the exogenous stochastic flow of fruits at time t. Since there is no source of the consumption good other than the fruit, which is perishable, market clearing implies that $C_t = D_t$.

As in Lucas, the agent has CRRA preferences, i.e.

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$ is the coefficient of risk aversion. Since γ is positive, the agent in the economy is risk-adverse.

The price of the asset is determined by the first order conditions as follows:

$$P_t = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (P_{t+1} + C_{t+1}) \right]. \tag{1}$$

To link the asset pricing variables and the welfare cost of fluctuations, it is useful to rewrite (1) in terms of the price-dividend ratio $V_t = P_t / D_t$:

$$V_t = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} (V_{t+1} + 1) \right]. \tag{2}$$

As in Lucas, I first assume that the logarithm of consumption, $c_t = \log(C_t)$, is an i.i.d. process around a linear trend, i.e.:

$$C_t = (1+\mu)^t e^{z_t - \frac{1}{2}\sigma_z^2} \quad z_t \sim N\left(0, \sigma_z^2\right).$$

In this case, a first order approximation implies that the welfare gain from eliminating fluctuations, expressed in consumption compensation terms, is:

$$\lambda = \frac{1}{2}\gamma\sigma_z^2, \tag{3}$$

and the approximated expected value of the equity premium is

$$E\left(R^{EP}\right) = \left\{\beta^{-1}(1+\mu)^{-\alpha} - 1 + \gamma\right\}(1+\mu)\gamma\sigma_z^2. \tag{4}$$

where $\alpha = 1 - \gamma$. Given equation (3) and equation (4), it is evident that the model prediction about the equity premium is tightly related to the welfare cost. In fact, the two variables are proportional since:

$$\lambda = \frac{1}{2}\left\{\beta^{-1}(1+\mu)^{-\alpha} - 1 + \gamma\right\}^{-1}(1+\mu)^{-1} E\left(R^{EP}\right).$$

Moreover, notice that both the equity premium and the welfare cost are tied to the coefficient of risk aversion, γ.

Alternatively, assume that the growth rate of the consumption, $\Delta\log(C_t)$, is distributed as a normal with mean μ and standard deviation σ. Then the following expression for the price-dividend ratio holds[10]:

$$V = \frac{\beta\exp\left(\alpha\mu + \frac{1}{2}\alpha^2\sigma^2\right)}{1 - \beta\exp\left(\alpha\mu + \frac{1}{2}\alpha^2\sigma^2\right)}. \tag{5}$$

Under this assumption, the present value of the lifetime expected utility is a function of the expected price-dividend ratio:

$$U_0 = \frac{1 + E(V)}{\alpha}. \tag{6}$$

As stated in the previous section, the standard deviation of the growth rate of consumption has declined 46 percent in the Great Moderation period. What would the model predict about the effects of a significant decline of σ?

Clearly equation (5) implies that a decline of the volatility of the growth rate of consumption, σ, would lead to a reduction of the price-dividend ratio. In fact, the derivative of V with respect to σ is given by

$$\frac{\partial V}{\partial \sigma} = \alpha^2\sigma\beta\frac{e^{\frac{1}{2}\alpha^2\sigma^2+\mu\alpha}}{\left(\beta e^{\frac{1}{2}\alpha^2\sigma^2+\mu\alpha} - 1\right)^2} > 0,$$

and it is positive. Using equation (6) and the fact that $\frac{\partial V}{\partial \sigma} > 0$, I conclude that a decline in the variance of consumption growth leads to a welfare improvement in the economy, as long as $\gamma > 1$, $(\alpha < 0)$.

3.2 Mehra and Prescott's Calibration

In the second step, I show that the specification of the consumption process affects the welfare computation. I follow Mehra and Prescott (1985), modeling the exogenous process as a first order autoregressive process for consumption growth:

$$\Delta\log(C_t) = \mu(1-\rho) + \rho\Delta\log(C_{t-1}) + \sigma\varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0,1),$$

where the mean μ is calibrated to be 0.0179, the autoregressive coefficient ρ is calibrated to -0.139, and the standard deviation of the error term σ is calibrated to 0.0347. This model best fits the

[10] See Altug and Labadie (1994), p.83

aggregate consumption data observed in Mehra and Prescott's (1985) sample period, 1889-1978. Using the same preference parameters as in the previous step ($\beta = 0.95 \;\; \gamma = 2$), the welfare cost of business cycles is now 0.65 percent, 30 times larger than Lucas' (1987) estimate. Obstfeld (1994) reaches similar conclusions: under the unit-root assumption, innovations in growth have cumulative effects, which greatly affect welfare. However, the cost of the business cycle is still modest and, as Table 6 displays, the equity premium predicted by the model is still small.

Finally, in the third step I maintain the assumption of the autoregressive process for consumption growth, but I calibrate the preference parameters such that the model predicts a risk-premium of 6 percent and a risk-free return of close to 1 percent, the average values observed in Mehra and Prescott's (1985) sample. Table 6 shows that a coefficient of risk-aversion equal to 17 and a discount parameter greater than unity are able to generate asset returns whose first moments reasonably match the data[11]. In this scenario, the welfare cost of the fluctuations is large, about 7.4 percent. This result suggests that the welfare cost of business cycle fluctuations implied by a model is tightly related to the ability of that model to generate a large price for risk.

However, as shown in the previous section, none of these calculations is appropriate for thinking about the effects of the Great Moderation because it did not lead to a decline in volatility at all frequencies. There was an increase in volatility at medium-frequencies coincident with the reduction in volatility at high frequencies. Therefore, a basic exercise in which consumption growth is the exogenous stochastic process, and the variance of its innovation declines, is completely silent about the effects of medium frequencies on welfare and asset pricing.

In this paper I propose a particular solution to this problem, introducing a model in which the consumption process is flexible enough to capture the behavior of the data at both high and medium frequencies, and in which preferences are such that the model can match the key moments of asset prices in the post-war period. With this model I compute the effect of the Great Moderation on welfare, and conclude that the gain implied by the Great Moderation is rather small, even though the model predicts a large equity premium and a small risk-free rate.

4 An Asset Pricing Framework

In this section I introduce a model which is able to match some of the basic asset pricing moments, that, given the analysis in Section 3, seem particularly relevant to welfare calculations: the risk-free rate, the equity return, and the price-dividend ratio. The model describes an endowment economy, in which the law of motion is sufficiently rich that it captures both the high and medium frequency features of the data emphasized in Section 2.

[11] Kocherlakota (1990) obtains similar results in an analogous exercise.

4.1 The Model

The economy is similar to the one described above, but I assume that utility is time-nonseparable, by introducing external habits. The adoption of habits in asset price models was introduced by Abel (1990), and Constantinides (1990), and used by Campbell and Cochrane (1999) in their models. The representative agent maximizes his lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta U\left(C_t, X_t\right).$$

The agent's instantaneous utility depends on the consumption surplus, which is the difference between present consumption, C_t, and the habit stock, X_t:

$$U\left(C_t, X_t\right) = \frac{\left(C_t - X_t\right)^{1-\gamma} - 1}{1-\gamma}, \tag{7}$$

where C_t is the agent's consumption at time t, X_t is the external stock of habit, and γ governs the curvature of the utility function. In this setting the coefficient of relative risk aversion is time-dependent and is affected by the magnitude of the consumption surplus. The local coefficient of relative risk aversion is defined as

$$CRRA_t \equiv -\frac{C_t U_{cc}}{U_c} = \gamma \frac{C_t}{C_t - X_t}.$$

When consumption is close to the habit stock, the agent's utility declines and his aversion to risk increases, for any given γ.

