

Underdevelopment of Financial Markets and Excess Consumption Growth Volatility in Developing Countries

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Abstract

This paper aims at explaining, both qualitatively and quantitatively, why consumption growth is substantially more volatile in developing countries than in developed countries. I propose an infinite-horizon stochastic growth model with endogenous financial development, à la Acemoglu and Zilibotti (1997). In this model, micro-level project indivisibility and aggregate savings determine the degree of diversification in financial markets. In addition, countries are subject to TFP shocks with different means, capturing differences in technology, but with equal variance and persistence. On average, less technologically advanced economies have lower income and savings, translating into lower financial development. When the financial market is underdeveloped, shocks to investments and TFP endogenously have more persistent effect on future output. Thus, consumption responds more to those shocks, and the volatility of consumption relative to the volatility of output is higher in poorer than in richer countries. I also show that a calibrated version of the model is consistent with a number of features of the data, without relying on exogenous differences in the variance and persistence of TFP shocks.

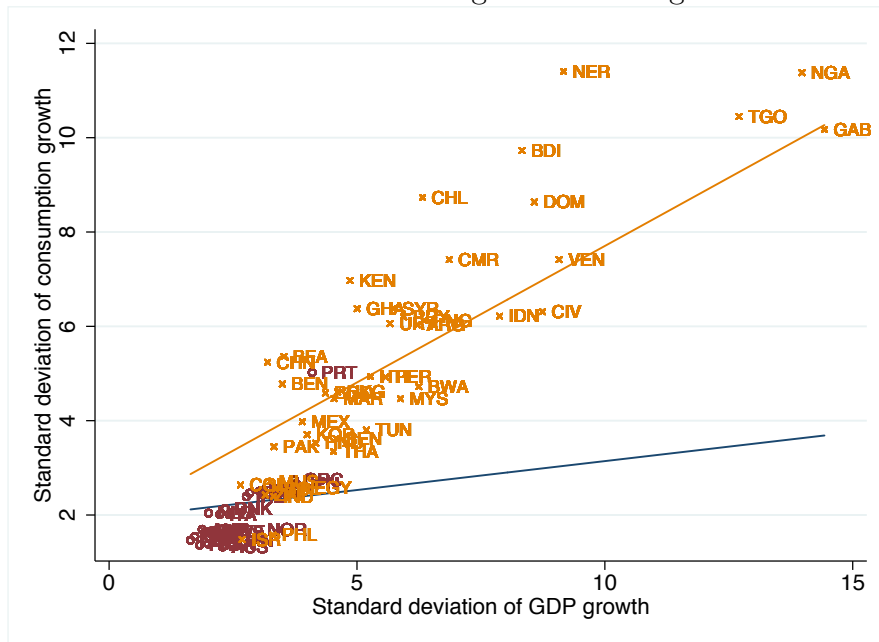
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1 Introduction

This paper aims at explaining the excess consumption growth volatility puzzle in developing countries. The data suggest that output growth is generally more volatile in those countries. More interestingly and puzzlingly, the negative relationship between volatility and development is even more pronounced in the case of consumption growth volatility. In other words, consumption growth volatility in developing countries is disproportionately higher than in developed countries, relative to output growth volatility (Kose, Prasad, and Terrones 2003). The purpose of this paper is to construct a theory that is consistent with these observations.

The focus on consumption growth volatility is well justified. The extent to which high volatility is a first-order problem for developing countries depends on the extent to which output growth volatility translates into consumption growth volatility. If, for instance, it were the case that poor countries can insure themselves through international risk-sharing, consumption growth can be fairly stable and the welfare costs of output fluctuation would be less significant. However, that is not the case in reality. Evidence shows (e.g. Lewis 1996) that international consumption risk sharing is quite limited. This implies that reducing volatility in developing countries would potentially entail substantial welfare gains.

Figure 1: The regression



Source: WDI data, 1960-2007. Regression of the standard deviation of consumption per capita growth on the standard deviation of GDP per capita growth.

Using WDI data from 1960 to 2007, I regress the standard deviation of consumption per capita growth on the standard deviation of GDP per capita growth and country group

Table 1: The pattern

	σ_c	σ_y	σ_c/σ_y
Developed countries	2.155 (0.46)	2.403 (0.31)	0.896 (0.07)
Developing countries	5.385 (0.34)	4.503 (0.19)	1.197 (0.05)
Difference	3.23 (0.57)	2.101 (0.36)	0.302 (0.08)

Source: WDI (1960 - 2007). All the numbers are reported in percentage. σ_c and σ_y are standard deviation for consumption growth and output growth, respectively. σ_c/σ_y is their ratio. Standard errors are in parenthesis.

dummy.¹ Figure 1 shows the regression lines for developed and developing countries, respectively. Developed countries cluster around the lower left corner, which means that both consumption and GDP growth volatilities are low. The picture for developing countries is quite different: Most of them spread out towards the upper right corner, which means that both volatilities are higher in developing countries. Moreover, consumption growth seems to present excess volatility: Consumption growth volatility increases much more in response to GDP growth volatility. The positive slope of the regression line for developing countries is significantly higher.

To more clearly identify this pattern, I analyze the ratio of consumption per capita growth volatility to GDP capita growth volatility. Table 1 gives the average standard deviations of consumption and GDP growth as well as their ratios in developing and developed countries, respectively. In the second column, the negative relationship between output growth volatility and income level is obvious, while the first column shows that the same relationship also holds for consumption growth. The third column gives the mean ratios in each group and shows that the average ratio is disproportionately higher in developing countries. The gap between the two averages, roughly 0.3, is large and statistically significant.

Similar exercises have been conducted using different data in terms of sample countries, time interval and frequency.² Kose, Prasad, and Terrones (2003) document a similar

¹More than twenty industrial economies are referred to as developed countries and the remaining countries of the sample, which have a lower income level, are labeled as developing countries. Note that very small countries, countries with clearly unreliable data and oil producers are excluded from the analysis. Consumption and GDP are both in real per capita terms and in constant local currency unit.

²Aguiar and Gopinath (2007) also lend support to this finding with a relatively small sample of emerging and industrial economies. Their data suggest that emerging economies exhibit relatively volatile consumption at business-cycle frequencies, even though the already high income volatility is controlled for. Resende (2006) studies a sample of 41 small open economies. His findings are well consistent with previous research. Similarly, De Ferranti et al. (2000) show that the volatility of the growth rate of real

pattern, although the gap that they find is relatively smaller than mine.³

The existing literature tries to explain why consumption is substantially more volatile in lower income countries by relying either on different properties of exogenous shocks (e.g. Aguiar and Gopinath (2007)) or on international channels (e.g. Levchenko (2005), Resende (2006) and Neumeyer and Perri (2005)). In contrast, this paper shows that the frictions in domestic financial markets can help explain the empirical puzzle.

I propose an otherwise standard stochastic growth model with an infinite horizon. The new element is that the financial market is explicitly modeled, à la Acemoglu and Zilibotti (1997). I assume that agents have access to a large number of imperfectly correlated risky projects in the intermediate sector which transform savings into capital goods, only one of which succeeds each periods, i.e. delivers a positive return. Those projects receive savings from households by issuing securities in the financial market. When the uncertainty has unraveled, the productive project distributes output to security holders. In addition, some of the risky projects are required to raise a certain amount of savings from individuals, before being productive. If there are not enough savings in the economy, some projects are not funded and thus, not all securities are available in the financial market. In contrast, if there are enough savings in the economy, minimum size requirements are irrelevant and the financial market is complete.

If the financial market is incomplete, the return to investment in risky securities is stochastic. It is a good (bad) draw if the productive project is (not) funded and yields (does not yield) a return. Good draws result in more capital goods being brought forward to the next period. It implies higher savings, which helps to better diversify the risks in the intermediate sector. This, in turn, increases the chances of receiving good draws in the following periods, hence increasing expected future income.⁴ In other words, shocks in the financial market *amplify* themselves through capital accumulation.

Unlike Acemoglu and Zilibotti (1997), this model allows exogenous TFP shocks and their *interaction* with endogenous shocks from the financial market. I assume countries to be subject to TFP shocks, which have exactly the same variance and persistence. The only exogenous difference between developing and developed economies is the long-run mean TFP, which captures the difference in technological or productivity levels.

Importantly, the difference in development levels translates into the difference in diversification of the financial market. In this model, the developed economy behaves similarly to

GDP in Latin American countries is twice as high as in industrial economies, while consumption growth volatility is three times higher than in industrial economies.

³They construct an income measure based on GNP. The standard deviation of income growth is higher than that of output growth in both types of economy. They report the gap between the within-group medians.

⁴Similarly, bad draws do not only decrease the capital stock, but also reduce the probability of good draws in the following periods, and further reduce the future expected income.

a standard stochastic growth model. Its steady-state level of capital is sufficiently high to afford a fully diversified financial market and all idiosyncratic risks are diversified. Most of the time, it is fluctuating around the (deterministic) steady state, with the complete financial market. The interesting difference as compared to standard stochastic growth models is that a sequence of bad TFP shocks could shift the fully diversified economy away from the steady state and back to the situation where the financial market is less complete and the economy could be hit by bad draws. Thus, the model predicts both frequent “small recessions” and rare deep and persistent recessions in developed countries. On the other hand, since the developing economy is less productive and the “steady-state” level of capital is so low, the fully diversified financial market is not affordable. The economy is always subject to shocks to investment from the intermediate sector, so that the volatility in both consumption and output will be higher than in the developed economy. The model also predicts that the output gains during expansion can be larger in lower income countries. To understand this, suppose that the economy is hit by a sequence of good TFP shocks. They lead to higher savings and allow the economy to expand, which improves the diversification opportunities in the financial market. This, in turn, implies a higher chance of getting good draws from the financial market. Booms are reinforced and stronger. This prediction is well consistent with the empirical findings in Caldern and Fuentes (2006).⁵

The two important mechanisms (amplification and interaction) imply more volatile consumption (relative to output) in developing countries than in developed countries. First, while output just keeps track of the capital level, consumption responds even more to endogenous shocks from the financial market, since these shocks have persistent effects on future output and consumption opportunity through the amplification channel. Since the developing economy is in a less complete financial market most of the time, this effect is stronger in the developing economy. It implies that the ratio concerned should be relatively higher in developing countries.