All output in the economy is derived from an asset that produces a stochastic endowment of a single perishable good for each unit of the asset that the agent owns at the beginning of time t.

The budget constraint is

$$C_t + P_t A_{t+1} = \left(P_t + D_t\right) A_t,$$

where A_t is the quantity of asset owned at time t, P_t is the price of the asset, and D_t is the dividend generated by the asset.

The Euler equation governing the agent's optimal choice of consumption is

$$P_t \left(C_t - X_t\right)^{-\gamma} = \beta E_t \left(C_{t+1} - X_{t+1}\right)^{-\gamma} \left(P_{t+1} + D_{t+1}\right),$$

which can be rewritten in terms of the price-dividend ratio, $V_t = P_t / D_t$, as:

$$V_t D_t \left(C_t - X_t\right)^{-\gamma} = \beta E_t \left(C_{t+1} - X_{t+1}\right)^{-\gamma} \left(V_{t+1} + 1\right) D_{t+1}.$$

Since, in equilibrium, $D_t = C_t$, we have

$$V_t = E_t \left[M_{t+1} \left(V_{t+1} + 1 \right) \frac{C_{t+1}}{C_t} \right], \tag{8}$$

where M_{t+1} is the stochastic discount factor, defined as:

$$M_{t+1} = \beta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right)^{-\gamma}. \tag{9}$$

I now specify the nature of the process of consumption C_t and of the external habit stock X_t. I assume that the level of consumption is the product of a deterministic time trend, e^{gt}, and a stationary component, $\tilde{C}_t$, that governs the fluctuations around the trend:

$$C_t = e^{gt} \tilde{C}_t.$$

Here g is the mean growth rate of consumption. Although one of the basic consumption-based models presented in the previous section and other habit-models, like Campbell and Cochrane (1999), parameterize consumption as a difference stationary process, my choice of a trend stationary process is motivated by the ability of the stationary component $\tilde{C}_t$ to capture the medium-frequency properties of the U.S. consumption time-series.

Denoting with lower-case the logarithm of a variable, i.e. $z_t = \log(Z_t)$, it follows that

$$c_t = gt + \tilde{c}_t. \tag{10}$$

I assume that the stock of habit is an infinite geometric average of the aggregate level of consumption, C_t^a i.e.

$$X_t = \left[\prod_{i=0}^{\infty} \left(C_{t-1-i}^a \right)^{\phi^i} \right]^{1-\phi}.$$

In logarithms we have:

$$x_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i c_{t-1-i}^a.$$

In equilibrium identical individuals choose the same level of consumption, therefore $C_t = C_t^a$. Thus, I drop the superscripts in what follows, since they are not essential.

Applying the decomposition in (10), we have:

$$x_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i \left[g(t - i - 1) + \tilde{c}_{t-1-i} \right] = gt - \omega + \tilde{x}_t,$$

where $\tilde{x}_t = (1 - \phi) \sum_{i=0}^{\infty} \phi^i \tilde{c}_{t-1-i}$, and $\omega = (1 - \phi) g \sum_{i=0}^{\infty} \phi^i (i + 1)$.

The constant ω can be analytically computed as

$$\omega = (1-\phi)\, g \sum_{i=0}^{\infty} \phi^{i} (i+1) = \frac{g}{1-\phi}.$$

The stochastic discount factor in (9) can be rewritten in terms of stationary variables:

$$M_{t+1} = \beta \left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right)^{-\gamma} = \beta e^{-g\gamma} \left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma},$$

where $\Omega = e^{-\omega}$.

Finally, the Euler equation in (8) becomes:

$$V_t = \beta e^{(1-\gamma) g} E_t \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} (V_{t+1} + 1) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right]. \tag{11}$$

Numerical methods, to be described below, allow me to solve this equation to obtain a pricing function that expresses the price-dividend ratio V_t as a function of the relevant state variables, which depend on the parametric specification of the exogenous process for the stationary component of consumption, $\tilde{c}_t$.

4.2 The Law of Motion of the Endowment

In order to close the model, I define a process for the stationary component of consumption, $\tilde{c}$. A simple first-order autoregressive $AR(1)$ process, which is commonly used in quantitative macroeconomic models, is not suitable for this purpose since its spectral shape, which is a function of only two parameters, is not flexible enough to match the reduction of the volatility at high-frequencies and the increase of the volatility at the medium frequencies observed in my sample period.

Therefore, I consider a higher-order autoregressive process, whose spectrum is more flexible since it is a function of a larger number of parameters. The Schwarz Information Criterion suggests that a fourth-order autoregressive, $AR(4)$, process of the form

$$\tilde{c}_t = \theta_1 \tilde{c}_{t-1} + \theta_2 \tilde{c}_{t-2} + \theta_3 \tilde{c}_{t-3} + \theta_4 \tilde{c}_{t-4} + \sigma_\varepsilon \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0,1),$$

is the best candidate among all the autoregressive processes, in the sense that it maximizes a penalized likelihood. Thus, I estimate the parameters of an $AR(4)$ process for consumption in each of the two subsamples. Define Θ^1 the set of parameters of the $AR(4)$ process estimated using data from Sample 1 (1947:1-1982:4) and define Θ^2 the set of parameters of the $AR(4)$ process estimate in Sample 2 (1983:1-2007:4):

$$\Theta^1 = \begin{bmatrix} \theta_1 = 1.000 \\ \theta_2 = 0.030 \\ \theta_3 = -0.068 \\ \theta_4 = -0.097 \\ \sigma_\varepsilon = 0.0055 \end{bmatrix} \quad \Theta^2 = \begin{bmatrix} \theta_1 = 1.147 \\ \theta_2 = -0.127 \\ \theta_3 = 0.294 \\ \theta_4 = -0.360 \\ \sigma_\varepsilon = 0.0029 \end{bmatrix}.$$

As Table 7 shows, these estimated processes are able to match the pattern of the sample moments of consumption at different frequencies. The Sample 2 process is characterized by a large reduction in high frequency volatility and an increase in medium frequency volatility, relative to the Sample 1 process. As a result the standard deviation of the combined high and medium frequencies is similar for the two processes. Although the magnitude of the medium frequency volatility is smaller than in the data, the relative change of the implied standard deviations in the two subsamples is identical to their sample counterpart.

The ability of these processes to capture the changes in the shape of the spectrum of consumption relies on two factors, as Table 8 suggests. First, the decline of the standard deviation of the innovation, σ_ε, from Sample 1 to Sample 2 implies a proportional downward shift of the spectrum, resulting in a decline of the volatility at all frequencies. Second, the changes in the estimates of the autoregressive parameters imply an increasing persistence of the process from the first to the second subsample, as suggested by the higher largest root of the lag polynomial estimated in Sample 2. Such increased persistence generates a redistribution of the mass of the spectrum from higher to lower frequencies. Therefore, the effect of the decline of the variance of the error term is offset by the increase of the persistence at medium-frequencies, whereas the two effects go in the same direction at higher frequencies.

Finally, in some of my quantitative experiments I consider an additional counterfactual process for consumption in which the autoregressive parameters are held constant at the estimates obtained in Sample 1, but the variance of the innovation is calibrated such that the resulting process has the same high frequency variance as in Sample 2. Let us define as Θ^C the parameters of the $AR(4)$ process that generates this counterfactual scenario:

$$\Theta^C = \begin{bmatrix} \theta_1 = 1.000 \\ \theta_2 = 0.030 \\ \theta_3 = -0.068 \\ \theta_4 = -0.097 \\ \sigma = 0.0034 \end{bmatrix}.$$

This counterfactual scenario is of interest because it allows me to assess what the welfare gain of the Great Moderation would have been, had there been an across-the-board decline in volatility at all frequencies. This helps me highlight the extent to which my welfare calculations depend on the spectral shape of consumption, not just the degree of volatility at high frequencies. Figure 5 plots the spectral density of the three different processes. In order to show more clearly the differences of the processes at medium frequencies, I truncate the x-axes to the frequency 0.6, since the three spectra have low power at higher frequencies. Note that the area below the spectrum in a particular interval of frequencies measures the variance of the process attributable to those frequencies. Figure 6 plots the log-spectra of the three processes in the support of frequencies $\left[0, \frac{\pi}{4}\right]$.

5 Solution Method: Parameterized Expectations

In order to solve the model presented in Section 4, I use the Parameterized Expectations approach[12]. I assume that the price dividend ratio can be approximated by a parametric function of the 21 state variables defined by the complete set of polynomials of total degree 2 in the five variables, $\tilde{C}_t$, $\tilde{X}_t$, $\tilde{C}_{t-1}$, $\tilde{C}_{t-2}$, $\tilde{C}_{t-3}$, i.e.

$$V_t \simeq \psi\left(s_t; \Phi\right),$$

where s_t is a vector containing the constant, the five variables listed above, their square values, and all the possible cross-product of degree 2, and Φ is a set of parameters. I assume that ψ is a linear function of Φ and s_t :

$$\psi\left(s_t; \Phi\right) = \Phi' s_t.$$

The Euler equation in (11) implies that

$$\psi\left(s_t; \Phi\right) \simeq \beta e^{(1-\gamma)g} E_t \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} \left(V_{t+1} + 1\right) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right].$$

Define

$$\vartheta\left(s_t, s_{t+1}, \Phi\right) = \beta e^{(1-\gamma)g} \left[\left(\frac{\tilde{C}_{t+1} - \Omega \tilde{X}_{t+1}}{\tilde{C}_t - \Omega \tilde{X}_t} \right)^{-\gamma} \left(\psi\left(s_{t+1}; \Phi\right) + 1\right) \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right].$$

The solution for Φ is the vector of parameters that minimizes the distance between $\psi\left(s_t; \Phi\right)$ and $E_t\left\{\vartheta\left(s_t, s_{t+1}, \Phi\right)\right\}$. In particular, given a vector of parameters $\Phi^{(n-1)}$, I can obtain successive $\Phi^{(n)}$s using the recursion:

$$\Phi^{(n)} = \arg\min_{\Phi} M\left(\Phi, \Phi^{(n-1)}\right),$$

[12] See Judd (1998), Den Haan and Marcet (1990), Marcet and Marshall (1994), and Marcet and Lorenzoni (1998).

with

$$M\left(\Phi,\Phi^{(n-1)}\right)=E\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)-\psi\left(s_t;\Phi\right)\right\}^2.$$

The first order conditions for this optimization problem imply

$$E\left(\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)-\psi\left(s_t;\Phi\right)\right\}\frac{\partial\psi\left(s_t;\Phi\right)}{\partial\Phi_j}\right)=0, \tag{12}$$

for $j=1,..,J$, where J is the dimension of Φ.

Since $\psi\left(s_t;\Phi\right)$ is linear in Φ, we have that:

$$\frac{\partial\psi\left(s_t;\Phi\right)}{\partial\Phi_j}=s_{t,j},$$

where $s_{t,j}$ denotes the j-th element of s_t.

Therefore, the first order condition can be rewritten as

$$E\left(\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)-\psi\left(s_t;\Phi\right)\right\}s_{t,j}\right)=0\ \text{ for }\ j=1,...,J,$$

or, equivalently,

$$E\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)s_{t,j}\right\}=E\left\{\psi\left(s_t;\Phi\right)s_{t,j}\right\}\ ,$$

and since $\psi\left(s_t;\Phi\right)=\Phi' s_t$, we have

$$E\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)s_{t,j}\right\}=E\left\{\left[\Phi' s_t\right]s_{t,j}\right\}.$$

Defining a $J\times 1$ vector b whose j-th element is

$$b_j=E\left\{\vartheta\left(s_t,s_{t+1},\Phi^{(n-1)}\right)s_{t,j}\right\},$$

and a $J\times J$ matrix A whose ij-th element is

$$A_{ij}=E\left\{s_{t,i}s_{t,j}\right\},$$

then the optimality condition is simply

$$b=A\Phi,$$

or

$$\Phi^{(n)}=A^{-1}b.$$

The procedure can be recursively iterated until $\Phi^{(n)}\approx\Phi^{(n-1)}$.

In order to compute the matrix A and the vector b we need to solve the expectations. For this purpose I use a simulation-based numerical approximation. Note that

$$b_j = E\left[\beta e^{(1-\gamma)g}\left[\left(\frac{\tilde{C}_{t+1}-\Omega\tilde{X}_{t+1}}{\tilde{C}_t-\Omega\tilde{X}_t}\right)^{-\gamma}\left(\psi\left(s_{t+1};\Phi\right)+1\right)\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right]s_{t,j}\right].$$

The expectation depends on the values of s_{t+1} and their probabilities, and the values of s_t and their probabilities. In order to calculate the unconditional expectations, I approximate the expectations with the mean of M Monte Carlo simulations for the process $\tilde{c}_t$.

Therefore, we have

$$b_j = \frac{1}{M}\sum_{m=1}^{M}\left\{\beta e^{(1-\gamma)g}\left[\left(\frac{\tilde{C}^m_{t+1}-\Omega\tilde{X}^m_{t+1}}{\tilde{C}^m_t-\Omega\tilde{X}^m_t}\right)^{-\gamma}\left(\psi\left(s^m_{t+1};\Phi\right)+1\right)\frac{\tilde{C}^m_{t+1}}{\tilde{C}^m_t}\right]s^m_{t,j}\right\}.$$

Analogously, the elements of the matrix A are defined as

$$A_{ij} = \frac{1}{M}\sum_{m=1}^{M}\left(s^m_{t,i}s^m_{t,j}\right).$$

6 Welfare Gain from the Great Moderation

Throughout this paper I have emphasized that the welfare cost of the fluctuations computed from a model is related to its asset price implications. Thus, in order to compute a plausible welfare gain from the Great Moderation, the model needs to be calibrated to match key asset pricing moments. However, here I face a difficult challenge. Not only should the model do well at explaining asset prices, on average, in the post-war period, but it should also do well in the separate pre- and post-Great Moderation subsamples. Moreover, to isolate the effects of the different exogenous processes of consumption in the two periods, the preference parameters of the model must be assumed to be unchanged throughout the post-war period.

I first estimate an $AR(4)$ process for consumption in the entire sample 1947:1-2007:4. Let Θ^{ALL} denote my parameter estimates:

$$\Theta^{ALL} = \begin{bmatrix} \theta_1 = 1.072 \\ \theta_2 = 0.011 \\ \theta_3 = -0.020 \\ \theta_4 = -0.143 \\ \sigma = 0.0047 \end{bmatrix}.$$

Throughout the paper I set the value of the average growth rate of consumption, g, equal to its full-sample estimate 0.0052, and I assume the parameter governing the persistence of the habit stock is $\phi = 0.91$. Finally, I calibrate the value of the discount factor, β, and the parameter of the curvature of the utility function, γ, to match the average risk-free rate and average equity-premium in the post-war period. The resulting values of the preference parameters are $\beta = 0.996$, and $\gamma = 1.40$. Table 9 shows that the model is able to perfectly match these moments. However, most importantly, the sample average returns in the two subsamples belong to the 95 percent confidence band computed for the full sample calibration of the model by Monte Carlo simulations.