Second, exogenous TFP shocks are amplified by endogenous shocks from the financial market. Therefore, the persistence effect on output of exogenous TFP shocks is endogenously higher. Consumption also responds to this effect and becomes more volatile. Once more, this type of interaction plays a larger role in the developing economy. It is almost absent in the developed economy, since the financial market is complete most of the time.

This paper also sheds some light on the link between frictions in the financial market and observed differences between *measured* TFP shock processes (e.g. Aguiar and Gopinath (2007) hypothesize that the TFP shock properties are different across groups.)⁶. I assume

⁵They show that expansions are, on average, stronger in lower income countries (e.g. Asian developing and Latin-Americans countries) than in industrial ones. In particular, they show that Colombia and Malaysia achieved the largest output accumulation during the expansion phases.

⁶Measured TFP processes is constructed from Solow residual, where the capital stock is measured as

there to be no exogenous difference in shock processes between the two types of economy. Instead, I study how a standard stochastic growth model, enhanced by the friction of micro-level project indivisibility, could endogenously deliver the observed differences between *measured* TFP shock processes.

The quantitative results show that the model can replicate the empirical pattern pretty well. The ratio of consumption growth volatility to output growth volatility is substantially higher in the developing economy case. The gap generated by the model accounts for a substantial part of the data. The model also predicts that an increase in the technological level is associated with a decrease in both consumption and output growth volatilities. Moreover, consumption growth volatility should drop even more quickly. The results from simulation data also confirm this prediction.

The paper relies on the endogenous diversification channel, proposed by Acemoglu and Zilibotti (1997). Apart from the fact that the main focus is the consumption volatility puzzle, there are noteworthy differences between this model and their work. First, I model an economy with an infinite horizon which is better suited for studying high-frequency phenomena, in contrast to the two-period OLG framework in their paper, which is appropriate for development issues. Second, exogenous TFP shocks are included, so that it is possible to quantitatively assess the model economy with the data. More importantly, the interaction between endogenous and exogenous shocks arises in this model. Third, I impose more general assumptions on preferences and depreciation, which yield important new insights and turn out to be critical for solving the puzzle.⁷ Finally, the general setup of this model poses technical challenges. The numerical analysis of the paper provides a functional and successful algorithm for solving the general framework.

This paper finds its place in the growing literature on consumption volatility. One group of research relies on the international sector to address the question of why increasing international financial integration is associated with higher consumption volatility in more financially integrated developing countries.

For example, Resende (2006) hypothesizes that developing countries are borrowing constrained and therefore, the lack of ability to smooth their consumption renders the ratio higher than in developed countries. He finds that this mechanism alone has a rather limited explanatory power.⁸ Neumeyer and Perri (2005) propose that shocks to the country risk premium could provide another source of uncertainty and also amplify the exoge-

the sum of past investment, assuming that one unit of saving translates into one unit of investment in a closed economy.

⁷They assume logarithm utility and full capital depreciation in their model, which allows them to derive analytical solutions. However, the simplicity comes at a cost: Substitution effect, income effect and wealth effect cancel out exactly. The savings rate is constant and therefore, the relationship between consumption growth volatility and output growth volatility cannot be properly studied.

⁸He suggests that the reason why consumption volatility cannot exceed income volatility is due to the lack of permanent shocks in his model.

nous TFP shocks, if the default risk premium is negatively correlated with TFP shocks. They claim that through this channel, consumption can be more volatile than output in emerging economies. Levchenko (2005) adopts the Kocherlakota (1996) framework of risk sharing subject to limited commitment to explain why consumption volatility can be higher, if lower income countries are better integrated into the international market.

While this line of research is successful to different degrees, no explanation is offered as to why the relative consumption growth volatility differential still exists in less financially integrated developing countries. This empirical fact can be readily explained by this model.

Another line of research focuses on the different properties of TFP shocks in emerging countries. Aguiar and Gopinath (2007) argue that industrial and lower income countries undergo different underlying income processes. Their hypothesis is that there are two components in the productivity shock process, transitory and permanent. In industrial economies, the transitory shocks are relatively more important, while in poorer emerging economies, the permanent component plays a larger role. Their theory implies that consumption is relatively more volatile in lower income countries.

Although they also point out that the difference in TFP processes might be a manifestation of deeper frictions in the financial market, they do not focus on how the financial frictions translate into the observed differences in TFP processes. This paper attempts to provide a link between these two.

This paper is also related to research focusing on the relationship between diversification and macroeconomic volatility (Acemoglu and Zilibotti (1997), Imbs and Wacziarg (2003), Koren and Tenreyro (2007a), Koren and Tenreyro (2007b) and Kalemli-Ozcan, Sorensen, and Volosovych (2009)). In contrast to previous research which puts emphasis on output growth volatility, this paper tries to explain the consumption volatility pattern. It also stresses the importance of the interaction between aggregate shocks and the diversification channel, which is absent in the previous literature.

The rest of the paper is organized as follows. The next section presents the basic model and characterizes the equilibrium. A numerical example is used to explain the basic mechanisms in the model. Section 3 explains the calibration and simulation strategy and Section 4 presents the basic findings. The empirical pattern found in the data is compared with the numerical results. The model is shown to be consistent with a number of features of the data, without relying on any exogenous differences in the variance and persistence of the TFP shock process. Section 5 concludes the paper.

2 The Model

2.1 Environment

The decentralized model economy is populated by infinitely lived agents. A constant relative risk aversion utility function is assumed to parameterize their preferences. Agents maximize their expected life time utility, which is defined by

$$U = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\sigma}}{1-\sigma}$$

where c_t is consumption in period t , σ is the coefficient of relative risk aversion and β is the discount factor. The population is constant and normalized to be one. Therefore, labor supply is also constant.

The production side consists of two sectors, the final good sector and the intermediate sector. The final good sector uses capital and labor to produce a final output. The production function in the final good sector is assumed to be Cobb-Douglas with capital K_t and labor L_t as inputs

$$Y_t = A_t K_t^\eta L_t^{1-\eta}$$

where $\eta \in (0, 1)$ is the elasticity of output to capital and A_t is productivity in period t . Productivity is subject to an aggregate shock.⁹ Formally, $A_t = e^{z_{it}}$ and z_{it} follows an $AR(1)$ process

$$z_{it} = (1 - \rho) \mu_i + \rho z_{it-1} + \varepsilon_t$$

where $|\rho| < 1$ and ε_t is a serially uncorrected normally distributed random variable with zero mean and constant variance, that is $\varepsilon_t \sim \mathcal{N}(0, \sigma_z)$. μ_i is a constant and i is the country type dummy: 0 stands for developing countries and 1 for developed countries. e^{μ_i} gives the long-run average productivity level and μ_i characterizes the difference between developing and developed countries: $\mu_0 < \mu_1$.¹⁰

⁹Note that the growth trend shock is an important source of volatility in output and consumption growth in developing countries, which has been studied by Aguiar and Gopinath (2007). Since my goal is to explore and highlight the underdevelopment of financial markets and its effects on consumption growth volatility, I assume away the growth trend of productivity or, in other words, assume the exogenous productivity growth to be zero. This can be considered as a de-trended version of a more general model. I provide a version of this model with a deterministic trend in the Appendix and show that it is not essential for the results.

¹⁰Note that it is the only exogenous difference I assume between these two groups. In a more general setup, I could assume there to be a stochastic type-switching process: Each type of economy has some probability of switching to the other type, governed by an exogenous transition matrix. The switching

Agents work in the final goods sector and earn a competitive wage and also receive capital income through the competitive renting market. Prices, precisely wage rate and return to capital, are competitively determined by aggregate capital in the economy, K_t , and the productivity level, A_t . Agents decide how much to consume and save every period. They are also allowed to decide on the allocation of their savings in the financial market. Following Acemoglu and Zilibotti (1997), I assume there to be an intermediate sector, which transforms savings into capital goods brought forward to the next period without using any labor. There is uncertainty which is represented by a continuum of equally likely states $state \in [0, 1]$. The transformation technology takes two forms: Safe and risky projects. The safe project gives the non-stochastic return r . There is a continuum of risky projects, corresponding to the states of nature. Risky project j pays a positive return only in state $j \in [0, 1]$ and zero otherwise.

Risky projects are financed by issuing securities in the financial market. Output from the risky projects is entirely distributed to the holders of securities. No profit is retained. The payoff to security holders in state of nature j is $R \cdot F_j$, if security holders invest F_j (density) in security indexed by j . It is assumed that $R > r$, which is consistent with the intuition that risky assets give a higher return. Note that not all the projects are necessarily funded, and therefore not all the securities are available in the economy. The measure of available securities, n_t , is determined in equilibrium.

In addition to deciding on savings (and consumption) in each period, agents are also allowed to decide how they allocate their savings in the financial market, i.e. the portfolio decision. They can invest in a set of available risky securities ($i \in [0, n_t]$), which consists of state-contingent claims to the output of the risky projects, and the safe asset, which consists of claims to the output of a safe technology. The assets portfolio is defined by α , which is the percentage of savings invested in the safe asset. It is assumed that $\alpha \in [0, 1]$, which means that the agent is not allowed to borrow to invest in risky or safe assets.

The agents invest an equal amount of savings in risky securities, F , due to the symmetry of risky assets: The expected return to each risky security is exactly the same. Moreover, they would invest in *all* available securities, so that the variance in the payoff from risky investment is minimized, while the expected return is the same. That is, $F_j = F_i = F$, $\forall i, j \in [0, n_t]$. This is called “balanced portfolio” .¹¹

In this model, only one type of friction is introduced, namely micro-level project indivisibility or minimum requirement of investment: The project, indexed by $j \in [0, 1]$, is productive only if it attracts at least a minimum amount of savings from individuals (see

probability is usually quite low. To keep the model simple and the results sharp, I assume that the switching probability is zero.

¹¹It can be shown the expected return r_E is constant. $r_E = F \cdot n \cdot R + (1 - n) \cdot F \cdot 0 = (1 - \alpha) \cdot s \cdot R$, which is the same, independent of n . The variance is decreasing in n , the measure of risky securities in which agents choose to invest, $Var = [(1 - \alpha) \cdot s \cdot R]^2 \left[\left(1 - \frac{2}{R}\right) + \frac{1}{R^2 n} \right]$.

Figure 2), $M(j)$. One example is railway production: Building a railway requires a large amount of investment before the project becomes useful and productive. To capture the heterogeneity in minimum size requirement across projects, it is normalized to zero for projects $j < \gamma$, while the minimum size of the rest is linearly increasing in their index.¹² Formally, the minimum size is specified by

$$M(j) = \max \left\{ 0, \frac{D}{1-\gamma} (j - \gamma) \right\}$$

where D is the highest minimum requirement in the economy.

To appreciate the importance of this friction, consider the following case where $D = 0$ or $\gamma = 1$. Given this assumption, the micro-level project indivisibility is absent and all projects will be funded. Agents would invest an equal amount in all risky securities. The return to this portfolio becomes deterministic. Intuitively, with the assumption that $D > 0$ and $\gamma < 1$, it is not necessarily the case that all projects could attract enough savings to meet their minimum requirements.

Aggregate savings and associated portfolio choice, together with micro-level project indivisibility, help determining the measure of open projects in equilibrium. Intuitively, if savings in the economy are less sufficient, agents would invest in the safe asset to seek insurance and invest even less in risky securities. Based on the balanced portfolio, each open project would raise the same amount of savings to fund its production in the intermediate sector. In equilibrium, given the aggregate amount of savings allocated to risky projects, the maximum possible measure of projects will be less than one in the economy.¹³ Suppose, on the other hand, that the savings in the economy are sufficiently high and all projects can raise enough savings to overcome the minimum requirement. The maximum possible measure of risky securities is one and the market is complete.

2.2 Recursive Formulation: Decentralized Equilibrium

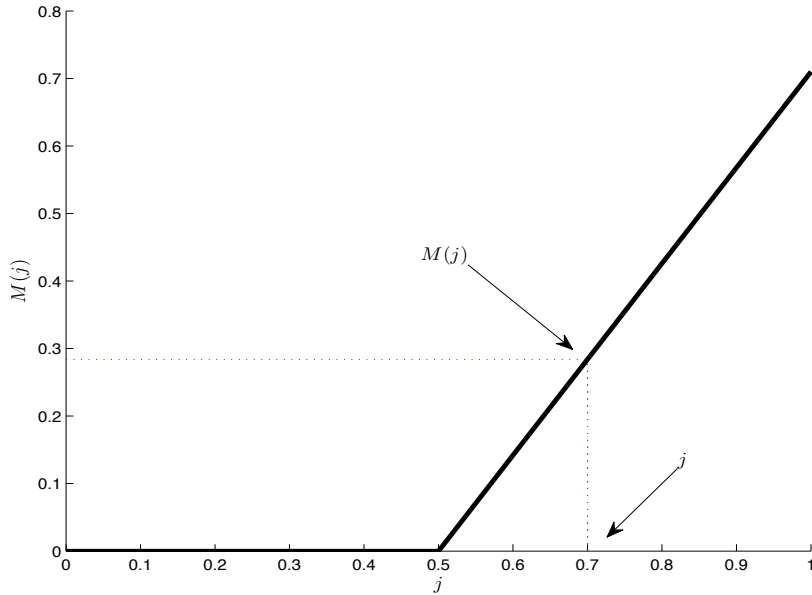
Formally, the problem solved by the representative agent can be restated in the following recursive formulation. The measure of available securities, $n(K, A)$, is a function of aggregate variables. The agent takes this as given, and solves the following problem:

$$V(K, k, A) = \max_{s \geq 0, 1 \geq \alpha \geq 0} \{u(c) + \beta \cdot E_{K,A} V(K', k', A')\}$$

¹²The results are not driven by the specification of the linear form. Parameters γ and D will also be calibrated.

¹³Acemoglu and Zilibotti (1997) have an interesting micro foundation for justifying the mapping from aggregate resources to the maximum measure of securities. A similar mechanism applies in this model. To avoid a repetition of their analysis, I skip the static equilibrium determination in the financial market and focus on the dynamic aspect of the model.

Figure 2: Minimum Requirement of Investment



Note: The case where minimum investment is assumed.

The value function of the representative agent is a function of aggregate capital,¹⁴ K , his own capital k , and aggregate productivity, A . The right-hand side of the Bellman equation consists of utility derived from current consumption and the discounted expected continuation value. The expectation is conditional on both A and K . First, the agent needs information about A to compute the distribution of A' in the next period. Second, the agent also needs to know the probability of good draws at the end of the period, since there are two possible realizations. The probability is computed using both aggregate variables, K and A , and $n(K, A)$. In other words, the forecasted continuation value must be conditional on K . It reflects the additional source of uncertainty in the economy, namely the endogenous shocks from the financial market.

The representative agent chooses saving and portfolio optimally. The representative agent's choice is subject to the budget constraint

$$c + s = w(K, A) + \varphi(K, A) \cdot k$$

where $w(K, A)$ is the wage rate, $\varphi(K, A)$ is the return to capital and k is his own capital. The representative agent takes factor prices as given and makes the savings decision, s ,

¹⁴Aggregate capital information is important for the agent to solve the problem. First, in the decentralized economy, the agent acts as a perfectly competitive price taker, and factor prices are pinned down by aggregate variables. In a central planner version of this model, the agent's portfolio decision is different. The central planner trades off between opening more projects to diversify risks and a higher expected return. In the end, available aggregate resources help to determine the measure of active risky projects. Second, the measure of available securities, n , is necessary information for her to solve for decision rules. It is jointly determined by aggregate variables A and K .

and thus the consumption decision, c .

The total amount invested in the safe asset is ϕ ,

$$\phi = \alpha \cdot s$$

The total amount of investment in risky securities is $(1 - \alpha) \cdot s$. Recall the “balanced portfolio” : 1) The agent invests in each risky security with F and 2) the measure of securities, in which she invests, is the measure of available ones, $n(K, A)$. Therefore, the following relationship holds

$$n \cdot F = (1 - \alpha) \cdot s$$

The following discussion describes the law of motion for the three state variables. The individual capital accumulation function takes two forms, depending on the realization of state of nature (see Figure 3). Suppose that the state of nature j is realized at the end of the period. If $j < n$, project j is both funded and productive. The agent must have invested in risky securities indexed by j (once more, recall the balanced portfolio). The agent collects returns from both safe and risky assets. In this case, the capital in the next period, k' , consists of three components: Return from safe asset, $r \cdot \alpha \cdot s$, return from risky asset, $R \cdot \frac{(1-\alpha)}{n} \cdot s$, and undepreciated capital $(1 - \delta)k$, where δ is the depreciation rate in the economy. In this case, I denote k' as k^g . Conversely, if $j > n$, i.e. project j is not funded, the agent’s risky portfolio gives no return. Capital at the end of the period only consists of return from the safe asset and undepreciated capital. Similarly, in this case I denote k' as k^b .

Since all states of nature have equal chances of being realized, the measure of available risky securities, n , is also the probability for the agent of receiving a “good draw” , or $k' = k^g$. The probability of a “bad draw” is therefore $(1 - n)$ (see Figure 3).

The individual capital accumulation function is therefore as follows,

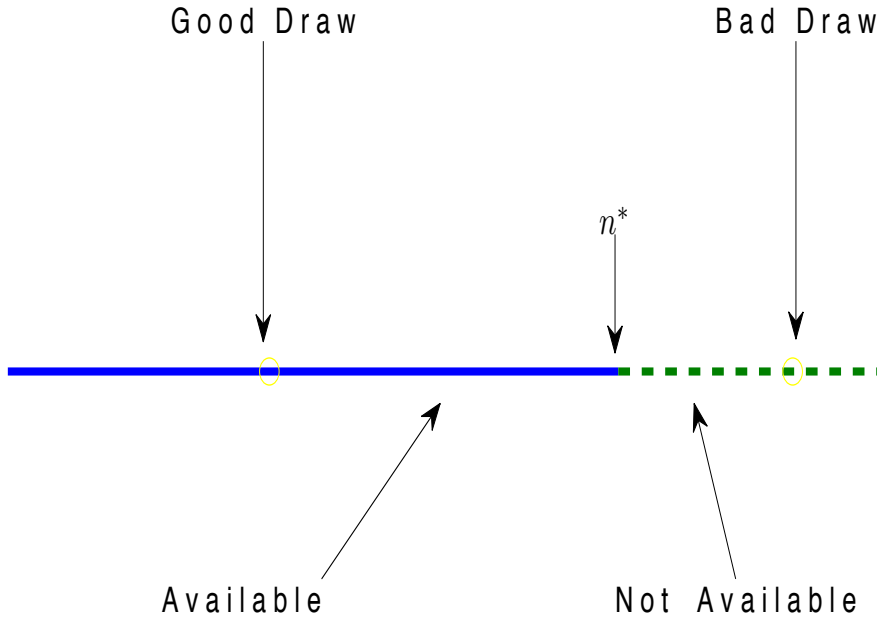
$$k' = \begin{cases} r \cdot \alpha \cdot s + (1 - \delta) \cdot k & \text{if } j > n \text{ with prob } 1 - n \\ r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s + (1 - \delta) \cdot k & \text{if } j \leq n \text{ with prob } n \end{cases}$$

The law of motion of aggregate capital is needed for the agents to optimize.¹⁵

$$K' = \Psi(K, A)$$

¹⁵The agent could choose an arbitrary belief in the law of motion of aggregate capital. In equilibrium, it must satisfy the “consistency condition” . See the equilibrium definitions.