These results allow me to assume that the preference parameters, β and γ, did not change with the Great Moderation. In fact, the average returns from the data in the pre- and post-Great Moderation are plausibly generated by the model with a finite number of observations. This property is necessary to isolate the effects of a different law of motion of consumption during the Great Moderation on welfare.

Using the proposed model, which is able to match the average risk-premium before and during the Great Moderation, I calculate the percentage of consumption that the representative agent would be willing to give up, in order to have an alternative law of motion of consumption. This approach is equivalent to Lucas' calculation of the welfare cost of the business cycle. Let $\mathbf{c}$ and $\tilde{\mathbf{c}}$ be two different processes for consumption. Let U be the present value of the lifetime expected utility under the exogenous process $\mathbf{c}$, i.e.

$$U = E\left[U_0\left(\mathbf{c}\right)\right].$$

I define the parameter λ such that the following equation holds

$$\bar{U} = E\left[U_0\left(\left[1-\lambda\right]\bar{\mathbf{c}}\right)\right] = E\left[U_0\left(\mathbf{c}\right)\right] = U,$$

which means that λ is the fraction of his consumption that an agent with income stream $\bar{\mathbf{c}}$ would be willing to give up to avoid the fluctuations associated with income stream $\mathbf{c}$.

The lifetime expected utility depends both on the set of preference parameters and on the law of motion of the exogenous process, Θ. To compute the cost or the gain brought about by the Great Moderation, I compute the cost in consumption terms, λ, when the agent's endowment evolves according to the law of motion estimated for the first subsample, Θ^1, rather than the law of motion estimated for the second subsample, Θ^2. As shown in the first row of Table 10, the Great Moderation causes a very modest gain in the agent's welfare, equal to about 0.6 percent in consumption-equivalent terms. This result might be surprising, since we observed a large reduction in the high-frequency volatility of macroeconomic variables during the Great Moderation. I attribute the small welfare gain to the role of the medium frequencies. In fact, as pointed out above, the exogenous process for consumption, when governed by Θ^2, has more volatility at medium frequencies, as well as less volatility at high frequencies, compared to Θ^1. Since in the proposed habit model the agent cares about lower frequency fluctuations because they drive the behavior of the habit-stock, the welfare loss due to the increase of the medium frequency volatility offsets the welfare gain due to the decline in high frequency volatility. This intuition is supported by the second row of Table 10, which shows the welfare gain implied by the counterfactual exogenous process, Θ^C. If the agent faced a consumption process characterized by the same decline of high-frequency variation as in the second subsample as well as by a reduction of variance at all frequencies, then the welfare gain would be larger, equal to 2 percent in consumption-equivalent terms. This highlights the importance of medium frequency fluctuations in welfare calculations, if we adopt a model of habit formation. Indeed, we can also calculate the change in welfare if the agent faced a change in endowment process, from one governed by Θ^C to one governed by Θ^2. Here the volatility of the endowment increases, but only at medium frequencies. The third row of Table 10 illustrates that this increase in medium frequency volatility has a large negative impact on welfare, equal to -1.45 percent in consumption-equivalent terms. Finally, the literature on the welfare cost of economic fluctuations usually considers the welfare gain that an agent would obtain from eliminating all the fluctuations in the economy. The fourth row of Table 10 computes the welfare benefit from completely eliminating the variance of the processes of consumption in the pre-Great Moderation period. It is equal to 3.10 percent in consumption-equivalent terms. This result indicates that the agent's welfare in the model is potentially largely affected by the volatility of the consumption process, since the welfare gain from having a deterministic path for consumption is considerable.

A natural question to ask is, how precise are these estimated welfare gains and losses? To address this question I use a bootstrapping procedure to compute confidence bands. The point estimate of 0.57 percent has a large 95 percent confidence band, whose bounds are -2.39 percent and 3.50 percent. The wide confidence interval is a consequence of the difficulty of obtaining precise estimates of the parameters that determine the importance of the medium frequency component of consumption in the two subsamples, and the fact that the welfare calculations are very sensitive to these parameters.

7 Medium Frequency Fluctuations and Asset Prices

I have shown that how we model the law of motion of consumption has a big impact on welfare calculations. In this section, I more thoroughly explore the role played by the medium frequency features of the law of motion of consumption in determining asset prices. The exercise runs as follows. Keeping the preference parameters β, γ, and ϕ as previously calibrated, I derive the expected value for the risk-free rate, the equity return, and the equity risk premium, when the three sets of parameters for the consumption process, Θ^1, Θ^2, and Θ^C are considered.

Table 11 reports the moments predicted by the model. The model cannot replicate the average returns in the two subsamples when the different processes for consumption are considered as exogenous processes, as Table 11 displays. In fact, the 7.70 percent equity premium in the pre-83 period is not included in the 95 percent confidence interval for its model counterpart, although the mean-equity return (8.68 percent in the data) and the mean risk-free rate (0.98 percent in the data) are close to the model predictions.

The performance of the model is more problematic when I consider the process estimated for the Great Moderation period, Θ^2. In this case the model-predicted equity premium is very low (2.82 percent) compared to the data (8.11 percent). The small risk premium is a result of the large increase in the average risk-free rate and the contemporaneous decline of the equity return. Most importantly, the third row of Table 11 shows that the contribution of medium frequency fluctuations to asset prices is rather small, since the average returns implied by the counterfactual process, Θ^C, are very similar to the average returns implied by the process Θ^2. Since the two processes are distinguishable only by medium frequency volatility, I conclude that the asset pricing variables in this model are mainly driven by the variance of the unpredictable component of consumption, rather than the variance of the consumption process at medium frequencies.

This result is in contrast to the welfare implications of the model, where the medium frequencies play a big role. Therefore, the habit model does not share the close links between welfare calculations and asset prices that we saw for the simple consumption-based model. In addition, the asset prices moments implied by the model in the two sub-samples should be interpreted as steady-state values resulting from two different processes for consumption. On the other hand, the data moments do not have the same interpretation. In fact, it would not be realistic to think about the Great Moderation as of a sudden shift to a new steady-state. Moreover, since in the model the agent forms habit that depends on the past value of consumption, these calculations do not take into account the transitional dynamics of having a new process for consumption while forming habits driven by the old process of consumptions. The effects of this transition are not take into account in the moments of the asset prices presented in the table and are interesting material for future research.

8 Alternative Models

The inability of my model to match the behavior of the financial variables across the two subsamples might raise the question of whether my estimated small gain brought on by the Great Moderation is robust to alternative modeling choices. In this section I analyze the predictions of three alternative models that are successful in solving the equity premium puzzle: the habit model of Campbell and Cochrane (1999), the rare disaster model of Barro (2009) and the long-run risk model of Bansal and Yaron (2004).

8.1 Campbell and Cochrane Model

My model shares several features with the habit model introduced by Campbell and Cochrane (1999), which is successfully able to reconcile model predictions for many financial variables with their sample counterparts. However, one crucial difference distinguishes the two models, namely, the relationship between preference parameters and the parameters of the exogenous laws of motion. In conducting welfare calculations we generally want to hold preference parameters fixed, while experimenting with the law of motion of consumption. This is impossible with Campbell and Cochrane's model, and motivates the design of my model of habits.