Figure 3: Good Draws and Bad Draws



Note: The probability of good draws and the availability of risky securities. The solid line shows the measure of available risky securities and the probability of good draws. The dashed line shows unavailable risky securities and the probability of bad draws.

Finally, the exogenous shock process is AR(1) ¹⁶,

$$\log A' = (1 - \rho) \mu_i + \rho \log A + \varepsilon$$

Given the model described above, the definition of a competitive equilibrium is stated as follows:

1. $V^*(K, k, A)$, $\alpha^*(K, k, A)$ and $s^*(K, k, A)$ solve the individual's maximization problem, taking $n^*(K, A)$ as given.
2. Prices, namely wage rate, $w^*(K, A)$ and capital return, $\varphi^*(K, A)$, are both competitively determined.
3. Consistency conditions: The law of motion of aggregate capital is consistent with the aggregation of individual capital, $K' = \Psi(K, A) = \int k' di$.
4. Financial market equilibrium: Given $n^*(K, A)$, the associated $\alpha^*(K, k, A)$, $s^*(K, k, A)$ and the implied $F^*(K, k, A) = \frac{(1-\alpha^*)}{n^*} \cdot s^*$, the following conditions hold:

$$F^* = \frac{D}{1 - \gamma} (n^* - \gamma) \text{ if and only if } 0 < n^* < 1$$

¹⁶I drop subscript i on A_i , when there is no confusion.

$F^* \geq D$ if and only if $n^* = 1$

5. $K = k$.

The consistency conditions need to be elaborated. The agent knows the law of motion for the aggregate shock. She also needs to conjecture the law of motion of aggregate capital to make her decision. The conjecture, $\Psi(K, A)$, turns out to be correct and equal to the aggregation of individual capital in equilibrium.

The equilibrium conditions reflect the fact that aggregate resources and micro-level project indivisibilities jointly determine the measure of available securities. Agent i 's investment in the available security indexed by j , $F(i, j)$, depends on his savings, portfolio choice and the availability of risky securities. That is, $F(i, j) = \frac{(1-\alpha)}{n} \cdot s$. Therefore, $\int_0^n F(i, j) dj$ gives the total amount of risky investment by agent i . The aggregate risky investment in the whole economy is $\int_0^1 \left(\int_0^n F(i, j) dj \right) di$. The equilibrium is a mapping from aggregate resources to the *possible maximum* measure of available securities, and the following condition holds in equilibrium:

$$\int_0^1 \left(\int_0^{n^*} F^*(i, j) dj \right) di \geq \int_0^{n^*} \frac{D}{1-\gamma} (n^* - \gamma) dj$$

where the backward inequality holds, if $n^* = 1$, and the equality holds, if $n^* < 1$. The equilibrium conditions are derived using a balanced portfolio rule.

Finally, the economy always remains on the equilibrium path. Therefore, only the case where $K = k$ is of interest.¹⁷

2.3 Optimization

Taking $n(K, A)$ as given, the agent solves the optimization problem, which reduces to two Euler equations (see the Appendix for a detailed solution),

$$U'(c) = \beta \cdot E \left[U'(c_g) \cdot R \cdot \left(\eta \cdot A' \cdot K^{g(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right] \quad (1)$$

$$U'(c) \geq \beta \cdot E \left[U'(c_b) \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \left(\eta \cdot A' \cdot K^{b(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right] \quad (2)$$

¹⁷ k and K need to be distinguished when posing the decision problems of the household and firms. The equilibrium that $K = k$ is imposed *after* firms and the agent has optimized. I only need to solve for decision rules on the equilibrium path and ignore information outside the equilibrium path

where $c_g(K^g, k^g, A')$ and $c_b(K^b, k^b, A')$ are consumption choices, given the capital stock k^g, k^b and the aggregate capital level K^g, K^b in the next period, while n is the probability of good draws,¹⁸ given the state (K, A) . The equality holds in equation (2), if and only if $n < 1$ and the inequality is strict, if and only if $n = 1$. The two equations are the Euler equations relating current and future marginal utilities.

In this model, the diversification opportunity is endogenous. There are two important cases. First, given a certain state, $(\widehat{K}, \widehat{A})$, $n(\widehat{K}, \widehat{A})$ can be equal to one and the backward inequality holds in the second equation. Only the first Euler equation is relevant.¹⁹ In this case, all risks from the intermediate sector can be diversified.²⁰ The model behaves similarly to a standard stochastic growth model: Only exogenous shocks provide uncertainty. Second, in the other states, $n(\widehat{K}, \widehat{A})$ can be less than one and equality holds in the second equation. Both equations are relevant. In this case, the economy is subject to shocks arising from the financial market.

After imposing the equilibrium conditions, the solution is a combination of $s^*(k, A)$ and $\alpha^*(k, A)$, which satisfies the two functional equations and $n^*(k, A)$, which guarantees the financial market equilibrium condition.

2.4 Analytical Special Case

In general, the model has no closed-form solution. Interestingly, there is one special case, which can be solved by paper and pencil and it is the case where the analytical solution is also obtained in standard stochastic growth models. I study the special case where $\delta = 1$ and $U(c) = \log(c)$. I use the method of “guess and verify” to solve this case (see the Appendix for the detailed solution). Decision rules are solved for:

$$\alpha^*(k, A) = \frac{R \cdot (1 - n^*(k, A))}{R - r \cdot n^*(k, A)}$$

$$s^*(k, A) = \beta \cdot \eta \cdot A \cdot k^\alpha$$

Using the equilibrium condition, the measure of available risky securities is also solved for:

¹⁸The agent uses the given function $n(K, A)$ to compute the probability n at a certain state. Note that the agent can compute the decision rules, even though she takes an “incorrect” belief of $n(K, A)$. The financial market equilibrium condition is violated in that case. In other words, in equilibrium, the agent will hold a correct belief of $n(K, A)$.

¹⁹Given this state, the probability of a bad draw is 0. Moreover, $\alpha(\widehat{K}, \widehat{A}) = 0$ and only the savings decision, s , is of importance for the agent.

²⁰If I further assume $R = 1$ and $r = 1$, the model exactly simplifies to the Euler equation in the standard stochastic growth model.

$$n^*(k, A) = \frac{(R + \gamma \cdot r) - \sqrt{(R + \gamma \cdot r)^2 - 4 \cdot r \left(\frac{(R-r) \cdot (1-\gamma)}{D} \cdot s^* + \gamma \cdot R \right)}}{2 \cdot r}$$

A few comments are in order. In this special case, the saving rate is constant, which means that exogenous shocks and capital levels do not affect the agent's saving rate. The result is the same as in the standard stochastic growth model with these two special assumptions. The portfolio choice and the equilibrium measure of risky securities are the same as in Acemoglu and Zilibotti (1997), which means that their two-period OLG model is a special case of this general framework.

The implications of this solution are that 1) the consumption rate is constant over time and it is the same in both developing and developed countries; 2) the ratio of consumption growth volatility to output growth volatility will be one, in both types of economy. The full depreciation assumption severs one crucial channel of persistence. Moreover, the substitution and income effects of inter-temporal prices cancel out under log preference. In this case, the relationship between consumption and output volatility will be trivial, due to the unrealistic assumptions. This is one of the reasons why I must analyze the general model. On the other hand, this analytical solution provides a good starting point for the numerical computation of the general model, where the rooting finding method must be used and requires an exact initial guess (see the Appendix for a detailed solution).

2.5 Analysis

2.5.1 Decision Rules and Equilibrium

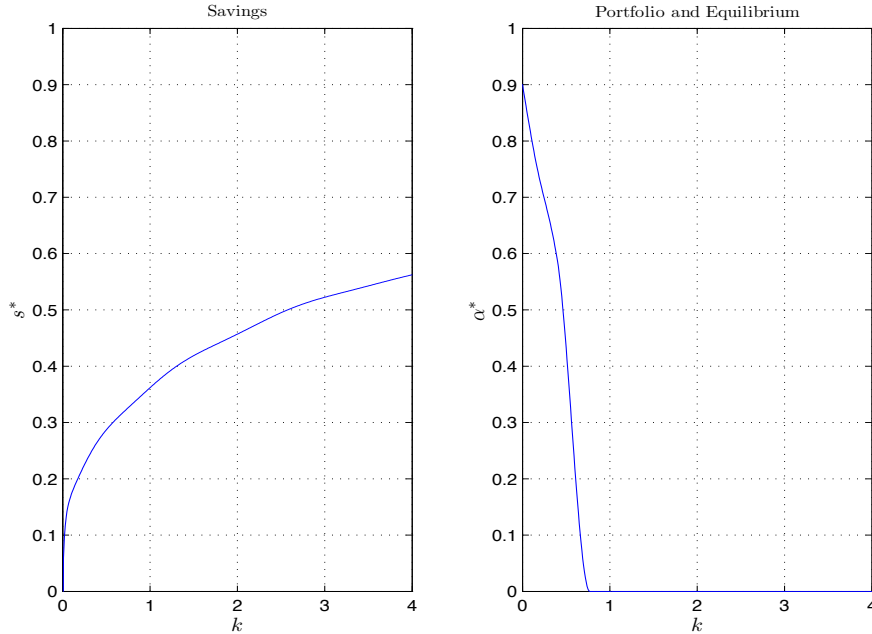
Policy functions, specifically the savings decision and the portfolio choice, are both two-dimensional functions of capital, k and productivity, A . The same applies for the equilibrium measure of available securities or the probability of good draws, $n(k, A)$. I present decision rules and equilibrium from a special numerical case,²¹ where the exogenous shock is absent, namely, $\rho = 0$ and $\sigma_z = 0$. In this special case, the difference between the two economies is the long-run productivity levels, A_i , where A_0 stands for the developing economy case and A_1 for developed economy. The solutions are denoted by $n^*(k; A_i)$, $\alpha^*(k; A_i)$ and $s^*(k; A_i)$.

The decision rules are similar in both economies. I plot the decision rules for the developing economy case with A_0 in Figure 4 and the developed economy case with A_1 in 5. The left-hand plot gives the savings decision, which is concave and increases in capital. The decreasing curve in the right-hand plot is the portfolio decision, α . For a given productivity level, the higher is the capital, the less do agents invest in the safe asset.

²¹See the next section for the solution algorithm.

Up to a certain point, the investment in the safe asset is positive. After the threshold, agents invest nothing in the safe asset. The intuition is that agents with low capital would invest in the safe asset to seek insurance. If agents are sufficiently rich, they invest all their savings in risky securities to seek a higher return.

Figure 4: Decision Rules with A_0

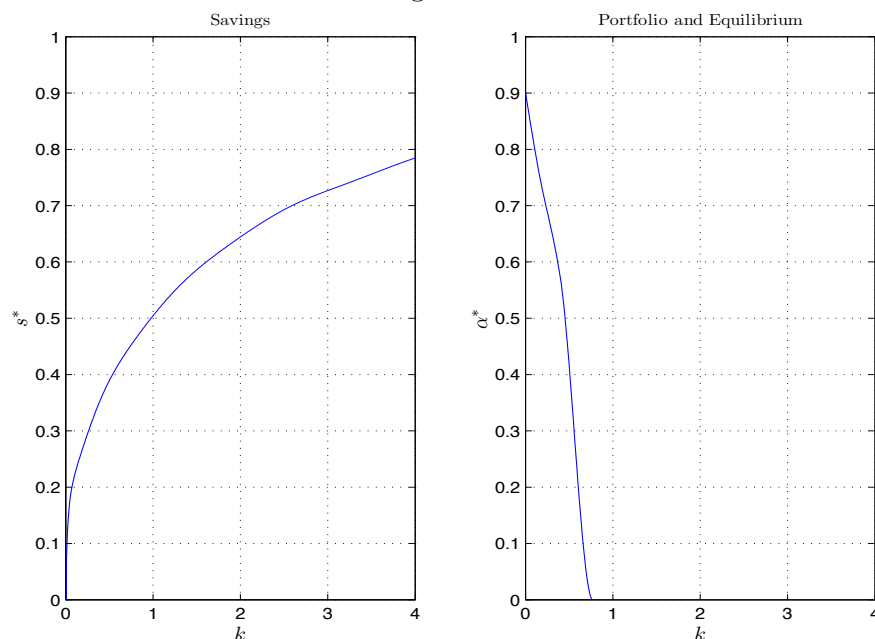


Note: Decision rules in the developing country case. The left-hand plot gives the savings decision, while the right-hand plot presents the portfolio decision.

Figure 6 shows the dynamics of capital and equilibrium in the developed economy case with A_1 . The upper part of Figure 6 gives the probability of good draws in equilibrium, n^* . It increases in k and approaches 1 from below. As shown in the decision rules, savings increase in capital and the portfolio shifts toward risky securities, if k becomes higher. The total amount of savings invested in risky securities increases in k , and based on the equilibrium condition, there is an expansion in the equilibrium measure of available securities; hence, the higher probability of good draws. If capital k is sufficiently high in this economy, full diversification is achieved, that is $n^* = 1$.

The lower plot of Figure 6 gives two possible realizations of capital, k_g and k_b . The plot for k_g is increasing in capital, which describes the case of good draws. It does not only consist of undepreciated capital but also of returns to risky securities and the safe asset. k_b increases in k , when k is very low and decreases until a threshold where $\alpha^* = 0$ and then increases, following the line of $(1 - \delta) \cdot k$. As analyzed before, with the increase in k , the diversification opportunities improve and investment in the safe asset decreases until zero. Below that threshold, k_b comprises both undepreciated capital and return to the safe asset. Above that threshold, k is sufficiently high, and k_b only consists of

Figure 5: Decision Rules with A_1



Note: Decision rules in the developed country case. The left-hand plot gives the savings decision, while the right-hand plot presents the portfolio decision.

undepreciated capital. In this situation, if bad shocks emerge in the intermediate sector, only undepreciated capital works as a buffer and becomes capital in the next period.²²

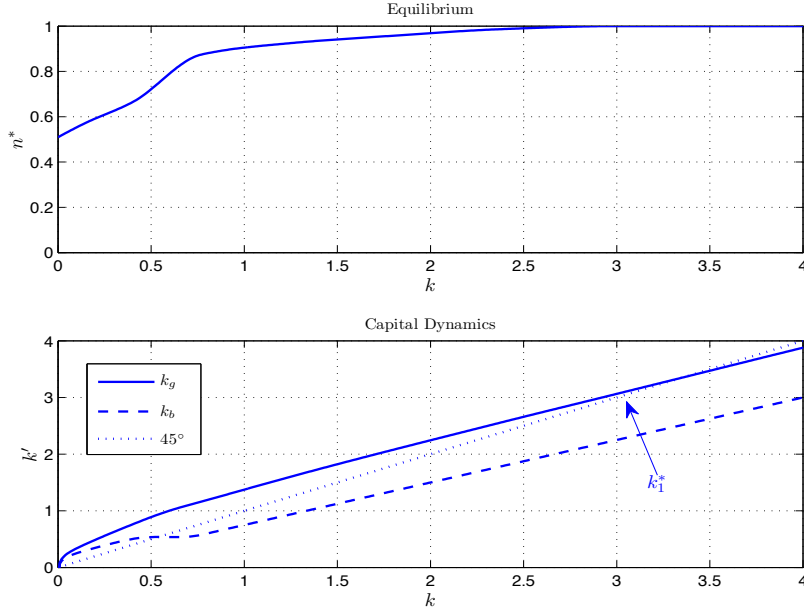
The intercept of curve k_g with the 45 degree line gives the steady-state of the economy, k_1^* . In this steady state, full diversification is obtained and the probability of good draws is one. It means that if the economy reaches k_1^* , its capital is k_1^* in the next period with probability one. That is the situation which mimics standard neoclassical growth models.

Similarly, in the developing economy case with A_0 , the probability of good draws in the intermediate sector increases in k and both k_g and k_b in Figure 7 present a similar shape as in Figure 6.

The intercept of curve k_g with the 45 degree line in Figure 7 gives the “pseudo steady-state” of the developing economy, k_0^* . In this case, the pseudo steady-state level of capital is lower than its counterpart in the developed economy with higher productivity, A_1 . More importantly, full diversification is not achieved at this pseudo steady state. The probability of good draws is lower than one. With a positive probability, k' shifts onto the curve of k_b .

²²That is a scenario which does not occur in Acemoglu and Zilibotti (1997). In their model, full depreciation implies that agents invest all their savings in risky assets, only if the financial market is fully developed. Therefore, another implication of their model is that a reasonably rich economy could revert back to a really poor economy right before it becomes fully developed, with a small probability. Introducing undepreciated capital helps us provide the economy with an additional buffer and avoid the unrealistic implication.

Figure 6: Developed Economy with A_1



Note: Developed country case without exogenous shocks. The upper plot gives the equilibrium probability of a good draw. The lower plot gives two possible realizations at the end of the period and a 45 degree line. The dashed line stands for k_b , and the solid line for k_g .

A comparison between Figure 7 with lower A_0 and Figure 6 with higher A_1 shows that the increase from A_0 to A_1 shifts the curves of $n^*(k; A)$ and $k'(k; A)$ upwards. The following subsection shows that the dynamics of the two economies differ substantially.

2.5.2 Consumption Dynamics and Amplification Effect

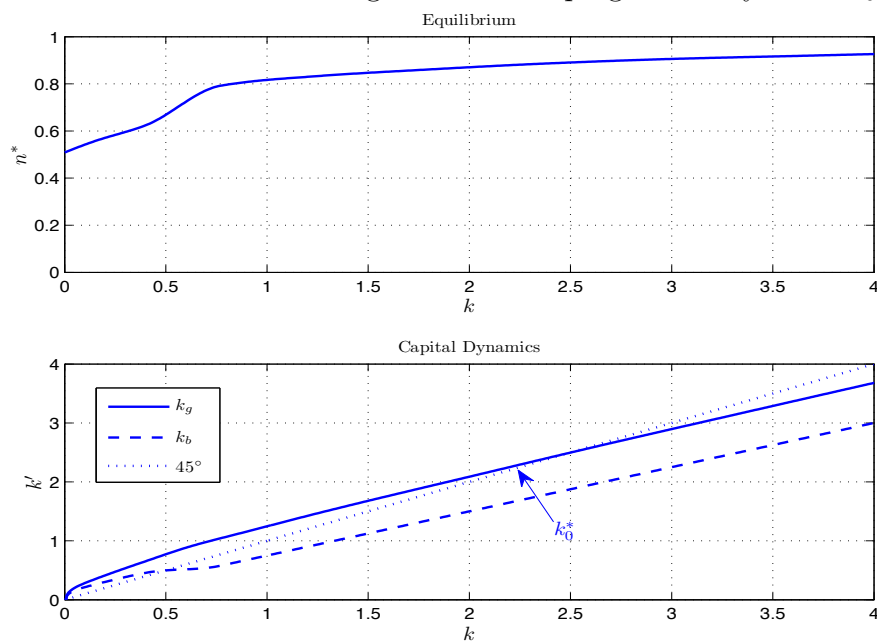
The developing economy analyzed in the last subsection most of the time exists in a world with a less fully diversified financial market. Most of the time, it also holds that $k \leq k_0^*$,²³ because there is always a positive probability that the economy is hit by bad shocks. Even without exogenous TFP shocks, the capital stock switches between the curves of k_g and k_b . Therefore, the output also jumps up and down substantially as does consumption.

The previous subsection also shows that without any TFP shocks, the developed economy would achieve full diversification and remain in the steady-state, where both consumption and output are constant.

It is interesting to compare the developed economy case with standard stochastic growth models. It is not difficult to imagine that with exogenous shocks, the developed economy fluctuates around the deterministic steady state and behaves similarly to a standard

²³If the economy starts with a capital $k_0 < k_0^*$, its capital will never exceed k_0^* .