There are three important features of the Campbell-Cochrane model. First, external habit formation, second, a slow response of habit to consumption, and third, a non-linear relationship between habit and consumption. In particular, the agent's instantaneous utility has the same form as in (7). Define the surplus consumption ratio, S_t as

$$S_t = \frac{C_t - X_t}{C_t}.$$

The law of motion of the habit stock is modeled specifying a heteroskedastic AR(1) process for the log surplus consumption ratio, s_t, i.e.

$$s_{t+1} = (1 - \phi)\,\bar{s} + \phi s_t + \lambda\,(s_t)\,(c_{t+1} - c_t - g)\,,$$

where $\bar{s}$, g, and ϕ are parameters, and $c_{t+1} - c_t - g = v_{t+1}$ is an i.i.d. normal process with mean 0 and standard deviation σ. The function $\lambda\,(s_t)$ is the *sensitivity function* specified as follows

$$\lambda\,(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2\,(s_t - s)} - 1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{if } s_t > s_{\max} \end{cases},$$

with $s_{\max} = \bar{s} + \frac{1}{2}\left(1 - \bar{S}^2\right)$. The parameter $\bar{S}$ is the steady state surplus consumption ratio and is defined as follows, imposing some useful conditions on the sensitivity function[13]:

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1 - \phi}}.$$

The sensitivity function measures the response of the surplus consumption ratio to innovations in consumption growth. Notice that since $\bar{S}$ is proportional to σ, a less volatile consumption growth process, such as that experienced in the Great Moderation, implies a lower steady-state surplus consumption ratio. Moreover, the functional form of the sensitivity function indicates that a less volatile consumption growth process is associated with higher values of the sensitivity function, holding γ and ϕ constant. As a result, fixing the percentage deviation of the log-surplus consumption ratio from its steady state, $s_t - s$, the distribution of s_t associated with a less volatile consumption growth process shifts to the left and does not change its variance, since the lower volatility of the v_t process is amplified by a larger sensitivity function.

Although this mechanism helps to reconcile the model predictions with several otherwise puz-

[13]See Campbell and Cochrane (1999), p.213.

zling asset pricing data moments, namely, the average risk-free return and the average equity premium as well as their volatilities, and the Sharpe ratio of equity returns, it also creates some counter-intuitive welfare implications. In fact, a reduction of the volatility of consumption growth leads to a decline of the surplus consumption ratio, which is the variable from which the agent gains utility. Thus, a less volatile growth rate of consumption has a negative effect on utility.

In the previous section I showed that the volatility of consumption growth has declined from 0.62 percent in the pre-1983 sample to 0.34 percent in the post-1983 sample. When these estimates are applied to the Campbell and Cochrane model, the welfare loss implied by the less volatile consumption process is 5 percent in consumption-equivalent terms. This result seems paradoxical since we usually expect that a risk-averse agent would prefer a smoother consumption process. However, the left-ward shift on the distribution of the surplus consumption ratio is equivalent to a change in the preference parameters of the agent, or, in other words, to a re-scaling of the variable from which the agent gains utility Therefore the decline in utility implied by this calculation is mainly due to the cardinal value of the utility function, which has no meaning in terms of welfare.

8.2 Rare Disaster Model

In the rare disaster model the equity premium is generated by two components; the first one is proportional to the variance of consumption growth, and the second one depends on the probability and magnitude of the rare disaster. In particular, the expression for the equity premium is

$$R^{RP} = \gamma\sigma^2 + pE\left(b\left[(1-b)^{-\gamma} - 1\right]\right),$$

where γ is the coefficient of relative risk aversion, σ^2 is the variance of consumption growth, p is the probability that a disaster occurs, and b is the magnitude of the disaster. Using Barro's calibration, the risk-premium implied by the model is 5.9 percent. However, only 0.16 percent is due to the first component. Therefore, the impact of the volatility of consumption growth is negligible if compared to the contribution of the rare disaster. This observation suggests that a reduction in the volatility of consumption growth will have a small impact on welfare calculations based on this model. In fact, assuming that the probability and the magnitude of a disaster did not change in the Great Moderation, a 50 percent decline of the standard deviation of consumption growth, as experienced in the post-1983 sample, implies a welfare gain of 0.84 percent. Since the agent in this model is mainly concerned about disaster risk, and this is what is being priced in the equity markets, a change in day-to-day "normal" volatility has only a limited effect on welfare.

8.3 Long-Run Risk Model

The Long-Run Risk model, introduced by Bansal and Yaron (2004), is an alternative model which is able to successfully predict several stylized facts about asset prices. The two main features of the model are the adoption of recursive preferences, and the presence of a small but very persistent component that drives the long-run behavior of consumption growth. Tallarini (2000) analyzes the welfare cost of fluctuations using recursive preferences and concludes, using an Epstein-Zin utility function, that the welfare cost of fluctuations is much higher than in Lucas' calculation. [14]

Croce (2006) separates the welfare effects of the short run component, which depends on the volatility of consumption growth, and the welfare effects of the long-run component. Assuming an intertemporal elasticity of substitution equal to unity to be consistent with Tallarini's calibration, Croce (2006) finds that the largest fraction of the welfare cost of fluctuations depends on the long-run component, since it increases the amount of risk and it increases the effective discount factor. In addition, the long-run risk component is the predominant source of the expected risk premium. In fact, the risk premium in this model is given by the sum of two components:

$$R^{RP} = c\sigma^2 + c_x\sigma_x^2,$$

where c and c_x are constants, σ^2 is the variance of the error term of consumption growth, and σ_x^2 is the variance of the long-run component of consumption growth. Using Croce's calibration to match a 6 percent annual risk premium, the first component accounts for 0.35 percentage points of the premium, while the second component accounts for the remaining 5.65 percentage points. Moreover, Croce (2006) finds that the total welfare cost of fluctuations is large, but its largest fraction (80 percent) depends on the long-run risk component.

What would be the implied gain from the Great Moderation in this model? I assume that the variance of the error term of consumption growth decreases by 50 percent, as in the data, whereas the volatility of the long-run component does not change. Although changes in the long-run risk component are not directly observable in the data, my assumption is supported by two facts: first, the medium frequency component of consumption did not display a decline in its volatility, and second, the equity premium stayed approximately unchanged across the pre- and post-Great Moderation periods. Therefore, a Great Moderation characterized by a decline in the volatility of the growth rate of consumption and by an unchanged long-run risk component implies an unchanged risk-premium and a small effect on welfare, since it is the long-run risk component that mainly affects the two variables.

9 Conclusions

In this paper I estimated the welfare improvement brought about by the Great Moderation, the reduction in the high frequency volatility of macroeconomic variables after the early 1980s. Using simple consumption-based asset pricing models, I showed that the welfare estimates and the moments of asset prices are very sensitive to the time-series properties of the consumption processes

[14] See Epstein and Zin (1989, 1991), Kreps and Porteus (1978), and Weil (1990).

that are fed into these calculations.

The contribution of this paper is to take very seriously the need for welfare calculations to be based on plausibly calibrated laws of motion of consumption, and on models which have reasonable predictions for asset prices. I document that the reduction in volatility in the Great Moderation period is a high frequency phenomenon, since medium frequency volatility did not change significantly after 1983. Therefore, I develop an asset pricing model with habit in which the law of motion of consumption captures the different behavior of consumption at high and medium frequencies. With a set of calibrated preference parameters, the proposed model delivers sensible asset price behavior over the full sample. The implied welfare gain brought about by the Great Moderation is modest, and equal to 0.6 percent in consumption-equivalent terms. This result is not surprising, given that the welfare gain generated by the reduction in high frequency volatility is offset by the loss caused by the increasing persistence of the consumption process.

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Table 1: Variability of components of consumption

Standard Deviations (Percent)

		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log(C_t)$	0.62 [0.06]	0.34 [0.04]
High-Frequencies	C_t^{HF} (2-32Q)	0.90 [0.09]	0.51 [0.04]
Medium-Frequencies	C_t^{MF} (32-80Q)	0.91 [0.09]	1.14 [0.17]
High-to-Medium Frequencies	C_t^{HM} (2-80Q)	1.32 [0.06]	1.24 [0.15]

Note: Consumption is defined in real per-capita terms, measured as non durable goods plus services from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 2: Variability of components of output

Standard Deviations (Percent)

		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log(Y_t)$	1.18 [0.08]	0.56 [0.05]
High-Frequencies	Y_t^{HF} (2-32Q)	1.89 [0.18]	0.88 [0.07]
Medium-Frequencies	Y_t^{MF} (32-80Q)	2.05 [0.15]	1.73 [0.28]
High-to-Medium Frequencies	Y_t^{HM} (2-80Q)	2.84 [0.17]	1.92 [0.23]

Note: Output is defined in real per-capita terms, measured as Gross Domestic Product from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 3: Variability of components of investment

Standard Deviations (Percent)

		Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
First Differenced	$\Delta \log (I_t)$	3.02 [0.27]	1.79 [0.14]
High-Frequencies	I_t^{HF} (2-32Q)	5.53 [0.67]	3.55 [0.31]
Medium-Frequencies	I_t^{MF} (32-80Q)	3.04 [0.36]	6.83 [1.07]
High-to-Medium Frequencies	I_t^{HM} (2-80Q)	6.65 [0.83]	7.67 [0.93]

Note: Investment is defined in real per-capita terms, measured as private investment from NIPA. The cyclical components are isolated using a band-pass filter. Heteroskedasticity consistent standard errors computed with the Newey-West (1987) procedure in brackets.

Table 4: Moments of Asset Prices

Annualized Mean Return and Standard Deviations (Percent)

	Whole Sample	Sample 1	Sample 2	Chow Statistic
	1947Q1-2007Q4	1947Q1-1982Q4	1983Q1-2007Q4	
Equity Return: Mean	8.68	7.74	10.03	0.30
	[1.76]	[2.43]	[2.64]	
Risk Free Asset Return: Mean	0.98	0.33	1.92	24.41
	[0.45]	[0.49]	[0.60]	
Risk Premium: Mean	7.70	7.41	8.11	0.03
	[1.68]	[2.48]	[2.33]	
Risk Premium: Standard Deviation	15.91	15.85	15.99	
	[1.17]	[1.53]	[1.92]	

Note: The Chow Statistic in the fifth column tests the null hypothesis that the mean returns are equal in Sample 1 and Sample 2. The critical value of the Chow test at 5% is 3.84. Heteroskedasticity consistent standard errors computed with the Newey-West procedure in brackets.

Table 5: Mean Level of Price-Dividend and Price-Consumption ratios

	Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4	Chow Statistic Sample 1 vs Sample 2	Bubble-Free Scenario	Chow Statistic Sample 1 vs Bubble-Free
Price-Dividend ratio	1.20 [0.10]	2.21 [0.36]	155.7	1.90 [0.23]	158
Price-Consumption ratio	1.76 [0.22]	2.24 [0.35]	23.6	1.89 [0.21]	2.75

Note: The Chow Statistic in the forth column tests the null hypothesis that the mean levels of the two ratios are equal in Sample 1 and Sample 2. The Chow Statistic in the sixth column tests the null hypothesis that the mean levels of the two ratios are equal in Sample 1 and in the "Bubble Free" scenario. The critical value of the Chow test at 5% is 3.84.Heteroskedasticity consistent standard errors computed with the Newey-West procedure in brackets.

Table 6: Asset Returns and Welfare implications of a time-separable model with alternative specifications of the consumption process

	Linear trend + iid	iid Consumption Growth	AR(1) Consumption Growth	
Discount factor (β)	0.95	0.95	0.95	1.13
Risk aversion (γ)	2	2	2	17
	Annualized Mean Return (Percent)			
Equity Return	11.8	11.8	9.1	7.4
Risk Free Asset Return	11.7	11.8	8.8	1.4
Risk Premium	0.1	0.0	0.3	6.0
	Welfare change in terms of consumption compensation (Percent)			
Welfare gain from eliminating fluctuations	0.02	0.1	0.65	7.4

Note: The model assumes a CRRA utility function. The "linear trend +iid" consumption process and the utility parameters in the second column are as in Lucas (1987). The "iid Consumption Growth" process in the third column is calibrated to match the post war data. The $AR(1)$ consumption growth process and utility parameters in the fourth column are as in Mehra and Prescott (1985). The $AR(1)$ consumption growth process in the fifth column has the same specification as in Mehra and Prescott (1985). However, the utility parameters are calibrated to match a 6 percent risk premium and a 1.4 percent risk free asset return

TABLE 7: MODEL AND DATA VARIABILITY OF COMPONENTS OF CONSUMPTION

Standard Deviations (Percent)

Law of motion of consumption	Standard Deviations (Percent)					
	High-Frequencies		Medium-Frequencies		High-to-Medium Frequencies	
	Model	Data	Model	Data	Model	Data
AR(4) estimated in Sample 1 (Θ^1)	0.93	0.91	0.74	0.91	1.19	1.32
AR(4) estimated in Sample 2 (Θ^2)	0.57	0.52	0.94	1.14	1.10	1.24
Counterfactual AR(4) $\left(\Theta^C\right)$	0.57		0.45		0.73	

Note: The model standard deviations are computed as the average standard deviations of the one thousand simulated series. The length of the each simulated series is two thousand quarters and its cyclical component is extracted using the appropriate band-pass filter.

Table 8: Properties of the Laws of Motion of consumption

Law of motion of consumption	Largest Root	Standard deviation of innovations
AR(4) estimated in Sample 1 (Θ^1)	0.76	0.0055
AR(4) estimated in Sample 2 (Θ^2)	0.90	0.0029
Counterfactual AR(4) $\left(\Theta^C\right)$	0.76	0.0034

Table 9: Model and Data Returns

Mean Annualized Returns (Percent)

	Model	Data		
		Whole Sample 1947Q1-2007Q4	Sample 1 1947Q1-1982Q4	Sample 2 1983Q1-2007Q4
Equity Return	8.68 [6.75 ; 11.38]	8.68	7.74	10.03
Risk Free Asset Return	0.98 [-1.17 ; 3.00]	0.98	0.33	1.92
Risk Premium	7.70 [5.76 ; 10.94]	7.70	7.41	8.11

Note: The 95 percent confidence bands of the model mean returns are computed with Monte Carlo simulations (10000 repetitions).