Figure 7: Developing Economy with A_0



Note: Developing country case without exogenous shocks. The upper plot gives the equilibrium probability of a good draw. The lower plot gives two possible realizations of capital at the end of the period and a 45 degree line. The dashed line stands for k_b , and the solid line for k_g .

stochastic growth model. The subtle and non-trivial difference is that a sequence of bad TFP realizations might possibly decrease capital to such a low level that full diversification cannot be afforded and the economy could be hit by bad draws. Thus, on one hand, the model predicts frequent “small recessions” , which are driven by the exogenous TFP shocks around steady-state; on the other hand, it also predicts that deep and persistent recessions could happen in developed countries. Those rare events are driven by the combination of a sequence of low realizations of TFP shocks and the bad draws emerging from the financial market.

The endogenous diversification channel does not only provide an important source of uncertainty, which makes output more volatile, it also amplify the uncertainty in a certain period through capital accumulation. Thus, it has interesting implications for consumption behavior as well. The mechanism of *amplification* is that the probability of good draws is an increasing function of k . It means that, for a given level of capital k , if the economy happens to be hit by a good draw, the capital will be higher in the next period and the diversification opportunities will expand. Accordingly, there will be a higher probability of once more being hit by a good draw in the period after the next. This results in higher output in the next period and possibly an even higher output in the following periods. In contrast, if the economy happens to be hit by a bad draw, the capital will be substantially reduced in the next period and the degree of diversification will also shrink. The probability of being hit by a good draw decreases further. It results

in a lower output in the next period and possibly even lower outputs in future periods.

This amplification channel is absent in the developed economy most of the time, since the market is fully diversified and $n(k; A_1) = 1$. In the case of the developing economy, in contrast, this channel plays an important role in consumption dynamics. As analyzed above, output becomes more volatile in this case, because of the shocks from incomplete financial market. Consumption will be even more volatile in the developing economy, in response to both current income of higher volatility and to the endogenous diversification channel, which affects future consumption opportunities. In other words, since capital k contains additional information about future output, consumption would respond to the amplification effect, while output would just keep track of changes in capital levels.

Moreover, TFP shocks, although assumed to be exogenous, would interact with endogenous shocks from the financial market. The mechanism of *interaction* works as follows. A good realization of the TFP shock enhances the productivity of the economy. It results in higher savings, which implies more aggregate investment in risky securities. It helps the economy improve the diversification opportunities, and therefore increases the probability of good draws. Therefore, even though the TFP shock itself is transitory, it could be amplified by interacting with the endogenous diversification channel and have a persistent effect on future output. In this sense, the TFP shocks would endogenously have a higher persistence.

The model predicts that the output gains during expansion can be larger in the developing economy: A sequence of good TFP shocks could interact with the diversification channel and lead to substantial output gains in the developing economy. In other words, booms are reinforced and become stronger through the interaction channel. This prediction is well consistent with the empirical findings in Caldern and Fuentes (2006), which find that output gains are larger in lower income countries, during the trough-to-peak phase.

Since the exogenous TFP shocks are endogenously more persistent through the interaction channel, consumption in the developing economy is expected to respond to this and becomes more volatile. Once more, this channel plays a minor role in the developed counterpart since the financial market is complete most of the time. It implies that the ratio concerned should be relatively higher in developing countries.

3 Numerical Analysis

In this section, I will outline the solution algorithm and the simulation procedure (see the Appendix for a detailed algorithm). Similarly to most stochastic dynamic general equilibrium models, the system of equations (equations (1) and (2)) does not have any

analytical solution, except the special case. I solve the model numerically by exploring the recursive formulation.

The numerical exercise posts interesting challenges. First, the function which maps aggregate states to the equilibrium measure of available securities needs to be solved endogenously in equilibrium.²⁴ Second, the endogenous shocks could substantially shift the capital stock in the economy within a broad ergodic set. Methods with local approximations cannot be used. Decision rules must be solved for over a large range in which the curvature of the decision rules changes dramatically. Third, another difficulty is due to the kink-shape of the portfolio decision and thus, I have to choose the approximation methods wisely. Finally, the inequality in one of the Euler equations brings in an additional difficulty.

Two steps are taken to solve the general equilibrium problem. First, I take an educated guess for $n(k, A)$ and solve the two functional equations with a root-finding (Broyden) method. The solved decision rules, together with the guess, would imply whether the financial market equilibrium condition holds. Second, if the equilibrium condition does not hold, update the function of $n(k, A)$ until the equilibrium condition is met. I simulate the economy for 11000 periods to compute the average statistics.

3.1 Calibration

In this subsection, I outline the calibration strategy. Table 2 summarizes the values and describes the parameters.

I calibrate the model economy with different average productivity parameters, that is μ_0 and μ_1 , to match certain moments in the data for developing and developed countries. I normalize the average productivity to be 1 for developing countries, or $\mu_0 = 0$. μ_1 , which characterizes the average productivity level in developed countries, will be calibrated. Except for that, all other parameters are common to both groups of countries.

The remaining parameters are those characterizing preference and technology and those related to the financial market.

Regarding preference and technology, I parameterize the model using standard data in the growth literature. I use the standard *CRRA* utility function, where the risk aversion parameter is chosen to be 1.5. The discount rate β is standard from the literature, 0.96. The capital share is set to be 0.3, which is also common in growth models. I choose the annual depreciation rate to be 0.10. Values of $\rho = 0.95$ and $\sigma_z = 0.02$ are widely used in the literature. The AR(1) shock process is approximated by a Markov chain, using the Tauchen method.

²⁴Unlike the Aiyagari-type model, where only one factor price needs to be updated to ensure the equilibrium, the whole function in this model must be updated until the equilibrium condition is met.

Table 2: Baseline Parameters

Parameter	Economic interpretation	Value
σ	CRRA risk aversion	1.5
β	Annual discount rate	0.96
η	Capital share	0.30
δ	Depreciation rate	0.10
ρ	Shock persistence	0.95
σ_z	Shock standard deviation	0.02
r	Return to safe asset	1.00
R	Return to risky securities	1.055
D	Largest minimum size	1.70
γ	Minimum size parameter	0.16
μ	Log of average of productivity	0.605

Source: Standard and calibrated parameters

Regarding the parameters characterizing the financial market, the gross return to safe assets, r , is normalized to 1, and the gross return to risky projects, R , and the minimum requirement parameters, γ and D , will be calibrated.

I choose to match long run average saving rates (s/y) and output growth volatilities (σ_y) in both developing and developed countries by choosing these four need-to-be-calibrated parameters (see Table 3).²⁵ I compute the average saving rates for developing and developed countries and find there to be a statistically significant difference between these two groups. It is also interesting to let the model deliver the correct output growth volatilities and observe if consumption growth volatilities are sufficiently close to the data.

The parameters are jointly mapped to the targets. An increase in D or a decrease in γ would shift the curve of minimum requirement upwards and thus, the economy is more likely to be constrained by micro-level project indivisibility. The volatility of output growth in both economies will be higher. A higher R implies that the return to the risky portfolio is higher, which would induce a higher saving rate in both economies. A larger value of μ_1 would only make the developed economy less constrained and reduce its volatility level. A higher μ_1 also implies that the risky portfolio is “safer” and its savings rate is therefore higher, since the financial market is more complete, everything else equal.

²⁵The saving rate refers to the “gross domestic saving rate” in The World Development Indicators data. I compute the average saving rate for each country from 1960 to 2007 and compute the group mean for each group.

Table 3: Moments to match

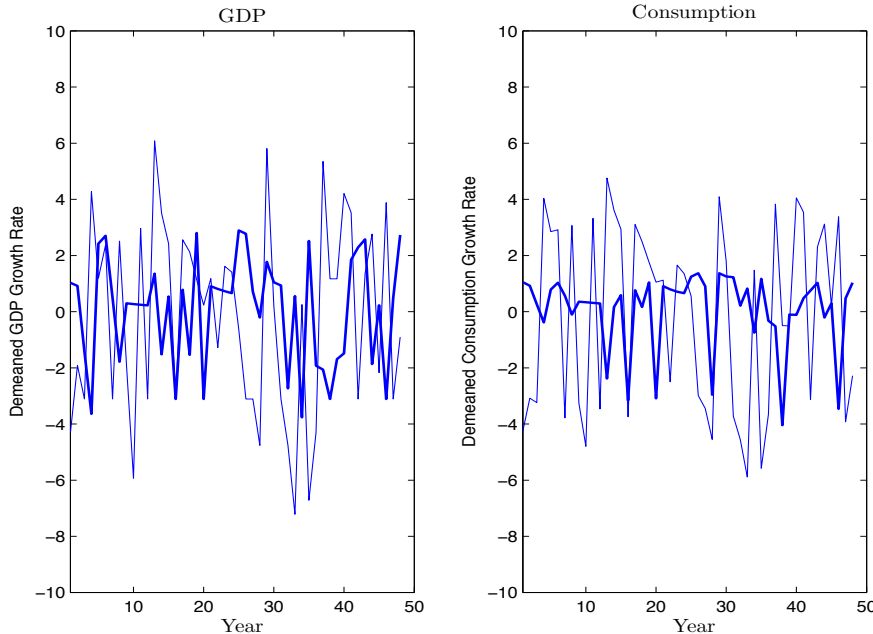
	Saving Rate	Model
Developed countries	$s/y = 0.25$	0.29
Developing countries	$s/y = 0.19$	0.176
	Growth Volatility	Model
Developed countries	$\sigma_y = 2.403$	2.43
Developing countries	$\sigma_y = 4.503$	4.48

Source: WDI (1960 - 2007) and simulations

4 Results

This section discusses the basic results from the simulations. I present the representative simulation series for the experiments of developing and developed countries in Figure 8.²⁶ In both experiments, the exogenous shock processes are set to be exactly the same.

Figure 8: Representative Simulation



Source: Simulation data. Demeaned growth rates of output and consumption are presented on the left-hand side and the right-hand side, respectively. The thicker curves are from the experiment of the developed country case while the regular curves are demeaned growth rates from the experiment of the developing countries. In both experiments, the persistence and variance of the exogenous shock processes are the same.