Table 10: Welfare Change

Consumption Compensation for the different laws of motion (Percent)

		Welfare Change
From Sample 1 to Sample 2	$U(\Theta^1) \Longrightarrow U(\Theta^2)$	0.57
From Sample 1 to Counterfactual	$U(\Theta^1) \Longrightarrow U\left(\Theta^C\right)$	2.00
From Counterfactual to Sample 2	$U\left(\Theta^C\right) \Longrightarrow U(\Theta^2)$	-1.45
From Sample 1 to Deterministic	$U(\Theta^1) \Longrightarrow U(0)$	3.10

Note: The welfare change is computed with Monte Carlo simulation (100000 repetitions).

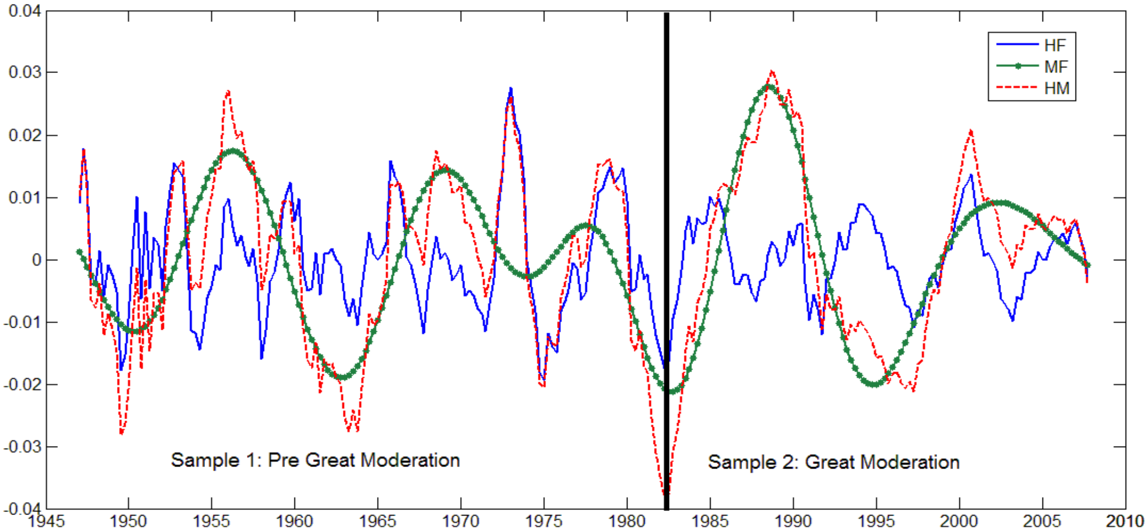
Table 11: Model Asset Pricing Returns

Annualized Returns (Percent)

Law of motion of consumption	Equity Return		Risk-Free Return		Risk-Premium	
	Model	Data	Model	Data	Model	Data
AR4 estimated in Sample 1 (Θ^1)	9.88 [8.08 ; 12.52]	7.74	−0.37 [−2.38 ; 1.68]	0.33	10.25 [7.90 ; 13.77]	7.41
AR4 estimated in Sample 2 (Θ^2)	6.20 [4.77 ; 8.13]	10.03	3.38 [1.84 ; 5.05]	1.92	2.82 [2.13 ; 4.00]	8.11

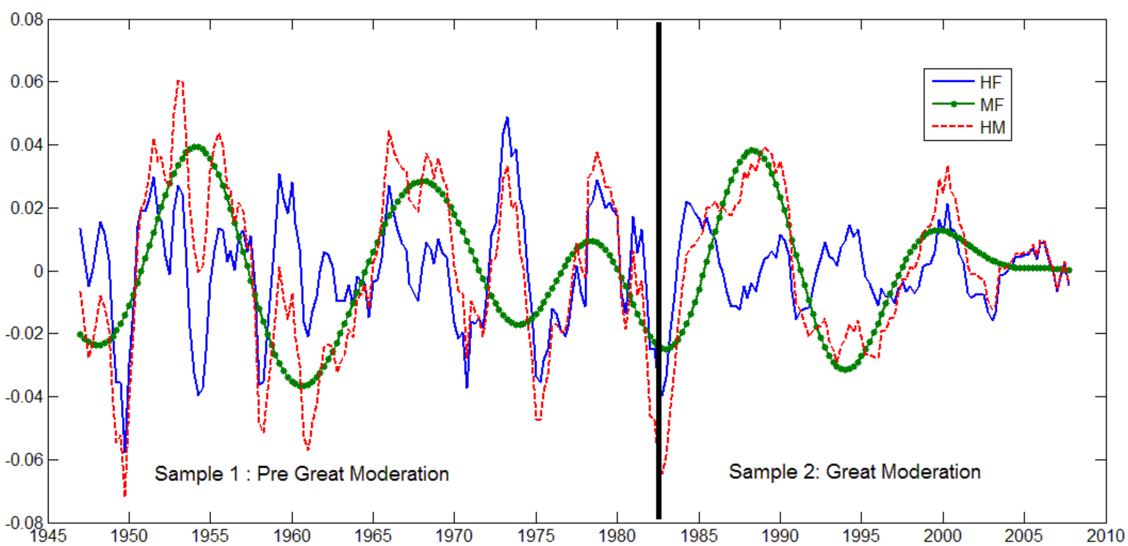
Note: The 95 percent confidence bands of the model mean returns are computed with a Bootstrap procedure (1000 repetitions).

Figure 1: Cyclical components of consumption



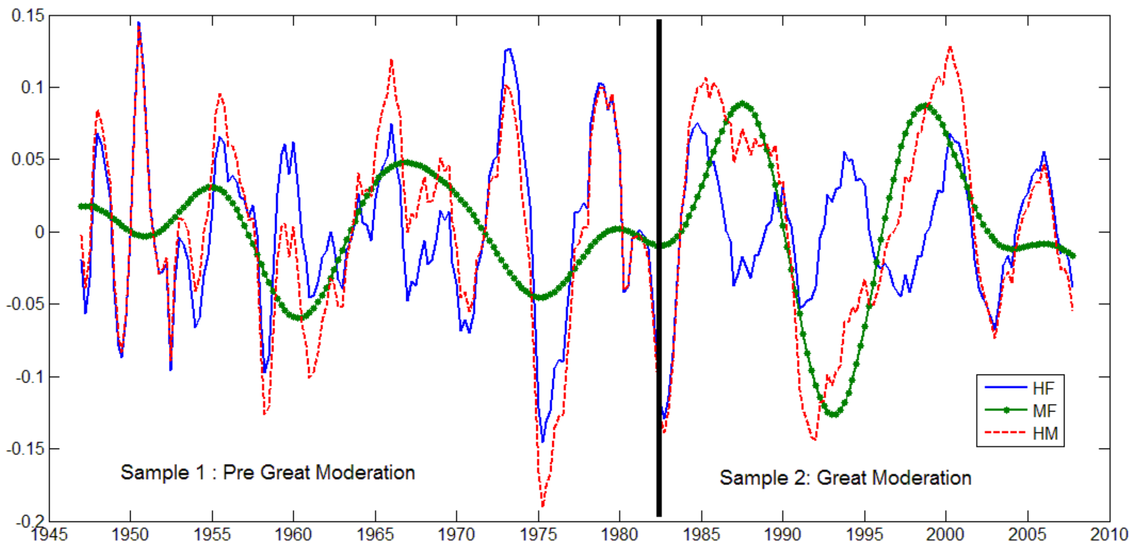
Note: Consumption is defined in real per-capita terms, measured as non durable goods plus services from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4 The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

Figure 2: Cyclical components of output



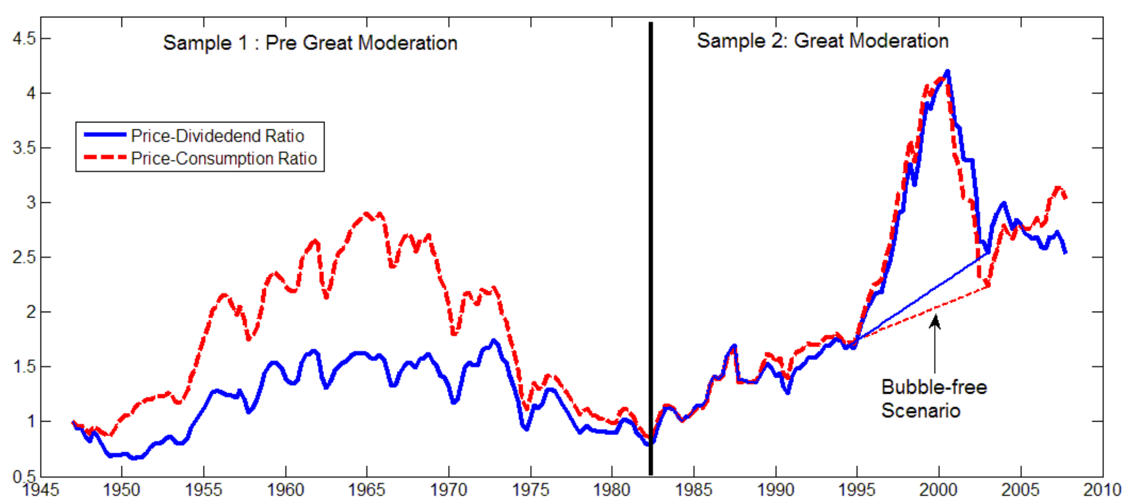
Note: Output is defined in real per-capita terms, measured as Gross Domestic Product from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4 The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

Figure 3: Cyclical components of investment



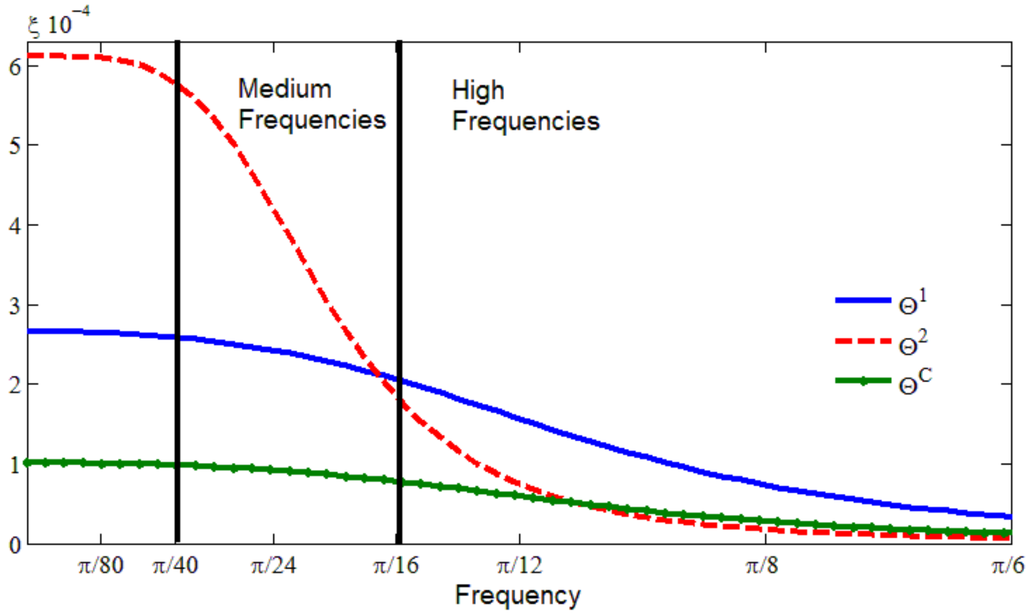
Note: Investment is defined in real per-capita terms, measured as private investment from NIPA. The sample includes quarterly observation from 1947:1 to 2007:4 The cyclical components, which are the High-Frequencies (HF, solid line), Medium-Frequencies (MF, dotted line), and High-to-Medium Frequencies (HM, dashed line) are isolated using a band-pass filter.

Figure 4: Price-Dividend and Price-Consumption Ratios



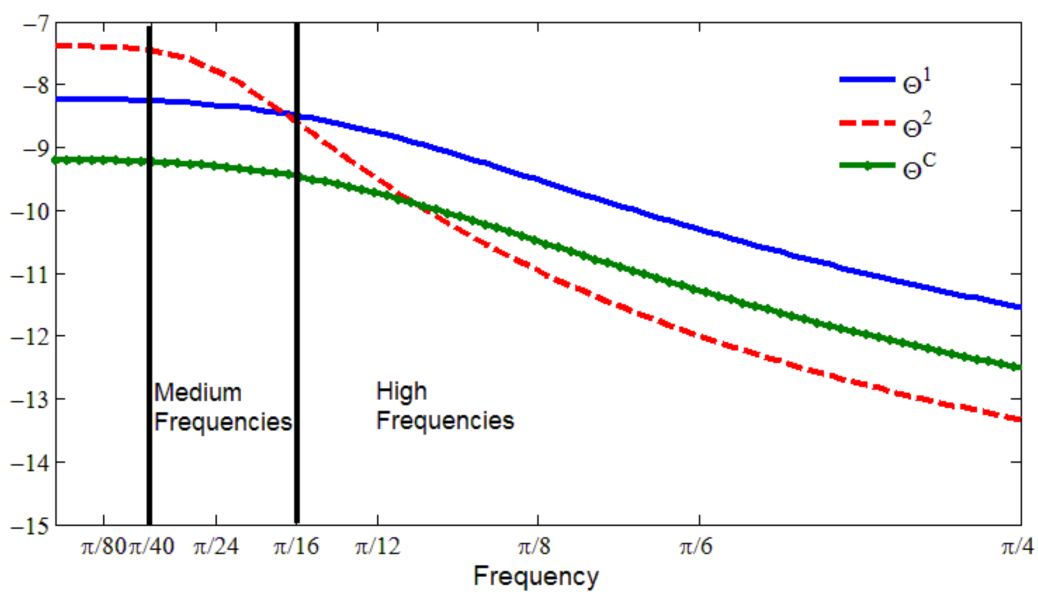
Note: The Price-Dividend ratio (solid blue line) and Price-Consumption ratio (dashed red line).are normalized to one in the first observation of the sample 1947:1-2007:4. The straight lines during the period 1995-2003 represent the bubble-free scenario for the evolution of the two variables.

Figure 5: Spectra of the AR4 processes for consumption



Note: The figure shows the spectral density of the three $AR(4)$ processes of consumptions within the frequencies 0 and $\pi/6$. The Medium Frequencies are defined in the interval $\left[\frac{\pi}{16}, \frac{\pi}{40}\right]$, the High-Frequencies are defined in the interval $\left[\pi, \frac{\pi}{16}\right]$.

Figure 6 Log-Spectra of the AR4 processes for consumption



Note: The figure shows the log-spectral density of the three AR4 processes of consumptions within the frequencies 0 and $\pi/4$. The Medium Frequencies are defined in the interval $\left[\frac{\pi}{16}, \frac{\pi}{40}\right]$, the High-Frequencies are defined in the interval $\left[\pi, \frac{\pi}{16}\right]$.