The excess consumption growth volatility pattern is quite clear. The left-hand side graph presents the demeaned output growth rates of these two experiments. The thicker line

²⁶A series of realizations of 47 periods is randomly picked from each simulation.

Table 4: Excess Volatility

Type	σ_c	σ_y	σ_c/σ_y	$(\sigma_c/\sigma_y)^{data}$
Developed countries	1.87	2.43	0.768	0.896
Developing countries	4.53	4.48	1.01	1.197

Source: Simulation data and WDI data(1960-2007).

represents the developed economy experiment, while the regular line stands for the developing economy experiment. It is observed that the developing economy experiences a more volatile output growth. It is even more obvious that the volatility of the consumption growth rate is higher in the developing economy than in the developed economy. As compared to output growth rate volatility, consumption growth rates seem to be more smooth in the developed case. In the experiment for the developing economy, the consumption growth rate is much less smooth.

4.1 Basic Results

The main results are summarized in Table 4. I compute the average standard deviation of consumption growth rates from the simulation data. The first column reports the results from the two experiments. I also compute the ratio of consumption growth volatility to output growth volatility in the third column.

First, the first and second columns show that the negative relationship between development and growth volatility exists in the model and it is even more pronounced in the case of consumption growth volatility.

Second, the third column reveals that, in this model, the ratio of consumption growth volatility to output growth volatility is substantially higher in the developing economy. It is approximately 1 in the experiment for the developing economy, while the ratio in the developed economy case is below 0.77. The pattern found in the model is well consistent with the empirical findings in the data and in Kose, Prasad and Terrones (2003).

This result confirms the intuition that the higher is the development level, characterized by a higher μ in the model, the less likely would the micro level constraint be to bind. The financial market will be more complete, shocks from the intermediate sector would play a minor role, and the ratio in question would be lower. In this model, it is observed that the lower productivity level is associated with both higher output and consumption growth volatilities, keeping the variance and persistence of exogenous shocks unchanged. More interestingly, consumption growth volatility is disproportionately higher, relative to output growth volatility. These results are not expected in the standard stochastic growth models.

Table 5: Development level

	μ_0	μ_a	μ_b	μ_c	μ_1
μ	0.00	0.25	0.45	0.55	0.605
σ_c/σ_y	1.01	0.96	0.93	0.84	0.768

Source: Simulation data.

The model also predicts that if an economy switches from a lower level of development to a more advanced level, both consumption and output growth volatility should be lower and consumption growth volatility should drop more quickly. Table 5 gives results from a series of experiments, which use intermediate values of μ , that is μ_a , μ_b and μ_c , between μ_0 , the normalized value for the developing group, and μ_1 , the calibrated value for the developed group. It is shown that the ratio of σ_c/σ_y decreases steadily with the increase in the value of μ .

Finally, although this model successfully generates the empirical pattern, there are still notable differences between the model and the data. Volatilities of consumption growth of both types of economy are lower in the model than their counterparts in the data. The model under-predicts the consumption growth volatility in both types of economy, since there are only two sources of uncertainty in the model.²⁷

4.2 Measuring TFP Properties

With the standard growth accounting method, information about the properties of TFP shocks can be extracted from the Solow residual, by assuming one unit of savings in each period to be transformed into one unit of gross investment. The model assumes there to be an intermediate sector where the transformation process is stochastic, due to the fact that the financial market is not necessarily fully diversified. If I make use of information about capital which is actually used in the final goods production, the Solow residual could correctly help identify the TFP shock process. Therefore, the properties of the TFP shock should be identical in simulations for developing and developed economies, since I impose the same exogenous shock process on both experiments. However, if constructing the capital sequence by ignoring the incomplete financial market structure, the results will be biased.

Suppose that the model proposed in the paper exactly captures the reality, but that one tries to understand the data produced by the model, with a “misspecified” model, where

²⁷However, it is not hard to imagine that other channels of uncertainty would increase the volatility level of consumption growth. Empirical evidence (e.g. Easterly, Islam, and Stiglitz (2000)) also shows that fiscal policy, public consumption and nominal shocks all help to increase the volatility in both developing and developed countries. The international sector can contribute another source of uncertainty, which exposes the economy to external shocks (e.g. Neumeier and Perri (2005)).

an exogenous difference between the TFP shock processes for the two types of economy is assumed. If the estimation concludes there to be a difference between the TFP processes of the two groups, the observed difference should be attributed to the ignored endogenous diversification channel.

One of the important findings in Aguiar and Gopinath (2007) is that permanent shocks are relatively unimportant in industrial economies, as compared to lower income emerging economies.²⁸ This section evaluates if this model with an endogenous diversification channel has the ability to generate the observed difference. Toward this end, I use the same approach, proposed by Aguiar and Gopinath (2007) to analyze the Solow residual from simulation data.

I conduct this exercise in the following way. Suppose that one tries to analyze the simulation data, with an otherwise standard stochastic growth model, augmented by exogenous permanent shocks. The production function is assumed to be Cobb-Douglas, producing final goods with capital and one unit of labor.

$$Y_t = e^{z_t} K_t^\eta (\Gamma_t L_t)^{1-\eta}$$

There are two types of shocks. The first is z_t , which is a standard $AR(1)$ process.

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

The second, Γ_t , is used to represent the cumulative product of white noise. Specifically,

$$\Gamma_t = e^{\varepsilon_t^g} \Gamma_{t-1} = \prod_{s=0}^t e^{\varepsilon_s^g}$$

where ε^g is an innovation from a normal distribution with zero mean and standard deviation σ_g .

To directly estimate the underlying shock process, one wants to make use of information from Solow residuals. Savings in simulation data are assumed to be converted into investment one for one in a close economy. Given the initial capital k_0 from the simulation, a sequence of pseudo capital can be constructed. With information on output sequence, the sequence of the pseudo Solow residual can be backed out from the final goods production function.

²⁸In their model, they assume the permanent shock to be an accumulative product of “growth shocks” and growth shocks to be positively correlated. They find that the Solow residual in lower income countries is more volatile and the permanent component is larger than it is in developed countries. They further argue that, between the two groups, the ratio of the standard deviation of permanent shocks to the standard deviation of mean reverting shocks is different. In the misspecified model, the permanent shock to be identified is an accumulative product of white noise, which can be interpreted as uncorrelated growth shocks.

To extract information on permanent shocks from the pseudo Solow residual, one makes use of the standard method proposed by Beveridge and Nelson (1981), which implies that if following an $I(1)$ process, the log of the Solow residual can be decomposed into a random walk component and a transitory component.

$$sr_t = \tau_t + z_t$$

where sr_t is the log of the Solow residual. Moreover, τ_t represents random walk and follows the below process,

$$\tau_t = \tau_{t-1} + (1 - \eta) \varepsilon_t^g$$

Following the method proposed by Cochrane (1988), the variance of the permanent component is estimated with the below approximation,

$$\lim_{M \rightarrow \infty} M^{-1} Var(sr_t - sr_{t-M}) = \sigma_{\Delta\tau}^2$$

where sr_{t-M} is the M period lag of log of Solow residual. The estimation is sufficiently accurate, conditional on M being sufficiently large. In practice, it is difficult to choose M to be sufficiently large due to the limitation of the data length. However, this exercise is not constrained by data length, since I can simulate the model for a sufficiently long period of time. I choose M to be unusually large, higher than what would be chosen in practice. I choose M to be 1000, 2000, 3000, respectively, in different estimations.

The critical point of this exercise is to discover the difference of permanent shocks between experiments of developing and developed economies. Towards this end, I look at the ratio of the standard deviation of permanent shocks in the developing economy to that in the developed economy. The essential finding is that the ratio is quite stable and roughly 1.8, despite the choice of M .

$$\frac{\sigma_{\Delta\tau}^{developing}}{\sigma_{\Delta\tau}^{developed}} \approx 1.8$$

This finding shows that if using the standard growth accounting method to construct the Solow residuals, it would be concluded that there is a difference in the measured TFP processes. However, the data are generated by the true model, where no difference in the TFP process is exogenously assumed. In other words, the diversification channel in this model could at least partially explain the observed difference in the measured TFP processes of developing and developed countries.

5 Conclusion

Mounting evidence shows that consumption growth exhibits an excess volatility in developing countries. This pattern has been documented in several recent research papers.

This research shows that the lack of diversification opportunity in developing countries could account for the empirical regularity. This paper models this connection based on Acemoglu and Zilibotti's classical contribution (1997) in this field. Numerical results show that the model is well in line with the empirical regularity documented in different datasets, although the volatility of consumption growth in both groups is obviously lower in the model than in the data. Interestingly, in this model, even though both types of economy are subject to exactly the same exogenous shock process, the only difference in average TFP translates into the excess consumption growth volatility pattern.

In this research, it is argued that the endogenous difference in the financial development of both types of economy could help explain observed differences in TFP processes. When the financial development is low, stochastic shocks to investments and TFP are endogenously more volatile and more persistent. This model also has an implication for estimating TFP processes in developing countries: The estimation might be biased if we ignore the endogenous diversification channel.

6 Appendix

6.1 Euler Equations

Euler equations are derived by following the procedure:

1. Use budget constraint to express consumption choice with state and other choice variables and substitute it for consumption c in the Bellman equation.
2. Rearrange the expected continuation value. $V(K^{g(b)}, k^{g(b)}, A')$ is the continuation value if the good (bad) shock is realized and the exogenous shock turns out to be A' . $[n \cdot V(K^g, k^g, A') + (1 - n) V(K^b, k^b, A')]$ is the expected continuation value, given A . Since A' is also stochastic, the expected continuation value, only conditional on A , is

$$E_A [n \cdot V(K^g, k^g, A') + (1 - n) V(K^b, k^b, A')]$$

Therefore, the Bellman equation becomes the following,

$$V(K, k, A) = \max_{s \geq 0, 1 \geq \alpha \geq 0} \left\{ \begin{array}{c} U(w + \varphi k - s) + \\ \beta \cdot E_A [n \cdot V(K^g, k^g, A') + (1 - n) V(K^b, k^b, A')] \end{array} \right\}$$

3. Assume the interior solution (or $n < 1$) and derive the first-order conditions (from now on, E_A is replaced by E for simplicity)

the first-order condition with respect to s :

$$U'(c) = \beta \cdot E \left[\begin{array}{c} n \cdot \left(r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n} \right) \cdot V_k(K^g, k^g, A') \\ + (1 - n) \cdot r \cdot \alpha \cdot V_k(K^b, k^b, A') \end{array} \right]$$

the first-order condition with respect to α :

$$0 = E \left[\begin{array}{c} n \cdot \left(r \cdot s - R \cdot \frac{1}{n} \cdot s \right) \cdot V_k(K^g, k^g, A') \\ + (1 - n) \cdot r \cdot s \cdot V_k(K^b, k^b, A') \end{array} \right]$$

4. Rearrange the two first-order conditions and the system becomes

$$U'(c) = \beta \cdot R \cdot E [V_k(K^g, k^g, A')]$$

$$U'(c) \left(\frac{1}{r} - \frac{n}{R} \right) = \beta \cdot (1 - n) \cdot E [V_k(K^b, k^b, A')]$$

5. Derive the Envelope condition

$$V_k(K, k, A) = U'(c) \cdot \left(\eta \cdot A \cdot K^{\eta-1} + (1 - \delta) \cdot \frac{1}{r} \right)$$

6. Use the Envelope condition and update it one period forward. Replace $E [V_k(K^g, k^g, A')]$ and $E [V_k(K^b, k^b, A')]$ on the right-hand side of the equation system. The equation system becomes two intertemporal equations,

$$U'(c) = \beta \cdot R \cdot E \left[U'(c_g) \cdot \left(\eta \cdot A' \cdot K^{g(\eta-1)} + (1 - \delta) \cdot \frac{1}{r} \right) \right]$$

$$U'(c) = \beta \cdot \frac{(1 - n)}{\left(\frac{1}{r} - \frac{n}{R} \right)} \cdot E \left[U'(c_b) \cdot \left(\eta \cdot A' \cdot K^{b(\eta-1)} + (1 - \delta) \cdot \frac{1}{r} \right) \right]$$

where

$$c_g = c(K^g, k^g, A') \text{ and } c_b = c(K^b, k^b, A')$$

Note that in the states where $n = 1$, backward inequality holds in the second equation. It corresponds to the case where the financial market is fully diversified.

6.2 Special Case

The analytical solution can be derived from a special case, where $\delta = 1$ and $U(c) = \log(c)$. Using the guess that consumption is a function of aggregate capital and is not dependent on individual capital or, more precisely, consumption is a constant fraction of aggregate output. That is, $c = \tau \cdot A \cdot K^\eta$. Moreover, the conjecture of the law of motion of aggregate capital is the same as the individual capital.

1. Replace consumption in the inter-temporal equations with this guess,

$$\frac{1}{c} = \beta \cdot R \cdot E \left[\frac{1}{\tau \cdot A' \cdot K^{g\eta}} \cdot (\eta \cdot A' \cdot K^{g(\eta-1)}) \right]$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot E \left[\frac{1}{\tau \cdot A' \cdot K^{b\eta}} \cdot (\eta \cdot A' \cdot K^{b(\eta-1)}) \right]$$

2. Rearrange both equations. Then, the expectation operator drops out,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot K^g}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{\tau \cdot K^b}$$

3. Impose the equilibrium condition, $K = k$.

4. Replace k^g and k^b with their law of motion,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot \left(r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s \right)}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{\tau \cdot r \cdot \alpha \cdot s}$$

5. Savings are replaced by $(1-\tau) \cdot A \cdot K^\eta$, since savings are also a constant fraction of output,

$$\frac{1-\tau}{\tau} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot \left(r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n} \right)}$$

$$\frac{1-\tau}{\tau} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R} \right)} \cdot \frac{\eta}{\tau \cdot r \cdot \alpha}$$

6. Solving for τ and α from this equation system leads to the solution in the text. It is obvious that τ is indeed a constant. The implied investment in each risky security is

$$F(n) = \frac{R-r}{R-r \cdot n} s$$

7. Impose the equilibrium condition, $F^*(n^*) = \frac{D}{1-\gamma} (n^* - \gamma)$, and the equilibrium $n^*(k, A)$ is thus obtained.

6.3 The Model with Deterministic Trend

In this section, a similar model with deterministic trend is provided. One of the critical assumptions is that the minimum size parameter D is growing with the economy at the same growth rate. This assumption captures the idea that the effect of micro-level project indivisibility (or non-convexity of production feasible set) cannot be eliminated by growth alone²⁹.

I assume that labor productivity grows constantly at the rate of g , that is $\Gamma_t = \Gamma_{t-1}e^g$, (where Γ_t is the labor productivity level in period t). Thus, the production function at period t is

$$Y_t = A_t K_t^\eta \cdot (\Gamma_{t-1} L_t)^{(1-\eta)}$$

The system can be normalized by the growth factor Γ_{t-1} . I use the hat version to denote the normalized variables:

$$\hat{x}_t = \frac{x_t}{\Gamma_{t-1}}$$

I start with the definition of the value function in sequential form,

$$V_t = E_t \sum_{s=t+1}^{\infty} \beta^s u(c_s)$$

²⁹Intuitively, one can justify this assumption by observing that the size of the largest project that a developed economy could afford, is much bigger now than its counterpart a hundred years ago.

1. Rearranging this with hat variables yields

$$V_t = (\Gamma_{t-1})^{(1-\sigma)} \frac{(\widehat{C}_t)^{1-\sigma}}{1-\sigma} + \beta \cdot \left(E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{C}_s)^{1-\sigma}}{1-\sigma} \right)$$

2. Divide both sides by $(\Gamma_{t-1})^{(1-\sigma)}$

$$\frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}} = \frac{(\widehat{C}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}} \beta \cdot \left(E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{C}_s)^{1-\sigma}}{1-\sigma} \right)$$

3. Denote $\widehat{V}_t = \frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}}$, so that

$$\widehat{V}_t = \frac{(\widehat{C}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}} \beta \cdot \left(E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{C}_s)^{1-\sigma}}{1-\sigma} \right)$$

4. Use $\widehat{V}_{t+1} = \frac{V_{t+1}}{(\Gamma_t)^{(1-\sigma)}}$, so that

$$\widehat{V}_t = \frac{(\widehat{C}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}} \beta \cdot \left((\Gamma_t)^{(1-\sigma)} \cdot E_t \widehat{V}_{t+1} \right)$$

5. Rewrite the resource constraint, capital accumulation and portfolio definition with the hat version

$$\widehat{c} + \widehat{s} = w(\widehat{K}, A) + \varphi(\widehat{K}, A) \cdot \widehat{k}$$

$$\widehat{\phi} = \alpha \cdot \widehat{s} \text{ and } n \cdot \widehat{F} = (1 - \alpha) \cdot \widehat{s}$$

$$\widehat{k}' = \begin{cases} r \cdot \alpha \cdot \widehat{s} + (1 - \delta) \widehat{k} & \text{if } j > n \text{ with } 1 - n \\ r \cdot \alpha \cdot \widehat{s} + R \cdot \frac{(1-\alpha)}{n} \cdot \widehat{s} + (1 - \delta) \widehat{k} & \text{if } j \leq n \text{ with } n \end{cases}$$

where $\widehat{\beta} = e^{g(1-\sigma)}\beta$ and it is easily shown that $w(\widehat{K}, A) = w(K, A)$ and $\varphi(\widehat{K}, A) = \varphi(K, A)$, respectively. The function form of decision rules will be the same as those in the text.

6.4 Algorithm and Simulation

I take two steps to solve the general equilibrium problem. First, I take an educated guess of the function form of $n(k, A)$ and discretize this function with a two-dimension piece-

wise linear function, which is increasing in both k and A ³⁰. The choice of piecewise linear approximation is based on the observation that $n(k, A)$ is increasing and reaches 1, if k and A are sufficiently large. Taking the guess as given, I solve the two functional equations with a root-finding method. I discretize A and k in a two dimensional space with Chebyshev nodes and then interpolate the consumption rule $c(k, A)$ with two dimensional *Chebyshev approximation* with $nk \times nA$ coefficients, where nk and nA are the numbers of collocation points of capital and productivity level, respectively. Chebyshev approximation is used to take advantage of the high accuracy of the solution. However, if the policy function displays a kink-shape, the scheme may deliver a poor approximation. To avoid this, I must interpolate the portfolio decision $\alpha(k, A)$ with a *shape-preserving piecewise approximation* (with $nk \times nA$ coefficients). The following steps are taken sequentially.

Step 1: Compute the left-hand side value at each collocation point of the $k - A$ mesh;

Step 2: Compute k^b and k^g , according to the law of motion of capital;

Step 3: Compute the right-hand side value at each collocation point of the $k - A$ mesh, which requires computation of the expectation;

Step 4: Compute the expectation using numerical integration with a *modified Tauchen algorithm*;

Step 5: Solve the $2 \times nk \times nA$ nonlinear equation system with Broyden's method and obtain the two policy functions.³¹

Second, I set up an outer loop to solve for $n(k, A)$

Step 1: Guess a pseudo function for $n(k, A)$;

Step 2: Take the function as given and solve for $s(k, A)$ and $\alpha(k, A)$ (see above);

Step 3: Compute $F(k, A)$ and $\frac{D}{1-\gamma}(n(k, A) - \gamma)$ at each state;

Step 4: If the difference is below the tolerance level, $n^*(k, A)$ is found;

Step 5: If not, update $n(k, A)$ with a *generalized bisection method*;

Step 6: Return to Step 2 until the equilibrium condition is satisfied in Step 4.

³⁰I use lower case k in this subsection to represent capital stock. It does not contradict the previous section, since I am solving numerically the case, where the condition $K = k$ is imposed.

³¹Care has to be taken that strict inequality in the second Euler equation may hold.

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